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Essays on Ecological and Economic Viability of Agricultural Systems

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Résumé

Dans cette thèse, composée de trois essais, nous utilisons la théorie de la viabilité pour aborder les problèmes de restauration des sols soumis à certaines contraintes environnementales et socio-économiques telles que l'atteinte d'une qualité du sol acceptable sur le plan environnemental et la garantie de revenus acceptables pour les agriculteurs. Notre objectif est d'étudier ces problèmes dans différents cadres qui prennent en compte certains des facteurs les plus influents tels que les incertitudes (incertitudes climatiques) et les pratiques agricoles (la parcellation) afin de mieux comprendre comment les systèmes agronomiques réagissent à ces facteurs et dans quelle mesure ils y sont sensibles. Et ce, afin d'avoir une idée plus précise des stratégies pouvant être mises en place pour atteindre les objectifs de restauration et de préservation des sols. Dans le premier essai, intitulé "**A Survey of Applications of Viability Theory to the Sustainable Exploitation of Renewable Resources**", nous fournissons une revue complète de la littérature sur les applications de la *théorie de la viabilité* aux problèmes de gestion durable des ressources renouvelables, y compris les écosystèmes et les populations tels que les pêcheries et les espèces non marines, l'environnement (avec un accent sur le changement climatique et la concentration des GES) et d'autres ressources telles que les forêts et le sol.

Dans le deuxième essai, intitulé "**Viability of Agroecological Systems Under**

Climatic Uncertainty", nous examinons le problème de l'exploitation durable d'une seule ferme, tant du point de vue physique (qualité du sol) qu'économique (revenus des agriculteurs) tout en tenant compte des incertitudes induites par les événements climatiques majeurs (ouragans). Compte tenu de certaines contraintes économiques et de qualité des sols, et en prenant en compte la survenance possible d'événements climatiques, nous souhaitons déterminer quelles sont les rotations de cultures viables à privilégier sur un horizon de planification prédéfini, quelle est la sensibilité de ces solutions par rapport aux valeurs des paramètres, et quels systèmes ou secteurs agricoles sont les plus vulnérables face aux incertitudes climatiques?

Dans le troisième essai, intitulé "**Viability of a Multi-Parcel Agroecological System**", nous traitons de la gestion à long terme des exploitations agricoles et nous examinons les avantages potentiels que pourrait offrir la parcellation. Étant donnée une exploitation agricole d'une certaine taille et un horizon de temps déterminé, nous voulons déterminer les choix de cultures que l'agriculteur pourrait faire afin d'atteindre simultanément des objectifs de durabilité écologique (qualité du sol) et économique (revenu). Nous considérons une série de cas représentant différents systèmes agricoles pratiqués dans les Antilles françaises, et analysons leur impact sur la qualité des sols et le bien-être économique de l'agriculteur. En outre, nous effectuons une analyse de sensibilité approfondie pour évaluer l'impact des principaux paramètres sur les résultats, à savoir, la qualité initiale du sol, l'horizon temporel et les différents coûts.

Mots clés: Théorie de la viabilité; Agriculture; Incertitude climatique; Multi-Parcelle; Durabilité; Ressources renouvelables.

Méthodes de recherche: Programmation mathématique; Théorie de la viabilité.

Abstract

In this thesis, composed of three essays, we use viability theory (VT) to address soil restoration problems subject to some environmental and socio-economic constraints such as reaching an environmentally acceptable quality of the soil and ensuring acceptable incomes for the farmers. Our objective is to study these problems in different frameworks that take into account some of the most important factors that influence them such as uncertainties (climatic uncertainties) and farm practices (parcellation) in order to gain insight into how the agronomic systems react to these factors and to what extent they are sensitive to them and thus have a more precise idea of the strategies that can be put in place to achieve the objectives of soil restoration and preservation.

In the first essay titled "**A Survey of Applications of Viability Theory to the Sustainable Exploitation of Renewable Resources**" we provide a comprehensive review of the literature on applications of *viability theory* to the sustainable management of renewable resources including ecosystems and populations such as fisheries and non-marine species, the environment (with a focus on climate change and GHG concentration), and other resources (e.g., forests and soil).

In the second essay titled "**Viability of Agroecological Systems Under Climatic Uncertainty**", we consider the problem of the long-term sustainable operation of a single farm, from both the physical (soil quality) and economic (farmer's revenues) perspectives while taking into account uncertainties induced by major climatic events (hurricanes). Given some economic and soil quality constraints, and taking into account the possible occurrence of climatic events, we are interested in finding out what are the viable crop rotations to grow over a predefined planning horizon, how sensitive are these solutions with respect to parameter values, and which farming systems or sectors are the most vulnerable to climatic uncertainties.

In the third essay, titled "**Viability of a Multi-Parcel Agroecological System**", we deal with long-term management of farms and we examine the potential advantages that could be offered by parcellation given a farm of a predetermined size and a planning horizon, we aim to determine what choices should the farmer make in order to simultaneously achieve ecological (soil's quality) and economic (revenue) sustainability objectives. We consider a series of cases representing different farming systems practiced in the French West Indies, and analyze their impact on soil quality and economic well-being of the farmer. Also, we conduct an extensive sensitivity analysis to assess the impact of main parameters on the results, namely, initial soil quality, planning horizon and different costs.

Key Words: Viability Theory; Agriculture; Climatic Uncertainty; Multi-parcel; Sustainability; Renewable resources.

Research methods: Mathematical programmation; Viability Theory.

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To my parents, Dida and Sofiane
for your endless love and support.

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Preface

This thesis is created based on three research articles. The list of published and submitted articles is provided below:

Oubraham, A., & Zaccour, G. (2018). A survey of applications of viability theory to the sustainable exploitation of renewable resources. Ecological economics, 145, 346-367.

Oubraham, A., Saint-Pierre, P., & Zaccour, G. (2020). Viability of Agroecological Systems under Climatic Uncertainty. Sustainability, 12(15), 5880.

Oubraham, A., Saint-Pierre, P., & Zaccour, G. (2021). Viability of a Multi-Parcel Agroecological System. (Submitted for publication)

General Introduction

It is not new that societies care about their environment and resources and take actions to protect them.[1] What is however of recent vintage is the awareness that (i) immoderate human activity, e.g., burning fossil fuels, over fishing or excessive deforestation, have direct undesirable consequences, such as loss of biodiversity and deterioration in environmental quality, and (ii) some concerted actions are urgently needed to preserve these resources. A pivotal date in first gaining this awareness was probably the publication of *Limits of Growth* in 1972 (Meadows et al. [12]), a study that triggered fervent debate and stroked the popular imagination, since some of the simulated growth scenarios predicted the collapse of the global system. Later in the same decade, it was argued that economic development could be sustained indefinitely, but only if it were to take into account its ultimate interaction with the natural environment. This marked the advent of the concept of *ecological management*, which paved the way for the notion of *sustainable development*, which was coined by the International Union for the Conservation of Nature and Natural Resources (IUCN) in 1980; see (Allen et al. [1]). Since then the idea of sustainable development very quickly gained in popularity among scientists, decision makers and activists. Gradually, with more and more evidence supporting the need to protect natural

[1] The following website offers an environmental history timeline with a list of events and actions related to environmental protection: `http://environmentalhistory.org`

resources, actions are starting to be taken for better and more sustainable management of all natural resources including soil.

It is self-evident to state that agricultural land is essential to life and a valuable resource for most, if not all, countries. Still, its importance seems to only be noticed when productivity declines and our food security is at stake. It is well-established now that soil has been poorly protected and overexploited for decades, which has resulted in its deterioration worldwide.

One of the most important human-induced factor of soil's deterioration is modern agriculture and farming. Indeed, to meet an ever growing demand, agronomic systems have had to drastically change during last decades and migrated to agricultural practices based on chemical fertilization of soil and intensive and specialized farming practices. This has increased soil productivity in the short term, but in the long term has caused serious ecological drawbacks (degradation of soil quality, pollution of water and air, loss of biodiversity, erosion, etc.) and even reversed the trend of the agricultural productivity.

In contrast with the traditional agricultural systems based on natural fertilization and diversified crops production, the modern agriculture relies on intensive single crop production that progressively degrades the soil quality and reduces its productivity by changing its physical, chemical and biological composition. Indeed, fertilization and low-quality irrigation water alters the soil's chemical makeup. Further, plowing, tillage, removal of vegetative cover, and overgrazing make soil more vulnerable to wind and water erosion, and intensive and specialized cultivation exhausts some minerals and water from the soil and damages its microfauna (Blanco and Lal [7]). Modern agricultural systems have since fallen in a vicious cycle, where increasing chemical fertilizers are used to compensate for the loss of productivity, causing more damage to the soil and the environment (pollution of water and air) (Trautmann et al. [14], Conway [10]).

Given the state of affairs we just described, it becomes urgent to take actions to replace actual agricultural practices by more eco-responsible ones based on crop rotations and mixed crop-livestock associations that are healthier for the soil. We need to establish an agroecological transition that would, in the medium term, allow to return to more environmentally friendly agricultural practices and to restore soil quality to an acceptable level, while taking into account the socio-economic aspects related to the sector. Implementing eco-responsible agricultural practices, based on crop rotations and mixed crop-livestock associations that are less harmful for the soil, is one way to achieve resource (soil) sustainability (Altieri [2], Kremen et al. [11]). At a macro level, this transition is a long-term process that must take populations' food needs and farming profitability into account. Such a transition is particularly needed in island regions, given the importance of agriculture to their economies (Angeon et al. [3]).

In this thesis, we are precisely interested in soil restoration problems subject to some environmental and socio-economic constraints such as reaching an environmentally acceptable quality of the soil and ensuring acceptable incomes for the farmers. Our objective is to study these problems in different frameworks that take into account some of the most important factors that influence them such as climatic uncertainties and farm practices (parcellation) in order to gain insight into how the agronomic systems react to these factors and to what extent they are sensitive to them and thus have a more precise idea of the strategies that can be put in place to achieve the objectives of soil restoration and preservation.

To do so, we adopt viability theory (Aubin [4]) as our methodological framework. In a nutshell, a viability problem involves a dynamical system whose evolution depends on state and control variables, and possibly of some random events. Given a set of constraints and initial state of the system, one looks for viable solutions, that is, evolutions (or trajectories) of these variables that satisfy these constraints. Given our problem statement, VT seems a very relevant choice especially that it was successfully applied in many fields, including economics (Aubin [5]), finance (Aubin et al. [6]), demography and genetics (Bonneuil and Saint-Pierre [8] and [9]), aerospace (Tomlin et al. [13]) and in renewable resources management, which is our topic.

We start by surveying the literature in the field to get an idea of what has already been done since, to the best of our knowledge, no such literature review existed yet. Then, we look at the soil restoration problem in a stochastic framework by introducing climatic uncertainty. Finally, we look at the potential advantages that could be brought by parcellation in a deterministic framework.

More specifically, in the first essay titled "**A Survey of Applications of Viability Theory to the Sustainable Exploitation of Renewable Resources**" we provide a comprehensive review of the literature on applications of *viability theory* to the sustainable management of renewable resources including ecosystems and populations such as fisheries and non-marine species, the environment (with a focus on climate change and GHG concentration), and other resources (e.g., forests and soil).

In the second essay titled "**Viability of Agroecological Systems Under Climatic Uncertainty**", we adopt a micro point of view and consider the problem of the long-term sustainable operation of a single farm, from both the physical (soil quality) and economic (farmer's revenues) perspectives while taking into account uncertainties induced by major climatic events (hurricanes). Given some economic and soil quality constraints, and taking into account the possible occurrence of climatic events, we are interested in finding out what are the viable crop rotations to grow over a predefined planning horizon, how sensitive are these solutions with respect to parameter values, and which farming systems or sectors are the most vulnerable to climatic uncertainties? Our empirical study concerns the archipelago of Guadeloupe, located in the French West Indies.

In the third essay, titled "**Viability of a Multi-Parcel Agroecological System**", we deal with long-term management of farms and we examine the potential advantages that could be offered by parcellation. Given a farm of a predetermined size and a planning horizon, we aim at determining what choices should the farmer make in order to simultaneously achieve an ecological (soil's quality) and economic (revenue) sustainability objectives. We consider a series of cases representing different farming systems practiced in the French West Indies and analyze their impact on soil quality and economic well-being of the farmer. Also, we conduct an extensive sensitivity analysis to assess the impact of main parameters on the results, namely, initial soil quality, planning horizon and different costs. Our contribution is at three levels. First, we develop a new multi-parcel model that provides additional flexibility in the search for sustainable solutions. Second, we design a novel algorithmic approach for computing viable solutions. Finally, we contribute empirically by answering questions that are on the agenda of farmers and decision makers in the French West Indies.

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Chapter 1

A Survey of Applications of Viability Theory to the Sustainable Exploitation of Renewable Resources*

Abstract

In this paper, we survey the literature applying viability theory to the sustainable management of renewable resources. After a refresher on the main concepts of viability theory, we provide a general map of the contributions and next discuss them by area of application, including ecosystems and population biology, climate change, forestry and others. We conclude by pointing out issues that deserve more attention and should be part of a research agenda.

Key Words: Viability theory; Sustainability; Renewable resources; Fisheries; Climate change; Forests.

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Introduction

It is not new that societies care about their environment and resources and take actions to protect them.[1] What is however of recent vintage is the awareness that (i) immoderate human activity, e.g., burning fossil fuels, over fishing or excessive deforestation, have direct undesirable consequences, such as loss of biodiversity and deterioration in environmental quality, and (ii) some concerted actions are urgently needed to preserve these resources. A pivotal date in first gaining this awareness was probably the publication of *Limits of Growth* in 1972 (Meadows et al. [101]), a study that triggered fervent debate and stroked the popular imagination, since some of the simulated growth scenarios predicted the collapse of the global system. Later in the same decade, it was argued that economic development could be sustained indefinitely, but only if it were to take into account its ultimate interaction with the natural environment. This marked the advent of the concept of *ecological management*, which paved the way for the notion of *sustainable development*, which was coined by the International Union for the Conservation of Nature and Natural Resources (IUCN) in 1980; see Allen et al. [2]. Although at that time a precise definition of sustainable development was lacking, the idea itself very quickly gained in popularity among scientists, decision makers and activists.[2] A second notable date is the publication in 1987 of the Brundtland Report, which provided a unifying definition of sustainable development:

> "Sustainable development is development that meets the needs of the present without compromising the ability of future generations to meet

[1] The following website offers an environmental history timeline with a list of events and actions related to environmental protection: `http://environmentalhistory.org`

[2] For a list of some definitions of sustainable development used between 1980 and 1988, see the Appendix in Pezzey [107].

their own needs."(Brundtland et al. [43])

This definition has since been adopted by all stakeholders, although refinements have occasionally been considered, implicitly or explicitly, in some studies (see for example Pezzey [107], Neumayer [105], Heal [82] and Klauer [85] for an overview of some characterizations and operationalizations of sustainability that have been proposed). For example, Fleurbaey [75] proposed to define sustainability in terms of leaving the possibility for future generations to sustain certain defined targets. Martinet et al. [96] defined sustainability as a combination of biological, economic and social constraints which need to be met. Baumgärtner and Quaas [17] conceptualized strong sustainability under uncertainty as ecological-economic viability. Durand et al. [67] and Doyen and Martinet [62] considered the notion of intergenerational equity in defining sustainability.

This paper provides a comprehensive review of the literature on applications of *viability theory* to the sustainable management of renewable resources including ecosystems and populations such as fisheries and non-marine species, the environment (with a focus on climate change and GHG concentration), and other resources (e.g., forests and soil). In a nutshell, "Viability theory is an area of mathematics that studies the evolution of dynamical systems under constraints on the system's state and control (Aubin [7], Aubin et al. [14]). It was developed to formalize problems arising in the study of various natural and social phenomena, and has close ties with the theories of optimal control and set-valued analysis."[3] As in optimal control, the basic ingredients of viability theory (VT) are control and state variables, and a dynamical system whose evolution is governed by differential (or difference) equations, which are functions of the state and control variables and some parameters.

[3] https://en.wikipedia.org/wiki/Viability_theory

The system evolution can be deterministic or not, and is subject to some (viability) constraints. A notable difference with optimal control is the absence of an objective functional to be optimized. As we will see, main objects of viability theory are sets, hence the link made above to set-valued analysis. The theory was initiated by Jean-Pierre Aubin in the late 1970s and the fundamental results established in the 1980s (see Haddad [78]).

In Aubin [6], viability theory is described as a mathematical theory based on three main features, namely: (i) non-determinism of evolutions; (ii) viability constraints; and (iii) inertia principle. The two first features concern the state trajectory of the studied system and reflect the fact that a system can evolve in many different and possibly unpredictable ways depending on its initial state, its past evolution, the environment in which it evolves or anything else (non determinism), and also the fact that, for many reasons, the evolution of a system is restrained by some constraints that must be satisfied at each instant of time [4]. These are the two founding pillars of viability theory models.[5] The last feature (inertia principle) concerns the control variables and stipulates that these controls are changed only when required for maintaining viability. To find a viable solution (or a set of viable solutions), VT follows a backward (or inverse) method, that is, starting from a set of given viability constraints, one looks for the set of initial states from which the system can be indefinitely viable.

[4]When the model is stochastic, satisfying the constraints at each instant of time has to be interpreted in a stochastic or robust-control sense

[5]Besides, Aubin et al. [14] present this theory as a mathematical translation of Jacques Monod's *Chance and Necessity* ([102]) in which there appears a quotation from Democritus stating that "the whole universe is but the fruit of two qualities, chance and necessity." Chance refers to the non-determinism of trajectories, and necessity expresses the need to meet certain conditions or criteria, which results in viability constraints.

Viability theory was successfully applied in many fields, including economics (Aubin [8]), finance (Aubin et al. [13]), demography and genetics (Bonneuil and Saint-Pierre [36] and [38]), aerospace (Tomlin et al. [126]) and in renewable resources management, which is our topic. Other approaches than VT are of course available to determine sustainable exploitation of a renewable resource, in particular the so-called *policy optimization* and *policy evaluation* (Weyant et al. [130]). In the former, as the name suggests, one defines an objective function that typically measures the relevant costs and benefits of possible decisions, and the optimization is carried out subject to a series of constraints. In policy evaluation, some feasible scenarios are assessed and eventually the best one is selected. While these approaches have obvious merits, they often involve trade-offs between the different environmental, economic and social facets of sustainability, which may not be desirable. As mentioned above, there is no (intertemporal) objective to be optimized in a VT model, and sustainability is addressed through the viability constraints. Therefore, a VT model avoids the contentious issue of weighting different sustainability facets, or making trade-offs between short- and long-term considerations. Writing down an intertemporal objective requires an assessment of future options. In a VT model, such knowledge of the future is not mandatory because the choice of controls at any given initial time is not final, and can be adapted to eventual changes in the system's environment (Aubin [5]). However, unless the model is every simple, the viable controls or strategies cannot be obtained in closed form but only be numerically approximated. This is somehow similar to what is done in the *policy guidance approach* (PGA), which was recently proposed and has been referred to by different names in different areas, e.g., *tolerable window approach* in climate change and GHG management ([121], [41], [42] and [127]), *population viability analysis* in conservation biology ([19], [74], [71], [39], [20], [122] and [18]) or *safe minimum standards* in fisheries ([27], [28] and [30]).

Indeed, the basic idea behind the PGA is to maintain the system as long as possible within some predefined bounds (De Lara and Doyen [54]). Finally, we note that determining feedback control maps when solving a VT model is similar to what is done when solving a dynamic optimization problem using dynamic programming.

The rest of the paper is organized as follows: In Section 2, we provide a short refresher on viability theory. In Section 3, we show how viability theory is used to address sustainable development problems, and give a general map of the contributions. In Section 4, we review the applications of viability theory to the management of renewable resources, which is the main block of interest. In Section 5, we briefly conclude. A table summarizing all reviewed papers is given in the Appendix.

1.1 A refresher on viability theory

In this section, we recall some concepts of viability theory that are useful for appreciating its applications in renewable resources. For a rigorous introduction to viability theory, the interested reader may consult the books by Aubin (1991), Aubin [6], Aubin et al. [14], Aubin [7] and De Lara and Doyen [54].

We shall distinguish in the sequel between deterministic viability and stochastic viability. Although in both settings the main questions are the same, e.g., how to remain viable, to reach a target or to restore viability if lost during the process, the concepts and techniques used to answer these questions will be different, at least to a certain extent.

Denote by $x(t)$ the state of a system of interest at time $t \in [0, +\infty)$, and let

$X \subset \mathbb{R}^n$ be the state space. The evolution of the state is described by

$$\mathcal{F}\begin{cases} x'(t) = f(x(t), u(t)) \\ u(t) \in U(x(t)) \end{cases} \tag{1.1}$$

where $u(\cdot)$ is the control variable and $U(x(t))$ is the set of admissible controls at time t, which depends on the state of the system at that time. We shall refer to $\mathcal{F}$ as the controlled-evolution system.

At each time t and starting from any state x, the system can follow different trajectories depending on the applied control u and other parameters. We denote by $\mathcal{S}$ the set of all solutions of the system (1.1) and $\mathcal{S}(x) \subset \mathcal{S}$ the set of all admissible trajectories starting from x and governed by (1.1), that is,

$$\mathcal{S}(x) = \{x(\cdot) | x(0) = x \text{ and (1.1) satisfied}\}.$$

Where $x(\cdot)$ are absolutely continuous functions.

Let $K \subset X$ be the set of (viability) constraints. In its simplest expression, this set would involve lower and upper bounds on the state variables, i.e.,

$$K = \left\{ x \in X | \begin{array}{c} \underline{x}_1 \leq x_1 \leq \overline{x}_1 \\ \vdots \\ \underline{x}_n \leq x_n \leq \overline{x}_n \end{array} \right\},$$

but of course, in general, the constraints can be more complex, i.e., of the form:

$$K = \left\{ x \in X | \begin{array}{c} g_1(x) \geq 0 \\ \vdots \\ g_m(x) \geq 0 \end{array} \right\}.$$

Let $C \subset K$ be a target.

1.1.1 Viability kernel

The *viability kernel* is a cornerstone of viability theory. To define it, we first need to recall what is meant by a viable trajectory. A trajectory of the system is said to be viable on a time interval if it satisfies the viability constraints at each moment of this time interval. A mathematical definition follows.

Definition 1 (Viable trajectory). *A trajectory $x(\cdot)$ is said to be viable in K on the time interval $[0,T)$ $(T \leq +\infty)$ if*

$$\forall t \in [0,T)\,, x(t) \in K.$$

The set of all viable trajectories in K on $[0,T)$ $(T \leq +\infty)$ is

$$\mathcal{V}(K) = \{x(\cdot) \in \mathcal{S} | \forall t \in [0,T)\,, x(t) \in K\}.$$

We shall later on give an overview of the viability constraints that have been considered in the context of the sustainable exploitation of renewable resources, and the list of these constraints in each contribution.

The viability kernel is the set of all initial states from which **at least one** viable trajectory starts.

Definition 2 (Viability kernel). *The viability kernel of K for the system $\mathcal{F}$ is the set*

$$Viab_{\mathcal{F}}(K) = \{x_0 \in K | \exists\ x(\cdot) \in \mathcal{S}(x_0) \text{ such that } \forall t \geq 0, x(t) \in K\}.$$

The viability kernel is a tool that allows us to check whether a system is viable, and in particular if the current state (as initial state) is viable. If the current state does not belong to the viability kernel, then a first conclusion is that the system is not sustainable. A natural follow-up question is then: can viability be restored? We will come back to this below.

A more restrictive notion is the *invariance kernel*, which corresponds to the set of all initial states such that **all** trajectories starting from these states are viable.

Definition 3 (Invariance kernel). *The invariance kernel of K for the system $\mathcal{F}$ is the set*

$$Inv_{\mathcal{F}}(K) = \{x_0 \in K | \forall\ x(\cdot) \in \mathcal{S}(x_0), \forall t \geq 0, x(t) \in K\}.$$

Clearly, the invariance kernel is a subset of the viability kernel.

1.1.2 Capture basin

In some problems, the aim is to reach a target in finite time rather than to maintain the state in a viable set at each instant of time. In this case, the relevant concept is the *capture basin*, and the following three definitions are the corresponding alternatives to the above three definitions.

In presence of a target, we will be interested by the so-called "capturing trajectories " rather than viable ones. A trajectory of the system captures a target if it permanently satisfies the viability constraints before reaching the target in finite time.

Definition 4 (Capturing trajectory). *The trajectory $x(\cdot)$ captures the target C if*

$$\exists\ T < +\infty | \forall t \in [0, T)\,, x(t) \in K \ \&\ x(T) \in C.$$

The set of all capturing trajectories of C is

$$\mathcal{K}(K, C) = \{x(.) | \exists\ T < +\infty \textit{ such that } x(.) \in \mathcal{V}(K) \textit{ on } [0, T] \textit{ and } x(T) \in C\}.$$

The alternative notion to the viability kernel when a target is involved is the capture basin, which is the set of all initial states from which **at least one** capturing trajectory starts.

Definition 5 (Capture basin). *The capture basin of C for system $\mathcal{F}$ is the set*

$$\begin{aligned} Capt_{\mathcal{F}}(K, C) &= \{x_0 \in K | \exists\ (x(\cdot), T) \in \mathcal{S}(x_0) \times \mathbb{R}_+ \\ &\qquad \textit{such that } \forall t \in [0, T]\,, x(t) \in K \textit{ and } x(T) \in C\}\,. \end{aligned}$$

Finally, equivalently to the notion of the invariance kernel, we define the *absorption basin* of a target, which corresponds to the set of all initial states such that **all** trajectories starting from these states capture the target.

Definition 6 (Absorption basin). *The absorption basin of C for system $\mathcal{F}$ is the set*

$$\begin{aligned} Abs_{\mathcal{F}}(K,C) &= \{x_0 \in K | \forall x(\cdot) \in \mathcal{S}(x_0), \exists\, T \leq +\infty \\ &\quad \textit{such that } \forall t \in [0,T]\,, x(t) \in K \textit{ and } x(T) \in C\}\,. \end{aligned}$$

We note that the absorption basin is a subset of the capture basin.

1.1.3 Restoring viability

As alluded to above, it may well be the case that viability is not at hand, which occurs when, e.g., the viability kernel is empty or the initial state of the system is not viable. In such cases, one may wonder how much time will elapse before the constraints are violated, whether the system's viability is compromised definitively and, if it is possible to restore it, how can it be restored and how long will it take? The *exit function* and the *crisis function* [64] are the starting points for such an analysis. The exit function measures the maximum time during which the system evolution can satisfy the constraints. The crisis function measures the minimum time that an evolution starting from a given state spends outside the viability kernel.

Definition 7 (Exit function). *The exit function associates to a state $x \in X$ its maximum exit time $\tau_K(x)$:*

$$\begin{array}{rcl} \tau_K : X & \to & \mathbb{R}_+ \cup \{+\infty\}, \\ x & \mapsto & \tau_K(x) = \sup\limits_{x(.)\in\mathcal{S}(x)} \inf\{t \geq 0 | x(t) \notin K\}. \end{array}$$

Definition 8 (Crisis function). *The crisis function associates to a state $x \in X$ its minimum crisis time $\mathcal{C}_K(x)$:*

$$\begin{array}{rcl} \mathcal{C}_K : X & \to & \mathbb{R}_+ \cup \{+\infty\}, \\ x & \mapsto & \mathcal{C}_K(x) = \inf\limits_{x(.)\in\mathcal{S}(x)} \lambda_l(t \geq 0 | x(t) \notin K), \end{array}$$

where λ_l is the Lebesgue measure.

One can easily deduce that a viable state will have an infinite exit time and a crisis time equal to zero, while a non-viable one will have a finite exit time and positive (finite or not) crisis time.

To restore viability, we can for example apply the *viability multiplier* to change the initial dynamics, use *reset mapping* (impulse controls) to change the initial conditions of the system, and other methods. For more details, see Aubin et al. [14], chapter 12.

1.1.4 Non-deterministic viability

In many problems, the evolution of the system of interest may depend on some uncertain parameters. In such cases, the dynamics of the system will involve some random variables describing the uncertainty. System $\mathcal{F}$ (1.1) then becomes

$$\mathcal{F}\begin{cases} x'(t) = f(x(t), u(t), \zeta) \\ u(t) \in U(x(t)) \end{cases}, \tag{1.2}$$

where ζ is a vector of random variables representing the different uncertainties considered in the model and each following a probability distribution that can be known or unknown.

Stochastic viability or robust viability can be used to deal with such contexts. In the stochastic viability framework, the assumption is that the uncertain events obey a probability law, which is inferred from some historical observations, experiences, etc. Here, the satisfaction of the viability constraints is stated in terms of a given confidence level. (Of course, one can conduct a sensitivity analysis that varies this level.) Robust viability is a special case of stochastic viability in the sense that the confidence level is set at 100%, i.e., the constraints must be satisfied whatever the uncertainties. This approach is related to the concept of *ambiguity* and is preferred when the probability law of the uncertain event is unknown, or the decision maker is seeking a strategy against the worst-case scenario. Both approaches have been considered in many other areas and are by no means limited to viability theory. However what is particular here is the adaptation of the above definitions to a non-deterministic setting. To illustrate, the next two definitions give the viability kernel in the context of stochastic and robust approaches.

Definition 9 (Stochastic viability kernel). *The stochastic viability kernel of K under system $\mathcal{F}$(1.2) to the confidence level of m% is the set*

$$Viab^m_{\mathcal{F}}(K) = \left\{x_0 \in K | \exists\ x(.) \in \mathcal{S}(x_0) \text{ such that } \forall t \geq 0, \mathbb{P}(x(t) \in K) \geq \frac{m}{100}\right\},$$

where $\mathbb{P}(x(t) \in K)$ *is the probability of realization of the event* $x(t) \in K$.

Definition 10 (Robust viability kernel). *The robust viability kernel of environment* K *under system* $\mathcal{F}$ *(1.2) is the set*

$$Viab^R_{\mathcal{F}}(K) = \{x_0 \in K | \exists \cdot x(.) \in \mathcal{S}(x_0) \text{ such that } \forall t \geq 0, \mathbb{P}(x(t) \in K) = 1\}.$$

1.2 Applications of viability theory

Devising a VT model to study the sustainability of a system essentially involves the following inputs:

A description of the dynamical system. The ingredients here are state variables (e.g., stock of fish, size of a forest, pollution stock, population), control variables (e.g., fishing effort; deforestation and reforestation efforts; emissions; birth, death and migration rates), some uncontrollable factors (weather, epidemics, state of the economy, etc.), and their interrelationships.

An operationalization of sustainability. In the context of renewable resources, and as implied by the definition in the Brundtland Report, environmental, economic and social variables are needed to construct the validity of sustainable management (or exploitation of a resource). Practically speaking, the sustainable domain is described by a series of (viability) constraints that are imposed on the state variables (and possibly on their velocities), on the control variables, and on some joint constraints involving both types of variables. The satisfaction of the constraints is one way of handling the multi-criteria feature of sustainability, without, however, having to aggregate these facets into one index.

Depending on the context, the output is the viability kernel or the capture basin, or their more restrictive versions, that is, the invariance kernel or the absorption basin. Also, we obtain the controls that must be exerted to remain in one of these sets. These controls are interpreted as policy guidance.

We make the following remarks:

1. Sustainability must in some way refer to intergenerational equity to account for the principle stated in the Brundtland Report, namely, of meeting "the needs of the present without compromising the ability of future generations to meet their own needs." This intergenerational equity is inherently preserved in VT because the constraints must be satisfied at *each* instant of time, independently of which generation is living at that instant, which means that all generations are treated equally.

2. Irreversibility is an important notion when it comes to managing some types of renewable resources like animals or atmosphere for which overexploitation can lead to the point of non return, e.g., extinction of species or irreversible changes in a climate system. VT is particularly efficient for managing this type of problems. Indeed, the risk of falling into an irreversible situation can be monitored through the crisis function or can be totally avoided using adequate viability constraints.

3. As VT proceeds numerically, the functions describing the dynamical system and the constraints can be of any form. This huge flexibility comes at the cost that the controls needing to be exerted to remain viable can only rarely be described in closed form.

4. A VT model can have as many control variables as the situation dictates. The number of state variables is not restricted in theory, but in practice, it is very hard to go beyond a four-dimensional state. In fact, in applications of VT to renewable resources, the dimension of the state space is generally less than three. Of course, some models with high dimensions exist; see, e.g., Cissé et al. [50] and [51], Gourguet et al. [76] and [77], Mouysset et al. [103] and Hardy et al. [79]. In these references, the authors typically avoid the numerical complexity by choosing to identify only some viable states and trajectories instead of identifying the whole viability kernel. Note that all other alternative methodologies that involve dynamic optimization also suffer from this curse of dimensionality.

In this paper we reviewed the literature applying VT to the sustainable management of renewable resources. We adopted the following "algorithm" to select the list of papers to be included in our survey:

Step 1: We searched Google and three databases (ScienceDirect, SpringerLink and Wiley Online Library) using several combinations of keywords. "Viability theory" as the main key word combined with one or more secondary keywords, i.e., "Renewable resource", "Sustainability", "Fishery", "Population", "Forest", "Climate" and "Agriculture". The searches were done in English and French, without excluding any types of documents or years. We retained only peer-reviewed papers (published, online, accepted or in proceedings).[6]

Step 2: The bibliography in each of retained papers in Step 1 was examined to check if we did not miss any paper, and indeed few were discovered here.

[6] To be very rigorous, two Ph.D. thesis are also included.

Step 3: Each paper was scanned to verify that it does fit our topic, that is, applications of viability theory to management of renewable resources. This means that the paper must be methodologically and topic relevant.

Step 4: The list of papers resulting from above was sent to eight active researchers in the field asking them to add any reference that we could have missed. Only few additions were made.

Table 1.1 reports the number of papers applying VT to renewable resources by area. The main takeaway is that ecosystems and population biology are by far the most studied areas, with fisheries accounting for almost half of all applications of VT to renewable resources (49%).

Table 1.1: Viability theory applications by area

	Number of articles	%
Ecosystems and population biology:		
Fisheries	38	49
Other non-marine species	14	18
Farming and agro-ecology	9	11
Climate change	6	8
Forests	4	5
Renewable resources (general)	3	4
Other	4	5

From Table 1.2, we learn that most models have infinite time horizons, that discrete-time models are slightly more popular then continuous-time models and that two-thirds of publications assumed a deterministic world. Stochastic viability is used slightly more often than robust viability when uncertainty is considered.

From Table 1.3, we notice that most articles involve a practical numerical application, with 49% using empirically estimated values from real situations. The other studies

Table 1.2: Type of model

	%
Discrete time model	53
Continuous time model	47
Infinite time horizon	61
Finite time horizon	39
Deterministic viability	66
Stochastic viability	20
Robust viability	14

either give a numerical illustration using some suitable values or do not provide any numerical examples.

Table 1.3: Type of numerical application or illustration

	%
Practical application	49
Arbitrary values	28
No numerical result reported	23

As Figure 1.1 shows, early publications applying viability theory on sustainable management of renewable resources problems started in 1991. The publications were then few, irregular and restrained to applications in fisheries and population biology until 2004 where we observe a significant increase in the number and rhythm of publications over time as well as a diversification of topics addressed with applications on farming and agro-ecological problems, on climate change and on management of renewable resources in general. Applications on forestry started to appear only since 2011.

Finally, we note that of the 78 papers selected for this survey, 42 (or 54%) were published during the period from 2010 to 2015.

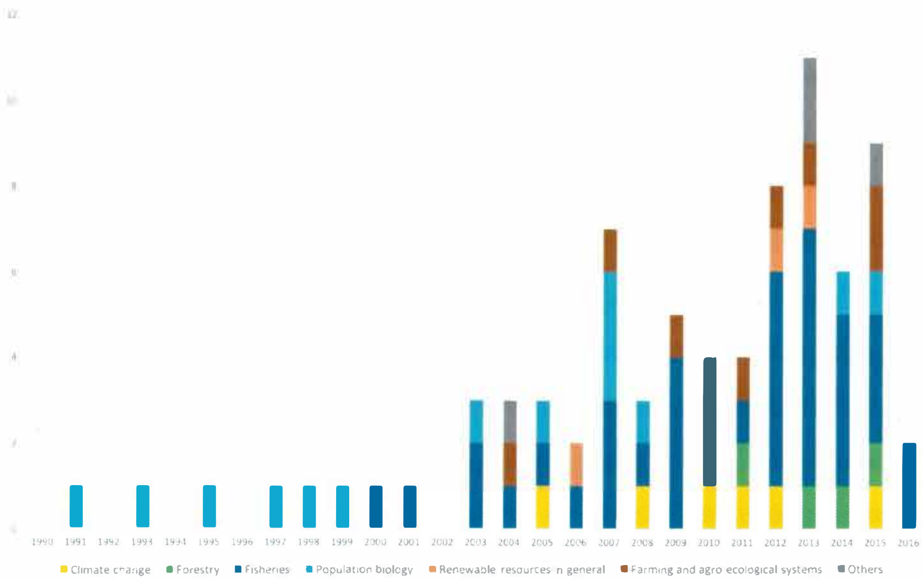

Figure 1.1: Publications applying VT on renewable resources management problems over time

1.2.1 Viability studies in ecosystems and population biology

Early contributions of viability theory in renewable resources are related to ecosystems and population biology; see Křivan ([88], [89], [90]) and Bonneuil [31]. Křivan was mainly interested in the following question: "How can we modify a dynamical system to make it viable, (i.e., having solutions that do satisfy the constraints), knowing the dynamical behavior of the system without the state constraint?" ([88]). Bonneuil's contribution, in [31], was to revisit the Malthus-Boserup explanatory framework of population biology using the point of view of viability theory.

