HEC MONTRÉAL
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Essays on interest-rate derivatives: model risk, exercise strategies and credit risk protection

par
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Résumé

Cette thèse traite du risque de saut dans les dérivés de taux d'intérêt et examine aussi les implications du nouveau système de compensation des swaps de défaillance (CDS).

Le premier chapitre présente un cadre général d'évaluation de produits dérivés de type bermudéen reposant sur la programmation dynamique associée à la transformation de processus de diffusion affine avec saut proposée par Duffie, Pan & Singleton (2000). Ce cadre général est appliqué à l'évaluation d'obligations sans risque comportant des options d'achat ou de vente implicites, de swaptions de type Européen et Bermudien, et de contrats à terme sur obligations comportant des options implicites de livraison. Des résultats numériques sont proposés dans le cadre du modèle de Vasicek (1977) augmenté de sauts exponentiels à la hausse et à la baisse.

Le deuxième chapitre est consacré à l'évaluation de l'erreur de modèle résultant de l'utilisation d'un modèle de diffusion pure par rapport à un modèle de diffusion avec sauts. Cette évaluation est faite pour divers dérivés de taux d'intérêt en utilisant le cadre général décrit dans le premier chapitre. Elle permet d'identifier des situations pour lesquelles la possibilité de sauts joue un rôle important dans l'évaluation du prix d'un produit. Nous montrons que le choix d'un modèle avec sauts calibré sur la courbe des taux peut conduire à des différences de prix significatives, plus particulièrement dans les circonstances suivantes: un environnement à faible volatilité, des taux d'intérêt élevés et une faible vitesse de retour à la moyenne, surtout lorsque la structure à terme des taux d'intérêt est croissante ou inversée.

Le troisième chapitre se concentre sur la stratégie de livraison des contrats à terme sur bons du Trésor, notamment sur le contrat transigé au Chicago Board of Trade (CBOT). Ce contrat est l'un des plus négociés au monde pour couvrir le risque de taux d'intérêt. L'existence de plusieurs options implicites de livraison complique le choix de la date d'exercice. Nous comparons les stratégies de livraison appliquées historiquement par les détenteurs d'une position courte à la stratégie optimale calculée selon l'approche décrite dans le premier chapitre. Notre évaluation à posteriori de la stratégie de livraison sur la période 2005-2015 montre que les détenteurs du contrat ont généralement attendu la fin du mois de livraison, optant de ce fait pour une position plutôt risquée par rapport à la stratégie optimale. Nous constatons cependant que le bénéfice moyen réalisé est proche de celui qui aurait été obtenu en appliquant la stratégie préconisée par le programme dynamique.

Le quatrième chapitre examine l'impact du nouveau système volontaire de compensation centrale sur le marché des CDS. L'approche utilisée est celle des différences de différences généralisée. Pour pallier le problème d'endogénéité dû à l'adhésion volontaire, nous proposons une méthodologie reposant sur un appariement dynamique du score de propension. Nos résultats empiriques montrent que la compensation centrale entraîne une légère augmentation (estimée à 19 points de base) des écarts de taux des CDS. Par contre, nous ne trouvons aucune évidence significative ni d'amélioration de la liquidité ou de l'activité sur le marché, ni d'une détérioration du risque de crédit de l'obligation sous-jacente. Ces résultats suggèrent que l'augmentation des écarts de taux des CDS peut être principalement attribuée à une réduction du risque de contrepartie.

Mots-clés: Taux d'intérêt, sauts, dérivés bermudéens, swaps de défaillance, chambre de compensation, risque de crédit, liquidité, stratégie d'exercice, erreur de modèle.

Méthodes de recherche: Recherche quantitative, recherche empirique, programmation dynamique, différence-des-différences.

Abstract

The aim of this thesis is to provide various contributions on the subjects of the jump risk in interest-rate derivatives and the new central clearing scheme for single name credit default swaps (CDS).

The first chapter presents a general pricing framework for Bermudan-style derivatives relying on dynamic programming combined with the transform analysis for affine jump-diffusion processes proposed in Duffie, Pan & Singleton (2000). This general framework is used to price risk-free bonds with embedded call or put options, European and Bermudan swaptions, and T-bond futures with embedded delivery options. Numerical results are provided under the Vasicek (1977) model augmented with upward and downward exponential jumps.

The second chapter is devoted to the assessment of the model error resulting from using a pure diffusion model against a jump-diffusion. This assessment is done for various interest-rate derivatives using the general pricing framework described in the Chapter 1. This allows to identify situations where the possibility of jumps plays an important role in the derivative price. We show that ignoring jumps and relying on a pure diffusion model calibrated to the yield curve could lead to large price differences, more specifically in the following circumstances: a low volatility environment, high interest rates and a low speed of mean reversion, and more so when the term structure is upward sloping or when it has a humped shape.

The third chapter focuses on the delivery strategy of bond futures, namely on the Chicago Board of Trade (CBOT) T-Bond futures contract. This contract is one of the most actively traded in the world in order to hedge interest-rate risk. However, the existence of several embedded options complicates the choice of the exercise strategy. We compare the delivery strategies actually applied historically by the short traders to the optimal strategy, obtained using the approach described in Chapter 1. Our post-trade evaluation of the deliveries over the period 2005-2015 shows that the traders generally waited until the end of the delivery month, thus taking a riskier position with respect to the optimal strategy. However, we find that the average profits they realized is close to what they would have gained had they used the strategy computed by the dynamic programming.

The fourth chapter revisits the issue of the impact of the voluntary central clearing scheme on the CDS market using a generalized difference-in-differences (DID) methodology. In order to address the endogeneity issue due to voluntary adhesion, we propose a methodology relying on a dynamic propensity-score matching. Our empirical findings show that central clearing results in a small increase (estimated at 19 bps) in CDS spreads, while there is no evidence of an associated improvement in CDS market liquidity and trading activity, or of a deterioration in the default risk of the underlying bond. These results suggest that the increase in CDS spreads can be mainly attributed to a reduction in CDS counterparty risk.

Keywords: Interest rates, jumps, Bermudan derivatives, credit default swaps, central clearing, counterparty risk, liquidity, exercise strategy, model error.

Research methods: Quantitative research, empirical research, dynamic programming, difference-in-differences.

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List of acronyms

BIS Bank of Internation Settlements
CBOT Chicago Board of Trade
CCP Central Counterparty
CDS Credit Default Swaps
CTD Cheapest-To-Deliver
CVA Credit Value Adjustment
DID Difference-In-Differences
DP Dynamic Programming
DTCC Depository Trust and Clearing Corporation
EJ Exponential Jumps
GJ Gaussian Jumps
ICECC Intercontinental Exchange Clear Credit
OTC Over the Counter
PECS Par Equivalent CDS Spread
YTM Yield-To-Maturity
YTW Yield-To-Worst

Notation

t	Time
t_m	Possible exercise dates
τ_m	Decision dates
T	Maturity
r_t	Short-term interest rate process
$\mathcal{F}_t$	Filtration generated by (r_t)
$\mathbb{E}_t\left[\cdot\right]$	Expectation under the risk-neutral measure, conditional on the filtration $\mathcal{F}_t$
e_m	Exercise payoff at date t_m
Δ_m	Notice period
c_m	Received cash flow at date t_m
$P_t(r,T)$	Price at t of a zero-coupon bond maturing at $T \geq t$ when the interest is r
v_m^e	Exercise value
v_m^h	Holding value
v_m	Value function
$\mathcal{G}$	Set of grid points
p	Number of grid points
F_j	Family of basis functions
$\mathbb{I}$	Indicator function
α	Speed of mean reversion
ζ	Long-term mean

s	Value of the swap
σ	Diffusion volatility
W	Wiener process
$N(\lambda)$	Poisson process with intensity λ
λ_u	Intensity of upward jumps
λ_d	Intensity of downward jumps
η_u	Parameter of the exponential distribution of the magnitude of upward jumps
η_d	Parameter of the exponential distribution of the magnitude of downward jumps
J	Jump size
ρ	Discount factor
Λ_m	Call prices
Γ_m	Put prices
κ	Swap rate
K	Swaption strike price
Φ	Standard normal cumulative distribution function
c	Credit default swap premium
l	Bond loss given default
ξ_t	Default intensity process
γ_{1i}	Firm fixed effects
γ_{2t}	Time fixed effects
β	Vector of the parameters of the difference-in-differences

To my beloved parents,

To my beloved wife Myriam.

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Chapter 1

Introduction

Fixed-income securities represent a large proportion of the overall traded securities in the financial markets. These securities are widely used by practitioners to get access to the debt market or to hedge interest-rate risk exposure. According to the BIS, the total notional amount outstanding in fixed-income securities was $437,837$ billion USD by the end of 2018. This thesis focuses on fixed-income securities, more specifically on the pricing and exercise strategy of those securities that include some form of optionality, and on the market for their protection from credit risk.

The evaluation of fixed-income securities is often straightforward, consisting of evaluating the value of a stream of future payoffs under some given theoretical model for the interest-rate dynamics. However, many fixed-income securities include implicit embedded options, which requires the use of more advanced approaches. These problems are generally addressed by either directly solving the partial differential equation characterizing the value of the option, given its boundary values, or by computing the expectation, under the risk-neutral measure, of the future potentiality of the contract under an optimal exercise strategy. The former approach generally involves quasi analytical solutions or finite differences methods, while the second is usually done through dynamic programming.

The dynamic programming approach has been widely used thanks to its simplicity, efficiency and ability to handle pricing problems involving intermediate decisions, as in the case of Bermudan options. Dynamic programming is a general approach, encompassing trees and other lattice methods, that finds the optimal exercise strategy and the corresponding value of a derivative, at all possible dates, and for all possible states of the world, provided the *state variable* is observable and is a sufficient statistic describing all the relevant information. Dynamic programs are usually solved by backward recursion, from the known value of the derivative at maturity.

Clearly, the model used to represent the dynamics of the underlying asset is an important ingredient of the valuation process. Simple models are appealing, but may not be flexible enough to represent market expectations and may result in pricing errors, while more involved models may increase the computational complexity of a pricing algorithm. One interesting question is to what extent the use of a complex model is granted, given the additional computational burden implied. In this thesis, we consider two popular class of interest-rate dynamics, that is, pure diffusion and jump-diffusion models. The inclusion of jumps in interest-rate dynamics has been advocated to account for discontinuous movements that frequently occur following Central Banks announcements and macroeconomic events. In fact, numerous studies find that jumps do occur and that they cannot be captured by pure diffusion models. A first contribution of this thesis is the examination of the opportunity of using jump-diffusion models for the evaluation of a variety of interest-rate derivatives, possibly including optionality clauses.

It is worthwhile noting that products that do not contain optional clauses can be evaluated in a straightforward manner, either analytically or by Monte-Carlo simulation. In the same way, one can easily compute the expected payoff of a security corresponding to a given exercise strategy. Accordingly, another important ingredient of the valuation process is the determination of the optimal exercise strategy. Products involving interrelated options, as for instance T-bond futures and callable and putable bonds, are difficult to price because of the complexity of their exercise strategy. Another interesting question about the pricing of interest-rate derivatives is the relative sensitivity of the value of the instrument to the exercise strategy of its implicit options. A second contribution of this thesis is the analysis, using historical data, of the impact of differences in the exercise strategies of complex implicit options.

Pricing methodologies and models represent only one part of the picture concerning fixed-income securities. Regulations should be in place to help organise the trades and reduce the overall risk in the market. A large part of the fixed-income security market consists of debt instruments (e.g. bonds and notes), that are subject to credit risk. The credit default swap (CDS) acts as an insurance contract against the default of the issuer of an underlying bond. The large size of the CDS market and its interconnectedness with the bond market makes this product a particularly interesting research topic. Initially, CDS were solely exchanged in the over-the-counter (OTC) market, until they were heavily criticized for their lack of transparency and for the role they consequently played in the recent financial crisis. By the end of 2009, voluntary clearing operations began in North America and in Europe for single-name CDS, with the objective of mitigating counterparty risk of single-name CDS. A third contribution of this thesis is to evaluate the impact of this central clearing scheme on the CDS and bond markets.

The rest of the thesis is organised as follows. Chapter 2 presents a general dynamic programming pricing framework and shows how this framework can be used to price various fixed-income derivatives including callable bonds, Bermudan swaptions and T-bond futures, under different interest-rate models. Chapter 3 assesses the importance of model error by comparing the prices of several types of interest-rate derivatives under pure diffusion and jump-diffusion models and identifies situations where the modeling choice could lead to large pricing differences. Chapter 4 performs a post-trade evaluation of the delivery strategy of the U.S. T-bond futures, comparing the actual delivery behavior of short traders to the optimal strategy and assessing the impact of using a sub-optimal strategy. Chapter 5 evaluates the impact of the voluntary central clearing scheme on the liquidity and trading volume of the CDS market and on the credit risk in the bond market. Chapter 6 concludes the thesis and proposes avenues for future research.

Chapter 2

Pricing American-style interest-rate derivatives under affine jump diffusions

2.1 Introduction

Numerical methods for the valuation of interest-rate derivatives are frequently implemented under the assumption that the underlying security's movements are described by a diffusion process. Several methodologies have also been proposed in order to incorporate potential jumps in the valuation procedure. Bouziane (2008) applies Fourier transform techniques to one- and multi-factor models to price European interest-rate options. Beliaeva, Nawalkha & Soto (2008) use recombining multinomial trees constructed assuming various jump-size distributions and rely on the jump-extended Vasicek model. Beliaeva & Nawalkha (2012) rely on the same approach but consider instead the jump-extended constant-elasticity-of-variance (CEV) models. Lim & Linetsky (2012) use an eigenfunction expansion approach to price sovereign bonds with embedded options. Jaimungal & Surkov (2013) combine Fourier transform with a sequence of measure changes to solve the partial integro-differential equation. More recently, Park & Kim (2015) use Monte Carlo simulation to price caplets under short and forward rate models augmented with normally-distributed jumps.

The inclusion of jumps not only helps accounting for macroeconomic news announcements, but also alleviates the need for stochastic volatility and regime switching models when pricing interest-rate derivatives. In fact, shifts in monetary policy that are usually captured by regime switching models could also be captured by appropriately calibrating jump parameters. For instance, in a low (high) interest-rate regime, we should have a low (high) probability of occurrence and magnitude of jumps. Moreover, Bali and Wu (2006) show that regime switching models can compensate for not incorporating a nonlinear drift specification to the short rate process. Das (2002) shows that jump models also mitigate the nonlinear drift specification since nonlinearity is caused by information effects, which is taken into account by jumps. On the other hand, stochastic volatility models better capture persistence in volatility across periods as well as fat-tailed distribution. Johannes (2004) shows that jump-diffusion models can easily generate patterns of conditional and unconditional kurtosis. In addition, they generate a non-negligible proportion of the conditional variance of interest rates. Hence, jump-diffusion models help address the drawbacks associated with pure diffusion models and also add improvements that are usually associated with regime switching and stochastic volatility models.

In this chapter, we propose a general numerical pricing approach, under a jump-diffusion model, for interest-rate derivatives that could offer multiple optional exercise opportunities. We then investigate the impact of jumps on the behavior of such derivatives.

The procedure is based on a dynamic programming (DP) formulation, combined with Fourier transform, to price American-style derivatives under affine jump-diffusion models (Duffie, Pan & Singleton 2000). The formulation is flexible and can be applied to many instruments with a low-dimensional state space. As illustrations, we price risk-free bonds with embedded call and put options, European and Bermudan swaptions, as well as T-Bond futures with delivery options. Starting from known exercise value at maturity, the derivative is evaluated by backward induction at all decision dates, and for all possible states of the world. We rely on the Vasicek (1977) model, augmented with exponential jumps (Vasicek-EJ) to describe the interest-rate dynamics. Discontinuities are represented using two Poisson processes, one controlling the upward movements, and the other the downward movements. The jump size in either directions is exponentially distributed.

The organization of this chapter is as follows. Section 2.2 provides a description of a general DP framework for the evaluation of interest-rate derivatives with early-exercise opportunities. Section 2.3 explains how the dynamic program is solved. Section 2.4 describes how to implement the DP procedure under the Vasicek-EJ model. Section 2.5 specializes this DP model to various interest-rate derivatives, Section 2.6 provides some numerical illustrations and Section 2.7 concludes.

2.2 The general dynamic programming valuation framework

The pricing of interest-rate derivatives with early-exercise features requires the use of lattice methods as they cannot generally be priced using analytical formulas. In this thesis, we use a dynamic programming formulation where the stage variable corresponds to decision or monitoring dates and the state variable is the underlying asset value (here, the interest rate). The derivative contract is thus priced by backward induction until the contract's inception date. The exercise decision of the option holder, at each possible exercise opportunity, and for each possible value of the state variable, is based on a comparison between the *exercise value*–which is the outcome earned from immediate exercise–and the *holding value*–representing the discounted value of the expected future potentialities of the contract, should the holder decide to wait at least until the next exercise date. At a given state and date, the *value function* is the maximum between the exercise and holding values, which allows to determine the optimal exercise strategy. We now briefly present the setting and corresponding notation.

We consider an interest-rate derivative contract with inception date $t = 0$ and maturity T. Denote by $(r_t)_{0\leq t\leq T}$ the short-term interest rate, which evolves in continuous time. We assume that the process r_t is Markovian and denote by $\mathcal{F}_t$ the filtration generated by $(r_t)_{0\leq t\leq T}$. In the sequel, depending on the context, the notation $\mathbb{E}_t[\cdot]$ (*resp.* $\mathbb{E}_m[\cdot]$) represents the expectation, under the risk-neutral measure, conditional on the filtration $\mathcal{F}_t$ (*resp.* $\mathcal{F}_{t_m}$).

According to the contract, exercise is possible at dates $t_1, ..., t_n$, where $t_n = T$ and $[0, t_1)$ is the *protection period.* The exercise payoff at date t_m, $m = 1, ..., n$ is contingent on the underlying interest rate and is denoted by $e_m(r_{t_m})$.

We also assume that the contract allows for a notice period $\Delta_m \in [0, t_m - t_{m-1})$ at exercise date t_m, that is, the exercise decisions are made at dates $\tau_m = t_m - \Delta_m$ while the exercise payoff is obtained at t_m for $m = 1, ..., n$. Define $\tau_0 \equiv t_0 \equiv 0$.

Finally, a fixed cash flow c_m (which could be positive, negative or 0) is received by the option holder at date t_m, $m = 1, ..., n$, if the option has not been exercised at t_m or earlier.

For a given $m = 0, ..., n-1$, denote by

$$\rho_m \equiv \exp\left(-\int_{\tau_m}^{\tau_{m+1}} r_s ds\right)$$

and define

$$P_t(r, u) = \mathbb{E}_t\left[\exp\left(-\int_t^u r_s ds\right)\right],$$

where $P_t(r, u)$ is the price at $(t, r_t = r)$ of a zero-coupon bond maturing at $u \geq t$.

The exercise value at decision date τ_m, $m = 1, ..., n$ is then given by

$$v_m^e(r) = \mathbb{E}_m\left[\exp\left(-\int_{\tau_m}^{t_m} r_s ds\right) e_m(r_{t_m})\right]. \tag{2.1}$$

The holding value $v_m^h(r)$ at date τ_m when $r_{\tau_m} = r$ is defined by

$$v_m^h(r) = \mathbb{E}_m\left[\rho_m v_{m+1}(r_{\tau_{m+1}})\right] + c_m P_{\tau_m}(r, t_m),\ m = 0, ..., n-1. \tag{2.2}$$

The value function $v_m(r)$ at date τ_m when $r_{\tau_m} = r$ is the maximum between the exercise value and the holding value when exercise is allowed:

$$v_n(r) = v_n^e(r) \tag{2.3}$$

$$v_m(r) = \max\left\{v_m^e(r), v_m^h(r)\right\}, \ m = 1, ..., n-1 \tag{2.4}$$

$$v_0(r) = v_0^h(r). \tag{2.5}$$

The framework presented here is general as it can accommodate various specifications of interest-rate instruments (i.e. exercise payoffs and schedules), as well as different specifications for the interest-rate dynamics, including the possibility of jumps, provided the expected values in (2.1) and (2.2) can be obtained.

2.3 Solving the dynamic program

Starting from the known function v_n, the dynamic program (2.1-2.5) yields the value of v_m for $m = 0, ..., n$ by backward recursion, providing the value of the derivative contract at all decision dates τ_m as a function of the interest rate at that date. In general, the value function cannot be obtained in closed-form, and numerical approximation techniques are required.

Assume that zero-coupon bond prices $P_t(r, T)$ can be obtained in closed form. One possible approach is to interpolate the value function to a form chosen in such a way that the expected value in (2.2) is analytical (see Breton & de Frutos 2012). More precisely, consider a family $\mathcal{V}$ of functions such that $\mathbb{E}_t\left[\exp\left(-\int_t^T r_s ds\right) v(r_T)\right]$ can be obtained analytically for $v \in \mathcal{V}$ and assume that $e_m \in \mathcal{V}$ for $m = 1, ..., n$.

Now suppose that $v_{m+1} \in \mathcal{V}$. This implies that Equations (2.1), (2.2) and (2.4) can be used to obtain the value of v_m on a finite grid of points. We then use these results to obtain an interpolation function $\hat{v}_m \in \mathcal{V}$ approximating v_m. Starting from the known function v_n, this process yields, by backward induction, analytical interpolation functions $\hat{v}_m(r) \in \mathcal{V}$ for all evaluation dates τ_m.

Define a set $\mathcal{G} = \{r_j, j = 0, ..., p\}$ of p grid points, such that

$$0 = r_0 < r_1 < r_2 < ... < r_p < \infty$$

where r_p is chosen so that $\mathbb{P}[r_t > r_p | r_0] < \varepsilon$ for $t \in [0, T]$, with ε a very small number. An interpolation function $\hat{v}_m$ for v_m is defined by

$$\hat{v}_m(r) = \sum_{j=0}^{p} c_j^m F_j(r) \text{ if } r \in [r_0, r_p]$$

where $(F_j)_{j=0,...,p}$ is a family of basis functions and where the coefficients c_j satisfy the linear system

$$v_m(r_i) = \sum_{j=0}^{p} c_j^m F_j(r_i),\ i = 0, ..., p.$$

In the numerical experiments reported in this thesis, we use finite-element interpolation with linear splines (piecewise linear interpolation) of the value function, where

$$F_j(r) = \begin{cases} \frac{r-r_{j-1}}{r_j-r_{j-1}} \text{ if } r_{j-1} < r \leq r_j \\ \frac{r_{j+1}-r}{r_{j+1}-r_j} \text{ if } r_j < r \leq r_{j+1} \\ 0 \text{ otherwise} \end{cases},\ j = 1, ..., p$$

$$F_0(r) = \begin{cases} \frac{r_1-r}{r_1-r_0} \text{ if } r_0 < r \leq r_1 \\ 0 \text{ otherwise} \end{cases}$$

$$F_p(r) = \begin{cases} \frac{r-r_{p-1}}{r_p-r_{p-1}} \text{ if } r_{p-1} < r \leq r_p \\ 0 \text{ otherwise} \end{cases}$$

so that

$$\hat{v}_m(r) = \begin{cases} \sum_{j=0}^{p} v(r_j) F_j(r) \text{ if } r \in [r_0, r_p] \\ 0 \qquad\qquad \text{otherwise.} \end{cases}$$

As a consequence, the dynamic program (2.1) and (2.2) can be solved for any specification of the interest-rate dynamics such that $P_t(r, T)$, $\mathbb{E}_m\left[\rho_m \mathbb{I}(\underline{r} \leq r_{\tau_{m+1}} < \overline{r})\right]$ and

$\mathbb{E}_m\left[\rho_m r_{\tau_{m+1}}\mathbb{I}(\underline{r} \leq r_{\tau_{m+1}} < \overline{r})\right]$ can be obtained in closed form, and for any piecewise linear exercise payoff function, where $\mathbb{I}$ is the indicator function

$$\mathbb{I}(\mathcal{S}) = \begin{cases} 1 \text{ if } \mathcal{S} \text{ is true} \\ 0 \text{ otherwise.} \end{cases}$$

In the next section, we show how to use the transform analysis proposed in Duffie, Pan & Singleton (2000) to apply this numerical approach to an affine jump-diffusion interest-rate model.

2.4 Valuation under the Vasicek-EJ interest-rate model

We consider the following jump-diffusion dynamics for the instantaneous spot interest rate r_t (Chacko & Das 2002):

$$dr_t = \alpha(\zeta - r_t)dt + \sigma dW_t + J_u dN_u(\lambda_u) - J_d dN_d(\lambda_d)$$

where N_u and N_d are two independent Poisson processes with intensities λ_u and λ_d, respectively. J_u (resp. J_d) represents the magnitude of the upward (*resp. downward*) jump and is exponentially distributed with positive mean $(1/\eta_u)$ (*resp.* $1/\eta_d$), α is the speed of mean reversion, ζ is the long-term mean and σ is the volatility.

Such dynamics for the interest rate, that is a Vasicek (1977) model with double exponential jumps, allows for separate exponential distributions for the upward and downward jumps. The advantage of this representation is that the probability of obtaining negative interest rates can be significantly lowered by reducing the size and frequency of downward jumps. Nonetheless, given the continuously observed negative interest rates in the European market during the last years, what was widely known as a disadvantage of the Vasicek model may no longer be valid. The model also allows to take into account different possible interest-rate regimes (e.g., in a regime characterized by low interest rates, the size and frequency of upward jumps may be lower than those associated with downward jumps, to reflect the lower magnitude and probability of occurrence). In addition, according to Andersen & Andreasen (2001), the Wall Street practice of using continuously re-calibrating single factor models (instead of using multiple-factor models) does not lead to significant mispricing. One other remarkable advantage of the Vasicek-EJ model is that it is straightforward to move from jump-diffusion models to pure diffusion models, simply by setting both jump intensities to zero.

In the sequel, we use the notation

$$\begin{aligned} u &\equiv \alpha\eta_u + 1 \\ d &\equiv \alpha\eta_d + 1 \\ \delta &\equiv \exp\left(-\alpha(T-t)\right). \end{aligned}$$

Under this model, Chacko & Das (2002) and Beliaeva et al. (2008) derive a closed-form solution for the zero-coupon bond price that is expressed as follows under the risk neutral measure

$$P_t(r,T) = \exp\left(C(T-t) - (r-\zeta)D(T-t) - H(t,T)\right) \tag{2.6}$$

where

$$\begin{aligned} H(t,T) &= \zeta\,(T-t)\,, \\ D(T-t) &= \frac{1-\delta}{\alpha}, \\ C(T-t) &= (T-t-\frac{1-\delta}{\alpha})\frac{\sigma^2}{2\alpha^2} - \sigma^2\frac{(1-\delta)^2}{4\alpha^3} - (\lambda_u+\lambda_d)\,(T-t) \\ &\quad + \frac{\lambda_u\eta_u}{u}\log\left|\frac{u-\delta}{\alpha\delta\eta_u}\right| + \frac{\lambda_d\eta_d}{d}\log\left|\frac{d-\delta}{\alpha\delta\eta_d}\right|. \end{aligned}$$

This interest-rate model is a particular case of the general affine jump diffusion model proposed by Duffie et al. (2000). Using the Duffie et al. (2000) transform and extended transform, the following expectations can be computed under Vasicek-EJ (details are provided in Appendix 2.8):

$$\begin{aligned}&\mathbb{E}_m\left[\rho_m \mathbb{I}(\underline{r} \leq r(\tau_{m+1}) < \overline{r})\right] \\ =\ & \frac{1}{\pi}\int_0^\infty \frac{\mathrm{Im}[\psi^\chi(i\omega, r_{\tau_m}, \tau_m, \tau_{m+1})(\exp(-i\omega\underline{r}) - \exp(-i\omega\overline{r}))]}{\omega} d\omega\end{aligned}$$

and

$$\begin{aligned}&\mathbb{E}_m\left[r(\tau_{m+1})\rho_m \mathbb{I}(\underline{r} \leq r(\tau_{m+1}) < \overline{r})\right] \\ =\ & \frac{1}{\pi}\int_0^\infty \frac{\mathrm{Im}[\phi^\chi(1, i\omega, r_{\tau_m}, \tau_m, \tau_{m+1})(\exp(-i\omega\underline{r}) - \exp(-i\omega\overline{r}))]}{\omega} d\omega\end{aligned}$$

where

$$\begin{aligned}\psi^\chi(i\omega, r, t, T) &= \exp(\Omega_t + \Pi_t r) \\ \phi^\chi(i\omega, r, t, T) &= \psi^\chi(i\omega, r, t, T)(A_t + B_t r).\end{aligned}$$

Denote

$$b \equiv i\omega + \frac{1}{\alpha}$$

After solving four ordinary differential equations, we obtain the following expressions for Ω_t, Π_t, A_t and B_t:

$$\Pi_t = b\delta - \frac{1}{\alpha}$$

$$\begin{aligned}\Omega_t &= b(\zeta - \frac{\sigma^2}{\alpha^2})(1-\delta) + \frac{\sigma^2 b^2}{4\alpha}(1-\delta^2) \\ &\quad +(\frac{\sigma^2}{2\alpha^2} + \alpha\frac{\lambda_u\eta_u}{u} + \alpha\frac{\lambda_d\eta_d}{d} - \lambda_u - \lambda_d - m)(T-t) \\ &\quad +\frac{\lambda_u\eta_u}{u}\ln\left(\frac{u - b\alpha\delta}{u - b\alpha}\right) + \frac{\lambda_d\eta_d}{d}\ln\left(\frac{d - b\alpha\delta}{d - b\alpha}\right) \\ B_t &= \delta \\ A_t &= (\zeta - \frac{\sigma^2}{\alpha^2})(1-\delta) + \frac{\sigma^2 b}{2\alpha}(1-\delta^2) \\ &\quad +\alpha b\lambda_u\eta_u\frac{1-\delta}{(u-b\alpha)(u-b\alpha\delta)} + \alpha b\lambda_d\eta_d\frac{1-\delta}{(d-b\alpha)(d-b\alpha\delta)}.\end{aligned}$$

2.5 Application to the pricing of various American-style interest-rate derivatives

In this section, we illustrate how to adapt the general dynamic programming model presented above to evaluate various interest-rate instruments. The instruments considered here include risk-free bonds embedding call and put features, European and Bermudan interest-rate swaptions, as well as futures contracts written on Treasury bonds.

2.5.1 Bonds with embedded call and put options

A *callable bond* allows the issuer to buy his debt back at some time before its maturity. Because this call feature is at the option of the issuer and the bondholder is exposed to the risk of redemption, callable bonds generally trade at lower prices compared with similar option-free bonds. When interest rates are significantly high, callable bonds behave like their straight counterparts, as there is no incentive to exercise the option. However, when interest rates are low, the embedded call option becomes valuable because of the high probability of redemption. A *putable bond*, on the other hand, is a security that allows its holder to ask for an early payment of the principal. The embedded put option is more likely to be exercised when interest rates are high, and the price of a putable bond should be higher than that of the corresponding option-free bond. Because of these early-exercise features, callable and putable bonds are considered American-style interest-rate derivatives.

We now briefly describe the adaptation of the dynamic program (2.2)-(2.5) for the pricing of options embedded in risk-free bonds, as proposed in Ben-Ameur et al. (2007).[1]

[1]Ben-Ameur et al. (2007) price options embedded in bonds under the Vasicek (1977), Cox-Ingersoll-Ross (1985), and Hull & White (1990) interest-rate models.

The security to be priced is a risk-free bond embedding at the same time the option to call when interest rates are low, so that the issuer refinances at lower rates, and the option to put when interest rates are high, so that the holder can invest the proceeds of early exercise at higher rates. The bond principal is normalized at 1. The bond pays a fixed coupon, denoted by c, at regular discrete dates. The exercise dates, after the protection period, correspond to the coupon dates. The call and put prices at each exercise date are denoted by Λ_m and Γ_m, respectively, with $\Lambda_m > \Gamma_m > 0$.

Note that the call and put options are held by two decision makers with opposing interests: the investor, who maximizes the bond value, and the bond issuer, who minimizes it. More specifically, the issuer should call the bond when the continuation value is higher than the call price, while the investor should put the bond when the continuation value is lower than the put price. Neither option should be exercised in the remaining cases. At the maturity date, if neither option has been exercised, the bond value is equal to the principal to which the coupon payment is added.

The dynamic program (2.2-2.5) is adjusted to account for the presence of two decision makers and of payments during the protection period, where the value function $v_m(r)$ represents the value of the bond with its embedded option at date τ_m:

$$
\begin{aligned}
v_n(r) &= 1 + c \\
v_m^h(r) &= \mathbb{E}_m[\rho_m v_{m+1}(r_{\tau_{m+1}})] + cP_{\tau_m}(r, t_m), m = 1, ..., n-1 \\
v_m(r) &= \max\left\{P_{\tau_m}(r, t_m)\Gamma_m, \min\left\{P_{\tau_m}(r, t_m)\Lambda_m, v_m^h(r)\right\}\right\}, m = 1, ..., n-1 \\
v_0(r) &= \mathbb{E}_0[\rho_0 v_1(r_{\tau_1}) + S]
\end{aligned}
$$

where S is the value, discounted to date 0, of all the coupons due during the protection period.

