HEC MONTRÉAL
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Vehicle Scrappage Subsidies in the Presence of Strategic Consumers

par
Hosain Zaman

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Présentée par :

Hosain Zaman

a été évaluée par un jury composé des personnes suivantes :

Pierre-Olivier Pineau
HEC Montréal
Président-rapporteur

Georges Zaccour
HEC Montréal
Directeur de recherche

Mehmet Gümüş
McGill University
Membre du jury

Sudheer Gupta
Simon Fraser University
Examinateur externe

Verena Gruber
HEC Montréal
Représentant du directeur de HEC Montréal

Résumé

Dans cette thèse constituée de trois essais, nous étudions les programmes de mise à la casse de véhicules dans un contexte où les consommateurs agissent d'une manière stratégique. Ces programmes, très populaires dans de nombreux pays, visent à accélérer le remplacement des voitures âgées et polluantes par de nouvelles moins gourmandes en carburant. Bien que l'objectif environnemental de réduction des émissions de polluants est toujours présent, notons que parfois ces programmes ont été mis en place afin de stimuler le marché de l'automobile, comme c'était le cas après la crise financière globale en 2007-2008.

Dans le premier essai, intitulé "*Vehicle Scrappage Incentives to Accelerate the Replacement Decision of Heterogeneous Consumers*", nous supposons que la subvention est offerte à des consommateurs hétérogènes quant à leur désir de remplacer leur voiture.

Alors que des programmes de mise à la casse ont été mis en œuvre pour inciter au remplacement des véhicules plus tôt, certains consommateurs l'auraient fait même en l'absence du programme. En conséquence, afin d'avoir une estimation juste des avantages d'un tel programme, il est important de tenir compte de l'hétérogénéité des consommateurs relative à la valeur de reprise de leur véhicule.

Nous démontrons théoriquement que, bien que l'augmentation du niveau de subvention incite les consommateurs ayant une faible valeur de reprise nette à remplacer plus tôt, elle incite également les consommateurs ayant une plus grande valeur à remplacer plus tard afin de devenir éligibles à la subvention. En fait, les consommateurs qui attachent une grande valeur à leur voiture, auraient remplacée avant d'être admissible reporteront

le remplacement jusqu'à ce le véhicule devienne admissible au programme. Ces résultats montrent l'importance de tenir compte de l'hétérogénéité des consommateurs lors de la mesure des avantages d'un tel programme de subvention. La principale contribution de cet essai réside dans la démonstration qu'ignorer l'hétérogénéité des consommateurs, induit à une exagération des bénéfices d'un programme de mise à la casse de vieilles voitures.

Le deuxième essai caractérise les niveaux de subvention d'équilibre fixés par le gouvernement face à des consommateurs stratégiques. Dans cet essai, intitulé "*Optimal Government Scrappage Subsidies in the Presence of Strategic Consumers*", le gouvernement agit de manière stratégique en minimisant le coût de subvention tout en ayant un niveau cible de remplacement de voitures âgées.

Le jeu est non coopératif à la Stackelberg, où le gouvernement joue le rôle de leader et le fabricant de suiveur. En considérant un modèle dynamique à deux périodes, nous montrons que les niveaux de subvention dans les deux périodes sont positifs et augmentent avec le temps. Aussi, nous obtenons que lorsque le niveau cible augmente, le gouvernement augmente également l'écart entre les deux niveaux de subvention.

Dans le troisième essai, intitulé "*Subsidies and Pricing Strategies in a Vehicle Scrappage Program with Strategic Consumers*", nous considérons un jeu impliquant un gouvernement et un constructeur automobile. Nous étudions le problème de design de subventions dans le temps par un gouvernement, dans plusieurs contextes, c'est-à-dire quand le manufacturier implémente une stratégie de prix constant (ou non constant) dans le temps et le gouvernement peut choisir entre offrir la même subvention dans le temps ou adopter une formule flexible.

Nous caractérisons les stratégies d'équilibre dans les divers scénarios. Dans tous les cas, nous supposons que le consommateur agit stratégiquement en décidant la date de remplacement de son véhicule en tenant compte du prix actuel et futur. Nous analysons comment ces stratégies affectent les différentes parties du jeu, c'est-à-dire le gouvernement, le fabricant et les consommateurs.

Mots-clés

Subventions gouvernementales, programmes de mise au rebut de véhicules, consommateurs hétérogènes, consommateurs stratégiques, stratégies de tarification, théorie des jeux, programmation dynamique

Méthodes de recherche

Recherche quantitative, analyse numérique

Abstract

In this dissertation, presented in three essays, we study vehicle scrappage programs in the presence of strategic consumers. Vehicle scrappage programs are offered to accelerate replacement of cars older than a certain age (program's eligibility age) with new ones.

In the first essay, entitled "*Vehicle Scrappage Incentives to Accelerate the Replacement Decision of Heterogeneous Consumers*", we assume that the subsidy is offered to consumers who are heterogeneous in terms of their willingness to replace their cars.

Whereas scrappage programs have been implemented to provide motivation for replacing vehicles earlier, some consumers would replace anyway, even without the program. As a result, in order to have a correct estimation about the program's benefits, it is important to account for variations in consumers' willingness to replace or equivalently, their net tarde-in valuation.

We theoretically demonstrate that, although increasing the subsidy level does motivate consumers with a low net trade-in valuation to replace earlier, it also induces high-value consumers to replace later in order to become eligible for the subsidy program. In fact, those high-value consumers who would have replaced their car under the eligibility age without the subsidy, may postpone their car replacement until it becomes eligible for the program. These results show the importance of considering consumer heterogeneity when measuring the subsidy program benefits, which is the main contribution of this essay: when consumer heterogeneity is neglected, net benefits of scrappage program in terms of pollution reduction and car sales are exaggerated.

The second essay characterizes the equilibrium subsidy levels set by government when

facing strategic consumers. In this essay entitled "*Optimal Government Scrappage Subsidies in the Presence of Strategic Consumers*", we let the government to act strategically by minimizing its subsidy cost subject to a car-sales target level.

The game is played non-cooperatively à la Stackelberg, with the government acting as leader and the manufacturer as follower. Considering a two-period dynamic model, we show that subsidy levels in both periods are positive and are increasing over time. We find that, when the target level increases, the government increases the differential between the two subsidy levels too.

In the third essay, entitled "*Subsidies and Pricing Strategies in a Vehicle Scrappage Program with Strategic Consumers*", we consider a channel formed of a government and a car manufacturer. We investigate the problem of designing subsidies over time by a government, when the prices are set by a (representative) car manufacturer (or a car dealer). On the demand side, to capture the strategic behavior of consumers, we assume that the decision to replace or not a car is based on current and future subsidies.

Various subsidy and pricing strategies are analyzed and compared. In particular, the manufacturer/government can opt for a constant price/subsidy, or let them vary over time. We investigate how these strategies affect different parties of the game, i.e., the government, the manufacturer, and consumers.

Keywords

Government Subsidies, Vehicle Scrappage Programs, Heterogeneous Consumers, Strategic Consumers, Pricing Strategies, Game Theory, Dynamic Programming

Research methods

Quantitative Research, Numerical Analysis

Contents

List of Tables

List of Figures

List of acronyms

CSI California Solar Initiative

CARS Car Allowance Rebate System

HEC Hautes études commerciales

HVAC Heating, Ventilation, and Air Conditioning

KKT Karush–Kuhn–Tucker

LCA Life Cycle Analysis

LCO Life Cycle Optimization

MPG Miles Per Gallon

PhD Doctorat

VEI Vehicle Efficiency Incentive

RHS Right Hand Side

To my parents,

for their unconditional and endless love and support.

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Last but not least, I would like to express my deepest gratitude to my parents for their emotional support and love. This journey would not have been possible without their encouragement, moral support, and care.

Preface

This thesis is written based on three research articles. The list of published and submitted papers are as follows:

- Zaman, H., Zaccour, G. , 2020 Vehicle Scrappage Incentives to Accelerate the Replacement Decision of Heterogeneous Consumers. *Omega*, 91, p.102016.
  **Awarded the Esdras−Minville best student paper award in* 2019, *HEC Montreal.*

- Zaman, H., Zaccour, G. , 2021 Optimal Government Scrappage Subsidies in the Presenceof Strategic Consumers. *European Journal of Operational Research*, 288(3), pp. 829-838.

- Zaman, H., Zaccour, G. , Subsidies and Pricing Strategies in a Vehicle Scrappage Program with Strategic Consumers

General Introduction

Environmental subsidy programs are usually offered by governments to promote adoption of green durable products or technologies. Since environment-friendly products are usually more expensive than the same regular products, subsidies are offered to offset the higher price paid by consumers. One of the well-known examples of these subsidy programs in the US is the federal tax credit for plug-in electric vehicles. Introduced in 2009, this program provided consumers who purchase an electric vehicle with a subsidy of $7500. Similar program in Canada offers $2500 to $5000 to Canadians who purchase electric vehicles or plug-in hybrids. While these subsidy plans are offered to aid *purchasing* green products, some incentive programs subsidize *replacing* polluting products with less polluting ones. These programs, which are known as trade-in policies, are applied in different markets from smartphone market to car industry. In this thesis, we study scrappage subsidy programs which subsidize consumers who replace their old car with a new one.

Many European countries launched scrappage programs to increase car sales after the global recession in 2008. These programs were introduced with the dual aim of reducing emission and stimulating the car market. For example, scrappage program in France was introduced in January 2009 where cars older than ten years were eligible for €1000 subsidy. Another example of this type of subsidy programs is the scrappage program in Germany, which is the largest one so far. In this program every vehicle aged older than nine is eligible for a subsidy, should it be replaced by a new car. While scrappage programs are designed to stimulate the car market, they are also costly: the German scrappage program is estimated to cost 2,5 billion euros (Ewing, 2009). Therefore, in order to avoid wasting governmental funds, it is essential to design these programs in the most cost-efficient way.

According to our survey, problems and issues related to scrappage programs are categorized in three groups: subsidy cost, environmental impact, and consumer targeting. In the first category, the main focus is on the economic gains of the subsidy, based on cost-benefit analyses; see Lavee et al. (2014); Hahn (1995); Lavee and Becker (2009); Lorentziadis and Vournas (2011). The second group analyzes the efficiency of scrappage programs in terms of environmental improvement. In this regard, the main argument is that, when measuring the program's pollution reduction, it is important to do a Life Cycle Analysis (LCA); see Kim et al. (2004); Kim et al. (2006); Lenski et al. (2010). In the LCA approach, all stages of a product's life including production, usage and disposal are taken into account. Basically, scrappage programs result in higher new car productions and more used cars scrappage. Therefore, while a scrappage program reduces the emission by cars in their usage phase, it could impact the environment negatively in the manufacturing and disposal phases. The last category, consumer targeting, highlights the importance of grouping consumers based on their behavioural characteristics. This thesis contributes to the last category; however, we investigate how our new setting impacts subsidy program's costs and its environmental effects too.

This three-essay thesis studies the interaction between government and consumers when a vehicle scrappage subsidy program is announced.

In the first essay, we investigate how behavioural complexities of consumers affect their replacement decision. Consumer behaviour is a recent concern regarding trade-in subsidy programs. In a study on air conditioners, Kleine et al. (2011) highlighted that consumers differ in their willingness to replace, and that, there is a need for further research to understand their behavior. More specifically, they argued that the design and assessment of incentive programs for air conditioner replacement must take into account those individuals who would replace even without a subsidy. Such consumers are called "free riders" in Blumstein (2010) and Skumatz et al. (2009), and they are excluded when calculating the net benefits of the incentive program. Hoekstra et al. (2015) justified the need to account for free riders in their conclusion that 60% of subsidies from the Cash for Clunkers program (scrappage program in the US) went to households that would have purchased regardless while the program was ongoing. Such observations constitute an invitation to account for consumer heterogeneity, which seems a crucial feature when designing an incentive program.

Consumer heterogeneity in scrappage policies has been taken into account in a few previous works. Schiraldi (2011) used a dynamic discrete-choice model in which heterogeneous consumers with different tastes decide whether to replace their automobile. He evaluated different incentive policies and checked whether or not the beneficiaries of such policies would have replaced their vehicle without the subsidy. Li and Wei (2016) retained the same discrete-choice setting to investigate vehicle scrappage programs with consumer heterogeneity. They indicated that targeting marginal consumers, i.e., those who would not have replaced without a subsidy, is the key factor in designing scrappage programs. Unlike these two papers, where the focus is on the effect of the scrappage program on heterogeneous consumers' vehicle choice, in the first essay of this thesis, we concentrate on the impact of the subsidy level on the timing of the replacement decisions. In this regard, the closest paper to this essay is Langer and Lemoine (2017), which analyzes the reaction of strategic consumers to government subsidies for new products. However, they investigated the effect of subsidies on purchasing decisions rather than on replacement decisions as we do here.

More specifically, the first essay entitled "*Vehicle Scrappage Incentives to Accelerate the Replacement Decision of Heterogeneous Consumers*", studies the impact of scrappage programs on different consumer groups. A dynamic program model is considered, where at each period, consumers decide whether to replace their vehicle or to keep it until the next period. According to the subsidy program's rule, consumers owning a car older than the program's eligibility age would receive an incentive payment, if they replace their car with a new one. The key assumption is that consumers are heterogeneous in terms of their net trade-in valuation. In line with Huang et al. (2014), the net trade-in valuation is measured by the difference between the environmental valuation of a new car minus the valuation of a retained car. Consumers with low trade-in valuation are identified as low-value consumers and those who give higher valuation to new cars compared to their used car are called high-value consumers.

Considering the problem described above, the first essay answers two specific questions. The first question is whether or not the subsidies affect different consumer groups in the same way. In particular, we wanted like to investigate the reaction of low-value and high-value consumers to the subsidy program. The second question is related to the importance of the heterogeneity and the consequences of its ignorance. In fact, we compare the results of our setting with those of homogeneous setting, the benchmark case where all consumers are assumed homogeneous.

Our results show that, intuitively, the subsidy motivates low-value consumers to replace their cars earlier. However, increasing the subsidy value induces those with high valuation, to delay their replacement until their vehicle becomes eligible for the program. Note that some high-value consumers would have replaced without subsidy. This implies that investing in high-value consumers is a waste of funds. Therefore, in such circumstances, one challenging task of the government is to manage spending subsidy on high-value consumers. However, direct price discrimination between different consumer groups is not possible for the regulator. Instead, she must determine the subsidy level considering both groups, i.e., high-value and low-value consumers. On the one hand, the subsidy must be high enough to accelerate purchases by low-value consumers. On the other hand, it should be lower than the threshold above which high-value consumers delay replacing their vehicles in order to qualify for the subsidy.

Another important result of the first essay is the comparison of the heterogeneous and homogeneous settings in terms of the net benefits of the scrappage program. When consumer heterogeneity is taken into account, the estimated benefits of the scrappage program are lower than when all consumers are assumed homogeneous. Specifically, the pollution reduction and new vehicle purchases resulting from the program are overestimated when consumer heterogeneity is ignored. This finding reveals the real benefits of scrappage programs. This can be useful when assigning budgets for future subsidy programs. Also, with a better understanding of advantages of subsidy programs, policy-makers are able to choose among different options to reach their objective in the most efficient way.

In the first essay, the main focus is on the replacement decisions of consumers, assuming that the subsidy has a given, constant value. In fact, we solve the optimization problem of consumers, while considering no strategic role for the government. However, in practice, governments design subsidy programs to determine optimal subsidy levels, which may vary or to remain constant over time. Jørgensen and Zaccour (1999) and Janssens and Zaccour (2014) determine the equilibrium path of consumer subsidies in dynamic games involving a government and a firm. Similarly, Lobel and Perakis (2011) characterize the trajectory of consumer subsidies for photovoltaic technology. In these papers, dynamic subsidies are offered for purchasing new technologies (or products), but, scrappage programs provide incentives for replacing old products with new ones. In this sense, scrappage programs are similar to trade-in subsidies; see Zhu et al. (2018); Miao et al. (2018); Zhang and Zhang (2018). Likewise, in the second essay of this thesis, we determine optimal subsidies offered to consumers to trade-in their cars.

The problem to solve in trade-in programs is more complicated than the one arising in a typical purchasing decision by strategic consumers. Indeed, when replacing an old car with a new one, the consumer considers the value of both cars, not only the value of the new one. This implies that, in a replacement problem, consumers are not only heterogeneous in terms of their perceived valuation of the new product, but also in the age of their currently owned cars. This led us to include two heterogeneities in the second essay: in consumer valuation (the consumer's valuation of driving a car) and in used car age (quality of the car). Another important factor when designing scrappage subsidy policies is the program's eligibility age. This distinguishes vehicle scrappage policies from trade-in programs, where the consumer can trade in a product for a new one, without any age constraint. Hence, it is necessary to differentiate between consumers who are eligible at a given period and those who can get the subsidy only in future periods.

More precisely, in the second paper entitled "*Optimal Government Scrappage Sub-*

sidies in the Presence of Strategic Consumers", we characterize incentive equilibrium strategies for a game between consumers and government. The game is played à la Stackelberg, with the government acting as leader and consumers as follower. We assume that the government wishes to satisfy a car-sales target level. Governments often assign an adoption target to their subsidy programs. For example, President Obama announced in 2011 a target of "one million electric vehicles on the road by 2015". Another example is the "1940 megawatt of new solar generation capacity" target assigned by the California Solar Incentive program (Cohen et al., 2016). Along these lines, we suppose that the government's objective is to minimize the total cost of the program, subject to a preset target. Clearly, one can imagine other objectives such as minimizing greenhouse gas emissions or maximizing social welfare. However, Cohen et al. (2016) argue that these optimization problems are equivalent and yield the same results.

We show that the optimal subsidy policy depends on the car replacement target level. Clearly, a low target can be met with no subsidy. For medium target levels, subsidies in both periods are positive and increasing in the target level, and the second-period subsidy is higher than the first-period one. In this case, the government applies an "age-based price discrimination" when setting optimal subsidy levels. More precisely, consumers who are eligible for the subsidy only in the second period may replace in the second periods, in the first period, or to never replace at all. However, those who are eligible in both periods either replace in the first period, or they do not replace at all. As the target increases to a high level, the second-period subsidy level increases too. As a results, consumers who are eligible in both periods may consider replacing in the second period too.

In the third part of this thesis, we extend the second essay by accounting for another key player in the context of scrappage programs, i.e., a representative manufacturer. Scrappage subsidies aim at increasing the willingness to pay of consumer to replace her car. One concern is that this boost may not fully materialize because manufacturers may increase their prices when a subsidy program is established. Kaul et al. (2009) investigated how much of the €2,500 subsidy in German vehicle scrappage program is actually captured by consumers, and showed that subsidized buyers paid a little more than comparable buyers who did not receive the subsidy. Jiminez et al (2016) obtained that car manufacturers increased vehicle prices by €600 on average, after a scrappage program has been announced in Spain. Such observations constitute an invitation to account for the manufacturer role when designing an incentive program.

While some researchers studied subsidy programs from the government's viewpoint, others investigated the problem from the manufacturer's perspective. Hirte and Tscharaktschiew (2013) determined subsidy rates for electric vehicles that maximize social welfare. Lobel and Perakis (2011) characterized the optimal trajectory of subsidies for photovoltaic systems that optimizes the government's subsidy cost, instead of social welfare. Whereas in Hirte and Tscharaktschiew (2013) and Lobel and Perakis (2011) the government decides the subsidy rates without considering manufacturers' role, Ding et al. (2015) and Yu et al. (2016) let the manufacturers determine the price and quantity assuming the subsidy is given, that is, the government does not play a strategic role. In He et al. (2019), and Janssens and Zaccour (2014) both the government and manufacturers act strategically. In this essay, we too consider the government and manufacturer as players, and add the important feature that consumers also behave strategically.

Continuing the investigation of the optimal vehicle scrappage subsidy policies, in the third part of this thesis, we analyze and compare two following strategic options adopted by the government: (i) a commitment strategy in which it announces (and sticks to) a subsidy plan at the beginning of the first period; and (ii) a flexible strategy where the government sets the subsidy level at the beginning of each period. In the parlance of dynamic games, the pre-announcement strategy is referred to as commitment or open-loop strategy, whereas a strategy that depends on the state of the system is referred to as feedback (or Markovian) strategy. For example, the subsidy level in German feed-in-tarif program for solar electricity adoption was adjusted multiple times per year depending on the installed capacity of photovoltaics (International Energy Agency, 2014). In another solar subsidy case, the California Solar Initiative (CSI), there was a planned decrease in the subsidy level that was pre-announced from the outset (Chemama et al., 2019).

In the third essay of this thesis, entitled "*Subsidies and Pricing Strategies in a Vehicle*

Scrappage Program with Strategic Consumers", we consider a two-period game between government and a manufacturer. The game is played à la Stackelberg with the government acting as leader and the manufacturer as follower. Our results show that, if consumers act strategically, then the equilibrium price levels will be higher than in the scenario where they behave myopically. We also analyze and compare various subsidy and pricing strategies. In particular, we investigate scenarios where the manufacturer/government apply constant and dynamic pricing/subsidy strategies.

Chapter 1

Vehicle Scrappage Incentives to Accelerate the Replacement Decision of Heterogeneous Consumers[1]

Abstract

Vehicle scrappage subsidy programs have been widely applied by governments to replace old cars by newer, more fuel-efficient ones. While these programs have been implemented to provide motivation for replacing vehicles earlier, they may not be as effective as expected. From a cost-benefit perspective, the consumers who would have replaced anyway, even without the program, must be considered when evaluating the net benefits of the program. This requires accounting for variations in consumers' willingness to replace. Considering consumer heterogeneity in net trade-in valuation, this study investigates a dynamic vehicle-replacement problem based on a life cycle optimization (LCO) approach. We theoretically demonstrate that although increasing the subsidy level does motivate low-value consumers to replace earlier, it also induces consumers with a high net trade-in valuation to replace later in order to become eligible for the subsidy pro-

[1]This paper is published in Omega: The International Journal of Management Science

gram. We have also developed a simulation program based on real data, to demonstrate the application of our general model. According to the simulation results, ignoring consumer heterogeneity could result in an overestimation of the net benefits of the scrappage program.

Keywords: Vehicle scrappage program; Heterogeneous consumers; Dynamic programming; Life cycle optimization

1.1 Introduction

Following the global recession in 2008, many countries introduced car scrappage incentive programs with the dual aim of stimulating the car market and reducing emissions, as new cars typically pollute less than older ones. Unlike some subsidy programs that provide financial incentives for retailers and recyclers (Liu et al. (2016) and Zhang et al. (2015)), scrappage subsidies are offered directly to consumers. The CARS (Car Allowance Rebate System) program in the US was introduced in 2009, and provided subsidies of up to $ 4,500 to individuals trading in their car for a more fuel-efficient one. The program was so successful that its $1 billion budget was used up well ahead of the expected date (Huang et al., 2014). In Canada, the Vehicle Efficiency Incentive (VEI) was introduced in 2007, and offered a $2,000 rebate to consumers purchasing a new fuel-efficient vehicle (Walsh, 2012). Similar scrappage programs have also been adopted in European countries to incentivize consumers to take cars over a certain age off the road.

Scrappage incentive programs have been reported to be very successful at stimulating car sales, but their effectiveness in terms of environmental impact and cost is debatable (Smit, 2016). A recent concern regarding scrappage and other subsidy programs used to accelerate durable products replacement is consumer targeting. In a study on air conditioners, Kleine et al. (2011) highlighted that consumers differ in their willingness to replace, and that there is a need for further research to understand their behavior. More specifically, they argued that the design and assessment of incentive programs for air conditioner replacement must take into account those individuals who would replace even without a subsidy. Such consumers are called "free riders" in Blumstein (2010) and Skumatz et al. (2009), and they are excluded when calculating the net savings from the incentive program. Hoekstra et al. (2015) justified the need to account for free riders in their conclusion that 60% of subsidies from the Cash for Clunkers program (scrappage program in the US) went to households that would have purchased regardless while the program was ongoing. Such observations constitute an invitation to account for consumer heterogeneity, which seems a crucial feature when designing an incentive program.

We consider a dynamic life cycle optimization model (LCO) to study the vehicle scrappage problem. In the LCO approach, the environmental impact of a vehicle is determined by taking into account all the stages in the product's life cycle, including material production, manufacturing, use, maintenance, and end-of-life disposal. However, we concentrate only on the usage phase, which accounts for the largest share of pollution in a car's life cycle. At each period, heterogeneous consumers decide whether to replace or keep their vehicle. In order to accelerate replacement, the government offers a subsidy to consumers who replace their vehicle with a new one.

In this work, it is assumed that a new vehicle pollutes less than an older one, as a result of technological progress as well as increases in vehicle emissions levels due to age (Zachariadis et al., 2001; Pastramas et al., 2014; Trusts, 2010; Zyl et al., 2015). We develop a vintage model in which different generations of cars are differentiated in terms of their usage emissions. In such a setting, since the pollution generated from vehicle use differs for each car model, the total pollution at each period can be calculated only by knowing the age distribution of the current vehicle fleet. In other words, one needs information on the number of vehicles in use for each car model. In terms of modeling, such an information requirement creates a high burden on the dimension of the state space, making the problem of *pollution from use* much more complex than the one of *pollution from production*. When pollution is seen as an outcome of production, then the emissions in each period are usually assumed to be a function of the quantity of production in the same period (Jorgensen et al., 2010). On the other hand, pollution caused by consumption at each period is the aggregation of emissions from the use of all products (e.g., vehicles) of different ages. Consequently, in a dynamic setting, many state variables are required to keep track of the pollution generated from all products still in use in each period.

1.1.1 Literature background

Problems and issues related to environmental benefits of scrappage programs are studied widely in the literature. For instance, Dill (2004) showed that scrapped vehicles would have been driven less compared to those of the same vintage that are kept by their owners, and concluded that the reduction in emissions due to a scrappage program is lower than expected. Further, since some of the scrapped vehicles might have been retired even without the program, the subsidy in such cases is a waste of public funds. Such observations triggered cost-benefit studies of scrappage programs aimed at finding the optimal subsidy level. Assuming that a consumer will participate in a scrappage program when the subsidy exceeds the price of the old vehicle, Hahn (1995) determined the number of vehicles scrapped for different subsidy levels. Under the same assumption in Hahn (1995), Lavee and Becker (2009) also estimated a supply curve of vehicles for retirement as a function of the subsidy level and used car value. Using a cost-benefit analysis, they next computed the subsidy level that maximizes the net benefit of the scrappage program. Lavee et al. (2014) extended the study in Lavee and Becker (2009) by adding age and maintenance cost in the participation rate function, and estimated its parameters using real data.

Unlike the static models in the above-cited papers, Lorentziadis and Vournas (2011) proposed a dynamic model where the demand for new vehicles is equal to the number of scrapped old vehicles and depends on a time-varying subsidy. However, the authors did not account for the impact of vehicle age in the decision to replace. We shall also consider a dynamic model where the decision to replace is endogenous and depends on, among other things, the vehicle's age: intuitively, this factor is an important driver of such decisions.

As mentioned above, we apply an LCO approach to model the vehicle replacement problem. Kim et al. (2003) introduced a novel LCO approach and applied it to an automobile replacement problem. In addition to its application in single-vehicle analysis, the LCO approach has been used to find the optimal vehicle fleet conversion (Kim et al., 2004; Figliozi et al., 2013). Kim et al. (2004) studied a vehicle fleet optimization problem from a life cycle perspective, where the objective was to minimize the total environmental burden of vehicles. For a comprehensive review of fleet-based life cycle approaches, see Garcia and Freire (2017). Considering such an approach, some studies, e.g., Kim et al. (2004, 2006) and Lenski et al. (2010), have raised doubts about the environmental efficiency of scrappage programs, stating that they may increase pollution emissions due to the increased production of new cars. Further, while the decisions to replace in the above-cited papers are related only to the environment, other studies, —see, e.g., Kim et al. (2006), Spitzley et al. (2005) and Kleine et al. (2011)— used an LCO approach with a focus on both the environment and the owner's cost. Spitzley et al. (2005) showed that when ownership cost is accounted for, the replacement intervals (number of years a vehicle is kept) are larger than when one considers an emissions-damage minimization problem. Also, when the pollution damage cost is combined with ownership cost, the results are identical to those of the ownership cost optimization.

