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Essays on Omnichannel Fulfillment Strategies

par
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Résumé

Par rapport au commerce traditionnel, le commerce omnicanal offre aux clients la commodité d'interagir avec les produits via plusieurs canaux. Cela leur permet de débuter sans heurt leur parcours d'achat sur un canal et de le terminer facilement sur un autre. Les détaillants omnicanals offrent aux clients une variété d'options, y compris la livraison directe en ligne, la livraison en magasin (STS), l'expédition depuis les magasins (SFS), l'achat en ligne avec retrait en magasin (BOPS) et le retrait depuis un emplacement tiers. Dans cette thèse, qui se divise en trois essais, nous cherchons à explorer les défis stratégiques et opérationnels que les détaillants rencontrent lors de la mise en œuvre de stratégies de réalisation omnicanal. En utilisant des modèles stylisés d'entreprises opérant à la fois en ligne et hors ligne, nous examinerons comment le commerce omnicanal peut être mis en œuvre efficacement.

Le premier essai offre des perspectives intéressantes aux détaillants envisageant d'intégrer des services de retrait en magasin. Il étudie l'impact des différentes stratégies de retrait sur le comportement des clients et la gestion des stocks, et identifie les approches optimales basées sur la structure des coûts du détaillant. L'essai utilise un modèle pour représenter les caractéristiques clés des BOPS et STS dans le commerce omnicanal, afin de déterminer les meilleures stratégies de retrait en magasin pour les entreprises, en tenant compte de la segmentation de la demande et de la structure des coûts.

Nos conclusions suggèrent que le BOPS est plus favorable pour les détaillants

avec des marges bénéficiaires élevées en magasin, indiquant des coûts d'exploitation et de détention en magasin faibles. BOPS peut également être plus rentable pour les détaillants dans les villes à faible densité où la réalisation de la livraison directe est coûteuse. D'autre part, le STS peut être plus bénéfique pour les détaillants avec des marges bénéficiaires faibles en magasin et des coûts d'exploitation élevés en magasin, car il permet des allocations d'inventaire plus petites pour les magasins physiques, réduisant ainsi les coûts de détention en magasin. STS peut également être plus rentable pour les détaillants dans les villes à forte densité où la réalisation de la livraison directe est moins coûteuse. Mettre en œuvre simultanément les deux stratégies est uniquement recommandé pour les détaillants avec des coûts d'exploitation modérés en magasin. Les conclusions de la recherche fournissent des perspectives sur les facteurs qui influencent la sélection des stratégies de retrait en magasin et peuvent aider les détaillants à optimiser leurs stratégies de réalisation omnicanale pour améliorer leur rentabilité.

Le second essai examine le problème de la conception d'une politique d'attribution de crédit de vente appropriée dans le commerce omnicanal. Nous cherchons à déterminer les quantités de commande optimales pour chaque canal et à identifier les destinataires du crédit de vente les plus méritants. Nous utilisons une méthodologie de théorie des jeux non coopérative pour analyser quatre scénarios distincts et établir des quantités de commande d'équilibre pour y parvenir. En comparant les stratégies et les résultats obtenus, nous offrons des perspectives précieuses sur les meilleures pratiques pour chaque stratégie de réalisation, en tenant compte des préférences d'achat des clients.

Notre recherche souligne que l'allocation du profit des ventes inter-canaux est un facteur critique qui détermine les avantages de la mise en œuvre des stratégies omnicanal. De plus, la proportion de clients visitant les magasins sur le marché est cruciale pour déterminer l'allocation du profit des ventes inter-canaux et si un détaillant bénéficiera des stratégies omnicanales. Nous démontrons que lorsque le marché comprend un mélange égal de clients visitant les magasins et de clients préférant la livraison directe, il est plus avantageux pour l'entreprise d'allouer le crédit de vente également ou d'utiliser la solution d'arbitrage de Nash.

Le troisième essai explore si un fournisseur devrait investir dans des services omnicanals pour soutenir un détaillant avec l'investissement dans l'amélioration des opérations pour offrir des services omnicanals. Nous analysons une chaîne d'approvisionnement impliquant un fabricant leader et un détaillant hors ligne indépendant en utilisant un modèle de jeu de Stackelberg. Notre étude examine le soutien du fabricant et son impact sur les stratégies d'équilibre et les profits, identifiant les conditions qui permettent aux fournisseurs et aux détaillants de bénéficier des services de retrait en magasin. Nous constatons que la mise en œuvre de services de retrait en magasin peut bénéficier à la fois aux fabricants et aux détaillants, mais le niveau d'investissement dans ces services joue un rôle crucial dans la détermination de leur rentabilité. Les fabricants et les détaillants devraient collaborer pour établir des services de retrait en magasin réussis, et en identifiant les facteurs clés qui conduisent au succès de ces services, ils peuvent élaborer des stratégies efficaces pour la collaboration.

Mots-clés

Omnicanal, Opérations de vente au détail, Théorie des jeux, Service de retrait en magasin, Comportement du consommateur, Gestion des canaux, Allocation de crédit de vente, Coopération

Méthodes de recherche

Optimisation; Théorie des jeux; Analyse numérique

Abstract

Compared to traditional retail, omnichannel retail provides customers with the convenience of interacting with products through multiple channels. This allows them to seamlessly begin their purchasing journey on one channel and easily complete it on another. Omnichannel retailers offer customers a variety of options, including direct online shipping, shipping to stores (STS), shipping from stores (SFS), buy-online-pickup-in-store (BOPS), and pick-up from a third-party location. In this thesis, which is divided into three essays, we aim to explore the strategic and operational challenges that retailers face when implementing omnichannel fulfillment strategies. By using stylized models of firms that operate both online and offline, we will examine how omnichannel retail can be effectively executed.

The first essay, offers valuable insights for retailers who are considering incorporating in-store pickup services. It examines the impact of various pickup strategies on customer behavior and inventory allocation, and identifies optimal approaches based on the retailer's cost structure. The essay employs a model to represent the key features of BOPS and STS in omnichannel retailing to determine the best in-store pickup strategies for firms, considering demand segmentation and cost structure. Our findings suggest that BOPS is more favorable for retailers with high in-store profit margins, indicating low store operating costs and holding costs. BOPS may also be more cost-effective for retailers in low-density cities where direct delivery fulfillment is expensive. On the other hand, STS may be

more beneficial for retailers with low in-store profit margins and high store operating costs because it allows for smaller inventory allocations to brick-and-mortar stores, reducing in-store holding costs. STS may also be more cost-effective for retailers in high-density cities where direct delivery fulfillment is less expensive. Implementing both strategies simultaneously is only recommended for retailers with moderate store operating costs. The research findings provide insights into the factors that influence the selection of in-store pickup strategies and can assist retailers optimize their omnichannel fulfillment strategies to improve profitability.

The second essay examines the problem of designing a proper sales-credit allocation policy in omnichannel retailing. We aim to determine the optimal order quantities for each channel and identify the most deserving sales credit recipients. We employ a noncooperative game theory methodology to analyze four distinct scenarios and establish equilibrium order quantities to achieve this. By comparing the resulting strategies and outcomes, we offer valuable insights into the best practices for each fulfillment strategy, considering customers' shopping preferences. Our research highlights that cross-channel sales profit allocation is a critical factor that determines the advantages of implementing omnichannel strategies. Furthermore, the market's proportion of store-visiting customers is crucial in determining the allocation of cross-channel sales profit and whether a retailer will benefit from omnichannel strategies. We demonstrate that when the market comprises an equal mix of store-visiting and direct-shipping customers, it is more advantageous for the firm to allocate sales credit equally or use the Nash bargaining solution.

The third essay explores whether a supplier should invest in omnichannel services to support a retailer with the investment in upgrading operations to offer omnichannel services. We analyze a supply chain that involves a leading manufacturer and an independent offline retailer utilizing a Stackelberg game model. Our study examines the manufacturer's support and its impact on equi-

librium strategies and profits, identifying the conditions that allow suppliers and retailers to benefit from in-store pickup services. We find that implementing in-store pickup services can benefit both manufacturers and retailers, but the level of investment in these services plays a crucial role in determining their profitability. Manufacturers and retailers should collaborate to establish successful in-store pickup services, and by identifying the key factors that drive the success of these services, they can develop effective strategies for collaboration.

Keywords

Omnichannel, Retail operations, Game theory, In-store pickup service, Consumer behavior, Channel management, Sales credit allocation, Cooperation

Research methods

Mathematical optimization; Game theory; Numerical analysis

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List of acronyms

BM Brick and mortar

STS Ship-to-store

SFS Ship-from-store

BOPS Buy-online-pickup-in-store

DC Distribution center

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Preface

This thesis is composed of three essays which are listed below:

- Mahboob Ghodsi, M., Gümüs, M., and Ertekin, N. BOPS, STS, or both: How should omnichannel retailers choose an in-store pickup fulfillment model? *Manuscript in preparation to be submitted.*

- Mahboob Ghodsi, M. and Zaccour, G. Omnichannel fulfillment strategies and sales credit allocation. *Manuscript under second round of review in EJOR.*

- Mahboob Ghodsi, M. and Zaccour, G. Should suppliers support retailer's omnichannel investments? *Manuscript has been submitted.*

General Introduction

A retailer operating in a multi-channel environment discovers the following statistics while trying to attract customers and retain them:

- Retailers that provide a seamless omnichannel experience see a 30% higher lifetime customer value than those that don't (Deloitte, 2018).
- Nearly 80% of customers prefer omnichannel strategies due to the seamless communication experience (HBR, 2017).
- Retailers with an omnichannel strategy experience a 15-35% increase in average transaction value compared to retailers with a single channel strategy (JDA, 2016).
- Retailers that offer in-store pickup services saw a 48% increase in online sales in 2020, compared to the previous year (NRF, 2021).
- Businesses that adopt omnichannel strategies see 91% higher year-over-year customer retention rates compared to businesses that don't (Margalit, 2020).
- 87% of customers expect retailers to provide a seamless omnichannel experience, and 70% of customers said they would switch brands if a retailer offered a better omnichannel experience (Salesforce, 2020).
- 44% of retailers currently offer in-store pickup services, while 56% plan to implement it in the next three years (OrderDynamics, 2019).

The statistics highlight an escalating trend in adopting omnichannel strategies among retailers. This trend has been driven by the need to meet consumers' evolving preferences who increasingly seek seamless integration between physical stores and digital platforms. The retail industry is rapidly evolving, and traditional product selling methods are no longer sufficient to satisfy customers' needs. Instead, retailers are turning to omnichannel sales methods. In omnichannel retailing, brick-and-mortar stores, e-commerce platforms, and mobile applications are integrated to offer customers a seamless and consistent shopping experience.

The evolution of omnichannel retailing can be traced back to traditional and multichannel frameworks. Retailing traditionally relies on brick-and-mortar stores and in-store interactions, with factors like store layout and product availability affecting the customer experience. The rise of the internet and e-commerce led to multichannel retailing, which utilizes multiple independent channels, such as physical stores and online platforms. This approach, however, often results in inconsistent shopping experiences across channels due to poor integration. Omnichannel retailing emerged as a response to the limitations of multichannel retailing and is characterized by channel integration, customer journey mapping, and seamless retailing. These elements ensure a consistent, smooth, and cohesive customer experience across all retail channels.

The ongoing transformation of the retail industry, driven by digital advancements and shifting consumer preferences, has given rise to the omnichannel retailing paradigm. As part of this approach, retailers have increasingly adopted in-store pickup fulfillment options to bridge the gap between online and offline shopping experiences, enhancing customer convenience and satisfaction. In-store pickup fulfillment options, such as Buy Online, Pick Up In-Store (BOPS) and Ship-to-Store (STS), have gained popularity in recent years due to their potential to improve customer satisfaction, reduce delivery costs, and drive foot traffic to physical stores. BOPS allows customers to purchase items online and pick them up at a nearby brick-and-mortar store, often within the same day. This method offers the advantage of faster fulfillment, as customers can collect their orders without waiting for delivery, while retailers can leverage their existing inventory and store locations.

STS, on the other hand, involves shipping items ordered online to a designated store location for customer pickup. This approach enables retailers to consolidate their inventory at centralized warehouses, reducing in-store holding costs and offering customers access to a wider range of products. In addition, in-store pickup services offer customers a convenient and efficient way to shop, avoiding shipping fees, getting items quickly, and easily returning products. Another omnichannel fulfillment strategy, Ship-From-Store (SFS), has emerged to prevent stockouts for online orders. Omnichannel retailers utilize their full inventory by shipping orders directly from the distribution center to the customer upon acceptance by the online channel. If the DC cannot fulfill the order due to a stockout, the SFS strategy allows the order to be delivered to the customer from the brick-and-mortar store (Bayram and Cesaret, 2017). While the advantages of embracing omnichannel approaches are numerous, retailers need to approach the adoption of omnichannel strategies with caution. Implementing omnichannel methods can incur significant costs, necessitating investments in technology, supply chain management, and employee training. It is crucial to investigate whether integrating these strategies is consistently beneficial for retailers or if certain circumstances may diminish their effectiveness or even prove detrimental.

Retailers must thoroughly assess the costs and benefits to guarantee an adequate return on investment. Furthermore, omnichannel strategies may pose challenges for inventory management, as retailers must maintain inventory levels across multiple channels, resulting in escalated inventory carrying costs and the possibility of stockouts. Retailers need to make strategic and operational choices to ensure the successful execution of these approaches. Identifying the operating conditions that contribute to effectively implementing and managing omnichannel strategies is also essential. Such insights will empower retailers to make well-informed decisions regarding the development and execution of their omnichannel strategies.

The growing popularity of omnichannel retailing has led to a surge in scholarly interest. Numerous studies explore how retailers can integrate online and offline channels to offer a seamless shopping experience for customers (Bell et al., 2014; Gallino et al., 2017; Aflaki and Swinney, 2021). Research on omnichannel operations management primarily investigates the influence of various fulfillment strategies on customer behavior and the subsequent effects on retailers' operations. Key strategies examined include buy-online-pickup-in-store (BOPS) (Gallino and Moreno, 2014; Cao et al., 2016; Song et al., 2020), ship-to stores (STS) (Ertekin et al., 2022; Akturk et al., 2018), ship-from-store (SFS) (Li, 2020; He et al., 2021), and others, all to understand their impact on customer behavior. Despite the wealth of research, several critical questions remain unanswered. For instance, it is not yet clear which fulfillment strategy is most profitable for retailers under diverse circumstances.

Furthermore, when multiple channels are employed to fulfill an order, the allocation of credit between channels remains a pressing issue. A majority of retailers (55%) currently do not incentivize store employees for fulfilling omnichannel orders (DigitalCommerce, 2016). As sales attribution across various channels can be challenging, determining the appropriate credit allocation is crucial for overall profitability and warrants further investigation. This thesis attempts to fill this gap because these seemingly beneficial fulfillment strategies can undermine the firm's profitability if sales credit is not distributed appropriately among the channels. Additionally, implementing omnichannel fulfillment strategies can be costly for retailers, raising the question of whether suppliers should assist retailers in upgrading their operations to provide omnichannel services to customers. Through three essays, this thesis contributes methodologically and substantively to the literature by offering novel insights into the strategic and operational decisions that retailers face when implementing omnichannel fulfillment strategies.

The first essay examines the impact of in-store pickup services on customer behavior and inventory management, focusing on the two primary fulfillment strategies: buy-online-pickup-in-store (BOPS) and ship-to-store (STS). Both strategies offer the convenience of in-store pickup, but there are significant differences in the fulfillment processes and costs associated with each. While some retailers offer only one strategy, others provide both simultaneously. Previous studies have found that BOPS can lead to decreased online sales, increased in-store traffic, and potential loss of profit margin due to channel cannibalization (Gallino and Moreno, 2014; Cao et al., 2016). Additionally, BOPS has been shown to positively affect offline purchase frequency and online purchase amounts (Song et al., 2020), while posing a threat to competition (Akturk and Ketzenberg, 2022).

Although some research suggests that BOPS can attract new customers and improve store fill rates (Hu et al., 2022), it may not be optimal for products that sell well in physical stores (Gao and Su, 2017), and retailers could benefit from reducing the number of physical stores under BOPS (Gao et al., 2022). Additionally, another part of the stream empirically and analytically investigate STS functionality. Gallino et al. (2017) found that STS increases sales dispersion, while Akturk et al. (2018) observed customer channel switching. Ertekin et al. (2022) explored merchandising strategies and product availability types.

This essay aims to clarify the optimal in-store pickup strategy for retailers by investigating factors affecting profitability and providing valuable insights for informed decision-making. Key research questions include whether it is always better for retailers to offer in-store pickup, the preferred approach under different operating conditions, and the potential benefits of offering both strategies simultaneously. We employ empirical and analytical research to examine the conditions under which one strategy should be prioritized or if both should be implemented simultaneously. We develop a stylized model to represent the fundamental features of BOPS and STS functionality in an omnichannel retail environment.

The impact of in-store pickup strategies on customer behavior is studied, and the results suggest that the retailer's demand is impacted in two ways: market expansion can increase profits, but shifting demand to a less profitable strategy can hurt profits. We establish conditions under which each fulfillment strategy is most beneficial and characterize when each policy is optimal for the retailer. Our findings indicate that the retailer's cost structure determines whether they should implement in-store pickup strategies and, if so, which approach they should pursue. Overall, this essay contributes to the field by helping companies choose their in-store pickup strategy according to their unique circumstances, considering demand fulfillment and inventory allocation between brick-and-mortar stores and distribution centers.

The second essay addresses the challenge of sales attribution in omnichannel retailing, as retailers struggle with determining which channel should receive credit for a sale. The complexity of sales attribution increases when stores fulfill online orders, rendering conventional store performance metrics insufficient. Although numerous retailers have adopted omnichannel strategies, only a small fraction allocate revenues between channels. (ForresterResearch, 2014) reports that merely 16% of retailers allocate revenues between channels, while 31% and 21% attribute revenue solely to online or store channels, respectively. Furthermore, Benes (2019) finds that over 40% of merchants still assign credit to a single touchpoint. Retailers must decide whether to credit the sale to the online channel, where the order originated, or the brick-and-mortar store where it was fulfilled. Inventory management policies are further influenced by the relationship between brick-and-mortar stores and online channels, complicating appropriate credit assessment for omnichannel sales. Accurate credit attribution for sales involving online and offline channels remains challenging, potentially resulting in under-ordering or channel conflicts.

This study employs a game theoretic approach to explore sales-credit allocation policies in omnichannel retailing, addressing key interrelated questions previously unexplored in marketing literature. Building upon the limited attention given to ship-from-store (SFS) and store-to-store (STS) fulfillment options in prior research (Li, 2020; Bayram and Cesaret, 2021), our paper investigates inventory decisions under different sales allocation policies in a decentralized setting, where online and offline channels are managed by separate teams. We examine the challenge of designing an appropriate sales-credit allocation policy in omnichannel retailing and develop an inventory game between the store and the online channel.

We consider four scenarios for fulfilling customer orders: benchmark, STS, SFS, and Hybrid scenarios while evaluating the impact of sales credit allocation policies on retailer profit under various omnichannel strategies. Our analysis accounts for specific omnichannel factors, including asymmetric demands, fulfillment processes, and costs for both store and online channels. Additionally, we address the under-explored issue of revenue allocation between channels after implementing omnichannel strategies, which is crucial for understanding the potential impact of these fulfillment options on a firm's overall profit. Our findings offer valuable insights for companies intending to launch fulfillment services by promoting collaboration rather than competition between channels.

In the third essay, we examine the challenges faced by retailers in balancing costs for omnichannel retailing and competitiveness. As an example, Walmart and Loblaw in Canada have introduced new fees for suppliers to fund operational upgrades for handling pickups, with Walmart imposing a 1.25% "infrastructure fee" on the cost of goods sold and an additional 5% on online items to support its $3.5 billion investment in omnichannel strategies (WalmartCanada, 2020; FinancialPost, 2020). While large retailers can enforce such fees on suppliers, smaller retailers may struggle due to their limited authority over suppliers. Since retailer profitability partially relies on funding from suppliers or manufacturers, we examine whether suppliers should bear some costs related to upgrading in-store and digital operations.

This essay contributes to the literature by exploring the cooperative efforts between retailers and manufacturers in implementing omnichannel strategies. Utilizing a game-theoretic modelling approach, we demonstrate that collaboration in boosting operations to enable in-store pickup services can lead to manufacturers incentivizing offline retailers to maximize their investment in implementing omnichannel strategies. The results reveal that retailers benefit from BOPS sales if manufacturers contribute to the investment costs. Still, manufacturers are only willing to offer support if the investment level exceeds a specific threshold. Factors influencing this willingness include last-mile delivery cost, product type, and cross-selling potential. Decision-makers must understand these factors for successful in-store pickup implementation. Retailers should carefully consider the fees they charge for in-store pickup services, as increasing fees may deter manufacturers from compensating them for their investments, potentially harming profitability.

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Chapter 1

BOPS, STS, or Both: How should omnichannel retailers choose an in-store pickup fulfillment model?

Abstract

In-store pickup services have become increasingly popular among omnichannel retailers, offering customers the convenience of purchasing online and picking up in person. Typically, retailers use three approaches to offer such services: buy-online-pickup-in-store (BOPS), ship-to-store (STS), or both. Each strategy has advantages and disadvantages, which retailers should consider when implementing in-store pickup services. In this research, we develop a stylized model representing the three strategies' fundamental features. We find that in-store pickup services may either benefit or hurt retailers depending on their operating costs. STS can incentivize customers to switch from online shopping to in-store pickup, prompting retailers to allocate more inventory to their distribution center (DC) warehouses. However, this approach may not benefit retailers with low store operating costs, as smaller inventory allocations to the brick-and-mortar store can

shift sales towards online orders fulfilled from the distribution center, potentially hurting the retailer's profitability.

Conversely, BOPS is not advantageous for retailers with high store operating costs. By comparing BOPS and STS, we identified the conditions under which retailers should choose one strategy over the other. Our research indicates that BOPS is more beneficial for retailers with higher in-store profit margins, implying lower store operating and holding costs. BOPS may also be more cost-effective for retailers in low-density cities where direct delivery fulfillment is more expensive. In contrast, STS may be more advantageous for retailers with lower in-store profit margins and higher store operating costs, as it allows for smaller inventory allocations to physical stores, reducing in-store holding costs. STS may also be more cost-effective for retailers in high-density cities where direct delivery fulfillment is less expensive. The research findings provide valuable insights into the factors that influence the selection of in-store pickup strategies, enabling retailers to optimize their omnichannel fulfillment strategies and improve their profitability.

1.1 Introduction

In today's ever-changing retail industry, it's crucial for businesses to meet the diverse needs of their customers. A significant aspect of this is providing various order delivery options, such as in-store, direct delivery, or in-store pickup (Bell et al., 2018). In-store pickup has become a popular choice among consumers, with 67% preferring buy-online-pickup-in-store (BOPS) and 36% selecting ship-to-store (STS), as per a survey conducted by the National Retail Federation (NRF, 2019). A seamless omnichannel experience is desired by nearly 80% of shoppers, and a staggering 87% expect retailers to offer it (Clerk.io, 2021). Not only can offering in-store pickup exceed customer expectations, but it can also benefit multichannel retailers. Those with an omnichannel approach typically see a 15-35%

increase in average transaction value compared to those with a single channel strategy (Morganti et al., 2014). Additionally, in-store pickup can prove to be a cost-effective option as store inventory can be utilized, which lowers last-mile delivery expenses.

As e-commerce continues to grow, and consumer preferences evolve, in-store pickup services are expected to play a crucial role in retailers' omnichannel strategies. Insider Intelligence predicts that North American click-and-collect sales will reach 140.96 billion $USD by 2024, with click-and-collect buyers expected to rise from 143.8 billion $USD in 2020 to over 160 billion $USD in 2024 (BusinessInsider, 2021). The COVID-19 pandemic has further fueled the demand for in-store pickup, as customers look for faster, safer, and more convenient delivery options (Toneguzzi, 2021). Digital Commerce 360's survey conducted in August 2020 revealed that 43.7% of the top 500 retailers with physical stores offer pickup services, up from 6.9% before the pandemic (Berthene, 2020).

Retailers can fulfill in-store pickup orders using two different methods: Buy-online-pickup-in-Store (BOPS) and ship-to-store (STS). BOPS enables customers to purchase items online and collect them at a brick-and-mortar (BM) store, where store associates have already prepared the order for pickup. This method leverages in-store inventory to satisfy customer demand and offers real-time product availability information at the store level (Bell et al., 2014). Conversely, STS is an alternative in-store pickup strategy in which a customer's online order is shipped from a distribution center to the designated store for collection (Gallino et al., 2017). In this approach, items are generally shipped from the DC, even if they are already available at the store. As a result, STS relies on centralized fulfillment rather than in-store inventory. Customers complete their purchase transactions online and await notifications regarding the arrival of their purchased items at the local store, which are typically delivered without additional shipping fees.

Although both BOPS and STS are in-store pickup services, there are significant differences between the two when it comes to fulfillment points and order processing time. BOPS allows customers to order online and pick up their items on the same day, making it a faster option. In contrast, STS orders can only be picked up after they have been shipped from the central warehouse to the local BM store, which usually takes several days. By using a hybrid strategy that combines both STS and BOPS, retailers can fulfill in-store pickup orders at the store when local inventory is available. If inventory is not available, they can send stock from the distribution center to the store for pickup. Among retailers offering in-store pickup services, we observe three distinct groups. The first group provides only BOPS, such as Lowe's, Macy's, and Eddie Bauer. The second group offers both BOPS and STS simultaneously, such as Target, BestBuy, and The Home Depot. The third group provides only STS, such as Decathlon, Forever 21, and Lane Bryant. Each of these fulfillment strategies offers different operating benefits and costs, and it remains unclear from the existing literature how multichannel retailers planning to offer in-store pickup services should choose among them.

Retailers must consider several factors when deciding between BOPS and STS as omnichannel fulfillment strategies. For example, if a retailer chooses BOPS and uses store inventory for both online and in-store purchases, tighter inventory integration between online and offline channels is needed to avoid stockouts for walk-in customers. Additionally, shipping costs should be considered, as products purchased through an STS must be efficiently delivered to the store from the distribution center to take advantage of economies of scale in middle-mile logistics. If the retailer does not have daily delivery trucks delivering items to BM stores, offering STS may result in additional costs. Therefore, retailers must consider their inventory management, delivery logistics, and other operational factors when selecting between BOPS and STS.

Retailers must carefully consider their pickup strategy to efficiently manage inventory, allocate between the distribution center and the store, and optimize order processing and customer experience. By assessing the costs and benefits of in-store pickup services and identifying the most effective approach, retailers can enhance efficiency and improve the customer experience. This paper examines whether in-store pickup services are always advantageous for retailers and compares the three strategies- BOPS, STS, or both- to determine which is most efficient. The following research questions are addressed:

1. Is offering in-store pickup services always beneficial for retailers?
2. What are the operating conditions that influence the preference for BOPS or STS?
3. When is it beneficial for retailers to offer both services, and when should they avoid them altogether?

Our research aims to guide retailers in optimizing their implementation of in-store pickup services and improving overall operational efficiency. To achieve this, we develop a stylized model that captures the essential features of BOPS and STS in an omnichannel retail environment. We consider a retailer that sells products through two channels: online and BM stores. Customers strategically make channel choices to maximize their utility when an in-store pickup option is offered. We classify customers into two types: walk-in customers who only consider purchasing from the BM store and omnichannel customers who prefer online fulfillment options. Our primary objective is to investigate the impact of in-store pickup strategies on customer behavior. We find that in-store pickup services can expand a retailer's market coverage by encouraging non-shoppers to purchase through BOPS or STS. Additionally, some existing customers may switch from direct delivery to in-store pickup, affecting the retailer's demand in two ways. While market expansion can increase profits, shifting demand to a less profitable strategy can negatively impact a retailer's profits.

Our second objective is to evaluate the advantages of implementing in-store pickup options and determine which strategy, STS, BOPS or both, is most beneficial. Initially, we establish a benchmark model without any omnichannel strategy. Then, we compare each in-store pickup strategy with the benchmark model to determine which fulfillment strategy is most advantageous under specific conditions. Our analysis indicates that the retailer's cost structure significantly influences the decision of whether to implement in-store pickup strategies. Specifically, store operating costs are crucial in determining whether retailers will benefit from in-store pickup and which policy they should adopt. We found that the STS fulfillment strategy is not profitable for retailers with low in-store operating costs. Although STS reduces direct delivery shipping costs, smaller inventory allocations to BM stores can lead to unsatisfied walk-in customers and lost sales. Moreover, retailers with low in-store operating costs may be negatively impacted by implementing STS because they will shift sales to a less profitable strategy, and the expanded sales may not surpass this loss.

We find that BOPS may not always be the optimal strategy for retailers, especially if the local BM store has high operating costs. While BOPS encourages retailers to allocate more inventory to BM stores, overstocking can occur, leading to decreased profits. Furthermore, the shift in customer behavior from direct delivery to in-store pickup can hurt retailers' profits if in-store sales profit margins are already low. We compare BOPS and STS and determine the optimal strategy for retailers under varying operating costs. We also investigate the advantages of implementing both BOPS and STS simultaneously and find that offering both options is more profitable for retailers when store operating costs are moderate.

Our research also investigates the impact of service region density, holding cost, and lost sale cost on the selection of in-store pickup strategies. By numerical analysis, we determine that the choice between BOPS and STS depends on several factors. BOPS is more cost-effective for retailers in low-density cities where direct delivery fulfillment is expensive. STS may be more beneficial for retailers in high-density cities where direct delivery fulfillment is less expensive and for those facing higher store operating costs. Operating costs for BM stores can vary depending on various factors, including store size, location, and type of products sold (HQ, 2021). To support our findings, we conducted a descriptive analysis on a comprehensive data set from the 578 largest retailers in the United States. Our results highlight the product type's significance in determining the optimal in-store pickup strategy.

In support of the analytical results, our analysis reveals that retailers of large, heavy items do not prefer STS for in-store pickups. This is because such items are not easily transported to a store for pickup, and customers may have difficulty transporting them, making STS less convenient. Instead, these retailers typically offer hybrid or direct delivery as their primary fulfillment strategies. Jewelry retailers prefer the STS fulfillment strategy due to the high value and small size of their products. Retailers dealing with smaller, less expensive products like clothing, footwear, and accessories, have low operating costs, and prefer BOPS as their primary in-store pickup strategy. Department stores, meanwhile, tend to offer both BOPS and STS as part of their omnichannel strategy. Retailers need to consider product characteristics and operating costs when deciding on the most efficient in-store pickup strategy.

Implementing in-store pickup services requires a significant investment, as it involves integrating physical stores and online marketplaces. Financial performance is also a critical factor in determining the optimal in-store pickup strategy. Larger, financially stable retailers with higher sales volumes may adopt both STS and BOPS strategies as they can handle the associated costs and complexity. However, implementing both strategies may not be feasible for smaller retailers with limited resources and sales volumes.

Our findings provide valuable insights for retailers on optimizing their in-store pickup strategies to improve operational efficiency and enhance the customer experience. The rest of the paper is organized as follows. In Section 1.2, we review the related literature. In Section 1.3, we formalize our model, analyze the model, and derive several analytical and numerical results. In Section 1.4. we conclude by discussing the managerial implications of our results.

1.2 Literature

Academic literature has increasingly focused on omnichannel retailing as it grows in practice. In a series of papers, researchers discuss how retailers can integrate online and offline channels to give customers a seamless shopping experience (Bell et al., 2014; Gallino et al., 2017; Aflaki and Swinney, 2021). This paper studies the management of in-store pickup services as one of the omnichannel fulfillment strategies and their profitability conditions. Our primary focus is on omnichannel management, specifically on two in-store pickup services: BOPS and STS.

One part of this stream focuses only on BOPS fulfilment and investigates the impact of BOPS on customers' choice behavior. For example, empirical studies such as Gallino and Moreno (2014) analyze the impact of BOPS on a retailer's online and offline sales and find that BOPS results in a reduction in online sales and an increase in-store traffic and sales. They explain these findings due to channel switching incidents after BOPS implementation. Online customers show channel switching behaviour after BOPS implementation because they obtain information about product availability at stores during their search process. Cao et al. (2016) demonstrate that implementing BOPS could cannibalize the existing offline channel, resulting in a loss of profit margin due to BOPS operations. Using customer usage data, Song et al. (2020) quantify the impact of BOPS usage on subsequent customer purchase behaviors and find that BOPS positively affects offline purchase frequency and online purchase amounts. Akturk and Ketzenberg (2022) evaluate the competitive impact of BOPS. They argue that omnichannel services pose a direct and immediate threat to competition. A competitor's launch of a BOPS service adversely impacts both online and store sales at a focal retailer.

All of the above studies on BOPS assume that the decision has already been made to implement such a strategy. Consequently, these studies focus on examining the impact of BOPS on customer behaviour and retailers' product fulfillment. This literature mainly focuses on the operational implications of BOPS implementation for businesses. Only a limited number of studies in the literature address the profitability of omnichannel strategies. Among them, Gao and Su (2017a) find that BOPS can attract new customers, but it may not be optimal for products that already sell well in physical stores. Hu et al. (2022) study the "demand pooling" and "demand depooling" effects brought by the BOPS strategy on store operations and show that retailers can take advantage of the additional demand induced by the BOPS to improve the fill rate of their stores. Lastly, the findings of Gao et al. (2022) suggest that retailers may benefit from reducing the number of physical stores they maintain under BOPS.

Another part of the stream empirically and analytically investigate STS functionality. Among them Gallino et al. (2017) find that launching STS increases a retailer's overall sales dispersion and may shift sales from high-selling products to low-selling products. Additionally, Akturk et al. (2018) shows the channel switching behavior of customers from the online to the BM store when the retailer launches STS. Ertekin et al. (2022) is the only study that empirically and analytically examine channel merchandising strategies for STS implementation. They recognize two types of availability for products (i) online-exclusive and (ii) hybrid, products available both online and offline. They show that STS increases sales more for products that are available only online than for hybrid products. Even though STS may increase sales, it may also result in a loss of customers if they are exposed to alternatives at nearby competitors. They suggest that when implementing STS, retailers should make easy-to-substitute products available online only while difficult-to-substitute products available in stores and online both.

We aim to contribute to the existing literature by examining the conditions under which the BOPS strategy is beneficial for retailers compared to another in-store pickup strategy known as STS. To this end, we develop a benchmark model that does not implement an omnichannel strategy. By comparing each fulfillment strategy with this benchmark model, we can identify scenarios in which each fulfillment approach is profitable to the retailer. It remains unclear which is the best in-store pickup strategy for retailers, since little guidance is provided in the existing literature. While studies have examined each strategy in isolation, there is a lack of research on the conditions that determine when one strategy should be prioritized over the other or if both should be implemented simultaneously. This study aims to address this gap in the literature by conducting empirical and analytical investigations into the factors that affect the profitability of different in-store pickup strategies. Our findings offer valuable insights for retailers considering the implementation of in-store pickup, aiding them in making informed decisions on demand fulfillment and inventory allocation between BM stores and distribution centers. In summary, our contribution to the field lies in assisting companies in selecting the most appropriate in-store pickup strategy based on their unique circumstances.

1.3 Theoretical Model

In this section, we create develops a stylized model to characterize key features of in-store pickup. The following subsections describe our modelling framework, initially introducing a No Store-Pickup (NSP) model that forms the basis of comparison without the inclusion of in-store pickup services. Following this, we examine three in-store pickup fulfillment strategies STS, BOPS, and Hybrid which expand upon the basic model.

1.3.1 Modeling Framework

In this study, we model an omnichannel retailer that sells one product at price p through her BM store and online store. Consistent with previous research on in-store pickup services (Cao et al., 2016; Hu et al., 2022), we assume the retailer serves to two customer types: (i) walk-in customers who comprise $(1-\lambda)$ of the customer base and only purchase in the BM store, and (ii) omnichannel customers, who make up λ of the customers and prefer to purchase online. Omnichannel customers face uncertainty regarding product fit and resolve this uncertainty after the purchase. We assume that the customer's valuation for the product can be either high ($\bar{v}$) or low ($\underline{v}$) with probabilities of α and $(1-\alpha)$, respectively. Customers who receive a high valuation keep the product, whereas those who experience a misfit receive a low valuation and return the product for a refund. Without loss of generality, we normalize $\underline{v}$ to 0. The BM store allows customers to physically inspect the product before purchasing, and thus, we assume that product-fit related returns only occur online, not in BM stores. We model customers' probabilities of receiving a high valuation as uniformly distributed across $\alpha \sim U[0,1]$, reflecting their heterogeneity in the product's perceived value.

Omnichannel customers select a fulfillment option that maximizes their utility. To facilitate the exposition, we assume exogenous walk-in demand (Gao and Su, 2017b). The retailer possesses an inventory of K products that are distributed between the local BM store and the distribution center. In the BOPS strategy, the BM store's inventory is utilized to fulfill both in-store pickup and walk-in orders. Under the STS strategy, the DC's inventory is employed to fulfill direct delivery and in-store pickup orders. This paper does not posit a fulfillment strategy priority. Instead, we implement the proportional rationing rule (Maskin, 1986; Anderson and Bao, 2010)[1].

In this study we analytically investigates the impact of implementing BOPS and STS on the optimal allocation of inventory and retailer's profit, taking into account the retailer's omnichannel strategy and customer preferences. Prior to random demand being realized, the retailer determines the inventory levels to

[1]The proportional rationing rule assumes that customers arrive randomly (i.e., the arrival process is independent of willingness-to-pay) and are served on a first-come, first-served basis.

Table 1.1: Summary of Notation

Notation related to the customer	
λ	Proportion of the omnichannel customers in the market
$\alpha \sim U[0,1]$	Probability that product valuation is high
$\bar{v}$	Valuation for the product
h_o	Hassle cost of an online order (e.g. browsing the web and online payment)
h_p	Hassle cost of store visit for in-store pickup
t_p	Hassle cost of waiting for store-pickup (we normalize t_p to 0 for BOPS)
h_s	Hassle cost of STS purchase $= h_p + t_p$
h_b	Hassle cost of BOPS purchase, $h_b = h_p$
h_d	Hassle cost of using direct delivery (e.g., shipping cost and delivery time)
r_d	Return cost if product does not fit after online purchase
ζ	Customer's a-priory belief about in-store product availability at the time of BOPS order
p	Price of the product
Notation related to the retailer	
c_s	Handling cost for each in-store fulfillment (e.g. procurement costs, storage costs, transportation costs, store operations cost)
c_d	Handling cost for each direct-delivery order (e.g., direct shipping cost)
c_h	Holding cost per unit
c_g	Cost of lost walk-in sales
c_p	Handling cost for fulfilling each STS order (e.g., cost of shipping to the pickup location)

maintain at the BM store and the distribution center. The model accounts for potential in-store holding and lost sales due to random demand. Specifically, the cost of lost sales is only considered for walk-in customers, while lost online sale costs are normalized to zero. In view of lower rental and storage fees, without loss of generality, the holding cost at the DC is assumed to be negligible and therefore set equal to zero. The retailer forms a priori belief to determine the proportion of consumers likely use each channel, after which inventory allocation between the channels is optimized. Our analysis employs the rational expectations framework, under which all participants are considered to act strategically. Under the rational expectations equilibrium, actual outcomes must align with stated beliefs. We demonstrate that both stated expectations and purchase decisions are consistent with the rational expectations equilibrium. A comprehensive list of notations used in the theoretical model is presented in Table 1.1.

Section 1.3.2 establishes a baseline policy by examining a multichannel retailer that does not offer an in-store pickup service. We subsequently explore an omnichannel retailer that offers STS services in Section 1.3.3 and BOPS services in Section 1.3.4. Finally, we discuss the Hybrid fulfillment scenario in Section 1.3.5, which entails the operation of an omnichannel retailer using both BOPS and STS services.

1.3.2 Baseline Policy: No Store-Pickup (NSP)

In the absence of in-store pickup services, the online retailer operates with a BM store and an online channel that provides direct delivery. This is referred to as the No Store-Pickup (NSP) policy, where ℓ_s represents the proportion of customers who make walk-in purchases, and $D_s \sim U[0, \ell_s M]$ denotes the exogenous demand for walk-in customers. When the BM store is out of stock, walk-in customers leave, leading to a lost sales cost of c_g for the retailer. Customers who opt for direct delivery receive a utility of $U_d = -h_d + \alpha(v-p) - (1-\alpha)r_d$, where the subscript d indicates direct-delivery fulfillment. The hassle cost h_d takes into account online search, shipping expenses, and the customer's waiting time for delivery. In the event of a mismatch, the customer can return the product and incur a loss of r_d. Customers choose to order online with direct delivery when $U_d > 0$. The retailer's belief about the fraction of customers who choose direct delivery is denoted by $\hat{\ell}_d$. Figure 1.1 depicts the market segmentation for omnichannel customers under the No Store-Pickup policy.

