HEC MONTRÉAL

École affiliée à l'Université de Montréal

Essays on the valuation of syndicated loans in the presence of covenants : Term loans, loans with performance pricing , and revolving credit lines

par

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Thèse présentée en vue de l'obtention du grade de Ph. D. en administration
(spécialisation Ingénierie financière)

Décembre 2023

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Cette thèse intitulée :

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Résumé

L'objectif de cette thèse est d'apporter diverses contributions à l'évaluation des crédits syndiqués ayant des clauses financière restrictives et d'analyser l'efficacité de leur tarification.

Après les chapitres introductifs et contextuels, le troisième chapitre présente les spécifications générales et les hypothèses des modèles que nous avons développés.

Dans le chapitre 4, nous construisons un modèle dynamique de jeu stochastique visant à reproduire les négociations successives des termes du contrat de prêt syndiqué assorti d'une clause financière restrictive. Le modèle intègre plusieurs observations pratiques du marché, notamment le droit du prêteur de prendre des mesures punitives en cas de défaut technique (violation de la clause restrictive). Il tient également compte de la flexibilité de l'emprunteur dans le refinancement du prêt en rejetant les mesures punitives du prêteur, ou en se conformant aux nouvelles conditions tout en modifiant éventuellement sa stratégie de prise de risque.

Le chapitre 5 présente les expériences numériques. Bien qu'une clause restrictive améliore la valeur du prêt dans la plupart des situations, elle peut avoir un effet indésirable lorsque le risque de défaut devient important. Des analyses supplémentaires montrent que le prêteur peut tolérer de manière optimale certains défauts techniques pour prévenir cet effet indésirable. Nous calculons la valeur de marché de la clause restrictive dans un monde neutre au risque et dans un monde avers au risque. La valeur risque neutre de la clause est nulle pour le preteur et négative pour l'emprunteur. Dans un monde avers au

risque, la valeur de la clause restrictive pour le prêteur est positive dans les états de risque élevé et celle de l'emprunteur est possive seulement quand the risque est faible.

Le chapitre 6 présente une approche novatrice pour évaluer les prêts syndiqués avec une clause de tarification liée à la performance. Cette approche repose sur un modèle de jeu dynamique stochastique décrivant les réactions successives du suiveur aux variations du coût du crédit. Nous évaluons certains contrats existants extraits de DealScan et comparons l'utilisation de tarification liée à la performance à l'utilisation d'une clause restrictive. Les résultats suggèrent que la flexibilité apportée par la clause restrictive confère une valeur plus élevée pour les emprunteurs à risque élevé et moyen. Dans une deuxième partie, nous introduisons trois approches pour construire une grille de tarification liée à la performance. Selon le levier initial et la volatilité des actifs, la structure de tarification optimale implique un mélange de type d'augmentation et diminution du spread. En comparant les approches, les deux approches basées sur le modèle de jeu stochastique fournissent une meilleure structure de tarification en termes de compensation du risque.

Le chapitre 7 est consacré à l'évaluation d'un portefeuille syndiqués comprenant un prêt à terme et une ligne de crédit renouvelable. Nous étendons le modèle développé dans le chapitre 4 en permettant à l'emprunteur de puiser dans la ligne de crédit chaque fois qu'il choisit d'adopter une stratégie agressive. Nous avons testé différentes options de gestion de la facilité, comprenant un prélèvement complet ou partiel pour l'emprunteur, avec ou sans le droit du prêteur de geler la facilité. La présence de la ligne de crédit renouvelable ajoute de la valeur à la facilité uniquement pour un emprunteur avec un levier relativement faible. En revanche, la flexibilité offerte par la ligne de crédit renouvelable améliore la valeur de l'emprunteur dans tous les états. Avec le droit de geler la facilité, les résultats montrent que le prêteur peut permettre de manière optimale l'utilisation de la facilité même lorsque l'emprunteur se trouve dans une région de défaut technique.

Mots-clés

Prêt syndiqués, clauses restrictives, jeu dynamique stochastique, tarification de la performance, risque de crédit.

Méthodes de recherche

Théorie des jeux, programmation dynamique.

Abstract

The aim of this thesis is to provide various contributions to the valuation of syndicated loans in the presence of covenants and analyze their pricing efficiency.

After the introductory and contextual chapters, the third chapter presents the general specifications and the assumptions of our models.

In Chapter 4, we develop a dynamic stochastic game model to replicate the successive contract negotiation in a syndicated loan with a covenant. The model incorporates several practical observations from the market. The lender's right to take some punitive actions in case of technical default (breach of the covenant), the borrower's flexibility in refinancing the loan by rejecting the lender's punitive actions, or complying with the new terms while eventually changing its risk-taking strategy.

Chapter 5 presents the numerical experiments. While a safety covenant improves the loan value in most states, it can have an adverse effect when bankruptcy risk becomes important. Additional investigation shows that the lender can optimally tolerate some technical default to prevent this adverse effect. We compute the covenant value in a risk-neutral and a risk-averse world. Risk-neutral covenant value is null for the leader and negative for the follower. In a risk-averse world, covenant value is positive in states of high risk for the lender and all states of low risk for the follower.

Chapter 6 introduces a novel approach to value syndicated loans with a performance pricing clause. This approach is based on a stochastic dynamic game model of the follower's successive reaction to the change of the credit cost. We evaluate some exist-

ing contracts extracted from DealScan and compare a performance pricing clause to a covenant clause. The results suggest that the flexibility that brings the covenant has a higher value for high- and medium-risk borrowers. In the second part, we introduce three approaches to derive a performance pricing grid. Depending on the initial leverage and the asset volatility, the optimal pricing structure is a mix of interest-increasing/decreasing. In comparing the pricing approaches, the two approaches based on the stochastic game model provide a better pricing structure in terms of risk compensation.

Chapter 7 is devoted to the valuation of a syndicated facility package that includes a term loan and a revolving credit line. We extend the model developed in Chapter 4 by allowing the borrower to draw from the credit line whenever it opts to play aggressively. We tested different facility management options, including full take-down or partial take-down for the borrower, with and without the lender's right to freeze the facility. The presence of revolving credit adds value to the facility only for a borrower with relatively low leverage. On the other hand, the flexibility provided by the RCL improves the borrower value in all states. With the right to freeze the facility, the results show that the lender can optimally allow the utilization of the facility even when the borrower is in a technical default region.

Keywords

Syndicated loans, covenant, loan monitoring, stochastic dynamic game, performance pricing, covenant, credit risk.

Research methods

Game theory, dynamic programming.

Contents

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List of Acronyms

ATC: Agency Theory of Covenant

EBITDA: Earnings Before Interest, Taxes, Depreciation and Amortization

CDS: Credit Default Swaps

DP: Dynamic Programming

PP: Performance Pricing

LIBOR: London Interbank Offered Rate

EURIBOR: Euro Interbank Offer Rate

LSTA: Loan Syndication & Trading Association

PP: Performance Pricing

SSE: Strong Stackelberg Game

WSE: Weak Stackelberg Game

IVADO: Institut de Valorisation des Données

NSERC: Natural Sciences and Engineering Research Council

LPC: Loan Pricing Corporation

EBIT: Earnings Before Interest and Taxes

Capex: Capital expenditures

LC: Loan Commitment

RCL: Revolving Credit Line

MAC: Material Adverse Change

EAD: Exposure at the default instant

SLLs: Sustainability-linked loans

ESG: Environmental, social, or governance

List of Symbols

State variables

s	current level of the process S_t
d	current level of the debt (Chapter 7)

Parameters

α	contractual physical default parameter
$\hat{\alpha}$	contractual technical default parameter
c_m	periodic payment due at date t_m, $m = 1, \dots, n$
D_t	contractual outstanding debt at date t
r	risk-free rate
δ	the continuous dividend payout rate
i	contractual periodic interest rate
$\beta = \exp(-r\Delta)$	periodic discount factor
γ	proportional bankruptcy costs
g	immediate net dividend resulting from an aggressive investment strategy
δ	is the continuous dividend payout rate
T :	Maturity of the loan
$\tau(S)$	Stopping time modeling the default date
B_t and $B_t{}'$	Brownian motion
P	risk-neutral probability measure
$\mathbb{P}$ and $\mathbb{P}'$	physical probability measure

Decision variables

p : coupon increase imposed by the leader in case of technical default, due at the next monitoring date.

κ : default threshold increase imposed by the leader, $0 \leq \kappa \leq 1$, where $\kappa = 0$ indicates no change with respect to the contractual value.

λ : indicator of the follower's response, $\lambda \in \{-1, 0, 1\}$, where $\lambda = -1$ corresponds to refinancing the loan, $\lambda = 0$ to filing for bankruptcy, and $\lambda = 1$ corresponds to accepting the terms of the loan.

θ : indicator of the follower's investment strategy, $\theta \in \{0, 1\}$, where $\theta = 0$ corresponds to a conservative strategy and $\theta = 1$ corresponds to an aggressive strategy.

B_j : threshold of the PP grid, $j = 1, ..., J$ (Chapter 6)

p_j : spread applied by the leader when the performance indicator is in $[B_j, B_{j+1})$ (Chapter 6)

Functions

$b(t) = \alpha(1+\kappa)D(t)$: physical default barrier at date t

$\hat{b}(t) = \hat{\alpha}(1+\kappa)D(t)$: contractual technical default barrier at date t

$\mathbb{I}_x(i)$: indicator function of a discrete variable x

$$\mathbb{I}_x(i) = \begin{cases} 1 \text{ if } x = i \\ 0 \text{ if } x \neq i. \end{cases}$$

$\mathbb{I}(A)$: indicator function of a condition A

$$\mathbb{I}(A) = \begin{cases} 1 \text{ if } A \text{ is true} \\ 0 \text{ if } A \text{ is false.} \end{cases}$$

$1_m(\alpha(1+\kappa))$, $0_m(\alpha(1+\kappa))$: the probability of a physical default (no physical default) in the time interval $(t_m, t_{m+1}]$, given the information available at t_m. Note that this probability depends on s and θ, which are observable at t_m.

$R_t^L(s,D), R_t^F(s,D)$: Leader and follower recovery given default at date t when $S_t = s$ and $D_t = D$,

$$
\begin{aligned}
R_t^L(s,D) &= \min\{(1-\gamma)s, D\} \\
R_t^F(s,D) &= (1-\gamma)s - R_t^L(s,D).
\end{aligned}
$$

$\omega_m(s)$: refinancing cost at date t_m when $S_{t_m} = s$. The expression and calibration of this function is detailed in Section 4.6.1.

$\vartheta(s)$: terminal payoff function of the borrower.

w_m^j : Immediate expected payoff of a player j

δ_m^j : feedback strategy function for a player j

$V_m^j(.)$: functions used by the players to evaluate a strategy

Φ : standard normal cumulative distribution function

$\mathbb{E}_m[\cdot]$: expectation conditional on the information available at t_m.

To my beloved parents,

To my beloved wife Djata,

To my beloved daughters Dahlia, Malaïka, and Malia,

To my brothers and sisters Nabassaga,

To the cherished memories of my late mother Azeta, my late grandmother Zeinab, and my late aunt Lizeta, whose inspirations continue to be a guiding light.

Acknowledgements

I express my profound gratitude to Professor Michèle Breton for her unlimited moral and financial support, motivation, and invaluable guidance. Without her, this journey would not have been possible.

I extend my thanks to Professor Frédéric Godin and Professor Ramzi Ben-Abdallah for their roles as members of my phase III committee.

I appreciate the contributions of the participants at the 2019 Mathematical Finance Days, the Fin-ML conference in Montreal, the HEC Student seminar, and the optimization day in Montreal. Their comments and suggestions were very constructive.

My thanks also go to HEC Montréal, the J.A. DeSève Foundation, the Fin ML program (IVADO and CREATE), and the Natural Sciences and Engineering Research Council of Canada (NSERC) for their generous financial support.

A special thanks to my wife Djata and my three daughters Dahlia, Malaïka Zeinab, and Malia Fati for their care, tolerance, patience, and support.

Finally, I express gratitude to my friends in Montreal, my colleagues at BNP Paribas, my former colleagues at EY, and at the African Development Bank for their moral support.

Chapter 1

Introduction

There are generally three financial channels that corporate firms can use to finance their operations; borrowing, free cash flow, and selling additional shares. Among these three sources of financing, borrowing is predominant. Borrowing can be private, with one or several financial institutions, or public, through the issuance of bonds or commercial papers in the debt market. The issuance process of public loans is standardized and has to satisfy some market regulations. Private loans can be bilateral, involving a single lender (e.g. a bank), or multilateral. Multilateral loans can take different forms, including club loans and syndicated loans.

The syndicated loan market plays an important role in corporate firms financing (see M. Campbell and Weaver (2019), Simons et al. (1993), Thomas and Z. Wang (2004)). According to the Bank of International Settlement (BIS), syndicated loans represent roughly one-third of the overall international financing, including bonds, equity issuance, and commercial papers. The most active borrowers in the syndicated loans market are large corporate firms and, to a lesser extent, sovereign countries. Corporations need syndicated loans to finance large projects such as acquisitions and business expansion or as bridge loans. Sovereign countries mostly use syndicated loans to finance large infrastructure projects. On the lender side, the most active participants are large banks, mutual funds, private equities, and large insurance companies. The syndication process can bring together different types of lenders from various countries or regions. This is particularly interesting as it gives more flexibility in the deal design (e.g. multi-currency tranches) to meet the borrower's needs.

This thesis focuses on syndicated loans, whose essential purpose is to finance projects requiring a large amount of funding, and more specifically on safety covenants, which are contractual clauses that are commonly found in syndicated loan contracts because of the specific issues raised by the size of the loans and the decentralized nature of their management.

Safety covenants have been the main tool suggested to address the agency problem between lenders and borrowers (see, for instance, Smith Jr and Warner (1979), Aghion and Bolton (1992), Roberts (2015), Rajan and Winton (1995), Sufi (2007), and Freudenberg et al. (2017)).

An agency problem arises when one party (the agent) is expected to behave or act in the interest of another party (the principal), but has some personal interest or motivation not to do so. This problem becomes important when there is an information asymmetry between the principal and the agent. In the syndicated loan contract design, the agency problem raised in Smith Jr and Warner (1979) and further discussed in Aghion and Bolton (1992) stems from the fact that a lender can hardly have all the information needed to design an efficient loan contract.

One illustration of this agency problem is the risk-shifting issue, also known as the substitution problem. Asset substitution arises when the borrower deliberately switches from a low-risk project to a high-risk one after receiving the funding. In this thesis, we will designate this effect as playing aggressively to earn immediate dividends beyond the usual performance. Its impact on the market value of the loan will depend on the loan pricing model.

Private loans have been traditionally designed using a fixed or a variable interest rate, where a variable rate is generally determined as a variable base rate (e.g. LIBOR or lender's prime rate) plus a fixed spread over the contract life. From a risk-sharing perspective, a fixed interest rate bestows both the interest rate and credit risks to the lender, while a variable rate transfers the interest-rate risk to the borrower while credit risk remains with the lender. In the two pricing structures, asset substitution remains problematic as it increases the credit risk component which is not shared and remains with the lender in both cases. Over the past four decades, credit risk has gained greater significance in determining capital costs, as evidenced by frameworks like Basel II and III. It has also emerged as a central aspect of comprehensive risk management, thereby prompting the development of novel strategies for sharing loan-related risks. For instance, Performance pricing that links the credit spread to the financial performance of the borrower initiates the spirit of credit risk sharing, with the potential effect of reducing the risk-shifting problem. Another example is the introduction of financial covenants with the embedded right to the lender to increase the spread under some circumstances of a material change in the borrower's credit quality.

This thesis calls upon a vast literature related to the agency problem in loan contracting (Smith Jr and Warner (1979); Aghion and Bolton (1992)), the use of covenants in optimal contract design (Watts and Zimmerman (1986); Rajan and Winton (1995)), the effect of restrictive covenants on credit cost (Jensen and Meckling (1976); Smith Jr and Warner (1979); Reisel (2014); Simpson and Grossmann (2017)), the use of performance-sensitive debt to address information asymmetry problems at the loan inception (Asquith, Beatty, and Weber (2005); Sarkar and Zhang (2015)), and, on the theoretical angle, the use of game theory to explain empirical observations about the corporate debt market (Thakor (1991); Allen and Morris (2014); Annabi, Breton, and François (2012)).

There are numerous applications of game theory in finance (see, for instance, the surveys in Thakor (1991), Allen and Morris (2014), and Breton (2018)), encompassing a large variety of corporate and investment finance issues, with the aim of explaining empirically observed agents' decisions by incorporating strategic considerations. Among these applications are bankruptcy games and option games, which can both be used to model credit risk in the syndicated loan market.

Bankruptcy games (Thomson (2003), Curiel, Maschler, and Tijs (1987)) address how assets are distributed among various claimants in case of default, specifically when the default can be considered as a strategic decision leading to changes in the contract's term. In the credit risk literature, two main avenues are used to characterize the default stopping time. The intensity-based (or reduced-form) approach considers default to be governed by an exogenous process, while, according to the structural approach, the default event is driven by the evolution of some structural variable, for instance, the relative value of the borrower's assets and liabilities. In the structural approach case, one can also assume that default is decided by one of the strategic players (optimal or strategic default), as in Leland (1994), François and Morellec (2004) and Broadie, Chernov, and Sundaresan (2007).

Option games appear when a contingent claim gives interacting optional rights to more than one holder. A typical application occurs with debt instruments that are callable and/or redeemable, allowing one of the parties to exit from its contractual obligations under some conditions (see for instance Brennan and Schwartz (1980) and Ben-Ameur et al. (2007)). Credit risk in that case relates to the opportunity cost incurred by the other party as a result.

This thesis proposes a model of the syndicated loan process, where the inclusion of a covenant gives rise to a dynamic game between the syndicate and the borrowing entity, incorporating characteristic features from both the bankruptcy and option game literature, and relating it to the agency problem literature.

Although several empirical studies support that financial covenants are implicitly priced in the credit spread of syndicated loans (see for instance Chang and Ross (2016) and Bradley and Roberts (2015)), no contribution has yet been dedicated to estimating the contract value adjusted for the presence of covenants. We are proposing to contribute to the literature by building a model to evaluate the effect of the covenant on the loan value. Some of the research questions we examine include: What is the market value of the covenant? How efficient is performance pricing compared to pricing with a covenant?

How can the use of covenants improve the financial performance of syndicated revolving credit lines?

Our general approach consists of using a game model to characterize the interactions between the borrower and the lender in the context of loan contracts containing covenants. We develop a stochastic dynamic Stackelberg game model that accounts for the lender's and the borrower's strategic instruments and for their interactive impact on the contract's value. In our model, the lender monitors the loan and can punish any covenant violation, while the borrower can choose to reject the punitive action by refinancing the loan or comply with the new terms of the contract while eventually altering the risk level of its operations. This strategic interaction can lead to successive adjustments to the terms of the contract. Depending on the form of the covenant, we consider both feedback and open-loop strategies. Our credit-risk model is based on the structural approach, using the value of the borrower's structural variables to compute the probability of a covenant breach or default. In all cases, we compute the lender's optimal strategy, given the borrower's optimal response, and characterize the equilibrium value of the contract as a function of the remaining time to maturity and of the state of the world.

Our numerical investigations show that the presence of a covenant may have a significant impact on the loan value, particularly in regions where default risk is high, according to the form of the restrictive clauses (safety covenant, performance pricing, discretionary credit).

The rest of the thesis is organized as follows: After a brief exposition of the salient characteristics of the syndicated loan market in Chapter 2, Chapter 3 presents the main assumptions about the firm's capital structure and the default model. Chapter 4 develops a stochastic dynamic-game model of the interactions between the lender and the borrower, which serves as the basis for computing the market value of a covenant. Numerical results pertaining to the implementation of the model are presented and discussed in Chapter 5. The basic model is extended to the valuation of a performance pricing syndicated loan contract in Chapter 6, and to revolving credit lines in Chapter 7. The last chapter concludes the thesis.

Chapter 2

Syndicated loans and covenants

Syndicated lending is an important component of the private credit market. It is used in order to provide a loan to a single borrower when, for various reasons, the amount cannot be funded by a single lender. In this chapter, we present the syndication process, covenants as a specific instrument used in syndicated loans, and some statistics pertaining to credit risk in the syndicated loan market.

2.1 The syndicated loan market

2.1.1 Some definitions

The syndicated loan market offers various financial instruments, the most common being the *term loan*, the *revolving credit facility*, and the *standby facility*. A term loan consists of lending a fixed amount to be either reimbursed by a bullet payment at maturity (*bullet term loan*), or, alternatively, amortized over the contract life (*amortizing term loan*). The borrower can draw down the full amount at once or in tranches; in the latter case, commitment fees are paid by the borrower on the non-used part. Any early repayment by the borrower can not be drawn down again. The general setup used in Chapters 4, and 6 is that of the term loan. The revolving credit facility is similar to the term loan, except for the flexibility offered to the borrower to pay and redraw any amount, provided that the total drawn amount does not exceed a fixed *credit line*. This is also the case for the standby facility, which is, however, not expected to be used for working capital, but is kept as a backup funding solution when other sources become inaccessible. Chapter 7 considers a revolving facility setup.

In addition to these popular types, *bridge loans* and *leverage loans* are increasingly used. A bridge loan is a short-term loan used to fill an urgent need for financing while waiting for more stable financing. It is typically used in mergers and acquisitions transactions. Leverage loans are used to finance already heavily indebted borrowers.

Syndicated loans generally have moderate maturity, with a term varying between 6 months and 10 years. The size of the deal varies from $20 million to $55 billion, according to the Loan Market Association. Interest rates are mostly variable, indexed to LIBOR or EURIBOR.

2.1.2 Size and important trends

The size of the syndicated credit market has been steadily increasing since the financial crisis of 2007-2008. For the year 2018, Bloomberg's global syndicated loans database records a total worldwide volume of 4.9 trillion U.S. dollars with 8,359 deals, which represents a volume growth of 9.36% compared to 2017. Figure 2.1 depicts the trend for the borrowers located in U.S. The total loan amount (including term loans, drawn amount from credit lines, and all other credit facilities) has increased from 900 billion in 2009 to around 2 trillion in 2021. We observe a peak in the amounts drawn from syndicated credit lines during the economic shutdown in Q1 and Q2 of 2020, the drawn amount increasing from 651 billion in Q4 2019 to 1065 billion in Q1 2020 and 801 billion in Q2 2020, before reverting to the historical trend from Q3 2020. According to the empirical investigations in Bosshardt and Kakhbod (2021) and Acharya and Steffen (2020), this peak is due to precautionary measures against future liquidity risk from COVID-19, and the drawn amount has been used to repay the credit line during the recovery phase of the COVID-19 crisis.

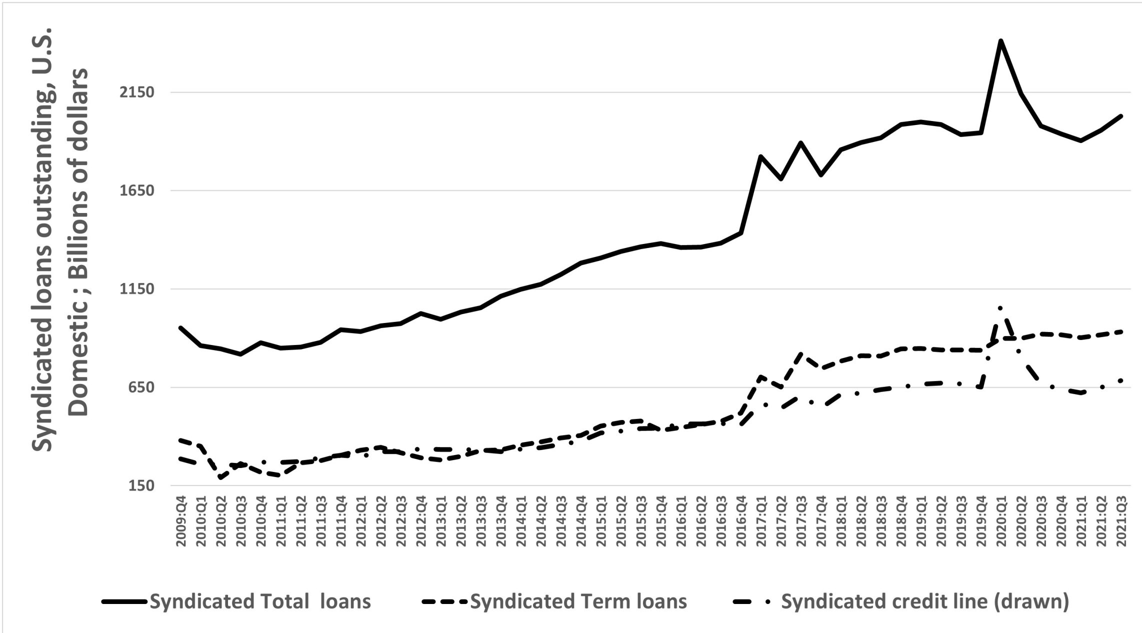


Figure 2.1: Total syndicated credit amount outstanding. Source: Federal Reserve, Shared National Credit Program

2.1.3 The syndication process and important players

The syndicated credit market involves many players, among which the two most important groups are the lenders (the syndicate members) and the borrowers (corporations, financial institutions, and sovereign borrowers). Over the last twenty years, corporate borrowers have been the most active, accounting for more than 80% of the total debt volume, while financial institutions make up around 15% of it. Over that period, sovereign borrowers' share in the total volume has been decreasing in favor of that of corporate borrowers. General corporate purpose, refinancing previous loans, and mergers and acquisitions are the most predominant reasons corporations call for a syndicated loan. Some other purposes for syndicated loans include working capital and project financing, the latter being the predominant use by the sovereign borrowers. On the lenders' side, the primary market (the deal initiation) is dominated by banks, a few of them playing the lead role. Private equities and institutional investors are also active in this market.

The syndicated loan process starts with an underwriting commitment between one or a group of banks (denoted as lead or co-lead arrangers) and a borrower. The lead group has the responsibility of providing the needed amount by calling for syndication. After some administrative conditions are met, the lead arrangers distribute the loan amount among interested lenders, which can include banks and non-bank entities like private equity, hedge funds and institutional investors. The process of underwriting and loan distribution among the initial participants represents the primary market.

2.1.4 Secondary market and syndicated credit derivatives

In addition to the primary market, in which banks and other financial institutions share the initial loan amount, there is a growing secondary market, in which the initial lenders can sell a part of their tranche to investors who cannot normally participate as syndicate members (bond investors, insurance companies, pension funds, mutual funds, etc.). This secondary market has been steadily increasing since 1990; according to the Loan Syndications & Trading Association (LSTA), the annual trading in this market has reached $824 billion in 2022, with an increase of 6% on an annual basis. The expansion of the secondary market has an important impact on the development of the overall syndicated credit market, effectively increasing liquidity, as it becomes easy for an initial participant to reduce or close its position at any time. There are three main reasons why a syndicate member would want to sell its tranche: first, to release capital for new lending; second, to manage credit risk by diversification over industries and/or geography; a third important reason is to comply with regulatory capital requirements.

Overall, the expansion of the secondary market allows the size of the primary market to increase, as the initial lenders can take a larger tranche with the intention of redistributing it among their network in the secondary market. On the demand side, there are two main motives for investors to participate in the syndicated loan secondary market: the first is, for small lenders that are excluded from the primary market, to participate in a syndicated deal; the second is to develop connections with lenders and borrowers in the hope of participating or arranging future deals. Finally, the secondary market provides an opportunity for borrowers to manage their credit in the same way as in the bond market, by selling and buying back their own loan to improve their creditworthiness.

2.1.5 Pricing structure

The syndicated loan pricing structure includes upfront fees, at the contract inception, and annual (or periodic) fees. Upfront fees include the *arrangement*, *underwriting*, and *participation* fees. Arrangement and underwriting fees are paid to the lead arrangers for the commitment to obtain the needed amount. Participation fees are distributed among all the participant banks.

The periodic fees generally encompass a *commitment* fee for the non-used amount in the facility, *utilization* fees for the used part of the facility, and *agency* fees. The *agency* fee is paid to the agent bank in charge of the administrative and monitoring activities. Utilization and commitment fees are usually expressed as a base rate (generally the LIBOR) plus a spread. Some financial options are also frequently included in the pricing.[1] Many syndicated loans use *performance pricing* (PP) to define the spread over the base rate as a function of a covenant indicator. Under PP, the spread changes–up or down–each time the covenant indicator crosses a predefined threshold. Performance pricing will be analyzed in detail in Chapter 6.

Table 2.1 presents some descriptive statistics about the relative size of the various fees, obtained from a sample of contracts extracted from the Dealscan database.

2.2 Covenants, performance measures and credit risk

Two of the main differences between syndicated loans with respect to bilateral private loans or bonds are the size of the loan and the presence of multiple lenders. These char-

[1] For instance, recent innovations allow the inclusion of a multi-currency option in the contract, which gives the right to the borrower to switch from one currency to another in a specified list, without additional fees.