In our paper, we focus on the usage phase of the LCO approach when evaluating the environmental performance of the subsidy program, and the decisions to replace is studied from the consumer's point of view. That is, the individual's objective is to minimize her ownership cost, while the environmental consequences are evaluated ex-post.

In all the papers mentioned above, the decisions to replace a vehicle are made by homogeneous consumers. Consumer heterogeneity in scrappage policies has been taken into account in a few recent papers. Miao et al. (2017) and Li and Xu (2015) considered consumer heterogeneity in trade-in models. Shiraldi (2011) used a dynamic discrete-choice model in which heterogeneous consumers with different tastes decide whether to replace their automobile. He evaluated different incentive policies and checked whether or not the beneficiaries of such policies would have replaced their vehicle without the subsidy. Wei and Li (2016) retained the same discrete-choice setting to analyze vehicle scrappage programs with consumer heterogeneity. They stated that targeting marginal consumers, i.e., those who would not have replaced without a subsidy, is the key factor in designing scrappage programs. Further, they concluded that programs that do not have an environmental objective and that target marginal consumers are more cost-effective than those with explicit environmental objectives and no targeting of marginal consumers. Unlike these two papers, where the focus is on the effect of the scrappage program on heterogeneous consumers' vehicle choice, in this paper we concentrate on the impact of the subsidy level on the timing of the replacement decisions. In this regard, the closest paper to ours is Langer and Lemoine (2017), which studies the reaction of strategic consumers to government subsidies for new products. However, they investigated the effect of subsidies on purchasing decisions rather than on replacement decisions as we do here.

In the literature on scrappage programs, a subsidy has always been considered an incentive to accelerate replacement. However, interestingly, we show that it can motivate consumers to postpone replacing, so they can become eligible for the incentive program. Similar consumer behavior has been observed in the dynamic adoption of new products and new technologies. Here, consumers wait for cheaper options and for the entry of other firms onto the market (Gowrisankaran and Rysman, 2012). Similarly, firms postpone investing in new technologies to wait for new inventions when technological progress is rapid (Feichtinger et al., 2006). Likewise, our results imply that, besides borrowing future demand, increasing the subsidy can lead some consumers to postpone their demand too, affecting net benefit calculations of scrappage subsidy programs.

1.1.2 Research questions and contributions

To the best of our knowledge, this paper is the first to study the impact of the level of incentive payment on the supply of vehicles retired for scrappage in the presence of consumer heterogeneity. More specifically, our objective is to answer the following research questions:

1. Does the subsidy affect all consumers' replacement decisions in the same way?

2. What is the effect of a subsidy on the environment when consumer heterogeneity is taken into account?

3. What is the impact of varying the parameter values on the results?

We believe that answering these questions can help governments design a more efficient policy while also providing a better understanding of the real benefits of scrappage programs. Our main results can be summarized as follows:

1. Subsidies affect high-value consumers differently than low-value consumers. While the subsidy induces low-value consumers to replace their vehicle earlier, it motivates high-value consumers to delay replacing until their cars become eligible for the subsidy. This implies that investing in high-value consumers is a waste of funds. However, direct price discrimination between different consumer groups is not possible for the regulator. Instead, she must determine the subsidy level considering both groups, i.e., high-value and low-value consumers. On the one hand, the subsidy must be high enough to accelerate purchases by low-value consumers. On the other hand, it should be lower than the threshold above which high-value consumers delay replacing their vehicles in order to qualify for the subsidy.

2. When consumer heterogeneity is taken into account, the estimated benefits of the scrappage program are lower than when all consumers are assumed homogeneous. Specifically, the pollution reduction and new vehicle purchases resulting from the program are overestimated when consumer heterogeneity is ignored. This finding clarifies the real benefits of a scrappage programs. This can be useful when assigning budgets for future subsidy programs. Also, with a better understanding of advantages of subsidy programs, policy makers are able to choose among different options to reach their objective in the most efficient way.

The remainder of the paper is organized as follows: In Section 2, we introduce our general model, and we analytically investigate two particular instances in Section 3. In Section 4, we report and discuss our empirical results. In Section 5, we briefly conclude.

1.2 The Model

Denote by t the time (year) index, with $t \in \mathscr{T} = \{0,\ldots,T^s,\ldots,T\}$, where T^s corresponds to the terminal date of the subsidy program, which is assumed to be shorter than the planning horizon T. Let $j, j = 0,\ldots\omega$, be the vehicle's age, where ω is the maximum life span of a vehicle. We assume ω to be constant for all vehicles, irrespective of their date of introduction onto the market. At time period t a vehicle of age j is of the year model $t-j$. Consumers are heterogeneous in terms of their valuation of the environment and are indexed by $i \in I$, where I represents the set of types. Denote by $n_t, t \in \mathscr{T}$, the exogenously given number of consumers buying a car for the first time, and by N_t the number of new cars purchased at t. The difference $N_t - n_t$ is the demand for replacement vehicles at year t and will be determined endogenously.

Denote by p_t the price of a new car at time t, and let θ_i be the "utility" that a consumer of type i derives from replacing her used car by a new one. We shall refer to θ_i as the net trade-in valuation, which is measured by the difference between the environmental valuation of a new car minus the valuation of a retained car. Consequently, we define by $p_t - \theta_i$ the effective price of a vehicle sold at p_t to a consumer of type i. This definition of an effective vehicle-replacement price is in line with Huang et al. (2014), where a parameter θ is added to the consumer's utility function. The probability P_i of a first-time buyer being of type i is constant over time. We denote the distribution of first-time buyers by $(P_i)_{i\in I}$. The distribution of all buyers, including those replacing their vehicle at period t, is denoted by $(\pi_{i,t})_{i\in I, t\in\mathscr{T}}$, and will be determined endogenously.

In deciding whether or not to replace a vehicle one must consider an investment cost (effective price of the new vehicle) and the usage cost. We assume that the usage cost of a vehicle of age j at time t can be well approximated as follows:

$$C_t(j) = f(t-j) \cdot D \cdot O_t + m(j), \tag{1.1}$$

where D is the number of kilometers traveled per year (km/y), $f(t-j)$ is the per-kilometer fuel consumption, O_t is the unit fuel cost at time t, and $m(j)$ is the maintenance cost per year of a car of age j. In our formulation, fuel consumption varies by car vintage (given by $t-j$), while maintenance cost is only affected by age j. Also, consumers are assumed to be homogeneous in terms of their yearly travel demand.

The government (or the regulator) can affect the vehicle replacement decision through a scrappage subsidy. The subsidy, denoted by r, is given to any consumer who scraps a vehicle older than a given age η (measured in years), independently of the type of consumer and of the actual age of the vehicle (assuming eligibility, of course, that is, $j \geq \eta$).

We let q_t be the environmental quality index of a new vintage vehicle at time t, i.e., vintage $t-0$, defined by

$$q_t = 1 - \left(\frac{(f(t) - f(T))}{f(T)}\right). \tag{1.2}$$

This index reflects the empirical observation that the per-kilometer fuel consumption increases over time and reaches its maximum value at the terminal date, i.e., $q_T = 1$. This means that consumers give more value to newer, more fuel-efficient vehicles. Note that in practice, $\frac{f(t)}{f(T)} \leq 2$ which means that the fuel efficiency of new vehicles would at most double in the short and mid terms. Consequently, q_t is non-negative.

Given the usage cost, new vehicle price, and government policy, the replacement decision problem of consumer i owning a vehicle of age j can be modeled by the following dynamic optimization program:

$$V_{i,t}(j) = \begin{cases} C_t(j) + \beta \cdot V_{i,t+1}(j+1) & \text{if } 0 \leq j < \zeta, \\ \min\{C_t(j) + \beta \cdot V_{i,t+1}(j+1), (p_t - q_t \cdot \theta_i) + C_t(0) + \beta \cdot V_{i,t+1}(1)\} & \text{if } \zeta \leq j < \eta, \\ \min\{C_t(j) + \beta \cdot V_{i,t+1}(j+1), (p_t - q_t \cdot \theta_i) - r + C_t(0) + \beta \cdot V_{i,t+1}(1)\} & \text{if } \eta \leq j \leq \omega \text{ and } 0 \leq t \leq T^s, \\ \min\{C_t(j) + \beta \cdot V_{i,t+1}(j+1), (p_t - q_t \cdot \theta_i) + C_t(0) + \beta \cdot V_{i,t+1}(1)\} & \text{if } \eta \leq j \leq \omega \text{ and } t > T^s, \end{cases} \tag{1.3}$$

where $V_{i,t}$ is consumer i's cumulative cost of keeping or of replacing the vehicle from period t to T, and where $\zeta = 1, \ldots, \eta$ is the minimum car age below which no individual will consider replacing her vehicle. The terminal condition of the problem is $V_{i,T+1}(j) = 0$ for all consumers and all vehicles. The above program has four parts. If the car is still brand new, that is, if its age is below ζ, then replacing is not considered, and the total cost is the sum of the current cost and the discounted future cost. In the second case ($\zeta \leq j < \eta$), no subsidy is yet available. Therefore, the consumer of type i compares the total cost of replacing her vehicle to the cost of keeping the same car. The replacement cost is given by the sum of the effective price of a new car $(p_t - q_t \cdot \theta_i)$ and its usage cost. The same type of comparison is done for $\eta \leq j \leq \omega$: however, here, the price includes the subsidy. In other words, the comparison is given by $(p_t - q_t \cdot \theta_i) - r$. Recall that the subsidy is only available in periods $t = 0, \ldots, T^s$. The case $\zeta < \eta$ allows us to assess the government program's impact on those consumers who would have replaced their vehicle before becoming eligible for a subsidy, but who postpone their purchase until they become eligible. As alluded to in the introduction, one issue with subsidies being offered to all individuals satisfying a given condition is that some of them would have purchased the product anyway, without the subsidy. Our framework will allow us to assess the size of this group.

Let $d_{i,t}(j)$ be the decision made at time t by consumer i owning a vehicle of age j, defined as follows:

$$d_{i,t}(j) = \begin{cases} 0 & \text{if keep,} \\ 1 & \text{if replace.} \end{cases}$$

In order to keep track of consumers corresponding to each vehicle vintage at each period, let $K_t(j-1)$ be the set of consumers who have a vehicle of age $j-1$ at time t and let $R_t(j-1)$ be the set of consumers who replace their vehicles of age $j-1$ at time t. Consequently, we have $K_{t+1}(j) = K_t(j-1) \backslash R_t(j-1)$. In other words, the set of individuals who have a vehicle of age j at period $t+1$ is made up of those who had a vehicle of age $j-1$ at period t, minus those who replaced their vehicles at period t. Further, let $W_t(i)$ be the set corresponding to the age of vehicles that are replaced by consumers of type i at time t .

Using the above-defined sets, the expected total number of used cars being replaced at time t is then given by

$$\sum_{j=1}^{\omega} \sum_{i \varepsilon R_t(j)} \pi_{i,t-j} \cdot N_{t-j}.$$

The expected total number of new vehicles purchased at t is

$$N_t = n_t + \sum_{j=1}^{\omega} \sum_{i \varepsilon R_t(j)} \pi_{i,t-j} \cdot N_{t-j},$$

where $\pi_{i,t}$ is the probability that new car buyers (all buyers including first-time buyers as well as those who replace their car) at time t will be of type i. This probability is determined as follows:

$$\pi_{i,t} = \frac{P_i \cdot n_t + \sum_{j \varepsilon W_t(i)} \pi_{i,t-j} \cdot N_{t-j}}{N_t}.$$

To determine the pollution dynamics, denote by e the emissions in kilograms generated by the consumption of one liter of fuel, and let E_t be the accumulated stock of pollution (a state variable) by time t. To capture the fact that the vehicle emissions rate increases with age (Zyl et al., 2015), we introduce the increasing deterioration factor $\mu(j)$. The dynamics of pollution accumulation are given by the following difference equation:

$$
\begin{aligned}
E_{t+1} &= E_t + \sum_{j=1}^{\omega} \sum_{i \varepsilon K_t(j) \backslash R_t(j)} \pi_{i,t-j} \cdot N_{t-j} \cdot e \cdot f(t-j) \cdot D \cdot \mu(j) \\
&+ \sum_{j=1}^{\omega} \sum_{i \varepsilon R_t(j)} \pi_{(i,t-j)} \cdot N_{t-j} \cdot e \cdot f(0) \cdot D + n_t \cdot e \cdot f(0) \cdot D.
\end{aligned}
$$

The three terms in the above equation represent the stock of pollution from the last period, the pollution from vehicles kept in the current period, and the emissions from new cars, respectively. As we are considering a short-term planning horizon, we do not account for emissions absorption into nature. As car sales depend on the subsidy program, so does the stock of pollution. Consequently, the government can easily assess the environmental impact of the subsidy.

Table 1.1 summarizes the notations used throughout the paper.

1.3 A Simplified Model

The dynamic program defined in (1.3) cannot be solved analytically in its full generality. We shall solve the model numerically in the next section, using real data. To gain qualitative hints about its solution, we analytically investigate a simplified version of the model. Specifically, instead of having the detailed cost function in (1.1), we omit specifying $C_t(j)$ and simply assume that it is an increasing concave function in j. We also assume that the pollution emitted per year by a vehicle of age τ is e_τ, which is increasing in τ.

1.3.1 Two-period setting

First, we consider a two-period setting, with the subsidy being available in both periods. In the first period, the vehicles that can either be kept or replaced by a new one are of age 0 to ω. Vehicles of age j that are replaced in the first period are vehicles of age 1 in the second period, and those that are kept will be of age $j+1$. The terminal value of all

Notation	Description
Related to Consumers	
i	Index of consumers
θ_i	Net trade-in value of consumer type i
P_i	Distribution of new buyers
$\pi_{i,t}$	Distribution of consumers who buy a vehicle of age 0 at time t and who are affected by replacement decisions
$V_{i,t}(j)$	Value function related to consumer i who has a vehicle of age j at period t
$d_{i,t}(j)$	Decision variable related to consumer i who has a vehicle of age j at period t
$K_t(j)$	Set of consumers who have a vehicle of age j at period t
$R_t(j)$	Set of consumers who replace a vehicle of age j at period t
s_i	Incremental threshold related to the subsidy's adverse effect
I	Set of all consumers
n_t	Number of new buyers at time t
N_t	Total number of consumers who buy a new car at time t including those who replace their car
Related to Vehicles	
j	Index of vehicles (age)
$C_t(j)$	Ownership cost of a car of age j at time t
D	Travel demand per year
$f(t-j)$	Per-kilometer fuel use for a car of age j at time t
O_t	Unit fuel price at time t
$m(j)$	Maintenance cost of a car of age j
ω	Maximum life span of a vehicle
$W_t(i)$	Set of vehicles (ages) replaced at time t by consumer type i
p_t	Price of the vehicle at time t
$\mu(j)$	Parameter related to emissions deterioration by age
q_t	Fuel economy coefficient of a new car produced at time t
ζ	The age after which replacement is allowed
η	Eligibility age threshold
r	Subsidy level
Other Parameters	
t	Index for each period
T	The last period
T^s	The last period of the subsidy program
β	Discount rate
τ	Number of periods remaining till the end of the horizon
E_t	Accumulated pollution at time t
δ	Parameter representing absorption of pollution by nature

Table 1.1: Notation for the general model

vehicles at the end of the second period is assumed to be zero. The value functions in the first and second periods are as follows:

$$V_{i,1}(j) = \begin{cases} C_1(j)+\beta\cdot V_{i,2}(j+1) & \text{if } 0\le j<\zeta, \\ \min\{C_1(j)+\beta\cdot V_{i,2}(j+1),\ p-q_1\cdot\theta_i+C_1(0)+\beta\cdot V_{i,2}(1)\} & \text{if } \zeta\le j<\eta, \\ \min\{C_1(j)+\beta\cdot V_{i,2}(j+1),\ p-q_1\cdot\theta_i+C_1(0)-r+\beta\cdot V_{i,2}(1)\} & \text{if } j\ge\eta, \end{cases} \quad (1.4)$$

$$V_{i,2}(j) = \begin{cases} C_2(j) & \text{if } 0\le j<\zeta, \\ \min\{C_2(j),\ p-q_2\cdot\theta_i+C_2(0)\} & \text{if } j<\eta, \\ \min\{C_2(j),\ p-q_2\cdot\theta_i+C_2(0)-r\} & \text{if } j\ge\eta. \end{cases} \quad (1.5)$$

In the first period, strategic consumers make their vehicle replacement decision based not only on the cost in the first period, but also on the cost in the second period. Consequently, the effect of the subsidy on different groups of consumers may vary. On the one hand, offering a subsidy to those who replace only after age η in the absence of a subsidy induces them to replace their vehicle earlier. This result is quite intuitive since higher subsidies make the purchase of new vehicles cheaper, thereby motivating the consumer to replace her vehicle when it is younger. On the other hand, increasing the government payment level may have the opposite effect on the replacement decision of those who would have replaced before η if the subsidy did not exist. This adverse effect occurs because forward-looking consumers find it more cost-efficient to delay their purchase so they can become eligible for the subsidy in the next period, rather than replacing the vehicle in the current period and receiving no subsidy.

To analyze this adverse effect in the subsidy, consider the consumers who replace their vehicle of age $\eta-1$ in the first period. Assuming $\zeta>1$ such consumers satisfy the following inequality:

$$C_1(\eta-1)+\beta\cdot\min\{C_2(j),\ p-q_2\cdot\theta_i+C_2(0)-r\}\ge p-q_1\cdot\theta_i+C_1(0)+\beta\cdot C_2(1), \quad (1.6)$$

where the left-hand side (LHS) is the cost of keeping a vehicle of age $\eta-1$ in the first period plus the discounted cost-to-go of the vehicle of age η in the second period. The right-hand side (RHS) is the cost of replacing the vehicle in the first period plus the discounted cost-to-go of the vehicle of age 1 in the second period. Next, we show that, for some parameter values, increasing the subsidy level incites consumers to postpone the replacement of their vehicles aged $\eta - 1$, when they would have replaced them if the subsidy level had been lower.

Proposition 1.1. Let $\tilde{r}$ be a subsidy level satisfying

$$\tilde{r} \geq p - q_2 : \theta_i + C_2(0) - C_2(\eta),$$

that is, a consumer of type i replaces her vehicle of age η in period 2. Suppose also that the inequality in (1.6) holds true for such $\tilde{r}$.

If the subsidy level is set at $\tilde{r} + s_i$, where

$$s_i \geq \frac{1}{\beta}(C_1(\eta - 1) + \beta(p - q_2 \cdot \theta_i + C_2(0) - \tilde{r} - C_2(1)) - p + q_1 \cdot \theta_i - C_1(0)), \qquad (1.7)$$

then the same consumer would no longer replace the vehicle aged $\eta - 1$ in the first period but would postpone the replacement from period 1 to period 2.

Proof. Suppose that for a given subsidy, there are consumers who replace their vehicle of age η in the second period. Also, assume that there is a subgroup of these customers who replace their vehicle aged $\eta - 1$ in the first period with the same subsidy, that is, the inequality in (1.6) holds true. We need to show that when the subsidy exceeds a certain level, some consumers in this subgroup prefer to keep their vehicle aged $\eta - 1$ in the first period and to replace it at age η in the second period.

Suppose that the subsidy is $\tilde{r}$. For consumers who replace their vehicle at age η in period 2 at this subsidy level $\tilde{r}$, it holds that

$$C_2(\eta) \geq p - q_2 \cdot \theta_i + C_2(0) - \tilde{r},$$

or equivalently,

$$\tilde{r} \geq p - q_2 \cdot \theta_i + C_2(0) - C_2(\eta). \tag{1.8}$$

Note that if, in the second period, the replacement happened for a subsidy level r, then it would also happen for higher levels. Therefore, (1.8) holds for all $r \geq \tilde{r}$.
Besides, since vehicle η is replaced at subsidy level $\tilde{r}$, then $V_{i,2}(\eta)$ reads

$$V_{i,2}(\eta) = p - q_2 \cdot \theta_i + C_2(0) - \tilde{r}.$$

Therefore, inequality (1.6) turns into the following:
$C_1(\eta - 1) + \beta(p - q_2 \cdot \theta_i + C_2(0) - \tilde{r}) \geq p - q_1 \cdot \theta_i + C_1(0) + \beta \cdot V_{i,2}(1)$.

If the value of $\tilde{r}$ increases enough—say, to $\tilde{r} + s_i$— the direction of the above inequality changes. In other words, the consumer will no longer replace the vehicle aged $\eta - 1$ in the first period. The value of s_i can be simply characterized as follows:

$$s_i \geq \frac{1}{\beta}(C_1(\eta - 1) + \beta(p - q_2 \cdot \theta_i + C_2(0) - \tilde{r} - C_2(1)) - p + q_1 \cdot \theta_i - C_1(0)).$$

□

This result illustrates the idea that a subsidy program may have the opposite effect than the one intended. It may also benefit those who would have replaced their vehicles without the subsidy.[2] This opposite effect of the subsidy may impact estimates of the net benefits of the scrappage program, both in terms of the environment and the car market. In order to investigate this issue, let us call the consumers mentioned in Proposition 1.1 "high-value" consumers. In fact high-value consumers are those whose trade-in valuations are high enough to replace their car even in the absence of the subsidy program. In Proposition 1.1, these consumers would replace their vehicle aged $\eta - 1$ in the first period without cashing in the subsidy, $\tilde{r}$. Assume that the number of vehicles aged $\eta - 1$, and aged η

[2]This result is familiar in the area of price promotions on frequently purchased products. Indeed, a discount that is offered to attract non-users of the product or buyers of competitive brands also benefits regular users, who would have bought the product anyway. What is more, these regular users can also buy during the promotion for later consumption, inflicting a double loss in revenues to the firm.

or older, owned by these consumers at the beginning of the first period is $v_{\eta-1}$ and $v_{\geq\eta}$, respectively. The subscript $_{\geq\eta}$ represents vehicles aged η or older. We disregard cars younger than $\eta-1$, because they are not eligible for the subsidy in either of the two periods under study.

Next, we show that increasing the subsidy does not necessarily result in more car replacements and in less pollution by high-value consumers.

Proposition 1.2. Increasing the subsidy level from $\tilde{r}$ to $\tilde{r}+s_i$ in Proposition 1.1 increases the pollution emitted by high-value consumers by $v_{\eta-1}\left(e_{\eta-1}-e_1\right)$. Also, it has no effect on the total number of cars replaced by these consumers.

Proof. The proof is straightforward. When the subsidy level is $\tilde{r}$, consumer i replaces vehicles aged η and $\eta-1$ in the first period. The total pollution emitted by these consumers in both periods is

$$(v_{\geq\eta}+v_{\eta-1})e_0+(v_{\geq\eta}+v_{\eta-1})e_1. \tag{1.9}$$

When the subsidy increases to $\tilde{r}+s_i$, consumers with cars aged $\eta-1$ delay their car replacement to the second period. In this case, the total pollution is

$$v_{\geq\eta}e_0+v_{\eta-1}e_{\eta-1}+v_{\geq\eta}e_1+v_{\eta-1}e_0. \tag{1.10}$$

The difference between (1.9) and (1.10) is $v_{\eta-1}(e_{\eta-1}-e_1)$. Also, in both cases, the total number of replacements in both periods is $v_\eta+v_{\eta-1}$. □

This result shows that offering a higher subsidy to high-value consumers may degrade the environment as a result of delayed replacements. More precisely, increasing the subsidy only shifts the purchase of cars by high-value consumers from the first period to the second period. Therefore, while the total number of car replacements remains constant, the environment is degraded, since old vehicles stay on the road for one more period.

This adverse effect of the subsidy program would not have been identified if all the consumers were assumed to be homogeneous. In a homogeneous setting, high-value consumers who would have replaced their car even without the program are ignored, and all consumers have the same valuation. In other words, car owners are all treated as "low-value" consumers who need the subsidy to scrap their old car. In order to examine the problem in such a setting, we study homogeneous consumers' reaction to the subsidy, as well as its impact on the environment and on car replacements.

Assume that the net trade-in valuation, i.e., θ_i in our model, is equal to value x, which is the same for all consumers. Since all consumers are the same, the index i is omitted in value functions (1.4) and (1.5). Proposition 1.2 describes the effect of the subsidy program on the environment and on total car replacements in a homogeneous setting.

Proposition 1.3. Suppose that the net trade-in valuation of homogeneous consumers satisfies the following inequality:

$$x \leq \frac{1}{q_1}(p + C_1(0) + \beta \cdot C_2(1) - C_1(\eta - 1) - \beta \cdot \min\{C_2(\eta),\ p - q_2 \cdot \theta_i + C_2(0) - r\}), \tag{1.11}$$

that is, consumers do not replace their vehicle aged $\eta - 1$ in the first period. Also, assume that consumers do not replace their vehicle aged $\tau \in [\eta, \omega]$ at subsidy $\hat{r}$. Increasing the subsidy from $\hat{r}$ to $\hat{r} + s$ reduces pollution and increases car replacements by at least $v_\tau(e_\tau - e_1)$ and v_τ, respectively, where

$$s = p - q_1 \cdot x + C_2(0) + \beta \cdot C_2(1) - \hat{r} - C_1(\tau) - \beta \cdot \min\{C_2(\tau + 1),\ p - q_2 \cdot \theta_i + C_2(0) - r\}.$$

Proof. Condition (1.11) can be transformed into

$$C_1(\eta - 1) + \beta \cdot V_2(\eta) \leq\ p - q_1 \cdot x + C_1(0) + \beta \cdot C_2(1),$$

that is, consumers do not replace their vehicle aged $\eta - 1$ in the first period. Note that, since these cars are not eligible for the subsidy program, increasing the subsidy from $\hat{r}$ to a higher level has no effect on the decision to replace them.

Suppose that consumers keep the vehicles aged $\tau \in [\eta, \omega]$ at subsidy level $\hat{r}$. The following inequality holds:

$$C_1(\tau) + \beta \cdot V_2(\tau + 1) \leq\ p - q_1 \cdot x + C_1(0) + \beta \cdot C_2(1) - \hat{r}.$$

If the value of $\hat{r}$ increases enough—say, to $\hat{r}+s$—the direction of the above inequality changes. In other words, the consumer will no longer keep the vehicle aged τ in the first period. The value of s can be simply characterized as follows:

$$s = p - q_1 \cdot x + C_2(0) + \beta \cdot C_2(1) - \hat{r} - C_1(\tau) - \beta \cdot V_2(\tau+1).$$

Therefore, vehicles aged τ in the first period are replaced at level $\hat{r}+s$. This results in the following inequality:

$$C_1(\tau)+\beta \cdot \min\{C_2(\tau+1),\ p-q_2 \cdot \theta_i + C_2(0) - r\} \geq p - q_1 \cdot x + C_1(0) + \beta \cdot C_2(1) - (\hat{r}+s). \tag{1.12}$$

Note that, while the LHS of (1.12) is increasing in τ, the RHS is constant with respect to τ. Consequently, vehicles older than τ are replaced at level $\hat{r}+s$, if they have not already been replaced at level $\hat{r}$. Therefore, at least v_τ more vehicles are replaced at level $\hat{r}+s$, as compared to $\hat{r}$. Also, in a similar way to the argument in Proposition 1.2, one can easily obtain that the pollution decreases by at least $v_\tau(e_\tau - e_1)$. □

Condition (1.11) ensures that consumers would not replace their vehicle aged $\eta-1$ in the first period. In other words, consumers whose cars are not eligible for the scrappage program in the first period (aged $\eta-1$ or younger) would not replace their car in the first period. Also, note that x is the government's belief regarding the average valuation of all consumers, which is as low as it is characterized in (1.11). This assumption basically implies that the government only launches a scrappage program because it believes that consumers would not replace their vehicle without a subsidy. Unless if (1.11) did not hold, the government would not lunch the program as consumers would already replace cars aged $\eta-1$ (and apparently older) without receiving any subsidy. Therefore, (1.11) is the necessary condition for the existence of the scrappage program in the homogeneous setting. The result in Proposition 1.3 illustrates that, in a homogeneous setting, unlike in a heterogeneous setting, increasing the subsidy always benefits the environment and the car market. Hence, ignoring heterogeneity could result in an overestimation of the net benefits of the program. In the next section, using an example, we will clarify how the advantages of the program could be exaggerated if consumers are assumed homogeneous.

