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Asset Prices in Granular Economies

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Résumé

Les investisseurs sur les marchés d'actions et d'obligations se soucient de l'état de la granularité de l'économie. En tant que variable d'état corrélée à des ensembles d'opportunités d'investissement futures, la granularité commande une prime dans la section transversale des rendements des actions et des obligations. De plus, en tant que facteur de risque commun aux marchés des actions et des obligations, la granularité correspond à des corrélations actions-obligations plus élevées. Dans cette thèse, nous construisons tout d'abord un cadre théorique et montrons que l'effet "faible risque" apparaît naturellement lorsque l'économie est granulaire. Ensuite, nous fournissons de nombreuses preuves empiriques que la granularité est un prédicteur négatif des excès de rendement futurs des actions et des obligations d'entreprises.

Mots-clés

Granularité, effet de faible risque, parier contre le bêta, CAPM conditionnel, obligations d'entreprise, corrélation actions-obligations, primes de risque, contagion financière.

Méthodes de recherche

Analyse transversale, analyse de portefeuille, développement de modèles théoriques.

Abstract

Investors in equity and bond markets care about the state of granularity of the economy. As a state variable that correlates with future investment opportunity sets, granularity commands a premium in the cross section of equity and bond returns. Further, as a common risk factor in equity and bond markets, granularity corresponds to higher equity-bond correlations. In this thesis, first, we build a theoretical framework and show that the 'low-risk effect' arises naturally when the economy is granular. Second, we provide ample empirical evidence that granularity is a negative predictor of future excess returns on equity and corporate bonds.

Keywords

Granularity, Low-risk effect, Betting against Beta, Conditional CAPM, Corporate Bonds, Equity-Bond Correlation, Risk Premia, Financial Contagion

Research methods

Cross-sectional analysis, Portfolio Analysis, Theoretical model development

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List of acronyms

ABBR Abréviation

BAA Baccalauréat en administration des affaires

DESS Diplôme d'études supérieures spécialisées

HEC Hautes études commerciales

MBA Maîtrise en administration des affaires

MSc Maîtrise

PhD Doctorat

To my mother and my late father

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General Introduction

A growing strand in literature examines the micro origins of macro fluctuations (Gabaix, 2011; Acemoglu et al., 2012; Herskovic et al., 2020). Contrary to the longstanding assumption that firm-specific risk will eventually average out in the aggregate (Lucas, 1977), the nascent literature uncovers ample evidence that firm-specific risk could propagate throughout the economy and lead to aggregate fluctuations when the linkages between firms nurture a network effect and/or when the distribution of firms' size is significantly heavy-tailed. The second channel is known as granularity. This thesis comprises two chapters that investigate the implications of granularity for asset prices.

Granularity portrays a state of the economy where a small number of firms represent a relatively substantial, hence incompressible, fraction of the economy. The state of granularity is measured similar to market concentration. We focus on one such measure that considers the market value of the largest 20 firms in the economy as fraction of total market value of all firms.

Throughout the first chapter, we shed fresh light on the low-risk effect in a granular economy. The low-risk effect refers to the inability of the Capital Asset Pricing Model (CAPM) to explain expected asset returns (Black, 1972). By the CAPM, expected returns are proportionate to the degree asset returns co-move with the market portfolio. Hence, higher expected returns must correspond to higher market betas. Empirical findings, however, reveal an inverse relation between betas and expected asset returns, known as the low-risk effect.

We argue that the low-risk effect arises naturally in a granular economy. To motivate this claim, we develop a theoretical economy with two firms, and derive equity betas as function of granularity. Higher granularity corresponds to higher betas for larger firms. On the other hand, in a more granular economy, the market portfolio, the value-weighted return on individual firms, is more representative of larger firms, which in turn indicates that these firms are more systematic. As a result, despite higher betas, investors view large firms safer, and ask a lower premium to hold them in their portfolios. The relation reverses for smaller firms that have lower betas while are perceived less systematic, therefore higher expected returns pair with lower market betas for these firms.

To corroborate the theoretical predictions of the model, we run a battery of tests based on US equity markets, and show that the slope of the Security Market Line (SML) is negatively related to news about the state of granularity of the economy. In months when granularity increases, the SML is flat even negatively slopes, hence the low-risk anomaly. In months when the state of granularity decreases, the SML is upward sloping and the predictions of the CAPM hold.

While the empirical findings of the first chapter are statistically persuasive, it is imperative to acknowledge the divergence between the predictions of the model where the level of granularity is the main driver of equity betas and expected returns, and the empirical analyses where we establish that changes in granularity matter to equity valuation. We hope that future explorations of the topic will bridge this limitation.

In the second paper, we study the cross section of corporate bond returns in a granular economy. We argue that as a state variable, granularity is negatively related to future investment and consumption opportunity sets. By Merton's ICAPM, any state variable that is correlated with future opportunity sets is priced in the capital markets (Merton, 1973).

The empirical investigation in this paper relies on daily transaction data from Enhanced TRACE, and bond characteristics from Mergent FISD. We compute monthly returns on individual corporate bonds throughout 2002 to 2019. The empirical tests in this paper are twofold, Fama-Macbeth cross sectional regressions, and portfolio analysis.

For the cross-sectional analysis we proceed by estimating individual bonds' betas with respect to granularity, measured as the market value of largest 20 firms as fraction of total market value of all firms where the value of the firm is the value of firm equity plus that of firm debt. The granularity measure is orthogonalized on common risk factors in the bond market. This procedure yields a panel of estimated betas.

Then, every month, we regress future excess returns on corporate bonds on granularity betas while controlling for other risk factors as well as firm characteristics. Repeating this procedure yields a time series of risk premia. We verify that the time series average of the granularity premium is always negative and economically significant. In other words, investors ask for a premium to hold bonds with negative granularity betas in their portfolios. These bonds are perceived riskier when the economy grows more granular.

A granularity strategy takes a long (short) positions in bonds with negative (positive) granularity betas. We form granularity portfolios by sorting individual bonds into deciles. The first (last) decile comprises bonds with negative (positive) granularity betas, which are estimated based on a similar procedure used in cross sectional analysis.

When we regress the excess return on the granularity strategy on risk factors in bond, equity, and bond and equity markets, the alpha of the strategy is always positive and economically significant, 6% on an annual basis. This finding is robust to weighting scheme, and it holds in double sorts with respect to credit ratings and time to maturity.

We also study the conditional correlation between equity and bond returns, and show that in months when the state of granularity of the economy increases, equity-bond correlations are on average 50% larger. Cross-sectional tests corroborate the hypothesis that higher equity-bond correlations are because granularity is similarly priced in the cross section of corporate bond as well as equity returns.

Chapter 1

Equity Prices in a Granular Economy

Abstract

This paper[1] revisits the properties of the conditional CAPM when the economy is granular. When some firms are more like 'grains' than atoms, shocks to such firms are not diversified away. When a large firm becomes larger, the economy becomes more granular, as the large firm represents a greater share of the market. This increase in granularity translates into a higher cross-sectional difference in equity betas, which reduces the slope of the Security Market Line (SML). We provide empirical support for the negative relation between the slope of the SML and various granularity measures from the U.S. equity market. When granularity decreases, portfolio betas are strongly and positively related to average equity returns. In contrast, the relation turns negative when granularity increases, thereby explaining the relatively 'flat' SML observed unconditionally.

1.1 Introduction

Modern asset pricing theory dictates that investors should be rewarded for bearing systematic risk. More than half a century after the seminal contributions of Sharpe (1964) and Lintner (1965), the literature on stock returns still offers no definitive answer regard-

[1] This article is co-authored with Harjoat Bhamra, Christian Dorion, and Alexandre Jeanneret

ing the nature of systematic risk. It might be a 400-dimensional animal, as suggested by the factor census of Harvey and Liu (2019). Or, as suggested by Kozak et al. (2018), a few factors might suffice, the "market" return being, comfortingly, the central one. Our work suggests that the distribution of firm sizes has profound implications on how one relates stock returns to this market factor. Many anomalies could arise from overlooking those implications.

As opposed to what is implicitly assumed in most theoretical asset pricing models, firms are not all atomistic. As documented by Axtell (2001) and Gabaix (2011), the distribution of firm sizes is fat-tailed, which implies that the world is best characterized as a granular economy. When some firms are more like grains than atoms, their impact on aggregate measures do not vanish as it may be the case for atomistic firms under the central limit theorem. We show that nontrivial impacts of firm-specific risk on equity prices arise endogenously in a granular economy.

We consider a two-firm economy, in the spirit of Cochrane et al. (2007), to study how the distribution of firm size generates asset pricing implications for all firms. Investors price firm-specific risk, as the aggregate shock to the economy is a combination of the firms' shocks. We analyze how the stock returns of the two firms interact with the market index composed of these stocks. The model helps understand the properties of the conditional CAPM. When firms are identical, they have the same market beta, and their alpha is zero. However, as firms are hit by idiosyncratic shocks, the economy deviates from this initial condition. The beta of a firm directly relates to the weight of this firm in the economy. A larger firm contributes more to the market, which increases its beta and reduces its alpha. In equilibrium, the opposite applies to the other firm as the value-weighted average beta must be one. So if the larger firm has a beta larger than one (and a negative alpha), the smaller firm must have a beta less than one (and a positive alpha). A shock to one firm thus affects the other firm in the opposite direction through market clearing conditions, which increases the dispersion in their systematic exposure to the market. The cross-sectional relationship between individual stocks' expected excess returns and their market beta thus weakens with the level of granularity in the economy, that is when a large firm becomes a greater contributor to the market.

Our empirical analysis confirms the prediction that the slope of the security market line (SML) is negatively related to the level of granularity in the equity market. We consider straightforward measures of granularity, such as the market capitalization of the (100 or 50) largest firms relative to that of the entire market. Our main finding is summarized in Figure 1.1. We plot the average returns of various test portfolios against their conditional market betas, conditioning on the monthly change in granularity. The slope of the SML is positive in times of lower granularity (red diamonds) and negative in times of lower granularity (blue points). We show using the Fama-MacBeth approach that the slope of the SML is statistically different across both subsamples. We obtain similar findings with alternative test assets, namely 25 equally-weighted and value-weighted beta portfolios, 48 industry portfolios, or 25 size and book-to-market portfolios. Overall, the negative relation between the conditional slope of the SML and granularity is statistically significant and robust to the choice of portfolios and granularity measures.

We verify that our results are not capturing alternative explanations suggested by the existing literature. First, we control for market (excess) returns, as Savor and Wilson (2014) find that the slope of the SML is particularly strong when macroeconomic news is scheduled for announcement, which correspond to large market return days. Second, we control for investor sentiment, as Antoniou et al. (2015) show that the slope of the SML is positive during pessimistic sentiment periods and negative during optimistic periods. Third, we control for inflation as money illusion, intensified by high inflation rates, affects the slope of the SML according to Cohen et al. (2005). Finally, we account for the impact of funding liquidity conditions on the slope of the SML using the TED spread following Antoniou et al. (2015). In all cases, the negative relation between the slope of the SML and our measures of granularity remains significant after controlling for these dimensions.

Building on our theoretical and empirical findings on the negative relation between the SML slope and granularity, we revisit one of the most studied implications of the 'too-flat' slope of the SML: the betting-against-beta (BAB) strategy. Frazzini and Pedersen (2014) show that a long position in low-beta assets and a short position in high-beta

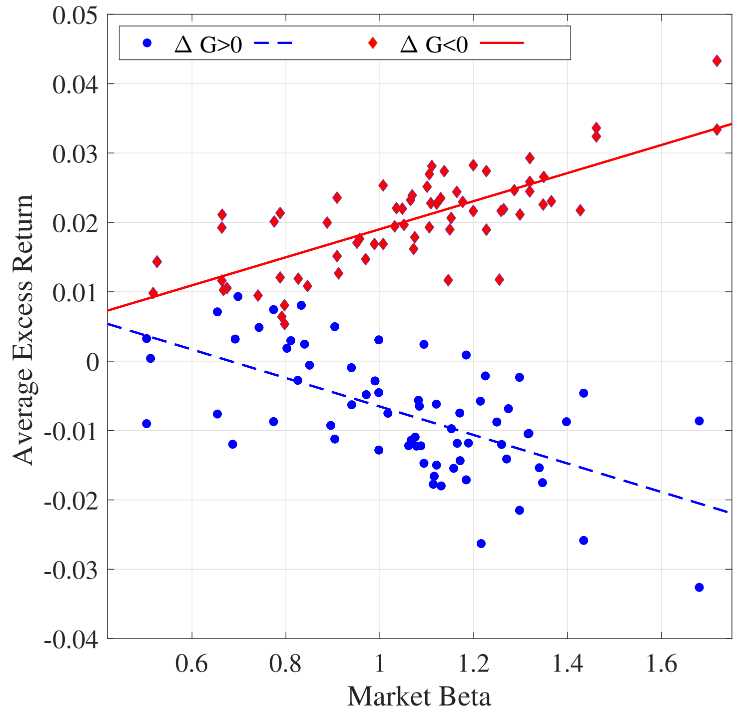

Figure 1.1: Variations in Granularity and the Conditional SML.

This figure shows the average conditional monthly returns against the previous month's average conditional market betas for 10 value-weighted and 10 equal-weighted beta portfolios, and 48 industry portfolios. We separate portfolio returns by months of increases ($\Delta G_t > 0$) and decreases ($\Delta G_t < 0$) in granularity, where granularity, G_t, is the market value of top 100 firms as fraction of total market capitalization. Monthly data span 1973-2018.

assets produces significant positive risk-adjusted returns. Our analysis predicts that such returns should be particularly high when granularity increases (and the slope of the SML decreases), while they should be reduced when granularity decreases. We provide strong evidence for this prediction and find that BAB returns are significantly and positively related to changes in granularity, even after controlling for alternative predictors. This analysis sheds new light on the conditional performance of the BAB strategy.

The contribution of this paper is to shed light on the asset pricing implications of granularity in the equity market. Firms are subject to specific shocks, i.e., news that are independent of other firms, and it is intuitive to believe that these shocks should diversify away. That is, such shocks should not have any global impact on stock market prices.

However, this view does not hold in a granular economy. For sufficiently large firms, a firm-specific shock becomes systematic in nature, as a large positive shock implies a greater market capitalization of the firm and, thus, an increase in the value of the stock market index. For example, the 5 largest firms dominating the tech sector in the U.S. (Amazon, Apple, Facebook, Netflix, and Microsoft) have recently represented more than a fifth of the total stock market capitalization of the S&P500. These tech giants have therefore contributed largely to evolution of the U.S. stock market in 2020, thereby providing a clear illustration of a granular equity market. The comovement between such firms' stock returns and the index return would suggest an increase in their systematic risk, as measured by the equity beta. As the average beta of the market must be one, the remaining firms in the market should experience a reduction in their beta. This mechanism illustrates how firm-specific shocks can propagate through the economy and potentially affect all firms.

Overall, both our model and our empirical analysis suggest that this propagation of shocks can help explain the failure of the CAPM, as captured by an empirical SML that is unconditionally "too flat" compared to what the CAPM predicts. We find that this feature of the data is an artifact of an economy in which firm-specific shocks are not diversified away and, thus, end up affecting asset prices globally through fundamental market clearing conditions. This paper thus revisits the conditional CAPM through the lens of a granular economy, providing novel insights on the fundamental relation between a firm's equity risk premium and systematic risk.

This paper proceeds as follows. Section 1.2 provides a summary of the literature, Section 1.3 describes the model and its predictions. Section 1.4 presents the data and the methodology, while Section 1.5 discusses the empirical results. Section 1.7 contains the concluding remarks.

1.2 Related Literature

This paper relates to two strands of the literature which study, first, the failure of the conditional CAPM and, second, the implications of firm-specific risk in a granular economy. We now discuss each body of the literature.

Stocks with lower systematic exposure tend to have a larger CAPM alpha, a phenomenon first documented by Black (1972) and referred to as the 'low-risk effect'. Several studies reconcile this observation with leverage constraints. When borrowing is constrained, investors willing to invest more in risky opportunities increase their exposure to systematic risk by tilting their portfolios toward high beta assets. These assets tend to underperform and involve lower alphas.

Based on this insight, Frazzini and Pedersen (2014) construct a model with constrained investors who bid up high beta assets to address their limited leverage to invest in rewarding opportunities. The authors document a significant return on a strategy that shorts high beta and holds low beta assets, the betting against beta (BAB) strategy. Similarly, Boguth and Simutin (2018) show that the tightness of leverage constraints is reflected in the average market beta of mutual funds. Jylhä (2018) finds that exogenous changes in the margin requirement corroborate the pricing implications of the constrained leverage story, predicting a flatter SML. Malkhozov et al. (2018) show how a measure of international illiquidity predicts BAB returns worldwide.

A parallel strand of literature associates the low-risk effect to behavioral explanations. Bali et al. (2017) show that the buying pressure exerted towards high beta stocks arises from lottery preferences of investors, while Liu et al. (2018) argue that this phenomenon only appears among over-priced stocks. Antoniou et al. (2015) associate this effect to investor sentiment: In optimistic periods, bullish trades distort prices, while prices are in line with the CAPM in pessimistic periods. The low-risk effect can also be attributed to aggregate disagreement among investors, which affects speculative demand for financial assets (Hong and Sraer, 2016).

From a different perspective, Buchner and Wagner (2016) link the presence of BAB returns to measurement errors. They argue that, due to optionality of equity, hence its non-linearity, equity returns should not be explained by a linear CAPM regression. Providing a review of different explanations of the low-risk effect, Asness et al. (2020) decompose the BAB strategy into Betting-against-Correlation (BAC) and Betting-against-Volatility (BAV). They show that BAC is more related to funding constraints, while BAV is more tied to sentiment. Finally, firm-specific risk can be priced when information is costly to acquire, as pointed by Merton (1987). Building on this insight, Campbell and Kassa (2018) argue that costly acquisition of information develops segmented markets. They show that returns on the BAB strategy are consistent with their story.

A direct consequence of the low-risk effect is that the SML is flatter than what the CAPM predicts. Therefore, the low-risk effect directly relates to the literature examining the drivers of the slope of the SML. Such determinants include money illusion among investors (Cohen et al., 2005), arbitrage (Huang et al., 2016), sensitivity of asset prices to macroeconomic announcements (Savor and Wilson, 2014), or the informational gap between investors and the econometrician (Andrei et al., 2018). We contribute to this literature by showing how the distribution of firm sizes, which we refer to as granularity, affects individual stocks' market beta, alphas, and eventually the slope of the SML.

The idea of considering a granular economy is not new. As firm size in the economy is not normally but power law distributed, the law of large numbers does not apply and firm-specific risk of relatively large firms (grains) becomes incompressible (Gabaix, 2011; Gabaix and Koijen, 2020). Granularity also implies that country size and trade affect macroeconomic volatility (di Giovanni and Levchenko, 2012), that large firms drive business cycles (Carvalho and Grassi, 2019), and that firm-specific shocks propagate through production networks and affect firms' sales growth and stock prices (Barrot and Sauvagnat, 2016).

Several studies focus on the asset pricing implications of granularity. Herskovic et al. (2016) find that systematic versus firm-specific risk are not easily distinguished, suggesting that the two are fundamentally linked and driven by a common component. Herskovic et al. (2020) explore the channels through which firm size distribution determines how firm-specific shocks propagate and affect firm volatilities. Granularity also offers a fresh angle and a theoretically sound basis to revisit the literature on whether firm-specific risk is priced (e.g. Ang et al., 2006; Fu, 2009; Bali et al., 2011). Building on this literature, we show that granularity in the U.S. stock market plays a fundamental role in driving the conditional CAPM.

1.3 Theory

We consider a simple economic environment to illustrate how granularity shapes the relation between the expected excess return of an asset and its market beta, which lies at the core of the capital asset pricing model (CAPM). In the spirit of Gabaix (2011), we refer to granularity as to an economy in which firms are not atomistic, i.e., firm-specific risk is not fully diversifiable.

1.3.1 Environment

The economy consists of two Lucas trees, which can be viewed as two individual firms (or industries). Each firm $i = \{A, B\}$ generates a stream of dividends X_i in the form of a consumption good, whose dynamics satisfies

$$\frac{dX_{i,t}}{X_{i,t}} = \mu_i dt + \sigma_i dZ_{i,t}, \tag{1.1}$$

where μ_i and σ_i reflect the expected growth and volatility, while $dZ_{i,t}$ is the incremental change of a standard Brownian motion defined on the probability space $(\Omega, \mathscr{F}, \mathbb{P})$. We assume that the processes are independent across firms.

We assume there exists an exogenous stochastic discount factor (SDF) π_t:

$$\frac{d\pi_t}{\pi_t} = -rdt - \eta_A \sigma_A dZ_A - \eta_B \sigma_B dZ_B, \tag{1.2}$$

where r is a constant risk-free rate and $\eta_i \sigma_i$ reflects the market price of risk for the shocks dZ_i, i.e., the agent prices both sources of shocks in the economy.[2] All parameters of the

[2] An economy with a representative agent consuming a Cobb-Douglas basket of the two goods can generate a stochastic discount factor of this form (see Appendix A.1.1).

economy are common knowledge.

1.3.2 Equity valuation

In the absence of debt, the equity value of firm i is given by

$$E_{i,t}(X) = \mathbb{E}^{\mathbb{Q}}\left[\int_t^{\infty} e^{-r(s-t)} X_{i,s} ds\right] = \frac{X_{i,t}}{r-\tilde{\mu}_i}, \tag{1.3}$$

where $\tilde{\mu}_i = \mu_i + \mathbb{E}\big(\frac{d\pi_t}{\pi_t}\frac{dX_{i,t}}{X_{i,t}}\big) = \mu_i - \eta_i \sigma_i^2$ is the expected growth rate under the risk-neutral probability measure $\mathbb{Q}$. Firm i's equity is then governed by the following dynamics:

$$\frac{dE_{i,t}}{E_{i,t}} = \mu_i dt + \sigma_i dZ_{i,t}. \tag{1.4}$$

and its expected excess return is given by

$$\mathbb{E}\big(\frac{dE_{i,t}}{E_{i,t}} - rdt\big) = -\mathbb{E}\left(\frac{d\pi_t}{\pi_t}\frac{dE_{i,t}}{E_{i,t}}\right) = \eta_i \sigma_i^2, \tag{1.5}$$

which implies an equity risk premium equal to the product between the market price of risk and the volatility of a firm's equity returns. Note that the equity risk premium is constant over time but varies across firms.

1.3.3 Equity Betas

In the spirit of the CAPM, we compute the equity beta of each firm, which corresponds to the exposure of its equity returns to the market index returns.