Within this group of studies, fishery is by far the most popular topic. One possible explanation for this is that optimal-control models, which share a number of commonalities with VT models, were already widely used in fisheries, and therefore, the transition from one methodological framework to the other was somewhat easy. In their survey on the assessment of economic viability in small-scale fisheries, Schuhbauer and Sumaila [120] point out that viability theory is a popular methodology in this area.[7]

Whatever the precise objective being pursued, e.g., protection of an endangered species or preservation of biodiversity, this literature will typically have a population state space $X \subset \mathbb{R}^n$ where $n \geq 1$ is the number of different species considered, or age classes in the case of age-structured populations, and $x(t) = (x_i(t))_{i=\overline{1,n}}$ is the biomass or stock level of each species $i \in \{1, ..., n\}$ at time t. Of course, other state variables may be considered, such as biodiversity or economic indicators. In the continuous-time case, the evolution of the (population) state variables is described by the following system of differential equations:

$$\begin{aligned} x'(t) &= f(x(t), u(t)), \\ u(t) &\in U(x(t)), \end{aligned}$$

where function f captures the evolutionary characteristics of each species (e.g., reproduction and fertility, natural mortality) as well as the interactions with other species (e.g., predation, mutualism, etc.).

The literature can be divided along different lines. One is multi-species studies (e.g., Béné and Doyen [23], De Lara and Martinet [55], Martinet et al. [97], Gourguet et al. [76], Lercari and Arreguín-Sánchez [91], Krawczyk et al. [87], Mullon et al. [104], Doyen et al. [66], Maynou [100], Martinet and Blanchard [95], Hardy et al.

[7] Although there is an overlap between Schuhbauer and Sumaila [120] and our survey (19 fishery papers are common and we cover 21 additional papers), we note that Schuhbauer and Sumaila [120] survey is topic driven, that is, small-scale fisheries, whereas ours focuses on applications of VT to all renewable resources.

[79], Mouysset et al. [103], Cissé et al. [50] and [51]) versus single-species studies (e.g., Chavas [49], Doyen and Béné [61], Eisenack et al. [70], Péreau et al. [106], De Lara et al. [58], Ferchichi et al. [73], Sanogo et al. [119], Curtin and Martinet [52], Alais et al. [1]) or age structured population studies (e.g., De Lara et al. [56], De Lara and Martinet [55], Doyen et al. [65], Gourguet et al. [76], Curtin and Martinet [52], Maynou [100], De Lara et al. [58], Alais et al. [1], De Lara et al. [57], Chavas [49]) or even sex-structured population studies (e.g., Gourguet et al. [77], Ferchichi et al. [73]). In a multi-species context, the focus is on marine (and sometimes non-marine) ecosystems and food webs. The resulting models are, generally speaking, more complex than in single-species models, as all relevant interactions between the different species must be taken into account. Each control variable may concern one or many of these species, and all of them may be involved in the economic or environmental viability constraints. In the single-species category, only one resource stock is considered, and the (often implicit) assumption is that the effect of the other species on this stock is captured by the mortality and fertility parameters, while the effect of variations in the considered species on the others can be captured through some biodiversity indicators.

A second distinction can be made between studies that consider human intervention (Béné and Doyen [22], Cissé et al. [50] and Eisenack et al. [70]), and those that do not (see, e.g., Bonneuil [33], Křivan [88], Křivan [89], Bonneuil and Müllers [35], Křivan and Colombo [90], Bonneuil [31], Rougé et al. [113], Bonneuil and Saint-Pierre [37], Aubin and Saint-Pierre [11]). When human action is absent, the long-term evolution of the system will depend only on inter-species interactions and possibly some unforeseen events, and can then be considered a benchmark for assessing the impact of human intervention.

A third distinction is between deterministic models and those where some form of uncertainty is considered. In population-biology and fisheries models, this uncertainty can be of a biological nature, for instance uncertainties in the population's rate of reproduction, inter-species relationships and rate of predation, or the initial biomass stock size; see, e.g., Regnier and De Lara [111], Chapel et al. [48] and Křivan and Colombo [90]. It can also be related to the environment, e.g., the uncertainty related to climate change or the effect of pollution on the species; see, e.g., Doyen et al. [66], Křivan [89] and Martinet et al. [98]. Finally, the uncertainty can be related to market conditions (demand and price) or to the evolution of technology; see, e.g., Gourguet et al. [76].

Although this literature is dense, it is interesting to note that the different contributions share a lot of common features when it comes to selecting the control and state variables and defining the viability constraints. With the following list of variables and constraints, we account for a large extent of what has been considered in this literature:

State variables: The most common variables are (i) the biomass stock of the species; and (ii) some biodiversity indicators. See, e.g., Doyen and Béné [61], Hardy et al. [79], Cisse et al. [51] and De Lara et al. [59].

Control variables: The most frequently considered variables are (i) the harvest level (e.g., De Lara et al. [56], Béné and Doyen [22], Doyen and Béné [61], De Lara et al. [57] and Curtin and Martinet [52]); and (ii) the catching effort (e.g., Doyen et al. [66], De Lara and Martinet [55] and De Lara et al. [58]).

Viability constraints: Ecological viability constraints are somehow linked to the principles of population viability analysis (PVA), which focus on extinction processes of populations and seek to avoid irreversible situations and extinction of species under uncertainty. In VT, the ecological viability constraints translate the same objective of preserving species, that is, they typically refer to the non-extinction of species (e.g.,. Bonneuil [33]), minimum biomass stock of the resources (used by a large majority of studies), or minimum levels for some biodiversity indicators (Doyen et al. [66], Hardy et al. [79], Cisse et al. [51], Béné and Doyen [23] and Cissé et al. [50]). Economic viability constraints include the satisfaction of demand or guaranteeing food security (Eisenack et al. [70], Cissé et al. [50], Hardy et al. [79], Regnier and De Lara [111], Hardy et al. [80], Thébaud et al. [123], Cisse et al. [51] and De Lara et al. [57]), or minimum revenue or productivity level (e.g., Doyen et al. [66], Meadows et al. [101], Béné and Doyen [22] and Doyen et al. [65]). Social constraints are rarely addressed, but still, a few examples are available, e.g., limiting the number of layoffs per period, which in a fishery context requires to lower-bound the fleet size (Meadows et al. [101]) or maintaining a minimum level of activity for fishermen (Lercari and Arreguín-Sánchez [91], Martinet et al. [97], Sanogo et al. [118], Péreau et al. [106], Sanogo et al. [119], Krawczyk et al. [87], Ferchichi et al. [73] and Alais et al. [1]).

1.2.2 Viability studies in forestry

For the Food and Agriculture Organization, "[Forests] are to provide renewable raw materials and energy, maintain biological diversity, mitigate climate change, protect land and water resources, provide recreation facilities, improve air quality and help alleviate poverty" (see *Global Forest Resources Assessment* 2005 [109]). The world's forests cover nearly one-third of the Earth's surface, but are shrinking at an alarming rate, with an area equivalent to the size of Costa Rica being deforested every year

(FAO 2010 [72]). The main reason for deforestation is agriculture, which brings revenues but by the same token eliminates some of the benefits listed above.

A viability model for forestry essentially aims at preserving the forest while balancing its competing uses. In the few available studies, the state variable is typically the forest's size (Bernard and Martin [25] and Andrés-Domenech et al. [4]) or the number of trees (Mathias et al. [99]), although other variables have also been considered, such as forest biodiversity indicators or the size of the population whose life quality depends on the forest and their wealth or the stock of timber (Mathias et al. [99], Bernard and Martin [25] and Andrés-Domenech et al. [4]). Examples of control variables are forestation and deforestation rates, frequency of these activities, monetary transfers to forest owners to incentivize them to protect their forests, or measures to control the size of a population living around the forest (as suggested in Andrés-Domenech et al. [4], Bernard and Martin [25], Mathias et al. [99]). Environmental viability constraints include imposing a minimum forest size (Andrés-Domenech et al. [3], Mathias et al. [99] and Andrés-Domenech et al. [4]), minimum level of biodiversity, maximum level of deforestation or constraints related to the composition of the forest in terms of species or age of the trees (Mathias et al. [99]). Typical economic constraints are the satisfaction of the demand for timber (Andrés-Domenech et al. [3] and Andrés-Domenech et al. [4]) and a minimum revenue from forest exploitation (Andrés-Domenech et al. [3], Mathias et al. [99] and Andrés-Domenech et al. [4]).

1.2.3 Viability studies in farming and agro-ecological management

The applications in agro-ecological and farming problems mostly relate to herd- and grazing-management systems (Tichit et al. [124], Baumgärtner and Quaas [17], Sabatier et al. [114], Sabatier et al. [115], Tichit et al. [125], Martin et al. [94] and Sabatier et al. [116]). The state variables are the grass biomass or height (as in Baumgärtner and Quaas [17], Sabatier et al. [114], Sabatier et al. [115], Tichit et al. [125], Martin et al. [94] and Sabatier et al. [116]), the herd composition or size(Tichit et al. [124] and Baumgärtner and Quaas [17]) or the abundance of some protected wildlife leaving in the grassland (Mouysset et al. [103], Sabatier et al. [114], Tichit et al. [125] and Sabatier et al. [115]). The control variables are grazing frequency and intensity (Tichit et al. [125], Baumgärtner and Quaas [17], Sabatier et al. [114], Martin et al. [94], Sabatier et al. [115] and Sabatier et al. [116]) or breed composition within the herds (Tichit et al. [124]). Examples of viability constraints include the preservation of the grassland (as in Baumgärtner and Quaas [17] and Martin et al. [94]), the satisfaction of cattle feeding requirements (like in Sabatier et al. [114], Sabatier et al. [115], Tichit et al. [125] and Sabatier et al. [116]), the guarantee of a minimum income to the farmers (Tichit et al. [124], Sabatier et al. [115], Mouysset et al. [103], Baumgärtner and Quaas [17], Martin et al. [94] and Sabatier et al. [116]), maintain acceptable level of biodiversity (Mouysset et al. [103]) and protect wildlife leaving or breeding in the grassland (Tichit et al. [125], Mouysset et al. [103], Sabatier et al. [114] and Sabatier et al. [115]).

Few models dealt with agriculture and cropping. For example, Mouysset et al. [103] uses incentives to encourage certain crop and grass activities as control variables and Durand et al. [68] addresses soil preservation problems through agricultural based models. In the latter, the soil quality (state variable) is measured by a composite index involving physical, chemical and biological characteristics. The control variables refer to the choice of activities (agriculture, cattle breading, etc.), the sequence of plantation, the type of agricultural practices (traditional or intensive), the investment in green technologies, etc. In the same reference ([68]), the main ecological viability constraint is a lower bound on soil quality, and the economic constraints are related to the cash balance, the total revenue from agricultural activity, or investments.

In this farming and agro-ecology category, the principal types of uncertainty that have been considered are related to climatic risk (Tichit et al. [124], Baumgärtner and Quaas [17], Mouysset et al. [103] and Sabatier et al. [116]) or to uncertainty about parameter values (Sabatier et al. [114]).

1.2.4 Viability studies in climate change and GHG management

The Earth's atmosphere (or more commonly air) is composed of different gases whose concentration determines its efficiency to properly complete its tasks, e.g., warming the Earth's surface through the greenhouse effect, protection from solar and cosmic radiations, regulation of the day/night temperature.[8] This means that the Earth's atmosphere can be considered as a natural resource that is essential for life and human activities, which provides a rationale to include in our survey VT studies that dealt with climate change.

Schematically, the main question when it comes to climate change and GHG management is how to limit the rise in temperature to below a given threshold (two

[8] https://en.wikipedia.org/wiki/Atmosphere_of_Earth

degrees is the most cited number) by a certain date (the end of the century). The assumption is that surpassing two degrees will lead to a long series of problems such as loss of biodiversity, rise in the sea level, and droughts, with considerable negative impacts on all living species and their ecosystems. Any attempt to answer this question requires that a dynamical system be defined that adequately describes the evolution of the environment as a function of some control variables and uncontrollable factors. It then suffices to introduce relevant constraints to have a viability model. Actually, this viability theory philosophy is embedded in the Tolerable Window Approach proposed in the nineties by the German advisory council on global change (Scientific Advisory Council on Global Change [121]), even though the viability study per se only began to appear ten years later.

In the few published papers that use the tools of viability theory, the state variables are the same as those used in other methodological frameworks, namely, GHG concentration (Bernardo and Saint-Pierre [26], Aubin et al. [12], Von Bloh et al. [128], Andrés-Domenech et al. [3] and Aubin [9]), mean global temperature (Bernardo and Saint-Pierre [26]) and, a novelty, emission flows (Aubin [9] and Andrés-Domenech et al. [3]). The rationale behind seeing emissions flows as a state variable rather than a control variable lies in the fact that emissions are a by-product of the production of goods and services, and thus, modifying emissions cannot be easily feasible for technological or economic reasons. However, their rate of change can be controllable.

Commonly considered control variables include GHG-emissions (or -abatement) rates (Bernardo and Saint-Pierre [26] and Aubin et al. [12]), investments in green technologies, or intensity of industrial activities (Aubin [9]) and emission rights allocations (Aubin et al. [15]). In models where forests are included as carbon sinks that reduce GHG concentrations in the atmosphere, deforestation and reforestation rates are also retained as decision variables (Andrés-Domenech et al. [3]). Control variables are often lower- and/or upper-bounded to account for some hard technological and economic constraints. For instance, one should not decrease emissions beyond a certain level to avoid massive short-term economic losses or because it is impossible to take too many cars off the road overnight.

It is not surprising that environmental viability constraints take the form of an upper bound on GHG concentrations in the atmosphere (Bernardo and Saint-Pierre [26], Aubin et al. [12] and Andrés-Domenech et al. [3]) or an upper bound on the global mean temperature (Bernardo and Saint-Pierre [26]). Popular economic viability constraints are either limits imposed on the cost that can be borne when changing emissions levels, which can be operationalized by an upper bound on the velocity of the cumulative emissions, or they can constrain the minimum revenues from industrial activities responsible for GHG emissions to be no lower than a given vital threshold (see, e.g., Bernardo and Saint-Pierre [26], Andrés-Domenech et al. [3]).

Finally, we observe that, with few exceptions (see, e.g., Aubin et al. [15]), not much has been done to incorporate uncertainty in VT climate models.

1.2.5 Other applications

In this miscellaneous category of applications of VT, we have contributions dealing with water resources (Martin [93], Rougé et al. [112] and Alais et al. [1]) and ecotourism-based systems (Wei et al. [129]).

Martin [93] considered a lake eutrophication deterministic model, where the quantity of phosphorus in water (state variable) results from some human activities. Rougé et al. [112] extend the model to account for uncertainty, which is related to the capacity of soil storage. Alais et al. [1] considered a hydroelectric dam management problem under uncertainty. In addition to the satisfaction of economic profitability constraint, a tourism-based constraint, which depends on the water storage level during the tourism season, was introduced.

Finally, Wei et al. [129] proposed a VT model aiming at finding an acceptable balance between the protection of the environment and the economic benefits from tourism activities.

1.3 Concluding remarks

We surveyed in this paper the applications of viability theory to the sustainable exploitation of renewable resources, i.e., population biology and ecosystems (fisheries and other species), climate change, farming, forests and other resources. We wish to conclude by pointing out some issues and topics that deserve some attention from researchers.

Gaps and promising developments: One conclusion of our survey is that VT is a useful methodological framework to study sustainability of renewable resources and that many gaps still need to be filled. In particular, forest, soil, and climate-change VT models are still at their infancy, and there is clearly a need to expand their scope and to test them empirically. To give a hint, VT applications to sustainable management of forests have often ignored number of important aspects such as biodiversity and recreational services that forests offer to society. Correcting this specification bias (to use the language of econometrics), would most likely lead to different exploitation policies. Similarly, the first attempts to apply VT to soil management and agriculture have made some simplifying assumptions, which is typical in early stages of any research program, such as considering only one parcel at a time or assuming deterministic crops yields. Multi-parcel models would allow farmers to assess the value of a diversified agriculture and also to hedge some risks. One expects different qualitative (and of course quantitative) insights from multi-parcel models than from one-parcel models. Even in the highly researched area of fisheries where a lot has already been achieved, we still need to improve our models to better account for the impact of climate uncertainties on fisheries and also of multi-species interactions. Finally, we believe that VT can be part of a research program dealing with environment and climate change. Indeed, VT offers the tools to adequately formulate what is tolerable or sustainable and the impact of violating some constraints. Further, some clearly relevant interactions between systems such as forests and climate change, soil preservation and protection of some animal species, etc., have not yet attracted the attention they deserve. Finally, we mention the challenging need to account for spatial issues that pop up each time the control exerted in a region affects the viability of the species (birds, fish, etc.) in another region. The few available contributions, see, e.g., Mouysset et al. [103], Thébaud et al. [123] and Jerry and Cartigny [83] are a good place to look for a start.

Computation (algorithms): A series of algorithms for solving a viability problem, e.g., determining its viability kernel, are available; see, e.g., Saint-Pierre [117], Bonneuil [34], Deffuant et al. [60], Aubin et al. [14], Krawczyk and Pharo [86] and Maidens et al. [92]. Except in some special cases, all these algorithms are subject to the curse of dimensionality. (The size of the state space matters here, rarely the number of control variables is an issue.) Facing this problem, the strategy in the applied literature was to limit the number of state variables to less than three or four (of course some applications included more than that). Developing efficient algorithms using machine learning techniques, see, e.g., Deffuant et al. [60], Chapel et al. [48], and approximate dynamic programming, see, e.g., Bertsekas [29], Powell [108], will allow dealing with practical problems in higher dimensions and improving computational time.

Computation (users): One reason why some methods (think of statistical methods and linear programming) are more used than others is clearly the availability of (friendly users) software. An economist or an ecologist looking for a viable solution to his/her system will be very much reluctant to jump in the area unless he/she could easily have access, if not to a fully canned software, to at least some programs written in, e.g., Matlab, Python or in other widely available computational environment. This is to say that popularity of viability theory is bounded to increase with the availability of computational tools. In some sense, we launch a call to the community to share its resources in order to converge in the long term to developing a platform for solving viability problems that could be accessible to researchers (and eventually) practitioners in this area.

Viability and strategic interactions: We mentioned before that viability theory offers a framework that accommodates for the presence of more than one stakeholder and more than one objective. This works quite well as long as these stakeholders can "coordinate" in some sense when drawing the list of viability constraints and they are not playing strategically. In many situations, the resource considered is of open access, that is, more than one agent can exploit the stock of resources, e.g., exploitation of high-sea fisheries and the environment. Here, coordination is much harder to achieve because the players are seeking their individual interests and competing. Some attempts have been made to account for both viability and strategic issues. For instance, Doyen and Péreau [63] proposed a model combining coalitional games and viability approach to study a renewable resource harvest system involving multiple agents. A certain form of commitment from the coalition's members is supposed since the aim of the latter is to sustain the global rent rather than optimize it and the possibility of a negative individual rent within the coalition is not discarded. A similar work has been done in Hardy et al. [80] in the context of small scale fisheries management. Another example is Péreau et al. [106], where a form of strategic choice is considered in a transferable quota system, in the presence of an authority deciding on the initial allocation of harvesting quotas. More generally, the question is whether viability theory can help in addressing problems involving strategic interactions. If a regulator can impose that the dynamical system describing the evolution of the resource remains viable, then the answer is yes. The resulting noncooperative game is then played à la Rosen (1965), that is, the players are subject to coupling constraints and a generalized (or normalized) Nash equilibrium is sought. For an introduction to coupling constraint noncooperative games, see, e.g., Haurie et al. [81]. Further, it is worth mentioning that some mathematical analysis and numerical methods links have been established between zero-sum two-player differential games and viability theory, see, e.g., Cardaliaguet et al. [46, 47], Cardaliaguet and Plaskacz [45].

Appendix

1.A Summary table

In the following table we present a brief summary of the papers reviewed in this survey. The papers in the table are first sorted according to the domain of application and next ordered by publication year.

The first column gives the reference, and the second column provides information about the following elements:

1. The studied model or problem and its characteristics.

2. The viability theory tools used, with some details on their use.

3. Information about the numerical applications, if any.

The third and fourth columns give the state and control variables, while the fifth column displays the list of viability constraints. Each one of these constraints is tagged with a specific sign according to its type: “🍃” for environmental constraints, “$” for economic constraints and “# ” for social ones. Combinations of these symbols are used to mark constraints of more than one type (for example “$# ” designates socio-economic constraints). It is important to mention that there are some physical constraints (like non negativity of a physical stock, carrying capacities, etc.) that are used in some studies but are not listed in this table because they are obvious constraints and are explicitly or implicitly considered in all concerned studies.

The information about the model's horizon (finite, infinite) and time (discrete, continuous) appears in the sixth column. The last column indicates if uncertainty has been considered in the model, and eventually the approach used (robust or stochastic).

To simplify the table, the abbreviations listed below are used:

VT: Viability theory.

VK: Viability kernel.

SVK: Stochastic viability kernel.

RVK: Robust viability kernel.

VD: Viability domain.

VP: Viability probability.

CB: Capture basin.

IK: Invariance kernel.

TCF: Time of crisis function.

IF: Inertia function.

RA: Robust approach.

SA: Stochastic approach.

RS: Regulated system.

SS: Stabilized system.

NA: Numerical application.

UB: Upper bound.

LB: Lower bound.

ULB: Upper and Lower bounds.

C/D: Continuous / Discrete (time).

F/ ∞: Finite / Infinite (time).

Ref	Details	State variables	Control variables	Viability constraints	T	U
			Climate change			
[12] 2005	1)GHG accumulation model. 2)Uses VK and associated feedbacks to study the sustainability of the system and choose the sustainable management strategies with minimum transition cost. 3) NA with chosen parameters' values.	-GHG concentration.	-Short-term pollutant emissions.	UB on GHG concentration level in the atmosphere.	C ∞	No

[128] 2008	1)Atmospheric CO_2 concentration model parametrization problem. 2)Uses VK and associated feedbacks to estimate the values of the unknown parameters of the model (biogenic enhancement factor of weathering). 3)NA on empirical data.	-Atmospheric CO_2 concentration.	-None (RS). The regulon is the biogenic enhancement factor of weathering.	-UB on error degree (Coherence of the state variable with observed data) -UB on the velocities of the regulon (No big changes in the parameters' value in a small time interval).	D ∞	No
[9] 2010	1)Economic-Climatic coupled system. 2)Uses the IF to measure the transition cost of the industrial activity necessary to maintain the GHG concentration at acceptable level. 3)No NA.	-GHG Concentration. -Short-term pollution rate.	-None (RS).The regulon is the intercity of the industrial activities.	UB on GHG concentration level.	C ○●	No

[3] 2011	1)GHG Accumulation-Deforestation coupled system. 2)Uses VK to study the sustainability conditions of the system in different situations and assess the sustainability of the world's forests to limit CO_2 concentration. 3)NA on estimated parameters' values.	-CO_2 emissions. -CO_2 concentration in the atmosphere. -Forest surface.	-Deforestation rate. -Reforestation rate. -Speed of CO_2 emission adjustment. -Monetary transfers.	UB on CO_2 stock. $ULB on emission rate. $LB on revenue. $LB on wood quantity production.	C ∞	No

[15] 2012	1)Emission rates allocation problem. 2) Uses VK and the associated feed-backs to determine the sustainable initial emission rates and their corresponding dynamical allocation knowing the maximum emission growth rates of polluters. 3) NA on chosen parameters' values.	-Global emissions. -Individual emissions (by polluter).	-Emission rights allocations.	UB on global emissions level. UB on each polluter's allowed emission level. $LB on each polluter's emission right. $UB on each polluter's emission rate growth.	C F	RA

[26] 2015	1) Climate change problem. 2) Proposes VT based measurement tools and climate indicators and uses VK and associated feedbacks to study the sustainability of the system and the viable management strategies. 3) NA on estimated parameters' values.	-CO_2 concentration. -Global mean temperature. -Cumulative CO_2 emission.	-Anthropogenic CO_2 emission.	UB on CO_2 concentration. UB on global temperature. UB on cumulative emissions. $ULB on state variables' velocities. $ULB on the emission level.	C ∞	No
Forest protection						
[44] 2011	1) Single agent savanna management problem. 2) Uses VK to study the viability and resilience of the model. 3) NA on estimates parameters' values.	-Density of trees in the savanna.	-Grazing pressure (More grazing = less grass = more trees).	ULB on the density of trees. $ULB on grazing pressure. $UB on grazing pressure variation.	C ∞	No

[25] 2013	1) Forest urbanisation management problem. 2) Uses VK and associated feedbacks to study the system's sustainability conditions and best management strategies. Also shows the importance of monetary transfers to achieve viability. 3) NA on the rain forest in the corridor of Fianarantsoa (Madagascar).	-Size of built area. -Population size. -Total wealth of the population.	-Urbanizing effort. -External workers proportion. -Monetary transfers. -Demographic growth rate.	LB on forest size (UB on built area). $LB on capital/capita value. $Increasing individual wealth over time. $ULB on the control variables. # LB on population size.	C ∞	No

[4] 2014	1) Single agent single species forest management system. 2) Uses VK to show the unsustainability of the current state and practices in the Androy forest and to derive some possible ways to recover sustainability. 3) NA on the Androy forest in Madagascar.	-Size of forest area. -Population size. -Physical capital of zebu.	-Forestation. -Births rate. -Per capital consumption. -Deforestation rate for agriculture. -Deforestation rate for wood. -Deforestation rate for cattle breeding. -Monetary transfers.	$Non decreasing per capita level of consumption. $Non decreasing absolute and relative levels of capital. $# Covering population's basic need of wood at any time.	D ∞	No

[99] 2015	1) Single agent single species forest management system. 2) Discusses the efficiency of different forest management strategies (bounds' values in the constraints) using VK and the corresponding values of the flexibility indicator. 3) NA on the univen-aged silver fir forest in "Quatre montagnes" (France).	-Number of trees in both strata of the forest. -Volume of deadwood. -Timber stock.	-Intensity and frequency of harvesting wood in upper stratum and tree recruitment in lower stratum. -Deadwood retention volume.	ULB on trees quantity in each stratum. ULB on per hectare deadwood quantity. $ULB on timber stock level.	C ∞	No

Fisheries

[22] 2000	1) Single agent single species fishery subject to resource and market seasonal oscillation. 2) Uses VK to study the role of storage regulation in maintaining the system's viability. 3) NA on the French Guyana shrimp fishery.	-Storage volume of the harvested resource.	-Export flow (Fishing flow).	$Positive profit. $Catches bounded by demand. $LB on storage level. $Limited storage capacity.	C ∞	No
[24] 2001	1) Single species single agent bio-economic marine system. 2) Uses VK and associated feedbacks to identify overexploitation situations preventing regulation controls. Uses also TCF to study the reversibility of overexploitation situations. 3) No NA.	-Biomass stock level. -Fishing effort.	-Time variation of fishing efforts (Velocity of the fishing effort).	LB on biomass stock level. $ULB on fishing effort level. $Positive global and net benefit at any time.	C ∞	No

[61] 2003	1) Single agent single renewable resource protected area. 2) Uses IK to study the efficiency of marine reserves in protecting resources and its sensitivity to uncertainty. 3) NA on chosen parameter values.	-Biomass stock level.	-Harvest rate.	LB on biomass stock level.	D ∞	RA
[69] 2003	1) Co-managed single species fishery. 2) Uses VT modelling approach combined to a qualitative approach to study the sustainability of the system. 3) No NA.	-Biomass stock. -Capital accumulated in the fishery.	-Catch recommendation.	LB on biomass stock level. $# LB on total harvest (for acceptable employment level, food safety and economic profitability).	C ∞	No

[104] 2004	1) Single agent multi-species marine ecosystem. 2) Uses VK calculated on different scenarios (with or without exploitation) to study the sustainability of the system. 3) NA on the Benguela ecosystem.	-Biomass of each species.	-global mortality and interspecies consumption as regulons. -Catches as control variable (scenarios with exploitation).	ULB on each species biomass level.	D ∞●	RA
[53] 2005	-Explains how viability theory can be applied to study the sustainability of ecosystem based fisheries.	X	X	X	X	X
[70] 2006	1) Co-managed single species fishery (one decision maker). 2) Uses VD to study the sustainability of the system and the efficiency of three control strategies. 3) No NA.	-Biomass stock level	-Catches recommendation.	LB on biomass stock level. $# LB on total harvest (Food safety).	C ∞	No

[56] 2007	1) Single agent single species age structured fishery. 2) Uses VD to study the efficiency of the spawning-stock biomass and fishing mortality as indicators of sustainability in the precautionary approach. 3) NA on the northern hake and Bay of Biscay anchovy.	-Vector of abundance of the stock at each age.	-Fishing mortality multiplier.	LB on spawning-stock biomass. UB on fishing mortality over predetermined age range.	D ∞	No
[66] 2007	1) Single agent exploited food-web with marine reserves. 2) Uses VK to study the influence of protected areas upon environmental and economic sustainability of the system. 3) NA on the Aboré coral reef reserve in New Caledonia.	-Biomass stock for each species. -State of the habitat.	-Harvesting effort for each species.	Preservation of all the spaces. LB on a biodiversity indicator value. $LB on utility from catches.	D F	SA

[101] 2007	1) Single agent single species fishery. 2) Uses VK to identify the viable states of the system and the TCF to study the recovery possibilities of the non-viable ones. 3) NA on the bay of Biscay Nephrops fishery.	-Biomass stock level.	-Fleet size and fishing effort.	LB on biomass stock. $LB on per vessel benefice. # LB on the fleet size. # UB on size changing speed (velocity of fleet size)	D ∞	No
[48] 2008	1) Single agent multi-species ecosystem fishery. 2) Uses VK to study the effect of fishing some species of fish and determines sustainable yield policies. 3) NA on the southern Benguela ecosystem.	-Biomass stock level of each species.	-Yields on harvested fish (Pelagic fish and Demersal fish).	ULB on biomass stock levels. $LB on the yield.	C ∞	RA

[55] 2009	1) Single agent multi-species age structured fishery with one exploited species and one non exploited one. 2) Uses VK and associated feedbacks to determine the fishing strategies maximising the viability probability. 3) NA on the nephrops-hake fisheries on the Bay of Biscay.	-Biomass stock of each species (Nephrps and hakes) at different ages.	-Harvesting effort for Nephrops fish.	LB on abundance level of mature hakes fish. $Profitability of the fishery.	D F	SA
[95] 2009	1) Single agent multi-species exploited ecosystem with one exploited species (Shrimp) and one non exploited species (Frigate bird feeding on fishery discards). 2) Uses VK to study the sustainability of the system. 3) NA on the French Guiana shrimp fishery.	-Biomass of the shrimp stock.	-Fishing effort.	LB on fishing discards level (To feed and conserve the Frigate bird population). $LB on catches level per unit of effort.	D ∞	No

[21] 2009	1)Multi-agent multi-species fishery. 2) Uses VT modelling combined with simulation to compare different fishing scenarios. 3) NA on chosen parameter values.	-Biomass stock of the different species.	-Fishing effort.	Preservation of all species. $Profitability of the fishing activity for all the fishers.	D ∞	No
[91] 2009	1) Single agent multi-species fisheries. 2) Uses VT modelling combined with simulation to study the sustainability of the system and determine viable harvesting strategies. 3) NA on the Northern Gulf of California ecosystem.	-Biomass levels of the different species.	-Fishing effort.	UB on ecosystem deterioration level. LB on biomass recovery level for endangered species. $Profitability of the fisheries. # Maintain fishermen jobs. # Respect the government regulation plans of fisheries.	C ∞	No

[97] 2010	1) Single agent single species fishery. (Fleet composed of multiple vessels: single decision maker). 2) Uses VK to study the sustainability of the system and TCF to determine acceptable recovery paths from non-viable states. 3) NA on the Bay of Biscay nephrops fishery.	-Biomass stock of the exploited resource. -Fleet size.	-Fishing effort. -Changes in the fleet size.	LB on biomass level. $LB on profit per vessel. #LB on the fleet size.	D ∞	No
[83] 2010	1) Two models for single species single agent fishery one with and the other without protected area. 2) Uses the VK to study the sustainability of the systems in order to investigate the benefits of protected areas. 3) No NA.	-Stock level of the resource in each area considered in the model.	-Harvesting effort in the non protected areas.	LB on stock level. $LB on fishermen income.	C ○●	No

[58] 2011	1) Single agent single species age structured monotone harvest fishery. 2) Uses VK to study the sustainability of the system. 3) NA on two Chilean fisheries (Sea bass and Alfonsino).	-Abundance of population at different ages.	-Fishing effort.	LB on spawning stock biomass of the resource. $LB on yield from fishing.	D ∞	No
[84] 2012	1) Two single agent single species commercial fishing models (with and the other without a price state variable). 2) Uses VK to study the sustainability of the systems and determine the best exploitation strategies (combinations of resource stock, price, capital and investment). 3) No NA.	-Density of fish population. -Capital investment in fishing activity. -Price	-Investment rate.	LB on resource stock level. $LB on catches level. $LB on capital investment	C ∞	No

[65] 2012	1) Multi-agent multi-species age structured fishery. 2) Uses the SVK to determine the exploitation strategies maximizing the viability probability of the fishery. 3) NA on the nephrops and hake fisheries in the bay of Biscay (France).	-Abundance of the species at different ages.	-Fishing mortality associated with the fleets (Target species for each fleet).	LB on abundance level of each species at each age. $LB on each fleet income.	D F	SA
[118] 2012	1) Two-agent single species fishery. 2) Uses VK to study the sustainability of the system. 3) No NA.	-Biomass of the exploited resource.	-variation rate of fishing effort of the two fleets. (investments)	LB on biomass stock level. $LB on each fleet income. $# ULB on the fishing efforts.	C ∞	No

[106] 2012	1) Multi-agent single species transferable quota based management fishery. 2) Uses VK to study the sustainability of the system with asymmetric agents. 3) NA on the Bay of Biscay nephrops fishery.	-Biomass level of the resource.	-Total allowable catches (the sum of all the quotas attributed to the agents).	LB on biomass level. $Profitability of the fishery. # LB on the number of active fishers.	D ∞	No
[59] 2012	1) Single agent multi-species ecosystem based fishery. 2) Uses VK to study the sustainability of the system. 3) NA on the Hake-Anchovy couple in the Peruvian Upwelling ecosystem.	-Biomass level of each species.	-Harvesting effort for each species.	LB on each species biomass. $LB on catch levels (yield).	D ∞●	No

[50] 2013	1) Multi-fleet multispecies ecosystem based management fishery. 2) Uses VT modelling approach combined with simulation to evaluate the sustainability of a set of management strategies. 3) NA on the coastal fishery of French Guiana.	-Biomass of each species.	-Fishing effort of each fleet.	LB on the Species richness indicator (SR) (biodiversity level). LB on the trophic marine index indicator (MTI) (total biomass level). LB on the Simpson diversity index(SI). $LB on harvest (food security). $Positive profit for each fleet.	D F	No
[76] 2013	1) Multi-agent multi-species age structured fishery. 2) Uses VK to compare the efficiency of different management strategies in sustainability. 3) NA on the demersal fishery in the Bay of Biscay.	-Abundance of the species at different ages.	-Fishing efforts multipliers (allocation of the vessels).	LB on species abundance levels. $Positive profit for each vessel.	D F	SA

[52] 2013	1) Single species age structured regulated transboundary fishery (2 countries with different technology). 2) Uses VK to study the sustainability of the system and assess the viable management strategies. 3) NA on the France-Spain Bay of Biscay anchovy fishery.	-Biomass stock of fish at each age.	-Annual total catches for each age class of fish and the fishing quota allocation.	$LB on each country's profit. # Fairness between the countries in the quota allocation.	D ∞	No
[119] 2013	1) Single agent single species fishery. 2) Uses VK to study the viability of the system and assess the sustainable management options. 3) No NA.	-Biomass stock. -Available catching effort.	-Investments rate in catching efforts.	LB on biomass stock level. # ULB on catching effort levels.	C ∞	No

[87] 2013	1) Multi-agents multispecies by-catch fishery. 2) Uses VK to study the sustainability of the system. 3) NA on chosen data.	-Biomass stock. -Catching effort	-The catching effort variation.	LB on biomass level. $Positive profits for the fishery's fleets. # ULB on catching efforts.	C ∞	No
[79] 2013	1) Multi-agent multispecies small scale fishery. 2) Uses VT modelling combined with simulation to identify the system's sustainability conditions. 3) NA on the Solomon islands' small scale fisheries.	-The biomass stock of each species.	-Vector of fishing efforts allocated to each fleet.	LB on "Species richness" and "Simpson index" ecological indicators (biodiversity). $# Food and cash security.	D ∞●	No