1.3.2 T-period setting

The focus of this subsection is to characterize the threshold in Proposition 1.1 for more than two periods. We extend the model to T periods, with the subsidy being available until $T^s < T$. Suppose that in period t, consumer i has a vehicle of age $j < \eta$—say, $j = \eta - \tau$—where τ is the number of periods the consumer must wait to become eligible for the subsidy program, with $\tau = 1, \ldots, \eta - 1$. Assume that the consumer's valuation is so high that she replaces the vehicle in period t, while she is not eligible to cash in the subsidy, i.e., she is a high-value consumer. Also, assume that a subsidy is available in period $t + \tau$ and that the consumer is forward-looking, meaning that she anticipates her eligibility for the subsidy, $r = \check{r}$, in period $t + \tau$. We are interested in characterizing a threshold for the subsidy at which the consumer postpones replacing her vehicle from period t to $t + \tau$ in order to benefit from the subsidy.

When the consumer replaces her vehicle at period t, her current and discounted cost-to-go from period t to $t + \tau$ is as follows:

$$C_t(0) + p - q_t \cdot \theta_i + \sum_{a=1}^{\tau} \beta^a C_{t+a}(a) \tag{1.13}$$

On the other hand, if she decides to wait until period $t + \tau$, to be able to cash in the subsidy when she replaces at age η, then she will incur the following cost:

$$\beta^{\tau}(p - q_{t+\tau} \cdot \theta_i - \check{r} + C_{t+\tau}(0)) + \sum_{a=0}^{\tau-1} \beta^a C_{t+a}(\eta - \tau + a), \tag{1.14}$$

where the first part is the discounted cost of purchasing a new vehicle in period $t + \tau$ and the second part is the discounted cost of keeping the vehicle for $\tau - 1$ periods. Note that at period $t + a$, $a = 1, \ldots, \tau - 1$, the age of the vehicle is $\eta - \tau + a$. Since we assumed that the consumer will replace the vehicle at period t when the subsidy is $\check{r}$, the cost in (1.13) is lower than in (1.14). However, this changes if the government increases the subsidy from $\check{r}$ to $\check{r} + s(i, \tau)$ where $s(i, \tau)$ is given by

$$s(i,\tau) \geq \frac{1}{\beta^\tau}\left(\beta^\tau(p - q_{t+\tau}\cdot\theta_i - \check{r} + C_{t+\tau}(0)) + \sum_{a=0}^{\tau-1}\beta^a C_{t+a}(\eta-\tau+a)\right. \quad (1.15)$$

$$\left. -C_t(0) - p + q_t\cdot\theta_i - \sum_{a=1}^{\tau}\beta^a\cdot C_{t+a}(a)\right). \quad (1.16)$$

Then, the consumer keeps the vehicle for τ periods and replaces at period $t+\tau$.

To recapitulate, the above discussion shows that in a T-period setting, as in the two-period case, increasing the subsidy triggers a delay in vehicle replacement which in turn, leads to higher emissions and lower car sales. In a T-period setting, the threshold (1.7) in Proposition 1.1 (two-period model) is replaced by (1.16) for $\tau = 1,\ldots,\eta-1$. Similarly, having the information about the number of vehicles with age $1,\ldots,\eta-1$, we can rewrite Proposition 1.2 for the general case. However, here, in order to be able to rank $s(i,\tau)$ for different values of τ, we need the full information about the shape of the function $C_t(j)$. In addition, since the value function is not monotone in age, the RHS of (1.11) is also not monotone in $\tau = 1,\ldots,\eta-1$. This adds to the complexity of characterizing an upper bound for x in the T-period setting. Nevertheless, Proposition 1.3 holds for different values of τ separately. In a nutshell, all the qualitative results obtained with 2 periods seem to carry over to T periods.

1.4 Empirical Results

In this section, we illustrate our general model using available data, and run a sensitivity analysis to assess the impact of the main model's parameter values on the results.

1.4.1 Parameters setting

To run our model, we need to estimate some parameter values and to fit some functions. To begin with, we shall solve the problem for a base-case scenario characterized by the following parameter values:

$$\begin{aligned} T^s &= 3,\ T = 10,\ \omega = 14,\ \beta = 0.9,\ \eta = 10, \\ p &= 33.5,\ O_t = 1,\ e = 2.6,\ n_t = 0,\ D = 25000, \end{aligned}$$

where we let the subsidy rate and trade-in valuation have different values in order to capture the impact of the subsidy program on different market segments, that is,

$$r \in \{0, 100, \ldots, 4,000\},\ \ \theta \in \{22, 23, \ldots, 38\}.$$

In order to account for consumer heterogeneity, we follow Huang et al. (2014), where the net trade-in valuation is assumed to be normally distributed, with a mean $E(\theta)$=30,000 and standard deviation σ =4,000. We assume that θ is deterministic and constrain its values to $E(\theta) \pm 2\sigma$ and, also as in Huang et al. (2014), we fix $p = 33,500$. In accordance with scrappage programs in some European countries such as France, Spain, and Ireland, we assume that the vehicle's age of eligibility for the subsidy is 10 years. Note that, while the duration of the subsidy program is 3 years, the (consumer's and government's) planning horizon is longer, i.e., 10 years. Further, in order to identify the number of vehicles of different ages at the initial date, we use the density distribution of vehicles aged 1 to 14 provided in Engers et al. (2012).

To run the model, we need to give values to $f(t-j), m(j)$ and $\mu(j)$. To estimate $f(t-j)$, we follow Spitzley et al. (2005), where the miles per gallon (MPG) for passenger cars is assumed to increase 1% yearly with respect to 22.8 MPG in 1995. Then, we use the MPG data to estimate $f(t-j)$ as follows:

$$f(t-j) = \frac{3.8}{1.6}\left(\frac{1}{MPG(t-j)}\right),$$

which results in fuel consumption in liters per kilometer. To estimate the maintenance cost $m(j)$, we take data from YourMechanic (2016), which gives this cost per block of 25,000 miles driven (see Table 1.2).

Mileage	Total Maintenance Cost per 25k Miles
0-25,000	$1,400
25,000-50,000	$2,200
50,000-75,000	$3,000
75,000-100,000	$3,900
100,000-125,000	$4,100
125,000-150,000	$4,400
150,000-175,000	$4,800
175,000-200,000	$5,000

Table 1.2: Maintenance cost for each 25,000 miles driven

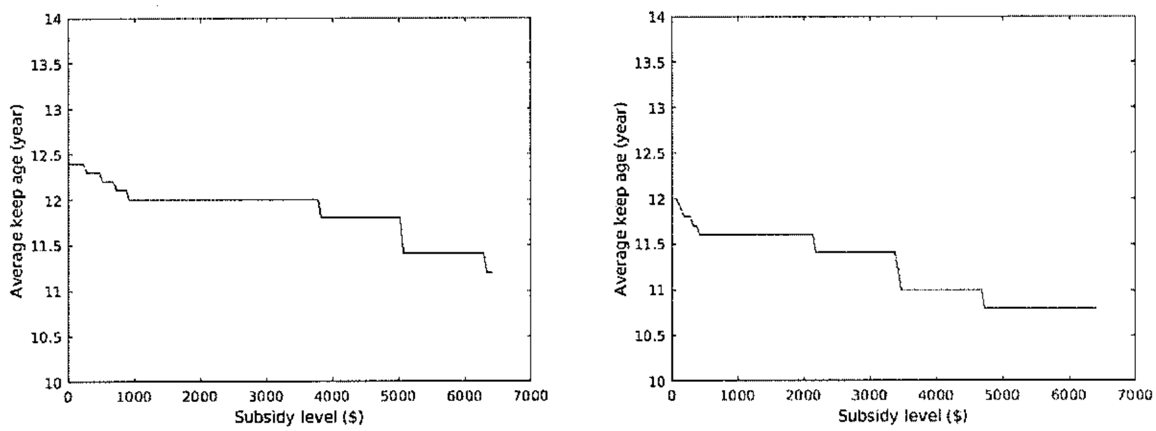
The emissions deterioration factor is assumed to increase by 0.09 from age 1 to 10 and then to remain constant for the rest of the vehicle's life (Zyl et al., 2015):

$$\mu(j) = \begin{cases} 1 & \text{for } j = 0, \\ \mu(j-1) + 0.09 & \text{for } 1 \leq j \leq 10, \\ \mu(10) & \text{for } j \geq 11. \end{cases}$$

1.4.2 Heterogeneous consumers' decisions

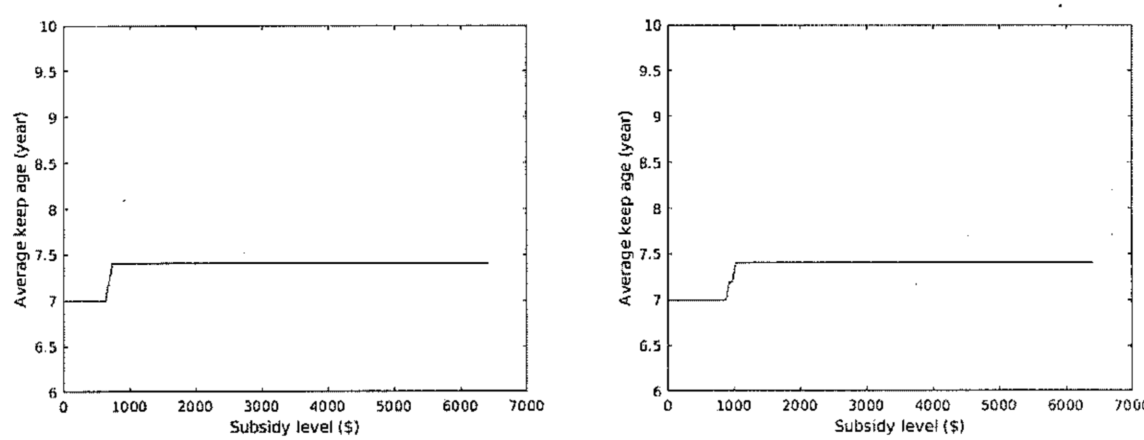
Considering the functions and parameters described above, we evaluate the effect of the subsidy on the decision to replace, by solving the dynamic program. Figures 1.1 and 1.2 show how the subsidy affects the replacement decisions of the different groups of consumers. The average keep age demonstrates how long vehicles are kept by consumers on average during the horizon.

One main takeaway is that increasing the subsidy does not necessarily result in earlier replacements for all groups of consumers. This surprising result stems from the subsidy program having an indirect effect on some consumers, who will prefer to postpone replacing their car in order to benefit from the subsidy, instead of paying the full price themselves. By now we are able to answer our first research question, namely, what effect does the subsidy have on heterogeneous consumers' decisions? The answer is different for consumers whose trade-in valuation is high than for those with a low valuation. When



(a) (b)

Figure 1.1: Average keep age decreases with subsidy for consumers with a valuation of (a) 25,000 and (b) 27,000.



(a) (b)

Figure 1.2: Average keep age decreases with subsidy for consumers with a valuation of (a) 35,000 and (b) 37,000.

consumers have a low valuation—say 25,000 or 27,000 (as in Figure 1.1)—increasing the subsidy induces earlier replacements, which results in a decreasing trend in the average keep age. This intuitive result holds true for all "low-value" consumers whose trade-in valuation ranges between 22,000 and 31,000. In contrast, "high-value" consumers with a trade-in valuation higher than $31,000—say 35,000 or 37,000 (as in Figure 1.2)—respond to an increase in the subsidy by replacing later. Tables 1.3 and 1.4 clarify this behavior in high-value consumers more precisely. Consumers with a valuation of 35,000 replace vehicles aged 8 or 9 at subsidy levels under 1,230 and 680, respectively. However, above the thresholds, they keep these vehicles until age 10 so they can benefit from the subsidy. Similarly, consumers with a valuation of 37,000 postpone replacing, as shown in Table 1.4.

Vehicle age	Subsidy range to replace	Subsidy range to keep
8	$r < 1230$	$r \geq 1230$
9	$r < 680$	$r \geq 680$

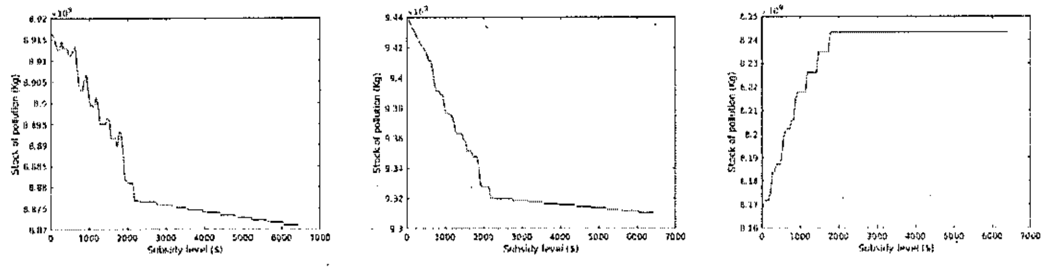
Table 1.3: Replacement decision of consumers with a valuation of 35,000, for a vehicle aged 8 and 9

Vehicle age	Subsidy range to replace	Subsidy range to keep
8	$r < 1830$	$r \geq 1830$
9	$r < 930$	$r \geq 930$

Table 1.4: Replacement decision of consumers with a valuation of 37,000 for a vehicle aged 8 and 9

1.4.3 Stock of pollution and total car replacements

So far, we have studied separately the reaction of different groups of consumers to a variation in the subsidy level. The next step is to mix consumer groups to find the subsidy's environmental effect and market impact in the presence of heterogeneity. These effects are shown in Figures 1.3 and 1.4. According to Figure 1.4.3, the effect of increasing the subsidy on the total amount of pollution by all consumer groups is decreasing, de-



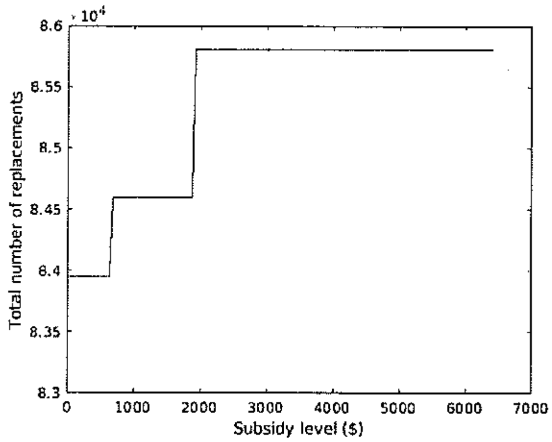
(a) (b) (c)

Figure 1.3: Effect of subsidy on stock of pollution, considering consumers with (a) both high and low valuations, (b) only a low valuation and (c) only a high valuation.

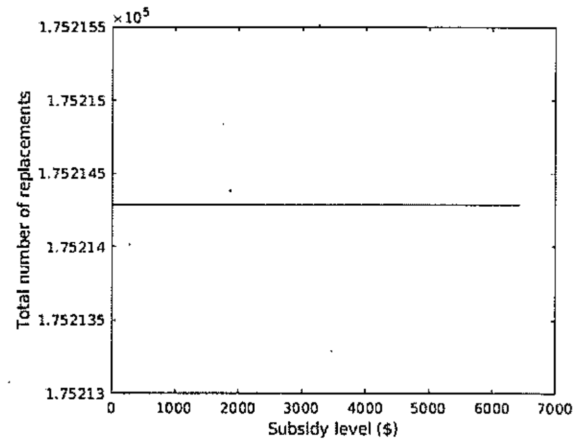
spite some ups and downs. When consumers are divided into low-value and high-value groups, this result does not necessarily hold true. More precisely, when consumers have a low valuation, ranging between 22,000 and 31,000, the trend is downward (Figure 1.4.3). However, when we only take into account high-value consumers, whose valuation is between 32,000 and 38,000, the subsidy has a negative effect on the environment (Figure 1.4.3).

Similarly, the effect of the subsidy on the total number of replacements during the planning horizon is not the same for high-value and low-value consumers. While increasing the subsidy increases replacements in low-value consumers (Figure 1.4.3), it has no effect on new vehicle purchases by those with a high valuation (Figure 1.4.3). Indeed, the subsidy only delays new vehicle purchases by high-value consumers and does not have any impact on the total number of cars replaced by them. Consequently, older vehicles remain on the roads longer, which is environmentally harmful.

Now, we turn to the paper's second question, namely, what is the effect of the subsidy on the environment when consumer heterogeneity is taken into account? The rationale behind this question is both methodological and practical. If consumers are indeed heterogeneous, then retaining a homogeneous representation would lead to, at best, a very rough approximation of their behavior. At worst, it would lead to a completely wrong result. Practically speaking, predicting numbers of replacements when assuming a homogeneous market will be erroneous and misleading to the public decision-maker at-



(a)



(b)

Figure 1.4: Effect of subsidy on total replacements during the horizon, considering consumers with (a) only a low valuation and (b) only a high valuation.

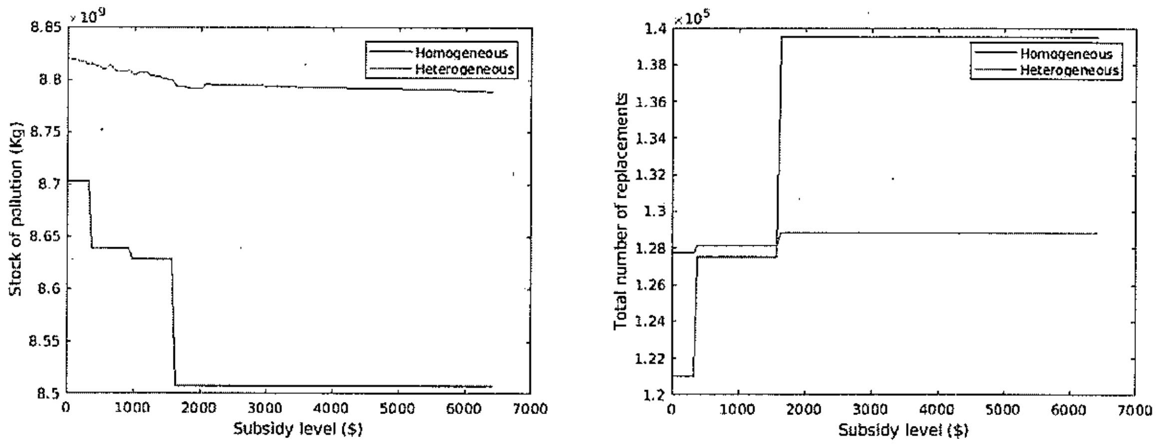
tempting to design a subsidy program. To obtain a hint of how the results differ under a homogeneous and a heterogeneous market, we run our model under the two scenarios. To do so, suppose that heterogeneous consumers are uniformly distributed over the set $\Theta \in \{22{,}000; 23{,}000; \ldots; 38{,}000\}$ of trade-in values. Also assume that, in a homogeneous setting, the only consumer group has a net trade-in value of $\tilde{\theta} = 30{,}000$, which is the middle point in the set Θ. In Figure 1.5, we plot the values of the stock of pollution (panel (a)) and the number of vehicle replacements (panel (b)) for different subsidy level values. From Figure 1.4.3, we see that giving a $2,000 subsidy leads to a reduction in pollution of 2×10^8 kilograms (from 8.7×10^9 to 8.50×10^9) when we assume consumer homogeneity. It also leads to a much lower decrease, of 3×10^7 kilograms (from 8.82×10^9 to 8.79×10^9) under the heterogeneity assumption. Such a large difference is observed for all other subsidy values. Similarly, we obtain very different estimates for the number of vehicle replacements under the two market compositions (see Figure 1.4.3). In particular, for a subsidy of $2,000, the number of additional new vehicle purchases, as compared to a no-subsidy case, is estimated at 18,500 and 1,088 when consumers are assumed to be homogeneous and heterogeneous, respectively. By any measure, this difference is sizeable. The immediate implication of these comparative results is that the reduction in pollution and the number of new vehicle purchases resulting from a scrappage program would be highly exaggerated if the regulator used a mean trade-in valuation to predict the impact of the program.

We also compared the cost of the program under both assumptions: the results are exhibited in Figure 1.6. As previously discussed, accounting for heterogeneity makes it possible to assess the magnitude of high-value consumers who, even without the program, would replace their vehicle before it reached the program eligibility age. This enables us to measure the money wasted on these consumers who do not need the subsidy as an incentive to replace their vehicle. Figure 1.4.3 illustrates that, in our case study, up to 40% of the total subsidy spending is invested on consumers with a high valuation. This result is in line with those in Hoekstra et al. (2015) and Li and Linn (2013), which empirically show that 60% and 45%, respectively, of the subsidies in Cash for Clunkers went to households that would have replaced their vehicle even without the program. As stated in Dill (2004), Kim et al. (2004), and Lenski et al. (2010), although scrappage programs improve both the environment and the car market overall, they might not be as efficient as could be imagined. These papers identified pollution from car production as the main cause of scrappage program inefficiency. Here, by considering consumer heterogeneity, we are able to better estimate how such programs positively impact pollution emissions from vehicle usage. This provides scholars and policy makers with a greater understanding of scrappage program efficiency.

1.4.4 Sensitivity analysis

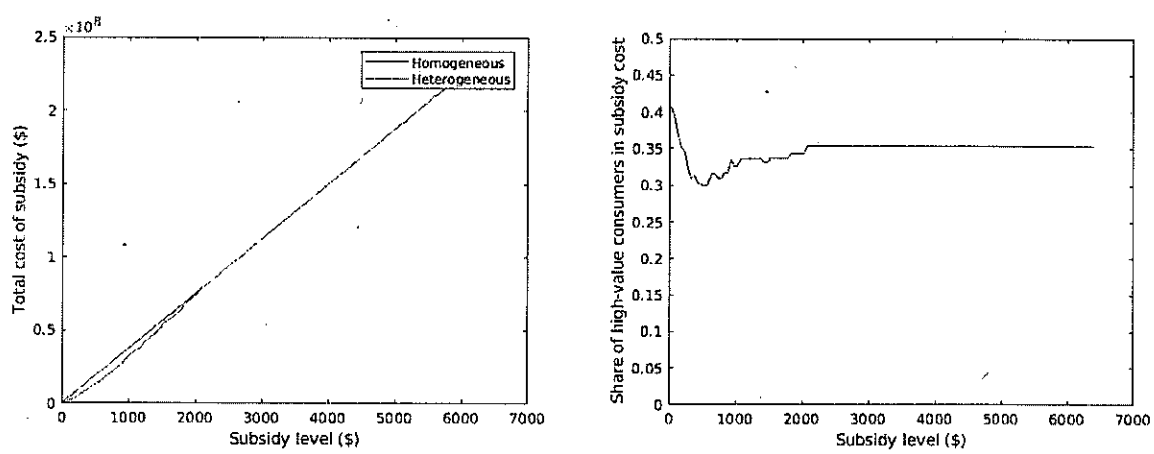
Finally, we conduct a sensitivity analysis to assess the robustness of our results with respect to changes in some of the parameter values. The retained parameters and their values are listed below:

$$T \in \{9,10,11\}, T^s \in \{2,3,4\}, \beta \in \{0.85,0.9,0.95\}, \eta \in \{9,10,11\},$$
$$\zeta \in \{7,8,9\}, D \in \{24000,25000,26000\}, MPG(t-j) \in \{0.008,0.01,0.012\}.$$



(a) (b)

Figure 1.5: Effect of subsidy on a) the stock of pollution and b) total replacements, for homogeneous and heterogeneous consumers.