We first determine the dynamics of the market index, denoted by I_t, whose return is the value-weighted average of each firm's equity returns:

$$\frac{dI_t}{I_t} = w_{A,t}\frac{dE_{A,t}}{E_{A,t}} + w_{B,t}\frac{dE_{B,t}}{E_B} \tag{1.6}$$

$$= \mu_{I,t} dt + \sigma_{I,t} dZ_{I,t}, \tag{1.7}$$

where $w_{i,t}$ denotes the date-t weight of firm i's equity value into the market, which is defined as $w_{i,t} = \frac{E_{i,t}}{E_{A,t}+E_{B,t}}$. The expected return of the index is $\mu_{I,t} = w_{A,t}\mu_A + w_{B,t}\mu_B$, while its volatility is given by $\sigma_{I,t}$ and $dZ_{I,t}$ is the Brownian motion that characterizes shocks on the index, such that $\sigma_{I,t} dZ_{I,t} = w_{A,t}\sigma_A dZ_{A,t} + w_{B,t}\sigma_A dZ_{B,t}$.

The conditional equity beta of firm i's is given by

$$\beta_{i,t} = \frac{cov_t(\frac{dI_t}{I_t}, \frac{dE_{i,t}}{E_{i,t}})}{var_t(\frac{dI_t}{I_t})} = \frac{w_{i,t}\sigma_i^2}{w_{A,t}^2\sigma_A^2 + w_{B,t}^2\sigma_B^2}, \tag{1.8}$$

which implies that the beta of a firm fluctuates over time through variation in firms' market weights, $w_{A,t}$ and $w_{B,t}$. A change in a firm's weight in the market directly affects the covariance between individual equity and market returns, $cov_t(\frac{dI_t}{I_t}, \frac{dE_{i,t}}{E_{i,t}})$, and the variance of the market index returns, $var_t(\frac{dI_t}{I_t})$.

Equity betas, by construction, result in the market clearing conditions, that is at any date t, we have

$$w_{A,t}\beta_{A,t} + w_{B,t}\beta_{B,t} = 1, \tag{1.9}$$

as the beta of the market index must always be equal to one, by definition. Hence, all equity betas in the economy cannot jointly increase (or decrease) in equilibrium. That is, if a firm becomes more exposed to the market, the other firm must become less exposed, although its riskiness may not change fundamentally. This is a specific property of a granular economy, i.e., when firm weights are non-negligible, shocks on one firm affect the whole market. We hereafter exploit this property to shed new light on the relation between expected excess returns and betas, which is at the core of the CAPM paradigm.

1.3.4 Conditional CAPM

According to the CAPM, the market risk exposure of every firm is captured by its equity beta. In this case, the expected excess returns of individual assets are simply their equity beta times the market risk premium. We can thus express the conditional CAPM as follows:

$$\mathbb{E}\left(\frac{dE_{i,t}}{E_{i,t}} - rdt\right) = \alpha_{i,t} + \beta_{i,t}\mathbb{E}\left(\frac{dI_t}{I_t} - rdt\right), \tag{1.10}$$

where $\alpha_{i,t}$ denotes firm i's equity alpha, which is equal to zero if the CAPM holds perfectly. Put it differently, the central cross-sectional implication of the CAPM is that the individual firms' equity risk premia only vary with their betas.

The premise that investors price systematic market risk only, as in the CAPM, is at the core of modern asset pricing. We now discuss how the conditional CAPM no longer holds in an economy that consists of non-atomistic firms, in the spirit of Gabaix (2011). We first study the implications of such an economy on firms' equity alphas and betas, and eventually on the slope of the security market line (SML).

1.3.5 Predictions

In this section, we study the asset pricing implications of the CAPM in a granular economy. We hereafter assume that $X_A > X_B$ and $\eta_A > \eta_B$, such that the larger firm (A) displays a higher equity risk premium (Equation 1.5), has a higher weight in the market index, and thus has a higher beta (Equation 1.8) than the other firm (B). Firms are otherwise identical.

Non-zero equity alphas

In the economy, the agent prices securities by their exposure to the SDF, which is determined by a linear combination of the firm shocks. The first expectation of Equation (1.10), which is the expected excess equity return for firm i, is

$$\mathbb{E}\big(\frac{dE_{i,t}}{E_{i,t}} - rdt\big) = \eta_i \sigma_i^2, \tag{1.11}$$

whereas the expectation on the right-hand side of Equation (1.10), which is the expected excess return of the market index, equals

$$\mathbb{E}\big(\frac{dI_t}{I_t} - rdt\big) = \eta_A \sigma_A^2 w_{A,t} + \eta_B \sigma_B^2 w_{B,t}. \tag{1.12}$$

Substituting Equations (1.11) and (1.12) in Equation (1.10) yields the following date-t equity alphas of firm i:

$$\alpha_{i,t} = \eta_i \sigma_i^2 - \beta_{i,t}\big(\eta_A \sigma_A^2 w_{A,t} + \eta_B \sigma_B^2 w_{B,t}\big), \tag{1.13}$$

which shows that a firm's equity alpha, $\alpha_{i,t}$, will generally differ from zero.

The mechanism driving non-zero alphas is as follows. While the CAPM predicts that the expected excess equity return of a firm i is increasing in its beta, the 'true' equity risk premium is in fact constant, being equal to $\eta_i \sigma_i^2$. Hence, a relatively high-beta firm would be characterized as a negative-alpha firm, such that the expected excess equity return remains unchanged. In equilibrium, a firm i's equity alpha therefore varies negatively with its equity beta, both over time and in the cross-section. In addition, market clearing conditions imply that if one firm has a positive alpha, the other firm must have a negative alpha given that that the market index, which is the value-weighted average of the two firms, has no alpha by definition. Hence, there exists a tight connection across firms' alphas.

Conditional equity betas and alphas

We now discuss the implications for the conditional equity betas and alphas. The beta of a firm directly relates to its weight in the market. Following Equation (1.8), the beta of a firm i becomes zero as its equity weight in the market portfolio converges to zero, as the shocks are independent to those of the market index. Observe that, in this case, the alpha of this firm equals exactly $\eta_i \sigma_i^2$, which corresponds to the equity risk premium.

As the firm becomes larger, it naturally contributes more to the total market and its beta begins to increase correspondingly, as illustrated by Panel A of Figure 1.2. The equity beta first increases and then declines until it equals one again, i.e., when this firm becomes the entire market. Similarly, the firm's alpha decreases with the firm's market weight, becoming negative, and then reverts to increase to 0 as the weight converges to 1, as illustrated by Panel B of Figure 1.2. Hence, both a firm's alpha and beta are non-monotonically related to the weight of the firm in the market.

FIGURE 1.2 ABOUT HERE

In equilibrium, the opposite applies to the other firm as the value-weighted average beta must be one. So if a firm has a beta larger than one, the other firm must have a beta less than one. Similarly if a firm has a positive alpha, the other firm must have a a negative alpha. A shock to one firm thus affects the other firm in the opposite direction through market clearing conditions, as illustrated in Figure 1.2. Observe that the cross-sectional difference in equity alphas and betas increases as the economy becomes dominated by a single firm.

Conditional Slope of the SML

We now discuss how the slope of the SML varies with the relative weights of firms in the market, that is when the granularity of the economy changes. One can directly interpret the weight of the larger firm as a measure of granularity (Gabaix, 2011). Granularity increases when the larger firm experiences a positive dividend shock and thus contributes relatively more to the market index.

The conditional slope of the SML, denoted by S_t, is given by

$$S_t = \frac{\mathbb{E}\left(\frac{dE_{A,t}}{E_{A,t}} - rdt\right) - \mathbb{E}\left(\frac{dE_{B,t}}{E_{B,t}} - rdt\right)}{\beta_{A,t} - \beta_{B,t}}, \tag{1.14}$$

$$= \frac{\left(\eta_A \sigma_A^2 - \eta_B \sigma_B^2\right)\left(w_{A,t}^2 \sigma_A^2 + w_{B,t}^2 \sigma_B^2\right)}{w_{A,t} \sigma_A^2 - w_{B,t} \sigma_B^2}, \tag{1.15}$$

which indicates that the slope of the SML varies over time depending on the firms' relative equity betas. It is interesting that the slope of the SML is solely driven by the dispersion of equity betas, hence the denominator of the equation. Note that if both firms have identical risk premia ($\eta_A \sigma_A = \eta_B \sigma_B$), the SML is exactly flat, as the numerator of Equation (1.15) is zero. By contrast, an economy with $\eta_A \sigma_A > \eta_B \sigma_B$ and $\beta_A > \beta_B$ generates a positive slope of the SML, as the firm with the higher expected excess return also has a higher beta.

Panel A of Figure 1.3 shows that the SML slope decreases with the weight of firm A, while the opposite applies to the intercept of the SML, equal to $\left(\eta_A \sigma_A^2 w_{A,t} + \eta_B \sigma_B^2 w_{B,t}\right) - S_t$ (Panel B). As the larger asset (firm A) becomes a greater contributor to the market due to a positive dividend shock, its equity beta increases, whereas the equity beta of firm B decreases through market clearing conditions. However, fundamentally, the 'true' expected excess returns of these firms are independent of their equity betas. As a result, a positive shock on the weight of firm A amplifies the cross-sectional difference in betas ($\beta_{A,t} - \beta_{B,t}$) but not the difference in expected excess returns, thus flattening the SML. Hence, when granularity increases (the larger firm becomes larger), the relation between expected excess equity returns and equity betas weakens, as illustrated in Figure 1.4.

FIGURES 1.3 AND 1.4 ABOUT HERE

In sum, we show that the slope of the SML decreases with the weight of the larger firm in the economy, which reflects a more granular market. This firm's specific shocks become a greater determinant of the market returns, which affects the levels of systematic risk of all firms in equilibrium. The cross-sectional relationship between individual assets' expected excess returns and their exposure to the market thus weakens. The central theoretical prediction is therefore that the slope of the SML becomes negatively related to the level of granularity in the economy, which we aim to verify empirically in the following sections.

1.4 Data & Methodology

This section first details the data we employ in our empirical study and then describes the main methodology.

1.4.1 Portfolio Returns

We obtain stock and Treasury bond return data, spanning January 1973 to December 2018, from the Center for Research in Security Prices (CRSP).[3] The value-weighted index of

[3]CRSP starts including firms traded on NYSE American and NASDAQ in 1962 and 1972, respectively. The latter almost doubles the number of firms in the CRSP universe. We consider data from 1973 to avoid having our analysis contaminated by substantial changes in the number of firms.

all listed shares (NYSE, Amex, and Nasdaq) is our stock market proxy. To obtain returns on beta-sorted portfolios, we first estimate (pre-formation) betas for each individual stock using 60 months of monthly returns and sort stocks into 25 portfolios according to their beta. We then compute returns on value-weighted and equal-weighted portfolios. We also retrieve returns on alternative test assets (e.g., 25 size and book-to-market, 48-industry portfolios) from Kenneth French's website.

1.4.2 Granularity

We consider various measures of granularity in the equity market. Our approach builds on Gabaix (2011), who defines granularity in the economy as the sum of sales of the top 100 firms as a fraction of the GDP. We adapt this measure to the U.S. equity market and measure granularity, denoted by G_t, as the market capitalization value of the top 100 firms as a fraction of the total market capitalization from CRSP. We alternatively consider the market capitalization value of the top 50 and top 1% of firms as fraction of total market capitalization. In addition, we compute a measure of market concentration using the excess Herfindahl-Hirschman Index (HHI) proposed by Gabaix and Koijen (2020). These measures are defined in Table 1.1, while Figure 1.5 plots their time series. A higher value of an index means that the U.S. market is more granular.

TABLE 1.1 AND FIGURE 1.5 ABOUT HERE

Panel A in Table 1.2 presents the descriptive statistics for our measures of granularity. The market weight of the top 100 firms represents 45% of the total market, on average, and ranges between 39% and 55%. There is a high correlation between these measures, as reported in Panel A of Table 1.3.[4] We can thus conclude that these alternative measures capture similar information.

[4]Panel B of Table 1.3 shows correlation coefficients between granularity and various variables that are known to predict the slope of the SML. Granularity appears to be weakly correlated with investor sentiment (0.03), inflation (-0.12), market returns (-0.27), and the TED spread (0.27), whose roles are discussed in Section 1.5.

TABLES 1.2 AND 1.3 ABOUT HERE

1.4.3 Methodology

We test the theoretical prediction that the slope of the SML decreases with positive shocks to granularity. We provide three types of analysis. First, we separate times of increases and decreases in granularity and plot the conditional SML. Second, we conduct a Fama-MacBeth estimation and test the difference in slope across both subsamples. Third, we use a regression analysis to exploit the time-series of the SML slope and study how the slope varies with granularity after controlling for alternative explanations.

For each portfolio, we compute the conditional (post-formation) betas over rolling a 60-month-windows using monthly returns. Specifically, we estimate the CAPM beta $\beta_{p,t}$ of portfolio p in month t by estimating the regression

$$R_{p,\tau} - R_{f,\tau} = \alpha_{p,t} + \beta_{p,t}(R_{mkt,\tau} - R_{f,\tau}) + \varepsilon_{p,\tau}, \tag{1.16}$$

where $R_{p,\tau}$ is the return on portfolio p at time $\tau \in \{t-59, t\}$, $R_{f,\tau}$ is the risk-free rate given by the 1-month Treasury bill return, and $R_{mkt,\tau}$ is the market return. This procedure yields a times series of conditional beta estimates for each portfolio, $\hat{\beta}_{p,t}$.

As a first exercise, we compute the average conditional betas for every test asset, $\hat{\beta}_p^H$ and $\hat{\beta}_p^L$, where the superscript H and L denote the months when granularity increases ($\Delta G_t > 0$) and decreases ($\Delta G_t < 0$), respectively. We then compute the corresponding average conditional portfolio returns over the following month, R_p^H and R_p^L. Figure 1.6 plots R_p^H against $\hat{\beta}_p^H$ and R_p^L against $\hat{\beta}_p^L$ for different test assets.

FIGURE 1.6 ABOUT HERE

We then present results using the classic two-step testing procedure for the CAPM. For the second-stage regressions, we adopt the Fama-MacBeth procedure and compute coefficients separately by estimating, for each month t, the following cross-sectional regressions:

$$R^H_{p,t} - R^H_{f,t} = a^H + \gamma^H \hat{\beta}^H_{p,t-1} + \varepsilon^H_p \tag{1.17}$$

and

$$R^L_{p,t} - R^L_{f,t} = a^L + \gamma^L \hat{\beta}^L_{p,t-1} + \varepsilon^L_p, \tag{1.18}$$

where H (L) denotes months with increases (decreases) in granularity, i.e., $\Delta G_t > 0$ ($\Delta G_t < 0$). We calculate the sample coefficient estimates, $\bar{\gamma}^H$ and $\bar{\gamma}^L$, as the average across time of the cross-sectional estimates, while their standard error equal the time series standard deviation of the cross-sectional estimates divided by the square root of the respective sample lengths. We can thus test whether the difference in coefficient estimates is statistically significant by applying a simple t-test for a difference in means. Table 1.4 reports the results for different test portfolios in Panel A and using various granularity measures in Panel B.

TABLE 1.4 ABOUT HERE

As a last exercise, we quantify the conditional slope of the SML and study its relation with granularity after controlling for alternative explanations. Specifically, we first estimate, for every month t, the slope of the SML with a cross-sectional regression of portfolio excess returns on their beta obtained in the previous month:

$$R_{p,t} - R_{f,t} = a_{0,t} + \gamma_t \hat{\beta}_{p,t-1} + \varepsilon_{p,t}. \tag{1.19}$$

We then regress the estimates of the slope of the SML, denoted by $\hat{\gamma}_t$, on changes in granularity ΔG_t, controlling for existing predictors, such as market returns (Savor and Wilson, 2014), inflation (Cohen et al., 2005), or funding liquidity (Frazzini and Pedersen, 2014). Table 1.5 reports the results, using t-statistics based on Newey-West standard errors with 12 lags.

TABLE 1.5 ABOUT HERE

1.5 Main Results

This section presents and discusses the main empirical results of the paper. We study the conditional slope of the SML, conduct a robustness analysis using different test portfolios, and discuss the results when we control for alternative explanations.

1.5.1 Conditional SML slope

Figure 1.6 plots average realized excess returns for 25 beta-sorted (Panel A) and 48 industry (Panel B) portfolios against their conditional portfolio betas separately for increases ($\Delta G_t > 0$) and decreases ($\Delta G_t < 0$) in granularity. The slope of the SML is positive in times of increasing granularity and negative in times of decreasing granularity, whether we consider beta-sorted portfolios or industry portfolios. Figure 1.7 shows that the results are robust to using alternative granularity measures to condition the analysis. Consistent with our theoretical prediction, the slope of the SML is negatively related to granularity.

FIGURE 1.7 ABOUT HERE

Table 1.4 reports the average of the conditional slope of the SML, estimated with Equations (1.17) and (1.18), and the difference of the two, with the t-statistics reported in parentheses. Panel A reports the results for different test assets. Column 1 uses 25 equally-weighted beta portfolios (our benchmark case), Column 2 uses 25 value-weighted beta portfolios, Column 3 uses 48 industry portfolios, Column 4 uses 25 size and book-to-market portfolios, while Column 5 uses a mix of 10 equally-weighted, 10 value-weighted beta, 10 industry, and 6 size and book-to-market portfolios. We find that the slope of the SML is always positive when the U.S. market becomes less granular (i.e., the largest firms playing a smaller role in terms of market capitalization), but the slope of the SML turns negative in times of higher granularity. A test for the difference, based on the t-test comparing means between months of increase vs. decrease in granularity, indicates that the slope of the SML is statistically different across both subsamples.

Panel B reports the conditional slope of the SML using different measures of granularity. The difference in the conditional means is always significantly different from zero and with the expected sign. Hence, the negative relation between the conditional slope of the SML and granularity is statistically significant and robust to the choice of test assets and granularity measures.

1.5.2 Controlling for alternative explanations

We now consider a regression specification to study how the slope of the SML varies with granularity once we account for alternative explanations. We first estimate the conditional slope of the SML by regressing equally-weighted monthly returns of 25 beta portfolios at time t on the market betas of the same portfolios at time $t-1$, according to Equation (1.19). We then regress the SML slope estimates on changes in granularity and report the results in Table 1.5. Column 1 presents the results without controls. In Column 2 through 4, we increment the specification by including various control variables. All variables are standardized to facilitate the interpretation of their coefficients.

The results indicate that the slope of the SML decreases with granularity, computed as change in the market capitalization of the top 100 firms as a fraction of the total market capitalization in the U.S. The regression coefficient equals -0.445 with a t-statistic of -7.80, which is both statistically and economically significant. A one-standard-deviation increase in granularity implies a decrease in the slope of the SML by almost one half (0.445) of a standard deviation in the slope.

We now verify that the role of granularity is not subsumed by alternative explanations, as suggested by the existing literature. First, we control for market (excess) returns, as Savor and Wilson (2014) find that the slope of the SML is particularly strong when macroeconomic news is scheduled for announcement, which corroborate with large market return days. Column 2 of Table 1.5 shows that the effect of granularity decreases by almost one half but remains highly statistically significant. Hence, we can safely rule out the possibility that variations in granularity are merely capturing return fluctuations of the market. Note that we do not need separate days with and without macroeconomic announcements, following Savor and Wilson (2014), as our analysis uses monthly data.

Another explanation for the time-variation in the SML slope is money illusion. Modigliani and Cohn (1979) argue that inflation, by driving a wedge between nominal versus real discount rates, brings about major errors in how investors price equity. Based on the same argument, Cohen et al. (2005) hypothesize that money illusion, intensified by high inflation rates, affects the slope of the SML. They show that the slope of the SML preceded by low inflation months is steeper than the slope of the SML preceded by high inflation months. Following Cohen et al. (2005), we control for lagged inflation using monthly changes in the producer price index. Yet, the impact of granularity remains unchanged, as indicated by Column 3 of Table 1.5.

Finally, we account for changes in funding liquidity conditions. Frazzini and Pedersen (2014) use the TED spread as a measure for funding conditions and show that it is negatively correlated with contemporaneous returns on the betting-against-beta (BAB) strategy. We include the TED spread in our set of control variables to account for the potential impact of funding conditions on the slope of the SML. This control has no effect on the role of granularity, as evidenced by Column 4 of Table 1.5.

Alternatively, Antoniou et al. (2015) show that the slope of the SML is positive during pessimistic sentiment periods and negative during optimistic periods. Optimism attracts equity investment by less sophisticated traders in risky opportunities (high beta stocks), while such traders stay along the sidelines during pessimistic periods (see, e.g., Grinblatt and Keloharju (2001); Lamont and Thaler (2003)). Thus, high beta stocks become overpriced in optimistic periods, which induces the negative slope of the SML. Following Antoniou et al. (2015), we use the Baker and Wurgler (2006)'s index of investor sentiment, which we obtain from the authors' website. In Table 1.6 (1.7) we regress the SML slope estimates on changes in granularity when the investor sentiment index is positive (negative). The sign and statistical significance of estimated coefficients remain consistent with Table1.5. These findings suggest that granularity does not reflect high versus low sentiment periods, as the coefficient of interest remains very similar across the three tables.

Overall, the negative relation between the slope of the SML and granularity remains significant after controlling for existing explanations such as aggregate market fluctuations, money illusion, investor sentiment, and funding liquidity conditions. In addition, the results are robust to using the SML slope estimated with the cross-section of individual stocks (Table 1.8–1.10) instead of the cross-section of 25 portfolios.

1.6 Revisiting Betting Against Beta

In this section, we revisit one of the most studied implications of the 'too-flat' slope of the SML, which is known as the betting-against-beta (BAB) strategy. Frazzini and Pedersen (2014) show that a long position in low-beta assets and a short position in high-beta assets produces significant positive risk-adjusted returns. Our theoretical analysis predicts that such returns should be particularly high when granularity increases, while they should be reduced when granularity decreases. We now test this prediction and shed new light on the conditional performance of the BAB strategy.

1.6.1 Conditional Beta-sorted Portfolio Alpha

We start by studying the conditional performance of 10 beta-sorted portfolios with respect to granularity. Consistent with the rest of the paper, we first use the (pre-formation) betas estimated for each individual stock using 60 months of monthly returns. We then sort stocks into 10 portfolios according to their beta and compute the equally-weighted returns for each portfolio. Following prior work, we exclude stocks with prices below $5 to ensure that results are not driven by small, illiquid stocks.

The unconditional CAPM alpha of a portfolio is the intercept of a regression of a portfolio's excess return on the market excess return over the whole sample. For the conditional analysis, we first split the portfolio returns into two subsamples corresponding to months when granularity decreases ($\Delta G_t < 0$) and increases ($\Delta G_t > 0$). Then, we estimate the intercept of the regression based on each subsample.