[80] 2013	1) Multi-agent single species artisanal fishery. 2) Uses TCF to study the resilience of the system in case of cooperation or non-cooperation between the agents. 3) NA on the Solomon Islands' small scale fisheries.	-Biomass stock of the resource. -The number of fishermen.	-Fishing effort allocation among the agents.	$# LB on each agent catches (food security and acceptable cash income for each agent).	D ∞	No
[73] 2014	1) Single species hermaphrodite maturity stage structured population fishery (3 stages: Juvenile, male, female). 2) Uses VK to study the viability domain and sustainable management strategies of the system. 3) No NA.	-Resource density at each maturity stage. -The number of fishermen.	-Fishing effort for each class of the resource.	LB on female density. $LB on fishermen revenue. # LB on fishing activity at any time.	C ∞	No

[100] 2014	1) Two fleets multispecies and age structures fishery. 2) Uses VP to study, compare and rank some management scenarios. 3) NA on the main western Mediterranean Spanish fisheries.	-Abundance of each species at each age.	-Strategies for fishing mortality reduction.	🍃 LB on spawning stock biomass level for all species. $Positive economic profit for each fleet.	D F	SA
[123] 2014	1)Single agent single species fishery with several fishing regions. 2) Proposes a VT based approach to the evaluation of fisheries management strategies. 3) NA on the Ningaloo marine park of western Australia.	-Biomass level of the exploited resource at each region.	-Exploitation strategies.	🍃 LB on regional and global spawning biomass level. $# LB on catches level.	D F	SA

[98] 2014	1) Single agent single species age structured fishery. 2) Uses VP to study and compare the effort based and quota based fishing strategies. 3) NA on the Chilean Jack-mackerel fishery.	-Biomass stock of the resource at each age class and spawning stock biomass level.	-Fishing effort.	LB on the spawning stock biomass indicator (SSB). $# LB on fishery yield. $# LB on fishing activity level.	D F	SA
[111] 2015	1) Single agent two species exploited ecosystem. 2) Uses RVK to study the effect of different types of uncertainty on the sustainability of the system. 3) NA on the anchovy-hake couple in the Peruvian upwelling ecosystem.	-Biomass of species.	-Harvesting effort for each species.	LB on each species biomass. $LB on catch level for both species.	D F	RA

[77] 2015	1) Single agent multispecies sex-structured fishery in an ecosystem composed of 4 species (3 targeted and one non fished species). 2) Uses SVK to compare different management strategies and harvesting efforts allocations. 3) NA on the Australian northern prawn fishery.	-Biomass stock of each species.	-Harvesting effort for each targeted species. -Fishing management strategy.	🍃 LB on spawning stock for all species (targeted or not). $LB on annual net benefit from fishing.	D F	SA
[51] 2015	1) Multi-agent multispecies small scale fishery. 2) Uses VT modelling framework combined with simulation to study the sustainability of the system under three fishing scenarios. 3) NA on the coastal fishery of French Guiana.	-Biomass level of each species.	-Fishing effort of the fleets.	🍃 LB on the Species richness indicator (SR) (biodiversity). 🍃 LB on the trophic marine index indicator (MTI). $LB on harvest level (food security). $Positive profit for each fleet.	D F	SA

[40] 2016	1) a-Multispecies population growth model. b-Single agent multi-species by catch fishery model. 2) Proposes a VK algorithm and applies it the models. 3)NA on chosen parameters' values.	a-Size of each species population and their evolution rate. b-Biomass of each species.	a-None (RS) The regulon is the evolution rate velocity. b-Fishing effort of the targeted species.	model a: ULB on the populations sizes. model b: LB on species biomass. $ULB on fishing effort. $LB on fishery profit. $# UB on fishing effort variation.	D F	No
Ecosystems and population biology						
[88] 1991	1) Food web in an ecosystem composed of n species. 2) Proposes a VT model of population biology studies its sustainability using the G-projection method. 3) No NA.	-Biomass of each species.	-None (RS). The regulons are the choice of resource used by each species to feed.	Preservation of all species.	C F	No

[31] 1993	1) Boserupian system for population growth. 2) Uses VK to study the properties of the system subject to the possible technological changes. (For different bounds on the technological changes). 3) No NA.	-Population size. -Level of technological advance.	- None (RS). The regulon is the level of technological change (Velocity of technological evolution).	-ULB on technological changes.	C ∞	No
[89] 1995	1) Prey predator ecosystem composed of two areas where live one predator and two pray species. 2) shows how the viability theory is useful to model the interactions between populations competing for space in presence of uncertainties in an ecosystem. 3) No NA.	-Abundance of each type of population.	- None (RS) The regulons are the fractions of predator population in each area of the system and strategies of the populations.	Space limitation.	C F	RA

[35] 1997	1) Prey predator system with one predator and one prey. 2) Uses VK and associated feedbacks to study the sustainability of the system according to the preservation objectives (Preservation of one or both species). 3) NA on chosen data values.	-The density of the prey and predator species.	-None (RS) The regulons are the species' survival strategies.	LB on density of one or other species (accordingly to the considered situation).	C ∞	No
[90] 1998	1) Single species extinction problem. 2) Uses VT modelling to study the extinction possibilities of the population and estimate its extinction time. 3) NA on the grizzly-bear female population in the Yellowstone National Park.	-Abundance of the population.	None (RS). The regulon is the growth rate of the population (The uncertainty).	Reaching the extinction threshold at the final time. (endangered species: those which will reach their extinction threshold in finite time.)	C F	RA

[32] 1999	1) Explains how some game theory models of population growth and fishery can be reinterpreted through application of viability theory. Model a) Bassori population-cattle interaction. Model b) Norwegian fishery: Multiagent fishery. 2)Uses VT modelling and VK to study the sustainability of the systems. 3) NA on predefined parameter values.	Model a: -Households' sizes in the Bassouri nomad organisation. -Households' herd sizes. Model b: -Agents' capital. -Agents' possible catches. -Probability of bad catching level.	Model a: -Sedentarization rate. -Predation rate. Model b: -Level of risk taking (probability of acting individually and not following the group).	Model a: 🍃 Food security of the population. 🍃 ULB on the predation levels. # ULB on the sedentarisation levels. Model b: $UB on each agent's ruin probability.	C ∞	No

[33] 2003	1) Prey predator ecosystem with one predator and one prey species. 2) Studies the effect of additive and multiplicative viability multipliers on the viability of the system. 3) No NA.	-Density of the predator and prey species.	-None (SS). Looks for states ensuring sustainability through natural equilibrium of the system.	Non-extinction of both predator and prey species.	C ∞	No
[37] 2005	1) Multispecies ecosystem composed of a 3-level food chain (prey, predator and super predator species). 2) Uses VK to study the sustainability conditions of the system. 3) NA with chosen parameters' values.	-Density of each species.	-None (RS) the regulons are the predation and competition strategies of the different species.	LB on each species density (Non extinction of the species).	C ∞	No

[10] 2007	1) Malthus population growth model. 2) Explains how to use VT and its tools to study renewable resources management problems in general with illustrations on population growth model. 3) No NA.	-Population size. Population's growth rate.	-None (RS) the regulon is the growth rate variation.	ULB on population size. -ULB on growth rate variation.	C ∞	No
[57] 2007	1) Single agent single species age structured population growth system. 2) Exploits the monotonicity properties to estimate the VK and study the sustainability of the system. 3) No NA.	-Abundance of the population at each age.	-Harvesting level.	LB on the population's abundance level at each age. $LB on harvest level.	D ∞	No

[11] 2007	1) Verhulst model for population dynamics. 2) Illustrates the main concepts of VT by revisiting the Verhulst type models for population dynamics. 3) No NA.	-Stock level of the resource.	-None(RS). The regulon is the growth rate.	ULB on the resource stock.	C	No
[23] 2008	1) Ecosystem with multiple species competing for one resource. 2) Uses VP to study the sustainability of the system in case of non-exploitation (No harvesting and without the economic constraint) and in the case of exploitation. 3) NA on chosen parameter's values.	-Abundance of each species. -Resource level.	-Harvesting intensity for each species.	LB on the Shannon biodiversity index. $LB on the utility derived from the exploitation activity.	D F	SA

[113] 2014	1) Single species population growth model. 2) Uses SVK to study the sustainability and resilience of the system. 3) NA on chosen parameters' values.	-Population density. -Growth coefficient.	-None(RS). The regulon is the changes in growth coefficient.	ULB on population density level.	D F	SA
[49] 2015	1) Single species age structured population. 2) Explains how VK and CB can be useful to study the sustainability of the system and management strategies. 3) No NA.	-Abundance of the population at each age.	-Harvesting strategies (harvest for each age class).	LB on the total population abundance level.	D F	No

Renewable resources in general

[110] 2006	1) Single agent single age-structured renewable resource with one mature harvestable age. 2) Uses VK and associated feedbacks to study the sustainability of the system and the harvesting strategies. 3) NA on chosen parameters' values.	-Available resource quantities at each age.	-Harvesting quantity.	$# LB on harvest at each time.	D ∞	No
[63] 2012	1) Multi-agent single renewable resource harvest system in presence of cooperation between the agent. 2) Uses VK to analyse the conditions under which cooperation promotes the sustainability of the system. 3) No NA.	-Resource stock level.	-Harvesting effort for agents inside and outside the coalition.	LB on the resource stock. $Positive total rent for the coalition.	D F	No

[16] 2013	1) Single agent multispecies renewable harvest system. 2) Uses RVK to study the sustainability of the system. 3) NA with chosen data values.	-Stock resource of the species. -The global harvest.	-None (RS) The regulon is the share of each species in the global harvest.	LB on the stock resource of the species. $Satisfy the total and by species harvest demand.	C F	RA

Farming and agro-ecological systems

[124] 2004	1) Single agent mixed herd composed of two species (llama and Sheep). 2) Uses RVK to study the sustainability of the system. 3) NA on the Bolivian highlands llama-sheep mixed herd.	-Wealth of the owners of the herds.	-Breeds management decisions: (rate of female's offtake and herd composition)	$LB on income and wealth level at any time.	C ∞	RA

[125] 2007	1) Single agent grassland ecosystem which is the breeding habitat of 3 wader species and feeding resource 2 species suckling cattle (cow/calves). 2) Uses VK to study the sustainability of the system and the efficiency of different grazing strategies. 3) NA on measured and estimated data from European grasslands.	-Grass mass (live grass and standing dead grass).	-Grazing intencity.	Suitable sward state for the reproduction of the birds during their breeding period. $# Satisfy the cattle feeding requirement.	D F	No
[17] 2009	1) Single agent livestock grazing management system in semi-arid rangelands. 2) Uses VP to study the sustainability of the system and the management strategies. 3) NA on chosen parameters' values.	-Grass biomass (reserve and green biomass). -Herd size.	-grazing management strategy.	LB on grass biomass level. $LB on income.	C F	SA

[114] 2010	1) Single agent grassland ecosystem with 2 wader species and cattle. 2)Uses VK to study the sustainability of the system and grazing practices as well as the effect of grazing on the conservation of wader species. 3) NA on the Ouest-du-Lay marsh (France).	-Biomass of grass (Alive and dead grass).	-Grazing intensity (cattle density and grazing rhythm).	Suitable sward state for the reproduction of the birds during their breeding period. Eggs survival (UB on cattle density to limit the trampling impact on eggs). $# Satisfaction of cattle feeding requirement.	D F	RA
[94] 2011	1) Rangeland management model. 2) Uses VK, CB and associated feedbacks to study the sustainability and resilience of the system. 3) NA using parameters' values from the literature.	-Grass biomass	-grazing pressure.	LB on grass biomass. $LB on grazing pressure.	D ∞	No

[115] 2012	1) Single agent grassland ecosystem. 2) Uses VK and associated feedbacks to study the sustainability of the system under different ecological constraints and determine the sustainable management strategies. 3) Application to the conservation of lapwing birds in the wet grasslands in France.	-Grass biomass (Live and standing grass). -Bird population size.	-Timing and intensity of grazing.	Conservation of the bird population (several constraints studied). $Satisfaction of cattle feeding requirements. $LB on productivity level (LB on grazing time)	D F	No

[103] 2013	1) Multispecies agro-ecological ecosystem. 2) Uses VP to identify sustainable management scenarios. 3) NA on bird population in small agricultural regions in metropolitan France.	-Abundance of each bird species at each region.	-Incentives (Subsidies and taxes) to encourage specific crop or grass activities in the different agricultural regions.	LB on 3 biodiversity indicators. $LB on income from the farming activities. $LB on the budget allocated for farming activities.	D F	SA
[116] 2015	1) Single agent grassland agro system. 2) Uses RVK and associated feedbacks to study the sustainability of the system and of management strategies 3) NA on the cool-season grassland of south-central Wisconsin (USA).	-Grass biomass. -Production of the grassland system.	-Stocking rate and grazing sequences.	Satisfy the cattle daily needs of grass. $LB on the system's production level.	D F	RA

[68] 2015	1) Single agent single parcel agro ecological system. 2) Uses CB to study the possibility of restoring the soil quality within the time horizon while maintaining acceptable economic performance. 3) NA on French West Indies.	-Soil quality indicator. -Cash balance.	-Agricultural strategy (planting sequences, agricultural activity and techniques).	Bring back the soil quality indicator to an acceptable level at the end of the exploitation period. (Target) $Positive cash balance at any time.	D F	No
			Others			
[93] 2004	1) Single agent lake eutrophication model. 2) Uses VK and TCF to study the sustainability of the system and find the best management strategies. 3) NA with parameters' values from the literature.	-Phosphorus quantity in water. -Annual phosphorus input from human activity.	-Variation of the annual phosphorus input.	UB on phosphorus quantity in water. UB on the phosphorus total input from human activity. $LB on the total input level (LB on activity level).	C F	No

[129] 2013	1) Single agent socio-ecological tourism based system. 2) Uses VK to identify the sustainable situations, then uses CB calculated for different time horizons to estimate the required time to reach a sustainable state. 3) NA on chosen parameters' values.	-Tourist activity. -Quality of nature. -Capital (infrastructure)	-Investments in tourism. -Advertisement campaigns (to control the effect of competition).	ULB on the nature quality level. $ULB on the tourism activity level. $ULB on the capital value.	C ∞	No
[112] 2013	1) Single agent lake eutrophication model. 2) Uses SVK to study the sustainability and resilience of the system. 3) NA with parameters' values from the literature.	-Quantity of phosphorus in water. -Annual phosphorus input from human activity.	-Variation of the annual phosphorus input.	UB on phosphorus quantity in water. UB on the phosphorus total input. $LB on the total input level (LB on activity level).	C F	SA

[1] 2015	1) Single hydroelectric dam under uncertainty and tourism constraints. 2) Uses VP to study the system's management strategies. 3) NA on data provided by the French electricity provider Electricité France.	-Water storage in the dam. -Dam inflow. -Electricity price	-Dam turbined flow.	LB on the guaranteed gain from the electricity production. $# LB on water storage level during the tourism season.	D F	SA

Features of the retained papers in the survey

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Chapter 2

Viability of Agroecological Systems Under Climatic Uncertainty*

Abstract

To cope with ever-increasing demand and ensure food security, agronomic systems have shifted over time from traditional agriculture, based on the organic fertilization of soils, to intensive and specialized farming that use chemical fertilization. This resulted in increased soil productivity in the short term, but caused serious ecological drawbacks over time (degradation of soil quality, pollution of water and air, loss of biodiversity, erosion, etc.), and even reversed the trend of agricultural productivity. In this paper, we propose a viability theory–based model to study the sustainability of an agricultural system subject to climate uncertainty. Our objective is to determine what farming practices and activity sequences restore soil quality to a desired level while ensuring an acceptable level of productivity in the presence of the risk of

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major climatic disasters. The model is applied to Guadeloupe, an island in the West French Indies. We found that the results are highly sensitive to the direct effect of hurricanes on the soil's quality, which, in turn, strongly affects the impact of the other parameters and that the export oriented sector is more vulnerable and less resilient to climatic uncertainties than the sector aimed at the local market.

Key Words: agriculture; viability theory; agriculture; farming; climatic uncertainty.

Introduction

It is self-evident to state that agricultural land is essential to life and a valuable resource for most, if not all, countries. Still, its importance seems to only be noticed when productivity declines and our food security is at stake. Soil has been poorly protected and overexploited for decades, which has resulted in its deterioration worldwide (see Figure 2.0.1).

Modern agriculture and farming are important human-induced factors in this degradation. Whereas traditional practices are based on organic fertilization and diversified crop production, modern agriculture relies on intensive single-crop production and chemical fertilizers, which together progressively degrade soil quality and reduce its productivity by changing its physical, chemical, and biological composition. Indeed, fertilization and low-quality irrigation water alters the soil's chemical makeup. Further, plowing, tillage, removal of vegetative cover, and overgrazing make soil more vulnerable to wind and water erosion, and intensive and specialized cultivation exhausts some minerals and water from the soil and damages its microfauna (Blanco and Lal [6]).

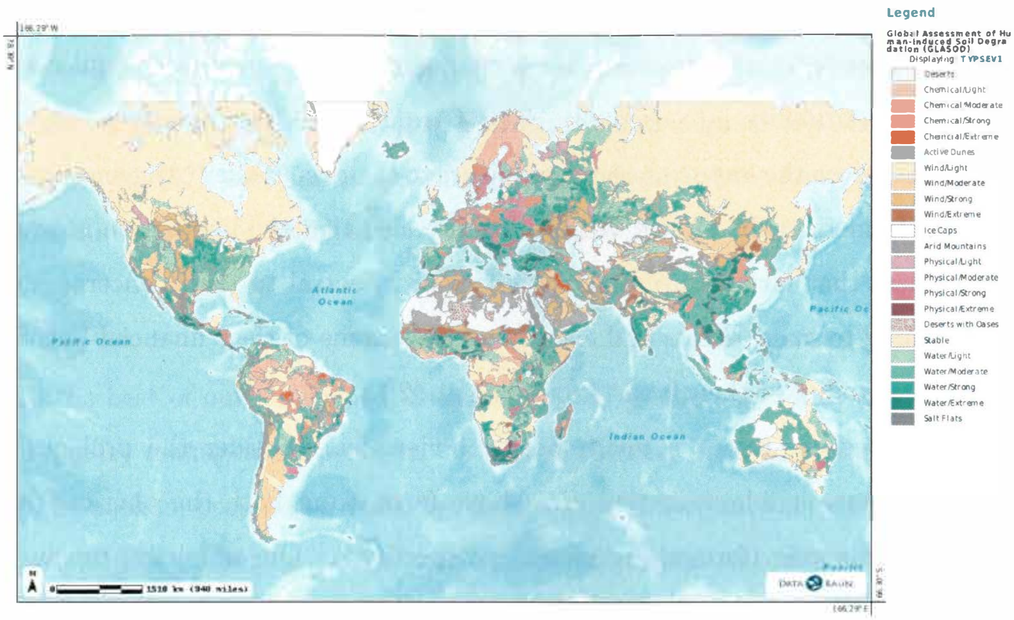

Figure 2.0.1: Global assessment of human-induced soil degradation. Source: `https://databasin.org/datasets/7254137cabb042298cae0b769cba589f`.

The shift from traditional practices to highly specialized agricultural systems was, in part, a result of population growth, which increased demand. This shift increased soil productivity in the short term, but generated serious ecological drawbacks in the long term (degradation of soil quality, pollution of water and air, loss of biodiversity, erosion, etc.) and even reversed the trend of agricultural productivity. Modern agricultural systems have since fallen in a vicious cycle, where increasing chemical fertilizers are used to compensate for the loss of productivity, causing more damage to the soil and the environment (pollution of water and air) (Trautmann et al. [22], Conway [8]).

Implementing eco-responsible agricultural practices, based on crop rotations and mixed crop-livestock associations that are less harmful for the soil, is one way to achieve resource (soil) sustainability (Altieri [1], Kremen et al. [11]). At a macro level, this transition is a long-term process that must take populations' food needs and farming profitability into account. Such a transition is particularly needed in island regions, given the importance of agriculture to their economies (Angeon et al. [3]). A survey of farmers in the French West Indies revealed that the population is aware of the problem that farmers are making soil quality central to their concerns, and they are willing to make efforts and even to sacrifice some of their financial benefits to restore the quality of their land (Angeon et al. [2]).

In this paper, we adopt a micro point of view and consider the problem of the long-term sustainable operation of a single farm, from both the physical (soil quality) and economic (farmer's revenues) perspectives. Our empirical terrain is Guadeloupe, an island that is part of the French West Indies. In this region of the world, the restoration of soil quality depends (as everywhere else) on the soil's inherent properties, the type of agricultural practice adopted by the farmer and on recurrent climatic events such as hurricanes.

Our research questions are as follows:

1. Given some economic and soil quality constraints, and taking into account the possible occurrence of climatic events, what are the viable crop rotations to grow over a predefined planning horizon?

2. How sensitive are these solutions with respect to parameter values?

3. Which farming systems or sectors are the most vulnerable to climatic uncertainties?

Given our problem statement, we adopt viability theory (Aubin [4]) as our methodological framework. There is now a long tradition of applying this theory to address problems that are related to the exploitation of renewable resources, e.g., fisheries and forests (see Oubraham and Zaccour [14] for a literature review). A few have applied viability theory to farming; see, e.g., Tichit et al. [20], Sabatier et al. [15], Sabatier et al. [16], Mouysset et al. [13], Tichit et al. [21], Baumgärtner and Quaas [5], Martin et al. [12], Sabatier et al. [17], and Durand et al. [9]; most of which have focused on herd and grassland management problems.

To the best of our knowledge, Durand et al. [9] is the only study that looked at a soil-quality management problem. When considering a single plot of agricultural land, the authors proposed a viability theory-based deterministic model, and looked for agricultural strategies and crop rotations that would restore soil quality to an acceptable level while preserving the farm's economic profitability. In this paper, we extend their model to a stochastic setting to account for the impact of uncertainty from major climatic events, such as hurricanes, on the system's evolution. During one season of the year, it is almost a certainty that the West Indies region will be affected by climatic events. The uncertainty relates to their force and the damage they will cause. Given this state of affairs, we believe our model adds a level of realism to what was done in Durand et al. [9].

The rest of the paper is organized, as follows: in Section 2.1, we extend the model in Durand et al. [9] by introducing climatic uncertainty. In Section 2.2, we define so-called emergency control, that is, what needs to be done after a climatic event. Section 2.3 is dedicated to the formal definition of all the elements related to climatic events. The solution method is introduced in Section 2.4, and the empirical application is presented in Section 2.5. Finally, we briefly conclude in Section 2.6.

2.1 A Stochastic Bio-Economic Model

In this section, we extend the model in Durand et al. [9] to take into account the uncertainty related to major climatic events, e.g., cyclones and hurricanes. As in Durand et al. [9], we consider a one-hectare parcel farm managed over time by a single agent. It is a commercial farm whose production is intended for sale. The planning horizon is T and the current time is denoted by $\tau \in \mathcal{T} = \{0, ..., T\}$. The state of the farm is described by two variables: (i) the cash flow $\mathcal{W} \in \mathbb{R}$, i.e., a continuous variable characterizing the financial situation; and, (ii) the soil quality of the parcel, which is an agronomic metric. Evaluating the quality of a soil is a complex operation, as it involves a series of physical, chemical, and biological characteristics. Here, we adopt the General Indicator of Soil Quality (GISQ), which has values in the interval $[0, 1]$ and is based on 54 variables measuring these characteristics (Velásquez et al. [23] and Camacho et al. [7]). Let $\mathcal{I}$ be the GISQ value of the parcel, with $\mathcal{I} \in \mathscr{I} = \{0, ..., N_{\mathcal{I}} - 1\}$, which is, $\mathcal{I}$ can take a finite number of values.

The evolution of the soil quality depends on its current value $\mathcal{I}$ and on the following two control variables: (i) the crop σ grown on the parcel, with $\sigma \in \Sigma = \{\sigma_1, ..., \sigma_{N_\sigma}\}$; and, (ii) the agricultural practice $\pi \in \Pi = \{\pi_1, ..., \pi_{N_\pi}\}$. To illustrate, in the case studies to follow, the set of crops includes either all or some of the following: plantain, export banana, sugar cane, yam (yellow), yam (Grosse Caille), tomato, eggplant, lettuce, carrot, green bean, cabbage, cassava, melon, cucumber, and turban squash. The set of agricultural practices only includes two elements: (i) conventional practice, which is based on modern practices/tools and chemical fertilization; and, (ii) agroecological practice, i.e., traditional practice based on organic fertilization of the soil or other practices for improved management of agroecosystems (Wezel et al. [24]).

Let the evolution of the soil quality be described by the following function:

$$\begin{aligned} \phi \ : \ \mathscr{I} \times \Sigma \times \Pi \ &\rightarrow \qquad \mathscr{I}, \\ (\mathcal{I}, \sigma, \pi) \ &\mapsto \ \phi(\mathcal{I}, \sigma, \pi). \end{aligned}$$

Denote by $\delta(\sigma, \pi)$ the duration of the whole agricultural cycle of a crop σ using agricultural practice π. Subsequently, to each pair (σ, π), we can associate production cycles ψ, defined as follows:

$$\begin{aligned} \psi \ : \ \Sigma \times \Pi \ &\rightarrow \ [0, \delta(\sigma, \pi)], \\ (\sigma, \pi) \ &\mapsto \quad \psi(\sigma, \pi). \end{aligned}$$

Moreover, to each crop and practice (σ, π), we associate its first harvest time $f_c(\sigma, \pi)$ and the time duration between two successive harvests $p_c(\sigma, \pi)$. The duration of the first production cycle is $f_c(\sigma, \pi)$, and the following cycles last $p_c(\sigma, \pi)$. If a crop has a single harvest, then $f_c(\sigma, \pi) = \delta(\sigma, \pi)$ and $p_c(\sigma, \pi) = 0$. Additionally, we suppose that the produce is sold right away after it is harvested.

The revenue from each harvest depends on the crop and the agricultural practice, as well as on the soil quality at the beginning of the cycle. Denote by ℓ the revenue function defined by

$$\begin{aligned} \ell \ : \ \mathscr{I} \times \Sigma \times \Pi \ &\rightarrow \qquad \mathbb{R}, \\ (\mathcal{I}, \sigma, \pi) \ &\mapsto \ \ell(\mathcal{I}, \sigma, \pi). \end{aligned}$$

Denote by $s \in \mathcal{S} = \{1, ..., 12\}$ the current month. This discrete variable is needed to deal with the seasonality of some crops, which can be planted or sown only at specific times of the year. Let $\Sigma(s) \subset \Sigma$ be the set of all crops that can be planted or sown during season s. In the deterministic model presented in Durand et al. [9], the only event that must be accounted for over time is the beginning/end of a production cycle. If we use n to refer to an event, and $\tau(n)$ to its timing, then the next event will happen at

$$\tau(n+1) = \tau(n) + \psi(\sigma(n), \pi(n)). \tag{2.1}$$

The above equation assumes that once an agricultural cycle starts, it will reach its end. Here, we suppose that a production cycle may be randomly interrupted by a climatic event, which can occur during a specific period of the year, i.e., hurricane season. Consequently, (2.1) needs to be modified to account for such an event. We introduce the following additional variables:

- e_1: the remaining time before the next end/start of a production cycle,
- e_0: the remaining time before the next potential hurricane strikes, and
- $\epsilon = \min\{e_0, e_1\}$: the remaining time before the next change in the system's state,

Denote by $e = (e_0, e_1, \epsilon)$ the vector of events.

Denote by $H(t)$ the intensity of a climatic event at time t. We suppose that $H(t)$ takes its values from a finite and discrete set $\mathcal{H} = \{0, 1, ..., N_h\}$,where 0 represents the absence of a climatic event and N_h is the highest possible intensity. In the case studies, we assume that $\mathcal{H} = \{0, 1, 2\}$, where 0 corresponds to no hurricane, 1 to a minor hurricane, which is, a hurricane of intensity 1 or 2 on the Saffir-Simpson hurricane wind scale, and 2 to a major hurricane, that is, a hurricane of intensity 3, 4, or 5 on the Saffir-Simpson scale. In line with what is typically observed in the West Indies, which is the area targeted by our case studies, we assume that a climatic event can happen at most once a year during the hurricanes season, starting at $\beta \in \mathcal{S}$ and ending at $\gamma \in \mathcal{S}$. Denote by $\mathcal{P}(s, h)$ the probability that a hurricane of intensity $h \in \mathcal{H}$ happens during season $s \in \mathcal{S}$.

Depending on its intensity, on the type and state of the planted crops, and on the geological characteristics of a parcel, a hurricane happening at step n can interrupt a production cycle and totally or partially destroy the crop growing on that parcel. Let $\theta(n) \in [0, 1]$ be the level of damage, with 0 and 1 corresponding to no damage to crops and total destruction of crops, respectively. We specify $\theta(n)$, as follows:

$$\theta(n) = \Theta(b, \sigma(n), a(n), E),$$

where E is a vector of fixed and known parameters describing the geological characteristics of the soil parcel, e.g., its slope and wind exposure; $a(n)$ is the age of the crop occupying the parcel at step n; and, b is the intensity of the hurricane it was exposed to.

If a hurricane strikes during a production cycle, then the revenues and the GISQ will be affected. The revenue loss will depend on the age of the crop at the time the hurricane hits and on the level of damage. Let $a(n)$ be the age of the crop, and b the intensity of the hurricane, respectively. If no hurricane happens during the production cycle, then the parameter b is set equal to zero. The emergency control $v(n)$ on the parcel at step n will then be defined by

$$v(n) = (\sigma(n), \pi(n), a(n), b, c) \in \Sigma \times \Pi \times [0, \delta_{max}] \times \mathcal{H} \times [0, 1].$$

where $c \in [0, 1]$ is the GISQ of the soil at the beginning of the agronomic cycle of σ (when planted or sown). The new parcel's GISQ value is given by

$$\mathcal{I}(n+1) = \zeta\left(H(n), \mathcal{I}(n), v(n), \epsilon(n), E\right).$$

Further, cleaning and rehabilitation work have to be carried out on the farm before normal agricultural activities resume, which takes time and involves costs. The duration of the rehabilitation work after a hurricane that happened at step n is defined by

$$T(v(n)) = f(H(n), \sigma(n), a(n), \theta(n)).$$

The earnings generated by the emergency control $v(n)$ are defined by

$$\mathcal{L}(\mathcal{I}, v(n), \epsilon(n)) = g(H(n), \sigma(n), a(n), \theta(n), \epsilon(n)),$$

that is, it depends on the hurricane intensity $H(n)$, the crop type $\sigma(n)$ and age $a(n)$, the level of damage $\theta(n)$, and the time elapsed since the last event $\epsilon(n)$.

2.1.1 Dynamical System

We let the hurricane intensity $H(t)$ be a state variable, which adds a new event, e.g., a potential hurricane strike, to the only one considered in the deterministic model, which is, the end/start of a production cycle. The state of the farm (cash flow and soil quality) is determined by the vector

$$x = (\mathcal{I}, \mathcal{W}, H, e, s, \tau). \tag{2.2}$$

We recall that the control variable is

$$v = (u, a, b, c) \in V,$$

where $u = (\sigma, \pi) \in U = \Sigma \times \Pi$ is the control applied to the parcel.

To account for crop seasonality, we define by

$$U(s) \subset U, \quad U(s) = \Sigma(s) \times \Pi,$$

the set of possible controls on the parcel during a particular season s.

The evolution of the farm is then governed by the following discrete-time dynamical system $\mathcal{G}$:

$$
\mathcal{G} = \begin{cases}
\tau(n+1) = \tau(n) + \epsilon(n), & (2.3a) \\
s(n+1) = (s(n) + \epsilon(n)) \ mod \ 12, & (2.3b) \\
H(n+1) = \begin{cases} \text{Generated following } \mathcal{P}, & \text{if } \epsilon(n) = e_0(n), \\ 0, & \text{otherwise,} \end{cases} & (2.3c) \\
\mathcal{I}(n+1) = \varsigma(\mathcal{I}(n), v(n), \epsilon(n), E), & (2.3d) \\
v(n+1) \in \begin{cases} \{(\sigma(n), \pi(n), a(n) + \epsilon(n), H(n+1), c(n))\}, & \text{if } \epsilon(n) = e_0(n), \\ \{(\sigma(n), \pi(n), a(n) + \epsilon(n), 0, c(n))\} & \text{if } \epsilon(n) = e_1(n) \text{ and } \theta(n) < \theta_{max} \\ & \text{and } a(n) < \delta(\sigma(n), \pi(n)), \\ \{(\sigma, \pi, 0, 0, \mathcal{I}(n+1)) | (\sigma, \pi) \in U(s(n+1))\} & \text{otherwise,} \end{cases} & (2.3e) \\
\mathcal{W}(n+1) = \mathcal{W}(n) + \mathcal{L}(\mathcal{I}(n), v(n), \epsilon(n)), & (2.3f) \\
e_0(n+1) = \begin{cases} 12 - (s(n) - \beta) & \text{If } H(n) > 0 \text{ Or } s(n) = \gamma, \\ 1 & \text{If } H(n) = 0 \text{ And } \beta \leq s(n) < \gamma, \\ e_0(n) - \epsilon(n) & \text{Otherwise} \end{cases} & (2.3g) \\
e_1(n+1) = \begin{cases} T(v(n)), & \text{if } \epsilon(n) = e_0(n), \\ p_c(\sigma(n), \pi(n)), & \text{if } \epsilon(n) = e_1(n) \text{ and } \theta(n) < \theta_{max} \\ & \text{and } a(n) < \delta(\sigma(n), \pi(n)), \\ f_c(\sigma(n+1), \pi(n+1)) & \text{otherwise,} \end{cases} & (2.3h) \\
\epsilon(n+1) = \min\{e_0(n+1), e_1(n+1)\} & (2.3i)
\end{cases}
$$

The initial condition is $x(0) = (\mathcal{I}(0), \mathcal{W}(0), H(0), e(0), s(0), \tau(0))$, with $\tau(0) = 0$, $s(0) = 1$, $e_0(0) = \beta - 1$, $H(0) = 0$, $e_1(0) = 0$, $\epsilon(0) = \min_{i=0,1} e_i(0)$, $\delta(0) = 0$, $\mathcal{I}(0)$ and $\mathcal{W}(0)$ are given and $c(0) = \mathcal{I}(0)$.

Equation (2.3i) serves to update the event vector at each step. Equation (2.3a) sets the clock on time at each step $(n+1)$ by advancing it with $\epsilon(n)$ (the time elapsed between steps n and $n+1$). Equation (2.3b) does the same thing with the season, while taking into account its cyclicity. The other variables are updated according to the type of current event. At any step n, if the remaining time until the next event is equal to the remaining time until the next hurricane strike ($\epsilon(n) = e_0(n)$), meaning that the event in question (at step $n+1$) is a hurricane, and then the hurricane intensity is generated following the probability distribution $\mathcal{P}$ (see (2.3c)). The emergency control $\{(\sigma(n), \pi(n), a(n) + \epsilon(n), H(n+1), c(n))\}$ has to be applied using (2.3e). The evolution of the GISQ is given in (2.3d). The remaining time until the next hurricane is set to the hurricane period of next year, since only one hurricane can occur per year (2.3g). Otherwise, a production cycle ends on the parcel, leading to the modification of the soil quality following (2.3d). If the production cycle that just ended is not the last one ($a(n) < \delta(\sigma(n), \pi(n))$, then the crop and practice remain the same. We merely update the age of the crop and reset the value of the parameter b to 0. We supposed that a crop with multiple production cycles (harvests) entirely recover after one production cycle. The effect of the hurricane is only felt on the harvest following the hurricane (see (2.3e)), and a new production cycle starts right away, with the remaining time until the next event on the parcel being $p_c(\sigma(n), \pi(n))$ (see (2.3h)). Otherwise, if the whole agricultural cycle ends (interruption by total distruction from the hurricane or the end of the last production cycle of $\sigma(n)$), then a new crop and agricultural practice $c(n+1)$ has to be chosen from the set $U(s(n+1))$ for this parcel, as expressed in (2.3e). The new time until the next event on the parcel ($e_1(n+1)$) is equal to the length of the first production cycle of the new crop, which is, $f_c(\sigma(n+1), \pi(n+1))$; see (2.3h).

Finally, (2.3f) updates the economic state of the farm by accumulating at each step the earnings generated by the parcel when a production cycle ends and any expenses resulting from a hurricane strike.