Table 2.1: Statistics on syndicated contract fees (bps)

	Commitment fees	Agency fees	Upfront fees	Utilization fees
Average	40.00	84.16	75.29	146.15
Min	0.2	5	0.14	20.25
Max	750	150	2750	1490.02
STD	27.12	45.43	95.78	143.69
N	3308	11299	1460	14467

Source: Dealscan Database.

acteristics led to the wide use of covenants to control the borrower over the life of the contract.

2.2.1 Definition

A covenant can be literally defined as a formal agreement between two parties to do (positive covenant) or not to do (negative covenant) something specific. Safety covenants in loan agreements are used to address the agency problem between lenders and borrowers. In the literature (see for instance Chen, Mao, and Hu (2015); Prilmeier (2017); Freudenberg et al. (2017)), the optimal debt contract is derived by considering the introduction of safety covenants, with a possibility of negotiation throughout the life of the contract as additional information on the borrowers is gradually acquired by the lenders.

2.2.2 Agency theory

The literature on covenants and credit costs can be traced back to William H, Michael, et al. (1976) and Smith Jr and Warner (1979). These papers propose an *Agency Theory of Covenant* (ATC) that translates into a tradeoff between yields and the inclusion of a covenant in a loan contract, arguing that covenants reduce the expected loss of investors, and, therefore, lead to lower returns.

The ATC theoretical model motivated a number of empirical studies on the impact of covenants on the yield spread of loans. The ATC model is tested empirically in

Bradley and Roberts (2015), confirming that bond covenants are indeed priced. More precisely, Reisel (2014) shows that the inclusion of covenants that restrain the borrower's investment decisions lowers the corresponding loan cost by 35 to 75bps. Bradley and Roberts (2015), using a large sample of corporate debts, finds a negative relation between the expected yields and the type of covenant included in the contract. In Chang and Ross (2016), survey and bond data from China is used to investigate the impact of various types of covenants, classified according to their purpose –bankruptcy protection, addressing information asymmetry or preventing financial distress. The authors find that covenants against bankruptcy have the most important impact on loan valuation, followed by covenants reducing informational asymmetry. Deng et al. (2016) also find a negative correlation between the inclusion of a covenant and the loan margin. Simpson and Grossmann (2017), analyzing the period after the financial crisis, confirm the result of Reisel (2014) and estimate that the impact of covenants restricting the borrowers' investment decisions lower the spread by 60 to 72 bps for investment-grade loans and by 141 to 150 bps for non-investment-grade loans.

2.2.3 Covenants in syndicated loans

In syndicated loans, covenants are typically implemented using one or more performance indicators and threshold values for these indicators that should not be crossed. Examples of covenant indicators and their definition are presented in Appendix 2.4. This list is obtained from a sample of 5,000 syndicated loan contracts concluded between 2000 and 2012, extracted from the Loan Pricing Corporation (LPC) DealScan database.

This database collects individual deals around the world, recording information at the facility level or at the package with several facilities (e.g. covenant information, package composition, type of loans within the package, the existence or not of performance pricing clause, etc.)

2.2.4 Credit risk

Figure 2.2 presents the trend of the defaulted loans amount, as a percentage of the total outstanding debt for the U.S. syndicated loan market between 2009 and 2021. While this ratio was unusually high in the period following the global financial crisis, reaching 35% for the term loans in Q2 2010, it has remained below 5% since Q2 2014. We observe that the ratio is generally higher for term loans, as compared to lines of credit.

Note that the defaulted amount does not correspond to actual losses, since the recovery rate, in the percentage of the due amount, is relatively higher for the syndicated loan market than for other types of loans (Altman and Suggitt (2000); Emery, Cantor, and Arner (2004)). However, even then, the loss given default, in absolute terms, is generally relatively high in the syndicated loan market, due to the large size of syndicated loans.

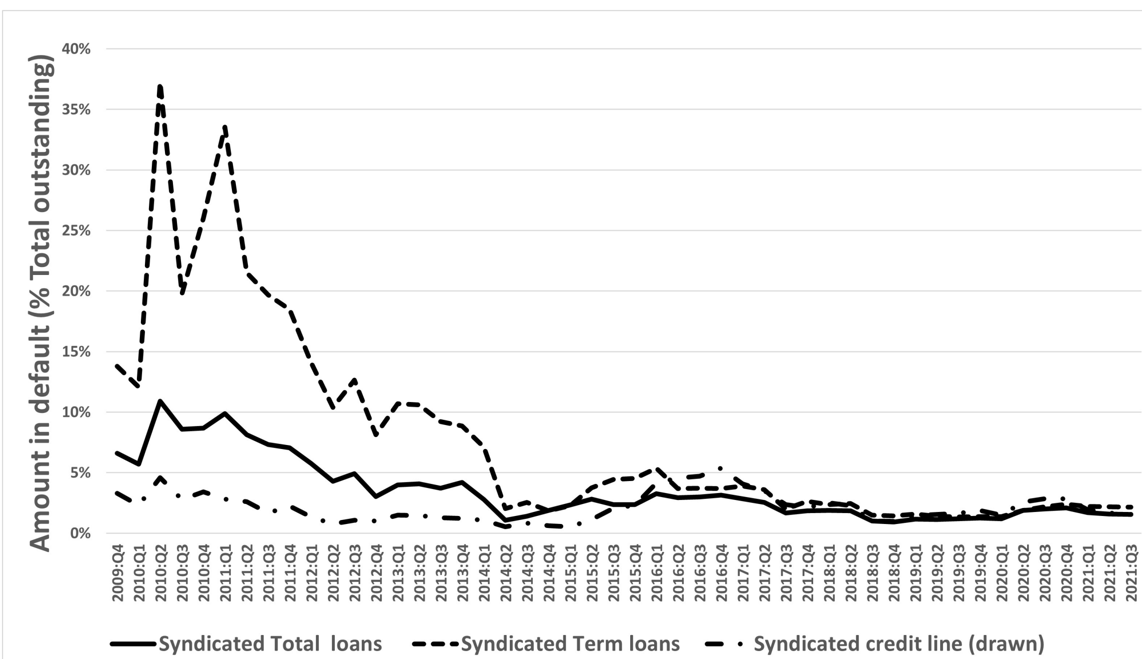


Figure 2.2: Amount in default (% Total outstanding). Source: Federal Reserve, Shared National Credit Program

2.3 Conclusion

The syndicated loan market plays an important role in corporate finance. The large size of these loans makes their monitoring a challenging task. Covenants are the main tools used by the syndicated members to closely monitor the borrowers over the contract life.

There are several reasons why estimating the covenant-adjusted value of syndicated loans is relevant. The first reason is contract design: covenants are important components of syndicated loan contracts and their type and restrictiveness, in light of the borrower's creditworthiness and informational opacity, have an important effect on the loan value (Jing Wang (2017); Prilmeier (2017)). Another important reason is the development of secondary markets and, more specifically, of loan securitization. Building credit derivation requires the ability to estimate the value of the underlying loan contract at any time, for trading and risk management purposes.

The objective of the next chapters is to propose valuation models for three of the most common types of syndicated loan contracts, focusing on the impact of covenants on the strategic decisions of the lenders and borrowers, and their implication on the loan's value and default risk.

2.4 Appendix: List of the most frequently used covenant indicators

- EBITDA: Earnings before interest tax, depreciation and amortization
- Debt to EBITDA: Ratio of total debt to EBITDA
- Interest coverage: Ratio of EBITDA to the interest payments due
- Fixed charge coverage: Ratio of the earnings before interest and taxes (EBIT) to fixed charges before tax (debt payments, interest and equipment lease expenses)
- Capex: Capital expenditures
- Debt to tangible net worth: Ratio of total debt to total assets less liabilities and intangible assets
- Debt service coverage: Ratio of net operating income to total debt service
- Senior debt to EBITDA: Ratio of total senior debt to EBITDA
- Quick ratio: Ratio of assets that can be sold through standard business operations within one year to current liabilities
- Leverage ratio: Ratio of total debt to total asset
- Senior leverage: Ratio of total senior debt to total asset
- Debt to equity: Ratio of total debt to equity
- Loan to value: Ratio of total debt to total book value
- Net debt to assets ratio: Ratio of total short and long term debt, less any cash and any liquid financial instruments or assets that could be easily converted to cash, to total asset value
- Equity to assets: Ratio of equity to total asset value

Chapter 3

Capital structure and default model

This chapter presents the general specifications and assumptions about the borrower's default and capital structure models that are used in this thesis.

3.1 Capital structure model

We consider a firm with a capital structure that includes equity and debt. For simplicity, we assume that the debt is entirely financed from a syndicated loan facility. As long as the firm operates, it generates a stochastic pre-tax cash flow and pays a continuous dividend at a constant rate. In a risk-neutral context, the firm's asset value is the expected sum of the future cash flows, discounted at the risk-free rate. Following Merton (1974) and Leland (1994), we assume that the firm's asset value follows a geometric Brownian motion whose dynamics is expressed as

$$dS_t = (r_s - \delta)S_t dt + \sigma_s S_t dB_t, \tag{3.1}$$

where B_t is a standard Brownian motion,σ_s is the asset volatility, r_s is the instantaneous risk-free rate, δ is the continuous dividend payout rate. The drift $(r_s - \delta)$ can eventually be adjusted for the coupon rate.

Under some simplifying assumptions about the default barrier and debt structure, it is possible to obtain the value of the various components of the capital structure analytically. For instance, Appendix 3.5 recalls the details of the computation of the value of the firm under the assumptions used in Leland (1994). The determination of the value of equity will be used to set the terminal conditions for the borrower at the maturity of the syndicated contract, depending on how the borrower's debt will be refinanced.

3.2 Credit risk models

Credit risk modeling can be summarized according to two general classes of approaches: the structural approach (Merton (1974); Black and Cox (1976); Leland (1994)), and the intensity approach (Darrell Duffie, Pan, and K. J. Singleton (1996); Darrel Duffie and K. Singleton (1998); Elhiwi (2014)). One of the aims of credit risk models is to determine the probability of a default by a borrower in a given period. To this end, the structural approach consists of defining an explicit relationship between the default probability and one of the borrower's structural variables, while the reduced form approach assumes default events are driven by an exogenous random process.

In this thesis, we adopt the structural approach. This approach has been introduced by Merton (1974), adapted and complemented by Black and Cox (1976), Brennan and Schwartz (1978), Longstaff and Schwartz (1995), and Leland (1994). In a typical structural model of default, the default events happen when the value of the borrower's assets falls below a given threshold called the *default boundary*.

In Merton (1974), default can only happen at the loan maturity, when the value of the assets falls below the outstanding debt value, which stands for the default boundary. Default results in liquidation, the lender receiving the firm's asset value while the equity holders receive nothing. In the Black and Cox (1976) model, default can happen at any time before the contract maturity; a safety covenant determines the default barrier, which is equal to the present value of the outstanding loan value (assuming that the debt is reimbursed in a single payment at maturity). Brennan and Schwartz (1978) and Longstaff and Schwartz (1995) also assume that the default barrier is equal to the outstanding loan value. In Leland (1994), the default barrier is endogenous, corresponding to a stopping time optimizing the equity value.

In this thesis, we assume an exogenous default barrier determined by a financial safety covenant, which is expressed as a fraction of the loan balance. Accordingly, the default barrier at date t takes the form

$$b(t) = \alpha D(t), \tag{3.2}$$

where α is a contractual parameter and $D(t)$ is the outstanding loan balance at date t.

This formulation encompasses various possible forms for the default barrier, depending on the terms of the loan: fixed barrier (non-amortizing term-loan), linear barrier (amortizing term-loan or zero-coupon loan), or stochastic barrier (line of credit). The computation of default probabilities according to these different cases is provided in the next section.

3.3 Default probability model

In a general specification, we consider a probability space $(\Omega, F, \mathbb{P})$. Let B_t and S_t denote respectively a standard Brownian motion and a Markovian process defined in the probability space. According to our syndicated loan default risk model, a default time in $(0,T]$, where T is the maturity of the loan, can be defined as a stopping time $\tau(S)$,

$$\tau(S) = \inf\{t \in (0,T] : S_t \leq b(t)\},$$

where S_t is a structural variable of the borrower and $b(t)$ is a deterministic time-dependent default barrier determined from the loan outstanding value. In the following paragraphs, we characterize the probability of a default event in a time interval $(t_1, t_2]$,

$$\mathbb{P}(\tau(S) \leq t_2 | \tau > t_1),$$

for various dynamics of S_t and $b(t)$.

3.3.1 Standard Brownian motion and constant barrier

We start with the simple case where the process S_t is a standard Brownian motion $S_t = B_t$ and the default barrier is constant, where $S_0 = 0$ and $b(t) = b < 0$,

$$\tau(B) = \inf\{t > 0 : B_t \leq b\}.$$

Note that because a Brownian motion has a continuous path, this stopping time can also be defined as

$$\tau(B) \quad = \quad \inf\{t > 0 : B_t = b\}.$$

The reflection principle of the Brownian motion implies that

$$\mathbb{P}\left(\tau(B) \leq t\right) = 2\mathbb{P}\left(B_t \leq b\right). \tag{3.3}$$

Using (3.3),

$$\mathbb{P}\left(\tau(B) \leq t\right) = 2\Phi\left(\frac{b}{\sqrt{t}}\right)$$

where Φ is the standard normal cumulative distribution function,

$$\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{y^2}{2}} dy,$$

so that

$$\begin{aligned}
\mathbb{P}\left(\tau(B) \leq t\right) &= 2\mathbb{P}\left(B_t \leq b\right) = 2\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\frac{b}{\sqrt{t}}} e^{-\frac{y^2}{2}} dy \\
&= \frac{2}{\sqrt{2\pi t}} \int_{-\infty}^{b} \exp\left(-\frac{y^2}{2t}\right) dy,
\end{aligned}$$

which leads to

$$\mathbb{P}\left(\tau(B) \in (0,t]\right) = \frac{|b|}{\sqrt{2\pi t^3}} \exp\left(-\frac{b^2}{2t}\right).$$

3.3.2 Drifted Brownian Motion and constant barrier

We now consider S_t as a drifted Brownian motion (where a_0 is constant)

$$S_t = S_0 + B_t + a_0 t,$$

with $b(t) = b$ and $S_0 > b$, so that

$$\tau(S) = \inf\{t > 0 : a_0 t + B_t \leq b - S_0\}.$$

Let $B_t^{'} = a_0 t + B_t$. Using Girsanov's theorem, there exists a measure $\mathbb{P}'$ on (Ω, F), equivalent to $\mathbb{P}$, such that $B_t^{'}$ is a $(\Omega, F, \mathbb{P}')$-Brownian motion without drift. Define for any continuous variable x and interval A, the indicator function

$$\mathbb{I}\,(A) = \begin{cases} 1 \text{ if } x \in A \\ 0 \text{ otherwise} . \end{cases}$$

The Radon-Nikodym theorem states that

$$\mathbb{P}'\,(\tau(S) \leq t) = \mathbb{E}^{\mathbb{P}'}\left(\frac{d\mathbb{P}'}{d\mathbb{P}}\mathbb{I}\left(\{\tau(B^{'}) \in (0,t]\}\right)\right),$$

where

$$\frac{d\mathbb{P}'}{d\mathbb{P}} = \exp(a_0 B^{'}_{\tau(B')} - a_a^2 \tau(B^{'})),$$

which satisfy the Girsanov's conditions below:

- $\frac{d\mathbb{P}'}{d\mathbb{P}} \geq 0$
- $E^{\mathbb{P}'}[\frac{d\mathbb{P}'}{d\mathbb{P}}] = E^{\mathbb{P}'}[\exp(a_0 B^{'}_{\tau(B')} - a_0^2 \tau(B^{'}))] = \exp(a_0(b - S_0)) E^{\mathbb{P}'}[\exp(-a_0^2 \tau(B^{'}))] = 1$, because $E^{\mathbb{P}'}[\exp(-a_0^2 \tau(B^{'}))] = \exp(-a_0(b - S_0))$ (Borodin et al. (2002) , p 204)

Since $\exp(\frac{1}{2}\int_0^T \mu^2 du) = \frac{\mu^2 T}{2} < \infty$, the default probability can be computed as:

$$\mathbb{P}\left(\tau(B^{'}) \leq t\right) = E^{\mathbb{P}'}\left(\mathbb{I}\left(\{\tau(B^{'}) \in (0,t]\}\right)\exp(a_0 B^{'}_{\tau(B')} - a_0^2 \tau(B^{'}))\right).$$

This expectation can be computed using the density function of $\tau(B^{'})$ and the fact that $E^{\mathbb{P}'}\left(B^{'}_{\tau(B')}\right) = b - S_0$ (Borodin et al. (2002) , p 223).

Finally, the default probability in $(t_1, t_2]$ can be expressed by

$$\begin{aligned} \mathbb{P}\,(\tau(S) \in (t_1, t_2]) &= 1 - \Phi\left(\frac{-(b - S_0) - a_0(t_2 - t_1)}{\sqrt{t_2 - t_1}}\right) & (3.4) \\ &+ \exp(2(b - S_0)a_0)\Phi\left(\frac{(b - S_0) - a_0(t_2 - t_1)}{\sqrt{t_2 - t_1}}\right). & (3.5) \end{aligned}$$

3.3.3 Geometric or arithmetic Brownian Motion and constant default barrier

Assume that the borrower's structural variable S_t follows a geometric Brownian motion

$$S_t = S_0 \exp\left((r_s - \frac{\sigma^2}{2})t + \sigma B_t \right),$$

and that the default barrier is constant, $b(t) = b$. The default time is

$$\tau(S) = \inf\{t > 0 : S_t \leq b\} = \inf\{t > 0 : \frac{1}{\sigma}(r_s - \frac{\sigma^2}{2})t + B_t \leq \frac{1}{\sigma}\log(\frac{b}{S_0})\},$$

which reduces to the drifted Brownian motion with constant barrier case by identifying $b - S_0 \equiv \frac{1}{\sigma}\log(\frac{b}{S_0})$ and $\frac{1}{\sigma}(r_s - \frac{\sigma^2}{2})t$ as the drift.

Replacing these parameters into Equation (3.5), we find the default probability in Black and Cox (1976)

$$\begin{aligned}
\mathbb{P}(\tau \in dt) &= 1 - \Phi\left(\frac{-(\frac{1}{\sigma}\log(\frac{b}{S_0})) - (\frac{1}{\sigma}(r_s - \frac{\sigma^2}{2}))dt}{\sqrt{dt}} \right) \\
&+ \left(\frac{b}{S_0}\right)^{(\frac{2r_s}{\sigma^2}-1)} \Phi\left(\frac{(\frac{1}{\sigma}\log(\frac{b}{S_0})) - (\frac{1}{\sigma}(r_s - \frac{\sigma^2}{2}))dt}{\sqrt{dt}} \right).
\end{aligned}$$

The case of arithmetic Brownian motion is straightforward and can be solved using Equation (3.5):

$$\tau(S) = \inf\{t > 0 : S_t \leq b\} = \inf\{t > 0 : \frac{r_s}{\sigma}t + B_t \leq b - S_0\}.$$

3.3.4 Geometric Brownian Motion and exponential default barrier

Assume that the default barrier takes the form $b(t) = Ke^{rt}$. This would be the case for, e.g., bullet or zero-coupon loans, where the default barrier is proportional to the discounted value of the loan balance at maturity. Explicitly, $b(t) = \alpha D_T e^{-r(T-t)} = Ke^{rt}$ where r could be the risk-free rate and $K = \alpha D_T e^{-rT}$. We then have

$$\tau = \inf\{t > 0 : S_0 e^{(r_s - \frac{\sigma^2}{2})t + \sigma B_t} \leq K e^{rt}\},$$

or, equivalently,

$$\tau = \inf\{t > 0 : \sigma^{-1}(r_s - \frac{\sigma^2}{2} - r)t + B_t \leq \sigma^{-1} \log(\frac{K}{S_0})\}.$$

This can be solved as in the drifted Brownian motion with constant default barrier case by using $\sigma^{-1}(r_s - \frac{\sigma^2}{2} - k$ as the drift and $b = \sigma^{-1}\log(\frac{K}{S_0})$) in Equation (3.5), yielding

$$\begin{aligned}
\mathbb{P}(\tau \in dt) &= 1 - \Phi\left(\frac{-(\frac{1}{\sigma}\log(\frac{K}{S_0})) - (\frac{1}{\sigma}(r_s - k - \frac{\sigma^2}{2}))dt}{\sqrt{dt}}\right) \\
&+ \left(\frac{K}{S_0}\right)^{(\frac{2(r_s - k)}{\sigma^2} - 1)} \Phi\left(\frac{(\frac{1}{\sigma}\log(\frac{K}{S_0})) - (\frac{1}{\sigma}(r_s - k - \frac{\sigma^2}{2}))dt}{\sqrt{dt}}\right).
\end{aligned}$$

3.3.5 Geometric Brownian and linear barrier

Now consider a linear default barrier $b(t) = a + bt$. This would be the case for an amortizing-term loan, where the outstanding loan value decreases linearly over time, $D_t = D_0(\frac{T-t}{T})$, where D_0 is the initial loan amount and $\frac{t}{T}$ is the periodic amortizing rate. The default barrier is then $b(t) = a + bt$ with $a = \alpha D_0$ and $b = \alpha D_0 \frac{t}{T}$.

This case is simple when the borrower's structural variable is described by an arithmetic Brownian motion. The stopping time can then be expressed as

$$\tau = \inf\{t > 0 : S_t \leq b(t)\} = \inf\{t > 0 : S_0 + \frac{r_s}{\sigma}t + B_t \leq D_0\alpha(\frac{T-t}{T})\}$$

and reduces to the case of a drifted Brownian motion with a constant default barrier.

However, this is no longer the case when the underlying structural variable is a geometric Brownian motion. The stopping time is then

$$\tau = \inf\{t > 0 : \sigma^{-1}(r_s - \frac{\sigma^2}{2})t + B_t \leq \sigma^{-1}\log(\frac{D_0\alpha(T-t)}{S_0 T})\},$$

and an analytical solution is no longer available. If T is large enough, one can approximate $\log(\frac{T-t}{T})$ by $\approx -\frac{t}{T}$. The approximate probability can then be computed as for a drifted Brownian motion and constant barrier as follows:

$$\begin{aligned}\tau &= \inf\{t > 0 : \sigma^{-1}(r_s - \frac{\sigma^2}{2})t + B_t \leq \sigma^{-1}(log(\frac{\alpha D_0}{S_0}) - \frac{t}{T})\} \\ &= \inf\{t > 0 : \sigma^{-1}(r_s - \frac{\sigma^2}{2} + \frac{1}{T})t + B_t \leq \sigma^{-1}(\log(\frac{D_0\alpha}{S_0}))\}\end{aligned}$$

and using Equation (3.5) with $b = \sigma^{-1}(\log(\frac{D_0\alpha}{S_0}$ and $\sigma^{-1}(r_s - \frac{\sigma^2}{2} + \frac{1}{T})$ as the drift.

3.4 Conclusion

In this chapter, we presented the borrower capital structure and the default probability models under various assumptions about the evolution of the firm's structural variables and the form of the default barrier. For all the default probability models, the default barrier is one of the important parameters. Everything being equal, a higher default barrier leads to a higher default probability. These default models will be used in the rest of the thesis, where we assume that safety covenants characterize the default barrier of a syndicated loan.

3.5 Appendix: Terminal value and optimal leverage

3.5.1 Value of the firm

Black and Cox (1976) and Leland (1994) provide the necessary tools to determine the total firm value and the optimal leverage for a perpetual-coupon debt, assuming that the default barrier is chosen to optimize the equity value.

Given a default barrier B, Leland (1994) defines the default probability at s, that is, the probability that the asset value hits the default barrier as a function of the initial asset level, as

$$p_B = \left(\frac{B}{s}\right)^{\varsigma},$$

with

$$\varsigma = \frac{1}{2}\frac{2\left(r-\delta\right)-\sigma^2+\sqrt{4\left(r-\delta\right)\left(r-\delta-\sigma^2\right)+\sigma^2\left(8r+\sigma^2\right)}}{\sigma^2},$$

where δ is the continuous dividend payout rate and σ is the asset volatility.

The value of the firm at s is then given by the sum of the current asset value, the expected tax saving benefit from credit cost, minus the expected bankruptcy cost in case of default:

$$V^F(s) = s + \frac{\nu c}{r}\left(1-\left(\frac{B}{s}\right)^{\varsigma}\right) - \gamma B\left(\frac{B}{s}\right)^{\varsigma},, \tag{3.6}$$

where c is the perpetual coupon rate, ν is the tax rate and γ is the proportional bankruptcy cost.

In the particular case of zero debt ($B = 0$), the value of the firm is equal to the equity value.

Similarly, the debt value is defined as the sum of the expected perpetual coupon plus the expected recovery in case of default:

$$V^D(s) = \frac{c}{r}\left(1-\left(\frac{B}{s}\right)^{\varsigma}\right) + (1-\gamma)B\left(\frac{B}{s}\right)^{\varsigma}.,\tag{3.7}$$

The equity value at s can be obtained from the equality $V^F(s) = V^E(s) + V^D(s)$, yielding

$$V^E(s) = s - \frac{C(1-\nu)}{r} - \left(\frac{C(\nu-1)}{r} + B\right)\left(\frac{B}{s}\right)^{\varsigma}. \tag{3.8}$$

3.5.2 Default barrier

Default barriers can be exogenous or endogenous.

Exogenous default barrier

Exogeneous default barriers are mostly imposed by adding covenants. The most used are net positive equity (Brennan and Schwartz (1978); Longstaff and Schwartz (1995)) and zero cash flow (Kim, Ramaswamy, and Sundaresan (1993)). The positive equity condition will force default when the firm value falls below the equity value. The zero cash flow triggers force default when the firm is not generating enough revenue to cover the coupon payment.

Endogenous default barrier

When default is decided by the equity owner, the endogenous default barrier is obtained by maximizing the equity value under a non-negativity equity constraint (Black and Cox (1976); Mello and Parsons (1992); Leland (1994)), leading to the following analytical expression for the endogenous default barrier:

$$B^{endo} = \frac{\varsigma}{r(\varsigma+1)}(1-\nu)c.$$

Replacing B with its endogenous expression (B^{endo}) and developing, the firm total value at s becomes

$$V(s) = s + \frac{c}{r}\nu - \left(c\frac{\varsigma(1-\nu)}{rs(\varsigma+1)}\right)^{\varsigma} c\frac{\nu(\varsigma+1) - \gamma\varsigma(1-\nu)}{r(\varsigma+1)}$$

Optimal coupon

The optimal coupon can be derived by optimizing the total firm value. By differentiating the expression of $V^F(s)$ in Equation 3.6, with respect to c, yields the expression below using the first-order condition.

$$c^*(s) = s\left(\frac{\nu}{rX(1+\varsigma)}\right)^{\frac{1}{\varsigma}},$$

where

$$X \equiv \left(\frac{\varsigma}{r(\varsigma+1)}\right)^{\varsigma} \frac{\nu r(\varsigma+1)+\gamma\varsigma r}{r(\varsigma+1)}.$$

Replacing c^* in Equation 3.6 yields the value of the optimally levered firm at s.

Chapter 4

A stochastic-game model of the syndicated loan process

This chapter outlines the game-theoretical interpretation, using a feedback Stackelberg information structure, of the management of a syndicated loan contract that includes a safety covenant.

4.1 Introduction

The seminal model in Black and Cox (1976) is the first contribution to the pricing of safety covenants in loan contracts. In this model, built under the popular Merton (Merton (1974)) structural model assumptions, the covenant consists of maintaining a net positive equity value over the life of the contract. A covenant violation leads to forced bankruptcy, transferring the firm's ownership to the bondholders. Using some important simplification in the covenant monitoring, the authors derive a closed-form solution, adapted from Merton (1974), and analyze the impact of the covenant on the value of the borrower's securities.

Most of the simplifications used in Black and Cox (1976) are not consistent with the current practice on covenant monitoring. First, financial covenants are based on observable financial indicators like debt to equity or asset ratio (see Section 2.4). In addition, recent empirical findings (Roberts and Sufi (2009); Nini, Smith, and Sufi (2012); Roberts (2015); Prilmeier (2017)) show that most lenders do not exercise their right to force bankruptcy. Instead, they use it as bargaining power to renegotiate the loan contract's terms, which can lead to an increase in interest rates (repricing), a reduction of the size of the credit line, or to a request for more collateral. On the other hand, borrowers are not passive actors in the covenant monitoring process, as the call for terms renegotiation can come from the borrower when its financial performance is better than expected at the loan contract inception (Denis and Jing Wang (2014)).

These successive re-negotiations can be interpreted as a dynamic stochastic game between the lender and the borrower, over a finite horizon corresponding to the loan contract's life.