The difference between $v_m(r)$ and the value of the associated straight bond at τ_m, $m = 0, ...n$, yields the combined value of the two embedded options. The value of a callable bond is obtained by setting $\Gamma_m = 0$ and the value of a putable bond is obtained by setting $\Lambda_m = \infty$. Note that, because exercising one's option cancels the option of the other holder, the combined value of two embedded options is smaller than the sum of their independent values.

2.5.2 European and Bermudan interest-rate swaptions

Under a swap contract, the two counterparties agree to exchange interest payments based on a notional amount, at fixed *installment dates.* The payer swap pays a fixed rate, also known as the *swap rate*, whereas the receiver swap pays a floating rate. The swap rate is determined by setting the value of the contract at zero at inception (no money transfer between the two counterparties). The position of a payer (*resp. receiver*) swap is equivalent to a short (*resp. long*) position in a bond with a fixed rate along with a long (*resp. short*) position in a bond with a floating rate.

A receiver swaption gives the holder the right, but not the obligation, to enter into a swap agreement as the floating-rate payer and fixed-rate receiver. This position is equivalent to a call option with a strike K on a coupon bond with rate κ and notional M.

A payer swaption gives the holder the right to enter into a swap agreement as the fixed-rate payer and floating-rate receiver. Similarly, the position of a payer swaption is equivalent to a put option, with a strike price K, on a coupon bond with a rate κ and notional M.

We consider Bermudan swaptions that offer the option holder the right to enter into an underlying swap contract with inception date t_1 and maturity t_n at any of a fixed set of dates t_m, $m = 1, ..., n-1$, that coincide with instalment dates.[2] This type of Bermudan swaption, where the underlying swap tenor changes over time, is the most common instrument variety used in practice (see Andersen & Andreasen 2001).

[2]Swaption can also involve a notification period before exercise. Here, we assume that $\Delta_m = 0$ to alleviate the notation. The adjustment to allow for notification is straightforward.

For instance, consider a Bermudan t_{n-1} noncall t_1 receiver swaption with rate κ, which offers the holder the option to enter at any date t_m, $m = 1, ..., n-1$, into a interest-rate receiver swap with a swap rate of κ. For a swaption notional value M normalized to 1, the adaptation of the general model (2.2-2.5) is then as follows, where the value function $v_m(r)$ represents the value of the Bermudan swaption at date t_m

$$
\begin{aligned}
v_n(r) &= 0 \\
v_m^e(r) &= \max\left\{\kappa \sum_{k=m+1}^{n} P_{t_m}(r, t_k)\,(t_k - t_{k-1}) + P_{t_m}(r, t_n) - K; 0\right\}, m = 1, ..., n-1 \\
v_m^h(r) &= \mathbb{E}_m[\rho_m v_{m+1}(r_{t_{m+1}})], m = 1, ..., n-1 \\
v_m(r) &= \max\left\{v_m^e(r), v_m^h(r)\right\}, \; m = 1, ..., n-1 \\
v_0(r) &= \mathbb{E}_0[\rho_0 v_1(r_{\tau_1})].
\end{aligned}
$$

Note that, since there is no notice period, $\tau_m = t_m$ for $m = 0, ..., n$.

European swaptions are priced by assuming that the only exercise date is t_1 $(n - 1 = 1)$.

2.5.3 Treasury Bond futures

The Chicago Board of Trade T-Bond futures is one of the most actively traded futures contracts in the world. It is mainly used to hedge interest-rate risk. According to the contract's specifications, short traders are afforded the timing option, which allows them to deliver the cheapest-to-deliver bond (CTD) on any business day during the delivery month. Because of this implicit early exercise feature, the U.S. T-Bond futures contract belongs to the category of American-style interest-rate derivatives. The decision dates τ_m correspond to the position days, during which the short trader can declare his intention to deliver at dates t_m.

The DP algorithm (2.2-2.5) needs to be adjusted to account for various specific features of this derivative contract. One important feature that needs to be accounted for is the so-called *quality option*, that is, the option to choose which issue to deliver among a set of deliverable bonds. A system of conversion factors is used by the CBOT to adjust the price received by the short trader upon delivery according to the quality (coupon and maturity) of the delivered issue. In addition, the delivery month where exercise is allowed (from t_1 to t_n) is further divided into two sub-periods, that is the *switch period* (from t_1 to t_s), during which the futures contract is traded, and the *end-of-month period* (from t_{s+1} to t_n), during which the futures market is closed but delivery remains possible, where $t_1 < t_s < t_n$. During the switch period, parties are required to make payments in a marking-to-market account to reflect the variations in the quoted futures price.

To alleviate notation, in the following adaptation of the general model (2.2-2.5), we assume that $\tau_m = t_m$.[3] The value function $v_m(\lambda, r)$ represents the value (for the short trader) of the futures contract at date t_m, as a function of the futures price and interest rate observed at t_m.

The exercise value at t_m, $m = 1, ..., n$ is given by

$$v_m^e(\lambda, r) = \max_{(c,M)\in\Theta} \{\lambda f_{cM} - b_m(r, c, M)\}, \ m = 1, ..., n,$$

where Θ is the set of bonds that are eligible for delivery, f_{cM} is the CBOT conversion factor for a bond with a coupon rate c and maturity $M > n$, and $b_m(r, c, M)$ is the price at t_m of a bond with a coupon rate c and maturity M when $r_{t_m} = r$.

Note that the futures settlement price at date t_m, $m = 1, ..., s$, should be such that the value to both parties is 0, taking into account the timing and quality options. Accordingly, the holding value is given by

$$\begin{aligned} v_m^h(\lambda, r) &= \begin{cases} \mathbb{E}_m\left[\rho_m v_{m+1}(\lambda, r_{t_{m+1}})\right] & \text{for } m = s+1, ..., n-1 \\ \mathbb{E}_m\left[\rho_m(\lambda - \lambda_{m+1}(r_{t_{m+1}}))\right] & \text{for } m = 0, ..., s. \end{cases} \\ v_0^h(\lambda, r) &= \mathbb{E}_0\left[\rho_0(\lambda - \lambda_{m+1}(r_{t_{m+1}}))\right] \end{aligned}$$

where $\lambda - \lambda_{m+1}(r_{t_{m+1}})$ is the variation of the marking-to-market account of the short trader. The function $\lambda_m(r)$ characterizes the dependence between the futures

[3] We refer the reader to Ben-Abdallah, Ben-Ameur, and Breton (2012) for more details about way to account for the three-day delivery sequence in the DP model.

price and the interest rate, taking into account the value of the options embedded in the futures contract. Since the value of the contract is 0 at all settlement dates, $\lambda_m(r)$ is obtained by solving the implicit equation

$$v_m(\lambda_m(r), r) = 0,\ m = 0, ..., n.$$

Finally, the value function $v_m(\lambda, r)$ at date t_m when $r_{\tau_m} = r$ and $\lambda_m = \lambda$ is the maximum between the exercise value and the holding value when exercise is allowed:

$$v_n(\lambda, r) = v_n^e(\lambda, r) \tag{2.7}$$

$$v_m(\lambda, r) = \max\left\{v_m^e(\lambda, r), v_m^h(\lambda, r)\right\},\ m = 1, ..., n-1 \tag{2.8}$$

$$v_0(\lambda, r) = v_0^h(\lambda, r). \tag{2.9}$$

2.6 Numerical illustrations

In this section, we present illustrative numerical results of the DP procedure applied to the pricing of risk-free bonds with embedded options, European and Bermudan swaptions, and U.S. T-Bond futures under the Vasicek-EJ model.

2.6.1 Callable coupon bonds

We consider the 4.25% callable bond issued by the Swiss Confederation for the period 1987-2012. This security was used as a reference by several papers to compare their numerical pricing methods (Buttler & Waldvogel 1996, d'Halluin, Forsyth, Vetzal & Labahn 2001, Ben-Ameur et al. 2007, Lim et al. 2012). The pricing date $t_0 = 0$ is December 23, 1991 and the Vasicek parameter values as well as the call prices are presented in Table 2.1.

Numerical experiments are performed using 600 grid points. When jump intensities are set to 0, we obtain prices under the Vasicek (1977) model. We perform a sensitivity analysis of the embedded bond call value with respect to jump param-

Table 2.1 – Call prices and parameter values for the Vasicek model used to price the Swiss callable bond.

ζ	α	σ	C_{11}	C_{12}	C_{13}	C_{14}	C_{15}	$C_{16} = ... = C_{21}$
0.098397028	0.44178462	0.13264223	1.025	1.02	1.015	1.01	1.005	1

eters. The value of the embedded option (for the issuer) is the difference between the price of the callable bond and that of the corresponding option-free bond.

In Figure 2.1, we plot the value of the embedded option as a function of the interest rate for different values of λ_u and λ_d. We observe that the value of the embedded call option decreases with λ_u, decreases with λ_d, and increases with the interest rate. In fact, when rates are sufficiently high, the probability that the issuer calls the bond becomes extremely small, and the value of the embedded option converges to zero.

In Figure 2.2 we study the sensitivity of the embedded call option to the jump size parameters. The size of the jump is measured by the quantities $1/\eta_u$ and $1/\eta_d$. We observe that the value of the embedded call option is increasing with η_u, decreasing with η_d, and decreasing with the interest rate. Increasing η_u (*resp.* η_d) is equivalent to decreasing (*resp. increasing*) the jump size, and implicitly decreasing (*resp. increasing*) the interest rate.

2.6.2 Bermudan swaptions

We now provide illustrative results by pricing European and Bermudan swaptions under Vasicek-EJ using the dynamic programming procedure.[4] Parameter values, provided in Table 3.1, are obtained by calibrating to the term structure of interest rates on December 1^{st}, 2015.

Tables 2.3 and 2.4 report the prices of, respectively, European and Bermudan receiver swaptions for various strikes, swap tenors and option maturity. Swap pay-

[4]Beliaeva et al. (2008) price a payer European swaption under the Vasicek-EJ model using multinomial trees and the cumulant expansion approximation of Collin-Dufresne and Goldstein (2002).

Figure 2.1 – Impact of the jump intensity parameters on the value of the embedded call option. In the top panel, $\lambda_d = 0$ and $\eta_u = 100$. In the bottom panel, $\lambda_u = 0$ and $\eta_d = 100$. The remaining model parameters are in Table 2.1.

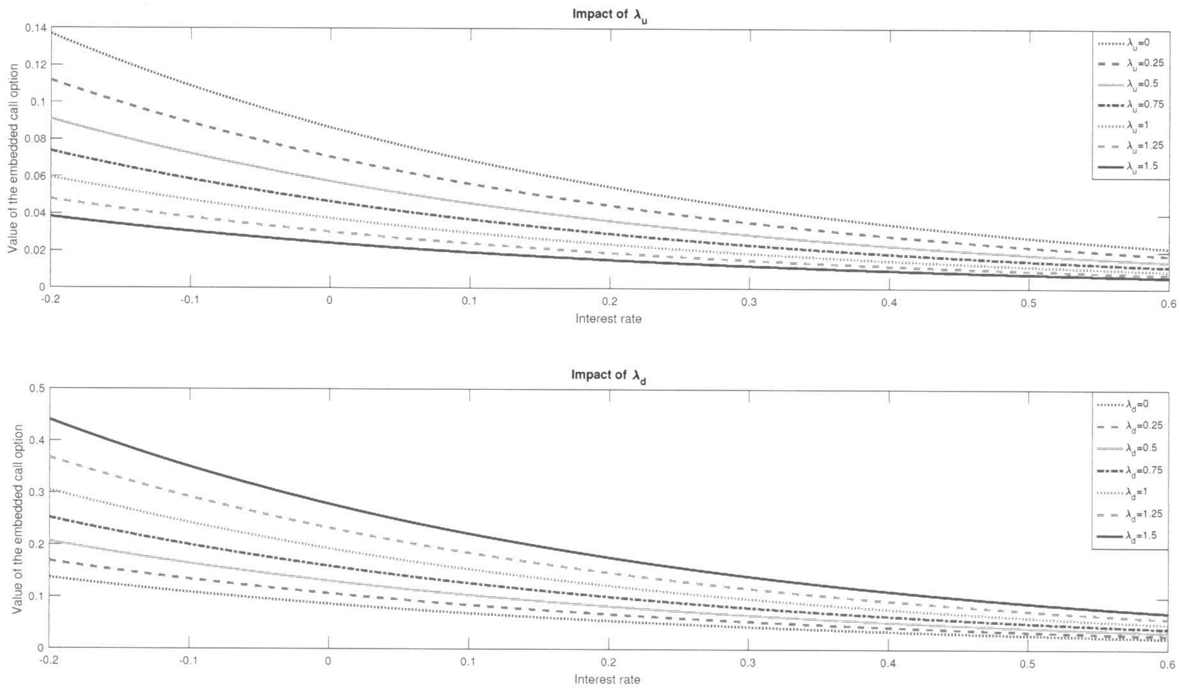
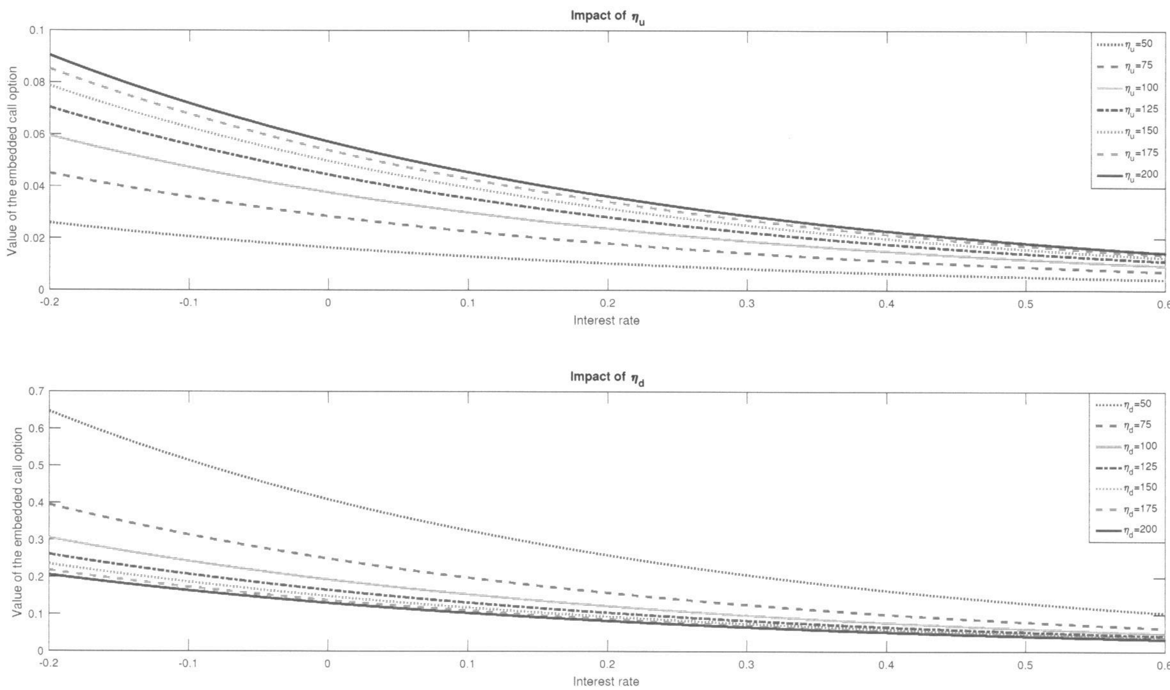


Table 2.2 – Parameter values for the Vasicek-EJ model on December 1st, 2015.

α	ζ	σ	λ_u	λ_d	η_u	η_d
0.175	0.051	0.034	0.034	4.364×10^{-5}	491.013	-5.533

Figure 2.2 – Impact of the jump size parameters on the value of the embedded call option. In the top panel, $\lambda_u = 1$ and $\lambda_d = 0$. In the bottom panel, $\lambda_u = 0$ and $\lambda_d = 1$. The remaining model parameters are in Table 2.1.



ments are exchanged on a semi-annual basis. Obviously, the price of a Bermudan swaption is higher than the corresponding European swaption, where the option holder benefits from multiple exercise dates. In fact, the Bermudan swaption could be considered as a portfolio of European swaptions with expiration dates corresponding to the exercise dates.

In Table 2.5, we report the Bermudan premium as a percentage of the corresponding European swaption price. Depending on maturity, tenor and strike, the multiple exercise premium value ranges between 1.53% and 176.18% of the swaption price. As expected, the premium is an increasing function of the swap tenor and a decreasing function of the earliest exercise date.

In Figures 2.3 and 2.4, we analyze the sensitivity of the Bermudan payer swaption to the jump parameters λ_u and λ_d as a function of the interest rate for different

Table 2.3 – European receiver swaptions. The first row is the swap tenor in years. The first column is the option maturity. Parameter values are given in Table 3.1. The underlying swap pays a 5% fixed rate on a semi-annual basis.

	Tenor t_n - t_1									
	1	2	3	4	5	6	7	8	9	10
t_1					K=0.99					
1	0.015577	0.024105	0.031808	0.038894	0.045579	0.052014	0.058315	0.064549	0.070767	0.077001
2	0.018548	0.029934	0.040022	0.049103	0.057442	0.065237	0.072638	0.079749	0.086648	0.093392
3	0.020154	0.033142	0.044592	0.054826	0.064129	0.072721	0.080772	0.088404	0.095712	0.102766
4	0.021056	0.035004	0.047277	0.058211	0.068101	0.077175	0.085616	0.093556	0.101099	0.108327
5	0.021527	0.036040	0.048801	0.060149	0.070383	0.079736	0.088395	0.096501	0.104163	0.111469
					K=1					
1	0.010356	0.019140	0.026871	0.033910	0.040501	0.046820	0.052981	0.059063	0.065120	0.071186
2	0.013589	0.025162	0.035271	0.044317	0.052588	0.060298	0.067598	0.074598	0.081379	0.087999
3	0.015375	0.028519	0.039987	0.050191	0.059438	0.067959	0.075925	0.083464	0.090673	0.097623
4	0.016427	0.030512	0.042802	0.053709	0.063548	0.072561	0.080926	0.088784	0.096240	0.103377
5	0.017033	0.031671	0.044447	0.055771	0.065958	0.075255	0.083846	0.091877	0.099461	0.106683
					K=1.01					
1	0.006446	0.014909	0.022477	0.029372	0.035819	0.041983	0.047980	0.053892	0.059772	0.065657
2	0.009591	0.020926	0.030915	0.039856	0.048022	0.055620	0.062800	0.069676	0.076328	0.082816
3	0.011401	0.024346	0.035712	0.045828	0.054987	0.063414	0.071281	0.078716	0.085817	0.092657
4	0.012511	0.026418	0.038618	0.049448	0.059209	0.068138	0.076416	0.084182	0.091543	0.098582
5	0.013188	0.027664	0.040358	0.051611	0.061728	0.070948	0.079460	0.087408	0.094904	0.102038

Table 2.4 – Bermudan receiver swaptions. The first row is the swap tenor in years. The first column is the earliest exercise date. Parameter values are given in Table 3.1. The underlying swap pays a 5% fixed rate on a semi-annual basis.

	Tenor t_n - t_1									
	1	2	3	4	5	6	7	8	9	10
t_1					K=0.99					
1	0.017276	0.029920	0.042990	0.056224	0.069433	0.082484	0.095288	0.107784	0.119932	0.131709
2	0.019879	0.034528	0.048934	0.063036	0.076781	0.090129	0.103055	0.115547	0.127601	0.139220
3	0.021292	0.037094	0.052295	0.066922	0.080985	0.094494	0.107463	0.119909	0.131851	0.143308
4	0.022068	0.038534	0.054179	0.069076	0.083273	0.096812	0.109729	0.122062	0.133845	0.145110
5	0.022449	0.039262	0.055112	0.070098	0.084294	0.097759	0.110549	0.122713	0.134296	0.145339
					K=1					
1	0.011257	0.023704	0.036708	0.049926	0.063144	0.076219	0.089055	0.101587	0.113774	0.125593
2	0.014265	0.028672	0.042970	0.057024	0.070754	0.084105	0.097049	0.109566	0.121649	0.133300
3	0.015945	0.031510	0.046596	0.061165	0.075204	0.088709	0.101688	0.114153	0.126117	0.137602
4	0.016932	0.033174	0.048706	0.063544	0.077714	0.091246	0.104170	0.116518	0.128321	0.139610
5	0.017493	0.034098	0.049841	0.064770	0.078938	0.092396	0.105191	0.117368	0.128969	0.140034
					K=1.01					
1	0.006843	0.018477	0.031148	0.044198	0.057323	0.070347	0.083158	0.095682	0.107872	0.119701
2	0.009886	0.023631	0.037623	0.051508	0.065138	0.078431	0.091342	0.103844	0.115925	0.127581
3	0.011651	0.026641	0.041447	0.055855	0.069796	0.083241	0.096186	0.108633	0.120592	0.132079
4	0.012732	0.028459	0.043735	0.058422	0.072499	0.085973	0.098863	0.111193	0.122989	0.134279
5	0.013391	0.029526	0.045032	0.059820	0.073901	0.087304	0.100067	0.112227	0.123821	0.134887

Table 2.5 – Bermudan receiver premium as a percentage of the European Swaption price. Parameter values are given in Table 3.1. The underlying swap pays a 5% fixed rate on a semi-annual basis.

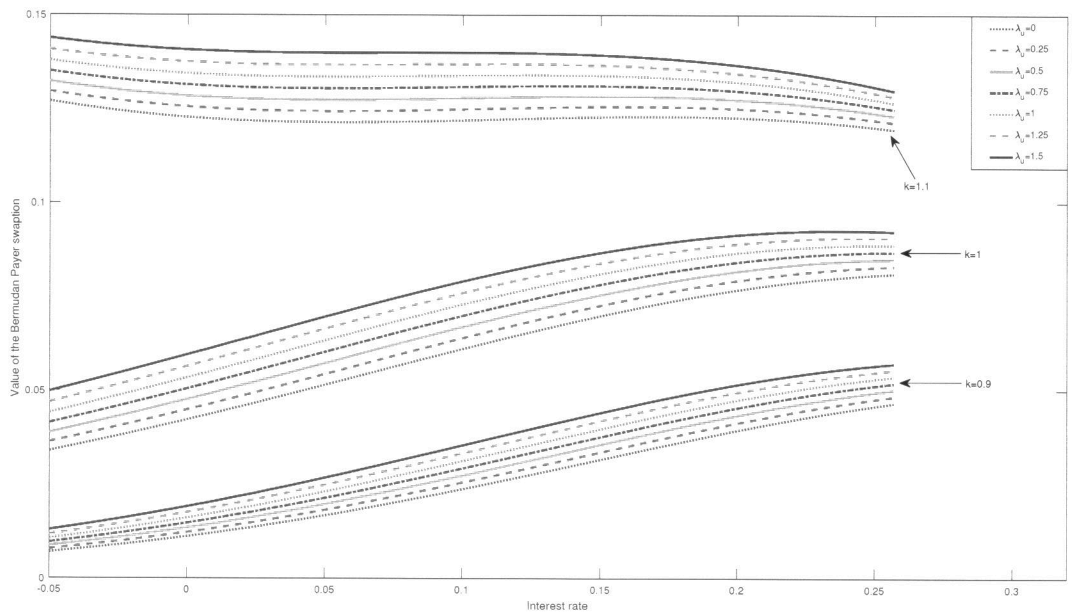
	Tenor t_n - t_1									
	1	2	3	4	5	6	7	8	9	10
t_1					K=0.99					
1	10.91	24.12	35.16	44.56	52.33	58.58	63.40	66.98	69.47	71.05
2	7.18	15.35	22.27	28.38	33.67	38.16	41.87	44.89	47.26	49.07
3	5.65	11.92	17.27	22.06	26.28	29.94	33.04	35.64	37.76	39.45
4	4.81	10.09	14.60	18.67	22.28	25.44	28.16	30.47	32.39	33.96
5	4.28	8.94	12.93	16.54	19.77	22.60	25.06	27.16	28.93	30.39
					K=1					
1	8.70	23.85	36.61	47.23	55.91	62.79	68.09	72.00	74.71	76.43
2	4.97	13.95	21.83	28.67	34.54	39.48	43.57	46.88	49.48	51.48
3	3.71	10.49	16.53	21.86	26.53	30.53	33.93	36.77	39.09	40.95
4	3.07	8.72	13.80	18.31	22.29	25.75	28.72	31.24	33.33	35.05
5	2.70	7.66	12.14	16.14	19.68	22.78	25.46	27.74	29.67	31.26
					K=1.01					
1	6.15	23.93	38.58	50.48	60.04	67.56	73.32	77.54	80.47	82.31
2	3.08	12.93	21.70	29.24	35.64	41.01	45.45	49.04	51.88	54.05
3	2.19	9.43	16.06	21.88	26.93	31.27	34.94	38.01	40.52	42.55
4	1.77	7.73	13.25	18.15	22.44	26.17	29.38	32.09	34.35	36.21
5	1.53	6.73	11.58	15.91	19.72	23.05	25.93	28.39	30.47	32.19

strikes. The value of the swaption increases with λ_u and with the strike and decreases with λ_d.

2.6.3 T-Bond futures

Figure 2.5 illustrates the sensitivity of futures prices to the jump parameter values as a function of the interest rate under the Vasicek-EJ model, using the parameters in Table 3.1 and the set of conversion factors and basket of deliverable bonds on December 1^{st}, 2015 (see Table 2.6). We observe that the value of futures contracts is decreasing in λ_u: when upward jumps in the interest-rate movements are more likely to occur, the value of the futures contract decreases as a result of a decrease in the price of the deliverable bonds. The impact of the upward and downward jump intensities is more important when interest rates are low. The opposite effect on the value of the futures contract is observed when we change the values of λ_d, i.e. the

Figure 2.3 – Impact of λ_u on the value of the Bermudan payer swaption. Parameter values are given in Table 3.1. The underlying swap pays a 5% fixed rate on a semi-annual basis. $t_1 = 5$ and $t_n = 10$.



futures prices increases when downward jumps are more likely to occur. [5]

2.7 Conclusion

In this chapter, we present a general dynamic programming procedure combined with Fourier transform to price American-style interest-rate derivatives under affine jump-diffusion models. Specifically, we price risk-free bonds with embedded options, European and Bermudan swaptions, as well as the U.S. T-Bond futures under the Vasicek-EJ interest-rate model. Numerical illustrations are provided, allowing to assess the sensitivity of various interest-rate derivatives to interest-rate jumps parameter values.

One interesting extensions to this work would be to adapt the model to account

[5]Similar results are obtained for the other jump parameters η_u and η_d.

Figure 2.4 – Impact of λ_d on the value of the Bermudan payer swaption. Parameter values are given in Table 3.1. The underlying swap pays a 5% fixed rate on a semi-annual basis. $t_1 = 5$ and $t_n = 10$.

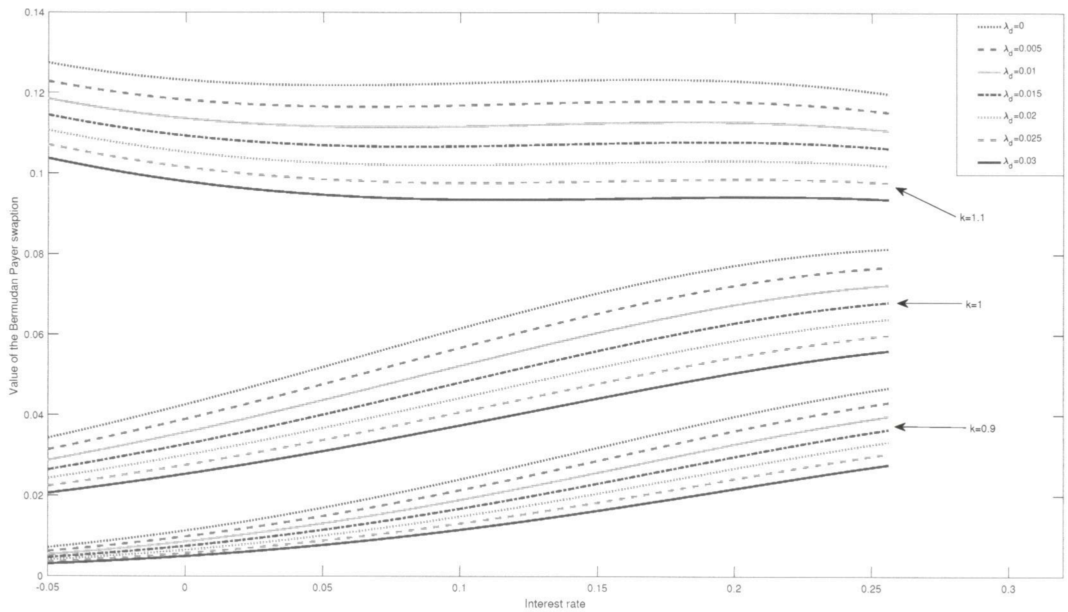
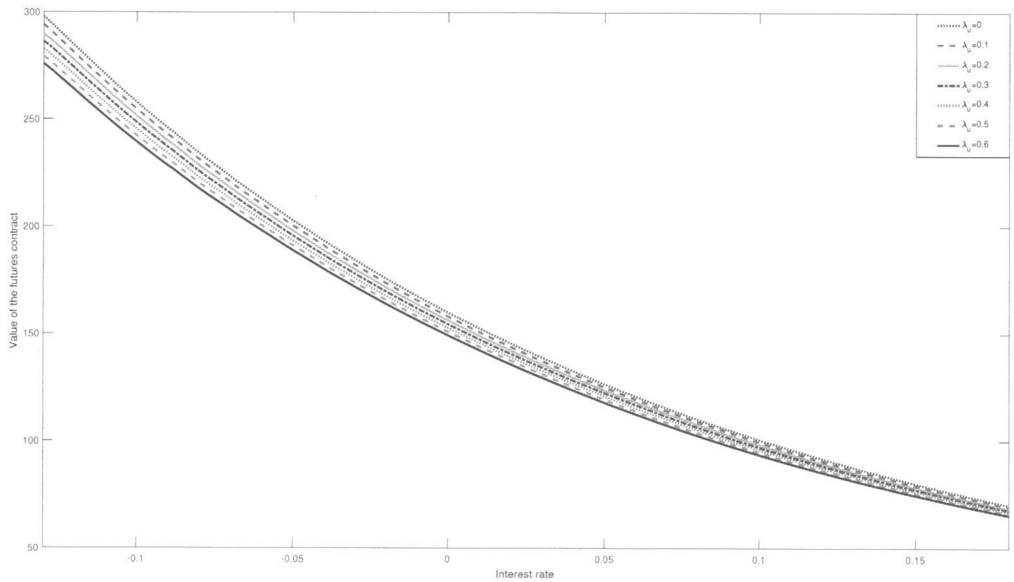


Table 2.6 – Basket of deliverable bonds on December 1st, 2015.

Coupon	**Maturity**	**Conversion factor**
4.5	02/15/36	0.8266
4.75	02/15/37	0.8519
5	05/15/37	0.8807
4.375	02/15/38	0.8029
4.5	05/15/38	0.8170
3.5	02/15/39	0.6903
4.25	05/15/39	0.7820
4.5	08/15/39	0.8123
4.375	11/15/39	0.7956
4.625	02/15/40	0.8263
4.375	05/15/40	0.7937
3.875	08/15/40	0.7290
4.25	11/15/40	0.7758

Figure 2.5 – Impact of λ_u on futures prices. Parameter values are given in Tables 3.1 and 2.6.



for default risk, in order to price corporate bonds and other credit-sensitive instruments. This would most probably require an additional state variable indicating the level of a relevant risk factor.

2.8 Appendix

The Duffie, Pan and Singleton (2000) transform analysis for affine jump-diffusion processes

Duffie et al. (2000) provide closed-form solutions for an extended transform of a state vector X that follows an affine jump-diffusion process and with realizations $\{X_t, 0 \leq t < \infty\}$ in some state space $D \subset \mathbb{R}^q$. The proposed extended transform of X is the expectation at time t of a terminal payoff function of X_T, $T > t$, discounted at a stochastic discount rate $R(X_t)$, and is expressed as follows

$$\mathbb{E}_t \left(\exp\left(- \int_t^T R(X_s)ds \right) (\upsilon_0 + \upsilon_1 X_T) \exp\left(uX_T\right) \right)$$

where υ_0, υ_1 and u are scalars.

In the following paragraphs, we provide a brief description of the transforms proposed by Duffie et al. (2000) for affine jump-diffusion state vectors. The authors obtain closed-form solutions for these transforms thanks to the use of a Fourier-Stieltjes transform and a Lévy inversion. We refer to the original paper for the proofs and mathematical details related to these transforms.