2.1.2 Viability Constraints and Capture Problem

As in Durand et al. [9], we assume that the farmer aims at achieving the following two main objectives:

1. Ensuring an acceptable income throughout the whole planning horizon.

2. Restoring the quality of the soil to an acceptable level $\mathcal{I}^*$ by the end of the planning horizon.

The first objective is economic. Clearly, it can take different forms depending on how one defines an "acceptable" income. One practical option is to retain a minimum threshold W_{min}, below which cash flow should not fall at any step. Consequently, we have the following admissible set:

$$\mathcal{K} = \{(\mathcal{I}, \mathcal{W}, H, e, s, \tau) \| \tau \leq T, \mathcal{W} - W_{min} \geq 0\}. \tag{2.4}$$

The second objective is bio-ecological and it can be translated into a target to reach, i.e.,

$$\mathcal{C} = \{(\mathcal{I}, \mathcal{W}, H, e, s, \tau) \| \tau \geq T, \mathcal{I} \geq \mathcal{I}^*\}. \tag{2.5}$$

An alternative to (2.5) could be to restore the soil quality to at least a certain level $d\%$ of the initial value $\mathcal{I}(0)$, which is,

$$\mathcal{C}_d = \{(\mathcal{I}, \mathcal{W}, H, e, s, \tau) \| \tau \geq T, \mathcal{I} \geq \min\{\frac{d}{100}\mathcal{I}(0) \ ; \ 1\}\}. \tag{2.6}$$

With the uncertainty that is related to climatic events, we end up with a stochastic capture problem, where we look for initial states of system $\mathcal{G}$ defined by (2.3a–2.3i), for which there exists at least one viable evolution in $\mathcal{K}$ (2.4) until it reaches the target $\mathcal{C}$ (2.5), or $\mathcal{C}_d$ (2.6), with a probability exceeding a certain threshold p (the confidence level). This set is called the stochastic capture basin and it is defined as follows:

$$Capt_{\mathcal{G},\mathcal{P}}(\mathcal{K},\mathcal{C}) = \{X_0 \in \mathcal{K} \mid \exists x(.) \text{ s.t. } x(0) = X_0 \ \& \ \exists t > 0 \text{ s.t. } \mathcal{P}(x(t) \in \mathcal{C} \ \& \ \forall n \in [0,t], x(n) \in \mathcal{K}) \geq p\}. \quad (2.7)$$

2.2 The Emergency Control

The emergency control $v = (\sigma, \pi, a, b, c)$ includes everything that must be done after a climatic event. The duration, cost, and earnings that result from the emergency control depend on the hurricane's intensity and the damage it caused. In principle, a large variety of cases could occur, but, for tractability, we focus on two of them:

Total destruction of the crops. In this case, the climatic event destroys the totality of the crops on the parcel ($\theta(n) = 1$). If this happens, then the farmer has to carry out cleaning and rehabilitation work on the farm, and remove the damaged crops in order to be able to farm the parcel again. We suppose that the cleaning and rehabilitation work directly depends on the hurricane's intensity. The corresponding time and cost are denoted $t_c(H(n))$ and $c_c(H(n))$, respectively. The time and cost depend on the type of crop occupying the parcel $\sigma(n)$ and its age $a(n)$. Denote the time and cost by $t_r(\sigma(n), a(n))$ and $c_r(\sigma(n), a(n))$, respectively. Consequently, the duration of the emergency control $v = (\sigma, \pi, a, b, c)$ is

$$T(v(n)) = t_c(b) + t_r(\sigma(n), a).$$

The associated earnings correspond to the total financial flow generated during the time elapsed since the last update $\rho(I(n), v(n), \epsilon(n))$ minus the total amount incurred for the cleaning and rehabilitating of the parcel ($c_c(H(n))$ and $c_r(\sigma(n), a(n))$).

The term $\rho(I(n), v(n), \epsilon(n))$ includes the fixed costs and the costs for maintenance, labour, etc., and any other nonrecoverable cost. Therefore, we have

$$\mathcal{L}(\mathcal{I}, v, \epsilon) = \rho(\mathcal{I}, v, \epsilon) - \Big[c_c(b) + c_r(\sigma, a)\Big].$$

Partial degradation of the crops. In this case, the climatic event only destroys a portion of the crops on the parcel ($\theta(n) < 1$). Here, the farmer must decide, based on a given criterion, whether to bring the crop to maturity, or abandon it and start a new agricultural cycle instead. In practice, this decision will depend on the type of crop, its age, and the earnings it will potentially generate. To keep the model parsimonious, we consider a threshold θ_{max} above which degradation is considered to be total and the farmer decides to drop the crop. To be more realistic, one could let the threshold θ_{max} be a function of all other parameters.

If crop degradation caused by the hurricane is sufficiently low ($\theta(n) < \theta_{max}$), then the emergency control must last as long as necessary for the surviving crops to reach maturity and for the cleaning work to be completed. Because these two processes evolve simultaneously, the duration of the emergency control $v = (\sigma, \pi, a, b, c)$ is given by

$$T(v(n)) = Max\{\mathbb{1}_{\{t_c(b) \geq \delta(\sigma(n), \pi(n)) - a(n)\}} t_c(b) \ , \ e_1(n) - \epsilon(n)\}.$$

The earnings that are associated with $v = (\sigma, \pi, a, b, c)$ correspond to the gain from the surviving crops minus the total cost (cleaning cost, fixed and set-up costs). We suppose that the earnings from a production cycle are inversely proportional to its level of degradation θ. Knowing that a totally healthy production cycle ($\theta = 0$) generates earnings

$$\ell(\mathcal{I}(n), \sigma(n), \pi(n)) = m(\mathcal{I}(n), \sigma(n), \pi(n)) - h_c(\sigma(n), \pi(n)).$$

where $m(\mathcal{I}(n), \sigma(n), \pi(n))$ represents the income from selling the harvest and $h_c(\sigma(n), \pi(n))$ the harvesting cost, then the earnings from the surviving crop after a hurricane that caused a level θ of degradation are equal to

$$(1-\theta)m(\mathcal{I}(n), \sigma(n), \pi(n)) - h_c(\sigma(n), \pi(n)).$$

Consequently, we have the following expression for the earnings from $v = (\sigma, \pi, a, b)$:

$$\mathcal{L}(\mathcal{I}, v, \epsilon) = (1-\theta)\ell(\mathcal{I}, \sigma, \pi) - \theta h_c(\sigma, \pi) + \rho(\mathcal{I}, v, \epsilon) - c_c(b).$$

Subsequently, the general form of the function $T(v(n))$ for $v(n) = (\sigma, \pi, a, b, c)$ is given by

$$T(v(n)) = \begin{cases} t_c(b) + t_r(\sigma(n), a), & \text{if } \theta(n) \geq \theta_{max} \\ Max\{\mathbb{1}_{\{t_c(b) \geq \delta(\sigma(n), \pi(n)) - a(n)\}} t_c(b) \ , \ e_1(n) - \epsilon(n)\}, & \text{otherwise} \end{cases} \tag{2.8}$$

and for $\mathcal{L}(\mathcal{I}, v, \epsilon)$ by

$$\mathcal{L}(\mathcal{I}, v, \epsilon) = \begin{cases} \rho(\mathcal{I}, v, \epsilon) - \Big[c_c(b) + c_r(\sigma, a)\Big], & \text{if } \theta \geq \theta_{max}, \\ (1-\theta)\ell(\mathcal{I}, \sigma, \pi) - \theta h_c(\sigma, \pi) + \rho(\mathcal{I}, v, \epsilon) - c_c(b), & \text{otherwise.} \end{cases} \tag{2.9}$$

Note that this expression of revenues is also valid for any other control than the emergency control. Indeed, in the absence of a hurricane, $b = 0$ so $\theta(n) = 0$ and $c_c(b) = 0$, which leads to $\mathcal{L}(\mathcal{I}, v, \epsilon) = \ell(\mathcal{I}, \sigma, \pi) + \rho(\mathcal{I}, v, \epsilon)$, which is the earnings from a normally completed production cycle.

2.3 The Climatic Events

This section presents all of the relevant information regarding the climatic events and their related uncertainties.

2.3.1 Types of Climatic Events

A tropical cyclone is a rotating weather system that forms over the ocean when specific water temperature and air humidity conditions arise simultaneously. A significant amount of ocean water then evaporates quickly, causing an atmospheric disturbance. This results in winds revolving around an axis, under the effect of the earth's rotation, ultimately causing very strong winds, spiral thunderstorms, and heavy rain. When the maximum sustained surface winds of such formations are less than 39 miles per hour (mph), the cyclone is called a tropical depression, and between 39 and 74 mph, it is called a tropical storm. Beyond 74 mph, the event is referred to as a hurricane if it forms over the North Atlantic or Northeast Pacific Ocean, a typhoon if it forms over the Northwest Pacific Ocean, and simply a cyclone if it forms over the South Pacific or Indian Ocean (Source: NOAA website, Wikipedia, and others.)

The intensity of a hurricane is measured by the Saffir–Simpson Hurricane Wind Scale (SSHWS). It has a 1 to 5 categorization based on the hurricane's intensity at an indicated time (see Table 2.3.1 Source: `https://www.nhc.noaa.gov/aboutsshws.php`.) We talk about a major hurricane for events of categories 3 to 5.

Table 2.3.1: Saffir–Simpson Hurricane Wind Scale (SSHWS).

Category	**Winds**	**Damage**
1	74–95 mph 119–153 Km/h	Minimal
2	96–110 mph 154–177 Km/h	Moderate
3 Major	111–129 mph 178–208 Km/h	Intense
4 Major	130–156 mph 209–251 Km/h	Extreme
5 Major	$\geq$157 mph $\geq$252 Km/h	Catastrophic

2.3.2 Climatic Uncertainty

As the Caribbean Basin is one of the most hurricane-prone regions in the world, many meteorological studies are available. Surprisingly, however, the French West Indies seem to be an exception, with relatively few studies.

One of the few dedicated studies of this region is Garnier et al. [10]. The authors obtained data from the French archives in France and the islands of Guadeloupe and Martinique to survey all of the hurricanes that hit the French West Indies between 1635 and 2007. They used the data to build a long-term chronology of hurricane severity and damage in this area and to draw some statistical conclusions from these events, in particular yearly probabilities of hurricanes by category, which are of direct interest to our study. We make use of the results of this work because of its geographical relevance and the reliability of its data sources. In addition, the authors only included hurricanes that actually hit the islands of Guadeloupe and Martinique, which makes the results extremely relevant geographically. Table 2.3.2 presents the yearly hurricane probabilities extracted from [10], which will be used in our numerical illustrations.

Table 2.3.2: Yearly probability of a hurricane strike in the French West Indies, by category.

SSHWS Category	Yearly Probability
5	0.02
4	0.05
3	0.08
2	0.12
1	0.16

2.3.3 Hurricane Effects

The effects of a hurricane on soil and agriculture depend strongly on the type of agriculture and crops, and the soil's type, relief, etc. Strong winds may affect the physical characteristics of the soil itself. Strong rainfall may induce flooding that can change the chemical composition of the soil and cause landslides in sloped areas. Ocean wind pressure may result in storm surges that can cause severe salt contamination of the soil in coastal areas. All of these factors may disturb the microfauna and ultimately change the biological characteristics of the soil ([19]).

There are currently no studies estimating the effect of hurricanes on the GISQ. Therefore, in our numerical applications, we will look at three arbitrary situations: no effect, moderate effect, and significant effect of a hurricane on the GISQ (the precise meaning of these effects is given in Section 2.5.1).

Strong winds can cause significant structural damage to or uproot crops, and flooding may cause certain crops (especially those growing underground) to gorge with water or to rot, and changes to soil composition can compromise the normal growth of the crops. Spencer and Polachek [18] studied the effects of hurricanes on local crop production in Jamaica and found that crops grown above ground suffer greater damage than those grown below ground. In fact, the only below-ground crops that experience a drop of productivity are yams and potatoes. The vast majority of hurricanes that were considered in this study were minor (category 1 and 2) and their effects on crops are displayed in the first row of Table 2.3.3.

We consider, not unrealistically, that the losses are total after a major hurricane strike. Accordingly, our choice to consider a degradation of more than 60% ($\theta_{max} = 0.6$) as a total loss will have no impact on the results as studies indicate that hurricanes of category 3 or more cause at least an 80% loss of production.

Table 2.3.3 displays the percentage loss in productivity caused by certain types of climatic events on three categories of crops: the above-ground category, which includes all the crops growing above ground; the water-damaged category, which includes above-ground crops that are vulnerable to flooding; and the below-ground category, which includes crops growing underground that are flood resistant.

Table 2.3.3: Damage caused by hurricanes to crops. (Damages based on [18] and probabilities aggregated from Table 2.3.2).

Climatic Event	**Yearly Probability**	**Above Ground**	**Below Ground**	**Water Damaged**
Minor Categories 1–2	0.28	9.2 %	0%	60.6%
Major Categories 3–5	0.15	100 %	100%	100%

2.4 Solution Method

To compute the stochastic capture basin defined in (2.7), we adopt a dynamic programming approach. We exploit the following property established in Proposition 11 and Corollary 12: if it is possible (not possible) to reach the target with a certain initial treasury, then it is possible (not possible) to reach it with a higher (lower) treasury.

The stochastic capture basin defined in equation (2.7) is the set of initial states

$$X_0 = (\mathcal{I}(0), \mathcal{W}(0), H(0), e(0), s(0). \tau(0)),$$

satisfying the constraints stated in (2.7), meaning that there exists an evolution starting from X_0 that remains in the set $\mathcal{K}$ during the entire planning horizon and reaches the target $\mathcal{C}$ at the end of that time horizon with a probability greater than or equal to the confidence level.

As $\tau(0) = 0$, $s(0) = 1$, $H(0) = 0$, and $e(0) = 0$ are given data, we can characterize any initial state X_0 only by its $\mathcal{I}(0)$ and $\mathcal{W}(0)$ values, and write

$$X_0 = (\mathcal{I}(0), \mathcal{W}(0)).$$

Proposition 11. *Let $(\mathcal{I}, \mathcal{W})$ be an initial state from the stochastic capture basin $Capt_{\mathcal{G},\mathcal{P}}(\mathcal{K}, \mathcal{C})$; then any initial state $(\mathcal{I}, \mathcal{W}')$ with $\mathcal{W}' \geq \mathcal{W}$ belongs to $Capt_{\mathcal{G},\mathcal{P}}(\mathcal{K}, \mathcal{C})$.*

$$(\mathcal{I}, \mathcal{W}) \in Capt_{\mathcal{G},\mathcal{P}}(\mathcal{K}, \mathcal{C}) \Rightarrow (\mathcal{I}, \mathcal{W}') \in Capt_{\mathcal{G},\mathcal{P}}(\mathcal{K}, \mathcal{C}) \quad \forall \;\; \mathcal{W}' \geq \mathcal{W}.$$

Proof. Let $B(x, I, p)$ be the minimum budget needed for the evolution $x(.)$ to be viable:

$$B(x, \mathcal{I}, p) = \min\{\mathcal{W} \| x(0) = (\mathcal{I}, \mathcal{W}) \;\&\; \exists t > 0 \text{ s.t. } \mathcal{P}(x(t) \in \mathcal{C} \;\&\; \forall n \in [0, t], x(n) \in \mathcal{K}) \geq p\}$$

We have,

$$(\mathcal{I}, \mathcal{W}) \in Capt_{\mathcal{G},\mathcal{P}}(\mathcal{K}, \mathcal{C}) \quad \Leftrightarrow \quad \exists \; x(.) \text{ s.t. } B(x, \mathcal{I}, p) \leq \mathcal{W}.$$

Therefore,

$$\forall\ \mathcal{W}' \geq \mathcal{W}, \quad B(x, \mathcal{I}, p) \leq \mathcal{W}',$$

and consequently

$$(\mathcal{I}, \mathcal{W}') \in Capt_{\mathcal{G},\mathcal{P}}(\mathcal{K}, \mathcal{C}).$$

□

Corollary 12. *If it is not possible to reach the target from an initial state* $(\mathcal{I}, \mathcal{W})$, *then it is not possible to reach it with less initial treasury.*

$$(\mathcal{I}, \mathcal{W}) \notin Capt_{\mathcal{G},\mathcal{P}}(\mathcal{K}, \mathcal{C}) \Rightarrow (\mathcal{I}, \mathcal{W}') \notin Capt_{\mathcal{G},\mathcal{P}}(\mathcal{K}, \mathcal{C}) \quad \forall\ \ \mathcal{W}' \leq \mathcal{W}$$

The results presented in Proposition 11 and Corollary 12 imply that, in order to completely characterize the capture basin, it is sufficient to identify its border or, in other words, to find for each initial GISQ the minimum initial treasury $\mathcal{W}_{inf}(\mathcal{I}, p)$ needed for the restoration, i.e.,

$$\mathcal{W}_{inf}(\mathcal{I}, p) = \inf_{(\mathcal{I},\mathcal{W}) \in Capt_{\mathcal{G},\mathcal{P}}(\mathcal{K},\mathcal{C})} \mathcal{W} = \min_{x} B(x, \mathcal{I}, p).$$

Therefore, the problem amounts to finding a viable evolution of the system $x(\cdot)$ with the minimum possible restoration budget $B(x, \mathcal{I}, p)$. This problem can be formulated as a stochastic shortest-path problem, whose objective is to minimize the expected restoration budget, and which can be solved by dynamic programming.

A feedback solution takes the form of a decision tree that gives a control to apply for each state of the system from where it is possible to reach the target. Knowing that each sequence of controls applied in the past, combined with the realization of the uncertainties (the climatic events that occurred) during that period, leads the system to a specific state, the feedback solution gives a sequence of controls to apply during the next periods. Each branch of the decision tree representing the solution describes the evolution of the system for one possible realization of the uncertainties during the exploitation period.

2.5 Numerical Results

In this section, we consider some farming systems, to which we will refer as cases in a single-parcel setting. Table 2.5.1 lists the crops considered in these cases. Each crop can be used either with a conventional or agroecological practice. Additionally, farmers can choose to leave their land fallow. When coupled with the agroecological practice (short fallow and long fallow), the land is left in simple fallow. When coupled with the conventional practice (improved short fallow and improved long fallow), the fallow is improved with the use of chemical additives or fertilizers.

Table 2.5.1: Description of the different cases.

Case	Crops
Case 1 Banana	Plantain, Export banana
Case 2 Banana & sugar cane	Plantain, Export banana, Sugar cane
Case 3 Tomato and yam	Yam(Yellow), Yam(Grosse Caille), Tomato
Case 4 Multicrop	Yam (Yellow), Yam(Grosse Caille), Tomato, Eggplant, Lettuce, Carrot, Green bean, Cabbage, Cassava, Melon, Cucumber, Turban squash
Case 5 All crops	Plantain, Export banana, Sugar cane, Yam (Yellow), Yam(Grosse Caille), Tomato, Eggplant, Lettuce, Carrot, Green bean, Cabbage, Cassava, Melon, Cucumber, Turban squash

Each one of the cases listed above represents a type of farming system practiced in the French West Indies: the export sector, specialized in banana and sugar cane (cases C1 and C2); the local market sector, based on diversified vegetable farming (C3 and C4); and, a theoretical case (C5) that combines all of the crops.

Computations were made for various hurricane impacts on the GISQ, initial GISQ levels, time horizons, confidence levels, and fixed costs in order to be able to analyze the effect of each parameter. We always require that the farm remain self-sufficient on average, i.e., we set the economic constraint to 0 ($W_{min} = 0$) and impose that the retained solution(s) allow for a positive mean treasury.

All of the case studies concern the Guadeloupe archipelago. For the common elements, we use the same data and parameter values, as in Durand et al. [9]. The other required data and parameter values are estimated in Section 2.3 and/or displayed in Appendices 2.B and 2.C.

The rest of the section is divided into two parts. In the first part (Section 2.5.1), we focus on the capture basins. In particular, we analyze the impact of hurricanes on the GISQ (in Section 2.5.1), the impact of the time horizon (in Section 2.5.1), the impact of crop diversification (in Section 2.5.1), the impact of fixed charges (in Section 2.5.1), and the impact of the initial GISQ (in Section 2.5.1).

In the second part (Section 2.5.2), we look at viable evolutions. More specifically, we discuss the main characteristics of such evolutions and how they are affected by such features as direct hurricane impact, time horizon, fixed charges, and GISQ improvement level.

2.5.1 Analysis of Capture Basins

Denote by $Capt_{C_i,T}(I^*,p)$ the capture basin of Case i for a time horizon T, where the GISQ target is I^* with a confidence level p. Concretely, the capture basin is the set of all couples of initial GISQ I_0 and budget (initial treasury) that make it possible to reach the target while respecting the imposed constraints.

Each panel in Figure 2.5.1 represents a superposition of capture basins of the same case for different confidence levels. For example, in the upper left panel of Figure 2.5.1, the surface in represents the capture basin $Capt_{C_1,40}(0.8, 0.9)$, whereas represents the capture basin $Capt_{C_1,40}(0.8, 0.2)$.

In all cases, as expected, the capture basins get smaller when the confidence level is increased, which is,

$$Capt_{C_i,T}(I^*,p_1) \subseteq Capt_{C_i,T}(I^*,p_2), \forall p_1 \geq p_2.$$

Indeed, if it is possible to reach the target starting from a certain GISQ and using a certain budget with a confidence level p_1, then it is possible to reach it with a confidence level $p_2 \leq p_1$

The higher is the initial GISQ I_0, the lower is the minimum budget needed to reach the target. However, the budget is always strictly positive, even when the initial GISQ I_0 is larger than the target I^*, due to the fixed costs and the money needed for planting the first crop. In fact, even if nothing is done (i.e., leaving the parcel in free fallow), the farmer needs to cover the fixed cost that must be paid, regardless of activity.

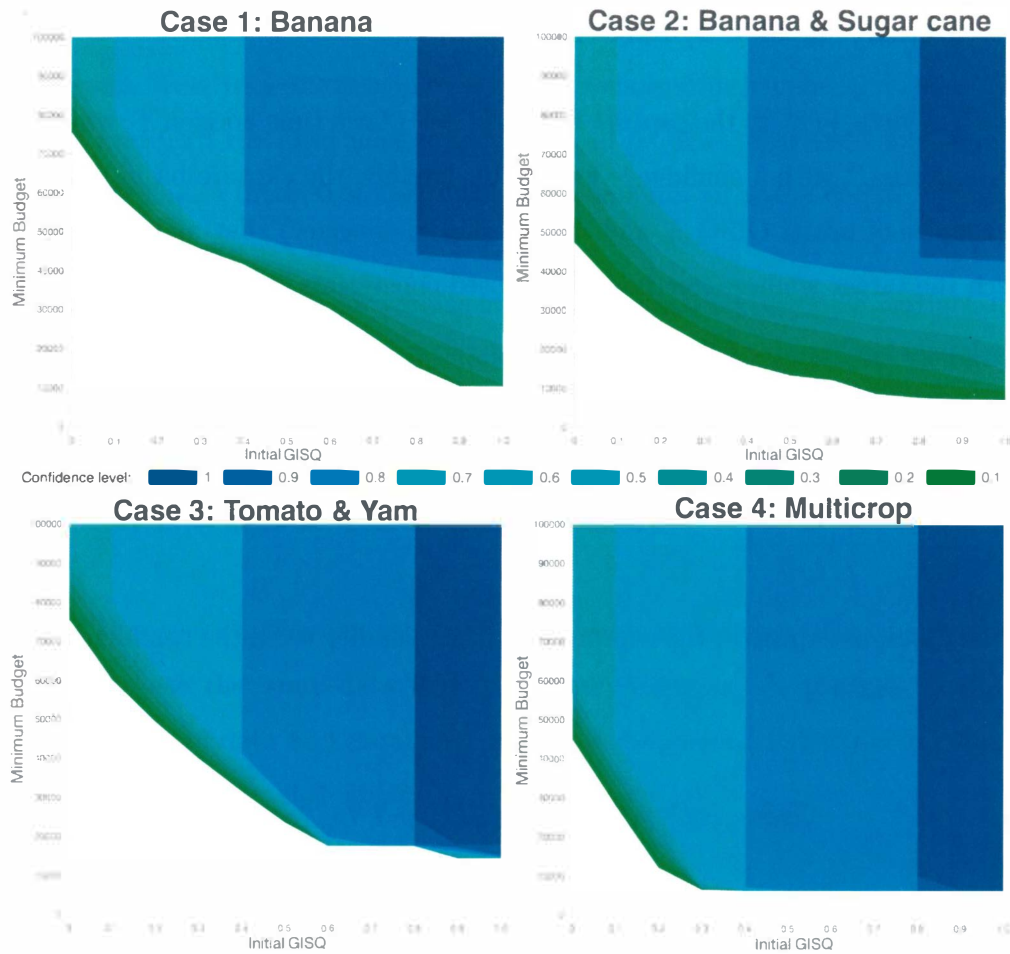

Figure 2.5.1: Capture basins by confidence level ($T = 40$, $I^* = 0.8$).

Direct Effect of Hurricanes on the GISQ

The GISQ is a quite recent index describing the soil quality and, thus, there are no studies establishing how its value is affected by hurricanes. To work around this lack of information, our numerical simulations retained the following three arbitrary contexts:

1 - No effect: regardless of its intensity, a hurricane's impact on the GISQ is set equal to 0.

2 - Moderate effect: a minor hurricane decreases the GISQ value by 0.01, while a major hurricane reduces the GISQ by 0.02. This case is considered to be the default one, which is, when the effect is not specified in a figure or in the discussion, then it is a moderate effect.

3 - Large effect: as above, a minor hurricane decreases the GISQ by 0.01, but a major hurricane reduces the GISQ by 0.05.

Figure 2.5.2 displays the capture basins for a series of cases, where the time horizon is 40 years and the GISQ target $I^* = 0.8$ for the three different hurricane effects introduced above. A first observation is that when a hurricane has no direct impact on the GISQ (see the first column of Figure 2.5.2), then it is always possible to reach the desired level of soil restoration with probability 1, provided that the time horizon is long enough (see Section 2.5.1) and a budget is available. Note that the confidence level only affects the minimum budget needed for restoration, due to the assumption that the climatic event does not deteriorate the GISQ.

Now, if a hurricane affects the GISQ, then we notice that the greater the impact, the more difficult it is to restore the soil to the desired level. In particular, for some realizations of the uncertain event, the cost of restoring the soil to the desired level is high, and restoration itself may become infeasible for certain confidence levels. To illustrate, for all displayed cases in Figure 2.5.2, it is possible to restore the soil with an initial GISQ of 0.2, with a confidence level of 0.6, when we suppose that the hurricane have no direct impact on the GISQ. Restoration is also possible when we assume a moderate effect, but it would take a higher minimum budget. However, restoration is no longer possible if the hurricane has a large impact on the GISQ.

Finally, we note that the capture basin for a large hurricane impact on the GISQ is included in the capture basin with a moderate effect, which is, in turn, included in the no-effect capture basin.

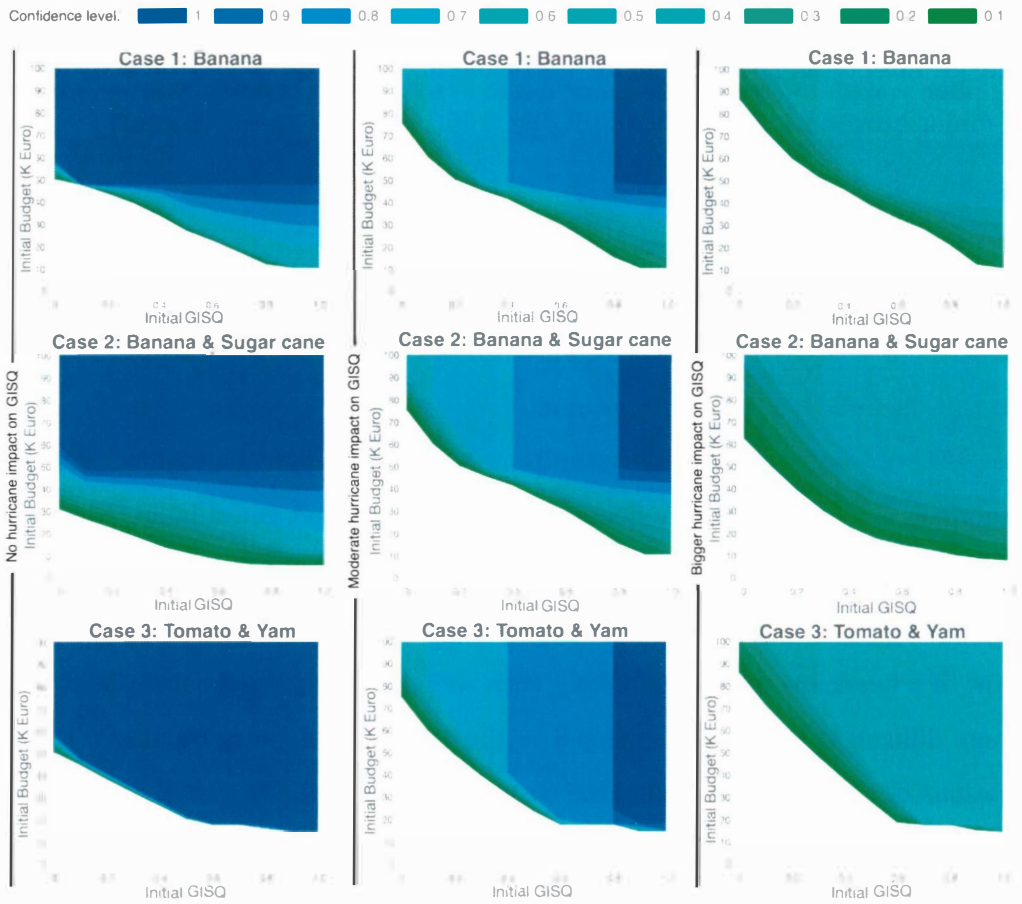

Figure 2.5.2: Capture basins for different hurricane impacts on the General Indicator of Soil Quality (GISQ) ($T = 40$, $I^* = 0.8$).

Impact of the Time Horizon

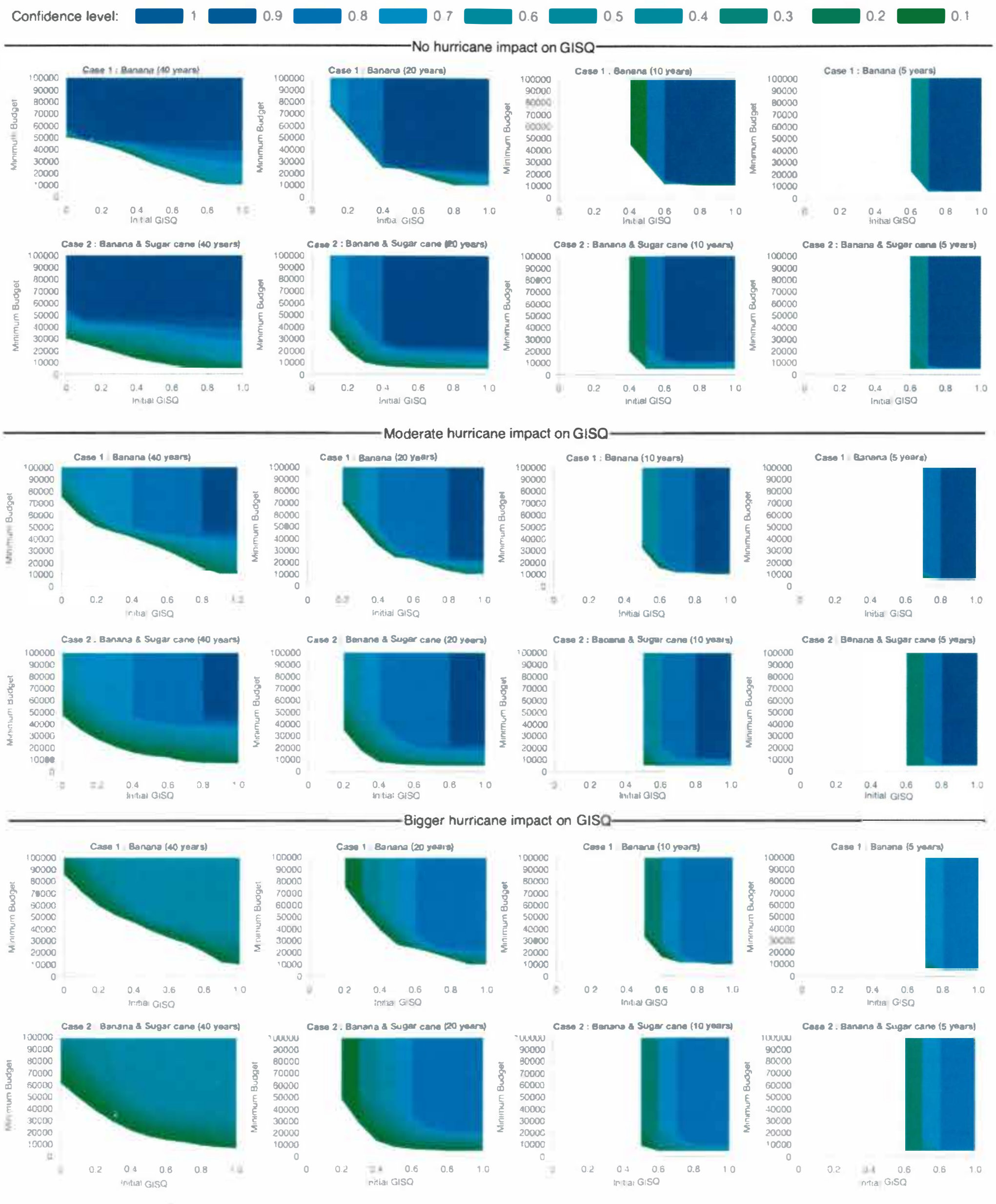

Figure 2.5.3: Capture basins for different time horizons ($I^* = 0.8$).

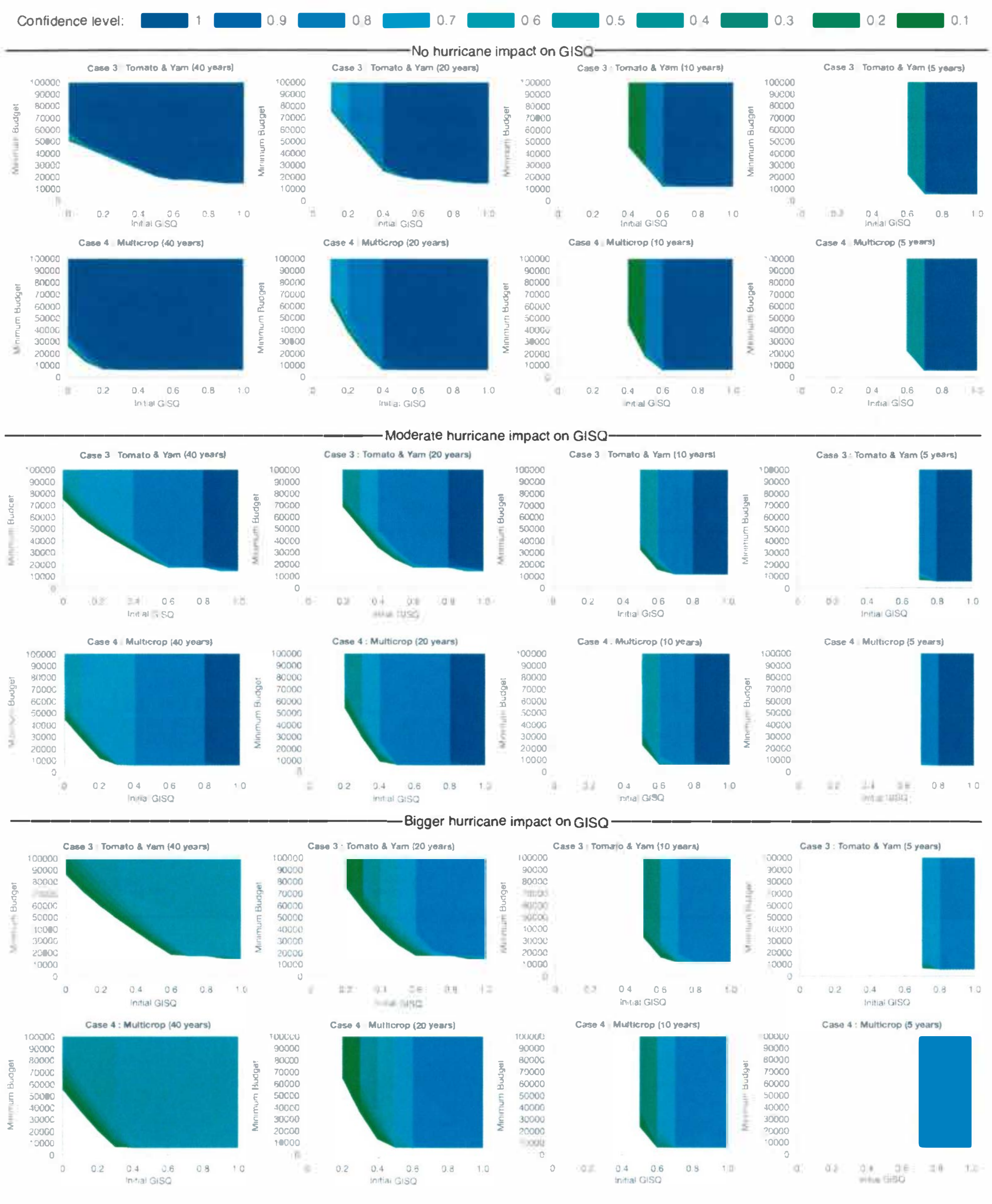

Figure 2.5.4: Capture basins for different time horizons ($I^* = 0.8$).