4.2 The model

We consider a loan contract between a *borrower* and a group of banks, identified in the sequel as the *lender,* with inception date $t = 0$ and maturity T. The loan contract includes a financial safety covenant, in the form of a constraint that needs to be respected by the borrower. Two levels are identified for the covenant constraint, corresponding respectively to a *physical default* and a *technical default* barrier. The borrower's default state at a given date t is determined by the value at t of the borrower's asset value process $\{S_t\}$ (assumed to be observable by both parties) with respect to these barriers. According to the terms of the contract, the lender forces bankruptcy whenever the borrower is in physical default. Moreover, the contract allows the lender to take some corrective actions (e.g. modify the default barrier or increase the interest rate of the loan) when the borrower is in technical default. On the other hand, the borrower has the possibility to refinance the loan, modify the risk level of their investment strategy, or file for bankruptcy.

4.2.1 Information structure

Both the lender and the borrower can take actions over the life of the contract that may alter the evolution of the borrower's firm process and their respective payoffs. The lender can take corrective actions when the covenant is breached, which amounts to changes in the terms of the contract. On the other hand, the borrower has some control over the evolution of their firm's structural variable, which amounts to changes in its risk level, drift, and/or volatility.

This process is modeled as a sequential two-player leader-follower game in discrete-time, where the lender is the leader and the borrower is the follower, and where the state variable is the current level of the process S_t.

4.2.2 Time line

At some fixed contractual dates $t_1,...,t_n$, where $t_n = T$ and $[0,t_1)$ is the *protection period*, the lender will perform an audit to verify whether the covenant constraint has been breached or not. To simplify notation, we assume that audit dates are equally spaced, $t_{m+1} - t_m = \Delta$ for $m = 0,...,n$ and coincide with the dates of the periodic payment of interest and capital amortizing, if applicable.

At a given monitoring date t_m, the lender, identified as the leader of the game (Player L) chooses among a set of available actions, and the borrower, identified as the follower (Player F), observing the choice made by the leader, reacts by choosing among a set of available responses.

The lender's set of actions

In case of technical default, the lender can choose to implement:

- A temporary interest increase, due at the next payment date;
- A temporary increase of the physical default barrier. This action is equivalent to a

request for more collateral as a percentage of the outstanding debt, which results in a reduction of the loss given default;

- A combination of both corrective actions.

Note that when the borrower is not in technical default, or when the forced bankruptcy conditions are met, the lender has no available option. Moreover, in the absence of a covenant violation, any previous increase of the coupon and/or a temporary increase of the default barrier is canceled, which brings the spread and the default thresholds back to the initial contractual values.

The borrower's set of responses

At any monitoring date, the borrower can choose to:

- Refinance the loan;
- Accept the conditions of the loan, eventually including the corrective actions implemented by the lender;
- File for bankruptcy;
- Modify the risk level of their investment strategy.

To schematize, we assume that the borrower can adopt either a conservative or an aggressive investment strategy between two monitoring dates and that an aggressive investment strategy results in an immediate additional net dividend, at the expense of higher volatility for the process S_t.

4.2.3 Notation

This section defines the notation that will be used throughout the thesis.

State variable

s : current level of the process S_t

Parameters

α : contractual physical default parameter

$\hat{\alpha}$: contractual technical default parameter

c_m : periodic payment due at date t_m, $m = 1, \ldots, n$

D_t : contractual outstanding debt at date t; this deterministic series can be constant, linear, or exponential in time according to the contract's debt amortization.

r : risk-free rate

δ : continuous dividend payout rate

i : contractual periodic interest rate

Δ : time interval between two successive monitoring dates, assumed constant

$\beta = \exp(-r\Delta)$: periodic discount factor

γ : proportional bankruptcy costs

ν: tax rate, also known as net tax advantage of debt

g : immediate net dividend resulting from an aggressive investment strategy, as a percentage of the current asset value s.

Decision variables

p : coupon increase imposed by the leader in case of technical default, due at the next monitoring date.

κ : physical default threshold increase imposed by the leader, $\kappa \geq 0$, where $\kappa = 0$ indicates no change with respect to the contractual value.

λ : indicator of the follower's response, $\lambda \in \{-1, 0, 1\}$, where $\lambda = -1$ corresponds to refinancing the loan, $\lambda = 0$ to filing for bankruptcy, and $\lambda = 1$ corresponds to accepting the terms of the loan.

θ : indicator of the follower's investment strategy, $\theta \in \{0, 1\}$, where $\theta = 0$ corresponds to a conservative strategy and $\theta = 1$ corresponds to an aggressive strategy.

Functions

$\mathbb{E}_m[\cdot]$: expectation conditional on the information available at t_m

$b(t) = \alpha(1+\kappa)D_t$: physical default barrier at date t

$\hat{b}(t) = \hat{\alpha}D_t$: contractual technical default barrier at date t

$\mathbb{I}_x(i)$: indicator function of a discrete variable x

$$\mathbb{I}_x(i) = \begin{cases} 1 \text{ if } x = i \\ 0 \text{ if } x \neq i \end{cases}$$

$\mathbb{I}(A)$: indicator function of a condition A

$$\mathbb{I}(A) = \begin{cases} 1 \text{ if } A \text{ is true} \\ 0 \text{ if } A \text{ is false} \end{cases}$$

$1_m(\alpha(1+\kappa))$ *resp.* $0_m(\alpha(1+\kappa))$: the probability of a physical default (*resp.* no physical default) in the time interval $(t_m, t_{m+1}]$, given the information available at t_m. Details are given in Section 3.2. Note that this probability depends on s and θ, which are observable at t_m

$\hat{D}(\tau) = D_{t_m} + (iD_{t_m} + p)\frac{\tau - t_m}{\Delta}$: outstanding debt between two monitoring dates, accounting for accrued coupon payment, where $\tau \in (t_m, t_{m+1}]$

$R_t^L(s,D), R_t^F(s,D)$: leader and follower recovery given default at date t when $S_t = s$ and $D_t = D$,

$$\begin{aligned} R_t^L(s,D) &= \min\{(1-\gamma)s, D\} \\ R_t^F(s,D) &= (1-\gamma)s - R_t^L(s,D) \end{aligned}$$

$\omega_m(s)$: refinancing cost at date t_m when $S_{t_m} = s$. The expression and calibration of this function is detailed in Appendix 4.6.1

$\vartheta(s)$: terminal payoff function of the borrower. The terminal value function depends on assumptions about the way the debt is eventually refinanced. See Appendix 4.6.2 for more details.

4.2.4 Immediate payoffs and state dynamics

At a given monitoring date t_m, $m = 1, ..., n$, the level s of the state variable is observable by both players. At t_m, conditional to the borrower not being in physical default, each player will receive an immediate (expected) payoff that depends on the value of the state variable and on the decisions of both players. In addition to the payoffs secured at monitoring dates, players may receive intermediate payoffs if physical default happens between two monitoring dates.

The lender's payoff function

The immediate expected payoff of the lender (Player L), denoted by w_m^L, is given by

$$\begin{aligned} w_m^L(s,p,\kappa,\lambda,\theta) &= c_m + R_{t_m}^L(s,D_m)\mathbb{I}_\lambda(0) + D_m\mathbb{I}_\lambda(-1) + 0_m(\alpha(1+\kappa))\beta p\mathbb{I}_\lambda(1) \\ &+ \quad 1_m(\alpha(1+\kappa))\mathbb{E}_m\left[\exp(-r(\tau - t_m))R_\tau^L(b(\tau),\hat{D}_\tau)\right], \\ & \quad m = 1,...,n-1 \end{aligned} \tag{4.1}$$

$$w_n^L(\cdot) = c_n \tag{4.2}$$

$$w_0^L(s,\cdot) = 1_0(\alpha)\mathbb{E}_0\left[\exp(-r(\tau - t_0))R_\tau^L(b(\tau),\hat{D}_\tau)\right]. \tag{4.3}$$

The first term is the contractual coupon payment due at t_m; the second term is the lender's recovery given default at (t_m, s). The third term is the outstanding debt, which is received if the borrower decides to refinance the loan. The fourth term is the discounted expected value of the coupon increase penalty imposed by the lender, which is due at t_{m+1} if the borrower accepts the terms of the loan, provided that physical default does not occur before the next monitoring date. Finally, the last term is the expected value of the recovery by the lender if physical default happens before the next monitoring date. This last term depends on the exposure at the default instant (EAD), which is given by function $\hat{D}_\tau$.

We assume that the lender cannot impose a penalty or a physical default threshold modification at $t = 0$.

The borrower's payoff function

In the same way, the immediate expected payoff of the borrower (Player F) at t_m, denoted by w_m^F, is defined by

$$\begin{aligned} w_m^F(s, p, \kappa, \lambda, \theta) &= gs\mathbb{I}_\theta(1) - \omega_m(s)\mathbb{I}_\lambda(-1) + R_{t_m}^F(s, D_m)\mathbb{I}_\lambda(0) \\ &+ 1_m(\alpha\kappa)\mathbb{E}_m\left[\exp(-r(\tau - t_m))R_\tau^F\left(b(\tau), \hat{D}_\tau\right)\right], \\ & \quad m = 1, \ldots, n-1 \end{aligned} \tag{4.4}$$

$$w_n^F(s, \cdot) = \vartheta(s) \tag{4.5}$$

$$w_0^F(s, \cdot) = 1_0(\alpha)\mathbb{E}_0\left[\exp(-r(\tau - t_0))R_\tau^F\left(b(\tau), \hat{D}_\tau\right)\right]. \tag{4.6}$$

The first term corresponds to the immediate additional dividend resulting from an aggressive investment strategy during the period $(t_m, t_{m+1}]$. The second term corresponds to the refinancing costs. The third term is the recovery of the borrower given default at (t_m, s), while the fourth term is the recovery of the borrower if physical default happens before the next monitoring date. Finally, the function $\vartheta(s)$ represents the value of the borrower's going concern at s. We assume that the follower has no available option at $t = 0$ (e.g. the initial investment strategy is fixed).

Dynamics

Assuming that physical default does not happen in $(t_{m-1}, t_m]$, the value of the state vector s' at date t_{m+1} depends on the actions chosen by the lender and the borrower at t_m and satisfies

$$s' = S_{t_{m+1}} | (s, \sigma_\theta) - (c_{m+1} + p)(1 - \nu) \tag{4.7}$$

where the distribution of the random variable $S_{t_{m+1}}$, given (s, σ_θ), is obtained from Equation (3.1). The second component is the credit cost (coupon plus punishment), adjusted for the tax impact, assuming that the punishment amount p is part of the credit cost and tax deductible.

Note that both the lender's and the borrower's decisions impact the evolution of the stochastic variable S_t. The punishment amount p adds to the contractual coupon and thus reduces the asset level. On the other hand, the borrower's decision to adopt a conservative or aggressive investment strategy determines the volatility of the process.

4.3 Equilibrium strategies and value functions

A feedback strategy for a player $j \in \{L, F\}$ is a function δ_m^j indicating the action(s) taken by Player j at monitoring date t_m, $m = 1, \ldots n-1$, as a function of the information available to this player. Accordingly,

$$\begin{aligned} \delta_m^L &: \quad s \to \mathscr{L} \\ \delta_m^F &: \quad (s, p, \kappa) \to \mathscr{F} \end{aligned}$$

where $\mathscr{L}$ (*resp.* $\mathscr{F}$) is the set of admissible (p, κ) for Player L (*resp.* admissible (λ, θ) for Player F), given the available information at t_m.

The functions used by the players to evaluate a strategy vector $\delta \equiv \left(\delta^L, \delta^F\right)$, $\delta^j \equiv$

$\left(\delta_m^j\right)_{m=1,\ldots,n-1}$, satisfy the functional equations

$$\begin{aligned} V_m^j(s;\delta) &= w_m^j(s,\delta) + 0_m(s,\delta)\,\beta\mathbb{E}_m\left[V_{m+1}^j\left(s';\delta\right)\right],\ j\in\{L,F\}, \\ &\quad m=0,\ldots n-1 \qquad (4.8) \\ V_n^j(s;\delta) &= w_n^j(s,\cdot), \qquad (4.9) \end{aligned}$$

subject to the dynamics (4.7). Given (4.7) and (4.8)-(4.9), we seek a dynamic Stackelberg equilibrium in feedback strategies between the lender (leader) and the borrower (follower).

One of the crucial issues in Stackelberg games is the indeterminacy that can arise when the follower has multiple optimal responses to a given action of the leader. In the context of a multi-stage feedback Stackelberg game, Breton et al. (1988) characterize the concepts of Strong Stackelberg and Weak Stackelberg equilibria (SSE and WSE), where a lexicographic order, based on the leader's outcome, is assumed when the follower has multiple best responses. The SSE case corresponds to a situation where the payoff of the leader is maximal, under the constraint that the follower's strategy is the best response. The WSE gives rise to a security strategy for the leader, optimizing the worst case with regard to the follower's best response.

In our model of the leader-follower interaction, we opt for the SSE assumption: when facing a tie, the follower will select an action that maximizes the outcome for the leader. This assumption is motivated by the fact that the two players' interests are not exactly opposed, both preferring strategies avoiding costly physical default. This assumption does not preclude the existence of multiple equilibria, but it ensures that the follower's response function is well-defined.

Accordingly, for a given function $v:\mathbb{R}\to\mathbb{R}$, define the reaction set of the follower to a decision pair (p,κ) at $t_m, s=S_{t_m}$ for $m=1,\ldots,n-1$:

$$\mathscr{R}_m^F(s,p,\kappa,v) = \left\{(\lambda^*,\theta^*)\in\arg\max_{\lambda,\theta\in\mathscr{F}}\left\{\begin{array}{c} w_m^F(s,p,\kappa,\lambda,\theta)+\beta 0_m(\alpha\kappa)\,\mathbb{E}_m[v(s')] \\ \text{s.t. } (4.7)\end{array}\right.\right\}$$

.

Definition 1 *A strategy vector δ^* is a strong feedback Stackelberg equilibrium and $v_m^L(\cdot) \equiv V_m^L(\cdot;\delta^*)$ (resp. $v_m^F(\cdot) = V_m^F(\cdot;\delta^*)$) is the corresponding equilibrium value function if the following conditions are satisfied for all s and for $m = 1,...,n-1$:*

$$\delta_m^{*F} \in \mathscr{R}_m^F(s,p,\kappa,v_{m+1}^F) \tag{4.10}$$

$$V_m^L(s;\delta^*) = \max_{p,\kappa\in\mathscr{L}} \left\{ \max_{(\lambda,\theta)\in\mathscr{R}_m^F(s,p,\kappa,v_{m+1}^F)} \left\{ \begin{array}{c} w_m^L(s,p,\kappa,\lambda,\theta) + \beta 0_m(\alpha\kappa)\mathbb{E}_m\left[v_{m+1}^L(s')\right] \\ \text{s.t. (4.7)} \end{array} \right\} \right\} \tag{4.11}$$

As shown in Breton, Alj, and Haurie (1988), a feedback Stackelberg equilibrium in a multi-stage game is equivalent to a feedback Nash equilibrium in an associated game with twice the number of stages, where decisions are taken by the leader in odd stages and by the follower in even stages (a switching-controller game), and where the state variable in even stages includes the decisions announced by the leader, which are observable by the follower. A feedback Stackelberg equilibrium can then be characterized by the following dynamic program, which can be solved by backward induction from the known terminal value $\left(v_n^L, v_n^F\right)$.

$$
\begin{aligned}
v_n^L(s) &= c_n && (4.12)\\
v_n^F(s) &= \vartheta(s) && (4.13)\\
y_m^F(s,p,\kappa) &= \max_{\lambda,\theta\in\mathscr{F}} \left\{w_m^F(s,p,\kappa,\lambda,\theta)+\beta 0_m(\alpha\kappa)\mathbb{E}_m\left[v_{m+1}^F(s')|\theta\right]\right\},\\
&\quad m=1,...,n-1 && (4.14)\\
\mathscr{R}_m^F(s,p,\kappa) &= \left\{(\lambda^*,\theta^*)\in\arg\max_{\lambda,\theta\in\mathscr{F}}\left\{w_m^F(s,p,\kappa,\lambda,\theta)+\beta 0_m(\alpha\kappa)\mathbb{E}_m\left[v_{m+1}^F(s')|\theta\right]\right\}\right\},\\
&\quad m=1,...,n-1 && (4.15)\\
y_m^L(s,p,\kappa) &= \max_{\lambda,\theta\in\mathscr{R}_m^F(s,p,\kappa)} \left\{w_m^L(s,p,\kappa,\lambda,\theta)+\beta 0_m(\alpha\kappa)\mathbb{E}_m\left[v_{m+1}^L(s')|\theta\right]\right\},\\
&\quad m=1,...,n-1 && (4.16)\\
v_m^L(s) &= \max_{p,\kappa\in\mathscr{L}}\left\{y^L(s,p,\kappa)\right\}, m=1,...,n-1 && (4.17)\\
\mathscr{R}_m^L(s) &= \left\{(p^*,\kappa^*)\in\arg\max_{p,\kappa\in\mathscr{L}}\left\{y^L(s,p,\kappa)\right\}\right\}, m=1,...,n-1 && (4.18)\\
v_m^F(s) &= y^F(s,p^*,\kappa^*),(p^*,\kappa^*)\in\mathscr{R}_m^L(s), m=1,...,n-1 && (4.19)\\
v_0^L(s) &= w_0^L(s)+\beta 0_0(\alpha)\mathbb{E}_0\left[v_1^L(s')|\theta_0\right] && (4.20)\\
v_0^F(s) &= w_0^F(s)+\beta 0_0(\alpha)\mathbb{E}_0\left[v_1^F(s')|\theta_0\right] && (4.21)
\end{aligned}
$$

The functions $v_m^j(s)$, $j\in\{L,F\}$ yield the equilibrium total expected discounted cash flows received by the lender (Player L) and the borrower (Player F) resulting from the loan contract, from date t_m until maturity, as a function of the level of the state variable at date t_m. Note that multiple equilibria may exist if one of the sets $\mathscr{R}m^L(s)$ contains more than one element. However, this does not affect the equilibrium value function, which is well-defined.

Finally, recall that the leader has no available option when the follower is not in technical default, that is, the set of admissible actions of the leader is a singleton ($p=0, \kappa=1$) at t_m, $S_{t_m}=s$ when $s>(1+\kappa)\alpha D_{t_m}$.

4.4 Numerical implementation

Since the state variable s is continuous and since the value function cannot be obtained in closed form, some form of approximation is needed. For the numerical experiments presented in this thesis, we use a cubic spline interpolation approach (see Breton and Frutos (2011) for a survey of interpolation methods for finite-horizon stochastic dynamic programs). The interpolation of v_m^j, $j \in \{L, F\}$, $m = 0, ..., n$, is denoted by $\widehat{v}_m^j$.

The dynamic programming algorithm is as follows.

1. Initialisation: Read the parameters. Define the discretization grids G_s, G_p, and G_κ for the state variable s and for the decision variables p and κ. Set v_n^L and v_n^F on G_s using (4.12)-(4.13). Interpolate and store the coefficients of $\widehat{v}_n^L$ and $\widehat{v}_n^F$.

2. For $m = n-1, .., 1$

   a) Follower's stage

      On $G_s \times G_p \times G_\kappa$:

      i) Using Equation (4.14), compute $y_m^F(s, p, \kappa)$.

      ii) Record the set $\mathscr{R}_m^F(s, p, \kappa)$ of optimal reactions of the follower.

      iii) Using Equation (4.16), compute $y_m^L(s, p, \kappa)$ and record the corresponding follower's equilibrium strategy $\delta_m^{*F}(s, p, \kappa)$

   b) Leader's stage

      On G_s :

      i) Using Equation (4.17), compute $v_m^L(s)$.

      ii) Record the leader's equilibrium strategy $\delta_m^{*L}(s) \in \mathscr{R}_m^L(s)$.

      iii) Using Equation (4.18), compute $v_m^F(s)$.

   c) Interpolation

      Interpolate and store the coefficients of v_m^L and v_m^F.

3. At $t = 0$, the loan contract's value for both players is given by (4.20)-(4.21).

As mentioned above, multiple equilibria may exist whenever the set $\mathscr{R}_m^L(s)$ is not a singleton at some m and $s \in G_s$. In our implementation, we assume that the leader chooses a strategy in $\mathscr{R}_m^L(s)$ that maximizes the follower's value function at (m, s).

4.5 Conclusion

We have developed a dynamic stochastic game model to replicate the successive contract negotiation in a syndicated loan that includes a covenant. The model incorporates several practical observations from the market. This includes the borrower's investment strategy (aggressive and conservative) and provides the lender the possibility to put a default barrier that gives a floor value to the loan (as in Black and Cox (1976)). In addition, the model includes the lender's right to punish any contractual breach.

This valuation model is the first in the literature to encompass these two rights of the lender, while also granting the borrower the flexibility to optimally adjust its investment strategy. Note that the equilibrium value function computed using our model specifies the expected equilibrium outcome for the leader (the lender) and the follower (the borrower), at all monitoring dates, as a function of the current asset value.

Our model introduces the first approach to evaluate the loan contract in ex-ante taking into account the effect of the possible contract renegotiation from the covenant monitoring. It goes beyond the case of a protective covenant (see Black and Cox (1976)), allowing dynamic pricing to adjust the spread over the course of the contract, depending on the covenant violation and the market conditions (e.g. the refinancing cost). The model can be used for numerous applications. One can particularly analyze the possible price deviation in the secondary market from the contract's intrinsic value. Another interesting application could be an efficiency analysis of the performance pricing compared to a flexible covenant monitoring as resented in this paper. Some interesting extensions and improvements are possible; among others, the inclusion of a reputation as a non-monetary reward for the two parties, the flexibility to monitor or not as an additional decision variable for the lender, or more extensively the possibility to combine the covenant monitoring game with a bankruptcy game under a judge's intervention as presented in Annabi, Breton, and François (2012). This shortlist constitutes a future research avenue. In the following chapter, we will use this model to estimate the value of a syndicated term-loan contract including a covenant and we will analyze the impact of the presence of a covenant and of various parameters on the value of a loan, in the spirit of covenant design.

4.6 Appendix

4.6.1 Refinancing cost function

The refinancing cost includes all the upfront fees paid by the borrower to the lender at the loan initiation. This cost differs from interest payments, as it consists of arrangement, underwriting, and participation fees.

Modeling the refinancing cost requires the identification of the determinants of upfront fees in the syndication process. Two drivers have been identified in the literature: the syndication risk and the syndication cost (Gadanecz (2004)). The *syndication risk* is defined as the risk that the underwriters will have to absorb any unallocated commitment amount in case of an insufficient number of lenders to raise the commitment amount. The *syndication cost* encompasses the expenses that the lead arranger will incur in organizing the syndication. These include legal, administrative, and roadshow costs. The syndication risk and cost can be linked to the borrower's credit risk through three channels. First, risky borrowers attract fewer lenders and increase the syndication risk, thus increasing the underwriting fee. Second, syndicate participants in a risky deal will ask for relatively higher participation fees. Finally, legal and administrative expenses may be higher for risky deals, thus increasing the arrangement fee required by the lead arranger. Consequently, as confirmed by Berg, Saunders, and Steffen (2016), risky deals tend to be associated with higher upfront fees.

To identify this relation, we correlate the total upfront fee to the leverage ratio of the borrower using data from the DealScan database. One can identify two regions in the scatter plot represented in Figure 4.1; for borrowers with leverage below a given level (identified l_0), there is virtually no correlation between the leverage and the upfront fee, while a significant correlation is apparent when leverage is above l_0.

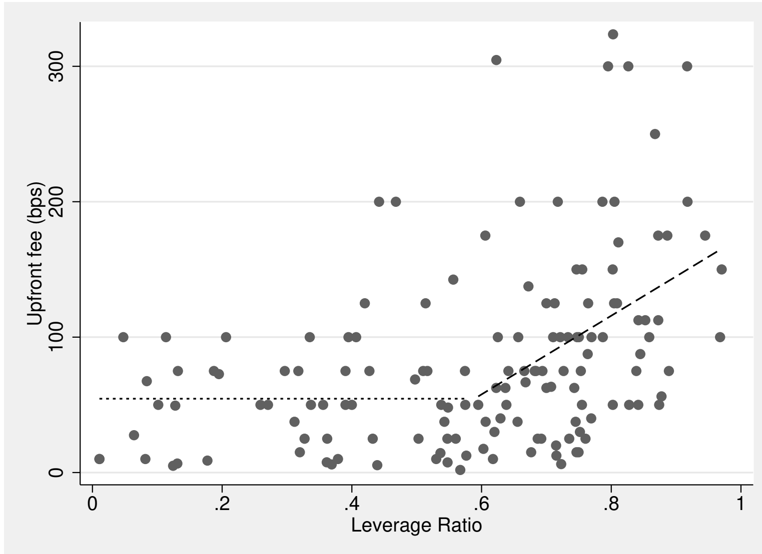


Figure 4.1: Upfront fee as a function of leverage

Based on this correlation and the empirical literature, we model the refinancing cost (expressed in percentage of the syndicated debt) as a linear function of the borrower's leverage ratio (computed over the total debt) for values above l_0, and a constant cost for any borrower with leverage below l_0. This can be expressed as

$$\frac{\omega_j}{D_j} = \left(c_0 + \left(c_1 + c_2 \frac{D_j^T}{s_j - D_j^T}\right) \mathbb{I}\left(\frac{D_j^T}{s_j - D_j^T} > l_0\right)\right), \tag{4.22}$$

where s_j is the asset value of the borrower j, D_j the syndicated loan amount at the initiation, D_j^T its total outstanding debt value (including syndicated loans and bilateral debts prior to the syndicated loan inception), l_0 is the threshold separating the two regions, and c_0, c_1, c_2 are the equation parameters (expressed in bps) to be determined in the calibration.

Using a sample from Dealscan, for each of the borrowers in the sample, we computed the variable in Equation 4.22 using the information below:

- The total upfront fees ω at the loan inception (in millions of USD), used as a depend variable

- Total asset value s as of the date of the syndicated contract inception (in million USD).

- Total syndicated debt amount value D (in million USD) used to compute the leverage

- Total outstanding debt D^T (million of USD) including other non-syndicated loans.

We run the regression for different values of l_0 and report the results in Table 4.1.

Table 4.1: Regression of upfront fee on leverage ratio

Ind. var.: upfront fee	$l_0 = 0.2$	$l_0 = 0.3$	$l_0 = 0.4$	$l_0 = 0.5$	$l_0 = 0.6$
c_2	146.7***	165.2***	184.8***	273.8***	289.0***
	(31.29)	(33.80)	(45.43)	(53.40)	(74.42)
c_1	-52.14*	-70.61***	-83.20**	-161.2***	-169.9***
	(26.45)	(26.99)	(33.94)	(39.90)	(57.03)
c_0	44.52***	49.62***	47.61***	57.57***	54.46***
	(16.25)	(14.36)	(10.72)	(9.490)	(7.930)
adj. R-sq	0.155	0.164	0.166	0.193	0.195
N. Observation	143	143	143	143	143
Note: Standard errors in parentheses, * p<0.10, ** p<0.05, *** p<0.01					

Based on these estimations, the highest adjusted R squared is obtained with a threshold of $l_0 = 0.6$. This gives the following values for the parameters ($c_0 = 0.005446$, $c_1 = -0.01699$,$c_2 = 0.0289$) when converted in decimal.

4.6.2 Terminal value

The choice of a terminal value may have a significant impact on the value of a loan, and this choice depends on the specific circumstances of the borrower. The simplest case (and the one that will be used in the subsequent chapter) happens when the borrower reimburses all their debt at maturity. In that case, one can assume that the terminal value is the value of the equity, which, if there is no debt, is equal to the asset value. As an alternative, one can assume that the follower chooses the optimal leverage, given the asset value at T, and refinances the firm using bonds with a perpetual coupon. In that case, one can use the developments in Appendix 3.5 and the terminal value can be obtained by determining the optimal coupon and deriving the firm value. Another option is to suppose that the borrower refinances their debt in the syndicated loan market, using, for instance, the refinancing cost function estimated above. Finally, one could embed the model presented in this chapter to determine a fixed point value corresponding to the reconduction of the loan over a finite horizon, with the inclusion of a covenant.

Chapter 5

Numerical experiments

This chapter discusses the results of numerical experiments obtained for a term loan using the stochastic game model presented in Chapter 4. Specifically, we use our model to illustrate the impact of various parameters on the value of the loan and of the covenant, and to develop insight on the equilibrium strategies of the lender and the borrower, as a function of the borrower's circumstances.

5.1 Base-case parameter values

We assume that the borrower has no outstanding debt at inception and consider a loan for a normalized amount of \$100 over one-year maturity, with monthly auditing dates, under three contrasting coupon scenarios: Zero-coupon, constant debt level, and amortizing debt. The results reported in this chapter are for the zero-coupon case. Results pertaining to the constant debt and amortizing debt assumptions are reported in Appendix 5.9.2.