The authors consider a state vector X that follows the Markov process

$$dX_t = \mu(X_t)dt + \sigma(X_t)dW_t + dZ_t \tag{2.10}$$

where W is a standard Brownian motion in $\mathbb{R}^q$; $\mu : D \to \mathbb{R}^q$; $\sigma : D \to \mathbb{R}^{q\times q}$ and Z is a pure jump process whose jumps have a fixed probability distribution υ on $\mathbb{R}^q$ and arrive with intensity $\{\lambda(X_t) : t \geq 0\}$, for some $\lambda : D \to [0, \infty)$.

For the process X_t to be affine, Duffie et al. (2000) impose the following affine structures on the parameters and the stochastic discount rate $R(X_t)$ at $x = X_t$:

$\mu(x) = K_0 + K_1 x$, for $K = (K_0, K_1) \in \mathbb{R}^q \times \mathbb{R}^{q\times q}$

$(\sigma(x)\sigma(x)^T)_{ij} = (H_0)_{ij} + (H_1)_{ij}x$, for $H = (H_0, H_1) \in \mathbb{R}^{q\times q} \times \mathbb{R}^{q\times q\times q}$

$\lambda(x) = l_0 + l_1 x$, for $l = (l_0, l_1) \in \mathbb{R} \times \mathbb{R}^q$

$R(x) = \rho_0 + \rho_1 x$, for $\rho = (\rho_0, \rho_1) \in \mathbb{R} \times \mathbb{R}^q$

The jump transform is defined as $\theta(c) = \int_{\mathbb{R}^q} \exp(c \cdot z) dv(z)$ for $c \in \mathbb{C}^q$ and determines the jump-size distribution.

The transform

For any real number y, any a, b, d in $\mathbb{R}^q$ and set of characteristics $\chi = (K, H, l, \theta, \rho)$, we define the following transform :

$$\begin{aligned} G_{a,b}(y, X_t, T, \chi) &= \mathbb{E}^{\chi}\left(\exp\left(-\int_t^T R(X_s)ds\right)\exp\left(a \cdot X_T\right)\mathbb{I}\left(b \cdot X_T \leq y\right)|\mathcal{F}_t\right) \\ &= \frac{\psi^{\chi}(a, X_t, t, T)}{2} - \frac{1}{\pi}\int_0^{\infty} \frac{\text{Im}[\psi^{\chi}(a + i\omega b, X_t, t, T)\exp\left(-i\omega y\right)]}{\omega} d\omega \end{aligned}$$

where

$$\begin{aligned} \psi^{\chi}(a, X_t, t, T) &\equiv \mathbb{E}^{\chi}\left(\exp\left(-\int_t^T R(X_s)ds\right)\exp\left(a \cdot X_T\right)|\mathcal{F}_t\right) \\ &= \exp\left(\Omega_t + \Pi_t \cdot x\right) \end{aligned}$$

Ω_t and Π_t are the solutions to two complex-valued ordinary differential equations and lm(c) denotes the imaginary part of $c \in \mathbb{C}^q$. $\mathbb{E}^{\chi}$ denotes the expectation under the distribution of X determined by χ.

The extended transform

$$\begin{aligned} G_{a,b,d}(y, X_t, T, \chi) &= \mathbb{E}^{\chi}\left(\exp\left(-\int_t^T R(X_s)ds\right)\left(a \cdot X_T\right)\exp\left(d \cdot X_T\right)\mathbb{I}\left(b \cdot X_T \leq y\right)|\mathcal{F}_t\right) \\ &= \frac{\phi^{\chi}(a, d, X_t, t, T)}{2} - \frac{1}{\pi}\int_0^{\infty} \frac{\text{Im}[\phi^{\chi}(a, d + i\omega b, X_t, t, T)\exp\left(-i\omega y\right)]}{\omega} d\omega \end{aligned}$$

where

$$\begin{aligned} \phi^{\chi}(a, d, X_t, t, T) &\equiv \mathbb{E}^{\chi}\left(\exp\left(-\int_t^T R(X_s)ds\right)\left(a \cdot X_T\right)\exp\left(d \cdot X_T\right)|\mathcal{F}_t\right) \\ &= \psi^{\chi}(a, x, t, T)(A_t + B_t \cdot x) \end{aligned}$$

and A_t and B_t satisfy two linear ordinary differential equations.

The Vasicek-EJ interest-rate model

The Vasicek-EJ interest-rate model is a particular case of the general affine jump diffusion model (2.10) where the coefficient vector $\chi = (K, H, l, \rho)$ is set as follows:

$$\begin{array}{llll} K_0 = \alpha\zeta & H_0 = 0 & l_0 = 0 & \rho_0 = 0 \\ K_1 = -\alpha & H_1 = \sigma^2 & l_1 = 1 & \rho_1 = 1 \end{array}$$

and where the jump transform (i.e., the moment generating function of the jumps) is given by

$$\begin{aligned} \theta_u(c_t) &= \frac{\eta_u}{\eta_u - c_t} \text{ for } c_t < \eta_u \\ \theta_d(c_t) &= \frac{\eta_d}{\eta_d - c_t} \text{ for } c_t < \eta_d. \end{aligned}$$

The Vasicek-GJ interest-rate model

The same general setting could be used under the Vasicek model with normally distributed jumps:

$$dr(t) = \alpha(\zeta - r_t)dt + \sigma dW_t + JdN(\lambda)$$

where J is the jump size and is assumed to be normally distributed with mean μ and variance σ^2 and $N(\lambda)$ is a Poisson process with intensity λ.

Under this model, the closed-form solution of the zero-coupon bond price is the same as the one under Vasicek-EJ model except for the term $C(T-t)$, which becomes:

$$C(T-t) = (T-t-\frac{1-\delta}{\alpha})\frac{\sigma^2}{2\alpha^2} - \sigma^2\frac{(1-\delta)^2}{4\alpha^3} + \lambda\int_0^{T-t}(\exp(-\mu D(\omega) + \frac{1}{2}\sigma^2(B(\upsilon))^2) - 1)d\omega$$

The dynamic program expectations are also computed using the Duffie et al. (2000) transform, yielding the following functions:

$$\Pi_t = b\delta - \frac{1}{\alpha}$$

$$\begin{aligned}\Omega_t &= b(\zeta - \frac{\sigma^2}{\alpha^2})(1-\delta) + \frac{\sigma^2 b^2}{4\alpha}(1-\delta^2) \\ &+(\frac{\sigma^2}{2\alpha^2} - \lambda - \zeta)(T-t) + \lambda \int_t^T \theta(\Pi_s) ds\end{aligned}$$

$$B_t = \delta$$

$$A_t = (\zeta - \frac{\sigma^2}{\alpha^2})(1-\delta) + \frac{\sigma^2 b}{2\alpha}(1-\delta^2) + \lambda \int_t^T \Theta(\Pi_s) B_s ds$$

where θ is the jump transform:

$$\begin{aligned}\theta(c) &= \int_{\mathbb{R}^n} \exp(cz)\, dv(z) \\ &= \exp(\mu c + \frac{\sigma^2 c^2}{2}) \text{ for } c \in \mathbb{C}\end{aligned}$$

and

$$\begin{aligned}\Theta(c) &= \int_{\mathbb{R}^n} z \exp(cz)\, dv(z) \\ &= (\mu + c\sigma^2) \exp\left(\mu c + \frac{\sigma^2 c^2}{2}\right).\end{aligned}$$

The main drawback of this model is that the probability of having negative interest rates could be significant since jumps with the same magnitude and frequency could occur in both directions.

Chapter 3

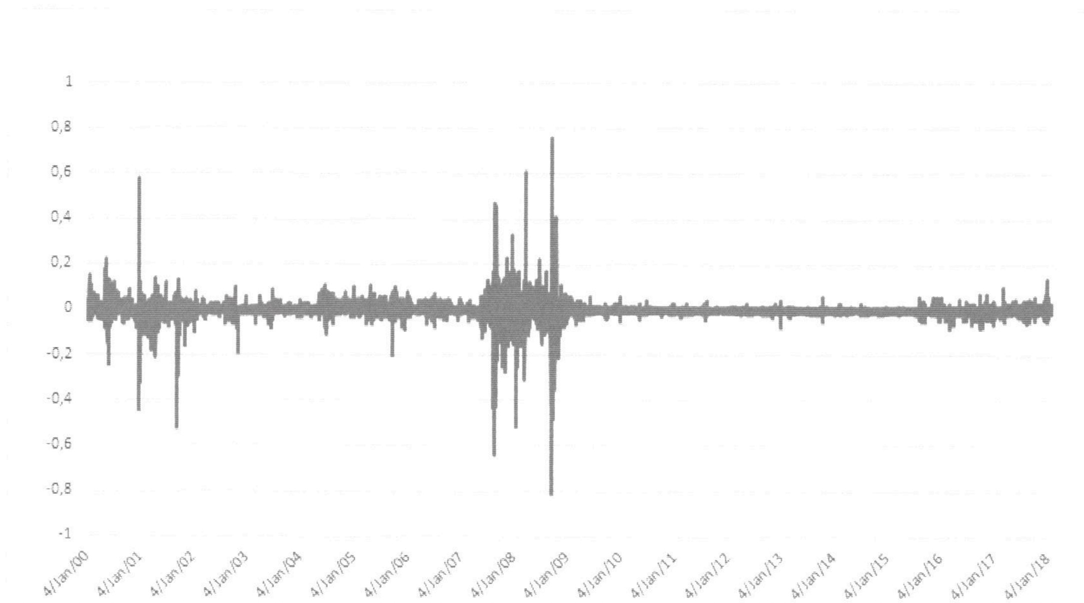
Impact of interest-rate jumps

3.1 Introduction

The importance of including jumps in the dynamics of the underlying asset has been well established for equity derivatives, particularly after the emergence of Merton (1976) jump-diffusion model. For instance, Campolongo, Cariboni & Schoutens (2006) show that, for European options, jumps drive most of the uncertainty in the option price, so that their inclusion can improve the model accuracy, particularly when the strike price is high. Using S&P 100 American options data, Kang, Nikitopoulosa, Schlogl & Taruvinga (2019) show that including jumps improve the model's ability to fit market data as well as the overall pricing performance compared to pure diffusion models. Moreover, the inclusion of jumps in asset prices could alter the optimal exercise decision and increase the free boundary.

However, the literature about the impact of jumps on the value of interest-rate derivatives is still scarce, even though jumps in interest rates are not rare events. Empirical studies show that jumps in the interest-rate movements do occur and that they cannot be captured by pure diffusion models. In fact, several papers find strong evidence that interest rates contain unexpected discontinuous changes (see, for instance, Farnsworth & Bass 2003; Johannes 2004; Andersen, Benzoni & Lund

Figure 3.1 – Daily changes (in bps) in the 3-month treasury bill rate from Jan 2000 to Jan 2018



2004; Piazzesi 2005; Jiang & Yan 2009; and Juneja & Pukthuanthong 2016). These movements generally occur before or after major economic events, Central banks announcements, as well as during particular economic regimes. As an illustration, Figure 3.1 shows the daily variation in the 3-month Treasury bill rate from January 2000 to January 2018. Large spikes are clearly observable, and can even reach 80 basis points during the period 2007-2009 corresponding to the last financial crisis.

Few papers examine the impact of jumps on the valuation of fixed-income instruments, due to the lack of flexible methods that incorporate unexpected discontinuous changes in interest-rates. Jarrow, Li & Zhao (2007) find that adding a jump component to the interest-rate dynamics is necessary to explain the volatility smile of interest rate caps. The authors show that, even though a three-factor stochastic volatility model is able to price at-the-money caps fairly accurately, the smile cannot be captured without including jumps. Feldhutter, Trolle & Schneider (2008) use a dataset of Eurodollar futures and options and show that interest-rate jumps significantly affect the tails of both the conditional risk-neutral and physical distributions of interest rates, making jumps important, not only for pricing purposes, but also for risk management. Wright & Zhou (2009) use high-frequency Treasury bond futures data and augment the standard regressions of excess bond returns on the term structure of forward rates with jump related measures. The authors find that the R^2 of the forecasting regression significantly increases, almost doubling in some cases, which indicates that incorporating unanticipated jumps in the movement of interest rates helps better understand the yield curve future behaviour as well as its risk premium. More recently, Lund (2019) shows that the jump-risk premium has a significant magnitude, and that term structure models that incorporate jumps can better explain the interest-rate risk premium and are more suitable for option pricing.

The above-mentioned studies focus on demonstrating the importance of jumps and their role in explaining returns. However, they do not address the issue of the instruments' potential mispricing if these discontinuous changes were not taken into account. While all models are approximation of reality, one interesting question is to what extent the assumption of continuous movement of interest rates may lead to evaluation and hedging errors.

The aim of this chapter is to contribute to this literature by assessing the model error resulting from using a pure diffusion model against a jump-diffusion one when pricing interest-rate derivatives. We evaluate the impact of the presence of interest-rate jumps on the value of the derivatives, both numerically and empirically, using the general pricing framework described in Chapter 2, which allows us to test several types of interest-rate instruments. To the best of our knowledge, this is the first study providing a complete analysis of the pricing differences that could occur if interest-rate jumps were not taken into account and identifying the different circumstances under which the model specification would have a larger impact on the prices of interest-rate derivatives.

Specifically, we price a sample of various types of interest-rate derivatives under the Vasicek and the Vasicek-EJ models, and we analyse the difference between the prices obtained under the two specifications.

We focus on actively traded instruments with various payoff functions that could be linear or convex, namely, payer and receiver interest-rate swaps and swaptions, straight and callable bonds, and T-bond futures. These instruments are used on a daily basis by fixed-income traders as well as other market participants for funding or hedging purposes. According to the BIS, most OTC interest-rate derivatives activity is coming from swaps, while futures and options dominate the exchange-traded markets given their high liquidity and capacity to cover different types of exposures to interest-rate risk. As a matter of fact, the total notional amount outstanding of interest-rate swaps reached $326,690$ Billion USD by the end of 2018, representing approximately 75% of the total notional amount of all interest rate derivatives[1]. Likewise, the demand for Treasury futures remains always high due to their important role in hedging interest-rate risk, especially during major economic events; for instance, post UK referendum and post U.S. election in 2016, the total number of traded contracts during the day reached 5.9 and 8.9 millions respectively[2].

Our analysis allows us to conclude that jumps have a higher impact in the following particular circumstances: a low volatility environment, a high level of interest rates and a low speed of mean reversion. We show that a small increase in the probability of occurrence of jumps could result in significant changes in derivatives prices, and consequently alter the exercise boundary for instruments with early exercise features. In addition, we find that interest-rate jumps play an important role when the term structure is steep and upward sloping or when it has a humped shape. In these cases, the derivatives prices could be significantly different when using Vasicek or Vasicek-EJ models.

The rest of this chapter is organized as follows. In Section 3.2, we perform a sensitivity analysis to evaluate how the jump parameters interact with the other model

[1]Source: Bank for International Settlements (BIS).
[2]Source: Chicago Mercantile Exchange (CME).

parameters. Section 3.3 sets up different theoretical shapes for the term structure of interest rates and evaluates the corresponding model error. Finally, in Section 3.4, we price Treasury bond futures using historical yield curves to investigate whether including jumps could have made a significant difference. Section 3.5 is a conclusion.

3.2 Sensitivity to parameter values

The objective of this section is to illustrate the impact of incorporating unanticipated changes in the interest-rate model, not only on derivative prices, but also on the exercise frontier of American-style instruments. We show that fixed-income instruments are not insensitive to the inclusion of jumps, even when their probability of occurrence is relatively low. We find that changes in prices are not linear in the variation of jump intensity, so that a relatively small increase in λ_u, for instance, can cause a significant increase in a derivative's price. We also find that there is an interaction between the jump parameters and other model parameters, so that, in some cases, high volatility levels or low levels of the long-term mean can mitigate the impact of jumps.

For illustration purposes, we first price an interest-rate receiver swap with a swap rate κ, maturity $t_n = T$ and principal equal to K. The swap value at t_m, $r_{t_m} = r$ can be expressed in closed form:

$$s_m(r) = K\left(1 - \kappa \sum_{k=m+1}^{n} P_{t_m}(r, t_k)(t_k - t_{k-1}) - P_{t_m}(r, T)\right)$$

where the fixed and floating payments are exchanged on the same dates $t_m, m = 1, ..., n$, and

$$P_t(r, u) = \mathbb{E}_t\left[\exp\left(-\int_t^u r_s ds\right)\right]$$

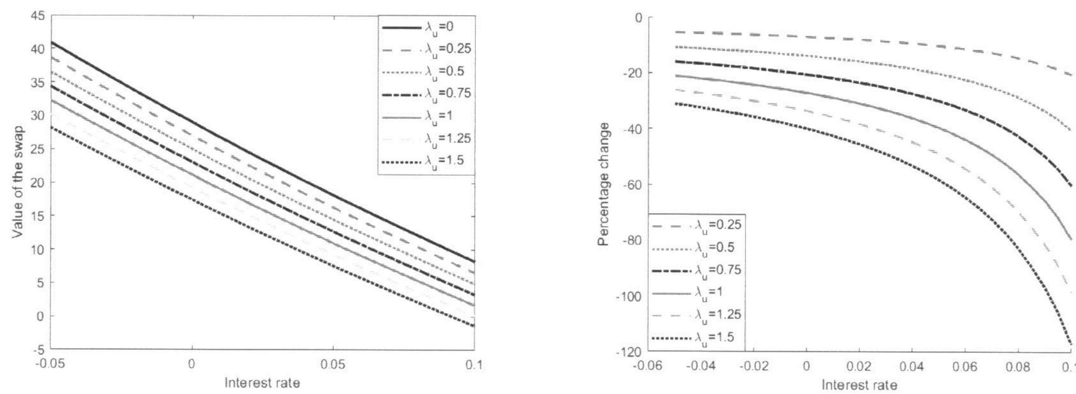
is the price at t of a zero-coupon bond maturing at $u \geq t$.

We price a 10-year swap with a swap rate of 5% and semi-annual payments. The base-case Vasicek-EJ parameters are given in Table 3.1.

Table 3.1 – Parameter values for the sensitivity of swaps in the Vasicek-EJ model.

α	ζ	σ	λ_d	η_u	η_d	n	κ	$t_m - t_{m-1}$	r	K
0.05	0.05	0.05	0	100	0	20	2.5%	0.5	5%	100

Figure 3.2 – Sensitivity of the swap value to the interest rate r for various values of upward jump intensities λ_u. Other parameter values are given in Table 3.1. Left panel: value of $s_0(r)$. Right panel: percentage change with respect to the case where $\lambda_u = 0$.



In Figures 3.2 to 3.7, we plot the value of the swap as a function of each one of the model parameters for values of the upward jumps intensities varying from 0 to 1.5 when $\lambda_d = 0$. The thick line corresponds to the case where $\lambda_u = 0$, which corresponds to the valuation under the Vasicek model without jumps. The right panel in each case shows the percentage change of the value with respect to the base Vasicek model without jumps. Similar results are obtained for the downward jump parameter λ_d.

In all graphs, it is clear that the receiver swap value is decreasing with λ_u; this is expected since an increase in λ_u implies that the interest rate is more likely to increase. Note that the impact of jump intensity can be important, particularly when the interest rate is high. For instance (see Figure 3.2) for $\lambda_u = 0.25$, a relatively low value, the value of the swap could change by up to 20% relative to the price obtained without jumps in a very high interest-rate environment. When $\lambda_u = 1.5$,

Figure 3.3 – Sensitivity of the swap value to the number of coupon dates n for various values of upward jump intensities λ_u. Other parameter values are given in Table 3.1. Left panel: value of $s_0(n)$. Right panel: percentage change with respect to the case where $\lambda_u = 0$.

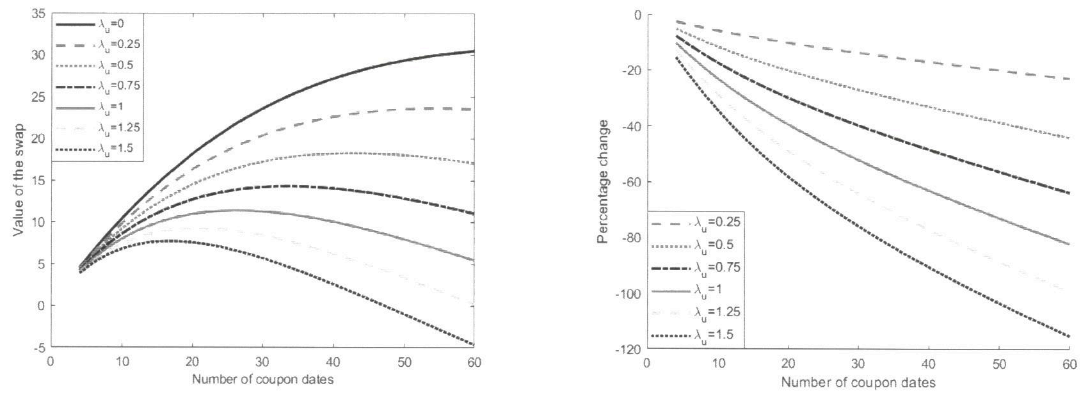


Figure 3.4 – Sensitivity of the swap value to the volatility σ for various values of upward jump intensities λ_u. Other parameter values are given in Table 3.1. Left panel: value of $s_0(\sigma)$. Right panel: percentage change with respect to the case where $\lambda_u = 0$.

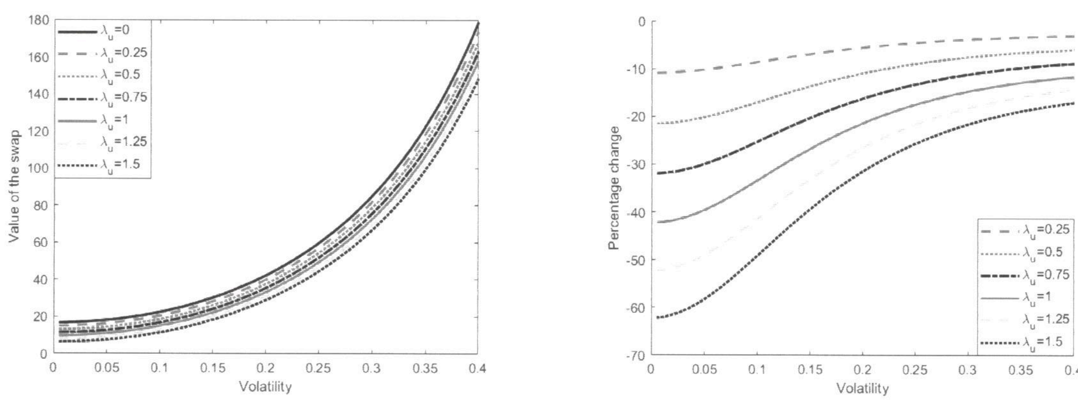


Figure 3.5 – Sensitivity of the swap value to the long-term mean ζ for various values of upward jump intensities λ_u. Other parameter values are given in Table 3.1. Left panel: value of $s_0(\zeta)$. Right panel: percentage change with respect to the case where $\lambda_u = 0$.

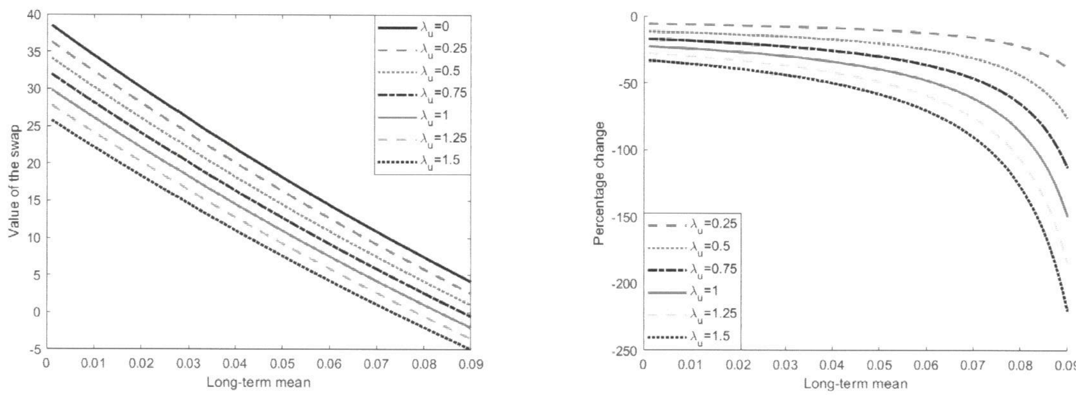


Figure 3.6 – Sensitivity of the swap value to the speed of mean reversion α for various values of upward jump intensities λ_u. Other parameter values are given in Table 3.1. Left panel: value of $s_0(\alpha)$. Right panel: percentage change with respect to the case where $\lambda_u = 0$.

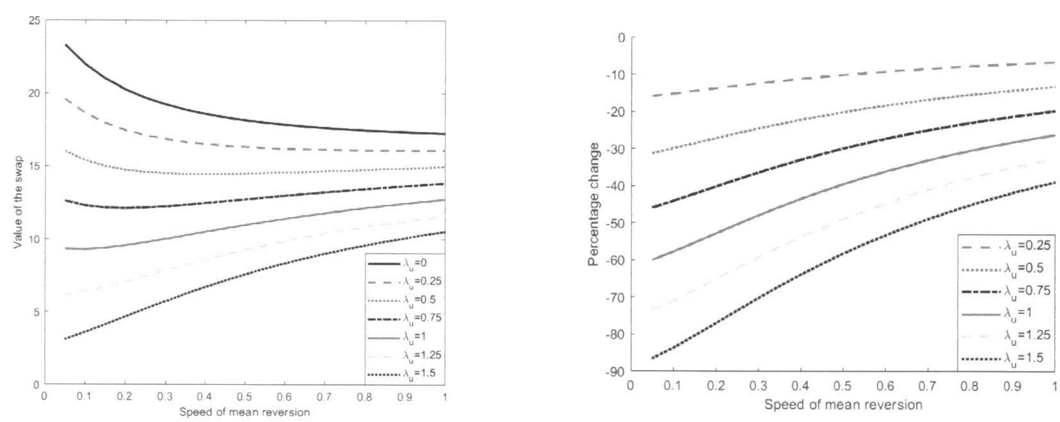
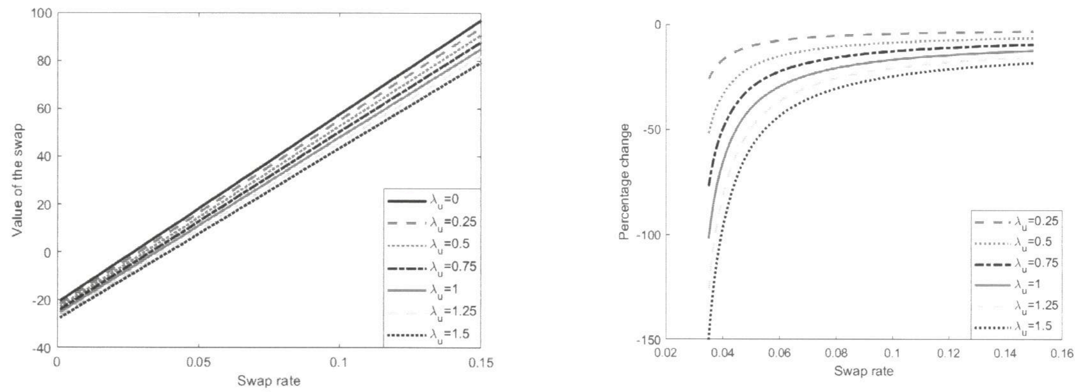


Figure 3.7 – Sensitivity of the swap value to the swap rate κ for various values of upward jump intensities λ_u. Other parameter values are given in Table 3.1. Left panel: value of $s_0(\kappa)$. Right panel: percentage change with respect to the case where $\lambda_u = 0$.



still a moderate value, the percentage change reaches 60% at $r = 4\%$.

Figure 3.3 shows that the negative impact of jump intensity become increasingly important as the maturity and number of coupons increase, so that the value of a swap may decrease with maturity when maturity and jump intensity become sufficiently high. Again, we observe that for moderate values of the jump intensity, the percentage change in swap prices relatively to a pure diffusion model could reach 100% for long-maturity swaps.

Figure 3.4 shows how the swap value is increasing with volatility, so that a higher jump intensity could be offset by a high volatility level, and Figure 3.5 illustrates the sensitivity of the swap value to the long-term mean, which is similar to that of the interest rate.

Figure 3.6 shows that the impact of the speed of mean reversion parameter α depends on the intensity of jumps: while α has a negative impact on the swap value in a pure diffusion model, its impact becomes positive when jumps are present and when α is sufficiently high. The negative impact of the jump parameter value is decreasing with α, and can reach 80% when $\alpha = 0.1$. This can be explained by the fact that when an upward jump in the interest rate occurs, it takes more time to revert to normal levels in an environment characterized by a low speed of mean reversion.

Finally, Figure 3.7 shows that the swap value is slightly more sensitive to the jump intensity parameter when the swap rate is low.

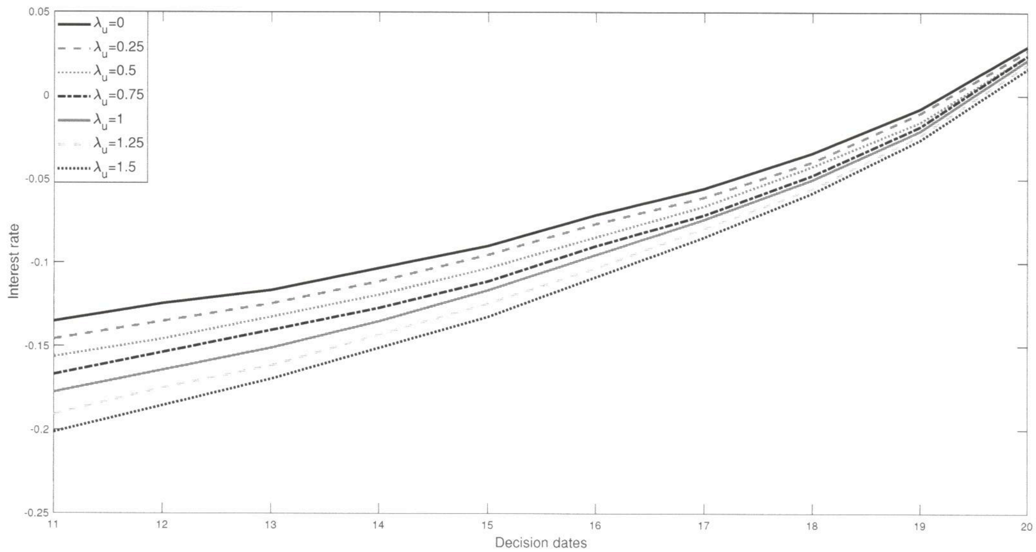
We now investigate the impact of jump intensity on the value of different instruments, namely straight and callable bonds, and European interest-rate payer swaption.

The bonds used for the illustration are the Swiss callable bond used in Chapter 2 (see bond parameters in Table 2.1) and its straight version. Sensitivity analyses pertaining to the straight bond are qualitatively similar to those of a swap, which is expected given the close relationship between the two instruments. Detailed results are provided in Figure 3.13 in Appendix 3.6. For callable bonds, Figure 3.8 plots the exercise frontier over time for various values of the jump intensity parameter λ_u. The value at which the embedded call option should be exercised is decreasing with the upward jump intensity, and the impact of jump intensity decreases with time to maturity. These results are intuitive since it becomes less interesting for the issuer to call his debt when the interest rate is more likely to increase.

The parameters of the European swaption used for the illustration are described in Table 3.1, with $t_1 = 3$ and $t_n = 5$. For a payer swaption, the value should increase with the intensity of upward jumps. Detailed results on the impact of all parameters are provided in Figure 3.14 in Appendix 3.6. These results indicate that the impact of jumps is more important for swaptions with shorter maturities and smaller swap rates and for lower volatility and mean reversion rate environments. All these results are intuitive. An interesting observation is that the convexity of the exercise payoff makes optional instruments' sensitivity to jumps more complex than that of instruments with linear payoffs, such as swaps and bonds.

These illustrative examples show that interest-rate jumps can affect the prices of fixed-income derivatives, as well as their exercise frontier, in a significant way, even when their probability of occurrence is small. In some situations resulting from com-

Figure 3.8 – Callable bond exercise frontier. The call dates and prices as well as the Vasicek-EJ parameters are described in Table 2.1.



binations of parameters, the percentage difference in the value of a derivative with or without jumps could be very high, for instance when the volatility is low or when the long-term mean is high. In addition, depending on the product specifications, prices of derivatives with long or short maturities may become very sensitive to the presence of jumps, even at a low intensity.

It is worth noting that these sensitivity analyses do not yet address the issue of model error, as presumably the parameters of a pure diffusion model and of a models including jumps would differ when calibrated on the same set of data. This issue is the object of the analyses presented in the next sections.

3.3 The term structure of interest rates

The term structure of interest rates can take various shapes depending on the economic environment and policies. One obvious observation is that jump-diffusion models, having more parameters than pure diffusion models, should provide a closer fit to the term structure. In this section, we investigate various typical shapes of the term structure in order to determine how these would influence the calibration by either a pure diffusion model or a jump-diffusion model.