Figures 2.5.3 and 2.5.4 display the capture basins for cases with different planning horizons (40, 20, 10, and five years) and levels of hurricane impact on the GISQ.

A first takeaway from these figures is that the capture basins shrink and slide to the right as the time horizon gets shorter in all cases, regardless of the level of hurricane impact on the GISQ. This indicates that the less time we have, the more difficult it is to restore the poorest soils, which is quite intuitive. For example, it is possible to bring a soil with a GISQ of 0.2 to a GISQ of 0.8 in 40 or 20 years, but not if we only have 10 or 5 years.

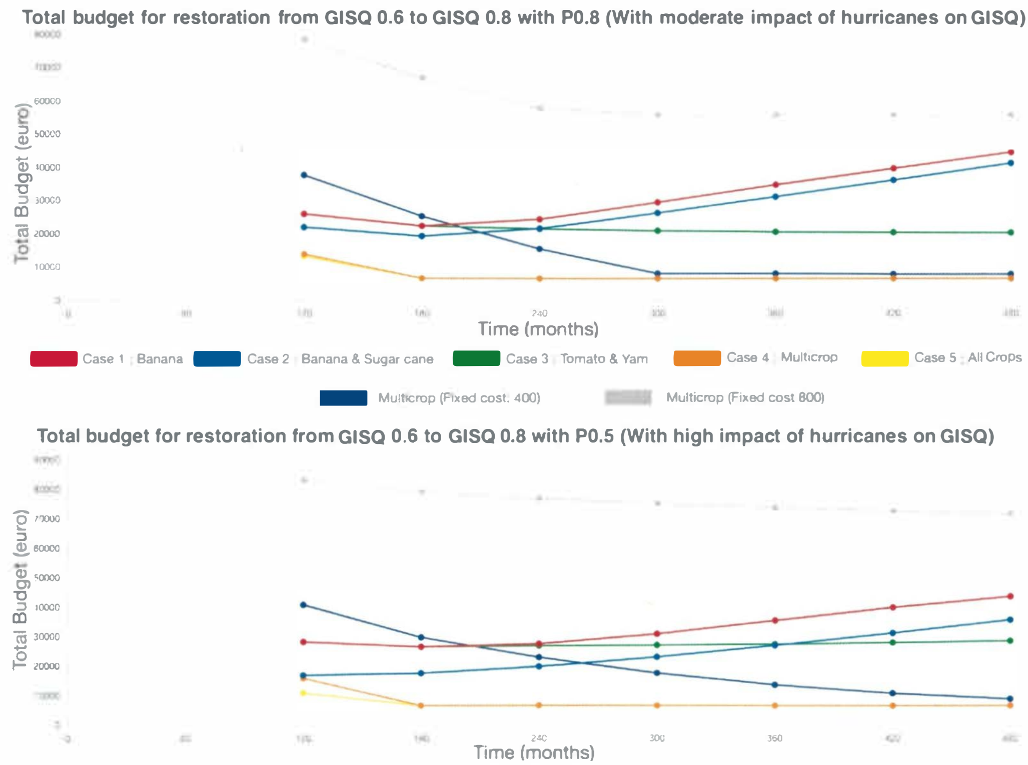

Figure 2.5.5: Total restoration cost for different time horizons.

When the improvement is feasible, the minimum restoration budget, for different time horizons, varies across scenarios. To better visualize this behavior, we display in Figure 2.5.5 a sample of the total restoration costs as a function of the time horizon for a soil restoration from GISQ 0.6 to GISQ 0.8 in the case of a moderate hurricane effect on the GISQ with a probability of 0.8 (upper part) and in the case of a bigger hurricane impact on the GISQ with a probability of 0.5 (lower part).

As we would expect, for the same conditions (same initial GISQ and same confidence level), the total restoration budget decreases with the time horizon; see case C4 and its variants (with fixed cost 400 and 800) as well as C3 and C5. Indeed, the less time is available to restore the soil, the quicker and more efficient the work must be, and the higher is the cost. Moreover, there is a threshold time horizon above which the minimum budget stabilizes and becomes constant. In particular, this happens when the farm is profitable enough: when it just needs a certain amount to start the first production cycle and then it becomes self-sufficient. The time horizon at which the minimum budget stabilizes is the minimum time it takes for farming to become totally self-sufficient.

The minimum time horizon for self-sufficiency depends on the farm's efficiency level. That is, when the farm faces fewer charges and/or achieves higher incomes, then the timespan for reaching self-sufficiency is shorter. For example, the minimum budget in C4 with a fixed cost of 100 stabilizes before those with fixed costs of 400 and 800. However, for other cases, e.g., C1 and C2, we observe the opposite, which is, globally, the longer the planning horizon, the more expensive it becomes to restore the soil, which seems to be pretty counterintuitive. In fact, this happens when the farm is not profitable enough, i.e., it does not generate sufficient cash to cover all of the operational expenses. Therefore, tautologically, the longer is the time horizon, the higher are the costs to be covered; as a result, the minimum restoration budget increases. This is confirmed by the treasury evolution in the different cases. In those cases with a decreasing restoration budget over time (the profitable cases), the treasury trend is increasing and ends up with a surplus. On the other hand, the cases with an increasing restoration budget over time exhibit a decreasing treasury trend and end up with an almost zero final treasury. To illustrate, we show in Figure 2.5.6 the GISQ and treasury over the restoration time for two cases: a profitable one (C4) in the upper part of the figure and an unprofitable one (C1) in the lower part. Note that the two chosen evolutions are computed with the same realization of the uncertainty.

In terms of confidence level, one would expect that, if it is possible to reach the target with a given probability in a certain time horizon, then it would be possible to reach this target with that same probability in a longer time horizon. This intuition is confirmed for no or moderate hurricane impact on the GISQ (see the relevant parts in Figures 2.5.3 and 2.5.4). However, this result does not hold when the impact of hurricanes on the GISQ is large. Indeed, the lower parts of Figures 2.5.3 and 2.5.4 show that it is possible to reach the target from an initial GISQ of 0.8 with probability 0.8 when we have 5, 10, or 20 years available, but not when the planning horizon is 40 years. The reason is that when hurricanes' impact on the GISQ is large, then the damage is also large; having a longer planning horizon does not help, as the costs accumulate and eventually exceed the restoration capability of the feasible controls in this case. Consequently, it becomes impossible to reach the target with the desired level of confidence.

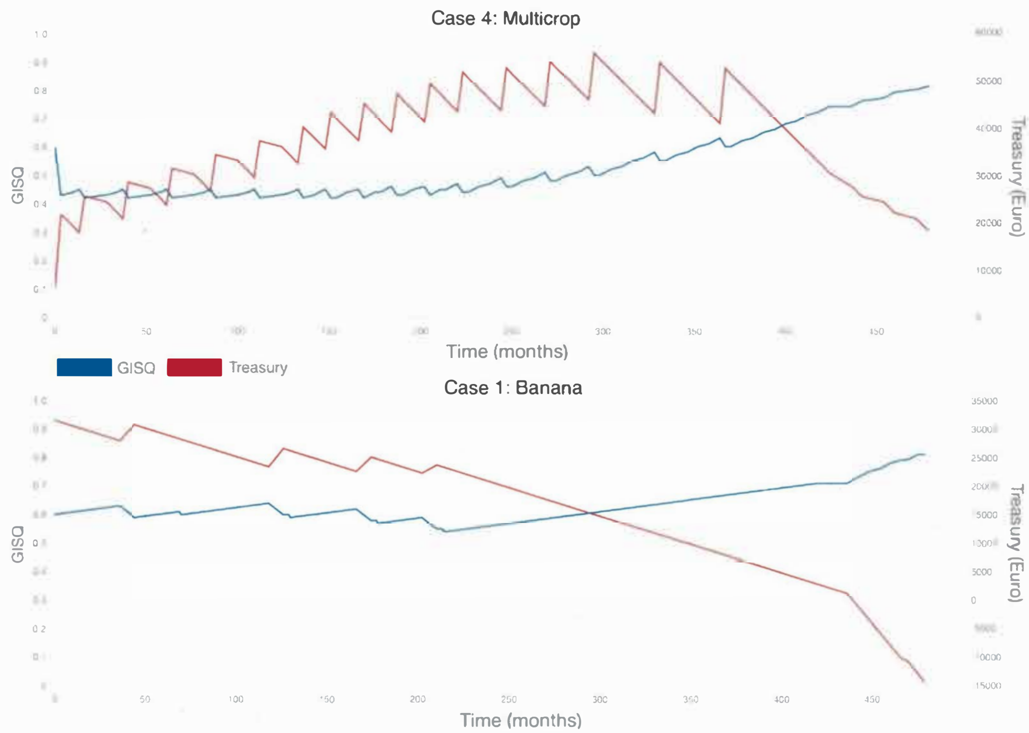

Figure 2.5.6: Examples of GISQ and treasury evolution over restoration period ($T = 40$, $I = 0.6$, $I^* = 0.8$).

Impact of Crop Diversification

The comparison of cases C1 and C2 to C3 and C4 in Figures 2.5.3 and 2.5.4 reveals that the capture basins of scenarios with more crops are always at least as large as those with fewer crops, which is,

$$Capt_{C_1,T}(I^*, p) \subseteq Capt_{C_2,T}(I^*, p) \quad \text{and} \quad Capt_{C_3,T}(I^*, p) \subseteq Capt_{C_4,T}(I^*, p).$$

In fact, adding a control (a degree of freedom) can only help in meeting the objectives. The comparison of C5, a case with all crops, to the other cases confirms this observation. We can then conclude that, if the set of possible controls U_i in case

C_i is included in the set U_j of controls in case C_j, then the capture basin of C_i is contained in that of C_j, i.e.,

$$U_i \subset U_j \Rightarrow Capt_{C_i,T}(I^*, p) \subseteq Capt_{C_j,T}(I^*, p).$$

Figure 2.5.5 shows that the restoration budget in a case with more crops (a larger set of possible controls) is at least as low as the restoration budget of a case with fewer crop choices, i.e.,

$$U_i \subset U_j \Rightarrow Budget_{C_i,T}(I^*, p) \geq Budget_{C_j,T}(I^*, p)$$

The above inclusion holds for all planning horizons and hurricane impacts on the GISQ.

It is true that crop diversification makes it easier to achieve the soil restoration objective, but adding new crops to the set of possible controls of a scenario does not necessarily decrease the restoration budget. If the initial set already contains the right mix of crops, adding new crops will certainly not deteriorate the solution, but neither will it improve it. For example, the performance of C5 is, in almost all cases, identical to the performance of C4, even if it has more crop choices.

Impact of Fixed Charges

Figure 2.5.7 displays the capture basins of C4 with different fixed costs (100, 400, and 800) and different hurricane impacts on the GISQ. First, we note that these results are consistent with the conclusions reached in Section 2.5.1. Second, the overall shape of the capture basins remains the same when the fixed cost changes. This means that, if it is possible to reach the target from a certain initial GISQ with a certain probability, then it remains possible to reach it if the fixed cost is increased. However, the minimum budget for restoration significantly changes, i.e., it gets higher for cases with higher fixed costs. Further, the difference in budget when considering two fixed-cost values is more pronounced when the initial GISQ is low. Indeed, if we compare the minimum restoration budgets for the same level of improvement within the same time horizon and with the same probability (Figure 2.5.8), we see that the budgets for cases with higher fixed costs are higher than those with lower fixed costs, and also that the budget increment is much larger when the initial GISQ is low. This difference can be explained by the fact that the crops planted in a better quality soil yield more and, therefore, generate more income, which can be used to cover a larger portion of the fixed costs.

Comparing the minimum restoration budget of these three cases for different time horizons (Figure 2.5.5), we confirm that the minimum budget increases with the fixed cost. Additionally, we notice that the difference in budgets becomes smaller as the time horizon gets longer. The budget difference gradually decreases and stabilizes after a certain value of time horizon.

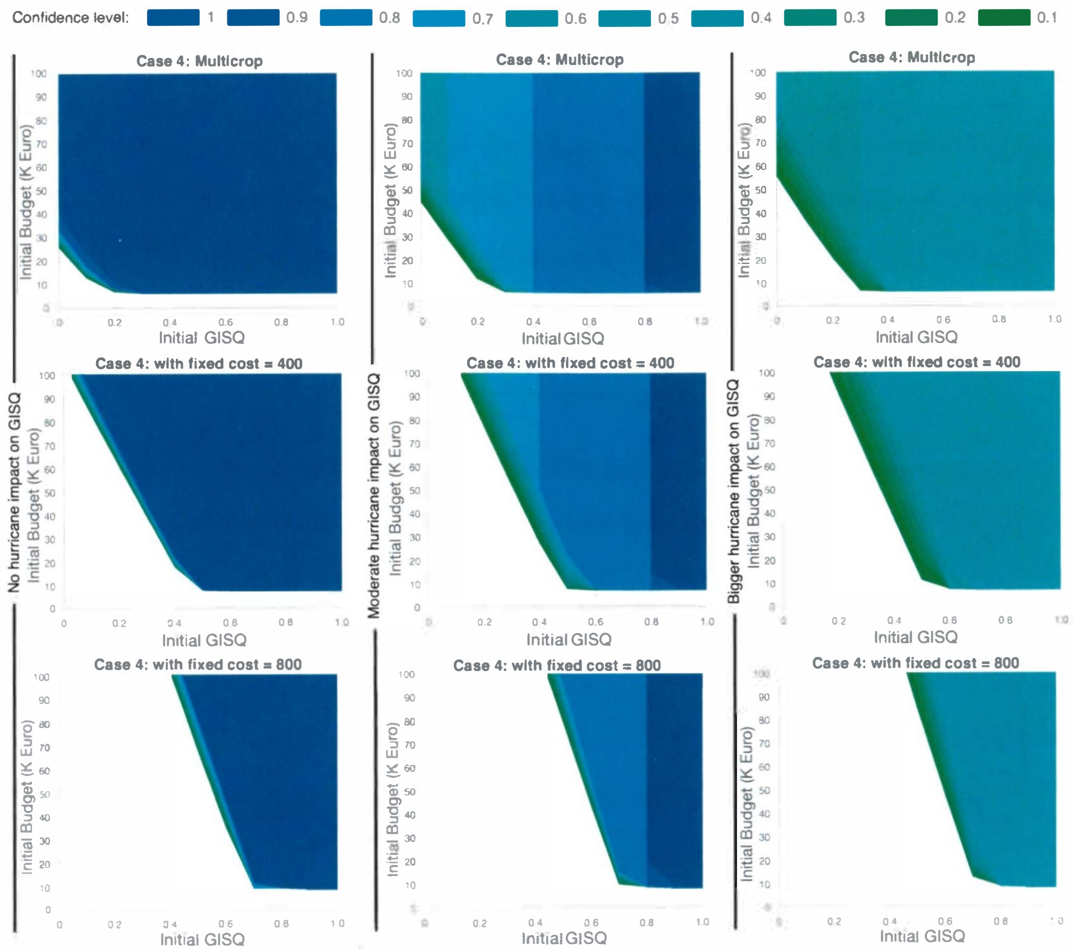

Figure 2.5.7: Capture basins of C4 with different values for fixed cost ($T = 40$, $I^* = 0.8$).

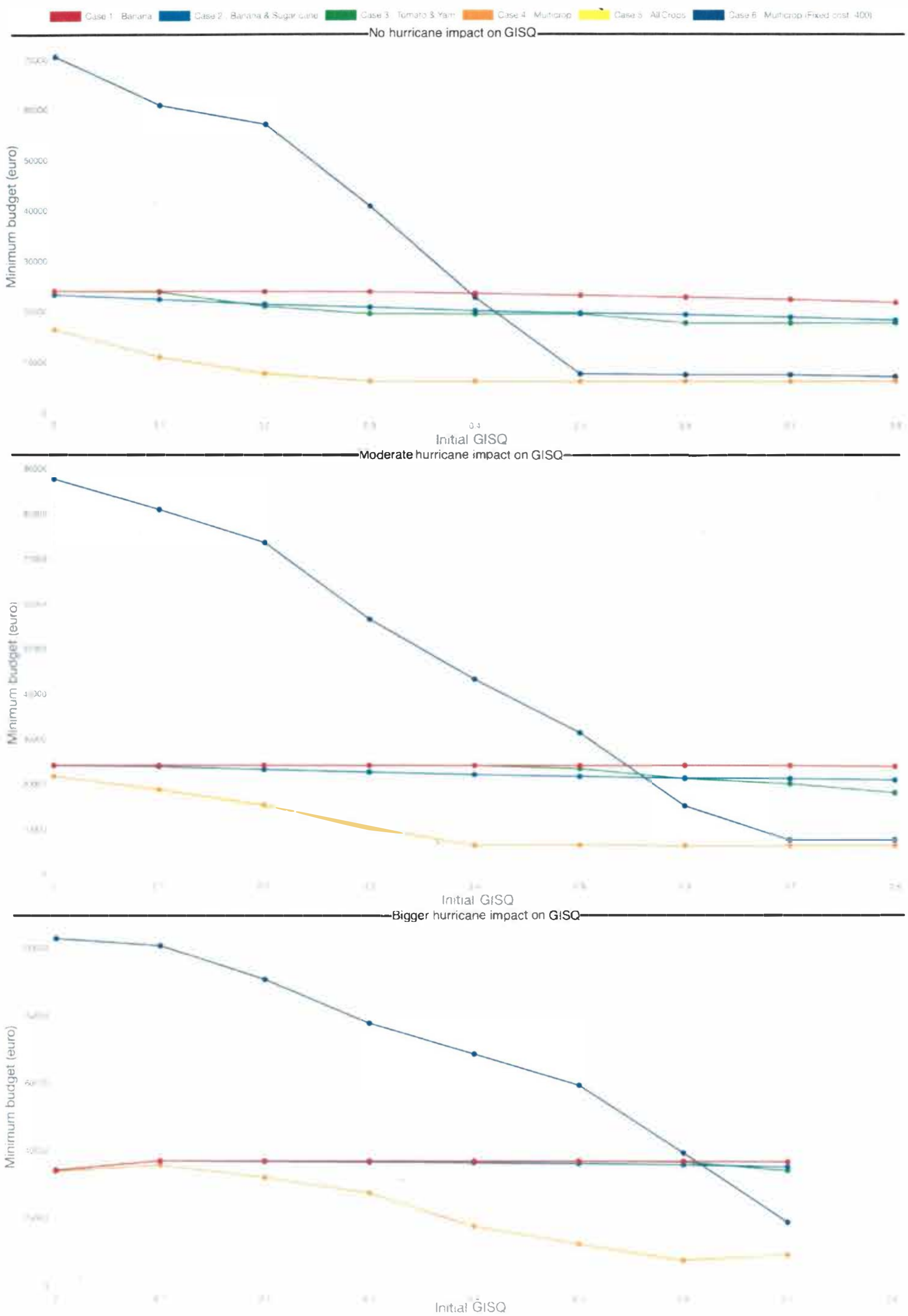

Figure 2.5.8: Minimum budget for improvement of 0.2 ($T = 20$).

Impact of the Initial GISQ

Figure 2.5.8 displays the evolution of the minimum budget needed to raise the GISQ by 0.2 within 20 years and with a probability of 0.8, as a function of the initial GISQ, for different hurricane impact levels on the GISQ. In a nutshell, a soil quality improvement of the same level does not necessarily require the same budget when the initial GISQ is low or high.

The first two plots in Figure 2.5.8, corresponding to no and moderate hurricane impact on the GISQ, show that the higher the initial GISQ, the lower is the restoration cost. The reason is that a crop planted in better soil generates a higher income than a crop grown in poor-quality soil. Consequently, a lower minimum budget is needed, as the income covers a larger portion of the exploitation costs. Note that the budget may be roughly constant regardless of the initial GISQ value, as in C2. This occurs when the crop's yield is not very sensitive to the GISQ value.

When the hurricane impact on the GISQ is large (lower part of Figure 2.5.8), the above result remains valid for mid-interval initial GISQ levels. However, the opposite behavior is observed for values that are either close to 0 or 1. For instance, in C4 and C2, the minimum budget curves are increasing near initial GISQ values of 0 and 1. One explanation for this is that, when the GISQ is too low, the degradation caused by a hurricane, regardless of the storm's severity, cannot be very high. On the other hand, starting with a soil of medium quality leaves room for improvements that exceed the target, in anticipation of future hurricanes and compensating for their damage. However, if the initial GISQ is too high, then such an opportunity is not available, which increases the farm's sensitivity to hurricane strikes.

2.5.2 Viable Evolutions

In this section, we exhibit (viable) solutions with the minimum initial budget that lies on the boundary of capture basins. These solutions are not the result of an optimization of, e.g., income or another criterion, they are simply viable. In the case of multiple solutions, we chose the one with the highest final treasury. Further, we recall that the problem of determining viable evolutions is represented by a decision tree, with each branch corresponding to the evolution of the state for one possible realization of the uncertainties during the exploitation period.

Figure 2.5.9 displays one branch of the solution for case C4 over 40 years ($T = 480$ months), for a GISQ target $I^* = 0.8$ with an initial GISQ of 0.6, and a confidence level $p = 0.8$. The upper part of the figure shows the evolution of the GISQ and the treasury over the time horizon, while the lower part exhibits the sequence of controls to apply. Finally, the red vertical lines indicate the instants of the hurricane's strikes and their intensity ("m" stands for a minor hurricane and "M" for a major one).

A first observation is that each increase in the treasury comes with a decrease of the GISQ level and vice versa. This illustrates the conflict between the criteria of farm profitability and resource preservation, which is at the root of our problem. The GISQ level goes from its initial value I^0 at time 0 to reach $I^T \geq I^*$ at the terminal date. The treasury starts at the minimum restoration budget, goes up and down over time, and remains positive throughout the planning horizon.

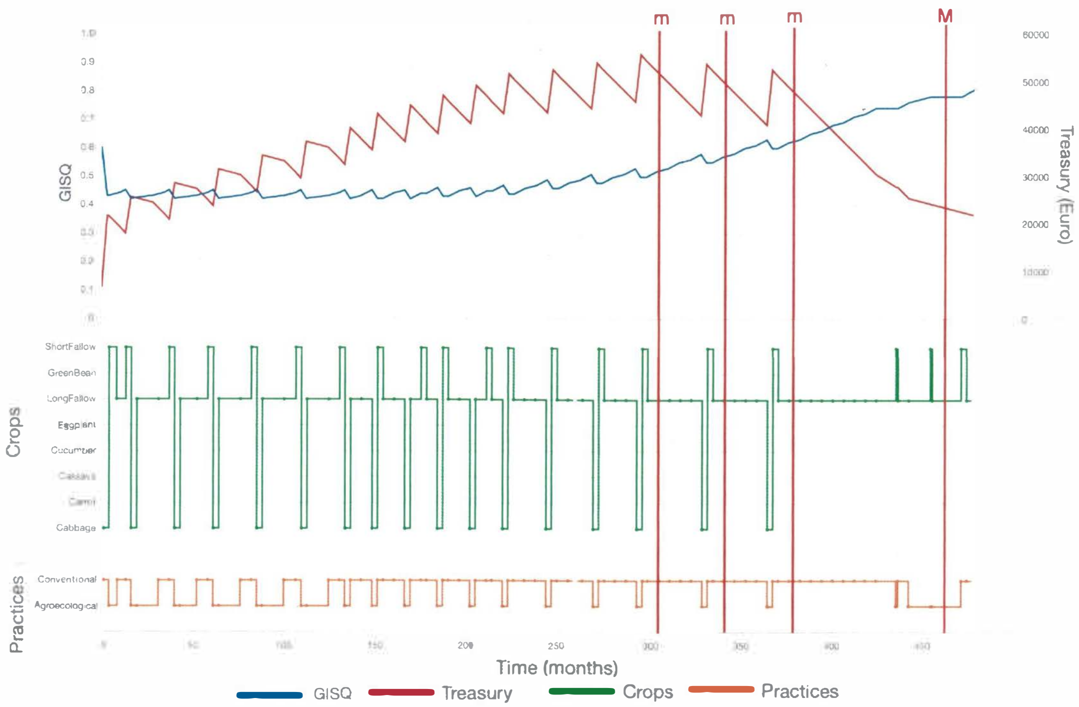

Figure 2.5.9: Evolution of the system for a particular realization of the uncertainty (Case C4, $T = 40, I^0 = 0.6, I^* = 0.8, p = 0.8$).

We make the following remarks on the viable evolutions and the effect of some parameters:

Crop recommendations: in all viable evolutions, there is a prevalence of fallow as a recommended control. This is due to the fact that the solutions lie on the boundary of the capture basin. Even if it were possible to obtain higher revenues by investing a bit more, the algorithm returns the solution with the minimum budget. As the cheapest option is fallow, this is most likely to be chosen.

Direct hurricane impact on the GISQ: when the GISQ is more sensitive to hurricane strikes, we once again notice that fallow controls are very prevalent. This is intuitive as they offer the best GISQ improvement to price ratio. When the impact of hurricanes on the GISQ is low, the degradation over time becomes less significant and thus we can afford the use of more profitable crops even if they are less performant on the GISQ improvement side.

Time horizon: the impact of the planning horizon depends on the hurricane impact on the GISQ. When this impact is low, fallows are less recommended when the horizon is longer. However, when the direct effect of hurricanes on the GISQ is high, then fallow controls are recommended more often, due to the large accumulation of damage on the GISQ over longer periods of time, as alluded to in Section 2.5.1.

Fixed charges: higher fixed costs call for more profitable crops, that is, other than fallow. This is expected, as a higher income helps to cover this cost. In fact, it is more appealing to increase revenues than to keep budget to cover fixed costs. However, this result no longer holds when the hurricane impact on the GISQ is too high. Indeed, in this case, achieving the GISQ target becomes harder, if even possible, using productive crops. Consequently, there may be no other choice than fallow controls to improve the GISQ, which adds the fixed cost to the initial budget.

Level of improvement: the comparison of Figures 2.5.9–2.5.11, which respectively show a branch of the feedback solution tree for an initial GISQ of 0.6, 0.1, and 1, shows that a higher level of improvement in the GISQ involves a fallow control. The intuition is very clear: a small level of improvement in the GISQ requires less effort and time to achieve, which leaves more time to improve the treasury.

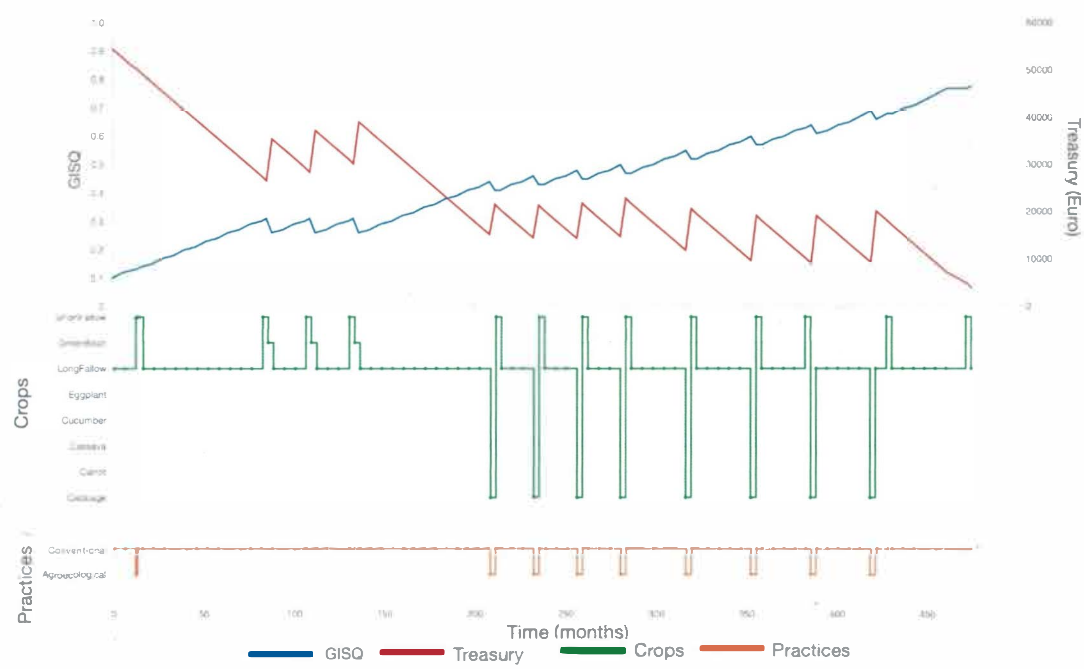

Figure 2.5.10: Evolution of the system for a particular realization of the uncertainty (Case C4, $T = 40$, $I^0 = 0.1$, $I^* = 0.8$, $p = 0.8$).

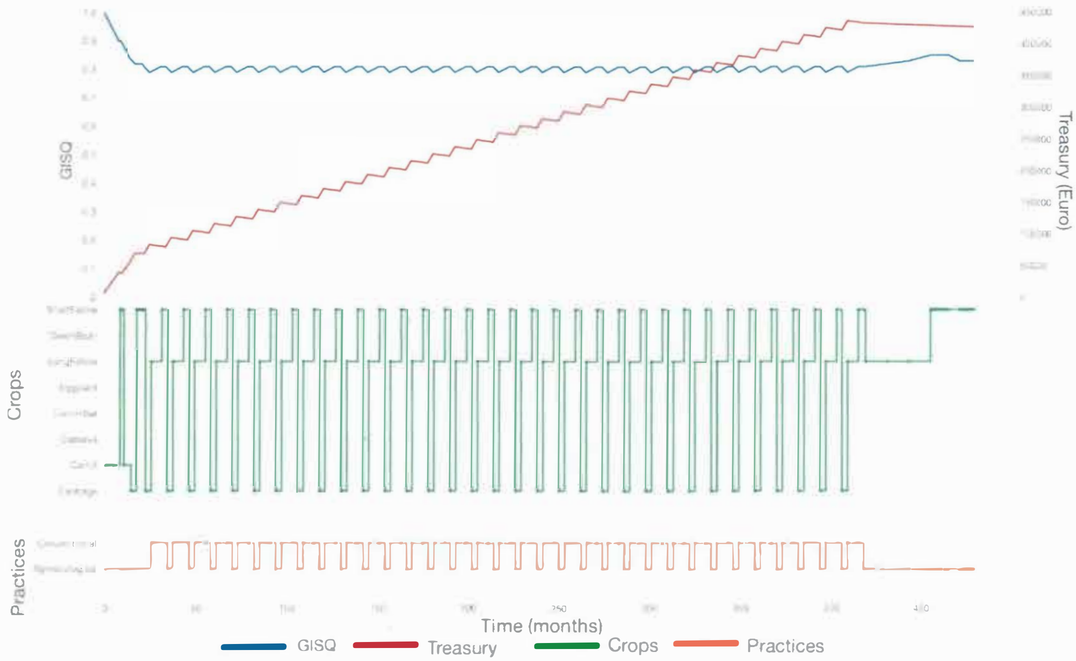

Figure 2.5.11: Evolution of the system for a particular realization of the uncertainty (Case C4, $T = 40$, $I^0 = 1$, $I^* = 0.8$, $p = 0.8$.)

2.6 Conclusions

The main qualitative takeaways of our study are as follows:

1. The results seem to be highly sensitive to the direct effect of hurricanes on the GISQ, which in turn strongly affects the impact of the other parameters.

2. Cases C1 and C2 are globally less efficient than the others, which suggests that the export-oriented sector is more vulnerable and less resilient to climatic uncertainties than the sector aimed at the local market. This result is quite

worrisome, given that the export-oriented sector represents a fairly large portion of the economy of small tropical islands such as Guadeloupe and Martinique.

3. Because it conflicts with profitability and food-security objectives, preserving the soil is far from an easy task. This difficulty is accentuated by climatic uncertainties, which are the most damaging hazards to soil preservation, not only in terms of cost, but also due to their direct impact on the soil itself. Our results highlight that restoration possibilities and cost are very sensitive to the direct impact of hurricanes on soil quality. Consequently, it is crucial to have studies assessing the impact of hurricanes on the GISQ (and hence on soil quality).

Future developments include collecting additional data and refining the estimations of hurricane-related parameters (impact on the GISQ, recovery time, cost, and occurrence probabilities). A second interesting extension would be to consider a multi-parcel setting to assess the potential advantages of multi-cropping, both in terms of efficiency and risk management. Finally some cropping systems, such as monocultures, often lead to gradual soil degradation, but this can be decreased or even reversed to a certain extent with no or minimum tillage, cover cropping and organic amendment strategies. It would be then interesting to take into account, in future research, the effect of successive use of the same crop or type of crop.

Appendix

2.A Lexicon

Capture basin: The set of all initial states of the system from which there exists at least one evolution that satisfies the viability constraints and reaches the target at the end of the time horizon with a certain probability.

Agricultural practice / type of agriculture: The type of practices adopted for the agricultural activities. We considered two types: **conventional** and **agroecological**.

Conventional practice: Agricultural practice based on modern practices/tools and on chemical fertilization.

Agroecological practice: Traditional practice based on organic fertilization of the soil or other practices for improved management of agroecosystems. Wezel et al. [24]

Fallow: Leaving land fallow consists in leaving it unsown for a certain period of time. We considered two types of fallow: the one coupled with the conventional practice (**conventional fallow**) and the one coupled with the agroecological practice (**agroecological fallow**).

Agroecological fallow: (short fallow and free long fallow) the land is left in simple fallow.

Conventional fallow : (improved short fallow and improved long fallow) the fallow is improved with chemical additives and fertilizers.

2.B Assumptions

We made the following assumptions in our study:

A1: There can be at most one hurricane per year in September ($\beta = \gamma = 9$).

A2: The probabilities of yearly hurricane occurrence are given and memoryless (independent of the hurricane occurrence history).

A3: Due to lack of information about hurricanes, we chose arbitrary values for some parameters, that is,

- The rehabilitation time after a hurricane is 2 months after a minor hurricane and 6 months after a major one. (Since the resilience of crops is not at the center of our study, these parameters are chosen such that only the crops with multiple harvest and whose degradation is not total can recover and that the effects of a hurricane are only felt at the harvest following it)
- The rehabilitation costs after a hurricane are set to 0, i.e., these costs are covered by insurance policies whose price is already included in the fixed costs.
- The degradation caused by a hurricane on a crop with a single harvest is nonreversible, while a crop with multiple harvests recovers after the harvest that follows the hurricane strike.

A4: When a production cycle is interrupted by a hurricane, the effect of the crop on the GISQ is proportional to the crop's age (to the time the crop spent on the parcel).

A5: The maximum degradation of all the crops is $\theta_{max} = 60\%$ (the one from which the farmer decides to replace the crop). With the data considered in the numerical applications, it corresponds to the degradation caused by a major hurricane.

A6: The direct hurricane impact on the GISQ can have three levels: no effect, moderate effect, and large effect.

2.C Parameter Values and Data

2.C.1 Agronomic Data and Parameters

For any control $(\sigma, \pi) \in \Sigma \times \Pi$, we have listed the agronomic parameters and transition functions in this section. The values of all the parameters used in this section are displayed in the tables of Section 2.C.3.

Crops Yield

- $R_M = R_M(\sigma, \pi)$: Mean yield (tons/Ha). Corresponds to the mean yield of the crop when the soil is of average quality (GISQ $= 0.5$).

- $r_i = r_i(\sigma, \pi)$: The control's sensitivity to the quality of the soil.

- $R(I, \sigma, \pi)$: The yield of the control in the absence of hurricanes.

If $r_i \neq 0.5$ the yield is given by

$$R(I,\sigma,\pi) = \begin{cases} 2R_M \left[\frac{I(1-2r_i)+r_i-\sqrt{(r_i)^2+2I(1-2r_i)}}{2r_i-1}\right] & \text{If } I \leq 0.5, \\ R_M\left[1+2r_i\frac{(1-2r_i)(I-0.5)-(1-r_i)+\sqrt{(1-r_i)^2-2(I-0.5)(1-2r_i)}}{2r_i-1}\right] & \text{Otherwise} \end{cases} \tag{2.10}$$

If $r_i = 0.5$ the yield is given by

$$R(I,\sigma,\pi) = \begin{cases} 2IR_M & \text{If } I \leq 0.5, \\ R_M(I+0.5) & \text{Otherwise} \end{cases} \tag{2.11}$$

- $RE(I,\sigma,\pi)$: The effective yield of the control.

$$RE(\sigma,\pi) = \begin{cases} 0 & \text{If } \theta \geq \theta_{max}, \\ (1-\theta)R(I,\sigma,\pi) & \text{Otherwise} \end{cases} \tag{2.12}$$

GISQ Transition Functions

- r_d: The sensitivity of the soil to the loss of organic matter by crop.
- r_p: The damage or improvement to the soil caused by the agricultural practice (effects that are dependent on the initial soil quality).
- r_{ap}: The damage or improvement to the soil caused by the agricultural practice (effects that are independent of the initial soil quality).
- $\Delta I_b(I,\sigma,\pi)$: The change in GISQ induced by the crop σ

$$\Delta I_b(I,\sigma,\pi) = \begin{cases} \frac{R(I,\sigma,\pi)}{2R_M(\sigma,\pi)} r_d(\sigma,\pi) & \text{If } R_M(\sigma,\pi) \neq 0, \\ 0 & \text{Otherwise} \end{cases} \tag{2.13}$$

- $\Delta I_p(I,\sigma,\pi)$: The change in GISQ induced by the agricultural practice π

$$\Delta I_p(I,\sigma,\pi) = I.r_p(\sigma,\pi) + r_{ap}(\sigma,\pi) \tag{2.14}$$

- $\phi(I,\sigma,\pi)$: The transition function of the GISQ in the absence of hurricanes.

$$\phi(I,\sigma,\pi) = \min\{\max\{I - \Delta I_b(I,\sigma,\pi) + \Delta I_p(I,\sigma,\pi), 0\}, 1\} \tag{2.15}$$

- $\delta(\sigma,\pi)$: The cycle length of control (σ,π) (the normal cycle duration when not interrupted).