Next, we examine the retailer's decision problem. The retailer forms expectations regarding the proportion of each demand segment. At the outset, the retailer assumes that a fraction $\hat{\ell}_d = \lambda[1 - a_{2n}]^+$ of omnichannel customers will opt for direct delivery, resulting in an online demand with direct delivery of

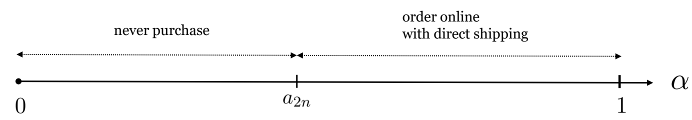


Figure 1.1: Market Segmentation under the NSP Policy

$D_d \sim U[0, \hat{\ell}_d M]$. To ensure the existence of a participatory equilibrium, we assume that the profit margin for online sales is less than that for offline sales, i.e., $p - c_d < p - c_s$. With respect to these beliefs, the retailer allocates inventory capacity K between the BM store and DC. The retailer's expected profit is then expressed as:

$$\begin{aligned}\Pi_n(q) =& p \underbrace{\mathbb{E}\min(D_s, q) - c_s q}_{\text{profit from walk-in sales}} + \underbrace{(p - c_d)\mathbb{E}\min(D_d, K - q)}_{\text{profit from direct delivery}} \\ & - \underbrace{c_h \mathbb{E}[q - D_s]^+}_{\text{store holding cost}} - \underbrace{c_g \mathbb{E}[D_s - q]^+}_{\text{walk-in lost sale cost}} \\ & \text{Subject to } 0 \leq q \leq K\end{aligned} \tag{1.1}$$

In Equation 1.1, the first term of the equation represents the expected profit of the retailer from sales to walk-in customers, while the second term represents the expected profit from omnichannel customers who opt for direct delivery. The last two terms denote the in-store holding and lost sale costs incurred by the retailer due to unsatisfied in-store demand. The optimal order quantity for the BM store is denoted as q_n, which is determined by maximizing the profit function $\Pi_n(q)$. Given the concavity of $\Pi_n(.)$ with respect to q, it can be inferred that a Lagrange multiplier exists. By incorporating a multiplier, denoted as γ, and formulating a Lagrangian function, the constrained maximization problem presented in Equation 1.1 can be solved. (See Appendix for details)

To explore the strategic interaction between the retailer and customers, we adopt the rational expectations equilibrium theory. This theory assumes that the retailer's beliefs about customer behavior align with the actual outcomes observed (Gao and Su, 2017a; Hu et al., 2022). In the No Store-Pickup (NSP) model, the rational expectations equilibrium entails that the retailer's belief about customers' choices is consistent with the realized choices, denoted as $\hat{\ell}_d = \ell_d$. When the retailer expects that omnichannel customers will opt for direct delivery, the optimal stock level for the BM store will be q_n. The proofs for all the results presented in this paper can be found in the Appendix.

Proposition 1. *Under the No Pickup policy and at the RE equilibrium, a fraction $\ell_d = \lambda[1 - a_{2n}]$ of omnichannel customers place orders online with direct delivery, and the retailer allocates $q_n = \frac{(1-\lambda)(\ell_d(c_d+c_g-c_s)M+(p-c_d)K)}{\ell_d(p+c_g+c_h)+(1-\lambda)(p-c_d)} > 0$ and $K - q_n$ to the BM store and the DC, respectively.*

Proposition 1 shows that if the hassle cost of direct delivery is low enough, then omnichannel customer would order the product with direct delivery. In contrast, if the hassle cost of direct delivery is high, no omnichannel customer purchase the product and leave the market. This proposition is supported by a comparison of retailer profits across diverse segments, as depicted in Figure 1.1.

1.3.3 Ship To Store (STS)

We proceed by examining the scenario in which the STS fulfillment strategy is implemented. In addition to the purchase options available in the previous scenario, customers are also afforded the opportunity to order online and have their items shipped to the store. STS fulfillment requires the shipment of items from the central warehouse to the local BM store. As affirmed by prior research (Morganti et al., 2014), the logistics costs for direct delivery are significantly higher than those for STS, aggregated into a single delivery. Hence, $c_p \leq c_d$. Figure 1.2 depicts the decision tree for omnichannel customers who may choose between direct delivery or STS. These customers evaluate their expected utility from purchasing with each option and select the one that provides greater utility. Customer who prefer BOPS earn a utility of $U_p^s = -h_o - h_s + \alpha(v - p)$. If they opt for the STS option, they incur a hassle cost of h_o for ordering online. Let $h_s = t_p + h_p$ denote the hassle cost of purchasing with STS, which includes the waiting time for shipping the order (t_p) and the hassle of picking up the item in-store (h_p). Since most retailers offer free shipping for the STS option, we assume no shipping costs for STS. The touch-and-feel experience of visiting the BM store enables customers to evaluate the product's fit before making a purchase and return it promptly if it does not meet their expectations. Thus, we assume no return hassle costs for in-store pickups. The decision tree provided in Figure 1.2 outlines all the alternatives available to customers.

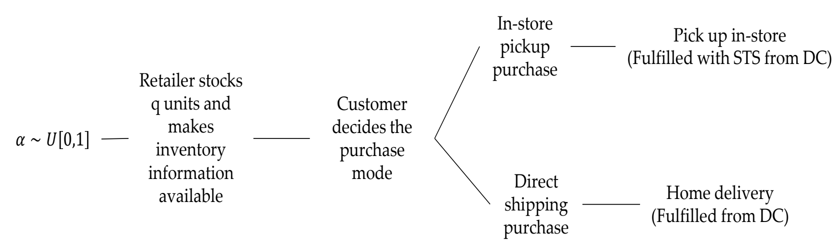


Figure 1.2: Customers' Decision Tree under the Omnichannel Retailer with STS Fulfillment

The market segmentation under the STS fulfillment strategy is illustrated in Figure 1.3. The retailer anticipates that a proportion of omnichannel customers will order using the STS approach, denoted by $\hat{\ell}_p^s$, and a fraction will opt for direct-delivery, denoted by $\hat{\ell}_d$. Consequently, D_s, D_p^s, and D_d follow the uniform distributions $U[0,(1-\lambda)M]$, $U[0,\hat{\ell}_p^s M]$, and $U[0,\hat{\ell}_d M]$ respectively. Following the implementation of the STS fulfillment strategy, consumers who were previously deterred from shopping online due to high direct shipping costs will now make purchases through online channels and subsequently pick up their orders in-store. This is represented by the equation $\lambda[a_{2n} - a_{1s}]^+$ and referred to as the market expansion effect induced by the STS fulfillment strategy. Additionally, omnichannel customers who previously ordered online with direct shipping will now opt for STS due to the convenience of in-store pickups, as indicated by the equation $\lambda[a_{2s} - a_{2n}]^+$. This is referred to as the demand shift effect.

Proposition 2. *The STS strategy results in a market expansion effect, inducing $\lambda[a_{2n} - a_{1s}]^+$ non-shopper omnichannel customers to adopt STS. Additionally, a demand shift effect is observed, with $\lambda[a_{2s} - a_{2n}]^+$ omnichannel customers switching to STS from direct delivery.*

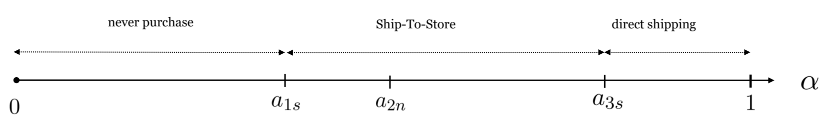


Figure 1.3: Market Segmentation under the STS

Utilizing the probability of a high valuation among omnichannel customers, denoted as α and demonstrated in Figure 1.3, three segments can be derived: the Never purchase segment with $\alpha \leq a_{1s}$ where omnichannel customers are disinclined to make purchases regardless of STS implementation, the STS segment with $a_{1s} \leq \alpha < a_{2s}$ where omnichannel customers consistently prefer to utilize STS for ordering, and the direct-delivery segment with $a_{2s} \leq \alpha \leq 1$ where direct delivery is always the preferred option for omnichannel customers. Our analysis considers a heterogeneous consumer population where all behaviour segments are present.

To have non-zero demands for both STS and direct delivery fulfillment options, we must ensure that both channels are attractive to omnichannel customers. First, in order to ensure that omnichannel customers show a willingness to utilize the STS option, the following inequality must hold: $v - p > \frac{r_d(h_s+h_o)}{(h_d+r_d)-(h_s+h_o)}$, meaning that consumer valuation must be higher than a certain threshold. Otherwise, without this condition, no omnichannel customer will order via the STS method.

Similarly, the inequality $h_o + h_s > h_d$ is necessary to guarantee that a fraction of omnichannel customers is willing to employ the direct-delivery option. Next, we investigate the impact of these demand changes on the retailer's inventory allocation decision and associated expected profits, as well as whether the STS fulfillment strategy is always beneficial for the retailer. Given the retailer's belief about omnichannel customers' choices, the aggregate expected profit subject can be expressed as follows:

$$\begin{aligned}
\Pi_p^s(q) = & \underbrace{p\mathbb{E}\min(D_s, q) - c_s q}_{\text{Profit from walk-in sales}} \\
& + \underbrace{(p - c_p)\frac{\hat{\ell}_p^s}{\hat{\ell}_p^s + \hat{\ell}_d}\mathbb{E}\min\left(D_p^s + D_d, (K - q)\right)}_{\text{Profit from STS}} + \underbrace{(p - c_d)\frac{\hat{\ell}_d}{\hat{\ell}_p^s + \hat{\ell}_d}\mathbb{E}\min\left(D_p^s + D_d, (K - q)\right)}_{\text{Profit from direct delivery}} \\
& - \underbrace{c_h\mathbb{E}[q - D_s]^+}_{\text{Store holding cost}} - \underbrace{c_g\mathbb{E}[D_s - q]^+}_{\text{Walk-in lost sale cost}} \\
& \text{Subject to } q \leq K, q \geq 0
\end{aligned} \tag{1.2}$$

In this section, we present the profit function for the retailer, which comprises five terms. The first two terms correspond to the expected profits generated from serving walk-in customers, while the subsequent two terms represent the profits from fulfilling STS and direct delivery orders, respectively. We assume that the distribution center fulfills STS and direct delivery orders in the order they are received, with the proportional rationing rule determining the retailer's belief about the proportion of STS and direct delivery sales, denoted as $\frac{\hat{\ell}_p^s}{\hat{\ell}_p^s+\hat{\ell}_d}$ and $\frac{\hat{\ell}_d}{\hat{\ell}_p^s+\hat{\ell}_d}$, respectively. The last two terms refer to the retailer's in-store holding costs and lost sales.

To solve the constrained maximization problem as depicted by Eq. (1.2), we formulate a Lagrangian function by introducing a multiplier. Under the STS strategy, RE equilibrium implies that the retailer's belief about customers' choices corresponds to the actual outcome, such that $\hat{\ell}_p^s = \ell_p^s$ and $\hat{\ell}_d = \ell_d$. If the retailer anticipates that a fraction of omnichannel customers equal to $\hat{\ell}_p^s$ and $\hat{\ell}_d$ will opt for

STS and direct delivery, respectively, she will stock q_p^s in the BM store and $K - q_p^s$ in the DC warehouse.

Definition 1.3.1. Strategy $(\ell_d, \ell_p^s, \hat{\ell}_d, \hat{\ell}_p^s, q_p^s)$ is an RE equilibrium if and only if the following conditions are satisfied:

i Given $\hat{\ell}_d$, $\hat{\ell}_p^s$, then $q_p^s = \underset{q}{\text{argmax}} \Pi_p^s(q)$, where $\Pi_p^s(q)$ is given in (eq. 1.2).

ii $\hat{\ell}_p^b = \ell_p^s = \lambda[a_{3s} - a_{1s}]^+ > 0$ and $\hat{\ell}_d = \ell_d = \lambda[1 - a_{3s}]^+ > 0$.

Definition 2.1's first condition ensures that the retailer's decision regarding optimal stocking quantity is profit-maximizing, accounting for consumer purchase behavior. The second condition necessitates coherence between expectations and outcomes. The following proposition establishes the existence of a rational expectations equilibrium.

Proposition 3. *When the retailer offer STS, the retailer allocates $q_p^s > 0$ to the BM store, while the remaining amount, $K - q_p^s$, is allocated to the distribution center.*

$$q_p^s = \frac{B - \sqrt{B^2 - 4AC}}{A}$$

Where

$$A = \frac{\hat{\ell}_p^s(p - c_p) + \hat{\ell}_d(p - c_d)}{2\hat{\ell}_d\hat{\ell}_p^s(\hat{\ell}_d + \hat{\ell}_p^s)M^2}$$

$$B = \frac{p + c_g + c_h}{M(1-\lambda)} + \frac{(\hat{\ell}_p^s(p - c_p) + \hat{\ell}_d(p - c_d))K}{\hat{\ell}_d\hat{\ell}_p^s(\hat{\ell}_d + \hat{\ell}_p^s)M^2}$$

$$C = p + c_g - c_s + \frac{(K^2 - 2\hat{\ell}_d\hat{\ell}_p^s M^2)(\hat{\ell}_p^s(p - c_p) + \hat{\ell}_d(p - c_d))}{2\hat{\ell}_d\hat{\ell}_p^s(\hat{\ell}_d + \hat{\ell}_p^s)M^2}$$

Corollary 0.1. The STS fulfillment strategy results in the allocation of less inventory to the BM store compared to the No Store-Pickup strategy, $q_p^s \leq q_n$.

Recall that q_n and q_p^s are the retailer's optimal inventory allocation to the BM store in the NSP and STS models. Corollary 0.1 reveals that the implementation of STS enables the use of DC warehouse stock to fulfill orders for both STS and direct delivery. Consequently, the retailer is able to increase the inventory allocation to the DC warehouse following STS implementation due to market expansion. We compare the expected profits under the STS and NSP strategies, in Proposition 4.

Proposition 4. *There exists a lower-bound $\underline{c}_s$ for the store operating cost c_s, such that if $c_s < \underline{c}_s$, then implementing STS decreases the retailer's expected profit $\Pi_p^s < \Pi_n$ compared to the No Store-Pickup (NSP) model.*

Proposition 4 demonstrates the importance of store operating cost, c_s, in determining the STS fulfillment strategy's profitability. The adoption of STS incites a transformation in sales distribution, with a portion of omnichannel consumers favoring this approach over direct delivery. Additionally, omnichannel customers who used to avoid direct delivery due to high inconvenience costs choose STS. As a result, the retailer allocates more inventory to the DC warehouse and diminishes it in the BM store. This sales reallocation may yield advantages for the retailer by decreasing shipping costs ($c_p < c_d$) and in-store holding costs (c_h).

However, the reduction of in-store inventory heightens the risk of losing unsatisfied walk-in customers who visit the BM store and encounter stockout situations. In other words, the retailer loses $(p - c_s + c_g)$ for each walk-in stockout. Proposition 4 reveals that implementing STS may shift sales to a less profitable strategy for lower store operating costs, and the expanded sales may not compensate for this loss. The STS fulfillment strategy is more beneficial when the in-store operating costs are high. This is because the cost savings associated with STS and the shift in sales composition outweigh the risk of losing walk-in customers due to inventory reduction. Therefore, the retailer needs to consider the cost structure of the BM store, shipping costs, and holding costs before deciding whether to implement the STS strategy.

1.3.4 Buy Online Pick-up in Store (BOPS)

In this section, we investigate the implementation of the BOPS fulfillment strategy by an omnichannel retailer. BOPS offers customers the convenience of ordering and picking up products on the same day, provided that the item is available in the local BM store inventory. When the item is not in the store inventory at the time of order, the retailer cancels the order and refunds the customer. Given that omnichannel customers are presumed to leave in the event of an order cancellation, they are assumed to be always aware of in-store product availability. However, the challenge of up-to-date inventory tracking for retailers has been noted in previous research (Fosco, 2019).

Notably, in cases where a walk-in customer purchases the last item in the BM store while an omnichannel customer concurrently places an order through BOPS, the latter would receive notification that the item is unavailable in-store and that their order has been cancelled. Herein, we denote the belief of omnichannel customers regarding the probability of store inventory availability at the time of order and receipt of an order confirmation email as $\hat{\zeta}_b$.

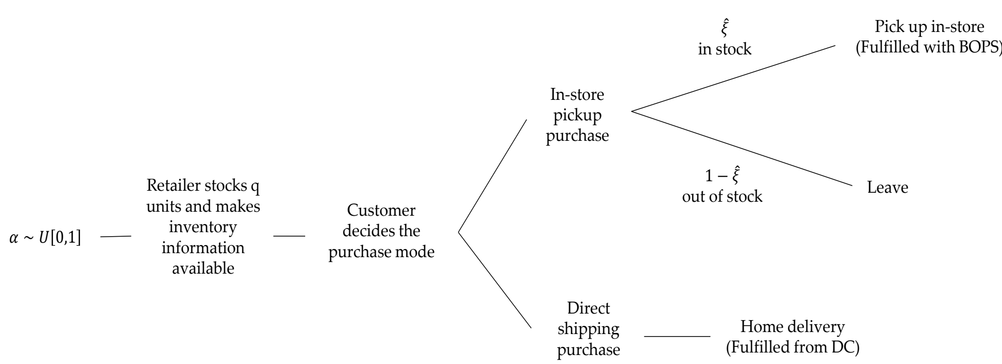


Figure 1.4: omnichannel Customers' Decision Tree Under an Omnichannel Retailer with Buy Online and Pick up in Store (BOPS)

Customer who prefer BOPS earn a utility of $U_p^b = -h_o + \hat{\zeta}_b(\alpha(v-p) - h_p)$.

Where the sub-superscript ${}^{b}_{p}$ denotes BOPS policy. In the event that $U_p^b > U_d$, omnichannel customers favor ordering via BOPS and incurring the inconvenience costs associated with online ordering h_o and product pickup at the store h_p. Assuming that $h_s = h_p + t_p$, we presume that the hassle cost for STS fulfillment surpasses that of BOPS fulfillment on account of the supplementary shipping time (t_p) from the distribution center to the BM store. This condition ensures that, in the event that an item is available in-store, customers will invariably opt for BOPS over the STS fulfillment option, signifying the normalization of t_p to 0 for BOPS. Additionally, we discount the customer's surplus from purchase due to the possibility of a stockout, which may arise due to walk-in customers.

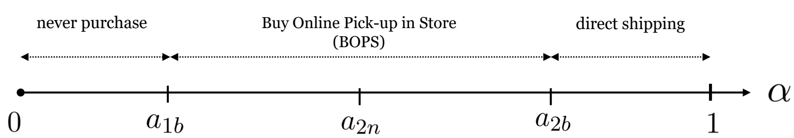


Figure 1.5: Market Segmentation Under an Omnichannel Retailer with Buy Online and Pick up in Store (BOPS)

Proposition 5. *BOPS implementation leads to a market expansion effect, resulting in a fraction of non-shopper omnichannel customers ordering through BOPS denoted by* $\lambda[a_{2n} - a_{1b}]^+$. *Furthermore, the demand shift effect induces a fraction of omnichannel customers to switch from direct delivery to BOPS, represented by* $\lambda[a_{2b} - a_{2n}]^+$.

In the presence of BOPS fulfillment option, as shown in Figure 1.5, the market is divided into three segments: never purchase $0 \leq \alpha \leq a_{1b}$, always purchase through BOPS $a_{1b} \leq \alpha \leq a_{2b}$, always order with direct-delivery $a_{1b} \leq \alpha \leq 1$. The implementation of BOPS expands the market coverage by attracting new customers, and a portion of the omnichannel customers previously ordered online with direct delivery switch to BOPS.

This study analyzes a heterogeneous population encompassing all previously mentioned segments. For both BOPS and direct delivery to effectively engage omnichannel customers, these options must be appealing enough for them to utilize both methods. The following condition must be satisfied to guarantee that the surplus for omnichannel customers is high enough, encouraging them to order via BOPS and direct delivery: $v - p > \max\{\frac{(\zeta h_b + h_o) r_d}{\zeta(h_d + r_d - h_b) - h_o}, \frac{h_d - \zeta h_b - h_o}{1-\zeta}\}$. Otherwise, omnichannel customers will not be willing to utilize either BOPS or direct delivery when placing orders.

By adopting the BOPS strategy, the retailer expects that fractions $\hat{\ell}_p^b$ and $\hat{\ell}_d$ of omnichannel customers op for BOPS and direct delivery respectively. Consequently, D_s, D_p^b and D_d follow the uniform distributions $U[0,(1-\lambda)M]$, $U[0,\hat{\ell}_p^b M]$, and $U[0,\hat{\ell}_d M]$ respectively. Given the retailer's belief about omnichannel customers' preferences, the expected profit can be expressed as follows:

$$\begin{aligned}\Pi_p^b(q) = &\underbrace{p\mathbb{E}\min(D_s + D_p^b, q) - c_s q}_{\text{Profit from walk-in and BOPS}} + \underbrace{(p - c_d)\mathbb{E}\min(D_d, (K-q))}_{\text{Profit from direct delivery}} \\ &- \underbrace{c_h \mathbb{E}[q - (D_s + D_p^b)]^+}_{\text{Store holding cost}} - \underbrace{c_g(\frac{\ell_s}{\hat{\ell}_p^b + \ell_s})\mathbb{E}[(D_s + D_p^b) - q]^+}_{\text{Walk-in lost sale cost}} \qquad (1.3) \\ &\text{Subject to } q \leq K, q \geq 0\end{aligned}$$

The first two terms represent the aggregate expected profits from arising from the fulfillment of walk-in and BOPS orders, both of which are satisfied from the store's inventory. The third term shows the profit of the retailer from fulfilling the orders with direct delivery. The fourth term reflects the holding cost incurred by the retailer for the remaining inventory that is not utilized after fulfilling the walk-in and BOPS demand. The final term captures the cost of lost sales incurred by walk-in customers. Given the proportional rationing assumption, the retailer serves both walk-in customers and BOPS demand simultaneously by utilizing the store's inventory. Thus, $\frac{\ell_s}{\hat{\ell}_p^b + \ell_s}$ represents the proportion of the potential walk-in sales.

The BOPS model assumes that the retailer's beliefs align with observed customer choices, i.e., $\hat{\ell}_p^b = \ell_p^b$ and $\hat{\ell}_d = \ell_d$. The retailer stocks inventory accordingly, with q_p^b units in the BM store and $K - q_p^b$ units in the DC. In addition, omnichannel customers' belief regarding the likelihood of an item being available at the BM store at the time of ordering, denoted by $\hat{\zeta}_b$, must align with the probability of the store confirming the order. To establish the RE equilibrium, we evaluate how the retailer's inventory allocation impacts the probability of order confirmation and subsequently influences customer expectations. We assume that this confirmation probability is determined by $\hat{\zeta}_b = \frac{\left(\frac{\ell_p^b}{\ell_p^b+\ell_s}\right)\mathbb{E}\min(D_s+D_p^b,q)}{E(D_p^b)}$. Specifically, we define the RE equilibrium as follows.

Definition 1.3.2. Strategy $(\ell_d, \ell_p^b, q, \hat{\zeta}, \hat{\ell}_d, \hat{\ell}_p^b)$ is an RE equilibrium if and only if the following conditions are satisfied:

i Given $\hat{\ell}_d$, $\hat{\ell}_p^b$, then $q_p^b = \underset{q}{\text{argmax}} \Pi_p^b(q)$, where $\Pi_p^b(q)$ is given in (eq. 1.3).

ii Given $\hat{\zeta}$, then $\ell_p^b = \lambda[a_{2b} - a_{1b}]^+ > 0$ and $\ell_d = \lambda[1 - a_{2b}]^+ > 0$.

iii $\hat{\zeta}_b = (\frac{\ell_p^b}{\ell_p^b+\ell_s})\frac{\mathbb{E}\min(D_s+D_p^b,q)}{E(D_p^b)} > 0$

iv $\hat{\ell}_p^b = \ell_p^b$ and $\hat{\ell}_d = \ell_d$.

The first two conditions, (i) and (ii), ensure that both the retailer and omnichannel customers make optimal decisions based on their anticipation of $\hat{\ell}_d$, $\hat{\ell}_p^b$, and $\hat{\zeta}_b$. The third condition guarantees consistency between the retailer's belief and the actual outcome in the equilibrium, resulting in $\hat{\ell}_p^b = \ell_p^b$ and $\hat{\ell}_d = \ell_d$. Proposition 6 establishes the Rational Expectations (RE) equilibrium.

Proposition 6. *When the retailer offers BOPS,*

- *The retailer will allocate* $q_p^b = \frac{-B+\sqrt{B^2+4AC}}{2A} > 0$ *units to the BM store, while allocating* $K - q_p^b$ *to the DC warehouse.*

$$A = \frac{\frac{c_g}{1+\ell_p^b-\lambda} + \frac{p+c_h}{1-\lambda}}{2\ell_p^b M^2}, B = \frac{p-c_d}{\hat{\ell}_d M}, C = \frac{(1-\lambda)c_g}{1+\ell_p^b-\lambda} + \frac{(p-c_d)K}{\ell_p^b M} + c_d - c_s$$

- *The equilibrium probability of fulfilling a BOPS order at the BM store is* $\zeta_b = \frac{(6\ell_p^b(1-\lambda)M^2-(q_p^b)^2)q_p^b}{3{\ell_p^b}^2(1-\lambda)(1-\lambda+\ell_p^b)M^3} > 0.$

Corollary 0.2. Compared with the No Store-Pickup scenario, the BOPS strategy allocates a higher portion of inventory to BM stores, $q_p^b \geq q_n$.

According to corollary 0.2, the retailer experiences an increase in inventory allocation to the BM store upon implementing BOPS. This outcome can be attributed to the channel switching and market expansion effects arising from adopting BOPS. The utilization of BM store inventory to fulfill in-store pickup orders may reduce in-store holding costs associated with unsold products, thereby incentivizing retailers to allocate a greater amount of inventory to BM stores.

Proposition 7. *There exist an upper-bound $\bar{c}_s$ for the store operating cost, c_s, such that when $c_s > \bar{c}_s$, implementing BOPS may not benefit the retailer, $\Pi_p^b < \Pi_n$.*

Proposition 7 reveals that the expected profit of a retailer may decrease upon implementing BOPS, particularly when the local BM store has high operating costs. This is attributed to the reduction in direct delivery sales and a subsequent increase in pickup in-store sales due to BOPS implementation. While the in-store pickup fulfillment can result in a reduction in logistics costs for the retailer, the overall effect on profit margin must be taken into account. Specifically, our findings indicate that implementing BOPS may be detrimental for the retailer when the profit margin of BOPS and walk-in sales is low. In the next section, we conduct a comparative analysis of BOPS and STS fulfillment strategies, evaluating the expected profits of each method to determine when the retailer should prioritize one over the other.

Comparing BOPS and STS Strategies. In the previous section, we demonstrated that implementing the STS strategy may only be advantageous for the retailer if the operating cost of the BM store is high. Conversely, the BOPS strategy may prove detrimental to the retailer with higher store operating costs. In light of these findings, we compare the BOPS and STS fulfillment strategies and evaluate the corresponding expected profits. By doing so, we aim to assist retailers in determining which approach to adopt.

To answer this question, we present the following proposition, which characterizes the trade-offs involved in the choice between the two strategies. We assume that consumer valuation is high enough so that omnichannel customers are willing to order with in-store pickup options (STS and BOPS) and direct delivery. To ensure non-zero demand for each fulfillment alternative, the conditions must satisfy:

- $v - p > \max\left\{\frac{r_d(h_s+h_o)}{(h_d+r_d)-(h_s+h_o)}, \frac{(\zeta h_b+h_o)r_d}{\zeta(h_d+r_d-h_b)-h_o}, \frac{h_d-\zeta h_b-h_o}{1-\zeta}\right\}$
- $h_o + h_s > h_d$.

Proposition 8. *There exists a threshold, denoted by $\tilde{c}_s$, for the store operating cost, c_s, such that it is beneficial for the retailer to adopt the BOPS strategy when $c_s \le \tilde{c}_s$, and to implement the STS strategy when $c_s \ge \tilde{c}_s$.*

Proposition 8 states that the choice between the BOPS and STS strategies depends on the level of store operating cost. Specifically, when the store's operating cost is relatively low, the retailer benefits from adopting the BOPS strategy, which leverages the store's inventory to fulfill in-store pickup orders. On the other hand, the STS strategy is preferable for higher store operating costs, as it allows for centralized fulfillment from a distribution center. Figure 1.6 visually illustrates the Propositions 4, 7, and 8. This illustration is conditioned on the value of c_s since it plays a crucial role in determining the retailer's optimal in-store pickup fulfillment strategy.

We further investigate the sensitivity of the optimal policy by examining other parameters such as the last-mile delivery cost, denoted as c_d. Our findings indicate that BOPS becomes more profitable for the retailer as the direct-delivery

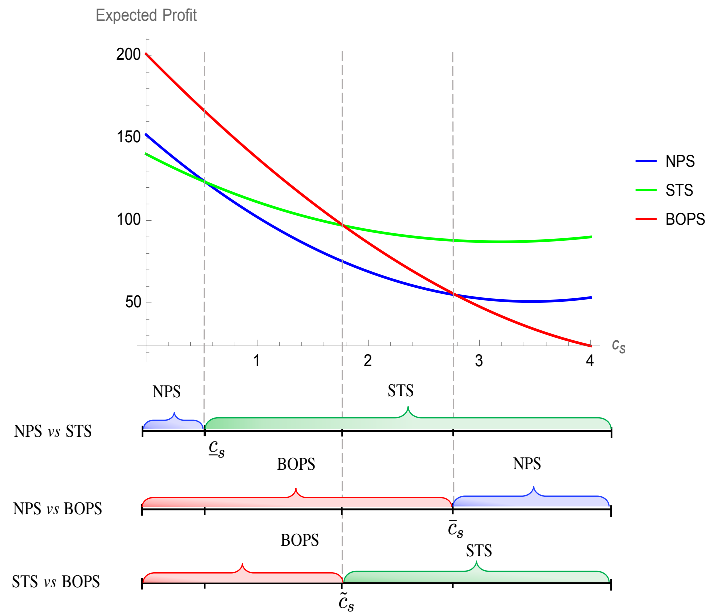


Figure 1.6: Optimal Policy for Pick-up in-Store Fulfillment
Note: $p = 4$, $M = 250$, $K = 60$, $c_p = 0.8$, $c_0 = 2$, $\lambda = 0.6$

fulfillment cost increases. However, this profitability is dependent on the level of c_s, where a high c_d and a low c_s make BOPS a less costly and more profitable option for in-store fulfillment, while STS becomes more expensive and less profitable. Additionally, we use the Continuous Approximation model, assuming that the direct-delivery fulfillment cost per delivery is inversely proportional to the population density. In high-density cities, the direct-delivery fulfillment cost per delivery tends to be lower due to economies of scale resulting from the concentration of demand in a smaller geographical area. As a result, direct delivery becomes a more cost-effective option in such areas.

When the STS strategy is employed, the retailer can allocate a more significant proportion of inventory to the distribution center to minimize inventory costs, while the fulfillment of orders is done using direct delivery to the customer. Thus, our research findings suggest that retailers earn a higher profit under the STS (BOPS) strategy in areas with high (low) density cities with high (low) operating costs. Thus, the optimal in-store pickup fulfillment strategy depends on the population density and operating costs of the area in which the retailer operates. Further details on this aspect of our study can be found in Appendix D.

1.3.5 Hybrid Fulfillment: Joint Implementation of BOPS and STS

So far, our analysis of in-store pickup strategies has focused on two separate models: BOPS and STS. We conduct a comparative analysis of these strategies, assessing the circumstances in which each approach most benefits retailers. In this section, we focus on a scenario in which retailers implement a hybrid fulfillment strategy that combines STS and BOPS approaches. Under the Hybrid method, retailers fulfill in-store pickup orders using available inventory from the BM store. If the desired item is not available in-store, the retailer will dispatch stock from the distribution center warehouse to the designated pickup location. The primary objective of this section is to identify the optimal conditions under which retailers should adopt both STS and BOPS fulfillment strategies.

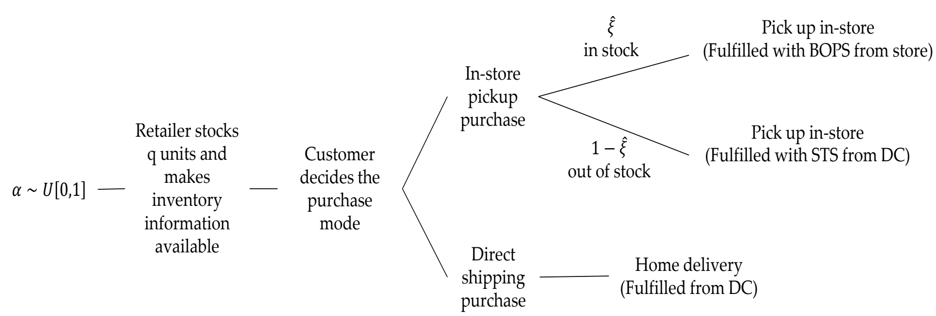


Figure 1.7: Customers' Decision Tree under the Omnichannel Retailer with Hybrid Fulfillment

As depicted in Figure 1.7, we assume the omnichannel customers who seek to maximize their utility will select either in-store pickup or direct delivery. Moreover, it is assumed that omnichannel customers are aware that when the product they order is not immediately available in the store, the retailer will arrange for the item to be delivered to the store for later pickup by the customer. When customers opt for the in-store pickup strategy, they derive a utility of U_p^h as shown in Equation (1.4), with the subscript $_p^h$ referring to the hybrid in-store pickup option.

$$
\begin{aligned}
U_p^h &= -h_o + \hat{\zeta}_h[\alpha(v-p) - h_b] + (1-\hat{\zeta}_h)[\alpha(v-p) - h_s] \\
&= -h_o - \hat{\zeta}_h h_b - (1-\hat{\zeta}_h)h_s + \alpha(v-p)
\end{aligned}
\tag{1.4}
$$

With this fulfillment option, customers incur the hassle of purchasing with BOPS and obtain the order immediately with probability $\hat{\zeta}_h$ and the hassle cost of purchasing with STS with probability $(1-\hat{\zeta}_h)$. Figure 1.8 illustrates the market segmentation resulting from this approach, which consists of three distinct segments: those who never make purchases ($0 \leq \alpha \leq a_{1h}$), those who make purchases through the Hybrid model ($a_{1h} \leq \alpha \leq a_{2h}$), and those who exclusively use direct-delivery ($a_{2h} \leq \alpha \leq 1$).

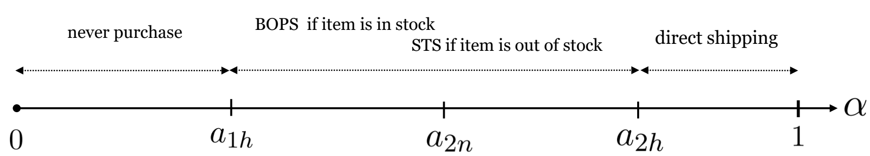


Figure 1.8: Market Segmentation under Hybrid Fulfillment

We consider a heterogeneous population including all mentioned customer segments. To effectively engage omnichannel customers, both hybrid and direct delivery options must be sufficiently attractive to encourage their utilization. In this regard, we establish a condition that guarantees a high enough surplus for omnichannel customers, incentivizing them to place orders through hybrid and direct delivery channels. Specifically, the following condition must hold:

$$(v-p)(h_d+r_d) > (v-p-r_d)(\zeta^h h_b + (1-\zeta^h)h_s + h_o) \tag{1.5}$$

$$h_d < \zeta^h h_b + (1-\zeta^h)h_s + h_o \tag{1.6}$$

If conditions (1.5) and (1.6) are met, then omnichannel customers will order through hybrid in-store pickup and direct delivery. On the other hand, if this condition is not met, omnichannel customers are unlikely to utilize either hybrid or direct delivery channels.

Proposition 9. *The Hybrid approach induces a market expansion effect, where $\lambda[a_{2n} - a_{1h}]^+$ non-shopper omnichannel customers adopt in-store pickup, and a demand shift effect, where $\lambda[a_{2h} - a_{2n}]^+$ omnichannel customers switch to the in-store pickup approach.*

By offering both BOPS and STS, retailers can cater to a wider range of customer preferences, resulting in increased customer satisfaction. Corollary 0.3 suggests that the simultaneous implementation of BOPS and STS strategies by the retailer results in a more significant market expansion and channel switching effect compared to when only one of these strategies is offered.

Corollary 0.3. Compared to BOPS and STS, the Hybrid strategy shows greater market expansion and channel switching effects

After offering Hybrid option, the retailer expects that fractions $\hat{\ell}_p^h$ and $\hat{\ell}_d$ of omnichannel customers order with in-store pickup and direct delivery respectively. Therefore D_p^h follows the uniform distributions $U[0, \hat{\ell}_p^h M]$. The retailer expect that a fraction of customers opt for in-store pickup and another fraction will opt for direct delivery, denoted as $\hat{\ell}_p^h$ and $\hat{\ell}_d$ respectively. The demand for in-store pickup, denoted as D_p^h, follows a uniform distribution with parameters of 0 and $\hat{\ell}_p^h M$. Given the retailer's belief about omnichannel customers' choices, the aggregate expected profit can be expressed as follows:

$$
\begin{aligned}
\Pi_p^h(q) = & \underbrace{p\mathbb{E}\min(D_s + D_p^h, q) - c_s q}_{\text{Profit from walk-in and BOPS}} \\
& \underbrace{+ (p - c_p)\frac{\hat{\ell}_p^h}{\hat{\ell}_p^h + \hat{\ell}_d} E\min\left(\frac{\hat{\ell}_p^h}{\hat{\ell}_p^h + \hat{\ell}_s}[(D_s + D_p^h) - q]^+ + D_d, (K - q)\right)}_{\text{Profit from STS}} \\
& \underbrace{+ (p - c_d)\frac{\hat{\ell}_d}{\hat{\ell}_p^h + \hat{\ell}_d} E\min\left(\frac{\hat{\ell}_p^h}{\hat{\ell}_p^h + \hat{\ell}_s}[(D_s + D_p^h) - q]^+ + D_d, (K - q)\right)}_{\text{Profit from direct shipping}} \qquad (1.7) \\
& - \underbrace{c_h\mathbb{E}[q - (D_s + D_p^h)]^+}_{\text{Store holding cost}} - \underbrace{c_g(\frac{\ell_s}{\hat{\ell}_p^h + \ell_s})\mathbb{E}[(D_s + D_p^b) - q]^+}_{\text{Walk-in lost sale cost}} \\
& \text{Subject to } q \leq K, q \geq 0
\end{aligned}
$$

The first term represents the store's expected profits from fulfilling walk-in and hybrid orders from the store's inventory. The second term represents the retailer's profit from fulfilling the remaining hybrid orders from DC warehouse's inventory through STS. The third term shows the profit of the retailer from fulfilling the orders with direct delivery. The forth and fifth terms show the in-store holding and walk-in lost sale costs. $(\frac{\hat{\ell}_p^h}{\hat{\ell}_p^h + \ell_s})[(D_s + D_p^h) - q]^+$ demonstrates the proportion of in-store pickup sales that cannot be fulfilled by the BM store's inventory. According to the RE equilibrium states that the retailer's belief about customers' choices is consistent with the realized proportions of customers $\hat{\ell}_d = \ell_d$, $\hat{\ell}_p^h = \ell_p^h$. Furthermore, in RE equilibrium, the omnichannel consumer's expectation of receiving their order confirmed immediately must be consistent with the outcome.

Definition 1.3.3. Strategy $(\ell_d, \ell_p^h, q_p^h, \hat{\zeta}^h, \hat{\ell}_d, \hat{\ell}_p^h)$ is an RE equilibrium if and only if the following conditions are satisfied:

i Given $\hat{\ell}_d$ and $\hat{\ell}_p^h$, then $q = \underset{q}{\text{argmax}}\Pi_p^h(q)$, where $\Pi_p^h(q)$ is given in eq. 1.7.

ii Given $\hat{\zeta}^h$, then $\ell_p^h = \lambda[a_{2h} - a_{1h}]^+ > 0$, and $\ell_d = \lambda[1 - a_{2h}]^+ > 0$.

iii $\hat{\zeta}^h = \left(\frac{\ell_p^h}{\ell_p^h+\ell_s}\right)\frac{\mathbb{E}\min(D_s+D_p^h, q_p^h)}{E(D_p^h)}$

iv $\hat{\ell}_p^h = \ell_p^h$ and $\hat{\ell}_d = \ell_d$.

Condition (i) and (ii) ensure that the retailer and omnichannel customers are choosing optimal decisions in anticipation of $\hat{\ell}_d$, $\hat{\ell}_p^h$ and $\hat{\zeta}^h$. The last condition ensures consistency, as in equilibrium, the retailer's belief matches the outcome, thus $\hat{\ell}_p^h = \ell_p^h$ and $\hat{\ell}_d = \ell_d$. Proposition 10 gives the RE equilibrium.

Proposition 10. *The retailer allocates $q_p^h > 0$ to the BM store and the $K - q_p^h$ to the DC respectively. Here $\zeta^h = \frac{(6\ell_p^h(1-\lambda)M^2 - {q_p^h}^2)q_p^h}{3{\ell_p^h}^2(1-\lambda)(1-\lambda+\ell_p^h)M^3} \geq 0$ is the equilibrium probability of fulfilling a in-store pickup order from BM store's inventory.*

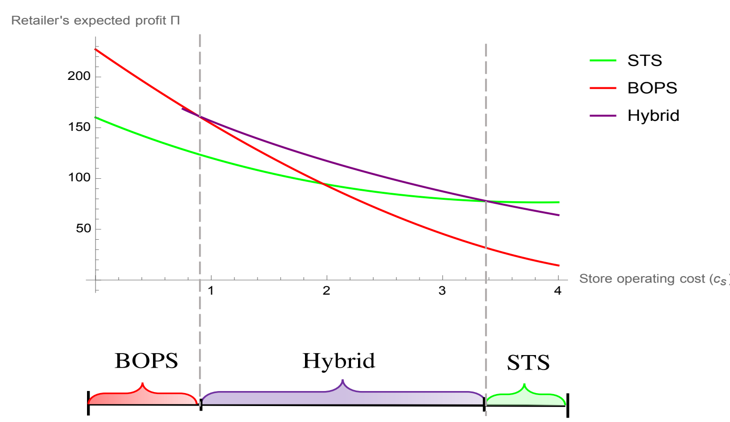


Figure 1.9: Impact of Store Operating Cost (c_s) on the Optimal Pick-Up In-Store Strategy

Note: p=4, M=250, K=60, c_p=0.8, c_0=2, λ=0.6

The objective of this study is to identify the optimal conditions for retailers to implement both BOPS and STS fulfillment strategies simultaneously. Analyzing the comparison between the Hybrid strategy and the BOPS and STS strategies is challenging using an analytical approach. Therefore, we conducted a numerical analysis to examine how retailers select among the three in-store pickup strategies based on the impact of store operating costs. According to the results presented in Figure 1.9, the hybrid fulfillment strategy is more profitable for the retailer when in-store operating costs are moderate.