We use the Nelson-Siegel model (Nelson & Siegel 1987) to generate various different hypothetical term structures. The Nelson-Siegel formula for the spot rate corresponding to a maturity T is given by

$$r(T) = \beta_0 + \beta_1 \frac{(1-\exp(-T/\theta))}{T/\theta} + \beta_2 \left(\frac{1-\exp(-T/\theta)}{T/\theta} - \exp(-T/\theta) \right)$$

where $\beta_0, \beta_1, \beta_2$ and $\theta > 0$ are constants. The constant β_0 represents the long-term level of interest-rates; β_1 is the coefficient of the decay component, a positive (*resp. negative*) value generating a downward (*resp. upward*) slope; the coefficient β_2 is a shape parameter, generating either a hump ($\beta_2 > 0$) or a through ($\beta_2 < 0$); finally, θ is a second shape parameter characterizing the steepness of the slope and the location of the hump/through.

We focus on the five following scenarios characterizing the shape of the term structure (see Figure 3.15 in Appendix 3.6):

S1 The yield curve is upward sloping with decreasing short-term steepness. The corresponding Nelson-Siegel parameters are $\beta_0 = 2$, $\beta_1 = -2$, $\beta_2 = 1$ and $\theta \in [0.5, 25]$.

S2 The yield curve is downward sloping with decreasing short-term steepness. The corresponding Nelson-Siegel parameters are $\beta_0 = 2$, $\beta_1 = 2$, $\beta_2 = -2$ and $\theta \in [0.1, 15]$.

S3 The yield curve is upward sloping with increasingly pronounced humps. The corresponding Nelson-Siegel parameters are: $\beta_0 = 2$, $\beta_1 = -1$, $\beta_2 \in [0.5, 15]$ and $\theta = 1.5$.

S4 The yield curve is downward sloping with decreasingly pronounced throughs. The corresponding Nelson-Siegel parameters are: $\beta_0 = 3.7$, $\beta_1 = 2$, $\beta_2 \in [-15, -0.5]$ and $\theta = 1.5$.

S5 The yield curve is flattening, from an upward to a downward trend. The corresponding Nelson-Siegel parameters are: $\beta_0 = 4$, $\beta_1 \in [-3, 3]$, $\beta_2 = 0$ and $\theta = 1$.

We fit both the Vasicek and the Vasicek-EJ models to these term structures using a non-linear optimisation approach. Specifically, we follow Zeytun and Gupta (2007) and find the model parameters reproducing the prices of zero-coupon bonds by minimizing the sum of squared differences between the discount factors implied from the theoretical term structure and the ones implied by each model for 11 different maturities (1 month, 3 months, 6 months, 1 year, 2 years. 3 years, 5 years, 7 years, 10 years, 20 years and 30 years).The model-implied rate, obtained from Equation (2.6), is given by

$$r(T) = \frac{(r_0 - \zeta)D(T) + \zeta T - C(T)}{T}$$

where r_0 is fixed at the shortest-maturity rate.

Finally, we price both a 5-year and a 10-year receiver interest-rate swaps, using the fitted parameters under each model, in order to evaluate the pricing difference due to the model specification. Results, in terms of the percentage difference with respect to the Vacisek-EJ model, are presented in Figure 3.9.

Results reported in Figure 3.9 show that, even though the two interest-rate models are calibrated to the same term structure, the pricing differences resulting from using a pure diffusion model against a jump-diffusion model can be sizable, depending on the shape of the term structure.

Specifically, the difference in prices obtained under the two model can reach 20% to 35% when the yield curve is steeply upward sloping (see Panels a and e of Figure 3.9). The difference can be even higher (up to 120%) when the curve is humped, specially when the hump is pronounced, so that short-term rates become larger than long-term rates. In this case, the price difference can reach 12% of the swap notional value. We note that pricing differences are generally positive (the Vasicek-EJ price is generally lower) and generally increasing with the swap maturity, except for the case where the yield curve is humped.

It is interesting to point out that the most common situation (upward sloping yield curve) is also the one where the difference in prices is the most important. Moreover, the present situation belongs to one of the cases where pricing differences can be sizable, (short-term rates are presently higher than long-term rates).

3.4 Empirical observation of model error for Treasury bond futures

In this section, we calibrate both the Vasicek and the Vasicek-EJ model to the historical term structure of interest rates. More specifically, we fit both the Vasicek and the Vasicek-EJ models to the observed U.S. yield curves downloaded from the U.S. Department of the Treasury website, on the first business day of March, June, September and December, from 2005 to 2017. Figure 3.10 is a graphical representation of the term structure over this period. We find the model parameters minimizing the sum of squared differences between observed prices and the corresponding zero-coupon prices, obtained using Equation (2.6), for maturities of one month, three months, six months, one year, two years, three years, five years, seven years, ten years, twenty years and thirty years. Detailed results are provided in Tables 3.2 - 3.3 and Figures (3.16)-(3.17)- in Appendix 3.6. Observation of Figure (3.16) shows high values of α and σ in 2006, which could be explained by the restoration of the 30-year U.S. treasury bonds that started in February 2006 and changed the shape of the term structure. The volatility parameter is relatively stable after this period due to the Federal Reserve near zero short rate policy, which continued even after 2009, maintaining the volatility at a very low level. On the other hand, observation of Figure (3.17) shows that parameter values are reasonably stable over time, with high values of α and σ in 2006-2007 and a relatively stable long term mean ζ, and

Figure 3.9 – Percentage difference in swap prices under various scenarios for the term structure of interest rates. Swap prices are evaluated under the Vasicek and Vasicek-EJ models fitted to the term structure. The 5- and 10-year swaps have a 5% swap rate and coupons are exchanged on a semi-annual basis.

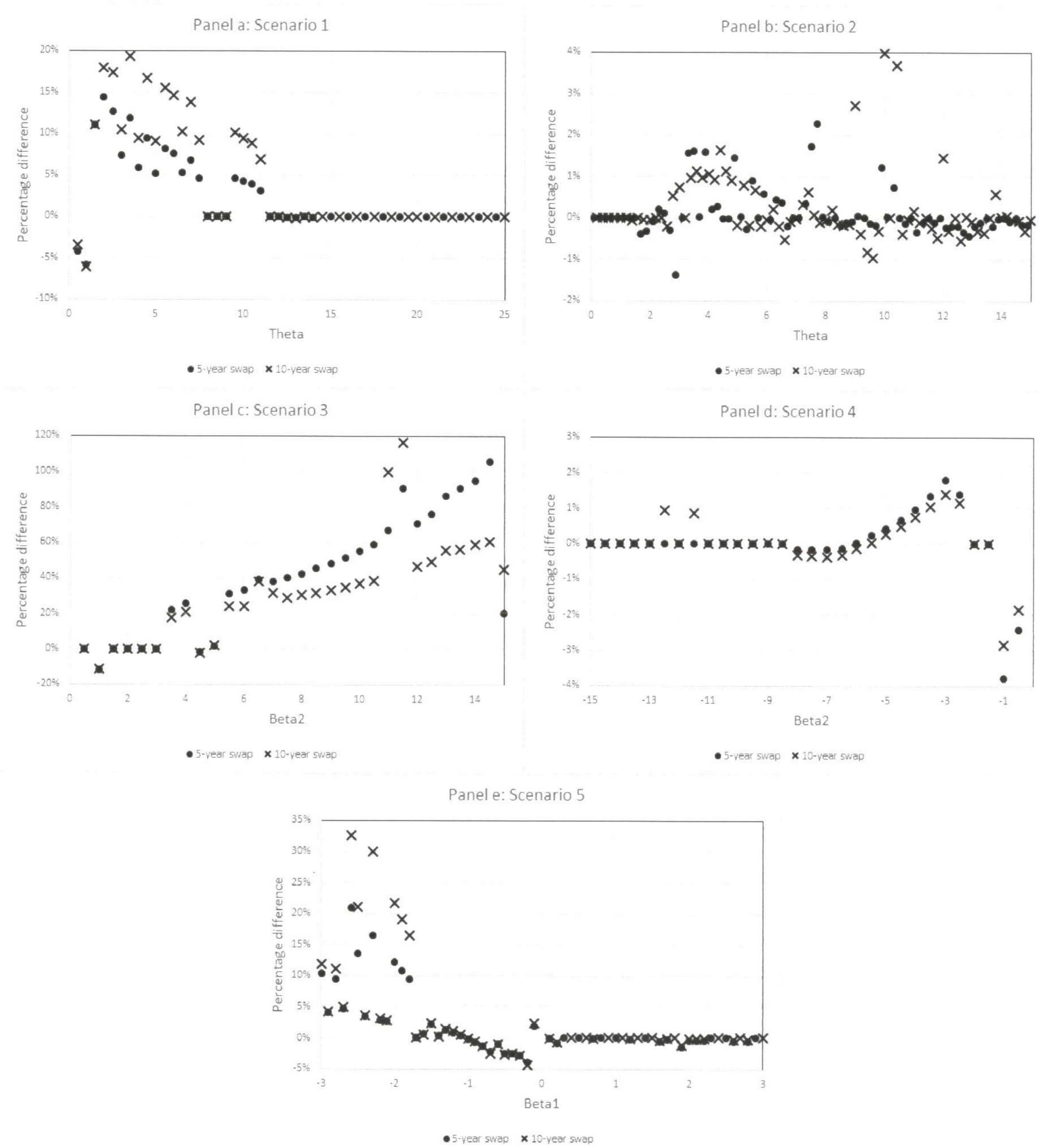
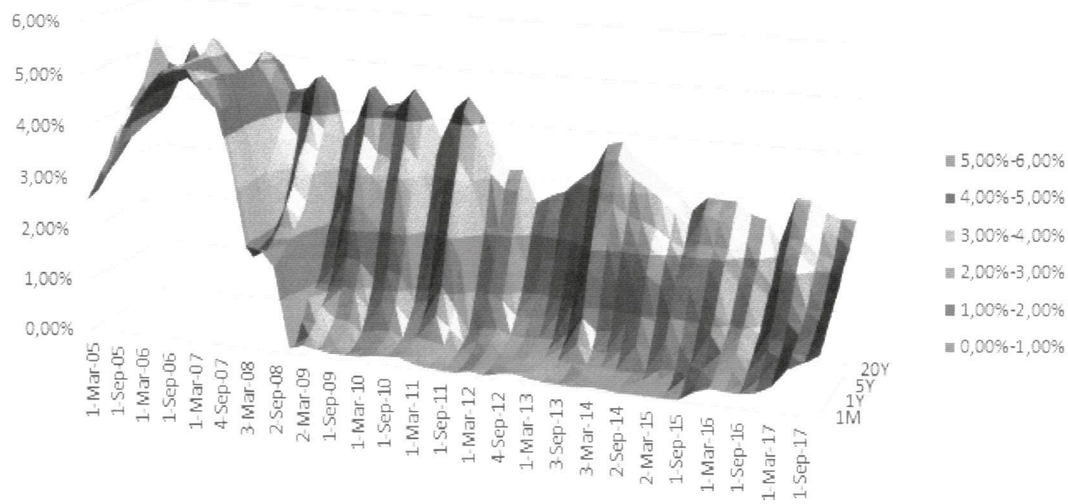


Figure 3.10 – Historical term structure 2005 - 2017



with a high intensity and amplitude of upward jumps in 2005-2006, a high intensity of small amplitude upward jumps between 2007 and 2014, and the occurence of downward jumps since 2016.

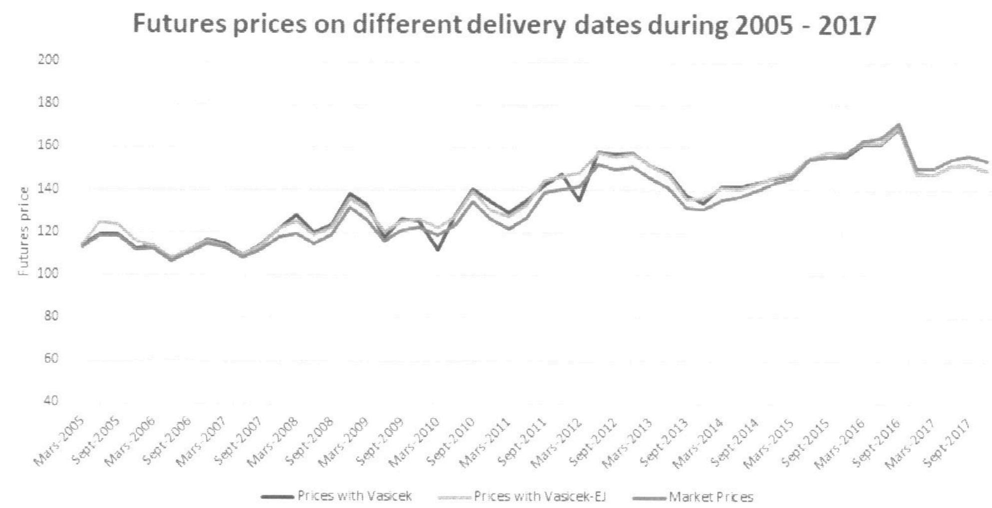
The next step consists of pricing a derivative contract under the Vasicek and Vasicek-EJ models, using the dynamic programming model presented in Chapter 2 and the calibrated parameter values, and comparing these prices with the observed market prices.

The observations of the CBOT T-bond futures market prices are available from Bloomberg, along with the deliverable baskets and conversion factors for each of the 52 future contracts traded during the time period 2005-2017. Theoretical prices are computed using the dynamic program (2.7)-(2.9) on the first day of each delivery month.

Figure 3.11 is a plot of the prices obtained under each of the two models and of the market prices of each contract on the first day of each delivery month during the period 2005-2017. A first observation is that the theoretical prices, under both models, are close to observed prices.

Figure 3.12 plots the percentage difference between the model-implied theoretical

Figure 3.11 – Comparison of futures theoretical prices and market prices over the period 2005-2017. Market prices are obtained from Bloomberg. Theoretical prices are obtained using the parameters in Table 3.3 for the Vasicek-EJ model and Table 3.2 for the Vasicek pure diffusion model.



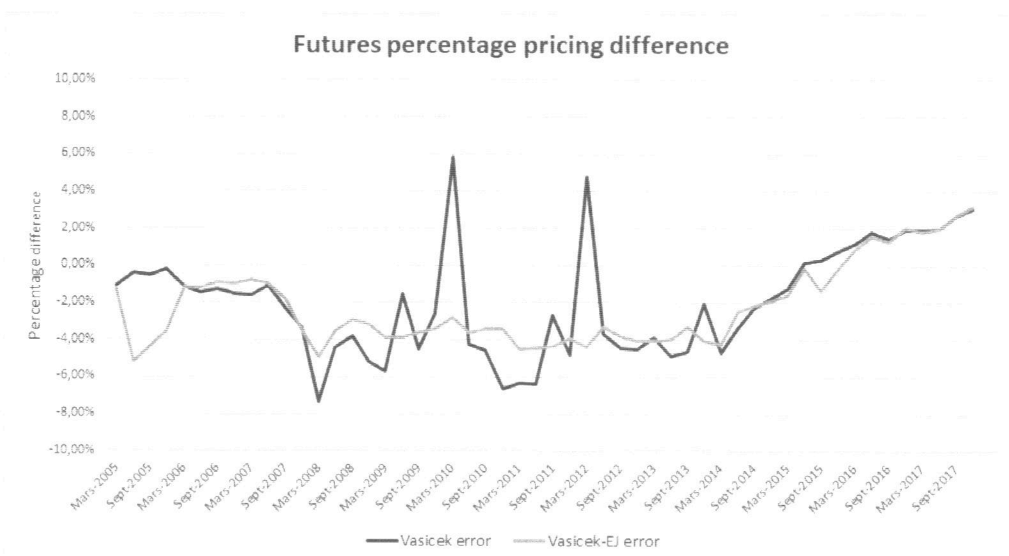
price and the market price under the two models. We observe that differences are generally positive (theoretical futures price are higher than market prices by more than 2 bps on average) and that the Vasicek-EJ model seems to perform slightly better in predicting T-bond futures market prices, with a RMSE of 4.11, compared to 4.67 for the pure diffusion model.

3.5 Conclusion

In this chapter, we investigated the impact of model specification on the valuation of interest-rate derivatives by assessing the impact of jump intensity on the price of derivatives and by comparing prices implied by a pure diffusion and a jump-diffusion Vasicek model, calibrated to the same data, namely the term structure of interest rates.

We rely on the flexible general dynamic programming procedure combined with Fourier transform presented in Chapter 2 to study the impact interest-rate jumps

Figure 3.12 – Comparison of the percentage difference between observed futures market prices and model-implied theoretical prices. Market prices are obtained from Bloomberg. Theoretical prices are obtained using the parameters in Table 3.3 for the Vasicek-EJ model and Table 3.2 for the Vasicek pure diffusion model.



on several fixed-income instruments.

By pricing actively traded products under both the Vasicek and Vasicek-EJ interest-rate models, we find that the contribution of jumps can be non-negligible, particularly when the term structure is steep and upward sloping or when it has a humped shape. In addition, depending on the derivative, upward or downward jumps can be more important and have a greater impact on the prices of interest-rate derivatives. We notice also that under some particular situations such as a low a volatility environment , price differences can become very large.

3.6 Appendix

Table 3.2 – Parameter values for the Vasicek pure diffusion model calibrated to the Treasury bond yield curve on the first day of March, June, September and December from 2005 to 2017.

Date	alpha	zeta	sigma
Mars-2005	0.530	0.050	0.010
June-2005	0.280	0.050	0.022
Sept-2005	0.192	0.050	0.016
Dec-2005	0.386	0.050	0.010
Mars-2006	1.121	0.055	0.150
June-2006	1.939	0.054	0.139
Sept-2006	2.175	0.050	0.150
Dec-2006	2.193	0.047	0.150
Mars-2007	2.196	0.048	0.150
June-2007	1.736	0.052	0.099
Sept-2007	0.200	0.047	0.010
Dec-2007	0.071	0.052	0.010
Mars-2008	0.093	0.060	0.010
June-2008	0.206	0.055	0.010
Sept-2008	0.240	0.050	0.010
Dec-2008	0.259	0.040	0.010
Mars-2009	0.232	0.046	0.010
June-2009	0.250	0.056	0.010
Sept-2009	0.258	0.051	0.010
Dec-2009	0.192	0.055	0.010
Mars-2010	0.214	0.057	0.010
June-2010	0.208	0.053	0.010
Sept-2010	0.129	0.052	0.010
Dec-2010	0.184	0.050	0.010
Mars-2011	0.247	0.050	0.010
June-2011	0.184	0.050	0.010
Sept-2011	0.100	0.055	0.010
Dec-2011	0.104	0.050	0.010
Mars-2012	0.082	0.057	0.010
June-2012	0.080	0.046	0.010
Sept-2012	0.070	0.052	0.010
Dec-2012	0.070	0.053	0.010
Mars-2013	0.080	0.055	0.010
June-2013	0.099	0.054	0.010
Sept-2013	0.171	0.050	0.010
Dec-2013	0.135	0.055	0.010
Mars-2014	0.149	0.049	0.010
June-2014	0.078	0.100	0.028
Sept-2014	0.084	0.100	0.031
Dec-2014	0.076	0.100	0.029
Mars-2015	0.102	0.083	0.034
June-2015	0.084	0.096	0.031
Sept-2015	0.126	0.070	0.034
Dec-2015	0.172	0.052	0.034
Mars-2016	0.149	0.043	0.023
June-2016	0.189	0.040	0.028
Sept-2016	0.181	0.034	0.025
Dec-2016	0.218	0.053	0.043
Mars-2017	0.258	0.047	0.044
June-2017	0.169	0.041	0.023
Sept-2017	0.145	0.040	0.020
Dec-2017	0.374	0.035	0.046

Table 3.3 – Parameter values for the Vasicek-EJ model calibrated to the Treasury bond yield curve on the first day of March, June, September and December from 2005 to 2017.

Date	alpha	zeta	sigma	lambda_up	lambda_down	1/eta_up	1/eta_down
Mars-2005	0.0886	0.0500	0.0100	0.0136	0.00119	1.11128	-5.29282
June-2005	0.0718	0.0500	0.0100	0.0080	0.00137	2.47507	-0.10615
Sept-2005	0.0820	0.0500	0.0100	0.0035	0.00099	5.62487	-0.11353
Dec-2005	0.4906	0.0438	0.0486	0.0081	0.00001	1.38444	-0.49094
Mars-2006	1.5000	0.0002	0.1831	0.9124	0.00013	0.09270	-12.72789
June-2006	1.2284	0.0300	0.1492	0.2047	0.00002	0.18420	-1.23410
Sept-2006	1.4220	0.0300	0.1401	0.1032	0.00001	0.33018	-1.42511
Dec-2006	1.4295	0.0300	0.1064	0.0561	0.00001	0.46564	-1.43293
Mars-2007	0.9991	0.0277	0.0100	0.0460	0.00010	0.42553	-1.01616
June-2007	0.7952	0.0500	0.1092	0.1230	0.00010	0.04952	-0.82172
Sept-2007	0.7713	0.0415	0.0100	1.0000	0.00010	0.00012	-0.78012
Dec-2007	0.8998	0.0302	0.0100	0.9869	0.00010	0.00017	-0.90509
Mars-2008	0.0100	0.0500	0.0100	1.0000	0.00644	0.00183	-0.25588
June-2008	0.0201	0.0500	0.0189	0.9788	0.00135	0.00472	-1.24267
Sept-2008	0.0324	0.0500	0.0218	0.9853	0.00110	0.00493	-5.21815
Dec-2008	0.0100	0.0499	0.0205	1.0000	0.00211	0.00593	-4.70042
Mars-2009	0.0455	0.0398	0.0268	0.9974	0.00009	0.00703	-0.06110
June-2009	0.0661	0.0492	0.0352	1.0711	0.00242	0.00797	-0.43281
Sept-2009	0.0577	0.0500	0.0317	1.0000	0.00236	0.00733	-0.62415
Dec-2009	0.0420	0.0500	0.0265	0.9999	0.00271	0.00657	-0.41868
Mars-2010	0.0564	0.0500	0.0309	1.0000	0.00268	0.00753	-0.42875
June-2010	0.0360	0.0500	0.0250	0.9961	0.00269	0.00638	-0.60457
Sept-2010	0.1614	0.0413	0.0100	0.0010	0.00261	0.00017	-0.34382
Dec-2010	0.0315	0.0500	0.0211	0.9963	0.00330	0.00518	-0.27777
Mars-2011	0.2034	0.0496	0.0100	0.0174	0.00281	0.00010	-0.50259
June-2011	0.0232	0.0400	0.0188	0.4827	0.00290	0.01187	-0.39065
Sept-2011	0.0200	0.0500	0.0140	1.0000	0.00383	0.00313	-0.25828
Dec-2011	0.0357	0.0800	0.0178	0.7391	0.00316	0.00241	-0.26804
Mars-2012	0.1053	0.0424	0.0100	0.0110	0.00272	0.00010	-0.29793
June-2012	0.0119	0.0120	0.0109	0.7414	0.00141	0.00376	-0.23737
Sept-2012	0.0200	0.0500	0.0105	0.9976	0.00259	0.00177	-0.24621
Dec-2012	0.0200	0.0500	0.0100	1.0000	0.00267	0.00174	-0.24448
Mars-2013	0.0010	0.0600	0.0116	0.8971	0.00283	0.00362	-0.24020
June-2013	0.1191	0.0425	0.0100	0.0010	0.00238	0.00018	-0.31263
Sept-2013	0.0448	0.0300	0.0235	0.9981	0.00309	0.00577	-0.41462
Dec-2013	0.1815	0.0402	0.0100	0.0303	0.00353	0.00022	-0.35807
Mars-2014	0.0353	0.0600	0.0205	0.9762	0.00317	0.00386	-40.52654
June-2014	0.0678	0.0984	0.0256	0.1855	0.00292	0.00053	-0.43682
Sept-2014	0.0647	0.0886	0.0246	0.2178	0.00349	0.00231	-0.64567
Dec-2014	0.0760	0.0778	0.0249	0.9995	0.00269	0.00022	-0.64122
Mars-2015	0.1354	0.0536	0.0315	0.0008	0.00237	0.00193	-0.71776
June-2015	0.0958	0.0722	0.0283	0.1750	0.00187	0.00061	-0.68071
Sept-2015	0.1291	0.0607	0.0295	0.2514	0.00079	0.00284	-0.13316
Dec-2015	0.1756	0.0514	0.0344	0.0350	0.00004	0.00204	-0.18073
Mars-2016	0.0651	0.0372	0.0100	0.0115	0.07133	0.53755	-0.00011
June-2016	0.0714	0.0293	0.0100	0.0146	0.00000	0.55134	-0.00091
Sept-2016	0.0755	0.0293	0.0103	0.0104	0.51853	0.59795	-0.00025
Dec-2016	0.1903	0.0009	0.0271	0.1517	0.06016	0.07551	-0.00020
Mars-2017	0.2087	0.0177	0.0389	0.3811	0.00044	0.01921	-9.49846
June-2017	0.1681	0.0521	0.0100	0.0016	0.89438	9.61703	-0.00353
Sept-2017	0.1449	0.0224	0.0176	0.1574	0.03341	0.01680	-0.00070
Dec-2017	0.2037	0.0267	0.0180	0.0066	0.00112	2.09226	-0.00016

Figure 3.13 – Sensitivity analysis of the straight bond value with respect to the interest-rate parameters and upward jumps. The model parameters are given in Table 2.1.

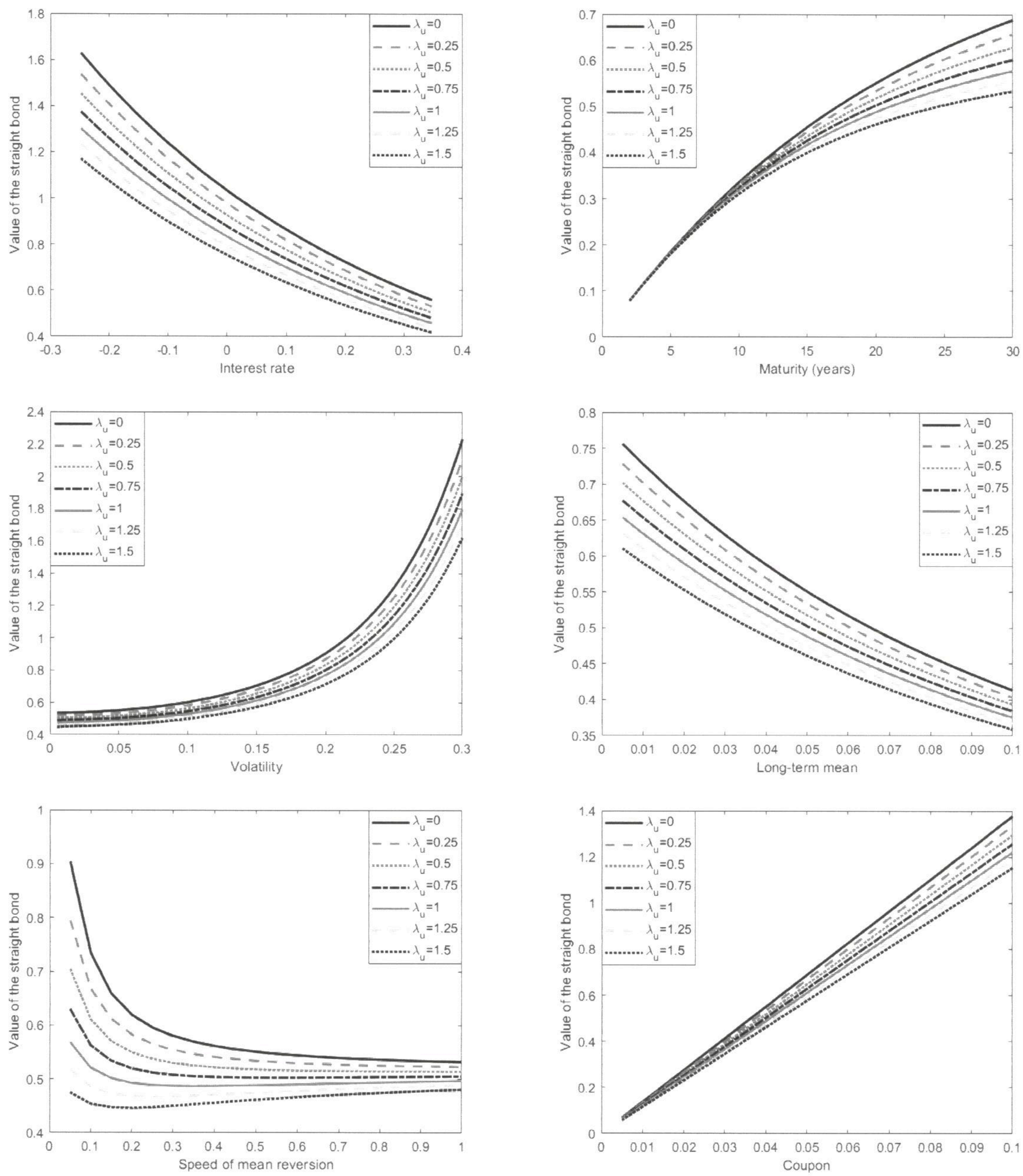


Figure 3.14 – Sensitivity analysis of the European payer swaption value with respect to the interest-rate parameters and upward jumps. The model parameters are given in Table 3.1.

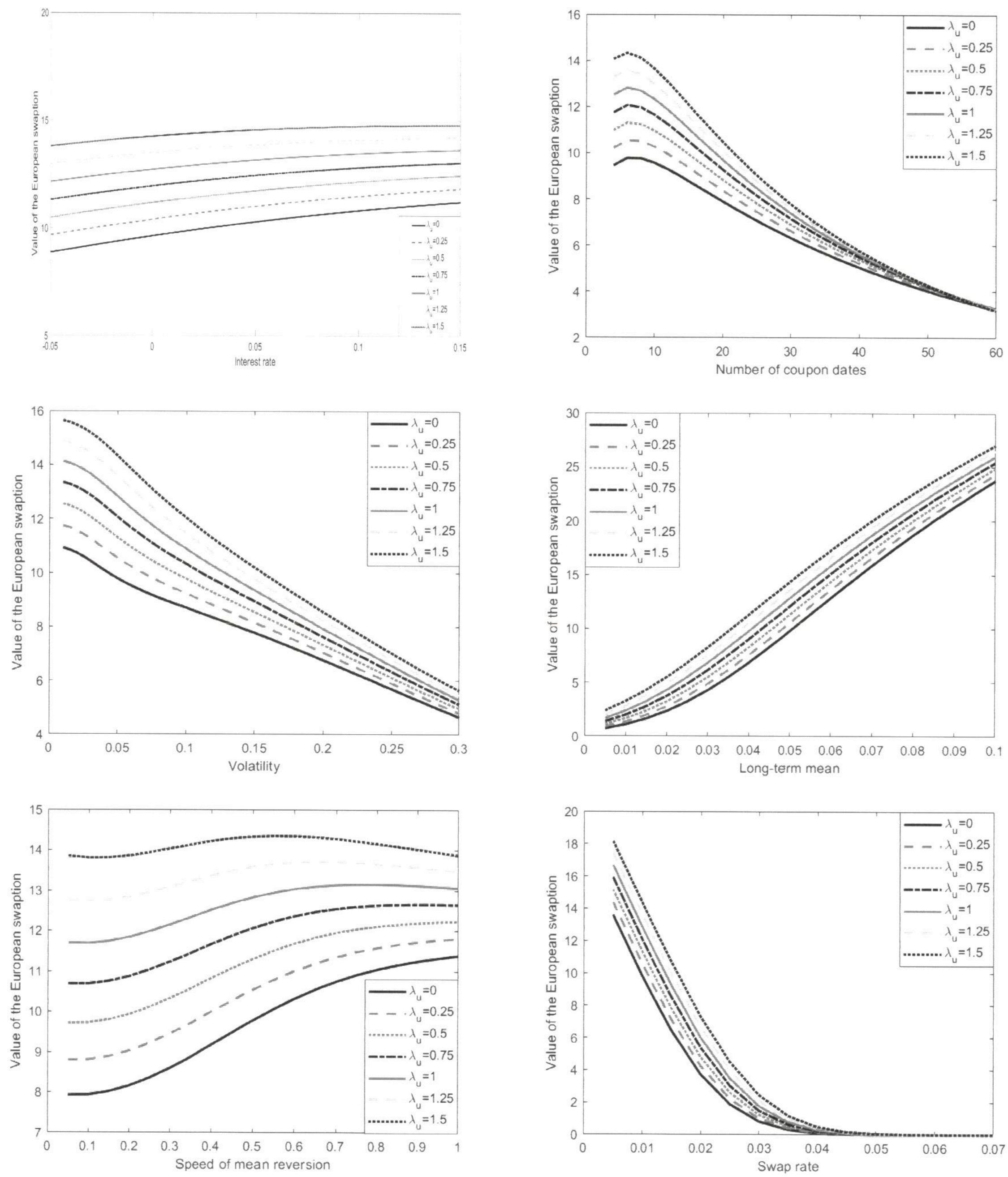


Figure 3.15 – Various scenarios representing possible shapes of the term structure of interest rates. Parameter values are given in Section 3.3.