- $D(h)$: The direct impact of a hurricane of intensity h on the GISQ.

- $\zeta(I,v,\epsilon,E)$: The transition function of the GISQ if a hurricane of intensity b hits a parcel on which the control (σ,π) is applied when the time elapsed since the last change in the GISQ is ϵ ($v = (\sigma,\pi,a,b,c)$).

$$\zeta(I,v,\epsilon,E) = \min\{\max\{I + \frac{\epsilon}{\delta(\sigma,\pi)}\phi(c,\sigma,\pi) - D(b), 0\}, 1\} \tag{2.16}$$

2.C.2 Economic Data and Parameters

The parameters values used in this section are displayed in the tables of Section 2.C.3.

For any control $(\sigma,\pi) \in \Sigma \times \Pi$, we have the following economic parameters and transition functions:

- S_a: Subsidy for the crop (€/month).

- S_p: Subsidy for the quantity produced (€/ton).

- C_t: Monthly labor cost (€/ton).

- C_i: Installation input cost (seeds, fertilizer, pesticides) (€/Ha).

- C_e: Other expenses (€/Ha).

- C_c: Harvesting cost (€/Ha).
- C_m: Maintenance cost (€/Ha/month).
- C_f: Fixed cost that are not related to a particular crop (€/Ha/month).
- P: Selling price (€/ton).
- f_c: Date of the first sale.
- p_c: The time between successive sales.
- ρ_0: The monthly recurring payoff.

$$\rho_0 = S_a - C_f - C_t - C_m \tag{2.17}$$

- $\rho(I, v, \epsilon)$: The cash flow generated during the time that elapsed since the last event.

$$\rho(I, v, \epsilon) = \begin{cases} -C_i, & \text{if } a = 0, \\ \epsilon\rho_0, & \text{otherwise,} \end{cases} \tag{2.18}$$

- $\ell(I, \sigma. \pi)$: The earning generated by each harvest sale.

$$\ell(I, v, \epsilon) = m(I, \sigma, \epsilon) - h_c(I, v, \epsilon) \tag{2.19}$$

where,

- $m(I, \sigma, \epsilon)$: The income from selling the harvest.

$$m(I, \sigma, \epsilon) = \begin{cases} RE(I, \sigma, \pi)(P + S_p) & \text{if } a = f_c \text{ or } (a - f_c) \mod P_c = 0, \\ 0 & \text{otherwise,} \end{cases} \tag{2.20}$$

- $h_c(I, v, \epsilon)$: The harvesting cost.

$$h_c(I, v, \epsilon) = \begin{cases} C_c \frac{RE(I,\sigma,\pi)}{R_M(\sigma,\pi)} & \text{if } a = f_c \text{ or } (a - f_c) \mod p_c = 0, \\ C_c \frac{RE(I,\sigma,\pi)}{R_M(\sigma,\pi)} + C_e & \text{if } a \geq \delta(\sigma, \pi), \\ 0 & \text{otherwise}, \end{cases} \tag{2.21}$$

2.C.3 Crop Data

All the parameters are normalized for one parcel of unit area (1 ha); production data are in tons per ha; economic data are provided by experts in euros per ha or per tons produced (fixed and variable input costs, depreciations, subsidies, labor, sale prices) or per month (labor). Data are extracted from Durand et al. [9] and displayed in the tables below.

Table 2.C.1 displays the planting/sowing seasons of the seasonal crops.

Table 2.C.2 displays the values of the agronomic parameters and Table 2.C.3 those of the economic parameters.

Table 2.C.1: Planting months of crops.

Crop	01	02	03	04	05	06	07	08	09	10	11	12
Plantain			x	x	x	x	x					
Export Banana			x	x	x	x	x					
Eggplant										x	x	
Tomato										x	x	

Table 2.C.2: Agronomic parameter values.

Crop and Practice	$\delta(\sigma, \pi)$	r_i	r_d	r_p	r_{ap}	f_c	p_c	R_M
Export banana Agro	60	0.75	0.2	0.12	0.02	12	12	24
Export banana Conv	60	0.3	0.5	−0.08	0	12	12	30.6
Plantain Agro	60	0.7	0.19	0.15	0	6	3	5
Plantain Conv	24	0.25	0.45	−0.02	−0.01	6	3	7.85
Cabbage Agro	3	0.5	0.21	0.15	0	3	0	13
Cabbage Conv	3	0.25	0.3	−0.02	0	3	0	18
Carrot Agro	4	0.65	0.14	0.07	0	4	0	11
Carrot Conv	4	0.05	0.15	−0.03	−0.005	4	0	14
Cassava Agro	12	0.5	0.15	0.07	0	12	0	22
Cassava Conv	12	0.1	0.2	−0.01	−0.01	12	0	25
Cucumber Agro	2	0.8	0.28	0.15	0	2	0	13
Cucumber Conv	2	0.05	0.25	−0.05	−0.025	2	0	18
Eggplant Agro	7	0.8	0.2	0.12	0	7	0	12
Eggplant Conv	7	0.15	0.2	−0.02	−0.075	7	0	20
Green beans Agro	4	0.6	0.09	0.01	0.002	4	0	10
Green beans Conv	3	0.05	0.1	−0.01	0	3	0	13
Lettuce Agro	2	0.6	0.18	0.12	0	2	0	7
Lettuce Conv	2	0.3	0.25	−0.01	0	2	0	13
Melon Agro	3	0.7	0.17	0.1	0	3	0	9
Melon Conv	2	0.2	0.35	−0.02	−0.015	2	0	16
Sugar cane Agro	60	0.3	0.08	0.08	0.035	12	12	60.5
Sugar cane Conv	60	0.1	0.05	−0.01	−0.01	12	12	63.75
Tomato Agro	5	0.8	0.2	0.13	0.002	5	0	11
Tomato Conv	4	0.1	0.4	−0.05	−0.06	4	0	15
Turban squash Agro	3	0.7	0.17	0.099	0	3	0	14
Turban squash Conv	3	0.1	0.15	−0.05	−0.02	3	0	22
Yam (yellow) Agro	8	0.6	0.18	0.11	0.002	8	0	11.7
Yam (yellow) Conv	8	0.15	0.4	−0.01	0	8	0	15
Yam (Grosse Caille) Agro	9	0.7	0.13	0.08	0.004	9	0	11
Yam (Grosse Caille) Conv	8	0.14	0.25	-0.02	-0.02	8	0	14
Short fallow	1	0	0	0	0	1	0	0
Free long fallow	12	0	0	0	0.025	12	0	0
Improved short fallow	3	0.1	0	0.01	0.01	3	0	0
Improved long fallow	6	0	0	0	0.025	6	0	0

Table 2.C.3: Economic parameter values.

Crop and Practice	S_a	S_p	C_t	C_i	C_m	P	C_c	C_e
Export banana Agro	0	400	400	15,143	250	600	8680.7	1840
Export banana Conv	0	400	392.9	17,143	366.83	550	8680.7	1840
Plantain Agro	0	0	270	5000	465	800	452	120
Plantain Conv	0	0	260	5500	465	600	452	160
Cabbage Agro	0	0	1000	2500	0	1300	1200	500
Cabbage Conv	0	0	850	2200	50	1200	1200	500
Carrot Agro	0	0	550	1000	24	1300	2800	503
Carrot Conv	0	0	500	150	54	1200	28,500	503
Cassava Agro	0	0	220	530	10	650	3390	0
Cassava Conv	0	0	221.5	528	14	610	3393	0
Cucumber Agro	0	0	1800	2800	200	850	1800	0
Cucumber Conv	0	0	1300	2500	650	790	1832	0
Eggplant Agro	0	0	600	3500	0	1300	1900	500
Eggplant Conv	0	0	300	3500	50	1100	1600	500
Green beans Agro	0	0	900	3000	0	2000	3000	500
Green beans Conv	0	0	700	2500	50	1800	6000	500
Lettuce Agro	0	0	1700	4000	0	2200	1000	500
Lettuce Conv	0	0	1700	3200	50	2000	1000	500
Melon Agro	0	0	600	5600	200	1200	387	0
Melon Conv	0	0	450	5600	750	1000	397	0
Sugar cane Agro	0	13.23	40	1928	30	60	1464	150
Sugar cane Conv	0	14.76	40	1872	50	55	1362.5	200
Tomato Agro	0	0	1800	5300	0	1600	2200	0
Tomato Conv	0	0	1100	5300	400	1300	2200	0
Turban squash Agro	0	0	850	6000	70	1000	876	0
Turban squash Conv	0	0	800	6500	130	900	876	0
Yam (yellow) Agro	0	0	1500	14,000	10	2700	2500	0
Yam (yellow) Conv	0	0	900	13,200	100	2500	2500	0
Yam (Grosse Caille) Agro	0	0	1406	11,550	0	2100	3192	0
Yam (Grosse Caille) Conv	0	0	1000	7815	82	2000	2400	0
Short fallow	0	0	0	0	0	0	0	0
Free fallow	0	0	0	0	0	0	0	0
Improved short fallow	0	0	150	400	50	0	0	0
Improved long fallow	0	0	100	800	50	0	0	0

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Chapter 3

Viability of a Multi-Parcel Agroecological System

Abstract

To satisfy an ever-rising demand, agriculture practices shifted over time from organic fertilization of soils to intensive and highly specialized farming that use chemical fertilization. The resulting short-term increase in soil productivity lead to some serious ecological drawbacks over time, e.g., degradation of soil quality, pollution of water and air and loss of biodiversity. Given this state of affairs, it is urgent to find alternative practices that preserve soil quality and at the same time ensure acceptable revenues to farmers. In this work, we rely on viability theory to determine a set of policies that allow to reach this dual objective. The proposed multi-parcel land model is applied to data from the archipelago of Guadeloupe, located in the French West Indies.

Key Words: Viability theory; Agriculture; Farming; Multi-parcel.

Introduction

This paper deals with long-term management of farms and their ability to maintain or restore the quality of their soil. The evolution over time of this quality depends on numerous factors, with some of them being controllable for by the farmer, e.g., the choice of crops and farming method, while others are not, e.g., climatic events. Our main research question can be framed as follows: given a farm of a predetermined size and a planning horizon, what choices should the farmer make in order to achieve simultaneously some ecological and economic sustainability objectives. Roughly speaking, the ecological objective corresponds to the soil's quality, while the economic one refers to revenues.

To answer our question, we rely on the mathematical theory of viability (Aubin [3]). In a nutshell, a viability problem involves a dynamical system whose evolution depends on state and control variables, and possibly on some random events. Given a set of constraints and initial state of the system, one looks for viable solutions, that is, evolutions (or trajectories) of these variables that satisfy these constraints.

Viability theory (VT) has been successfully used to determine sustainable policies in the management of ecosystems and renewable resources, e.g., fisheries and forests; see Oubraham and Zaccour [11] for a literature review. In the specific context of farming and agroecological systems, most applications of VT dealt with herds and grassland management problems (see, e.g., Tichit et al. [16], Sabatier et al. [13], Sabatier et al. [14], Mouysset et al. [10], Tichit et al. [17], Baumgärtner and Quaas [4], Martin et al. [9], Sabatier et al. [15] and Durand et al. [8]), and very few with soil quality management problems. Durand et al. [8] proposed a deterministic viability theory based model describing the evolution of a singleparcel land and looked for agricultural strategies and crop planting sequences allowing to restore the soil quality to an acceptable level, while preserving the economic profitability of the farm. Oubraham et al. [12] extended the setup to a stochastic environment to account for major climatic events that affect the evolution of the dynamical system describing the state of the farm. In this paper, we extend the deterministic model in Durand et al. [8] to a multi-parcel case to assess the potential advantages offered by a multi-cropping strategy. Our empirical study concerns the archipelago of Guadeloupe, located in the French West Indies.

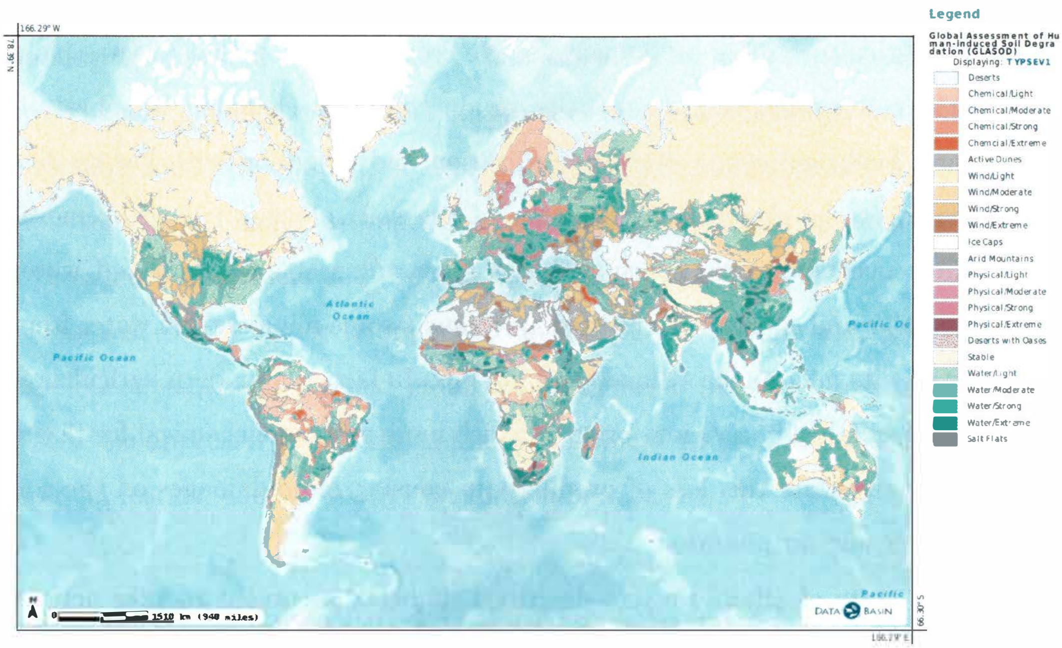

Figure 3.0.1: Global assessment of human-induced soil degradation. Source: `https://databasin.org/datasets/7254137cabb042298cae0b769cba589f`.

It is well-established that soil has been poorly protected and overexploited for decades, which has resulted in its deterioration worldwide (see Figure 3.0.1).

One of the most important human-induced factor of soil's deterioration is modern agriculture and farming. Indeed, to meet an ever growing demand, agronomic systems have had to drastically change during the last decades and have migrated to agricultural practices based on chemical fertilization of soil and intensive and specialized farming practices. This has increased soil productivity in the short term, but in the long term has caused serious ecological drawbacks (degradation of soil quality, pollution of water and air, loss of biodiversity, erosion, etc.) and even reversed the trend of the agricultural productivity. In contrast with the traditional agricultural systems based on natural fertilization and diversified crops production, the modern agriculture relies on intensive single crop production that progressively degrades the soil quality and reduces its productivity by changing its physical, chemical and biological composition. Fertilization and irrigation with low quality water imbalances the chemical composition of the soil; plowing, tillage, removal of vegetative cover and over-grazing makes it more vulnerable to wind and water erosion; intensive and specialized cultivation exhausts some minerals and water from it and damages its microfauna (Blanco and Lal [5]). In fact, the modern agricultural systems has then fallen in a vicious cycle in which more and more chemical fertilizers are used to compensate this loss of productivity causing more damages on the soil and more water and air pollution.

Given the state of affairs we just described, it becomes urgent to take actions to replace actual agricultural practices by more eco-responsible ones based on crop rotations and mixed crop-livestock associations that are healthier for the soil. We need to establish an agroecological transition that would, in the medium term, allow to return to more environmentally friendly agricultural practices and to restore soil quality to an acceptable level, while taking into account the socio-economic aspects related to the sector. This is particularly true for island regions given the importance of agriculture in their economies (Angeon et al. [2]). A survey of farmers in French

West Indies revealed that the population is aware of the problem and that farmers are now placing the soil quality at the center of concerns and are willing to make efforts, and even sacrifice some of their financial benefits, to restore the quality of their land (Angeon et al. [1]).

In our study, we consider a series of cases representing different farming systems practiced in French West Indies, and analyze their impact on soil quality and economic well-being of the farmer. Also, we conduct an extensive sensitivity analysis to assess the impact of main parameters on the results, namely, initial soil quality, planning horizon and different costs. Our contribution is at three levels. First, we develop a new multi-parcel model that provides additional flexibility in the search for sustainable solutions. Second, we design a novel algorithmic approach for computing viable solutions. Finally, we contribute empirically by answering questions that are on the agenda of farmers and decision makers in French West Indies.

The rest of the paper is organized as follows: In Section 3.1, we extend the single-parcel model in Durand et al. [8] to multiple parcels. In Section 3.2, we present the approach for obtaining viable solutions. In Section 3.3, we present our empirical results, and we briefly conclude in Section 3.4.

3.1 A multi-parcel bio-economic model

In this section, we extend the model in Durand et al. [8] to a multi-parcel land.

Consider a farm with N_P land parcels managed over time by a single agent. The planning horizon is T and the current time is denoted by $\tau \in \mathcal{T} = \{0, ..., T + T_\Delta\}$ where T_Δ is an extra time period by which the planning horizon can be extended to complete an ongoing agricultural cycle.

Each parcel $p \in \{1, ..., N_P\}$ is characterized by its area α_p (in Ha) and its soil quality which is an agronomic measure. Evaluating the quality of a soil is a complex operation, as it involves a series of physical, chemical, and biological characteristics. Here, we adopt the *General Indicator of Soil Quality* (GISQ), which has values in the interval $[0, 1]$ and is based on 54 variables measuring these characteristics (Velásquez et al. [18] and Camacho et al. [6]). Let $\mathcal{I}_p$ be the GISQ value of parcel p, with $\mathcal{I}_p \in \mathscr{I} = \{0, ..., N_{\mathcal{I}} - 1\}$, that is, $\mathcal{I}$ can take a finite number of values.

The main variables describing the farm's state are: (i) the cash flow $\mathcal{W} \in \mathbb{R}$, i.e., a continuous variable characterizing the financial situation; and (ii) the soil quality of all its parcels $\mathcal{I} = (\mathcal{I}_1, ..., \mathcal{I}_{N_P})$. The evolution of the soil quality of parcel $p \in \{1, ..., N_P\}$ depends on its current quality $\mathcal{I}_p$ and the following two control variables: (i) the crop σ_p grown on the parcel, with $\sigma_p \in \Sigma_p$; and (ii) the agricultural practice $\pi_p \in \Pi_p$. In the rest of the paper, we consider that the crop and practice choices are the same for all parcels that is $\Sigma_p = \Sigma = \{\sigma_1, ..., \sigma_{N_\sigma}\}$ and $\Pi_p = \Pi = \{\pi_1, ..., \pi_{N_\pi}\}$ $\forall p \in \{1, ..., N_P\}$.[1] To fix ideas, in the case studies to follow, the set of crops include either all or some of the following crops: plantain, export banana, sugar cane, yam (yellow), yam (grosse Caille), tomato, eggplant, lettuce, carrot, green bean, cabbage, cassava, melon, cucumber, and turban squash. The set of agricultural practices only includes two elements, i.e., conventional and agroecological practices.

Let the evolution of the soil quality be described by the following function:

[1] Differences in choice sets can be due to geographical or physical differences between the parcels. For instance, the slope or exposure to wind can make it impossible to grow some crops, and parcel accessibility can make it impossible to use some equipment.

$$\begin{aligned} \phi \ : \ \mathscr{I} \times \Sigma \times \Pi &\rightarrow \mathscr{I}, \\ (I, \sigma, \pi) &\mapsto \phi(I, \sigma, \pi). \end{aligned}$$

Denote by $\delta(\sigma, \pi)$ the duration of the whole agricultural cycle of a crop σ using agricultural practice π. Then, to each pair (σ, π), we can associate production cycles ψ defined as follows:

$$\begin{aligned} \psi \ : \ \Sigma \times \Pi &\rightarrow [0, \delta(\sigma, \pi)], \\ (\sigma, \pi) &\mapsto \psi(\sigma, \pi). \end{aligned}$$

Moreover, to each crop and practice (σ, π), we associate its first harvest time $f_c(\sigma, \pi)$ and the time duration between two successive harvests $p_c(\sigma, \pi)$. The duration of the first production cycle is $f_c(\sigma, \pi)$ and the following cycles last $p_c(\sigma, \pi)$. If a crop has a single harvest, then $f_c(\sigma, \pi) = \delta(\sigma, \pi)$ and $p_c(\sigma, \pi) = 0$. Also, we suppose that the produce is sold right away after it is harvested.

The revenue from each harvest depends on the crop and the agricultural practice, as well as on the soil quality at the beginning of the cycle. Formally, the revenue function ℓ is defined by

$$\begin{aligned} \ell \ : \ \mathscr{I} \times \Sigma \times \Pi &\rightarrow \mathbb{R}, \\ (I, \sigma, \pi) &\mapsto \ell(I, \sigma, \pi). \end{aligned}$$

Denote by $s \in \mathcal{S} = \{1, ..., 12\}$ the current month. This discrete variable is needed to deal with the seasonality of some crops, which can be planted or sown only at specific times of the year. Let $\Sigma(s) \subset \Sigma$ be the set of all crops that can be planted or sown during season s.

In the single-parcel case studied in Durand et al. [8], the only event that must be accounted for over time is the beginning or end of a production cycle. If we refer by n an event and by $\tau(n)$ its timing, then the next event will happen at

$$\tau(n+1) = \tau(n) + \psi(\sigma, \pi). \tag{3.1}$$

This formulation does not hold when we have multiple parcels as each one of them has its own crop sequence and agricultural practice. This implies that at any time, there are as many production cycles (independent, simultaneous and potentially of different duration) as parcels in the farm, and the next evolution of the system should coincide with the nearest event on the parcels. Consequently, (3.1) needs to be modified to account for such events. To do so, we introduce the following new variables:

- $e_p, p \in \{1, ..., N_P\}$: the remaining time before the next event on parcel $p \in \{1, ..., N_P\}$.
- $\epsilon = \min_{p \in \{1,...,N_P\}} e_p$: The remaining time before the next change in the system's state. (The remaining time before the nearest event on the parcels).

Denote by e the vector of events $e = (e_1, ..., e_{N_p}, \epsilon)$.

Let $a_p(n)$ be the age of the crop on parcel p at step n and $b_p(n)$ the state of the soil at the beginning of the agricultural cycle of $\sigma(n)$. The control variable on parcel p at that step is

$$v_p(n) = (\sigma_p(n), \pi_p(n), a_p(n), b_p(n)) \in \Sigma \times \Pi \times [0, \delta(\sigma_p, \pi_p)] \times \mathscr{I}.$$

The new parcel's GISQ value is given by

$$\mathcal{I}_p(n+1) = \varsigma\left(\mathcal{I}_p(n), v_p(n), \epsilon(n)\right),$$

and the earnings generated by control $v(n)$ on a parcel during the time lapse $\epsilon(n)$ is defined by

$$\mathcal{L}(\mathcal{I}, v(n), \epsilon(n)) = g(\sigma(n), \pi(n), a(n), b(n), \epsilon(n)),$$

that is, the earnings depend on the type of crop $\sigma(n)$, its age $a(n)$, the state of the soil when it was planted $b(n)$, the agricultural practice $\pi(n)$, and the time elapsed since the last event $\epsilon(n)$.

3.1.1 Dynamical system

Denote by $v = (v_1, ..., v_{N_P}) \in V^{N_P}$ the control vector, where the control variable is given by

$$v_p(n) = (u_p(n), a_p(n), b_p(n)), v_p(n), \quad \text{for } p \in \{1, ..., N_P\},$$

with $u_p = (\sigma_p, \pi_p) \in U = \Sigma \times \Pi$. To account for crop seasonality, we define by

$$U(s) \subset U, \quad U(s) = \Sigma(s) \times \Pi,$$

the set of possible controls on parcels during a particular season s.

Denote by $x = (\mathcal{I}, \mathcal{W}, e, s, \tau)$ the state vector. The evolution of the farm is then governed by the following discrete-time dynamical system $\mathcal{F}$:

$$
\mathcal{F}:\begin{cases}
\tau(n+1)=\tau(n)+\epsilon(n) & \text{(3.2a)}\\
s(n+1)=(s(n)+\epsilon(n))\ mod\ 12 & \text{(3.2b)}\\
\mathcal{I}_p(n+1)=\begin{cases}\zeta(\mathcal{I}_p(n),v_p(n),\epsilon(n)) & \text{If } \epsilon(n)=e_p(n)\\ \mathcal{I}_p(n) & \text{Otherwise}\end{cases} & \text{(3.2c)}\\
\mathcal{W}(n+1)=\mathcal{W}(n)+\sum_{p=1}^{N_P}\mathcal{L}(\mathcal{I}_p(n),v_p(n),\epsilon(n)) & \text{(3.2d)}\\
v_p(n+1)\in\begin{cases}\{(\sigma,\pi,0,\mathcal{I}(n+1))|(\sigma,\pi)\in U(s(n+1))\} & \text{if } \epsilon(n)=e_p(n) \text{ and}\\ & a_p(n)=\delta(\sigma_p(n),\pi_p(n)).\\ \{(\sigma_p(n),\pi_p(n),a_p(n)+\epsilon(n),b_p(n))\} & \text{otherwise,}\end{cases} & \text{(3.2e)}\\
e_p(n+1)=\begin{cases}p_c(\sigma_p(n),\pi_p(n)) & \text{If } \epsilon(n)=e_p(n) \text{ and}\\ & a_p(n)<\delta(\sigma_p(n),\pi_p(n))\\ f_c(\sigma_p(n+1),\pi_p(n+1)) & \text{If } \epsilon(n)=e_p(n) \text{ and}\\ & a_p(n)=\delta(\sigma_p(n),\pi_p(n))\\ e_p(n)-\epsilon(n) & \text{Otherwise}\end{cases} & \text{(3.2f)}\\
\epsilon(n+1)=\min_{p\in\{1,\dots,N_P\}} e_p(n+1) & \text{(3.2g)}
\end{cases}
$$

The initial state is given by $x(0)=(\mathcal{I}_1(0),...,\mathcal{I}_{N_P}(0),\mathcal{W}(0),e(0),s(0),\tau(0))$,where $(\mathcal{I}_p(0))_{p\in\{1,\dots,N_P\}}$, $\mathcal{W}(0)$ and $s(0)$are given parameters, $\epsilon(0)=\min_{p\in\{1,\dots,N_P\}} e_p(0)$, $\tau(0)=e_p(0)=0$ and $b_p(0)=\mathcal{I}_p(0)$ for all $p\in\{1,\dots,N_P\}$.

Equation (3.2a) sets the clock on time at each step $(n+1)$ by advancing it with $\epsilon(n)$ (the time that elapsed between steps n and $n+1$). Equation (3.2b) does the same thing with the season while taking into account its cyclicality. Equation (3.2g) serves to update the event vector at each step.

The update of the other variables depends on the places where current events take place. At any step n, if the remaining time until the next event on a certain parcel p is equal to the remaining time until the next event on the whole system ($e_p(n) = \epsilon(n)$), then it means that the event in question (at step $n+1$) will occur on that parcel, which indicates the end of a production cycle on it and leads to the modification of the soil quality at that place (see 3.2c). If the production cycle that just ended on the parcel is not the last one ($a_p(n) < \delta(\sigma_p(n), \pi_p(n))$), then the crop and practice applied on that parcel remain the same. We only update the age of the crop in the control variable (see 3.2e) and a new production cycle starts right away, with the remaining time until the next event on that same parcel being $p_c(\sigma_p(n), \pi_p(n))$ (see 3.2f). Otherwise, if it is the whole agricultural cycle that ended ($a_p(n) = \delta(\sigma_p(n), \pi_p(n))$), then a new crop and agricultural practice $v(n+1)$ have to be chosen from the set $U(s(n+1))$ for this parcel as expressed in (3.2e). The new time until the next event on that parcel is equal to the length of the first production cycle of the new crop, that is, $f_c(\sigma_p(n+1), \pi_p(n+1))$; (see 3.2f).

Now, if the event does not occur on that parcel ($e_p(n) \neq \epsilon(n)$), then the cycle already in progress cannot be interrupted. Consequently, the control and the soil quality of this parcel remain the same and only the age of the crop is updated (see 3.2c and 3.2e). The remaining time until the next event just has to be reduced by the time that elapsed between steps n and $n+1$ (see 3.2f).

Finally, equation (3.2d) updates the economic state of the farm by accumulating at each step the earnings generated by the parcels where a production cycle ends.

3.1.2 A viability problem

When making the choice of crops and agriculture practices on the different parcels, the farmer aims at achieving the following two objectives:

1. Ensuring an acceptable income throughout the whole planning horizon.

2. Restoring the quality of the soil to a desired level by the terminal date of the planning horizon.

The first objective is economic, and it can clearly take different forms depending on how one defines "acceptable" income. One practical option is to define a minimum threshold W_{min} below which cash flow should not fall at any step. Consequently, we have the following admissible set:

$$\mathcal{K} = \{(\mathcal{I}, \mathcal{W}, e, s, \tau) \| \tau \leq T, \mathcal{W} - W_{min} \geq 0\}. \tag{3.3}$$

In the numerical applications, we set $W_{min} = 0$ and then require to have a nonnegative treasury $\mathcal{W}$ over time.

The second objective is bio-ecological and embeds a long-term concern of maintaining the resource (land) in an acceptable state. For instance, the objective may consist in restoring the quality of all the farm's parcels to a certain desired level $\mathcal{I}^*$ by the end of the time horizon. This can then be translated into a target to reach, i.e.,

$$\mathcal{C} = \{(\mathcal{I}_1, ..., \mathcal{I}_{N_P}, \mathcal{W}, e, s, \tau) \| \tau \geq T, \mathcal{I}_p \geq \mathcal{I}^* \ \forall p \in \{1, ..., N_P\}\}. \tag{3.4}$$

Restoring the quality of all parcels to a desired level may be too ambitious in some situations, especially when the initial qualities of some parcels are very low and the time horizon is not long enough. In such case, one can adopt a less demanding target that requires improving the quality of all parcels by a certain level (percentage) d. The target set is then defined as follows:

$$\mathcal{C}_d = \left\{ (\mathcal{I}_1, ..., \mathcal{I}_{N_P}, \mathcal{W}, e, s, \tau) \| \tau \geq T, \mathcal{I}_p \geq \min\{d.\mathcal{I}_p(0) \ ; \ 1\} \ \forall p = \overline{1, N_P} \right\}. \qquad (3.5)$$

Finding the controls that satisfy some constraints taking into account the evolution of the dynamical system can be framed as a viability problem. More specifically, we aim at determining the initial states

$$X_0 = (\mathcal{I}_1(0), ..., \mathcal{I}_{N_P}(0), \mathcal{W}(0), e(0), s(0), \tau(0)),$$

of system $\mathcal{F}$ for which there exists at least one viable evolution, i.e., that remains in $\mathcal{K}$ (3.3) during the entire planning horizon, and reaches the target $\mathcal{C}$ (3.4) or $\mathcal{C}_d$ (3.5) at the end of that time horizon. This set is the ***viability kernel***, and is formally defined as follows:

$$Ker_{\mathcal{F}}(\mathcal{K}, \mathcal{C}) = \{X_0 \in \mathcal{K} \| \exists x(.) \text{ s.t. } x(0) = X_0 \ \& \ \exists t > 0 \text{ s.t. } x(t) \in \mathcal{C} \ \& \ \forall n \in [0, t], x(n) \in \mathcal{K}\}. \qquad (3.6)$$

3.2 Solution method

To compute the viability kernel defined in (3.6), we exploit the following property established in Proposition 13 and Corollary 14: if it is possible (not possible) to reach the target with a certain initial treasury, then it is possible (not possible) to reach it with a higher (lower) treasury.

As $\tau(0) = 0$, $s(0) = 1$, and $e(0) = 0$ are given data, we can characterize any initial state X_0 only by $\mathcal{I}(0) = (\mathcal{I}_1(0), ..., \mathcal{I}_{N_P}(0))$ and $\mathcal{W}(0)$ values, and write

$$X_0 = (\mathcal{I}(0), \mathcal{W}(0)).$$

Proposition 13. *Let $(\mathcal{I}, \mathcal{W})$ be an initial state from the viability kernel $Ker_{\mathcal{F}}(\mathcal{K}, \mathcal{C})$, then any initial state $(\mathcal{I}, \mathcal{W}')$ with $\mathcal{W}' \geq \mathcal{W}$ belongs to $Ker_{\mathcal{F}}(\mathcal{K}, \mathcal{C})$. Formally,*

$$(\mathcal{I}, \mathcal{W}) \in Ker_{\mathcal{F}}(\mathcal{K}, \mathcal{C}) \Rightarrow (\mathcal{I}, \mathcal{W}') \in Ker_{\mathcal{F}}(\mathcal{K}, \mathcal{C}) \quad \forall \;\; \mathcal{W}' \geq \mathcal{W}.$$

Proof. Let $B(x, I)$ be the minimum budget needed for the evolution $x(\cdot)$ to be viable, i.e.,

$$B(x, \mathcal{I}) = \min\{\mathcal{W} \| x(0) = (\mathcal{I}, \mathcal{W}) \;\&\; \exists t > 0 \text{ s.t. } x(t) \in \mathcal{C} \;\&\; \forall n \in [0, t], x(n) \in \mathcal{K}\}$$

We have

$$(\mathcal{I}, \mathcal{W}) \in Ker_{\mathcal{F}}(\mathcal{K}, \mathcal{C}) \quad \Leftrightarrow \quad \exists \; x(\cdot) \text{ s.t. } B(x, \mathcal{I}) \leq \mathcal{W}.$$

Therefore,

$$\forall \; \mathcal{W}' \geq \mathcal{W}, \quad B(x, \mathcal{I}) \leq \mathcal{W}',$$

and consequently

$$(\mathcal{I}, \mathcal{W}') \in Ker_{\mathcal{F}}(\mathcal{K}, \mathcal{C}).$$

$\square$

Corollary 14. *If it is not possible to reach the target from an initial state $(\mathcal{I}, \mathcal{W})$, then it is not possible to reach it with less initial treasury.*

$$(\mathcal{I}, \mathcal{W}) \notin Ker_{\mathcal{F}}(\mathcal{K}, \mathcal{C}) \Rightarrow (\mathcal{I}, \mathcal{W}') \notin Ker_{\mathcal{F}}(\mathcal{K}, \mathcal{C}) \quad \forall \;\; \mathcal{W}' \leq \mathcal{W}$$

The results in Proposition 13 and Corollary 14 imply that in order to completely characterize the viability kernel, it is sufficient to identify its border or, in other words, to find for each initial GISQ vector $\mathcal{I}$ the minimum initial treasury $\mathcal{W}_{inf}(\mathcal{I})$ needed for the restoration, i.e.,

$$\mathcal{W}_{inf}(\mathcal{I}) = \inf_{(\mathcal{I},\mathcal{W}) \in Ker_{\mathcal{F}}(\mathcal{K},\mathcal{C})} \mathcal{W} = \min_{x} B(x, \mathcal{I}).$$

Therefore, the problem amounts at finding a viable evolution of the system $x(\cdot)$ with the minimum possible restoration budget $B(x, \mathcal{I})$. This problem can be formulated as a shortest path problem, where the objective is to minimize the restoration budget, and which can be solved by mixed-integer programming (MIP). Now, we introduce the network describing the system and the MIP modelling. We use Cplex software with a column generation approach to solve the MIP.

3.2.1 The network

The network representing the possible viable evolutions of a parcel $p \in \{1, ..., N_p\}$ is an oriented graph $G^p = (V^p, A^p)$ where:

V^p is the set of vertices. Each vertex $v^p = (t, i, u, e) \in V^p$ is identified by:

- its time : $T(v^p) = t$.
- its GISQ : $I(v^p) = i$.
- its control: $U(v^p) = u = (\sigma(v^p), \pi(v^p)) \in \Sigma \times \Pi$.
- its type: $E(v^p) = e \in \{\alpha, \beta, \gamma, SN, TN\}$, where
  - α : start of an agricultural cycle (the start of the first production cycle).