Figure 1.8 illustrates the annualized CAPM alphas of each portfolio in the unconditional case (Panel A), when granularity increases (Panel B), and when granularity decreases (Panel C). The results reproduce the typical betting-against-beta pattern in the unconditional estimation: the low-beta portfolios exhibit positive alphas, while the high-beta portfolios exhibit negative alphas. However, the relation varies according to granularity, as we find a negative (positive) relation between CAPM alphas and corresponding betas in Panel B (Panel C). Note that alphas are on average negative in Panel B, which indicates that relatively-small stocks underperform the market when granularity increases, i.e., when the larger firms become even larger and thus overperform the market. The opposite applies to Panel C, when granularity decreases.

FIGURE 1.8 ABOUT HERE

We present the results of the 10 beta-sorted portfolios and assess their statistical significance in Table 1.11. The first two rows report the post formation market betas and time series averages of monthly excess returns for each of the portfolios. We then report the unconditional (Panel A) and the conditional (Panel B) portfolio alphas. The rightmost column presents the difference in estimates between the top beta and the bottom beta portfolios, i.e., P_{10}-P_1. In Panel C, we conduct a robustness analysis when we orthogonalize changes in granularity to excess market returns, thus avoiding that our conditioning analysis merely reflects good vs. bad market days. The conditional results reported in Panels B and C of Table 1.11 are qualitatively similar. In both cases, we find that the high-beta portfolio (P10) alpha is statistically lower than the low-beta portfolio (P1) alpha when granularity increases, but the relation becomes statistically insignificant (and of the opposite sign) when granularity decreases. That is, the classic BAB pattern appears to be concentrated in times of increasing granularity.

TABLE 1.11 ABOUT HERE

1.6.2 Explaining BAB Returns with Granularity

In this section we examine the conditional performance of the long-short BAB strategy, which we construct as buying the low-beta portfolio (P_1) and selling the high-beta portfolio (P_{10}). That is, the long portfolio is an equal-weighted average of the bottom decile stocks, when stocks are ranked according to their beta, whereas the short portfolio is an equal-weighted average of the top decile stocks.

We first compute the conditional average return of the BAB strategy. The first two rows in Table 1.12 show the average BAB return for months when granularity increases ($\Delta G_t > 0$) and decreases ($\Delta G_t < 0$), respectively. We use four different granularity measures corresponding to each of the columns and report, in each case, the average BAB return (with t-statistics in parentheses). The different granularity measures are based on the market value of the top 100, 50, and 1% largest firms as fraction of total market capitalization, as well as the excess Herfindahl-Hirschman Index (exHHI). The last row reports the difference between the conditional averages and the corresponding t-statistics. In all cases, the BAB strategy yields a positive average return when granularity increases and a negative average return when granularity decreases. Hence, the performance of the BAB strategy appears to be highly related to granularity.

TABLE 1.12 ABOUT HERE

We then examine how the return on this BAB strategy relates to changes in granularity by estimating the following regression:

$$r_{BAB,t} = b + \beta_G \Delta G_t + \mathbf{X}_t' \beta_C + \varepsilon_t, \tag{1.20}$$

where the BAB return $r_{BAB,t}$ is regressed on granularity changes ΔG_t, while $\mathbf{X}_t$ is a vector of financial conditions that we use as control variables. The set of controls includes excess market returns, lagged inflation, lagged BAB returns, and the TED spread. Table 1.13

presents the results, which indicate that BAB returns are significantly and positively related to changes in granularity, even after controlling for alternative predictors. The effect is statistically significant and of the expected sign.

In sum, we provide evidence that the performance of the BAB strategy appears to be particularly high when granularity in the U.S. stock market increases, which is when the slope of the SML decreases, as predicted by our theory.

TABLE 1.13 ABOUT HERE

1.7 Conclusion

This paper shows that granularity drives the joint dynamics of asset prices. According to the CAPM, firm-specific risk is fully compressed and systematic risk is the sole determinant of the equity risk premium. However, investors consider the granular context of financial markets to form their expectations about the risk-return trade-off. They ask for a premium to hold assets with incompressible firm-specific exposure. We construct a two-firm economy and show that the equity beta are functions of the relative weight of the firms in the economy.

We show that assuming that the CAPM holds reveals an alpha different from zero. Larger firms are more correlated with the market, have larger betas, and, hence have smaller alphas. Unless the pricing kernel compensates firm-specific risk, we find that the CAPM alpha is always different from zero. This mechanism arises naturally in a granular economy. Accordingly, the empiricist's CAPM-implied SML is always flatter than what theory predicts and the slope of the SML is negatively correlated with granularity. We develop an empirical analysis that confirms the predictions of our theory. The data suggest a significant and negative relation between various measures of granularity and the slope of the SML.

1.8 Tables

Table 1.1
Definition of Variables

This table defines the variables underpinning this study and the corresponding data sources. All series are retrieved monthly.

Variable	Definition	Source
G	Degree of granularity measured as the total market capitalization value of the largest firms in the U.S. in percentage of the total market capitalization of the CRSP universe. We use either the top 50, 100, or 1% of the firms with the largest market capitalization. Alternatively, we consider the excess Herfindahl-Hirschman Index proposed by Gabaix and Koijen (2020), defined as exHHI = $\sqrt{-\frac{1}{N}+\sum_1^N w_i^2}$, where w_i the market value of firm i to total market capitalization and N is the total number of firms.	Wharton Research Data Services
R_m - R_f	Market excess return, computed as the value-weighted return on the CRSP universe.	Wharton Research Data Services
Sentiment	Sentiment composite index based on the common variation in six underlying proxies for sentiment: the closed-end fund discount, NYSE share turnover, the number and average first-day returns on IPOs, the equity share in new issues, and the dividend premium.	Baker and Wurgler (2006)'s website
TED	The TED spread, measured by difference between the three-month Treasury bill and the three-month LIBOR rates based in US dollars.	Federal Reserve Bank of Saint Louis
Inflation	Inflation measured as the exponentially weighted average (36-month window) of the log growth rates on the monthly Producer Price Index (PPI) for all commodities, following Cohen et al. (2005).	Federal Reserve Bank of Saint Louis

Table 1.2
Summary Statistics

This table presents the summary statistics of the main variables. Panel A considers different measures of granularity, including the market capitalization value of the 100, 50, and 1% largest firms in fraction of total market capitalization of the CRSP universe, and the excess Herfindahl-Hirschman index (exHHI) defined in Gabaix and Koijen (2020). Panel B reports the estimated slope and intercept of the security market line (SML), while Panel C presents the statistics for the control variables used in our regression analysis. The slope and intercept of the SML are estimated from Fama-MacBeth regressions based on 25 equally-weighted beta portfolios. Beta portfolios and granularity measures are computed using monthly individual stock information from CRSP. Detailed description of the control variables is provided in Table 1.1. Data span January 1973 to December 2018.

Measure	Min	Max	Mean	Med	Std	1%	25%	75%	99%	Skw	Kur
Panel A: Granularity measures											
Top100	0.39	0.55	0.45	0.44	0.04	0.40	0.42	0.48	0.54	0.74	2.47
Top50	0.28	0.43	0.34	0.33	0.04	0.29	0.31	0.37	0.42	0.63	2.41
Top1%	0.32	0.52	0.39	0.37	0.04	0.32	0.36	0.40	0.51	1.28	4.31
exHHI	0.05	0.09	0.06	0.06	0.01	0.05	0.06	0.07	0.09	0.75	2.27
Panel B: Slope and intercept of the SML											
Slope	-0.23	0.22	-0.00	-0.00	0.06	-0.14	0.04	0.03	0.18	0.12	4.33
Intercept	-0.15	0.12	0.01	0.01	0.03	-0.09	-0.01	0.03	0.08	-0.63	4.99
Panel C: Control variables											
R_m - R_f	-0.22	0.17	0.01	0.01	0.04	-0.11	-0.01	0.04	0.12	-0.41	4.89
Sentiment	-2.44	3.20	0.01	0.05	0.89	-2.25	-0.30	0.52	2.34	-0.29	4.18
Inflation	-0.64	0.69	0.04	0.03	0.12	-0.30	-0.02	0.08	0.35	-0.28	9.71
TED(%)	0.12	3.15	0.58	0.46	0.44	0.13	0.26	0.72	2.30	2.10	9.33

Table 1.3
Correlation between Variables

This table presents the correlations among the main variables. Panel A shows the correlations between changes in different measures of granularity, including the market capitalization value of the 100, 50, and 1% largest firms in fraction of total market capitalization of the CRSP universe, and the excess Herfindahl-Hirschman index (exHHI) defined in Gabaix and Koijen (2020). Panel B displays the correlations between changes in granularity (using the top 100 firms), the excess market return (R_m - R_f), the investor sentiment of Baker and Wurgler (2006), inflation as in Cohen et al. (2005), and the TED spread following Antoniou et al. (2015), which are used as control variables in the regression analysis. Detailed description of the control variables is provided in Table 1.1. We obtain individual stock information from CRSP. Monthly data span January 1973 to December 2018.

Panel A: Change in granularity measures				
Δ Measure	ΔTop100	ΔTop50	ΔTop1%	ΔexHHI
ΔTop100	1	0.97	0.96	0.78
ΔTop50		1	0.97	0.84
ΔTop1%			1	0.80
ΔexHHI				1

Panel B: Granularity and control variables					
Variable	ΔG	R_m - R_f	Sentiment	Inflation	TED
ΔG	1	-0.27	0.02	-0.12	0.24
R_m - R_f		1	-0.09	0.06	-0.15
Sentiment			1	-0.08	0.06
Inflation				1	-0.16
TED					1

Table 1.4
Conditional SML Slope from Fama-MacBeth Regressions

This table reports estimates of the conditional slope of the security market line (SML) from Fama-MacBeth regressions. Every month, excess portfolio returns are regressed on post-formation market betas of the same portfolios from the previous month. The table reports the average of the SML slope coefficient estimates when granularity increases ($\hat{\gamma}^H$) or decreases ($\hat{\gamma}^L$) in a given month. The difference in estimates is reported in the last row, where t-statistics (in parentheses) are calculated using the standard deviation of the time series of the coefficient estimates. In Panel A, the average slope of the SML is estimated across five different test assets: 25 equally-weighted (Column 1) and 25 value-weighted (Column 2) beta portfolios, 48 industry portfolios (Column 3), 25 size and book-to-market portfolios (Column 4), and a portfolio composed of 10 value-weighted, 10 equally-weighted beta portfolios, 10 industry portfolios, and 6 size and book-to-market portfolios (Column 5). Panel B reports results for different granularity measures when the test asset is the 25 equally-weighted beta portfolio. The granularity measures are the market capitalization value of the top 100 (Column 1), top 50 (Column 2), and top 1% (Column 3) largest firms in fraction of total market capitalization, and the excess excess Herfindahl-Hirschman index (exHHI) defined in Gabaix and Koijen (2020) (Column 4). Beta portfolios are formed based on monthly individual stock returns obtained from CRSP. All other portfolio returns are from Kenneth French's data library. Monthly data span January 1973 to December 2018. *, **, and *** indicate statistical significance at the 10%, 5%, and 1% level, respectively.

Panel A: Conditional mean of SML slope across test assets

	(1)	(2)	(3)	(4)	(5)
$\hat{\gamma}^H$	-0.265***	−0.135***	−0.169***	−0.332***	−0.202***
	(-6.25)	(−2.93)	(−4.32)	(−6.27)	(−4.67)
$\hat{\gamma}^L$	0.211***	0.316***	0.169***	0.244***	0.257***
	(5.10)	(7.40)	(4.51)	(5.10)	(6.36)
$\hat{\gamma}^H - \hat{\gamma}^L$	-0.476***	−0.452***	−0.337***	−0.575***	−0.458***
	(-8.03)	(−7.18)	(−6.24)	(−8.08)	(−7.76)

Panel B: Conditional mean of SML slope across granularity measures

	(1)	(2)	(3)	(4)
$\hat{\gamma}^H$	-0.265***	−0.242***	−0.220***	−0.250***
	(-6.25)	(−5.70)	(−5.19)	(−5.89)
$\hat{\gamma}^L$	0.211***	0.197***	0.191***	0.196***
	(5.10)	(4.74)	(4.61)	(4.74)
$\hat{\gamma}^H - \hat{\gamma}^L$	-0.476***	−0.438***	−0.411***	−0.446***
	(-8.03)	(−7.40)	(−6.93)	(−7.53)

Table 1.5
Slope of the SML and Granularity – Portfolio Level

This table reports the effect of changes in granularity on the conditional slope of the security market line (SML). The dependent variable is the slope of the SML, which is obtained from regressing equally-weighted monthly returns of 25 beta portfolios on the previous month's market betas of the same portfolios. Conditional betas are estimated by rolling a 60-month window. The variable of interest is the change in granularity (ΔG_t), measured by the market capitalization value of the 100 largest firms in fraction of total market capitalization of the CRSP universe. The control variables include excess market return ($R_{m,t}$ - R_f), investor sentiment of Baker and Wurgler (2006), lagged inflation as in Cohen et al. (2005), and the TED spread following Antoniou et al. (2015). Details about the variables are provided in Table 1.1. Newey-West t-statistics with 12 lags are reported in parentheses. Monthly data span January 1973 to December 2018, obtained from CRSP. All variables are normalized. *, **, and *** indicate statistical significance at the 10%, 5%, and 1% level, respectively.

	(1)	(2)	(3)	(4)
ΔG_t	-0.445***	−0.237***	−0.233***	−0.244***
	(-7.80)	(−5.17)	(−5.16)	(−4.47)
$R_{m,t}$-$R_{f,t}$		0.704***	0.711***	0.765***
		(11.29)	(11.61)	(11.47)
Inflation_{t-1}			0.088***	0.039
			(3.17)	(0.95)
TED_t				0.118***
				(3.97)
Adj. R^2(%)	0.20	0.65	0.66	0.67
Observations	551.00	551.00	551.00	387.00

Table 1.6
Slope of the SML and Granularity – Portfolio Level, High Sentiment Months

This table reports the effect of changes in granularity on the conditional slope of the security market line (SML). The dependent variable is the slope of the SML, which is obtained from regressing equally-weighted monthly returns of 25 beta portfolios on the previous month's market betas of the same portfolios. Conditional betas are estimated by rolling a 60-month window. The variable of interest is the change in granularity (ΔG_t), measured by the market capitalization value of the 100 largest firms in fraction of total market capitalization of the CRSP universe. The control variables include excess market return ($R_{m,t}$ - R_f), investor sentiment of Baker and Wurgler (2006), lagged inflation as in Cohen et al. (2005), and the TED spread following Antoniou et al. (2015). Details about the variables are provided in Table 1.1. Newey-West t-statistics with 12 lags are reported in parentheses. Monthly data span January 1973 to December 2018, obtained from CRSP. All variables are normalized. *, **, and *** indicate statistical significance at the 10%, 5%, and 1% level, respectively.

	(1)	(2)	(3)	(4)
ΔG_t	-0.352***	−0.191***	−0.184***	−0.230***
	(-6.83)	(−3.36)	(−3.21)	(−3.40)
$R_{m,t}$-$R_{f,t}$		0.718***	0.724***	0.743***
		(9.37)	(9.44)	(7.85)
Inflation$_{t-1}$			0.091*	0.073
			(1.88)	(1.44)
TED_t				0.109***
				(3.86)
Adj. R^2(%)	0.13	0.61	0.61	0.60
Observations	300.00	300.00	300.00	237.00

Table 1.7
Slope of the SML and Granularity – Portfolio Level, Low Sentiment Months

This table reports the effect of changes in granularity on the conditional slope of the security market line (SML) when the investor sentiment index of Baker and Wurgler (2006) is negative. The dependent variable is the slope of the SML, which is obtained from regressing equally-weighted monthly returns of 25 beta portfolios on the previous month's market betas of the same portfolios. Conditional betas are estimated by rolling a 60-month window. The variable of interest is the change in granularity (ΔG_t), measured by the market capitalization value of the 100 largest firms in fraction of total market capitalization of the CRSP universe. The control variables include excess market return ($R_{m,t}$ - R_f), lagged inflation as in Cohen et al. (2005), and the TED spread following Antoniou et al. (2015). Details about the variables are provided in Table 1.1. Newey-West t-statistics with 12 lags are reported in parentheses. Monthly data span January 1973 to December 2018, obtained from CRSP. All variables are normalized. *, **, and *** indicate statistical significance at the 10%, 5%, and 1% level, respectively.

	(1)	(2)	(3)	(4)
ΔG_t	-0.545***	−0.294***	−0.293***	−0.278***
	(-6.87)	(−4.68)	(−4.71)	(−5.22)
$R_{m,t}$-$R_{f,t}$		0.678***	0.681***	0.800***
		(7.42)	(7.54)	(11.28)
Inflation_{t-1}			0.032	−0.060
			(1.10)	(−0.97)
TED_t				0.161***
				(2.60)
Adj. R^2(%)	0.29	0.71	0.71	0.80
Observations	251.00	251.00	251.00	150.00

Table 1.8
Slope of the SML and Granularity – Individual Stock Level

This table reports the effect of changes in granularity on the conditional slope of the security market line (SML). The dependent variable is the slope of the SML, which is obtained from regressing individual monthly stock returns on the previous month's market betas of the stocks. Conditional betas are estimated by rolling a 60-month window. The variable of interest is the change in granularity (ΔG_t), measured by the market capitalization value of the 100 largest firms in fraction of total market capitalization of the CRSP universe. The control variables include excess market return ($R_{m,t}$ - R_f), lagged inflation as in Cohen et al. (2005), and the TED spread following Antoniou et al. (2015). Details about the variables are provided in Table 1.1. Newey-West t-statistics with 12 lags are reported in parentheses. Monthly data span January 1973 to December 2018, obtained from CRSP. All variables are normalized. *, **, and *** indicate statistical significance at the 10%, 5%, and 1% level, respectively.

	(1)	(2)	(3)	(4)
ΔG_t	-0.443***	−0.237***	−0.237***	−0.243***
	(-7.35)	(−5.00)	(−5.02)	(−4.59)
$R_{m,t}$-$R_{f,t}$		0.697***	0.697***	0.783***
		(10.74)	(10.82)	(10.19)
Inflation_{t-1}			0.010	−0.036**
			(0.29)	(−2.10)
TED_t				0.113***
				(3.92)
Adj. R^2(%)	0.19	0.64	0.64	0.68
Observations	551.00	551.00	551.00	387.00

Table 1.9
Slope of the SML and Granularity – Individual Stock Level, High Sentiment Months

This table reports the effect of changes in granularity on the conditional slope of the security market line (SML) when the investor sentiment index of Baker and Wurgler (2006) is positive. The dependent variable is the slope of the SML, which is obtained from regressing individual monthly stock returns on the previous month's market betas of the stocks. Conditional betas are estimated by rolling a 60-month window. The variable of interest is the change in granularity (ΔG_t), measured by the market capitalization value of the 100 largest firms in fraction of total market capitalization of the CRSP universe. The control variables include excess market return ($R_{m,t}$ - R_f), lagged inflation as in Cohen et al. (2005), and the TED spread following Antoniou et al. (2015). Details about the variables are provided in Table 1.1. Newey-West t-statistics with 12 lags are reported in parentheses. Monthly data span January 1973 to December 2018, obtained from CRSP. All variables are normalized. *, **, and *** indicate statistical significance at the 10%, 5%, and 1% level, respectively.

	(1)	(2)	(3)	(4)
ΔG_t	-0.331***	−0.177***	−0.178***	−0.223***
	(-6.09)	(−3.39)	(−3.43)	(−3.65)
$R_{m,t}$-$R_{f,t}$		0.686***	0.684***	0.725***
		(7.89)	(7.90)	(6.68)
Inflation_{t-1}			−0.042	−0.053*
			(−1.48)	(−1.72)
TED_t				0.106***
				(3.33)
Adj. R^2(%)	0.12	0.60	0.60	0.61
Observations	300.00	300.00	300.00	237.00

Table 1.10
Slope of the SML and Granularity – Individual Stock Level, Low Sentiment Months

This table reports the effect of changes in granularity on the conditional slope of the security market line (SML) when the investor sentiment index of Baker and Wurgler (2006) is negative. The dependent variable is the slope of the SML, which is obtained from regressing individual monthly stock returns on the previous month's market betas of the stocks. Conditional betas are estimated by rolling a 60-month window. The variable of interest is the change in granularity (ΔG_t), measured by the market capitalization value of the 100 largest firms in fraction of total market capitalization of the CRSP universe. The control variables include excess market return ($R_{m,t}$ - R_f), lagged inflation as in Cohen et al. (2005), and the TED spread following Antoniou et al. (2015). Details about the variables are provided in Table 1.1. Newey-West t-statistics with 12 lags are reported in parentheses. Monthly data span January 1973 to December 2018, obtained from CRSP. All variables are normalized. *, **, and *** indicate statistical significance at the 10%, 5%, and 1% level, respectively.

	(1)	(2)	(3)	(4)
ΔG_t	-0.564***	−0.307***	−0.306***	−0.286***
	(-6.80)	(−4.39)	(−4.42)	(−5.03)
$R_{m,t}$-$R_{f,t}$		0.694***	0.694***	0.861***
		(7.57)	(7.70)	(11.19)
Inflation_{t-1}			0.010	−0.023
			(0.24)	(−0.84)
TED_t				0.155**
				(2.13)
Adj. R^2(%)	0.29	0.70	0.69	0.79
Observations	251.00	251.00	251.00	150.00

Table 1.11
Beta-sorted Portfolio Returns and Granularity

This table shows the beta-sorted portfolio returns by granularity. The first two rows report the market beta and the time series average of 10 equally-weighted beta-sorted portfolios (Columns P1 to P10). The rightmost column corresponds to the difference between the high-beta portfolio and the low-beta portfolio (P10-P1). CAPM alpha is the intercept from a regression where excess portfolio returns are regressed on excess market returns. Panel A presents the unconditional results. Panel B presents results conditional on granularity increases ($\Delta G_t > 0$) and decreases ($\Delta G_t < 0$), where granularity is measured by the market value of the 100 top firms as fraction of total market capitalization. Panel C conditions the analysis on ΔG_{orth}, which is given by ΔG orthogonalized to excess market returns. Robust Newey-West t-statistics with 12 lags are reported in parentheses. Monthly data span January 1973 to December 2018, obtained from CRSP. *, **, and *** indicate statistical significance at the 10%, 5%, and 1% level, respectively.