(a) (b)

Figure 1.6: a) Total subsidy cost for homogeneous and heterogeneous consumers, and b) Share of total subsidy spent on high-value consumers

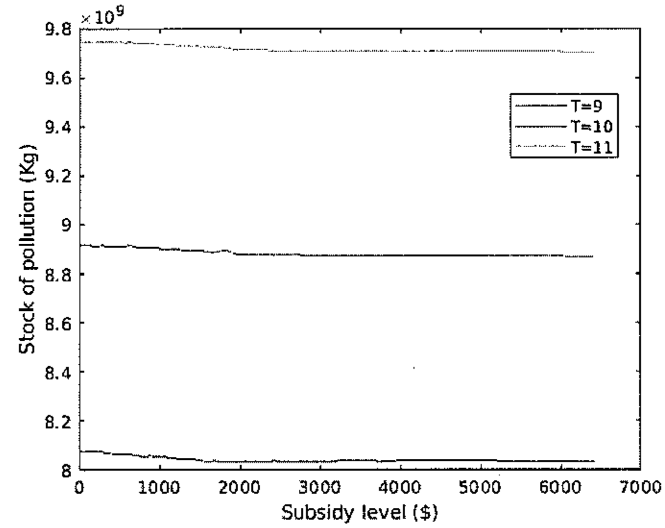


Figure 1.7: Effect of subsidy on stock of pollution for different values of T

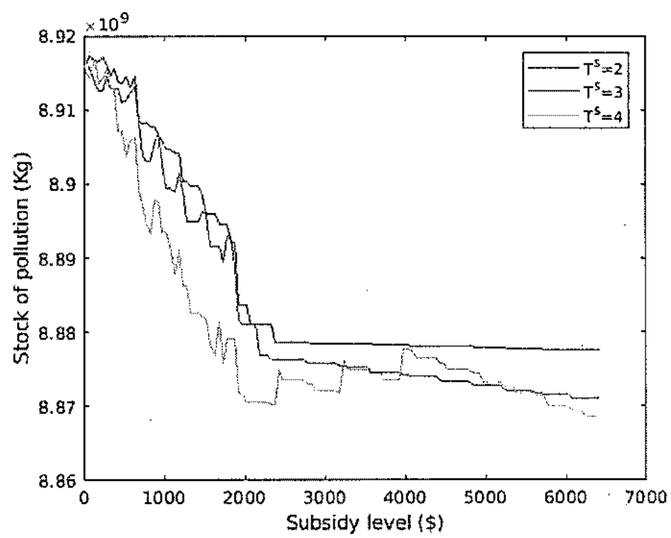


Figure 1.8: Effect of subsidy on stock of pollution for different values of T^s

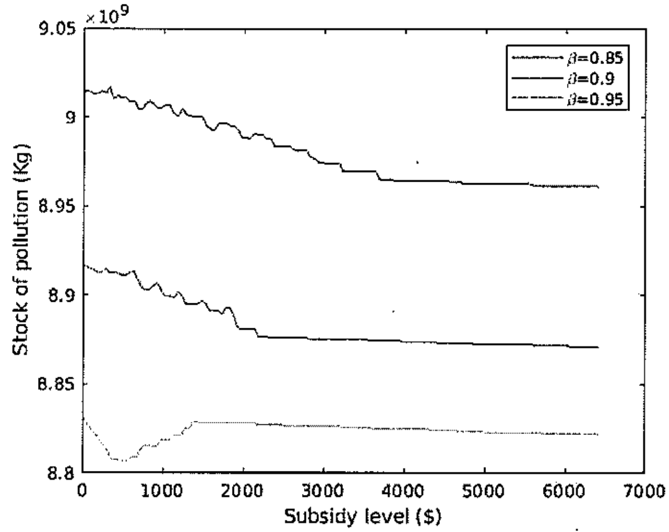


Figure 1.9: Effect of subsidy on stock of pollution for different values of β

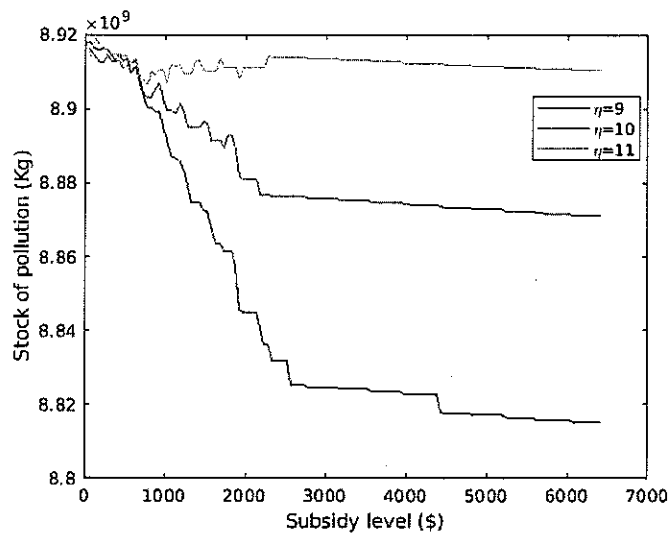


Figure 1.10: Effect of subsidy on stock of pollution for different values of η

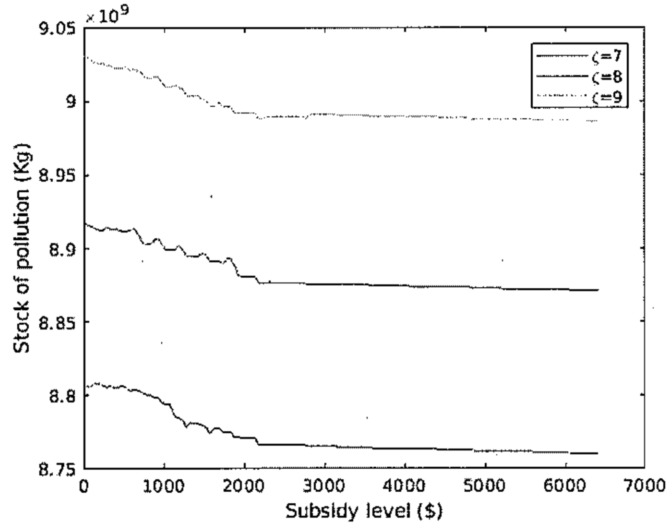


Figure 1.11: Effect of subsidy on stock of pollution for different values of ζ

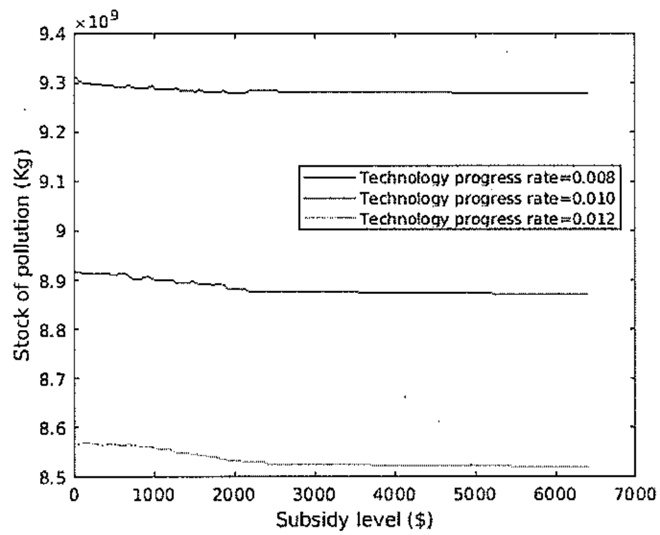


Figure 1.12: Effect of subsidy on stock of pollution for different values of $f(t-j)$

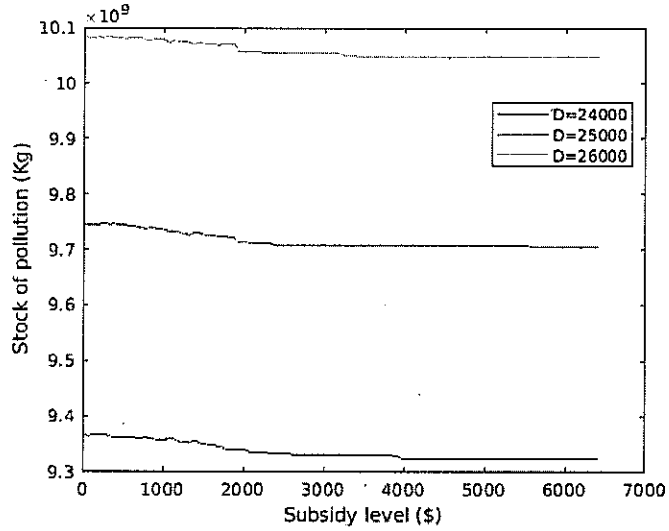


Figure 1.13: Effect of subsidy on stock of pollution for different values of D

The results can be summarized as follows:

1. Increasing the value of T leads to an increase in the stock of pollution (see Figure 1.7). This figure calls for two observations. First, the incremental pollution is almost the same for all subsidy levels. From a subsidy-program design perspective, this result tells the regulator that the main effects of these two parameters are not correlated, and hence, can be considered independently. Second, adding one year to the planning horizon, i.e., shifting T from 9 to 10 or from 10 to 11 (an increase of 11%) leads to an increase in the pollution stock of roughly 0.9×10^9 kg or 8.65%. Although this is far from negligible, the increase in pollution is of a lower magnitude than the increase in period length. The intuition behind this result is as follows: old vehicles are scrapped at the beginning of the planning horizon (subsidy period), and for the rest of the horizon, the vehicle fleet is new. As pollution from a new, fuel-efficient vehicle fleet is lower than from older vintages, the impact on pollution of adding one period is not as high. The parallel curves in Figure 1.7 mean that varying T only has a quantitative impact on the stock of pollution, without affecting the qualitative behavior.

2. Varying the duration of the subsidy program seems to have very little effect on the stock of pollution (see Figure 1.8). This is because old vehicles, which are the main source of pollution, are eligible for the subsidy program in the first or second period of the subsidy program. Therefore, prolonging the subsidy period would have little effect on the replacement of cars, because the old ones are already off the road. This result should be of great interest to policymakers wishing to design scrappage programs. Indeed, it states that the duration of the subsidy program itself has little importance when it comes to environmental performance.

3. Despite some non-monotonic behavior for low subsidy values, varying the discount factor β essentially has an impact of the same magnitude on the pollution stock (see Figure 1.9). When consumers attach a higher importance to future costs, they are more inclined to purchase earlier to benefit from a new fuel-efficient car longer. In turn, the stock of pollution is lower.

4. From Figure 1.10, we see that varying the eligibility age η (of course naturally within our given range) has no effect on the stock of pollution when the subsidy is low. For higher levels of subsidy, the impact of η on this stock is very low in relative terms. Indeed, adding one year (an increase of 11%) leads to an increase in the pollution stock of at most 1%.

5. The relative impact of increasing the minimum age of replacement on the pollution stock is rather small. Indeed, decreasing ζ from 9 years to 7, which is a large decrease, leads to a reduction in the stock of pollution of less than 3% (see Figure 1.11). This result may seem counterintuitive, but it is not. Indeed, the result means that lowering further the minimum age of replacement does not help in reducing pollution. The reason is that consumer is anyway not interested in changing her vehicle if it is still "too" young.

6. Increasing the technology progress rate by 50%, that is, from 0.008 to 0.012, leads to an increase in the stock of pollution of roughly 10% (see Figure 1.12). On the

one hand, a higher technological progress results in a newer vehicle fleet, which in turn, reduces pollution. On the other hand, it takes time for a whole fleet to be changed. This result is along the same line as the one just above.

7. Finally, for all subsidy levels, a larger travel demand induces an increase in pollution of the same magnitude (see Figure 1.13).

In summary, the main takeaway of our sensitivity analysis is that varying the parameter values does not much affect the qualitative results: we only see parallel shifts in the curves. Quantitatively speaking, the impact on pollution is at most of the same magnitude as the change in the parameter values. The conclusion is that our results are clearly robust, and should therefore be helpful for designers of scrappage programs.

1.5 Conclusion

In this paper, we applied a dynamic programming model to investigate the effect of vehicle scrappage programs on strategic consumers' decision to replace their vehicle. Assuming that consumers are heterogeneous in terms of their net trade-in valuation, we examined the impact on different consumer groups of changing the government payment level. We theoretically show that under certain circumstances, increasing the government subsidy may have an adverse effect on some consumers. In particular, when the government payment exceeds a certain threshold, forward-looking consumers with a high net trade-in valuation will prefer to delay their replacement until their vehicle become eligible for the program.

Our computational experiments also demonstrate that, while a scrappage program induces low-value consumers to discard their old vehicle earlier, it may also motivate high-value consumers, who would have replaced their vehicle even without the subsidy, to replace it later. Therefore, in addition to wasting the subsidy, encouraging high-value consumers to delay replacing their car is also harmful to the environment. These results emphasize the importance of accounting for heterogeneity in consumer valuations when a subsidy program is designed. We find that the net benefits of scrappage programs could be overestimated when consumers are taken to be homogeneous. More precisely, ignoring heterogeneity and relying only on the mean group of consumers results in higher estimates of pollution reduction and of new vehicle purchases when calculating program benefits. These results are in line with Li and Linn (2013), where 45% of the Cash for Clunkers program's expenditure is estimated to be spent on those who would have replaced even without the program.

Two challenging extensions to our work are worth considering. First, our approach aimed at providing the regulator with results describing the impact of different subsidy levels on the outcomes (pollution and replacements). It would be interesting to give the regulator a more strategic role by assuming, for instance, that it plays as the leader in a non-cooperative game, where consumers are followers reacting to the leader's subsidy announcement. The main challenge when computing Stackelberg equilibria is in solving the resulting high-dimensional dynamic program. Second, it is worth attempting to relax the assumption that the subsidy is constant over time and to introduce a time-varying one. Where needed, this would give the regulator a tool to smooth replacements over time.

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Chapter 2

Optimal Government Scrappage Subsidies in the Presence of Strategic Consumers[1]

Abstract

Many countries have introduced vehicle scrappage programs to motivate consumers to replace their old cars earlier. Since these programs are offered during a given period of time, policy makers need to plan for inter-temporal subsidies. Considering a two-period game between strategic consumers and the government, we determine the optimal scrappage subsidy levels. Our results demonstrate that the subsidy level in the second period is higher than the first period's, allowing the government to discriminate on price (or subsidy) between consumers with different valuations. In addition, we show that subsidy levels increase with the government's targeted replacement level. However, when the government target level changes from intermediate to high levels, the first-period subsidy drops while the second-period subsidy remains unchanged.

Keywords: Vehicle scrappage program; Strategic consumers; Stackelberg game; Gov-

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ernment subsidies.

2.1 Introduction

It is a common practice for governments to design incentive programs to accelerate the adoption of new green (or less-polluting) durable products or technologies. For instance, the US government gives consumers who buy a new electric vehicle $2,500 to $7,500 in tax credits (Shao et al., 2017), and the California Solar Incentive (CSI) program invested $2.167 billion on installing solar panels (Cohen et al., 2016). Other programs offer consumers a subsidy for replacing their aging appliances. In Canada, Heating, Ventilation, and Air Conditioning (HVAC) rebates offset the cost of upgrading to a new heating and cooling system. These financial incentives motivate households to replace their old, high-energy-consuming appliance with a more energy-efficient one. One of the most prevalent replacement incentive policies is the vehicle scrappage program, variations of which have been implemented in many countries. In the US, the Cash for Clunkers program provided up to $4,500 to consumers who replaced their old car with a new, more fuel-efficient one. Similar subsidy programs have also been adopted in Canada, Asia, and the European Union (Huang et al., 2014).

While these subsidy programs have both environmental and financial benefits for consumers and suppliers, they are also costly. Therefore, governments should aim at designing subsidy programs that achieve their objectives in the most cost-efficient way. When an incentive program lasts over time, the decision-maker must carefully plan the subsidy schedule. Indeed, the impact of having a constant subsidy versus a varying (decreasing or increasing) subsidy over time is not neutral. For instance, if the subsidy is decreasing over time, then some consumers may accelerate their vehicle replacement, which would most likely result in a smaller environmental gain than if these consumers had waited a while before replacing. In practice, both decreasing and increasing trends are observed in different subsidy programs: while the US federal government subsidy for renewable energy was increasing over time, a vehicle scrappage program in Denmark had a decreasing trend (Jorgensen and Zaccour, 1999).

In this paper, we investigate the problem of designing a subsidy program over time. To keep our model as parsimonious as possible, while still being able to shed an interesting light on subsidy schedules, we retain a two-period game model. The game is played à la Stackelberg, where the government (leader) announces the subsidy program, and consumers (followers) make their replacement decisions. Consumers are assumed to be strategic, that is, they decide whether to keep or replace their car based on their current and future utilities. The problem to solve in our context is more complicated than the one arising in a typical purchasing decision by strategic consumers. Indeed, when replacing an old car with a new one, the consumer takes into account the value of both cars, not only the value of the new one. Moreover, a strategic consumer purchasing a new product accounts for the perceived value of the product at different periods. This implies that, in a replacement problem, consumers are not only heterogeneous in terms of their perceived valuation of the new product but also in the age of their currently owned cars. This led us to including two heterogeneities in our model: in consumer valuation (the valuation of a consumer of driving a car) and in used car age (quality of the car). Another difference with the typical timing of purchasing decisions is the fact that scrappage programs usually include an eligibility age. This distinguishes vehicle scrappage policies from trade-in programs, where the consumer can trade in a product for a new one, without any age constraint. Hence, it is necessary to differentiate between consumers who are eligible at a given period and those who can get the subsidy only in future periods.

Governments often assign an adoption target to their subsidy programs. For example, in 2011, President Obama announced in 2011 a target of "one million electric vehicles on the road by 2015". Another example is the "1,940 MW of new solar generation capacity" target assigned by the California Solar Incentive program (Cohen et al., 2016). Along these lines, we suppose that the government's objective is to minimize the total cost of the program, subject to satisfying a preset target. Clearly, one can imagine other objectives such as minimizing greenhouse gas emissions or maximizing social welfare. However, Cohen et al. (2016) argue that these optimization problems are equivalent and yield the same results. In this paper, we are also interested in finding out how the target level affects the government's optimal policy.

2.1.1 Brief literature background

Vehicle scrappage subsidy programs, which were designed to protect the environment and boost the car market, have been the topic of numerous studies over the past two decades. Some have investigated the problem using a static model. In a static framework, Hahn (1995) determines the number of scrapped cars as a function of the subsidy level. In a similar setting, Dill (2004) illustrates that scrapped vehicles are usually driven less than other of the same age. Lavee et al. (2014) analyze the effectiveness of the scrappage program by estimating a supply curve of retired vehicles as a function of the subsidy level. Li and Linn (2013) calculate the number of new vehicle sales resulting from the Cash-for-Clunkers program. They show that 45% of the spending went to consumers who would have replaced their car even without the subsidy.

Unlike the above-cited contributions, some papers have developed dynamic models to design subsidy programs. In the context of the adoption of a new technology with learning-by-doing, Jorgensen and Zaccour (1999) and Janssens and Zaccour (2014) determine the equilibrium path of consumer subsidies in dynamic games involving a government and a firm. Jorgensen and Zaccour (1999) also take into account guaranteed buys by the government, while assuming linear cost learning. Janssens and Zaccour (2014) retain hyperbolic cost learning and compute the total subsidy budget needed to reach a desirable price by the end of the subsidy program. Lobel and Perakis (2011) characterize the trajectory of consumer subsidies for photovoltaic technology. In these papers, dynamic subsidies are offered for purchasing new technologies (or products), but scrappage programs provide incentives for replacing old products with new ones. In this sense, scrappage programs are similar to trade-in subsidies. Zhu et al. (2017) consider a two-period model where a firm makes new products in the first period and collects used products in the second period through a trade-in program. Using a similar setting, Miao et al. (2017) develop three supply chain structures with trade-ins. They also characterize conditions under which trade-ins can promote the environment. Zhang and Zhang (2018) address how consumer purchase behavior affects the economic and environmental benefits of trade-in programs. Likewise, we consider a two-period model where strategic consumers decide whether to trade in their car. However, our paper differs from these contributions by accounting for an eligibility age in the scrappage programs.

Strategic consumer behavior has been extensively studied in the literature. For a comprehensive review of the effect of strategic consumers on pricing, inventory, and information, see Wei and Zhang (2017). A few recent papers have accounted for consumer behavior in subsidy policies. Shiraldi (2011) proposes a dynamic discrete choice model to study automobile replacement decisions by heterogeneous consumers. He argues that some beneficiaries of the scrappage subsidy program would have replaced even without the subsidy. Using a similar setting, Wei and Zhang (2017) show that targeting consumers who would not have replaced their car without a subsidy is the key factor in designing scrappage programs. While in Shiraldi (2011) and Wei and Zhang (2017), the vehicle choices of heterogeneous consumers are studied, Langer and Lemoine (2017) focus on the timing of the decision of strategic consumers. In this regard, Langer and Lemoine (2017) is similar to our paper except that it investigates the effect of the subsidy on new product purchases, rather than on replacing old products as we do here. The closest paper to ours is Zaman and Zaccour (2018), where the effect of consumer heterogeneity on the vehicle replacement decision is addressed. Specifically, consumers who need subsidy to replace their car and those who replace in the absence of the subsidy program are characterized. In Zaman and Zaccour (2018), the authors solve the optimization problem of the consumer, and the effects of the subsidies are analyzed ex-post, assuming no strategic role for the government. In this paper, we relax this assumption and let the government act strategically when designing its subsidy program.

2.1.2 Research questions and contributions

To the best of our knowledge, this paper is the first to study the equilibrium scrappage subsidy policies in the presence of strategic consumers. Our aim is to answer the following questions:

1. How do strategic consumers owning cars of different ages react to current and future subsidies?
2. What are the most cost-efficient policies to reach different replacement target levels?
3. Under what conditions should the government reduce the eligibility age?

We believe that answering these questions can help the government design cost-efficient scrappage programs. Our results are summarized as follows:

1. A consumer's decision to replace depends on both periods' subsidy levels and on the difference between them. When this difference is low, consumers who are eligible in both periods will either replace in the first period or never replace. However, some consumers who are only eligible in the second period wait to become eligible for the program and cash in the subsidy. When the second-period subsidy is large enough compared to the first-period subsidy, some consumers who are eligible in both periods would be better off replacing in the second period rather replacing in the first period or never replacing at all. In fact these consumers (who are eligible in both periods) need more incentive in the second period (i.e., higher second-period subsidy) to opt for replacement in that period rather than for no replacement at all.
2. As one might expect, the optimal subsidy policy depends on the car replacement target level. Clearly, a low target can be met with no subsidy. For medium target levels, subsidies in both periods are positive and increasing in the target level, and

the second-period subsidy is higher than the first-period one, allowing for price discrimination between high- and low-value consumers (consumers with high and low valuations, respectively). While high-value consumers replace in the first period with a lower subsidy, those with a lower valuation are given higher incentives in the second period. As the target level increases, there is a threshold at which the first-period subsidy is dropped to a lower value while the second-period subsidy is increased. However, after this threshold, the two subsidy values again increase in the target level. In fact, for a higher target than this threshold, the government needs to induce more low-value consumers to replace in the second period. Consequently, it offsets the higher subsidy cost in the second period by reducing the first-period subsidy level.

3. The government's eligibility age is based on two factors, namely, its target level and the incremental number of cars that become eligible if eligibility age is reduced. Generally, the government is better off to decrease the eligibility age when these two factors are high enough.

2.2 Model

Consider a two-period model where the government subsidizes consumers who replace their car that are older than a given age by new ones. Denote by p, s_1, and s_2, the price of the new car and the subsidies in the first and second period, respectively. Let v_i be the valuation of consumer i of driving a car. We assume v_i to be independent of the car's quality and age and uniformly distributed between 0 and 1. We also assume that each consumer can only have one car. Denote by $Q_\tau, \tau = 0, \ldots \omega$, the quality of a car of age τ, where ω is the maximum age in the vehicle fleet. We suppose, not unrealistically, that $Q_\tau \geq Q_{\tau+1}$, i.e., that car quality decreases with age. The utility of consumer i owning a car of age τ is postulated to be given by $v_i Q_\tau$. In line with Barahona et al. (2016), this multiplicative form implies that utility is higher for newer cars and for higher-value consumers. Let η be the minimum age to become eligible for the scrappage subsidy program. In our two-period framework, a vehicle can be eligible either in both periods or only in the second one. (There is no point in considering cars that are not eligible in either period, as the program has no effect on their replacement.) Consumers whose vehicles are eligible in both periods can choose between not replacing, replacing in the first period and receiving subsidy s_1, or replacing in the second period and receiving subsidy s_2. Consumers who are not eligible in the first period can benefit from the subsidy program only if they replace their vehicles in the second period. To keep the problem tractable, without much loss of qualitative insight, we make the simplifying assumption that all vehicles aged $\eta - 1$ or older are of low quality, denoted Q_l, and can be replaced by new cars of high quality Q_h.

Remark 2.1. As in Zaman and Zaccour (2019) and Huang et al (2014), the subsidy is understood to cover a possible salvage value of the old car. Alternatively, one can assume that the salvage value has been normalized to zero or it is a given constant, which does not affect the optimization problem of the consumer.

A consumer owning a vehicle aged $\tau \geq \eta$ and replacing it in the first period, enjoys the total discounted utility given by $v_iQ_h - p + s_1 + \delta v_iQ_h$. If instead she replaces in the second period, then her total discounted utility is $v_iQ_l + \delta(v_iQ_h - p + s_2)$. Never replacing yields the total discounted utility $v_iQ_l + \delta v_iQ_l$. To keep it simple, we assume that the quality of a new vehicle remains equal to Q_h after driving it for one period. This assumption, which is not unrealistic, can be justified on the following two grounds. First, it allows us to keep the model parsimonious. Second, it seems reasonable to think that once a consumer replace her old car with a new one, she perceives its quality as a brand-new car in both periods.

The utility of consumers having a vehicle aged $\eta - 1$ is computed in a similar way, while keeping in mind that they are not eligible for the subsidy program in the first period. Table 2.1 gives the utility in all relevant cases.

	$\tau = \eta - 1$	$\tau \geq \eta$
Replacement in the first period	$v_iQ_h - p + \delta v_iQ_h$	$v_iQ_h - p + s_1 + \delta v_iQ_h$
Replacement in the second period	$v_iQ_l + \delta(v_iQ_h - p + s_2)$	$v_iQ_l + \delta(v_iQ_h - p + s_2)$
No replacement	$v_iQ_l + \delta v_iQ_l$	$v_iQ_l + \delta v_iQ_l$

Table 2.1: The utilities of replacing in the first period, second period, and never

The government's objective, which will be stated formally later on, is to minimize the subsidy budget, given a target replacement level. The game is played à la Stackelberg, with the government acting as leader. It first announces a subsidy schedule (s_1, s_2), and next, consumers, as followers, react to this announcement by deciding whether or not to replace their vehicles. As is usual in this setting, the equilibrium is obtained by solving the game in reverse order. That is, we start by determining the consumer's reaction to the subsidy schedule and then optimize for the government.

Table 2.2 summarizes the notations used throughout the paper.

2.3 Results

In this section, we solve for equilibrium, starting by computing the consumer's reaction function.

2.3.1 Consumer best response

Given the utility functions and considering vehicles of age $\tau \geq \eta$, the value for which a consumer is indifferent between replacing her car in the first or the second period is given by

$$v_\tau^{1,2} = \frac{(1-\delta)p + \delta . s_2 - s_1}{\tilde{Q}},$$

where $\tilde{Q} = Q_h - Q_l$, and the superscript $(1,2)$ stands for being indifferent between the first and second periods. Similarly, the value of being indifferent between replacing in

Notation	Description
p	Price of a new car
s	Subsidy value
i	Index of consumers
v_i	Consumer valuation for driving a car i
τ	Age of a car
Q_τ	Quality of a car aged τ
ω	Maximum age of a car
η	Eligibility age of a car for the scrappage subsidy program
δ	Discount factor
$v^{1,2}$	Consumer who is indifferent between replacing in the first and the second period
$v^{1,n}$	Consumer who is indifferent between replacing in the first and to not replacing in both periods
$\tilde{Q}$	Difference between the quality of an old car and that of a new one
Q_h	Quality of a new car
Q_l	Quality of an old car
Γ	Car replacement target level of the scrappage program
β	The portion of cars aged $\eta-1$
α	The portion of cars aged $\eta-2$

Table 2.2: Notations used throughout the paper

the first period and never replacing is given by

$$v_\tau^{1,n} = \frac{p-s_1}{(1+\delta)\tilde{Q}},$$

where the superscript $(1,n)$ refers to being indifferent between replacing in period 1 and never replacing. Finally, the value of being indifferent between replacing in the second period and never replacing is given by

$$v_\tau^{2,n} = \frac{p-s_2}{\tilde{Q}},$$

where the superscript $(2,n)$ refers to being indifferent between replacing in period 2 and never replacing. Lemma 2.1 describes the reaction function of consumers owning a car aged τ to any subsidy plan (s_1,s_2) announced by the government.

Lemma 2.1. Given any announced subsidy plan, the reaction function of consumer i owning a car aged $\tau \geq \eta$ is as follows:

- If $s_2 \leq \frac{s_1+\delta p}{1+\delta}$, then consumers with $v_i \geq v_\tau^{1,n}$ replace in the first period, and the rest of the consumers never replace.

- If $s_2 > \frac{s_1+\delta p}{1+\delta}$, then consumers with $v_i \geq v_\tau^{1,2}$ replace in the first period, those with $v_\tau^{2,n} \leq v_i < v_\tau^{1,2}$ replace in the second period, and the rest of the consumers never replace.

Proof. See Appendix. □

According to Lemma 2.1, when s_2 is lower than the threshold $\frac{s_1+\delta p}{1+\delta}$, then consumers only replace in the first period. Any consumer whose utility for the first-period replacement is higher than no replacement purchases in the first period. The rest of the consumers who have not replaced in the first period would not replace in the second period either. In other words, s_2 is not sufficiently larger than s_1. To interpret further this result, note that in such circumstances there are two possibilities: a consumer may replace in the first period or she may wait until the second period to make a replacement decision. In the later case, the consumer has already refused the subsidy in the first period, i.e., she has found the subsidy not high enough for scrapping her car. Consequently, she would refuse the subsidy in the second period too because the second-period subsidy is also not high enough. Therefore, consumers either replace in the first period or do not replace at all. Otherwise, when s_2 is larger than $\frac{s_1+\delta p}{1+\delta}$, consumers can replace in either period. In particular, those who have not replaced their car in the first period do replace in the second period, provided that their utility of replacement in the second period is higher than of never replacing.