The base-case parameter values are presented in Table 5.1. Some of the parameters are taken from the literature, while others are estimated from a sample of 5,000 syndicated loan contracts concluded in the period between 2000 and 2012, extracted from the Loan Pricing Corporation (LPC) DealScan database. The asset value at inception is set at $S_0 =$ \$150, which corresponds to a debt-to-total asset ratio of $\frac{2}{3}$ — the average in our sample of syndicated loans. In our sensitivity analyses, we use initial asset values varying between \$130 and \$180. The minimum value rules out the possibility of the borrower being in technical default at the loan inception, thus excluding leverage loans from our analysis. The tax rate is set at 35%, which is the average in the U.S. and Canada. Following the empirical literature, we assume a bankruptcy cost of 20% of the asset value at the default date.

The value of g (net dividend when adopting an aggressive investment strategy) is set at $\frac{1}{12}$ of the current asset value. This value is obtained by correlating historical annual dividend rates to the annual asset volatility and computing the elasticity of dividends to the volatility and corresponds to the additional dividend when moving from 15% to 25% volatility.

We set the contractual physical default parameter α to 1, which means that the physical default is triggered when the asset value attains the outstanding loan value. This is also known as the *non-negative equity condition* (Black and Cox (1976)). We set the technical default parameter $\hat{\alpha}$ at 1.3.

Finally, in our base-case specification, the terminal value for the follower is set to the asset value.

Table 5.1: Base-case model parameter values

Parameters	Notation	base-case value	Min	Max
Debt to equity ratio at inception	$\frac{D_0}{S_0}$	$\frac{2}{3}$	0.55	0.77
Loan maturity	T	12 months		
Risk-free rate	r	3%	1%	12%
Loan spread over the base rate	$i-r$	150 bps	130	180
Net tax advantage of debt	ν	35%		
Bankruptcy costs	γ	20%	5%	30%
Asset volatility, conservative (annual)	σ_0	15%	5%	50%
Asset volatility, aggressive (annual)	σ_1	25%	15%	65%
Dividend payout rate	ς	1%	0%	1.5%
Immediate dividend	g	$\frac{1}{12}$ %	$\frac{0.5}{12}$ %	$\frac{2.5}{12}$ %
Physical default parameter	α	1	0.7	1.5
Technical default parameter	$\hat{\alpha}$	1.3 α	0.7 α	1.5 α
Default threshold increase	κ	10%	5%	20%
Revolving credit size	M	40		
commitment interest rate	i_c	0.5%		

5.2 Comparative statics

In this section, we analyze the behavior of the value of the loan contract for the two players, with or without a covenant.

We first compare the equilibrium value of the contract at inception, for both players, assuming a fixed loan spread over the base rate. Figures 5.1 and 5.2 represent respectively the value (normalized value) of a loan for the leader and the value of the follower as a function of the asset level at the loan inception date, other parameter values being the same as in the base case (see Table 5.1).

In Figure 5.1, the loan value is compared across three scenarios: one without covenant, where the leader is not using any punishment while the borrower keeps all their strategic actions; a scenario with a covenant allowing the leader to increase either of both the default barrier and the coupon in the event of technical default; and a scenario with a limited covenant where the leader can only ask for an increased coupon when technical default occurs.

As expected, the presence of a financial safety covenant, at the same spread level, improves the loan value for the leader. Figure 5.1 also shows that, in the interval of values used in our implementation, the loan value with covenant is a decreasing function of the initial asset value (or, equivalently, an increasing function of leverage): the possibility of increasing the spread in case of technical default improves the loan value. In contrast, the loan value without covenant is an increasing function of the initial asset value (decreasing with leverage), driven by the physical default risk.

As the initial leverage improves, the loan values with or without covenant converge, which suggests that covenants may not add value for low-risk borrowers. This is consistent with empirical findings, which support that fewer covenants are imposed on riskless borrowers (Reisel (2014)). In that case, the cost of the covenant (e.g. monitoring cost, which is not accounted for in our model) may be higher than the protection gain it provides.

Similarly, Figure 5.2 compares the borrower value under the three scenarios described above.

Recall that the terminal value for the follower corresponds to the equity value at maturity. Throughout the contract, the borrower accrues immediate dividends as payoffs whenever they opt for aggressive play. Therefore, the value depicted in Figure 5.2 represents the total of expected discounted cash flows (including dividends) received by the borrower over the course of the contract, plus the discounted expected terminal value, at equilibrium.

The result shows that the presence of the covenant reduces the follower's value in regions of high leverage (higher technical default risk). As the risk of technical default becomes smaller, the follower values, with or without covenant, converge. When the lender uses only interest punishment, the follower value improves compared to the case where the lender can also increase the default barrier.

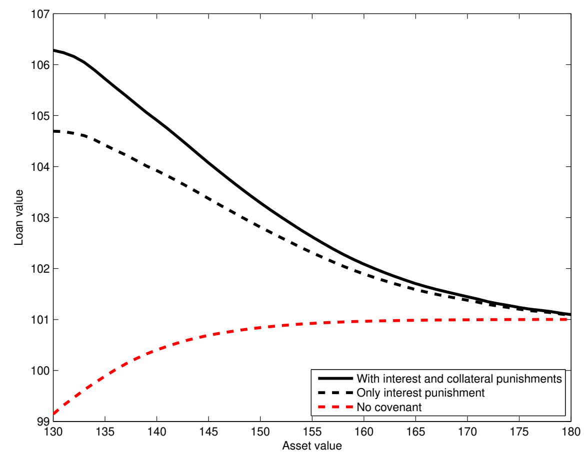


Figure 5.1: Loan value as a function of the initial asset value for a fixed loan amount at the loan inception date, for three alternative scenarios under the base-case specification given in Table 5.1

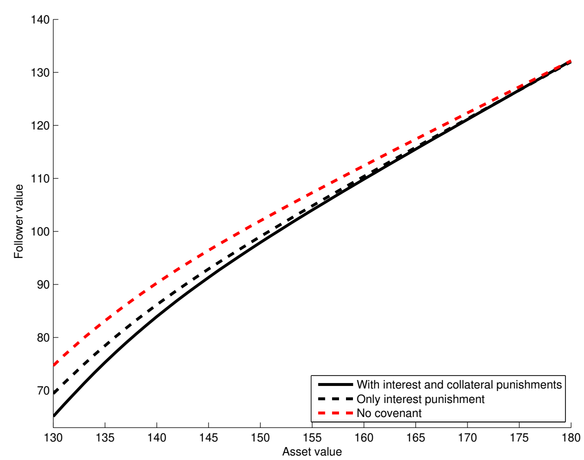


Figure 5.2: Follower value as a function of the initial asset value for a fixed loan amount at the loan inception date, for three alternative scenarios under the base-case specification given in Table 5.1

5.3 Sensitivity analysis

We now perform sensitivity analyses around the base case values of various parameters of the stochastic game model presented in Chapter 4.

Impact of asset volatility

Figure 5.3 depicts the impact of the borrower's asset volatility on the equilibrium loan value at the contract inception, keeping all the other parameters at their base-case values (Table 5.1), with a debt face value 0f \$100. It is important to recall that all experiments are performed using the same loan spread.

In the absence of a covenant, the loan value is a decreasing function of the initial asset volatility. However, when a covenant is present, the loan value initially increases with the asset volatility, to subsequently decrease when the volatility surpasses a given threshold. Across various initial debt-to-equity ratios, the covenant improves the loan value for all levels of initial asset volatility. Nevertheless, the impact of the covenant, indicated by the difference between the loan value with and without a covenant, is notably more important for high-leverage borrowers (represented by solid lines) as opposed to low-leverage borrowers (depicted by dotted lines).

Figure 5.4 presents the corresponding results for the follower. Across all scenarios, the follower's value decreases as the initial asset volatility increases. The follower's value, whether with or without a covenant, tends to converge for lower initial asset volatility.

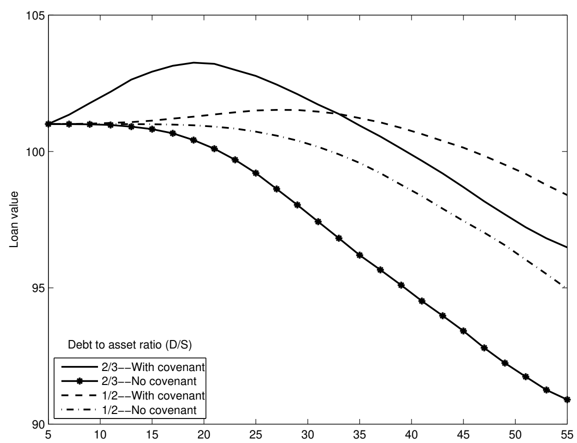


Figure 5.3: Loan value as a function of the asset volatility, for various initial leverage ratios, other parameter values are given in Table 5.1

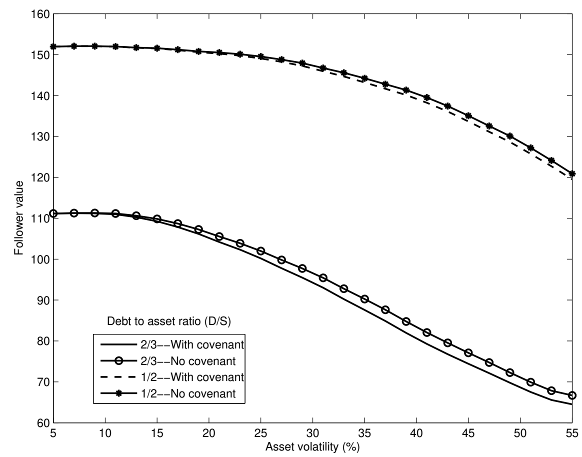


Figure 5.4: Follower value as a function of the asset volatility, for various initial leverage ratios, other parameter values are given in Table 5.1

Impact of the physical default barrier parameter

In our model, the covenant specifies the physical default barrier through the parameter α. The value of this parameter determines the borrower's asset value at which the default will be forced by the lender. Through the covenant's monitoring, the lender can change the value of this parameter to increase or reduce the default barrier when the borrower is in technical default. Although a higher default barrier increases the probability of default, it can also protect the lender as it can reduce the loss given default and provide a higher minimum value (floor value) to the loan. The loss-given default also depends on the level of the bankruptcy cost, which can impact the relationship between the default probability and the level of the default barrier Black and Cox (1976). We tested this relationship by considering different levels of bankruptcy costs. Figure 5.5 reports the equilibrium value for the lender and the borrower, as a function of the physical default barrier parameter, for various levels of bankruptcy costs. The result supports that when the bankruptcy cost is relatively high (e.g., 30%), the loan value becomes a convex function of the default parameter (default barrier). In that case, the lender should either choose a barrier much higher than the outstanding debt value to cover the bankruptcy cost, or a lower default barrier to lower the default probability. For any level of bankruptcy cost, the equilibrium value for the borrower is relatively stable for $\alpha < 1$, and then decreases with the probability of physical default. Figure 5.5 also illustrates a scenario without a covenant for a low level of bankruptcy cost (5%). In the absence of a covenant and with low bankruptcy costs, the loan value for the leader remains constant, adhering to its minimum value. Conversely, the follower's value is decreasing in areas where the default parameter exceeds 1.

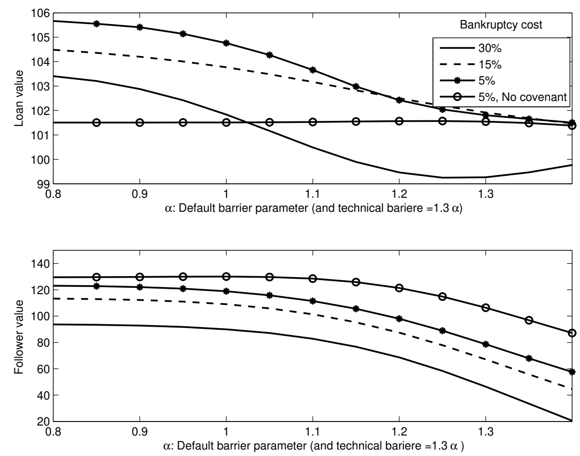


Figure 5.5: Loan value and follower value as a function of the default barrier parameter α for various levels of bankruptcy cost, expressed as a percentage of the asset value at the time of default, other parameter values are given in Table 5.1

Impact of dividend

Figure 5.6 and 5.7 depict how the dividend rate influences both the loan value and the follower value in relation to the asset level at loan inception. A higher dividend rate exerts a negative impact on the loan value specifically for high-leverage borrowers.

Impact of the borrower's earning opportunities when using an aggressive investment strategy

One of the significant innovations in our model is the consideration of the asset substitution problem. Asset substitution is one of the most important agency problems in finance. By shifting from low-risk to high-risk projects with higher earning opportunities, the borrower increases their overall risk and negatively impacts the loan value (the lender value). In our setting, the main factor that drives the borrower's decision to opt for aggressive play is the earning opportunity, designated by the immediate dividend rate g.

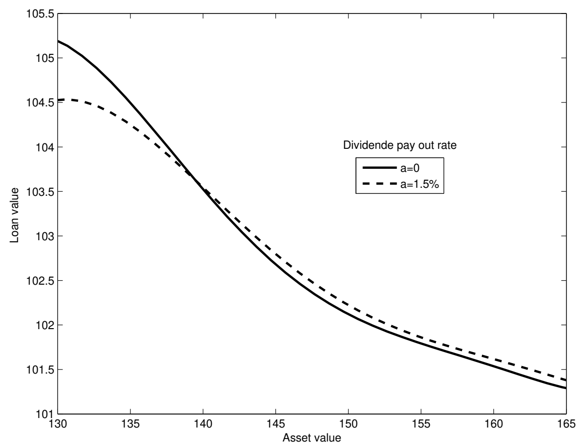


Figure 5.6: Loan value as a function of the asset value for two dividend rates, other parameter values are given in Table 5.1

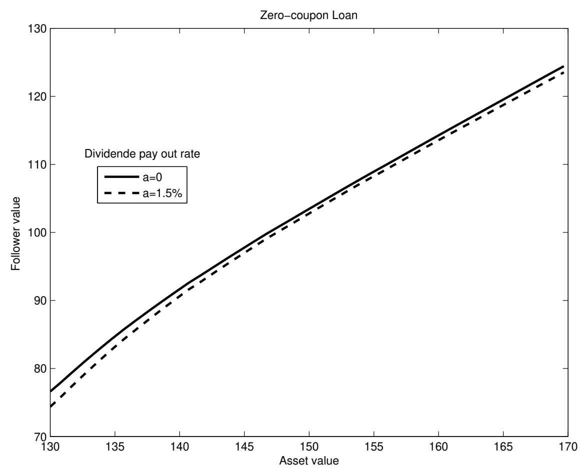


Figure 5.7: Follower value as a function of the asset value for two dividend rates, other parameter values are given in Table 5.1

Figure 5.8 illustrates the impact of a change in the earning opportunity on both the loan and the follower values. Maintaining all other parameters constant as per the base-case specification, the follower value increases, and the loan value decreases when the immediate dividend rate surpasses 1.25. Prior to this threshold, changes in the earning opportunity do not significantly impact the two values. It is important to note that this threshold will vary with other factors, such as the physical and technical default barrier, the cost of refinancing, etc.

This result is noteworthy for two reasons: firstly, it indicates that the effectiveness of covenants in preventing the impact of asset substitution is limited to a certain threshold of earning opportunity. Secondly, given that earning opportunity is an observable factor and may change over the course of the contract, it is practically impossible to eliminate asset substitution using only financial covenants (per design).

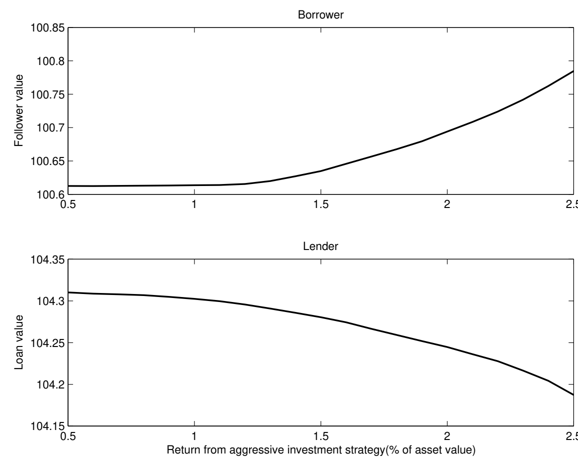


Figure 5.8: Loan and follower value as a function of the borrower's incentive to use an aggressive investment strategy, other parameter values are given in Table 5.1

Impact of auditing frequency

Figures 5.9 and 5.10 illustrate the influence of the covenant audit frequency on the loan and follower values, respectively, by depicting the value functions for three auditing frequencies (bi-weekly, monthly, and bi-monthly). As anticipated, the audit frequency positively affects the loan value while exerting a negative impact on the follower value. Higher audit frequencies are associated with increased punishment possibilities and the potential for early forced bankruptcy, aiming to mitigate larger losses. This results in increasing the loan value for the lender and reducing the follower value due to reinforced control. Note however that our model does not account for monitoring costs, which could alter the presented results.

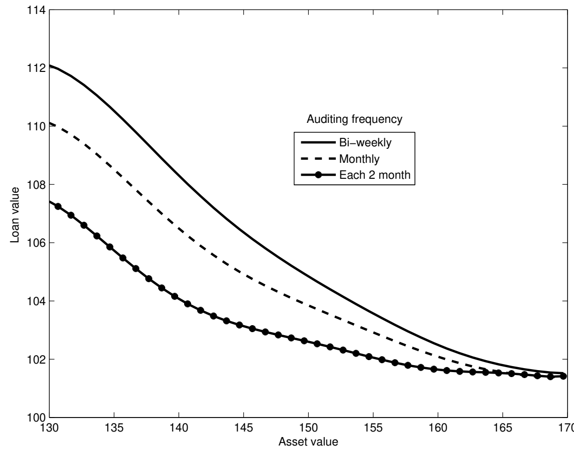


Figure 5.9: Loan value as a function of the asset value for three auditing frequencies, other parameter values are given in Table 5.1

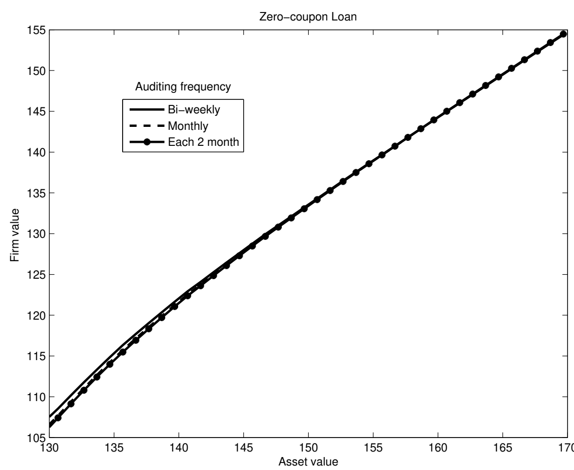


Figure 5.10: Loan value and follower value as a function of the asset value for three auditing frequencies, other parameter values are given in Table 5.1

5.4 Equilibrium values and strategies over time

5.4.1 Value functions

The equilibrium value for the leader and the follower are functions of both the asset value (state variable) and time to maturity. These value functions are represented in Figures 5.11 and 5.12, at chosen auditing dates, for the base-case parameter values given in Table 5.1. The solid and dashed vertical lines represent, respectively, the physical and technical default barriers, which move over time as they are functions of the outstanding debt. The range of the possible asset values at an auditing date t_m is determined by the initial asset value, the volatility, and the elapsed time since inception.

At a given date, the loan value is generally increasing for asset values ranging from the physical default barrier up to an asset value that is smaller than the technical default barrier, and subsequently decreasing (Figure 5.11). This behavior illustrates that the effect of a covenant is not limited to the punishment region (lying between the physical and the technical default barriers); Indeed, the loan value is decreasing outside of this range, contrary to the standard form of fixed-income asset values, which generally increase with the borrower's creditworthiness.

The borrower value function is presented in Figure 5.12. The main repercussion of the covenant can be observed in the area around the punishment region. In that area, the follower value is not consistently increasing with the asset value, reflecting the combined effect of the covenant and the default risk. Out of the punishment region, the follower value has steadily increasing trends, which is aligned with the conventional form.

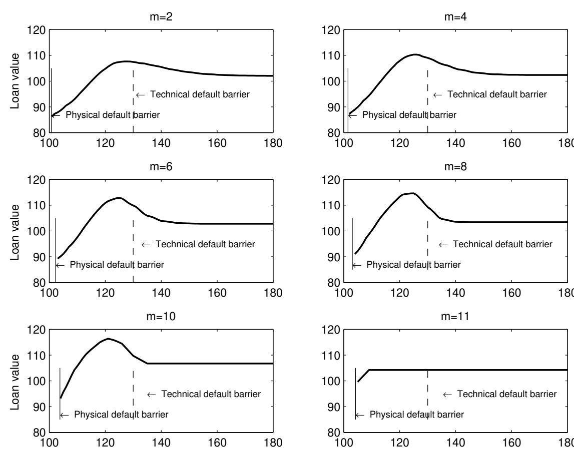


Figure 5.11: Equilibrium loan value (with covenant) as a function of the asset level at various auditing dates, other parameter values are given in Table 5.1

5.4.2 Lender's punishment strategy

The lender's equilibrium punishment strategy is a function of time and of the asset level (state variable). Recall that the lender can only intervene when the borrower is in technical

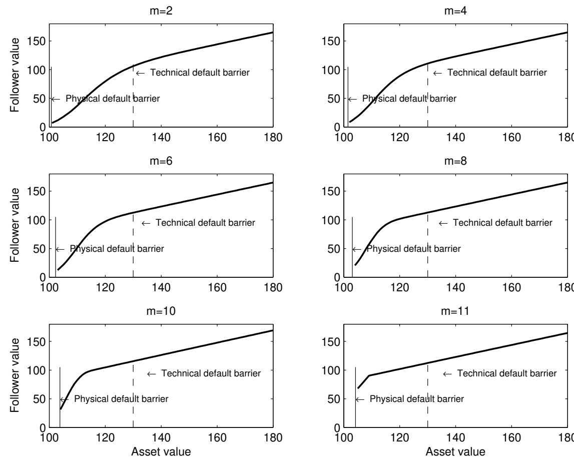


Figure 5.12: Equilibrium borrower value (with covenant) as a function of the asset level at various auditing dates, other parameter values are given in 5.1

default, by deciding on a temporary increase in the interest payment and/or a temporary increase of the default barrier parameter. The equilibrium strategy of the lender is represented in Figure 5.13. The curve represents the equilibrium interest punishment rate, as a function of the borrower's asset value at various auditing dates, while the colored region indicates asset values where the lender will increase the default barrier.

These results show that the equilibrium punishment rate is a bell-shaped function of the borrower's asset level, with the punishment rate increasing and then decreasing after reaching a maximum value. At equilibrium, the lender will initiate punishment with a progressively increasing interest rate, even as the probability of default decreases. However, as the asset value rises, the borrower's creditworthiness improves and this makes them eligible for refinancing at a lower cost; at that point, the equilibrium punishment rate decreases in order to avoid losing the contract. It is interesting to note that the punishment rate is not positive all through the technical default area, starting from an asset value that is slightly higher than the physical default barrier, which we will refer to as the *punishment barrier*. Within the range between the physical default barrier and this punishment barrier, the penalty associated with a covenant breach becomes null. This suggests that in this interval, the lender is strategically tolerating some technical default with respect to interest punishment. Instead, they will opt for an increase in the default barrier (as illustrated by the colored region). This can be related to the strategic debt servicing theory, for which the lender may have an interest in avoiding bankruptcy by reducing the coupon, particularly when bankruptcy costs are important (Breton (2018);Mella-Barral and Perraudin (1997)).

Over the course of the contract, this strategic approach is observable with certain adaptations. Notably, as the contract approaches maturity, there is an observed increase in the punishment barrier, signifying that it is optimal for the lender to tolerate more technical defaults occurring later after the contract's inception.

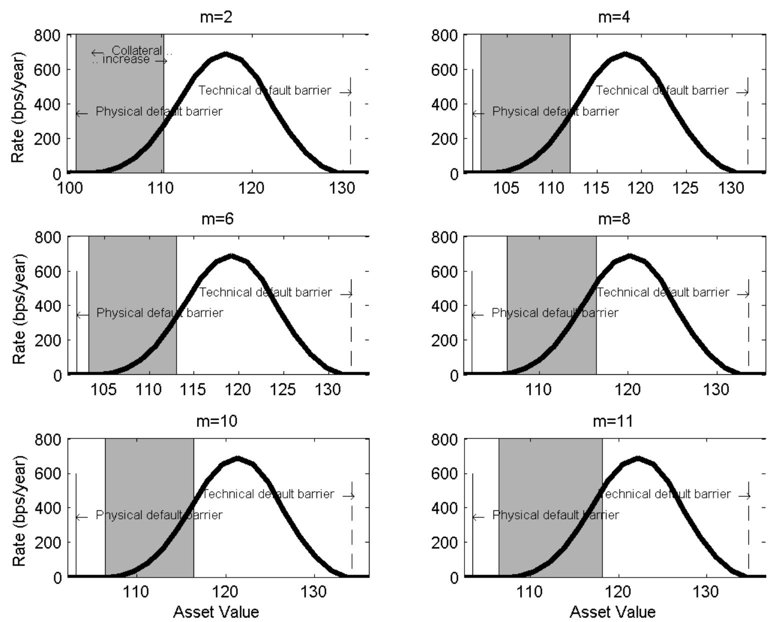


Figure 5.13: Leader's interest and barrier punishment strategies giving the punishment rate (in bps/year) as a function of the asset value at some selected auditing time under the base-case specification given in Table 5.1

5.4.3 Follower's response

As presented in Chapter 4, the borrower's response is a discrete-valued function (δ_m) of the asset value, the lender's decisions, and time. For simplicity, we discretize the lender's interest-punishment decisions into four categories (no punishment, low, medium, and high). Note that the regions with low asset value concurrent with low or no interest punishment correspond to situations where the lender will increase the default barrier (see Figure 5.13). Figure 5.14 presents the borrower strategy as a function of the asset value for the four levels of the lender's punishment rate, at some audit dates. As in the previous section, the two vertical lines indicate the two default barriers (physical and technical).

We distinguish three regions: the physical default region, the technical default region, and the region above the technical default barrier. Despite the apparent overlap of the curves (the follower decision), they are not actually overlapping. This discrepancy can be attributed to the reduction in the size of the graph.

When asset values fall below the physical default barrier, the follower is forced to file for bankruptcy ($\delta_m(s,p,\kappa) = -2$) as per the covenant conditions.

In the area of technical default (between the two default barriers), the follower is playing either conservatively, aggressively, or refinancing. For asset values close to the physical default barrier, the borrower is playing conservatively across all levels of punishment. They start playing aggressively as the asset value increases, particularly in the higher punishment regions. Results show that it can be optimal for the borrower to refinance the loan in the technical default region.

Outside the technical default region, the follower is playing either conservatively ($\delta_m(s,p,\kappa) = 0$) - when technical default risk is still important - or aggressively ($\delta_m(s,p,\kappa) = 1$) - when the technical default risk become smaller.

5.5 Risk-neutral covenant value for the follower

In this section, we use a two-step approach to estimate a risk-neutral value of the covenant for the follower at the contract inception. This approach is based on the reasonable assumption - verified empirically - that the inclusion of a covenant should lower the spread of the loan, all things being equal. We therefore seek to compare the value, for the borrower, of two contracts (with and without a covenant), assuming that the spread makes the lender indifferent between the two.

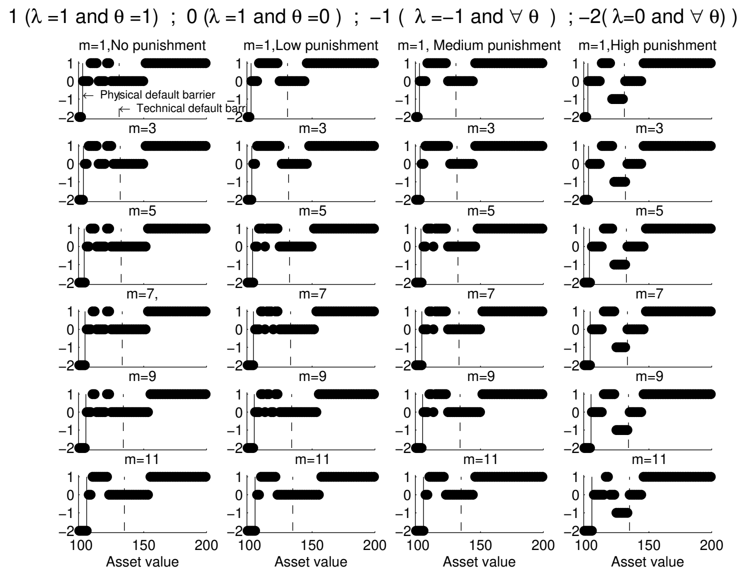


Figure 5.14: The borrower strategy (λ, θ) as a function of the asset value and the lender punishment rate p under the base-case specification given in Table 5.1

In the first step, we use our model to compute a spread that would make the value of the loan equal to the value of the amount invested initially, in both cases (with and without covenant). When there is no covenant, the lender has no action available, but the follower still has the option of refinancing or investing aggressively. The interest rates i^*_C and i^*_{NC} solve

$$v^L_0(s; i^*_C) = D_0 \tag{5.1}$$

$$v^{LNC}_0(s; i^*_{NC}) = D_0, \tag{5.2}$$

where v^{LNC}_0 solves the dynamic program (4.12)-(4.21) with the restriction that the lender is a dummy player (they have no decision variables), while there is no constraint on the borrower's decisions. The values of the interest rates i^*_C and i^*_{NC} are obtained by successive iteration. Figure 5.15 reports the results corresponding to various values of the default barrier and the interest punishment rates.