	P1 (Low β)	P2	P3	P4	P5	P6	P7	P8	P9	P10 (High β)	P10-P1
Panel A: Unconditional											
Beta	0.428	0.577	0.718	0.821	0.925	1.018	1.111	1.213	1.360	1.675	1.247
Ex. Ret	0.749	0.870	0.987	1.015	1.016	1.075	1.093	1.108	1.204	1.429	0.680
CAPM α	0.246	0.193	0.145	0.052	−0.069	−0.120	−0.210*	−0.315**	−0.391***	−0.536***	−0.783***
	(1.60)	(1.45)	(1.09)	(0.41)	(−0.52)	(−0.99)	(−1.84)	(−2.40)	(−2.83)	(−2.66)	(−2.82)
Panel B: Conditional on ΔG											
CAPM α, $\Delta G > 0$	-0.317*	−0.446**	−0.587***	−0.780***	−0.983***	−1.077***	−1.278***	−1.555***	−1.804***	−2.252***	−1.935***
	(-1.67)	(−2.42)	(−3.66)	(−5.17)	(−6.15)	(−7.47)	(−10.17)	(−11.88)	(−13.37)	(−10.80)	(−6.12)
CAPM α, $\Delta G < 0$	0.939***	0.931***	0.995***	1.023***	0.982***	0.935***	0.969***	1.021***	1.076***	1.137***	0.198
	(5.05)	(5.48)	(5.41)	(5.60)	(5.51)	(5.53)	(5.60)	(5.75)	(5.60)	(4.26)	(0.56)
Panel C: Conditional on ΔG_{orth}											
CAPM α, $\Delta G_{orth} > 0$	-0.388*	−0.449**	−0.586***	−0.761***	−0.986***	−1.078***	−1.310***	−1.576***	−1.809***	−2.303***	−1.915***
	(-1.90)	(−2.34)	(−3.44)	(−4.82)	(−5.97)	(−7.28)	(−10.07)	(−11.98)	(−13.23)	(−12.33)	(−5.82)
CAPM α, $\Delta G_{orth} < 0$	0.951***	0.899***	0.945***	0.944***	0.939***	0.929***	0.995***	1.062***	1.157***	1.385***	0.434
	(5.82)	(5.98)	(5.42)	(5.06)	(5.33)	(5.56)	(5.70)	(5.62)	(5.67)	(4.91)	(1.29)

Table 1.12
Conditional BAB returns

This table presents the conditional mean of betting-against-beta (BAB) returns by changes in granularity. We construct BAB returns from the strategy that is long the low-beta stocks and short the high-beta stocks. The long portfolio is an equally-weighted average of the bottom decile stocks, when stocks are ranked according to their beta, whereas the short portfolio is an equally-weighted average of the top decile stocks. The conditioning criteria correspond to changes in four measures of granularity G_t: the market value of the top 100, 50, and 1% largest firms as fraction of total market capitalization, as well as the excess Herfindahl-Hirschman Index (exHHI). The first two rows show the average BAB return for months when granularity increases ($\Delta G_t > 0$) and decreases ($\Delta G_t < 0$), respectively. The last row reports the difference between the conditional means and the test of the difference. Robust Newey-West t-statistics with 12 lags are reported in parentheses. Equal weighted beta portfolios are formed based on CRSP. Monthly data span January 1973 to December 2018. *, **, and *** indicate statistical significance at the 10%, 5%, and 1% level, respectively.

	ΔTop100	ΔTop50	ΔTop1%	ΔexHHI
(1) $r_{BAB,\,\Delta G>0}$	0.024***	0.022***	0.019***	0.022***
	(6.25)	(5.73)	(5.29)	(5.53)
(2) $r_{BAB,\,\Delta G<0}$	-0.019***	-0.018***	-0.017***	-0.017***
	(-4.82)	(-4.32)	(-3.93)	(-4.47)
(1) - (2)	0.043***	0.039***	0.036***	0.039***
	(7.81)	(7.07)	(6.43)	(7.09)

Table 1.13
BAB returns and Granularity

This table presents results of a regression of betting-against-beta (BAB) returns on changes in granularity. We construct BAB returns from the strategy that is long the low-beta stocks and short the high-beta stocks. The long portfolio is an equally-weighted average of the bottom decile stocks, when stocks are ranked according to their beta, whereas the short portfolio is an equally-weighted average of the top decile stocks. The explanatory variables include contemporaneous changes granularity, excess market return, lagged inflation, the lagged return of the long-short strategy, and the TED spread. Granularity is the market value of the 100 largest firms as a fraction of total market capitalization. Robust Newey-West t-statistics with 12 lags are reported in parentheses. Monthly data span January 1973 to December 2018. All variables are normalized. *, **, and *** indicate statistical significance at the 10%, 5%, and 1% level, respectively.

	(1)	(2)	(3)	(4)	(5)
ΔG_t	0.409***	0.200***	0.197***	0.196***	0.225***
	(7.43)	(4.76)	(4.70)	(4.69)	(4.10)
$R_{m,t} - R_{f,t}$		−0.705***	−0.711***	−0.710***	−0.833***
		(−9.45)	(−9.60)	(−9.58)	(−8.91)
Inflation_{t-1}			−0.072***	−0.072***	−0.065
			(−2.67)	(−2.67)	(−1.13)
$r_{BAB,t-1}$				0.006	0.010
				(0.29)	(0.44)
TED_t					−0.122***
					(−3.27)
Adj. R^2(%)	16.54	61.88	62.26	62.20	64.89
Observations	552	552	551	551	387

1.9 Figures

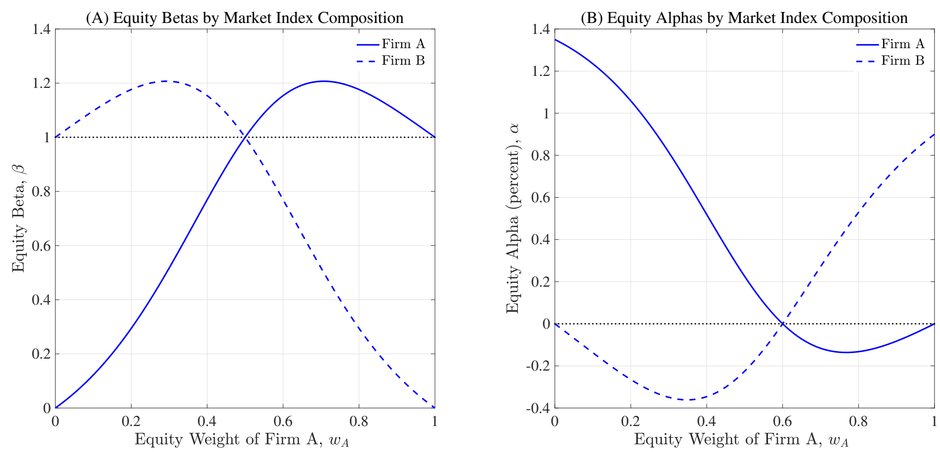

Figure 1.2: Equity beta and alpha by market index weight.

This figure shows how the equity beta and alpha of two firms vary with the market index composition, measured by with one firm's equity weight in the market index. Panel A reports the equity betas of firms A and B, which correspond to the exposure of a firm's equity excess return to the market excess return. Panel B reports the equity alphas of firms A and B, which correspond to the expected excess return less the compensation for the market exposure. The weight of firm *A*, denoted by w_A, reflects the equity value of firm *A* as a fraction of total market value (sum of equity values of firms A and B). The weight w_A varies endogenously by changing firm A's dividend level, maintaining firm B unchanged. Parameters are set to $\mu_B = \mu_A = 0.05$, $\sigma_A = \sigma_B = 0.15$, $\eta_A = 0.60$, $\eta_B = 0.40$, $X_B = 1$, and $r = 0.05$.

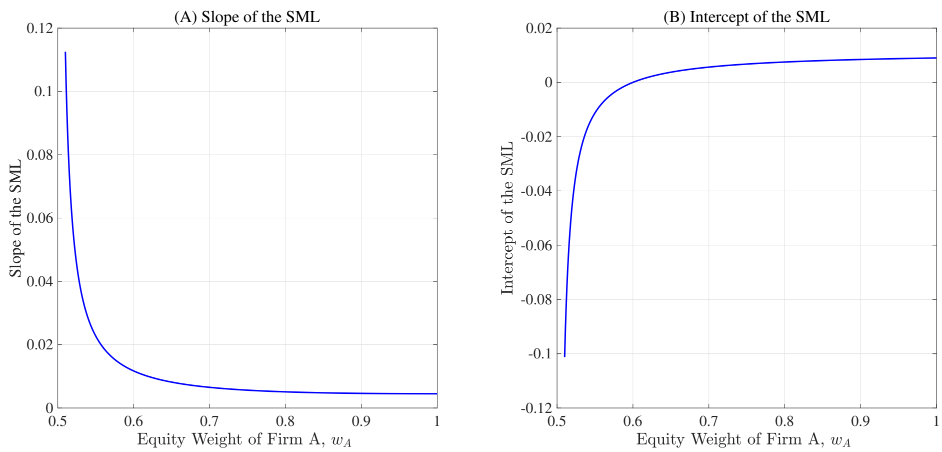

Figure 1.3: Security market line by market index weight.

This figure shows how the security market line (SML) changes with the market index composition, measured by one firm's equity weight in the market index. Panel A reports the slope of the SML, which corresponds to the difference in firms' equity excess return divided by the the difference in firms' equity betas. Panel B reports the intercept of the SML. The weight of firm A, denoted by w_A, reflects the equity value of firm A as a fraction of total market value (sum of equity values of firms A and B). The weight w_A varies endogenously by changing firm A's dividend level, maintaining firm B unchanged. Parameters are set to $\mu_B = \mu_A = 0.05$, $\sigma_A = \sigma_B = 0.15$, $\eta_A = 0.60$, $\eta_B = 0.40$, $X_B = 1$, and $r = 0.05$.

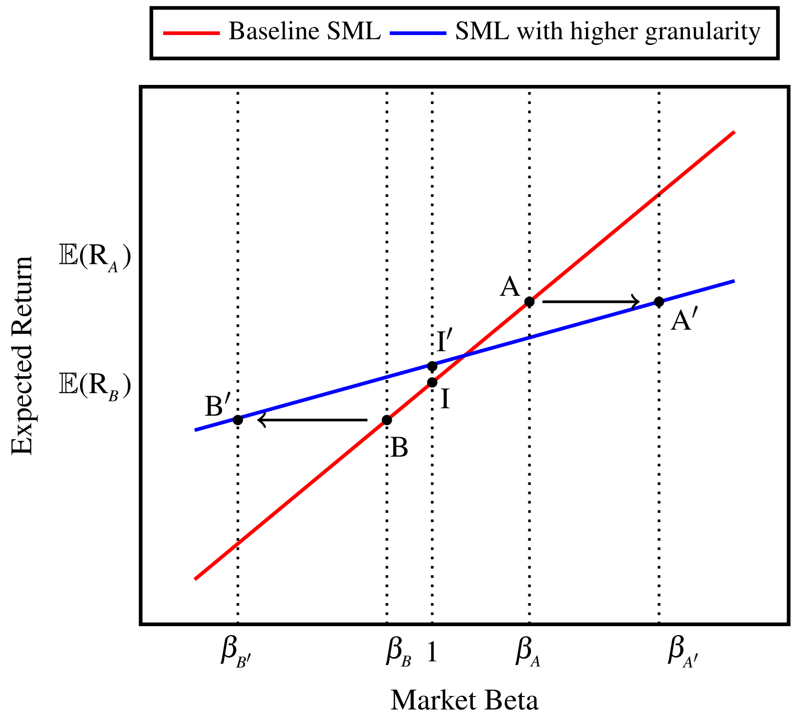

Figure 1.4: Security market line and granularity.

This figure illustrates the security market line (SML) in a two-firm economy for different levels of granularity. The red line corresponds to the benchmark case, while the blue line reflects an increase in granularity. Granularity increases when the larger firm becomes larger and the smaller firm smaller, which implies that the market is more exposed to the larger firm. In this example, a positive dividend shock to firm A, which has the higher weight in the market index ($w_A > w_B$), increases its equity beta ($\beta_{A'} > \beta_A$) but decreases the other firm's equity beta ($\beta_{B'} < \beta_B$) by market clearing conditions. As a result, the slope of the SML decreases when the economy becomes more granular.

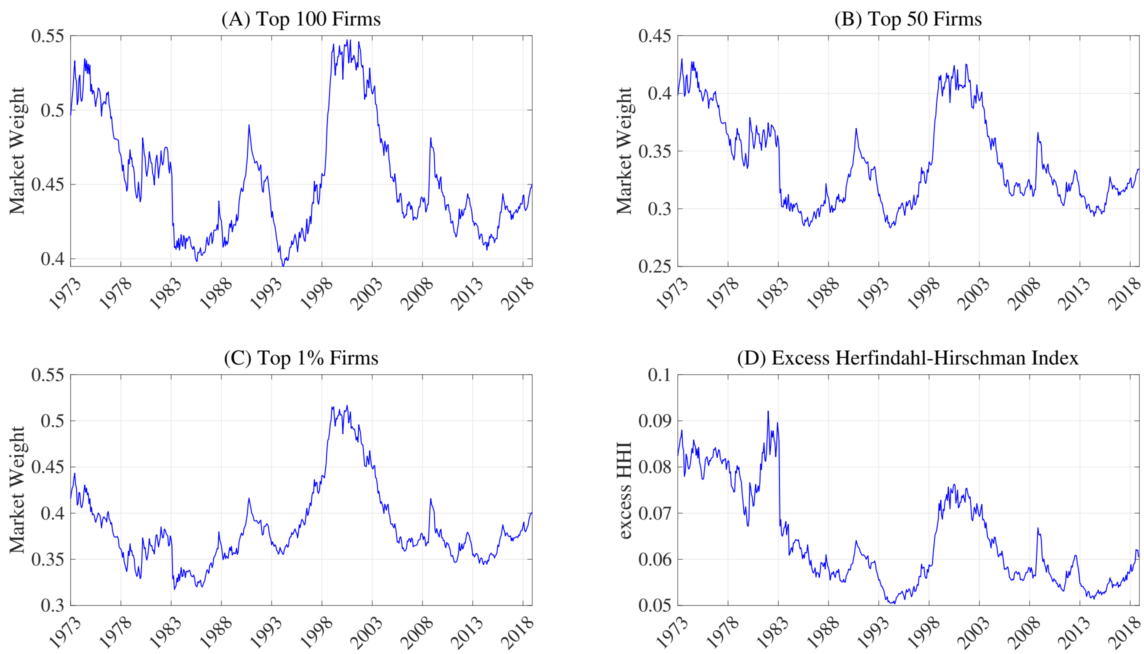

Figure 1.5: Time series of granularity.

This figure shows the time series of four different measures of granularity. Panel A, B, and C, respectively, plot the market value of the 100, the 50, and the 1% largest firms as a fraction of total market capitalization. Panel D displays the excess Herfindahl-Hirschman Index, defined in Table 1.1. Monthly data span January 1973 to December 2018 and are obtained from CRSP.

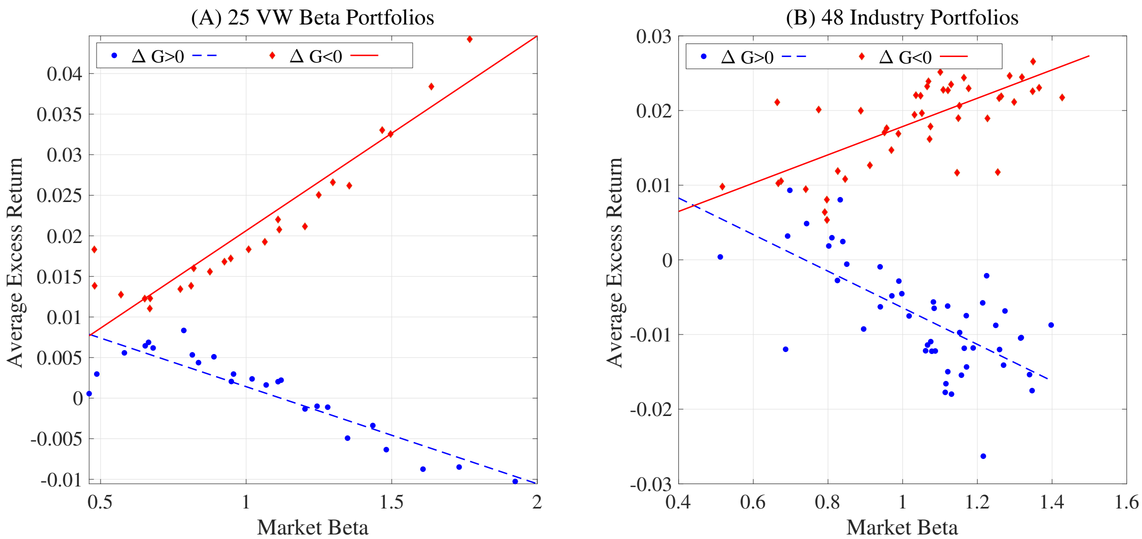

Figure 1.6: Conditional SML and granularity – Alternative portfolios.

This figure shows the average conditional monthly returns against the average conditional market betas, computed in the previous month, of 25 value weighted beta portfolios (Panel A) and 48 industry portfolios (Panel B). We separate portfolio returns and corresponding betas for increases ($\Delta G_t > 0$) versus decreases ($\Delta G_t < 0$) granularity months. Granularity, denoted by G_t, is the market value of top 100 firms as a fraction of total market capitalization. Monthly data span January 1973 to December 2018. Beta portfolios are formed based on monthly individual stock returns obtained from CRSP, while industry portfolio returns are from Kenneth French's data library.

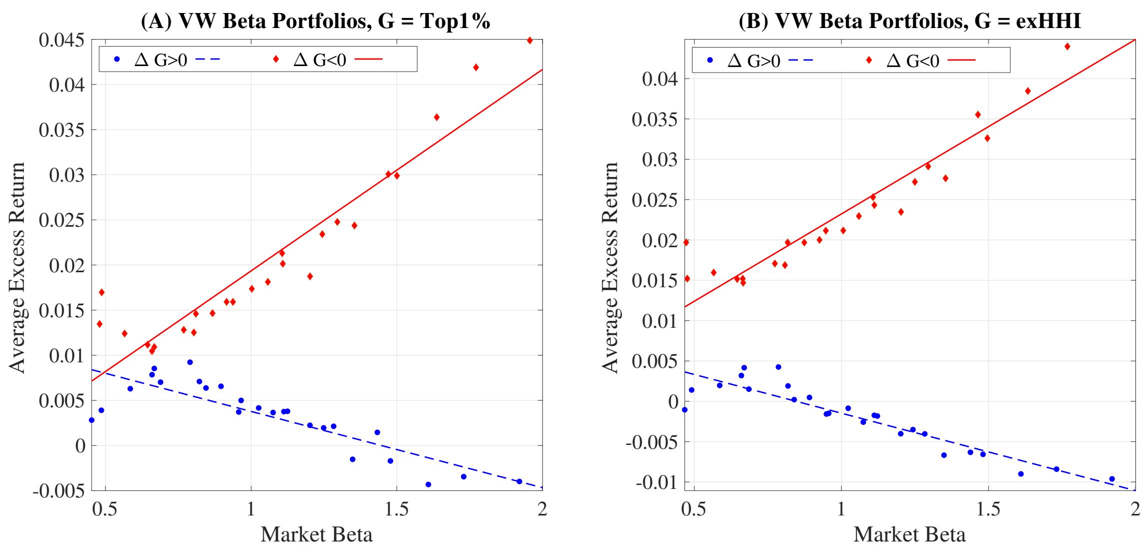

Figure 1.7: Conditional SML and granularity – Alternative granularity measures.

This figure shows the average conditional monthly returns against the average conditional market betas, computed in the previous month, of 25 value weighted beta portfolios. We separate portfolio returns and corresponding betas for increases ($\Delta G_t > 0$) versus decreases ($\Delta G_t < 0$) granularity months. Granularity, denoted by G_t, is the market value of top 100 firms as a fraction of total market capitalization in Panel A and the excess Herfindahl-Hirschman Index in Panel B. Detailed description of the granularity measures is provided in Table 1.1. Monthly data span January 1973 to December 2018 and are obtained from CRSP.

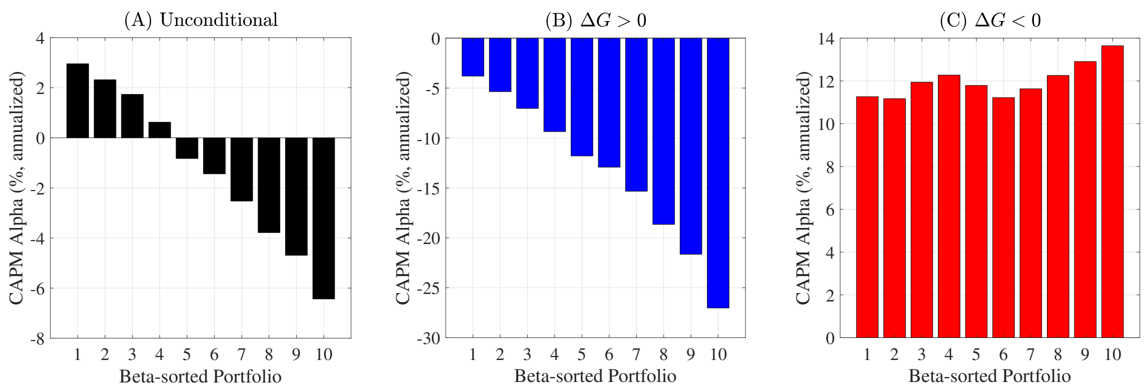

Figure 1.8: Conditional Alpha of Beta-sorted Portfolios.

This figure plots the CAPM alphas of 10 equally-weighted portfolios based on beta-sorted stocks. The unconditional alpha of a portfolio is the intercept of a regression of a portfolio's excess return on the market excess return over the whole sample. For the conditional analysis, we first split the portfolio returns into two subsamples corresponding to months when granularity decreases ($\Delta G_t < 0$) and increases ($\Delta G_t > 0$), where granularity G_t is the market value of top 100 firms as a fraction of total market capitalization. We then estimate the intercept of the regression based on each subsample. Panel A displays the unconditional portfolio alphas, while Panels B and C report results for increases and decreases in granularity, respectively. Beta portfolios are formed based on monthly individual stock returns obtained from CRSP. Monthly data span January 1973 to December 2018.

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Chapter 2

Granular Gravity: Equity-Bond Returns & Correlation

Abstract

Investors care about the state of granularity, the heavy tail of firms' size distribution. I adapt and extend an existing granularity measure, and show that changes in granularity are negatively related to the cross-section of corporate bond returns. I verify that the risk-adjusted return on a strategy that provides effective hedge against granularity shocks is economically significant. I further document that the correlation between firms bond and equity returns is 50% higher when the economy becomes more granular than atomistic. Data supports the hypothesis that this arises from granularity being a priced factor in the cross-sections of both equity and corporate bond returns.