So far, we have characterized the reaction of consumers owning a vehicle aged $\tau \geq \eta$ to any subsidy plan. To get a picture of the overall vehicle fleet replacements, we need to account for cars aged $\eta - 1$ too. Let

$$v_{\eta-1}^{1,2} = \frac{(1-\delta)p + \delta . s_2}{\tilde{Q}},$$

$$v_{\eta-1}^{1,n} = \frac{p}{(1+\delta)\tilde{Q}},$$

and

$$v_{\eta-1}^{2,n} = \frac{p-s_2}{\tilde{Q}},$$

where $v_{\eta-1}^{1,2}$, $v_{\eta-1}^{1,n}$ and $v_{\eta-1}^{2,n}$ represent consumers who are indifferent between replacing in the first and the second period, those who are indifferent between replacing in the first period and never replacing, and those who are indifferent between replacing in the second period and never replacing respectively. Corollary 2.1 describes this situation.

Corollary 2.1. Given any announced subsidy plan, the reaction function of consumer i owning a car aged $\eta - 1$ is as following:

- If $s_2 \leq \frac{\delta p}{1+\delta}$, then consumers with $v_i \geq v_{\eta-1}^{1,n}$ replace in the first period, and the rest of the consumers never replace.
- If $s_2 > \frac{\delta p}{1+\delta}$, then consumers with $v_i \geq v_{\eta-1}^{1,2}$ replace in the first period, those with $v_{\eta-1}^{2,n} \leq v_i < v_{\eta-1}^{1,2}$ replace in the second period, and the rest of the consumers never replace.

Proof. See Appendix. □

Corollary 2.1 and lemma 2.1 show that when s_2 is lower than the threshold $\frac{\delta p}{1+\delta}$, then consumers owning vehicles aged $\eta - 1$ and also those owning cars aged $\tau \geq \eta$ replace only in the first period. If s_2 is larger than this threshold but still less than $\frac{\delta p + s_1}{1+\delta}$, then some low-value consumers, with vehicles aged $\eta - 1$ and who have not replaced in the first period $\left(\frac{\mathbf{p}-\mathbf{s_2}}{\tilde{\mathbf{Q}}} \leq \mathbf{v_i} < \mathbf{v_{\eta-1}^{1,2}}\right)$, find it profitable to replace in the second period. However, s_2 is still too low to induce those low-value consumers who have not replaced their vehicle aged $\tau \geq \eta$ in the first period, to replace in the second period. So, they need an even greater subsidy in the second period $\left(\mathbf{s_2} > \frac{\delta \mathbf{p}+\mathbf{s_1}}{1+\delta}\right)$ to replace in this period rather than never replacing. This intuitive result stems from the fact that the owners of cars aged $\eta - 1$ are eligible only in the second period, but those with cars aged $\tau \geq \eta$ are eligible in both periods. Therefore, while the earlier group only look forward the second-period subsidy, the latter one compares subsidies in both periods. Consequently, non-replacers in the first period who were also eligible for the first-period subsidy require a higher subsidy s_2 to prefer replacing in the second period over no replacement at all.

2.3.2 Government problem

As alluded to it before, the government aims at minimizing the subsidy cost, subject to a car-replacement target level Γ. (For clarity, Γ, p, s_1, s_2 and Q take their values in the interval $(0,1)$ in the theoretical part of the paper. This normalization will be relaxed in the numerical example.) We assume that $\tilde{Q} \geq p$, which ensures that the added value of a new car is at least equal to the price of the vehicle. The total number of vehicles is normalized to 1 and it is assumed that the numbers of cars with $\tau = \eta - 1$ and $\tau \geq \eta$ are β (≤ 1) and $1 - \beta$, respectively. Considering the consumer reaction function for the three different subsidy levels, that is, $s_2 \leq \frac{\delta p}{1+\delta}$, $\frac{\delta p}{1+\delta} < s_2 \leq \frac{\delta p + s_1}{1+\delta}$ and $s_2 > \frac{\delta p + s_1}{1+\delta}$, the government faces the three following optimization problems:

$$
\begin{aligned}
&\text{minimize } (1-\beta)s_1\left(1 - v_{\eta}^{1,n}\right) \\
&\text{subject to: } (1-\beta)\left(1 - v_{\eta}^{1,n}\right) + \beta\left(1 - v_{\eta-1}^{1,n}\right) \geq \Gamma, \\
&\qquad s_2 \leq \frac{\delta p}{1+\delta}, \\
&\qquad s_1, s_2 \geq 0.
\end{aligned}
\tag{2.1}
$$

$$
\begin{aligned}
\text{minimize } & (1-\beta)s_1\left(1-v_\eta^{1,n}\right)+\beta\cdot s_2\left(v_{\eta-1}^{1,2}-\frac{p-s_2}{\tilde{Q}}\right)\\
\text{subject to: } & (1-\beta)\left(1-v_\eta^{1,n}\right)\\
& +\beta\left((1-v_{\eta-1}^{1,2})+(v_{\eta-1}^{1,2}-\frac{p-s_2}{\tilde{Q}})\right)\geq\Gamma,\\
& \frac{\delta p}{1+\delta}-s_2\leq 0,\\
& s_2-\frac{\delta p+s_1}{1+\delta}\leq 0,\\
& s_1,s_2\geq 0.
\end{aligned}
\tag{2.2}
$$

$$
\begin{aligned}
\text{minimize } & (1-\beta)\left(s_1\left(1-v_\eta^{1,2}\right)+s_2(v_\eta^{1,2}-\frac{p-s_2}{\tilde{Q}})\right)+\beta\cdot s_2\left(v_{\eta-1}^{1,2}-\frac{p-s_2}{\tilde{Q}}\right)\\
\text{subject to: } & (1-\beta)\left((1-v_\eta^{1,2})+(v_\eta^{1,2}-\frac{p-s_2}{\tilde{Q}})\right)\\
& +\beta\left((1-v_{\eta-1}^{1,2})+(v_{\eta-1}^{1,2}-\frac{p-s_2}{\tilde{Q}})\right)\geq\Gamma,\\
& s_2\geq\frac{\delta p+s_1}{1+\delta},\\
& s_1,s_2\geq 0.
\end{aligned}
\tag{2.3}
$$

The objective functions are composed of the total subsidy paid in both periods for cars with $\tau \geq \eta$ plus the subsidy cost in the second period spent on cars aged $\eta - 1$. The first constraint states that the total number of scrapped vehicles must be at least equal to the target level. There are also other constraints corresponding to the relation between s_1 and s_2 as described in the previous subsection. In order to find the optimal subsidy levels we need to solve the above three problems. Proposition 2.1 describes the government's optimal policy.

Proposition 2.1. The unique equilibrium subsidy plan is given by

$$
(s_1,s_2)=\begin{cases}
(0,0), & \text{if } \Gamma<\frac{(1+\delta)\tilde{Q}-p}{(1+\delta)\tilde{Q}},\\
((1+\delta)(\Gamma-1)\tilde{Q}+p,\ (\Gamma-1)\tilde{Q}+p), & \text{if } \frac{(1+\delta)\tilde{Q}-p}{(1+\delta)\tilde{Q}}\leq\Gamma\leq\frac{(2+\delta)\tilde{Q}-2p}{(1+\delta)\tilde{Q}},\\
\left(\frac{1}{2}((1+\delta)\Gamma-\delta-2)\tilde{Q}+p,\ (\Gamma-1)\tilde{Q}+p\right), & \text{if } \Gamma>\frac{(2+\delta)\tilde{Q}-2p}{(1+\delta)\tilde{Q}}.
\end{cases}
\tag{2.4}
$$

Proof. See Appendix. □

The above proposition shows that the government's strategy is defined in steps that depend on four parameters, namely, the difference in quality $\tilde{Q}$, the car replacement target Γ, the discount factor δ and the vehicle's price p. We make the following comments:

1. When the target level is sufficiently low, that is, $\Gamma < \frac{(1+\delta)\tilde{Q}-p}{(1+\delta)\tilde{Q}}$, then the government does not need to offer any subsidy to reach its target. Also, all consumers with a high valuation ($v_i \geq v_{\eta-1}^{1,n}$ for owner of cars with age $\eta - 1$ and $v_i \geq v_{\eta}^{1,n}$ for those with cars aged η or older) purchase in the first period only and low-value consumers never replace.

2. When the target level is of intermediate value, i.e., $\frac{(1+\delta)\tilde{Q}-p}{(1+\delta)\tilde{Q}} \leq \Gamma \leq \frac{(2+\delta)\tilde{Q}-2p}{(1+\delta)\tilde{Q}}$, consumers with cars aged η or older either replace in the first period ($v_i \geq v_{\eta}^{1,n}$) or do not replace at all ($v_i < v_{\eta}^{1,n}$). This means that the second-period subsidy is not high enough to induce those who do not replace in the first period to do so in the second period. Further, although owners of cars aged $\eta - 1$ are not eligible for the subsidy in the first period, some of them can still replace their cars without cashing the subsidy. Now, s_2 is large enough for some of those who were not eligible in period 1 to replace in the second period $\left(\frac{p - s_2}{\tilde{Q}} \leq v_i < v_{\eta-1}^{1,2}\right)$. In fact, the government is implementing a price- (subsidy-) discrimination policy based on the car's eligibility. One advantage of such a policy is that the subsidy targets consumers with a medium valuation and who do not need a high incentive to replace, i.e., $\frac{p - s_2}{\tilde{Q}} \leq v_i < v_{\eta-1}^{1,2}$. In fact these consumers' valuation is, neither too high that they replace in the first period with no subsidy, nor too low that they ignore the second-period subsidy (never replacing at all). Another advantage is that, by offering a not-too-high second-period subsidy level, high-value consumers do not postpone replacement to free ride on the program.

3. For high target level, i.e., $\Gamma > \frac{(2+\delta)\tilde{Q}-2p}{(1+\delta)\tilde{Q}}$, as the government needs more owners to replace their cars, it intensifies its subsidy discrimination policy by increasing the differential between the two subsidy levels. In fact, unlike the intermediate Γ case, s_2 is high enough (compared to s_1) to convince some of the $\geq \eta$-car owners who rejected first-period subsidy to replace their car in the second period and cash in s_2. Therefore, consumers with cars aged $\geq \eta$ and valuation $v_i \geq v_\eta^{1,2}$ or $\frac{p-s_2}{\tilde{Q}} \leq v_i < v_\eta^{1,2}$ replace in the first or the second period, respectively. Besides, similar to intermediate Γ case, those owning cars aged $\eta - 1$ whose valuation is $v_i \geq v_{\eta-1}^{1,2}$ or $\frac{p-s_2}{\tilde{Q}} \leq v_i < v_{\eta-1}^{1,2}$ replace in the first or the second period, respectively.

4. The subsidy is non-decreasing over time. For low Γ, the value is zero in both periods, while for intermediate Γ, we have

$$s_2 - s_1 = \delta(1-\Gamma)\tilde{Q} > 0. \tag{2.5}$$

For high Γ, the difference in subsidies is given by

$$s_2 - s_1 = \frac{1}{2}\tilde{Q}(\Gamma(1-\delta)+\delta) > 0. \tag{2.6}$$

We mentioned in the Introduction that subsidies could be increasing or decreasing over time in practice. In our specific case of scrappage program, the result shows that the government prefers increasing trend when designing the subsidy schedule.

5. The non-decreasing trend of the subsidy allows attracting more consumers in the second period. The intuitive reasoning for this policy is as follows. First, there are more cars eligible in the second period than in the first period. Second, and more important, this subsidy plan allows the government to do discriminate among consumers. As discussed before, consumers with higher valuation replace in the first period. Therefore, with less first-period subsidy than the second-period subsidy, the government offers less subsidy to high value consumer than those with lower valuation. In fact, consumers with high valuation who would like to replace their

car sooner (first period) rather than later (second period) are subsidized with a low-value subsidy in the first period. This is in line with the government objective, i.e., subsidy cost minimization.

6. As expected, the subsidy is increasing in the target. Comparing the subsidy plans for intermediate and high target values, we note that the first-period subsidy drops from $(1+\delta)(\Gamma-1)\tilde{Q}+p$ to $\frac{1}{2}((1+\delta)\Gamma-\delta-2)\tilde{Q}+p$, while the second-period subsidy remains unchanged.

7. The subsidy is decreasing in the quality differential $\tilde{Q}$. That is, the larger Q_h with respect to Q_l, the lower is the subsidy. As the utility of consumer i owning a car of age τ is increasing in quality (recall the utility is given by $v_i Q_\tau$), this result is intuitive.

8. The larger the discount factor δ, the lower the subsidy in the first period. To interpret this result, let us look at the two extreme cases of $\delta=0$ and $\delta=1$. When the discount factor is zero, which means that consumers are myopic, that is, they are only considering current-period payoff when making a decision, then the equilibrium strategy of the government is to set equal subsidies in both periods when the target is of intermediate value (see (2.5)). Here, there is no need for intertemporal age-based discrimination to reach the target. Indeed, the same-level-subsidy ($s_1=s_2$) strategy means that the government equally subsidizes consumers with cars aged $\geq \eta$ and $\eta-1$. However, when the target is high, the subsidy in the second period is higher than the subsidy in the first period. The reason is that the government needs to attract those consumers that require a higher incentive to change their cars in the second period. As a result the government is better off (in terms of achieving its target) offering higher subsidies in the second period where all consumer groups are eligible for the subsidy program.

   When $\delta=1$, that is, when consumers attach the same importance to earned utilities in both periods, the main established results and relationships remain unaltered, qualitatively speaking. Unlike in the other extreme case of $\delta = 0$, it is not optimal to offer the same subsidy in the two periods for both high and intermediate Γ cases. Note that when Γ is high, the difference $s_2 - s_1$ becomes independent of Γ and is given by $\tilde{Q}/2$, that is half of the differential quality.

Since the optimal policy in Proposition 2.1 does not depend on parameter β (the portion of cars aged $\eta - 1$), one may wonder about extreme situations where $\beta \to 0$ or $\beta \to 1$. We argue that in these two extreme cases the optimal policy is reasonable too. When $\beta \to 0$, all cars are of age η or older, which means that all cars are eligible for the subsidy in both periods. For medium target level, i.e., $\frac{(1+\delta)\tilde{Q}-p}{(1+\delta)\tilde{Q}} \leq \Gamma \leq \frac{(2+\delta)\tilde{Q}-2p}{(1+\delta)\tilde{Q}}$, consumers only replace in the first period and only the first period subsidy is consumed. The second period subsidy is not high enough to attract any consumer to the second period. In other words, while the government offers subsidy in the second period, it keeps the subsidy low enough to avoid moving the demand to the second period. When the target level increases to $\Gamma > \frac{(2+\delta)\tilde{Q}-2p}{(1+\delta)\tilde{Q}}$, it is not optimal to fulfill the target level only in the first period. Therefore, the government intensifies the subsidy discrimination and consumers replace either in the first or the second period.

When $\beta \to 1$, all cars are of age $\eta - 1$, which means that all cars are only eligible for the subsidy in the second period. According to Proposition 2.1, for both medium and high target levels consumers replace either in the first or the second period. However, they can cash the subsidy only if they replace in the second period. This holds true for both cases (medium and high target levels) and therefore, the form of the optimal subsidy in the second period remains the same in both cases (although it is increasing in target level).

The above argument confirms that for the two extremes values of β, the optimal subsidy remains valid. Moreover, it is intuitive for moderate values of β, i.e., $0 < \beta < 1$, as it is discussed before. Hence, no matter what the level of β is, the result of Proposition 2.1 stands.

2.4 An example

In this section, we use an example to illustrate the application of our model in its general form. Here, rather than dividing vehicles into two groups, of old and new cars, we account for all cars of various qualities (ages). In addition, the normalization assumption from the previous section is relaxed, and the data are taken from either the literature or practical cases.

2.4.1 Parameter setting

Similarly to Zaman and Zaccour (2018), we let $\delta = 0.9$, $\omega = 14$, and $\eta = 10$. Further, to identify the number of vehicles of different ages at the beginning of the first period, we use the density distribution of vehicles aged 1 to 14 provided in Engers et al. (2012). In this distribution, most of the vehicles are between 2 and 8 years old. More precisely, out of almost 130,000 cars, there are around 80,000 between 2 and 8 years old. Accordingly, based on the number of vehicles in our setting, we set $\Gamma \in \{20{,}000; 21{,}000; \ldots; 28{,}000\}$. Also as in Zaman and Zaccour (2018), we let the consumer-valuation and government's decision variables have different values, that is,

$$s_1 \in \{0, 100, \ldots, 4{,}000\}, \quad s_2 \in \{0, 100, \ldots, 4{,}000\}, \quad v_i \in \{0, 0.1, \ldots, 1.0\}.$$

In addition, we adjust the price and quality of cars as follows:

$$\begin{aligned} p &= \$10{,}000,\ Q_0 = \$25{,}000,\ Q_9 = \$20{,}000,\ Q_{10} = \$15{,}000,\ Q_{11} = \$10{,}000, \\ Q_{12} &= \$5{,}000,\ Q_{13} = \$2{,}000,\ Q_{14} = \$1{,}500,\ Q_{15} = \$1{,}000. \end{aligned}$$

Since we have cars with different qualities, we need to adjust the model in Section 3. So, instead of $\tilde{Q}$, we introduce ΔQ_τ that represents the difference between qualities of cars aged τ and new cars. Moreover, for each level of target (Γ) we find the pair of subsidies (s_1, s_2) that minimize the objective function.

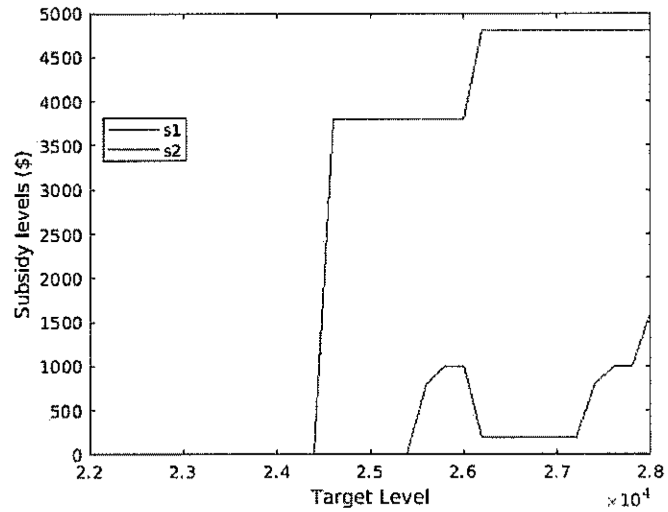

Figure 2.1: Effect of target level on the first- and second- period subsidy levels

2.4.2 Numerical results

Considering the above parameter values and ranges, we examine how the optimal subsidy values change with different levels of Γ. As shown in Figure 2.1, increasing the target level does not necessarily result in a higher s_1 and s_2. For example, increasing Γ from 25,000 to 26,000 results in a higher s_1 while s_2 remains at the same level. However, when the target level reaches 27,000 units, the government prefers to transfer purchases to the second period by reducing s_1 and increasing s_2 at the same time. This illustrates the idea that increasing the target level may induce the government to lower the first-period subsidy, which is perfectly in line with Proposition 2.1.

2.5 Extension: Changing the eligibility age

Increasing the subsidy value enables the government to reach higher target levels. Another potential way to attract more consumers to the subsidy program is to change the eligibility age. To investigate this, we need to do a sensitivity analysis on η. However, since η does not appear explicitly in the model's equations and expressions, the sensitivity analysis on the eligibility age cannot be done by simply taking first-order conditions. Instead, we would check whether the government is better off decreasing the eligibility age from η to $\eta - 1$. Assume that changing the eligibility age from η to $\eta - 1$ makes $\alpha(\leq 1)$ more vehicles eligible for the program. Note that these vehicles are of age $\eta - 2$ and are eligible for the subsidy only in the second period (when they will be aged $\eta - 1$). Also, β vehicles that were previously only eligible in the second period are now eligible in both periods. In summary, $\beta + (1 - \beta)$, that is, all vehicles in the previous setup are now eligible in both periods, and α vehicles are only eligible in the second period. Corollary 2.2 characterizes the equilibrium in this setting.

Corollary 2.2. The unique equilibrium subsidy plan, when the eligibility age is $\eta - 1$ is given by

$$(s_1, s_2) = \begin{cases} (0,0), & \text{if } \Gamma \leq \frac{(1+\delta)\bar{Q}-p}{(1+\delta)\bar{Q}}, \\ \left(\frac{(1+\delta)(\Gamma-1)\bar{Q}+p}{1+\alpha(1+\delta)}, \frac{\alpha\delta p+(\Gamma-1)\bar{Q}+p}{1+\alpha(1+\delta)}\right), & \text{if } \frac{(1+\delta)\bar{Q}-p}{(1+\delta)\bar{Q}} < \Gamma \leq \frac{\delta\bar{Q}+(2+\alpha(1+\delta))(\bar{Q}-p)}{(1+\delta)\bar{Q}}, \\ \left(\frac{(2+\alpha+\alpha\delta)(p-\bar{Q})+\bar{Q}(\Gamma(1+\delta)-\delta)}{2(1+\alpha(1+\delta))}, \frac{\alpha\delta p+(\Gamma-1)\bar{Q}+p}{1+\alpha(1+\delta)}\right), & \text{if } \Gamma > \frac{\delta\bar{Q}+(2+\alpha(1+\delta))(\bar{Q}-p)}{(1+\delta)\bar{Q}}. \end{cases} \tag{2.7}$$

Proof. The corollary is proved in the same way as Proposition 2.1. □

Considering the solutions in (2.4) and (3.2), we are interested in characterizing the conditions under which the government is better off reducing the eligibility age. Proposition 2.2 gives the results.

Proposition 2.2. Assume that the government can choose between ages η and $\eta - 1$ for a vehicle to be eligible for the scrappage program. The optimal policy of the government for different target levels is as follows:

1. When $\frac{(1+\delta)\tilde{Q}-p}{(1+\delta)\tilde{Q}} < \Gamma \leq \frac{(2+\delta)\tilde{Q}-2p}{(1+\delta)\tilde{Q}}$, then

$$\begin{array}{lr} \text{if } \alpha \geq \hat{\alpha} & \eta-1 \text{ is superior} \\ \text{if } \alpha < \hat{\alpha} \text{ and } \beta \geq \hat{\beta} & \eta \text{ is superior} \\ \text{if } \alpha < \hat{\alpha} \text{ and } \beta < \hat{\beta} \text{ and } \begin{cases} \Gamma > \hat{\Gamma} & \eta-1 \text{ is superior} \\ \Gamma \leq \hat{\Gamma} & \eta \text{ is superior} \end{cases} & \end{array}$$

where $\hat{\alpha}$, $\hat{\beta}$ and $\hat{\Gamma}$ are given by

$$\begin{aligned} \hat{\alpha} &= -\frac{\beta(1+\delta)(p-\tilde{Q})}{(\tilde{Q}+\beta(p-\tilde{Q}))\delta^2+(1+2\delta)(\beta-1)(p-\tilde{Q})}, \\ \hat{\beta} &= \frac{\alpha\left(\left(2+2\alpha+4\delta+5\alpha\delta+3\alpha\delta^2\right)(p-\tilde{Q})-\tilde{Q}\delta^2(1+\alpha(1+\delta))\right)}{(1+\delta)(p-\tilde{Q})(1+\alpha(1+\delta))^2}, \\ \hat{\Gamma} &= \frac{\alpha\delta\left(p\alpha+\tilde{Q}\right)-\beta\left(1+\alpha(1+\delta)\right)^2(p-\tilde{Q})}{\alpha\tilde{Q}(1+2\delta+\alpha(1+\delta)^2)} \end{aligned}$$

2. When $\frac{(2+\delta)\tilde{Q}-2p}{(1+\delta)\tilde{Q}} < \Gamma \leq \frac{(2+\delta)\tilde{Q}-2p}{(1+\delta)\tilde{Q}} + \frac{\alpha\left(\tilde{Q}-p\right)}{\tilde{Q}}$, there is a unique $0 < \check{\alpha} < 1$ where:

$$\begin{cases} \text{if } \alpha \geq \check{\alpha} & \eta-1 \text{ is superior} \\ \text{if } \alpha < \check{\alpha} & \eta \text{ is superior.} \end{cases}$$

3. When $\Gamma > \frac{(2+\delta)\tilde{Q}-2p}{(1+\delta)\tilde{Q}} + \frac{\alpha\left(\tilde{Q}-p\right)}{\tilde{Q}}$, $\eta-1$ is always superior.

Proof. See Appendix. □

Proposition 2.2 indicates that the comparison between the eligibility ages η and $\eta-1$ depends on all parameter values, and in particular on α, which measures the incremental number of vehicles that become eligible, and on the target Γ. In a nutshell, depending on the target level, there is a minimum threshold on α after which it is cost efficient to decrease the eligibility age from η to $\eta-1$. In particular, for a target level satisfying

$$\frac{(1+\delta)\tilde{Q}-p}{(1+\delta)\tilde{Q}} < \Gamma \leq \frac{(2+\delta)\tilde{Q}-2p}{(1+\delta)\tilde{Q}},$$

when $\alpha \geq \hat{\alpha}$, $\eta - 1$ is superior to η. Otherwise, the result is ambiguous and depends on the values of Γ and β. When Γ increases to higher levels, that is,

$$\frac{\delta\tilde{Q}+2(\tilde{Q}-p)}{(1+\delta)\tilde{Q}} < \Gamma \leq \frac{\delta\tilde{Q}+2(\tilde{Q}-p)}{(1+\delta)\tilde{Q}} + \alpha\frac{\tilde{Q}-p}{\tilde{Q}},$$

again there is a threshold for α after which the government is better off when changing η to $\eta - 1$. In this case, for α less than this threshold, the government always sticks to η. Finally, when

$$\Gamma > \frac{(2+\delta)\tilde{Q}-2p}{(1+\delta)\tilde{Q}} + \frac{\alpha(\tilde{Q}-p)}{\tilde{Q}},$$

$\eta - 1$ would always satisfy the target level at a lower cost than η.

2.6 Conclusion

In this paper, we use a two-period game framework to find the optimal vehicle scrappage subsidy policies offered to strategic consumers. We obtain that the government gives a higher subsidy in the second than the first period to price-discriminate between high-value and low-value consumers. Low-value consumers will not replace in the second period unless the subsidy in that period is high enough compared to that of the first period. We also show that the second-period subsidy is increasing in the government's car replacement target level. However, after a target level threshold, the first-period subsidy drops to a lower level once, while the second-period subsidy keeps increasing.

We also study the eligibility age set by the scrappage program from a parameter-design point of view. In particular, the policy maker may be better off decreasing the eligibility age of the subsidy program. According to our results, this depends on the number of new vehicles becoming eligible with a lower eligibility age and on the target level.

Two challenging extensions to our paper are worth looking. First, while in this work, we only focus on the interaction between the government and consumers, it would be also interesting to consider the strategic role of the manufacturer. Indeed, the manufacturer may adjust her prices in the presence of a subsidy program, thereby affecting the pass-through effect of the subsidy. Here, one important question to look at is whether the government should subsidize the consumer, the manufacturer, or both? Second, since one of the main aims of vehicle scrappage programs is to reduce car pollution, the environmental effect of the subsidy policies needs to be taken into account. Therefore, it is worth attempting to either analyze the environmental effect of the policies ex-post or account for them directly in the government objective.

2.7 Appendices

Proof of Lemma 2.1.