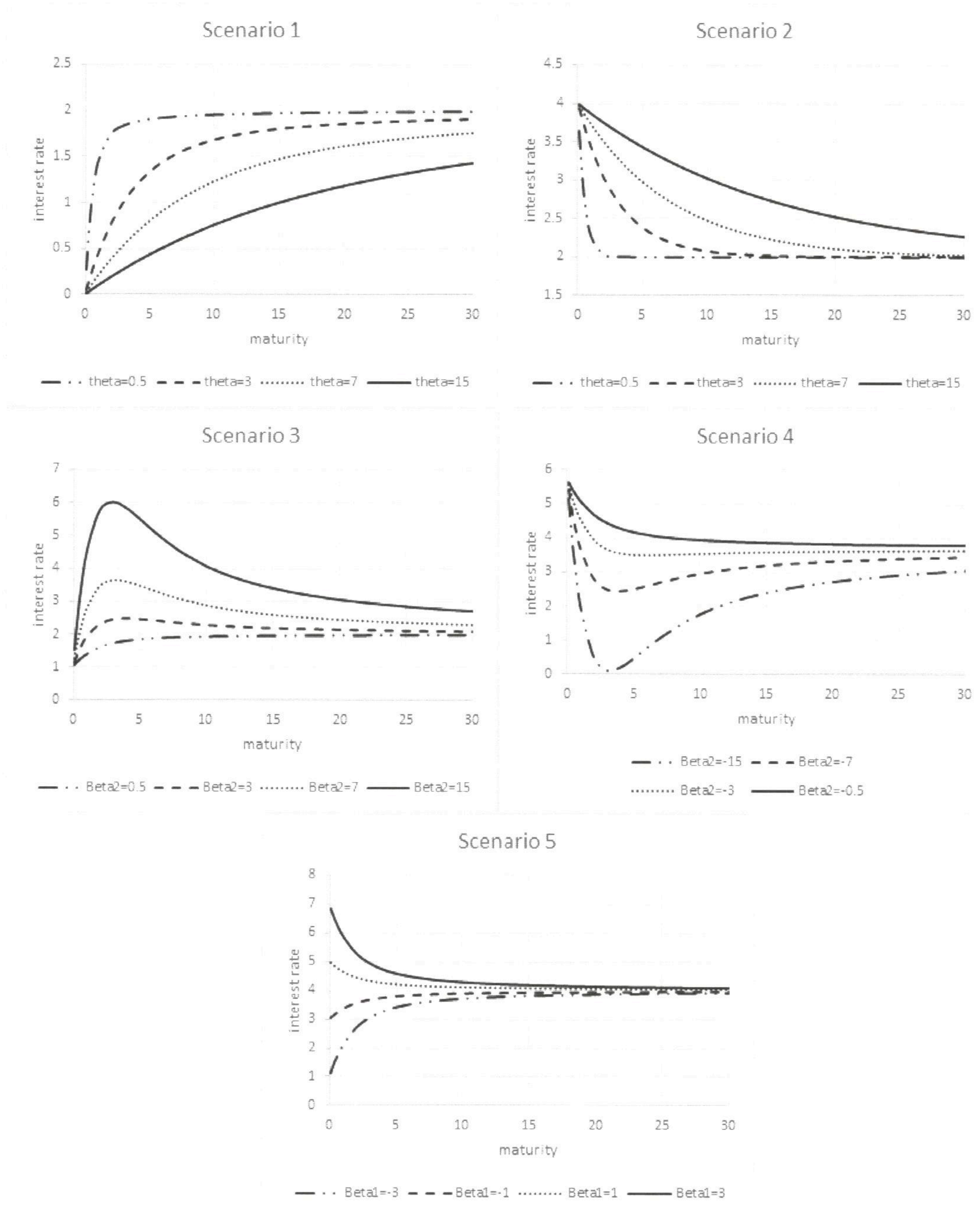


Figure 3.16 – Parameter values for the Vasicek model, calibrated to the Treasury bond yield curve on the first day of March, June, September and December from 2005 to 2017.

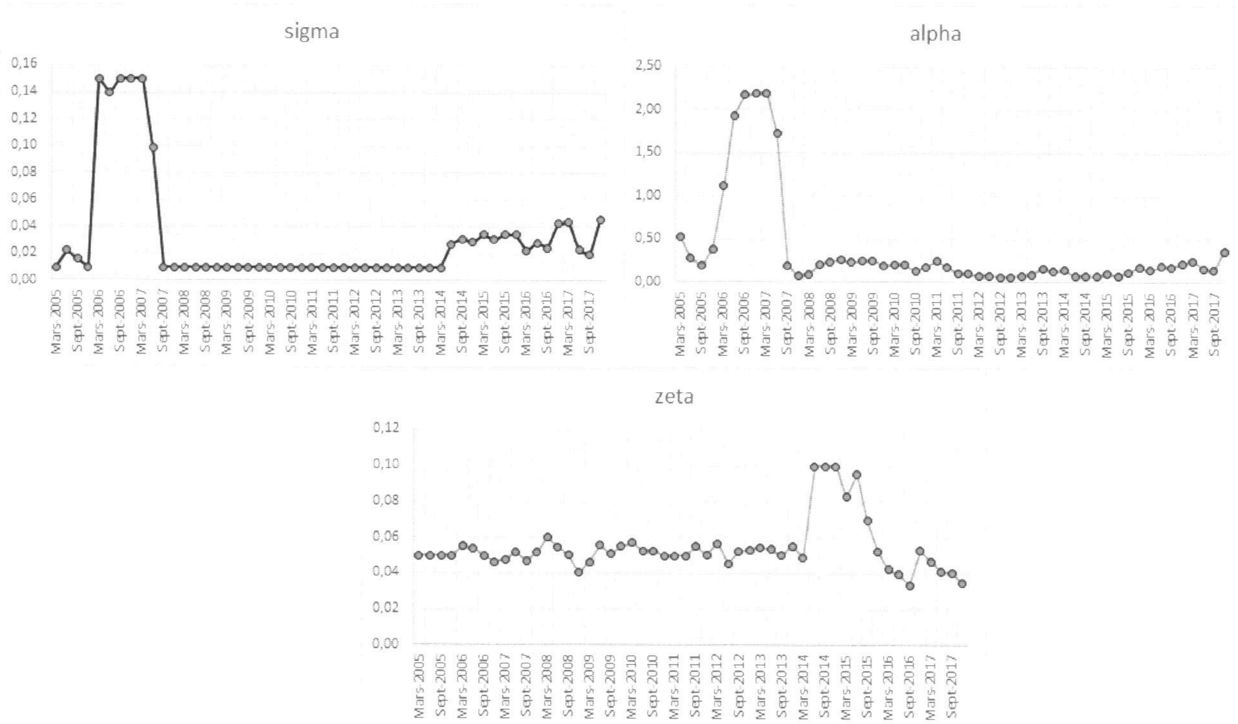
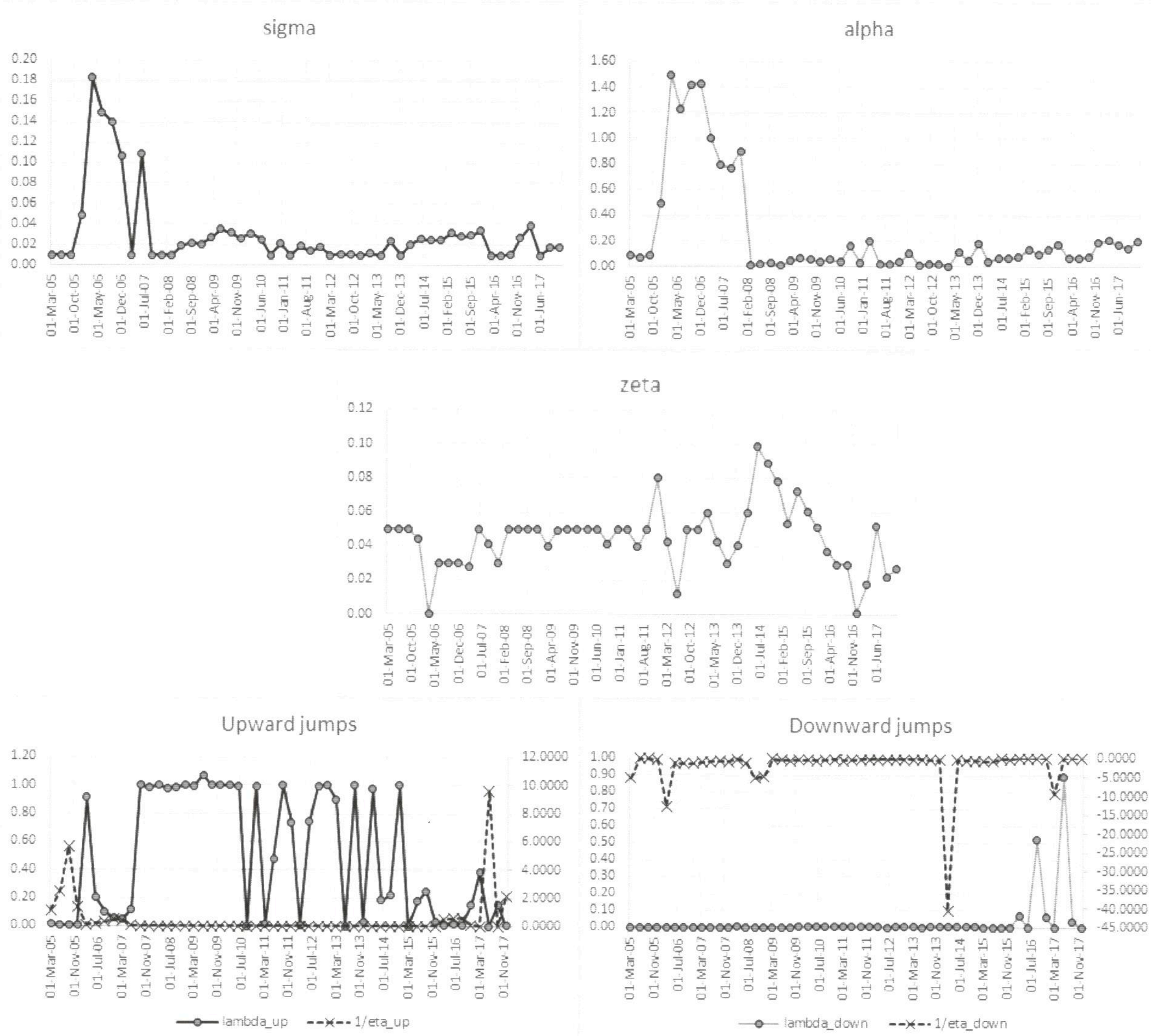


Figure 3.17 – Parameter values for the Vasicek-EJ model, calibrated to the Treasury bond yield curve on the first day of March, June, September and December from 2005 to 2017.



Chapter 4

Post-trade evaluation of the exercise strategy of CBOT Treasury bond futures

The CBOT Treasury bond (T-bond) futures is one of the most actively traded futures contract in the world; it is widely used to hedge long-term interest-rate risk. The seller of the contract (or short trader) is required to deliver a 20-year bond with a 6% coupon and a face value of 100, 000$ (the *notional bond).* However, as the notional bond is not necessarily traded in the market, the short trader can choose to deliver any bond from a defined basket (the *delivery basket*). In order to make the delivery fair for both parties, each bond of the delivery basket is assigned a *conversion factor* that takes into account the difference in coupon and maturity between this bond and the notional bond. The conversion factor system is not perfect, so that all eligible bonds are not equal for delivery; the bond that maximizes the short trader's profit is called the CTD. The flexibility of the short trader in the choice of the bond to be delivered is called the *quality option.*

Moreover, the seller has the option to deliver the bond on any day during the last trading month. This timing option allows the traders to profit from changes in the interest rate that will affect the price of the CTD, or even change the ranking of the bonds in the delivery basket, and thus their profit or loss. Recall that the delivery month, where exercise is allowed, is further divided into the *switch period*, during which the futures contract is traded, and the *end-of-month period*, during which the futures market is closed but delivery remains possible. Another timing option relates to the fact that the futures market closes and the futures price is settled at 2 p.m. each day, while the market for T-bonds stays open. The flexibility to use any new information regarding the bond prices after 2 p.m., before announcing delivery by 8 p.m., is known as the *wild card* option. The interaction between the timing and quality options makes the choice of the delivery day complex. As shown in Chapter 2, the determination of the optimal delivery strategy can be modeled as a dynamic program, where the strategy of the short trader depends on the value of two state variables, that is, the observed futures price and the current level of interest rates.

Most of the literature on the options embedded in the T-bond futures concerns the pricing of the quality and timing options (see, for instance, Ben-Abdallah, Ben-Ameur & Breton 2012 and Chen & Yeh 2015). Few papers focus on the timing strategy. Gay & Manaster (1986) show how the embedded options, and particularly the wild card option, can give positive results when skilfully exercised. The authors propose an exercise strategy that is based mainly on the movements of the spot price and the value of the conversion factor. The general rule is that, for a conversion factor greater than one, if the bond spot price becomes lower than the futures invoice price during the wild card period, then the short trader should deliver. An ex-post evaluation of this strategy shows that the wild card option was not used optimally by traders, resulting in non negligible losses. This simple delivery rule was later challenged by Kane & Marcus (1986), who develop a valuation model for the wild card option as well as rules for its optimal exercise. The authors argue that the rule proposed by Gay & Manaster (1986) is not value-maximizing for the short trader and that the spot price must decrease below a certain critical value before delivery becomes optimal. Other rules of thumb have been suggested, indicating situations where the short trader should deliver early (see Burghardt et al. 2005 and Choudry 2010).

More recently, Breton & Ben-Abdallah (2018) study the short traders delivery behavior observed in the CBOT T-Bond futures market from 1985 to 2016 and determine, using a regression analysis, the most important factors affecting the traders decisions, namely, the slope of the U.S. Treasury yield curve, the number of days where the CTD basis has been negative and the total amount of the outstanding Treasury bonds. With hindsight, the authors evaluate the exercise strategy actually used by the traders, comparing their profits with those made by delivering in the beginning versus in the end of the delivery month. They find that, even when positive profits could be made by delivering early, postponing delivery until the end of the delivery month generally resulted in higher profits. However, the authors do not evaluate traders decisions against the ex-ante optimal exercise strategy.

Our main contribution in this chapter is to evaluate the advantage of using an optimal exercise strategy, with respect to the delivery strategy that was actually used by traders over the period 2005 to 2015. The optimal exercise strategy is obtained, using the algorithm described in Chapter 2, on every day of the delivery month of each futures contract traded between 2005 and 2015. We find that traders generally used a commonly recognized rule of thumb that consists of relying on the slope of the term structure of interest rates to determine if one should postpone delivery until the end of the delivery month. Our results show that this simple rule of thumb resulted in a very volatile profit, as measured by the CTD bond gross basis. On the other hand, the optimal strategy, computed using market data and the Vasicek-EJ model, differs significantly from decisions taken by the majority of traders, and would have resulted in a comparable average profit, however with a significantly lower volatility.

The chapter is organized as follows. Section 4.1 describes the data set. Section 4.2 discusses the issues in choosing a delivery strategy and characterizes the nature of the timing decisions made by traders over the study period. Section 4.3 gives the details of the implementation used to compute the optimal exercise strategy. Section 4.4 compares the optimal strategy to the observed delivery behavior, and Section 4.5 is a conclusion.

4.1 Data

Our study covers 44 futures contracts (March, June, September and December) from 2005 to 2015. We elect to analyze this specific period because it covers a full economic cycle, including the financial crisis of 2008, while avoiding some outstanding episodes due to market anomalies during the period from 1985 to 1995.[1]

The physical-delivery data set is provided by the CBOT. For each contract, it includes the dates on which deliveries took place during the delivery month, the number of futures contract settled on each of these dates, as well as the T-bond actually delivered. This data allows us to obtain the distribution, during the delivery month, of the observed timing of deliveries (see Figure 4.1).

The price data is provided by Bloomberg. For each day during the delivery month of each futures contract, we obtain the prices of the futures contracts, along with the CTD bond and its corresponding conversion factor and spot price.

Finally, the daily observations of the interest rate are obtained from the U.S. Treasury website.

4.2 Delivery strategy: issues and market rules

The difference between the cash price of a T-bond and the futures invoice price (that is, the settlement price multiplied by the conversion factor) is called the bond's *gross basis*. The gross basis represents the immediate cost of delivering a T-bond into a futures contract at a given date, and, therefore, the CTD is the bond with the lowest gross basis in the delivery basket. The interest earned from holding a bond, that is,

[1]See Breton and Ben-Abdallah (2018) for a case-by-case analysis of early delivery episodes.

Figure 4.1 – Timing of actual deliveries from 2005 to 2015. Period 1 corresponds to the two first business days of the delivery month. Periods 3 and 4 correspond to the end of the month period (last seven business days), where Period 4 refers to the two last business days of the delivery month. Period 2 contains the rest of the delivery month.

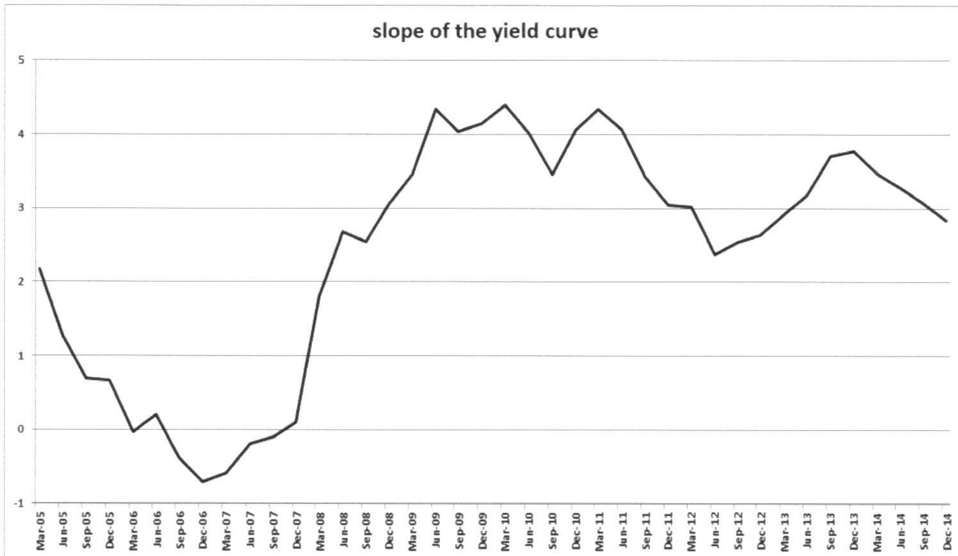
the difference between the coupon income and the financing cost, is called the bond *carry*.

While the basis indicates the profit or loss related to immediate delivery, it gives no indication of whether or not delivery should take place, as the basis could increase or decrease during the delivery month. Note however that a negative gross basis, corresponding to a delivery profit, gives rise to an arbitrage opportunity (sell the futures, buy the CTD and deliver it immediately) that should rapidly vanish.

A commonly used rule of thumb (Burghardt et al. 2005) consists of postponing delivery whenever the term structure of interest rates is upward sloping; by doing so, the short trader benefits from the positive carry of the CTD, while keeping possession of the futures embedded delivery options. According to this rule of thumb, early delivery is only advisable under extreme circumstances, when the slope of the term structure of interest rates is negative and market volatility is low.

Observation of Figure 4.1 shows that 98% of the deliveries during the period 2005-2015 took place during the last seven business days of the delivery month, with

Figure 4.2 – Slope of the yield curve, obtained by subtracting the spot from the 30 year Fed Fund rate on the first day of the delivery month. Rates are obtained from the Federal Reserve Statistical Release.



92% during the last two days. Deliveries during the two first business days occurred in December 2005 (3% of the total deliveries for this contract) and December 2006 (15% or total deliveries); in both cases, the slope of the yield curve (see Figure 4.2) was negative on the first day of the delivery month.

Figure 4.3 represents the evolution of the gross basis of the CTD during the delivery month of the future contracts traded between 2005 and 2015. This figure clearly shows the increase in volatility of the gross basis during the end of the delivery months, and specially during their last seven business days, where the trading of futures contract is suspended while the bond market remains open and delivery remains possible based on the last futures settlement price.

Figure 4.4 shows the value of the gross basis of the CTD, that is, the immediate loss resulting from the delivery, on the days where at least one delivery was made.

Figure 4.3 – Gross basis of the CTD on each business day of the delivery month of futures contracts from 2005 to 2015. The vertical line separates the switch period from the end-of-month period.

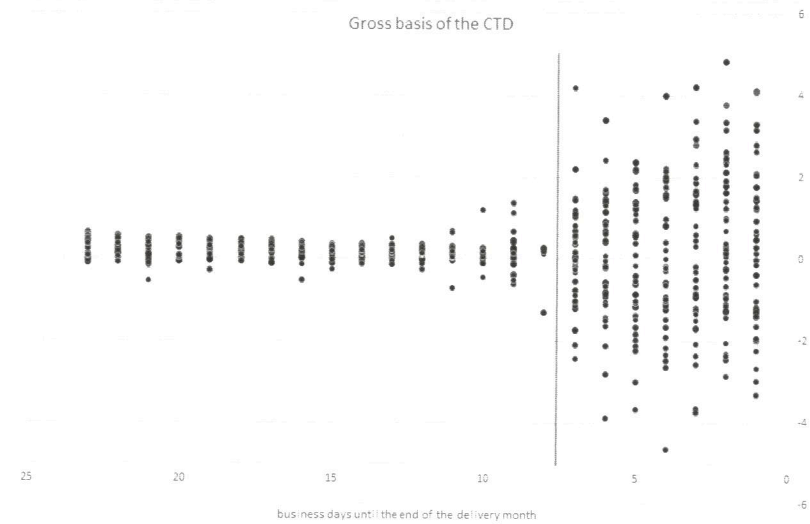
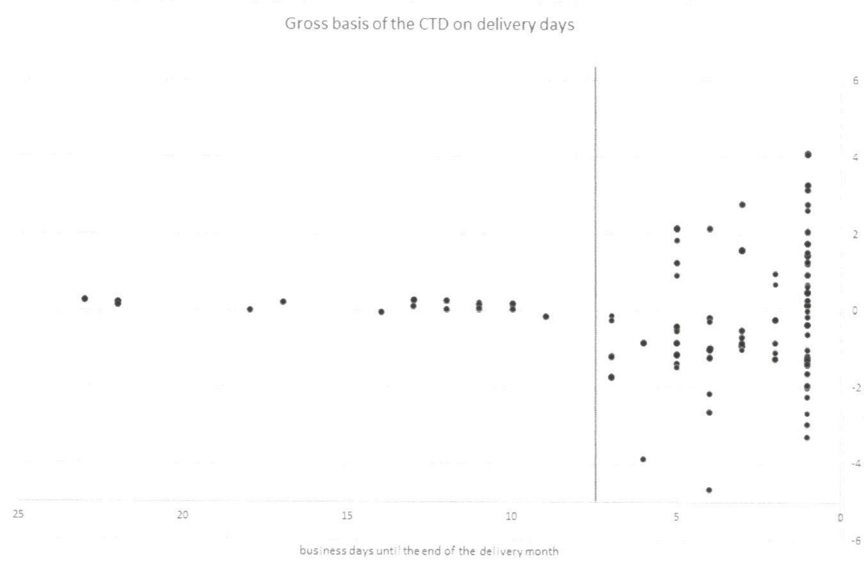


Figure 4.4 – Gross basis of the CTD on days where delivery occured from 2005 to 2015. The vertical line separates the switch period from the end-of-month period.



4.3 Delivery strategy implied by the dynamic programming algorithm

In this section, we describe how we determine the delivery day that would have been suggested by the dynamic programming algorithm described in Chapter 2, for each of the futures contract during the period 2005-2015. Specifically, we follow the following steps, for each futures contract:

1. Obtain the basket of deliverable bonds as well as their corresponding conversion factors.

2. Estimate the parameters of the Vasicek-EJ interest-rate model on the first day of the delivery month, by calibrating the model to the observed U.S. yield curve at that date.

3. Run the DP algorithm (2.7)-(2.9) and obtain the holding value function $v_m^h(\lambda, r)$ for all business days of the delivery month (indexed by m) on a grid of values for the futures price λ and spot interest rate r.

4. For each delivery day $m = 1, ..., n$:

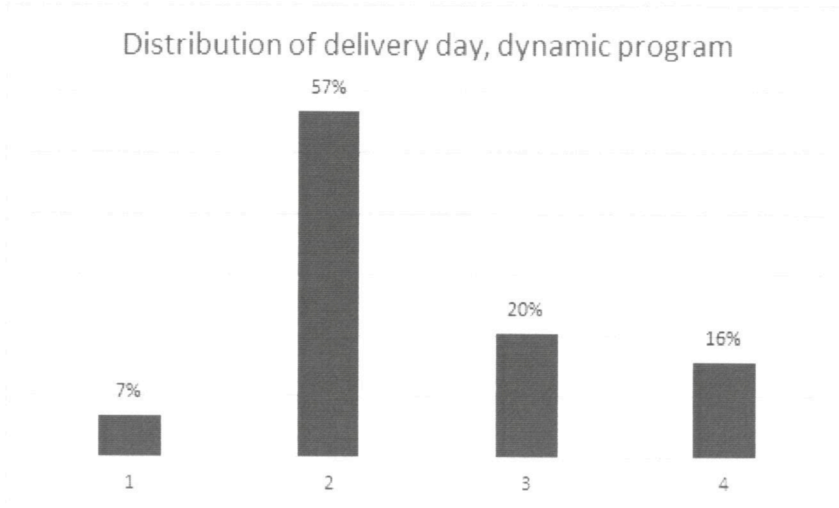
    a) Observe the interest r_m, the futures price λ_m, and the gross basis of the CTD e_m. Compute the holding value $v_m^h(\lambda_m, r_m)$ by interpolation.

    b) If $-e_m > v_m^h(\lambda_m, r_m)$ or if $m = n$, the last delivery day, stop and store m, the optimal delivery date, and e_m, the realised loss on delivery.

    c) Otherwise, set $m = m + 1$ and return to step 4 a.

Figure 4.5 shows the distribution of the delivery days according to the optimal exercise strategy under the Vasicek-EJ model, and Figure 4.6 compares actual and optimal delivery dates. We observe that the delivery strategy obtained using the

Figure 4.5 – Timing of deliveries from the dynamic program. Period 1 corresponds to the two first business days of the delivery month. Periods 3 and 4 correspond to the end of the month period (last seven business days), where Period 4 refers to the two last business days of the delivery month. Period 2 contains the rest of the delivery month.



dynamic program differs significantly from the timing of deliveries actually observed over the period 2005-2015.

In particular, early delivery is recommended for almost 60% of the contracts, and only 16% of deliveries would have occurred on the last two days of the delivery month according to the dynamic program. For many contracts, exercise is recommended whenever the gross basis becomes negative. Contracts where exercise is deferred to the last trading days are those where the gross basis remain positive during the whole delivery month. We notice however that the majority of early deliveries are recommended during the period from September 2005 to June 2008, a period encompassing the financial crisis. It is interesting to note that the strategy recommended by the dynamic program is in line with the numerical illustrations of Kane and Marcus (1986), in which they compute the critical bond price increase or decrease that will initiate the contract settlement. Depending on the number of trading days left in the month as well as the conversion factor, the authors find that a change of 0.1 to 1.5 in the bond spot price will trigger delivery. Indeed, for many

Figure 4.6 – Comparison of actual and optimal delivery days. Numbers in the table indicate the percentage of actual deliveries. Shaded cells correspond to the delivery day obtained by dynamic programming.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	-7	-6	-5	-4	-3	-2	-1
Mar-05												0.0	0.0	0.0						0.1	0.0		99.9
Jun-05																			0.1				99.9
Sep-05												0.0	0.0						0.0		0.0		99.8
Dec-05	2.8	0.0																					97.2
Mar-06																							100
Jun-06																							100
Sep-06																			0.1		0.1		99.9
Dec-06		14.5																	0.0	1.5		50.2	33.8
Mar-07												7.1					40.6		0.0			0.2	52.1
Jun-07										20.6	0.7	1.7	0.8										76.3
Sep-07																	86.5	4.5	0.1	2.0	1.1	1.4	4.3
Dec-07																			3.9				96.1
Mar-08																				0.2			99.8
Jun-08																							100
Sep-08						8.4											59.6						32.1
Dec-08																				0.2			99.8
Mar-09																						47.7	52.3
Jun-09																							100
Sep-09										0.1	0.1	0.1	0.1				0.1						99.7
Dec-09																				0.6			99.4
Mar-10																				0.0			100
Jun-10																			0.2		0.1		99.7
Sep-10																							100
Dec-10																							100
Mar-11																			0.5				99.5
Jun-11																							100
Sep-11																							100
Dec-11																							100
Mar-12																							100
Jun-12																						16.9	83.1
Sep-12																				0.0			100
Dec-12																							100
Mar-13																						100	
Jun-13																		0.3		50.6			49.0
Sep-13																							100
Dec-13																			9.8		2.7		87.5
Mar-14																			8.0				92.0
Jun-14																							100
Sep-14																							100
Dec-14																					0.1		99.9
Mar-15																							100
Jun-15																							100
Sep-15														25.3									74.7
Dec-15																							100

Figure 4.7 – Gross basis of the CTD on days where delivery is optimal according to the Vasicek-EJ model, from 2005 to 2015.

contracts, the DP algorithm suggests delivery after a sizable change in the CTD price or in its gross basis.

4.4 Comparison of the two delivery strategies

We now compare the outcome of the delivery strategy obtained by the dynamic programming approach to those actually obtained by the traders. Figure 4.7 shows the gross basis (the exercise loss) corresponding to the optimal delivery strategy during the period 2005-2015.

Comparison of Figures 4.4 and 4.7 show that the distribution of exercise gains or losses is quite different in terms of dispersion, not only in values, but also in chronology. We observe that the profits implied by the DP strategy are less volatile than those realized by the traders, who are clearly more agressive, resulting in either a severe loss or a high profit.

This observation is confirmed by the descriptive statistics in Table 4.1 below; over the period 2005-2015, the average profits of the two strategies are very close and not statistically different but with a higher standard deviation for the strategy followed by the traders (1.88 against 1.15) and interquartile range (2.65 against 0.21).

For instance, in September 2009, the realized profit obtained by exercising on the delivery day recommended by the DP approach is 52$ per contract against a loss of 206$ for traders who delivered in the last business day (99.7% of the contracts). In June 2013, the algorithm suggests delivering on day 19 to realize a profit of 72$; those who waited until the end (50% of the contracts) realised a profit of 297$; the others delivered on the 25 and realized a profit of 466$. In September 2014, the DP suggests delivering on day 24, incurring a delivery loss of 56$; all the contracts were settled on the last trading day, at a more severe loss of 176$. Finally, if we consider a short trader who held a single position in all the contracts from 2005 to 2015, we find that his cumulative loss would have been 1265$ by always delivering on the last trading day, against a loss of 1164$ by using the optimal delivery strategy.

Table 4.1 – Descriptive statistics of the gross basis under the two delivery strategies

	Mean	**Std**	**Min**	**P25**	**P50**	**P75**	**Max**
DP delivery day GB	0.26	1.15	-1.42	-0.15	-0.05	0.05	4.11
Realized GB (average)	0.23	1.88	-3.83	-1.21	0.38	1.44	4.11

Finally, to better understand, the difference between the two delivery strategies and assess whether it is related to interest-rate jumps, we evaluate again the dynamic program under the pure diffusion Vasicek model. We only find six contracts with a different delivery date compared to what is obtained under the jump-diffusion model. In addition, we verify that the delivery dates obtained with the dynamic program are not related to the major macro economic announcements such us FOMC meetings, unemployment rate, non-farm payrolls, etc. These results confirm that the difference between the two delivery strategies is mainly due to the timing effect and that traders rely on the simple rule of thumb to take their decision.

4.5 Conclusion

The timing of the delivery for T-bond futures contract is a complex decision due to the existence of multiple interacting embedded options. The rule of thumb used by the traders generally implies delivery in the last two business days of the trading month. In this chapter, we perform an ex-post evaluation of the observed delivery behavior and compare it to what is implied by the dynamic programming approach described in Chapter 2. Even though the profits realised under the two approaches are not statistically different, we find that the traders took a higher level of risk by being exposed to the volatility of the basis as opposed to delivering earlier as suggested by DP, in other words, traders could have realized the same average profit by being more prudent.

It is important to point out that the strategy computed by dynamic programming is optimal for the interest-rate model used to represent uncertainty. The optimal strategy discussed in this chapter was obtained using the Vasicek-EJ model, a specification where jumps in the interest rate are likely to happen. Under this specification, it is not surprising that the strategy suggested by the dynamic program is relatively prudent, often avoiding the end of the month period, where jumps in the interest rate would not be reflected in the futures price, when profits can be made during the switch period.