As expected, the spread (with or without a covenant) decreases as the initial asset value increases. As the creditworthiness of the borrower improves (lower initial leverage), the spreads applied to the two types of contracts converge. The spread of a loan contract including a covenant depends on the initial default barrier (α) and on the possible increases in case of technical default (κ). For a higher default barrier and/or with the possibility of increasing it ($\alpha = 1$, $\kappa = 10\%$), the loss given default for a loan including a covenant becomes lower (mostly reduced to the bankruptcy cost) and the optimal spread applied by the lender becomes almost constant. The results show that putting the initial default threshold to 1 (positive equity condition) without a possibility of future increases or putting a lower threshold (e.g. 0.9) with the possibility of future increases ($\kappa = 10\%$) yields the same result.

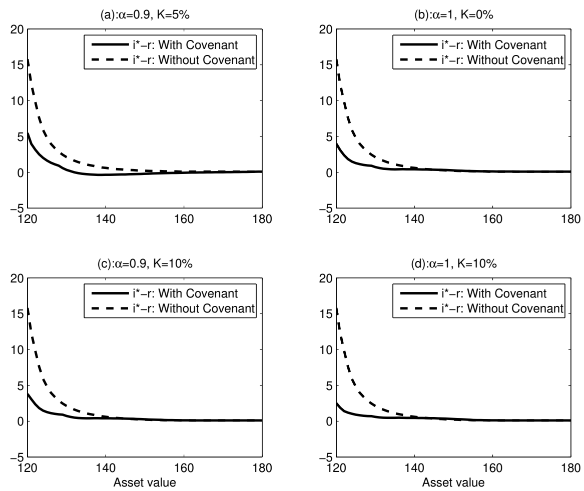


Figure 5.15: Equivalent spreads over the risk-free rate for a loan with and without covenant under the base-case specification in Table 5.1

In the second step, we use the two equivalent interest rates i_C^* and i_{NC}^* to derive a risk-neutral covenant value for the follower. To this end, we define the value of the covenant for the borrower as the difference between the equilibrium follower's value, with and without a covenant, using the equivalent spreads computed in the first step. Formally, this can be expressed as:

$$CV_0 = v_0^F(s; i_C^*) - v_0^{FNC}(s; i_{NC}^*) \tag{5.3}$$

Using the base-case specifications, we compute and plot the covenant value in Figure 5.16 as a function of the initial asset value. Results indicate that the covenant value is negative for the borrower in the regions where there is a risk of technical default. The value of the covenant converges to zero as the borrower's initial creditworthiness improves. This suggests that, for risk-neutral players, there is no value in adding a covenant to a loan contract: the lender is indifferent, as per the specification of the experiment, and the follower is never better off when the contract includes a covenant. This result may seem surprising, as the use of covenants in syndicated loans is prevalent. On the other hand, as the main purpose of a covenant is to mitigate risk, we release the risk-neutrality assumption in the next section.

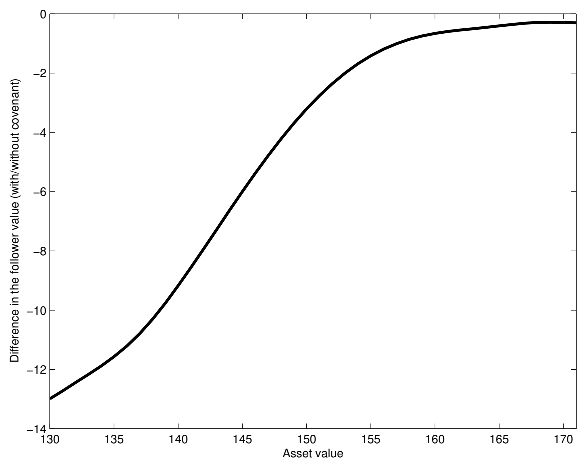


Figure 5.16: Covenant value for the follower as a function of the asset value under the base-case specification given in Table 5.1

5.6 Covenant value in a risk-averse world

The previous section introduced a covenant value under the assumption of risk neutrality for the two players. This value is computed as the difference in the follower value for a loan with and without a covenant, given that the interest rates are adjusted to make the lender indifferent between the two contracts.

In this section, we use the approach introduced in the previous section to re-compute the values of the two contracts under the assumption that the two players are risk-averse.

Risk aversion and utility functions

Incorporating a risk aversion assumption into the game between the lender and the borrower entails computing the utility of the instantaneous rewards of both the leader and the follower, using an appropriate utility function. For risk-averse players, the appropriate utility function should exhibit strict concavity. Such a utility function results in a positive Pratt's risk aversion parameter, as described by (Pratt and Zeckhauser (1987)).

The absolute risk aversion parameter AA is defined by:

$$AA(x) = -\frac{U''(x)}{U'(x)} \tag{5.4}$$

while the relative risk aversion parameter RA is defined by:

$$RA(x) = -\frac{U''(x)}{U'(x)}x \tag{5.5}$$

where U' and U'' are the first and second derivatives of the utility function with respect to x. For a risk-neutral investor, $U(x) = x$ and the second derivative is null. We implement two types of utility functions, from the family of constant absolute risk aversion (CARA) and constant relative risk aversion (CRRA) functions. We use the following specific forms:

$$U_1(x) = \frac{1 - e^{-a_1 x}}{a_1} \tag{5.6}$$

$$U_2(x) = \frac{x^{1-a_2}}{1 - a_2} \tag{5.7}$$

where x is the increase in the wealth , $a_1 > 0$ is the (constant) absolute risk-aversion parameter and $a_2 > 0$ is the (constant) relative risk-aversion parameter.

The assumption behind the CARA type of utility function is that investors are keeping constant the amount they invest in risky assets as their wealth changes. This is not the case for the CRRA type of utility functions, where investors rather keep the proportion of their wealth invested in risky assets constant as their wealth changes.

Value of the covenant

We implement the computation of the value of the covenant in the risk-averse case in two steps.

In the first step, we obtain the risk-neutral equivalent spreads, with the methodology used in the previous section, as presented in Figure 5.15.

In the second step, we replace the immediate reward functions by the expected utility of the immediate rewards and the terminal value by its utility. This implies applying the chosen utility function to the payoff functions for both the leader and the follower, possibly with different risk aversion parameters, denoted by a_L and a_F, respectively, for the leader and the follower. The covenant value for the follower is computed as the difference between the follower value with covenant and without covenant, under the assumption of risk aversion. The same approach is used to compute the leader value.

Our experiments cover three cases for each choice of the utility function ($a_L = a_F$, $a_L > a_F$,$a_L < a_F$), for a case where $\alpha = 1$ and $\kappa = 10\%$.

The combined results when both players have the same risk aversion parameter value are reported in Figures 5.17. We observe that the covenant value for the follower is negative when the initial asset value is low, and increases as the asset value increases. As the follower's creditworthiness improves, the covenant value converges to a positive value near zero, depending on the risk aversion. This suggests that, in a region with virtually no technical default risk, the follower becomes indifferent about having a contract with or without a covenant. As for the leader, the value of the covenant is positive for high-risk borrowers and decreases toward zero as the borrower's creditworthiness improves. This result supports the intuition that the value of a covenant is only positive for risky borrowers. This result seems to be robust to the choice of a type of (concave) utility function.

Figures 5.18 and 5.19 present the results when the lender and the borrower differ in their risk-aversion level. Not surprisingly, the value of the covenant is higher for the player who is more risk-averse than the other.

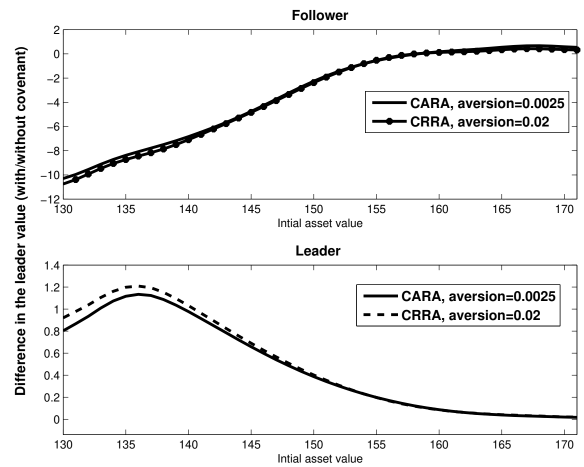


Figure 5.17: Value of the covenant as a function of the asset value under the base-case specification given in Table 5.1

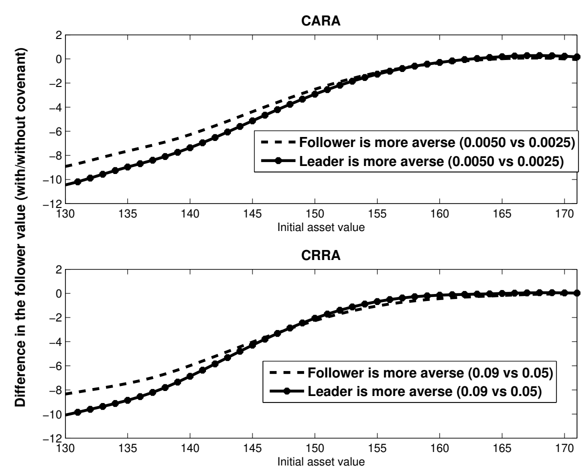


Figure 5.18: Covenant value for the borrower as a function of the asset value under the base-case specification given in Table 5.1

5.7 Discussion and possible extensions

Why do lenders use covenants and why do borrowers accept them? One of the prevalent answers is that covenants are important tools to address the agency problem, for both the lender and the borrower. By retaining the right to increase/decrease the interest over the life of the contract, the lender reduces the moral hazard post-contract inception. On the borrower's side, a covenant is a good signaling tool that can lead to lower initial credit spread and the possibility of future reduction.

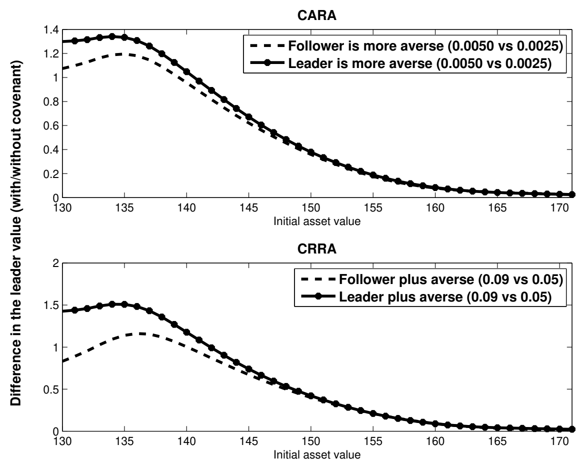


Figure 5.19: Covenant value for the lender as a function of the asset value under the base-case specification given in Table 5.1

From a risk-sharing perspective, by allowing a continual adjustment of the spread, the cost related to future changes in the borrower's creditworthiness is shared between the lender and the borrower. This cost is mainly in the form of lowering the market value of the loan. The adjustment of the pricing has the effect of reducing this cost by making the borrower pay part of it.

Conceptually, a syndicated loan contract design involves several procedures, one of which includes the selection of the covenants indicator, the determination of the technical default threshold, and the monitoring strategy, which basically answers the question of how technical default will be solved (M. Campbell and Weaver (2019); Ivanov, Ranish, and James Wang (2017)). In this chapter, we analyzed the impact of some parameters of the covenant on the loan value to determine how those parameters can be optimally set by the lender at the loan inception.

Optimal leverage is an important issue in the syndicated loan market as the amounts are generally large. In a general setting, syndicated loans can be classified as a leverage loan when the initial credit risk of the borrower is very high and requires a high spread over the base rate (more definitions can be found in Yago and McCarthy (2004)). The use of a covenant has been perceived as the best way of controlling high-leverage borrowers, as it gives decision power to the lender over the contract life. In this chapter, we investigate this question by analyzing the equilibrium strategy of the lender and borrower. Our results suggest that it is not necessarily optimal for a lender to use interest punishment as the exclusive covenant monitoring strategy for a leverage syndicated loan. Our findings support that, when the initial leverage is high, the lender is better off tolerating technical default (as for interest punishment) and requesting more collateral (thereby increasing the default barrier) to reduce the loss-given default. Alternatively, the lender can use non-financial covenants such as negative covenants that force the borrower to request his approval in all important economic decisions. This type of covenant can be regarded as a management right, offering greater protection than merely penalizing the borrower to discourage risky decision-making.

Two important extensions are possible from our model. The first is the inclusion of the lender and the borrower's reputation in the dynamic game. Reputation is an important factor for the two parties in the syndicated loan market as it can have a significant impact on their future participation as syndicate members or as lead arrangers. Borrower reputation in respecting contract terms including covenant compliance can give a good signal and lower its future borrowing cost. Instead of pursuing exclusively a cash profit, the two players may have their reputations improve over time. A breach of contract will negatively impact the borrower's reputation, but the effect should be lower if they comply with the punishment without adopting an aggressive investment strategy.

A second extension could be the inclusion of a monitoring cost, assuming that the lender can decide whether to monitor or not. While the lead arranger receives a monitoring fee, profit maximization may lead to the skipping of some planned monitoring activities. For instance, the lender may observe the trend of the distance to the covenant threshold to decide if an audit is necessary at the next monitoring date.

5.8 Conclusion

This chapter explores the possibility of our stochastic dynamic game model in correctly representing the interactions between a lender and a borrower in the context of covenant clauses. Our numerical implementation yields results that align with empirical observations. The presence of the covenant enhances the contract value in states with high default risk. Specifically, the ability to choose a dynamic default barrier and punish covenant violations helps protect the lender against the adverse effects resulting from the borrower's asset volatility and potential deterioration of creditworthiness after the contract's inception. We find that the value of covenants is highly related to the concept of risk aversion. As syndicated loans involve very large amounts, one cannot ignore risk preferences. The risk mitigating value of covenants is most probably the reason for their popularity in the syndicated loan market.

5.9 Appendix

This appendix reports on numerical experiments that are not discussed in the chapter, that is, considering various reimbursement schedules and a larger range of risk-aversion parameters. Note that the results reported in this appendix are qualitatively similar to the ones discussed in the chapter and are provided to show the robustness of the model.

5.9.1 Interest only loan

Figures 5.20 and 5.21 present the leader and follower's value for the case of an interest-only loan (the principal is reimbursed at maturity). These results are qualitatively similar to the case of a zero coupon loan (the principal and interest are reimbursed at maturity).

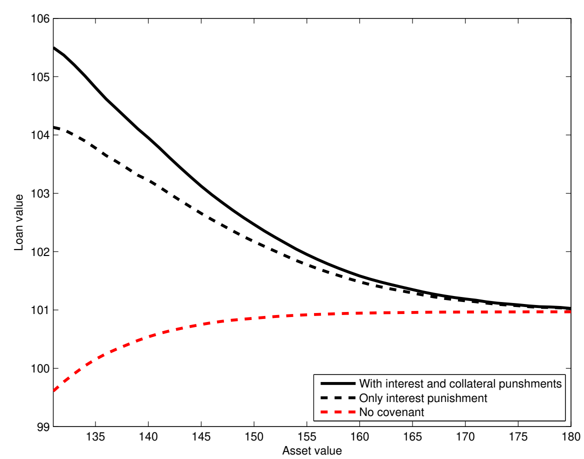


Figure 5.20: Interest-only loan value as a function of the initial asset value at the loan inception date for three alternative scenarios under the base-case specification given in Table 5.1

5.9.2 Amortizing loan

Figures 5.22 and 5.23 present the leader and follower's value for the case of an amortizing loan (the principal and interest are reimbursed over the life of the loan)

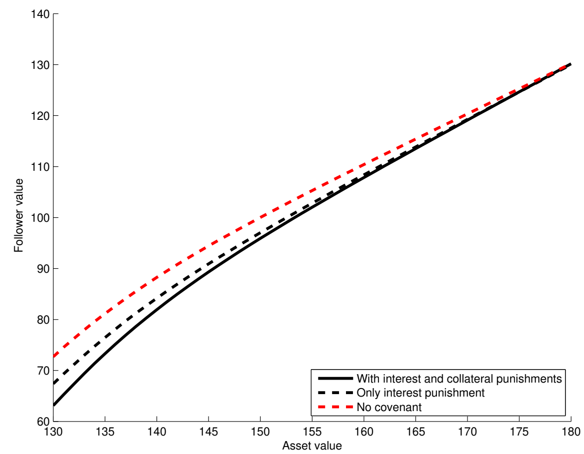


Figure 5.21: Follower value with interest-only loan as a function of the initial asset value for three alternative scenarios under the base-case specification given in Table 5.1

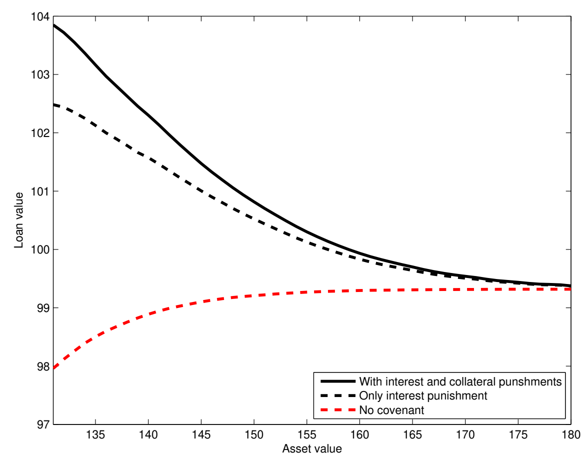


Figure 5.22: Amortizing loan value as a function of the initial asset value at the loan inception date for three alternative scenarios under the base-case specification given in Table 5.1

5.9.3 Real world covenant value for different aversion parameters

Figures 5.24 and 5.25 present the difference in the follower value with and without covenant, under the risk aversion assumption with two utility functions (CARA and CRRA) and for different aversion parameters. Figures 5.26 and 5.27 correspond to the case of the leader. In all the cases, the difference in each player's value between with covenant and without covenant increases with the risk aversion parameter.

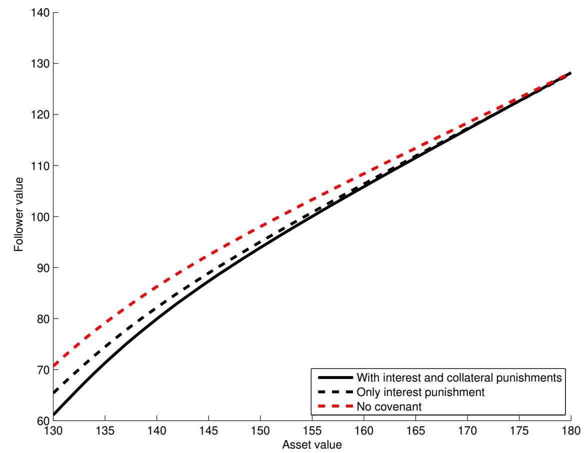


Figure 5.23: Follower value with amortizing loan as a function of the initial asset value for three alternative scenarios under the base-case specification given in Table 5.1

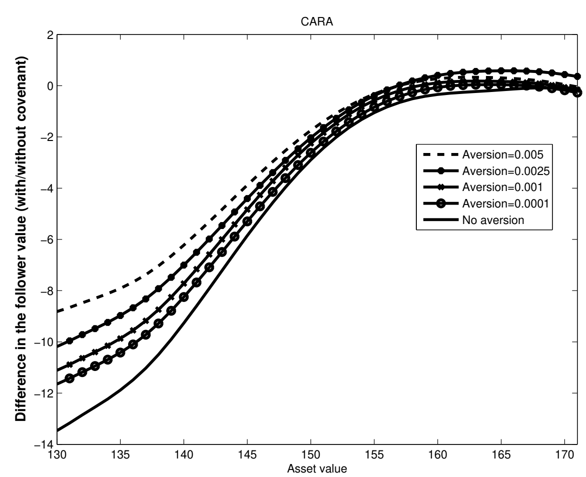


Figure 5.24: Difference in the follower value (with and without covenant) as a function of the asset value under the base-case specification given in Table 5.1

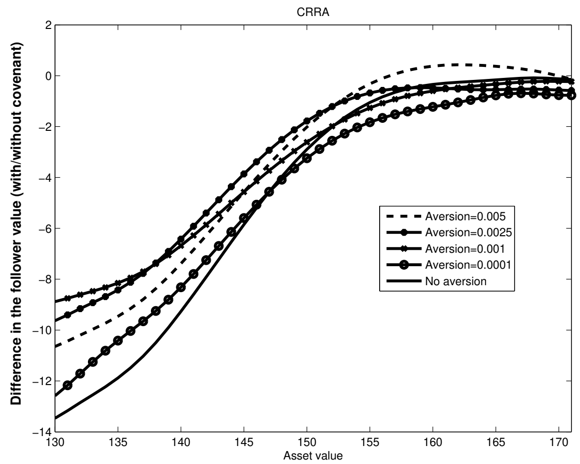


Figure 5.25: Difference in the follower value (with and without covenant) as a function of the asset value under the base-case specification given in Table 5.1

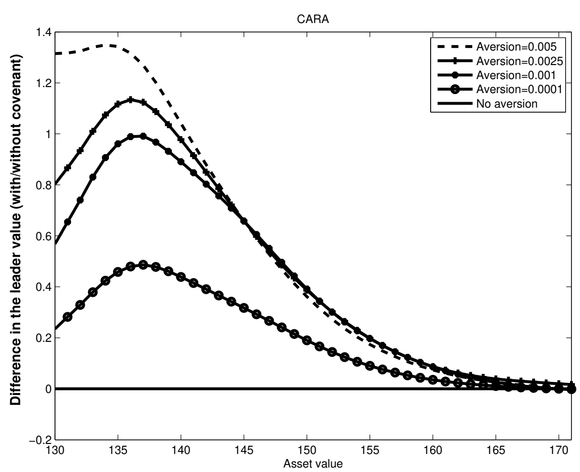


Figure 5.26: Difference in the leader value (with and without covenant) as a function of the asset value under the base-case specification given in Table 5.1

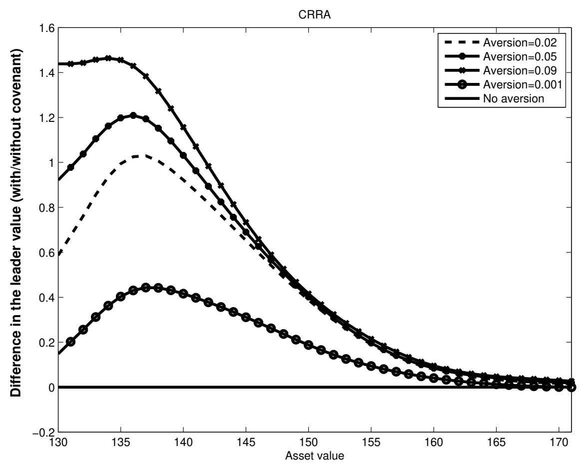


Figure 5.27: Difference in the leader value (with and without covenant) as a function of the asset value under the base-case specification given in Table 5.1

Chapter 6

Valuation and efficiency of performance pricing in syndicated loans

This chapter is devoted to performance pricing, a form of loan contract extensively used in the syndicated loan market. We approach the valuation, pricing, and design of such contracts using a novel approach based on stochastic game theory, leveraging the game model developed in Chapter 4. This allows us to allow for the borrower's response to change in their credit cost, and to compare the efficiency of performance pricing with respect to loans with covenant clauses.

6.1 Introduction

A fixed credit spread entails an important risk for the lender in periods of unstable business environment; any decrease in the borrower's creditworthiness makes the conditions of the loan contract unfair to the lender. On the borrower's side, a fixed spread without a prepayment option may not be the best option for financing projects or working capital. In fact, an improvement in the borrower's financial situation makes the contract's cost higher than what the borrower would bear if they were to tap the market again. The need of both parties for more flexible pricing, out of the conventional fixed and variable rates, has led to the introduction of cost-based covenants and performance-sensitive pricing in loan contracts.

A cost-based covenant allows both the lender and the borrower to renegotiate the credit cost in the advent of events that significantly impact the borrower's financial indicators. On the other hand, performance-sensitive pricing, or simply *performance pricing* (PP), allows for automatic spread changes as the borrower's risk indicator fluctuates. PP has been introduced by Loomis (1991) and is widely used in the syndicated loan market.

Performance pricing introduces two challenges for the lender: the determination of the pricing structure, and the valuation of the contract after its inception. The design of the pricing structure involves selecting an appropriate performance measure, determining the thresholds that will trigger a risk premium adjustment, and determining a fair spread for each interval. Assessing the value of the contract requires the development of a model that links the lender's cash flows to the borrower's performance measure, and takes into account credit risk and the possible reaction of the borrower to the change in the credit cost.

Building valuation and pricing models is essential for three main purposes: regulatory measures, risk management, and secondary market operations. From the regulatory perspective, capital adequacy requires the use of a valuation model that considers a comprehensive risk profile of the borrower. In risk management, a valuation model is needed to estimate the risk-adjusted value of loans. Finally, participants in the secondary market need to assess the value of a loan contract at any time, as a function of the borrower fundamentals.

The question of the evaluation, and, to a lesser extent, of pricing, have both been investigated in the literature (see for instance Manso, Strulovici, and Tchistyi (2003); Koziol and Lawrenz (2010); Sarkar and Zhang (2015), Sarkar and Zhang (2016)). These contributions use either the contingent claims approach developed by Black and Scholes (1973) and Merton (1974) or the stochastic real-option theory formalized in Myers (1977). One of the challenges lies in the fact that, under PP, the cash flows from the contract are stochastic; by changing the coupon value over time, PP alters the fixed-income nature of the contract. This makes the conventional way of discounting future cash flow adjusted for counterparty risk not practical. On the pricing issue, there exists no practitioner literature on how performance pricing is done in the industry, how the performance measure is selected, how the trigger thresholds are settled, and how the fair spread is computed for each interval of the performance measure.

From the existing literature, one important shortcoming is the lack of consideration of the borrower's possible reactions to the changes in its credit cost through performance pricing. Practical literature supports that borrowers are not passive and may continually adapt their risk-taking strategy and therefore increase or reduce their credit risk. Not considering this feedback effect can lead to an overestimation of the performance pricing type of loan.

We seek to contribute to both the valuation and pricing issues, using a different approach than what has been used in the literature, that is, dynamic stochastic game theory. Approaching PP as a dynamic game allows us to consider the borrower's response to changes in their credit costs. This is a key difference from previous works, where the borrower's investment and borrowing policies are assumed independent of the loan terms. We also address the issue of selecting an "optimal" pricing structure, taking the borrower's reaction function into account. Finally, we analyze the relative efficiency of PP with respect to pricing covenants. For this purpose, we use a selected list of contracts with a PP clause and compare their market value under PP to those obtained when using cost-based covenant clauses.

Our results suggest that the performance pricing grid should be determined by considering at least two risk measures: the leverage (alternatively the debt to cash flow or the rating), and the asset volatility, particularly for medium- and long-term facilities. In addition, the borrower's response to the coupon change can significantly increase its default risk and distort the lender's expectations in applying PP. It therefore appears that PP does not prevent default in an environment where the borrower can anticipate the coupon increase and adjust its strategy accordingly. Our previous findings suggest that performance pricing should at least be bi-dimensional and may include some protections against bad management decisions. This supports the existence of contracts that include PP and negative covenants to control management decisions.

The rest of the chapter is organized as follows. Section 6.2 is an extensive literature review, including a short background on PP and statistics on the characteristics of PP clauses used in the syndicated loan market. Section 6.3 details our valuation and pricing model. Numerical results are presented and discussed in Sections 6.4.3, 6.5.2, 6.5.3, and 6.5.3. Section 6.6 is a short conclusion.

6.2 Literature review

The literature on performance pricing covers the existence, valuation, and pricing of this type of contract. The first question has been addressed by Asquith, Beatty, and Weber (2005) and Sarkar and Zhang (2015), while the valuation and pricing are discussed in Manso, Strulovici, and Tchistyi (2010), Myklebust (2012), and Sarkar and Zhang (2015).