2.1 Introduction

A granular economy comprises a small number of very large, and a large number of atomistic firms. Such economy does not yield to classic arguments where firms, no matter how large, are always negligible relative to the whole economy (Lucas, 1977). On the contrary, in a granular economy risk inherent in large firms is not compressible (Gabaix, 2011). I argue that granularity is a state variable that correlates with the future investment opportunity set (Merton, 1973). As such, a typical investor (or consumer) cares about this state variable. Positive innovations in granularity correspond to a narrower future opportunity set, hence they deteriorate the utility from future consumption, and lower the expected return on financial assets. In this paper, I study the cross section of corporate bond returns, and provide ample empirical evidence that corporate bonds with more exposure to granularity pay less.

The law of large numbers (LLN) states that in an economy with N firms, firm-specific risk of portfolios decays at the order of $\frac{1}{\sqrt{N}}$. Gabaix (2011) argues that LLN applies when the distribution of firms' size is thin-tailed. However, the distribution of firms' size, like many other variables of interest in economics, is substantially heavy-tailed, and well approximated by a power law (Axtell, 2001). With a heavy-tailed distribution, the decay mechanism predicted by the LLN is hindered. Strictly speaking, in a granular economy, firm-specific risk decays at the order of $\frac{1}{\ln N}$ rather than $\frac{1}{\sqrt{N}}$ (Gabaix, 2011). For example, when $N = 10,000$, $\frac{1}{\ln N}$ implies a ten times slower decay than $\frac{1}{\sqrt{N}}$. The number of firms in the economy as well as the heaviness of firms' size distribution varies over time, so does the rate of decay (theoretically between $\frac{1}{\sqrt{N}}$ and $\frac{1}{\ln N}$). I adapt a measure for granularity to capture this time variation, and study the corresponding cross-sectional implication in the cross section of corporate bond returns.

Granularity is an economy-wide phenomenon, and it impacts debt markets as much as equity markets, as they are both exposed to negative firm-level shocks that do not average out in the aggregate. Imagine an earnings shock to largest firms in the economy. Such incompressible shock propagates and alters the riskiness of the market portfolio, hence becomes systematic in nature. Expected returns on securities across markets adjust accordingly. An investor quantifies the impact proportionate to the sensitivity of her portfolio to the state of granularity.

Notice that as a state variable, granularity represents the likelihood that full diversification, an assumption that CAPM investors take for granted, no longer holds. As such, I do not ask whether a particular shock brings about lasting effects on the risk-return relation. Instead, the focus is on whether heightened levels of granularity, corresponding to a state of the world where granularity shocks are expected to propagate with a higher probability, generate cross-sectional effects. This chapter compliments the findings in the first chapter of the thesis by documenting that not only is granularity a priced factor in the cross section of equity returns but also corporate bond returns.

As such, I hypothesize that exposure to state of granularity of the economy shows in the correlation between firms' security returns. In months when the economy grows more granular, it is more likely that firm-specific risk propagates across markets. As a result these months correspond to higher equity-bond correlations. On the other hand, in months when granularity decreases, it is less likely that shocks stemming from large firms matter in the aggregate. These months correspond to a narrower granularity channel where equity-bond returns correlate less.

In my empirical investigation, I document that positive innovations in granularity correspond to an increase in equity-bond correlation (henceforth EBC). Thereby, the state of granularity of the economy helps us better understand the conditional behaviour of security prices within markets, as well as the conditional co-movement of different financial securities across markets. A direct application of this finding would be for risk managers especially involving investment strategies in equity and corporate bond portfolios. Higher granularity translates into riskier equity-bond portfolios as they grow more correlated, lessening the diversification benefit expected from such strategies.

In the first chapter of this thesis, we constructed a granularity index, and provided empirical and theoretical evidence that granularity innovations are priced in the cross section of equity returns. I adapt and extend this index to reflect the relative size of the

g largest firms in the economy as a fraction of total market value of firms' equity and debt. The granularity index depicts the extent to which the economy is dominated by a certain number of firms. For instance, since the beginning of the millennium 20 firms accounted for an average of a quarter (between 18 to 28 percent) of the combined value of equity and corporate bond markets[1]. Hence, the higher my granularity index, the less likely it is to achieve full diversification of firm-specific risk. A positive shock to the index therefore reflects that the state of the economy deteriorated in that diversification benefits are more elusive.

Bali et al. (2017), and Bali et al. (2020b) use the economic uncertainty index of Jurado et al. (2015), and study the cross-sectional implication of innovations in economic uncertainty on expected equity and corporate bond returns. I contribute to this nascent literature and show that variation in granularity, similar to economic uncertainty, is linked to future returns on corporate bonds. To test this hypothesis, first, I estimate the conditional betas of corporate bond returns with respect to the granularity index using a rolling window regression. Then, every month, I estimate Fama-Macbeth cross-sectional regressions where I regress future excess bond returns on estimated betas from the prior period. I show that the time series average of the estimated slope coefficient is negative and economically significant. I provide evidence that results do not depend on choice of number of firms in the granularity index, and hold with betas with respect to past innovations in granularity.

Besides, I form a strategy that takes a long position in corporate bonds that are least sensitive to innovations in granularity (negative betas), and a short position in bonds that are most sensitive to these innovations (positive betas). Specifically, I estimate conditional granularity betas by estimating rolling window regressions. Every month, I sort bonds into ten deciles. The first (tenth) decile contains bonds with negative (positive) betas. Effectively shorting granularity shocks yields a risk-adjusted return that remains economically significant after controlling for established risk factors in the corporate bond and equity markets.

[1] As comparison, the smallest 100 firms in the S&P500 represent less than 4% of total market capitalization.

Bonds that are more sensitive to granularity shocks (i.e. with positive granularity betas) are perceived riskier when granularity increases. These bonds are relatively larger (measured by the amount outstanding), with better credit quality, and longer time to maturity. I ensure that granularity is not a proxy for size, credit quality, or time to maturity by including them in cross-sectional regressions. Besides, I sort bonds into granularity deciles only for short versus long time to maturity bonds, and investment grade versus non-investment grade bonds. I find that the risk-adjusted return on the granularity strategy remains economically significant in all cases.

Next, I examine whether exposure of firms' securities to this common source of risk explains conditional correlations between equity and corporate bond returns. I proceed by estimating conditional EBCs and show that firms' securities are 50% more correlated in months where granularity shocks are large and positive. Then, I construct a panel of correlation coefficients by estimating a rolling window correlation coefficient for every firm in the sample. Fixed effect regressions verify that EBCs are positively and significantly related to granularity.

The granularity index is a measure of market concentration, and it is imperative to examine the robustness of the empirical findings with regard to alternative measures. I use the Herfindahl-Hirschman Index (HHI), the excess HHI of Gabaix and Koijen (2020), and the granularity index of Abolghasemi et al. (2020) for different values of g and repeat the main asset pricing tests in this paper. I verify that the results hold regardless of the type of granularity measure.

The remainder of this paper is organized as follows. I provide a brief review of the literature in Section II, and describe the data in Section 2.3. I then proceed by describing the details of my empirical methodology and data analysis in Section 2.4, and presenting the concluding remarks in Section 2.6. In the Appendix, I provide a sketch of a theoretical framework that shows how granularity affects equity and bond returns.

2.2 Literature Review

This paper contributes to a number of streams in the literature: granularity, cross sectional predictors of corporate bond returns, and the drivers of the bond-equity conditional correlation. I provide a brief review of these streams in the following sections.

2.2.1 Granularity

The literature on granularity is one of two strands that examine micro foundations of macroeconomic fluctuations. Axtell (2001) documents that like many variables of interest in economics, the size of the firms is well approximated by the power law, hence it is substantially heavy-tailed. As a consequence, firm-specific risk of relatively large firms becomes incompressible (Gabaix, 2011). The granularity channel implies that country size and trade affect macroeconomic volatility (di Giovanni and Levchenko, 2012), and that large firms are potential drivers of business cycles (Carvalho and Grassi, 2019). Gabaix and Koijen (2020) build on the idea of granularity and propose a granular instrumental variable (GIV). This instrument is easy to construct, it is the difference between a value-weighted index and its equally-weighted counterpart, and it is used to address many questions in economics and finance where the endogeneity issue obscures empirical findings.

A parallel strand in the literature examines networks and linkages among firms. This literature does not consider a heavy-tailed distribution of firms' size. Rather, micro shocks propagate via linkages and networks among firms. Acemoglu et al. (2012) provide a theoretical framework for how these linkages aid micro-shocks to grow into macro fluctuations, and Barrot and Sauvagnat (2016) show how production networks affect firms' sales growth and stock prices.

Herskovic et al. (2020) explore the joint capacity of granularity and networks as channels through which firm size distriution determines how firm-specific shocks propagate and affect firm volatilities. From an asset pricing perspective, Abolghasemi et al. (2020) show that the slope of the SML (Security Market Line) is flat even negative, when granularity is high, hence revisiting the properties of the conditional CAPM. They show that the low-risk anomaly reflects the state of granularity of the economy.

2.2.2 Cross Section of Corporate Bond Returns

Traditional predictors of corporate bond returns, the term and default spreads, reflect economic conditions. The term spread captures unexpected changes in interest rates, and the default spread the likelihood of default (Fama and French, 1993). Other variables such as inflation (Elton et al., 1995; Kang and Pflueger, 2015), or uncertainty about the macroeconomy (Bali et al., 2020b), are shown to affect corporate bond returns. Bali et al. (2020b) and Bali et al. (2017) measure exposure of corporate bonds and equity, respectively, to the measure of economic uncertainty of Jurado et al. (2015). The current study follows a similar methodology to measure the exposure of corporate bonds with respect to a granularity index.

Elton et al. (2001) provide evidence that excess corporate bond returns are explained by common risk factors in the equity market. Subsequently, the literature substantiates the relevance of factors such as (il)liquidity (Chen et al., 2007; Covitz and Downing, 2007; Bao et al., 2011; Lin et al., 2011; Friewald et al., 2012; Acharya et al., 2013), volatility (Chung et al., 2019), investor sentiment (Guo et al., 2019), momentum (Jostova et al., 2013), and reversal (Bali et al., 2020a) to corporate bond returns.

Inspired by Daniel and Titman (1997), Gebhardt et al. (2005) compare the pricing implications of systematic risk (default and term) versus characteristics (e.g. ratings and duration) in the bond market. Corroborating the relevance of either approach, they find that systematic risk matters in the cross section of bond returns. Israel et al. (2018) consider, in tandem, the carry, quality, momentum, and value factors in the bond market. Israelov (2019) investigates the capacity of the option markets to explain corporate bond returns.

Mostly through a default channel, factors exclusive to bonds are also shown to claim a premium in the cross section of corporate bond returns. Driessen (2004) shows that default event risk is priced in the cross section of corporate bonds. Bai et al. (2019) develop three bond-market-specific measures of down-side risk that are priced in the cross section of corporate bonds. Not only default events, but also ambiguity about the credit rating, an indicator for default, predicts higher premium for the corporate bonds (Kim et al., 2018).

2.2.3 Equity-Bond Return Correlation

Correlation between bond and equity returns is examined in the literature via some information-based models. Kwan (1996) argues that information flows from the equity to the bond market, and Acharya and Johnson (2007) provide evidence that the equity market reacts to innovations in the CDS market, derivatives written on corporate bonds. In a similar investigation, Bao and Pan (2013) show that corporate-bond CSD returns volatilities exceed equity return volatilities and what the Merton model predicts. When it comes to the sign of the correlation, Back and Crotty (2014) use a Kyle model and argue that when information is about the asset means, the correlation is positive, and when it is about the asset risk, it is negative.

Aretz and Yang (2019) use a disinvestment model to explain the negative relation between bond returns and firm distress risk, measured by Campbell's measure of distress. The bond-equity correlation literature is richer for government issued rather than corporate bonds. Baele et al. (2010) develop a structural model for the Treasury bonds and equity returns correlation, estimated via the component model of dynamic correlations of Colacito et al. (2011).

In a more recent work, Baele et al. (2019) propose a regime-switching model to identify Flight-to-Safety (FTS) episodes where in distressed times, large and positive bond returns are accompanied by large and negative equity returns. They show that FTS represents a flight to quality, but not liquidity, in the corporate bond market, hence a potential explanation for equity-corporate-bond correlation.

Bethke et al. (2017) argue that bond-equity correlation stems from correlated risk factors. They show that investor sentiment is the main driver of bond-equity (positive and negative) correlation. Bektić (2018a) shows that high beta equity corresponds to low return in the equity market for European firms. Bektić (2018b) also documents momentum spill-over from the equity market to the global bond market.

2.3 Data

2.3.1 Corporate Securities

I construct my bond sample from the Enhanced TRACE (Trade Reporting and Compliance Engine) daily OTC transactions. TRACE transaction data is more prevalently used in recent corporate bond pricing studies. Bessembinder et al. (2006) document that bonds eligible for TRACE transaction reporting are less prone to external liquidity issues than bonds that are not eligible for TRACE reporting. I retrieve bond characteristics such as coupon information, time to maturity, and ratings from the Mergent Fixed Income Securities Database (FISD). The sample spans July 2002 to June 2019. To compute bond returns, I follow the methodology of Bai et al. (2019). I apply the procedure recommended by Nielsen (2014) to clean the Enhanced TRACE transaction data, then exclude all bonds with special features. Also, due to their peculiar properties, I exclude bonds with less than a year to maturity. For an exhaustive list of filters applied to the dataset see the Appendix.

The return on bond b in month t is defined as

$$r_{b,t} = \frac{P_{b,t} + AI_{b,t} + C_{b,t}}{P_{t-1} + AI_{b,t-1}} - 1, \tag{2.1}$$

where $\mathrm{P}_{b,t}$ is the bond price, $\mathrm{AI}_{b,t}$ is accrued interest, and $\mathrm{C}_{b,t}$ the coupon payment. This procedure yields 1.4 million month-return observations. The cross section of monthly bond returns contains between 2,600 to 8,000 observations. This sample is based on 47,900 unique bonds issued by 5,588 firms.

I use NCUSIP from CRSP and the Issuer CUSIP and Issue Id from TRACE and FISD, and match securities for 1,797 firms. Generally, there is a unique equity issue corresponding to each firm in the equity dataset. However, firms issue more than one bond (on average 6 and maximum 167 issues per firm). The final subset of matched securities contains 10,900 bonds, 23% of the initial bonds dataset. When I examine equity-bond correlations, I aggregate bond returns at firm level. Table 2.2 shows descriptive statistics of the final bond sample.

2.3.2 Granularity Index

Abolghasemi et al. (2020) construct a measure for granularity that is the market value of a certain number, g, of firms' equity as fraction of total capitalization of the equity market. I extend this measure by replacing the equity value with the combined market value of firm equity and debt (V_M). This measure is closer to capturing the value of firm assets by including the right hand side of the balance sheet, combined value of debt and equity. Since this measure follows the relative market value of the largest g firms in the market,

$$Top_g = \frac{\sum_{i=1}^{g} V_i}{\sum_{j=1}^{N} V_j} = \frac{\sum_{i=1}^{g} V_i}{V_M}. \tag{2.2}$$

$V_M = \sum_{j=1}^{N} V_j$ is the total market capitalization of firms. This measure is simple to construct, and intuitive too. As Top_g increases, the economy is relatively more dominated by the same number of firms and vice versa.

Top_g is relatively close to measures of market concentration, for instance the Herfindahl–Hirschman Index (HHI),

$$HHI = \sqrt{\sum_{i=1}^{N} w_i^2}, \tag{2.3}$$

or Gabaix and Koijen (2020)'s excess Herfindahl–Hirschman Index (exHHI), an adjustment to HHI,

$$exHHI = \sqrt{-\frac{1}{N} + \sum_{i=1}^{N} +w_i^2}, \tag{2.4}$$

where $w_i = \frac{V_i}{V_M}$, V_i is the market value of firm i.

Since any number of firms could be used to construct this measure, I show in Figure (2.1) that for $g \in \{10, ..., 20\}$, $Top_{g=20}$ is close to perfectly ($\rho = 0.98$) correlated with exHHI. As g increases, the correlation coefficient starts to decrease. In Panel A and B of Figure (2.2) I plot exHHI and Top_{20}, and in Panel C and D I display shocks to these measures.

Top_g is a persistent variable, hence I follow Bali et al. (2017) and use innovations in the granularity index in my empirical tests. Throughout the upcoming sections of the paper, I use ΔTop^t_{20} as measure of granularity innovations. In every month t, I consider corresponding change over the past three months, ΔTop^t_{20} = Top^t_{20} - Top^{t-3}_{20}. While changes over shorter horizons are significant in many of the analyses in the paper, changes over the past three months are consistently significant within and across equity and bond return datasets.

Top_g captures the relative appreciation in firm value for the top g firms. An average appreciation in firms value could correspond to average positive return on constituents of Top_g. The immediate question is whether granular innovations are a nonlinear proxy of the market return. I investigate this question in Figure (2.3). First, I compute return on two hundred portfolios. Consider a universe of firms composed of the largest 100 firms in the market. Proxy return on grains by return on the top g firms when $g = \{1, 2, ..., 100\}$. As well, proxy return on non-grains by returns on $100 - g$ firms. Then, I compute the correlation between different measures of granularity and these portfolio returns and plot the result.

It is evident that ΔTop_g is positively correlated with the value weighted return on the top g firm, and this result does not depend on measure of granularity nor on the number of firms in Top_g. After all, when Top_g firms occupy a bigger slice of the market pie, it means they have, on average, appreciated in value, hence offering positive returns (blue lines). On the other hand, ΔTop_g is always negatively correlated with return on bottom firms. I also produce similar figures based on equity and bond market valuation data only (Figure 2.4-2.5). These figures are suggestive of a similar story.

Notice that for any number greater than 20, the value weighted return on the Top g firms is highly correlated with the market return. Therefore, in the main tests conducted in the next section, I only control for market return. I verify that substituting the market return with return on Tog_g firms yield similar results.

I show the correlation structure between granular innovations and documented risk factors in the bond market in Table 2.3. Granular innovations are modestly negatively related to measures of market return in equity and bond markets. While they are positively correlated with the term spread, granular innovations are negatively correlated with the default spread. Granularity and economic uncertainty are positively correlated. This observation comes from the fact that both measures tend to increase in bad economic times. Finally, there is almost no correlation with the downside risk factor of Bai et al. (2019), but negative and modest correlation with their liquidity and credit risk factors.

2.3.3 Cross Sectional Predictors of Bond and Equity Returns

I examine the robustness of all empirical findings of his paper after controlling for established risk factors in the corporate bond market to better understand how exposure to granularity shocks is different from exposure to other sources of risk. Following Fama and French (1993), I control for default and term spreads. Default spread is the difference return between a government issued bond and a AAA corporate bond with the same maturity. I proxy default spread by taking the difference in return between 10-year corporate and Treasury bond returns. Term spread is the difference in return between a short term and a long term government-issued bond. I proxy the term spread by taking the difference between 10-year T-bonds and 3-month T-bills.

Following Daniel and Titman (1997), I control for firm characteristics such as size, time to maturity, and crediting ratings. Size is measured by the natural logarithm of the amount outstanding, the product of number of bonds outstanding and the price of the bond. Credit ratings are from rating agencies, assigned to firms based on the ability of the firm to service its debt. I combine credit rating information from Moody's and Standard and Poor's. When there are more than two ratings available for the same bond, I follow Bai, Bali, and Wen (2019) and consider the average of the two ratings. Time to maturity measures the remaining life of the bond before maturity time in years.

Bai et al. (2019) propose a model with three risk factors that explains the cross section of future bond returns. These factors are based on bond market variables, and measure credit, downside, and liquidity risks in the cross section of corporate bond returns. Bai et al. (2019) show that these factors, along with return reversal, explain most of the variation in the cross section of corporate bond returns. I retrieve these data from the authors' website.

Bali, Subrahmanyam, and Wen (2020b) argue that economic uncertainty affects the cross section of corporate bonds. Similar to Bali et al. (2017), the authors use the economic uncertainty measure of Jurado et al. (2015). An increase in economic uncertainty translates into a narrower investment opportunity set for investors. Since increase in granularity is correlated with an increase in economic uncertainty, I include this variable to distinguish between the two channels. Additionally, I control for financial uncertainty.

It is standard practice to control for firm-level characteristics in cross-sectional regressions (Bai et al., 2019; Bali et al., 2017). Accordingly, I control for short-term reversal, return from the prior month, size, the natural logarithm of bonds amount outstanding, time to maturity, the number of days before an issue matures, and credit ratings, the numerical equivalent of credit ratings assigned to bond issues by rating agencies.

Abolghasemi et al. (2020) show that granularity is priced in the cross section of common stocks. I reproduce their results and use the same set of control variables in the cross sectional regressions. Namely, I control for Fama and French SMB and HML factors, market return, momentum factor of Carhart (1997), aggregate liquidity (Pástor and Stambaugh, 2003), and economic uncertainty of Bali et al. (2017). I also control for size and reversal. Size is the natural logarithm of market capitalization. Reversal is equity returns from the past month.

2.4 Empirical Methodology and Analysis

I present two sets of results in this section. First, I perform standard asset pricing tests that examine the cross section of corporate bond returns. These tests include Fama-Macbeth cross-sectional regressions at individual bond level, portfolio sorts, and panel regressions at portfolio level.

Second, I show that the conditional correlation between firms' bond and equity returns is higher when the economy is more granular (either level or shocks). My findings further show that changes in the correlation coefficient are predicted by granularity innovations. Additionally, I present a more formal set of results to support the hypothesis that this co-movement is due to exposure of firms' securities to common risk factor in the cross-section of equity and bond returns.

2.4.1 Asset Pricing Implications

In this section, I show that granularity innovations influence expected corporate bond and equity returns. I conduct two analyses, a standard two-pass Fama-Macbeth regression as well as panel regressions. I concur the findings in ABDJ (2020), and focus the asset pricing implications of granularity innovations to the corporate bond market. Further, I estimate cross sectional regressisons to develop an understanding about how granularity betas relate to exposure to traded risk factors.

Fama-MacBeth Regressions

In this section, I perform the standard two-step Fama-Macbeth cross sectional regressions, and estimate the slope coefficient associated with granularity innovations in equity and bond markets. I proceed such that for each security (bond or equity) i, at a given time τ, I run the following time-series regression

$$R_{t,i} - R_{t,f} = a_i + \beta_{\tau,i}^{\Delta G} \Delta G_t + \sum_j \beta_{\tau,i}^{j} X_{t,j} + e_{t,i}, \qquad t \in \{\tau - 35, \dots, \tau\}, \tag{2.5}$$

where $X_{t,j}$ are aggregate risk factors in the corporate bond market. This procedure yields a cross-section of $\{\hat{\beta}_{\tau,i}\}$.