When $v_\tau^{1,2} \le v_\tau^{1,n}$, consumers either replace in the first period ($v_i \ge v_\tau^{1,n}$) or never replace ($v_i < v_\tau^{1,n}$). Otherwise, when $v_\tau^{1,2} > v_\tau^{1,n}$, consumers may replace in the first period ($v_i \ge v_\tau^{1,2}$), the second period $\left(\mathbf{\frac{p - s_2}{\tilde{Q}} \le v_i < v_\tau^{1,2}}\right)$ or never replace $\left(\mathbf{v_i < \frac{p - s_2}{\tilde{Q}}}\right)$.
If $v_\tau^{1,2} \le v_\tau^{1,n}$, that is,

$$\frac{(1-\delta)p + \delta . s_2 - s_1}{\tilde{Q}} \le \frac{p - s_1}{(1+\delta)\tilde{Q}},$$

or equivalently,

$$s_2 \le \frac{s_1 + \delta p}{1+\delta},$$

then consumers either replace in the first period or never replace. If $v_\tau^{1,2} > v_\tau^{1,n}$, that is,

$$s_2 > \frac{s_1 + \delta p}{1+\delta},$$

then consumers with a valuation higher than $v_\tau^{1,2}$ replace in the first period, and the rest go on to the second period to make their replacement decision.
Finally, those who have not replaced their car in the first period do replace in the second period, provided that their utility of replacement in the second period is higher than of never to replace that is,

$$v_i Q_\tau + \delta(v_i Q_0 - p + s_2) \ge v_i Q_\tau + \delta v_i Q_\tau,$$

which results in the following inequality:

$$v_i \ge \frac{p - s_2}{\tilde{Q}}.$$

Proof of Corollary 2.1.

Similar to Lemma 2.1, when $v^{1,2}_{\eta-1} \leq v^{1,n}_{\eta-1}$, consumers either replace in the first period ($v_i \geq v^{1,n}_{\eta-1}$) or never replace ($v_i < v^{1,n}_{\eta-1}$). Otherwise, when $v^{1,2}_{\eta-1} > v^{1,n}_{\eta-1}$, consumers may replace in the first period $\left(\mathbf{v_i \geq v^{1,2}_{\eta-1}}\right)$, the second period $\left(\mathbf{\frac{p-s_2}{\tilde{Q}} \leq v_i < v^{1,2}_{\eta-1}}\right)$ or never replace $\left(\mathbf{v_i < \frac{p-s_2}{\tilde{Q}}}\right)$.

Proof of Proposition 2.1. We need to solve the following three optimization problems:

$$
\begin{aligned}
\textbf{P1} &: \min(1-\beta)s_1\left(1-\frac{p-s_1}{(1+\delta)\tilde{Q}}\right), \\
\text{subject to} &: \\
\Gamma &\leq (1-\beta)(1-\frac{p-s_1}{(1+\delta)\tilde{Q}})+\beta(1-\frac{p}{(1+\delta)\tilde{Q}}), \\
s_2 &\leq \frac{\delta p}{1+\delta}, \\
s_1,s_2 &\geq 0.
\end{aligned}
$$

$$
\begin{aligned}
\textbf{P2} &: \min\left\{(1-\beta)\left(s_1\cdot\left(1-\frac{p-s_1}{(1+\delta)\tilde{Q}}\right)\right)\right. \\
&\quad \left.+\beta\cdot s_2\cdot\left(\frac{(1+\delta)s_2-\delta.p}{\tilde{Q}}\right)\right\} \\
\text{subject to} &: \\
\Gamma &\leq (1-\beta)(1-\frac{p-s_1}{(1+\delta)\tilde{Q}})+\beta\left(1-\frac{p+\delta(-p+s_2)}{\tilde{Q}}+\frac{p+\delta(-p+s_2)}{\tilde{Q}}-\frac{p-s_2}{\tilde{Q}}\right), \\
\frac{\delta p}{1+\delta}-s_2 &\leq 0, \\
s_2-\frac{\delta p+s_1}{1+\delta} &\leq 0, \\
s_1,s_2 &\geq 0.
\end{aligned}
$$

$$
\begin{aligned}
\textbf{P3} \quad : \quad & \min\left\{(1-\beta)\left(s_1\cdot\left(1-\frac{p+\delta(-p+s_2)-s_1}{\tilde{Q}}\right)+s_2\cdot\left(\frac{p+\delta(-p+s_2)-s_1}{\tilde{Q}}-\frac{p-s_2}{\tilde{Q}}\right)\right)\right.\\
& \left.+\beta\cdot s_2\cdot\left(\frac{p+\delta(-p+s_2)}{\tilde{Q}}-\frac{p-s_2}{\tilde{Q}}\right)\right\},\\
\Gamma \quad \leq \quad & (1-\beta)\left(1-\frac{p+\delta(-p+s_2)-s_1}{\tilde{Q}}+\frac{p+\delta(-p+s_2)-s_1}{\tilde{Q}}-\frac{p-s_2}{\tilde{Q}}\right)\\
& +\beta\left(1-\frac{p+\delta(-p+s_2)}{\tilde{Q}}+\frac{p+\delta(-p+s_2)}{\tilde{Q}}-\frac{p-s_2}{\tilde{Q}}\right),\\
s_2 \quad \geq \quad & \frac{\delta p+s_1}{1+\delta},\\
s_1,s_2 \quad \geq \quad & 0.
\end{aligned}
\tag{2.8}
$$

In the following, we provide the solution of each problem.

Problem **P1** : The objective function of (2.8) is increasing and convex in s_1 and independent of s_2. The first constraint is increasing in s_1 and independent of s_2. The second constraint is increasing in s_2 and independent of s_1. These conditions imply that the optimal solution of (2.8) is achieved when the first constraint is met. The solutions are as follows:

$$
s_1=\frac{(1+\delta)(\Gamma-1)\tilde{Q}+p}{1-\beta},\ s_2=0,\ \text{if}\ (1+\delta)(\Gamma-1)\tilde{Q}+p\geq 0\ \left(\text{i.e., }\Gamma\geq\frac{(1+\delta)\tilde{Q}-p}{(1+\delta)\tilde{Q}}\right).
\tag{2.9}
$$

When the condition in (2.9) is not satisfied, i.e., $\Gamma\leq\frac{(1+\delta)\tilde{Q}-p}{(1+\delta)\tilde{Q}}$, both subsidy values are zero.

Problem **P2** : The Lagrangian function is as follows:

$$
\begin{aligned}
&\mathscr{L}(s_1,s_2,\mu_1,\mu_2,\mu_3)=(1-\beta)\left((1-\beta)\left(s_1\cdot\left(1-\frac{p-s_1}{(1+\delta)\tilde{Q}}\right)\right)\right)\\
&+\beta\cdot s_2\cdot\left(\frac{(1+\delta)s_2-\delta.p}{\tilde{Q}}\right)+\mu_1\cdot\left(\Gamma-(1-\beta)\frac{\delta.\tilde{Q}-p+s_1+\tilde{Q}}{(1+\delta)\tilde{Q}}-\beta(1-\frac{p-s_2}{\tilde{Q}})\right)\\
&+\mu_2\cdot\left(\frac{\delta p}{1+\delta}-s_2\right)+\mu_3\cdot\left(s_2-(\frac{\delta p+s_1}{1+\delta})\right)
\end{aligned}
\tag{2.10}
$$

Considering Kuhn-Karush-Tucker conditions, the feasible solutions are as follows:

$$
\begin{aligned}
&\mu_1 = 0,\ \mu_2 = \frac{-(1+\delta)(-1+\beta)\tilde{Q}+p(-1+(1+\delta)\beta)}{\tilde{Q}},\\
&\mu_3 = -\frac{(-+\beta)(\delta\cdot\tilde{Q}-p+\tilde{Q})}{\tilde{Q}},\ s_1 = 0,\ s_2 = \frac{\delta\cdot p}{1+\delta},\\
&\text{If } (1+\delta)(\Gamma-1)\tilde{Q}+p<0\ \left(\text{i.e., } \Gamma \leq \frac{(1+\delta)\tilde{Q}-p}{(1+\delta)\tilde{Q}}\right).
\end{aligned}
$$

$$
\begin{aligned}
&\mu_1 = -(1+\delta)(\beta-2\Gamma+1)\tilde{Q}+p\cdot(\beta\cdot\delta+\beta+1),\ \mu_2 = 0, \mu_3 = -\frac{\beta\cdot(1+\delta)(-1+\beta)(p-\tilde{Q})}{\tilde{Q}},\\
&s_1 = (1+\delta)(\Gamma-1)\tilde{Q}+p,\ s_2 = \Gamma\cdot\tilde{Q}+p-\tilde{Q}\\
&\text{If } (1+\delta)(\Gamma-1)\tilde{Q}+p\geq 0\ \left(\text{i.e., } \Gamma \geq \frac{(1+\delta)\tilde{Q}-p}{(1+\delta)\tilde{Q}}\right).
\end{aligned}
\tag{2.11}
$$

Problem **P3** : The Lagrangian function is as follows:

$$
\begin{aligned}
&\mathscr{L}(s_1,s_2,\mu_1,\mu_2) = (1-\beta)\left(s_1\cdot\left(1-\frac{p+\delta(-p+s_2)-s_1}{\tilde{Q}}\right)+s_2\cdot\left(\frac{p+\delta(-p+s_2)-s_1}{\tilde{Q}}-\frac{p-s_2}{\tilde{Q}}\right)\right)\\
&+\beta\cdot s_2\cdot(\frac{p+\delta(-p+s_2)}{\tilde{Q}}-\frac{p-s_2}{\tilde{Q}})\\
&+\mu_1\cdot(\Gamma-(1-\frac{p-s_2}{\tilde{Q}}))\\
&+\mu_2\cdot\left((\frac{\delta p+s_1}{1+\delta})-s_2\right)
\end{aligned}
\tag{2.12}
$$

From the Kuhn-Karush-Tucker conditions, the only feasible solution is

$$
\begin{aligned}
&\mu_1 = \frac{1}{2}(1+\delta)\left((\Gamma-1)(-1+\beta)\cdot\delta+(\Gamma-2)\cdot\beta+3\Gamma-2\right)\cdot\tilde{Q}+p\cdot(\beta\cdot\delta+\beta+1),\ \mu_2 = 0,\\
&s_1 = \frac{1}{2}\left((1+\delta)\cdot\Gamma-\delta-2\right)\cdot\tilde{Q}+p,\ s_2 = \Gamma\cdot\tilde{Q}+p-\tilde{Q}\\
&\text{IF } \frac{1}{2}\left((1+\delta)\cdot\Gamma-\delta-2\right)\cdot\tilde{Q}+p\geq 0\ \left(\text{i.e., } \Gamma > \frac{(2+\delta)\tilde{Q}-2p}{(1+\delta)\tilde{Q}}\right).
\end{aligned}
\tag{2.13}
$$

We compare the subsidy cost for the three regions of Γ.

- When $\Gamma < \frac{(1+\delta)\tilde{Q}-p}{(1+\delta)\tilde{Q}}$, then $s_1 = s_2 = 0$ is the optimal subsidy.

- When $\frac{(1+\delta)\tilde{Q}-p}{(1+\delta)\tilde{Q}} \leq \Gamma \leq \frac{(2+\delta)\tilde{Q}-2p}{(1+\delta)\tilde{Q}}$, the difference between the cost in (2.9) and (2.11) is given by

$$\frac{\beta\cdot((1+\delta)(\Gamma-1)\tilde{Q}+p)(-(1+\delta)(\beta-\Gamma)\tilde{Q}+((-1+\beta)\cdot\delta+\beta)p)}{(-1+\beta)(1+\delta)\tilde{Q}},$$

which is non-negative for $\frac{(1+\delta)\tilde{Q}-p}{(1+\delta)\tilde{Q}} \leq \Gamma \leq \frac{(2+\delta)\tilde{Q}-2p}{(1+\delta)\tilde{Q}}$. Therefore, the optimal subsidy levels are

$$s_1 = (1+\delta)(\Gamma-1)\tilde{Q}+p, \quad s_2 = \Gamma\cdot\tilde{Q}+p-\tilde{Q}.$$

- When $\Gamma > \frac{(2+\delta)\tilde{Q}-2p}{(1+\delta)\tilde{Q}}$, the difference between the cost in (2.11) and (2.13) is given by

$$\frac{1}{4}(1-\beta)\cdot\tilde{Q}\cdot(\Gamma\cdot(1+\delta)-\delta)^2,$$

which is non-negative. Therefore, the optimal subsidy levels are

$$s_1 = \frac{1}{2}\Big((1+\delta)\cdot\Gamma-\delta-2\Big)\cdot\tilde{Q}+p,\ s_2 = \Gamma\cdot\tilde{Q}+p-\tilde{Q}.$$

Proof of Proposition 2.2 . We need to compare the subsidy costs in (2.4) and (3.2) for each region.

- When
$$\frac{(1+\delta)\tilde{Q}-p}{(1+\delta)\tilde{Q}} < \Gamma \leq \frac{(2+\delta)\tilde{Q}-2p}{(1+\delta)\tilde{Q}},$$
the difference between the costs in (2.4) and (3.2) is given by
$$\frac{(1+\delta)(\Gamma-1)\tilde{Q}+p}{(1+(1+\delta)\cdot\alpha)^2\cdot\tilde{Q}}\{(-(1+\delta)^2(\beta-\Gamma)\Delta Q+p\cdot(\beta\cdot\delta^2+(2\beta-1)\cdot\delta+\beta))\cdot\alpha^2$$
$$+(((-2\beta+2\Gamma-1)\cdot\delta-2\beta+\Gamma)\tilde{Q}+2p\cdot\beta\cdot(1+\delta))\cdot\alpha+\beta\cdot(p-\Delta Q)\},$$
which is the product of two expressions. The first expression is always non-negative. The first derivative of the second expression with respect to Γ is
$$(1+\delta)^2\cdot\tilde{Q}\cdot\alpha^2+(2\delta+1)\cdot\tilde{Q}\cdot\alpha,$$

which is non-negative. Also, $\Gamma = \hat{\Gamma}$ is the root of the second expression. When $\alpha \geq \hat{\alpha}$, $\hat{\Gamma} \leq \frac{(1+\delta)\tilde{Q}-p}{(1+\delta)\tilde{Q}}$, that is, the cost difference is positive and $\eta - 1$ is superior. When $\alpha < \hat{\alpha}$ and $\beta \geq \hat{\beta}$, $\hat{\Gamma} \geq \frac{(2+\delta)\tilde{Q}-2p}{(1+\delta)\tilde{Q}}$, that is, the cost difference is negative and η is superior. When $\alpha < \hat{\alpha}$ and $\beta < \hat{\beta}$,

$$\frac{(1+\delta)\tilde{Q}-p}{(1+\delta)\tilde{Q}} < \hat{\Gamma} \leq \frac{(2+\delta)\tilde{Q}-2p}{(1+\delta)\tilde{Q}},$$

that is, when $\Gamma > \hat{\Gamma}$, $\eta - 1$ is superior and when $\Gamma \leq \hat{\Gamma}$ η is superior.

- When

$$\frac{(2+\delta)\tilde{Q}-2p}{(1+\delta)\tilde{Q}} < \Gamma \leq \frac{((1+\delta)\alpha+\delta+2)\tilde{Q}-(2+(1+\delta)\alpha)p}{(1+\delta)\tilde{Q}},$$

the difference between the costs in (2.4) and (3.2) is as follows:

$$\begin{aligned} f(\alpha) \quad & \triangleq \frac{1}{4}(-1+\beta)\Big(((\Gamma-1)^2\cdot\delta^2+(-2\Gamma^2+2\Gamma)\cdot\delta-3\Gamma^2+4\Gamma)\tilde{Q}-4\Gamma\cdot p\Big) \\ & + \frac{\Big((1+\delta)(\Gamma-1)\cdot\tilde{Q}+p\Big)\Big((\Gamma-1)\tilde{Q}+p\Big)\beta}{\tilde{Q}} \\ & - \frac{\Big(((\Gamma+\delta)\cdot\alpha+\Gamma)\tilde{Q}+p\cdot\alpha^2\cdot\delta\Big)\Big((1+\delta)(\Gamma-1)\cdot\tilde{Q}+p\Big)}{\tilde{Q}\Big(1+(1+\delta)\cdot\alpha\Big)^2}. \end{aligned} \tag{2.14}$$

The function $f(\alpha)$ is continuous and its first derivative is positive, that is, (2.14) is increasing in α (for this range of Γ). Also, (2.14) is negative for $\alpha = 0$ and positive for $\alpha = 1$. Therefore, there is a unique $\alpha = \check{\alpha}$ that makes (2.14) equal to zero, where

$$\breve{\alpha} = \frac{-4(\frac{1}{2}\tilde{Q}(\Gamma-1)\delta+\frac{1}{2}\tilde{Q}.\Gamma+p-\tilde{Q})^2(1+\delta)\beta+\tilde{Q}^2(\Gamma-1)^2\delta^3-\tilde{Q}^2(\Gamma-1)^2\delta^2}{4(\frac{1}{2}\tilde{Q}(\Gamma-1)\delta+\frac{1}{2}\tilde{Q}.\Gamma+p-\tilde{Q})^2(1+\delta)^2\beta}$$
$$\frac{-4(\frac{3}{4}\Gamma^2.\tilde{Q}+(p-\frac{3}{2}\tilde{Q})\Gamma-\frac{1}{2}p+\frac{1}{2}\tilde{Q})\tilde{Q}.\delta-\Gamma^2.\tilde{Q}^2-2\tilde{Q}(p-\tilde{Q})\Gamma}{-\tilde{Q}^2(\Gamma-1)^2\delta^4+2\tilde{Q}^2(\Gamma-1)\delta^3}$$
$$\frac{+4((\frac{1}{4}(\delta(1+\delta)^2\beta-\delta^3+2\delta^2+3\delta+1)\tilde{Q}^2.\Gamma^2+\delta((-\frac{1}{2}\delta.\tilde{Q}+p-\tilde{Q})(1+\delta)\beta}{+(6\Gamma^2.\tilde{Q}^2-6\Gamma.\tilde{Q}^2+4p.\tilde{Q}-\tilde{Q}^2)\delta^2}$$
$$\frac{+\frac{1}{2}\tilde{Q}(\delta^2-\delta-1))\tilde{Q}.\Gamma+\delta((-\frac{1}{2}\delta.\tilde{Q}+p-\tilde{Q})^2\beta-\frac{1}{4}\delta.\tilde{Q}^2(\delta-1))}{+(8\Gamma^2.\tilde{Q}^2+(4p.\tilde{Q}-10\tilde{Q}^2)\Gamma-4p(p-\tilde{Q}))\delta}$$
$$\frac{(\tilde{Q}(1+\delta)\Gamma-\delta.\tilde{Q}+p-\tilde{Q})^2)^{\frac{1}{2}}}{+4(\frac{3}{4}\tilde{Q}.\Gamma+p-\tilde{Q})\Gamma.\tilde{Q}}$$

- When

$$\Gamma > \frac{((1+\delta)\alpha+\delta+2)\tilde{Q}-(2+(1+\delta)\alpha)p}{(1+\delta)\tilde{Q}},$$

the difference between the costs in (2.4) and (3.2) is as follows:

$$[0.8]\frac{1}{4}\frac{\Big((1+\delta)^2(2(-1+\beta)\delta^2+4(\beta+1)\cdot\delta+2\beta+6)\cdot\alpha^2+2(1+\delta)(2+(-1+\beta)\delta^2+4(\beta+1)\cdot\delta+2\beta+2)\cdot\alpha+2(1+\delta)^2\beta\Big)\tilde{Q}}{(1+\delta)^2\alpha^2},$$

which is a positive value.

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Chapter 3

Subsidies and Pricing Strategies in a Vehicle Scrappage Program with Strategic Consumers

Abstract

We consider a problem of a government that wishes to promote replacing old cars with new ones via a vehicle scrappage program. Since these programs affect the total price paid by consumers, manufacturers could adjust prices accordingly. In a two-period game between government and a manufacturer, we find equilibrium prices and subsidy levels. Our results demonstrate that price levels are increasing over time and are higher than in the benchmark case where there is no subsidy. Further, if consumers act strategically, then the equilibrium price levels will be higher than in the scenario were they behave myopically.
Keywords: Vehicle scrappage program; Strategic consumers; Pricing; Government subsidies.

3.1 Introduction

Vehicle scrappage programs have been introduced by governments in different countries to accelerate the replacement of old cars by less-polluting ones. Introduced in 2007, the Canadian Vehicle Efficiency Incentive program offered up to $2,500 for the purchase a new fuel-efficient car (Walsh, 2012). Another example is the German scrappage subsidy program, where every vehicle older than nine years is eligible for a subsidy should it be replaced by a new car (Ewing, 2009). While these scrappage programs are designed with the dual aim of stimulating the car market and taking old cars off the road, they are costly. The German scrappage program, so far the largest one ever, is estimated to cost 2,5 billion euros (Ewing, 2009). Whatever the amount involved, it is important to design these programs in the most cost-efficient way to avoid wasting governmental funds.

A scrappage subsidy aims at increasing the willingness to pay of consumer to replace her car. One concern is that this boost may not fully materialize because manufacturers may increase their prices when a subsidy program is established. Kaul et al. (2009) investigated how much of the 2,500 € subsidy in German vehicle scrappage program is actually captured by consumers, and showed that subsidized buyers paid a little more than comparable buyers who did not receive the subsidy. Jiminez et al (2016) obtained that car manufacturers increased vehicle prices by 600 € on average after a scrappage program has been announced in Spain. Such observations constitute an invitation to account for the manufacturer role when designing an incentive program.

In this paper, we investigate the problem of designing subsidies over time by a government, when the prices are set by a (representative) car manufacturer (or a car dealer). To capture the strategic interactions between these parties, while keeping the setup as parsimonious as possible, we retain a two-period model. The noncooperative game is played à la Stackelberg, with the government, as leader, announces the subsidy program and the manufacturer, as follower, determines the prices. On the demand side, to capture the strategic behavior of consumers, we assume that the decision to replace or not a car is based on current and future subsidies. Moreover, since scrappage programs usually include an eligibility age, it is necessary to group cars based on their age. This enables us to differentiate between consumers whose cars are eligible in a given period and those who can receive the subsidy only in the future.

Governments may apply different strategies when announcing a subsidy program. In particular, they can opt for a constant subsidy over time, or let it varies over time. Further, the government can commit to a fixed subsidy plan, that is, it pre-announces the subsidy levels for all (here two) periods, or adopt a flexible one by avoiding any commitment on the subsidy in later periods. In the parlance of dynamic games, the pre-announcement strategy is referred to as commitment or open-loop strategy, whereas a strategy that depends on the state of the system is referred to as feedback (or Markovian) strategy. For example, the subsidy level in German feed-in-tarif program for solar electricity adoption was adjusted multiple times per year depending on the installed capacity of photovoltaics (International Energy Agency, 2014). In another solar subsidy case, the California Solar Initiative (CSI), there was a planned decrease in the subsidy level that was pre-announced from the outset (Chemama et al., 2019). We will compare different subsidy strategies, i.e., commitment vs flexible, and constant vs varying over time subsidy plans. Moreover, we will investigate if the manufacturer is better off implementing dynamic pricing or constant price over time.

3.1.1 Brief literature background

Subsidy programs designed to accelerate replacement of existing products by greener ones have been studied extensively using both static and dynamic models. Using a static framework, Hahn (1995) performed a cost-benefit analysis to determine the number of scrapped cars for different subsidy levels. Similarly, Lavee and Becker (2009) suggested a function to estimate car replacements based on the subsidy amount and used car values. Unlike these papers, other researchers retained a dynamic model to investigate subsidy programs over time. For example, De Grrote and Verboven (2019) developed a dynamic model of new technology adoption to study a subsidy program to promote adoption of solar photo-voltaic systems. It is shown that consumers significantly discount the future benefits of the new technology. In another study on photo-voltaic distributed generation, He et al. (2018) investigated emission reduction benefits of the subsidy program. Taking into account cost learning, they showed that the net benefits of the subsidy program decreases with the reduction of unit cost.

While some researchers studied subsidy programs from the government's viewpoint, others investigated the problem from the manufacturer's perspective. Hirte and Tscharaktschiew (2013) determined subsidy rates for electric vehicles that maximize social welfare. Lobel and Perakis (2011) characterized the optimal trajectory of subsidies for photo-voltaic systems that optimizes the government's subsidy cost, instead of social welfare. Unlike Hirte and Tscharaktschiew (2013), and Lobel and Perakis (2011) where the government actively optimizes subsidy rates without considering manufacturers' role, in Zhu et al. (2016) and Miao et al. (2017) manufacturers determine price and quantities given a constant subsidy value, that is, the government does not play a strategic role. In Huang et al. (2019) and Janssens and Zaccour (2014) both the government and manufacturers act strategically. In this paper, we too consider the government and manufacturer as players, and add the important feature that consumers also behave strategically.

Strategic behavior of consumers and its effect on subsidy programs has been studied in few previous papers. Shiraldi (2011) assessed how vehicle choices of heterogeneous consumers with different tastes is affected by a scrappage subsidy program. He et al. (2018) showed that some beneficiaries of the subsidy program would have replaced their car without the subsidy. A similar result is obtained in Zaman and Zaccour (2020), where strategic consumers decide when to replace their cars, assuming that the government plays no strategic role. In Zaman and Zaccour (2021), this assumption is relaxed, that is, the government determines strategically the subsidy levels when designing its program. In this stream of literature, the price of a new car is given, as if car dealers do not react to the existence of a subsidy program. In this paper, the manufacturer is a strategic player in the game and decides upon the price of the subsidized new car.

3.1.2 Research questions and contributions

To the best of our knowledge, this paper is the first analysis of equilibrium scrappage subsidy and car prices in the presence of strategic consumers. Our aim is to answer the following questions:

1. Does a scrappage subsidy program affect the equilibrium pricing strategy of a car manufacturer?
2. What is the impact of consumer's strategic behavior on equilibrium prices and subsidies?
3. What is the impact of the manufacturer's pricing strategy choice on the equilibrium subsidies, prices and outcomes?
4. Is it in the best interest of the government to pre-announce its subsidy policy or to adopt a flexible one? What is the impact of its choice on the manufacturer and consumers?

Our results are summarized as follows:

1. Price levels in the presence of scrappage program are higher than in the benchmark case where no subsidy is offered. In addition, when subsidy is a given constant, prices are increasing over time.
2. Price and subsidy levels are higher when consumers act strategically then when they behave myopically. Moreover, manufacturer profit and subsidy cost are higher, when consumers are strategic.
3. When the manufacturer applies dynamic pricing, it realizes a higher profit than when adopting a constant price. Dynamic pricing increases the cost of the subsidy program.

4. Comparing the results obtained under pre-announced and flexible subsidies, we obtain that the government is better off, that is, the program cost is lower, and the manufacturer's profit is lower, when flexible subsidies are implemented.

3.2 Model

Consider a two-period model where the government subsidizes consumers who replace their car older than a certain age, η, with a new one. For example, the scrappage program in Germany subsidizes all cars older than 9 years should they are replaced by a new one. The total number of vehicles targeted by the government program is normalized to 1, with the proportions of cars with age $\eta - 1$ and $\geq \eta$ being $1-\beta$ and β, respectively. In each period, a (representative) manufacturer chooses the new-car price p_t, $t = 1,2$, and the government sets subsidy level s, assumed to be the same in both periods. Later on we consider a time-varying subsidy.

In our two-period framework, a vehicle can be eligible either in both periods (i.e., cars aged $\geq \eta$) or only in the second one (i.e., cars aged $\eta - 1$). We disregard cars that are not eligible in either periods, as the program has no effect on their replacement. Therefore, we assume that the demand for new cars is originated from replacement of cars older than $\eta - 1$ with new cars. Note that, while cars aged $\eta - 1$ are only eligible in the second period, those with age $\geq \eta$ are eligible in both periods. As a result we distinguish between demand functions representing the number of replacements of these two categories of cars. For cars aged $\geq \eta$, we assume that the number of replacements in the first period can be approximated by the following linear demand function:

$$d_1^{\geq \eta} = \beta - p_1 + \theta s, \tag{3.1}$$

where the superscript $\geq \eta$ stands for cars older than η, θ is a positive parameter measuring the consumer sensitivity to the subsidy, and β represents the market potential β.