Asquith, Beatty, and Weber (2005) empirically analyze interest-decreasing and interest-increasing PP, and discuss the existence of these two pricing methods. Their findings support that interest-decreasing PP is more used when prepayment probability is high. They provide proof that this type of pricing grid is a simplified version of a prepayment option, that allows a borrower to reduce the credit cost when market conditions change or their creditworthiness improves. PP has the advantage of lowering or simply eliminating the renegotiation and administrative costs that the borrower would bear by exercising a prepayment option. In addition, when it is well designed, performance pricing can address the information asymmetry problem by reducing the adverse selection due to borrower misclassification at the contract inception. The second important finding suggests that interest-increasing PP is predominant when there is an important information asymmetry with a high moral hazard cost. In such a case, the lender can price the contract at the lowest cost and add a clause to increase the risk premium in case of poor performance, thus significantly reducing the borrower's ex-post bad decisions, which may adversely impact the contract value.

Koziol and Lawrenz (2010) argues that PP can be a particularly good tool to reduce the asset substitution problem in the agency problem. Asset substitution arises when the borrower deliberately switches from a low-risk project to a high-risk one after receiving the funding. Interest-increasing PP would ultimately readjust the risk premium and reduce or eliminate the additional profit for shareholders.

Sarkar and Zhang (2015) completes the list of reasons to use PP by showing that PP can mitigate or eliminate the agency problem of under-investment presented in Myers (1977). An under-investment problem arises when a very leveraged borrower gives up on positive net present value projects because undertaking such a project would profit the lender more than the shareholder. In a fixed-coupon loan, a good project will improve the firm's creditworthiness, with a positive impact on the market value of the loan as the default risk decreases, while shareholders keep paying the same credit cost and may have little direct profit from the project.

Using a real-option model, Sarkar and Zhang (2015) shows that PP improves profit sharing and can alter the borrower's rational decision to delay a positive cash flow investment. The borrower will profit from a decrease in the coupon, while the lender will have a loan with better market value as the default risk decreases.

The theoretical literature on valuation and pricing is based on two general approaches; the stochastic real-option and the contingent claim theories. Manso, Strulovici, and Tchistyi (2010) uses the structural model developed by Leland (1994) to build a PP model and analyze the risk-compensating nature of different PP grids. A general model of a PP grid is defined as a function that links a performance measure of the borrower to the coupon. The authors implement an asset-based and a rating-based PP and introduce the notion of the relative efficiency of a PP grid. They show that if two different PP grids lead to the same amount of future cash, the more risk-compensating grid will be inefficient in an environment of high bankruptcy costs, as it will lead to an earlier default. More importantly, the authors prove that high-risk compensating PP is less efficient than fixed-spread loans, and attributes the use of such pricing to its screening effect on the agent problem at the contract inception.

Koziol and Lawrenz (2010) values interest-increasing PP, where the performance measure is based on the borrower's cash flow. For simplicity, the coupon increases irreversibly by a constant factor (for instance, each time the cash flow crosses the threshold, the coupon increases by a fixed and predetermined percentage). Assuming a positive equity value as the limit for physical default, and assuming that the asset can be sold to pay the coupon in case of insufficient cash flow, the authors derive a formula of the firm value and use it as the objective function of an optimization problem.

Myklebust (2012) uses a linear function linking the coupon value to the change in cash flow. The author derives two solutions, the first-best solution when the value function is the total firm value, and the second-best solution when the borrower maximizes the market value of equity.

More recently, Sarkar and Zhang (2015) uses a similar approach as Myklebust (2012) and shows how to design a PP to eliminate the underinvestment problem. In a second contribution,Sarkar and Zhang (2016) models and estimates the value of a loan commitment with a performance pricing clause, and analyzes the firm investment decision with a real-option approach.

Ming, Yang, and Song (2018) investigates the question on convertible performance-sensitive debt, a particular type of loan with PP clauses, and a conversion to equity options. The authors use a contingent claim approach to build a pricing model, and derive the optimal capital structure. Their main finding suggests that PP improves shareholder value and addresses the asset substitution problem.

In most of these theoretical models, the borrower is assumed to keep the same investment and risk-taking strategy over the contract life, without reacting to the coupon change. The modification of a borrower's risk-taking level is an important agency problem in finance (the previously mentioned asset substitution problem).

An exception appears in a few applications using a real-option approach, where the borrower's reaction to a coupon change is limited to the decision to delay some investments.

To address this limitation and add more flexibility, we build a valuation model based on a stochastic game approach considering the borrower's response to their credit cost change. In our implementation, we use the borrower's leverage as the performance measure. As in Merton (1974), we define the leverage as the ratio of the total debt to the total asset value,

$$L_t = \frac{D_t}{S_t}, \tag{6.1}$$

where D_t is the total outstanding debt and S_t is the borrower's total asset value, as defined in Chapter 3. The model can be easily adapted to other financial ratios, for instance, the total debt to cashflow ratio (see Koziol and Lawrenz (2010) and Sarkar and Zhang (2015)). In addition, the total outstanding debt can be equal to the syndicated loan amount for initial investment borrowers as in Sarkar and Zhang (2015) or may include previous outstanding debt balance with or without performance pricing as in Ming, Yang, and Song (2018) and Myklebust (2012).

6.3 The theoretical model

6.3.1 The contract

As in Chapter 4, we consider a syndicated term loan between a *borrower* and a group of banks, identified in the sequel as the *lender* with inception date 0 and maturity T. The contract includes a performance pricing clause that links the coupon to a performance indicator.

The performance pricing grid is defined by a set of J thresholds $\left\{B_j : j = 1, ..., J\right\}$.

We denote by p_j the spread (risk premium) added to the contractual rate when, at a given auditing date t_m, the performance indicator $\digamma_{t_m}$ belongs to the interval (B_j, B_{j+1}). This general formulation of the PP grid allows to encompass all types of PP (e.g. interest-decreasing, interest-increasing, etc.). In addition to the performance pricing clause, the contract includes a covenant that forces bankruptcy at the first time the performance measure reaches a given *physical default* barrier, denoted by $B^{\flat}(t)$.

6.3.2 The players' strategy space

Both players can take actions that impact their respective payoffs and the evolution of the system's state. The lender decides on the performance pricing grid, by establishing the set of thresholds and the corresponding spreads. This performance pricing function will apply over the contract's life, impacting the borrower's interest payments over time. On the other hand, the borrower can control the evolution of their structural variable, for instance by choosing their risk-taking strategy. The borrower can also decide to refinance their loan or to file for bankruptcy.

Since the leader can only act at inception, while the borrower can act at any time and observe the evolution of the state, we model the PP process as a sequential two-player leader-follower game in discrete time, where the lender is the leader, using an open-loop information structure, while the borrower is the follower, using a feedback information structure, and where the state variable is the current level of the process S_t.

More precisely, the lender's open-loop strategy consists of determining the performance pricing function, which is to provide the set of grid points and the corresponding spreads. This function can be constant or can depend on the date. The lender also decides on the physical default barrier, which forces bankruptcy the first time the performance indicator attains it.

Formally, the lender's strategy is defined as a function

$$\delta_m^L : (s_0) \to \mathscr{L}, \tag{6.2}$$

where $\mathscr{L}$ is the set of admissible actions (e.g. PP functions), as a function of time and of the value of the state variable at inception.

The borrower's feedback strategy consists of taking discrete actions (e.g. refinance the loan, use an aggressive investment strategy, file for bankruptcy), at discrete dates, after observing the current value of the state variable.

Formally, the borrower's strategy is defined as a function

$$\delta_m^F : (s_m) \to \mathscr{F}, \tag{6.3}$$

where $\mathscr{F}$ is the set of admissible vectors (λ, θ), as a function of the asset value.

As in Chapter 4, we assume that the asset value is observed by the lender and the borrower at some fixed contractual equally spaced dates $t_1, ..., t_n$, where $t_n = T$ in order to check the situation of the performance indicator with respect to the PP grid. We also assume that the borrower can only act upon these given monitoring dates.

6.3.3 Equilibrium strategies and value functions

Stochastic games where the leader has an open-loop information structure while the follower has a feedback information structure have been introduced and analyzed in Başar and Olsder (1998) (Chapter 7.5), where it is shown that such games can admit an equilibrium under some mild conditions. The equilibrium strategy pair is obtained in two steps. First, the optimal response of the follower is obtained by solving a dynamic program, yielding the optimal strategy and the value function of the follower, parametrized by the leader's strategy. Then the leader's optimization problem can be solved, using the best response of the follower to any of the leader's feasible strategy. The existence of an equilibrium relies on the existence of the best response function and of an optimal value for the leader's objective function. For a fixed PP schedule, denoted by δ^L, the determination of the follower's optimal response can be obtained by solving a dynamic program, where the value of a strategy vector $\delta^F \equiv \left(\delta_m^F\right)_{m=1,\dots,n-1}$ satisfies the functional equations

$$\begin{aligned} V_m^F(s;\delta_m^F,\delta^L) &= w_m^F\left(s,\delta_m^F;\delta^L\right)+0_m\left(s,\delta_m^F;\delta^L\right)+\beta\mathbb{E}_m\left[V_{m+1}^F\left(s';\delta^F,\delta^L\right)\right], \\ & m=0,\dots n-1 \end{aligned} \tag{6.4}$$

$$V_n^F(s;\delta^F,\delta^L) = w_n^F\left(s,\delta_n^F;\delta^L\right), \tag{6.5}$$

subject to the dynamics (4.7), where the immediate return functions w_m^F are adapted in a straightforward manner from (4.4)-(4.6) to account for the fact that the punishment p and the technical default barrier $\alpha\kappa$ are obtained from the PP schedule δ^L. Given (4.7), (4.4)-(4.6) and (6.4)-(6.5), we seek an optimal feedback strategy of the borrower.

To ensure that the follower's optimal response is well defined, we opt for the SSE assumption, as defined in Chapter 4: the borrower will select a response among their optimal actions that maximizes the outcome of the lender. For that reason, the dynamic program solved by the follower needs to keep track of the leader's value function.

Accordingly, define the set $\mathscr{R}_m^F(s,v;\delta^L)$ of optimal actions of the follower, given a PP schedule δ^L, for any function v at s:

$$\mathscr{R}_m^F(s,v;\delta^L)=\left\{(\lambda^*,\theta^*)\in\arg\max_{\lambda,\theta\in\mathscr{F}}\left\{\begin{array}{c} w_m^F\left(s,\lambda,\theta;\delta^L\right)+\beta 0_m\left(\delta^L\right)\mathbb{E}_m\left[v(s')\right] \\ \text{s.t. } (4.7)\end{array}\right\}\right\}. \tag{6.6}$$

We then have:

Definition 2 *A strategy vector δ^{*F} is a strong best response and $v_m^F\left(\cdot;\delta^L\right)\equiv V_m^F(\cdot;\delta^{*F},\delta^L)$ is the corresponding optimal value function of the follower if the following conditions are satisfied for all s and for $m=1,\dots,n-1$:*

$$\delta_m^{*F}\in\mathscr{R}_m^F(s,v_{m+1}^F;\delta^L), \tag{6.7}$$

and

$$V_m^L(s;\delta^{*F},\delta^L)=\max_{(\lambda,\theta)\in\mathscr{R}_m^F(s,v_{m+1}^F);\delta^L}\left\{\begin{array}{c} w_m^L\left(s,\lambda,\theta;\delta^L\right)+\beta 0_m\left(\delta^L\right)\mathbb{E}_m\left[V_{m+1}^L(s';\delta^F,\delta^L)\right] \\ \textit{s.t. (4.7)}\end{array}\right\}. \tag{6.8}$$

Using this definition, the optimal response to a given PP schedule δ^L is characterized by the following dynamic program, to be solved by backward induction from the known terminal value $\left(v_n^L, v_n^F\right)$.

$$
\begin{aligned}
v_n^L(s;\delta^L) &= 0 && (6.9)\\
v_n^F(s;\delta^L) &= \vartheta(s) && (6.10)\\
v_m^F\left(s;\delta^L\right) &= \max_{\lambda,\theta\in\mathscr{F}}\left\{w_m^F\left(s,\lambda,\theta;\delta^L\right)+\beta 0_m\left(\delta^L\right)\mathbb{E}_m\left[v_{m+1}^F(s';\delta^L)|\theta\right]\right\}, m=1,\ldots,n-1 && (6.11)\\
\mathscr{R}_m^F\left(s;\delta^L\right) &= \left\{(\lambda^*,\theta^*)\in\arg\max_{\lambda,\theta\in\mathscr{F}}\left\{w_m^F\left(s,\lambda,\theta;\delta^L\right)+\beta 0_m\left(\delta^L\right)\mathbb{E}_m\left[v_{m+1}^F(s';\delta^L)|\theta\right]\right\}\right\},\\
&\quad m=1,\ldots,n-1 && (6.12)\\
v_m^L\left(s;\delta^L\right) &= \max_{\lambda,\theta\in\mathscr{R}_m^F\left(s;\delta^L\right)}\left\{w_m^L\left(s,\lambda,\theta;\delta^L\right)+\beta 0_m\left(\delta^L\right)\mathbb{E}_m\left[v_{m+1}^L(s^{\prime;\delta^L})|\theta\right]\right\},\\
&\quad m=1,\ldots,n-1 && (6.13)\\
v_0^L\left(s;\delta^L\right) &= 0_0\left(\delta^L\right)\beta c_1+1_0\left(\delta^L\right)\mathbb{E}_0\left[\exp\left(-r\left(\tau-t_0\right)\right)R_\tau^L(s,\hat{D}_\tau)\right] && (6.14)\\
v_0^F\left(s;\delta^L\right) &= 1_0\left(\delta^L\right)\mathbb{E}_0\left[\exp\left(-r\left(\tau-t_0\right)\right)R_\tau^F(s,\hat{D}_\tau)\right] && (6.15)
\end{aligned}
$$

The functions $v_m^j(s;\delta^L),\ j\in\{L,F\}$ yield the total expected discounted cash flows received by the lender (Player L) and the borrower (Player F) corresponding to a given PP schedule δ^L, from date t_m until maturity, as a function of the level of the state variable at date t_m, under the borrower's best response strategy. To solve for the equilibrium strategy, one has to go through all feasible PP schedules in order to optimize the lender's total expected discounted cash flows.

While computing the value of a loan for a given PP schedule can be done by solving the dynamic program (6.9)-(6.15), the determination of an optimal PP schedule by the leader is clearly a challenging problem since the number of possible feasible schedules is infinite. In the next section, we use the dynamic program (6.9)-(6.15) to evaluate various instances using a sample of existing facilities.

6.4 Valuation of existing facilities including a performance pricing clause

In this section, we compute the value of various syndicated loans that include a performance pricing clause. For comparison purposes, we use the data to compute the value of a loan with the same characteristics, except for the PP clause, which is replaced by a covenant at the same initial coupon. The objective of this section is two-fold. First, we apply our model to the valuation of a PP schedule. Second, we use the comparison loans to evaluate the value of the flexibility provided by a covenant.

6.4.1 The data: DealScan and Compustat

Our sample of existing facilities is extracted from the DealScan database. We use a sample of 5,000 syndicated loan contracts concluded between 2000 and 2012. From this initial sample, we keep only the facilities where leverage is used as a performance measure. We also exclude all the revolving credit lines and standby credit facilities. We then select loans contracted by public borrowers with available ticker numbers. This allows us to obtain information about the borrower, such as the total asset, equity value, total outstanding debt, leverage, asset volatility, etc. Our final sample contains 41 credit facilities from four main countries (U.S., Canada, France, and U.K.). We obtain from the CRISP database the daily and monthly stock price files of the borrowers in the sample. We use the monthly stock return over five years before the facility's starting date in order to estimate the volatility parameter. We estimate the parameter g (immediate net dividend from an aggressive investment strategy) from the 80th percentile of the distribution of the monthly returns. To evaluate the increase in volatility associated with an aggressive investment strategy, we correlate the monthly value-weighted and volatility to compute the Pearson correlation coefficient and use linear interpolation to determine the volatility corresponding to a given value of g.

Statistics pertaining to our sample of 41 borrowers are recorded in Table 6.5 in Appendix 6.7.1.

6.4.2 Implementation

Loans with a performance pricing clause

The DealScan database provides the PP schedule (thresholds and corresponding spreads) for the 41 instances of our sample. We harmonize the contract terms in our sample by assuming monthly auditing dates coinciding with coupon payments. Accordingly, in our model, the coupon due at date t_{m+1} is the sum of two components: a deterministic fixed leg defined in the contract (e.g. interest on the outstanding loan at the contractual rate), denoted by c_{m+1}, and a variable leg that depends on the value of the performance indicator at the monitoring date t_m, so that the total coupon payment due at t_{m+1} is

$$\hat{c}_{m+1}(s) = c_{m+1} + \sum_j p_j D_m \mathbb{I}_{[B_j, B_{j+1}[}(s), \tag{6.16}$$

where D_m is the amount of the loan outstanding at date t_m. To compute the loan value, we run the dynamic program (6.9)-(6.15), using the coupons $\hat{c}_m$ in the immediate reward functions, and setting κ to 1.

Equivalent loans with a covenant clause

As discussed in Section 5.5, restrictive clauses in a loan contract have an impact on the initial spread. Consequently, the first step in comparing the value of two different restrictive clauses (here, a PP schedule to a covenant) would be to find an equivalent initial spread for the two types of loans. This is a challenging task when considering a variety of loans and PP grids and this is beyond the scope of this analysis. In this implementation, we assume that the loans under consideration are priced in the same way, under the assumption that risk-less borrowers would be charged the same interest rate. Accordingly, we set the initial spread for the loan including a covenant to the value in the PP grid that corresponds to the borrower's initial risk profile.

It is important to note that this choice is a simplification for illustrative purposes, and that we do not pretend to find a covenant loan equivalent to a loan with performance pricing. In particular, in opting for performance pricing instead of a covenant, the lender relinquishes the possibility to optimally adjust the risk premium over time, accounting for available information at the decision time. Hence, the initial spread for loans could differ according to the type of restrictive clause (PP or covenant).

Categories

Our analysis in Chapter 5 indicates that the value of a loan including a covenant is sensitive to two key parameters characterizing the loan: the borrower's asset volatility and initial leverage, both of them being an indication of the default risk. We therefore analyze the value of the loans in our sample by classifying them in three broad categories: Low-, Medium- and High-risk loans. The Low-risk category includes borrowers with low initial asset volatility and low initial leverage. The high-risk category includes the borrowers with high initial asset volatility and leverage. The medium-risk category includes all the other borrowers.

6.4.3 Numerical illustration

The results of our numerical investigation are presented in Figure 6.1. Overall, for all the risk categories, the value of the loans in our sample is never higher under PP than under a covenant. This is an expected result, since a lender using a covenant has more flexibility, allowing them to optimally choose the spread that better compensates for the changes in the borrowers' circumstances (provided the technical barrier is breached). For instance, the lender could have chosen to apply increases in the interest rates corresponding to the PP grid.

A closer examination shows that the differences in values are not uniform across categories. In the low-risk category (low volatility and low leverage), the loan values are very close, suggesting that the flexibility of the covenant has a low additional value. This could be the result of either a low default probability or simply a low probability of the covenant being breached or of a modification in the PP spread. On the other hand, the value added of a covenant clause is higher in the high-risk category. These combined results suggest

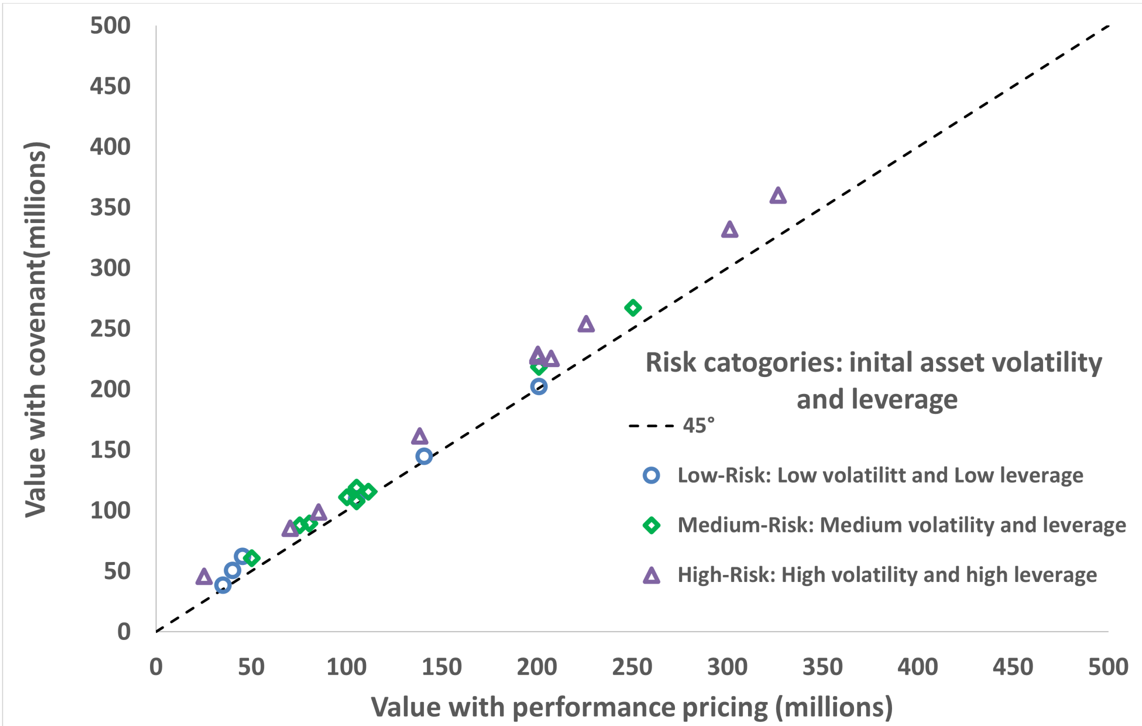


Figure 6.1: Loan value with performance pricing or with covenant.

that the choice of a PP schedule may be more critical for high- and medium-risk borrowers. In particular, how should a PP grid be chosen for a borrower with a specific set of parameters, and can performance pricing approach the efficiency of covenant clauses? These questions lead to the next section, where we will explore ways to determine the PP schedule optimally.

6.5 Determination of the pricing grid

In this section, we discuss the lender's problem, that is, the determination of the performance pricing grid at the inception of a loan contract. We investigate three approaches to this problem and discuss their advantages and limits. Illustrative examples, using the base case parameters, are provided in each case.

6.5.1 General setting

A first remark is that PP is predominantly used for term loans with coupon payments, and this is the setting considered in this section. In that case, pricing the spread and the coupon are equivalent. It is straightforward to adapt the approaches and notation used in this section to a case where the outstanding debt is not constant.

The pricing problem consists of choosing:

1. An observable performance indicator $\digamma_t$
2. An integer J representing the precision of the PP grid
3. A set of positive threshold values $\left\{B_j : j = 1, ..., J\right\}$, where $B_0 = 0$ and $B_J = \infty$
4. A function $\hat{c} : \mathbb{R} \mapsto \mathbb{R}_+$ defining the coupon value, as a function of the position of the performance indicator in the PP grid (or, equivalently, a function p defining the additional spread, as in Equation (6.16).

Note that, to simplify the notation and in accordance with the practice in the industry, we are assuming that both the function $\hat{c}$ and the set of thresholds $\left\{B_j : j = 1, ..., J\right\}$ are independent of time, but time-varying pricing grids could be considered.

Performance indicator

Various performance measures can be used in PP, and the most popular in the industry are based on the borrower's leverage ratio (debt to total asset, debt to equity, or debt to total book value). Other possibilities include debt to EBITDA, interest coverage ratio, credit score, etc. Many covenant indicators (see Appendix 2.4) are also used as performance measures. In the three approaches presented here, we use the debt-to-total-asset ratio (or its inverse) as the performance measure. This measure can be determined from the observation of the state variable $S_t = s$, and the deterministic value D_t.

Precision of the PP grid

We arbitrarily select the same number of intervals, and intervals of equal width, for the numerical implementations of the three approaches. We use six levels ($J = 6$), which is consistent with the majority of the loans in our sample. Under this assumption, the determination of the PP grid reduces to choosing two values, B_1 and B_5, determining the lower and the upper interval for the performance measure.

Coupon function

Practitioner literature and the DealScan database indicate that the most implemented coupon function is a step function that leads to a fixed coupon on each interval of the PP grid, and this is the approach we will take in the three implementations. Accordingly, at an evaluation date t_m, if $\mathcal{F}_{t_m} = f$, and provided the borrower is not in default, a coupon function c defines the coupon paid by the borrower and takes the form

$$c(f) = \hat{c}_j \text{ if } f \in [B_j, B_j + 1), j = 0, ...J - 1. \tag{6.17}$$

We now provide details on the three approaches to the determination of the coupon function. The first approach is based on a contingent-claim interpretation and consists of deriving a closed-form expression of the spread as a function of the leverage and other risk parameters. In the second and third approaches, we propose approximations to solve the dynamic stochastic game model presented in Section 6.3 by restricting the strategy space of the leader.

6.5.2 Contingent claim model

For some types of loans (e.g. bridge loans), a closed-form solution of the coupon function can be derived using the contingent claim approach developed by Black and Scholes (1973) and Merton (1974). The underlying model expresses the market value of the loan as a function of the borrower's financial indicators, under eight main assumptions:

1. There are no transaction costs, taxes, or problems with the indivisibilities of assets.

2. There are a sufficient number of investors with comparable wealth levels, so that each investor believes that they can buy and sell as much of an asset as they want at the market price.

3. There exists an exchange market for borrowing and lending at the same interest rate.

4. Short sales of all assets, with full use of the proceeds, is allowed.

5. Trading in assets takes place continuously in time.

6. The Modigliani-Miller theorem that the value of the firm is invariant to its capital structure obtains.

7. The term structure is "flat" and known with certainty, that is, the price of a riskless discount bond which promises a payment of one dollar at date T in the future is $P(T) = exp(-rt)$, where r is the (instantaneous) riskless rate of interest, constant over time.

8. The dynamics for the value of the borrower V, through time can be described by a diffusion-type stochastic process described by a stochastic differential equation.

As discussed in Merton (1974), most of the assumptions can be relaxed, and particularly 1 to 4 (perfect market conditions), and the solution below still holds. Assumption 7 is used to isolate the credit risk from the impact of the term structure of interest rates. In our model, we are assuming a short-term loan (12 months) and a flat-term structure, which is aligned with Assumption 7. Assumptions 5 and 8 are the most critical ones. Assuming continuous trading for syndicate credit assets can be challenging at first glance. However, given the recent growth of the secondary market, these assets may become open for trade most of the time. Lastly, Assumption 8 ensures market efficiency which, in the context dynamic secondary market, can be applicable in the context of syndicate credit assets.

Under the Merton (1974) model, for an outstanding loan value D_t and an asset value $S_t = s$ at t, the market value of the loan can be expressed as

$$V_t = s\left[\phi(-d_1) + \frac{D_T e^{-r(T-t)}}{s}\phi(d_2)\right], \tag{6.18}$$

where $\phi(.)$ is the cumulative standard normal distribution function,

$$\phi(x) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{x} e^{-\frac{1}{2}y^2}dy, \tag{6.19}$$

and the parameters d_1 and d_2 are defined by

$$d_1 \equiv -\frac{\left[(r-\delta-\gamma+\frac{\sigma^2}{2})(T-t) - log(\frac{D_T e^{-r(T-t)}}{s})\right]}{\sigma\sqrt{T-t}}, \tag{6.20}$$

$$d_2 = d_1 - \sigma\sqrt{T-t}. \tag{6.21}$$

Merton (1974) rewrites the loan value in the form

$$V_t^D \equiv D_T e^{-R(t)(T-t)}, \tag{6.22}$$

where $R(t)$ is the yield to maturity and determines the spread (risk premium) as a function of the parameters d_1 and d_2

$$R(t) - r = -\frac{s}{T-t}log\left[\phi(-d_1) + \frac{D_T e^{-r(T-t)}}{s}\phi(d_2)\right]. \tag{6.23}$$

The link between the risk premium and the leverage is obtained by identifying the leverage ratio $L_t = \frac{D_T e^{-r(T-t)}}{S_t}$ in the expression of d_1 and d_2.

Numerical illustration

Figure 6.2 illustrates the behavior of the spread (risk premium) as a function of the leverage ratio, for the base case (Table 5.1), varying the volatility parameter. The spread increases with the leverage up to a a certain value $L^*(\sigma)$ where spread is maximal, and then decreases. Both the level of the spread and the maximizing leverage $L^*(\sigma)$ depend on

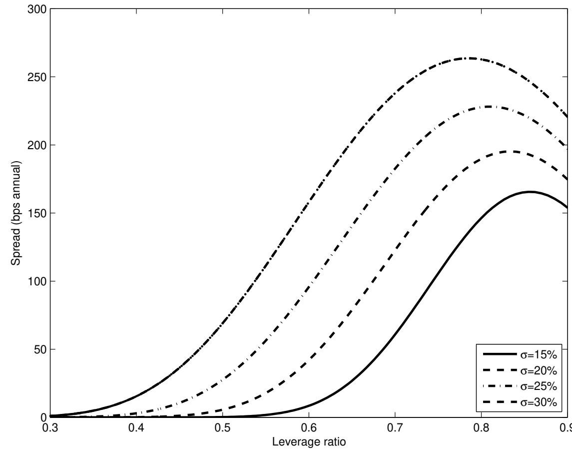


Figure 6.2: Spread as a function of the leverage ratio under the base-case specification given in Table 5.1

the asset volatility. As expected, higher volatility leads to higher spreads and lower L^*. The shape of the curve (Figure 6.2) shows an inflection point, smaller than $L^*(\sigma)$, where the concavity of the function changes, from convex (increasing positive slope) to concave (decreasing positive slope). The value of this inflection point ranges from 0.75 to 0.85, depending on the asset volatility. Note that performance pricing is not used in general for high-leverage loans. Therefore, we follow Doyle (2003) and posit that the performance grid should be somewhere in the convex region.