Then, I perform the cross-sectional regression where future excess returns on firms bond or equity in month $\tau + 1$ are regressed on estimated betas with respect to different risk factors, including granularity shocks, from month τ,

$$R_{\tau+1,i} - R_{\tau+1,f} = \alpha_{\tau+1} + \gamma_{\tau+1}^{\Delta G}\hat{\beta}_{\tau,i}^{\Delta G} + \sum_j \gamma_{\tau+1}^{j}\hat{\beta}_{\tau,i}^{j} + \sum_c \gamma_{\tau+1}^{c} C_{\tau,i} + \varepsilon_{\tau+1,i}, \quad i \in \{1,\dots,N\}, \tag{2.6}$$

where N is the number of securities in the sample, and $C_{\tau,i}$ represents firm-level characteristics such as size and reversal. Repeating the analysis at each time $\tau+1$ in turns yields a time-series of $\{\hat{\gamma}_{\tau+1}^{\Delta G}\}$. I report the average value of this slope coefficient, with the standard error around the mean.

One common criticism to the Fama-Macbeth cross-sectional regressions is erroneous estimations of factor betas in the first step. I observe that the level of granularity is higher in bad economic times, and granular innovations are modestly correlated with default and market returns. Based on these observations and common practice in the literature, I estimate granularity betas from

$$\begin{aligned} R_{t,i} - R_{t,f} = a_i &+ \beta_{\tau,i}^{\Delta G}\Delta G_t + \beta_{\tau,i}^{Mkt} Mkt_t + \beta_{\tau,i}^{Def} DEF_t + \beta_{\tau,i}^{Term} TERM_t \\ &+ \beta_{\tau,i}^{CRF} CRF_t + \beta_{\tau,i}^{LRF} LRF_t + \beta_{\tau,i}^{DRF} DRF_t \\ &+ \beta_{\tau,i}^{Unc_f}\Delta Unc_{f,t} + \beta_{\tau,i}^{Unc_e}\Delta Unc_{e,t} + e_{t,i}, \\ & t \in \{\tau-35,\dots,\tau\}. \end{aligned} \tag{2.7}$$

Thus, even in the nested specification of equation 2.5, $\beta_{\tau,i}^{\Delta G}$ is estimated after controlling for sources of bias in obtaining granularity betas. I report the average slope coefficients for individual bond returns in Table 2.4 and 2.5, and for firm equity returns in Table 2.6. Each table presents results for various nested versions of equation 2.6, corresponding to different columns.

In Table 2.4, the first column shows estimation results when only granularity innovations explain future excess bond returns. The time series average of granularity slope coefficients is negative and significantly different from zero. In the second column, I highlight the fact that changes in the relative size of grains, the largest firms in the economy, is different from changes in the relative size of large, but not the largest firms in the economy. In other words, a granularity index should capture the dynamics of firms that are far in the tail of the size distribution. Accordingly, I construct an index that captures the relative size the bottom 50 firms among the top 200 largest firms. The market value of these firms is not negligible, still they are not far in the tail of firms' size distribution, and the estimated slope coefficient is on average insignificant.

In the third column, I control for bond market betas, term spread betas, and default spread betas (Fama and French, 1993). Market beta risk premiums are positive and significantly different from zero in all nested estimations. Term spread and default spread premiums are not significant predictors of future returns. In the forth and the fifth columns, bond market risk factors of Bai, Bali, and Wen (2019), and innovations to economic and financial uncertainty measures of Jurado, Ludvigson, and Ng (2015) are included. In the full specification, the last column, granularity shocks remain a negative and economically significant predictor of expected corporate bond returns.

As represented in equation 2.6, I examine whether granularity is picking firms characteristics. In Table 2.5, I include short term reversal, size, credit worthiness, and time to maturity. Following Bai et al. (2019), I add returns from the prior month to control for short term reversal (Jegadeesh, 1990). Consistent with the literature, reversal, the return from prior month, is a negative and significant predictor of future bond returns. Size, the natural logarithm of the amount outstanding of bonds issued by firms, is also a negative and significant predictor of future corporate bond returns. Credit worthiness is proxied by credit ratings from Moody's and Standard and Poor's, with the average estimated slope coefficient that is not different from zero. Time to maturity is the number of days before each bond matures, and it is significant in the last column, with a positive coefficient. Regardless of the nested specification, the time series average of granularity slope coefficients remains negative, hence granularity is a significant predictor of future corporate bond returns.

Table 2.6 concurs the findings of Abolghasemi, Bhamra, Dorion, and Jeanneret (2020) who document that granularity innovations are negatively related to future common stock returns. Similar to the bond analysis, I start by including granularity innovations as the sole explanatory variable in Column 1. In the second column, similar to the bond analysis, I construct and include a measure that captures the size dynamics of large, but not the largest firms, and include shocks to this measure. Market betas, along with betas with respect to the Fama and French factor betas (SMB, HML, CMA, and RMW), are included in the third and fourth columns.

Bali et al. (2017) argue that uncertainty is a predictor of future stock returns. Therefore, I include innovations in both financial and economic uncertainty measures of Jurado et al. (2015) in column 5. In column 6, I add the aggregate liquidity measure of Pástor and Stambaugh (2003). In the remaining columns I augment the specification by including short-term reversal, and size. Granularity remains a negative and significant predictor of future equity returns.

In all the empirical tests described so far, standard errors are robust to heteroskedasticity and autocorrelation (Newey and West, 1987) with an optimal number of lags.

Granularity Betas versus Other risk Factors

It is constructive to develop an understanding of how granularity betas relate to traded risk factors in the cross section of corporate bond returns. Accordingly, I follow Bail et al. (2020) and estimate this cross sectional regression

$$\beta_{\Delta G_t} = c_0 + c_i \beta_{t,i} + c_j X_{t,j} + \varepsilon_t, \tag{2.8}$$

where granularity betas every month are regressed on betas with respect to aggregate risk factors ($\beta_{t,i}$) and firms characteristics ($X_{t,j}$). Table 2.13 reports the average slope coefficient for each predictor in equation 2.8. Columns correspond to estimation results from nested versions of of equation 2.8 where predictors are incrementally included in the specification. From Table 2.13 one learns that exposure to granularity is positively and significantly related to exposure to market, default spread, and aggregate liquidity and downside risk factors. Granularity betas are on average higher for firms that are more liquid. They are also negatively related to reversal, return from prior month.

Granularity Portfolios

In this section, I sort corporate bonds with respect to their granularity betas, and investigate the return on a strategy that provides effective hedge against granularity innovations. This strategy takes a long position in bonds that are least exposed to granularity (with lowest granularity beta), and a short position in bonds that are most exposed to granularity (with highest granularity beta). I document that the risk-adjusted return on this strategy is economically significant after controlling for established risk factors in the bond and equity markets. First, I follow the these steps to form granularity sorted portfolios,

- Obtain bond preformation betas over 36-month rolling window regressions, where excess returns on firm bonds from in period $\tau = \{t-35,...,t\}$ is regressed on granularity shocks (ΔG_t), market excess return ($R_{B,t} - R_{f,t}$), and the interaction between market and granularity shocks ($\Delta G_t \times (R_{B,t} - R_{f,t})$ over the same period,

  $$R_{t,i} - R_{t,f} = a_i + \beta_{\tau,i}^{\Delta G} \Delta G_t + \beta_{\tau,i}^{Mkt} (R_{B,t} - R_{f,t}) + \beta_{\tau,i}^{Mkt \times \Delta G_t} (\Delta G_t \times (R_{B,t} - R_{f,t})) + e_{t,i}, \tag{2.9}$$

  where similar to the first step in the Fama-Macbeth regressions, $t \in \{\tau - 35, \ldots, \tau\}$. Including market excess return and the interaction term ensures that the estimated granularity beta is not biased by potential common drivers of market returns and granularity shocks,

- For every month τ sort bonds into 10 portfolios based on the preformation betas in time $\tau - 1$ such that the first (last) decile comprises bonds with smallest (largest) granularity betas.

- Consider both equal and value weighting schemes to compute corresponding time series of returns on bonds included in each decile.

Table 2.7 presents the characteristics of granularity portfolios. The first column reports the average return for each decile. Columns 2-3 report the intercept of a regression where excess portfolio returns are regressed on a constant and bond market risk factors

(1), equity market risk factors (2), and both equity and bond market risk factors (3). T-statistics are computed by HAC standard errors.The last row reports the results for the L-H strategy. Columns 5-7 report portfolio betas, and columns 8-10 the average portfolio characteristics, credit ratings, time to maturity, and size.

The first decile is comprised of bonds with less favourable credit ratings, shorter time to maturity, and smaller amount outstanding. A credit rating of 11 is the threshold that separates investment grade (IG) from non-investment grade (non-IG) bonds. In the Appendix of the paper I show that the results hold even if portfolios are built on only IG or non-IG, or short versus long time to maturity bonds.

The granularity strategy takes a long position in the low beta decile and a short position in the high beta decile. I regress the return on the long-short strategy on cross sectional predictors of equity and corporate bond returns. I show the estimation results in Table 2.8. Each column corresponds to a nested specification. For example the first shows the alpha of the strategy with respect to traditional bond market risk factors. The first row reports the regression alphas with t-statistics (based on HAC standard errors) in parentheses. I repeat the same exercise for equal-weight returns (Table 2.9).

The average return on the long-short strategy is positive, and the excess return on the long-short strategy, after controlling for risk factors, is significantly positive too. Thus, being least exposed to granularity by keeping the first decile while shorting the tenth decile yields a risk-adjusted return that is economically significant. These findings are consistent with a negative slope coefficient, estimated in cross-sectional regressions. The quantity of granularity risk of the L-H strategy is negative ($\beta_l^{\Delta G}$-$\beta_h^{\Delta G} < 0$), the slope coefficient from Fama-MacBeth regressions is negative,$\hat{\gamma}^{\Delta_G} < 0$, hence the expected return on L-H strategy is positive.

Panel Regressions

In this section, I provide further evidence to support the hypothesis that granularity innovations matter in the cross section of portfolios of corporate bonds, sorted based on various criteria. I use an alternative estimation methodology, namely fixed effect regressions.

The cross section of bond portfolios is significantly narrower than the cross section of individual bonds. Compared to Fama-Macbeth two-pass approach, fixed effect estimations allow for a reliable estimation of the slope coefficient at portfolio level tests (Petersen, 2009).

I use rolling betas with respect to granularity shocks as well as all the bond market predictors mentioned Table 2.1. I then estimate

$$R_{b,\tau+1} - R_f = \gamma_g \beta_{g,\tau} + \sum_{j=1} \gamma_j \beta_{b,\tau} + \eta_{b,\tau+1}, \tag{2.10}$$

where excess bond returns in month $\tau + 1$ are regressed on betas from month τ.

I estimate fixed effect regressions for value and equal weighted returns on bond portfolios, including beta-sorted, credit rating sorted, and maturity sorted portfolios. Portfolio betas are estimated from regressing excess bond returns on excess bond marker return and a constant. For credit rating portfolios I combine Moody's and Standard & Poor's credit ratings. Where there is an overlap, I consider the average of credit scores reported by different rating agencies. Each column in Table 2.10 corresponds to estimation results for a specific portfolio, but the last column shows results for all portfolios in tandem. The first row reports the granularity coefficient, where as expected is always negative and significantly different from zero.

Results in these two tables concur with those of the Fama-Macbeth regressions. Granularity innovations are negatively related to the cross section of corporate bond returns.

2.4.2 Conditional Correlations

The empirical evidence from the previous section supports the hypothesis that granularity innovation are influential factors in the cross section of equity and corporate bond returns. In this section, I show that granularity is significantly related to the conditional co-movement of securities issued by the same firm, but traded in different markets. This significant relation supports the hypothesis that granularity is perceived similarly in equity and corporate bond markets.

As argued earlier, I argue that months when granularity increases (decreases) correspond to higher (lower) correlation between firms' equity and bond returns. To support this story, I propose two tests.

First, I follow a two-step procedure to examine the conditional EBC, estimated as

$$\begin{aligned}
\rho_h &= Corr(r_{b,t}, r_{e,t}) | \Delta G_t > Q_{\Delta G,70}, \\
\rho_l &= Corr(r_{b,t}, r_{e,t}) | \Delta G_t < Q_{\Delta G,30}.
\end{aligned} \tag{2.11}$$

$\rho_h(\rho_l)$ is the correlation between firm bond and equity returns when granularity is high (low). I consider the 70^{th} and 30^{th} percentiles of granularity shocks, $\Delta \mathrm{G}_t$, at time t as thresholds for high or low granularity, respectively, and measure granularity by the market value of the 20 largest firms as fraction of total market value of the firms. The latter, as in previous analyses, is the combined value of firm equity and bond(s). The histogram of the bootstrapped averages of conditional versus unconditional correlations is illustrative for the purpose of this section.

Considering the length of the sample, and the conditioning criterion, there remains a small number of firms to estimate EBCs. Notice that with a balanced panel of firm equity and bond return observations, the conditioning criterion yields estimates based on 50 months for high and 50 months for low granularity months. But the firm level bond and equity data are not balanced, hence estimations are not reliable. To address this problem, I form equity and bond micro portfolios (Barras, 2019) comprised of 20 randomly chosen firms. This approach yields a time series of returns for firms securities with a reliable number of observations to estimate conditional correlations. For every random draw of portfolios, I estimate corresponding EBC coefficients.

Portfolio construction based on characteristics or betas is not applicable here two reasons. First, the main purpose of portfolio formation in this section is to make sure there are enough observations in the equity versus bond returns time series to have a reliable estimate of conditional EBCs. Second, portfolio construction procedures usually require an estimation window, hence a less inclusive final sample. Besides, investigating correlations based on sorted securities forces the reader to wonder whether the underlying characteristic is crucial to the results.

In the second step, to construct the distribution of the mean for the EBC, I draw 50,000 samples from the estimated EBCs, and record the mean of the corresponding sample for each iteration. I plot the histogram of the bootstrapped means for both conditional correlations and the corresponding difference in Figure 2.6 where returns are conditioned on ΔexHHI. I also present the histograms in Figure 2.7 with respect to ΔTop_{20}.

In either figure, the average EBC is higher when granularity is higher. Panel A and C do not overlap, suggestive of a significant difference in conditional correlations. In absolute terms, conditional correlations are more than 50% higher in high versus low granularity months. The results are robust to including as few as five firms in the ΔTop_g measure. It is hard to claim that only five firms among thousands are representative of the market return. In other words, the ΔTop_g measure picks a different economic force. Still, I provide other tests.

A more formal empirical test should substantiate what the histograms show. Similar to previous tests, I obtain conditional coefficients by estimating a rolling window correlation measure for each firm,

$$\hat{\rho}_{e,b,t} = \frac{\sum_{s=t-w-1}^{t-1} r_{e,s}, r_{b,s}}{\left(\sum_{s=t-w-1}^{t-1} r_{e,s}^2\right)\left(\sum_{s=t-w-1}^{t-1} r_{b,s}^2\right)}. \tag{2.12}$$

where w is the length of the estimation window, 36 months. This procedure yields a panel of EBCs. I then test if changes in the rolling correlations relate to granularity shocks. Due to a relatively short span of my corporate bond returns (about 200 months), applying a more sophisticated methodology such as the Dynamic Conditional Correlation would not be reliable (Engle and Sheppard, 2001).

I estimate

$$\Delta\hat{\rho}_{f,t} = \beta^{\Delta G_t}\Delta G_t + \sum_j \beta_j Control_j + e_{f,t}, \tag{2.13}$$

where the evolution of the cross section of changes in firms EBC is regressed on changes in granularity as well as control variables. In Table 2.11, I report fixed effect results, and take into account, incrementally, potential predictors of EBC in the specification. I provide a detailed list of control variables underpinning the empirical analysis of this paper in Table 2.1. I compute t-statistics by clustering standard deviations at firm level.

The first row in Table 2.11 reports the granularity coefficient. It is always positive and significantly different from zero. Hence, these results support the hypothesis that granularity shocks lead to co-movement between firm securities.

Among other control variables, I include excess return on the top 20 firms, R_{top}-R_f, same firms used to construct the granularity measure. Unlike granularity, return on top firms could be positively, negatively, or insignificantly related to changes in correlations. This finding highlights the distinct nature of changes in granularity and return on top firms.

2.5 Granularity and Industry Competition

A strand of literature in industrial organization links expected equity returns to industry competition. Hou and Robinson (2006) document that firms in more concentrated, hence less competitive, industries are expected to deliver lower expected returns. The authors attribute this empirical finding to entry barriers and propensity of firms to embrace innovation. Entry barriers insulate the firms from aggregate demand shocks, and innovation, as a more prominent characteristic of a competitive industry, corresponds to more uncertain cashflows, hence riskier equity. Bustamante and Donangelo (2017) develop a theoretical framework and argue that assuming an only negative relation between industry competition and expected returns is incomplete for it does not consider the dynamic nature of competition nor the role of higher industry discount rates as barriers to entry.

While granularity is a market-wide phenomenon, industry competition is the concentration in the product markets of firms that belong to the same industry. I argue in this section that the industry competition premium is different from the granularity premium. First, as Bustamante and Donangelo (2017) indicate, the final effect of industry competition on expected equity returns, both in terms of magnitude and sign, depends on the operating leverage versus the threat of entry channels, and it could be insignificant, positive, or negative. Second, the empirical investigation in this literature is based on the level, rather than innovations in industry competition. Third, both portfolio sorts and cross-sectional tests reveal a positive risk premium on measures of industry competition while the granularity premium is negative (Hou and Robinson, 2006).

To investigate the relation and importance of industry competition to the granularity channel, I follow Hou and Robinson (2006) and Bustamante and Donangelo (2017) and proxy industry competition by an industry-level HHI index. Industries are recognized based on the Standard Industrial Classification (SIC) codes, leading to 86 industries in the cross section of firms in my sample. Rather than using sales data, which is available in quarterly frequency, I use the market capitalization of firms, consistent with other tests in this paper. This procedure yields a panel of industry concentration measures.

First, I verify that granularity innovations are insignificantly to modestly correlated with innovations in industry competitiveness indexes. The correlation coefficients range from -0.06 to 0.50 for the period 1973-2020. To further examine the role industry concentration measures, I focus on three industry HHIs that are at least 35% correlated with the granularity measure, hence potentially belonging to a similar economic force. In Table 2.12, I estimate cross sectional regressions where future excess returns on corporate bonds are regressed on granularity as well as industry concentration betas. Industry competition premium could be positive or negative but it does not provide any marginal explanation of cross-sectional variation after controlling for firm characteristics. The granularity premium remains negative and significant after including industry concentration measures.

2.6 Conclusion

In this paper I argue that time variation in the heavy tail of firms distribution is informative about corporate bond prices. Security returns are driven by the gravity towards the largest firms in the economy, the grains. I document that granularity is similarly perceived in equity and corporate bond markets, and it explains conditional correlations between firms' equity and bond returns. In months where the state of granularity is high, EBC is on average 50% higher. Throughout this paper, I adapt Abolghasemi et al. (2020)'s measure of granularity, Top_g, the total market value of largest g firms as fraction of total market capitalization. For example for $g = 20$, $\text{Top}_{20} = 25\%$ means that 20 firms account for 25% of the combined value of equity and corporate bond markets.

In the spirit of Merton (1973), granularity is a state variable that correlates with future investment and consumption opportunity sets. As such, innovations to state of granularity affect expected asset returns. I show that granularity is an influential factor in the cross section of bond returns, and corporate bonds that are more exposed to granularity pay less. Further, I concur the results of Abolghasemi et al. (2020) that granularity is an influential factor in the cross section of equity returns, and market and security risk premia are negatively related to granularity innovations. The empirical findings of this paper are robust to including an extensive set of established risk factors as control variables. Also, the findings are robust to using alternative measures for granularity such as the HHI, or current and past values of Top_g for $g = \{5, 10, 20\}$.

The negative sign of the slope coefficient comes from the fact that granularity is inversely related to the health of the investment opportunity set. A more granular economy corresponds to an adverse shift, and less diversification benefits for a typical investor. From the perspective of a consumer, the consumption basket grows less diversified with positive granular innovations, and an unfavourable shock to one of the firms disrupts smooth consumption even more. Hence, assets that are least sensitive to granularity will have a higher expected return. Assets that that are more exposed to granularity, including the market portfolio itself, deliver a lower expected return.

There are many questions that remain open to future research. Among others, it would be interesting to understand why, in the first place, some firms grow much larger in the economy. Is it a purely random phenomenon, or there is some predictability. This question aims at a multifaceted and endogenous phenomenon. Still, it is viable to examine potential relations such as institutional investment and how it contributes to mega-large firms.

Also, it would be interesting to understand whether granularity is informative about returns on derivative products written on firms securities. Culp et al. (2018) and Culp et al. (2021), in the spirit of Merton's credit risk model, derive the cross section of implied bond spreads, and shed light on the dynamics of option risk premia. The question is whether the dynamics of option implied credit spreads and option risk premia evolve differently with respect to the state of granularity of the economy.

A more challenging still exciting research path is building a theoretical framework that explains the empirical findings in this paper. The existing literature studies the dynamics of expected returns and conditional correlations in Lucas economies (see Cochrane et al. (2007) and Martin (2013)). However, solving these models is far from trivial. There are two dimension to improve in these models. First, starting with firms that finance their activities by issuing both equity and debt. The challenge is incorporating the endogenous decisions of firms to issue debt, with the stochastic and endogenous nature of relative firms' size, the granularity measure. Second, to incorporate a large number of firms in the model. While having two or three firms is a sizeable leap towards understanding the dynamics of prices with respect to granularity, the ultimate goal is solving for prices when the granular economy comprises a large number of firms.

2.7 Tables

Table 2.1
Definition of Variables

This table defines the variables underpinning this study and the corresponding data sources. All data are retrieved at monthly frequency.

Variable	Definition	Source
G	Degree of granularity measured as the total market capitalization value of the largest firms in the U.S. as fraction of the total market capitalization of the CRSP universe. We use 20 of the firms with the largest market capitalization. Alternatively, we consider the excess Herfindahl-Hirschman Index proposed by Gabaix and Koijen (2020), defined as exHHI = $\sqrt{-\frac{1}{N}+\sum_1^N w_i^2}$, where w_i the market value of firm i as fraction of total market capitalization, and N is the total number of firms.	Wharton Research Data Services
R_B - R_f	Bond market excess return, computed as the value-weighted return on the Enhanced TRACE universe.	Enhanced TRACE & Mergent FISD
Term	Term spread. The difference between long-term Treasury bond return and 3-month Treasury bill return.	Bloomberg
Def	Default spread. The difference in return between a corporate bond and treasury bond indices.	Bloomberg
BBW	Bond market risk factors of Bai et al. (2019), including Credit risk factor (CRF), downside risk factor (DRF), and liquidity risk factor (LRF).	Author's Website
Uncertainty	Measures of financial (Unc_f) and economic (Unc_e) uncertainty à la Jurado, Ludvigson, and Ng (2015).	Author's Website
Fama French Factors	Fama and French five factor model as control for the cross section of equity returns: Excess market return, SMB, HML, RMW, and CMA.	Kenneth French data library

Table 2.2
Summary Statistics: Individual Bonds

This table presents the summary statistics for individual bonds. Enhanced TRACE and Mergent FISD datasets are main sources of corporate bond price and characteristics information. Variables include excess monthly return (R_b-R_f), bond market betas (β_{bond}), time-to-maturity (TTM), credit ratings (Credit Rating), and Size (amount outstanding, in millions of dollars). Monthly data span July 2002 to June 2019.