Since cars aged $\eta - 1$ are not eligible for the subsidy in the first period, the demand function in (3.1) does not apply for these vehicles. One alternative is to assume that

$(\eta-1)$-car owners ignore the subsidy program in the first period as they cannot cash in the subsidy in this period. In this case, the demand in the first period linked to cars aged $\eta-1$ is obtained by omitting the term θs from equation (3.1). However, according to Zaman and Zaccour (2020), expectations for subsidies offered in future could affects the actual demand. In other words, consumers who are not eligible in current period but are in the next one, may consider postponing their car replacement to become eligible for the subsidy program. In our two-period model, some consumers owning a car aged $\eta-1$ who would have replaced in the first period when there is no subsidy program, could delay their car replacement to the second period when there is a subsidy program. Note that, while these consumers do not receive any subsidy in both periods when there is no subsidy program, they can benefit from a subsidy in the second period when a subsidy program is available. As a result, a part of the demand in the first period is moved forward, to the second period, which affects the first-period demand negatively. For cars aged $\eta-1$, we assume the number of replacements in the first period to be given by the following linear demand function:

$$d_1^{\eta-1} = 1 - \beta - p_1 - \gamma s, \tag{3.2}$$

where superscript $\eta-1$ stands for cars aged $\eta-1$ and γ is a nonnegative parameter measuring the consumer sensitivity to the future subsidy. A positive γ means that consumers are strategic (or farsighted), while $\gamma=0$ implies that consumers are myopic. A series of studies showed a negative impact of consumers' strategic behavior on a retailer's profit. In particular, some researchers argued that if firms sets their prices as if all consumers were myopic, they might end up with a revenue loss estimated to be between 20 and 60% (see, e.g., Aviv and Pazgal (2008), Besanko and Winston (1990)). These results constitute an invitation to sellers to account for strategic consumers when making their pricing and other marketing decisions.

We assume, not unrealistically, that consumers are more sensitive to present subsidy than to a future one, which we translate by setting $\gamma < \theta$. We can refer to θ and γ as the direct and indirect effect of the subsidy on demand, respectively.

Summing (3.1) and (3.2), we obtain the following total demand in period 1:

$$d_1^{\geq\eta} + d_1^{\eta-1} = 1 - 2p_1 + (\theta - \gamma)s. \tag{3.3}$$

Unlike the first period, in the second period all vehicles are eligible for the subsidy and it is not necessary to differentiate between cars with different ages. The second-period demand is given by

$$d_2 = d_2^{\eta} + d_2^{\eta-1} = (1 - d_1^{\eta-1} - d_1^{\geq\eta}) - p_2 + \theta s. \tag{3.4}$$

In this demand function, the market potential is the number of old cars remained after the first period, that is, the total number of cars minus the demand in the first period. The total demand in period 2 can be rewritten as

$$d_2 = 2p_1 + \gamma s - p_2,$$

which clearly shows that the demand in the second period depends on both prices and the subsidy. Note that if consumers were myopic, i.e., $\gamma = 0$, then the total demand in period 1 would be higher and the total demand in period 2 lower.

Assuming that the manufacturer maximizes its profit, Π, over the two periods, its optimization problem reads as follows:

$$\begin{aligned} \max_{p_1, p_2 \geq 0} \Pi &\triangleq \Pi_1 + \Pi_2 = p_1(d_1^{\eta-1} + d_1^{\geq\eta}) + p_2 d_2, & (3.5) \\ &= p_1(1 - 2p_1 + (\theta - \gamma)s) + p_2(2p_1 + \gamma s - p_2), & (3.6) \end{aligned}$$

where Π_1 and Π_2 are the profit in the first and second period, respectively.

Considering the best response of the manufacturer, the government sets the subsidy level. In order to formulate the government problem, we use the setting in Zaman and Zaccour (2021) where the government minimizes the total cost of subsidy subject to a car replacement target level, denoted by Γ. The government optimization problem is as follows:

$$\min_{s \geq 0} C \triangleq C_1 + C_2 = s.d_1^{\geq \eta} + s.d_2,$$
$$\text{subject to: } d_1^{\geq \eta} + d_1^{\eta-1} + d_2 \geq \Gamma, \tag{3.7}$$

where C_1 and C_2 are the subsidy costs in the first and second period, respectively. The objective function is composed of the total subsidy paid in the first period for cars aged $\geq \eta$ plus the subsidy cost in the second period spent on all vehicles replaced in that period. The constraint states that the total number of scrapped vehicles must be at least equal to the target level.

We make the following two remarks:

1. Throughout the rest of the paper, we assume that $\Gamma > \frac{1}{2}$; otherwise, the government will not offer a subsidy, which is a less interesting case.
2. As the government aims at minimizing the cost of its program, it is clear that it will never exceeds the target. Consequently, the inequality constraint can be written, without any loss of generality, as an equality constraint, i.e.,

$$d_1^{\geq \eta} + d_1^{\eta-1} + d_2 = \Gamma.$$

3.3 Results

In this section, we solve the Stackelberg game between government and manufacturer where the government leads the game and the manufacturer plays as follower. The following proposition characterizes the equilibrium subsidy and prices set by the government and manufacturer, respectively.

Proposition 3.1. Assuming an interior solution, the unique Stackelberg equilibrium strategies of the government and manufacturer are given by

$$
\begin{aligned}
s &= \frac{2\Gamma - 1}{\theta - \gamma}, \\
p_1 &= \frac{2\Gamma\theta - \gamma}{2(\theta - \gamma)}, \\
p_2 &= \frac{(\gamma + \theta)\Gamma - \gamma}{\theta - \gamma},
\end{aligned}
$$

and the manufacturer profit and subsidy cost by

$$
\begin{aligned}
\Pi &= \frac{(2T^2 - 2\Gamma + 1)\gamma^2 - 2\Gamma\gamma\theta + 2T^2\theta^2}{2(\theta - \gamma)^2}, \\
C &= \frac{(2\beta(\theta - \gamma) + (2\Gamma - 1)(2\theta + \gamma))(2\Gamma - 1)}{2(\theta - \gamma)^2}.
\end{aligned}
$$

Proof. See Appendix. □

First, we note that under the assumptions made earlier on the parameter values, i.e., $\Gamma > 1/2$ and $\theta > \gamma$, the equilibrium is indeed interior, that is, p_1, p_2 and s are strictly positive. Second, it is easy to verify that p_1, p_2 and Π are always increasing in γ for all admissible parameter values. Therefore, the price levels and manufacturer profit are higher than in the case where the strategic behavior of consumers is not taken into account, i.e., $\gamma = 0$. If all consumers were myopic, then the manufacturer's equilibrium pricing strategy would be to set $p_1 = p_2 = \Gamma$. To avoid that strategic consumers wait to make their purchase, which is how they behave when they expect future discounts, the manufacturer implements a penetration pricing strategy, that is, it sets $p_1 < p_2$.

Moreover, we have $\frac{\partial p_2}{\partial \gamma} > \frac{\partial p_1}{\partial \gamma}$, that is, the second-period price is more sensitive towards the propensity of farsightedness of consumers than the first-period price. In other terms, as the strategic behavior of consumers is intensified the second-period price increases with a higher rate than that of the first period. This intuitive result is in line with the fact that when $\gamma > 0$, a part of the demand originated from the replacement of cars aged $\eta - 1$ is moved forward from the first to the second period. Therefore, since the manufacturer is a profit maximizer, parameter γ has more influence on p_2 than p_1. In addition, we note that γ has a positive effect on subsidy cost too, i.e., $\frac{\partial C}{\partial \gamma} > 0$. Intuitively, the higher γ is, the more $(\eta - 1)$-car owners go to the second period to benefit from the subsidy, which results in higher subsidy cost.

It is also interesting to look at the equilibrium demands in the two periods. Since p_2 is higher than p_1 ($p_2 - p_1 = \dfrac{\gamma(2\Gamma - 1)}{2(\theta - \gamma)} > 0$), the replacement of cars aged $\geq \eta$ in the second period is equal to zero. Note that the demand for these cars has the same functional form in both periods. In contrast, the replacement of cars aged $\eta - 1$ is always positive in the second period, as these cars are only eligible for the subsidy in that period. Further, the replacement of cars aged η and $\eta - 1$ in the first period can be either positive or zero.

Remark 3.1. In this game, feedback (or Markov-perfect) and open-loop Stackelberg equilibrium coincide. Indeed, it is easy to verify that if the manufacturer solves simultaneously for p_1 and p_2, the result would be the same. The reason is that the leader (government) sets the same subsidy in both periods.

Corollary 3.1 describes how the first-period demand depends on the parameter values.

Corollary 3.1. The equilibrium number of cars with age $\geq \eta$ and $\eta - 1$ replaced in the first period are as follows:

$$d_1^{\eta-1} = \begin{cases} \dfrac{(1+2\beta-4\Gamma)\gamma + 2\theta(1-\beta-\Gamma)}{2(\theta-\gamma)}, & \text{if } \Gamma < \frac{2\theta(1-\beta)+\gamma(2\beta+1)}{2(\theta+2\gamma)}, \\ 0, & \text{otherwise.} \end{cases} \tag{3.8}$$

(3.9)

$$d_1^{\geq\eta} = \begin{cases} \dfrac{2\theta(\beta+\Gamma-1)+\gamma(1-2\beta)}{2(\theta-\gamma)}, & \text{if } \Gamma > \frac{2\theta(1-\beta)+\gamma(2\beta-1)}{2\theta}, \\ 0, & \text{otherwise.} \end{cases} \tag{3.10}$$

The total demand in the second period is given by

$$d_2 = d_2^{\eta} + d_2^{\eta-1} = \frac{\Gamma(\theta+\gamma)-\gamma}{(\theta-\gamma)}.$$

Proof. It suffices to insert the equilibrium strategies in Proposition 3.1 in the demand functions to get the results. □

According to Corollary 3.1, both demands in the first period are positive if

$$\frac{2\theta(1-\beta)+\gamma(2\beta-1)}{2\theta} < \Gamma < \frac{2\theta(1-\beta)+\gamma(2\beta+1)}{2(\theta+2\gamma)}.$$

Moreover, $d_1^{\eta-1}$ is equal to zero, unless the target level is less than a certain threshold, i.e., $\frac{2\theta(1-\beta)+\gamma(2\beta+1)}{2(\theta+2\gamma)}$. (Recall that these car holders are only eligible in the second period.) In addition, increasing the target leads to higher subsidy levels. As a result, when the target level is higher than a threshold, the subsidy is high enough to push all the demand related to cars aged $\eta-1$ to the second period. On the other hand, the number of replacements of cars aged $\geq \eta$ is positive only if the target level is higher than a threshold, i.e., $\frac{2\theta(1-\beta)+\gamma(2\beta+1)}{2\theta\gamma}$. Unlike cars aged $\geq \eta$, vehicles with age $\geq \eta$ are eligible in both periods. Besides, as mentioned before, the number of replacements of cars aged $\geq \eta$ in the second period is equal to zero. Consequently, as the target level increases, the first-period demand related to these cars increases too. So, it is intuitive to have a target level threshold after which the demand is positive.

3.4 Extensions

In this section, we extend the basic model in two ways. In the first extension, we assume that the manufacturer sets the same price in both periods, which will allow us to assess the benefit or loss resulting from dynamic pricing. In an optimization problem, it is obvious that constant pricing cannot be better than dynamic pricing. Indeed, if it were, then $p_1 = p_2$ would be the solution of the dynamic optimization problem. This result does not necessarily carry over to games, because of the strategic reactions of the players. In the second extension, we relax the assumption of constant subsidy over time.

3.4.1 Constant pricing

We compare two different pricing scenarios for the manufacturer, i.e., dynamic pricing and constant pricing. The constant pricing version of the model described in the previous section is obtained by setting $p_1 = p_2$ in the demand functions and manufacturer's profit.

Proposition 3.2. Assuming an interior solution, when the manufacturer implements a constant pricing strategy, the unique equilibrium price and subsidy levels, the manufacturer profit and the subsidy cost are given by

$$\begin{aligned}
\tilde{p} &= \Gamma, \\
\tilde{s} &= \frac{2\Gamma - 1}{\theta}, \\
\tilde{\Pi} &= \Gamma^2, \\
\tilde{C} &= \frac{(\theta(\beta + 2\Gamma - 1) + \gamma(2\Gamma - 1))(2\Gamma - 1)}{\theta^2}.
\end{aligned}$$

Proof. See Appendix. □

Proposition 3.3. If the manufacturer sets a constant price over time, then it achieves a lower profit than under dynamic pricing, i.e., $\Pi > \tilde{\Pi}$. The government's total program cost is lower under constant pricing, i.e., $C > \tilde{C}$.

Proof. It suffices to compute the following differences to get the result:

$$\begin{aligned}
\Pi - \tilde{\Pi} &= \frac{\gamma(2\Gamma - 1)(2\Gamma\theta - \gamma)}{2(\theta - \gamma)^2} > 0, \\
C - \tilde{C} &= \frac{\gamma(2\Gamma - 1)}{2\theta^2(\theta - \gamma)^2}\left(3\theta^2(2\Gamma - 1) + 2(\theta - \gamma)(\gamma(2\Gamma - 1) + \theta\beta)\right) > 0.
\end{aligned}$$

□

Proposition 3.3 shows that the manufacturer is better off with dynamic pricing, while the government is worse off. This result can be explained by the fact that the manufacturer sets higher prices under dynamic pricing than under constant pricing, while the government's subsidy is higher under dynamic pricing. Indeed, comparing the results in

Propositions 3.1 and 3.2, it is easy to verify that

$$\begin{aligned} p_2 &> p_1 > \tilde{p}, \\ s - \tilde{s} &= \frac{\gamma}{\theta}\frac{2\Gamma - 1}{\theta - \gamma} > 0. \end{aligned}$$

What about consumers who are the third party in this game? Comparing the net prices paid by consumers in the two scenarios, we have

$$\begin{aligned} (p_1 - s) - (\tilde{p} - \tilde{s}) &= \frac{\gamma(\theta - 2)}{2\theta}\frac{2\Gamma - 1}{\theta - \gamma} < 0, \\ (p_2 - s) - (\tilde{p} - \tilde{s}) &= \frac{\gamma(\theta - 1)}{\theta}\frac{2\Gamma - 1}{\theta - \gamma} < 0, \end{aligned}$$

that is, consumers pay a lower net price in each period under dynamic pricing. Consequently, the only party that would prefer constant pricing is the government.

It is also interesting to compare the two pricing strategies taking into account the consumer's type, i.e., strategic or myopic. In the dynamic pricing scenario, the prices, subsidies, manufacturer's profit and subsidy cost increase with γ, the parameter that measures the intensity of strategic behavior of consumers. Under constant pricing, the price and subsidy are independent of γ. In addition, manufacturer's profit is independent of γ because prices are equal in both periods, which makes farsightedness irrelevant. Interestingly, this result does not carry over to the subsidy cost. Indeed, while the subsidy levels are equal in both periods, consumers are not treated equally because of the program's eligibility age requirement. This differentiation between consumers is reflected in the subsidy cost function of the government.

3.4.2 Dynamic subsidies

In our base model, while the car prices set by the manufacturer vary over time, the government's subsidy remain constant over the two periods. However, the government might be interested in offering dynamic subsidies over time. Here, we consider a time-varying subsidy case where the government can set different subsidy levels in the two periods. In this setup, the demand functions become

$$\begin{aligned}
d_1^{\geq\eta} &= s_1\theta+\beta-p_1, \\
d_1^{\eta-1} &= -\gamma s_2-p_1+1-\beta, \\
d_2 &= 1-(d_1^{\eta-1}+d_1^{\geq\eta})+s_2\theta-p_2, \\
&= \gamma s_2+2p_1-s_1\theta+s_2\theta-p_2.
\end{aligned}$$

In such circumstances two possible options for the government can be imagined, namely: (i) A commitment strategy where the policy maker announces (and sticks to) a subsidy plan at the beginning of the first period; and (ii) A the flexible strategy where the government sets the subsidy level at the beginning of each period. The main difference between the commitment and flexible settings roots in the sequence of the events in the two strategies. In the commitment setting the government announces the subsidy levels for both periods and, then, the manufacturer decides on the prices in the two periods. However, in the flexible setting, the government announces the subsidy in the first period followed by the manufacturer who sets the first-period price. Afterwards, the government sets the second period subsidy, just before the manufacturer decides on her second-period price. In order to solve the commitment and flexible policies problems, we need to find open-loop and feedback equilibrium strategies, respectively. Propositions 3.4 and 3.5 describe dynamic pricing and dynamic subsidy equilibrium strategies set by the manufacturer and the government in commitment and flexible settings.

Proposition 3.4. Assume $\Gamma > \frac{1}{2}$ and $\beta > \frac{1}{2}$. Also, assume that θ is sufficiently larger than γ, i.e., $\theta > 2\gamma$. The open-loop Stackelberg equilibrium of the game between the government and manufacturer when they apply dynamic strategies are as follows:

$$\begin{aligned}
\check{s}_1 &= -\frac{1}{2}\frac{4\Gamma\theta+2\beta\gamma-\gamma-2\theta)}{\theta(2\gamma-\theta)}, \\
\check{s}_2 &= -\frac{1}{2}\frac{2\beta-5+8\Gamma}{2\gamma-\theta}, \\
\check{p}_1 &= \frac{(-2\beta-8\Gamma+3)\theta+4\gamma}{8\gamma-4\theta}, \\
\check{p}_2 &= \frac{(-2\beta-6\Gamma+3)\theta-4\gamma(\Gamma-1)}{4\gamma-2\theta}.
\end{aligned}$$

Proof. See Appendix. □

Proposition 3.5. Suppose $\Gamma > \frac{2\theta+\gamma}{3\theta+2\gamma}$. The unique Markov-perfect (feedback) Stackelberg equilibrium is given by

$$\hat{p}_1 = \frac{\theta+\gamma}{2\theta+\gamma}\Gamma, \quad (3.11)$$

$$\hat{p}_2 = 1-\Gamma, \quad (3.12)$$

$$\hat{s}_1 = \frac{6\Gamma\theta+4\Gamma\gamma-4\theta-2\gamma}{\theta(2\theta+\gamma)}, \quad (3.13)$$

$$\hat{s}_2 = 0. \quad (3.14)$$

Proof. See Appendix. □

The above propositions show that depending on the policy type, the government and manufacturer apply different strategies. We make the following comments:

1. Whether the conditions stated in Proposition 3.4 are reasonable or not is an empirical matter. The condition $\beta > \frac{1}{2}$ means that more than half of the cars aged $\geq \eta - 1$ are of age $\geq \eta$. Moreover, $\theta > 2\gamma$ implies that the present effect of the subsidy is at least twice the future effect of the subsidy.

2. The results in Proposition 3.4 show that subsidy levels under commitment are increasing in γ for all admissible parameter values. Therefore, both subsidy levels are higher than when consumer is myopic, i.e., $\gamma = 0$. Moreover, $\frac{\partial \check{s}_2}{\partial \gamma} > \frac{\partial \check{s}_1}{\partial \gamma}$, that is, the second-period subsidy is more sensitive towards the strategic behavior of consumers than the first-period subsidy. In addition, price levels are increasing in γ and $\frac{\partial \check{p}_2}{\partial \gamma} > \frac{\partial \check{p}_1}{\partial \gamma}$.

3. When the government commits, $\check{s}_2 > \check{s}_1$ and $\check{p}_2 > \check{p}_1$, i.e., prices and subsidies are increasing over time. This result shows that the pricing strategy of the manufacturer is in line with the government's subsidy plan. In fact, as the government increases the subsidy level from the first period to second period, the manufacturer responds by raising the price.

4. The results in Proposition 3.5 show that when the government applies a flexible strategy, subsidy and price in the first period are increasing in γ. However, both subsidy and price in the second period are independent of γ. Moreover, while the first-period price is increasing in Γ, the second-period price decreases when the target level increases.

5. In the flexible setting, while the first-period subsidy is positive, the subsidy in the second period is zero. The manufacturer responds to this subsidy plan by increasing its prices, i.e., $\hat{p}_1 > \hat{p}_2$. However, the net prices paid by consumers are decreasing over the two periods, that is, $\hat{p}_1 - \hat{s}_1 < \hat{p}_2 - \hat{s}_2$.

The above comments and observations show that price and subsidy strategies vary with the type of strategy, i.e., flexible or commitment. In the next proposition, we show which party in the game is better off in each setting.

Proposition 3.6. If the government commits to a subsidy plan, then it incurs a higher subsidy cost than under flexible subsidy strategy, i.e., $\check{C} > \hat{C}$. The manufacturer's total profit is higher under a government's commitment strategy, i.e., $\check{\Pi} > \hat{\Pi}$.

Proof. See Appendix. □

Proposition 3.6 shows that the government is better off with flexible setting, while the manufacturer is worse off. Note that in the feedback case, the government foresees the strategic implications of the subsidy on actual and future prices, which leads to lower subsidies than in the commitment case. In summary, it is easy to verify that

$$\begin{aligned} \check{p}_1 &> \hat{p}_1,\ \check{p}_2 > \hat{p}_2, \\ \check{s}_1 &> \hat{s}_1,\ \check{s}_2 > \hat{s}_2 \end{aligned}$$

Another interesting insight is that our result in the commitment setting verifies those provided in paper (Zaman and Zaccour, 2020) where it is shown that scrappage subsidies are increasing over time. Similar to the open loop equilibrium found in this paper, in

(Zaman and Zaccour, 2020), the government commits to its strategy. However, While in (Zaman and Zaccour, 2020) the price is a given constant parameter, in this paper we account for the strategic pricing of the manufacturer. We conclude that in both cases the government applies increasing subsidy strategy.

3.5 Conclusion

In this paper, we use a two-period game model to find equilibrium price and subsidy levels offered to strategic consumers. Various subsidy and pricing strategies are investigated and compared to each other. We obtain that when the subsidy is constant, the manufacturer applies an increasing price strategy, i.e., the second-period price is higher than that of the first period. It is also shown that, when consumers act strategically, price and subsidy levels are higher than in the case where they are myopic.

Our results show that, while the government incurs higher cost in dynamic pricing than under constant pricing, the manufacturer's profit is higher in dynamic pricing. Moreover, we compare two different subsidy strategies applied by the government, i.e., commitment vs flexible settings. It is illustrated that when the government uses commitment strategy, manufacturer profit and subsidy costs are both higher than the case of flexible strategy.

There are some challenging extensions to our work that are worth considering. One interesting direction is to investigate the effect of strategic behavior of consumers on similar subsidy programs such as those offered for adoption of electric vehicles (EVs). Currently, Transport Canada offers Canadians who purchase an EV an incentive of $2,500 to $5,000 based on the car type and lease duration. A specific problem to look at is to determine subsidy levels taking into account lease type and different EV categories while considering the strategic behavior of consumers.

3.6 Appendices

Proof of Proposition 3.1

We start by solving for the manufacturer to obtain its reaction to the subsidy announced by the government. Recall that the manufacturer's profit is given by

$$\Pi = \Pi_1 + \Pi_2 = p_1(1 - 2p_1 + (\theta - \gamma)s) + p_2(2p_1 + \gamma s - p_2). \tag{3.15}$$

First, we consider the second-period manufacturer's profit. Taking the derivative of Π_2 with respect to p_2, we get

$$\frac{\partial \Pi_2}{\partial p_2} = 0 \Leftrightarrow p_2 = \frac{2p_1 + \gamma s}{2}.$$

Substituting for p_2 in Π yields

$$\Pi = p_1(1 - 2p_1 + (\theta - \gamma)s) + \left(\frac{2p_1 + \gamma s}{2}\right)^2.$$

Differentiating with respect to p_1 gives

$$\Pi' = 1 - 2p_1 + \theta s = 0 \Leftrightarrow p_1 = \frac{1 + \theta s}{2}.$$

Substituting in p_2, we obtain the manufacturer's reaction functions, that is,

$$\begin{aligned} p_1 &= \frac{1}{2}(\theta s + 1), \\ p_2 &= \frac{1}{2}(\gamma s + \theta s + 1). \end{aligned} \tag{3.16}$$

To solve the government's problem, we write down the Lagrangian

$$\mathscr{L}(s, \mu) = s.d_1^{\geq \eta} + s.d_2 + \mu(\Gamma - d_1^{\geq \eta} - d_1^{\eta - 1} - d_2),$$

where μ is the Lagrange multiplier appended to the target constraint. Developing the above equation and inserting for p_1 and p_2 from (3.16), we get

$$\mathscr{L}(s, \mu) = \Gamma\mu - \frac{1}{2}\mu + s\beta + \frac{1}{2}s^2\gamma - \frac{1}{2}s\theta\mu + \frac{1}{2}s\gamma\mu.$$

Assuming an interior solution, the first-order optimality conditions are

$$\frac{\partial \mathscr{L}}{\partial s} = 2\beta + 2s\gamma + \mu(\gamma - \theta) = 0,$$
$$\frac{\partial \mathscr{L}}{\partial \mu} = 2\Gamma - 1 - s(\theta - \gamma) = 0.$$

Solving, we obtain

$$s = \frac{2\Gamma - 1}{\theta - \gamma},$$
$$\mu = \frac{2\left((2\Gamma - 1)\gamma + \beta(\theta - \gamma)\right)}{(\theta - \gamma)^2}.$$

The values of p_1 and p_2 can be found by inserting s into their formulas. Straightforward computations give Π and C.

Proof of Proposition 3.2

As in Proposition 3.1, we solve for the manufacturer and get the following reaction function:

$$p = \frac{1}{2}(\theta s + 1).$$

To solve the the government's problem, we introduce the Lagrangian

$$\mathscr{L} = s(s\theta + \beta - p_c) + s(\gamma s + p) + \mu(-s\theta + \Gamma + p - 1),$$

where μ is the Lagrange multiplier. Substituting for p and solving, we obtain the following unique subsidy and value of the Lagrange multiplier:

$$\begin{aligned} \tilde{s} &= \frac{2\Gamma - 1}{\theta}, \\ \mu &= \frac{2(4\Gamma\gamma + 4\Gamma\theta + \beta\theta - 2\gamma - 2\theta)}{\theta^2}. \end{aligned}$$

Substituting in the price, manufacturer's profit and government's cost, we obtain the $\tilde{p}$, $\tilde{\Pi}$ and $\tilde{C}$ in the statement of the Proposition. We note that the equilibrium is indeed interior.

Proof of Proposition 3.4

Same as proposition 3.1, we solve for the manufacturer and we get the following price functions:

$$\begin{aligned} p_1 &= \frac{1}{2}\theta s_2 + \frac{1}{2} \\ p_2 &= \frac{1}{2}\gamma s_2 + \theta s_2 - \frac{1}{2}\theta s_1 + \frac{1}{2}. \end{aligned}$$

The Lagrangian function of the government is given by:

$$\mathscr{L} = s_1(\theta s_1 + \beta - \frac{1}{2}\theta s_2 - \frac{1}{2}) + s_2(\frac{1}{2}\gamma s_2 - \frac{1}{2}\theta s_1 + \theta s_2 + \frac{1}{2}) + \mu(-s_2\theta + \Gamma + p_2 - 1),$$

which results in the following solution:

$$\begin{aligned}
\check{s}_1 &= -\frac{1}{2}\frac{4\Gamma\theta + 2\beta\gamma - \gamma - 2\theta)}{\theta(2\gamma - \theta)}, \\
\check{s}_2 &= -\frac{1}{2}\frac{2\beta - 5 + 8\Gamma}{2\gamma - \theta}, \\
\mu &= -\frac{1}{2}\frac{8\Gamma\gamma + 12\Gamma\theta + 4\beta\theta - 6\gamma - 7\theta}{\theta(2\gamma - \theta)}.
\end{aligned}$$

p_1 and p_2 can be found accordingly.