In order to produce a PP grid under the base-case specification given in Table 5.1, we use the results from the Merton model to determine the range of the grid in the convex region corresponding to each level of initial volatility. These regions are then uniformly divided into four sub-intervals and the spread value corresponding to each sub-interval is the average of the spreads computed over the sub-interval.

Table 6.1 reports the PP grid and the corresponding spreads in bps for various levels of volatility. Recall that these results are obtained using the base-case specification given in Table 5.1. The spreads reported in Table 6.1 are the total spreads applied to the base rate r.

Assuming a debt-to-total asset ratio of 0.67 (which corresponds to the average in our sample of syndicated loans), for a low-volatility borrower, the PP schedule is of an interest-increasing type. At a volatility of 15%, the initial leverage is on the lowest side of the grid, which means that the spread can only increase, in case of an increase in the leverage ratio. For a volatility of 20%, the PP schedule is a mix of increasing/decreasing interest. The initial leverage is in the middle of the grid, so it is possible that the borrower be rewarded by a decrease in the interest rate when their leverage ratio decreases. For higher volatility values (25% and 30%), the PP schedule tends to be an interest-decreasing type of structure. The initial leverage is high enough in the grid for the borrower to experience a reduction in their credit cost in the case of a decrease in their leverage ratio.

Table 6.1: PP grid corresponding to various asset volatilities. The first line contains the intervals for the leverage ratio. The second line reports the spread (bps).

Asset	**PP grid – Leverage (Spread in pbs)**					
Volatility	**Lower**	**level 2**	**level 3**	**level 4**	**level 4**	**Upper**
15%	0-0.63	0.63-0.68	0.68-0.72	0.72-0.76	0.76-0.81	0.81-0.85
	(28)	(57)	(95)	(132)	(159)	(165)
20%	0-0.55	0.55-0.60	0.60-0.65	0.65-0.70	0.70-0.75	0.75-0.80
	(29)	(60)	(101)	(144)	(177)	(188)
25%	0-0.50	0.50-0.55	0.55-0.61	0.61-0.66	0.66-0.72	0.72-0.77
	(41)	(79)	(125)	(173)	(210)	(216)
30%	0-0.45	0.45-0.51	0.51-0.57	0.57-0.63	0.63-0.69	0.69-0.75
	(54)	(101)	(157)	(210)	(247)	(256)

Source: Authors' computations using the Merton (1974) model

This first approach has the merit of being simple, providing a closed-form solution that only depends on the leverage, the (risk-free) interest rate, the volatility of the assets, and the time to maturity. Its main limitation is that it is not a solution to the lender's optimization problem, as it does not take into account the reaction function of the borrower. Literature shows that the borrower is not passive to coupon changes, and possible reactions include changing its risk-taking behavior, or refinancing the loan, when these alternatives are more interesting than accepting the coupon changes indicated by the performance pricing grid. Despite this limitation, this approach can be adapted in some cases to price performance pricing syndicated loan market. A practical case where this approach can be used is a bridge loan.

6.5.3 Dynamic game model

In this section, we address the limitations of the contingent claim model (section 6.5.2), with a dynamic game where the borrower reacts optimally to the change of the risk and coupon by considering their respective objective functions. Apart from the determination of the performance measure, the physical default barrier, and the precision of the grid, the determination of the lender's strategy entails choosing the range of the PP grid and the corresponding coupon function. In the two approaches tested here, we will concentrate on the last two components of the lender's strategy, which are the most challenging.

In order to reduce the space of feasible strategies available to the lender, we use the results from the covenant model present in Chapter 4, under the hypothesis that a good PP grid should give a value to the lender that is close to what they could attain with a covenant on the same performance indicator.

Determination of the PP grid region

We start by using the model developed in Chapter 4 to evaluate the sensitivity of the value of the loan to the level at which the borrower is considered in technical default, assuming that physical default is triggered when the ratio of assets to outstanding debt attains $\alpha = 1$. Figure 6.3 presents the results for various values of the asset volatility (15%, 20%, 25%). We observe that the loan value is an increasing function of the technical default parameter $\hat{\alpha}$. We identify three distinct regions characterizing the sensitivity of the value of the loan to $\hat{\alpha}$. For instance, for a volatility of 15% ,

1. for $\hat{\alpha} \in [1; 1.2]$, the the loan value is moderately sensitive

2. for $\hat{\alpha} \in [1.2; 1.75]$, the loan value is highly sensitive, almost linearly

3. for $\hat{\alpha} > 1.75$, the loan value stabilizes.

This suggests that, in the base case, the region where spreads are adjusted should lie between 1.2 and 1.75.

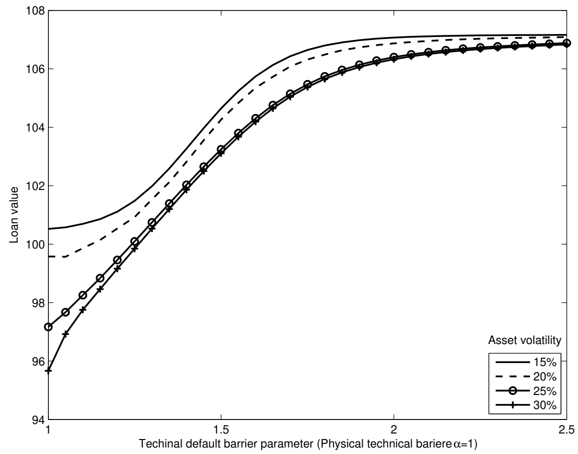


Figure 6.3: Loan value as a function of the technical default barrier parameter $\hat{\alpha}$ for various asset volatilities and under the base-case specification given in Table 5.1

Accordingly, Table 6.2 provides PP grids corresponding to various volatility levels. Assuming the base-case specification given in Table 5.1 with an initial loan amount of 100, one can then derive the lower and upper bounds of the PP grid in terms of leverage.

Table 6.2: Performance grid region using technical default barrier

Asset	**Technical default parameter**		**Asset value**		**Leverage**	
Volatility	**Lower**	**Upper**	**Lower**	**Upper**	**Lower**	**Upper**
15%	1.2	1.75	120	175	57%	83%
20%	1.15	1.75	115	180	56%	87%
25%	1.1	1.95	110	195	51%	91%
30%	1.07	1.99	107	199	50%	93%

Source: Authors computation using the model developed in Chapter 4

We then arbitrarily define five intervals of equal width between the lower and upper value of the leverage, to which we add two end-intervals to cover leverages ranging from 0 to α (defining the physical default barrier.

Restriction to linear risk compensation

A first approach consists of restricting the space of possible coupon functions. To this end, we follow the literature (Sarkar and Zhang (2015); Manso, Strulovici, and Tchistyi (2010); Myklebust (2012)) and restrict the coupon functions to the space of linear functions. For instance, in the case of a term loan and positive reward, the coupon function takes the form

$$c(f) = \mu_0(1 - \mu_1 f), \tag{6.24}$$

where $\mu_0 > 0$ and μ_1 are the parameters to be determined. Note that a linear risk-compensation function is not a step function. As in Section 6.5.2, we will first identify a linear function optimizing the value of the loan, and then discretize the spread over the PP grid.

Assuming that the coupon function takes the form in Equation 6.24, we optimize the value of the lender over a two-dimensional grid $G_0 \times G_1$ for the unknown parameters μ_0 and μ_1.

More precisely, for each point of $G_{c_0} \times G_{c_1}$, we use the dynamic program (6.9)-(6.15) to compute the value of the loan (the lender's value function). We then find the maximum value at $S_0 = 100$ on the grid and the corresponding μ_0^* and μ_1^* . As discussed in Manso, Strulovici, and Tchistyi (2010), different coupon functions could raise the same amount of cash flows adjusted for the default risk. Therefore, this approach could lead to multiple solutions optimizing the lender's value.

Denoting by $\mathscr{R}^L$ the set of pairs (μ_0^*, μ_1^*) optimizing the value of the loan, we adapt the relative efficiency defined in Manso, Strulovici, and Tchistyi (2010) to derive a partial order among the coupon functions.

Definition 3 *(Relative efficiency): A coupon function c_1 is efficient compared to a function c_2 yielding the same discounted total cash flows if c_1 gives a higher follower value than c_2.*

In our implementation, we retain as a final coupon function the one corresponding to the most efficient in the set $\mathscr{R}^L$.

Table 6.3 summarizes the results. On each pricing grid, the risk-compensating spread increase is constant as per the assumption of a linear coupon function (e.g., 2.5bps per 1%

change in the leverage for an initial asset volatility of 15%). Note that for all volatilities, the optimal pricing structure can lead to both increases or decreases in the coupon rate: the initial spread lies in the middle of the grid, with the possibility of reducing the interest to reward good performance or increasing it to punish bad performance in terms of the performance measure.

Moreover, we observe that the initial spread (corresponding to a leverage of 67%) is increasing at an average rate of 6bps per 1% change in the initial asset volatility, suggesting that the asset volatility at inception is an important factor to consider in determining the PP pricing schecule.

Table 6.3: Discretized grid and spread (bps)–Restriction to linear risk compensation

Asset	**PP grid –Leverage (spreads in pbs)**						
Volatility	**Lower**	**level 2**	**level 3**	**level 4**	**level 4**	**Upper**	**up to default**
15%	0-0.57	0.57-0.62	0.62-0.68	0.68-0.73	0.73-0.78	0.78-0.83	0.83-1
	(75)	(86)	(99,2)	(110)	(121)	(132)	(169)
20%	0-0.56	0.56-0.62	0.62-0.68	0.68-0.74	0.74-0.81	0.81-0.87	0.87-1
	(100)	(115)	(130)	(145)	(163)	(178)	(210)
25%	0-0.51	0.51-0.59	0.59-0.67	0.67-0.75	0.75-0.83	0.83-0.91	0.91-1
	(116)	(138)	(160)	(182)	(204)	(226)	(251)
30%	o-0.50	0.50-0.59	0.59-0.68	0.68-0.76	0.76-0.85	0.85-0.93	0.93-1
	(134)	(159)	(184)	(206)	(231)	(253)	(273)

Source: Authors' computations using the dynamic program (6.9)-(6.15)

The PP schedule determined using this approach has the advantage of accounting for the follower's best response function, while remaining computationally feasible as it involves solving the dynamic program (6.9)-(6.15) over a two-dimensional grid. The main limitation is the restriction of the coupon function to a linear function of the performance indicator, which excludes other types of risk-compensation functions, and does not depend on the specific form of the performance indicator.

In the next implementation, we drop the linearity assumption for the coupon function and directly optimize the spreads corresponding to the PP grid, therefore restricting the space of admissible coupon functions to the set of discrete-step functions.

Restriction to step functions

This third approach consists of determining the spread in each interval of the PP grid, without imposing any constraint on the form of the coupon step function. Using the same discretization for the leverage threshold as in the previous section, we consider spread values ranging from 0 to a maximum value, determined for instance by considering the maximum punishment obtained using one of the two first approaches (in our implementation of the base-case specification, we use a maximum spread of 10%). We then produce a set of admissible spread vectors randomly, therefore including various forms of PP grids (interest-decreasing, interest-increasing, or mixed). We apply a grid search on this set, using the follower's response provided by 6.9-6.15, in order to find the grid that provides the highest value to the leader, and, in case of a tie, the most efficient PP grid.

The results of our numerical investigation are presented in Table 6.4. We observe that the optimal PP schecule, as in the linear coupon-function case, can lead to increases or decreases in the coupon rate. Note that the optimal PP schecule is no longer linear. For instance, this leads to a non-constant risk-compensating rate with an average of 2.67bps per 1% change in the leverage for a volatility level of 15% and a higher rate for the initial spread.

Table 6.4: Discretized grid and spread (bps)–Restriction to step functions

Asset	PP grid –Leverage (Spread in pbs)						
Volatility	**Lower**	**level 2**	**level 3**	**level 4**	**level 4**	**Upper**	**Default**
15%	0-0.57	0.5-0.62	0.62-0.68	-0.68-0.73	0.73-0.78	0.78-0.83	0.83-1
	(78)	(108)	(118)	(120)	(134)	(149)	(185)
20%	0-0.56	0.56-0.62	0.62-0.68	0.68-0.74	0.74-0.81	0.81-0.87	0.87-1
	(103)	(124)	(141)	(169)	(185)	(187)	(233)
25%	0-0.51	0.51-0.59	0.59-0.67	0.67-0.75	0.75-0.83	0.83-0.91	0.91-1
	(141)	(159)	(164)	(193)	(207)	(234)	(260)
30%	0-0.50	0.50-0.59	0.59-0.68	0.68-0.76	0.76-0.85	0.85-0.93	0.93-1
	(160)	(164)	(206)	(216)	(256)	(274)	(280)

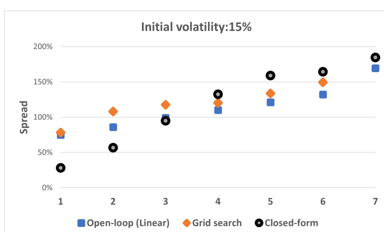
Source: Authors computations

While this third approach puts much less restriction on the coupon function space, it comes with some limitations. Even for given grid intervals, finding the optimal spread on each sub-interval of the grid can be very challenging computationally. The number of possibilities in the number of sub-intervals and in the possible spread vectors is huge. The mere choice of the search space may make this approach heuristic.

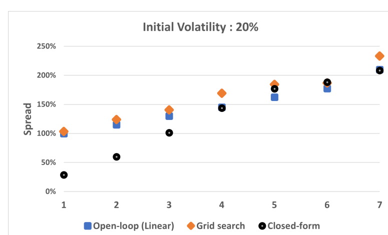
Comparison

Figure 6.4 compares the three pricing grids according to various asset volatilities. We aim to compare the trends of the pricing structures, the risk-compensating pace, and the overall level of spreads. Since the grid thresholds (the leverage thresholds) are not the same between the closed-form solution and the two other pricing from the dynamic game model, we can not compare point by point.

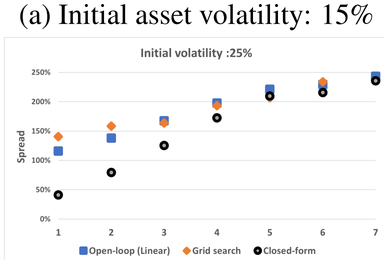
Overall, accounting for the follower's response tends to price with a higher initial spread and a lower risk-compensating slope. The closed-form solution starts with lower spreads for the lowest intervals and converges with the other approaches for the last two pricing intervals. Comparing the grids obtained from the dynamic game model, the overall trends are similar and the spreads are close in most sub-intervals, suggesting that linearity of the coupon function is a realistic assumption that simplifies the pricing without losing much of pricing efficiency.



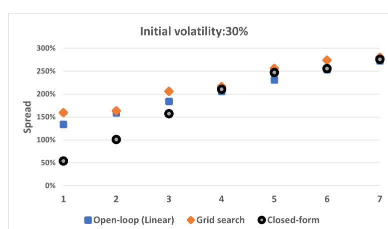
(a) Initial asset volatility: 15%



(b) Initial asset volatility: 20%



(c) Initial asset volatility: 25%



(d) Initial asset volatility: 30%

Figure 6.4: Comparing performance pricing grids for various initial asset volatilities.

6.6 Conclusion

We used a stochastic dynamic leader-follower game model to compute the value of the loan under performance pricing. Our model allows to account for the borrower's possible reactions to changes in their credit cost, through the four possible actions presented in Chapter 4.

This chapter offers two main contributions to the literature on performance pricing: the valuation of loans with a performance pricing clause, and the determination of an optimal PP schedule.

On the valuation point, we compute the value of a sample of contracts signed between 1995 and 2009 collected from the Dealscan database, by determining their covenant-equivalent value. Our results show that, depending on four main factors (initial leverage, asset volatility, maturity, and the incentive to play aggressively), the value of the loan with performance pricing is generally lower than the value under a covenant clause. In opting for performance pricing instead of a covenant clause, the lender relinquishes the possibility to adapt to factors other than the performance indicator, that may have a significant impact on the value of the loan (e.g. time to maturity and volatility).

In practice, performance pricing generally relies on a single performance measure, rarely considering other risk factors, such as the borrower's asset volatility.

On the pricing point, we compare three approaches to determining a PP schedule. The first approach, based on a contingent-claim interpretation, provides a closed-form solution but does not consider the follower's possible responses to the change in their credit cost. The two other approaches propose heuristic solutions to the stochastic game played by the lender and the borrower. We implement two versions, restricting the coupon function to the space of linear function or step function, respectively. Our results show that the contingent-claim approach provides a lower spread and lower loan value than the two other approaches and the direct search in the space of step functions provides the best PP grid to the lender. In all cases, we observe that asset volatility is an important factor that should be considered in determining the PP schedule.

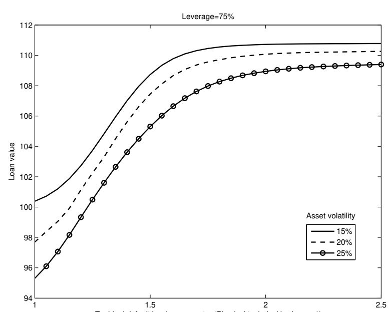
We suggest one possible extension of the work in this chapter. Since leverage and asset volatility are both important factors for the value of the loan, these two factors could be used to derive an average risk-compensating rate and evaluating by how much the spread should change for any unit change of the leverage and the volatility. This can be used to build an efficiency surface, expressing the optimal risk-compensating rate as a function of the leverage and asset volatility.

6.7 Appendix

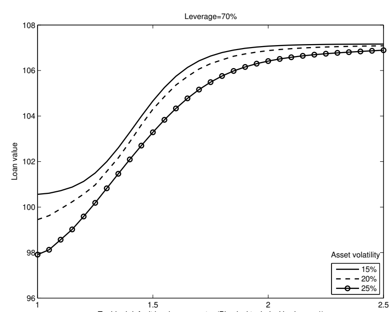
6.7.1 Dealscan data

Table 6.5 contains the description of the sample of facilities used in Section 6.4. The first column provides the asset value in millions of U.S. dollars at the contract inception. The estimated asset volatility varies from 16% to 30%. Total leverage (including other debt) is computed as the ratio of the total liability to the asset value. The additional profit due to a change in investment strategy represents our estimation, its value ranging from 5% to 14%. The remaining columns provide the facility information (size in millions of U.S. dollars, year of syndication, and maturity in months). We used the 1-year LIBOR rate as a proxy of the risk-free rate.

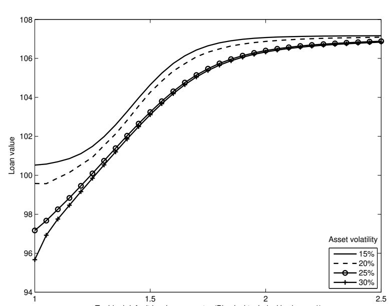
6.7.2 Loan value sensitivity to the level of the technical default barrier for various initial leverages



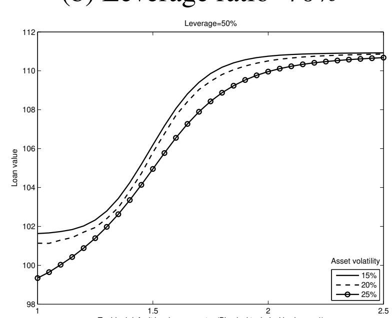
(a) Leverage ratio=75%



(b) Leverage ratio=70%



(c) Leverage ratio=67%



(d) Leverage ratio=50%

Figure 6.5: Loan value as function of technical default barrier

Table 6.5: Sample data

N.	Asset (millions)	Volatility (%,year)	leverage (total,%)	Extra profit (%)	Facility (million)	Year	Maturity (months)	LIBOR 1-Year, %
1	443.552	23.37	12.00	7.47	15	2003	60	1.3567
2	2183.909	16.51	58.47	5.59	100	2008	36	3.0909
3	4381.291	25.59	58.82	10.34	325	2007	12	5.1241
4	463.592	22.09	21.93	8.60	140	1997	12	6.0045
5	2633.984	23.12	49.28	6.37	113	2004	48	2.1210
6	2633.984	23.12	49.28	6.37	320	2004	32	2.1210
7	1919.288	23.12	52.35	6.37	100	2002	36	2.1973
8	232.969	24.00	7.14	9.00	80	1996	12	5.7802
9	1337.310	22.27	37.98	6.75	100	2005	36	4.0317
10	3372.851	30.85	57.23	6.73	106.7	2001	24	3.8547
11	2135.552	26.15	0.95	7.49	200	2004	12	2.1210
12	1900.502	26.15	1.30	7.49	200	2003	12	1.3567
13	1625.081	26.15	2.07	7.49	138	2002	11	2.1973
14	1625.081	26.15	2.07	7.49	200	2002	12	2.1973
15	1651.913	26.15	1.26	7.49	225	2001	12	3.8547
16	1298.831	26.15	1.60	7.49	200	1999	11	5.7194
17	142.878	25.71	40.68	8.39	50	2001	120	3.8547
18	9582.897	21.42	64.64	6.43	450	2003	36	1.3567
19	3929.672	16.53	17.87	5.16	200	1999	12	5.7194
20	13512.900	25.22	23.71	9.44	775	2001	12	3.8547
21	4161.024	22.95	10.09	10.40	105	2009	12	1.5689
22	1436.342	23.39	26.32	12.40	25	2002	38	2.1973
23	1369.014	23.39	39.08	12.40	75	1999	12	5.7194
24	110.382	30.26	15.40	13.95	9.7	1995	28	6.2344
25	774.786	20.74	38.69	7.54	40	2001	12	3.8547
26	758.659	20.74	40.89	7.54	45	2000	12	6.8687
27	4163.575	23.04	18.34	8.06	100	2003	12	1.3567
28	275.959	28.63	1.01	12.19	100	1996	24	5.7802
29	206.471	35.41	19.03	12.34	15	2002	24	2.1973
30	1205.012	25.02	37.73	11.80	300	2005	12	4.0317
31	5195.485	21.35	32.25	9.65	500	2008	40	3.0909
32	165.559	26.54	92.35	13.71	25	1999	12	5.7194
33	975.910	33.66	29.15	6.22	85	2001	12	3.8547
34	1094.177	23.40	5.56	7.44	30	2002	36	2.1973
35	1094.177	23.40	5.56	7.44	35	2002	12	2.1973
36	2081.790	25.29	16.29	9.66	100	2003	16	1.3567
37	753.025	22.14	17.38	10.06	100	2003	12	1.3567
38	1780.948	22.48	5.93	7.64	65	2003	60	1.3567
39	1049.486	25.67	36.04	11.54	100	1999	12	5.7194
40	781.614	33.44	28.93	10.06	70	2000	12	6.8687
41	5188.600	24.22	11.44	6.99	50	1999	12	5.7194

Source: Deal Scan and authors' computations

Chapter 7

Valuation of Syndicated Revolving Credit Line with restricting clauses

A revolving credit line (RCL) is a financial contract under which a lender grants a credit facility to a borrower under predetermined terms (maturity, facility size, interest rate spread, and other fees). Under such a contract, a borrower has the right but not the obligation to draw from the facility, pay partially or totally the drawn amount back, and redraw any time before the facility maturity and up to a fixed amount (facility size). While revolving credit lines often include a material adverse change (MAC) clause giving the lender the power to void the facility at any time, the loan acceleration (early repayment) right is rarely exercised in the syndicated loan context. Rather, financial covenants are used in almost all syndicated loan commitments. In this chapter, we adapt the dynamic stochastic game model introduced in Chapter 4 to value a syndicated facility package that includes both a term loan and a revolving credit line.

7.1 Introduction

A revolving credit line exposes the lender to various types of risks: interest rate risk, credit risk, and wrong-way risk. Interest rate risk arises mainly in the case of fixed-rate RCLs. In such a case, if interest rates increase in the market, the borrower will ultimately draw from the facility at a low cost instead of borrowing from the market. The case of fixed-interest loans is very limited in syndicated revolving credit facilities but remains an important risk in bilateral RCL contracts. The second and perhaps most important risk is the credit risk. This risk arises from the fact that the interest rate, or the spread over the base rate (LIBOR, prime rate, etc.), is determined at the contract inception, on the basis of credit information available at that time. Any subsequent negative change in the borrower's credit quality makes this pricing unfair for the lender. More importantly, the credit risk is magnified by the wrong-way risk: as the borrower's credit quality deteriorates, they will optimally draw from the facility instead of borrowing from the market. This makes the lender exposition to credit risk increase as the borrower's default risk increases. Revolving loan commitments are not generally traded in the secondary market, hence, it is not possible to use market-to-market valuation for this type of facility, which makes their valuation an interesting topic. In the literature, two families of approaches have been proposed to value revolving loan commitment: contingent claims (Hawkins (1982), Chateau (1990)), and game theory (Boot, Thakor, and Udell (1987), T. S. Campbell (1978)).

In this chapter, we seek to contribute to the methodology as well as the application point of view. We introduce a valuation approach adapted from the model developed in Chapter 4. The model accounts for the lender's right to change the spread or to refuse a drawn from the facility in certain circumstances, and for the borrower's flexibility in changing their risk-taking strategy (possibility of asset substitution). This is formalized as a stochastic dynamic game with a leader (the lender) and a follower (the borrower) using feedback strategies.

We use the model to value a syndicated facility package that includes a term loan and a revolving credit line. We consider various facility management options, including full or partial take-down for the borrower, with and without the right to freeze the facility for the leader. We find that the presence of revolving credit adds value to the facility when the borrower's leverage is relatively low. Our results also show that, at equilibrium, the lender may still allow the utilization of the facility when the borrower is in the technical default region.

The rest of the chapter is organized as follows: Section 7.2 provides the background on revolving credit lines in the syndicated market, and Section 7.3 is a brief literature review on the valuation of loan commitments. We develop the model in Section 7.4. Implementation and results are discussed in Section 7.5, while Section 7.6 concludes this chapter.

7.2 Background on revolving credit line

The revolving credit line is one of the most important credit facilities used by large enterprises for their future financial needs. The question of its existence has been widely discussed in the literature. On the demand side, enterprises use the RCL to minimize loan arrangement costs, and as insurance against future deterioration in their creditworthiness (Federal Reserve, the survey of the Board of Governance; Sofianos, Wachtel, and Melnik (1990)); Berger and Udell (1990)). On the supply side, banks provide revolving credit lines mainly to address information asymmetry prior to loan contract origination.

Figure 7.1 presents the trend of the total drawn and undrawn amount from revolving facilities of US financial institutions (depository institutions and other financial institutions). From around 1 trillion in Q4 2009, the unused amounts increased to more than 2.7 trillion in Q2 2023. The drawn amounts follow similar trends, moving from 285 billion in Q4 2009 to 930 billion in Q2 2023. Figure 7.1 also shows that the undrawn syndicated revolving credit is much higher than the outstanding amount of term loans.

Figure 7.2 depicts the default amount (and percentage) from the drawn part of the facility, compared to the term loan. The amount in default is historically higher in term loans compared to revolving. However, in percentage, the revolving default share has been higher since 2014.