- β : intermediate harvest (start/end of an intermediate production cycle).
- γ : final harvest (end of the agricultural cycle, i.e., the last production cycle).
- SN : source node.
- TN : sink node.

Remark 15. *There is no need to identify the γ vertices by their control. They are common to all cycles ending at the same time with the same GISQ. The source and sink nodes are particular vertices that do not have any of the characteristics of the other vertices.*

A^p is the set of arcs. Each arc $a^p_{i,j} \in A^p$ links the vertex $v^p_i \in V^p$ to $v^p_j \in V^p$ and has the following characteristics:

- $b^p_{i,j}(t_1, t_2)$: cost on arc $a^p_{i,j} \in A^p$ over the time period $[t_1, t_2]$.
- $m^p_{i,j}(t_1, t_2)$: wealth generated by arc $a^p_{i,j} \in A^p$ over the time period $[t_1, t_2]$.
- $d^p_{i,j}(t_1, t_2)$: change in the GISQ induced by arc $a^p_{i,j} \in A^p$ over the time period $[t_1, t_2]$

The network $G^p = (V^p, A^p)$ is constructed as follows:

Step 1. Initialize $V^p = \{v = (0, I_p(0), u, \alpha)$ such that $u = (\sigma, \pi) \in \Sigma(1) \times \Pi\}$.

Step 2. For each vertex $i \in V^p$, create its outgoing arcs $a_{i,j}$. An arc $a_{i,j}$ is created, if it satisfies one of the following conditions:

C1. $i \in V^p$ with $E(i) = \alpha$ and $p_c(U(i)) = 0$, and $E(j) = \gamma$, $T(j) = T(i) + f_c(U(i))$, and $I(j) = \min\{\max\{I(i) + d_{i,j}(T(i), T(j)) \,;\, 0\} \,;\, 1\}$.

C2. $i \in V^p$ with $E(i) = \alpha$ and $p_c(U(i)) \neq 0$, and $E(j) = \beta$, $T(j) = T(i) + f_c(U(i))$, $I(j) = \min\{\max\{I(i) + d_{i,j}(T(i), T(j))\ ;\ 0\}\ ;\ 1\}$, and $U(j) = U(i)$.

C3. $i \in V^p$ with $E(i) = \beta$ and $a(i) = \delta(U(i) - p_c(U(i)))$, and $E(j) = \gamma$, $T(j) = T(i) + p_c(U(i))$, and $I(j) = \min\{\max\{I(i) + d_{i,j}(T(i), T(j))\ ;\ 0\}\ ;\ 1\}$.

C4. $i \in V^p$ with $E(i) = \beta$ and $a(i) < \delta(U(i) - p_c(U(i)))$, and $E(j) = \beta$, $T(j) = T(i) + p_c(U(i))$, $I(j) = \min\{\max\{I(i) + d_{i,j}(T(i), T(j))\ ;\ 0\}\ ;\ 1\}$, and $U(j) = U(i)$.

C5. $i \in V^p$ with $E(i) = \gamma$ and $T(i) < T$, and $E(j) = \alpha$, $T(j) = T(i)$, $I(j) = I(i)$, $U(j) \in \Sigma(s(T(i))) \times \Pi$ and $T(i) + \delta(U(j)) \leq T + T_\Delta$.

Step 3. For each new arc $a_{i,j}$ added to A^p, if $j \notin V^p$ then add it to V^p and go to Step 2.

Step 4. The source and sink nodes are added to V^p. The source node is linked with arcs to all the initial vertices ($v \in V^p$ $s.t$ $T(v) = 0, I(v) = I_p(0)$ and $E(v) = \alpha$). All the terminal vertices that reach the target are linked with arcs to the sink node ($v \in V^p$ $s.t.$ $T(v) \geq T, I(v) \geq I_p^*$ and $E(v) = \gamma$).

Step 5. Delete from the network all the vertices from which there exist no path to the sink node and also their incoming and outgoing arcs.

Step 1 ensures that the evolution of a parcel starts at the beginning of an agricultural cycle at time $t = 0$ with initial GISQ $I_p(0)$. Steps 2 and 3 build the network forward to include all the possible evolutions of the parcel (viable and non viable). Conditions (C1-C3) ensures the proper functioning of the process: a transition from an agricultural cycle-start vertex (of type α) goes to an intermediate harvest vertex (of type β) if the crop has multiple harvests or to a cycle-end vertex (of type γ) if the control have a single harvest (see conditions C1 and C2). If it is the last harvest, the transition from an intermediate harvest node goes to an end of cycle; otherwise, the transition is to another intermediate harvest node (see conditions C3 and C4). Finally, at the end of a production cycle, if the time horizon is reached, then the process stops; otherwise, there will be a transition to any cycle start provided that the crop can be planted at that moment and that the remaining time (including the extension time T_{Δ}) is sufficient to complete the new production cycle.

To illustrate the network building process, we consider a simple example of a parcel with initial GISQ 0.5 and a time horizon of one year ($T = 12$ months) and $T_{\Delta} = 1$ month. Suppose that we have the choice between two possible controls:

- u_1: can be planted at any time except $s = 7$ and $s = 8$, $\delta(u_1) = 3$, $f_c(u_1) = 2$, $p_c(u_1) = 1$, and it improves the GISQ by 0.2 (0.1 after the first harvest and 0.1 after the second one).
- u_2: can be planted at any time, $\delta(u_2) = 2$, $f_c(u_2) = 2$, $p_c(u_2) = 0$, and it deteriorates the GISQ by 0.1.

In the illustrative figures below, the graphs are represented on a grid where the columns represent time and rows the GISQ. Thus, representing a vertex v at the intersection of column t and row i means that $I(v) = i$ and $T(v) = t$. Vertices of type α (start of an agricultural cycle) are represented by circles, those of type β (intermediate harvests) are represented by triangles, and finally those of type γ (end of agricultural cycle) are represented by squares (orange if the time horizon is reached, and grey otherwise). We use the red color to represent the control u_1 and green color to represent the control u_2.

Figure 3.2.1 displays the result of applying Steps 1 to 3 of the network building process. The first vertices (one for control u_1 and one for control u_2) are located at $t = 0$ and GISQ $I(0) = 0.5$, and they represent agricultural cycle-starts (type α). The node of u_1 goes to an intermediate harvest node located 2 units of time later ($f_c = 2$) and 1 unit of GISQ up (u_1 improves the GISQ by 0.1 after the first harvest). Then, the intermediate harvest transits to a cycle end one unit of time later ($p_c = 1$) and improves the GISQ by 0.1. The same thing is repeated for all the agricultural cycle-start vertices of control u_1. Vertices of control u_2 transit directly to a cycle-end vertex 2 time units later, and deteriorate the GISQ by 0.1. All the cycle-end vertices that did not reach the time horizon transit to cycle-start vertices of both controls u_1 and u_2 with no time or GISQ change. The only exceptions are for times $t = 6$, 7 and $t = 11$, because u_1 cannot be planted in seasons $s(6) = 7$ and $s(7) = 8$; and the remaining time horizon at $t = 11$ is not sufficient to grow control u_1 ($t + \delta(u_1) = 11 + 3 = 14 > 13 = T + T_\Delta = 12 + 1$).

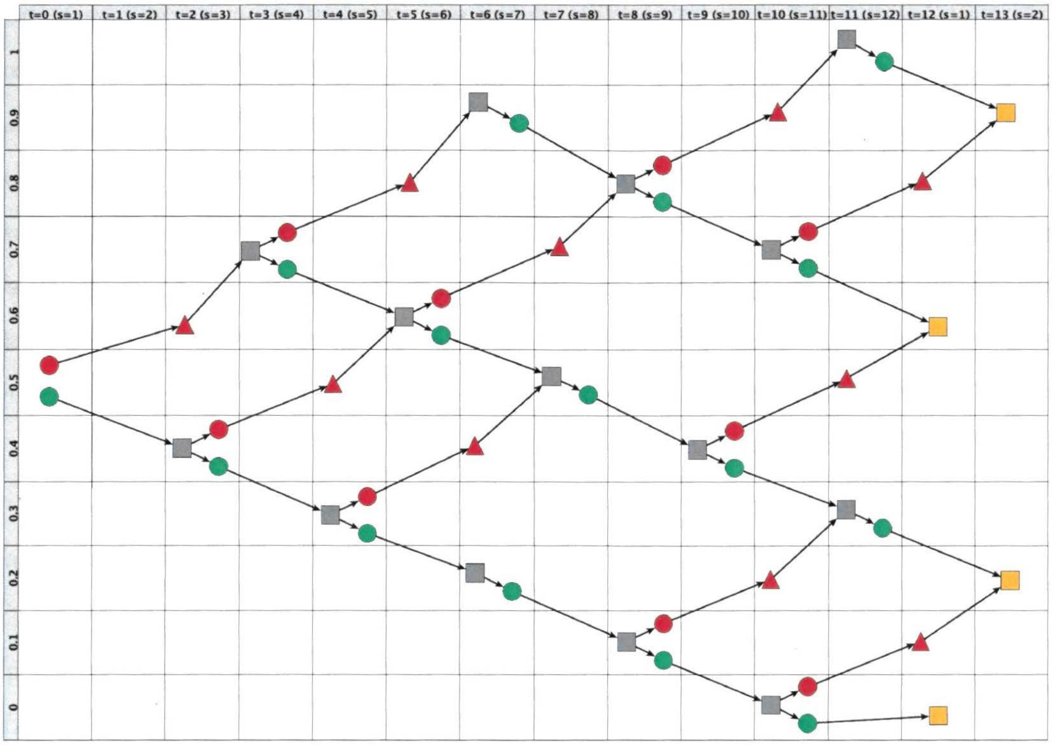

Figure 3.2.1: Steps 1 to 3

Figure 3.2.2 displays the result of applying Step 4. The source node is linked to the vertices of $t = 0$, and the end nodes that reached the target ($t \geq T$ and $GISQ \geq I^* = 0.6$) were linked to the sink node.

Finally, Figure 3.2.3 displays the actual network $G^p = (V^p, A^p)$ resulting from the application of Step 5 and only keeping the viable paths.

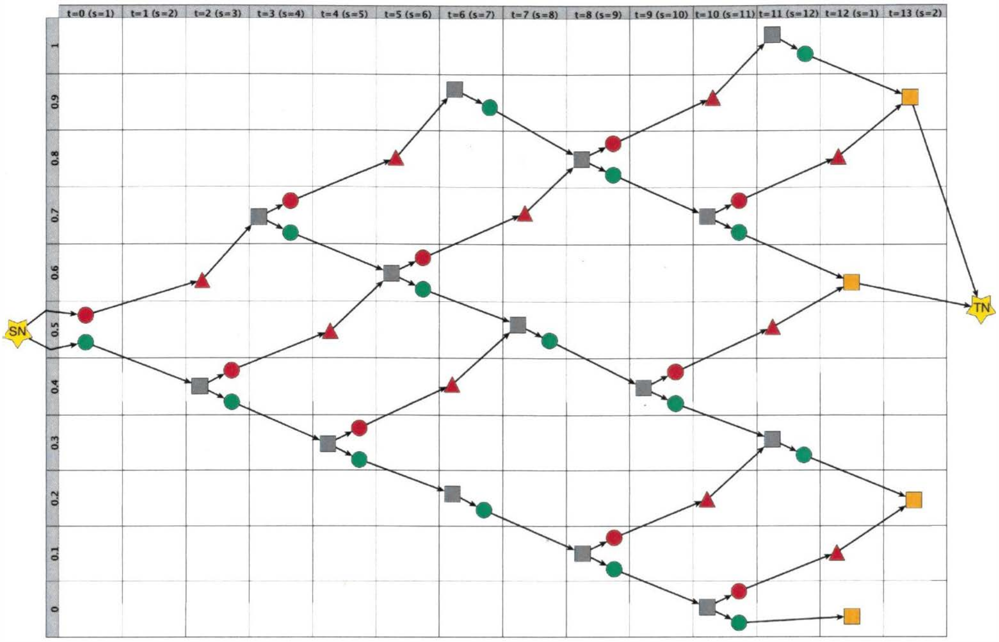

Figure 3.2.2: Step 4

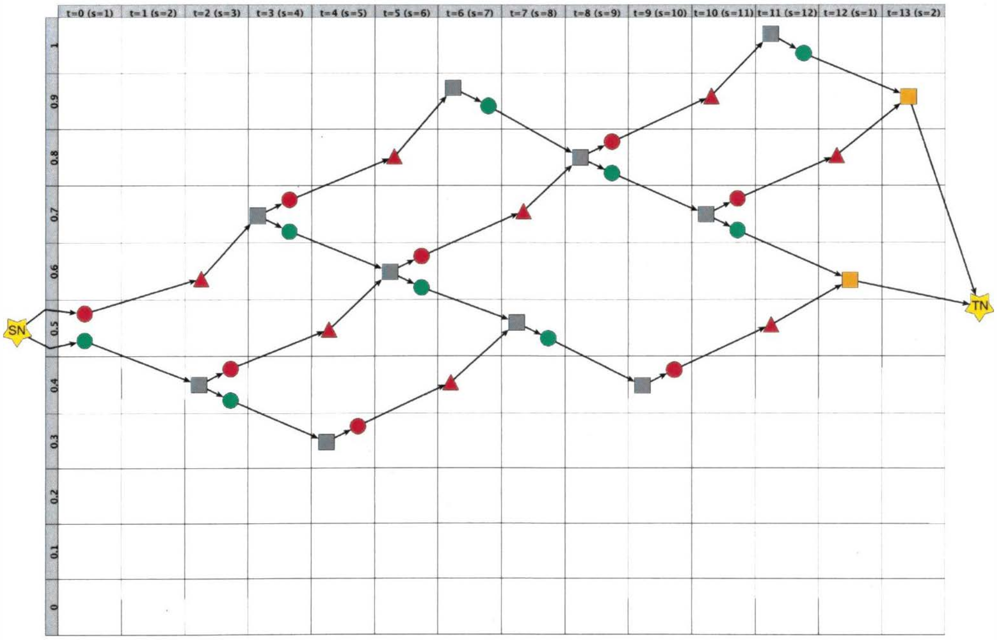

Figure 3.2.3: Step 5

3.2.2 The mixed-integer program

Let $G^p = (V^p, A^p)$ be the network representing parcel $p \in \{1, \dots, N_P\}$. Let A be the set of all arcs, and V the set of all vertices, that is,

$$A = \bigcup_{p \in \{1,\dots,N_P\}} A^p \qquad \text{and} \qquad V = \bigcup_{p \in \{1,\dots,N_P\}} V^p$$

Let N be the set of steps and $\tau(n)$ the timing of step $n \in N$

$$\tau(0) = 0 \qquad \text{and} \qquad \tau(n+1) = \min_{v \in V \text{ s.t } T(v) > \tau(n)} T(v).$$

Recall that in the dynamic system $\mathcal{F}$ (3.2a - 3.2g), each step $n \in N$ coincides with the nearest event on the parcels.

Introduce the following variables:

- $x^p_{i,j} = \begin{cases} 1, & \text{if arc } a^p_{i,j} \text{ is chosen,} \\ 0, & \text{otherwise.} \end{cases}$
- M_n: treasury available between steps $n-1$ and n, and defined by

$$M_n = M_{n-1} - S_{n-1} + \sum_p \sum_{(i,j) \in A^p_n} x^p_{i,j} m^p_{i,j}(\tau(n-1), \tau(n)), \quad \forall n \in N \text{ and } M_0 = 0.$$

- S_n: expenses during the period between steps $n-1$ and n, with the constraint

$$0 \le S_n \le M_n \quad \forall n \in N.$$

- C_n: extra budget needed to cover the expenses during the period between steps $n-1$ and n ($C_n \ge 0$, $\forall n \in N$), which is given by

$$C_n = \sum_p \sum_{(i,j) \in A^p_n} x^p_{i,j} b^p_{i,j}(\tau(n-1), \tau(n)) - S_n, \quad \forall n \in N.$$

- $A^p(t_1, t_2)$: The set of arcs of G^p that are active between t_1 and t_2. Formally,

$$A^p(t_1, t_2) = \{a^p_{i,j} \in A^p | T(i) \leq t_1 \text{ and } T(j) \geq t_2\}.$$

- A^p_n: The set of arcs of G^p that are active between $\tau(n-1)$ and $\tau(n)$, i.e.,

$$A^p_n = A^p(\tau(n-1), \tau(n)).$$

The mixed-integer program consisting in minimizing the total restoration budget over the entire planning horizon is given by

$$MIP \left\{ \begin{array}{ll} \min & \sum_{n \in N} C_n \\ s.t. & \left| \begin{array}{ll} \sum_{i|(i,j) \in A^p} x^p_{ij} - \sum_{k|(j,k) \in A^p} x^p_{jk} = \begin{cases} -1 & \text{if } j = \text{ SN}, \\ 1 & \text{if } j = \text{ TN}, \\ 0 & \text{otherwise}, \end{cases} & \forall p \text{ and } \forall j \in V^p \\ C_n = \sum_p \sum_{(i,j) \in A^p_n} x^p_{i,j} b^p_{i,j}(\tau(n-1), \tau(n)) - S_n & \forall n \in N \\ M_n = M_{n-1} - S_{n-1} + \sum_p \sum_{(i,j) \in A^p_n} x^p_{i,j} m^p_{i,j}(\tau(n-1), \tau(n)) & \forall n \in N \\ S_n \leq M_n & \forall n \in N \\ S_n \geq 0 & \forall n \in N \\ C_n \geq 0 & \forall n \in N \\ M_n \geq 0 & \forall n \in N \\ x^p_{i,j} \in \{0, 1\} & \forall p \text{ and } \forall a^p_{i,j} \in A^p \end{array} \right. \end{array} \right. \tag{3.7}$$

The first constraint represents the flow conservation over the network.

3.2.3 Column generation approach to solve the MIP

To solve the MIP defined in the previous subsection, we use Cplex with a column generation approach (Desaulniers et al. [7]). The column generation problem (CGP) is as follows:

$$
CGP \begin{cases}
\min \quad B_0 \\
s.t. \left| \begin{array}{l}
M_0 = B_0 - \sum_{i=0}^{N_P} p_j^i c_j^i(0,1) \\[2ex]
M_t = M_{t-1} - \sum_{i=0}^{N_P} p_j^i c_j^i(t,t+1) + \sum_{i=0}^{N_P} p_j^i m_j^i(t-1,t) \qquad \forall t \in \{1,2,\ldots T\} \\[2ex]
M_t \geq 0 \qquad \forall t \in \{0,1,2,\ldots T\} \\[2ex]
\sum_{j \in P(i)} p_j^i = 1 \qquad \forall i \in \{1,2,\ldots N_P\} \\[2ex]
p_j^i \in \{0,1\} \qquad \forall i \in \{1,\ldots,N_P\} \text{ and } \forall j \in P(i)
\end{array} \right.
\end{cases}
\tag{3.8}
$$

Here, we generate paths $j \in P(i)$ that respect the flow conservation constraints from the source node to the sink node in each parcel i. and minimize the initial budget required to reach the sink on all parcels. Note that p_j^i is a binary variable, which is equal to 1 if the path j on parcel i is chosen, and 0 otherwise.

$M(t)$ represents the state of the treasury at time t. Its evolution depends on the cost and revenue contributions (c_j^i and m_j^i, respectively) on the active arcs at any time. We want this treasury to be nonnegative at any time t.

3.3 Numerical results and discussion

In this section, we illustrate our model and the solution approach with a series of farming systems to which we will refer as cases. Table 3.3.1 lists the crops considered in these cases. Each crop can be used either with a conventional or agroecological practice. Also, farmers can choose to leave their land in fallow coupled with an agroecological practice (short fallow and free-long fallow) or a conventional practice (improved-short fallow and improved-long fallow). Each one of the listed cases represents a type of farming system practiced in French West Indies: the export sector is specialized in banana and sugar cane (cases C1 and C2); the local market oriented sector is based on diversified vegetable farming (C3 and C4); finally, C5 is a theoretical case that gathers all the crops.

Table 3.3.1: Description of the different cases.

Case	**Crops**
Case 1 Banana	Plantain, Export banana
Case 2 Banana & sugar cane	Plantain, Export banana, Sugar cane
Case 3 Tomato and yam	Yam(Yellow), Yam(Grosse Caille), Tomato
Case 4 Multicrop	Yam (Yellow), Yam(Grosse Caille), Tomato, Eggplant, Lettuce, Carrot, Green bean, Cabbage, Cassava, Melon, Cucumber, Turban squash
Case 5 All crops	Plantain, Export banana, Sugar cane, Yam (Yellow), Yam(Grosse Caille), Tomato, Eggplant, Lettuce, Carrot, Green bean, Cabbage, Cassava, Melon, Cucumber, Turban squash

In order to analyze the effect of each parameter, the computations have been made for different parcels sizes, initial GISQ levels and time horizons. We always require that the farm remains self-sufficient, i.e., we set the economic constraint to 0 ($W_{min} = 0$) and we impose that the retained solution(s) allows a positive treasury.

All the case studies concern the archipelago of Guadeloupe. For the common elements, we use the same data and parameter values in Durand et al. [8] and Oubraham et al. [12]. The other required data and parameter values have been estimated and displayed in Appendices 3.B and 3.C.

For the executions we considered two cases regarding the number of parcels: the **single-parcel case** where the whole land is considered as one parcel in which the farmer should apply one control at the time, and the **multi-parcel case** where the land is divided into two independent parcels on which the farmer can apply different controls. To have comprehensive comparisons, we impose that the total area in the multi-parcel case is equal the area in the single-parcel one. In the executions, we considered a 3Ha parcel in the single-parcel case and two parcels of 1Ha and 2Ha, respectively, in the multi-parcel case.

The rest of the section is divided into two parts. In the first part (Section 3.3.1), we focus on the viability kernel and the minimum restoration budgets. In particular, we analyze the impact of the time horizon (in Section 3.3.1), the impact of the initial GISQ (in Section 3.3.1) and the impact of crop diversification (in Section 3.3.1). In the second part (Section 3.3.2), we look at viable evolutions. More specifically, we discuss the main characteristics of such evolutions and how they are affected by such features as time horizon, and GISQ improvement level.

3.3.1 The viability kernels

Denote by $Ker_{C_i,T}(I^*, N)$ the viability kernel of case i for a time horizon T (in years), with I^* being the GISQ target and N the number of parcels. It represents the set of all pairs of initial GISQ I_0 and budget (initial treasury) that make it possible to reach the target while respecting the imposed constraints.

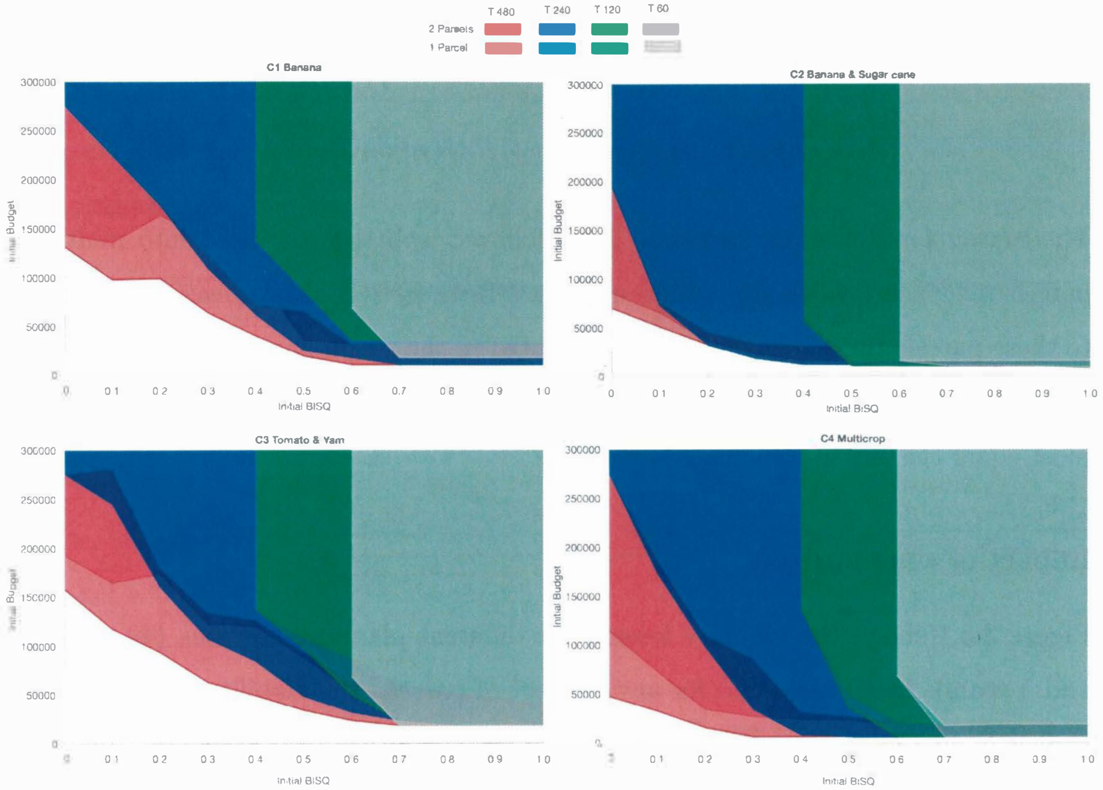

Figure 3.3.1: Viability kernels by scenario time horizons and number of parcels($I^* = 0.8$)

In Figure 3.3.1, each panel represents a superposition of viability kernels of the same problem case for different time horizons with one or two parcels. Each color represents a particular time horizon. The lightest and darkest shade of each color represents the single-parcel case and the two-parcel case, respectively.

In all cases, and for all time horizons, the viability kernel with a single parcel is always included in the two-parcel case, that is,

$$Ker_{C_i,T}(I^*, 1) \subseteq Ker_{C_i,T}(I^*, 2).$$

This result can be generalized to any number of parcels. Formally, we have

$$Ker_{C_i,T}(I^*, N_1) \subseteq Ker_{C_i,T}(I^*, N_2), \forall N_1 < N_2.$$

The reason for the above inclusion is as follows: splitting the land into multiple parcels does not involve any additional costs; further, it is always feasible to apply to these parcels the same controls implemented in the single-parcel case at the same cost. Therefore, the minimum restoration budget in the multi-parcel case will be at most equal the one in the single-parcel case.

Impact of time horizon

Figure 3.3.1 shows the viability kernels for different planning horizons (40, 20, 10, and 5 years). In all cases and for any number of parcels, the viability kernels shrink and slide to the right as the time horizon gets shorter. This indicates that the shorter the time, the more difficult and expensive it gets to restore the soil quality. Further, restoring a soil with a very low initial quality to a given desired level, may be impossible to achieve.

Intuitively, one expects the cost of soil restoration to be higher when the horizon is shorter. This is observed in the results, but not when the initial GISQ is very close to the target (or higher than the target). Indeed, in case C1 the kernel's border for

$T = 5$ is lower than that for $T = 10$. Similarly, in cases C2 and C4, the borders of the kernels for $T = 5$ and $T = 10$ are below that at $T = 20$ with a single parcel, and in case C3 the frontier of the kernel for $T = 5$ is lower than those for $T = 10$ and $T = 20$. One explanation is that when the difference in initial and target values of the GISQ is large, it requires a significant effort to improve the GISQ, and it is more difficult and pricey when the time horizon is short. However, when we start from a GISQ that is sufficiently close to or greater than the target, the effort is not to improve the GISQ but rather to maintain it over time. Maintaining the GISQ at a certain level over a short period of time is easier and cheaper than doing it over a long period.

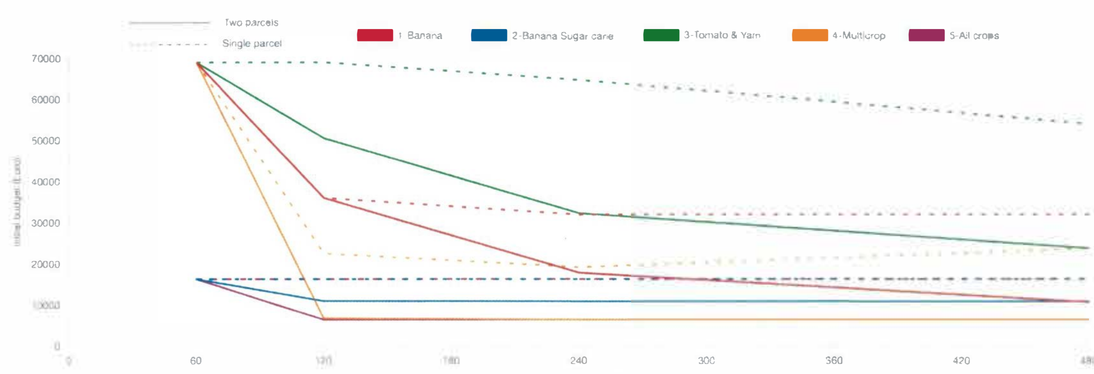

Figure 3.3.2: Total restoration cost for different time horizons

Further, we notice that the gap between the viability kernel of the single-parcel case and the multi-parcel case gets bigger as the time horizon is longer. To obtain some insight into this result, consider Figure 3.3.2 that represents the minimum budget needed for the restoration of a soil with initial GISQ of 0.6 to a GISQ target of 0.8 for different planning horizons. The solid lines represent the budgets for the multi-parcel case whereas the dotted lines those for the single-parcel case.

In all cases and any number of parcels, Figure 3.3.2 shows that the restoration budget first decreases with T and next stabilizes when the time horizon reaches a certain value. Moreover, the restoration budget in the multi-parcel case is always less or equal to the budget in the single-parcel case, which confirms the result established before, with the gap widening with the time horizon, until it stabilizes after a certain value of time horizon. This result indicates that having multiple parcels is never increasing the restoration costs, with the benefits being larger when the time horizon is longer.

To warp up, for each initial data set (problem case, initial and target GISQ), we can determine a threshold terminal date after which the benefits of multiple parcels are significant. Note that we assumed away any additional costs (e.g., management fees) induced by dividing a parcel into pieces. If such costs have to be paid, then the threshold will be more distant. In terms of computation effort, finding a solution in a single-parcel setting is much easier and faster than in the multi-parcel scenario.

Impact of initial GISQ

Figure 3.3.3 displays the evolution of the total cost of GISQ improvement by 0.2 for various initial GISQ values, for all cases in a single-parcel setting (dotted lines) and in a multi-parcel setting (solid lines), with $T = 40$ years.

The cost of improving the soil quality by a given level depends on the initial GISQ value. Figure 3.3.3 shows that, in general, the cost is lower when the initial GISQ is higher. The reason is that a crop planted in higher quality soil generates a higher income. Consequently, a lower minimum budget is needed, as the income covers a larger portion of the exploitation costs. However, we observe a different

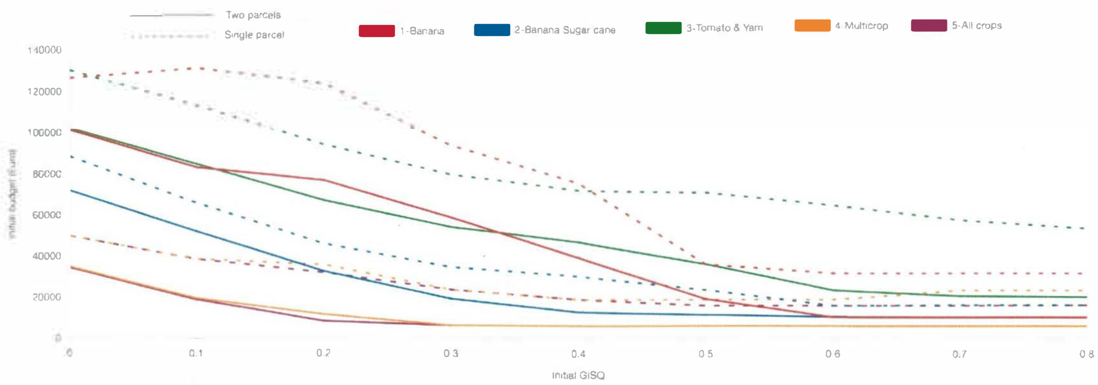

Figure 3.3.3: Total cost for GISQ improvement of 0.2 for different initial GISQ values and $T = 40$

behavior for GISQ values that are close to 0 and 1. For instance, in cases C1 and C4, the restoration cost curve is increasing near initial GISQ values of 0 and 1 for a single parcel. This result is even more visible in the representations of the viability kernels of cases C1 and C3 in Figure 3.3.1, where we observe that the costs increase a little for certain values of initial GISQ close to 0 before decreasing. One explanation is that, when the GISQ is too low, the degradation caused by a crop or a harming agricultural practice cannot be too high. On the other hand, starting with a soil of medium quality leaves room for improvements that exceed the target, in anticipation of future damage caused by more profitable crops. However, if the initial GISQ is too high, then such an opportunity is not available, which increases the restoration cost in some cases.

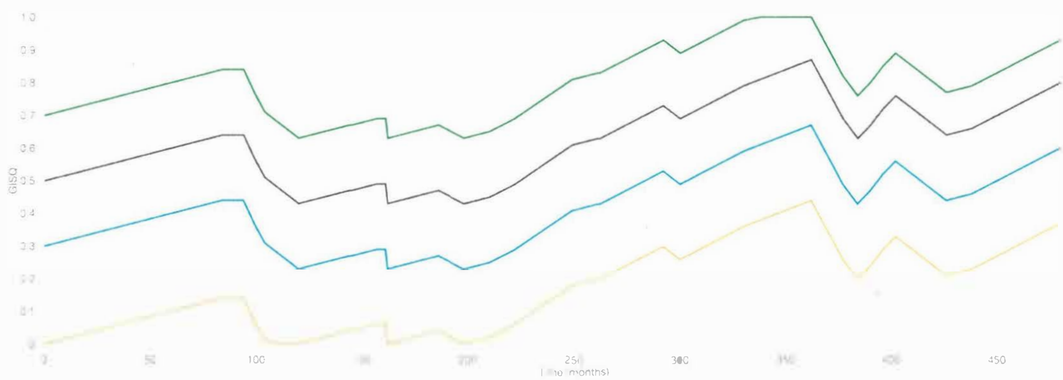

Figure 3.3.4: GISQ evolutions induced by the same control sequence for different initial GISQ

To illustrate the above findings, we report in Figure 3.3.4 the GISQ evolution induced by the same control sequence starting from different initial GISQ values (0.0 in orange, 0.3 in blue, 0.5 in grey and 0.7 in green) over 480 months. We can see that this strategy leads to an improvement of 0.3 of the GISQ (from 0.3 to 0.6 and from 0.5 to 0.8) when starting from an average initial GISQ level (0.3 and 0.5). However, if we start from a higher initial GISQ level (0.7) the improvement is only 0.23 units (from 0.7 to 9.3). On the other hand, the same strategy is more efficient when the initial GISQ is the lowest. Indeed, here the improvement is equal to 0.37 units (from 0.0 to 0.37). Finally, we note that when the initial GISQ is low, the GISQ values over time can exceed the target, which provides a buffer for possible future degradations.

Finally, we note that the results are independent of the number of parcels, as no significant difference is observed between single and multiple parcels. Moreover, the cost gap between the single parcel and multi-parcel settings is more or less the same regardless of the initial GISQ. Therefore, the benefits of having multiple parcels are invariable with respect to the initial GISQ level.

Impact of crop diversification

From Figures 3.3.1, 3.3.2 and 3.3.3 we observe that the viability kernel of a case with less crop options is always included in any viability kernel corresponding to a scenario with more crops. That is, if the set of possible controls U_i in case C_i is included in the set U_j in case C_j, then the viability kernel of C_i in contained in that of C_j for any planning horizon, i.e.,

$$U_i \subset U_j \Rightarrow Ker_{C_i,T}(I^*, N) \subseteq Ker_{C_j,T}(I^*, N), \quad \forall\, T \text{ and } N.$$

Furthermore, Figure 3.3.3 indicates that the restoration budget in a case with a larger number of crops is at most equal the budget with a lower number of crops. Formally,

$$U_i \subset U_j \Rightarrow Budget_{(C_i,T)}(I^*, N) \geq Budget_{(C_j,T)}(I^*, N), \forall\, T \text{ and } N.$$

In fact, adding a control can only help in meeting the objective. However, adding new crops to the set of possible controls does not necessarily enlarge the viability kernel or decrease the restoration budget. Indeed, if the initial set already contains the right mix of crops, adding new crops will certainly not deteriorate the solution, but not improve it either. For example, in almost all instances, the performance of C5 is identical to that of C4, even if it has more crop choices.

Finally, Figures 3.3.3 and 3.3.2 reveal that, the lower the number of crops, the more beneficial is the use of multiple parcels. Indeed, if we define by GAP_i the gap between the solid and dotted lines in case C_i , then we clearly have

$$U_i \subset U_j \Rightarrow GAP_i > GAP_j.$$

For instance, the gap between the red solid and dotted lines in these figures (C1) is larger than the gap between the blue solid and dotted lines (C2 : more crops).