Proof of Proposition 3.5

To solve for a Markov-perfect (feedback) Stackelberg equilibrium, we solve the game backward, that is, we start by the second stage.

$$\begin{aligned}
d_1^{\geq \eta} &= s_1\theta + \beta - p_1, \\
d_1^{\eta-1} &= -\gamma s_2 - p_1 + 1 - \beta, \\
d_2 &= 1 - (d_1^{\eta-1} + d_1^{\geq \eta}) + s_2\theta - p_2, \\
&= \gamma s_2 + 2p_1 - (s_1 - s_2)\theta - p_2,
\end{aligned}$$

Second-period equilibrium

For any given s_2 announced by the government, the manufacturer solves the following optimization problem:

$$\max_{p_2 \geq 0} \Pi_2 = p_2(\gamma s_2 + 2p_1 - (s_1 - s_2)\theta - p_2).$$

Introduce the manufacturer's Lagrangian

$$\mathscr{L}_{M_2}(p_2, \lambda_2) = p_2(\gamma s_2 + 2p_1 - (s_1 - s_2)\theta - p_2) + \lambda_2 p_2,$$

where λ_2 is the Lagrange multiplier appended to the constraint $p_2 \geq 0$. The first-order optimality conditions are

$$\begin{aligned}
\frac{\partial \mathscr{L}_{M_2}}{\partial p_2} &= \gamma s_2 + 2p_1 - (s_1 - s_2)\theta - 2p_2 + \lambda_2 = 0, \\
\lambda_2 &\geq 0, \; p_2 \geq 0, \; \lambda_2 p_2 = 0.
\end{aligned}$$

Solving the first equation, gives

$$p_2(s_2) = \frac{\gamma s_2 + 2p_1 - (s_1 - s_2)\theta + \lambda_2}{2}.$$

Now, we consider the second-period government's optimization problem, which is given by

$$\min_{s_2 \geq 0} C_2 = s_2.d_2.$$

Substituting for p_2 in d_2, the above optimization problem becomes

$$\min_{s_2 \geq 0} C_2 = s_2.\left(p_1 - \frac{1}{2}(\lambda_2 + \theta(s_1 - s_2) - \gamma s_2)\right).$$

Introduce the second-period government's Lagrangian

$$\mathscr{L}_2(s_2, \mu_2) = s_2\left(p_1 - \frac{1}{2}(\lambda_2 + \theta(s_1 - s_2) - \gamma s_2)\right) + \eta_2 s_2,$$

where η_2 is the Lagrange multiplier appended to the constraint $s_2 \geq 0$. The first-order optimality conditions are

$$\frac{\partial \mathscr{L}_2}{\partial s_2} = p_1 - \frac{1}{2}(\lambda_2 + \theta(s_1 - 2s_2) - 2\gamma s_2) + \eta_2 = 0,$$

$$\eta_2 \leq 0, \quad s_2 \geq 0, \quad \eta_2 s_2 = 0.$$

Solving the first equation, we obtain

$$s_2(s_1, p_1) = \frac{1}{\theta + \gamma}\left(\frac{1}{2}\lambda_2 - \eta_2 - p_1 + \frac{1}{2}\theta s_1\right). \tag{3.17}$$

Substituting in p_2 yields

$$p_2(s_1, p_1) = \frac{1}{4}(2p_1 + 3\lambda_2 - 2\eta_2 - \theta s_1). \tag{3.18}$$

The second-period demand is given by

$$d_2(s_1, p_1) = \frac{1}{4}(2p_1 - 2\eta_2 - \lambda_2 - \theta s_1). \tag{3.19}$$

To wrap up, in (3.17) and (3.18), we express the second-period strategies in terms of the first-period decision variables.

First-period equilibrium (or overall equilibrium problem)

The manufacturer overall optimization problem is as follows:

$$\max_{p_1 \geq 0} \Pi = p_1 (d_1^{\eta-1} + d_1^{\geq \eta}) + p_2 (s_1, p_1) d_2 (s_1, p_1), \tag{3.20}$$

where $p_2 (s_1, p_1)$ and $d_2 (s_1, p_1)$ have been determined in the previous step and the product $p_2 (s_1, p_1) d_2 (s_1, p_1)$ plays the role of a salvage value in the current optimization problem.

Substituting for the second-period equilibrium strategies, the above optimization problem becomes:

$$\max_{p_1 \geq 0} \Pi = p_1 \left(\frac{2(\theta+\gamma) - \gamma\lambda_2 + 2\gamma\eta_2 + (2\theta+\gamma)(\theta s_1 - 2p_1)}{2(\theta+\gamma)} \right) \tag{3.21}$$

$$+ \frac{1}{16} (2p_1 + 3\lambda_2 - 2\eta_2 - \theta s_1)(2p_1 - 2\eta_2 - \lambda_2 - \theta s_1). \tag{3.22}$$

The Lagrangian is given by

$$\begin{aligned} \mathscr{L}_M (p_1, \lambda_1) &= p_1 \left(\frac{2(\theta+\gamma) - \gamma\lambda_2 + 2\gamma\eta_2 + (2\theta+\gamma)(\theta s_1 - 2p_1)}{2(\theta+\gamma)} \right) \\ &\quad + \frac{1}{16} (2p_1 + 3\lambda_2 - 2\eta_2 - \theta s_1)(2p_1 - 2\eta_2 - \lambda_2 - \theta s_1) + \lambda_1 p_1, \end{aligned}$$

where λ_1 is the Lagrange multiplier appended to the constraint $p_1 \geq 0$.

The first-order optimality conditions are

$$\begin{aligned} p_1 (s_1) &= \frac{4(\theta+\gamma)(1+\lambda_1) + (\lambda_2 - 2\eta_2)(\theta - \gamma) + (3\theta+\gamma)\theta s_1}{14\theta + 6\gamma}, \\ p_1 &\geq 0, \quad \lambda_1 \geq 0, \quad \lambda_1 p_1 = 0. \end{aligned}$$

Now, we turn to the government's optimization problem. Substituting for $p_1 (s_1)$ in (3.17)-(3.19) and in the demands, we get

$$\begin{aligned} s_2 (s_1) &= \frac{-2(\theta+\gamma)(1+\lambda_1) + (3\theta+2\gamma)(\lambda_2 - 2\eta_2) + (2\theta+\gamma)\theta s_1}{7\theta^2 + 10\theta\gamma + 3\gamma^2}, \\ p_2 (s_1) &= \frac{2(\theta+\gamma)(1+\lambda_1) + (11\theta+4\gamma)\lambda_2 - 2(4\theta+\gamma)\eta_2 - (2\theta+\gamma)\theta s_1}{14\theta + 6\gamma}, \\ d_2 (s_1) &= \frac{2(\theta+\gamma)(1+\lambda_1) - (3\theta+2\gamma)\lambda_2 - 2(4\theta+\gamma)\eta_2 - (2\theta+\gamma)\theta s_1}{14\theta + 6\gamma}, \\ d_1^{\geq \eta} (s_1) &= \frac{-4(\theta+\gamma)(1+\lambda_1) + 14\theta\beta + 6\beta\gamma + (2\eta_2 - \lambda_2)(\theta - \gamma) + (11\theta+5\gamma)\theta s_1}{14\theta + 6\gamma} \\ d_1^{\eta-1} (s_1) &= -\frac{(((10\theta+6\gamma)(\beta-1) + 4\theta(\beta+\lambda_1))(\theta+\gamma) + (\lambda_2 - 2\eta_2)(\theta^2 + 3\gamma^2 + 6\theta\gamma) + (3\theta^2 + \cdots}{14\theta^2 + 20\theta\gamma + 6\gamma^2} \end{aligned}$$

Inserting for the above values in the following optimization problem

$$\min_{s_1 \geq 0} C = s_1 . d_1^{\geq \eta} + s_2 . d_2,$$
$$\text{subject to: } d_1^{\geq \eta} + d_1^{\eta - 1} + d_2 = \Gamma, \tag{3.28}$$

we get

$$\min_{s_1 \geq 0} C = s_1 \left(\frac{-4(\theta+\gamma)(1+\lambda_1) + 14\theta\beta + 6\beta\gamma + (2\eta_2 - \lambda_2)(\theta - \gamma) + (11\theta + 5\gamma)\theta s_1}{14\theta + 6\gamma} \right)$$
$$+ \left(\frac{-2(\theta+\gamma)(1+\lambda_1) + (3\theta+2\gamma)(\lambda_2 - 2\eta_2) + (2\theta+\gamma)\theta s_1}{7\theta^2 + 10\theta\gamma + 3\gamma^2} \right) \times \tag{3.29}$$
$$\left(\frac{2(\theta+\gamma)(1+\lambda_1) - (3\theta+2\gamma)\lambda_2 - 2(4\theta+\gamma)\eta_2 - (2\theta+\gamma)\theta s_1}{14\theta + 6\gamma} \right), \tag{3.30}$$
$$\text{subject to: } \frac{1}{14\theta^2 + 20\theta\gamma + 6\gamma^2} \left(8\theta^2 + 4\gamma^2 + 12\theta\gamma - \left(6\theta^2 + 2\gamma^2 + 8\theta\gamma\right)\lambda_1 - \right. \tag{3.31}$$
$$\left. \left(5\theta^2 + 4\gamma^2 - 11\theta\gamma\right)\lambda_2 - 2\left(2\theta^2 - \gamma^2 - \theta\gamma\right)\eta_2 + \left(6\theta^2 + \gamma^2 + 5\theta\gamma\right)\theta s_1\right) = \Gamma, \tag{3.32}$$

The Lagrangian is given by

$$\begin{aligned}
\mathscr{L}_1(s_1, \mu) \;=\; & s_1 \left(\frac{-4(\theta+\gamma)(1+\lambda_1) + 14\theta\beta + 6\beta\gamma + (2\eta_2 - \lambda_2)(\theta - \gamma) + (11\theta + 5\gamma)\theta s_1}{14\theta + 6\gamma} \right) \\
& + \left(\frac{-2(\theta+\gamma)(1+\lambda_1) + (3\theta+2\gamma)(\lambda_2 - 2\eta_2) + (2\theta+\gamma)\theta s_1}{7\theta^2 + 10\theta\gamma + 3\gamma^2} \right) \times \\
& \left(\frac{2(\theta+\gamma)(1+\lambda_1) - (3\theta+2\gamma)\lambda_2 - 2(4\theta+\gamma)\eta_2 - (2\theta+\gamma)\theta s_1}{14\theta + 6\gamma} \right) \\
& + \frac{\mu}{14\theta^2 + 20\theta\gamma + 6\gamma^2} \left(8\theta^2 + 4\gamma^2 + 12\theta\gamma - \left(6\theta^2 + 2\gamma^2 + 8\theta\gamma\right)\lambda_1 - \left(5\theta^2 + 4\gamma^2 - 11\theta\gamma\right)\right. \\
& \left. -2\left(2\theta^2 - \gamma^2 - \theta\gamma\right)\eta_2 + \left(6\theta^2 + \gamma^2 + 5\theta\gamma\right)\theta s_1 - \left(14\theta^2 + 20\theta\gamma + 6\gamma^2\right)\Gamma\right) \\
& + \eta_1 s_1,
\end{aligned}$$

where μ and η_1 are the Lagrange multipliers appended to the target constraint and $s_1 \geq 0$, respectively.

The first-order optimality conditions are

$$\frac{\partial \mathscr{L}_1}{\partial s_1} = -\frac{1}{14\theta+6\gamma}\left(4(\theta+\gamma)(1+\lambda_1) - 14\theta\beta - 6\beta\gamma + (\lambda_2 - 2\eta_2)(\theta-\gamma) - 22\theta^2 s_1 - 10\theta\gamma s_1\right)$$
$$+\frac{\theta(2\theta+\gamma)}{(7\theta+3\gamma)^2(\theta+\gamma)}\left(2(1+\lambda_1)(\theta+\gamma) - 3\theta\lambda_2 - 2\gamma\lambda_2 - \theta\eta_2 + \gamma\eta_2 - 2\theta^2 s_1 - \theta\gamma s_1\right)$$
$$-\theta\mu\frac{6\theta^2+5\theta\gamma+\gamma^2}{14\theta^2+20\theta\gamma+6\gamma^2}\left(6\theta^2\lambda_1 - 4\gamma^2 - 12\theta\gamma - 8\theta^2 + 5\theta^2\lambda_2 + 2\gamma^2\lambda_1 + 4\gamma^2\lambda_2 + 8\theta\gamma\lambda_1 - 11\theta\gamma\lambda_2\right)$$
$$+\eta_1 = 0$$

$$\frac{\partial \mathscr{L}_1}{\partial \mu} = \left(8\theta^2 + 4\gamma^2 + 12\theta\gamma - \left(6\theta^2 + 2\gamma^2 + 8\theta\gamma\right)\lambda_1 - \left(5\theta^2 + 4\gamma^2 - 11\theta\gamma\right)\lambda_2\right.$$
$$\left.-2\left(2\theta^2 - \gamma^2 - \theta\gamma\right)\eta_2 + \left(6\theta^2 + \gamma^2 + 5\theta\gamma\right)\theta s_1 - \left(14\theta^2 + 20\theta\gamma + 6\gamma^2\right)\Gamma\right) = 0$$

$$\eta_1 \leq 0, \quad s_1 \geq 0, \quad \eta_1 s_1 = 0.$$

From the second condition, we can get s_1 as function of the model's parameters and the Lagrange multipliers, that is,

$$\begin{aligned} s_1(\lambda_1,\lambda_2,\eta_2) &= \frac{1}{6\theta^3+\theta\gamma^2+5\theta^2\gamma}\left(-8\theta^2 - 4\gamma^2 - 12\theta\gamma + 14\Gamma\theta^2 + 6\Gamma\gamma^2 + 20\Gamma\theta\gamma + 2\left(3\theta^2+\gamma^2+4\right.\right. \\ &\quad \left.+\left(5\theta^2+4\gamma^2-11\theta\gamma\right)\lambda_2 + 2\left(2\theta^2-\gamma^2-\theta\gamma\right)\eta_2\right). \end{aligned}$$

Inserting for s_1 in p_1 we get

$$p_1(\lambda_1,\lambda_2,\eta_2) = \frac{\left(14\Gamma\theta^2 + 6\Gamma\gamma^2 + \left(14\theta^2+6\gamma^2+20\theta\gamma\right)\lambda_1 + \left(7\theta^2+3\gamma^2-12\theta\gamma\right)\lambda_2 + 20\Gamma\theta\gamma\right)}{28\theta^2+26\theta\gamma+6\gamma^2}. \tag{3.34}$$

Substituting for $s_1(\lambda_1,\lambda_2,\eta_2)$ in $s_2(s_1)$ and $p_2(s_1)$, we obtain

$$s_2(\lambda_2,\eta_2) = -\frac{\left(2\theta\gamma - 14\theta^2 - 6\gamma^2\right)\lambda_2 + \left(14\theta^2+6\gamma^2+20\theta\gamma\right)(1+\eta_2-\Gamma)}{21\theta^3+37\theta^2\gamma+19\theta\gamma^2+3\gamma^3}, \tag{3.35}$$

$$p_2(\lambda_2,\eta_2) = \frac{\left(7\theta^2+3\gamma^2+10\theta\gamma\right)(1-\Gamma) + (14\theta+17\gamma)\theta\lambda_2 - (14\theta+6\gamma)\theta\eta_2}{21\theta^2+16\theta\gamma+3\gamma^2} \tag{3.36}$$

We have four cases to consider:

1. $s_2 > 0$ and $p_2 > 0 \Rightarrow \eta_2 = \lambda_2 = 0$. Then,

$$s_2 = -\frac{\left(14\theta^2+6\gamma^2+20\theta\gamma\right)(1-\Gamma)}{21\theta^3+37\theta^2\gamma+19\theta\gamma^2+3\gamma^3}, \tag{3.37}$$

$$p_2 = \frac{\left(7\theta^2+3\gamma^2+10\theta\gamma\right)(1-\Gamma)}{21\theta^2+16\theta\gamma+3\gamma^2}. \tag{3.38}$$

   Clearly, s_2 is negative and therefore a contradiction.

2. $s_2 = 0$ and $p_2 > 0 \Rightarrow \eta_2 < 0$ and $\lambda_2 = 0$. Then,

$$\eta_2 = \Gamma - 1 < 0, \tag{3.39}$$

$$p_2(\lambda_2, \eta_2) = 1 - \Gamma > 0. \tag{3.40}$$

3. $s_2 = 0$ and $p_2 = 0 \Rightarrow \eta_2 < 0$ and $\lambda_2 > 0$. Then,

$$\eta_2 = \frac{\left(7\theta^2+3\gamma^2+10\theta\gamma\right)(\Gamma-1)}{11\theta\gamma} < 0, \tag{3.41}$$

$$\lambda_2 = \frac{\left(7\theta^2+3\gamma^2+10\theta\gamma\right)(\Gamma-1)}{11\theta\gamma} > 0. \tag{3.42}$$

   The fact that λ_2 is strictly negative is a contradiction.

4. $s_2 > 0$ and $p_2 = 0 \Rightarrow \eta_2 = 0$ and $\lambda_2 > 0$. Then,

$$s_2 = \frac{2(7\theta+3\gamma)(\Gamma-1)}{14\theta^2+17\gamma\theta} < 0, \tag{3.43}$$

$$\lambda_2 = \frac{(\Gamma-1)\left(7\theta^2+3\gamma^2+10\theta\gamma\right)}{\theta(14\theta+17\gamma)} < 0, \tag{3.44}$$

   a contradiction.

Consequently, the only admissible solution is

$$s_2 = 0, \;\; p_2 = 1-\Gamma, \;\;\; \eta_2 = \Gamma - 1, \;\; \lambda_2 = 0.$$

Substituting for these values in (3.33) and (3.34), we obtain

$$s_1(\lambda_1) = \frac{1}{6\theta^3+\theta\gamma^2+5\theta^2\gamma}\left(-8\theta^2-4\gamma^2-12\theta\gamma+14\Gamma\theta^2+6\Gamma\gamma^2+20\Gamma\theta\gamma+2\left(3\theta^2+\gamma^2+4\theta\gamma\right)\right. \tag{3.45}$$
$$\left.+2\left(2\theta^2-\gamma^2-\theta\gamma\right)(\Gamma-1)\right),$$

$$p_1(\lambda_1) = \frac{\left(14\Gamma\theta^2+6\Gamma\gamma^2+\left(14\theta^2+6\gamma^2+20\theta\gamma\right)\lambda_1+20\Gamma\theta\gamma\right)}{28\theta^2+26\theta\gamma+6\gamma^2}. \tag{3.46}$$

Clearly, $p_1(\lambda_1)$ is strictly positive and therefore λ_1 must be equal to zero. Therefore, the final value of p_1 is

$$p_1 = \frac{(\theta+\gamma)\Gamma}{2\theta+\gamma}, \tag{3.47}$$

and

$$s_1 = \frac{6\Gamma\theta+4\Gamma\gamma-4\theta-2\gamma}{\theta\,(2\theta+\gamma)},$$

which is positive for

$$\Gamma > \frac{2\theta+\gamma}{3\theta+2\gamma}.$$

To determine the Lagrange multiplier associated with the target constraint, it suffices to substitute for the equilibrium values in $\frac{\partial \mathscr{L}_1}{\partial s_1} = 0$ to obtain

$$\mu = -\frac{2\theta^3\,(32\Gamma-23+7\beta)+\theta^2\,(27\beta\gamma-93\gamma+137\Gamma\gamma)+\theta\left(91\Gamma\gamma^2-57\gamma^2\right)+\gamma^3\,(19\Gamma+3\beta-11)+16(}{2\theta\,(3\theta+\gamma)\,(\theta+\gamma)\,(2\theta+\gamma)^3}$$

Proof of Proposition 3.6 The manufacturer profits and subsidy costs in both settings are given by:

$$\check{C} = \frac{(8\beta^2+(64\Gamma-36)\beta+96\Gamma^2-112\Gamma+32)\theta^2+4(\beta^2 16\Gamma^2+\beta^2-24\Gamma-3\beta+\frac{37}{4})\gamma\theta-8(\beta-\frac{1}{2})^2\gamma^2}{8\theta(2\gamma-\theta)^2}$$

$$\check{\Pi} = \frac{(40\Gamma^2+(24\beta-36)\Gamma+4(\beta-\frac{3}{2})^2)\theta^2+16\gamma(2\Gamma^2+(\beta-\frac{9}{2})\Gamma-\beta+\frac{3}{2})\theta+(32(\Gamma^2-\Gamma+\frac{1}{2}))\gamma^2}{(2\gamma-\theta)^2}$$

$$\hat{C} = \frac{4((\frac{3}{2}\Gamma-1)\theta+\gamma(\Gamma-\frac{1}{2}))((2\beta+5\Gamma-4)\theta+\gamma(\beta+3\Gamma-2))}{(2\theta+\gamma)^2\theta}$$

$$\hat{\Pi} = \frac{(3\gamma+4\theta)\Gamma^2+(-3\gamma-5\theta)\Gamma+2\theta+\gamma}{2\theta+\gamma}$$

Considering the circumstances in propositions 3.4 and 3.5, $\check{C}-\hat{C}$ and $\check{\Pi}-\hat{\Pi}$ are increasing in Γ and positive at $\Gamma = \frac{2\theta+\gamma}{3\theta+2\gamma}$, that is to say, $\check{C} > \hat{C}$ and $\check{\Pi} > \hat{\Pi}$.

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General Conclusion

Sustainable subsidies are form of incentive programs to support environmental friendly activities. Among these program, trade-in subsidies are designed to reward replacement of old products with new ones. In this thesis, we concentrate on a specific type of trade-in programs called vehicle scrappage schemes which provides incentives to replace polluting cars with fuel-efficient ones. Scrappage programs are offered with the dual aim of reducing vehicles' pollution and improving the car market. While having environmental and economic benefits, these programs are also costly which necessitates designing the subsidy policy in the most cost-efficient way.

This thesis, composed of three essays, designs optimal vehicle scrappage programs and evaluates net benefits of the subsidies. Specifically, we consider a framework where government and consumers act strategically based on their interests in the presence of a vehicle scrappage program.

The first essay of this thesis entitled "*Vehicle Scrappage Incentives to Accelerate the Replacement Decision of Heterogeneous Consumers*", studies the impact of scrappage programs on different consumer groups. We consider a scrappage program which subsidizes car owners if their car is older than a certain age and if they replace it with a new one. Consumers are heterogeneous, that is, they have different net trade-in valuations. This is a key assumption which enables us to compare the results of this essay with that of a benchmark case of homogeneous consumers. Subsidies are offered during a finite time horizon where at each period consumers need to decide whether to replace their car with a new one or to keep it until the next period.

We find that subsidy does not have a similar effect on different consumer groups necessarily: It pushes low-value consumers to replace earlier; but, it may induce some high-value consumers to delay their car replacement. In fact, some high-value consumers may would have replaced their vehicle with a new one when there is no subsidy program. However, when a subsidy program is announced, they may decide to postpone their car replacement until it becomes eligible for the program. This is the opposite effect of the subsidy as the program is intended to accelerate rather than delay replacements. We argue that the opposite effect of the subsidy affects the net benefits of the subsidy program, and if ignored, may cause waste of the governmental funds. Our numerical results show that when consumer heterogeneity is taken into account economic and environmental effects of the subsidy program are exaggerated compared to the case where consumers are taken to be homogeneous. Specifically, it is shown that pollution reduction and increase in car sales are overestimated when consumer heterogeneity is neglected. In order to check the robustness of our numerical illustration, we conduct an extensive sensitivity analysis on different parameters of the general model in this research. The main takeaway of our sensitivity analysis is that varying the parameter values does not much affect the qualitative results: we only see parallel shifts in the curves.

This research is based on some assumptions whose relaxation may affect the results. First, it is assumed that subsidy is a given constant amount over the time. while constant subsidies are used in practice, governments may also apply increasing/decreasing subsidy plans. Second, in this essay, the problem is studied from consumers' point of view where their reaction to subsidy program is investigated. However, it is interesting to give a more strategic role to the government by allowing her to characterize optimal subsidy levels. These issues are addressed in the second essay.

In the second paper, entitled "*Optimal Government Scrappage Subsidies in the Presence of Strategic Consumers*", we characterize equilibrium subsidy levels set by the government. The game is played a la Stackelberg in two periods where the government and consumers play as leader and follower respectively. Consumers are divided into two groups based on their car age: consumers who are eligible for the subsidy in both periods, and those who are eligible only in the second period. All consumers face three options, that is, to replace in the first period, to replace in the second period, and to never replace at all. Finally, car owners are assumed to be strategic in terms of their replacement decisions, i.e., they make replacement decisions based on their current and future utilities.

We show that subsidy policy set by the government depends on the target level. Specifically, when the target level is low, subsidy levels are zero, that is to say, no subsidy program is offered. For medium target levels, subsidies in both periods are positive and increasing over time. In this case, consumers who are eligible in both periods, either replace in the first period or don't replace at all. In fact, car replacements in the second period are solely made by car owners who are not eligible in the first period, but in the second period. As a result, by using an increasing subsidy scheme, the government applies a price (-subsidy) discrimination among consumers based on cars age. When target level increases to a high level, the government intensifies the price discrimination by widening the gap between the first and second period subsidies. This is an intuitive policy: since the target level is high, the government increases the the second-period subsidy to attract more consumers to replace in the second period. Consequently, some consumers who are eligible in both periods and who do not replace in the first period, find the second-period subsidy large enough to replace in the second period.

This essay assumes that the government only accounts for the subsidy cost when characterizing its subsidy levels. In practice, policy makers may consider other parameters such as environmental impact of the subsidy program, and the social welfare. It is worth-attempting to consider other possible objective functions for the government and to check if it affects the results of this essay significantly. Another challenging extension to this paper is to involve a manufacturer in our game setting. In this research, government and consumers are playing strategically assuming that car prices remain unchanged. However, when a subsidy policy is announced manufacturers usually react by adjusting their prices. This issue is studies in the third essay.

In the third essay, entitled "*Subsidies and Pricing Strategies in a Vehicle Scrappage Program with Strategic Consumers*", we consider a Stackelberg game where the government leads the game and a profit-miximizer manufacturer plays as follower. Similar to the second paper, in a two-period game, we assume that the government wishes to minimize the subsidy cost with respect to a car replacement target level. While consumers' utilities are not explicitly considered in the model, the demand functions are carefully designed to capture the strategic behavior of car owners. The manufacturer can choose between dynamic and static pricing strategies. Moreover, the government may use a flexible subsidy policy or to announce the program in a commitment setting.

We show that price and subsidy levels are higher when consumers act strategically than the case where they are taken as myopic consumers. As a results, manufacturer profit and subsidy cost are higher, when the strategic behavior of consumers is taken into account. We compare different pricing and subsidy strategies set by manufacturer and government respectively. It is illustrated that dynamic pricing is a more profitable strategy for the manufacturer than the static pricing; but, the government incurs higher cost when the manufacturer applies dynamic pricing. In addition, we find that while the government is better off, that is, it incurs lower cost, the manufacturer is worse off in the flexible subsidy strategy.

In this paper, unlike the first two papers, consumer replace decisions are not explicitly included in the model. In fact, while the strategic behavior of consumers is partially considered in the demand functions, replacement decisions based on consumer utilities are not taken into account. An interesting extension is to model this problem using consumer utility functions rather than estimating demand functions. Since this may increase the complexity of the problem significantly, a numerical illustration may be required to solve the model.

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