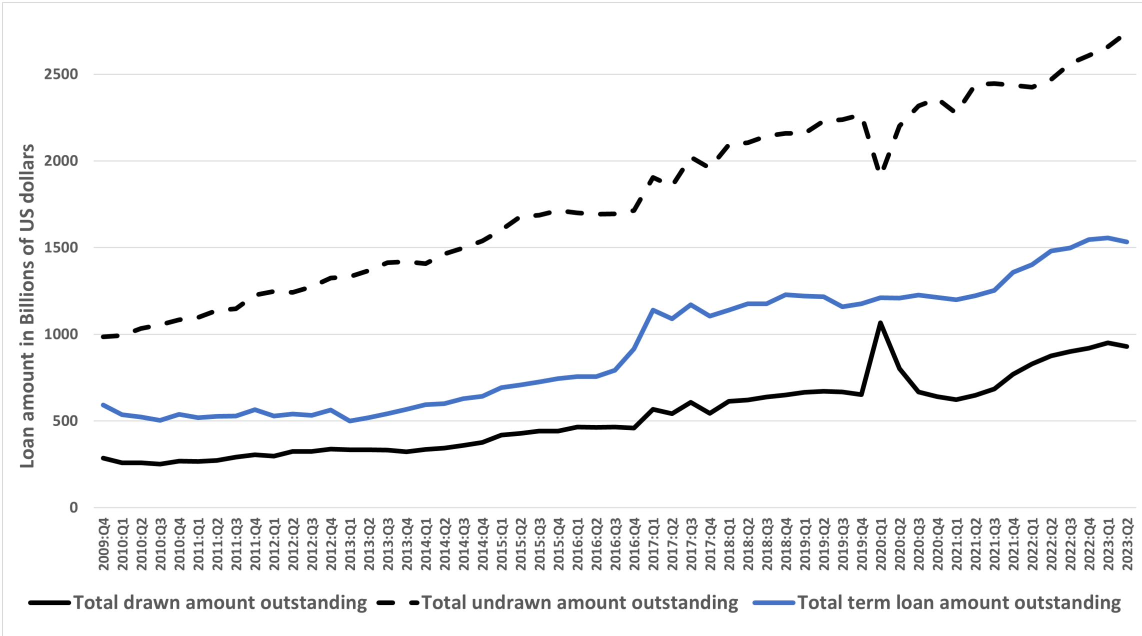


Figure 7.1: Total drawn amount outstanding. Source: Federal Reserve Bank of St. Louis

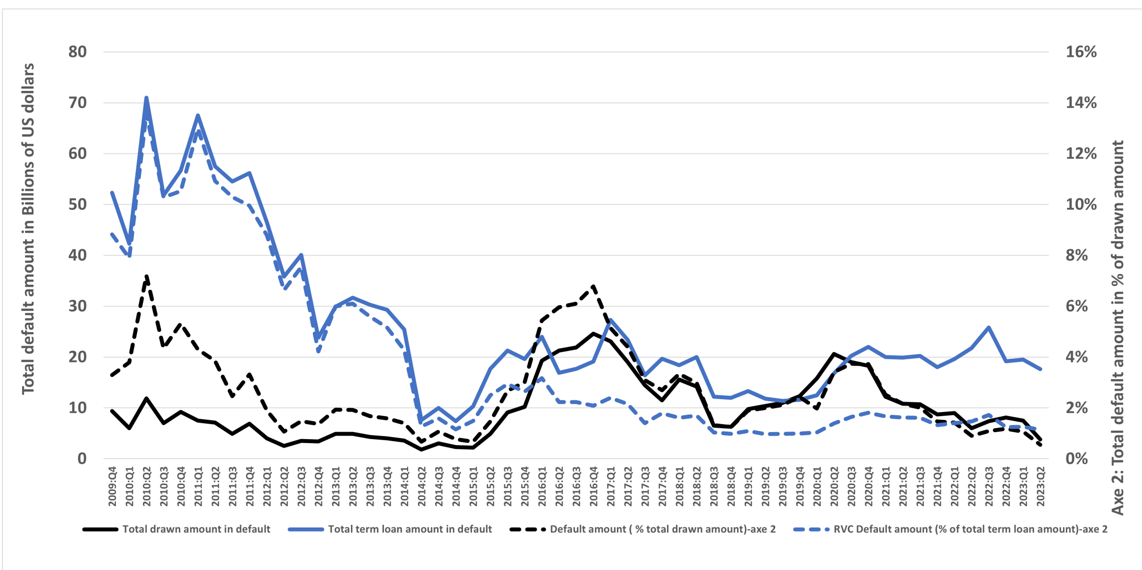


Figure 7.2: Total drawn amount in default. Source: Federal Reserve Bank of St. Louis

Revolving credit is recognized to have a complex pricing structure. In addition to the interest rate paid on the used amount from the facility, the borrower is required to pay a commitment fee on the unused part, along with the same upfront fees usually applied to the syndication (arrangement, underwriting, and participation fees).

7.3 Literature on revolving credit valuation

One of the most popular approaches to value or price revolving credit line is to consider it as a put option on the borrower's debt (Hawkins (1982), Thakor (1982), Ho and Saunders (1983), Rendleman Jr (1979)). Hawkins (1982) uses the contingent claim approach to evaluate a RCL, by considering the revolving credit facility as a portfolio of callable debt and options. When borrowing from the facility, the borrower increases the share of callable debt in the portfolio. The callable nature of the debt comes from the possibility of repaying any drawn amount. When not borrowing or paying back any drawn amount, the borrower increases the share of options in the portfolio. The commitment fee (on the undrawn amount) is equivalent to an option premium giving the right to the borrower to put callable debt to the bank any time before the facility maturity. The author characterizes the exercising boundary where the borrower switches from holding callable debt and put options as a function of the firm value. Using a set of assumptions similar to Black and Scholes (1973), the author derives a partial differential equation for the revolving credit line value in the two regions delimited by the exercising boundary. These equations are solved under an additional covenant assumption preventing dividend distribution.

A second approach accounts for strategic interactions between the lender and the borrower. Boot, Thakor, and Udell (1987) uses a non-cooperative game model in the context of an investment project. The borrower has some (costly) control over the expected payoff of the project, while the lender decides on the interest rate of the loan commitment. The model is solved in the case of information symmetry and asymmetry, and the author shows that two equilibria may arise under information asymmetry. T. S. Campbell (1978) uses a model based on random future cash needs to analyze the demand and supply of lines of credit. The author shows that the borrower will start using the fixed-rate line of credit when the marginal cost of borrowing from other sources is greater. On the supply side, the lender optimizes the borrower's utility function to determine the optimal cost structure of the facility.

Shockley and Thakor (1997) and Chava (2003) argue that loan commitment facilities are not priced as a put option in the market. Chava (2003) considers various risk factors modeled as random processes, calibrated with historical data.

The model in Chava (2003) addresses most of the limitations in the previous literature, by relaxing the simplistic hypothesis of a one-time total take-down at maturity, considering a random partial draw from the facility. This model is closer to the industry practice on RCL, however, it does not account for features that are specific to the syndicated revolving loan market, for instance, material adverse change and/or covenants clauses. These possibilities given to the lender are included in almost all the syndicated revolving credit facilities and should be considered in the pricing model. On the borrower's side, practical observations and literature (for instance, the literature pertaining to the contingent claim approach) show that borrowers are not passive players. Partial repayment (based on a combination of revolving and another source of financing) is very popular in the industry. More importantly, the theory on loan contracting and on agency problems (for instance, the asset substitution problem presented in Koziol and Lawrenz (2010)) suggest that the borrower could change their risk-taking strategy after securing funding. Finally, the majority of syndicated revolving credit facilities are provided in a package that includes another type of loan, for instance, a term loan.

In the following section, we consider the combination of a term loan with a revolving credit facility to derive the value of the RCL.

7.4 The theoretical model

7.4.1 The contract

We consider a facility package that includes both a term loan and a revolving credit line. Under the revolving credit contract, the borrower has the right, from the contract inception to its maturity T, to draw from the facility or reimburse it, provided that the cumulative drawn amount (used amount) does not exceed the facility size, denoted by M. The lender is committed to paying interest (i_u) on the used part of the facility, in addition to a commitment fee (i_c) on the unused part (remaining commitment amount). At maturity, all the outstanding balance, including the term loan amount and the takedown from the revolving is to be reimbursed. To protect the lender, the contract includes a financial and non-financial covenant, in addition to a MAC clause. More precisely, the lender has the right to refuse a drawing request (for instance as long as the borrower is in technical default). In addition, in case of technical default, the lender has the right to punish the borrower by increasing the fee on the used part of the facility over the next interest period and/or reducing the facility size by cutting an amount from the unused part of the facility.

On the other hand, the borrower can reimburse the outstanding loan balance at any time.

7.4.2 Timeline and players' actions

We model the game in discrete time, assuming that the interventions of the players occur at discrete (audit) dates where the state of the system is observed by both players. To simplify the exposition, we further assume that auditing dates $t_1, ..t_N = T$ correspond to the dates of the periodic payment of interest and capital amortizing if applicable.

At any auditing date, the players observe the level of the covenant indicator. In case of technical default, the lender has a choice of corrective actions, otherwise, the lender makes no move. The borrower can take action at any of the auditing dates, after observing the eventual action of the lender.

The set of actions available to the players at an audit date are the following:

Leader's (lender) actions

In case of technical default, the lender can

- Impose a temporary interest increase, due at the next audit date;
- Increase the technical default barrier (which is equivalent to asking for more collateral);

- Reduce the maximal amount of a draw from the revolving facility;
- Combine any of the above actions.

Follower's (borrower) actions

At any auditing date, the borrower can

- Accept the conditions of the loan, eventually including the corrective actions implemented by the lender;
- Choose the amount to draw from the facility;
- File for bankruptcy.

7.4.3 Modeling assumptions

Notice that the structure of the game model is very similar to the one presented in Chapter 4, except for the decision about the amount to draw from the RCL, which we will relate to the decision of choosing a risky vs. a conservative investment strategy, therefore modifying the volatility of the borrower's assets.

In Chapter 4, we assume that a change in the borrower's risk-taking strategy does not impact their debt level. The implicit assumption in that case is that the borrower can reallocate their existing resources to riskier projects.

In this chapter, we rather assume that the borrower uses the revolving facility to fund additional projects. Moreover, we assume that engaging in new (riskier) projects increases the borrower's overall asset volatility while providing an additional immediate dividend.

As in Chapter 4, we relate the additional dividend to the asset level. We analyze two possibilities to model this dependence.

In Case 1, we assume that the riskier activities are funded exclusively from the RCL. Accordingly, we set the additional dividend to a percentage of the amount drawn from the facility. In Case 2, we assume that existing resources can be used, along with the amounts drawn from the RCL, to fund the riskier activities, and we set the additional dividend to a percentage of the total asset value, including the amounts drawn from the RCL.

Moreover, we consider two different settings with respect to the borrower's possible actions when drawing from the RCL. In the full-take-down setting, the borrower does not have a choice in the level of the drawn amount: at a decision date, they can either draw nothing or the entire credit line. In the partial-take-down setting, we assume that the borrower can decide on the level they need, up to the size of the credit line.

7.5 Numerical results

In this section, we present the results of various experiments, performed using the base-case specification in Table 5.1. The covenant indicator is the asset-to-debt ratio, where the debt is the total of the outstanding debt about the term loan and the amount drawn from the RCL. To our base-case specification, we add the revolving credit size M, which corresponds to the maximum amount that the borrower can draw, the commitment interest rate i_c, and we set the utilization cost i_u equal to the term loan coupon rate.

7.5.1 Implementation

It is straightforward to adapt the dynamic program (4.12)-(4.21) to the case of a package including a term loan and a RCL under the assumptions described in Section 7.4.

For comparison purposes, we consider two debt portfolios. Portfolio 1 is the credit facility that includes both a term loan and a RCL, while Portfolio 2 includes only a term loan, where the loan value D_0 is fixed to the total of the term loan D_0 and the size M of the RCL in Portfolio 1.

In Portfolio 1, the borrower draws the full amount D_0 of the term loan at the contract inception and has the option to draw, pay back, and redraw from the revolving component, up to a maximum amount of M, at any auditing date before the maturity of the facility. Note that the technical and physical default barriers will change as the borrower draws from the revolving part, therefore decreasing their asset-to-debt ratio. When in technical default, the eventual interest rate increase is applied to the total of the term loan and the drawn part of the revolving facility.

In Portfolio 2, the borrower draws the full facility amount at the contract inception, so that the debt level remains constant at $D_0 + M$ over time.

7.5.2 Funding sources for risky activities

In a first experiment, we compare the value of Portfolio 1 under two assumptions regarding the amount engaged in risky activities (Case 1 vs. Case 2). Figure reports the value of the loan as a function of the initial asset level, for various values of the proportionality parameter g relating the dividend to the amount engaged. Examination of our results shows that Case 2 can be recovered from Case 1 by adjusting the proportionality parameter g. Accordingly, we will assume in the sequel that the additional dividend is proportional to the amount drawn from the RCL (Case 1).

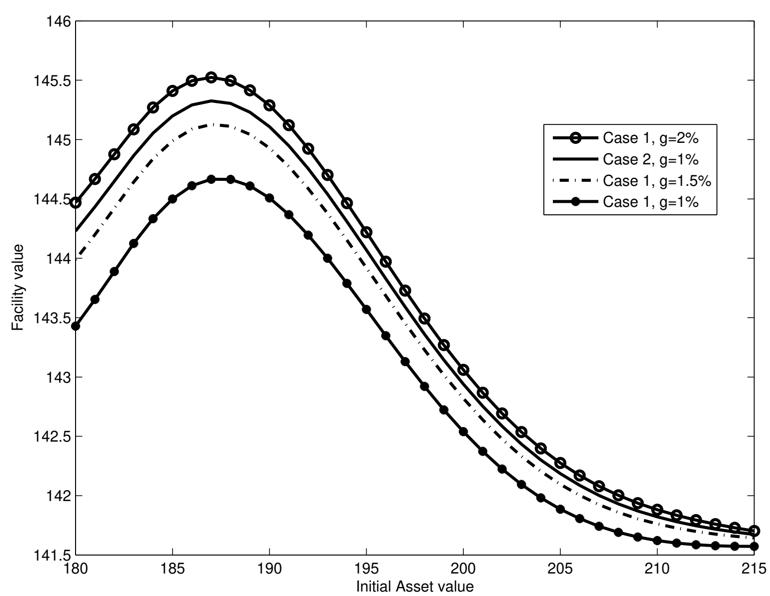


Figure 7.3: Facility value as a function of the initial asset level at inception under two contrasting assumptions regarding the amount engaged in risky activities. Case 1: Immediate dividend as a percentage of the drawn amount. Case 2: Immediate dividend as a percentage of the total asset value, including the drawn amount

7.5.3 Full take-down

In a second experiment, we compare the value of Portfolios 1 and 2 under a full-take-down setting: the follower has only two options, drawing either the maximal amount or nothing from the RCL to fund riskier activities. The values are compared for contracts including or not a covenant. Note that when the contract does not include a covenant, the leader is a dummy player, having no available action. Figures 7.4 and 7.5 report, respectively, the value of the lender and that of the borrower.

As expected, the covenant improves the facility value in the regions where the risk of technical default is high, converging to the facility value without covenant as the initial creditworthiness of the borrower improves. For the lender, the loan value of Portfolios 1 and 2 behaves similarly, with or without a covenant. Portfolio 1 provides a higher value to the lender, while Portfolio 2 (with a RCL), giving flexibility to the borrower to optimally adjust their debt level, yields a lower value for the lender.

Similar observations can be made for the value of the borrower (Figure 7.5); the follower value is higher for Portfolio 2, higher without covenant, and these values converge as the initial creditworthiness of the borrower improves.

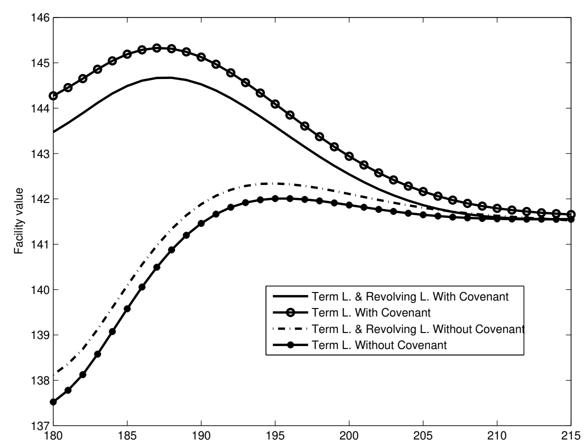


Figure 7.4: Facility (lender) value as a function of asset value (including the revolving amount) at inception date under the base-case specification given in Table 5.1. Portfolio 1 (Term L. plus Revolving L.) and Portfolio 2 (Term L.), with or without covenant.

Finally, Figure 7.7 reports the borrower's reaction function, and specifically, in what

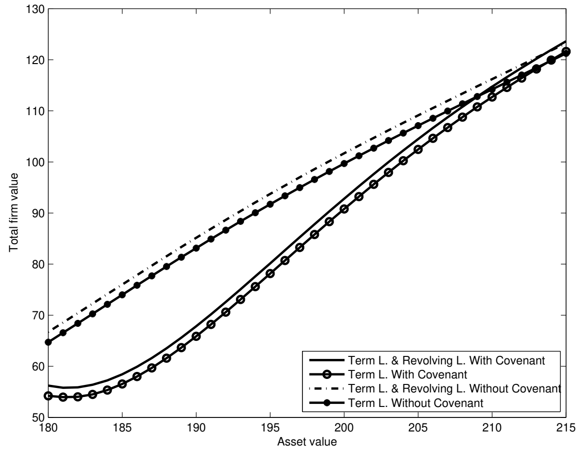


Figure 7.5: Follower value as a function of asset value (including the revolving amount) at inception date under the base-case specification given in Table 5.1. Portfolio 1 (Term L. plus Revolving L.) and Portfolio 2 (Term L.), with or without covenant.

circumstances they adopt an aggressive investment strategy, for Portfolio 1 with a covenant. The result for Portfolio 2 is the same as the reaction function in Chapter 5 (see Figure 5.15). The results show that the follower is using more aggressive investment in the case of revolving compared to the case without revolving.

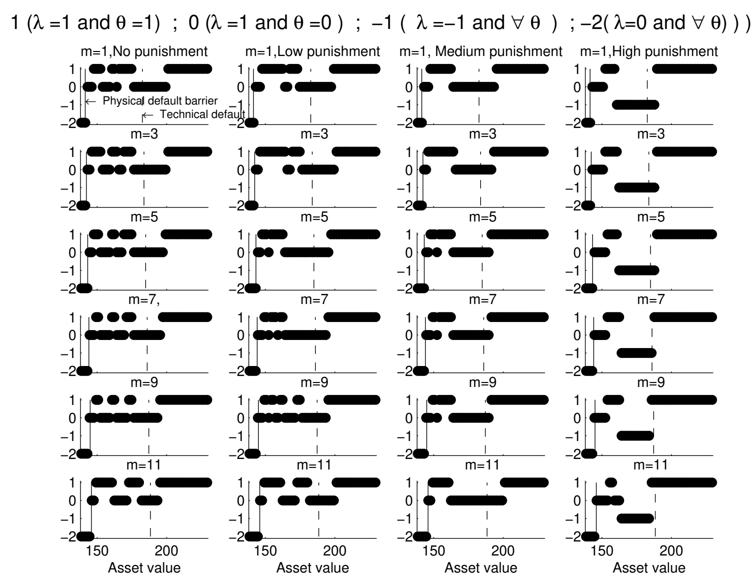


Figure 7.6: The borrower strategy (λ, θ) as a function of the asset value and the lender punishment rate p under the base-case specification given in Table 5.1

7.5.4 Partial take-down

In a third experiment, we enrich the strategy space of the borrower by allowing them to choose the amount invested in the risky activity (partial take-down). This is implemented by adding a decision variable corresponding to the amount drawn from the RCL, taking discrete values in the interval $[0,M]$. Figure 7.7 compares the the loan value obtained under the full and the partial take-down assumptions, along with the value of a loan with partial take-down, but without a covenant.

As expected, the additional flexibility granted to the follower reduces the loan value.

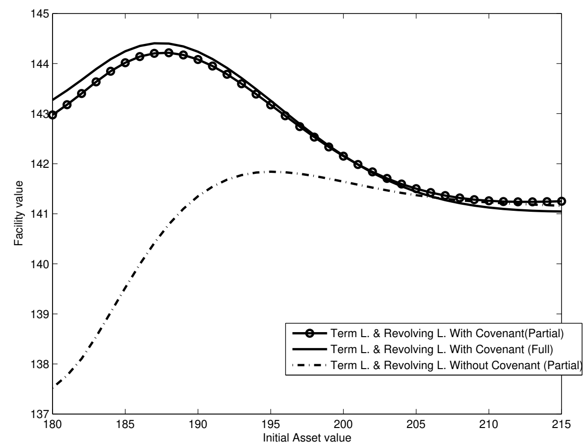


Figure 7.7: Facility value as a function of the asset value (including the revolving amount) at inception under the base-case specification given in Table 5.1

7.5.5 Adding a MAC clause

In the previous experiments, the lender had no control over the way the borrower was using the revolving facility. In practice, revolving credit usually comes with a non-financial covenant that gives the right to the lender to freeze the facility, rejecting any request from the borrower. In this fourth experiment, we are assuming that the lender has the option to use that right in order to limit the borrower from drawing from the revolving facility. In our leader-follower setting, this is modeled by assuming that the lender announces their decision to the borrower to request an immediate pay-back of any drawn amount and temporarily sets the RCL size to zero until the next audit date. This option is only available to the lender when the borrower is in technical default.

Results are presented in Figure 7.8. As expected, the loan value increases with this additional option available to the lender. We find that when the borrower's default risk is sufficiently low, the lender no longer uses their option to freeze the facility, as the use of the RCL by the borrower increases the debt amount and leads to a higher coupon.

Although the previous result is intuitive, it is interesting to observe that the lender does not systematically use their option to freeze the facility when the borrower is in technical default. Figure 7.9 shows that the lender can allow the borrower to draw from the RCL and play aggressively in the technical default region.

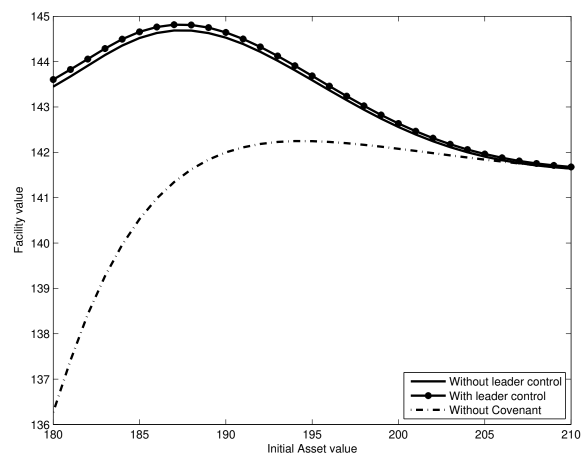


Figure 7.8: Facility (lender) value as a function of the asset value at inception under the base-case specification given in Table 5.1

7.5.6 Revolving credit line with performance pricing

As discussed in Chapter 6, performance pricing is an alternative to interest punishment through a covenant and is very frequently used for revolving credit lines in syndicated deals (Myklebust (2012)). In the literature, Sarkar and Zhang (2016) uses a linear coupon function to evaluate a loan commitment under a performance pricing clause.

In a last experiment, we use the valuation model developed in Chapter 6, changing the follower's action space by adding the possibility to draw from a revolving facility. Some illustrative results are reported in Table 7.1 for various initial asset volatility and corre-

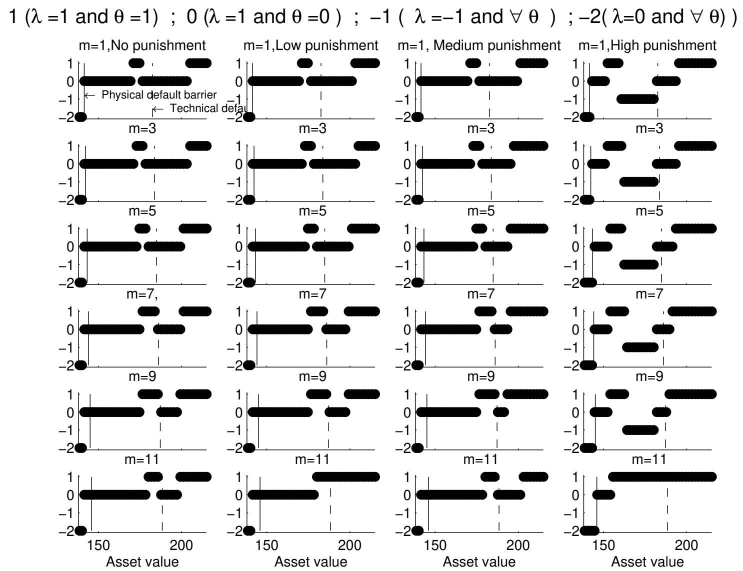


Figure 7.9: Borrower's best response (λ, θ) as a function of the asset value and the lender punishment rate p under the base-case specification given in Table 5.1

sponding performance pricing grid presented in Table 6.4. As in the case of covenant, we note that the value of the portfolio with term loan is higher than the portfolio combining term loan and revolving credit line.

Table 7.1: Revolving credit line with performance pricing

Asset Volatility	**Term L. & Revolving L.**	**Term L.Only**
15%	143.182	145.456
20%	143.093	143.204
25%	142.578	142.759
30%	142.017	142.328

Source: Authors computation using the model developed in Chapter 4

7.6 Conclusion

In this chapter, we adapted the model developed in Chapter 4 to build a valuation model for a facility that includes both a term loan and a revolving credit line. The model allows the borrower to extend the loan amount by drawing from the RCL when investing aggressively. We experimented with various facility management options, including full take-down or partial take-down for the borrower, with and without the right to freeze the facility for the leader.

Our results show that the presence of revolving credit (as compared to a full term loan) adds value for the lender for borrowers with relatively low leverage. For high-leverage borrowers, adding a credit extension negatively impacts the overall package value. When given the right to freeze the facility, the results show that the lender can still optimally allow the borrower to draw from it, even in the technical default region.

On the borrower's side, our experiments show that the follower with access to a RCL is more frequently adopting an aggressive investment strategy, as compared to the case of a facility with a full term loan, where there is no possibility of adjusting the credit level.

Conclusion

This thesis proposes three essays on the valuation of loans that include restrictive clauses, as is usually the case in syndicated loans. All three essays are based on a dynamic-stochastic game interpretation of the interactions between the syndicate (lender) and the borrower. We address the valuation of loans including a financial covenant, performance pricing as an alternative, and finally loans paired with a revolving credit line.

Chapter 4 proposes the dynamic stochastic game basic model that is used to replicate the successive changes of the contract terms through covenant monitoring. The model accounts for several practical observations of the behavior in the syndicated loan market. This includes the possibility of the borrower adjusting their investment strategy (aggressive or conservative) or refinancing the loan. It also accounts for the lender's right to punish any breach of the covenant by increasing the interest rate or increasing the default barrier, which provides a floor value to the loan. To the best of our knowledge, this is the first valuation approach to include these optional rights of the lender and borrower. Chapter 5 provides illustrative examples in the form of numerical experiments using the stochastic game model. We estimate the value of syndicated term-loan contracts with a covenant and analyze the impact of the covenant parameters in the spirit of covenant design. The findings from our numerical implementation support some empirical observations. The presence of the covenant, although reducing the initial pricing spread improves the contract value in some states of high default risk. Particularly, the possibility of choosing a dynamic default barrier and punishing the covenant violation can protect the lender against the adverse effects of the borrower's asset volatility and a possible deterioration of the borrower's creditworthiness after the contract's inception.

In Chapter 6, we introduce a new valuation model for a syndicated loan with a performance pricing clause. The model is based on a dynamic stochastic game to account for the borrower's successive reactions to the changes in their credit cost through four possible actions. We use our model to value a selected list of contracts signed between 1995 and 2009 and analyze their efficiency compared to alternative pricing using financial covenants. Our results suggest that performance pricing could result in a lower market value compared to their equivalent with a covenant, and that the decision to use covenant or performance pricing should depend on some of the loan's parameters (e.g. initial leverage, asset volatility, etc.) As an additional contribution, we determine an optimal performance pricing grid using a stochastic game model where the leader uses an open-loop strategy, while the follower uses a feedback strategy.

In Chapter 7, we adapt the model developed in Chapter 4 to value a syndicated facility that includes a revolving credit line. More specifically, we assume that the borrower is drawing from the revolving facility whenever he decides to play aggressively. We use this model to compare various settings in numerical experiments. Particularly, we investigate the value added, for the lender and the borrower, when adding a revolving facility in a loan package. We find that this value can be positive for both players, depending on the loan characteristics.

In addition to the possible extensions presented in Section 5.7, the model can be used in the valuation and pricing of sustainability-linked loans (SLLs). According to recent studies (Aleszczyk, Loumioti, and Serafeim (2022)), the issuance of SLLs has increased by 200% between 2017 and 2021. SLLs are performance-pricing loans where the performance indicator is related to the borrower's environmental, social, or governance (ESG) performance. The approaches developed in Chapter 6 can be used to estimate the market value of SLLs by replacing the financial performance indicator (e.g., the leverage) with ESG-related KPIs (e.g. CO2 emission, gender parity in board members or executive position, energy consumption, etc.). The model developed in Chapter 4 can be used to monitor ESG type of covenants, which are also gaining popularity in the SLLs market. The main challenge will be to build a model that can replicate the dynamic of the ESG performance indicators. This extension is important is two-fold: First, with the growing size of the market, pricing efficiency has become an important debate in the industry. By how much the spread should change for one percentage point change in the CO2 emission intensity, one more woman at executive positions, or a 1% decrease in energy consumption? In other words, what is the fair risk compensating rate? Secondly, the effectiveness of the pricing in providing an incentive to the borrowers to improve their sustainability outcomes has been widely questioned (LSTA, 2022). Adapting the model developed in Chapter 4 and 6 can help in gauging the value of SLLs and deriving their fair pricing structure.

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