As an extension to this work, it would be interesting to use intraday data and evaluate again the exercise strategy using the dynamic programming approach by taking into account intraday prices of the cheapeast-to-deliver between 2 p.m. and 8 p.m. (i.e. considering the impact of the wild card option).

Chapter 5

The impact of central clearing on the market for single-name credit default swaps

5.1 Introduction

Credit default swaps (CDS) are insurance contracts that act against the default of the issuer of an underlying bond. They were first introduced by J.P. Morgan in 1994 to meet the need for an instrument to manage and transfer credit risk. These contracts can also be used for speculation purposes, in order to benefit from a change in the credit quality of a particular reference entity. When they were first introduced, CDS were solely exchanged in the *over-the-counter* (OTC) market, until they were heavily criticized for their lack of transparency and for the role they consequently played in the 2007 financial turmoil (see Acharya, Philippon, Richardson & Roubini 2009 and Acharya, Engle, Figlewski, Lynch & Subrahmanyam 2009). In the aftermath of the 2007–2008 global financial crisis, the large size of the CDS market, as well as the amount of inherent risk associated with it, made market participants more cautious about their existing positions and pushed regulators to step in and announce reforms that mainly consisted of standardizing the CDS market and introducing central clearing.

After the introduction of the Dodd-Frank Wall Street Reform and the Consumer Protection Act, central clearing became an alternative for single-name CDS. By the end of 2009, clearing operations began in North America and Europe, conducted by the Intercontinental Exchange Clear Credit (ICECC). By stepping in as the buyer for every seller and the seller for every buyer, the clearinghouse plays the role of a counterparty to both traders. The introduction of central clearing was meant to reduce the counterparty risk of cleared contracts: while the default probability of the reference entity is normally not affected by the move to central clearing, the protection of the CDS holder should be enhanced, as long as the clearinghouse itself is well protected against default (see Acharya, Engle, Figlewski, Lynch, and Subrahmanyam 2009). Central clearing may also boost trading activity and attract new players to the market. However, to guarantee a good protection against default, the clearinghouse requires that its clients post daily margins in the form of cash or highly liquid assets in addition to paying administrative fees.

This paper is part of the ongoing research on the impact of introducing a central counterparty (CCP) that stands between buyers and sellers of default protection in the CDS market. In a generalized difference-in-difference setting, we revisit this impact on spreads, liquidity, and trading activity by considering CDS contracts written on North American reference entities over the 2009–2015 period. We also analyze this impact on the default risk of the underlying bonds during the same period. Our contribution is twofold. First, the propensity score matching approach that we propose addresses the endogeneity problem originating from the voluntary choice of adhering to central clearing. This matching approach also allows to account for a variety of treatment dates, as observed in the data set. We then show that the choice of a matching approach plays a significant role in the evaluation of the impact of the introduction of a CCP, and that differences between our results and those obtained in the previous literature are mainly explained by differences in methodology. Second, we find evidence that CDS spreads increase once a reference entity becomes centrally cleared. We show that this spread increase does not pertain to an improvement in CDS liquidity or trading activity, nor is the default risk of the underlying bond affected by CDS central clearing. We therefore argue that this increase in the CDS spreads provides an assessment of the magnitude of counterparty risk in the non-cleared CDS market.

The empirical literature on the impact of central clearing on the CDS market is still scarce. The papers focusing on this topic employ various methodologies and data sets, and reach different conclusions about the implications of introducing clearinghouses into the CDS market. Slive, Witmer, and Woodman (2012) use an event study and find that the new clearing mechanism slightly increases CDS liquidity. They argue that this improvement is the result of two opposite effects: an increase in collateral requirements, generating higher clearing costs, and an increase in transparency and operational facilities, leading to better competition and a more liquid market. They also find an improvement in trading activity as measured by gross notional amounts. Kaya (2017), using panel regression in a sample of non-financial firms, reports an increase in CDS spreads after central clearing. He argues that this rise is not the result of a reduction in counterparty risk, but is rather due to an increase in clearing costs that is passed on to end-users. Du, Gadgil, Gordy, and Vega (2018) investigate the impact of counterparty credit risk on the pricing of CDS using confidential data from the Depository Trust and Clearing Corporation (DTCC). They analyze the effects of the introduction of central clearing on CDS prices using a panel regression of cross-sectional variations of CDS spreads. Their findings show that counterparty risk has negligible effects on CDS spreads and that centrally-cleared trades have significantly lower spreads than uncleared interdealer trades. They argue that this latter result could be attributed to the impact of a more transparent centrally-cleared market on the competitive structure. They also conclude that this finding is consistent with market participants' managing counterparty risk, which would result in a modest impact of counterparty risk on the pricing of CDS contracts.

Loon and Zhong (2014) were the first to investigate the impact of central clearing on CDS spreads, as well as on liquidity and trading activities in the CDS market. They use an event study methodology and find that the spreads widen around the initiation of central clearing. This change is explained by a reduction in CDS counterparty risk and, to a lesser extent, by an improvement in CDS liquidity. They then combine a DID approach with a static propensity-score matching to provide evidence of an improvement in CDS liquidity as well as in trading activity.

While the framework of our paper is close to theirs, our methodology, scope, and findings are different. We focus on the changes in CDS spreads using the DID methodology. Our approach aims at eliminating the selection bias by proposing two improvements. First, we improve the matching technique, relying on firm data just prior to the move to central clearing, instead of using a fixed estimation period to match all the firms. Second, we estimate a generalized DID model including time and firm fixed effects. As in Loon and Zhong (2014), we obtain an increase in CDS spreads. The main difference between our results and theirs is that we do not find any significant impact of the move to central clearing on CDS liquidity, nor on trading activity. We show that this difference in results can be attributed to the improvements in the methodology. Finally, we also consider the effect of central clearing on bond default spread and find no effect.

Table 5.1 summarizes the main features of the literature dealing with the impact of central clearing on the CDS market, highlighting the differences in data sets, methodologies, and empirical results.

Table 5.1 – Impact of Central Clearing

This table summarizes the different empirical results about the impact of central clearing on the CDS market. Cells indicate the findings of each paper, the source and the range of data, and the employed methodology. Static PS matching consists of computing propensity scores during a fixed period whereas dynamic PS uses the period prior to each firm's clearing date.

	Slive et al. (2012)	Loon and Zhong (2014)	Kaya (2017)	Du et al. (2018)	This work
Period	11/2008 - 07/2011	01/2009 - 12/2011	01/2009 - 06/2013	01/2010 - 12/2013	01/2009 - 12/2015
CDS Data	DTCC, Markit, Bloomberg, ICECC	DTCC, Markit, CMA, ICECC	Bloomberg, ICECC	DTCC, Markit	DTCC, Markit, Bloomberg, CMA, ICECC
Number of cleared firms (market)	113 + 102 (North America & Europe)	132 (North America)	85 (North America)	142 (North America)	198 (North America)
CDS spreads (methodology)		Increase (Event study)	Increase (Panel regression)	Decrease (Panel regression)	Increase (Generalized DID + Dynamic PS matching)
Liquidity (methodology)	Slight Improvement (Event study + Static PS matching)	Improvement (Standard DID + Static PS matching)			No change (Generalized DID + Dynamic PS matching)
Trading activity (methodology)	Increase (Event study + Static PS matching)	Increase (Standard DID + Static PS matching)			No change (Generalized DID + Dynamic PS matching)
Bonds default risk (methodology)					No change (Generalized DID + Dynamic PS matching)
Counterparty risk (methodology)		Reduction (Panel regression)	No change (Panel regression)		Reduction (by elimination)
Costs (methodology)			Increase (Panel regression)		

The remainder of this chapter is organized as follows. Section 5.2 presents an overview of the CDS market and its regulatory reforms. Section 5.3 presents the framework and methodology applied in this paper. Section 5.4 is a description of the data. Section 5.5 reports our empirical results about the impacts of the introduction of central clearing on the CDS market and discusses other potential factors that may have affected the CDS spread. Section 5.6 is a conclusion.

5.2 The CDS market

5.2.1 CDS prices and their determinants

In a credit default swap contract, the buyer agrees to make regular payments, known as the *premium leg* of the contract, until the earliest between the contract maturity or the default event. The seller makes one contingent payment, known as the *protection leg*, when the default event occurs. This payment is considered as a compensation for the protection buyer's net loss. The most common methodology for pricing CDS contracts is to use a reduced-form setting and compute the fair spread, obtained by equalizing the values of the premium and protection legs, discounted at the inception date. As an illustration (see, e.g., Longstaff, Mithal, and Neis 2005), consider stochastic and independent interest-rate and default-intensity processes, denoted respectively by r_t and ξ_t. Given a bond with a unit par value, assume that the buyer pays a continuous premium c and receives an amount l upon default (l is the so-called *loss given default* of the bond). The present value of the premium leg can be expressed as follows:

$$cE\left[\int_0^T exp\left(-\int_0^t (r_u + \xi_u)\, du\right)\, dt\right], \tag{5.1}$$

where T is the maturity of the contract and t is the default date of the underlying bond. Similarly, the present value of the protection leg can be expressed as

$$lE\left[\int_0^T \xi_t exp\left(-\int_0^t (r_u + \xi_u)\, du\right)\, dt\right]. \tag{5.2}$$

The equilibrium premium c is obtained by equalizing (5.1) and (5.2):

$$c = l\frac{E\left[\int_0^T \xi_t exp\left(-\int_0^t (r_u + \xi_u)\, du\right)\, dt\right]}{E\left[\int_0^T exp\left(-\int_0^t (r_u + \xi_u)\, du\right)\, dt\right]}. \tag{5.3}$$

Formula (5.3) is obtained under the assumption that the price of the contract is not affected by liquidity, trading activity, or counterparty risk. Longstaff et al. (2005) mention that the premium s should be lower if the protection seller might not be able to honor its contractual obligations. The authors also argue that CDS spreads are less sensitive to liquidity risk than are corporate bonds because of their contractual nature, and they hence consider the spread to be a pure measure of default risk. This assumption was challenged after the 2007 financial crisis.

Recent papers provide empirical evidence that CDS spreads contain a non-negligible liquidity premium. Tang and Yan (2007) document that this premium is on average 13.2 basis points (bps). Buhler and Trapp (2009), relying on a reduced-form approach that includes a liquidity discount factor, find that the liquidity premium accounts for 5% of the mid quotes. Junge and Trolle (2015) develop an asset pricing model to extract liquidity from CDS data, and estimate that liquidity risk represents about 24% of CDS spreads. Many other papers, using various methodologies, confirm the existence of a liquidity premium in non-centrally-cleared markets (see, for instance, Chen, Fabozzi, and Sverdlove 2010; Bongaerts, Jong, and Driessen 2011; Qiu and Yu 2012; Lesplingart, Majois, and Petitjean 2012; Kuate Kamga and Wilde 2013; and Pires, Pereira, and Martins 2015). Since the premium varies cross-sectionally and over time, it is not straightforward to provide a general estimation for this component. In addition, numerous liquidity measures can be used, which may lead to different estimates. Nonetheless, our concern in this paper is not to measure how liquidity affects CDS spreads but rather to evaluate the relative magnitude of a potential liquidity premium between cleared and non-cleared markets.

On the other hand, trading-activity measures can disclose additional trading information that is not necessarily contained in liquidity measures. In fact, Kyaw and Hillier (2011) find that the relation between trading activity and liquidity is not always positive. They show that an increase in trading activity is associated with an improvement in liquidity for large stock portfolios, but with a reduction in liquidity for small stock portfolios. Moreover, Silva (2015) argues that the informational content of open-interest variables can be used as a predictor of CDS spread changes, by showing that open-interest measures contain private information that precedes CDS price movements. Hence, it is important to account for CDS trading-activity variables, since they may be used as an additional predictor of spreads.

Finally, the debate about the contribution of counterparty risk in the price of credit protection is still open, due to the difficulty of obtaining data that identifies the protection seller. Jarrow and Yu (2001) and Hull and White (2001) develop theoretical models that account for a possible correlation between the default of the reference entity and that of the seller of the credit protection (i.e., the so-called *wrong-way risk*), and show that CDS spreads decrease when this correlation increases. In their numerical illustrations, Hull and White (2001) find that an improvement in the credit rating of a protection seller, from BBB to AAA, increases CDS spreads by 5 to 36.1 bps, depending on the default correlation reflecting the counterparty risk in the CDS valuation. Empirically, Arora, Gandhi, and Longstaff (2012) document that the relation between the dealer's credit risk and the CDS spreads is statistically significant but economically very small. Specifically, they estimate that an increase of 645 bps in the dealer's credit risk results in a decrease of only 1 basis point in the price of protection. These results are supported by the analysis of Du et al. (2018), who also rely on panel regressions and argue that market participants manage counterparty risk by selecting dealers with a low credit risk. They estimate that a 100 bps increase in the dealer's credit spread reduces the CDS spread by about 0.6 bps.

Counterparty risk can also be analyzed from a different perspective, by quantifying the *Credit Value Adjustment* (CVA), which is defined as the difference between the value of a counterparty-risk-free portfolio and that of a comparable portfolio subject to counterparty risk. The CVA, an adjustment made to compensate one party for the other's default risk, also represents the market value of the counterparty risk. Brigo and Chourdakis (2009) evaluate the CVA of CDS contracts, taking into account default correlation and credit spread volatility. In their illustrations, the CVA of CDS contracts ranges from zero to 91 bps when the correlation is very strong. In the case of a moderate correlation of 20%, the CVA ranges between 15 and 25 bps, depending on the credit spread volatilities of the reference entity and of the counterparty. These estimates are in line with those of Gregory (2011), who finds a range of zero to 48 bps, where the CVA increases with the level of correlation. Brigo, Capponi, and Pallavicini (2014) evaluate the counterparty risk of collateralized agreements. They find that the CVA is an increasing function of the default correlation, ranging from 10 to 60 bps, with a maximum of 20 bps for a moderate correlation of 20%.

5.2.2 The principles of central clearing

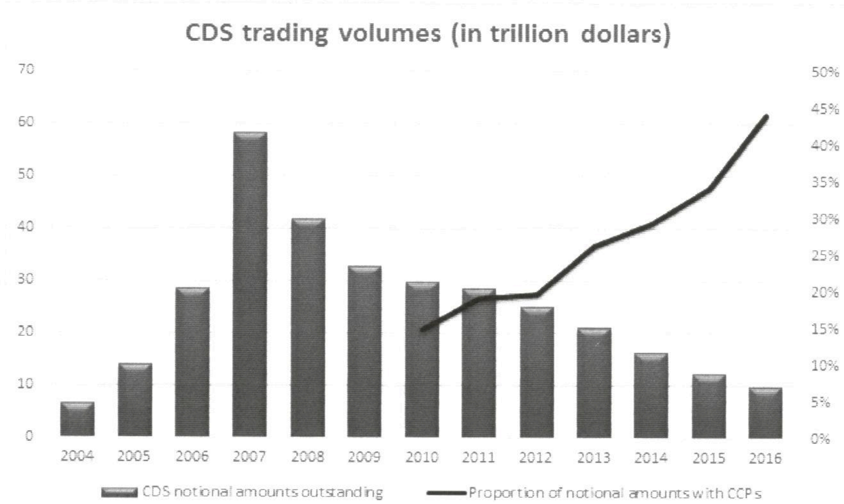
In recent years, CDS contracts have become very attractive tools to hedge a credit exposure or take a speculative position without having to purchase the underlying reference bond. The market grew dramatically after the beginning of the 2000s, reaching a peak in 2007, and then gradually declined afterwards. Figure 5.1 reports on the total notional amount outstanding in the CDS market, growing from \$6.4 trillion in 2004 to \$58.2 trillion in 2007, and dropping to \$9.9 trillion by the end of 2016[1].

Because of the large size of their market and because of their interconnectedness with other derivatives, CDS play an important role in the stability of the financial system; hence, the importance of monitoring the risks associated with CDS trading, and more specifically, counterparty risk. Following the 2007 financial crisis, regulatory authorities took new measures to control counterparty risk and increase market

[1]Source: Bank for International Settlements (BIS).

Figure 5.1 – CDS Trading Volumes.

This figure plots the notional amounts outstanding in trillion dollars for single-name CDS contracts (left axis) as well as the proportion of notional amounts cleared by central counterparties (right axis). The data is obtained from the Bank for International Settlements.



transparency. The most important regulatory change for CDS trades was the introduction of central clearing, as recommended by the Dodd-Frank Act in 2009. A clearinghouse acts as an intermediary between seller and buyer, and its main role is to mitigate counterparty risk. Once a trade is cleared, each party is unaffected by any default by the other. If a market participant defaults, the CCP honors its exposures and shares the losses with the other CCP members. The remaining counterparty risk is limited to the default of the CCP itself, which is highly unlikely, given the strong risk-management procedures it applies[2].

ICECC is the market leader in Europe and North America for clearing CDS trades. It started clearing CDS indices in March 2009 and single-name CDS in December 2009. Other clearinghouses, such as LCH Clearnet and CME, offer similar services but their market share is still small compared to that of ICECC. At present, the clearing of most CDS indices is mandatory, whilst that of single-name

[2]We refer the reader to Gregory (2014) for a detailed discussion of the structure and mechanics of clearinghouses.

CDS remains on a voluntary basis. The new system has become increasingly popular since its inception, and a growing number of reference entities have adhered to it. Investors are also increasingly aware of the benefits of trading through a clearinghouse. According to BIS data, the proportion of notional amount outstanding with CCPs increased from around 15% in 2010 to 44% in 2016 (see Figure 5.1).

The viability of a CCP is measured by its ability to absorb the losses caused by the default of one or more of its members. This is generally achieved by imposing strict collateral requirements in the form of margins or contributions to specific funds. Additionally, clearinghouses rely on a waterfall approach with several layers of protection, to be able to respond to extreme events. The first layer consists of the membership criteria. To become a cleared member, an entity must meet certain requirements of financial stability and operational capabilities. The second protection layer consists of margin requirements. Members must make an upfront payment, known as the *initial margin*, which may be used to close out the positions of a defaulting member without losses. Daily adjustments to this amount, or *variation margins*, are made to mark-to-market losses or gains. Intra-day margin calls can also be made in case of a large price movement. Under extreme market scenarios, clearinghouses rely on a third layer of protection, known as the *guaranty fund*. Members contribute to this fund by posting additional amounts of collateral, which help in mutualizing losses if the two first layers are insufficient. The CCP holds the assessment rights and may ask its members for additional contributions to the guaranty fund. All of the aforementioned measures are supposed to guarantee sufficient financial resources to bring confidence to the market and reduce the counterparty risk associated with bilateral trades.

5.3 Methodology

In order to study the impact of central clearing, we compare the spreads of single-name CDS contracts in two groups of firms, namely, cleared reference entities that are members of the clearinghouse and non-cleared reference entities; this comparison is undertaken before and after adhesion to the CCP, in a DID framework.

The DID methodology has been widely used in various application areas to evaluate the impact of an exogenous event or of a policy change. The classical two-by-two design uses data from a treatment group and from a control group, measured at two different dates: before treatment and after treatment. This methodology is flexible and can be generalized to the case of multiple groups and multiple time periods (see, e.g., Bertrand, Duflo, and Mullainathan 2004; Imbens and Wooldridge 2009; and Gormley and Matsa 2011). In our case, since we are dealing with multiple treatment (clearing) dates, we opt for a generalized DID framework with firm and time fixed effects.

Since its introduction in 2009, central clearing for single-name CDS has been conducted on a voluntary basis. Note that when subjects can choose to take the treatment or not, the two groups are more likely to differ and, therefore, estimates may be biased due to this endogeneity issue.[3]

Moreover, not all reference entities are eligible to become clearinghouse members; firms must meet some capital requirements and show sufficient financial strength in order to be accepted for central clearing.

To alleviate these endogeneity and heterogeneity concerns, we rely on propensity-score matching (see Rosenbaum and Rubin 1983; Heckman, Ichimura, and Todd 1997; and Dehejia and Wahba 2002) to construct treatment and control groups that have similar pre-clearing characteristics, before applying a generalized DID approach.

The combination of these two methodologies has been used in many fields, including finance (Greenaway and Kneller 2008; Lemmon and Roberts 2010; Hofmann 2013; Bandick, Gorg, and Karpaty 2014; Sari and Osman 2015; and Amiram, Beaver, Landsman, and Zhao 2017), but has not yet been applied to analyze the impact of

[3]We refer to Li and Prabhala (2005) and Roberts and Whited (2012) for a detailed discussion on this subject.

central clearing on CDS spreads.

5.3.1 Generalized DID with dynamic matching

To apply DID, we need to compose a treatment and a control group containing firms that have similar characteristics just before the treatment event. The first step consists of constructing a sample of candidate treatment and control entities, and computing their propensity scores on the basis of pre-clearing characteristics. Specifically, we consider the 29 clearing dates enumerated in Table 5.2 as the various possible times for adhering to a CCP. These treatment dates can be interpreted as hypothetical events for the control group. Each non-cleared firm thus generates up to 29 firm-event entities. The sample also contains the cleared firms, paired with the event corresponding to their clearing date.

We then estimate the following Probit model, using the sample of cleared and non-cleared firm-event entities and the corresponding observable variables that are relevant to clearinghouses:

$$Pr(Y = 1|X) = \Phi(X \cdot \beta), \tag{5.4}$$

where Y is a binary random variable that equals 1 if the firm is centrally cleared and 0 otherwise, Φ is the standard normal cumulative distribution function, X is the vector of regressors that influence the outcome Y, $\cdot$ is the inner product operator, and β is a vector of parameters. The vector β is obtained by maximum likelihood and is used to estimate the probability, for each firm-event entity, of being accepted for central clearing. This probability is the *propensity score* associated to a combination of a firm and a possible clearing date. We estimate the regressors by averaging them over a window of $[-8, -2]$ months before the relevant event, where the two months immediately prior to the clearing date are excluded so that the data does not contain any market anticipation. The propensity score of a given control firm-event entity thus indicates the joint probability of the firm being selected for central clearing and deciding to adhere to a CCP at the corresponding clearing date.

Table 5.2 – Clearing Dates.

This table presents the clearing dates and the number of cleared entities per date for North-American firms cleared from 2009 to 2015. This information is obtained from ICECC.

Clearing date	Number of cleared entities
21-Dec-09	2
11-Jan-10	3
01-Feb-10	2
15-Feb-10	14
08-Mar-10	9
29-Mar-10	15
19-Apr-10	8
10-May-10	12
07-Jun-10	1
06-Jul-10	1
09-Aug-10	7
30-Aug-10	8
28-Mar-11	9
11-Apr-11	8
02-May-11	7
13-Jun-11	9
14-Nov-11	3
09-Oct-12	5
22-Oct-12	6
05-Nov-12	8
19-Nov-12	1
30-Sep-13	7
23-Jun-14	9
07-Jul-14	9
21-Jul-14	11
04-Aug-14	12
20-Jul-15	9
03-Aug-15	10
17-Aug-15	7

The second step consists of matching cleared and non-cleared entities on the basis of the propensity scores. We match with replacement each cleared firm with its closest neighbor from the group of non-cleared firm-event entities. Our final sample is then composed of matched firm-event entities. A detailed example of the matching procedure is provided in Appendix A.

We then apply generalized DID regression to the matched sample in order to test for the presence of statistically significant impact factors. Using a generalized DID framework allows us to account for the different treatment times of CDS contracts. More specifically, to isolate the effect of central clearing on a given factor, we estimate the following DID equation:

$$Factor_{i,t} = \beta_0 + \beta_1 Cleared_{i,t} + \gamma_{1i} + \gamma_{2t} + \epsilon_{i,t}, \tag{5.5}$$

where subscript i denotes a firm-event entity and subscript t denotes a date in the event window. The dependent variable $Factor_{i,t}$ will take various definitions in order to investigate the impact of central clearing on CDS spreads, liquidity, trading activity, as well as on bond default spreads. The main explanatory variable $Cleared_{i,t}$ is a binary variable that indicates whether the reference entity i is centrally cleared or not on date t. This variable is the equivalent of the interaction term in the classic two-by-two DID design. The treatment effect is given by the corresponding coefficient β_1. The fixed effects of the generalized DID setting help control for unobserved heterogeneity across time and reference entities, thereby alleviating concerns about any omitted variables that might affect both groups in the same way. The firm fixed effect, γ_{1i}, captures differences across firms that are constant over time and replaces the dummy variable that indicates whether the firm is belongs to the treatment group or not under the classic DID design. The time fixed effect, γ_{2t}, captures differences over time that are common to all firms and replaces the dummy variable that indicates whether we are in the post-event period or not in the classic DID design. We deliberately do not control for specific time-varying variables to avoid confounding estimates of β_1, since these variables might also be affected by the move to central clearing. In all our regressions, the standard errors are clustered by firm.

5.3.2 Standard DID with static matching

In order to assess the robustness of our findings to the choice of methodology, we apply to our sample the methodology used in Loon and Zhong (2014) to analyze liquidity and trading activity. This methodology consists of using data from a fixed period (a window prior to December 2009) to estimate the Probit model (5.4) and then matching with replacement each cleared entity with the five noncleared entities that have the closest propensity score. The difference-in-difference is then evaluated by comparing, around the treatment date, the change in the relevant factor of a cleared firm with the average change in the corresponding matched firms. We call this procedure *static matching*; under this matching procedure, a firm cleared in 2011, for example, is matched with a control firm that had similar characteristics to it in 2009.

We argue that the period over which the independent variables are measured is important for the performance of the matching operation and the elimination of the selection bias. Clearly, a firm's financial situation can change considerably over time, making a good match in 2009 no longer valid two years later. The dynamic matching procedure outlined above matches cleared firms with firms that are similar to them at the moment of their decision to adhere to central clearing. Our experiments indicate that the differences between our findings and those of Loon and Zhong (2014) can be mainly explained by the difference in methodology.

5.4 Data

We use seven years of CDS data on North American firms, observed from January 2009 to December 2015, and compiled from different sources. From Markit, we obtain daily CDS spreads of five-year senior unsecured contracts denominated in USD (*CDS_Spr*). We follow the market convention for North American contracts since April 8, 2009, and focus on contracts with a no-restructuring clause (XR). We delete observations with a missing five-year spread and keep only reference entities with at least 20 observations. We also obtain from Markit the Composite Depth (*Comp_Dep*), which is the number of contributors whose CDS spreads have been used to calculate the five-year CDS spread.

5.4.1 Liquidity data

Our liquidity measures are mainly collected from Markit Liquidity. This database contains data that starts in April 2010 and was updated in November 2011 to include new variables. Specifically, we obtain bid-ask spreads from Markit Liquidity, and supplement the missing pre–April 2010 information from CMA and, where necessary, from Bloomberg to obtain a larger coverage. We then construct the Relative Quoted Spread (*RQS*), computed as the bid-ask spread divided by the spread midpoint. In addition, we rely on other liquidity measures from Markit Liquidity, depending on data availability. From April 2010 to December 2015, we use the Upfront five-year bid-ask spread (*Upf_BA*) and the Markit liquidity score (*Liq_sc*), defined on a scale from 1 to 5, where 1 indicates the highest liquidity. During this period, we also have the Quotes count (*Quotes*) and the Dealers count (*Dealers*), defined as the total number of unique quotes for a reference entity and the total number of distinct dealers quoting the reference entity across all available tenors, respectively. From November 2011 to December 2015, we have more detailed information about the quotes and dealers count. We obtain the Five-year quotes count (*5Y_Quotes*) and the Five-year dealers count (*5Y_Dealers*), defined respectively as the total number of unique quotes for a reference entity and the total number of distinct dealers quoting the reference entity for the five-year tenor. Data about the remaining tenors is given by the variables Non five-year quotes count (*Non_5Y_Quotes*) and Non five-year dealers count (*Non_5Y_Dealers*).

5.4.2 Trading activity data

The data on trading activity is obtained from the Depository Trust and Clearing Corporation, which covers approximately 98% of all credit derivative transactions in the global marketplace. The first available report is for the week that ended on October 31, 2008.

For each entity, we have weekly information on the gross and net notional amounts outstanding, i.e., the par amount of credit protection that is bought or sold, as well as the number of contracts outstanding. The gross notional amount includes all the contracts on a given firm, and thus increases with every trade, even if a new position offsets another. On the other hand, the net notional amount, which accounts for offsetting trades, indicates the actual amount insured by CDS contracts.

DTCC also discloses weekly data about market risk transfer activity in terms of gross notional value and number of contracts. This activity captures transaction types that result in a change in the market risk position of market participants, such as new trades, the termination of an existing transaction, and the assignment of an existing transaction to a third party. These measures exclude moving bilateral trades to CCPs, portfolio compression, and back-loaded trades, since all these trades do not change the risk profile. The market risk transfer activity data is available on a weekly basis, starting from the week that ended on July 16, 2010.

We end up with the five following variables defining the CDS trading activity: Gross notional amounts (*Gross_Not*), Net notional amounts (*Net_Not*), Contracts (*Contr*), Gross notional–Risk transfer (*Gross_Not_Risk*), and Contracts–Risk transfer (*Contr_Risk*).

5.4.3 Central clearing data

We identify the name of the entities that were centrally cleared as well as the corresponding clearing date by using the official list on the ICECC website and the regularly published circulars announcing the single-name CDS that are going to be cleared. We also check whether the entity has gone through any type of restructuring event that might affect its CDS spread. In such cases, the entity is excluded from the list, since we want to focus exclusively on the impact of central clearing. Reference entities that have experienced a merger or were acquired by another company are also eliminated. We keep entities that had a renaming event since this is unlikely to affect the spreads. We finally merge DTCC with Markit by name and then identify centrally-cleared entities with the Markit redcode. After this filtering and merging process, we obtain a total of 607 reference entities, of which 198 are centrally cleared. Our sample for the Probit estimation contains 7,102 firm-events. The final matched sample for the DID consists of the 198 cleared firms and their corresponding non-cleared firm-events.[4]

5.4.4 Bond data

To analyze the impact of central clearing on the default probability in the underlying bond market, we need to construct a bond default spread measure, since default spread is not observed in the market. To do so, we implement the J.P. Morgan Par Equivalent CDS Spread (PECS) methodology.

The data is mainly obtained from TRACE, which provides information about the prices, and FISD, which contains the different characteristics of the bonds. We keep only straight and redeemable bonds in FISD and we apply the Dick-Nielsen (2009) filter to TRACE data before merging the two datasets in order to eliminate reporting errors. Our objective is to have a unique bond for each issuer and therefore

[4]Because the match is made with replacement and because a firm can be matched at various event dates, the total number of control firms in the final sample is 100.

we choose, among bonds with maturities between three to ten years, the bond with the maturity closest to five years. We complement this dataset with the bond ratings obtained from S&P to be able to classify bonds into investment grade and high yield. To avoid losing observations, we replace any missing information with Moody's rating.

Since most of the bonds in the data set are callable, we need to apply a correction to the maturity in the computation of the PECS. For investment-grade bonds, we keep the original maturity since the bonds are not likely to be called. For callable high-yield bonds, we compute a new maturity based on the Yield-To-Worst (YTW), defined as the minimum between the Yield-To-Call, computed for each possible call date, and the Yield-To-Maturity (YTM), assuming no prior default. If one or more call dates have passed and the bond has not yet been called, then the calculation of the YTW is based on all the remaining call dates. This adjusted maturity reflects the worst scenario for a bondholder.

We then compute the daily default spread measure of the bond associated to each CDS contract using the following steps:

- Bootstrap default probabilities from the associated CDS market quotes.
- Compute the present value of the bond, using the implied default probabilities.
- Apply a parallel shift to the default probability curve so that the computed present value matches the bond's market price. The shift is obtained by solving a minimization problem.
- Compute survival probabilities using these implied probabilities and use the traditional CDS pricing equation (5.3) to compute an implied CDS spread. This spread is the variable $PECS$, which is a measure of bond default risk.

5.4.5 Variables

Table 5.3 provides the list of all the variables used in our analysis, along with the expected sign of the interaction term in the DID regression. Clearinghouses are expected to reduce counterparty risk, and boost liquidity and trading activity. Therefore, CDS contracts in the treatment group are anticipated to have higher spreads, liquidity and trading activity following central clearing, as compared to the control group. This translates into a positive sign for the coefficient β_1 in Equation (5.5) for the case of CDS spread, liquidity, and trading-activity variables and a negative sign for this coefficient in the case of illiquidity variables.

Table 5.3 – List of Variables.

This table presents the list of all the variables, their definitions and the prediction for the interaction term in the DID regression. The variables are obtained from Markit, Markit liquidity, CMA, Bloomberg, DTCC, TRACE, FISD, S&P and Moody's.