Measure	Mean	Med	Std	1%	5%	25%	75%	95%	99%
R_b-R_f	0.64	0.40	6.98	-11.86	-3.96	-0.53	1.59	5.30	14.67
β_{bond}	1.20	0.90	1.67	-0.40	0.10	0.46	1.54	3.03	7.39
TTM	9.84	6.73	9.06	1.12	1.56	3.69	13.20	27.62	30.26
Credit Rating	8.21	8.00	3.13	1.00	4.00	6.00	10.00	14	17.50
Size (m)	491.8	338.4	652.5	0.9	6.0	132.6	615.4	1543.1	2840.9

Table 2.3
Correlations with Granular Innovations

This table shows the correlation among the main variables used in the empirical analysis of this paper. The list of variables includes granularity innovations (based on largest 20 firms), excess bond market returns, term and default spreads, downside, credit, and liquidity risk factors of Bail, Bali, and Wen (2019), and innovations to financial (Unc_f) and economic (Unc_e) measures of uncertainty of Jurado et al. (2015). Data span July 2002 to June 2019.

	ΔG_t	R_B-R_f	Term	Def	DRF	CRF	LRF	Unc_f	Unc_e
ΔG_t	1	-0.16	0.23	-0.43	-0.02	-0.25	-0.21	0.30	0.26
R_B-R_f		1	0.36	0.22	0.34	0.49	0.48	-0.47	-0.41
Term			1	-0.62	0.25	-0.03	0.02	0.01	-0.07
Def				1	0.07	0.41	0.22	-0.30	-0.24
DRF					1	0.39	0.35	-0.42	-0.35
CRF						1	0.33	-0.55	-0.52
LRF							1	-0.39	-0.36
Unc_f								1	0.52
Unc_e									1

Table 2.4

Fama-Macbeth Regressions, Cross Section of Individual Bond Returns (2002 - 2019)

This table shows estimation results of Fama-Macbeth cross-sectional regressions where the dependent variable is individual bond excess returns. Granularity is the total market value of equity and debt for the 20 largest firms as fraction of total market value of equity and debt of all firms. Explanatory variables include past betas with respect to granularity, innovations to the relative market value of smallest 50 firms among the 200 largest firms, bond excess market returns, default and term spreads, the three factors of Bai et al. (2019), downside risk (DRF), credit risk (CRF) and liquidity risk (LRF), and measures of financial (Unc_f) and economic (Unc_e) uncertainty of Jurado et al. (2015). Bond data are from Enhanced TRACE and Mergent FISD datasets. Data span July 2002 to June 2019. *, **, and *** indicate statistical significance at the 10%, 5%, and 1% level, respectively.

	(1)	(2)	(3)	(4)	(5)
ΔG_t	-0.173^{***}	-0.181^{***}	-0.104^{***}	-0.134^{***}	-0.118^{***}
	(-3.12)	(-3.06)	(-2.70)	(-3.56)	(-3.07)
ΔB_t		0.005	0.004	0.004	0.006
		(0.92)	(0.98)	(0.97)	(1.02)
R_B - R_f			0.117^{**}	0.121^{*}	0.171^{**}
			(2.51)	(1.69)	(1.99)
Term			-0.105	-0.040	-0.014
			(-0.58)	(-0.21)	(-0.07)
Def			0.175	0.168	0.194
			(1.45)	(1.17)	(1.29)
DRF				0.124	0.317^{***}
				(1.58)	(2.92)
CRF				-0.043	0.081
				(-0.40)	(0.70)
LRF				0.065	0.142^{*}

Continued on next page

Table 2.4 – *Continued from previous page*

	(1)	(2)	(3)	(4)	(5)
				(0.88)	(1.69)
Unc_f					-0.456^{**}
					(-2.37)
Unc_e					-0.049
					(-0.46)
adj-R^2	1.04	2.08	9.54	15.35	17.53
Obs.	137.00	137.00	137.00	137.00	137.00

Table 2.5

Fama-Macbeth Regressions, Cross Section of Individual Bond Returns and Characteristics (2002 - 2019)

This table shows estimation results of Fama-Macbeth cross-sectional regressions where the dependent variable is individual bond excess returns. Granularity is the total market value of equity and debt for the 20 largest firms as fraction of total market value of equity and debt of all firms. Explanatory variables include past betas with respect to granularity, innovations to the relative market value of smallest 50 firms among the 200 largest firms, bond excess market returns, default and term spreads, the three factors of Bai et al. (2019), downside risk (DRF), credit risk (CRF) and liquidity risk (LRF), and measures of financial (Unc_f) and economic (Unc_e) uncertainty of Jurado et al. (2015). Columns 1-4 augment the list of risk factors with firm level characteristics including reversal (Rev), Size, credit ratings (Rating), and time to maturity (TTM), respectively. The last column reports results for the full specification. Bond data are from Enhanced TRACE and Mergent FISD datasets. Data span July 2002 to June 2019. *, **, and *** indicate statistical significance at the 10%, 5%, and 1% level, respectively.

	(1)	(2)	(3)	(4)	(5)
ΔG_t	-0.114^{***}	-0.130^{***}	-0.105^{***}	-0.119^{***}	-0.141^{***}
	(-2.92)	(-3.49)	(-4.29)	(-3.09)	(-4.04)
ΔB_t	0.005	0.006	-0.000	0.007	0.002
	(0.98)	(1.00)	(-0.06)	(1.26)	(0.53)
R_B - R_f	0.206^{**}	0.173^{**}	0.169^{**}	0.160^{**}	0.153^{**}
	(2.36)	(2.09)	(2.15)	(2.21)	(2.45)
Term	0.115	0.075	-0.047	-0.076	-0.034
	(0.54)	(0.37)	(-0.26)	(-0.43)	(-0.19)
Def	0.175	0.174	0.194^{*}	0.218	0.194^{*}
	(1.17)	(1.18)	(1.68)	(1.43)	(1.69)
DRF	0.387^{***}	0.379^{***}	0.382^{***}	0.294^{***}	0.424^{***}
	(3.63)	(3.41)	(2.76)	(3.05)	(3.01)
CRF	0.022	0.067	0.096	0.094	0.123
	(0.20)	(0.60)	(1.12)	(0.80)	(1.37)
LRF	0.146^{*}	0.129	0.129^{*}	0.143^{*}	0.127^{**}
	(1.66)	(1.55)	(1.81)	(1.76)	(2.03)
Unc_f	-0.394^{**}	-0.407^{**}	-0.425^{**}	-0.468^{**}	-0.453^{**}

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Table 2.5 – *Continued from previous page*

	(1)	(2)	(3)	(4)	(5)
	(-2.15)	(-2.17)	(-2.33)	(-2.51)	(-2.48)
Unc_e	-0.082	-0.083	-0.240^{**}	-0.038	-0.237^{**}
	(-0.80)	(-0.85)	(-2.44)	(-0.35)	(-2.33)
Rev	-0.078^{***}				-0.185^{***}
	(-4.38)				(-10.21)
Size		-0.055^{***}			-0.030^{*}
		(-2.99)			(-1.77)
Rating			0.001		-0.005
			(0.07)		(-0.25)
TTM				0.000	0.000^{**}
				(1.23)	(2.00)
adj-R^2	1.65	2.91	9.50	15.39	19.10
Obs.	137.00	137.00	137.00	137.00	137.00

Table 2.6

Fama-Macbeth Regressions, Equity Returns (1973 - 2019)

This table shows estimation results of Fama-Macbeth cross-sectional regressions where the dependent variable is individual equity excess returns. Granularity is the market capitalization of the 20 largest firms as fraction of total market value of all firms. Explanatory variables include past betas with respect to granularity, innovations to the relative market value of smallest 50 firms among the 200 largest firms, Fama and French five-factor model, measures of financial (Unc_f) and economic (Unc_e) uncertainty of Jurado et al. (2015), and aggregate liquidity (Liq). Also, firm level characteristics including reversal (Rev), and Size. Equity data is from CRSP. Data span July 1973 to June 2019. *, **, and *** indicate statistical significance at the 10%, 5%, and 1% level, respectively.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
ΔG	-0.018^{**}	-0.023^{**}	-0.045^{***}	-0.061^{***}	-0.066^{***}	-0.071^{***}	-0.056^{***}	-0.041^{**}
	(-2.19)	(-2.30)	(-2.90)	(-3.76)	(-3.97)	(-3.83)	(-2.86)	(-2.13)
ΔB		0.002	0.003	0.003	0.003	0.004	0.002	0.001
		(1.01)	(1.10)	(0.98)	(0.93)	(1.28)	(0.57)	(0.36)
R_E-R_f			0.031	0.072	0.123	0.159	0.112	0.186
			(0.39)	(0.71)	(1.16)	(1.38)	(0.90)	(1.49)
SMB			0.158^{***}	0.186^{***}	0.210^{***}	0.226^{***}	0.140^{**}	0.044
			(3.67)	(3.38)	(3.65)	(3.77)	(2.31)	(0.76)
HML			0.010	0.052	0.065	0.072	0.113	0.089
			(0.24)	(0.74)	(0.91)	(1.00)	(1.46)	(1.17)
RMW				-0.089^{*}	-0.093^{*}	-0.096^{*}	-0.050	-0.030
				(-1.89)	(-1.89)	(-1.93)	(-0.98)	(-0.58)
CMA				0.015	0.018	0.021	0.033	0.010
				(0.34)	(0.39)	(0.44)	(0.67)	(0.22)
Unc_f					-0.230^{***}	-0.241^{***}	-0.187^{***}	-0.170^{***}
					(-4.32)	(-4.49)	(-3.33)	(-3.08)
Unc_e					-0.037^{*}	-0.041^{*}	-0.019	-0.015

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Table 2.6 – *Continued from previous page*

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
					(-1.69)	(-1.67)	(-0.69)	(-0.53)
Liq						0.287^{**}	0.210^{*}	0.178
						(2.47)	(1.84)	(1.58)
Rev							-0.042^{***}	-0.042^{***}
							(-11.58)	(-11.55)
Size								-0.130^{***}
								(-5.09)
Obs.	504.00	504.00	504.00	504.00	504.00	504.00	504.00	504.00

Table 2.7

Granularity Sorted Portfolios, Characteristics (2002 - 2019)

This table shows intercept and slope coefficients of a regression where return on granularity sorted portfolios is regressed on bond and equity market risk factors. The first column is the intercept with respect to bond market risk factors, namely market, default and term spreads, and Bai et al. (2019)'s factors. The second column reports alphas with respect to the Fama and French five-factor model. The third shows the alpha with respect to all factors in the bond and equity market. Columns three to six report the average preformation betas, post-formation betas, and bond market betas. I obtain pre-formation betas by rolling a 36-month window. Data span July 2002 to June 2019. *, **, and *** indicate statistical significance at the 10%, 5%, and 1% level, respectively. Standard errors are HAC robust.

	Avg. Ret	α_{bond}	α_{equity}	α_{all}	β^{pre}	β^{post}	β^{Mkt}	Credit	TTM	Size (m)
Low	1.160	0.620***	0.860***	0.550***	−2.280	−1.090	1.290	10.619	9.328	379.322
		(4.22)	(2.81)	(3.46)						
2	0.710	0.220***	0.500***	0.170***	−0.770	−0.370	0.940	9.455	8.503	477.041
		(4.92)	(2.89)	(3.31)						
3	0.570	0.180***	0.430***	0.150***	−0.360	−0.120	0.700	8.797	7.425	514.653
		(4.03)	(2.79)	(3.25)						
4	0.460	0.190***	0.350***	0.160***	−0.130	−0.120	0.480	8.187	6.509	592.211
		(3.69)	(3.23)	(3.84)						
5	0.390	0.140***	0.300***	0.110***	0.040	0.010	0.420	7.863	6.043	644.027
		(3.47)	(3.14)	(3.44)						
6	0.360	0.130***	0.280***	0.100***	0.180	0.040	0.380	7.768	6.257	659.775
		(2.75)	(3.20)	(2.59)						
7	0.410	0.150***	0.300***	0.110**	0.330	0.030	0.460	7.883	7.303	658.453
		(3.14)	(2.84)	(2.51)						
8	0.470	0.150**	0.330***	0.100*	0.530	0.080	0.570	7.889	9.134	633.931
		(2.50)	(2.63)	(1.78)						
9	0.490	0.110	0.310*	0.030	0.870	0.110	0.720	7.808	12.639	636.757
		(1.46)	(1.88)	(0.45)						
H	0.560	0.030	0.290	−0.070	1.750	−0.060	1.060	7.994	15.605	555.392
		(0.35)	(1.34)	(−0.80)						
L-H	0.600	0.580***	0.570***	0.630***	−4.030	−1.030	0.240			

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Table 2.7 – *Continued from previous page*

	Avg. Ret	Bond Factor α	Equity Factor α	All Factor α	Pre β	Post β	Market β	Credit Rating	TTM (Year)
		(3.92)	(2.69)	(3.46)					

Table 2.8

VW Long-Short Granularity Strategy (2002 - 2019)

This table shows estimation results from a regression where the dependent variable is value-weighted excess return on a long-short portfolio of corporate bonds (D_1 - D_{10}) formed with respect to granularity betas. Each column corresponds to a set of explanatory variables included in the analysis. The first row reports the estimated alpha of the strategy. Granularity is the total market value of equity and debt for the 20 largest firms as fraction of total market value of equity and debt of all firms. Controls variables include bond and equity market risk factors, and measures of financial (Unc_f) and economic (Unc_e) of Jurado et al. (2015). More details about control variables is provided in Table I. Bond data are from Enhanced TRACE and Mergent FISD datasets. I obtain betas by rolling a 36-month window. Data span July 2002 to June 2019. *, **, and *** indicate statistical significance at the 10%, 5%, and 1% level, respectively. Standard errors are HAC robust.

	(1)	(2)	(3)	(4)
Constant(%)	0.592^{***}	0.568^{***}	0.659^{***}	0.497^{***}
	(4.15)	(2.69)	(4.26)	(4.11)
r_B - r_f	-0.185^{**}		-0.218^{**}	-0.281^{***}
	(−2.10)		(−2.26)	(−4.18)
Term	0.194		0.340^{*}	0.189
	(1.16)		(1.78)	(1.52)
Def	0.478^{***}		0.623^{***}	0.545^{***}
	(3.26)		(4.00)	(4.36)
r_E - r_f		0.131^{**}	-0.128^{***}	-0.170^{***}
		(2.44)	(−2.73)	(−2.92)
SMB		0.036	−0.009	0.009
		(0.36)	(−0.12)	(0.14)
HML		0.162	0.097	0.058
		(1.22)	(1.08)	(0.95)
CMA		−0.218	−0.082	−0.100
		(−1.36)	(−0.81)	(−0.99)
RMW		−0.043	0.081	0.045

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Table 2.8 – *Continued from previous page*

	(1)	(2)	(3)	(4)
		(-0.30)	(0.53)	(0.34)
DRF				0.282^{**}
				(2.30)
LRF				0.137
				(1.19)
CRF				0.154
				(0.98)
Adj. $\bar{R}^2(\%)$	0.42	0.15	0.43	0.52
Observations	172.00	169.00	169.00	169.00

Table 2.9

EW Long-Short Granularity Strategy (2002 - 2019)

This table shows estimation results from a regression where the dependent variable is equal-weighted excess return on a long-short portfolio of corporate bonds (D_1 - D_{10}) formed with respect to granularity betas. Each column corresponds to a set of explanatory variables included in the analysis. The first row reports the estimated alpha of the strategy. Granularity is the total market value of equity and debt for the 20 largest firms as fraction of total market value of equity and debt of all firms. Controls variables include bond and equity market risk factors, and measures of financial (Unc_f) and economic (Unc_e) of Jurado et al. (2015). More details about control variables is provided in Table I. Bond data are from Enhanced TRACE and Mergent FISD datasets. I obtain betas by rolling a 36-month window. Data span July 2002 to June 2019. *, **, and *** indicate statistical significance at the 10%, 5%, and 1% level, respectively. Standard errors are HAC robust.

	(1)	(2)	(3)	(4)
Constant(%)	0.369^{**}	0.405^{*}	0.442^{**}	0.553^{***}
	(2.20)	(1.65)	(2.44)	(3.26)
r_B - r_f	-0.727^{*}		-0.763^{*}	-0.440^{**}
	(-1.71)		(-1.91)	(-2.14)
Term	1.070^{***}		1.223^{***}	0.893^{***}
	(2.61)		(3.09)	(3.96)
Def	0.964^{***}		1.111^{***}	0.902^{***}
	(3.33)		(4.16)	(4.91)
r_E - r_f		0.153^{*}	-0.146^{**}	-0.222^{**}
		(1.67)	(-2.10)	(-2.54)
SMB		0.119	0.109	0.128
		(0.84)	(0.86)	(1.31)
HML		0.082	0.086	0.031
		(0.51)	(0.83)	(0.29)
CMA		-0.090	-0.038	-0.055
		(-0.55)	(-0.34)	(-0.44)
RMW		0.028	0.089	0.083

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Table 2.9 – *Continued from previous page*

	(1)	(2)	(3)	(4)
		(0.15)	(0.56)	(0.56)
DRF				0.165
				(1.13)
LRF				0.607^{***}
				(4.46)
CRF				-0.986^{***}
				(-3.17)
Adj. $\bar{R}^2(\%)$	0.27	0.07	0.28	0.48
Observations	172.00	169.00	169.00	169.00

Table 2.10

Fixed Effect Regressions, Bond Returns, Various Portfolios (2002 - 2016)

This table presents estimates of fixed effect regressions where the dependent variable is excess return on various portfolios of corporate bonds. These portfolios are sorted by market beta (P1 and P2, different weighting schemes), credit ratings (P3), and time to maturity (P4). The last column includes all these portfolios in tandem (P5). Granularity is the total market value of equity and debt for the 20 largest firms as fraction of total market value of equity and debt of all firms. Independent variables include bond market return, default and term spreads, the three factors of Bai et al. (2019), downside risk (DRF), credit risk (CRF) and liquidity risk (LRF), and measures of financial (Unc_f) and economic (Unc_e) uncertainty of Jurado et al. (2015). Bond data are from Enhanced TRACE and Mergent FISD datasets. I obtain betas by rolling a 36-month window. Data span July 2002 to June 2019. *, **, and *** indicate statistical significance at the 10%, 5%, and 1% level, respectively. Standard errors are clustered at portfolio level.

	(P1)	(P2)	(P3)	(P4)	(P5)
ΔG_t	-0.258^{***}	-0.298^{***}	-0.207^{***}	-0.239^{***}	-0.246^{***}
	(-13.81)	(-7.62)	(-7.96)	(-6.32)	(-17.00)
R_B - R_f	-0.085	0.079	0.033	0.088^{*}	0.052^{*}
	(-1.22)	(1.07)	(0.96)	(1.72)	(1.81)
Term	0.120^{***}	0.236^{***}	0.165^{***}	0.228^{***}	0.189^{***}
	(3.80)	(6.91)	(5.75)	(6.18)	(11.71)
Def	0.393^{***}	0.373^{***}	0.249^{***}	0.437^{***}	0.346^{***}
	(3.09)	(3.70)	(3.44)	(4.79)	(7.64)
DRF	-0.126^{***}	-0.229^{***}	-0.161^{***}	-0.134^{***}	-0.164^{***}
	(-4.80)	(-5.66)	(-9.09)	(-3.96)	(-10.70)
CRF	-0.104^{**}	-0.227^{***}	-0.084^{**}	-0.171^{***}	-0.146^{***}
	(-2.29)	(-5.08)	(-2.27)	(-5.05)	(-7.13)
LRF	-0.167^{***}	-0.069	-0.081^{***}	0.007	-0.071^{***}
	(-6.31)	(-1.30)	(-3.49)	(0.16)	(-3.73)
Unc_f	-0.113^{***}	-0.098^{**}	-0.184^{***}	-0.107^{**}	-0.129^{***}

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Table 2.10 – *Continued from previous page*

	(P1)	(P2)	(P3)	(P4)	(P5)
	(-3.05)	(-2.32)	(-5.87)	(-2.20)	(-5.59)
Unc_e	0.201^{***}	0.148^{***}	0.212^{***}	0.162^{***}	0.166^{***}
	(8.61)	(3.66)	(14.02)	(3.16)	(6.57)
adj-R^2(%)	15.614	12.325	13.058	12.068	12.709

Table 2.11

Fixed Effect Regressions, Equity-Bond Correlation (2002 - 2016)

This table presents estimates of fixed effect regressions where the dependent variable is value weighted return on 15 (market) beta-sorted portfolios. Granularity is the market value of total assets for the 20 largest firms to total market value of assets. Independent variables include return on top 20 firms, bond market return, default and term spreads, and the three factors of Bai et al. (2019), downside risk (DRF), credit risk (CRF) and liquidity risk (LRF), and measures of financial (Unc_f) and economic (Unc_e) uncertainty ofJurado et al. (2015). Bond data are from Enhanced TRACE and Mergent FISD datasets. I obtain betas by rolling a 36-month window. Data span July 2002 to June 2019. *, **, and *** indicate statistical significance at the 10%, 5%, and 1% level, respectively. Standard errors are clustered at portfolio level.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
ΔG_t	0.132**	0.128**	0.128**	0.125**	0.095**	0.119**	0.107*	0.114**	0.119**	0.118**
	(2.05)	(2.13)	(2.11)	(2.09)	(2.12)	(2.51)	(1.84)	(2.02)	(2.06)	(2.08)
R_B - R_f			0.001	−0.017	0.152	0.159	0.175	0.100	0.102	0.111
			(0.01)	(−0.24)	(1.22)	(1.37)	(1.60)	(0.98)	(1.02)	(1.10)
Term				0.066	−0.280**	−0.343**	−0.326**	−0.284**	−0.299**	−0.322**
				(0.79)	(−1.99)	(−2.56)	(−2.38)	(−2.27)	(−2.31)	(−2.19)
Def					−0.419**	−0.463***	−0.440**	−0.419**	−0.438**	−0.469**
					(−2.21)	(−2.59)	(−2.24)	(−2.33)	(−2.37)	(−2.24)
DRF						0.086	0.103*	0.067	0.061	0.059
						(1.20)	(1.78)	(1.17)	(1.14)	(1.15)
CRF							−0.059	−0.053	−0.066	−0.078
							(−0.59)	(−0.53)	(−0.68)	(−0.85)
LRF								0.110*	0.105	0.108*
								(1.67)	(1.62)	(1.74)
Unc_f									−0.037	−0.097
									(−0.83)	(−0.98)
Unc_e										0.067