Additionally, our research indicates that retailers favor BOPS over STS when store operating costs are moderate and $c_s \leq \tilde{c}_s$. This preference for BOPS is due to a lower in-store holding cost and a greater market expansion effect. However, the hybrid strategy outperforms BOPS due to its greater market expansion effect and lower in-store holding cost. Conversely, when $\tilde{c}_s \leq c_s$, STS is more profitable than BOPS. Nevertheless, the hybrid strategy still outperforms STS due to its greater market expansion effect and lower loss of sales at the store. In conclusion, retailers must carefully evaluate their store operating costs when selecting from the three fulfillment strategies.

1.3.6 Comparative Analysis

This section of the paper investigates the suitability of specific types of retailers or products for an optimal policy. We consider model parameters such as store operating cost (c_s), holding cost (c_h), lost sale cost (c_g), and direct delivery cost (c_d) to evaluate the effectiveness of three in-store pickup fulfillment strategies - STS, BOPS, and Hybrid. The benefit of STS increases with higher store operating and holding costs, as fulfilling in-store pickup orders only from the store's inventory becomes very costly for the retailer. Therefore, the retailer can earn a higher profit by implementing hybrid or STS and allocating more inventory to DC, fulfilling most in-store pickup orders through STS. Conversely, BOPS is more advantageous when store operating costs decrease and lost sale costs increase. Under such conditions, the retailer can earn higher profits by implementing hybrid or BOPS, fulfilling most in-store pickup orders from the BM store's inventory.

Additionally, we numerically examine the optimal fulfillment policy with respect to direct delivery cost (c_d) and population density (ρ). We assume that direct-delivery fulfillment cost is inversely related to population density. We find that as the direct-delivery fulfillment cost (c_d) decreases, the benefit of STS increases, making it a better option compared to hybrid when the population density is high. In contrast, the retailer can earn a higher profit with the hybrid fulfillment option in cities with lower population density. Figure 1.10 visually represents this finding.

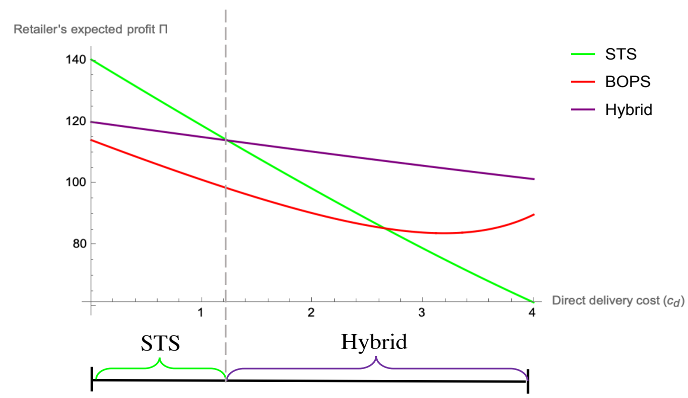


Figure 1.10: Impact of Direct Delivery Cost (c_d) on the Optimal In-Store Pickup Strategy

Note: p=4, M=250, K=60, c_p=0.8, c_s=1.5, λ=0.6

Regarding retailers with high store operating costs, high population density, and low in-store lost sale costs, our findings suggest that STS constitutes an optimal in-store pickup policy. Conversely, retailers with low store operating costs, low population density, and low in-store holding costs would be better served by BOPS. Meanwhile, a hybrid fulfillment strategy would be the most profitable for retailers with moderate store operating costs and direct-delivery expenses.

Moreover, we investigate the sensitivity of the optimal inventory allocation with respect to the product price (p), with technical details available in Appendix C. Our results indicate that the optimal inventory allocation to the BM store increases with increasing product price (p) under all fulfillment strategies. For NPS and BOPS, the fraction of customers opting for direct delivery decreases as prices rise, and the fraction of customers opting for BOPS increases.

BOPS mitigates some uncertainty associated with omnichannel customers purchasing higher-priced items online by enabling free and hassle-free returns if the customer is dissatisfied with the product during pickup. The retailer allocates more inventory to the BM store since walk-in demand is exogenous and not sensitive to pricing. In contrast, as prices rise under STS, fewer omnichannel customers opt for STS, and hence, the retailer assigns less inventory to the DC warehouse and more to the BM store. Overall, as summarized in Table 1.2, when the store operating costs are low and lost sale costs are high, BOPS or Hybrid strategies are the optimal choices for retailers with low direct delivery costs and high holding costs.

In contrast, when the retailer faces high direct delivery costs, STS is a more attractive option to minimize shipping expenses. STS and Hybrid strategies allow the retailer to optimize inventory allocation, minimize direct delivery costs, and maximize profits. On the other hand, when the retailer faces high direct delivery costs, making STS a more attractive option to reduce shipping expenses. Low store inventory holding costs enable the retailer to maintain inventory in the store with a minimal financial burden. Both STS and Hybrid strategies allow the retailer to optimize inventory allocation, minimize direct delivery costs, and maximize profits.

Table 1.2: Optimal In-store Pickup Strategies based on Retailer Characteristics

		c_s (c_g)		
		Low (High)	Moderate	High (Low)
c_d (c_h)	Low (High)	BOPS/Hybrid	Hybrid	STS
	High (Low)	BOPS	Hybrid	STS/Hybrid

1.3.7 Descriptive Analysis

In this section, to supplement the findings from our theoretical model and enhance our understanding of the retailer's decision-making process between in-store pickup strategies, we undertake an empirical investigation. For the purpose of this study, a comprehensive dataset was collected from the largest retailers in the United States with a total sales value exceeding $50 million. This dataset was obtained from Wharton Research Data Services (WRDS) and further cross-checked with the top 1000 online retailers list available on aftership.com. Following this extensive process, a final list of 578 retailers operating in the United States was generated. To rank the top retailers, their past 52/53-week annual retail sales were considered, with sales only taken into account for retail activity within the United States.

The dataset includes the name of each retailer, their product category, and their total annual sales. Additionally, information regarding each retailer's public or private status was gathered. To investigate the omnichannel fulfillment strategies adopted by the retailers, each retailer was individually examined to determine if they provide BOPS, STS, or both. Consequently, each retailer was categorized based on whether they solely operate with brick and mortar stores and provide direct delivery or if they offer omnichannel fulfillment strategies such as BOPS, STS, or both. This dataset is valuable for understanding the current state of omnichannel retailing in the United States. It allows us to analyze the prevalence of omnichannel fulfillment strategies among the largest retailers and to explore the relationship between these strategies and the type of retailer, their category of products , and their annual sales.

Out of the total 587 retailers in the dataset, 36% of retailers (210 retailers), have brick and mortar stores and solely offer direct delivery for online orders. BOPS is provided by 49% of retailers (289 retailers), while only 3% of retailers (18 retailers) provide only STS. Moreover, 12% retailers (70 retailers) offer both BOPS and

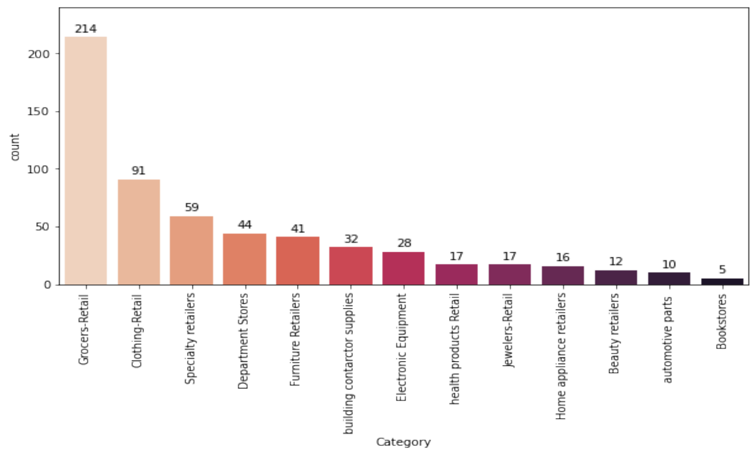


Figure 1.11: Number of Retailers in each Category of Products

STS fulfillment strategies simultaneously. This distribution of fulfillment strategies highlights the significance of BOPS in the retail industry as compared to STS. Retailers can leverage their existing stores for online order fulfillment, while customers benefit from a faster and more convenient fulfillment option. The analysis also revealed that out of the 587 retailers, 290 were public companies, while 297 were private companies. This information is crucial in understanding the impact of omnichannel fulfillment strategies on retailer profitability and decision-making, as the nature of ownership may affect the strategic decisions taken by the retailers. Fig 1.11 shows the distribution of retailers among different categories of products. The largest number of retailers are found in the Grocery, apparel category, followed by Department stores and furnishings category.

Figure 1.12reveals that retailers in higher sales deciles tend to prefer the hybrid fulfillment strategy. Larger and financially stable retailers with higher sales volumes may adopt both STS and BOPS fulfillment strategies simultaneously, as they can manage the costs and complexity associated with both strategies. These retailers have the resources to bear the costs of implementing these strategies, including investments in infrastructure, technology, and training, and may be better equipped to manage the logistical and inventory management challenges associated with operating both strategies. In contrast, the percentage of retail-

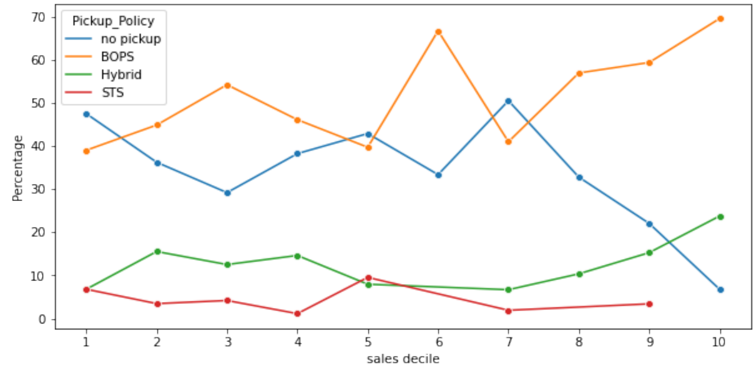


Figure 1.12: Distribution of Sales across Various Fulfillment Strategies

ers who only utilize direct delivery fulfillment decreases significantly as the sales volume increases. As sales volume increases, retailers may find it more feasible and cost-effective to adopt omnichannel fulfillment strategies that leverage their physical stores and distribution centers to fulfill orders.

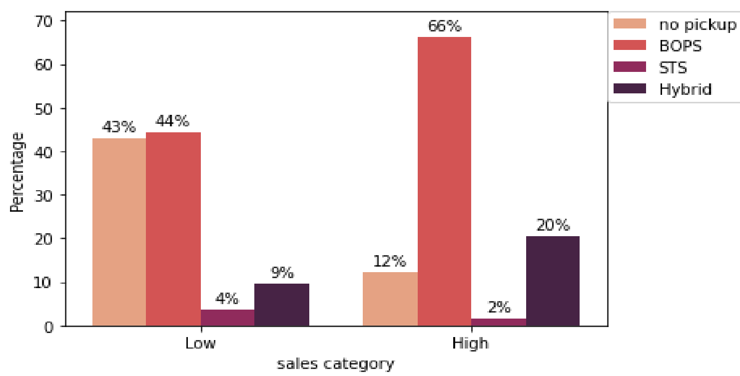


Figure 1.13: Number of Retailers in each Category of Products

Figure 1.14 presents a detailed analysis of adoption rates for BOPS and STS fulfilment options, categorized by various types of retailers, such as department stores, apparel and accessory stores, electronics stores, and others. The graph reveals that most retailers opt for BOPS fulfilment over STS fulfilment, with the exception of department stores and jewelry retailers. Grocery retailers, including wholesale clubs, tend to favor BOPS as an in-store pickup option. Moreover, beauty and health product retailers lean towards BOPS, given that beauty products are typically smaller and easier to transport for pickup. Notably, over 80%

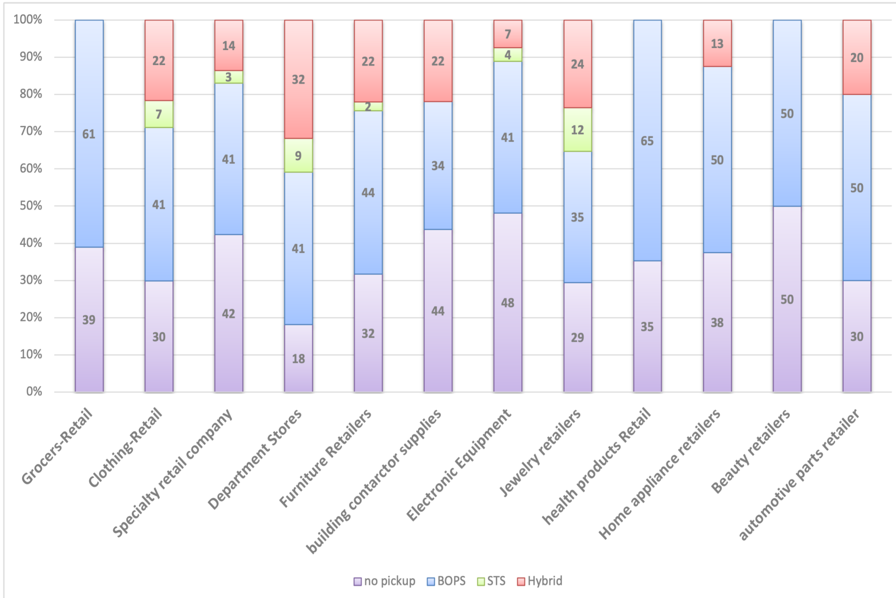


Figure 1.14: Pickup Strategies in each Category

of department stores offer in-store pickup fulfillment and over 30% of department stores offer both BOPS and STS fulfilment options. Apparel and electronic equipment have the lowest percentage of retailers offering either BOPS or STS fulfilment.

The perishable nature of products and the complexity involved in implementing in-house fulfillment infrastructure play a crucial role in the choice of in-store pickup strategy for grocery retailers. As Ship to Store (STS) involves delivering products from a warehouse to a store, the freshness of the products may be compromised during transit, making it unsuitable for grocery stores. Moreover, the study found that many grocery retailers outsource BOPS services to third-party partners, such as Instacart, instead of implementing their in-house omnichannel fulfillment services (e.g. Publix, Costco, Wegmans, Super King Markets, etc). This approach may be attributed to the complexity and cost involved in developing and maintaining an in-house omnichannel fulfillment infrastructure. By outsourcing BOPS, grocery retailers can leverage the expertise and infrastructure of established third-party providers and focus on core business operations.

Department stores mostly prefer STS or Hybrid fulfillment. This is because department stores often have a wide variety of products and sizes, making it difficult to maintain inventory levels for all items in-store. By utilizing STS, department stores can offer a wider selection of products to customers without the need to store them in-store. Additionally, STS allows for a more efficient use of space in the store, as products can be stored in a central warehouse and then shipped to the store as needed. This can help reduce the amount of space needed for storage in the store, allowing for more retail space to showcase products.

The majority of clothing retailers prefer Hybrid fulfillment options. Clothing items typically come in different sizes and colors, making it difficult to maintain large inventories without incurring high holding costs. Clothing items are usually tried on and purchased after an in-store visit. Hence STS is a convenient option for customers who prefer to try on the products before purchasing. By offering STS, retailers can avoid direct delivery costs and utilize warehouse inventory to fulfill online orders, thus reducing the risk of stockouts. In addition, with the help of BOPS, retailers can clear out inventory that may not be selling as well in-store but still has the potential for online sales. This is particularly important for seasonal items, where retailers in fast fashion industry need to make room for new products. The study found that for most product categories, BOPS fulfilment was more popular than STS fulfilment, with the exception of three categories: electronics, home goods, and sporting goods. In these categories, BOPS fulfilment was more popular than STS fulfilment.

Building supply, Furniture retailers and home appliance, as well as automotive parts retailers, do not typically offer STS as a fulfillment option due to several reasons. Firstly, these retailers usually sell larger and heavier items, which are not easily transportable to a store for pickup. Moreover, customers may face difficulty transporting these items themselves, making STS less convenient for them. Secondly, offering STS would require significant store space to accommodate the large items, potentially taking away from valuable retail space. Instead, these retailers tend to offer Hybrid or direct delivery as their primary fulfillment strategies. With the Hybrid option, retailers provide STS for smaller items and BOPS for larger items. The direct delivery option is particularly useful for larger items that cannot easily fit in a personal vehicle. Overall, these retailers aim to offer customers the convenience of shopping online and having their purchases delivered directly to their homes or job sites.

Luxury goods retailers tend to have higher operating costs than retailers who sell low-cost items. We find that jewelry retailers has the highest percentage (12%) who prefer pure STS as their fulfillment option. This is due to the fact that jewelry items are often small and valuable, making them more easily transportable to a store for pickup. Additionally, jewelry retailers may have a limited inventory of each item, making STS a good option for managing stock levels and avoiding stockouts. However, some jewelry retailers may also offer BOPS at the same time. By offering both STS and BOPS, jewelry retailers can provide a personalized and secure shopping experience for their customers while also reducing the risk of loss or damage during shipping.

1.4 Conclusion

With the advent of omnichannel retail, many retailers have recently started to offer pickup services so their customers can shop online and pick up their orders in-store. Although this business model continues to evolve, the literature does not yet suggest how multichannel retailers should choose between the different fulfillment modalities when planning to offer in-store pickup services. In order to fill this gap, this study focuses on the two in-store pickup strategies: BOPS (buy online, pick up in-store) and STS (ship-to-store) and analyzes their impact on customer behaviour.

This paper first derives a theoretical model to examine whether retailers should implement in-store pickup services and, if so, how to choose between the two strategies. We then examine the benefits of implementing the two types of in-store pickup strategies together. Based on utility-based customer choice, we assume that customers are heterogeneous in terms of how likely they are to return a product for misfitting. We find that the in-store pickup services induce a fraction of non-shopper customers to order or switch from direct delivery to in-store pickup option. Assuming that the retailer has a limited total inventory capacity, this study analyzes how inventory is allocated between the distribution center and the BM store before the random stream of demands are realized. We find that the retailer's cost structure determines whether they should implement in-store pickup strategies and, if so, which policy they should pursue.

We find that BOPS is particularly beneficial for retailers with a high in-store profit margin (those with low store operating costs and low holding costs) and a high direct-delivery fulfillment cost (those in low density cities). With lower store operating costs, the retailer stock more inventory in the BM store and save on fulfilling online orders from the local BM store's inventory, while avoiding walk-in lost sales. Whereas STS is recommended for retailers with low in-store profit margins (such as those with high store operating costs and lost sales costs) and low fulfillment costs (such as those in high-density cities). Due to high store operating costs, the retailer allocates more inventory to the DC warehouse and saves on fulfilling online orders by shipping orders from the DC warehouse to the store. We also find that only if a retailer has moderate store operating costs, it is best to implement both in-store pickup strategies simultaneously.

The results of our study provide several managerial implications for retailers seeking to introduce in-store pickup services. First of all, establishing an in-store pickup service might require significant investment and have a great impact on the company's systems and processes. The setting up of a pickup point in the store, assigning staff to take pickups, changing the order management systems, and integrating online and offline inventory are a few examples. Therefore, the retailer's initial investment varies for each in-store pickup strategy, with hybrid requiring the highest investment. In this paper, we demonstrate that retailers can profit substantially by choosing a pickup strategy that is in line with their cost structure, as opposed to following the herd and just implementing in-store pickup to remain competitive in the industry. Otherwise, a poor choice may result in costly investments, as well as an unexpected shift of sales to a less profitable channel.

Admittedly, more studies are needed to examine the growing trend of pickup in-store pickup strategies. We propose at least three potential avenues for future research. Firstly, examining the role of third-party logistics providers would be a valuable addition to the existing literature, which has yet to address their influence on the implementation of in-store pickup strategies. Further research could explore the impact of collaborating with third-party providers on the efficiency and effectiveness of in-store pickup services. Secondly, investigating the effects of competitive pressures on retailers' selection of in-store pickup strategies would be insightful. By analyzing the competitive landscape and taking into account factors such as market share, pricing strategies, and differentiation, researchers could help determine the optimal in-store pickup strategy for various competitive environments. Lastly, conducting empirical studies to validate and expand upon the theoretical findings of the current research would be advantageous. Such studies could provide real-world insights and support for the theoretical conclusions, thereby enhancing the overall understanding of in-store pickup strategies and their implications for retailers and customer behavior.

1.5 Appendix

1.5.1 The Lagrangian approach

We construct Lagrangian problem and the Lagrangian relaxation gives the following Lagrangian function and optimality conditions:

$$\pi_n^L(q) = p\mathbb{E}\min(D_s, q) - c_s q + (p - c_d)\mathbb{E}\min(D_d, K - q) + \gamma(K - q)$$
$$- c_h\mathbb{E}[q - D]^+ - c_g\mathbb{E}[D_s - q]^+$$

Where γ is the Lagrangian multiplier for the capacity constraint. One can easily verify that the expected profit is concave in q. We have $\frac{\partial^2 \pi_n^L(q)}{\partial q^2} \leq 0$, suggest that the profit function π_n^L is concave in q. Since the profit function is concave, the necessary and sufficient condition for optimality is given in the following system of equations.

$$\frac{\partial \pi^{L_n}}{\partial q} = 0,$$
$$\frac{\partial \pi^{L_n}}{\partial q} = \gamma,$$
$$\frac{\partial \pi^{L_n}}{\partial \gamma} = K - q = 0.$$

If the capacity constraint is binding such that $q_n = K$ and $\gamma_n > 0$ implies that retailer allocates full inventory to the BM store. Otherwise, if the capacity constraint is not binding such that $q_n < K$ and the Lagrangian multiplier $\gamma_n = 0$.

1.5.2 Proofs of propositions and corollaries

Proof of Proposition1. With a homogeneous population of customers, one can make the following observations:

- If $\alpha \leq a_{2n} = \frac{h_d + r_d}{v - p + r_d}$, the omni-customers exit the market and no one orders with direct shipping, $\hat{\ell}_d = \ell_d = 0$. Retailer's expected profit is

  $$\Pi_p^b(q) = p\mathbb{E}\min(D_s, q) - c_s q - c_h\mathbb{E}[q - D_s]^+ - c_g\mathbb{E}[D_s - q]^+.$$

  The equilibrium inventory allocation to the BM store is

  $$q^n = \min\{K, \frac{(1-\lambda)(p + c_g - c_s)}{p + c_g + c_h}\}.$$

- If $a_{2n} < \alpha < 1$, the omni-customers purchase online with direct shipping. If $v - p > h_d$, then customers will order with direct shipping and
$$\hat{\ell}_d = \ell_d = \lambda \left[1 - \frac{h_d + r_d}{v - p + r_d}\right].$$
Retailer's expected profit is
$$\Pi_n(q) = p\mathbb{E}\min(D_s, q) - c_s q + (p - c_d)\mathbb{E}\min(D_d, K - q) - c_h\mathbb{E}[q - D_s]^+ - c_g\mathbb{E}[D_s - q]^+.$$
The optimal inventory level in the BM store is
$$q_n = \frac{(1-\lambda)(\ell_d(c_d + c_g - c_s)M + (p - c_d)K)}{\ell_d(p + c_g + c_h) + (1-\lambda)(p - c_d)}.$$

With a heterogeneous consumer population, these segments of behavior may coexist. We assume that the omni-customer's valuation is sufficiently high $v - p > h_d$. Thus $\ell_d > 0$ which means there are omni-customers ordering with direct delivery. Note that for $v - p < h_d$ we end up to a trivial case where the omni-customer always leave the market.

Proof of Proposition 3. We first analyze the RE equilibrium in different regions in Figure 1.3 depending on the probability of having high valuation for omni-customers. We discuss the equilibrium by considering the following cases: $v - p > \frac{r_d(h_s+h_o)}{(h_d+r_d)-(h_s+h_o)}$ and $h_o + h_s > h_d$.

- If $\alpha \leq a_{1s} = \frac{h_o+h_s}{(v-p)}$, omni-customers never purchase and exit the market, $(\hat{\ell}_p^s, \hat{\ell}_d) = (0, 0)$. The retailer's expected profit is $\Pi_p^s(q) = \mathbb{E}\min(D_s, q) - c_s q - c_h\mathbb{E}[q - D_s]^+ - c_g\mathbb{E}[D_s - q]^+$. The equilibrium inventory level in the BM store is $q_p^s = \min\left\{K, \frac{(1-\lambda)(p+c_g-c_s)}{p+c_g+c_h}\right\}$.

- If $a_{1s} \leq \alpha < a_{2s} = \frac{r_d+h_d-h_s-h_o}{r_d}$, omni-customers prefer to purchase online and pick up in store once the order is ready. Retailer's expected profit is $\Pi_p^s(q) = pE\min(D_s, q) - c_s q + (p - c_p)E\min\left(D_p^s, K - q\right) - c_h E[q - D_s]^+ - c_g E[D_s - q]^+$. If $v - p > \frac{r_d(h_s+h_o)}{(h_d+r_d)-(h_s+h_o)}$, then $\hat{\ell}_p^s = \ell_p^s = \lambda[a_{3s} - a_{1s}]^+ > 0$ and in the equilibrium the inventory allocation to the BM store is $q_p^s = \frac{(1-\lambda)((p-c_p)K+\ell_p^s(c_g+c_p-c_s)M)}{(1-\lambda)(p-c_p)+\ell_p^s(p+c_g+c_h)}$.

- If $a_{2s} \leq \alpha \leq 1$ omni-customers only order online with direct delivery. The retailer's expected profit is

$$\begin{aligned}\Pi_p^s(q) = &\, p\mathbb{E}\min(D_s, q) - c_s q \\ &+ (p - c_d)\mathbb{E}\min(D_d, K - q) - c_h\mathbb{E}[q - D_s]^+ \\ &- c_g\mathbb{E}[D_s - q]^+.\end{aligned}$$

If $h_o + h_s > h_d$, then $\hat{\ell}_d = \ell_d = \lambda[1 - a_{3s}]^+ > 0$ and the equilibrium inventory level in the BM store is

$$q_p^s = q^n = \frac{(1-\lambda)(\ell_d(c_d + c_g - c_s)M + (p - c_d)K)}{\ell_d(p + c_g + c_h) + (1-\lambda)(p - c_d)}.$$

With a heterogeneous consumer population, these segments of behavior may coexist. Next, we analyze the RE equilibrium when omni-customers are heterogeneous in having a high valuation α. Retailer will stock $q_p^s > 0$ in the BM store and $K - q_p^s$ in DC. Since $0 < q \leq K$, we can obtain

$$\frac{\partial^2 \Pi_p^s(q)}{\partial q^2} = -\frac{p + c_g + c_h}{(1-\lambda)M} - \frac{(\ell_p^s(p - c_p) + \ell_d(p - c_o))(K - q)}{\ell_p^s \ell_d(\ell_d + \ell_p^s)M^2} < 0$$

One can verify that $\Pi_p^s(q)$ is a concave function for $0 < q \leq K$. Where $\frac{\partial \Pi_p^s(q)}{\partial q}$ is a quadratic function of q and have two roots as $q_1^s = B - \frac{\sqrt{B^2 - 4AC}}{2A} > 0$ and $q_2^s = B + \frac{\sqrt{B^2-4AC}}{2A} > 0$ where

$$\begin{aligned}A &= \frac{(\hat{\ell}_p^s(p - c_p) + \hat{\ell}_d(p - c_d))}{2\hat{\ell}_d\hat{\ell}_p^s(\hat{\ell}_d + \hat{\ell}_p^s)M^2} \\ B &= \frac{p + c_g + c_h}{M(1-\lambda)} + \frac{(\hat{\ell}_p^s(p - c_p) + \hat{\ell}_d(p - c_d))K}{\hat{\ell}_d\hat{\ell}_p^s(\hat{\ell}_d + \hat{\ell}_p^s)M^2} \\ C &= p + c_g - c_s + \frac{(K^2 - 2\hat{\ell}_d\hat{\ell}_p^s M^2)(\hat{\ell}_p^s(p - c_p) + \hat{\ell}_d(p - c_d))}{2\hat{\ell}_d\hat{\ell}_p^s(\hat{\ell}_d + \hat{\ell}_p^s)M^2}\end{aligned}$$

Since $A > 0$, we infer certain properties about the convexity of the function $\frac{\partial \Pi_p^s(q)}{\partial q}$. Specifically, we can show that for $q = 0$, $\frac{\partial \Pi_p^s(q)}{\partial q} > 0$ and for $q = K$, $\frac{\partial \Pi_p^s(q)}{\partial q} < 0$. Therefore, $q_p^s = q_1^s = B - \frac{\sqrt{B^2-4AC}}{2A} > 0$ is the only feasible root, which leads to $\frac{\partial \Pi_p^s(q)}{\partial q} = 0$ and $q_p^s =_q \Pi_p^s(q)$.

$$
\begin{aligned}
q_p^s =& \frac{\left(\frac{p+c_g+c_h}{M(1-\lambda)} + \frac{(\hat{\ell}_p^s(p-c_p)+\hat{\ell}_d(p-c_d))K}{\hat{\ell}_d\hat{\ell}_p^s(\hat{\ell}_d+\hat{\ell}_p^s)M^2}\right)}{\frac{(\hat{\ell}_p^s(p-c_p)+\hat{\ell}_d(p-c_d))}{2\hat{\ell}_d\hat{\ell}_p^s(\hat{\ell}_d+\hat{\ell}_p^s)M^2}} \\
&- \frac{\sqrt{\left(\frac{p+c_g+c_h}{M(1-\lambda)} + \frac{(\hat{\ell}_p^s(p-c_p)+\hat{\ell}_d(p-c_d))K}{\hat{\ell}_d\hat{\ell}_p^s(\hat{\ell}_d+\hat{\ell}_p^s)M^2}\right)^2 - 4\left(p+c_g-c_s+\frac{(K^2-2\hat{\ell}_d\hat{\ell}_p^sM^2)(\hat{\ell}_p^s(p-c_p)+\hat{\ell}_d(p-c_d))}{2\hat{\ell}_d\hat{\ell}_p^s(\hat{\ell}_d+\hat{\ell}_p^s)M^2}\right)\left(\frac{(\hat{\ell}_p^s(p-c_p)+\hat{\ell}_d(p-c_d))}{2\hat{\ell}_d\hat{\ell}_p^s(\hat{\ell}_d+\hat{\ell}_p^s)M^2}\right)}}{\frac{(\hat{\ell}_p^s(p-c_p)+\hat{\ell}_d(p-c_d))}{2\hat{\ell}_d\hat{\ell}_p^s(\hat{\ell}_d+\hat{\ell}_p^s)M^2}}
\end{aligned}
$$

In equilibrium, the retailer's beliefs is consistent with the outcome therefore $\hat{\ell}_d = \ell_d = \lambda[1-a_{3s}]^+$, $\hat{\ell}_p^s = \ell_p^s = \lambda[a_{3s}-a_{1s}]^+$. If $v - p > \frac{r_d(h_s+h_o)}{(h_d+r_d)-(h_b+h_o)}$, than $(a_{3s}-a_{1s}) > 0$ and a fraction of omni-customers order with STS. If $h_o + h_s > h_d$, then $(1-a_{3s}) > 0$ and a fraction of omni-customers order online with direct delivery.

Proof of corollary 0.1. By Eqs 1.1 and 1.2 and comparing the expected profit of retailer before and after implementing STS we have

$$
\begin{aligned}
\frac{\partial \Pi_n(q)}{\partial q} - \frac{\partial \Pi_p^s(q)}{\partial q} = &-\left(\frac{\ell_p^s}{\ell_p^s+\ell_d}\right)(c_d-c_p)\frac{\partial \mathbb{E}\min(D_d, K-q)}{\partial q} \\
&- \left(\left(\frac{\ell_p^s}{\ell_p^s+\ell_d}\right)(p-c_p) + \left(\frac{\ell_d}{\ell_p^s+\ell_d}\right)(p-c_d)\right)\frac{\partial \mathbb{E}\min(D_p^s, \max(K-q-D_d, 0))}{\partial q} > 0
\end{aligned}
$$

Since $\Pi_n(q)$ and $\Pi_p^s(q)$ are concave in q and $\frac{\partial \Pi_n(q)}{\partial q}$ and $\frac{\partial \Pi_p^s(q)}{\partial q}$ are decreasing in q, we can prove that $q_p^s \le q_n$.

Proof of Proposition 4. Since $\mathbb{E}\min\left(D_p^s + D_d^s, (K-q)\right) = \mathbb{E}[\min(D_d^s, (K-q))] + \mathbb{E}[\min(D_p^s, [(K-q)-D_d^s]^+)]$, then, we can rewrite the retailer's expected profit function as follows:

$$
\begin{aligned}
\Pi_p^s = \max_{q_p^s}\{&(p+c_h+c_g)([\min(D_s, q)] \\
&+ \left((p-c_d) + (\frac{\ell_d}{\hat{\ell}_p^s+\ell_d})(c_d-c_p)\right)[\min(D_d^s + D_d, (K-q))] \\
&- (c_s+c_h)q - c_g[D_s]\}
\end{aligned}
\tag{1.8}
$$

$$
\begin{aligned}
\Pi_n = &\max_{q_n}\{(p+c_h+c_g)[\min(D_s, q)] - (c_s+c_h)q - c_g[D_s] \\
&+ (p-c_d)\min(D_d, (K-q))\}
\end{aligned}
$$

Subject to $q \leq K$, $q \geq 0$. After substituting q_n and q_p^s and considering corollary 0.1 we have

$$\begin{aligned}
\Pi_p^s(q_p^s) - \Pi_n(q_n) = (p + c_h + c_g) &\underbrace{([\min(D_s, q_p^s)] - [\min(D_s, q_n)])}_{-} \\
&+ (p - c_d) \underbrace{(\min(D_d^s + D_d, (K - q_p^s)) - \min(D_d, (K - q_n)))}_{+} \\
&+ (\frac{\ell_d}{\hat{\ell}_p^s + \ell_d})(c_d - c_p)[\min(D_d, (K - q_p^s))] \\
&+ (c_s + c_h) \underbrace{(q_n - q_p^s)}_{+}
\end{aligned}$$

Using the Envelope Theorem, we have $\frac{\partial(\Pi_p^s(q_p^b) - \Pi_n(q_n))}{\partial c_s} = q_n - q_p^s > 0$, so we prove that $\Pi_p^s(q_p^s) - \Pi_n(q_n)$ increases in c_s. Suppose $c_g \to 0$, $c_h \to 0$ and $c_p, c_d \to p$, then

$$\Pi_p^s(q_p^s) - \Pi_n(q_n) = \underbrace{p([\min(D_s, q_p^s)] - [\min(D_s, q_n)])}_{-} + c_s \underbrace{(q_n - q_p^s)}_{+}$$

In this case, we have the following observations:

- If $c_s = 0$ then, $\Pi_p^s(q_p^b) - \Pi_n(q_n) = \underbrace{p([\min(D_s, q_p^s)] - [\min(D_s, q_n)])}_{-} < 0$
- If $c_s = p$ then, $\Pi_p^s(q_p^b) - \Pi_n(q_n) = \underbrace{p([\min(D_s, q_p^s)] - [\min(D_s, q_n)])}_{-} + p \underbrace{(q_n - q_p^s)}_{+} \geq 0$

Therefore we can conclude the result.

Proof of Proposition 6. Given retailer's belief about omni-channel customers' purchasing choices $(\hat{\ell}_d, \hat{\ell}_p^b)$, optimal inventory allocation to the BM store and the DC are $q_p^b > 0$ and $K - q_p^b$. We first investigate the RE equilibrium in different regions depending on the product valuation probability. With a homogeneous population of customers, we have the following observations in each segment below:

- If $\alpha < a_{1b} = \frac{\hat{\zeta}h_b+h_o}{\hat{\zeta}(v-p)}$, the omni-customers exit the market, $(\hat{\ell}_p^b, \hat{\ell}_d) = (0,0)$. Retailer's expected profit is $\Pi_p^b(q) = \min(D_s, q) - c_s q - c_h[q - D_s]^+ - c_g[D_s - q]^+$. The equilibrium inventory level in the BM store is $q_p^b = \frac{(1-\lambda)(p+c_g-c_s)}{(p+c_g+c_h)}$.

- If $a_{1b} \leq \alpha < a_{2b} = \frac{h_d+r_d-h_o-\hat{\zeta}h_b}{(1-\hat{\zeta})(v-p)+r_d}$ omni-customers order with BOPS and leave the market if their order doesn't get confirmed. $\hat{\ell}_p^b = \lambda[a_{2b} - a_{1b}]^+$ denotes the retailer's belief about the fraction of customers who order online and pick up in store. Retailer's expected profit is $\Pi_p^b(q) = p\min(D_s + D_p^b, q) - c_s q - c_h[q - (D_s + D_p^b)]^+ - c_g(\frac{\ell_s}{\ell_p^b+\ell_s})[(D_s + D_p^b) - q]^+$. The equilibrium inventory level in the BM store is $q_p^b = \min\{\sqrt{\frac{2\ell_p^b(1-\lambda)((1-\lambda)c_g+(1+b-\lambda)(p-c_s))M^2}{c_g(1-\lambda)+(1+\ell_p^b-\lambda)(p+c_h)}}, K\}$.

- If $a_{2b} \leq \alpha \leq 1$, the omni-customers only order online with direct shipping. $\ell_p^b = 0$ and $\hat{\ell}_d = \lambda[1 - a_{2b}]^+$ is retailer's belief about the fraction of omni-customers who purchase online with direct shipping. Retailer's expected profit is $\Pi_n(q) = p\min(D_s, q) - c_s q + (p - c_d)\min(D_d, K - q) - c_h[q - D_s]^+ - c_g[D_s - q]^+$. The equilibrium inventory level in the BM store is $q_p^b = q^n = \frac{(1-\lambda)(\ell_d(c_d+c_g-c_s)M+(p-c_d)K)}{\ell_d(p+c_g+c_h)+(1-\lambda)(p-c_d)}$.

With a heterogeneous consumer population, these segments of behavior may coexist. Next, we analyze the RE equilibrium when omni-customers are heterogeneous in having high valuation α. Since $0 < q \leq K$, we can obtain

$$\frac{\partial^2 \Pi_p^b(q)}{\partial q^2} = -\left(\frac{p}{1-\lambda} + \frac{p-c_d}{\ell_p^b M} + \frac{c_g q}{\ell_p^b(1+\ell_p^b-\lambda)M^2} + \frac{c_h q}{\ell_p^b(1-\lambda)M^2}\right) + \frac{(\ell_p^b M - q)p}{\ell_p^b(1-\lambda)M^2} < 0$$

One can verify that $\Pi_p^b(q)$ is a concave function for $0 < q \leq K$. Where $\frac{\partial \Pi_p^b(q)}{\partial q}$ is a quadratic function of q and have two roots as $q_1^b = \frac{B-\sqrt{B^2-4AC}}{2A} > 0$ and

$q_2^b = \frac{B+\sqrt{B^2-4AC}}{2A} > 0$ where

$$A = -\left(\frac{\frac{c_g}{1+\hat{\ell}_p^b-\lambda} + \frac{p+c_h}{1-\lambda}}{2\hat{\ell}_p^b M^2}\right)$$
$$B = \left(\frac{p-c_o}{\hat{\ell}_d M}\right)$$
$$C = \frac{(1-\lambda)c_g}{1+\hat{\ell}_p^b-\lambda} + \frac{(p-c_d)K}{\hat{\ell}_p^b M} + c_d - c_s$$

Since $A < 0$, $\frac{\partial \Pi_p^s(q)}{\partial q}$ is a concave function. because $B > 0$ and $c > 0$, $q_p^b = q_1^s = \frac{B-\sqrt{B^2-4AC}}{2A} > 0$ is the only positive and feasible root. Which makes $\frac{\partial \Pi_p^b(q)}{\partial q} = 0$ and $q_p^b =_q \Pi_p^b(q)$.

$$q_p^b = \frac{\left(\frac{p-c_o}{\hat{\ell}_d M}\right) - \sqrt{\left(\frac{p-c_o}{\hat{\ell}_d M}\right)^2 + 4\left(\frac{\frac{c_g}{1+\hat{\ell}_p^b-\lambda} + \frac{p+c_h}{1-\lambda}}{2\hat{\ell}_p^b M^2}\right)\left(\frac{(1-\lambda)c_g}{1+\hat{\ell}_p^b-\lambda} + \frac{(p-c_d)K}{\hat{\ell}_p^b M} + c_d - c_s\right)}}{-2\left(\frac{\frac{c_g}{1+\hat{\ell}_p^b-\lambda} + \frac{p+c_h}{1-\lambda}}{2\hat{\ell}_p^b M^2}\right)}$$

In equilibrium, the retailer's beliefs is consistent with the outcome therefore $\hat{\ell}_d = \ell_d = \lambda[1-a_{2b}]^+, \hat{\ell}_p^b = \ell_p^b = \lambda[a_{2b}-a_{1b}]^+$. If $v - p > \frac{(\zeta h_b+h_o)r_d}{\zeta(h_d+r_d-h_b)-h_o}$ then $(a_{2b}-a_{1b}) > 0$ and a fraction of omni-customers order with BOPS. If $v - p > \frac{h_d-\zeta h_b-h_o}{1-\zeta}$ then $(1-a_{2b}) > 0$ and a fraction of omni-customers order online with direct shipping. Retailer will stock $q_p^b > 0$ in the BM store and $K - q_p^b$ in DC. If $c_s \le c_d + \frac{(1-\lambda)c_g}{1+\ell_p^b-\lambda} + \frac{(p-c_d)K}{\ell_p^b M}$then $q_p^b > 0$. In the equilibrium, the omni-consumer's belief about BOPS order confirmation need to be consistent with the outcome. Where

$$\hat{\zeta} = \left(\frac{\ell_p^b}{\ell_p^b+\ell_s}\right)\frac{\min(D_s+D_p^b, q_p^b)}{E(D_p^b)} = \frac{(6\ell_p^b(1-\lambda)M^2 - {q_p^b}^2)q_p^b}{3{\ell_p^b}^2(1-\lambda)(1-\lambda+\ell_p^b)M^3} \ge 0$$

Which is indeed greater than 0. Thus, there are customers who are choosing to order with BOPS and the equilibrium inventory allocation is $q_p^b > 0$ and $K - q_p^b > 0$.