Variable	Symbol	Definition	Prediction for the interaction term
Five-year CDS spread	*CDS_Spr*	Composite spread for the five-year tenor	Positive
Composite depth	*Comp_Dep*	Number of contributors whose CDS spreads have been used to calculate the five-year CDS spread	Positive
Relative quoted spread	*RQS*	The bid-ask spread divided by the spread midpoint for the five-year tenor	Negative
Upfront five-year bid-ask spread	*Upf_BA*	Bid-ask spread in upfront points for the five-year tenor	Negative
Dealers count	*Dealers*	Total number of distinct dealers quoting the reference entity across all available tenors	Positive
Quotes count	*Quotes*	Total number of unique quotes for a reference entity, all tenors combined	Positive
Liquidity score	*Liq_sc*	Defined on a scale from 1 to 5 where 1 indicates the highest liquidity	Negative
Five-year dealers count	*5Y_Dealers*	Total number of distinct dealers quoting the reference entity for the five-year tenor	Positive
Five-year quotes count	*5Y_Quotes*	Total number of unique quotes for a reference entity for the five-year tenor	Positive
Non five-year dealers count	*Non_5Y_Dealers*	Total number of distinct dealers quoting the reference entity for the non five-year tenors	Positive
Non five-year quotes count	Non_5Y_Quotes	Total number of unique quotes for a reference entity for the non five-year tenors	Positive
Gross notional amounts	*Gross_Not*	Sum of all notional CDS contracts bought (or equivalently sold) for each reference entity	Positive
Net notional amounts	*Net_Not*	Sum of the net protection bought by net buyers (or equivalently sold by net sellers)	Positive
Contracts	*Contr*	Number of contracts outstanding for each CDS contract	Positive
Gross notional - Risk transfer	*Gross_Not_Risk*	Sum of all notional CDS contracts for transaction types that result in a change in the market risk position	Positive
Contracts - Risk transfer	*Contr_Risk*	Number of contracts involved in a market risk transfer activity	Positive
Par Equivalent CDS Spread	*PECS*	Bond default risk measure based on the J.P. Morgan methodology	Positive

5.5 Empirical results

5.5.1 Matching procedure

We first have to choose the appropriate variables to include in the Probit estimation. These variables should have an impact on the decision of a CCP about accepting a firm for central clearing. Intuitively, a CCP selects liquid contracts that have a low default risk, in order to be able to liquidate the position quickly and efficiently in the case of an undesirable event. Cleared contracts should therefore have lower CDS spreads and be traded more often than the other contracts. To support this intuition, Slive et al. (2012) conduct a Cox survival analysis and find that CCPs are more likely to accept contracts with larger notional amounts outstanding, higher liquidity, and smaller CDS spreads. Loon and Zhong (2014) also confirm that liquidity and open interests (available through DTCC data) are important criteria to accept obligors for central clearing. Hence, we take into account variables that fall into the above categories to construct the two groups.

In Table 5.4, we present four different specifications, including different combinations of variables, in order to select the best model. In all four specifications, variables are statistically significant and have the expected sign, in line with the ICECC requirements. The higher the CDS spread, the lower is the probability of being accepted for central clearing, because the firm has a higher default risk. Moreover, we confirm that reference entities with more liquid contracts and larger open interests have higher probabilities of being accepted by a CCP. We finally select Model 3, which has the highest log likelihood ratio and includes the important determinants of central clearing.

After matching with replacement each cleared entity with its nearest neighbor from the control group, we evaluate the quality of this matching and investigate whether a selection bias is present, using various statistics presented in Table 5.5. Panel A compares the mean of each variable included in the model in the treatment

Table 5.4 – Probit Estimation.

This table presents four probit estimations involving different combinations of variables and fitted on cleared and non-cleared entities, where the dependent variable is a binary variable that equals 1 if the firm is centrally cleared by ICECC during 2009-2015 and 0 otherwise, The vector of regressors' coefficients is estimated by maximum likelihood. We use data in the six-month period defined by the firm-event entity to compute the average of each regressor. *CDS_Spr* is the composite spread for the five-year tenor. *RQS* is the five-year relative quoted spread computed by dividing the bid-ask spread by the mid spread. *Comp_Dep* is the number of contributors whose CDS spreads have been used to calculate the five-year CDS spread. *Contr* is the number of contracts outstanding for each CDS contract. $Contr^2$ is the squared value of *Contr*. *Net_Not* is the sum of the net protection bought by net buyers (or equivalently sold by net sellers). Net_Not^2 is the squared value of *Net_Not*. Industry dummies are included in all the models and constructed based on the ten following sectors: telecommunications services, healthcare, technology, basic materials, utilities, industrials, financials, energy, consumer services and consumer goods. *N* is the number of firm-event entities. Numbers in brackets are standard errors. *, **, and *** denote statistical significance at the 10%, 5%, and 1% level, respectively.

Variables	Model 1	Model 2	Model 3	Model 4
CDS_Spr	-0.00145***	-0.00143***	-0.00141***	-0.00127***
	(0.000218)	(0.000218)	(0.000217)	(0.000208)
Contr	7.77e-05***	0.00157***	0.00152***	
	(2.39e-05)	(0.000151)	(0.000156)	
$Contr^2$		-2.70e-07***	-2.64e-07***	
		(2.96e-08)	(2.99e-08)	
RQS	-0.106***	-0.0904***	-0.0899***	-0.0946***
	(0.0127)	(0.0132)	(0.0132)	(0.0129)
Comp_Dep			0.0379	0.0913***
			(0.0316)	(0.0297)
Net_Not				7.91e-10***
				(1.24e-10)
Net_Not^2				-1.8e-19***
				(3.00e-20)
Constant	-0.597***	-2.391***	-2.571***	-1.620***
	(0.216)	(0.275)	(0.314)	(0.276)
Pseudo R^2	0.121	0.2155	0.2163	0.1645
LR Chi^2	219.54	391.12	392.56	298.67
Log likelihood	-797.80	-712.02	-711.30	-758.24
N	7,102	7,102	7,102	7,102

and control groups, both before and after the matching. We also compute the standardized bias, which is the difference between the means of the two groups, scaled by the average standard deviations. After the matching, and for all the variables, the means are closer for the matched sample, and the bias is reduced by more than 81%, which indicates that the characteristics of the two groups are very similar.

In Panel B, we perform additional tests to assess the matching quality. Specifically, we fit the Probit model again, this time on the matched sample. If the two groups are well matched, then we should obtain a bad fit. In fact, the variables that were useful for deciding if a company is eligible for central clearing should no longer be, since the non-cleared firms are similar to the cleared ones along the key dimensions relevant for central clearing. This intuition is confirmed by our results. We obtain a very low likelihood ratio and pseudo R^2, as shown in Table 5.5. Furthermore, we can no longer reject the null hypothesis that all the variables are jointly nonsignificant ($p\text{-}value = 0.905$). The mean and median biases (4.9 and 3.4, respectively) are also greatly reduced, compared to the Probit estimation with the unmatched sample (25.4 and 16.5, respectively). All the above results suggest that the selection bias is substantially reduced across the two samples.

In the next sections, we rely on the matched sample to study the implications of joining a CCP.

5.5.2 Impact of central clearing

Impact on CDS spreads

Here, we examine the impact of clearing on CDS spreads by using the generalized DID methodology. Specifically, we test the following hypothesis:

H1: CDS spreads increase when the reference entity becomes centrally cleared.

We start by plotting in Figure 5.2 the daily mean CDS spread for the treatment and control groups during a period of $[-250, 50]$ days around the central clearing

Table 5.5 – Balancing Tests.

This table presents balancing tests between the treated and control groups in the unmatched and matched samples. In Panel A we compare the means of the two groups and we compute the standard bias, that is, the difference between the means of the two groups scaled by the average standard deviations. *CDS_Spr* is the composite spread for the five-year tenor. *RQS* is the five-year relative quoted spread computed by dividing the bid-ask spread by the mid spread. *Comp_Dep* is the number of contributors whose CDS spreads have been used to calculate the five-year CDS spread. *Contr* is the number of contracts outstanding for each CDS contract. $Contr^2$ is the squared value of *Contr*. In panel B, we fit the Probit model first on the unmatched sample and then on the matched sample to test if variables that were useful in predicting the probability of a firm being eligible for central clearing in the full sample are still significant in the matched sample.

Panel A : Mean comparison

Variable	Sample	Mean Treated	Mean Control	% bias	% bias reduction
CDS_Spr	Unmatched	173.2	241.01	-13.6	81.9
	Matched	173.2	185.47	-2.5	
Contr	Unmatched	2260.2	1511.5	61.6	
	Matched	2260.2	2280.9	-1.7	97.2
$Contr^2$	Unmatched	6,00E+06	4,30E+06	20.2	
	Matched	6,00E+06	6,10E+06	-1	94.9
RQS	Unmatched	7.672	12.339	-74.3	
	Matched	7.672	7.459	3.4	95.4
Comp_Dep	Unmatched	6.3666	5.2254	81.2	
	Matched	6.3666	6.236	9.3	88.6

Panel B : Probit estimations

Sample	Pseudo R^2	Likelihood ratio Chi^2	$p>Chi^2$	Mean bias	Median bias
Unmatched	0.216	392.56	0.000	25.4	16.5
Matched	0.014	7.69	0.905	4.9	3.4

event (Day 0). We first note that both groups have the same pre-treatment trend, which confirms again the matching quality and makes it possible to graphically verify the parallel trend assumption of the difference-in-differences model. We also observe that the mean CDS spread of cleared entities is initially lower, and then increases following the event date. This is consistent with the notion of CCPs accepting entities with a lower default risk. Figure 5.2 also suggests that the spread of cleared entities gradually increases after the move to central clearing, and shows that the difference between the two groups reaches approximately 28 bps by the end of our event window. This behavior could be the result of increased confidence in the clearinghouse as an entity able to protect the investor against the seller's default and to mitigate counterparty risk, where market participants are willing to pay more to benefit from this advantage. We also observe that the trend increase in the spreads of cleared entities starts on average one week before the clearing date. This pattern could reflect an anticipatory effect by market participants.

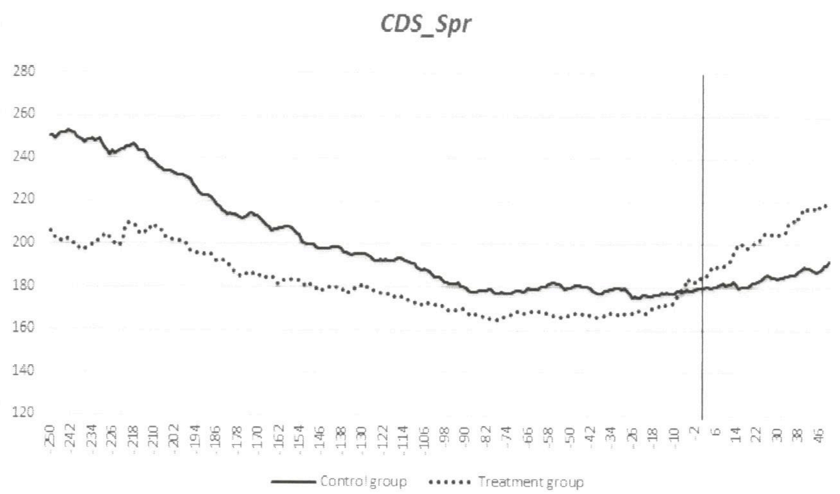
We then test hypothesis H1 by conducting a difference-in-differences analysis on the matched sample. We estimate Equation (5.5) with *CDS_Spr* as the dependent variable and we focus on the coefficient β_1 of the variable *Cleared*. In the first column of Table 5.6, we start with a large event window of [-250, 50] days and we find that the coefficient β_1 is positive and statistically significant. Our results show that moving a CDS contract from the OTC market to a clearinghouse increases its spread by 19.2 bps on average.

Despite the differences in methodology and sample size, this result is in accordance with most of the findings in the literature. Loon and Zhong (2014), using an event study with 132 cleared firms find that the spreads rise with the initiation of central clearing. Kaya (2017), using panel regression, estimates this increase to around 24 bps in a sample of 85 nonfinancial firms. Du et al. (2018) find a decrease in CDS spreads using panel regression with 142 cleared firms.[5]

[5] To check the robustness of our estimations, we consider different event windows of various lengths, including the [-250, 20] days considered by Du et al. (2018). All the specifications lead to a positive and statistically significant coefficient, with a lower magnitude for shorter event windows.

Figure 5.2 – Comparison of CDS Spreads.

This figure compares the CDS spreads of cleared and non-cleared entities. CDS_Spr is the composite spread for the five-year tenor and is obtained from Markit. The horizontal axis represents time in days where date 0 denotes the central clearing event. The dotted and solid lines represent the average daily CDS spread of the treatment group and the control group respectively. Both groups are constructed using dynamic propensity score matching.



Considering the main purpose of creating a central counterparty and the magnitude of the increase in CDS spreads, we can presume that this change is likely to be a reflection of a reduction in counterparty risk. The estimation of the coefficient β_1 is in the range provided by the papers that study the pricing of counterparty credit risk in CDS spreads. For instance, Brigo and Chourdakis (2009) find a range of 15 to 25 bps in the case of a moderate default correlation. The CCP provides several layers of protection that make the contract more reliable, and thus, more expensive. However, other factors, such as a possible improvement in liquidity or trading activity, or an increase in the underlying bond default risk, may also contribute to this observed rise in the CDS spreads. We assess the potential effects of the other factors in the following subsections.

Table 5.6 – Difference-in-Differences Analysis for CDS Spreads

This table presents the estimates of the coefficients in the generalized DID equation where the dependent variable *CDS_Spr* is the composite spread for the five-year tenor. Observations are pairs of firms (i) and dates (t). $Cleared_{i,t}$ is a binary variable that indicates if the firm i is centrally cleared at date t or not. The constant term includes firm and time fixed effects (α_i and γ_t respectively). In each column, we estimate the equation using a different estimation window around the clearing event date. In all the regressions, the standard errors (in brackets) are clustered by firm. *, **, and *** denote statistical significance at the 10%, 5%, and 1% level, respectively.

$$CDS\_Spr_{i,t} = \beta_0 + \beta_1 Cleared_{i,t} + \alpha_i + \gamma_t + \epsilon_{i,t}$$

CDS_Spr	[-250 , 50]	[-250 , 20]	[-100 , 50]	[-100 , 20]
Cleared	19.2**	18.2**	10.1*	8.37*
	(8.38)	(7.79)	(5.57)	(4.70)
Constant	385.4***	386.6***	166.4***	164.1***
	(73.2)	(72.2)	(18.4)	(18.8)
Observations	102,691	93,139	53,035	42,777
Number of firms	298	298	298	298
R-squared	0.224	0.224	0.260	0.247

Impact on liquidity

The introduction of central clearing may help improve CDS liquidity by attracting more market participants. In fact, the mitigation of counterparty risk, the increased transparency, and the reduction of operational risk may all incite more institutions to get involved in CDS trading. On the other hand, the demands of this new scheme, and particularly the margin requirements, could prevent some participants from having access to clearinghouses: not all investors can afford to pay collateral demands on a daily basis and to set aside a non-negligible amount of capital as a contribution to the default fund. According to Cont (2017), the collateral maintained by CCP members in the form of liquid assets was more than 400 billion USD in 2016. Hence, the overall impact of central clearing on market liquidity is still unclear. If the first effect prevails, then an improvement in CDS liquidity will widen CDS spreads. The second effect might also be sizeable and compensate for the benefits of the first improvement. We expect, however, an improvement in liquidity, as shown by Slive et al. (2012) and Loon and Zhong (2014).

By applying the same methodology as in the previous section, consisting of comparing groups matched on the basis of propensity scores, we empirically test the following hypothesis:

H2: Central clearing improves CDS liquidity.

We have a total of 10 liquidity measures, mainly obtained from Markit Liquidity. In Figure 5.3, we plot the evolution of the daily mean of the variables *RQS* and *Comp_ Dep* in the control group against that of the treatment group during a period of $[-250, 50]$ days around the central clearing event. Since liquidity is a key dimension for accepting a reference entity for central clearing, it is important to have similar pre-clearing trends for both groups. Figure 5.3 shows that the two graphs are very similar and have the same trend over the whole event window. Figures comparing the graphs for other liquidity measures are presented in Appendix B and show similar behavior. Unlike in the previous analysis of CDS spreads, where the cleared entities had a different behavior after the clearing event date, none of our liquidity measures exhibit a divergence in trend following the move to a clearinghouse. Overall, this preliminary investigation seems to indicate that CDS liquidity is not affected by the clearing event.

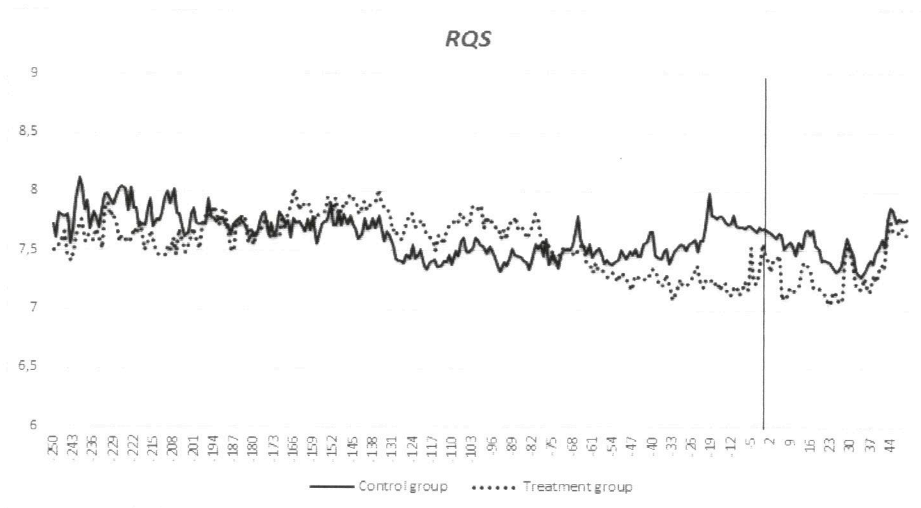
We now apply the difference-in-differences analysis to each liquidity measure, used as the dependent variable in Equation (5.5). We mainly focus on the *RQS* and *Comp_ Dep* variables because they fully cover our sample period. For these two measures, we fit the regression equation using various event windows. For all the specifications, presented in Table 5.7, none of the coefficients of the binary variable *Cleared* are statistically significant, suggesting that central clearing does not have any impact on CDS liquidity.

As a robustness check, we estimate the same equation using the remaining liquidity measures. The results for an event window of $[-250, 50]$ days are reported

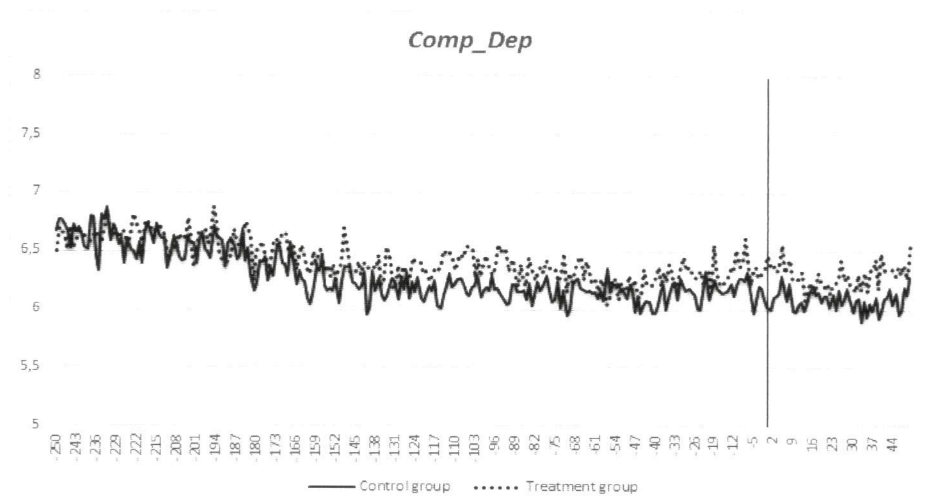
Figure 5.3 – Comparison of liquidity measures.

This figure compares the liquidity measures of cleared and non-cleared entities. *RQS* is the five-year relative quoted spread computed by dividing the bid-ask spread by the mid spread. *Comp_Dep* is the number of contributors whose CDS spreads have been used to calculate the five-year CDS spread. The horizontal axis represents time in days where date 0 denotes the central clearing event. The dotted and solid lines represent the average daily liquidity measure of the treatment group and the control group respectively. Both groups are constructed using dynamic propensity score matching.

Panel A : Comparison of Relative Quoted Spreads.



Panel B : Comparison of Composite Depths.



in Table 5.8.[6] All interaction coefficients β_1 are negligibly small and statistically nonsignificant, except for the coefficient of the variable Liq_sc, which is significant at the 10% level.

Our results suggest that cleared reference entities do not experience a significant improvement in their liquidity following central clearing. The only variable where the null hypothesis can be rejected (at the 10% level) does not have strong granularity (it takes integer values from one to five, while the coefficient $\beta_1 = 0.0553$) and may not have much discriminatory power with respect to the treatment event (see panel B of Figure B.1 in Appendix B).

The positive effects caused by the mitigation of counterparty risk and increased transparency may be counterbalanced by the inconvenience of daily margining. It might also be the case that the accepted contracts are already liquid, which makes them less likely to gain any additional liquidity benefit. Note that these results do not mean that liquidity is not priced in CDS contracts, but rather that liquidity is homogeneous among cleared and non-cleared contracts of firms with similar characteristics that would be eligible for central clearing.

[6] Results for smaller event windows are qualitatively the same.

Table 5.7 – Difference-in-Differences Analysis for the liquidity measures.

This table presents the estimates of the coefficients in the generalized DID equation. In panel A, the dependent variable RQS is the five-year relative quoted spread computed by dividing the bid-ask spread by the mid spread. In panel B, the dependent variable $Comp\_Dep$ is the number of contributors whose CDS spreads have been used to calculate the five-year CDS spread. Observations are pairs of firms (i) and dates (t). $Cleared_{i,t}$ is a binary variable that indicates if the firm i is centrally cleared at date t or not. The constant term includes firm and time fixed effects (α_i and γ_t respectively). In each column, we estimate the equation using a different estimation window around the clearing event date. In all the regressions, the standard errors (in brackets) are clustered by firm. *, **, and *** denote statistical significance at the 10%, 5%, and 1% level, respectively.

$$Liquidity\_measure_{i,t} = \beta_0 + \beta_1 Cleared_{i,t} + \alpha_i + \gamma_t + \epsilon_{i,t}$$

Panel A : Difference-in-Differences Analysis for the Relative Quoted Spread.

RQS	[-250 , 50]	[-250 , 20]	[-100 , 50]	[-100 , 20]
Cleared	-0.0194	-0.0161	0.0173	-0.00285
	(0.151)	(0.0612)	(0.149)	(0.0609)
Constant	5.21***	7.00***	9.95***	6.70***
	(0.763)	(1.67)	(1.18)	(0.410)
Observations	102,591	93,07	52,947	42,720
Number of firms	298	298	298	298
R-squared	0.159	0.019	0.174	0.031

Panel B : Difference-in-Differences Analysis for the Composite Depth.

Comp_Dep	[-250 , 50]	[-250 , 20]	[-100 , 50]	[-100 , 20]
Cleared	-0.0171	0.0264	0.00627	0.0376
	(0.0273)	(0.0295)	(0.0457)	(0.0327)
Constant	5.965***	6.651***	7.150***	5.686***
	(0.460)	(0.460)	(0.571)	(0.800)
Observations	102,691	93,139	53,035	42,777
Number of firms	298	298	298	298
R-squared	0.018	0.019	0.266	0.028

Table 5.8 – Difference-in-Differences Analysis for other liquidity measures.

This table presents the estimates of the coefficients in the generalized DID equation where, in each column, the dependent variable is a liquidity measures obtained from Markit Liquidity. *Upf_BA* is the bid-ask spread in upfront points for the five-year tenor. *Dealers* is the total number of distinct dealers quoting the reference entity across all available tenors. *Quotes* is the total number of unique quotes for a reference entity, all tenors combined. *Liq_Sc* is calculated by Markit and is defined on a scale from 1 to 5 where 1 indicates the highest liquidity. *5Y_Dealers* is the total number of distinct dealers quoting the reference entity for the five-year tenor. *5Y_Quotes* is the total number of unique quotes for a reference entity for the five-year tenor. Data about the remaining tenors is given by the variables *Non_5Y_Dealers* and *Non_5Y_Quotes*. Observations are pairs of firms (i) and dates (t). $Cleared_{i,t}$ is a binary variable that indicates if the firm i is centrally cleared at date t or not. The constant term includes firm and time fixed effects (α_i and γ_t respectively). The estimation window is [-250, 50] days around the clearing event date. In all the regressions, the standard errors (in brackets) are clustered by firm. *, **, and *** denote statistical significance at the 10%, 5%, and 1% level, respectively.

$$Liquidity\_measure_{i,t} = \beta_0 + \beta_1 Cleared_{i,t} + \alpha_i + \gamma_t + \epsilon_{i,t}$$

Liquidity measures	*Upf_BA*	*Dealers*	*Quotes*	*Liq_Sc*	*5Y_Dealers*	*5Y_Quotes*	Non_5Y_Dealers	Non_5Y_Quotes
Cleared	6.55e-05	-0.269	-1.114	0.0553*	-0.0655	-0.397	-0.118	0.0764
	(4.27e-05)	(0.181)	(1.578)	(0.0293)	(0.120)	(0.819)	(0.315)	(0.788)
Constant	0.00599***	7.428***	55.69***	0.974***	6.847***	26.81***	23.83***	32.87***
	(4.54e-05)	(0.208)	(2.268)	(0.0361)	(0.197)	(1.486)	(0.399)	(0.928)
Observations	74,167	74,167	74,167	74,167	39,699	39,712	34,577	34,578
Number of firms	212	212	212	212	123	123	123	123
R-squared	0.023	0.022	0.023	0.024	0.047	0.047	0.050	0.046

These results differ notably from those of Loon and Zhong (2014) who find a significant improvement in liquidity using standard DID with static matching. To show that the difference in our results may be explained by methodology, we first omit the fixed effects γ_{1i} and γ_{2t} from the DID estimation in Equation (5.5), and find a significant impact for the liquidity variables RQS and $Comp\_Dep$. We then apply the methodology used in Loon and Zhong (2014) to our data. The results, provided in Table 5.9, indicate a significant impact for the liquidity variables RQS and $Dealers$.

Table 5.9 – Impact of central clearing on CDS liquidity and trading activity using Loon and Zhong (2014) methodology

This table presents the results of the estimation of the impact of central clearing on CDS liquidity and trading activity using the same methodology as in Loon and Zhong (2014). For each measure, we compute the change between the event period average and pre-event period average. The pre-event and post-event windows are [-250, -21] and [0,20] days for liquidity measures and [-52, -5] and [0, 4) weeks for trading activity measures. The difference-in-differences is then computed as $DID = \Delta - \Delta_M$, where Δ is the change in liquidity or trading activity measures of a cleared firm around central clearing and Δ_M is the average change for the matched sample. We report the cross sectional mean (median) of Δ, Δ_M and DID and we test wether the mean (median) of DID is different from zero using t-tests (non parametric sign tests). The definition of all the liquidity and trading activity measures can be found in Table 5.3. *, **, and *** denote statistical significance at the 10%, 5%, and 1% level, respectively.

		Mean			Median		
Measure	*N*	Δ	Δ_M	*DID*	Δ	Δ_M	*DID*
Illiquidity measures							
RQS	174	-0,005	0,002	-0,007***	-0,004	0,004	-0,008***
Upf_BA	126	4,447e-05	-1,432e-04	1,877e-04	1,617e-04	-3,781e-05	1,995e-04
Liq_sc	126	0,079	0,114	-0,035	0,146	0,153	-0,007
Liquidity measures							
Comp_Dep	174	-0,128	-0,186	0,057	-0,138	-0,156	0,018
Dealers	126	-0,161	-0,463	0,301***	-0,155	-0,520	0,365**
Quotes	126	-1,870	-2,520	0,650	-1,246	-1,939	0,693
5Y_Dealers	69	-0,595	-0,637	0,041	-0,696	-0,588	-0,108
5Y_Quotes	69	-2,771	-3,226	0,455	-2,616	-3,085	0,469
Non_5Y_Quotes	69	-2,282	-2,327	0,046	-0,721	-1,612	0,892
Non_5Y_Dealers	69	-2,335	-1,313	-1,022***	-2,507	-1,284	-1,223
Trading activity measures							
Gross_Not	174	2,497e+08	-6,325e+08	8,823e+08***	4,210e+08	-5,067e+08	9,277e+08***
Net_Not	174	-7,056e+07	-8,427e+07	1,371e+07	-7,243e+07	-8,478e+07	1,236e+07**
Contr	174	-58,308	-89,003	30,694**	-51,962	-74,031	22,069*
Contr_Risk	107	0,750	0,100	0,651	4,093	2,571	1,523
Gross_Not_Risk	107	3,645e+06	-1,004e+06	4,649e+06	2,273e+07	1,400e+07	8,728e+06

Impact on trading activity

Since it has been shown that CCPs have a preference for contracts with large open interests, we find it interesting to check whether the introduction of central clearing affects trading-activity variables. Open interest indicates how much debt is insured with CDS, and could be considered a good measure of the market participants' demand. On the one hand, the trading activity could increase if participants wanted to benefit from the reduction in counterparty risk following central clearing. This behavior could raise the demand and exert an upward pressure on CDS spreads. On the other hand, informed traders may start looking for alternative derivatives and more opaque markets because of the increased transparency brought by clearinghouses. In such a case, demand for credit protection could decrease and drive CDS spreads down. We expect the first effect to dominate, given the numerous advantages of trading through a clearinghouse. Therefore, we propose the following hypothesis to test the overall impact of the introduction of central clearing on trading activity:

H3: Central clearing increases CDS trading activity.

We employ the same methodology to analyze the five trading activity variables provided by DTCC. We construct weekly means for these variables in the control and treatment groups matched with propensity scores over a period of $[-50, 10]$ weeks around the clearing event date. Figure 5.4, illustrating gross and net notional amounts, respectively, shows that the means in the two groups move together during the pre-treatment period.

Panel A of Figure 5.4 shows an increase of around 7% of the mean gross notional amount in the treatment group, while the control group maintains the same trend for the whole event window. Note that there is no such increase in the net notional amount, indicating that this increase is essentially due to the event itself, that is, the transfer of the contracts to a clearinghouse, rather than to an increase in the amount of risk managed in the CDS market.

For all other trading activity measures, we do not observe any change in trend following the clearing event (see Appendix B).

To confirm these preliminary observations, we perform difference-in-differences regressions, using each trading activity variable as the dependent variable in Equation (5.5). The results corresponding to a window of $[-50, 10]$ weeks around the clearing event are reported in Table 5.10.[7] We find that $Gross\_Not$ (the gross notional amount) is the only variable having a positive and statistically significant coefficient for the interaction term. For all the remaining variables, which represent better proxies for trading activity, coefficients of the interaction term are nonsignificant. Consequently, our results suggest that central clearing does not have any significant impact on trading activity.

Again, our findings differ from those of Loon and Zhong (2014) who find a significant increase in trading activity after a move to central clearing. To show that this difference is due to the difference in methodology and not to the difference in samples, we report in Table 5.9 the results obtained by applying the same methodology as in Loon and Zhong (2014) to our data. These results show a significant increase in the number of contracts and net notional amount, suggesting a positive impact on trading activity.

Impact on bond default spread

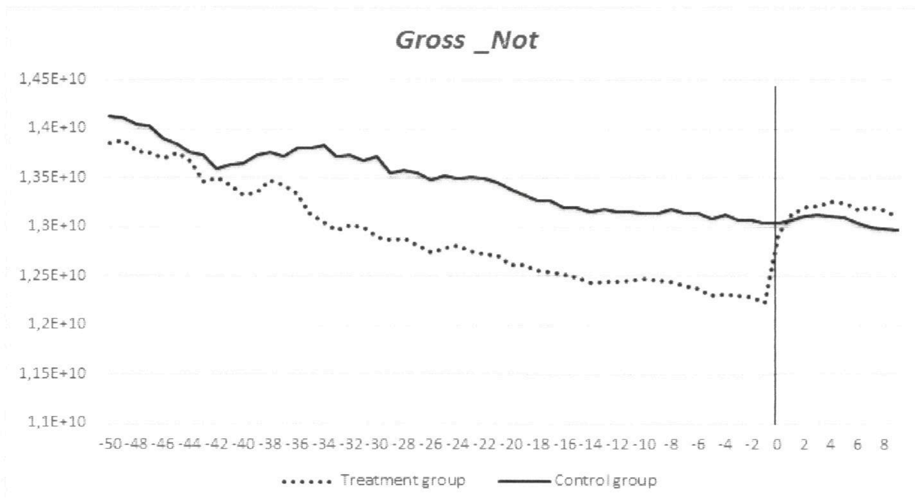
The CDS and bond markets are strongly related since the CDS contract is essentially used to hedge bond positions. Bond issuers may take riskier positions if they know that their associated CDS are protected against counterparty risk once they are centrally cleared. A similar moral-hazard situation was documented in the banking industry, where bank managers became less risk averse when their customers obtained a deposit insurance protecting them from a bank default event (Diamond and Dybvig, 1983). In fact, this moral-hazard effect is often used to justify banking regulations (Crouhy, Galai, and Mark, 2000).