3.3.2 Viable evolutions

In this section, we present the viable solutions with the minimum initial budget that lie on the boundary of the viability kernels. Figures 3.3.5, 3.3.6, 3.3.7 and 3.3.8 display the solutions for cases C1, C2, C3 and C4, respectively, for single and multiple parcels. The time horizon is $T = 40$ (480 months), initial GISQ is $I_0 = 0.6$ and the GISQ target is $I^* = 0.8$. The upper part of each figure shows the treasury and GISQ evolutions over time and the lower part displays the crop recommendations.

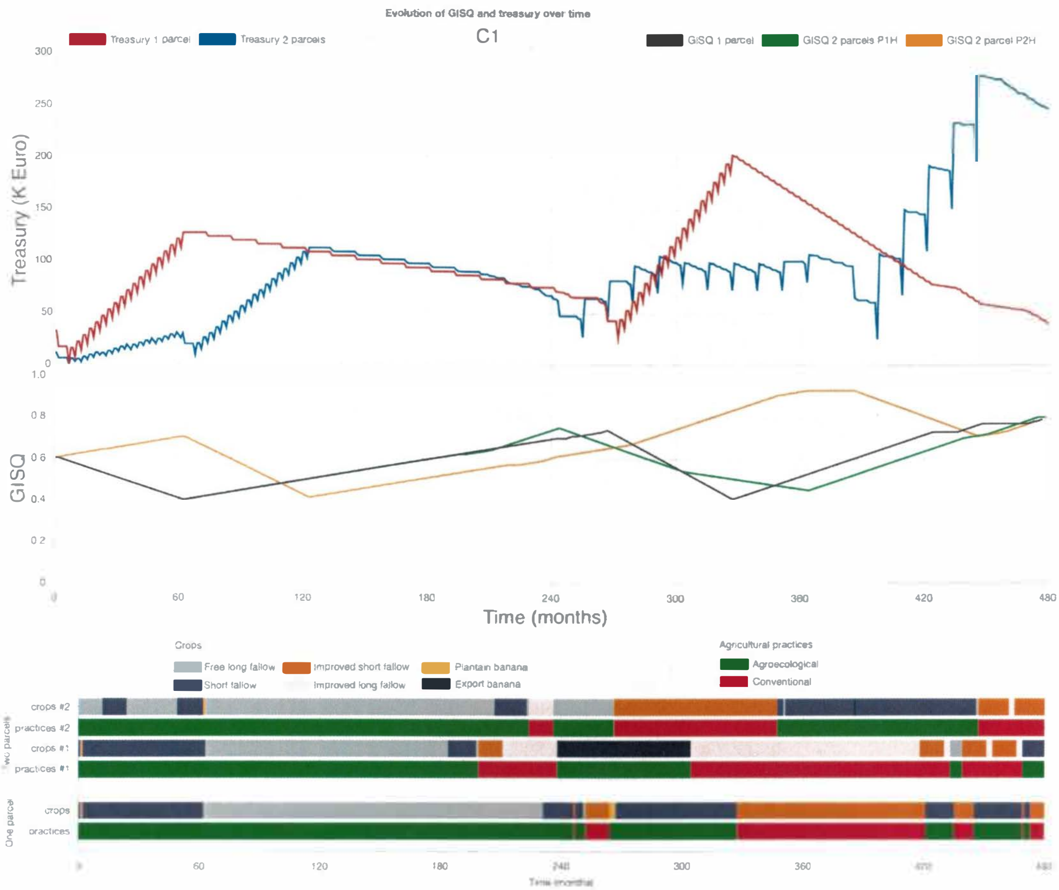

Figure 3.3.5: GISQ and treasury evolutions and crop recommandations for case C1

A first result is that, in almost all cases, the initial treasury (i.e., the restoration budget) is lower, and the final treasury significantly higher, for multiple parcels than the corresponding values for a single parcel. Also, in most cases, the GISQ evolution of one of the multiple parcels cases is quite similar to the one obtained for a single parcel. In terms of GISQ trajectories, we often observe that the GISQ of at least one of the parcels, in a multiple-parcel scenario, exceeds the target value at some intermediate dates before decreasing towards the target value at terminal date T. In

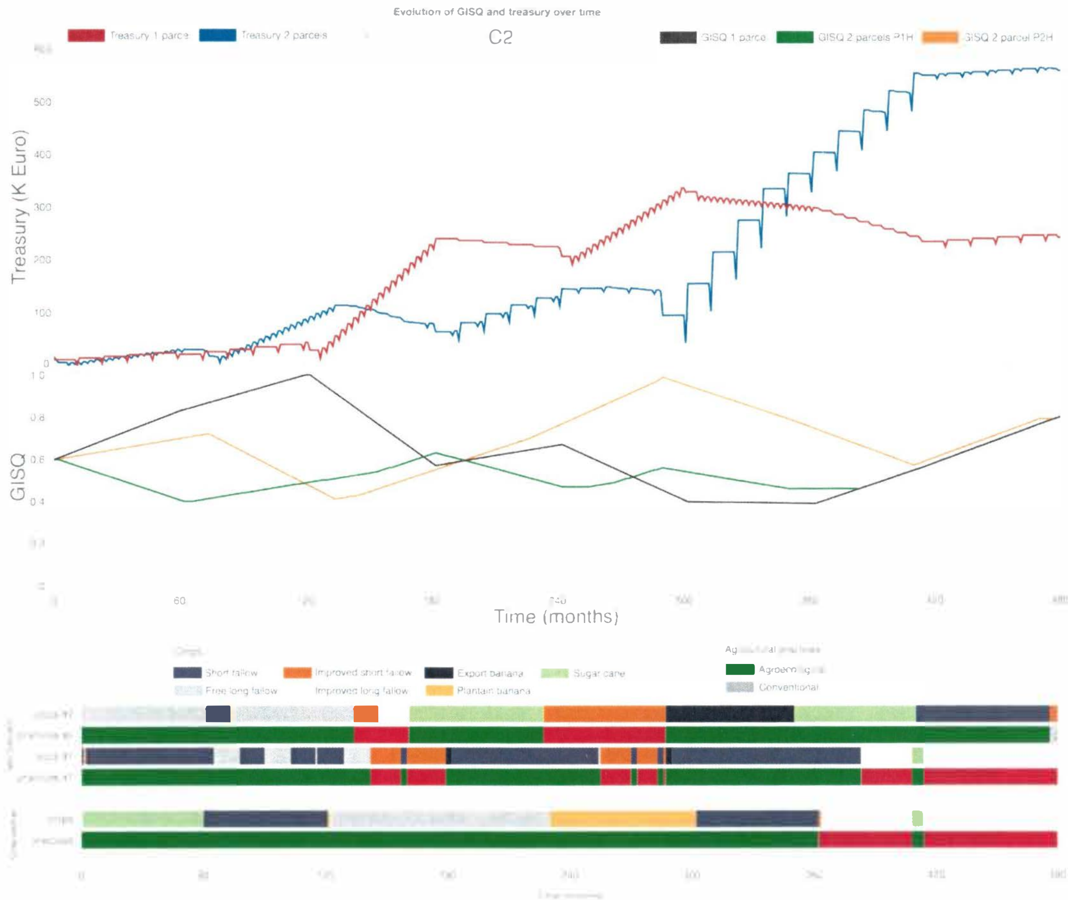

Figure 3.3.6: GISQ and treasury evolutions and crop recommandations for case C2

the single-parcel case, we see the opposite behavior, that is, in most cases the GISQ level first drops and next increases towards the target value at T. When the farm is a single parcel, then the whole land is initially used to boost the treasury needed for the restoration of the soil, before investing the accumulated amount to reach the GISQ target. In the multi-parcel setting, the farmer can dedicate one parcel to generate revenues, while the other parcel is restored. A portfolio of assets (parcels)

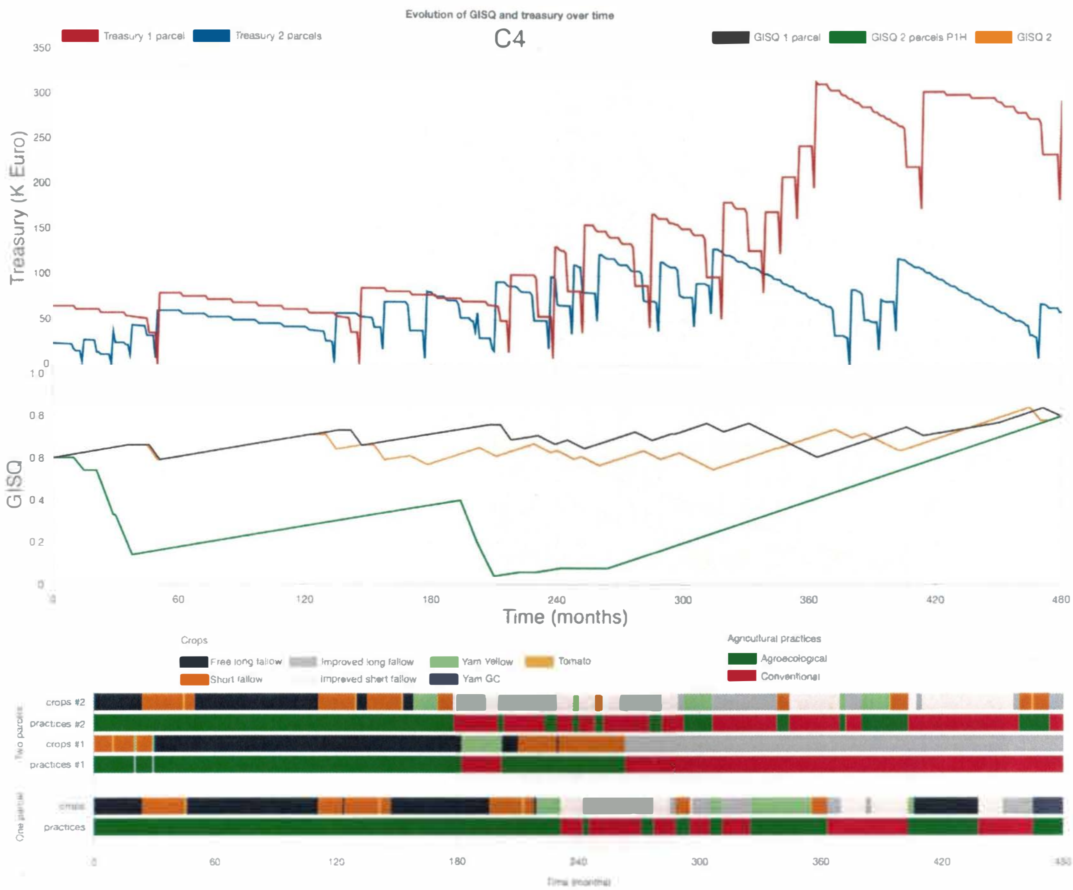

Figure 3.3.7: GISQ and treasury evolutions and crop recommandations for case C3

allows more flexibility than a single asset.

Finally, we make the following remarks on the viable evolutions and the effect of some parameters:

Crop recommendations: in all viable evolutions, there is a prevalence of fallow as a recommended control. This is due to the fact that the solutions lie on the boundary of the viability kernel. Even if investing a bit more could give

Figure 3.3.8: GISQ and treasury evolutions and crop recommandations for case C4

a higher revenue, the returned solution remains the one with the minimum budget. As the cheapest option is fallow, this is most likely to be chosen. However, we notice that in the multi-parcel setting, other controls than fallow are recommended. This result is due to the above-mentioned flexibility of parcelization.

Time horizon: when the time horizon is short, fallow becomes the prevalent recommended control (except when the initial GISQ is very close or higher than the target GISQ). Further, the recommended controls for both parcels in the multi-parcel setting are almost the same as the ones for a single parcel. As the time horizon gets longer, more non-fallow controls are recommended and the differences between the two scenarios (multiple parcels and one parcel) more pronounced.

Level of improvement: increasing the level of improvement in the GISQ has two main effects. First, it calls upon more frequent implementation of fallow controls. Second, the differences between the two scenarios (one and multiple parcels) are less visible. The reason is that a small level of improvement in the GISQ requires less effort and time to achieve, which leaves more possible paths to achieve and more time to improve the treasury. However, a higher level of improvement considerably reduces the number of possible paths to achieve it (generally only paths involving many fallow periods are left), which makes the recommendations in both settings quite similar, the initial budget high and final treasury low.

3.4 Conclusion

The main qualitative results obtained in our study are as follows:

1. The export-oriented sector (cases C1 and C2) is globally more sensitive to the number of parcels. The local-market-oriented sector also benefits from parcelization, but the gap between the results in one and two parcels is larger in the export-oriented sector.

2. Having multiple parcels leads to better results than one parcel in all cases. However, the benefit is only noticeable when the time horizon is long enough.

3. Parcelization's benefits are higher when the number of crop choices is lower.

4. If we take into consideration the final treasury generated, the multi-parcel setting is globally much better than the single-parcel one. Although we are not optimizing a specific performance index in this study (and in viability theory in general), this result can be used as a guideline for making a choice.

Future developments include collecting additional data and refining the estimations in particular the additional costs generated by parcelization (management fees for example) and other agricultural aspects like the effects of monocultures. A second extension would be to run executions on different cutouts of the land (varying the number of parcels and their areas). Finally, it would be interesting (and challenging) to take into consideration uncertainties related to market parameters, climatic events, crop diseases, ...

Appendix

3.A Lexicon

Viability kernel: the set of all initial states of the system from which there exists at least one evolution that satisfies the viability constraints and reaches the target at the end of the time horizon.

Agricultural practice / type of agriculture: the type of practices adopted for the agricultural activities. We considered two types: **conventional** and **agroecological**.

Conventional practice: agricultural practice based on modern practices/tools and on chemical fertilization.

Agroecological practice: traditional practice based on organic fertilization of the soil or other practices for improved management of agro-ecosystems. Wezel et al. [19]

Fallow: leaving land fallow consists in leaving it unsown for a certain period of time. We considered two types of fallow: the one coupled with the conventional practice (**conventional fallow**) and the one coupled with the agroecological practice (**agroecological fallow**).

Agroecological fallow: (short fallow and free long fallow) the land is left in simple fallow.

Conventional fallow: (improved short fallow and improved long fallow) the fallow is improved with chemical additives and fertilizers.

3.B Assumptions

We made the following assumptions in our study:

A1: the sum of the parcels areas in the multi-parcel setting is equal to the area of the single parcel.

A2: parcelization does not add any costs (management fees or others)

A3: the different parcels in the multi-parcel setting are completely independent. The only thing in common is the treasury that concerns the whole farm.

3.C Parameter values and data

3.C.1 Agronomic data and parameters

For any control $(\sigma, \pi) \in \Sigma \times \Pi$, we have listed the agronomic parameters and transition functions in this section. The values of all the parameters used in this section are displayed in the tables of section 3.C.4

Crops yield

- $R_M = R_M(\sigma, \pi)$: mean yield (tons/Ha). Corresponds to the mean yield of the crop when the soil is of average quality (GISQ = 0.5).

- $r_i = r_i(\sigma, \pi)$: the control's sensitivity to the quality of the soil.

- $R(I, \sigma, \pi)$: the yield of the control.

If $r_i \neq 0.5$ the yield is given by

$$R(I,\sigma,\pi) = \begin{cases} 2R_M\left[\frac{I(1-2r_i)+r_i-\sqrt{(r_i)^2+2I(1-2r_i)}}{2r_i-1}\right], & \text{if } I \leq 0.5, \\ R_M\left[1+2r_i\frac{(1-2r_i)(I-0.5)-(1-r_i)+\sqrt{(1-r_i)^2-2(I-0.5)(1-2r_i)}}{2r_i-1}\right], & \text{otherwise.} \end{cases} \tag{3.9}$$

If $r_i = 0.5$ the yield is given by

$$R(I,\sigma,\pi) = \begin{cases} 2IR_M, & \text{if } I \leq 0.5, \\ R_M(I+0.5), & \text{otherwise.} \end{cases} \tag{3.10}$$

GISQ transition functions

- r_d: the sensitivity of the soil to the loss of organic matter by crop.

- r_p: the damage or improvement to the soil caused by the agricultural practice (effects that are dependent on the initial soil quality).

- r_{ap}: the damage or improvement to the soil caused by the agricultural practice (effects that are independent of the initial soil quality).

- $\Delta I_b(I, \sigma, \pi)$: the change in GISQ induced by the crop σ

$$\Delta I_b(I,\sigma,\pi) = \begin{cases} \frac{R(I,\sigma,\pi)}{2R_M(\sigma,\pi)} r_d(\sigma,\pi) & \text{if } R_M(\sigma,\pi) \neq 0, \\ 0 & \text{otherwise} \end{cases} \tag{3.11}$$

- $\Delta I_p(I, \sigma, \pi)$: the change in GISQ induced by agricultural practice π

$$\Delta I_p(I, \sigma, \pi) = I.r_p(\sigma, \pi) + r_{ap}(\sigma, \pi). \tag{3.12}$$

- $\phi(I, \sigma, \pi)$: the transition function of GISQ (for the whole agricultural cycle)

$$\phi(I, \sigma, \pi) = \min\{\max\{I - \Delta I_b(I, \sigma, \pi) + \Delta I_p(I, \sigma, \pi), 0\}, 1\}. \tag{3.13}$$

- $\delta(\sigma, \pi)$: the cycle length of control (σ, π) (for the whole agricultural cycle).

- $\zeta(I, v, \epsilon)$: the transition function of GISQ at a step where the control $v = (\sigma, \pi, a, b)$ is applied.

$$\zeta(I, v, \epsilon) = \min\{\max\{I + \kappa(v), 0\}, 1\}, \tag{3.14}$$

where

$$\kappa(v) = \begin{cases} \frac{f_c(\sigma,\pi)}{\delta(\sigma,\pi)}\phi\ (b\sigma, \pi), & \text{if } a + \epsilon = f_c, \\ \frac{p_c(\sigma,\pi)}{\delta(\sigma,\pi)}\phi(b, \sigma, \pi), & \text{if } a + \epsilon > f_c \text{ and } (a + \epsilon - f_c) \mod p_c = 0, \\ 0, & \text{otherwise.} \end{cases}$$

3.C.2 Economic data and parameters

The values of all the parameters used in this section are displayed in the tables of section 3.C.4.

For any control $(\sigma, \pi) \in \Sigma \times \Pi$, we have the following economic parameters and transition functions:

- S_a: subsidy for the crop (€/Ha/month).

- S_p: subsidy for the quantity produced (€/ton).
- C_t: monthly labour cost (€/Ha).
- C_i: installation input cost (seeds, fertilizer, pesticides) (€/Ha).
- C_e: other expenses (€/Ha).
- C_c: harvesting cost (€/Ha).
- C_m: maintenance cost (€/Ha/month).
- C_f: fixed cost that are not related to a particular crop (€/Ha/month).
- P: selling price (€/ton).
- f_c: date of the first sale.
- p_c: the time laps between successive sales.
- ρ_0: the monthly recurring payoff, i.e.,

$$\rho_0 = S_a - C_0, \tag{3.15}$$

where C_0 is the monthly recurring cost

$$C_0 = C_f + C_t + C_m.$$

- $\ell(I, \sigma, \pi)$: the earnings generated by each harvest sale and given by

$$\ell(I, v) = R_h(I, u) - C_h(I, u), \tag{3.16}$$

where $R_h(I, u)$ is the revenue generated by the harvest and $C_h(I, u)$ is the harvesting cost, that is,

$$R_h(I, u) = R(I, \sigma, \pi)(P + S_p) \quad \text{and} \quad C_h(I, u) = C_c \frac{R(I, \sigma, \pi)}{R_M(\sigma, \pi)}.$$

- $\mathcal{L}(I, v, \epsilon)$ the earnings generated by v during the time lapse ϵ.

$$\mathcal{L}(I, v, \epsilon) = \begin{cases} \alpha_p(\epsilon\rho_0 - C_i), & \text{if } a = 0 \text{ and } a + \epsilon < f_c, \\ \alpha_p(\epsilon\rho_0 + \ell(I, v)), & \text{if } a = f_c \text{ or } (a - f_c) \mod p_c = 0, \\ \alpha_p \epsilon \rho_0, & \text{otherwise.} \end{cases} \tag{3.17}$$

3.C.3 MIP related data

Let ρ^+ be the monthly recurring income and ρ^- the monthly recurring cost of a control $u = (\sigma, \pi)$ in a 1Ha parcel:

$$\rho^+(u) = \max\{0 \ ; \ S_a(u) - C_0(u)\} \quad \text{and} \quad \rho^-(u) = \max\{0 \ ; \ C_0(u) - S_a(u)\}$$

For any arc $a^p_{i,j} \in A^p$, we have the following quantities:

- $b^p_{i,j}(t_1, t_2)$: the cost of arc $a^p_{i,j} \in A^p$ over the time period $[t_1, t_2]$, i.e.,

$$b^p_{i,j}(t_1, t_2) = \begin{cases} 0, & \text{if } E(i) \in \{\gamma, SN\} \\ \alpha_p\Big((t_2 - t_1)\rho^-(U(i)) + C_a\Big) & \text{otherwise,} \end{cases} \tag{3.18}$$

where $C_a(u)$ is the additional costs related to the following events:

if $E(i) = \alpha$ **and** $E(j) = \beta$**:**

$$C_a(u) = \begin{cases} C_i(U(i)) + C_h(I(i), U(i)), & \text{if } t_1 = T(i) \text{ and } t_2 = T(j), \\ C_i(U(i)), & \text{if } t_1 = T(i) \text{ and } t_2 < T(j), \\ C_h(I(i), U(i)), & \text{if } t_1 > T(i) \text{ and } t_2 = T(j), \\ 0, & \text{otherwise.} \end{cases}$$

if $E(i) = \alpha$ **and** $E(j) = \gamma$**:**

$$C_a(u) = \begin{cases} C_i(U(i)) + C_h(I(i), U(i)) + C_e(U(i)), & \text{if } t_1 = T(i) \text{ and } t_2 = T(j), \\ C_i(U(i)). & \text{if } t_1 = T(i) \text{ and } t_2 < T(j), \\ C_h(I(i), U(i)) + C_e(U(i)), & \text{if } t_1 > T(i) \text{ and } t_2 = T(j), \\ 0, & \text{otherwise.} \end{cases}$$

if $E(i) = \beta$**:**

$$C_a(u) = \begin{cases} C_h(I(i), U(i)), & \text{if } E(j) = \beta \text{ and } t_2 = T(j), \\ C_h(I(i), U(i)) + C_e(U(i)), & \text{if } E(j) = \gamma \text{ and } t_2 = T(j), \\ 0, & \text{otherwise.} \end{cases}$$

- $m^p_{i,j}(t_1, t_2)$: the wealth generated by arc $a^p_{i,j} \in A^p$ over the time period $[t_1, t_2]$, i.e.,

$$m^p_{i,j}(t_1, t_2) = \begin{cases} 0, & \text{if } E(i) \in \{\gamma, SN\}, \\ \alpha_p\Big((t_2 - t_1)\rho^+(U(i)) + R_h(I(i), U(i))\Big), & \text{if } E(j) \in \{\beta, \gamma\} \text{ and } t_2 = T(j), \\ \alpha_p(t_2 - t_1)\rho^+(U(i)), & \text{otherwise.} \end{cases} \tag{3.19}$$

- $d^p_{i,j}(t_1, t_2)$: the change on the GISQ induced by arc $a^p_{i,j} \in A^p$ over the time period $[t_1, t_2]$

$$d^p_{i,j}(t_1, t_2) = \begin{cases} \frac{f_c(U(i))}{\delta(U(i))}\phi(b, U(i)), & \text{if } E(i) = \alpha \text{ and } t_2 = T(j), \\ \frac{p_c(U(i))}{\delta(U(i))}\phi(b, U(i)), & \text{if } E(i) = \beta \text{ and } t_2 = T(j), \\ 0, & \text{otherwise.} \end{cases} \quad (3.20)$$

3.C.4 Crops data

All the parameters are normalized for one parcel of unit area (1 Ha); production data are in tons per Ha; economic data are provided by experts in Euros per ha or per tons produced (fixed and variables input costs, depreciations, subsidies, labor, sale prices) or per month (labor). Data are extracted from Durand et al. [8].

Table 3.C.1 displays the planting/sowing seasons of the seasonal crops.

Table 3.C.2 displays the values of the agronomic parameters and Table 3.C.3 those of the economic parameters.

Crop	01	02	03	04	05	06	07	08	09	10	11	12
Plantain			x	x	x	x	x					
Export banana			x	x	x	x	x					
Eggplant										x	x	
Tomato										x	x	

Table 3.C.1: Planting months of crops

Crop and practice	$\delta(\sigma, \pi)$	r_i	r_d	r_p	r_{ap}	f_c	p_c	R_M
Export banana Agro	60	0.75	0.2	0.12	0.02	12	12	24
Export banana Conv	60	0.3	0.5	-0.08	0	12	12	30.6
Plantain Agro	60	0.7	0.19	0.15	0	6	3	5
Plantain Conv	24	0.25	0.45	-0.02	-0.01	6	3	7.85
Cabbage Agro	3	0.5	0.21	0.15	0	3	0	13
Cabbage Conv	3	0.25	0.3	-0.02	0	3	0	18
Carrot Agro	4	0.65	0.14	0.07	0	4	0	11
Carrot Conv	4	0.05	0.15	-0.03	-0.005	4	0	14
Cassava Agro	12	0.5	0.15	0.07	0	12	0	22
Cassava Conv	12	0.1	0.2	-0.01	-0.01	12	0	25
Cucumber Agro	2	0.8	0.28	0.15	0	2	0	13
Cucumber Conv	2	0.05	0.25	-0.05	-0.025	2	0	18
Eggplant Agro	7	0.8	0.2	0.12	0	7	0	12
Eggplant Conv	7	0.15	0.2	-0.02	-0.075	7	0	20
Green beans Agro	4	0.6	0.09	0.01	0.002	4	0	10
Green beans Conv	3	0.05	0.1	-0.01	0	3	0	13
Lettuce Agro	2	0.6	0.18	0.12	0	2	0	7
Lettuce Conv	2	0.3	0.25	-0.01	0	2	0	13
Melon Agro	3	0.7	0.17	0.1	0	3	0	9
Melon Conv	2	0.2	0.35	-0.02	-0.015	2	0	16
Sugar cane Agro	60	0.3	0.08	0.08	0.035	12	12	60.5
Sugar cane Conv	60	0.1	0.05	-0.01	-0.01	12	12	63.75
Tomato Agro	5	0.8	0.2	0.13	0.002	5	0	11
Tomato Conv	4	0.1	0.4	-0.05	-0.06	4	0	15
Turban squash Agro	3	0.7	0.17	0.099	0	3	0	14
Turban squash Conv	3	0.1	0.15	-0.05	-0.02	3	0	22
Yam (yellow) Agro	8	0.6	0.18	0.11	0.002	8	0	11.7
Yam (yellow) Conv	8	0.15	0.4	-0.01	0	8	0	15
Yam (Grosse Caille) Agro	9	0.7	0.13	0.08	0.004	9	0	11
Yam (Grosse Caille) Conv	8	0.14	0.25	-0.02	-0.02	8	0	14
Short fallow	1	0	0	0	0	1	0	0
Free long fallow	12	0	0	0	0.025	12	0	0
Improved short fallow	3	0.1	0	0.01	0.01	3	0	0
Improved long fallow	6	0	0	0	0.025	6	0	0

Table 3.C.2: Agronomic parameter values

Crop and practice	S_a	S_p	C_t	C_i	C_m	P	C_c	C_e
Export banana Agro	0	400	400	15143	250	600	8680.7	1840
Export banana Conv	0	400	392.9	17143	366.83	550	8680.7	1840
Plantain Agro	0	0	270	5000	465	800	452	120
Plantain Conv	0	0	260	5500	465	600	452	160
Cabbage Agro	0	0	1000	2500	0	1300	1200	500
Cabbage Conv	0	0	850	2200	50	1200	1200	500
Carrot Agro	0	0	550	1000	24	1300	2800	503
Carrot Conv	0	0	500	150	54	1200	28500	503
Cassava Agro	0	0	220	530	10	650	3390	0
Cassava Conv	0	0	221.5	528	14	610	3393	0
Cucumber Agro	0	0	1800	2800	200	850	1800	0
Cucumber Conv	0	0	1300	2500	650	790	1832	0
Eggplant Agro	0	0	600	3500	0	1300	1900	500
Eggplant Conv	0	0	300	3500	50	1100	1600	500
Green beans Agro	0	0	900	3000	0	2000	3000	500
Green beans Conv	0	0	700	2500	50	1800	6000	500
Lettuce Agro	0	0	1700	4000	0	2200	1000	500
Lettuce Conv	0	0	1700	3200	50	2000	1000	500
Melon Agro	0	0	600	5600	200	1200	387	0
Melon Conv	0	0	450	5600	750	1000	397	0
Sugar cane Agro	0	13.23	40	1928	30	60	1464	150
Sugar cane Conv	0	14.76	40	1872	50	55	1362.5	200
Tomato Agro	0	0	1800	5300	0	1600	2200	0
Tomato Conv	0	0	1100	5300	400	1300	2200	0
Turban squash Agro	0	0	850	6000	70	1000	876	0
Turban squash Conv	0	0	800	6500	130	900	876	0
Yam (yellow) Agro	0	0	1500	14000	10	2700	2500	0
Yam (yellow) Conv	0	0	900	13200	100	2500	2500	0
Yam (Grosse Caille) Agro	0	0	1406	11550	0	2100	3192	0
Yam (Grosse Caille) Conv	0	0	1000	7815	82	2000	2400	0
Short fallow	0	0	0	0	0	0	0	0
Free long fallow	0	0	0	0	0	0	0	0
Improved short fallow	0	0	150	400	50	0	0	0
Improved long fallow	0	0	100	800	50	0	0	0

Table 3.C.3: Economic parameter values

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General Conclusion

In this three-essay thesis, we were interested in soil restoration problems and we used viability theory to address them while taking into account some environmental and socio-economic constraints such as reaching an environmentally acceptable quality of the soil and ensuring acceptable incomes for the farmers. Oure objective was to gain insight on how agronomic systems react and to which extent they are sensitive to some of the most influencing factors such as uncertainties (climatic uncertainties) and farm practices (parcellation) in different frameworks in order to have a more precise idea of the strategies that can be put in place to achieve the objectives of soil restoration and preservation.

In the first essay titled **"A Survey of Applications of Viability Theory to the Sustainable Exploitation of Renewable Resources"** we provide a comprehensive review of the literature on applications of *viability theory* to the sustainable management of renewable resources. After a refresher on the main concepts of viability theory, we provide a general map of the contributions and next discuss them by area of application. We conclude by pointing out issues that deserve more attention and should be part of a research agenda.

What popped out from this review is that viability theory is very suitable for addressing natural resources management problems and that there is now a long tradition of applying it to address renewable ressources exploitation related problems, e.g., fisheries and forests. However among the few that have applied it to farming, the overwhelming majority have focused on herd and grassland management problems; see, e.g., Tichit et al. [20], Sabatier et al. [16], Sabatier et al. [17], Mouysset et al. [13], Tichit et al. [21], Baumgärtner and Quaas [2], Martin et al. [12], Sabatier et al. [18], and Durand et al. [8]. To the best of our knowledge, Durand et al. [8] is the only study that looked at a soil-quality management problem. This observation confirms our choice of theme for the next articles as we are faced with a problem and a methodology that are very compatible but yet not sufficiently explored.

In the second essay titled "**Viability of Agroecological Systems Under Climatic Uncertainty**", we extend the model proposed in Durand et al. [8] by adding a level of realism through the introduction of climatic uncertainties. When considering a single plot of agricultural land, we use a stochastic viability theory-based model to look for agricultural strategies and crop rotations that would restore soil quality to an acceptable level while preserving the farm's economic profitability in presence of uncertainties resulting from major climatic events, such as hurricanes. And at the same time, we try to get some insight about the effect of these uncertainties on the performance of the agricultural systems both on the environmental and socio-economic sides.

Our empirical study concerns the archipelago of Guadeloupe, located in the French West Indies. Our analysis first reveals a high sensitivity of the results to the direct impact of hurricanes on the soil's quality indicator GISQ, which in turn strongly affects the impact of the other parameters. Then, that the export-oriented sector in Guadeloupe is more vulnerable and less resilient to climatic uncertainties than the local market oriented sector which is quite worrisome, given that this latter represents a fairly large portion of the economy of small tropical islands such as Guadeloupe and Martinique. Finally, we find out that the difficulty of sustainably managing soil, mainly because of the conflict between profitability and food-security objectives, is highly accentuated by climatic uncertainties, which are the most damaging hazards to soil preservation, not only in terms of cost, but also due to their direct impact on the soil itself. In fact, our results highlight that restoration possibilities and cost are very sensitive to the direct impact of hurricanes on soil quality. Consequently, it is crucial to have studies assessing the impact of hurricanes on the soil's quality indicators.

Given the high sensitivity of the results to the different parameter values, future developments to this paper include collecting additional data and refining the estimations of hurricane-related parameters (impact on the GISQ, recovery time, cost, and occurrence probabilities). A second interesting extension would be to develop more efficient resolution methods and algorithms for solving this type of problem. Indeed, introducing uncertainty to this type of problem inevitably leads to an explosion in the algorithm's memory consumption and execution time which greatly limits the range of possibilities for the frameworks that can be studied as well as the values of certain parameters such as the time horizon, the number of crops that can be taken at a time, etc. Finally some cropping systems, such as monocultures, often lead to gradual soil degradation, but this can be decreased or even reversed to a certain extent with no or minimum tillage, cover cropping and organic amendment strategies. It would be then interesting to take into account, in future research, the effect of successive use of the same crop or type of crop.

In the third essay, titled "**Viability of a Multi-Parcel Agroecological System**", we further extend the model of the second essay to examine the potential advantages that could be offered by parcellation. Given a farm of a predetermined size and a planning horizon, our objective was to get an idea about the effect of some parameters and especially parcellation on the ecologic and economic well-being of the farm and to determine the choices that could lead to simultaneously achieve the ecological (soil's quality) and economic (revenue) sustainability objectives. We rely on an extensive analysis on data and cases representing some farming systems practiced in the French West Indies to assess the impact of the main parameters on the results, namely, initial soil quality, planning horizon and different costs. These analysis reveal that parcellation is at least as good as non-parcellation in all cases. However, the benefit is only noticeable when the time horizon is long enough and these benefits are higher when the number of crop choices is lower. Furthermore, the export-oriented sector in Guadeloupe globally benefits more from parcellation than the local-market-oriented sector. Finally, in terms of final treasury, parcellation is globally much better than non-parcellation. Although we were not optimizing a specific performance index in this study (and in viability theory in general), this result can be used as a guideline for making a choice.

Again, the results being quite sensitive to the parameter values it would be very interesting for future developments to collect additional data and refine the estimations of the parameter values in order to get more precise and reliable results. In particular would be very interesting to use better estimations of the additional costs generated by parcellation (management fees for example) and other agricultural aspects like the effects of monocultures on the soil's quality. A second extension would be to run executions on different cutouts of the land (varying the number of parcels and their areas). However, this extension would be very complex and challenging due to the the curse of dimensionality to which resolution algorithms are undergoing.

More globally, it would be very interesting (yet challenging) to study a mix of what we have done in the second and third essays by taking into consideration uncertainties (those related to market parameters, climatic events, crop diseases, ...) in a multi-parcel framework. But once again the absence of faster and more efficient algorithms in terms of memory management represents a major obstacle to this. Overall, a series of algorithms for solving viability problems are available (e.g., Saint-Pierre [19], Bonneuil [4], Deffuant et al. [6], Aubin et al. [1], Krawczyk and Pharo [10] and Maidens et al. [11].) but, except in some particular cases, they are all subject to the curse of dimensionality. Facing this problem, the strategy in the applied literature was to limit the number of state variables to less than three or four. Besides, we are not an exception to this as it is, in a way, what we also did in the second and third essays by limiting the number of hurricane categories or the number of parcels or even by avoiding dealing with uncertainty and parcellation at the same time in our numerical applications. Thus, we always come back to the question of algorithms and resolution methods which confirms the importance of looking into this aspect and making efforts in developing more efficient algorithms for example using machine learning techniques, see, e.g., Deffuant et al. [6], Chapel et al. [5], and approximate dynamic programming, see, e.g., Bertsekas [3], Powell [15], that will allow dealing with practical problems in higher dimensions and improving computational time.

Finally, viability theory offers a framework that accommodates the presence of more than one stakeholder and more than one objectives as long as these stakeholders can "coordinate" in some sense when drawing the list of viability constraints and they are not playing strategically. In a farming model, strategic interactions between farmers are usually at the heart of the problem as prices on the market are influenced by offer and demand. Thus an other interesting further development would be to attempt to account for both viability and strategic issues somewhat in the style of what has been done in Doyen and Péreau [7] where a renewable resource harvest system involving multiple agents was studied using a model that combines coalitional games and a viability approach, or what has been done in Hardy et al. [9] in the context of small scale fisheries management, or even in Péreau et al. [14] where a form of strategic choice is considered in a transferable quota system, in the presence of an authority deciding on the initial allocation of harvesting quotas.

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