Proof of corollary 0.2. By Eqs 1.1 and 1.3, we have

$$\frac{\partial \Pi_n(q)}{\partial q} - \frac{\partial \Pi_p^b(q)}{\partial q} = -\left(p + c_h + c_g\left(\frac{\ell_s}{\ell_p^b+\ell_s}\right)\right)\frac{\partial[\min(D_p^b, [q-D_s]^+)]}{\partial q} - \left(2c_h + c_g\left(\frac{2\ell_s+\ell_p^b}{\ell_p^b+\ell_s}\right)\right)\frac{\partial[\min(D_s, q)]}{\partial q} < 0$$

Since $\Pi_n(q)$ and $\Pi_p^b(q)$ are concave in q and $\frac{\partial \Pi_n(q)}{\partial q}$ and $\frac{\partial \Pi_p^b(q)}{\partial q}$ are decreasing q, we can prove that $q_p^b \geq q_n$.

Proof of Proposition7. Since $\min(D_s + D_p^b, q) = [\min(D_s, q)] + [\min(D_p^b, [q - D_s]^+)$ then, The expected profit functions are as follows:

$$\begin{aligned}\Pi_n = &\max_{q_n}\{(p + c_h + c_g)[\min(D_s, q)] - (c_s + c_h)q - c_g[D_s] \\ &+ (p - c_d)\min(D_d, (K - q))\}\end{aligned}$$

$$\begin{aligned}\Pi_p^b = \max_{q_p^b}\{&(p + c_h + c_g(\frac{\ell_s}{\hat{\ell}_p^b + \ell_s}))[\min(D_s + D_p^b, q)] \\ &- (c_s + c_h)q - c_g(\frac{\ell_s}{\hat{\ell}_p^b + \ell_s})([D_s] + [D_p^b]) \\ &+ (p - c_d)\min(D_d^b, (K - q))\}\end{aligned}$$

Subject to $q \leq K$, $q \geq 0$. After substituting q_n and q_p^b and considering corollary 0.2 we have:

$$\begin{aligned}\Pi_p^b(q_p^b) - \Pi_n(q_n) = &\underbrace{(p + c_h + c_g(\frac{\ell_s}{\hat{\ell}_p^b + \ell_s}))\min(D_s + D_p^b, q_p^b) - (p + c_h + c_g)\min(D_s, q_n)}_{+,-} \\ &+ (c_s + c_h)\underbrace{(q_n - q_p^b)}_{-} + c_g\left((\frac{\hat{\ell}_p^b}{\hat{\ell}_p^b + \ell_s})[D_s] - (\frac{\ell_s}{\hat{\ell}_p^b + \ell_s})[D_p^b]\right) \\ &+ (p - c_d)\underbrace{\Big(\min(D_d^b, (K - q_p^b)) - \min(D_d, (K - q_n))\Big)}_{-}\end{aligned}$$

Using the Envelope Theorem, we have $\frac{\partial(\Pi_p^b(q_p^b) - \Pi_n(q_n))}{\partial c_s} = q_n - q_p^b < 0$, so we prove that $\Pi_p^b(q_p^b) - \Pi_n(q_n)$ decreases in c_s. Suppose $c_g \to 0$, $c_h \to 0$, then

$$\Pi_p^b(q_p^b) - \Pi_n(q_n) = \underbrace{p([\min(D_s + D_p^b, q_p^b)] - [\min(D_s, q_n)])}_{+} + c_s \underbrace{(q_n - q_p^b)}_{-}$$
$$+ (p - c_d) \underbrace{\Big(\min(D_d^b, (K - q_p^b)) - \min(D_d, (K - q_n))\Big)}_{-}$$

In this case we have the following observations:

- If $c_s = 0$ then,

$$\Pi_p^b(q_p^b) - \Pi_n(q_n) = \underbrace{p[\min(D_s + D_p^b, q_p^b)] - p[\min(D_s, q_n)]}_{+}$$
$$+ (p - c_d) \underbrace{\Big(\min(D_d^b, (K - q_p^b)) - \min(D_d, (K - q_n))\Big)}_{-} > 0$$

- If $c_s = p$ then,

$$\Pi_p^b(q_p^b) - \Pi_n(q_n) = \underbrace{p([\min(D_s + D_p^b, q_p^b)] - [\min(D_s, q_n)])}_{+} + p \underbrace{(q_n - q_p^b)}_{-}$$
$$+ (p - c_d) \underbrace{\Big(\min(D_d^b, (K - q_p^b)) - \min(D_d, (K - q_n))\Big)}_{-} < 0$$

Therefore we can conclude the result. By Eqs 1.2 and 1.3, we have

$$\begin{aligned}\frac{\partial \Pi_p^s(q)}{\partial q} - \frac{\partial \Pi_p^b(q)}{\partial q} =& c_g(\frac{\ell_p^b}{\ell_p^b + \ell_s})\frac{\partial[\min(D_s, q)]}{\partial q} - (p + c_h + c_g(\frac{\ell_s}{\hat{\ell}_p^b + \ell_s}))\frac{\partial \min(D_p^b, [q - D_s]^+)}{\partial q} \\ &+ (\frac{\ell_d}{\hat{\ell}_p^s + \ell_d})(c_d - c_p)\frac{\partial \min(D_d, [K - q]^+)}{\partial q} \\ &+ \left((p - c_d) + (\frac{\ell_d}{\hat{\ell}_p^s + \ell_d})(c_d - c_p)\right)\frac{\partial \min(D_p^s, [K - q - D_d]^+)}{\partial q} < 0\end{aligned}$$

Since $\Pi_p^s(q)$ and $\Pi_p^b(q)$ are concave in q and $\frac{\partial \Pi_p^s(q)}{\partial q}$ and $\frac{\partial \Pi_p^b(q)}{\partial q}$ are decreasing q, we can prove that $q_p^s \leq q_p^b$.

Proof of Proposition 8. The expected profit functions are as follows:

$$\begin{aligned}\Pi_p^s = \max_{q_p^s}\{&(p+c_h+c_g)\min(D_s,q)\\&+\left((p-c_d)+(\frac{\ell_d}{\hat{\ell}_p^s+\ell_d})(c_d-c_p)\right)\min(D_d^s+D_d,(K-q))\\&-(c_s+c_h)q-c_g[D_s]\}\end{aligned} \tag{1.9}$$

$$\begin{aligned}\Pi_p^b = \max_{q_p^b}\{&(p+c_h+c_g(\frac{\ell_s}{\hat{\ell}_p^b+\ell_s}))\min(D_s+D_p^b,q)\\&-(c_s+c_h)q-c_g(\frac{\ell_s}{\hat{\ell}_p^b+\ell_s})([D_s]+[D_p^b])\\&+(p-c_d)\min(D_d^b,(K-q))\}\end{aligned}$$

$$\begin{aligned}\Pi_p^s(q_p^s)-\Pi_p^b(q_p^b) = &\underbrace{(p+c_h+c_g)\min(D_s,q_p^s)-(p+c_h+c_g(\frac{\ell_s}{\hat{\ell}_p^b+\ell_s}))\min(D_s+D_p^b,q_p^b)}_{-}\\&+(p-c_d)\underbrace{\left(\min(D_d^s+D_d,(K-q_p^s))-\min(D_d,(K-q_p^b))\right)}_{+}\\&+(\frac{\ell_d}{\hat{\ell}_p^s+\ell_d})(c_d-c_p)\min(D_d^s+D_d,(K-q_p^s))\\&+(c_s+c_h)\underbrace{(q_p^b-q_p^s)}_{+}+c_g\left((\frac{\ell_s}{\hat{\ell}_p^b+\ell_s})[D_p^b]-(\frac{\hat{\ell}_p^b}{\hat{\ell}_p^b+\ell_s})[D_s]\right)\end{aligned}$$

Using the Envelope Theorem, we have $\frac{\partial(\Pi_p^s(q_p^s)-\Pi_p^b(q_p^b))}{\partial c_s} = q_p^b - q_p^s > 0$, so we prove that $\Pi_p^s(q_p^s)-\Pi_p^b(q_p^b)$ increases in c_s. Suppose $c_g \to 0$, $c_h \to 0$ and $c_d \to p$, then

$$\Pi_p^s(q_p^s)-\Pi_p^b(q_p^b) = \underbrace{p(\min(D_s,q_p^s)-\min(D_s+D_p^b,q_p^b))}_{-}+c_s\underbrace{(q_p^b-q_p^s)}_{+}$$

In this case we have the following observations:

- If $c_s = 0$ then, $\Pi_p^s(q_p^s)-\Pi_p^b(q_p^b) = \underbrace{p(\min(D_s,q_p^s)-\min(D_s+D_p^b,q_p^b))}_{-} < 0$

- If $c_s = p$ then, $\Pi_p^s(q_p^s) - \Pi_p^b(q_p^b) = p(\min(D_s, q_p^s) - \min(D_s + D_p^b, q_p^b) + \underbrace{(q_p^b - q_p^s)}_{+}) > 0$

Therefore we can conclude the result.

Sensitivity analysis for c_d Using the Envelope Theorem, we have

$$\frac{\partial(\Pi_p^s(q_p^s) - \Pi_p^b(q_p^b))}{\partial c_d} = -(\frac{\ell_p^s}{\ell_p^s + \ell_d}) \min(D_d^s + D_d, (K - q_p^s)) - \min(D_d, (K - q_p^b))) < 0$$

so we prove that $\Pi_p^s(q_p^s) - \Pi_p^b(q_p^b)$ decreases in c_d.

Proof of Proposition 10. We first investigate the RE equilibrium in different regions depending on the product valuation probability. With a homogeneous population of customers, we have the following observations in each segment below:

- If $\alpha < a_{1h} = \frac{h_o + \hat{\zeta} h_b + (1-\hat{\zeta})h_s}{(v-p)}$, omni-customers never purchase, $(\hat{\ell}_p^h, \hat{\ell}_d) = (0,0)$. Retailer expected profit is $\Pi_p^h(q) = p\min(D_s, q) - c_s q - c_h[q - D_s)]^+ - c_g[D_s - q]^+$. The equilibrium inventory level in the BM store is $q_p^h = \frac{(1-\lambda)(p+c_g-c_s)}{(p+c_g+c_h)}$.

- If $a_{1h} \le \alpha < a_{2h} = \frac{r_d + h_d - h_o - (1-\hat{\zeta})h_s - \hat{\zeta} h_b}{r_d}$, omni-customers order with the hybrid fulfillment option. $\hat{\ell}_p^h = \lambda[a_{2h} - a_{1h}]^+$ denotes the retailer's belief about the fraction of omnicustomers who order online and pick-up in store. Retailer's expected profit is

$$\begin{aligned}\Pi_p^h(q) =& p\min(D_s + D_p^h, q) - c_s q + (p - c_p)E\min\left(\frac{\hat{\ell}_p^h}{\hat{\ell}_p^h + \hat{\ell}_s}[(D_s + D_p^h) - q]^+, (K-q)\right) \\ &- c_h[q - (D_s + D_p^h)]^+ - c_g(\frac{\ell_s}{\hat{\ell}_p^h + \ell_s})[(D_s + D_p^b) - q]^+\end{aligned}$$

subject to $q \le K$ and $q > 0$. In the equilibrium the inventory allocation to the BM store is

$$
\begin{aligned}
q_p^h = &\frac{\frac{(p-c_p)(\ell_p^h-(1-\lambda))}{\ell^{h_p^2}M}}{\frac{(p+c_h)(\ell_p^h+(1-\lambda))(1-\lambda)c_g}{\ell_p^h(1-\lambda)(\ell_p^h+(1-\lambda))M^2}} \\
&+ \frac{\sqrt{\left(\frac{(p-c_p)(\ell_p^h-(1-\lambda))}{\ell^{h_p^2}M}\right)^2}}{\frac{(p+c_h)(\ell_p^h+(1-\lambda))+(1-\lambda)c_g}{\ell_p^h(1-\lambda)(\ell_p^h+(1-\lambda))M^2}} \\
&+ \frac{\sqrt{2\left(\frac{(p+c_h)(\ell_p^h+(1-\lambda))+(1-\lambda)c_g}{\ell_p^h(1-\lambda)(\ell_p^h+(1-\lambda))M^2}\right)}}{\frac{(p+c_h)(\ell_p^h+(1-\lambda))+(1-\lambda)c_g}{\ell_p^h(1-\lambda)(\ell_p^h+(1-\lambda))M^2}} \\
&\times \left(-c_s + c_p + \frac{(2K - M\ell_p^h)(p-c_p)(1-\lambda)}{2\ell^{h_p^2}M} + \frac{(1-\lambda)c_g}{\ell_p^h + (1-\lambda)}\right)
\end{aligned}
$$

where $\hat{\ell}_p^h = \ell_p^h = \lambda[a_{2h} - a_{1h}]^+ > 0$. The corresponding fill rate is $\zeta^h = (\frac{\ell_p^h}{\ell_p^h+\ell_s})\frac{\min(D_s+D_p^h, q_p^h)}{E(D_p^h)} = \frac{{q_p^h}^2(3\ell_p^h M - q_p^h)}{3{\ell_p^h}^2 M^3(1-\lambda)} > 0$.

- If $a_{2h} \leq \alpha < 1$, the omni-customers only order online with direct shipping. $\hat{\ell}_p^h = 0$ and $\hat{\ell}_d = \lambda[1 - a_{2h}]^+ > 0$ denotes retailer's belief about the fraction of omni-customers who purchase online with direct delivery in this segment.

$$
\begin{aligned}
\Pi_p^h(q) = &\; p\min(D_s, q) - c_s q - c_h[q - D_s]^+ - c_g[D_s - q]^+ \\
&+ \left((p - c_p)\frac{\hat{\ell}_p^h}{\hat{\ell}_p^h + \hat{\ell}_d} + (p - \varphi\bar{T}(\hat{\rho}_d))\frac{\hat{\ell}_d}{\hat{\ell}_p^h + \hat{\ell}_d}\right) E\min\left(D_p^h + D_d, (K - q)\right)
\end{aligned}
$$

subject to $q \leq K$ and $q > 0$. In the equilibrium the inventory allocation to the BM store is

$$
\begin{aligned}
q_p^h =& \frac{p+c_g+c_h}{(1-\lambda)M} \\
&+ \frac{(\ell_p^h(p-c_p)+\ell_d(p-c_o))(K-(\ell_p^h-\ell_d)M)}{\ell_p^h\ell_d(\ell_p^h+\ell_d)M^2} \\
&- \sqrt{\left(\frac{p+c_g+c_h}{(1-\lambda)M} + \frac{(\ell_p^h(p-c_p)+\ell_d(p-c_o))(K-(\ell_p^h-\ell_d)M)}{\ell_p^h\ell_d(\ell_p^h+\ell_d)M^2}\right)^2} \\
&- 2\frac{\ell_p^h(p-c_p)+\ell_d(p-c_o)}{\ell_p^h\ell_d(\ell_p^h+\ell_d)M^2} \\
&\times \left(p+c_g-c_s+\frac{(\ell_p^h(p-c_p)+\ell_d(p-c_o))(K^2-2\ell_p^h\ell_d M^2-2(\ell_p^h-\ell_d)KM)}{\ell_p^h\ell_d(\ell_p^h+\ell_d)M^2}\right)
\end{aligned}
$$

With a heterogeneous consumer population, these segments of behavior may coexist. Next, we analyze the RE equilibrium when omni-customers are heterogeneous in having high valuation α. Since $0 < q \leq K$, we can obtain

$$
\begin{aligned}
\frac{\partial^2\Pi_p^h(q)}{\partial q^2} = -\frac{1}{M^2}\Bigg(& \frac{3pM}{(1-\lambda)} \\
&+ \frac{6(\ell_p^s(p-c_p)+\ell_d(p-c_d)((1+\ell_p^s-\lambda)(K-q)-\ell_p^s\ell_d M)}{(\hat{\ell}_d\hat{\ell}_p^s(\hat{\ell}_d+\hat{\ell}_p^s))} \\
&+ \frac{3\left(\frac{c_g}{1+\ell_p^b-\lambda}+\frac{c_h}{1-\lambda}\right)q}{h} \\
&+ \frac{3p(q-\ell_p^b M)}{\ell_p^b(1-\lambda)}\Bigg) < 0
\end{aligned}
$$

One can verify that $\Pi_p^s(q)$ is a concave function for

$$
0 < q < \frac{2((1+\ell_p^s-\lambda)K-\ell_p^s\ell_d M)(\ell_p^s(p-c_p)+\ell_d(p-c_d))}{\ell_d\ell_p^s(\ell_d+\ell_p^s)\left(\frac{\ell_p^h p}{(1-\lambda)}+\ell_p^h\left(\frac{c_g}{1+\ell_p^b-\lambda}+\frac{c_h}{1-\lambda}\right)\right)} \leq K.
$$

Where $\frac{\partial\Pi_p^h(q)}{\partial q}$ is a quadratic function of q and has two roots as

$$
q_1^s = \frac{B-\sqrt{B^2-4AC}}{2A} > 0
$$

and

$$
q_2^s = \frac{B+\sqrt{B^2-4AC}}{2A} > 0,
$$

where

$$A = (\frac{(1+\ell_p^h-\lambda)(\ell_p^h(p-c_p)+\ell_d(p-c_o))}{\ell_d {\ell_p^h}^2(\ell_p^h+\ell_d)M^2} - (\frac{\frac{c_g}{1+\ell_p^h-\lambda}+\frac{p+c_h}{1-\lambda}}{2\ell_p^b M^2}) > 0$$

$$B = \frac{2\left((1+\ell_p^h-\lambda)K-\ell_p^h\ell_d M\right)\left(\ell_p^h(p-c_p)+\ell_d(p-c_o)\right)}{\ell_d {\ell_p^h}^2(\ell_p^h+\ell_d)M^2} > 0$$

$$C = p-c_s+\frac{(1-\lambda)c_g}{1+\ell_p^b-\lambda}+\frac{(\ell_p^h(p-c_p)+\ell_d(p-c_o))(2(1+\ell_p^h-\lambda)K^2-2\ell_d\ell_p^h KM-\ell_p^h\ell_d(1+2\ell_p^h-\lambda)M^2)}{2\ell_d {\ell_p^h}^2(\ell_p^h+\ell_d)M^2} > 0$$

Since $A>0$, $\frac{\partial\Pi_p^h(q)}{\partial q}$ is a convex function. We can show that for $q=0$, $\frac{\partial\Pi_p^h(q)}{\partial q} > 0$ and for $q=K$, $\frac{\partial\Pi_p^h(q)}{\partial q} < 0$. Therefore, $q_p^h = q_1^h = \frac{B-\sqrt{B^2-4AC}}{2A} > 0$ is the only feasible root. Which makes $\frac{\partial\Pi_p^h(q)}{\partial q} = 0$ and $q_p^h =_q \Pi_p^h(q)$.

$$\begin{aligned}
A &= (1+\ell_p^h-\lambda)K-\ell_p^h\ell_d M,\\
B &= \ell_p^h(p-c_p)+\ell_d(p-c_o),\\
C &= \ell_d {\ell_p^h}^2(\ell_p^h+\ell_d)M^2,\\
D &= \frac{c_g}{1+\ell_p^h-\lambda}+\frac{p+c_h}{1-\lambda},\\
E &= 2(1+\ell_p^h-\lambda)K^2-2\ell_d\ell_p^h KM-\ell_p^h\ell_d(1+2\ell_p^h-\lambda)M^2,\\
F &= p-c_s+\frac{(1-\lambda)c_g}{1+\ell_p^b-\lambda}+\frac{B\cdot E}{2C}.
\end{aligned}$$

$$q_p^h = \frac{2AB/C}{\left(\frac{AB/C-D}{2\ell_p^b M^2}\right)} - \frac{\sqrt{\left(\frac{2AB/C}{\left(\frac{AB/C-D}{2\ell_p^b M^2}\right)}\right)^2 - 4\left(\frac{AB/C-D}{2\ell_p^h M^2}\right)F}}{\left(\frac{AB/C-D}{2\ell_p^b M^2}\right)}.$$

In equilibrium, the retailer's beliefs is consistent with the outcome therefore $\hat{\ell}_d = \ell_d = \lambda[1-a_{2h}]^+, \hat{\ell}_p^h = \ell_p^h = \lambda[a_{2h}-a_{1h}]^+$. If $(v-p)(h_d+r_d) > (v-p-r_d)(\zeta^h h_b+(1-\zeta^h)h_s+h_o)$ then $(a_{2h}-a_{1h}) > 0$ and ℓ_p^h fraction of omni-customers choose the Hybrid option. If $h_d < \zeta^h h_b+(1-\zeta^h)h_s+h_o$, then ℓ_d fraction of omni-customers order online with direct delivery. Retailer will stock $q_p^h > 0$ in the BM store and $K-q_p^h$ in DC. If $c_s \leq p+\frac{(1-\lambda)c_g}{1+\ell_p^b-\lambda}+\frac{(\ell_p^h(p-c_p)+\ell_d(p-c_o))(2(1+\ell_p^h-\lambda)K^2-2\ell_d\ell_p^h KM-\ell_p^h\ell_d(1+2\ell_p^h-\lambda)M^2)}{2\ell_d {\ell_p^h}^2(\ell_p^h+\ell_d)M^2}$

then $q_p^h > 0$. In the equilibrium, the omni-consumer's belief about in-store pick-up confirmation need to be consistent with the outcome. Where

$$\hat{\zeta}^h = (\frac{\ell_p^h}{\ell_p^h + \ell_s})\frac{\min(D_s + D_p^h, q_p^h)}{E(D_p^h)} = \frac{(6\ell_p^h(1-\lambda)M^2 - {q_p^h}^2)q_p^h}{3{\ell_p^h}^2(1-\lambda)(1-\lambda+\ell_p^h)M^3} \geq 0$$

Which is indeed greater than 0. Thus, there are customers who are choosing to order through hybrid fulfillment and the equilibrium inventory allocation is $q_p^h > 0$ and $K - q_p^h > 0$. The retailer plays a best response given beliefs about consumers behavior and the consumers play a best response given beliefs about retailer behavior. Therefore, in RE equilibrium the omni-consumer's belief about BOPS order confirmation need to be consistent with the outcome. Note, in any participatory equilibrium, $\zeta^h > 0$, where $\hat{\zeta}^h = \zeta^h = (\frac{\ell_p^h}{\ell_p^h+\ell_s})\frac{\min(D_s+D_p^h, q_p^h)}{E(D_p^h)} = \frac{{q_p^h}^2(3\ell_p^h M - q_p^h)}{3{\ell_p^h}^2 M^3(1-\lambda)} > 0$ which is indeed greater than 0. Thus, there are customers who are willing to order with the hybrid option. If we check given ζ, under what condition omni-customers are willing to order BOPS, $U_h > 0$, gives us the condition $\alpha > \alpha' = \frac{r_d+h_d-h_o-(1-\zeta)h_s-\zeta h_b}{r_d}$, which is easy to check that this condition holds.

Proof of corollary 0.3. Based on the market segmentation under Hybrid strategy, we can conclude:

- The market expansion effect under Hybrid is greater than STS if $a_{1h} \leq a_{1s}$. We have $a_{1s} = \frac{h_o+h_s}{(v-p)}$ and $a_{1h} = \frac{h_o+\hat{\zeta}h_b+(1-\hat{\zeta})h_s}{(v-p)}$. It is easy to see that $\frac{h_o+\hat{\zeta}h_b+(1-\hat{\zeta})h_s}{(v-p)} < \frac{h_o+h_s}{(v-p)}$.

- The channel switching effect under Hybrid is greater than STS if $a_{2h} \geq a_{3s}$. We have $a_{2s} = \frac{r_d+h_d-h_s-h_o}{r_d}$ and $a_{2h} = \frac{r_d+h_d-h_o-(1-\hat{\zeta})h_s-\hat{\zeta}h_b}{r_d}$. It is easy to see that $\frac{r_d+h_d-h_s-h_o}{r_d} < \frac{r_d+h_d-h_o-(1-\hat{\zeta})h_s-\hat{\zeta}h_b}{r_d}$.

- The market expansion effect under Hybrid is greater than BOPS if $a_{1h} \leq a_{1b}$. We have $a_{1b} = \frac{\hat{\zeta}h_b+h_o}{\hat{\zeta}(v-p)}$. It is easy to see that $\frac{h_o+\hat{\zeta}h_b+(1-\hat{\zeta})h_s}{(v-p)} < \frac{\hat{\zeta}h_b+h_o}{\hat{\zeta}(v-p)}$.

- The channel switching effect under Hybrid is greater than BOPS if $a_{2h} \geq a_{2b}$. We have $a_{2s} = \frac{h_d+r_d-h_o-\hat{\zeta}h_b}{(1-\hat{\zeta})(v-p)+r_d}$. It is easy to see that $\frac{h_d+r_d-h_o-\hat{\zeta}h_b}{(1-\hat{\zeta})(v-p)+r_d} < \frac{r_d+h_d-h_o-(1-\hat{\zeta})h_s-\hat{\zeta}h_b}{r_d}$.

1.5.3 Details for sensitivity Analysis

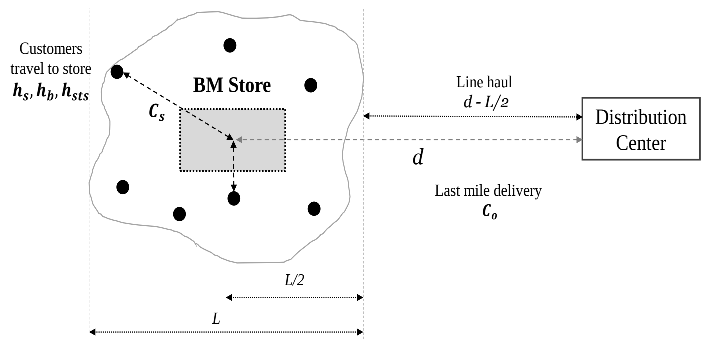
We first characterize how q_n, q_p^b,q_p^s and q_p^h are changing with respect to p. We can show that under all strategies $\frac{\partial q}{\partial p} > 0$ and therefore q_p^s, q_p^b and q_p^h are increasing in price p.

1.5.4 Details for Continuous Approximation model

In order to analyze the market, we are using a method called the continuum approximation approach, which has been used in previous studies by Cachon (2014), Belavina et al. (2017). Our assumption is that there are M potential customers evenly distributed throughout the service area, with a uniform density of demand points per square mile represented by the symbol ρ. The service region itself is homogeneous, with a total area of A. The total demand for the market, represented by the symbol D, can be estimated by multiplying the demand density ρ by the total area of the service region. Each customer will only purchase one product. To depict this, the retailer is located at the center of the circular area, as shown in figure 1.15.

The firm incurs transportation costs when delivering online orders to its customers and stores. These costs can be categorized into vehicle operating costs and driver wage costs, both of which are dependent on the length of the delivery routes. The cost for a direct shipment from the distribution center to the customer is determined by the number of units shipped and the distance between the two. To approximate travel distance, continuous approximation models use smooth functions such as a demand density function that varies slowly in time and location. Our analysis assumes that the service region is large relative to

Figure 1.15: The Circular Customer Distribution and Retailer's Logistics



the primary influence area of each distribution center, allowing us to formulate tractable models by considering densities instead of exact locations.

Daganzo (1984) shows that the travel distances for vehicle routing can be approximately proportional to the square root of the sizes of the area shapes. The length of the delivery routes is the sum of the trunk segment between the distribution center and routing zone, which must be traveled twice, and the routing segment. The retailer incurs a cost per unit of distance the truck travels, which includes fuel, labor, truck purchase, licensing, depreciation, etc. For every order delivered, the retailer incurs an average direct delivery cost. The average distance traveled to deliver an order can be approximated using the Euclidean distance metric and the number of orders delivered by one vehicle.

Belavina et al. (2017) proposed that the distance traveled to deliver orders in a region has two components: the average line-haul distance from the distribution center to the delivery region, which must be covered twice, and the optimal traveling salesman tour that visits each customer in the region. The effective density of potential customers for the retailer is $\hat{\rho}_d = l_d \rho$. The average distance between customers is approximated by the inverse of the square root of customer density. Belavina et al. (2017) define a metric-dependent constant as $\Lambda(k)$, which is approximately 0.67 for $k > 4$.

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Chapter 2

Omnichannel Fulfillment Strategies and Sales Credit Allocation

Abstract

Over the past few years, the world of retail has undergone a significant shift, integrating the physical and online shopping experience to create a seamless journey for customers. Ship-to-store (STS) and ship-from-store (SFS) are two omnichannel strategies that have emerged as essential tools for retailers. With STS, customers order online but pick up their items at a physical store, while SFS allows online orders to be fulfilled using the inventory of a physical store. By pooling inventory across channels, retailers can avoid stockouts and make the most of their entire inventory. However, integrating these strategies creates a complex problem of sales credit attribution, requiring a system that accurately assigns the proper recipient of sales credits. For instance, SFS is an effective way to avoid stockouts and improve the customer experience. However, it's essential to navigate sales credit allocation between the physical store and online channel to maximize the benefits of omnichannel retailing.

In this research, we investigate the optimal order quantity for each channel

and how to assign sales credit fairly and effectively. Using noncooperative game theory, we examine four scenarios: no cross-fulfillment, ship-to-store, ship-from-store, and hybrid fulfillment strategies. We also consider how customer preferences affect sales credit allocation across channels. Our analysis shows that inaccurate sales credit allocation can undermine the benefits of omnichannel strategies. Therefore, retailers need to optimize the sales credit allocation to determine the best approach for implementing the omnichannel fulfillment method. We consider various sales credit allocation rules and provide insights on the best practices for each fulfillment strategy, considering customers' shopping preferences.

2.1 Introduction

Omnichannel retailing is becoming increasingly important for retailers as customers are expecting a seamless shopping experience across all channels. To meet this demand, retailers need to ensure that their systems are integrated and that their customer service is consistent across all platforms. For instance, a customer should be able to buy a product in the store, pick it up at the store later, or have it delivered to their home, regardless of which channel they used to purchase it. While a lot has been studied about omnichannel strategies, there's a noticeable gap: how retail systems handle situations when products are out of stock. Addressing this is crucial because stockouts can affect customer satisfaction and business success in today's competitive retail environment.

According to a study by IHL Group, retailers worldwide lose more than $1.75 trillion due to overstocks and out-of-stocks (Ryan, 2015). Therefore retailers are increasingly using their inventory and order fulfillment capabilities to support omnichannel initiatives (Bell et al., 2014). In this light, omnichannel order fulfillment strategies, particularly ship-from-store (SFS) and ship-to-store (STS) have emerged as vital for enhancing this process and cultivating a productive omnichannel retail environment. These fulfillment strategies utilizes complete inventory and balances inventory levels across channels to provide a more flexible and responsive inventory management system that can react quickly to changing customer demands and prevent stockouts (Gao and Su, 2017a; Hu et al., 2022). As a result of the transition towards omnichannel retailing, GlobalData (2020) reports that the proportion of online sales supported by physical stores in the US reached 36% during the 2020 holiday season. In addition, online orders picked up from stores and online orders shipped from stores both increased by 103% and 80%, respectively (Unglesbee, 2021).

Implementing the SFS strategy enables physical retail stores to function as small-scale distribution centers. Online orders are ordinarily fulfilled from distribution center (DC), but if an item is stockout in the DC but available in the local store, the item can be dispatched from the store. This scenario prevents retailers from losing online sales due to warehouse stockouts and prevents customers from seeing discouraging "out of stock" notifications on websites, which usually redirect them to rival retailers (Bayram and Cesaret, 2021). Retailers such as Ulta Beauty Inc., Macy's Inc., and Tilly's Inc. are implementing SFS to expand their store fulfillment operations, joining merchants like Best Buy Co. and Nordstrom Inc. that have effectively used the strategy for years (Young, 2022).

Complementary to SFS is the ship-to-store (STS) approach, which involves customers ordering products online and having them shipped to a local store for pickup. In addition, when an item is not available in the local store, customers can choose to have it shipped to their preferred store. Companies like Target and Walmart have successfully implemented this model. If a product is unavailable in-store, Walmart associates can guide customers through the company's integrated system to order the item online for pickup at the same store or a different location (Walmart, 2018). Similarly, Target employees use handheld devices with an application called myCheckout to order out-of-stock items online for customers to pickup later at the store (Target, 2017).

Despite these advancements, one significant issue faced by retailers in the omnichannel retailing is the complex issue of omnichannel sales attribution. The retailer needs to determine whether credit for sales should be given to the online platform or the brick-and-mortar (BM) store that fulfilled the order. According to the annual Digital Commerce Survey conducted by (DigitalCommerce, 2016), many retailers (55%) do not (unfortunately) offer store employees incentives for fulfilling omnichannel orders. Offering incentives to store employees for fulfilling omnichannel orders, such as ship-from-store (SFS) or in-store pickup, is a strategy that can enhance the effectiveness of an omnichannel retail model.

In an omnichannel strategy, such as ship-from-store, store employees perform vital tasks like identifying, packaging, and shipping items, effectively transforming the traditional store into a mini-distribution center. These additional responsibilities require new skills and added effort. It is therefore important for retailers to consider providing incentives to boost their motivation and commitment to these strategies. According to a Forrester Consulting report, only 16% of retailers distribute revenues between channels, while 31% and 21% attribute revenue from such sales exclusively to either the online or store channel (ForresterResearch, 2014). Furthermore, Benes (2019) found that over 40% of merchants still assign credit to a single touchpoint, signifying that many retailers are yet to fully reconcile the interplay between online and offline channels in omnichannel retailing.

Formerly, sales attributing models were relatively simplistic and hinged on the belief that online shoppers exclusively used online platforms and offline shoppers only patronized physical stores. These attribution methods have obvious limitations. For instance, if the brick-and-mortar (BM) store gets credit for an STS sale, but the DC manager has to take on the expense of processing the order, and shipping the goods from the DC's inventory, the mismatched reward system could make tensions rise. If retailers fail to allocate sales to the appropriate channel or combination of channels, they risk creating disharmony in the goals and motives of the online and in-store management teams. When poorly resolved, channel conflicts can become a serious downside of an omnichannel approach. In most cases, this applied to situations where the online and offline channels were managed by separate departments (Gao and Su, 2017a) or by different companies with their own strategy and goals, such as Macy's, Saks, and Hudson's Bay (Kapner, 2021; Scott, 2021).

Inventory management policies will also be influenced by the relationship between the physical store and the online channel. It can be challenging to assess the appropriate credit for an omnichannel sale when it has some influence from both online and offline channels. This can result in channel managers under-ordering by some margin and in some cases, one channel sabotaging the sales of the other. In this research, we analyze the decision on a sales-credit allocation policy in omnichannel retailing. Sales attribution has only been studied in the marketing literature, and to the best of our knowledge, our research is the first to analyze this problem from a game theoretic perspective. We specifically seek to answer the following interconnected questions:

1. When an order is fulfilled using omnichannel strategies, e.g., STS or SFS, which channel should get credit for that sale? Is it the store where the customer picks up the order? Or is it the online channel that provides the information and processes the transaction? And what about the store that delivers an online order to the customer's address?

2. Under each fulfillment strategy, does the optimal sales allocation policy also reflect a fair allocation? What are the conditions under which the retailer's optimal credit allocation is also a fair sales allocation between the channels?

3. Are the various credit attribution policies implemented in practice, e.g., giving full credit to the BM store, giving full credit to the DC, or assigning equal credit to both, part of an optimal policy of a profit-maximizing omnichannel firm? What is the impact of the sales-credit allocation policy on retailer's profit under omnichannel strategies?

To answer these questions, we consider an inventory game between the store and the online channel of an omnichannel brand that sells a product to customers. The two channels are operated by two independent managers, and each channel's inventory decision affects the other channel's profit. We suppose that the market is made of two segments: (i) store-visiting customers who always prefer to visit the store to purchase or pick up the product; (ii) direct-shipping customers who prefer direct delivery to their address. As a benchmark, we first analyze an inventory game without any omnichannel strategy implementation. Next, we consider the STS, SFS, and hybrid fulfillment strategies, respectively, and analyze how the crediting of sales affects the channels' ordering decisions and the firms total profit. Our main contributions are the following:

- Prior work in omnichannel operations management is mostly based on a centralized supply chain, where the retailer decides the order quantities for the online and store channels. We focus on a decentralized system where each channel is managed by an independent team facing an uncertain demand and analyze how the order quantity of each channel is affected by the sales-credit attribution policies. The purpose of our work is not to determine whether a company should adopt STS, SFS or hybrid strategies, but to provide insights for companies that have decided to launch a fulfillment service on how to encourage collaboration rather than competition between channels.

  We observe that, when using ship-to-store and ship-from-store strategies, both channels tend to place lower orders compared to the order quantity in the benchmark model. This illustrates the enhanced efficiency in stock management achieved through omnichannel fulfillment strategies. By pooling inventory, retailers can utilize their online and physical store inventories interchangeably, reducing incidences of stockouts. Consequently, channels can place lower order quantities while still meeting customer demands efficiently.

- Multichannel attribution models in the marketing literature focus on how to allocate the advertising campaign budget to each marketing touchpoint, based on the role they play in the customer journey. These studies do not consider the impact of sales-credit allocation on the channels' decisions. In this research, we use a game theory approach with sales credit attribution between channels to investigate the optimal order-quantity decision of each channel under each omnichannel fulfillment strategy. We provide explanations on why each firm needs a different sales credit allocation based on the implemented fulfillment strategy and the combination of store-visiting and direct shipping customers in the market. An important insight is that the sales credit allocation is a key factor in determining whether the firm will benefit from the omnichannel strategies.

  The shopping preferences of customers have a significant impact on the profitability of implementing omnichannel fulfillments. When implementing the ship-to-store (STS) fulfillment strategy, an equal distribution of sales credits between online and in-store channels optimizes total retailer profit, regardless of market characteristics. For the ship-from-store (SFS) fulfillment strategy, it's still beneficial to allocate equal sales credits when there is a balance of customers preferring in-store visits and online shopping, or when the majority prefer online shopping. However, if the majority prefer in-store visits, it's wise to reduce the sales credits allocated to the store to avoid excessive inventory holding and associated costs. The Hybrid strategy combines elements of both STS and SFS, and it's most profitable for retailers to allocate higher credits to the demand fulfiller when the market has an equal balance of customer preferences or is store-dominant. Regardless of the strategy used, understanding and strategizing based on customer preferences helps streamline inventory levels and optimize sales credit allocation.

- In our game, we assume that the omnichannel firm first determines the sales credit policy, and next, the two channels make their decisions. Our third contribution is in providing the conditions under which the retailer's optimal sales credit allocation is also a fair allocation policy channels. We show that, in a market with equal proportions of store-visiting and direct-shipping customers, under all fulfillment strategies, the Nash bargaining solution (NBS) outcome is the most beneficial for the omnichannel company. We note that, under some conditions that will be explored, such an outcome is only implementable for a narrow range of parameter values.

The remainder of this paper is organized as follows. In Section 2.2, we review the most related literature. In Section 2.3, we provide preliminaries and introduce our key assumptions. In Section 2.4, we characterize the Nash equilibrium quantities under the STS, SFS, and hybrid fulfillment strategies, respectively, and analyze the NBS outcomes. In Section 2.5, we derive the optimal sales crediting policy and illustrate our results using realistic parameter values that represent a typical omnichannel setting. Concluding remarks are provided in Section 2.6.

2.2 Literature Review

This study examines the literature on omnichannel operations, sales compensation and attribution, and inventory transshipment. While inventory transshipment has been extensively studied in the past decade, omnichannel operations are a relatively new research stream. This article reviews each research stream, positions our work, and discusses our contributions to the literature.

2.2.1 Omnichannel Retailing

In the area of omnichannel retailing, there has been a recent surge of interest in operations management. Comprehensive reviews of the topic can be found in sources such as (Bell et al., 2016; Ishfaq and Bajwa, 2019; Davis-Sramek et al., 2020; Verhoef et al., 2015; Bijmolt et al., 2021). Empirical studies have shown that implementing various omnichannel fulfillment strategies, such as buy-online-pickup-in-store (Bell et al., 2014; Gallino and Moreno, 2014), ship-to-store (Ertekin et al., 2022), and offline showrooms (Bell et al., 2018), can have significant effects on sales. For example, Akturk et al. (2018) found that some customers switched from the online channel to the BM channel after the implementation of STS, which led to increasing BM store sales and lower online sales. Cao et al. (2016) also showed that in-store pickup provides customers with a new purchasing option, leading to an increase in sales. However, strategically allocating and using inventory across both online and offline channels remains a limited area of research. Hu et al. (2022) studied the positive pooling effect and negative depooling effect of BOPS, as well as its potential margin-loss and market-expansion effects.