[7]Results for smaller event windows are qualitatively the same.

Figure 5.4 – Comparison of trading activity measures

This figure compares the trading activity measures of cleared and non-cleared entities. *Gross_ Not* represents the total notional of CDS contracts bought (or equivalently sold) for each reference entity. *Net_ Not* is the sum of the net protection bought by net buyers (or equivalently sold by net sellers). The data is on a weekly basis and is obtained from DTCC. The horizontal axis represents time in weeks where date 0 denotes the central clearing event. The dotted and solid lines represent the weekly average trading activity measure of the treatment group and the control group respectively. Both groups are constructed using dynamic propensity score matching.

Panel A : Comparison of Gross Notional Amounts.



Panel B : Comparison of Net Notional Amounts.

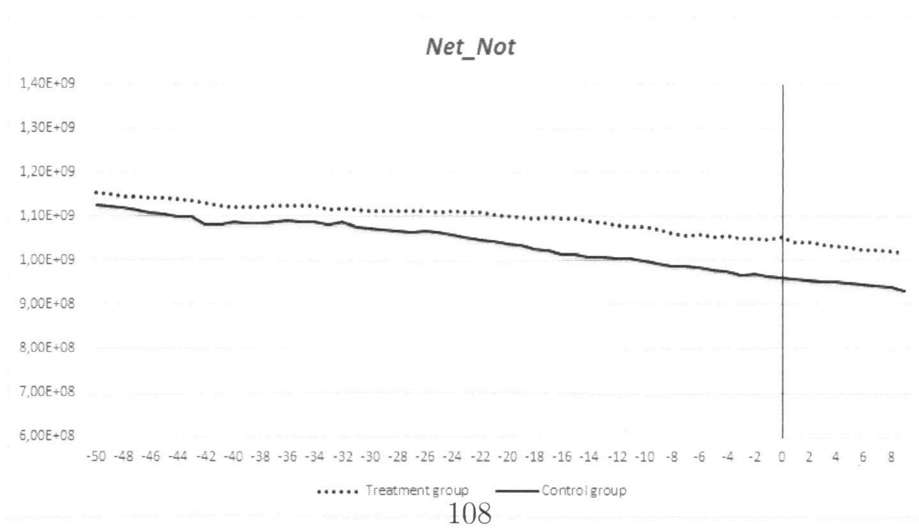


Table 5.10 – Difference-in-Differences Analysis for the trading activity measures.

This table presents the estimates of the coefficients in the generalized DID equation where, in each column, the dependent variable is a trading activity measure obtained from DTCC. *Gross_Not* is the sum of all notional CDS contracts bought (or equivalently sold) for each reference entity. *Net_Not* is the sum of the net protection bought by net buyers (or equivalently sold) by net sellers. *Contr* is the number of contracts outstanding for each CDS contract. *Gross_Not_Risk* Captures transaction types that result in a change in the market risk position. *Contr_Risk* is the number of contracts involved in market risk transfer. Observations are pairs of firms (i) and dates (t). $Cleared_{i,t}$ is a binary variable that indicates if the firm i is centrally cleared at date t or not. The constant term includes firm and time fixed effects (α_i and γ_t respectively). The estimation window is [-50, 10] weeks around the clearing event date. In all the regressions, the standard errors (in brackets) are clustered by firm. *, **, and *** denote statistical significance at the 10%, 5%, and 1% level, respectively.

$$Trading\_activity\_measure_{i,t} = \beta_0 + \beta_1 Cleared_{i,t} + \alpha_i + \gamma_t + \epsilon_{i,t}$$

Measure	*Gross_Not*	*Net_Not*	*Contr*	*Gross_Not_Risk*	*Contr_Risk*
Cleared	1.072e+09***	-1.303e+06	19.29	5.039e+06	-1.117
	(1.509e+08)	(1.539e+07)	(18.88)	(7.264e+06)	(1.386)
Constant	1.472e+10***	1.388e+09***	2,312***	9.570e+07***	21.71***
	(6.617e+08)	(5.805e+07)	(73.83)	(1.625e+07)	(3.545)
Observations	20,389	20,389	20,389	12,452	12,452
Number of firms	296	296	296	237	237
R-squared	0.403	0.386	0.433	0.301	0.317

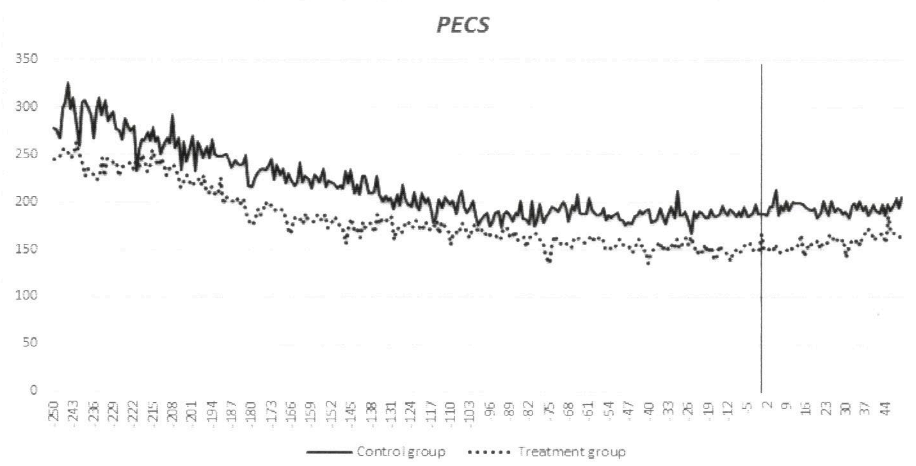
We now check whether the increase in CDS spreads may be due to a change in the default risk of the underlying bond, by testing the following hypothesis:

H4: Central clearing increases the bond default risk.

We compute the daily mean of the variable $PECS$ for the control and treatment groups over the period $[-250, 50]$ around the clearing event date. Figure 5.5 shows that there is no trend change after the event, suggesting that the default spread of bonds of cleared entities is the same before and after joining the clearinghouse. We confirm this observation by estimating the DID in Equation (5) using $PECS$ as a dependent variable. For all the estimation windows reported in Table 5.11, the coefficient of the interaction term is not statistically significant, which indicates that

Figure 5.5 – Comparison of Par Equivalent CDS Spreads.

This figure compares the Par Equivalent CDS Spreads ($PECS$) of cleared and non-cleared entities. This variable measures the default spread of the underlying bond and is computed using the J.P. Morgan methodology. The horizontal axis represents time in days where date 0 denotes the central clearing event. The dotted and solid lines represent the daily average PECS of the treatment group and the control group respectively. Both groups are constructed using dynamic propensity score matching.



the default risk of the underlying bond does not increase due to a move to central clearing,[8] confirming that the increase in CDS spreads is not caused by a change in the bond market.

In addition, we assess whether central clearing reduces the negativity of the CDS Bond basis computed as the CDS spread minus the PECS. If the negative basis persists, then arbitrage opportunities could arise due to counterparty risk, bond illiquidity or high funding risks. We apply the same previous methodology by estimating the DID and computing the daily means of the basis for the treatment and control groups. Figure 5.6 and table 5.12 show that there is no impact on the basis following the clearing event.

[8]Similar results are obtained when we add bond rating dummies as control variables.

Table 5.11 – Difference-in-Differences Analysis for the Par Equivalent CDS Spread.

This table presents the estimates of the coefficients in the generalized DID equation where the dependent variable *PECS* measures the bond's default risk using the J.P. Morgan methodology. Observations are pairs of firms (i) and dates (t). $Cleared_{i,t}$ is a binary variable that indicates if the firm i is centrally cleared at date t or not. The constant term includes firm and time fixed effects (α_i and γ_t respectively). In each column we estimate the equation using a different estimation window around the clearing event date. In all the regressions, the standard errors (in brackets) are clustered by firm. *, **, and *** denote statistical significance at the 10%, 5%, and 1% level, respectively.

$$PECS_{i,t} = \beta_0 + \beta_1 Cleared_{i,t} + \alpha_i + \gamma_t + \epsilon_{i,t}$$

PECS	[-250 , 50]	[-250 , 20]	[-100 , 50]	[-100 , 20]
Cleared	3.025	2.533	-6.317	5.900
	(8.503)	(8.311)	(4.476)	(3.943)
Constant	528.0***	526.3***	365.2***	369.9***
	(24.33)	(25.64)	(21.11)	(20.70)
Observations	36,659	33,292	19,263	15,568
Number of firms	121	121	121	121
R-squared	0.443	0.453	0.338	0.319

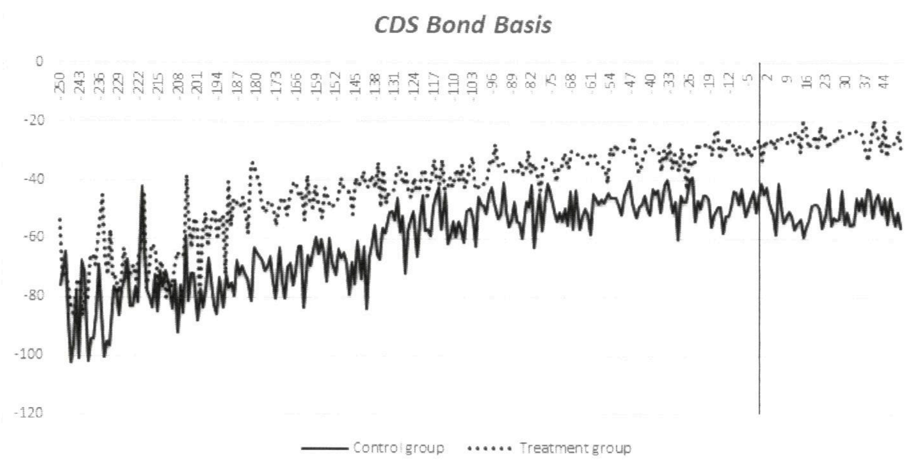
The issue of clearing costs

Reducing counterparty risk comes at the expense of higher margin requirements relative to OTC transactions. As argued by Kaya (2017), the increase in the CDS spreads following central clearing could be partially explained by an increase in clearing costs that is passed on to end-users. However, as documented in the literature, clearinghouses do not necessarily ask for larger collateral amounts and such a contribution to the increase in the CDS spread should be small. Evidence in that direction is provided, for instance, in the following publications:

- Duffie, Scheicher and Vuillemey (2015) take a snapshot on December 30, 2011 of the CDS bilateral exposures provided by DTCC and show that central clearing helps lower collateral demand relative to the OTC market, as long as there is no significant proliferation of central counterparties. In fact, the benefits of

Figure 5.6 – Comparison of the CDS Bond basis.

This figure compares the CDS Bond basis of cleared and non-cleared entities. The horizontal axis represents time in days where date 0 denotes the central clearing event. The dotted and solid lines represent the daily average basis of the treatment group and the control group respectively. Both groups are constructed using dynamic propensity score matching.



multilateral netting and diversifications outweigh the increased initial margin requirements.

- Mello and Parsons (2012) present a replication argument and show that the cost of initial margin requirements is insignificant. They find that there is no additional cost with the margin mandate but that the credit risk associated with the derivative is accounted for separately.

- According to the Committee on Global Financial System (2013), variation margin is not as costly as some argue. It represents a transfer of resources and does not affect the net demand for collateral.

Finally, clearing fees themselves should not represent a burden for those trading cleared contracts. The clearing fees charged by ICECC to its clients and members amount, respectively, to \$20 per million of notional for single-name CDS, and \$15

Table 5.12 – Difference-in-Differences Analysis for the CDS Bond basis.

This table presents the estimates of the coefficients in the generalized DID equation where the dependent variable *Basis* is the difference between the CDS spread and the PECS. Observations are pairs of firms (i) and dates (t). $Cleared_{i,t}$ is a binary variable that indicates if the firm i is centrally cleared at date t or not. The constant term includes firm and time fixed effects (α_i and γ_t respectively). In each column we estimate the equation using a different estimation window around the clearing event date. In all the regressions, the standard errors (in brackets) are clustered by firm. *, **, and *** denote statistical significance at the 10%, 5%, and 1% level, respectively.

$$Basis_{i,t} = \beta_0 + \beta_1 Cleared_{i,t} + \alpha_i + \gamma_t + \epsilon_{i,t}$$

Basis	[-250 , 50]	[-250 , 20]	[-100 , 50]	[-100 , 20]
Cleared	2.630	2.767	4.516	4.139
	(5.745)	(5.680)	(3.353)	(3.162)
Constant	-203.4***	-200.2***	-246.6***	-244.5***
	(10.89)	(11.46)	(9.365)	(9.083)
Observations	36,659	33,292	19,263	15,568
Number of firms	121	121	121	121
R-squared	0.251	0.252	0.142	0.160

per million of notional. We therefore argue that the clearing fees and additional margin requirements should not affect significantly the CDS spreads.

5.5.3 Summary of empirical results

Our empirical findings indicate that neither CDS liquidity, nor trading activity or bond default risk are significantly affected by the introduction of clearinghouses, while CDS spreads do increase. In addition, clearing costs are small and should not impact the spreads significantly. Consequently, our results suggest that the increase in CDS spreads following adhesion to a CCP can be mainly attributed to the reduction in counterparty risk. Assuming the counteparty risk of a CCP to be small (see, e.g., Cruz Lopez, Harris, Hurlin & Perignon 2017 on central counterparty risk), the magnitude of this increase could therefore be used as a measure of the counterparty risk present in the market before a reference entity joins central clearing. We find that this risk could reach up to 19 bps, which is in the range of what is found in the literature.

Participants in clearinghouses have higher trust in a central counterparty and less concern about the possibility of a default event. Hence, they could be willing to pay more to buy better credit protection. The ability of a CCP to prevent default contagion and to continuously monitor the risks arising from trading CDS contracts helps establish a safe and robust clearing environment. This was one of the main goals of the Dodd-Frank Wall Street Reform and Consumer Protection Act.

5.6 Conclusion

In this paper, we study the impact of central clearing on single-name CDS. The opportunity of voluntarily joining a CCP to trade these contracts has been effective since December 2009. This new scheme, mandated by the Dodd-Frank Act, aims at reducing the overall risk in the market and enforcing new regulations to avoid another financial crisis. The clearinghouse uses multiple layers of protection and strong risk-management strategies to prevent a potential domino effect.

Despite the economic importance of this regulatory change, little empirical evidence has been provided about its implications. In this work, we perform a generalized difference-in-differences analysis with fixed effects on samples matched with propensity scores computed just prior to the clearing event. This type of dynamic matching ensures that the cleared and non-cleared groups have similar pre-clearing characteristics, and alleviates the concern about the selection bias arising from the voluntary choice to adhere to central clearing. Our results indicate that the CDS spread increase resulting from a reference entity joining the clearinghouse could reach as high as 19 bps. We test whether this price change is due to various factors by separately analyzing the impact on liquidity and on trading activity, but we find that central clearing does not cause any significant change in these two factors. We also find that the clearing event has no significant impact on the underlying bond market. In addition, according to the recent literature it does not seem that important additional costs are passed on to end-users. Therefore, we argue that the change in CDS spreads can be used as an indication of the amount of counterparty risk that is reduced thanks to the clearinghouse.

5.7 Appendix A : Matching example

In this appendix, we present a detailed example of the dynamic propensity score matching procedure. For illustration purposes, consider a small sample of four firms, A, B, C, and D over the 2009–2015 period. Firms A was centrally cleared on December 21, 2009 and Firm B on March 28, 2011. Firms C and D were not centrally cleared during the sample period. Suppose that the event window is $[-8, -2]$ months before the clearing date, we consider data from 21/04/2009 to 21/10/2009 for firm A, and from 28/07/2010 to 28/01/2011 for firm B. We then assume that firms C and D had the possibility of being centrally cleared on December 21, 2009, or on March 28, 2011. Therefore, we create the following firm-event entities:

C_1: Data for Firm C from 21/04/2009 to 21/10/2009; the firm-event corresponds to the possibility of Firm C adhering to central clearing on December 21, 2009

C_2: Data for Firm C from 28/07/2010 to 28/01/2011; the firm-event corresponds to the possibility of Firm C adhering to central clearing on March 28, 2011

D_1: Data for Firm D from 21/04/2009 to 21/10/2009; the firm-event corresponds to the possibility of Firm D adhering to central clearing on December 21, 2009

D_2 : Data for Firm D from 28/07/2010 to 28/01/2011; the firm-event corresponds to the possibility of Firm D adhering to central clearing on March 28, 2011

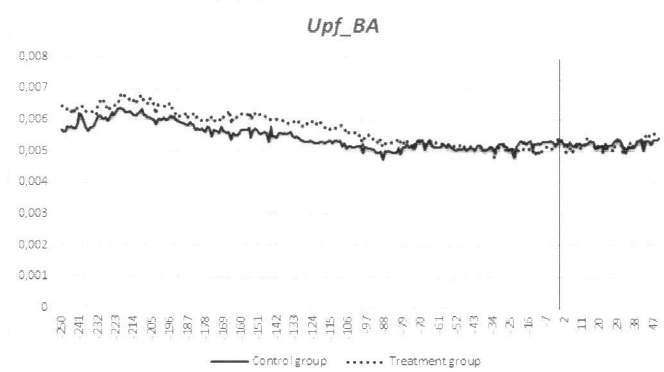
A and B constitute the treatment group and could be matched to any firm-event in the control group: C_1, C_2, D_1, or D_2. We apply the Probit model to the sample of six firm-events and match each firm in the treatment group to a firm in the control group having the closest propensity score. For instance, if A is matched to D_2 and B is matched to C_1, then the control group is D_2 and C_1. The firms in the control group that are not matched are dropped from the sample. This procedure allows us to construct a control group that exhibits pre-clearing characteristics that are similar to that of the treatment group, and thus eliminate the potential selection bias.

5.8 Appendix B : Additional figures

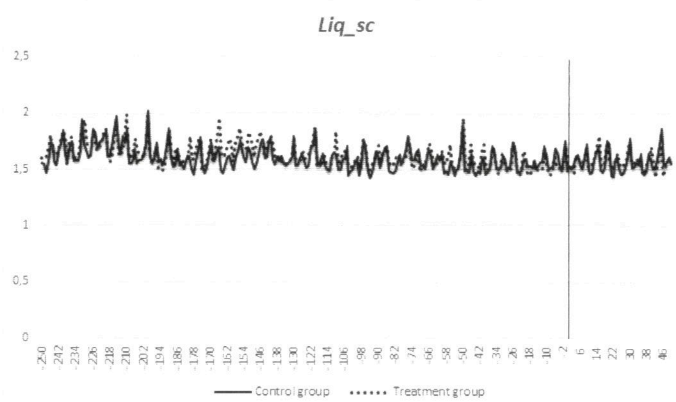
Figure B.1 – Comparison of other liquidity measures.

This figure compares other liquidity measures of cleared and non-cleared entities. All these variables are obtained from Markit Liquidity. *Upf_BA* represents the bid-ask spread in upfront points for the five-year tenor. *Liq_Sc* is a score defined on a scale from 1 to 5 where 1 indicates the highest liquidity. *Quotes* is the total number of unique quotes for a reference entity, all tenors combined. *Dealers* is the total number of distinct dealers quoting the reference entity across all available tenors. *5Y_Dealers* is the total number of distinct dealers quoting the reference entity for the five-year tenor. *5Y_Quotes* is the total number of unique quotes for a reference entity for the five-year tenor. *Non_5Y_Dealers* is the total number of distinct dealers quoting the reference entity for the non-five-year tenors. *Non_5Y_Quotes* is the total number of unique quotes for a reference entity for the non-five-year tenor. The horizontal axis represents the event time in days where 0 denotes the beginning of central clearing. The dotted and solid lines represent the daily average of the liquidity measure of the treatment group and the control group respectively. Both groups are constructed using dynamic propensity score matching.

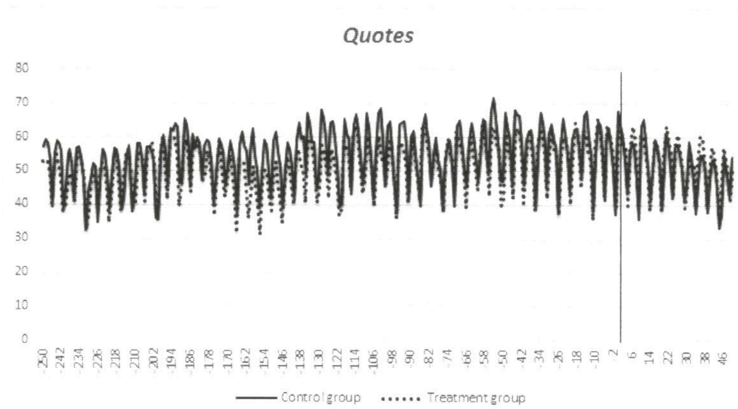
Panel A : Comparison of Upfront 5Y bid-ask spread.



Panel B : Comparison of Liquidity scores.

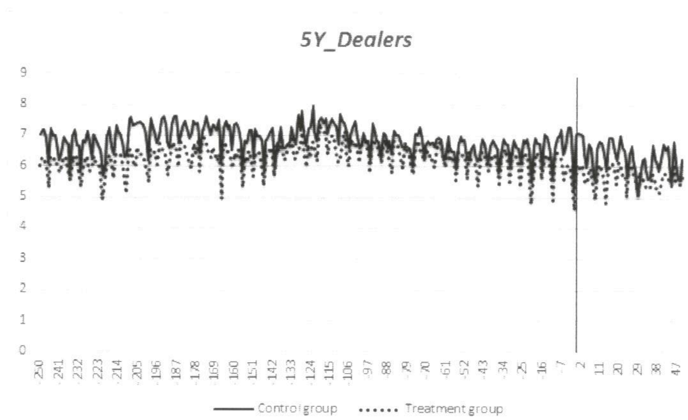


Panel C : Comparison of Quotes count.

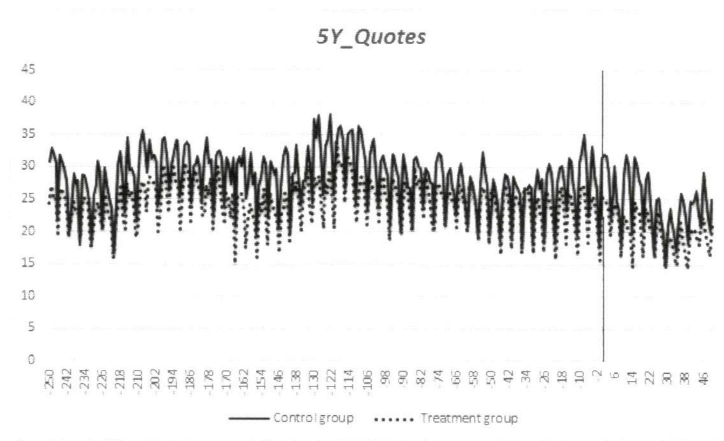


Panel D : Comparison of Dealers count.

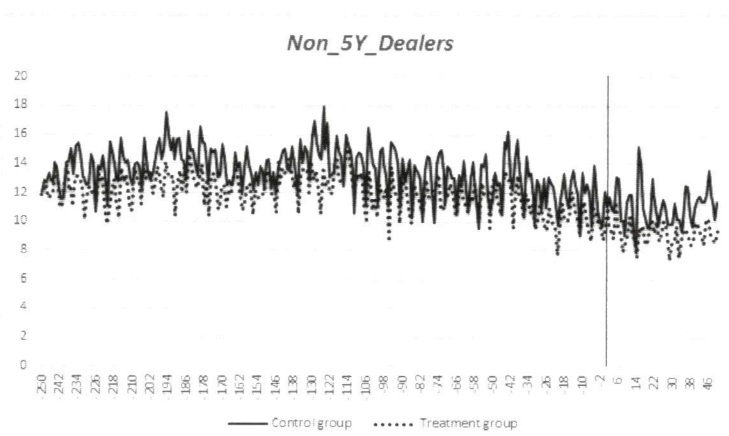
Panel E : Comparison of 5Y Dealers count.



Panel F : Comparison of 5Y Quotes count.



Panel G : Comparison of Non 5Y Dealers count.



Panel H : Comparison of Non 5Y Quotes count.

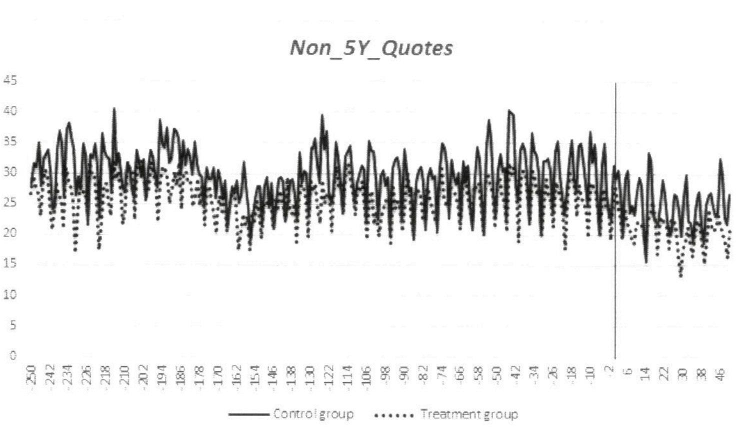
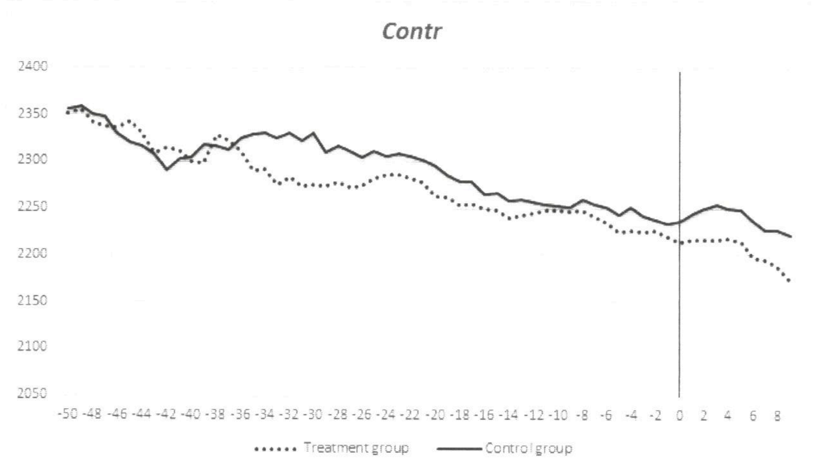


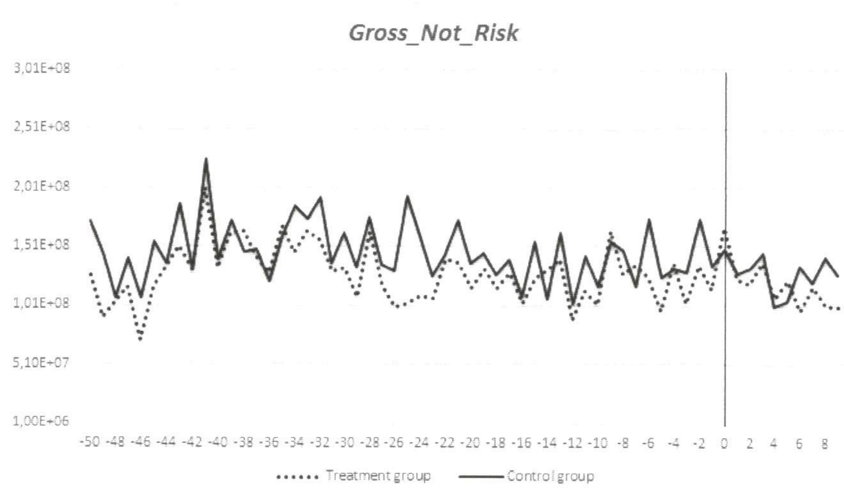
Figure B.2 – Comparison of other trading activity measures

This figure compares other trading activity measures of cleared and non-cleared entities, obtained from DTCC. *Contr* is the number of contracts outstanding for each CDS contract. *Gross_ Not_ Risk* is the sum of all notional CDS contracts bought (or equivalently sold) for each reference entity. It captures transaction types that result in a change in the market risk position. *Contr_ Risk* is the number of contracts outstanding for each CDS contract of cleared and non-cleared entities. It captures contracts involved in a market risk transfer activity. The horizontal axis represents time in weeks where date 0 denotes the clearing event date. The dotted and solid lines represent the weekly average number of the trading activity measure of the treatment group and the control group respectively. Both groups are constructed using dynamic propensity score matching.

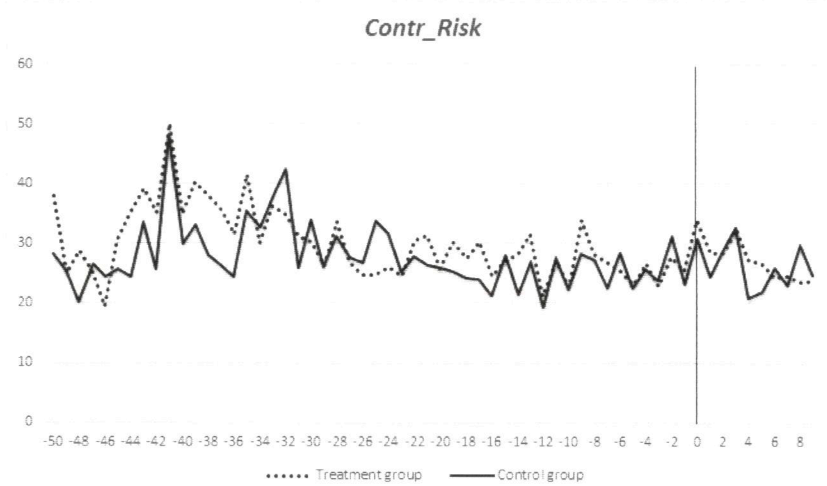
Panel A : Comparison of the number of contracts.



Panel B : Comparison of Gross notional amounts - Risk transfer.



Panel C : Comparison of the number of contracts - Risk transfer.



Chapter 6

Conclusion

This thesis proposes various contributions pertaining to the large family of interest-rate derivatives and securities. In a first part of the thesis, we address issues related to the evaluation and exercise strategies of interest-rate derivatives, while in the second part, we discuss questions raised by recent regulatory reforms concerning the CDS market.

A first contribution is a general pricing framework for low-dimensional derivatives under an affine jump-diffusion specification. We propose a flexible model that can be adapted to a large range of securities allowing for early-exercise opportunities. We show how to use transform analysis in order to adapt linear spline interpolation to the family of affine diffusion processes and how to adapt exercise and holding value functions to price a variety of products, namely bonds with embedded call or put options, European and Bermudan swaptions, and Treasury-bond futures. This general pricing framework can be used, for instance, to compare the value of derivatives under different model specification, or under different exercise strategies.

Accordingly, we use this pricing framework to analyse the sensitivity of derivative prices to the model specification. Specifically, we evaluate the pricing differences that could result from including or not jumps in the movement of interest-rates when pricing fixed-income securities. We perform numerical and empirical investigations in order to identify situations under which interest-rate jumps have a significant impact on derivatives prices. We consider various hypothetical shapes for the term structure and find that price differences can be particularly large when the yield curve is steep and upward sloping or when it has a humped shape. We also use historical observations of the yield curve to assess pricing differences with respect to observed market prices. Future research in that direction could be to investigate the impact of model specification on the sensitivity of calibrated parameters to option characteristics, such as maturity and moneyness.

We also evaluate to what extent the value of a derivative is sensitive to the use of sub-optimal strategies, particularly for complex products embedding multiple inter-related options. Specifically, we perform a post-trade evaluation of the exercise strategy of the U.S. T-bond futures. Since the contract involves several embedded options (quality option, timing option, wild card option, etc), choosing the delivery strategy is not straightforward. We compare the actual delivery behavior of the short traders to the optimal solution and find that the delivery strategies were historically very different. However, we find that the average gains and losses resulting for these different strategies over a 10-year period were very close, but that the optimal strategy implies a significantly lower outcome volatility. Future research arising from this essay could use intraday data for the spot price of the cheapest-to-deliver bond, allowing for a more accurate valuation of the wild card option.

Finally, we study the impact of the voluntary central clearing scheme on the market for single-name credit default swaps, which was introduced in order to mitigate counterparty credit risk and increase the market transparency. We revisit the conclusions obtained in the literature and find that some of these conclusions are reversed when the methodology is adapted to the dynamic nature of the data. Applying a dynamic propensity score matching allows us to better match the characteristics of cleared and un-cleared entities at the moment of adhesion to central clearing. We argue that the increase in CDS spreads following central clearing can be mainly attributed to a reduction in counterparty credit risk. An interesting future line of research related to this essay could be to investigate the impact of central clearing on other derivatives such as the interest-rate swaps and more importantly on the overall collateral demand in the market.

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