Unlike previous studies that focused on the potential impact of implementing omnichannel fulfillment strategies on market expansion and customers' switching behavior by considering customer utility, our study examines an inventory game between channels to determine the best way to allocate sales credit. Thus, considering customer valuation would not affect our results.

Several papers have examined the integration of online demand with a network of physical stores (Jalilipour-Alishah et al., 2015; Govindarajan et al., 2021). Other analytical studies have explored how omnichannel strategies impact retailers' operational decisions and profitability. These studies cover various aspects, such as the effect of customer information provision (Gao and Su, 2017b), the impact of implementing BOPS on customer channel choice (Gao and Su, 2017a), the optimal BOPS service area for retailers (Jin et al., 2018), and return policy strategies for omnichannel retailers (Jin et al., 2020; Mandal et al., 2021). Gao et al. (2022) investigates how adopting three omnichannel retailing strategies, namely, showrooms, flexible returns, and fulfillment flexibility, influence a retailer's decisions regarding the number and size of physical stores.

The SFS option in operations management literature has received little attention. In a study by Lin et al. (2021), the impact of the ship-from-store-to-store option on a retailer's multi-period inventory decisions was explored. He et al. (2020) investigated the effects of opening physical stores and implementing the SFS option on consumer shopping behaviors. Bayram and Cesaret (2021) analyzed dynamic SFS fulfillment decisions, specifically which store to fulfill an online order from when it arrives. They found that implementing ship-from-store is not practical when a large number of stores are included in the network.

Most of the studies in the literature have focused on a centralized supply chain where the retailer determines the order quantity for both online and offline channels. However, our study is interested in a scenario where separate and independent teams manage each channel. By examining the functionality of two omnichannel fulfillment options, STS and SFS, we contribute to the literature by considering how inventory decisions are made under different sales allocation policies.

Previous research has not examined the specific allocation of sales credit between brick-and-mortar and online channels when implementing omnichannel strategies. While Gao and Su (2017a) briefly explored the allocation of BOPS revenue between channels, they only considered the store's inventory under centralized and decentralized systems. They found that the store was typically either overstocked or understocked compared to the centralized benchmark. They also showed that a simple revenue-sharing contract between channels could coordinate the decentralized system and align the incentives of the brick-and-mortar store with the entire organization, but they did not consider inventory allocation for the online channel. Our study takes a fresh perspective on omnichannel inventory management by examining a game between the brick-and-mortar store management team and the DC's management team. We investigate profit and inventory allocation for both the store and the online channel, taking into account the effects of each player's inventory decision on the other channel's profit. Additionally, we consider the implementation of different omnichannel strategies (STS and SFS) and their related operating costs for both channels.

While there are similarities between our work and salesforce compensation studies in marketing literature, we focus on the interactions between inventory and sales effort in a centralized setting. Recently, a few papers have considered limited inventory (Dai et al., 2021), but the inventory decision is typically set exogenously before the sales agent makes the effort decision. Li et al. (2020) studied the problem of designing compensation contracts to incentivize retail store managers, who then choose the effort level and order quantity. They found that store managers exert more effort under target contracts than under profit sharing when all else is equal.

2.2.2 Inventory Transshipment

Our study looks into lateral transshipment, which involves effectively moving inventory across multiple locations to meet consumer demand. This strategy can be used in omnichannel retail to fulfill orders from either market. We are particularly interested in examining transshipment between decentralized independent players and their optimal stocking and sharing decisions within the game theory framework. While many studies have focused on two-symmetric-retailer systems with identical costs or market demands, our research centers on transshipment in a dual-channel retail setting, where the inventory pooling benefits or transshipment operating costs are not symmetric.

Several related studies shed light on this topic, such as Yang and Qin (2007), which analyzes a case where online demand can be fulfilled from a network of

BM stores without physically transferring inventory between locations. Zhao et al. (2016) determine the optimal order quantities of different channels under a lateral inventory transshipment strategy, where a manufacturer forwards its online orders to an offline retailer. He et al. (2014) propose a quantity-discount contract to achieve coordination in a manufacturer's dual channel, while Seifert et al. (2006) consider a supply chain with multiple independent retailers and a single virtual store.

Most research on transshipment in a dual channel assumes that a single retailer manages both channels and makes centralized decisions to maximize total profit. However, we assume that the channels are managed by separate teams with distinct profit objectives. Transfer price, which is a fee charged by one store for transshipping products to another, has been studied as a coordinating mechanism by various researchers, including Rudi et al. (2001), Hezarkhani and Kubiak (2010), and He et al. (2014). Hu et al. (2007) demonstrated that coordinating prices may only exist for a narrow region in the parameter space in a two-location inventory model with asymmetric locations. They also found that transshipment costs have the most direct effect on the existence of coordinating prices. Li and Wei (2018) discuss the impact of bargaining power in a two-echelon supply chain comprising a manufacturer and two symmetric retailers with bidirectional transshipment. Katok and Villa (2022) use behavioural laboratory experiments to investigate how transfer prices should be set when ordering decisions are made by two symmetric local retailers. They find a positive relationship between transfer prices and ordering decisions and show that having retailers negotiate transfer prices performs well.

Li et al. (2020) empirically study transfer prices from a behavioural perspective and examine how the decisions of decentralized retailers can be affected by commitments in transfer price and sharing. They show that ordering decisions are influenced by the transfer price when order quantities are set after the transfer price. Derhami et al. (2021) propose a data-driven model to estimate product availability in a network of interconnected retailers within the customer's acceptable time frame. It is the only study we know that considers inventory transshipment in an omnichannel environment.

Our analysis sets itself apart from previous research by taking into account specific features of the omnichannel setup, such as uneven demands, fulfillment processes, and costs for both the store and online channels. Unlike prior studies, we do not assume that all unsatisfied demand will be fulfilled with transshipped inventory, as varying fulfillment waiting times can lead to only some customers choosing cross-channel fulfillment strategies in the event of a stock out. This research gap is particularly significant given the widespread availability of omnichannel services. None of the works we reviewed address revenue allocation between the store and online channels after implementing omnichannel strategies. Our innovative model provides valuable insights into the decisions of an omnichannel firm, as incorrect sales credit allocation between channels can harm the firm's overall profit.

2.3 Setup and Assumptions

We consider a manufacturer/brand/firm that sells a product to customers through online (e-commerce site) and offline (BM store) channels at price p. In the proposed model, the total market size is denoted as M. Within this market, there are two types of consumers, namely, store-visiting customers, representing a proportion ϕ of the total, and the remaining $1-\phi$ being direct-shipping customers. Store-visiting customers prefer to make purchases in the BM store or pick up their orders there. In contrast, direct-shipping customers always prefer to have their orders shipped to their address. A store-visiting customer typically lives near a BM store, prefers traditional shopping, or simply needs the product immediately or want to physically interact with products before buying them. This segmentation is consistent with empirical studies (Skrovan, 2017; Nageswaran et al., 2020).

We assume that both in-store and online demands follow a uniform distribution. We use only the Uniform distribution in our analytical derivations because of the complexity of optimal policies. A uniform distribution assumption can capture the essential dynamics of demand split between online channel and BM store. Specifically, the in-store demand, D_s, is uniformly distributed between 0 and ϕM, $D_s \sim U[0, \phi M]$. Similarly, the demand for online or direct-shipping, denoted as D_o is uniformly distributed between 0 and $(1-\phi)M$, i.e., $D_o \sim U[0,(1-\phi)M]$. Denote by p the fixed retail price.

The store and the online channel are operated by two separate management teams, each maximizing its own profit. The firm may hold inventory in the store and in a DC. Orders q_s and q_o for the two channels are placed before the selling season starts. Throughout the paper, the subscript o stands for the *o*nline channel (distribution center) and the subscript s refers the *s*tore. When fulfilling orders, we assume that the BM store and DC prioritize in-store and online demand, respectively. We consider four scenarios for fulfilling consumers' orders, that is, benchmark, STS, SFS, and a hybrid scenario.

Benchmark scenario: Each channel fulfills its own demand and faces a newsvendor-type problem where any excess demand is lost. This scenario allows us to measure the benefit of the three others.

STS scenario: To prevent customers from leaving stores empty-handed when encountering a stockout, the retailer offer buy at store, ship-to-store option. Although they are given the STS option, not all are willing to wait the shipping time for product delivery. We assume that only a fraction α of in-store customers switch to STS in case of an in-store stock out. The STS sales are given by

$$T_t = \mathbb{E}\min[\alpha(D_s - q_s)^+, (q_o - D_o)^+],$$

where the subscript t stands for ship *t*o store. We suppose that there are no back orders and that lack of available inventory equals lost sales.

SFS scenario: The store prioritise and serves all in-store customers, and the excess stock, if any, may be used to fulfill unsatisfied online orders by shipping products directly to the customer's address. The SFS sales are given by the minimum between the store's excess inventory and the online channel's excess demand, that is,

$$T_f = \mathbb{E} \min[(D_o - q_o)^+, (q_s - D_s)^+],$$

where the subscript f stands for ship from store. Our implicit (reasonable) assumption is that online customers are not aware of where the product is shipped from. Also, as they are not sensitive to the delivery waiting time, even if delivery from the BM store takes more time than delivery from the DC, we assume the retailer decides to fulfill all unsatisfied online customers through SFS in case of a DC stock-out.

Hybrid scenario: Both STS and SFS options are available. We refer to this model by h for *h*ybrid.

Due to the differences in fulfillment processes, the handling costs vary across the two channels. Let c_s and c_o denote in-store procurement cost and the online channel's direct shipping fulfillment cost (shipping cost for last mile delivery), respectively. If the BM store fulfills online orders with SFS, a cost c_f is incurred for each unit delivered to the customers. With SFS, not only are items shipped twice, proper structure and store associates are required to pick, pack, and ship single items from stores to the customer's address. Compared to DCs, most BM stores are poorly positioned for picking and packing orders for delivery. Therefore, we assume that $c_f \geq c_o$. If the online channel fulfills an order via STS, a cost c_t is incurred for each unit sent to store. From the retailer standpoint, one can verify that the retailer's logistics cost for last mile deliveries c_o is much higher than selling through the STS channel, i.e., $c_t < c_o$. To avoid trivial cases, we assume that $p > \max\{c_s, c_o\}$, which implies that p is also larger than c_f and c_t. To save on notation, let $m_o = p - w - c_o$ denote the online profit margin. Further, denote by η the store's holding cost for unused inventory.

Due to the lower rent and storage fees, we assume that the DC's holding is negligible and set equal to zero. In the STS, SFS, and hybrid scenarios, the profit of the BM store and the DC will depend on the sales credit share, that is, the compensation that a channel gets for fulfilling a customer's order received by the other channel. Denote by k_j this sales credit share, with $k_j \in [0, p]$ for $j = f, h, t$. The actual value of k_j can be determined by some ad-hoc rules, e.g., the full credit goes to where the demand originated, demand is fulfilled or is split half-and-half. Here,we first assume that k_j is exogenously determined by the retailer and then by implementing the Nash bargaining solution, we examine whether the optimal sales allocation policy also reflect a fair allocation. Table 2.1 summarises the notations used in the paper.

Table 2.1: Notation.

Symbol	Definition
w	Wholesale price
M	Market size
p	Retail price
c_s	In-store fulfillment cost (wholesale price + store's operating cost)
c_o	DC's direct shipping fulfillment cost, e.g., packing and delivery cost
c_t	STS fulfillment cost
c_f	SFS fulfillment cost
k_j	Sales credit share, for $j = f, h, t$.
ϕ	Fraction of customers choosing to visit or pick up in-store
α	Fraction of the store demand switched to STS in case of stock out
η	Holding cost

2.4 Model Analysis

In this section, we characterize the Nash equilibrium order quantities in the different fulfillment scenarios, and determine the sales credit using the NBS in the relevant scenarios. In all, but the benchmark, the expressions of the equilibrium quantities are very large and are not shown. Instead, their properties are highlighted and they are illustrated numerically.

2.4.1 Benchmark

To evaluate the benefits of the omnichannel strategies, we start by examining a benchmark system in which the firm's online and offline channels are independently managed, and cross-channel sales are not possible. The expected profits are given by

$$\Pi_s^b(q_s) = p\mathbb{E}\min(D_s, q_s) - c_s q_s - \eta\mathbb{E}[q_s - D_s]^+, \tag{2.1}$$

$$\Pi_o^b(q_o) = (p - c_o)\mathbb{E}\min(D_o, q_o) - wq_o, \tag{2.2}$$

where Π_s^b and Π_o^b represent the store's and the online channel's expected profit, respectively. The first two terms in (2.1) represent the expected profit from selling the product in the store, while the third term measures the expected inventory holding cost for leftover inventory. To have $q_o^b > 0$, we make the intuitive assumption that $p > w + c_o$. In (2.2), the first term represents the revenues and the second one the purchasing cost.

To save on notation, let $m_o = p - w - c_o$ denote the online profit margin. From the first-order optimality conditions, it is easy to verify that the optimal order quantities are given by

$$q_s^b = \phi M\left(\frac{p - c_s}{p + \eta}\right) > 0,\ q_o^b = (1 - \phi)M\left(\frac{m_o}{p - c_o}\right) > 0.$$

Without any surprise, q_s^b decreases with the cost c_s, and q_o^b decreases with w and c_o. Also, as expected, both order quantities increase with the retail price. Finally, q_s^b increases with the fraction of customers who choose to visit or pick up in-store, whereas q_o^b decreases with this proportion. Inserting the equilibrium quantities in the profit functions, we obtain

$$\Pi_s^b = \frac{\phi M(p - c_s)^2}{2(p + \eta)},\ \Pi_o^b = \frac{(1 - \phi)\, M m_o^2}{2\,(p - c_o)}.$$

2.4.2 Ship to Store (STS)

To handle in-store stock outs, STS is an option retailers can use to provide in-store pickup services. If the store can get the items transferred from the DC, the customer is given the option of coming back later to pick up their order (Peinkofer et al., 2022). Although it is not as good as using in-store inventory to fulfill the order, STS is an improvement over not providing the service at all or losing sales. The number of orders fulfilled by STS is defined by

$$T_t = \mathbb{E}\min[\alpha(D_s - q_s)^+, (q_o - D_o)^+].$$

Let k_t be the credit allocation rule under STS. Then, the store and the online channel earn $(p - k_t)$ and $(k_t - c_t)$ per STS sale, respectively. The expected profits of the store and the online channel under the STS strategy are given by

$$\Pi_s^t(q_s) = p\mathbb{E}\min(D_s, q_s) - c_s q_s - \eta\mathbb{E}[q_s - D_s]^+ + (p - k_t)T_t, \tag{2.3}$$

$$\Pi_o^t(q_o) = (p - c_o)\mathbb{E}\min(D_o, q_o) - wq_o + (k_t - c_t)T_t, \tag{2.4}$$

where

$$T_t = \begin{cases} \alpha(D_s - q_s), & \text{if } (q_o - D_o) > \alpha(D_s - q_s) > 0, \\ q_o - D_o, & \text{if } \alpha(D_s - q_s) > (q_o - D_o) > 0, \\ 0, & \text{otherwise.} \end{cases}$$

In (2.3) and (2.4), the terms $(p - k_t)T_t$ and $(k_t - c_t)T_t$ represent the expected profit from the STS sales for the store and online channel, respectively. We assume that $k_t \in [c_t, p]$ to guarantee that the online channel is willing to fulfill STS orders. Given our assumption that the two channels are managed independently, we seek a Nash equilibrium. All proofs in this paper are presented in Appendix.

Proposition 11. *If $\alpha(k_t - c_t) \leq (p - c_o)$, then there exists a unique Nash-equilibrium order quantities under STS fulfillment.*

Proposition 11 shows that a unique STS Nash equilibrium exists when the online channel's profit margin from direct delivery sales $(p - c_o)$ is higher than the profit margin of fulfilling STS orders $\alpha(k - c_t)$. This condition ensures that it is not always beneficial to the DC to sell through the STS strategy. The best response functions of the two players are given by

$$q_s^t(q_o^t) = \frac{2\phi(1-\phi)(p-c_s)M^2 - (p-k_t)\left(q_o^t\right)^2}{2(1-\phi)M(p+\eta)}, \tag{2.5}$$

$$q_o^t(q_s^t) = \frac{2\phi(1-\phi)(p-w-c_s)M^2 + \alpha(k_t-c_t)(\phi M - q_s^t)^2}{2\phi M(p-c_o)}. \tag{2.6}$$

Remark 1. *The equilibrium quantities are obtained by solving the above system, which requires finding the roots of a fourth-degree polynomial. As the resulting expressions are very long and do not give any qualitative insight, we do not show them. In the numerical examples, we always obtain only one root that leads to positive quantities and demands. The same situation happens in all omnichannel scenarios.*

To characterize the strategic interaction between the two players, we compute the derivatives of the reaction functions to obtain

$$\begin{aligned}
\frac{dq_s^t(q_o^t)}{dq_o^t} &= -\frac{(p-k_t)q_o^t}{(1-\phi)M(p+\eta)} < 0, \\
\frac{dq_o^t(q_s^t)}{dq_s^t} &= -\frac{\alpha(k_t-c_t)(\phi M - q_s^t)}{\phi M(p-c_o)} < 0,
\end{aligned}$$

which implies strategic substitution, that is, each manager's order quantity is decreasing in the other manager's order.[1] Solving the nonlinear response functions (2.5)-(2.6) for some special cases leads to the following observations:

1. If the firm gives the full credit $k_t = p$ to where the demand is fulfilled (to the DC/online channel), we obtain $q_s^t = q_s^b = \phi M\left(\frac{p-c_s}{p+\eta}\right)$. Which means the store orders the same quantity as in the benchmark. Note that the upper bound of the BM store's equilibrium order quantity is equal to the optimal order quantity in the benchmark model when $k = p$; otherwise, the store always orders less than the benchmark's optimal order quantity $q_s^t \leq q_s^b$.

[1] We recall that when the strategy of each player decreases (increases, independent) in the other player's strategy, then we have strategic substitution (complementary, no interaction).

2. If the firm allocates the credit to the store ($k_t = c_t$), then in the equilibrium would be $q_o^t = q_o^b = (1-\phi)M\left(\frac{m_o}{p-c_o}\right)$. In this scenario, the online channel orders the same quantity as in the benchmark. The upper bound of the equilibrium order quantity for the online channel is equal to the optimal order quantity for the benchmark model when $k = c_t$; The intuition behind this result is that the store is somehow incentivized to free ride the online channel.

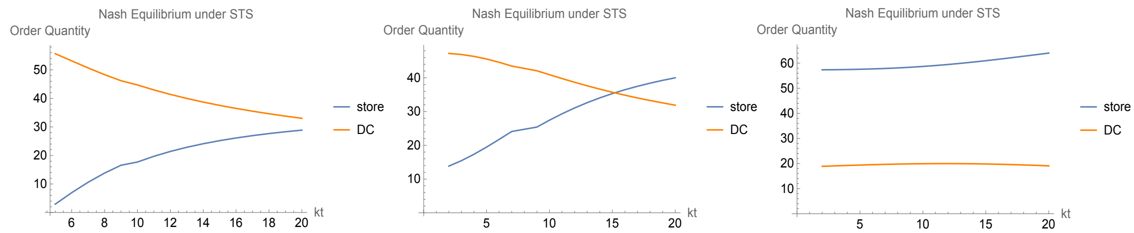
3. When $c_t < k_t < p$ in the intermediate case, both the store and online channels order less than the benchmark, resulting in a lower total order.

Given the very large expressions of the equilibrium quantities, any further analysis of how they vary with the credit allocation k_t must be done numerically. We use the following realistic parameter values that represent a typical omnichannel setting (Petersen, 2017), and satisfy the assumptions made previously:

$$w = 1, \;\; M = 100, \;\; p = 20, \;\; c_s = 2, \;\; c_o = 3, \;\; c_t = 1, \;\; c_f = 3, \;\; \alpha = 0.8, \;\; \eta = 1$$

Figure 2.1 shows the equilibrium orders as a function of the sales-credit allocation k_t for three different values of ϕ. We observe that the store's order quantities increase in the sales-credit allocation, whereas the DC's orders are decreasing in k_t. As k_t increases, the retailer gives higher sales credit to the DC for each STS order. As the retailer provides more sales credit to the DC, the BM store is incentivized to order more to reduce in-store stockouts.

Nash Bargaining Solution Under STS Strategy. Suppose that the retailer uses the Nash bargaining solution (NBS) to determine k_t. The rationale for doing so is the fairness property of the NBS, that is, both parties improve their outcomes equally with respect to the status quo point, which gives the payoffs the channels



(a) Online-dominant market $\phi = 0.3$

(b) Equal fraction of customers $\phi = 0.5$

(c) Store-dominant market $\phi = 0.7$

Figure 2.1: Comparison of Nash Equilibrium Order Quantities Under the STS Strategy for Various Fractions of Store-Visiting Customers

would obtain if there was no agreement. We let the players' benchmark profits to be the status quo. The determination of equilibrium quantities and the sales credit allocation involves the following two steps:

Operational stage: For any sales credit rule k_t set by the retailer, the store and the online channel simultaneously choose their order quantities q_s and q_o by solving the optimization problems

$$\max_{q_s} \Pi_s \left(q_s, q_o, k_t(q_s, q_o)\right),$$
$$\max_{q_o} \Pi_o \left(q_s, q_o, k_t(q_s, q_o)\right).$$

Denote by q_s^t and q_o^t the resulting optimal values.

Credit allocation stage: To determine k_t using the NBS, the retailer solves the following maximization problem:

$$\max_{k_t} \left[\Pi_s^t(k_t, q_s^t, q_o^t) - \Pi_s^b\right] \left[\Pi_o^t(k_t, q_s^t, q_o^t) - \Pi_o^b\right], \tag{2.7}$$

The objective of this maximization problem is to find the optimal value of k_t that maximizes the product of the two players' incremental profits with respect to the status quo which is their respective benchmark profits.

Next, the demands are realized and the two channels receive their sales credits.

Proposition 12. *Under the STS fulfillment strategy, if*

$$\frac{2p - c_s + \alpha c_t - \sqrt{(p - c_s + \alpha c_t) - \alpha^2 c_t p}}{p + 2(p - c_s + \alpha c_t) + \alpha^2 c_t} \leq \phi \leq \frac{2p(p - c_s)}{2p(p - c_s) + c_t(p - c_t)\alpha^2},$$

then there exists a unique NBS credit rule

$$\begin{aligned} k_t^*(q_s^t, q_o^t) = & \frac{p(\mathbb{E}\min(D_s, q_s^t) + T_t) - c_s q_s^t - \eta \mathbb{E}[q_s^t - D_s]^+ - \Pi_s^b}{T_t} \\ & - \frac{(p - c_o)\mathbb{E}\min(D_o, q_o^t) - w q_o - c_t T_t - \Pi_o^b}{T_t}, \end{aligned}$$

where $c_t < k_t^*(q_s^t, q_o^t) < p$.

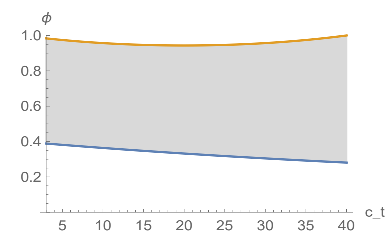
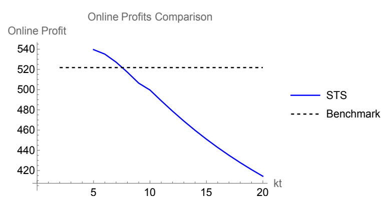


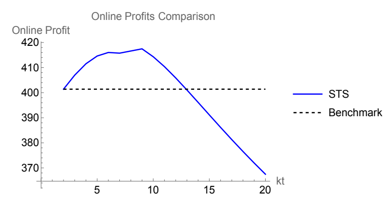
Figure 2.2: The Area Where a Unique NBS Exists Under STS (as a function of ϕ and c_t).

The proposition shows that the NBS exists only under some restrictions on the parameter values, written here in terms of the fraction of store-visiting customers. Using the values in (2.4.2), Figures 2.2 shows the area where the NBS exists for different values of ϕ and c_t. In particular, if ϕ is low enough, then the NBS no longer exists. also exhibits the NBS value when $\phi = 0.5$ and 0.7.

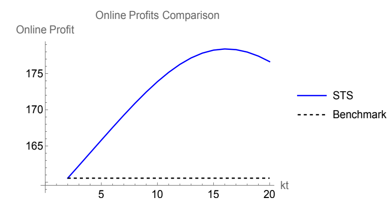
Impact of Sales Credit Allocation on Profits under STS. We examine how sales credit allocation (k_t) influences profitability for the online channel, the physical store and the retailer's total profit under the STS fulfillment strategy, considering varying market combinations. As obtaining a closed-form solution is impossible, we shall numerically determine the players' strategies and profits. To get a general sense of the results, we present in Figures 2.3 and 2.4 the impact of the STS fulfillment strategy on the profits of the online channel and the store for different fractions of store-visiting customers $\phi \in \{0.3, 0.5, 0.7\}$. Our analysis reveals that in predominantly online markets, where customers prefer to order online, the profit for the online channel decreases with an increase in sales credit allocation (k_t). Higher k_t values encourage stores to place larger orders, resulting in reduced stockouts in-store and reduced number orders fulfilled through STS. Conversely, in markets with high in-store shopping, the online channel earns more as k_t rises.



(a) Online-dominant market
$\phi = 0.3$



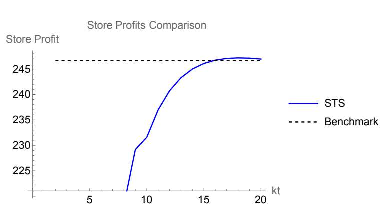
(b) Equal fractions of customers
$\phi = 0.5$



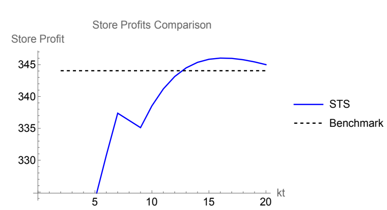
(c) Store-dominant market
$\phi = 0.7$

Figure 2.3: Expected Online Channel Profits Under STS Across Market Combinations

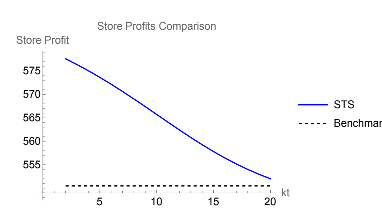
If online and in-store shoppers are evenly distributed, assigning more credit to physical stores will boost profits for the online channel. Conversely, channeling a greater portion of sales credit towards the online channel can enhance profits for brick-and-mortar stores. When the retailer increases the sales credit allocation to one channel, this effectively incentivizes the other channel to increase its inventory levels, which leads to increased sales. In a store-dominant market, the store's profit declines as k_t increases. The decline is primarily caused by increased order quantities in the store, which lead to higher holding costs.



(a) Online-dominant market
$\phi = 0.3$

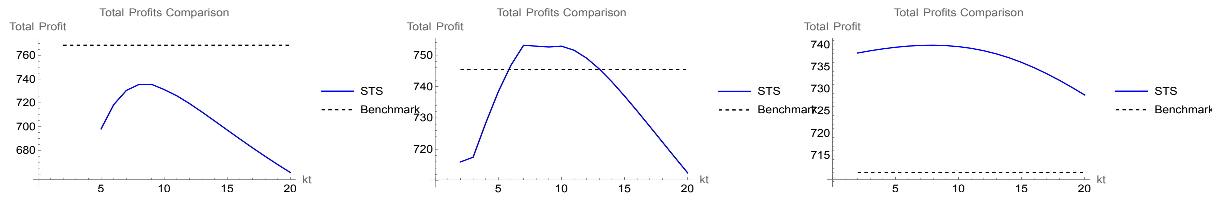


(b) Equal fraction of customers
$\phi = 0.5$



(c) Store-dominant market
$\phi = 0.7$

Figure 2.4: Expected Store Profits Under STS Across Market Combinations



(a) Online-dominant market $\phi = 0.3$

(b) Equal fraction of customers $\phi = 0.5$

(c) Store-dominant market $\phi = 0.7$

Figure 2.5: Expected Total Profits Under STS Across Market Combinations

Figure 2.5 shows that the total profit of the retailer is best optimized when an equal sales credit is given to both the online and store channels, regardless of the market combination. This highlights the importance of having a sales credit allocation policy that is strategically aligned to achieve maximum profitability under the STS fulfillment strategy.

2.4.3 Ship-from-Store (SFS)

In an SFS strategy, the BM store can accommodate unmet online orders, after satisfying in-store customers' orders. In this scenario, the store operates as a virtual distribution center and fulfills the online orders by shipping parcels to consumers. For instance, in addition to Walmart and Macy's, recently Zara and GAP, two apparel giants, have converted their physical outlets worldwide to fulfill online orders (Thau, 2018; Yang and Zhang, 2020). Most of the retailers do not deliver themselves, but rely on third party for shipping. Further, when the cost of staff handling SFS orders is factored in, SFS is more expensive than direct delivery from the DC (Reagan, 2017), that is, $c_f > c_o$. To ensure that the BM store is willing to fulfill SFS orders, we also assume that $k_f \in [c_f, p]$, where k_f is the allocation under SFS. The number of orders fulfilled via SFS is defined as

$$T_f = \begin{cases} (D_o - q_o), & \text{if } (q_s - D_s) > (D_o - q_o) > 0, \\ (q_s - D_s), & \text{if } (q_o - D_o) > (D_s - q_s) > 0, \\ 0, & \text{otherwise.} \end{cases}$$

The expected profit functions of the store and online channel under SFS are

$$\Pi_s^f(q_s) = p\mathbb{E}\min(D_s, q_s) - c_s q_s + (k_f - c_f)T_f - \eta\mathbb{E}[q_s - D_s - T_f]^+ \tag{2.8}$$

$$\Pi_o^f(q_o) = (p - c_o)\mathbb{E}\min(D_o, q_o) - wq_o + (p - k_f)T_f \tag{2.9}$$

The terms $(k_f - c_f)T_f$ and $(p - k_f)T_f$ are the expected profit from the SFS sales for each channel, while the last term in (2.8) represents one of the benefits of SFS, that is, the lower inventory-holding cost due to leveraging the store's inventory.

Proposition 13. *If $(p - k_f) \leq (p - c_o)$ and $k_f - c_f + \eta \geq 0$, then there exists a unique Nash equilibrium order quantities under STS fulfillment.*

The above proposition shows that the existence of a unique Nash equilibrium requires that (i) the online channel's marginal profit from direct delivery to be higher than the marginal profit from an SFS sale $\big(p - k_f \leq p - c_o\big)$, and (ii) the store's unit inventory holding cost be higher than the marginal cost of fulfilling an SFS order. The two inequalities can be rewritten in the compact form $k_f \in [\min\{c_o, c_f - \eta\}, p]$.

From the first-order equilibrium conditions, we obtain the players' best response functions, that is,

$$q_s^f(q_o^f) = \frac{(k_f + \eta - c_f)((1-\phi)M - q_o^f)^2 + 2\phi(1-\phi)(p - c_s)M^2}{2((1-\phi)(p+\eta)M)} \tag{2.10}$$

$$q_o^f(q_s^f) = \frac{2\phi(1-\phi)m_o M^2 - (p - k_f)(q_s^f)^2}{2\phi(p - c_o)M} \tag{2.11}$$

Their derivatives are given by

$$\begin{aligned}\frac{dq_s^f(q_o^f)}{dq_o^f} &= -\frac{(k_f+\eta-c_f)((1-\phi)M-q_o^f)}{(1-\phi)M(p+\eta)} < 0,\\ \frac{dq_o^f(q_s^f)}{dq_s^f} &= -\frac{(p-k_f)q_s^f}{\phi M(p-c_o)} < 0,\end{aligned}$$

which implies strategic substitution between the two decision variables. As in the STS scenario, each manager's order quantity is decreasing in the other manager's order. Solving (2.10)-(2.11) for some special cases leads to the following observations:

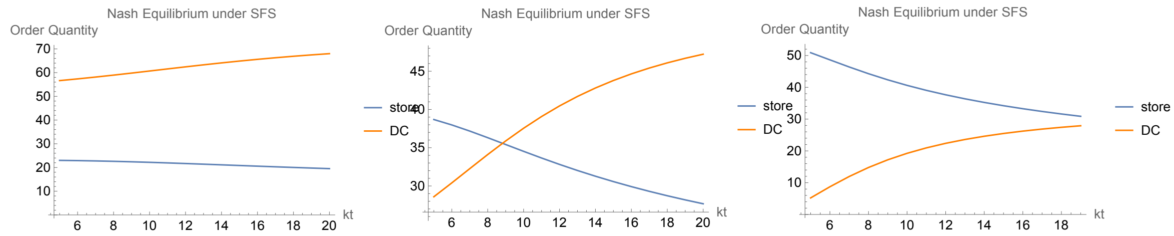
1. If the firm gives full credit to where the order was fulfilled (i.e., store), $k_f = p$, then we can observe that the upper bound of the DC's equilibrium order quantity is equal to the benchmark model's optimal order quantity, $q_o^f = q_o^b$. Otherwise, the DC always orders less than in the benchmark case $q_o^f < q_o^b$.

2. If $k_f = c_f - \eta$, that is, the DC gets the SFS sales credit, then the equilibrium quantity for the store becomes $q_s^f = q_s^b$. As SFS sales do not yield any marginal profit to the store, it orders the same quantity as in the benchmark.

3. Under an SFS strategy, if $c_f - \eta < k_f < p$, then the BM store (DC) always orders more (less) than the benchmark case. Thus, in an SFS scenario, the total order quantity is lower than in the benchmark scenario.

As illustrated in Figure 2.6, under an SFS fulfillment strategy, the DC's Nash equilibrium order quantities are increasing in the credit allocation k_f, whereas the store's quantities is decreasing in k_f. As k_f increases the retailer gives higher sales credit to the BM store for each SFS order. Thus, the DC increases inventory in order to reduce stockouts and lose profit.

Nash Bargaining Solution Under SFS Strategy. Similar to the previous scenario, the NBS allocation rule k_f is given by solving the following optimization problem:



(a) Online-dominant market
$\phi = 0.3$

(b) Equal fraction of customers
$\phi = 0.5$

(c) Store-dominant market
$\phi = 0.7$

Figure 2.6: Comparison of Nash Equilibrium Order Quantities Under SFS Strategy for Various Fractions of Store Visiting Customers

$$\max_{k_f} \left[\Pi_s^f(k_f, q_s^f, q_o^f) - \Pi_s^b\right] \left[\Pi_o^f(k_f, q_s^f, q_o^f) - \Pi_o^b\right].$$

Proposition 14. *Under the SFS fulfillment strategy, if*

$$\frac{(c_f - p)(c_f - \eta)}{(c_f - p)(c_f - \eta) - 2m_o(p - h)} \quad < \quad \phi \leq \frac{m_o + 2(c_f - \eta)}{2m_o + p - c_f + 4(c_f - \eta)} + \frac{\sqrt{(m_o + 2(c_f - \eta))^2 - (c_f - \eta)(2m_o + p - c_f + 4(c_f - \eta))}}{2m_o + p - c_f + 4(c_f - \eta)},$$

then there exists a unique NBS credit rule

$$k_f^*(q_s^f, q_o^f) = \frac{(p - c_o)\mathbb{E}\min(D_o, q_o^f) - wq_o + pT_f - \Pi_o^b}{T_f} - \frac{p\mathbb{E}\min(D_s, q_s^f) - c_s q_s^f - \eta E[q_s^f - D_s - T_f]^+ - c_f T_f - \Pi_s^b}{T_f}$$

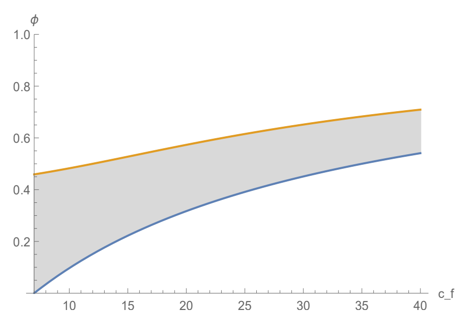
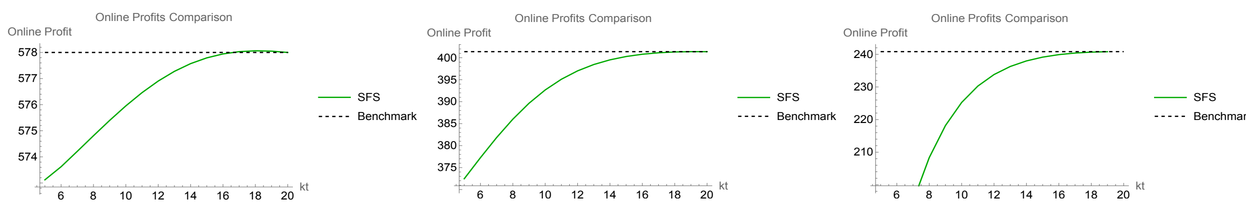


Figure 2.7: The Area Where a Unique NBS Exists Under SFS as a Function of the ϕ and Operating Costs c_f

Figure 2.7 shows the area where the NBS exists as a function of the fraction of store-visiting customers (ϕ) and the operating costs of SFS fulfillment strategy c_f. We observe that the higher the ϕ, the larger must be the value of c_f for the NBS to exist. Also, no solution exists for too high a value of ϕ.

Impact of Sales Credit Allocation on Profits under SFS. We examine how sales credit allocation (k_f) affects the expected profit under the SFS strategy. Our analysis will give us a better understanding of this impact in different market compositions. Figure 2.8 indicates that as the sales credit allocation for SFS orders, k_f, increases, the profit of the online channel also increases. However, implementing SFS for the online channel might not always be advantageous compared to the benchmark model. It seems to be beneficial only when the market is predominantly online.



(a) Online-dominant market $\phi = 0.3$

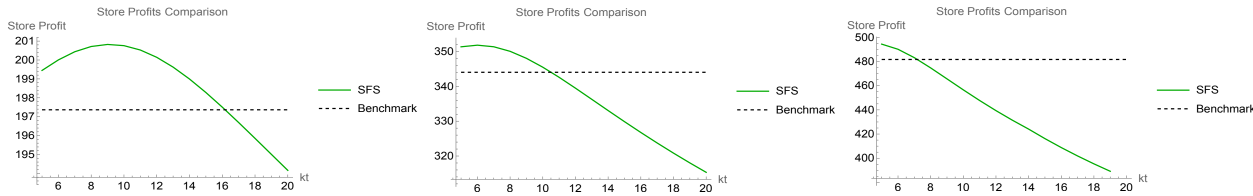
(b) Equal fraction of customers $\phi = 0.5$

(c) Store-dominant market $\phi = 0.7$

Figure 2.8: Expected Online Channel Profits Under SFS Across Market Combinations

According to Figure 2.9, the store's profit under SFS fulfillment is higher when the retailer allocates a lower sales credit (k_f) to the store for fulfilling SFS orders, particularly in markets predominantly consisting of store-visiting customers. Under the SFS strategy, the store's order quantity is inversely related to k_f. At lower k_f values, the store is incentivized to stock higher quantities. Since most customers visit the store, the store enjoys higher profits and sales.

Figure 2.10 shows that in a balanced market, the retailer benefit most from an equal distribution of sales credit between the online channel and the BM. Simi-

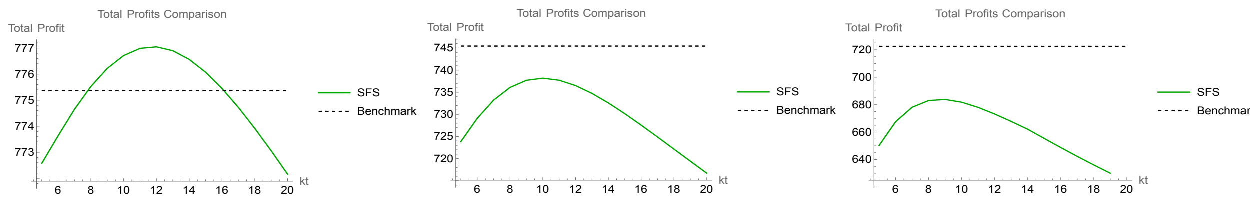


(a) Online-dominant market
$\phi = 0.3$

(b) Equal fraction of customers
$\phi = 0.5$

(c) Store-dominant market
$\phi = 0.7$

Figure 2.9: Expected Store Profits Under SFS Across Market Combinations



(a) Online-dominant market
$\phi = 0.3$

(b) Equal fraction of customers
$\phi = 0.5$

(c) Store-dominant market
$\phi = 0.7$

Figure 2.10: Expected Total Profits Under SFS Across Market Combinations

larly, when the market is primarily online-dominated, the figure shows that an equal sales credit allocation is still optimal. Our findings suggest that the retailer should give the online channel a higher share of sales credit when store-visiting customers dominate the market. Our findings suggest that retailers should give the online channel a higher share of sales credit when store-visiting customers dominate the market. Because of its dominant customer base, the store acts as a virtual DC, making order fulfillment more efficient online. To maximize profits, retailers must assess the market dynamics to determine the most profitable allocation scheme.

2.4.4 Hybrid Fulfillment Strategy

Consider now a hybrid strategy that consists in implementing both STS and SFS fulfilment options. Retailers can offer STS to deal with in-store stockouts and SFS to deal with DC stockouts without contradiction. As the name implies, hybrid strategy switches between STS and SFS depending on the situation. These strategies offer a better solution for both in-store stockouts and DC stockouts, resulting in a more cohesive customer experience. Recall that the number of orders fulfilled via STF and SFS are defined by $T_t = \mathbb{E}\min[\alpha(D_s - q_s)^+, (q_o - D_o)^+]$ and $T_f = \mathbb{E}\min[(D_o - q_o)^+, (q_s - D_s)^+]$. To incentivize both channels to fulfill the STS and SFS orders, we assume that $k_h \in [\max\{c_t, c_f\}, p]$, where k_h is the allocation in this scenario. The expected profits for the store and online channel under a hybrid strategy are as follows:

$$\Pi_s^h(q_s) = p\mathbb{E}\min(D_s, q_s) - c_s q_s + (p - k_h)T_t + (k_h - c_f)T_f - \eta\mathbb{E}[q_s - D_s - T_f]^+ \tag{2.12}$$

$$\Pi_o^h(q_o) = (p - c_o)\mathbb{E}\min(D_o, q_o) - wq_o + (k_h - c_t)T_t + (p - k_h)T_f \tag{2.13}$$

In (2.12) and (2.13), the third and fourth terms represent the expected profit from STS and SFS sales for each channel, respectively.

Proposition 15. *Under the following parameter restrictions,*

$$\begin{aligned} (p - k_h) &\leq (k_h + \eta - c_f), \\ (p - k_h) &\leq \alpha(k_h - c_t) \leq (p - c_o), \end{aligned}$$

there exists a unique Nash equilibrium order quantity for both the store and for the DC.

The first condition in Proposition 15 means that the store gains more from SFS fulfillment than it does from STS fulfillment. The second condition stipulates that the marginal gain of the online channel is higher under STS than under an SFS fulfillment strategy. From the first-order equilibrium conditions, we get the following response functions:

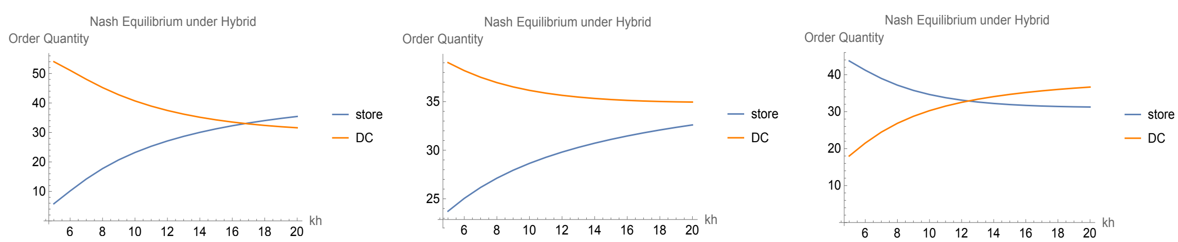
$$\begin{aligned} q_s^h(q_o) &= \frac{2\phi(1-\phi)(p - c_s)M^2 + (k_h + \eta - c_f)(M(1-\phi) - q_o)^2 - (p - k_h)q_o^2}{2M(1-\phi)(p + \eta)}, \\ q_o^h(q_s) &= \frac{2\phi(1-\phi)m_o M^2 + \alpha(k_h - c_t)(M\phi - q_s)^2 - (p - k_h)q_s^2}{2M\phi(p - c_o)}. \end{aligned}$$

Computing their derivatives, we obtain

$$\begin{aligned}
\frac{dq_s^h(q_o^h)}{dq_o^h} &= -\frac{(k_h+\eta-c_f)((1-\phi)M-q_o^h)+(p-k_h)q_o^h}{(1-\phi)(p+\eta)M} < 0, \\
\frac{dq_o^h(q_s^h)}{dq_s^h} &= -\frac{\alpha(k_h-c_t)(\phi M-q_s^h)+(p-k_h)q_s^h}{\phi(p-c_o)M} < 0.
\end{aligned}$$

As in the two other scenarios, we have strategic substitution between the two decision variables.

If $k = p$, meaning that the channel fulfilling the demand gets full credit, i.e., the store for SFS sales and the online channel for STS sales, then $q_s^h(q_o) = q_s^f(q_o)$ and $q_o^h(q_s) = q_o^t(q_s)$. We conclude that, under a hybrid fulfillment strategy, the store and the online channel always order lower quantities than in the benchmark, that is, $q_s^h < q_s^b$ and $q_o^h < q_o^b$. As illustrated in Figure 2.11, in an online dominated market, the BM store's order quantity increases with k_h, while DC's order quantity decreases. When compared to a store-dominant market, DC's order quantity is increasing in k_h.



(a) Online-dominant market $\phi = 0.3$

(b) Equal fraction of customers $\phi = 0.5$

(c) Store-dominant market $\phi = 0.7$

Figure 2.11: Comparison of Nash Equilibrium Order Quantities Under Hybrid Strategy for Various Fractions of Store-Visiting Customers

Nash Bargaining Solution Under a Hybrid Strategy. Under a hybrid system, the NBS sales credit is obtained by solving the following optimization problem:

$$\max_{k_h} \left[\Pi_s^h(k_h, q_s^h, q_o^h) - \Pi_s^h\right] \left[\Pi_o^h(k_h, q_s^h, q_o^h) - \Pi_o^b\right].$$

Proposition 16. *Under the hybrid fulfillment strategy, if*

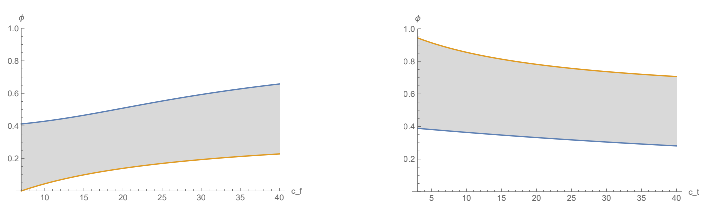
$$\phi \geq \frac{(2p + c_t\alpha - c_s) - \sqrt{(2p + c_t\alpha - c_s)(p + c_t\alpha - c_s) - p(p - c_s + c_t\alpha(1+\alpha))}}{(2p + c_t\alpha - c_s) + (p - c_s + c_t\alpha(1+\alpha))}, \tag{2.14}$$

$$\phi < \frac{m_o + 2(c_f - \eta) - \sqrt{(p - c_s - w + 2(c_f - \eta))^2 - (c_f - \eta)(2(m_o + c_f - \eta) + p + c_f - \eta)}}{2(p - c_o - w + c_f - \eta) + p + c_f - \eta} \tag{2.15}$$

then there exists a unique NBS allocation rule given by

$$\begin{aligned} k_h^*(q_s^h, q_o^h) = & \frac{p\mathbb{E}\min(D_s, q_s^h) - c_s q_s^h - \eta E[q_s^h - D_s - T_f]^+ + pT_t - c_f T_f - \Pi_s^b}{T_t - T_f} \\ & - \frac{(p - c_o)\mathbb{E}\min(D_o, q_o^h) - wq_o^h + pT_f - c_f T_f - \Pi_o^b}{T_t - T_f}. \end{aligned}$$

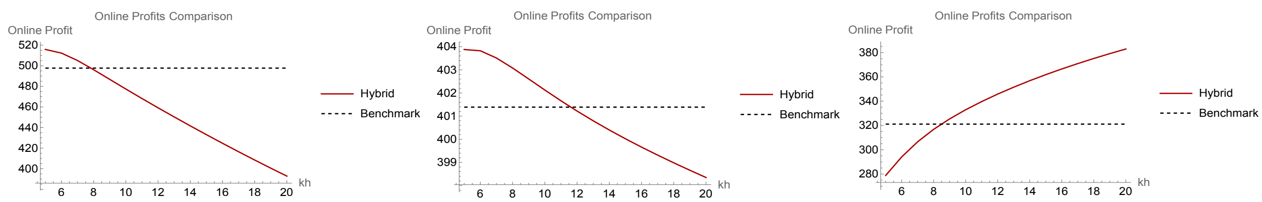
With the conditions in Proposition 16 we can obtain the area where the NBS exists is shown in Figure 2.12. We note that such a solution does not exist in markets with a very small or very large fraction of store-visiting customers. In such situations, the firm is better off allocating a higher sales credit to the channel fulfilling the order.



(a) c_f (b) c_t

Figure 2.12: The Area Where a Unique NBS Exist Under a Hybrid Strategy, as a Function of the ϕ and the Operating Costs c_f and c_t

Impact of Sales Credit Allocation on Profits under Hybrid Strategy. Figure 2.13 illustrates the asymmetric impact of sales credit allocation (k_h) on the profitability of the store and the online channel under the hybrid strategy. In markets with a low and balanced proportion of store-visiting customers, the online channel exhibits increased profitability when a larger sales credit allocation is allocated the origin of the demand (the DC in the case of SFS, or the store in the case of STS). Interestingly, the store experiences the opposite effect, its profitability diminishing under similar conditions.



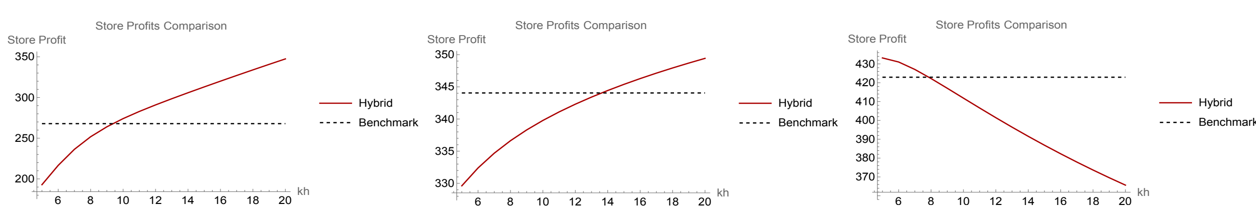
(a) Online-dominant market
$\phi = 0.3$

(b) Equal fraction of customers
$\phi = 0.5$

(c) Store-dominant market
$\phi = 0.7$

Figure 2.13: Expected Online Channel Profits Under Hybrid Across Market Combinations

According to 2.14 in a store-dominant market, the store's profit is significantly higher when the retailer attributes greater sales credit to the origin of the demand. Figure 2.15 indicates that in online-dominant markets, retailers can maximize their overall profitability by evenly distributing sales credit between their the store and online channel. However, retailers operating in balanced or store-dominant markets may be able to boost profitability by favoring demand fulfillers through biased allocation.

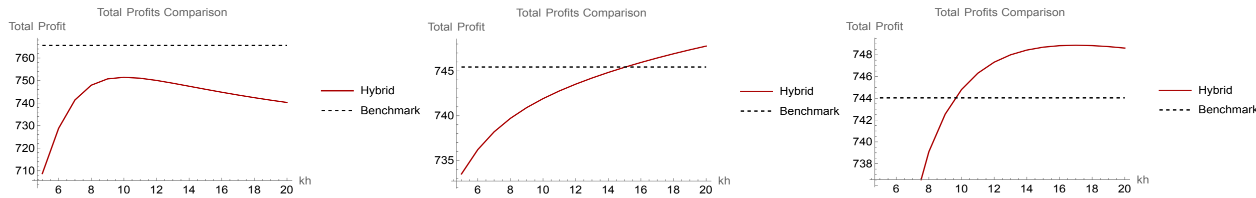


(a) Online-dominant market
$\phi = 0.3$

(b) Equal fraction of customers
$\phi = 0.5$

(c) Store-dominant market
$\phi = 0.7$

Figure 2.14: Expected Store Profits Under Hybrid Across Market Combinations



(a) Online-dominant market $\phi = 0.3$

(b) Equal fraction of customers $\phi = 0.5$

(c) Store-dominant market $\phi = 0.7$

Figure 2.15: Expected Total Profits Under Hybrid Across Market Combinations

2.5 Numerical Analysis

To complement our findings, we present a numerical study of the inventory game between channels using the parameter values in (2.4.2). Table 2.2 compares the equilibrium order quantities and expected profits when the credit allocation is given by the NBS, for $\phi = 0.5$. The total expected profit of the firm from both channels is given by $\Pi_s + \Pi_o$.

Table 2.2: Equilibrium Quantities, NBS, and Optimal Profits under different fulfillment strategies for various fractions of store-visiting customers

ϕ	Fulfillment	Credit allocation ratio (k/p)	NBS	q_s	q_o	Π_s	Π_o	$\Pi_s + \Pi_o$
0.3	Benchmark			28.33	61.38	240.83	521.80	762.63
	STS	0.45	0.53	16.56	46.20	229.17	506.40	735.58
	SFS	0.55		25.70	54.38	519.15	244.15	763.31
	Hybrid	0.5		20.78	42.25	249.09	496.73	745.82
0.5	Benchmark			40.47	47.22	344.04	401.389	745.43
	STS	0.45	0.56	24.09	43.48	337.37	415.75	753.13
	SFS	0.5	0.43	34.52	37.54	345.51	392.67	738.18
	Hybrid	1	0.63	32.60	34.95	349.42	398.34	747.76
0.7	Benchmark			64.76	18.88	550.47	160.55	711.03
	STS	0.4		58.09	19.78	569.03	170.89	739.93
	SFS	0.35		44.20	7.433	513.63	122.72	636.35
	Hybrid	1		31.32	34.14	405.74	343.97	749.71

Besides presenting the initial numerical results, we perform a series of experiments to investigate how the costs of operating a physical store and an online channel (such as c_o, c_s, η), as well as the expenses of fulfilling orders through STS and SFS (c_t, c_f), the size of the market (M), and the distribution of demand function, affect the optimal policy.

2.5.1 Sensitivity Analysis on the Distribution of Demands

Here we challenge the assumption that both in-store and online demand follows a uniform distribution. To address this, we're using the flexible Beta distribution to better capture variations in demand patterns and offer a more nuanced understanding of sales credit allocation. The probability density function (pdf) of the Beta distribution is defined as:

$$f(x;\alpha,\beta) = \frac{x^{\alpha-1}(1-x)^{\beta-1}}{B(\alpha,\beta)}$$

where α and β are the shape parameters, and $B(\alpha,\beta)$ is the Beta function defined as

$$B(\alpha,\beta) = \int_0^1 t^{\alpha-1}(1-t)^{\beta-1}dt$$

We use the Beta distribution to model demand, which is bounded between 0 and 1. However, when it comes to in-store customers, demand falls within the range of $[0, \phi M]$, while for online customers, the range is $[0, (1-\phi)M]$. To account for these differences, we apply a linear transformation to the variables and adjust the Beta distribution accordingly. For store customers, we use $X_{\text{s}} = X_{\text{beta}}\phi M$, and for direct-shipping customers, we use $X_{\text{o}} = X_{\text{beta}}(1-\phi)M$. X_{beta} is a random variable that follows the Beta distribution, and we can alter the distribution's shape by adjusting its parameters to model various demand scenarios. In Figure 2.16, we analyze three cases: (1) when $\alpha = \beta$, (2) when $\alpha < \beta$, and (3) when $\alpha > \beta$.

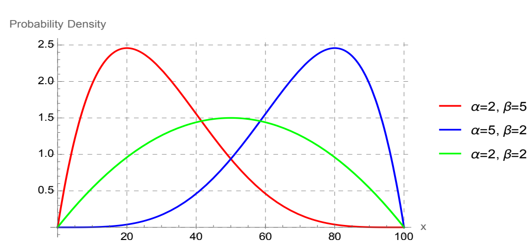
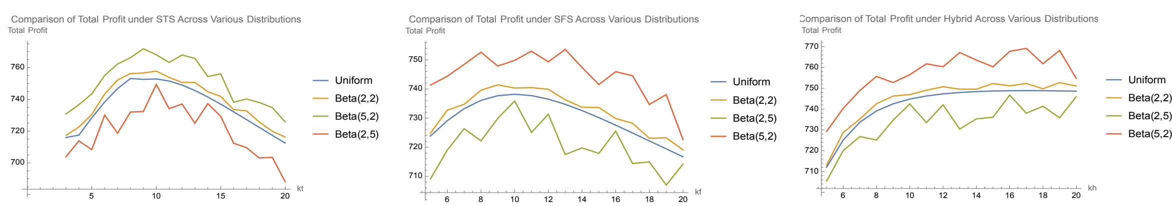


Figure 2.16: Probability density function of the scaled beta distribution with different parameter values.

We updated our computational model by replacing the uniform distribution with the scaled Beta distribution for both in-store and online demands. We also recalculated the expected value of sales and the expected value of transshipment using the new distributions. The Beta distribution introduces complexities that make it difficult to calculate the expected values analytically. In order to estimate the values of interest for each instance, we use Monte Carlo simulation. Gradient descent is then used to find the Nash equilibrium of ordering quantities, updating them until convergence is reached. Using the data from Figure 2.16, we examined the total profit for STS, SFS, and Hybrid scenarios. We found that a similar pattern emerged for all distributions. For instance under Hybrid strategy, when the market is balanced ($\phi = 0.5$), it is optimal to allocate full credit to the demand fulfiller in all distributions.

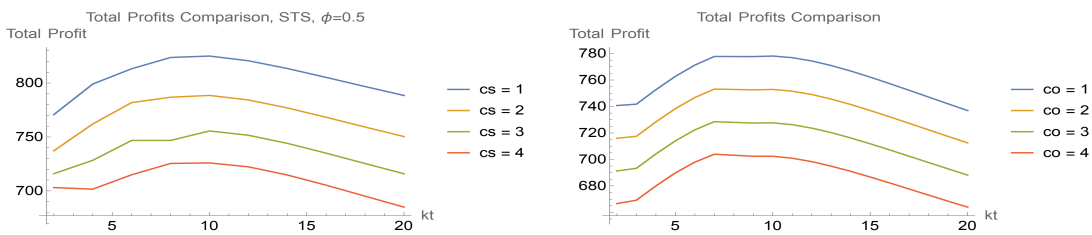


(a) STS (b) SFS (c) Hybrid

Figure 2.17: Total Profit with Various Distributions Functions when $\phi = 0.5$.

2.5.2 Sensitivity Analysis on the Value of Fulfillment Costs

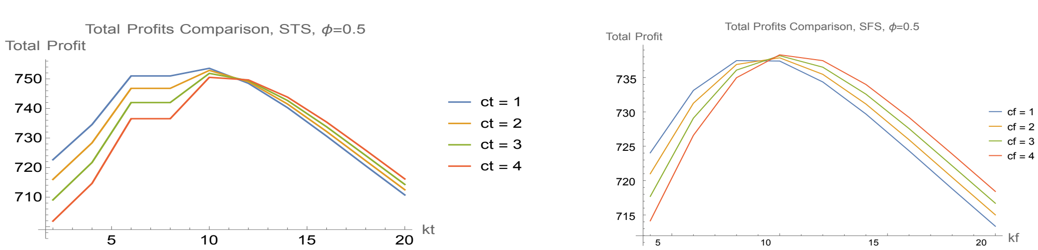
One of the main contributors to high operating costs for stores are their physical locations. Urban areas with large populations tend to have expensive rent and overhead costs. Additionally, if the store sells perishable goods or items that require specific storage conditions, holding costs can also be a significant expense. Figure 2.18 the solid lines indicate the overall profit gained through the STS strategy in a market with a balanced combination (ϕ) of store-visiting and direct shipping customers. As expected, profits decrease as operating costs increase. However, the optimal sales allocation policies remain constant across all scenarios.



(a) In-store fulfillment cost, c_s (b) direct shipping fulfillment cost, c_o

Figure 2.18: Sensitivity of Sales Credit Allocation Policy to Operating Costs.

The cost of omnichannel fulfillment presents a challenge for retailers. STS fulfillment can be more cost-effective, if it leverages economies of scale. For example, transfering STS orders with routine replenishments from the distribution center (DC) to the store can reduce transportation costs significantly. However, frequent transshipments, involving smaller and more frequent shipments, might erode the economies of scale. This is because smaller shipments often utilize more expensive transportation methods, inflating the costs from DC to stores. SFS can also be expensive if stores lack the infrastructure to facilitate direct deliveries to customers. In these cases, third-party services may be necessary, but they are generally more expensive. If stores rely heavily on third-party services, service charges can increase, leading to higher costs.



(a) STS fulfillment cost, c_t (b) SFS fulfillment cost, c_f

Figure 2.19: Sensitivity of Sales Credit Allocation Policy to Operating Costs.

Figure 2.19, examines the total profit of a retailer across a range of potential STS and SFS fulfillment costs, with a constant value of $\phi = 0.5$. The curve's consistent shape indicates that the ideal sales credit policy remains unchanged. However, it is possible for increased omnichannel fulfillment costs to shift the peak of the curve. Our findings suggest a slight tendency towards allocating more credit to the demand fulfiller as STS and SFS fulfillment expenses increase.

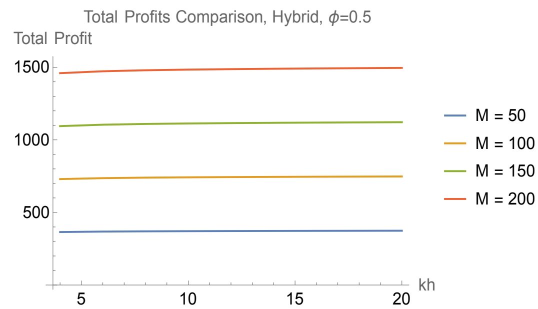


Figure 2.20: Sensitivity Analysis on Market size

We also analyze how market size affects the hybrid scenario in figure 2.20. It's evident that the retailer's profit increases with a larger market size as there are more sales. However, the slope of the curve remains consistent for different values of M.

2.5.3 Managerial Insights

In the current retail environment, it's crucial for retailers to understand and strategize based on customer preferences in implementing omnichannel strategies. Many retail businesses struggle to attribute sales and revenue in a multi-channel environment. In practice, many companies give annual performance bonuses to employees in various departments of the company (e.g., store operations, e-commerce). The bonus is primarily based on conventional metrics such as the sales and profit generated within the four walls of the store, without considering how that impact extends to other channels. Some sophisticated omnichannel retailers are designing innovative programs for incentivizing store associates. According to the 2016

Customer Experience/Unified Commerce Survey from BRP, here are some of the compensation plans used to reward cross-channel sales: (1) sharing credit for the sale equally across the channels involved; (2) adjusting store labor costs to compensate for orders fulfilled from a store; (3) a commission to the sales associate for all online sales, to the store closest to that customer's location; and (4) hiring separate employees to handle omnichannel fulfillment. While these methods simplify the allocation process, they may not always capture the nuances of sales generation and customer preferences, potentially leading to lost revenue and and incomplete customer insights.

In this paper we show that by carefully analyzing market preferences, retailers can choose the most suitable sales credit allocation strategy, which can significantly impact profitability. For the STS strategy, an equal distribution of sales credits between online and in-store channels optimizes total retailer profit, regardless of market characteristics. However, the optimal sales credit allocation for SFS strategy varies based on customer preferences and market dynamics. Employing a hybrid strategy allows for flexibility and the potential to maximize overall profitability through a balanced or slightly biased sales credit allocation, responding dynamically to the market composition. However, managers must be cautious while strategizing sales credit allocations in markets with high operating costs to ensure profitability.

Retailers face a challenge when it comes to the cost of omnichannel fulfillment. STS fulfillment can be more cost-effective when leveraging economies of scale. However, frequent transshipments may diminish these economies of scale. For SFS fulfillment, it can also be costly if stores lack the infrastructure to facilitate direct deliveries to customers. Our research indicates that as STS and SFS fulfillment expenses increase, there is a tendency to allocate more credit to the demand fulfiller. To succeed in this competitive market environment, managers must adopt a data-driven, flexible approach that takes customer preferences and market dynamics into consideration while balancing sales credits and fostering cooperation between online and offline channels.

2.6 Conclusion

This paper investigates the problem of designing a proper sales-credit allocation policy in omnichannel retailing. We study the impact of the sales credit allocation policy on the retailer's profit under different omnichannel strategies. We develop an inventory game between the store and the online channel, where each channel is managed by separate and independent teams. We consider four scenarios for fulfilling consumers' orders, that is, a benchmark, ship-to-store, ship-from-store, and hybrid scenarios. We assume that the omnichannel firm first determines on its sale-credit allocation policy, and next, the two channels make their ordering decisions. We characterize the Nash equilibrium of order quantities and find that, under all fulfillment strategies, each channel's order quantity is decreasing in the other's order.

For some special cases, we derive bounds on the values of equilibrium order quantities and discuss some insights. Our analysis shows that the fraction of store-visiting customers in the market plays a critical role in determining the allocation of the cross-channel sales profit, and consequently, whether a retailer will benefit from implementing omnichannel strategies. Later we consider the possibility of negotiation between the store and the online channel over the sales credit attribution and investigate a fair sales credit allocation policy across channels. We showed that a fair allocation by NBS is not guaranteed to exist and that it can only be implemented for a narrow range of parameters, such as fulfilling costs.

The results of our numerical analysis indicate that, when implementing the STS strategy, a balanced approach to sales credit allocation can increase a retailer's overall profitability, regardless of the market type. The profits of online and physical store channels vary depending on the degree of sales credit allocated. However, profitability increases when sales credits align with the predominant customer preference in the market, whether online or in-store. In addition, high sales credit allocation encourages stores to order larger quantities, which can increase holding costs in a store-dominant market and reduce the store's profit. Under to the SFS strategy, when the market is balanced, retailers benefit the most from distributing sales credit equally between their online channel and physical stores. Even in a market where online sales dominate, equal sales credit allocation is still the best approach. However, our research indicates that if customers predominantly visit physical stores, retailers should allocate a higher share of sales credit to their online channel.

For the hybrid strategy, an even distribution of credits between online and physical store channels optimizes overall profitability in online-dominant markets. Meanwhile, in balanced or store-dominant markets, a bias in allocation towards the demand fulfiller is more profitable. Finally, sensitivity analyses reveal challenges in omnichannel fulfillment arising from high operational costs influenced by factors such as geographical location and the nature of goods sold. The profitability tends to favour a slight bias towards the demand fulfiller in cases of rising STS and SFS fulfillment costs.

We believe that there are many directions in which to extend this work. Future research may introduce layers of complexity, such as capacity constraints or behavioral effects. Another interesting avenue is to study the inventory game when each channel has asymmetric information on the demand. In addition, in this paper, we only consider credit sharing between a DC and one BM store. It would be interesting to investigate ship-from-store-to-store sales credit allocation between DC and multiple stores (Li, 2020). Finally, empirical analysis could be useful to determine if our results can be confirmed in a much more realistic setting.

2.7 Appendix

Proof for Proposition 11. The expected profit function of the store and the online channel can be rewritten as

$$\begin{aligned}\Pi_s^t(q_s) &= \frac{p}{\phi M}\left(\int_0^{q_s} x dx + \int_{q_s}^{\phi M} q_s dx\right) - c_s q_s \\ &+ \frac{(p-k)}{\phi M(1-\phi)M}\left(\int_0^{q_o}\int_{q_s}^{\frac{(q_o-y)}{\alpha}+q_s} \alpha(x-q_s)dxdy + \int_0^{q_o}\int_{\frac{(q_o-y)}{\alpha}+q_s}^{\phi M}(q_o-y)dxdy\right) \\ &- \frac{\mu}{\phi M}\int_0^{q_s}(q_s-x)dx.\end{aligned}$$

$$\begin{aligned}\Pi_o^t(q_o) &= \frac{p-c_o}{(1-\phi)M}\left(\int_0^{q_o} y dy + \int_{q_o}^{(1-\phi)M} q_o dy\right) - wq_o \\ &+ \frac{k-c_t}{\phi M(1-\phi)M}\left(\int_{q_s}^{\phi M}\int_0^{q_o-\alpha(x-q_s)} \alpha(x-q_s)dydx + \int_{q_s}^{\phi M}\int_{q_o-\alpha(x-q_s)}^{q_o}(q_o-y)dydx\right).\end{aligned}$$

(i) Existence. For $p > c_o$, it is easy to check that $\frac{\partial^2\Pi_s^t}{\partial(q_s)^2} = -\frac{(p+\eta)}{\phi M} < 0$ and $\frac{\partial^2\Pi_o^t}{\partial(q_o)^2} = -\frac{(p-c_o)}{(1-\phi)M} < 0$, which means that Π_s^t is a concave function of q_s for any given q_o and Π_o^t is a concave function of q_o for any given q_s. Therefore, there exists at least one Nash equilibrium in pure strategies.

(ii) Uniqueness. To establish the uniqueness of the Nash equilibrium, it suffices to show that the reaction functions are monotone, and the absolute value of the slope is less than 1 (Cachon and Netessine, 2006). Under the parameter restriction $\alpha(k-c_t) \leq (p-c_o)$, we clearly have

$$\left|\frac{\partial q_s(q_o)}{\partial(q_o)}\right| = \frac{(p-k)}{(p+\eta)}\frac{q_o}{(1-\phi)M} < 1 \quad \text{and} \quad \left|\frac{\partial q_o(q_s)}{\partial(q_s)}\right| = \frac{\alpha(k-c_t)}{(p-c_o)}\frac{(\phi M-q_s)}{\phi M} < 1.$$

Proof for Proposition 12. The store's response functions can be written as

$$q_s^1(q_o) = \frac{-B-\sqrt{B^2-4AC}}{C},$$
$$q_s^2(q_o) = \frac{-B+\sqrt{B^2-4AC}}{C},$$

where

$$\begin{aligned}
A &= -(\phi M^2(2(p-c_s)(1-\phi)-c_t\phi\alpha^2)+(2c_t\alpha\phi M-pq_o)q_o),\\
B &= M((p+\eta)(1-\phi)-\alpha^2 c_t\phi)+\alpha c_t q_o,\\
C &= \alpha^2 c_t
\end{aligned}$$

- Since $C>0$, it suffices to prove that $A\leq 0$, then $q_s^1(q_o)\leq 0$ and $q_s^2(q_o)\geq 0$.
- Next, we show that there exist $q_o=q_o^{'}$ such that, if $q_o\leq q_o^{'}$, then $A\leq 0$. By solving $A=0$ we have

$$q_o^{'}=\frac{\alpha\phi c_t M+\sqrt{(\alpha\phi c_t M)^2+\phi p M^2(2(1-\phi)(p-c_s)-\phi\alpha^2 c_t)}}{p}$$

Since q_o is always smaller than the online market share, $(1-\phi)M$, then if we find under which conditions $(1-\phi)M\leq q_o^{'}$ then $q_o(q_s)\leq q_o^{'}$. As long as the market share fraction of store-visiting customers is such that the following condition holds, then, $q_o(q_s)\leq q_o^{'}$.

$$\frac{(2p-c_s+\alpha c_t)-\sqrt{(p-c_s+\alpha c_t)^{-}\alpha^2 c_t p}}{p+2(p-c_s+\alpha c_t)+\alpha^2 c_t}<\phi<\frac{2p(p-c_s)}{2p(p-c_s)+c_t(p-c_t)\alpha^2}$$

The DC's response functions can be written as

$$\begin{aligned}
q_o^1(q_s) &= \frac{-D-\sqrt{D^2-4FG}}{G},\\
q_o^2(q_s) &= \frac{-D+\sqrt{D^2-4FG}}{G},
\end{aligned}$$

where

$$F=-\alpha(2M^2\phi(1-\phi)(p-c_o-w)-\alpha c_t(\phi M-q_s)^2)$$

$$D=\alpha(pq_s-\phi c_o M)$$

$$G=p$$

- Since $G>0$, therefore it suffices to prove that $F\leq 0$, then we can show that $q_o^1(q_s)\leq 0$ and $q_o^2(q_s)\geq 0$.

- F is a convex quadratic function of q_s and $F(q_s = 0) < 0$, therefore it is easy to check that if F is also negative in the upper bound of q_s then, for all values of q_s, $F < 0$.

- Now considering the related parameter restrictions, we can show that, for all values of q_s, $q_o^2(q_s) - q_o^{'} < 0$, which implies that $A \leq 0$ hence, $q_s^1(q_o) \leq 0$ and $q_s^2(q_o) \geq 0$.

Proof for Proposition 13. Under the SFS fulfillment strategy, the expected profit function of the store and the online channel can be rewritten as

$$\begin{aligned}
\Pi_s^f(q_s, q_o) =& p\left(\frac{1}{\phi M}\right)\left(\int_0^{q_s} x dx + \int_{q_s}^{\phi M} q_s dx\right) - c_s q_s \\
&+ (k - c_f)\left(\frac{1}{\phi M}\right)\left(\frac{1}{(1-\phi)M}\right) \\
&\times \left(\int_{q_o}^{(1-\phi)M}\int_0^{q_s-(y-q_o)}(y - q_o)dxdy + \int_{q_o}^{(1-\phi)M}\int_{q_s-(y-q_o)}^{q_s}(q_s - x)dxdy\right) \\
&- \mu\left(\frac{1}{\phi M}\right)\left(\int_0^{q_o}\int_0^{q_s}(q_s - x)dxdy + \int_{q_o}^{(1-\phi)M}\int_0^{q_s-(y-q_o)}((q_s - x) - (y - q_o))dxdy\right)
\end{aligned}$$

$$\begin{aligned}
\Pi_o^f(q_o, q_s) =& (p - c_o)\left(\frac{1}{(1-\phi)M}\right)\left(\int_0^{q_o} y dy + \int_{q_o}^{(1-\phi)M} q_o dy\right) - w q_o \\
&+ (p - k)\left(\frac{1}{\phi M}\right)\left(\frac{1}{(1-\phi)M}\right) \\
&\times \left(\int_0^{q_s}\int_{q_o}^{(q_o-x)+q_s}(y - q_o)dydx + \int_0^{q_s}\int_{q_o+(q_s-x)}^{(1-\phi)M}(q_s - x)dydx\right)
\end{aligned}$$

Existence: For $p > c_o$, it is easy to check that the second-order partial derivatives of $\Pi_s^{sfs}(q_s)$ and $\Pi_o^{sfs}(q_o)$ are negative:

$$\frac{\partial^2 \Pi_s^{sfs}(q_s)}{\partial (q_s)^2} = -\frac{p + \eta}{M\phi} < 0,$$
$$\frac{\partial^2 \Pi_o^{sfs}(q_o)}{\partial (q_o)^2} = -\frac{p - c_o}{M(1-\phi)} < 0,$$

implying that Π_s^{sfs} is a concave function of q_s for any given q_o and Π_o^{sfs} is a concave function of q_o for any given q_s. Therefore, there exists at least one Nash equilibrium in pure strategies.

Uniqueness: For $c_o < k < p$, we clearly have

$$\left|\frac{\partial q_s^f(q_o)}{\partial(q_o)}\right| = \frac{(k+\eta-c_{sfs})}{(p+\eta)}\frac{((1-\phi)M-q_o)}{(1-\phi)M} < 1,$$
$$\left|\frac{\partial q_o^f(q_s)}{\partial(q_s)}\right| = \frac{(p-k)}{(p-c_o)}\frac{q_s}{M\phi} < 1.$$

Therefore, the equilibrium is unique.

Proof for Proposition 14. The BM store's response functions can be written as

$$q_s^1(q_o) = \frac{-B-\sqrt{B^2-4AC}}{C},$$
$$q_s^2(q_o) = \frac{-B+\sqrt{B^2-4AC}}{C},$$

where
$$A = -(2\phi(1-\phi)M^2(p-c_s) - (c_f-\eta)(M(1-\phi)-q_o^f)^2),$$
$$B = ((p-\eta)q_o^f + M\eta(1-\phi)),$$
$$C = p.$$

Since $B > 0$ and $C > 0$, therefore it suffices to prove that $A \leq 0$, then $q_s^1(q_o) \leq 0$ and $q_s^2(q_o) \geq 0$.

A is a convex quadratic function of q_o and $A(q_o = 0) < 0$, therefore it is easy to check that if A is also negative in the upper bound of q_o^f, then, for all values of q_o^f, $A \leq 0$. Therefore $q_s^2(q_o^f) \geq 0$.

The DC's response functions can be written as

$$q_o^1(q_s) = \frac{-D-\sqrt{D^2-4GF}}{G},$$
$$q_o^2(q_s) = \frac{-D+\sqrt{D^2-4GF}}{G},$$

where

$$
\begin{aligned}
&F = -((1-\phi)((2\phi(p-c_o-w)-(c_f-\eta)(1-\phi))M^2+2(c_f-\eta)q_s^f M)-p(q_s^f)^2),\\
&D = M((c_f-\eta)(1-\phi)-(p-c_o)\phi)-(q_s^f(c_f-\eta),\\
&G = (c_f-\eta).
\end{aligned}
$$

If $G > 0$ and $F \leq 0$, then $q_o^1(q_s) \leq 0$ and $q_o^2(q_s) \geq 0$. This implies the existence of a unique response function that leads to a unique Nash equilibrium.

F is a convex function of q_s^f. We can show that if $\phi > \frac{c_f-\eta}{2(p-c_o-w)+(c_f-\eta)}$, then $F(q_s^f = 0) < 0$. In addition, we can show that there exists $(q_s^f)' > 0$ such that, if $q_s < (q_s^f)'$, then $F < 0$. By solving $F = 0$, we find that if $\phi > \frac{(c_f-p)(c_f-\eta)}{(c_f-p)(c_f-\eta)-2(p-c_o-w)(p-h)}$, then $(q_s^f)' > 0$.

$$
(q_s^f)' = \frac{-(c_{sfs}-\eta)(1-\phi)+\sqrt{((c_{sfs}-\eta)(1-\phi))^2+p(1-\phi)\times}}{p}
$$
$$
\times(M^2(2(p-c_o-w)\phi-(c_{sfs}-\eta)(1-\phi)).
$$

Since q_s^f is always smaller than the store-visiting market share, ϕM, then if we find under which conditions $\phi M \leq (q_s^f)'$, then $q_s^f < (q_s^f)'$. We show that as long as the fraction of store-visiting customers is such that the following condition holds, then $q_s^f < (q_s^f)'$.

$$
\phi > \frac{(c_f-p)(c_f-\eta)}{(c_f-p)(c_f-\eta)-2(p-c_o-w)(p-h)}
$$

$$
\phi \leq \frac{(p-c_o-w)+2(c_f-\eta)+\sqrt{((p-c_o-w)+2(c_f-\eta))^2-(c_f-\eta)(2(p-c_o-w)+(p-c_f)+4(c_f-\eta))}}{2(p-c_o-w)+(p-c_f)+4(c_f-\eta)}
$$

Proof for Proposition 15.

$$\begin{aligned}
\Pi_s^h(q_s,q_o) &= p(\frac{1}{\phi M})\left(\int_0^{q_s} x dx + \int_{q_s}^{\phi M} q_s dx\right) - c_s q_s \\
&+ (p-k)(\frac{1}{\phi M})(\frac{1}{(1-\phi)M})\left(\int_0^{q_o}\int_{q_s}^{\frac{(q_o-y)}{\alpha}+q_s} \alpha(x-q_s)dxdy + \int_0^{q_o}\int_{\frac{(q_o-y)}{\alpha}+q_s}^{\phi M}(q_o-y)dxdy\right) \\
&+ (k-c_f)(\frac{1}{\phi M})(\frac{1}{(1-\phi)M})\left(\int_{q_o}^{(1-\phi)M}\int_0^{q_s-(y-q_o)}(y-q_o)dxdy + \int_{q_o}^{(1-\phi)M}\int_{q_s-(y-q_o)}^{q_s}(q_s-x)dxdy\right) \\
&- \mu(\frac{1}{\phi M})(\int_0^{q_o}\int_0^{q_s}(q_s-x)dxdy + \int_{q_o}^{(1-\phi)M}\int_0^{q_s-(y-q_o)}((q_s-x)-(y-q_o))dxdy).
\end{aligned}$$

$$\begin{aligned}
\Pi_o^h(q_o,q_s) &= (p-c_o)(\frac{1}{(1-\phi)M})\left(\int_0^{q_o} y dy + \int_{q_o}^{(1-\phi)M} q_o dy\right) - w q_o \\
&+ (p-k)(\frac{1}{\phi M})(\frac{1}{(1-\phi)M})\left(\int_0^{q_s}\int_{q_o}^{(q_o-x)+q_s}(y-q_o)dydx + \int_0^{q_s}\int_{q_o+(q_s-x)}^{(1-\phi)M}(q_s-x)dydx\right) \\
&+ (k-c_t)(\frac{1}{\phi M})(\frac{1}{(1-\phi)M})\left(\int_{q_s}^{\phi M}\int_0^{q_o-\alpha(x-q_s)}\alpha(x-q_s)dydx + \int_{q_s}^{\phi M}\int_{q_o-\alpha(x-q_s)}^{q_o}(q_o-y)dydx\right).
\end{aligned}$$

(i) Existence. For $p > c_o$, it is easy to check that $\frac{\partial^2 \Pi_s^h(q_s)}{\partial (q_s)^2} = -\frac{p+\eta}{M\phi} < 0$ and $\frac{\partial^2 \Pi_o^h(q_o)}{\partial (q_o)^2} = -\frac{p-c_o}{M(1-\phi))} < 0$, implying that Π_s^h is a concave function of q_s for each given q_o and Π_o^h is a concave function of q_o for each given q_s. Therefore, there exists at least one Nash equilibrium in pure strategies.

(ii) Uniqueness: We need to show that

$$\left|\frac{\partial q_s^h(q_o)}{\partial (q_o)}\right| = \frac{(k+\eta-c_f)}{(p+\eta)} - \frac{(k+\eta-c_f)-(p-k)}{(p+\eta)}\frac{q_o}{M(1-\phi)} < 1, \tag{2.16}$$

$$\left|\frac{\partial q_o^h(q_s)}{\partial (q_s)}\right| = \frac{\alpha(k-c_t)}{(p-c_o)} - \frac{\alpha(k-c_t)-(p-k)}{(p-c_o)}\frac{q_s}{M\phi} < 1. \tag{2.17}$$

Under the restrictions $p \geq k$ and $p > c_f$, we can conclude that, if $(k+\eta-c_f) \geq (p-k)$, then the inequality in (2.16) holds true. The inequality in (2.17) follows from $(p-k) \leq \alpha(k-c_t) \leq (p-c_o)$.

Proof for Proposition 16. The BM store's response functions can be written as

$$q_s^1(q_o) = \frac{-B - \sqrt{B^2 - 4AC}}{C},$$
$$q_s^2(q_o) = \frac{-B + \sqrt{B^2 - 4AC}}{C},$$

where

$$A = -\left(2(1-\phi)\phi(p-c_s) - (c_f - \eta)(1-\phi)^2 - c_t\alpha^2\phi^2\right)M^2,$$
$$(c_t\alpha\phi + (c_f-\eta)(1-\phi)) - 2Mq_o + (p + c_f - \eta)q_o^2,$$

$$B = M(\eta(1-\phi) - \phi\alpha^2 c_t) + q_o(p + \alpha c_t) > 0,$$
$$C = p + \alpha^2 c_t.$$

- Since $B > 0$ and $C > 0$, therefore it suffices to prove that $A \leq 0$, then $q_s^1(q_o) \leq 0$ and $q_s^2(q_o) \geq 0$.
- A is a convex quadratic function of q_o^h and $A(q_o^h = 0) < 0$, therefore we can show that if

$$\phi \geq \frac{(p-c_s) + (c_f - \eta) - \sqrt{(p-c_s)^2 + \alpha^2 c_t(\eta - c_f)}}{(p-c_s) + (c_f-\eta) + (p-c_s) + \alpha^2 c_t},$$

then, there exist $(q_o^h)' \geq 0$ such that if $q_o \leq (q_o^h)'$, then $A \leq 0$. By solving $A = 0$ we have

$$(q_o^h)' = \frac{M((c_f-\eta)(1-\phi) - c_t\alpha\phi)}{(p + c_f - \eta)} + \frac{\sqrt{M^2((c_f-\eta)(1-\phi) - c_t\alpha\phi)^2 + M^2(p+c_f-\eta)(2(p-c_s)\phi(1-\phi) - (c_f-\eta)(1-\phi)^2 - c_t\alpha^2\phi^2)}}{(p+c_f-\eta)} > 0.$$

Since q_o^h is always smaller than the online market share, $(1-\phi)M$, then if we find under which conditions $(1-\phi)M \leq (q_o^h)'$ then $q_o^h \leq q_o^{'}$. We find that, as long as the fraction of store-visiting customers is such that the following condition holds, then, $q_o(q_s) \leq (q_o^h)'$. This implies that $A \leq 0$, and, $q_s^1(q_o) \leq 0$ and $q_s^2(q_o) \geq 0$.

$$\phi \geq \frac{(2p + c_t\alpha - c_s) - \sqrt{(2p + c_t\alpha - c_s)(p + c_t\alpha - c_s) - p(p - c_s + c_t\alpha(1+\alpha))}}{(2p + c_t\alpha - c_s) + (p - c_s + c_t\alpha(1+\alpha))}.$$

The DC's response functions can be written as

$$q_o^1(q_s) = \frac{-D - \sqrt{D^2 - 4FG}}{G},$$
$$q_o^2(q_s) = \frac{-D + \sqrt{D^2 - 4FG}}{G},$$

where

$$F = -\left(M^2\alpha((1-\phi)(2(p - c_o - w)\phi - (c_{sfs} - \eta)(1-\phi)) - c_{sts}\alpha\phi^2\right)$$
$$+ 2q_s M\alpha((c_{sfs} - \eta)(1-\phi) + \phi c_{sts}\alpha) - \alpha(c_{sts}\alpha + p)q_s^2),$$

$$D = -\alpha(M((1-\phi)(c_{sfs} - \eta) + c_o\phi) - (p + c_{sfs} - \eta)q_s),$$
$$G = (p + \alpha(c_{sfs} - \eta)).$$

- Since $G > 0$, it suffices to prove that $F \leq 0$, then $q_o^1(q_s) \leq 0$ and $q_o^2(q_s) \geq 0$, which implies the existence of a unique response function and a unique Nash equilibrium.
- F is a convex quadratic function of q_s^h and $A(q_s^h = 0) < 0$, therefore we can show that if the following condition holds then there exist $q_s^{'} \geq 0$ such that if $q_s^h \leq q_s^{'}$, then $F \leq 0$. By solving $F = 0$ we have

$$(q_s^h)' = \frac{-M(c_f - \eta)(1-\phi)}{p + \alpha c_f}$$
$$+ \frac{\sqrt{M^2((c_f - \eta)(1-\phi))^2 + M^2(p + \alpha c_f)\left(2(p - c_o - w)(1-\phi)\phi - (c_f - \eta)(1-\phi)^2 - c_t\alpha\phi^2\right)}}{p + \alpha c_f}$$
$$> 0$$

- We find that if

$$\phi < \frac{(p - c_o - w) + 2(c_f - \eta)}{2(p - c_o - w + c_f - \eta) + p + c_f - \eta}$$
$$- \frac{\sqrt{(p - c_s - w + 2(c_f - \eta))^2 - (c_f - \eta)(2(p - c_o - w + c_f - \eta) + p + c_f - \eta)}}{2(p - c_o - w + c_f - \eta) + p + c_f - \eta}$$

then for all values of q_o^h, $q_s^2(q_o) - (q_s^h)' < 0$ (decreasing in q_o), which implies that $F \leq 0$ hence, $q_o^1(q_s) \leq 0$ and $q_o^2(q_s) \geq 0$.

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Chapter 3

Should Suppliers Support Retailer's Omnichannel Investments?

Abstract

Due to the increased use of omnichannel sales and distribution, retailers are under greater pressure than ever to deliver products at the right time, through the right channel, and at the right price. This paper aims to investigate if suppliers should assist retailers in upgrading their operations to provide omnichannel services. We use a Stackelberg game model to analyze the dynamics of a supply chain consisting of a manufacturer and an independent offline retailer. First, the manufacturer announces the online price and investment support rate offered to the offline retailer, who then sets the offline retail price. We examine how the manufacturer's support affects equilibrium strategies and related profits. In addition, we identify the conditions under which both the supplier and the retailer can benefit from the in-store pickup services. Our findings suggest that manufacturers can encourage retailers to maximize their level of investment in omnichannel strategy implementation by collaborating on the implementing in-store pickup services.

3.1 Introduction

During the pandemic, the practice of purchasing items online and picking them up in-store (BOPS) became a regular part of many consumers' shopping routines. According to a study, almost two thirds of these consumers plan to continue using BOPS due to its benefits (McKinsey and Company, 2020). By using BOPS, shoppers can avoid the hassle of searching for a product in-store and receive their purchases quicker than with home delivery. Retailers can also benefit from BOPS by implementing designated parking areas and pickup counters (Cao et al., 2016). Adopting an omnichannel business model that includes BOPS is a defensive strategy for retailers to prevent excessive demand loss to online shopping with direct delivery (Bell et al., 2014; Akturk and Ketzenberg, 2022). Additionally, retailers can increase foot traffic and engage with customers through BOPS (Gao and Su, 2017). However, retailers must carefully evaluate the profitability of BOPS before implementing it.

The omnichannel approach to retail requires businesses to restructure their operations, presenting both opportunities and challenges to the supply chain. In-store pickup services are beneficial, but offline retailers may face higher operating costs. As omnichannel retailing continues to grow, retailers must upgrade their infrastructure to support online orders and in-store pickups. After a customer places an order online, the retailer must select, pack, and place the items at the designated pickup location. Retailers need to invest in store infrastructure, including order management systems, accurate inventory tracking, and extra pickup space, to ensure a seamless pickup process for customers. Additionally, retailers should consider hiring or training additional staff to handle pickup orders and customer service demands.

Retailers are currently faced with the task of balancing costs for omnichannel retailing while remaining competitive. Recently, Walmart and Loblaw in Canada have introduced new fees for suppliers to cover operational upgrades for pickup handling. Walmart has imposed a 1.25% "infrastructure fee" on the cost of goods sold, as well as an additional 5% on online items to help fund its $3.5 billion investment in stores and e-commerce (WalmartCanada, 2020; FinancialPost, 2020). In order to remain competitive, retailers like United Grocers Inc. are also expecting the same terms from their suppliers. Although larger retailers have the authority to impose these fees on suppliers, it can be more difficult for smaller ones since they lack the power to enforce fees on suppliers. As a significant portion of retailer profitability depends on funding from suppliers or manufacturers, we will explore whether suppliers should be expected to cover some of the costs associated with upgrading in-store and digital operations.

When suppliers consider making investments in their business, they should weigh the potential benefits against the cost. These benefits could include increased sales, better customer experiences, and improved supply chain efficiency. In this paper, we explore whether manufacturers should participate in a retailer's investment to offer omnichannel services. For example, manufacturers can offer subsidies to retailers for advanced technology and infrastructure costs, or offer to cover the training costs needed for staff development. We also examine the impact of BOPS on both online and offline pricing strategies. To analyze the interactions between the two partners, we utilize game theory as a framework. Specifically, we look at a supply chain consisting of one manufacturer and one retailer that offer various buying and delivery options to consumers. We aim to answer the following To answer these questions, we construct stylized Stackelberg models of a dual-channel supply chain. We allow customers to differ in their convenience to order online with direct shipping or purchase in-store. The distribution of total demand among the various channels is determined by the consumer's maximization of utility. We consider the following three scenarios:

No in-store pickup: In this benchmark scenario, the manufacturer first announces the online price, and next the retailer decides its in-store price. In-store pickup service is not available.

In-store pickup: In this setup, the manufacturer first announces the online price, and next the retailer decides its retail price while providing in-store pickup services.

In-store pickup with manufacturer support: In this scenario, the manufacturer first determines the online retail price and the support rate for retailer's investments in pickup services; next, the retailer sets the retail price while providing in-store pickup services.

We find that in the absence of manufacturer's support, implementing BOPS hurts the retailer's profit for two reasons. First, BOPS requires the retailer to lower the offline price to compete with online channels, and the additional revenue generated from cross-selling is not enough to offset this. Secondly, for every purchase made through BOPS, retailers must pay an additional fulfillment cost on top of implementation expenses. However, the manufacturer benefits from BOPS as some customers switch from direct delivery to in-store pickup, which has a higher profit margin. This creates a conflict of interest between the offline retailer and the manufacturer when implementing in-store pickup services. Therefore, cooperation between manufacturers and retailers is essential for the successful launch of in-store pickup services.

One way for them to cooperate is for manufacturers to compensate retailers for their investment in the in-store pickup service. This can encourage retailers to maximize their investment in omnichannel implementation, leading to higher profits and the ability to cover the costs without affecting retail prices. We demonstrate that the retailer will benefit from BOPS sales if the manufacturer pays part of the retailer's investment cost in BOPS. In order to receive support from the manufacturer, the investment level must meet a certain threshold. Whether or not the manufacturer will compensate the retailer's investment in BOPS depends on a variety of factors, including the cost of last-mile delivery, product type, and the potential for cross-selling in offline retail. Understanding these factors is essential for decision-makers to ensure the successful implementation of in-store pickup services. While many retailers who offer pickup services currently charge fees or require a minimum purchase, suppliers cannot prevent retailers from charging customers for in-store pickup services. However, manufacturers can encourage retailers to keep their fees low by compensating them for their investment in implementing the service. According to a recent study, as retailers increase their fees for in-store pickup services, manufacturers are less likely to compensate them for their investment. Therefore, retailers should carefully consider the handling fees they charge for in-store pickup services to avoid negatively impacting their profitability.

The structure of the paper is as follows: Firstly, related work is reviewed in section 3.2. In section 3.3, model assumptions and notations are established. Next, the benchmark model is set up and the game equilibrium is calculated. The paper then delves into two in-store pickup implementation scenarios, namely 3.3.2 and 3.3.3, which are explored and compared. Our numerical analysis is presented in section 3.4, and the paper concludes with a summary in section 3.5. The appendix contains proofs for propositions.

3.2 Literature Review

Our paper draws from, and contribute to, three streams of literature in omnichannel retailing: In-store pickup services, cooperation in supply chain, and pricing in omnichannels.

In-Store Pickup Services. The growing popularity of in-store pickup services has generated significant interest among researchers. One area of focus is the practical implications of implementing a BOPS (Buy Online, Pick Up In-Store) fulfillment strategy. Gallino et al. (2014) discovered that while BOPS led to a decrease in online sales, it resulted in increased foot traffic and sales at physical stores. However, Cao et al. (2016) warned that in-store pickup may harm traditional sales channels, leading to a decrease in profit. Gao et al. (2017) found that BOPS is not always suitable for products that sell well in stores.

Other researchers have evaluated the BOPS strategy from various angles. Jin et al. (2018) analyzed the design of the service area, while Yan et al. (2018) examined how retailers can use BOPS to gain a competitive advantage. Akturk et al. (2022) demonstrated that the launch of a BOPS service by a competitor can adversely affect both online and in-store sales. Hu et al. (2022) showed that retailers can leverage BOPS to improve store fill rates. Finally, Gao et al. (2022) suggested that retailers may benefit from reducing their physical store presence under BOPS.

While much of the research on BOPS has focused on operational impacts, our paper explores how manufacturers can support retailers' investments in in-store pickup services to affect pricing strategies and related profits. Using game theory, we show that collaborative upgrades to implement BOPS can incentivize retailers to maximize their investment in omnichannel strategies.

Cooperation in Supply Chain. Major part of this literature focuses on the examination of coordination in traditional supply chains that involve suppliers, such as manufacturers and wholesalers, and retailers. A comprehensive review of this literature is provided by Cachon (2003), and Cachon (2004). A series of contracts are studied such as wholesale price contracts (Bernstein et al., 2006; Lariviere and Porteus, 2001), risk-sharing contracts (e.g., buy-back (Pasternack, 1985) and revenue-sharing contracts (Cachon and Lariviere, 2005)), and cooperative advertising contracts (Jørgensen and Zaccour, 2014; Huang and Li, 2001), and procurement contracts (Martínez-de Albéniz and Simchi-Levi, 2005). One critical aspect of successful in-store pickup service is the cooperation between manufacturers and retailers. Manufacturers can offer compensation to retailers for their investment in in-store pickup services to achieve this. However, to the best of our knowledge, this literature has not yet addressed cooperation between a retailer and a manufacturer in an omnichannel environment, which is the primary focus of this paper.

Pricing in Omnichannels. In line with the continuous improvement and development of the BOPS option, pricing and service research has attracted some attention. Several studies have explored how the implementation of BOPS fulfillment affects a retailer's pricing strategy. For instance, Kong et al. (2020) found that BOPS is more advantageous to the retailer when retail prices change after implementing BOPS strategy. Feng et al. (2022) suggested that retailers can benefit from strategically adjusting prices based on consumer store visiting costs after adopting the BOPS channel. They demonstrated that the firm can gain an advantage by reducing or boosting the price after implementing BOPS if the store visit cost is relatively low or high, respectively.

He et al. (2020) focus on the method of fulfilling orders known as ship-from-store, which involves a manufacturer using both physical and digital sales channels and relying on a brick-and-mortar retailer to handle deliveries for online purchases. Their study focuses on how this approach affects pricing and purchasing decisions. Li et al. (2019) analyze the effects of showrooming on a company's pricing and service strategies in a dual-channel supply chain. They found that showrooming benefits the supply chain the most when the retailer decides on its service level last.

In another study, Lin et al. (2021) investigate the impact of implementing the BOPS channel on the quality, prices, and profits of both the manufacturer and retailer. They discovered that offering the BOPS channel can lead to a decrease in selling price when shipping costs increase, but both parties can still benefit from it if the fulfillment cost is not too high. They also found that adjusting the product quality and wholesale price while raising the selling price can be beneficial under certain circumstances. Our research reveals that implementing in-store pickup services leads to a decrease in both online and offline retail prices when the cross-selling margin is not high. Additionally, we discovered that with manufacturer compensation, the retailer does not need to lower the offline retail price to attract customers to in-store shopping.

3.3 Models

We consider a dual-channel supply chain in which a manufacturer sells its products directly (online channel) to consumers at price p_o, and to an independent offline retailer (such as a brick-and-mortar store) at wholesale price w. The retailer resells the product to consumers at a price of p_s (hereafter, offline or store retail price). Following the literature, see, e.g., (Cao et al., 2016; He et al., 2020; Li et al., 2019), and examples from industry practices (Gerdeman, 2018; Mohammed, 2017), we do not constrain the online and store prices to be equal. We formulate the pricing problem as a sequential two-stage Stackelberg game where the manufacturer, as leader, decides first, and the retailer, as follower, acts second. We examine a scenario in which the manufacturer chooses to offer in-store pickup services (such as BOPS) to customers and encourages the retailer to invest in these services.

Customers make purchasing decisions based on their utility. The utility received by a consumer is v if the product meets their requirements and is 0 otherwise. Customers have heterogeneous hassle costs for ordering online and purchasing in-store. We use a Hotelling line, following Zhang and Choi (2021), in order to capture consumers' preferred channel (Hotelling, 1990).

Customers are uniformly distributed along the line, with $x = 0$ representing the offline channel and $x = 1$ the online channel. For a customer located at x, the cost of purchasing a product online with direct delivery is given by $t(1 - x)$, where $t\,(1 - x)$

measures the cost of shipping and waiting. Customers who visit physical stores incur a cost of tx, which includes the opportunity cost of time, travel expenses, and the inconvenience of searching for the product in-store.

The parameter $t > 0$ represents the strength of customer channel preference, with higher (lower) values indicating greater (lesser) channel heterogeneity. We assume that market demand is normalized to 1, and that the manufacturer has zero production cost. Thus, the manufacturer's direct shipping cost, denoted as c_o, only represents the additional unit cost of last-mile delivery. A summary of the notations used in this paper is shown in table 3.1.

Table 3.1: Summary of Notation

	Notation related to the customer
v	Product valuation
$1 - x$	Hassle cost of an online order with direct delivery
x	Hassle cost of store visit
θ	Product's matching probability
t	Customers' degree of channel heterogeneity
p_o	Online retail price
p_s	In-store retail price
	Notation related to the retailer
w	Wholesale price
c_o	Fulfillment cost for each direct-delivery order (e.g., direct shipping cost)
c_p	Fulfillment cost for fulfilling each BOPS order
m	Investment level for in-store pickup services

In the following subsections, we introduce a baseline case without the pickup services, and next consider two different pickup scenarios, without and with manufacturer's support. Firstly, we analyze the impact of in-store pickup services on the manufacturer and offline retailer's prices, demand, and profits. Secondly, we examine how the manufacturer's support of the retailer's investment in providing in-store pickup affects equilibrium strategies and profitability.

3.3.1 No In-Store Pickup

In this benchmark model, customers at location x can purchase a product by visiting the store or buying it online with direct delivery. If the customer purchases in-store, their utility is $U_s = v - p_s - tx$, where the subscript s refers to in-store purchases. The utility of purchasing online with direct delivery is given by $U_o = \theta(v - p_o) - (1 - x)t$, where the subscript o stands for online direct delivery. As noted in previous studies (Hsiao, 2012; Ertekin, 2021), the parameter $\theta \leq 1$ represents the utility discount for buying online, which is due to the inability to physically inspect the product and the potential hassle of returning it if it does not meet their expectations. Figure 3.1 illustrates consumer's purchasing decision in this benchmark model.

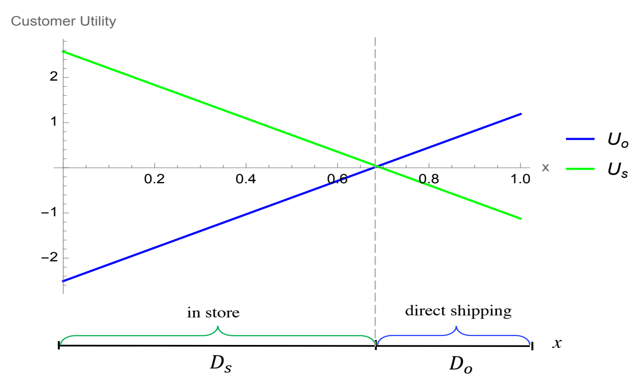


Figure 3.1: Customers' Purchasing Decision under No Pickup Scenario

Denote by D_o^n and D_s^n the online and in-store demand, respectively. The location of the indifferent consumer is determined by solving for $U_o = U_s$, which yields

$$D_s^n = \frac{t + (1-\theta)v + \theta p_o - p_s}{2t}, \quad D_o^n = \frac{t - (1-\theta)v - \theta p_o + p_s}{2t},$$

with $D_s^n + D_o^n = 1$.

Denote by c_o the fulfillment cost per direct-delivery order, e.g., direct shipping cost. The offline retailer and the manufacturer have the following profit functions:

$$\Pi_s^n = (p_s - w)D_s^n, \tag{3.1}$$

$$\Pi_o^n = wD_s^n + (p_o - c_o)D_o^n. \tag{3.2}$$

The first term in (3.2) represents the manufacturer's profit from selling the product to the retailer, while the second term measures the profit from direct delivery. A Stackelberg equilibrium is determined by first considering the retailer's optimization problem in response to the manufacturer's announcement, that is,

$$\max_{p_s} \Pi_s^n = (p_s - w)\left(\frac{t + (1-\theta)v + \theta p_o - p_s}{2t}\right).$$

Assuming an interior solution, the first-order optimality condition yields

$$p_s(p_o) = \frac{1}{2}(t + v + w - v\theta + \theta p_o). \tag{3.3}$$

As $p_s'(p_o) = \theta/2 > 0$, we conclude that the two prices are strategic complements. Substituting for p_s by its value from (3.3) into Π_o^n and maximizing, we get

$$p_o^n = \frac{1}{2\theta}(w + 3t - v + (v + w - c_o)\theta).$$

Substituting for p_o in the reaction function (3.3), we obtain

$$p_s^n = \frac{1}{4}(3w + 5t + v - (v - w - c_o)\theta).$$

We make three comments. First, we note that both prices are increasing in the wholesale price w and in t, and vary in opposite direction with respect to v and c_o. These results are intuitive. The variation with respect to θ is ambiguous as it depends on all other parameter values. Second, to avoid arbitrage by the retailer, that is, buying online to sell to consumers, we expect to have $w \leq p_s^n$, which is equivalent to $t \geq \frac{(w-v)(1-\theta)-\theta c_o}{5}$. As the right-hand side of this inequality is negative, this condition of absence of arbitrage is satisfied for all parameter values. Finally, the online price will be higher than the store price if t is sufficiently large, that is,

$$t \geq \frac{2+\theta}{6-5\theta}((1-\theta)(v-w) + \theta c_o) > 0.$$

3.3.2 In-Store Pickup

Now, we consider the case where the manufacturer offers and manages BOPS; that is, the profit from the in-store pickup orders belongs to the manufacturer. Retailers such as T-mall supermarkets and physical convenience stores adopt this strategy, eliminating the need to purchase in-store pickup orders at wholesale prices. However, offering in-store pickup services comes with additional costs, so retailers like Walmart, Costco, and Kroger charge a small handling fee to pass on the cost to their customers and make this option profitable (Ryan, 2022). As in these examples, we suppose the retailer charges a handling fee of f for the in-store pickup options.

Customers can purchase through three different channels: online, offline, or buy-online-pickup-in-store. Customers who choose the in-store pickup option with handling fee earn a utility $U_p = v - p_o - (1-m)tx - mf$, where the subscript p stands for the $p$ickup option. The variable m, with values ranging from 0 to 1, represents the retailer's service level for BOPS customers. The retailer can provide different levels of convenience for in-store pickup services, such as walk-in pickup, contactless curbside pickup, or self-service smart pickup lockers. The speed of preparation for in-store item pickup, such as in 2 hours, same day, or the next day, depends on how many employees are assigned to pick and pack.

In this paper, we suppose that customers who choose in-store pickup incur a hassle to collect their order, given by $(1-m)tx$, a decreasing function of m. Customers who prefer in-store pickup view it as more convenient than in-store shopping. Indeed, they save the time they would have otherwise spent in the store searching for their item or standing and waiting in line at the checkout counter.

Additionally, in-store pickup services often ensure customers that the item they need is available before they arrive at the store, thereby reducing frustration. We assume that m is an exogenous parameter that may be affected by factors such as the budget capacity of offline retailers. This assumption enables us to observe the impact of changes in the levels of investment following the implementation of in-store pickup services conveniently, which is one of the main purposes of this paper. To characterize the feature of the in-store pickup option, we assume that the online ordering cost (such as website browsing and e-payment risk) for customers who order from online platforms is normalized to zero. Additionally, we assume that the handling fee mf is higher for more convenient pickup options.

Let D_b^p denote the demand for in-store pickup option. Figure 3.2 illustrate the customers' purchasing behavior.

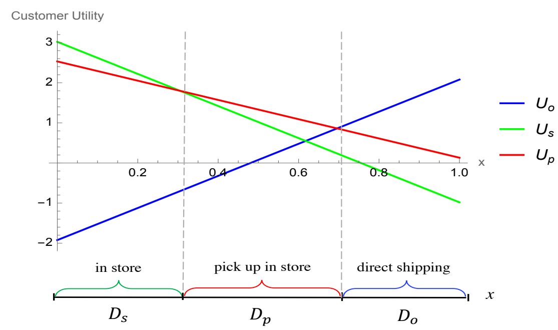


Figure 3.2: Customers' Purchasing Decision under Pickup in Store Services

When BOPS fulfillment strategy is available, then, the in-store, direct delivery, and BOPS demands are, respectively, as follows:

$$D_s^p = \frac{mf + p_o - p_s}{mt},$$
$$D_o^p = 1 - \frac{t + mf + (1-\theta)(v - p_o)}{(2-m)t},$$
$$D_b^p = \frac{t + mf + (1-\theta)(v - p_o)}{(2-m)t} - \frac{mf + p_o - p_s}{mt} = 1 - D_s^p - D_o^p.$$

Comparing the above demand system to the one we had in the benchmark model, we see that implementing in-store pickup services leads a proportion of customers who used to order online with direct delivery or in-store to switch to the in-store pickup option. We suppose that offering in-store services generates extra revenue for the offline retailer through cross-selling, measured by rD_b^p, where $r > 0$ is the marginal profit generated by BOPS demand. Therefore, the retailer's and the manufacturer's profits are as follows:

$$\Pi_s^p = (p_s - w)D_s^p + rD_b^p - m(c_p - f)D_b^p, \tag{3.4}$$

$$\Pi_o^p = wD_s^p + p_o D_b^p + (p_o - c_o)D_o^p, \tag{3.5}$$

Where c_p is the handling (or preparation) cost for fulfilling each BOPS order. The third term in (3.4) represents the offline retailer's cost for fulfilling in-store pickup orders when the offline retailer charges customers for handling fee f. The second term in (3.5) measures the manufacturer's profit from BOPS sales. It is easy to verify that the manufacturer's and retailer's profit functions in 3.4 and 3.5 are concave in p_o and p_s, respectively.

As in the benchmark scenario, the manufacturer first announces the online retail price, and next the retailer chooses the offline retail price. We start by considering the retailer's optimization problem to get its response to the manufacturer's announcement. Writing in full the retailer's optimization problem, we have

$$\begin{aligned}\max_{p_s} \Pi_s^p =& (p_s - w)\left(\frac{mf + p_o - p_s}{mt}\right) \\ &+ (r - m(c_p - f))\left(\frac{t + mf + (1-\theta)(v - p_o)}{(2-m)t} - \frac{mf + p_o - p_s}{mt}\right).\end{aligned}$$

Assuming an interior solution, the first-order optimality condition yields

$$p_s\left(p_o\right) = \frac{1}{2}\left(r + w + p_o - m(c_p - 2f)\right). \tag{3.6}$$

As $p_s'\left(p_o\right) = 1/2 > 0$, we conclude that the two prices are strategic complements. Substituting for p_s by its value from (3.6) in Π_o^p and maximizing, we get

$$p_o^p = w + mt + \frac{1}{2}(r - mc_p) - \frac{m(1-\theta)c_o}{(2-m)}. \tag{3.7}$$

Substituting for p_o in the reaction function (3.6), we obtain

$$p_s^p = \frac{1}{4}\left(4w + 3(r - mc_p) + 2m(2f + t) - \frac{2m(1-\theta)c_o}{(2-m)}\right). \tag{3.8}$$

The prices vary as follows with the different cost parameters

	c_o	c_p	f	m
p_o^p	$-$	$-$	0	?
p_s^p	$-$	$-$	$+$	?

The prices are decreasing in handling fee and delivery costs due to strategic complementarity. If these costs rise, online orders may become more expensive to counterbalance the increase, giving offline channels a competitive edge. The handling fee for BOPS consumers only affects offline prices, not online ones. A higher handling fee can lead to greater profits from BOPS sales. When offline prices go up, manufacturers tend to increase online prices due to strategic complementarity. A similar reasoning can be applied to the positive impact of r on the two prices. Like f, the retailers' revenues are increasing in r. Although higher offline prices might dampen in-store demand, they could drive up BOPS demand, another revenue source. The impact of m on the two prices depends on the other parameter values. Indeed, we have

$$\begin{aligned}\frac{\partial p_o^p}{\partial m} &< 0 \Leftrightarrow c_p > 2t - \frac{4c_o(1-\theta)}{(2-m)^2},\\ \frac{\partial p_s^p}{\partial m} &< 0 \Leftrightarrow c_p > \frac{2}{3}(2f+t) - \frac{4c_o(1-\theta)}{3(2-m)^2}.\end{aligned}$$

If the manufacturer's fulfillment cost of a BOPS order is high enough, then the higher the level of service provided by the retailer, the lower the two prices. Note that the above two inequalities are easier to satisfy when the direct-delivery cost is higher. We did not make any assumption on the ordering of c_p and c_0, but it is intuitive to suppose that c_o is higher than c_p; otherwise, there is no much incentive for the manufacturer to start a BOPS service.

Proposition 17. *The prices and profits with and without in-store pickup compare as follows:*

- *If* $r \le \tilde{r}_1$*, then* $p_s^p \le p_s^n$*; otherwise* $p_s^p > p_s^n$*, where*

$$\tilde{r}_1 = \frac{1}{3}\left(m(3c_p - 4f) + t(5-2m) + (v-w)(1-\theta) + \frac{(2\theta + m(2-3\theta))c_o}{2-m}\right)$$

- *If* $r \le \tilde{r}_2$*, then* $p_o^p \le p_o^n$*; otherwise* $p_o^p > p_o^n$*, where*

$$\tilde{r}_2 = mc_p - 2(mt+w) + \left(1 + \frac{2m(1-\theta)}{2-m}\right) + \frac{3t - v(1-\theta) + w(1+\theta)}{\theta}$$

- *If* $r \le \tilde{r}_3$*, then* $\Pi_s^p \le \Pi_s^n$*; otherwise,* $\Pi_s^p > \Pi_s^n$*, where*

$$\tilde{r}_3 = \frac{-B + \sqrt{B^2 - 4AC}}{2C}$$

$$\begin{aligned}
A =& \frac{1}{32}\left(8mt - 25t - 10\theta c_o - \frac{32mf(m(f+t(1-\theta))-t)}{(2-m)t} + \frac{2m(16(v-w)(1-\theta)(f-c_p) + (2+m(8\theta-9))c_p^2)}{(2-m)t}\right) \\
&- \frac{1}{32}\left(\frac{(v-w)^2(1-\theta)^2 + \theta^2 c_o^2 + 2(v-w)(1-\theta)(5t+\theta c_o)}{t} + \frac{8m(2(1-\theta)c_o + (2+m(4\theta-3))c_p)}{2-m}\right) \\
&\frac{1}{32}\left(\frac{8m((1-\theta)^2 c_o^2 + 2(2-m)mf(3-\theta)c_p - (1-\theta)c_o((2+m(3-4\theta))c_p - 4mf(1-\theta)))}{(2-m)^2 t}\right) \\
B =& \frac{2mf(3-\theta) - (2-3m)t - 4(v-w)(1-\theta) - 4m\theta(t-c_p)}{4t(2-m)} + \frac{(4-20m+9m^2)c_p - 2(1-\theta)(2+m(3-4\theta))c_o}{8(2-m)^2 t} \\
C =& \frac{2-9m+8m\theta}{16mt(2-m)}
\end{aligned}$$

- *If* $f \le \tilde{f}$*, then* $\Pi_o^p \ge \Pi_o^n$*; otherwise* $\Pi_o^p < \Pi_o^n$*, where*

$$\begin{aligned}
\tilde{f} =& \frac{1}{16mc_o}\left((2-m)\left[\frac{2r^2}{m} + 2m(4t^2 + c_p^2) - 4(r+2mt)c_p\right]\right) \\
&+ \frac{1}{\theta}\left((2-m)\left[\frac{t(8r\theta + 6(v-w)(1-\theta)) - 9t^2 - (v-w)^2(1-\theta)^2}{16mc_o}\right]\right) \\
&+ \frac{1}{16m(2-m)}\left[(4m(2-3\theta+2\theta^2) - \theta(4+m^2))c_o\right] \\
&+ \frac{1}{16m}\left[2(1-\theta)\left(4(mc_p - r) + (6+m)(v-w)\right) - 2t(2+m(3-8\theta))\right]. \qquad (3.9)
\end{aligned}$$

According to Proposition 17, if the profit margin from cross-selling is not very high, offering in-store pickup services can lower both online and offline retail prices. However, if the revenue from cross-selling is low, implementing in-store pickup can actually hurt the retailer's profitability. This is because the retailer is forced to lower their offline prices to compete with online retailers, but the additional revenue from cross sales is not enough to offset the decrease in demand caused by customers switching to in-store pickup. Additionally, for any purchase made through in-store pickup, the retailer must pay a fulfillment cost of mc_p.

For the manufacturer, the impact is the opposite. Some customers who previously opted for direct shipping may now prefer in-store pickup, which yields a higher profit margin. However, Proposition 17 indicates that if the retailer imposes a high fee for in-store pickup, it may not be advantageous for the manufacturer. Despite keeping the online price unchanged, charging a handling fee may deter customers from selecting in-store pickup, ultimately decreasing the manufacturer's profit margin. Therefore, the manufacturer should only consider implementing in-store pickup if the increase in profit from these sales and the reduction in last-mile delivery costs outweigh the decrease in profit from online retail price and offline sales. In addition, we can show that if

$$f < \frac{1}{16}\left(8(v-w)(1-\theta) + 8t(2\theta-1) + \frac{4(2+m)(1-\theta)^2 c_o}{2-m} + 8(1-\theta)c_p + \frac{(2-m)^2(2t-c_p)^2}{c_o}\right),$$

then $\frac{\partial(\Pi_o^p - \Pi_o^n)}{\partial m} > 0$, which implies that the manufacturer's profit of in-store pickup is strictly increasing in the investment level m if the handling fee is not high.

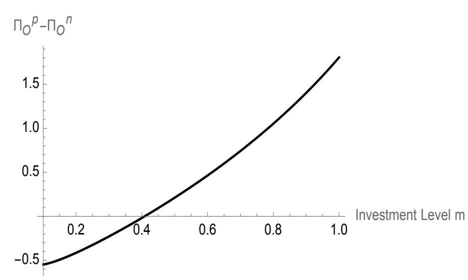


Figure 3.3: Manufacturer's profit change after implementing in-store pickup, $\Pi_o^p - \Pi_o^n$, as a function of m

Figure 3.3 illustrates the profit change for a manufacturer who implemented in-store pick-up with parameter values of $v = 9$, $\theta = 0.8$, $c_p = 0.5$, $r = 0.3$, $c_o = 3$, and $w = 2$. The graph indicates that at lower investment levels, the implementation of in-store pick-up services can have a negative impact on the manufacturer's profit. Therefore, it is advisable for the manufacturer to encourage the retailer to invest more in in-store pick-up.

In the subsequent corollary, we examine and compare the results when a handling fee cannot be charged to customers. Some customers may perceive it as unfair to be charged such a fee as they are putting in the same effort as those who shop offline. However, upon further investigation, we found that some retailers, such as Canadian Tire, Sprouts, and Stop and Shop, charge a fee for pick-up services based on the order size and pick-up time, while others like Walmart, Best Buy, and Low's offer free in-store pick-up.

Corollary 0.4. Charging handling fee for in-store pickup services increases the retailer's profit if $f < f'$

$$f' = \frac{2(t + (v - w - mt)(1 - \theta)) - (3 - \theta)(r - mc_p)}{2m} + \frac{(1 - \theta)^2 c_o}{2 - m},$$

and always decrease the manufacturer's profit.

According to Corollary 0.4, it is not profitable for retailers to charge high handling fees for in-store pickup. This is because the fee drives customers away from pickup and towards in-store shopping, which allows retailers to increase prices offline and maximize profits. However, if handling fees become too high, the increased prices can hurt in-store sales and ultimately harm the retailer's profits.

As previously discussed, implementing in-store pickup services can create a conflict of interest between the retailer and the manufacturer. In the next section, we will explore ways the manufacturer can create an agreement that encourages the retailer to invest more in this service, benefiting both parties.

3.3.3 In-store Pickup with Manufacturer's Support

Now, we consider a scenario in which the manufacturer supports the retailer's investment in in-store pickup service. The profit functions of the retailer and manufacturer are given by

$$\Pi_s^c = (p_s - w)D_s^p + rD_b^p - m((1-k)c_p - f)D_b^p, \tag{3.10}$$

$$\Pi_o^c = wD_s^p + (p_o - c_o)D_o^p + (p_o - kmc_p)D_b^p, \tag{3.11}$$

$$\text{subject to } 0 \leq k \leq 1, \tag{3.12}$$

Where k represents the manufacturer's support rate. In 3.10, kmc_p is the contribution made by the manufacturer towards the retailer's investment in in-store pickup services. The sequence of events is as follows: Given the investment level m, the manufacturer first announces the support rate k and the online retailer price p_o. Next, the retailer determines the offline retail price p_s. Finally, the consumer makes purchase decisions. To establish a Stackelberg equilibrium, we must first analyze the retailer's optimization problem to determine its response to the manufacturer's announcement. We have

$$\begin{aligned}\max_{p_s} \Pi_s^c =& (p_s - w)\left(\frac{mf + p_o - p_s}{mt}\right) \\ &+ \left(r - m((1-k)c_p - f)\right)\left(\frac{t + mf + (1-\theta)(v - p_o)}{(2-m)t} - \frac{mf + p_o - p_s}{mt}\right).\end{aligned}$$

Assuming an interior solution, the first-order optimality condition yields

$$p_s\left(p_o, k\right) = \frac{1}{2}\left(r + w + p_o - m(1-k)c_p + 2mf\right). \tag{3.13}$$

As $p_s'\left(p_o\right) = 1/2 > 0$, we conclude, as in the previous scenario, that the two prices are strategic complements. Since $p_s'\left(k\right) = \frac{mc_p}{2} > 0$, the offline price is increasing in the manufacturer's support rate. Considering the constraint in (3.12), we write the Lagrangian of the manufacturer's profit functions as follows:

$$\mathcal{L}_o^c = wD_s^p + (p_o - c_o)D_o^p + p_oD_b^p - kmc_pD_b^p + \lambda(1-k) + \beta k,$$

where λ and β are Lagrange multipliers associated, respectively, with $k \leq 1$ and the non-negativity constraint $k \geq 0$. Substituting for p_s by its value from (3.13) in $\mathcal{L}_o^c$, we get the following necessary conditions:

$$\frac{\partial \mathcal{L}_o^c(p_s(p_o))}{\partial p_o} = 0 \Leftrightarrow \tag{3.14}$$

$$p_o = \frac{(2-m)(r+2mt+2w) - 2m(1-\theta)c_o - m(2-2k(2-m\theta)-m)c_p}{2(2-m)} \tag{3.15}$$

$$\frac{\partial \mathcal{L}_o^c(p_s\,(p_o))}{\partial k} = 0 \Leftrightarrow \tag{3.16}$$

$$k = \frac{1}{2} + \frac{t(\beta-\lambda)}{mc_p^2} + \frac{2(m^2f - (r+2w) + (2-m\theta)p_o) - m(2(t+v(1-\theta)-w)-r)}{2(2-m)mc_p} \tag{3.17}$$

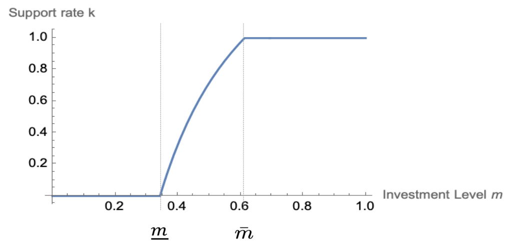
$$\lambda \geq 0, \quad (1-k) \geq 0, \quad \lambda(1-k) = 0, \tag{3.18}$$

$$\beta \geq 0, \quad k \geq 0, \quad \beta k = 0. \tag{3.19}$$

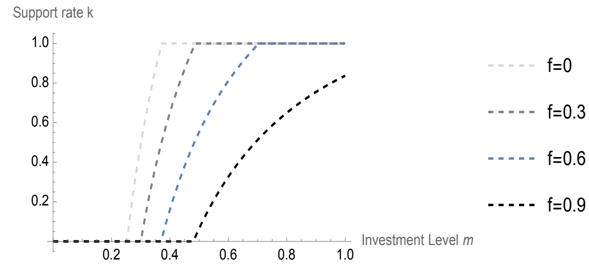
We explore different combinations of active and non-active constraints to identify equilibriums in detail (see Appendix for the detailed formulation and solutions).

Figure 3.4a demonstrates that unless the investment level exceeds $m > \bar{m}$, the manufacturer will not provide financial support to the retailer for their in-store pickup endeavor. But, if the retailer invests above this threshold value, the manufacturer will offer full compensation for the investment made in the in-store pickup services.

In Figure 3.4b, the change in support rate is displayed in relation to the pickup handling fee. An increase in handling fees results in a decrease in the support rate, as represented by the mathematical expression $\frac{\partial k}{\partial f} = -\frac{2-m}{(1-\theta)(4-m(1-\theta)c_p)} < 0$. If retailers charge high handling fees, the manufacturer may not recuperate their investment costs, and customers may opt for online or in-store shopping instead



(a)



(b)

Figure 3.4: The Change of Support Rate k with m

of in-store pickup services with high fees. Therefore, encouraging offline retailers to invest more may not benefit the manufacturer. The prices and the support rate vary as follows with the different parameter values:

	c_o	c_p	f	m
p_o^c	$+$	$-$	0	?
p_s^c	$+$	$-$	?	?
k^c	$+$	$-$	$-$	?

Compensation from the manufacturer to the retailer can impact prices based on several factors. Specifically, prices tend to rise with delivery costs (c_o), while prices tend to decrease with pickup fulfillment costs (c_p). If the manufacturer compensates the retailer and $\theta > \frac{1}{2}$, offline prices will decrease as f increases. Higher values of θ also incentivize customers to shop online. When the retailer's pickup fees are high, the manufacturer's compensation for that service decreases, leading the retailer to lower offline prices to attract customers to the physical store. As the handling fee (f) increases, the manufacturer's level of support decreases. We find that the impact of m on the two prices depends on the other parameters as follows:

$$\frac{\partial p_o^c}{\partial m} < 0 \iff c_p > \frac{A}{B},$$
$$\frac{\partial p_s^c}{\partial m} < 0 \iff c_p > \frac{t + f(1-2\theta)}{(1-\theta)},$$

Where

$$A = 2\left(2t(7-4m-3\theta) + m^2(1+\theta)(t-\theta f) - 8f(1-m\theta) - (1-\theta)^2(r+2(c_o-v+w))\right),$$

$$B = (1-\theta)\left(8(1-m) + m^2(1+\theta)\right).$$

Proposition 18 highlights how the resulting prices and profits compare to those in the in-store pickup without compensation case.

Proposition 18. *Comparing the prices and profits before and after manufacturer's supports, we find that:*

- *If $m \leq \underline{m}$, then $k = 0$, i.e., the manufacturer does not support retailer's investments, which leads to $p_s^c = p_s^p$ and $p_o^c = p_o^p$, $\Pi_o^c = \Pi_o^p < \Pi_o^n$.*
- *If $m > \underline{m}$, then $k > 0$ and $p_s^c > p_s^p$ and $p_o^c > p_o^p$.*
- *If $\underline{m} \leq m \leq \bar{m}$, then the manufacturer partially compensates for the retailer's investment m, with the support rate being increasing in m. In this case, $\Pi_o^c < \Pi_o^p$.*
- *If $m \geq \bar{m}$, then $k = 1$, that is, the manufacturer fully compensates for the retailer for its investment.*

- When the retailer's investment, denoted by m, is less than or equal to the lower threshold $\underline{\mathrm{m}}$, there's no manufacturer support. This results in equal prices when compared with the no-support scenario, and the profit under compensation is less than the no-support scenario.
- If the retailer's investment exceeds the lower threshold, manufacturer support kicks in, pushing prices upward.
- For investments lying between the two thresholds, $\underline{\mathrm{m}}$ and $\bar{m}$, the manufacturer provides only partial compensation. This support rate is positively correlated with the retailer's investment. However, in this scenario, profits under compensation are found to be less than profits in the prior no-support situation.

- Lastly, when the retailer's investment meets or exceeds the upper threshold, $\bar{m}$, the manufacturer takes on the complete investment. This is a clear indicator of the manufacturer's commitment to fully back the retailer's in-store pickup service when investments reach a certain magnitude.

According to Proposition 18, it may not be financially beneficial for the manufacturer to assist the retailer in covering their investment costs, especially when the investment level is low. This is because a low investment level usually results in less demand for in-store pickups, which leads to a lower retail price and a relatively small profit gain for the manufacturer. However, if the investment level surpasses a certain threshold, it becomes more advantageous for the manufacturer to compensate for the retailer's investment costs. If this happens, both online and offline retail prices can increase, resulting in higher profits for the manufacturer.

The retailer will benefit from in-store pickup sales when the manufacturer compensates the retailer's investment costs. By increasing profits, the retailer can cover the cost of investing and fulfilling in-store pickup orders. In this case, the retailer does not need to reduce the retail price in order to attract customers to the store. Therefore, if the level of investment is sufficiently high, the retail price may be higher than the benchmark retail price. We find that when the retailer's investment level is high, that is, $m > \bar{m}$, the manufacturer fully compensates the investment costs.

3.4 Numerical studies

In this section, we compare the manufacturer's profits using various strategies to determine the best compensation policy and implementation plan for in-store pickup. We also conduct sensitivity analysis to examine how important parameters such as c_o, r, f, and θ affect the manufacturer's profit gain from compensating the retailer.

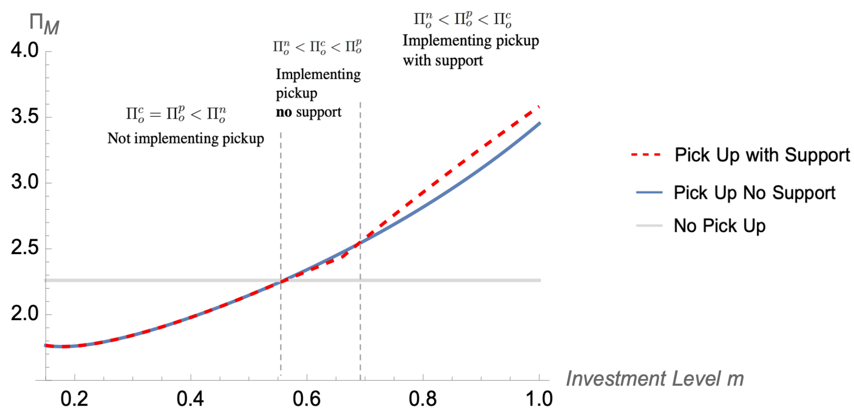
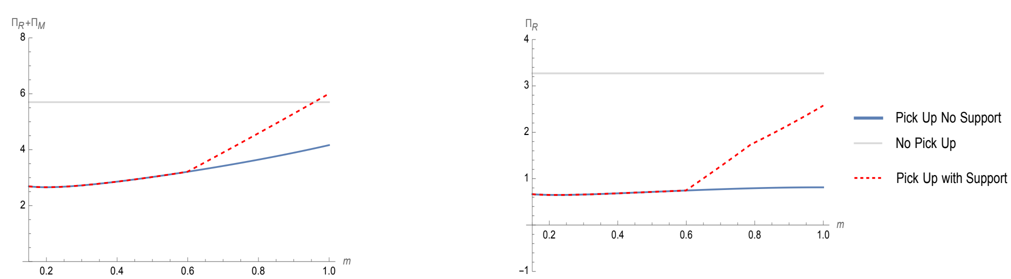


Figure 3.5: Manufacturer's Profit with and without Support for Retailer's Investment in In-store Pickup

Figure 3.5 illustrates how the profit of the manufacturer changes based on the investment level m, with and without support. It is observed that if a retailer's investment level is low, the implementation of in-store pickup services can harm the manufacturer's profit. However, implementing in-store pickup services without compensating the retailer for moderate investment levels allows the manufacturer to benefit. As the retailer invests more in in-store pickup services, the manufacturer benefits even more. If the investment level m is high, the manufacturer can further increase profit by fully compensating the retailer.



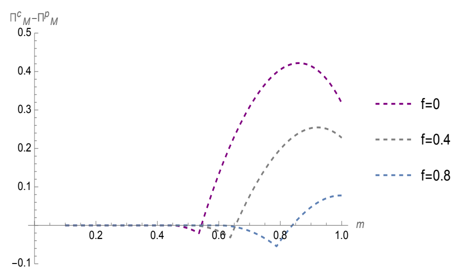
(a) $\Pi_M + \Pi_R$ (b) Π_R

Figure 3.6: Profit Change After Implementing In-store Pickup with and without Support

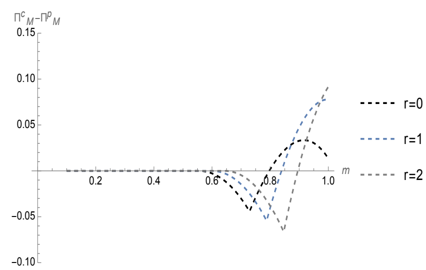
Figure 3.6a shows that the total profit of the retailer and the manufacturer benefits most from the in-store pickup service when the retailer has high investment levels and the manufacturer compensates the retailer.

3.4.1 Sensitivity analysis

Managers need to comprehend the various factors that can influence their decision-making process. Figure 3.7a demonstrates how compensating the retailer's investments in pickup services can impact profits. The threshold for investment level at which the manufacturer is willing to compensate the retailer increases as the retailer charges consumers more for pickup services. If the retailer offers free pickup services to consumers, the manufacturer will compensate the retailer, even for moderate investment levels.



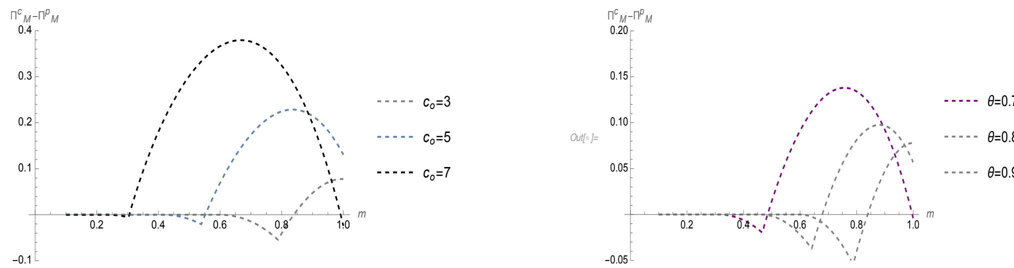
(a) Handling fee



(b) Cross-selling margin

Figure 3.7: Variation of $\Pi^c_M - \Pi^p_M$ with respect to m

The data shown in figure 3.7b suggests that when a retailer has a high cross-selling margin, they are less likely to receive compensation from the manufacturer. However, if the retailer has a high potential for cross-selling due to the product type (such as department stores, electronics retailers, home goods retailers, and sporting goods) or the pickup service type (such as walk-in in-store pickup), the manufacturer will compensate them if the investment level is very high. For pickup services like curbside pickup or locker pickup, the manufacturer's investment level threshold is lower since there is a lower chance of cross-selling in-store.



(a) Shipping Cost (b) Product's matching probability

Figure 3.8: Variation of $\Pi_M^c - \Pi_M^p$ with respect to m

We are also interested in how the direct shipping cost affects the manufacturer's profit gain from supporting the retailer's investment costs. If the direct shipping costs increase, it can have a negative impact on the manufacturer's profits. To address this issue, the manufacturer may choose to offer the retailer compensation for reducing their investment in the products. This incentivizes customers to pick up their purchases from the store instead, which is often more cost-effective for the manufacturer and more convenient for customers.

Our findings indicate that a manufacturer's decision to collaborate with a retailer on a buy online, pick up in-store (BOPS) system may also depend on the likelihood of a product match, particularly for fashion products. If the product match probability is low, the manufacturer may be more inclined to compensate the retailer for their investment in BOPS to improve the chances of selling the product. This is because it is more challenging to match fashion products with the right size and style preference. Conversely, for standardized products like books, the product matching probability is higher, and the manufacturer may not need to offer as much compensation to shift sales to in-store pickup.

Figures 3.7a-3.7b and 3.8a-3.8b numerically shows robustness of the result that the manufacturers can benefit more from in-store pickup services for higher levels of investment, particularly if the manufacturer fully compensates the retailer's investment costs.

3.5 Conclusion and Managerial Implications

In today's market, consumers hold high expectations for retailers to remain competitive. As a result, many retailers have adopted omnichannel fulfillment models. However, these models can be costly and require significant effort, which may lead to conflicts between channels. Therefore, it's crucial to evaluate the potential return on investment and align it with the overall strategy before making the decision. To successfully implement in-store pickup services, cooperation between manufacturers and retailers is essential. A way to foster collaboration is for the manufacturer to offer compensation to the retailer for implementing in-store pickup services. This incentive can encourage the retailer to invest in omnichannel implementation, leading to increased profits for both parties. Additionally, this can help retailers cover the cost of fulfilling in-store pickup orders without having to reduce retail prices.

This study explores the advantages of manufacturers collaborating with retailers to improve their in-store pickup services. Our research reveals that in-store pickup services can benefit both manufacturers and retailers, but the success of such services depends on the level of investment made in them. We analyzed the impact of online and offline pricing strategies on the manufacturer's equilibrium support rate using game theory models. In-store pickup services have the potential to attract customers who previously opted for direct shipping, which is believed to have a higher profit margin. The manufacturer benefits from in-store pickup when the increase in profit due to in-store sales and the reduction in last-mile delivery costs outweighs the decrease in profit due to a reduction in online retail prices and offline sales.

However, if the retailer charges high fees for this service, implementing in-store pickup services may not be profitable for manufacturers. Even if the online price remains unchanged, a handling fee can dissuade customers from choosing in-store pickup, ultimately lowering the manufacturer's profit margin.

According to our research, offering in-store pickup services may not be profitable for manufacturers with low investment levels. However, as the investment level increases, there are greater benefits to implementing such services, especially if the manufacturer compensates the retailer for their investment costs.

Retailers can also benefit from in-store pickup services, particularly when the manufacturer covers their investment costs. This can increase profits and allow the retailer to cover the costs of investing in and fulfilling in-store pickup orders without lowering the retail price. As such, when the investment level is high enough, the retail price can be set higher than the benchmark retail price.

In conclusion, in-store pickup services can be profitable for both manufacturers and retailers, but the investment level is a key factor in determining their success. Manufacturers should carefully evaluate the service level required before deciding whether to compensate retailers for implementing BOPS. The threshold for investment level at which the manufacturer is willing to compensate the retailer varies depending on delivery costs, product type, pickup service type, and cross-selling potential. When the investment level is high, manufacturers should compensate retailers for their investment costs. Retailers should also be mindful of the handling fee they charge for in-store pickup services to avoid negatively impacting in-store sales and profitability.

On the other hand, high-quality in-store pickup services can lead to increased sales, customer satisfaction, and return on investment. Poor-quality services can lead to frustration and dissatisfaction, which may cause customers to choose other retailers or opt for home delivery instead. Therefore, manufacturers should work closely with retailers to ensure that the service quality is consistently high.

Insights derived from these findings can aid retailers and manufacturers in enhancing their collaborative efforts, thereby boosting market share and revenue. Such insights are especially valuable for supply chain members operating dual-channel organizations seeking to provide omnichannel services. By gaining a deep understanding of the potential benefits and challenges associated with in-store pickup services, retailers and manufacturers can make more informed decisions about investing in this approach.

More studies are needed to examine the collaboration between manufacturers and retailers to establish successful in-store pickup services. By identifying the key factors that drive the success of these services, manufacturers and retailers can develop effective strategies for collaboration. We can think of at least three potential avenues for future research. (1) Investigating how the manufacturer might compensate multiple retailers for the cost of implementing BOPS by assessing the effect of competition between retailers. (2) Examining how manufacturers can assist retailers in promoting in-store pickup services through online marketing and in-store signage. (3) Additionally, evaluating the impact of different information sharing strategies on in-store pickup services and how it can affect the collaboration between manufacturers and retailers.

3.6 Appendix

Proof of Proposition 17.

- We compare the equilibrium prices under benchmark and in-store pickup strategies. The difference in online and offline prices are given by

$$p_s^n - p_s^p = \frac{1}{4}\left(t(5-2m) + 3(mc_p - r) + (v-w)(1-\theta) + \left(\frac{2m(1-\theta)}{2-m} + \theta\right)c_o - 4mf\right),$$
$$p_o^n - p_o^p = \frac{1}{2}\left(mc_p + t\left(\frac{3}{\theta} - 2m\right) + \left(1 + \frac{2m(1-\theta)}{2-m}\right)c_o - \frac{(v-w)(1-\theta)}{\theta} - r\right).$$

  It is easy to check that $p_s^p - p_s^n$ and $p_o^p - p_o^n$ is increasing in r since $\frac{\partial(p_s^p - p_s^n)}{\partial r} = \frac{3}{4} > 0$ and $\frac{\partial(p_o^p - p_s^n)}{\partial r} = \frac{1}{2} > 0$. Therefore we can obtain thresholds such as $\tilde{r}_1$ and $\tilde{r}_1$ such that when $r \leq \tilde{r}_1$ we have $p_s^p \leq p_s^n$, and when $r \leq \tilde{r}_2$ we have $p_o^p \leq p_o^n$.

$$\tilde{r}_1 = \frac{1}{3}\left(m(3c_p - 4f) + t(5-2m) + (v-w)(1-\theta) + \frac{(2\theta + m(2-3\theta))c_o}{2-m}\right),$$

$$\tilde{r}_2 = mc_p - 2(mt + w) + (1 + \frac{2m(1-\theta)}{2-m}) + \frac{3t - v(1-\theta) + w(1+\theta)}{\theta}.$$

- By substituting the related equilibrium values in 3.4 and 3.1 we have:

$$\begin{aligned}
\Pi_s^n &= \frac{\left(5t + (v-w)(1-\theta) + \theta c_o^2\right)^2}{32t}, \\
\Pi_s^p &= \frac{A}{16(2-m)^2 mt} + \frac{B}{16(2-m)^2 mt} + \frac{C}{16(2-m)^2 mt},
\end{aligned}$$

where

$$\begin{aligned}
A &= (9m^2 + 4 - 20m)r^2 + 16m^2(2 - 3m + m^2)ft + 4m(4 - 8m + 3m^2)rt \\
&\quad + 4(2-m)^2 m^2 t^2 + 8(2-m)m(mf + r)\left(2(v-w)(1-\theta) + r\theta + 2mt\theta\right), \\
B &= -8(2-m)m^2 f(2mf + 3r) + 4m(1-\theta)c_o\left(2(r + m^2(t + 2f(1-\theta))) + m\left(r(3-4\theta) + (1-\theta)c_o - 4t\right)\right) \\
&\quad + (2-m)m^2(2 - m(9-8\theta))c_p^2, \\
C &= 2mc_p\left((2-m)\left(-2r + 2m^2(2f(3-\theta) + t(3-4\theta)) - m\left(4t + 8(v-w)(1-\theta) - r(9-8\theta)\right)\right)\right. \\
&\quad \left. -2m(2-\theta)(2 + m(3-4\theta))c_o\right).
\end{aligned}$$

$$\Pi_s^p - \Pi_s^n = \underbrace{(p_s^p - w)D_s^p - (p_s^n - w)D_s^n}_{-} - (m(c_p - f) - r)D_b^p.$$

Using envelop theory, we have $\frac{\partial(\Pi_s^p - \Pi_s^n)}{\partial r} > 0$, and we can find a threshold $\tilde{r}_3$ such that when $r \leq \tilde{r}_3$, we have $\Pi_s^p \leq \Pi_s^n$.

$$\tilde{r}_3 = \frac{-B + \sqrt{B^2 - 4AC}}{2C},$$

where

$$\begin{aligned}
A &= \frac{1}{32}\left(8mt - 25t - 10\theta c_o - \frac{32mf(m(f + t(1-\theta)) - t)}{(2-m)t} + \frac{2m(16(v-w)(1-\theta)(f - c_p) + (2 + m(8\theta - 9))c_p^2)}{(2-m)t}\right) \\
&- \frac{1}{32}\left(\frac{(v-w)^2(1-\theta)^2 + \theta^2 c_o^2 + 2(v-w)(1-\theta)(5t + \theta c_o)}{t} + \frac{8m(2(1-\theta)c_o + (2 + m(4\theta - 3))c_p)}{2-m}\right) \\
&\frac{1}{32}\left(\frac{8m((1-\theta)^2 c_o^2 + 2(2-m)mf(3-\theta)c_p - (1-\theta)c_o((2 + m(3-4\theta))c_p - 4mf(1-\theta)))}{(2-m)^2 t}\right), \\
B &= \frac{2mf(3-\theta) - (2-3m)t - 4(v-w)(1-\theta) - 4m\theta(t - c_p)}{4t(2-m)} + \frac{(4 - 20m + 9m^2)c_p - 2(1-\theta)(2 + m(3-4\theta))c_o}{8(2-m)^2 t}, \\
C &= \frac{2 - 9m + 8m\theta}{16mt(2-m)}.
\end{aligned}$$

It is easy to show that $\tilde{r}_3$ is the unique positive root of $\Pi_s^p - \Pi_s^n = 0$.

- Considering 3.5 and 3.2, we have

$$\Pi_o^p - \Pi_o^n = \underbrace{w(D_s^p - D_s^n)}_{-} + \underbrace{p_o^p D_o^p - p_o^n D_o^n}_{-} + \underbrace{(D_o^n - D_o^p)c_o}_{+} + p_o^p D_b^p.$$

$$\Pi_o^p - \Pi_o^n \begin{cases} < 0 & \text{if} w(D_s^p - D_s^n) + p_o^p D_o^p - p_o^n D_o^n > (D_o^n - D_o^p)c_o + p_o^p D_b^p \\ > 0 & \text{if} w(D_s^p - D_s^n) + p_o^p D_o^p - p_o^n D_o^n < (D_o^n - D_o^p)c_o + p_o^p D_b^p \\ 0 & \text{if} w(D_s^p - D_s^n) + p_o^p D_o^p - p_o^n D_o^n = (D_o^n - D_o^p)c_o + p_o^p D_b^p \end{cases} \tag{3.20}$$

By substituting Stackelberg equilibrium values in 3.5 and 3.2 we obtain

$$\Pi_o^n = \frac{1}{16t\theta}\Big(9t^2 - 6tv(1-\theta) + (v-w)^2(1-\theta)^2 \\ + 2tw(3+5\theta) - 2\big(3t - (v-w)(1-\theta)\big)\theta c_o + \theta^2 c_o^2\Big),$$

$$\Pi_o^p = \frac{1}{8mt(2-m)^2}\Big(4m^2(1-\theta)^2 c_o^2 \\ - 4m(2-m)c_o\big(2(t + m(f - t\theta)) + (1-\theta)(r - mc_p - 2(v-w))\big) \\ + (2-m)^2\big(r^2 + m^2 c_p^2 + 4mt(r + mt + 2w) - 2m(r + 2mt)c_p\big)\Big).$$

It is easy to check that the profitability of in-store pickup for the manufacturer is decreasing in the handling fee $\frac{\partial(\Pi_o^p - \Pi_o^n)}{\partial f} = -\frac{mc_o}{(2-m)t} < 0$. Solving for $\Pi_o^p - \Pi_o^n = 0$, we obtain that $\Pi_o^p \geq \Pi_o^n$ if

$$f \leq \frac{(2-m)}{16mc_o}\left[\frac{2r^2}{m} + 2m(4t^2 + c_p^2) - 4(r + 2mt)c_p \right. \\ \left. + \frac{1}{\theta}\left(t(8r\theta + 6(v-w)(1-\theta)) - 9t^2 - (v-w)^2(1-\theta)^2\right)\right] \\ + \frac{1}{16m(2-m)}\left[(4m(2 - 3\theta + 2\theta^2) - \theta(4 + m^2))c_o\right] \\ + \frac{1}{16m}\left[2(1-\theta)(4(mc_p - r) + (6+m)(v-w)) - 2t(2 + m(3 - 8\theta))\right].$$

Proof of corollary 0.4.

From the equilibrium prices under in-store pickup with and without charging handling fees, we have

$$p_s^p - p_s^{p(f=0)} = mf > 0,$$
$$p_o^p - p_o^{p(f=0)} = 0.$$

Let $p_s^{p(f=0)}$ denotes the offline price when the handling fee is zero. We can obtain $p_s^p > p_s^{p(f=0)}$ and $p_o^p = p_o^{p(f=0)}$. Considering 3.4 we have

$$\Pi_s^p - \Pi_s^{p(f=0)} = \frac{mf\left(2m(1-\theta)^2c_o - (2-m)(2(mf-t) + (3-\theta)(r-mc_p) + 2(mt-v+w)(1-\theta))\right)}{2(2-m)^2t}.$$

We can show that $\frac{\partial(\Pi_s^p - \Pi_s^{p(f=0)})}{\partial f} < 0$. If we solve $\Pi_s^p - \Pi_s^{p(f=0)} = 0$ with respect to f we can find a threshold for handling fee such as f' where if $f \leq f'$ then $\Pi_s^p > \Pi_s^{p(f=0)}$.

$$f' = \frac{2(t + (v-w-mt)(1-\theta)) - (3-\theta)(r-mc_p)}{2m} + \frac{(1-\theta)^2c_o}{2-m}$$

Considering 3.5, we have

$$\Pi_o^p - \Pi_o^{p(f=0)} = -\frac{mfc_o}{t(2-m)} < 0$$

Thus, we can conclude that $\Pi_o^p < \Pi_o^{p(f=0)}$.

Proof of Proposition 18.

The difference in the retail prices without and with manufacturer's support can be written as $p_s^c - p_s^n$ and $p_o^c - p_o^n$. By solving $p_s^c - p_s^p = 0$ and $p_o^c - p_o^p = 0$ we can obtain a threshold such as m' such that if $m > m'$ then, $p_s^c > p_s^p = 0$ and $p_o^c > p_o^p$, where

$$\begin{aligned}
m' &= \frac{B - \sqrt{B^2 - 4AC}}{2C}, \\
A &= (4t - 2(1-\theta)(2(v-w+c_o) - r)), \\
B &= (4(t\theta - f) + 2t - (1-\theta)(2(v-w+\theta c_o - c_p) - r, \\
C &= (2(t\theta - f) + (1-\theta)c_p)
\end{aligned}$$

It is easy to verify that $m' = \underline{\mathrm{m}}$. Therefore if $m > \underline{\mathrm{m}}$, then $k > 0$and $p_s^c > p_s^p$and $p_o^c > p_o^p$.

- Comparing manufacturer's profit without and with support of retailer's investment, when investment level range is $\underline{\mathrm{m}} < m < \bar{m}$,we obtain

$$\begin{aligned}
\Pi_o^c = & \frac{2(1-\theta)(2-m\theta)c_o + (2-m)((1-\theta)(2(v-w)+mc_p-r)-2(t+m(n-t\theta)))}{2(1-\theta)(4-m(1-\theta))} \\
& \times \left(\frac{X+Y-mf}{mt} + \frac{t+(v+Y)(1-\theta)-mf}{(2-m)t}\right) \\
& + (w+Y)\left(\frac{mf-X-Y}{mt}\right) \\
& + c_o\left(\frac{t+(v+Y)(1-\theta)-mf}{(2-m)t}\right) \\
& - (c_o+Y).
\end{aligned}$$

where

$$X = \frac{1}{2}\left(r+v+w+c_o-mc_p\right) + \frac{mf(1-2\theta)-(1-m)t}{2(1-\theta)},$$
$$Y = \frac{2t(2+m^2-m(4-\theta))+2m((2-m\theta)f+(1-\theta)(w+v\theta))+(1-\theta)((2-m)(mc_p-r-2c_o)-4(v+w))}{2(1-\theta)(4-m(1+\theta))},$$

$$\Pi_o^c - \Pi_o^p = -\frac{(2(1-\theta)(2-m\theta)c_o-(2-m)(2(t+m(f-t\theta))+(1-\theta)(r-mc_p-2(v-w))))^2}{8(2-m)^2t(1-\theta)(4-m-m\theta)} < 0.$$

Note that the term on the right-hand side is always negative.

$$\begin{aligned}
\frac{\partial k}{\partial m} = & \frac{1}{2m^2(1-\theta)(4-m-m\theta)^2c_p}\Big[4m^2f+(8-4m+m^2)(r+2t) \\
& \qquad -2(4r+4mt+m^2(2n+t))\theta\Big] \\
& + \frac{1}{2m^2(1-\theta)(4-m-m\theta)^2c_p}\Big[-2(v-w)(m^2+4(1-\theta)(2-m(1+\theta))) \\
& \qquad + m((4-m)r+2m(2t+v-w))\theta^2 \\
& \qquad - 2(1-\theta)(8-m(1+\theta)(4-m\theta))c_o \\
& \qquad - 2m^2(1-\theta)^2c_p\Big].
\end{aligned}$$

$\frac{\partial k}{\partial m} > 0$ if $t \geq \frac{(1-\theta)\left(4(2-m(1+\theta))(v-w+c_o)+m^2(1+\theta)(v-w+\theta c_o)+m^2(1-\theta)c_p\right)}{8-4m(1+\theta)+m^2(1+2\theta^2-\theta)}$

Case-by-Case Examination of the Lagrangian Formulation.

In this part, we present the detailed mathematical derivations employed in the analysis of the Stackelberg equilibrium and the Lagrangian of the manufacturer's profit function. We present four such cases:

- If $\lambda = 0$ and $\beta = 0$,implying $(1-k) > 0$ and $k > 0$, then using 3.14 and 3.16 leads to

$$p_o^c = \frac{2mt(4-m-\theta)+(1-\theta)(2v(2-m\theta)+(2-m)(r-mc_p+2(w+c_o)))-2mf(2-m\theta)-4t}{2(1-\theta)(4-m(1+\theta))} \tag{3.21}$$

$$k^c = \frac{2(1-\theta)(2-m\theta)c_o+(2-m)((1-\theta)(2(v-w)-(r-mc_p))-2(t+m(f-t\theta)))}{2m(1-\theta)(4-m(1+\theta))c_p} \tag{3.22}$$

It is easy to see that the support rate is increasing in the offline retailer's investment level, if

$$t \geq \frac{(1-\theta)\left(4(2-m(1+\theta))(v-w+c_o)+m^2(1+\theta)(v-w+\theta c_o)+m^2(1-\theta)c_p\right)}{8-4m(1+\theta)+m^2(1+2\theta^2-\theta)}.$$

Considering the lower bound and upper bound of k we solve for $k^c = 0$ and $k^c = 1$ and obtain the interior solution for k in the range of $\underline{m} < m < \bar{m}$ where:

$$\begin{aligned}
\underline{m} =& \frac{-B+\sqrt{B^2-4AC}}{2C},\\
&A = (4t-2(1-\theta)(2(v-w+c_o)-r)),\\
&B = (4(t\theta-f)+2t-(1-\theta)(2(v-w+\theta c_o-c_p)-r)),\\
&C = -(2(t\theta-f)+(1-\theta)c_p),\\
\bar{m} =& \frac{-E+\sqrt{E^2-4DF}}{2F},\\
&D = -(4t-2(1-\theta)(2(v-w+c_o)-r)),\\
&E = -(4(t\theta-f)+2t-(1-\theta)(2(v-w)+6c_p+2\theta c_o-r)),\\
&F = -(2(t\theta-f)-(1-\theta)(1+2\theta)c_p).
\end{aligned}$$

Substituting for p_o and k in the reaction function (3.13), we obtain

$$p_s^c = \frac{mf(1-2\theta) + (1-\theta)(r - mc_p + v + w + c_o) - (1-m)t}{2(1-\theta)}$$

We must have $w \leq p_s^c$, which implies $t \leq \frac{(1-\theta)(v-w-m(c_p-f)+r+c_o)+(1-2\theta)mf}{1-m}$.

- If $\lambda = 0$ and $\beta > 0$, implying $k \leq 1$ and $k = 0$, then we have the previous pickup scenario without manufacturer's support.

- If $\lambda > 0$ and $\beta = 0$,implying $k = 1$ and $k \geq 0$, and

$$p_o^c = \frac{(2-m)(r+2(mt+w)) + m(2+m(1-2\theta))c_p - 2m(1-\theta)c_o}{2(2-m)},$$
$$p_s^c = \frac{(2-m)(3r+2mt+4w+4mf) + m(2+m(1-2\theta))c_p - 2m(1-\theta)c_o}{4(2-m)}.$$

- If $\lambda > 0$ and $\beta > 0$, then $k = 1$ and $k = 0$, which is infeasible.

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General Conclusion

The purpose of this thesis is to investigate the implementation of omnichannel fulfillment strategies and to develop optimal policies for multichannel retailers seeking to adopt such strategies. The research conducted in this thesis consists of three essays, each examining a specific aspect of omnichannel fulfillment, and featuring various operational challenges that retailers encounter when adopting such models. In this concluding section, we summarize the key findings of each essay and provide some insights into the broader implications of our research for the retail industry.

This thesis sheds light on the impact of omnichannel fulfillment strategies on consumer behavior. It highlights the need for retailers to adopt a customer-centric approach to remain competitive in the evolving retail landscape. We investigate the impact of omnichannel fulfillment strategies, specifically in-store pickup, on consumer behavior, focusing on the heterogeneity in product valuation. Our findings suggest that retailers who implement in-store pickup services benefit from an expanded market coverage, as non-shoppers may be encouraged to purchase through in-store pickup. Moreover, existing customers who previously preferred direct delivery may switch to in-store pickup, leading to a potential shift in the retailer's demand. These findings demonstrate the importance of considering the diversity in consumer behavior when implementing omnichannel fulfillment strategies, as different consumer segments may have varying order fulfillment preferences. By understanding how consumers respond to different fulfillment options, retailers can tailor their strategies to better meet the needs of their customer base and ultimately increase sales and profitability.

Implementation of an in-store pickup service as part of an omnichannel strategy demands a significant investment and can profoundly impact a company's systems and processes. It necessitates the establishment of a pickup point within the store, staff allocation for pickups, modifications to order management systems, and the integration of online and offline inventory systems, among other adjustments. Moreover, adopting a cross-channel fulfillment strategy such as in-store pickup compels retailers to reevaluate their existing operations and make numerous critical decisions to ensure efficient implementation. This thesis show that the potential benefits of adopting an omnichannel strategy depend on a multitude of factors, including the retailer's cost structure, which is influenced by store operating costs, population density in the store's location, logistics and last-mile delivery costs, and the nature of the products sold. As a result, it is imperative that retailers carefully consider whether the benefits of implementing an omnichannel strategy outweigh the associated costs and efforts.

Through this thesis, we have demonstrated that retailers can significantly profit by selecting a pickup strategy that aligns with their cost structure, rather than blindly following the industry trend of implementing in-store pickup. Making an informed decision based on a thorough analysis of the various factors involved allows retailers to maximize the return on their investment and prevent the potential negative consequences of an ill-advised strategy, such as wasted resources and a shift in sales to a less profitable channel.

In this thesis, we have explored the process of a multichannel retailer evolving into an omnichannel one, and the significant changes that ensue. A critical aspect of this transition lies in the redefinition of sales credit allocation strategies, particularly after implementing cross-channel fulfillment. Our research demonstrates that fostering cooperation between channels is crucial for maximizing the potential benefits of an omnichannel strategy. Our findings have significant implications for retailers aiming to optimize their omnichannel strategies, as they underscore the necessity of effectively allocating sales credits to avoid internal conflicts that can undermine the potential benefits of omnichannel initiatives.

The composition of the market, specifically the proportion of store-visiting customers relative to omnichannel customers who exclusively order online, has been identified as a vital factor in determining the allocation of cross-channel sales profit. This allocation directly influences whether a retailer will reap the rewards of implementing omnichannel strategies. Another key finding of our research is the potential risk associated with inappropriate sales credit allocation between channels. If not correctly established, implementing omnichannel initiatives may not yield the anticipated profit increase. This outcome may stem from internal conflicts and the phenomenon of "channel sabotage," whereby one channel actively undermines another due to perceived competition for sales credit. To mitigate this risk, it is essential for retailers to devise and implement a fair and transparent sales credit allocation strategy that incentivizes collaboration between channels, ensuring that the intended benefits of omnichannel strategies are realized.

Our research contributes to this area by demonstrating the importance of collaboration between suppliers and retailers and the impact of such cooperation on the success of omnichannel strategies. Our findings emphasize the critical role of interchannel collaboration in achieving more efficient omnichannel fulfillment. This is particularly relevant given the substantial investment required for fulfillment solutions such as in-store pickup. Smaller retailers and BM stores often lack the financial resources to shoulder these expenses independently. Therefore, fostering cooperation and shared investment between suppliers and retailers is essential to realizing the full potential of omnichannel strategies and maintaining a competitive edge in today's rapidly evolving retail landscape. Retailers should consider the factors influencing their suppliers' willingness to support and invest in these services to ensure a successful and profitable in-store pickup strategy.

The first essay specifically investigates the two popular in-store pickup strategies: Buy Online, Pickup in Store (BOPS) and Ship-to-Store (STS), assessing their influence on customer behavior. To facilitate retailers' choice between these strategies, a theoretical model is proposed, and the potential advantages of implementing both simultaneously are examined. The decision-making process between BOPS and STS is likened to a problem of allocating capacitated inventory between the BM store and the distribution center. The essay's analytical approach investigates the impact of implementing BOPS and STS on optimal inventory allocation and retailer's profit, factoring in the retailer's omnichannel strategy and customer preferences.

While our research contributes to understanding the complexities of in-store pickup strategies, conducting further empirical studies would be beneficial to validate and expand upon the theoretical findings of our research. These studies could offer real-world insights and corroborate the theoretical conclusions, ultimately enhancing our understanding of in-store pickup strategies and their implications for retailers and customer behavior. Future research could investigate the role of third-party logistics providers in implementing in-store pickup strategies, as their influence has yet to be extensively addressed in the existing literature. Analyzing the impact of collaborations with these providers on the efficiency and effectiveness of in-store pickup services would be a valuable addition to the field. Secondly, examining the effects of competitive pressures on retailers' choice of in-store pickup strategies would be insightful. A deeper analysis of the competitive landscape, considering factors such as market share, pricing strategies, and differentiation, could assist in determining the optimal in-store pickup strategy for various competitive environments.

In the second essay, we investigate the optimal and fair allocation of sales credit between online and physical store channels in an omnichannel retail environment. The study uses a game theory approach to analyze the effects of sales credit attribution policies on the ordering decisions of independent channel managers under different fulfillment strategies (STS, SFS, and hybrid). The research aims to provide insights for companies that have decided to launch a fulfillment service on how to encourage collaboration rather than competition between channels. We derive managerial policies via extensive numerical analysis even though we do not obtain our results analytically given the problem's complexity. The findings reveal that the choice of sales credit attribution policy plays a critical role in determining the success of omnichannel strategies. Depending on the fulfillment strategy and customer preferences, different credit allocation policies can lead to higher order quantities for both channels.

Moreover, the study identifies conditions under which a retailer's optimal sales credit allocation can also be considered fair for both channels. The insights this research provides can help omnichannel retailers make more informed decisions about sales credit allocation and inventory management, ultimately promoting collaboration between their online and physical store channels. This research serves as a foundation for further exploration of sales credit allocation in omnichannel retail environments, with several potential directions for future research. These could include introducing additional complexities, such as capacity constraints or behavioral effects, to understand the inventory game in omnichannel settings better. Another exciting avenue is to investigate the inventory game when each channel has asymmetric information on demand, offering insights into how information asymmetry impacts sales credit allocation and decision-making in retail environments. Expanding the scope of the study to consider credit sharing between a distribution center and multiple BM stores could provide valuable insights into more complex retail networks.

Lastly, the third essay investigates the impact of BOPS on the pricing decisions of manufacturers and retailers. It examines whether manufacturers should contribute to the costs associated with upgrading in-store and digital operations. Using stylized Stackelberg models, the study demonstrates that the retailer will benefit from BOPS sales if the manufacturer pays part of the retailer's investment cost. However, the manufacturer is willing to offer support only if the investment level is above a certain threshold. The findings suggest that cooperation between manufacturers and retailers is crucial for successfully implementing in-store pickup services. The willingness of the manufacturer to compensate the retailer's investment in BOPS will depend on several factors, such as last-mile delivery cost, product type, and the retailer's cross-selling potential.

This study focused on a supply chain model consisting of a single manufacturer and a single BM retailer. However, future research can extend this model by considering multiple retailers and manufacturers involved in implementing BOPS. By exploring the strategies manufacturers could adopt to distribute their financial support among various retailers, taking into account their diverse needs, locations, and target markets, we can gain valuable insights into optimizing manufacturers' investments and maintaining balanced relationships with multiple retail partners.

Moreover, future studies can delve into the possible cooperation between multiple manufacturers in compensating retailers for their investments in omnichannel strategies. This could provide a better understanding of the benefits and challenges of collaborative efforts in the supply chain and the potential synergies that can arise from such partnerships. Additionally, examining how manufacturers can assist retailers in promoting in-store pickup services can offer valuable perspectives on marketing and communication strategies that maximize the benefits of BOPS implementation for all parties involved. This exploration could help identify best practices for driving customer engagement, improving customer experiences, and enhancing the overall effectiveness of omnichannel retailing strategies.

In conclusion, the practical implications of this thesis underscore the importance of adopting a customer-centric approach and carefully evaluating the costs and benefits of implementing omnichannel fulfillment strategies. Retailers must also foster collaboration between channels and suppliers to ensure efficient implementation and maximize the potential benefits of omnichannel initiatives. By making informed decisions based on a thorough analysis of the various factors involved, retailers can meet the needs of their customers, increase sales, and maintain a competitive edge in today's rapidly evolving retail landscape.