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Table 2.11 – *Continued from previous page*

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
										(0.80)
adj-R^2(%)	1.723	1.957	1.950	2.332	3.893	5.269	5.427	6.025	6.106	6.241

Table 2.12

Granularity versus Industry Concentration (2002 - 2019)

This table shows estimation results of Fama-Macbeth cross-sectional regressions where the dependent variable is individual bond excess returns. Granularity is the total market value of equity and debt for the 20 largest firms as fraction of total market value of equity and debt of all firms. Other explanatory variables include ΔHHI for industries that are significantly correlated with the granularity measure, bond market return, default and term spreads, the three factors of Bai et al. (2019), downside risk (DRF), credit risk (CRF) and liquidity risk (LRF), measures of financial (Unc_f) and economic (Unc_e) uncertainty of Jurado et al. (2015), and changes in VIX. Firm level characteristics include reversal (Rev), Size, credit ratings (Rating), time to maturity (TTM), momentum (Mom), illiquidity (Illiq), and trade volume (Volume). The last column reports results for the full specification. Bond data are from Enhanced TRACE and Mergent FISD datasets. Data span July 2002 to June 2019. *, **, and *** indicate statistical significance at the 10%, 5%, and 1% level, respectively.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Constant	0.669***	0.568***	0.451**	0.452***	0.380***	0.545*	0.437*	1.023***
	(2.89)	(3.25)	(2.53)	(3.56)	(3.64)	(1.90)	(1.66)	(3.67)
ΔG_t	−0.095***	−0.086***	−0.082**	−0.106***	−0.117***	−0.097***	−0.101***	−0.216***
	(−2.89)	(−2.92)	(−2.55)	(−2.96)	(−2.81)	(−2.60)	(−2.78)	(−3.09)
ΔHHI_{Ind1}	0.170*	0.156*	0.106	0.017	0.007	0.055	0.079	−0.232
	(1.67)	(1.72)	(1.36)	(0.24)	(0.09)	(0.52)	(0.79)	(−1.32)
ΔHHI_{Ind2}	−0.013	−0.018**	−0.023**	−0.028***	−0.026**	−0.023*	−0.017	−0.049**
	(−1.50)	(−2.58)	(−2.56)	(−2.96)	(−2.50)	(−1.66)	(−1.33)	(−1.98)
ΔHHI_{Ind3}	−0.049*	−0.042	−0.025	−0.035	−0.020	−0.003	−0.039*	−0.036
	(−1.72)	(−1.62)	(−0.85)	(−1.31)	(−0.97)	(−0.07)	(−1.72)	(−0.71)
R_b - R_f		0.064*	0.116***	0.115*	0.168**	0.071	0.053	0.179**
		(1.96)	(2.67)	(1.84)	(2.01)	(1.10)	(0.78)	(2.34)
Term			0.040	0.001	0.007	0.017	0.010	−0.121
			(0.23)	(0.05)	(0.03)	(0.08)	(0.05)	(−0.49)

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Table 2.12 – *Continued from previous page*

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Def			0.055	0.104	0.168	0.071	0.046	0.246
			(0.67)	(0.92)	(1.11)	(0.45)	(0.31)	(1.32)
DRF			0.112^{*}	0.046	0.281^{***}	0.481^{***}	0.438^{***}	0.461^{***}
			(1.71)	(0.56)	(3.61)	(4.19)	(3.57)	(2.72)
CRF				−0.026	0.058	0.102	0.088	0.192
				(−0.25)	(0.55)	(0.79)	(0.71)	(1.53)
LRF				0.081	0.099	0.128	0.104	0.173^{**}
				(1.04)	(1.20)	(1.60)	(1.36)	(2.38)
Unc_{econ}					-0.377^{***}	-0.352^{*}	−0.270	-0.583^{**}
					(−2.62)	(−1.87)	(−1.47)	(−1.97)
Unc_{fin}					−0.129	−0.109	−0.104	−0.234
					(−1.58)	(−0.97)	(−0.97)	(−1.30)
ΔVIX					−23.337	5.180	13.753	−52.177
					(−1.01)	(0.21)	(0.53)	(−1.45)
Rev					-0.077^{***}	-0.080^{***}	-0.073^{***}	-0.182^{***}
					(−4.05)	(−4.22)	(−3.81)	(−9.91)
Size						−0.013	−0.007	−0.009
						(−1.11)	(−0.70)	(−0.72)
TTM						0.000^{*}	0.000^{*}	0.000
						(1.76)	(1.68)	(1.36)
Mom						0.216	0.287	−1.399
						(0.18)	(0.24)	(−1.15)
Illiq							1.923^{**}	3.967^{***}
							(2.05)	(3.18)
Rate								-0.052^{**}

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Table 2.12 – *Continued from previous page*

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
								(−2.35)
Volume								0.000
								(0.33)
adj-R^2	2.80	4.50	9.81	14.45	21.37	28.87	31.37	34.45
Obs.	137.00	137.00	137.00	137.00	137.00	137.00	137.00	137.00

Table 2.13

Granularity and Risk Factors (2002 - 2019)

This table shows estimation results of cross-sectional regressions where the dependent variable is granularity betas for individual corporate bond returns. Granularity is the total market value of equity and debt for the 20 largest firms as fraction of total market value of equity and debt of all firms. Other explanatory variables include betas with respect to bond market return, default and term spreads, the three factors of Bai et al. (2019), downside risk (DRF), credit risk (CRF) and liquidity risk (LRF), measures of financial (Unc_f) and economic (Unc_e) uncertainty of Jurado et al. (2015), and changes in VIX. Firm level characteristics include reversal (Rev), Size, credit ratings (Rating), time to maturity (TTM), momentum (Mom), illiquidity (Illiq), and trade volume (Volume). The last column reports results for the full specification. Bond data are from Enhanced TRACE and Mergent FISD datasets. Data span July 2002 to June 2019. *, **, and *** indicate statistical significance at the 10%, 5%, and 1% level, respectively.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Constant	−6.815***	−9.756***	−6.990**	−10.573***	−12.049	−24.858**	17.057
	(−3.08)	(−3.06)	(−2.17)	(−3.65)	(−1.05)	(−2.30)	(1.45)
β_{Mkt}	4.901*	7.052**	4.546	5.103*	12.791***	15.590***	20.968***
	(1.81)	(2.07)	(1.38)	(1.80)	(4.26)	(3.95)	(5.12)
β_{Term}		11.685	−1.388	7.593	29.053***	22.345*	2.272
		(1.27)	(−0.16)	(0.88)	(3.05)	(1.75)	(0.20)
β_{Def}			9.115***	11.272***	8.108***	16.746***	31.196***
			(3.65)	(5.92)	(3.57)	(4.74)	(6.17)
β_{DRF}			−6.959	−4.246	−1.837	0.470	−12.196*
			(−1.43)	(−1.20)	(−0.67)	(0.08)	(−1.66)
β_{CRF}			6.706	9.076**	7.353	18.134**	14.194*
			(1.09)	(2.51)	(1.63)	(2.46)	(1.86)
βLRF				13.535***	10.774***	15.034***	16.249***
				(3.87)	(2.89)	(3.02)	(3.11)
$\beta_{Unc_{econ}}$				−49.010***	−63.132***	−73.852***	−67.308***

Continued on next page

Table 2.13 – *Continued from previous page*

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
				(−3.41)	(−4.19)	(−4.95)	(−4.94)
$\beta_{Unc_{fin}}$					-21.485^{***}	-28.674^{***}	-35.054^{***}
					(−5.85)	(−4.71)	(−5.08)
$\beta_{\Delta VIX}$					-3249.742^{***}	-5716.270^{***}	-5179.221^{***}
					(−3.29)	(−4.18)	(−3.89)
Rev					-1.172^{***}	-1.327^{***}	-0.998^{***}
					(−2.94)	(−3.19)	(−3.48)
Size					−0.415	0.268	−0.929
					(−0.71)	(0.49)	(−1.39)
TTM						0.001	0.001
						(0.37)	(1.60)
Mom						−51.997	−34.409
						(−1.24)	(−0.86)
Illiq						-117.798^{***}	-246.767^{***}
						(−4.94)	(−4.88)
Rate							-3.397^{***}
							(−7.09)
Volume							-0.001^{***}
							(−2.63)
adj-R^2	6.56	13.18	22.56	31.39	35.40	46.19	50.80
Obs.	137.00	137.00	137.00	137.00	137.00	137.00	137.00

2.8 Figures

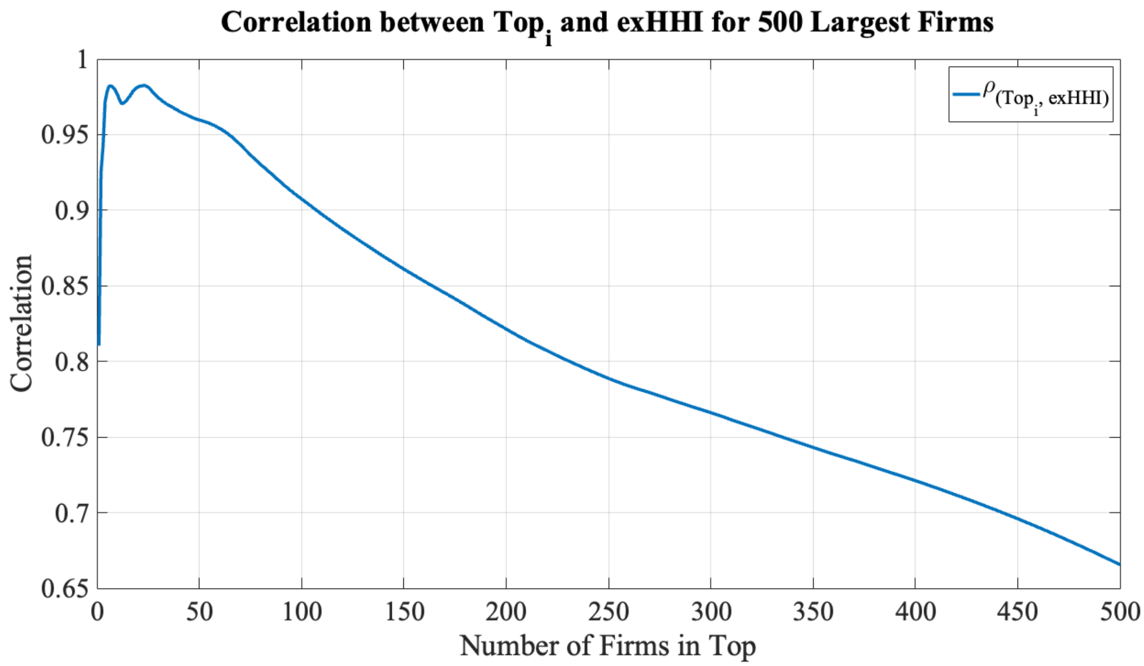

Figure 2.1: Compare exHHI with Top$_g$.

This figure shows the correlation coefficient between the exHHI measure and the Top$_g$ measure. The latter is constructed for the top i firms with $i = \{1, 2, ..., 500\}$. Enhanced TRACE and Mergent FISD datasets are main sources of corporate bond data. Equity data is retrieved from CRSP. Data span July 2002 to June 2019.

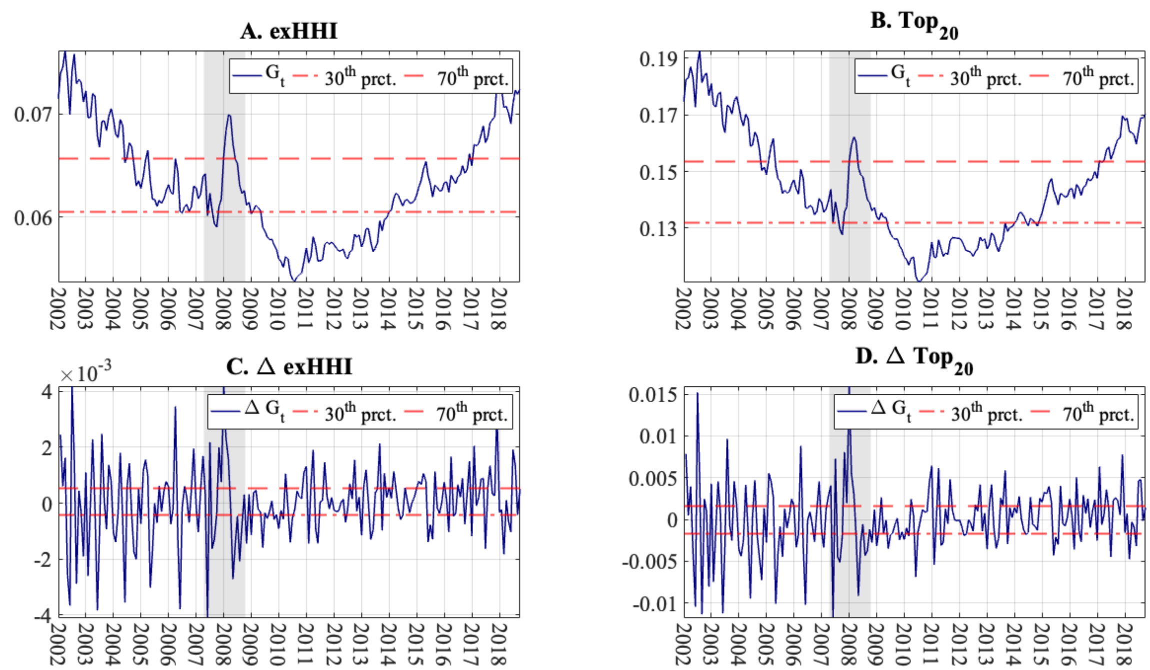

Figure 2.2: Model implied Risk Premia, Equity and Bond.

This figure shows two different measures of granularity. The excess HHI measure of Gabaix and Koijen (2020) versus Top_{20}, the total market value of equity and debt for the 20 largest firms as fraction of total market value of equity and debt of all firms. The top panels show the measures at level, the bottom panel plot the corresponding change. The red lines indicate the 30_{th} versus 70_{th} percentile of each measure. Enhanced TRACE and Mergent FISD datasets are main sources of corporate bond data. Equity data is retrieved from CRSP. Data span July 2002 to June 2019.

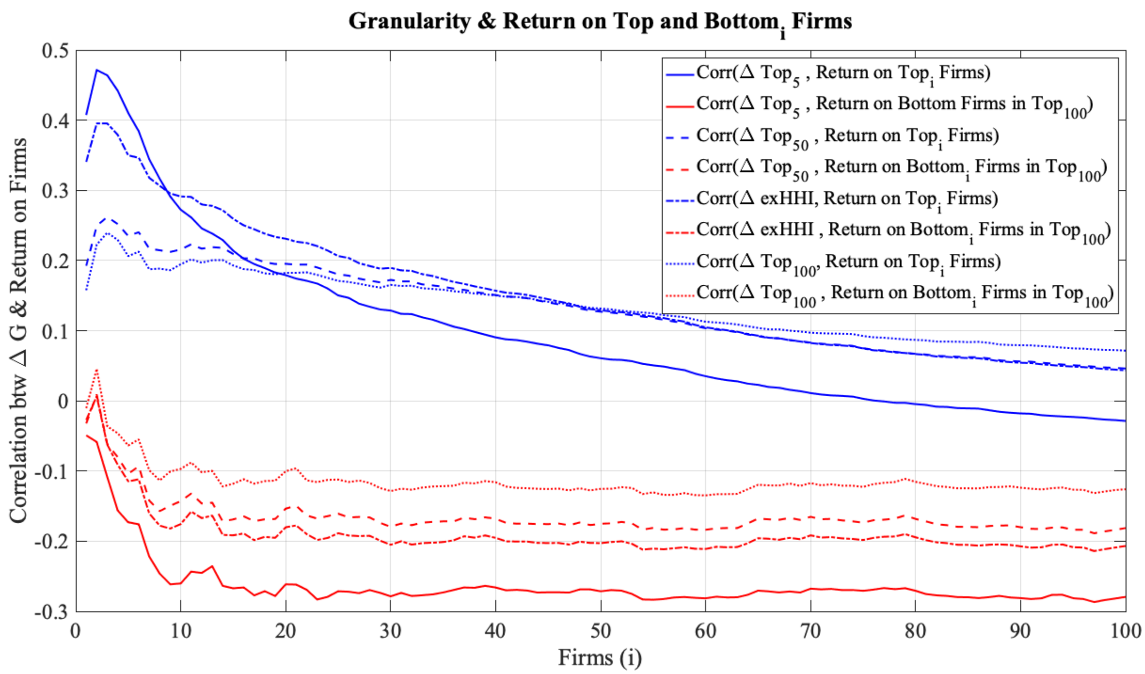

Figure 2.3: Changes in Firm Granularity versus Market Return.

This figure plots the correlation between changes in a granularity measure, and returns on the top i (blue lines) as well as the bottom 100-i firms (red lines) in the market. Granularity is measured at firm level. Enhanced TRACE and Mergent FISD datasets are main sources of corporate bond data. Equity data is retrieved from CRSP. Data span July 2002 to June 2019.

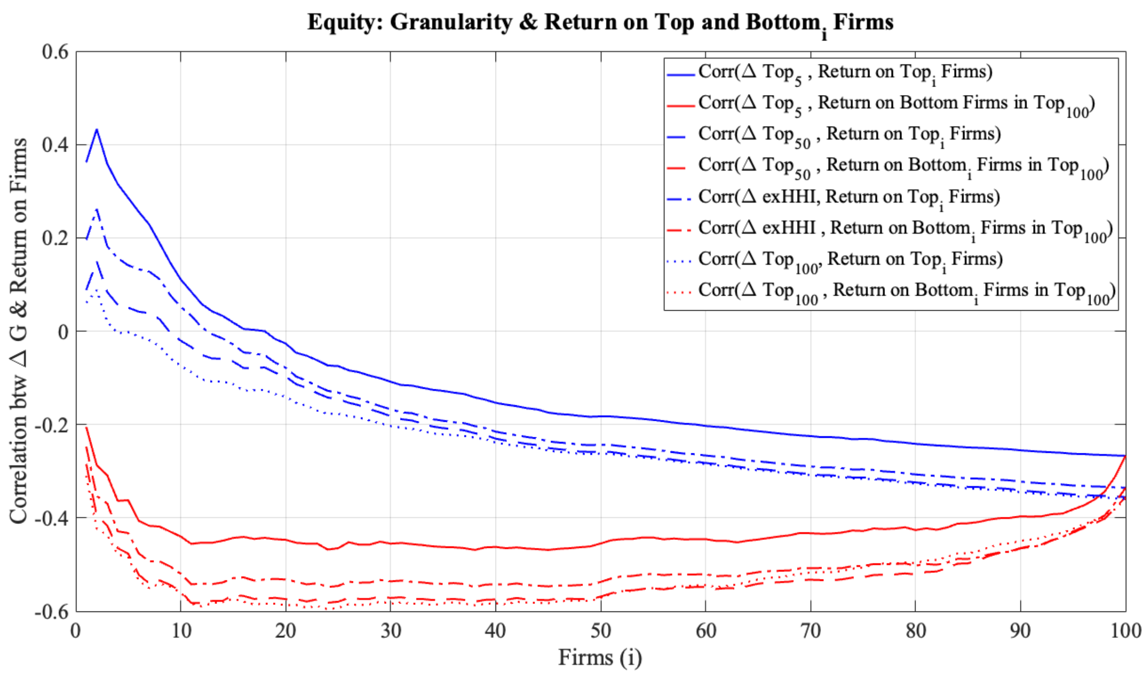

Figure 2.4: Changes in Equity Granularity versus Market Return.

This figure plots the correlation between changes in a granularity measure, and returns on the top *i* (blue lines) as well as the bottom 100-*i* firms (red lines) in the market. Granularity is measured at equity level. Enhanced TRACE and Mergent FISD datasets are main sources of corporate bond data. Equity data is retrieved from CRSP. Data span July 2002 to June 2019.

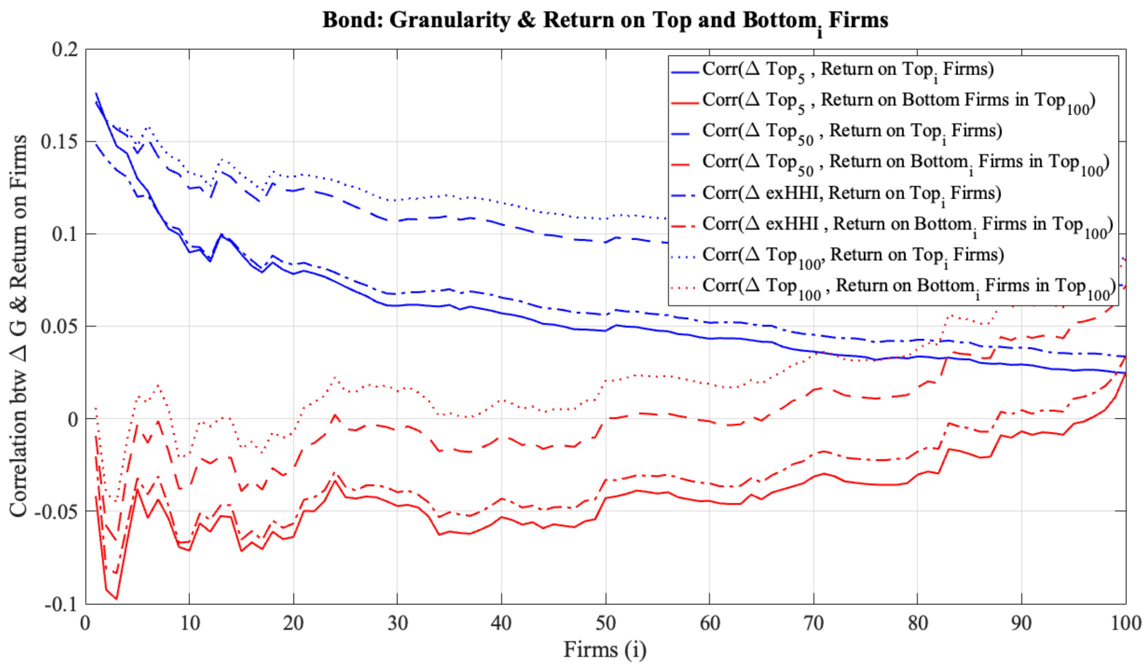

Figure 2.5: Changes in Bond Granularity versus Market Return.

This figure plots the correlation between changes in a granularity measure, and returns on the top *i* (blue lines) as well as the bottom 100-*i* firms (red lines) in the market. Granularity is measured at bond level. Enhanced TRACE and Mergent FISD datasets are main sources of corporate bond data. Equity data is retrieved from CRSP. Data span July 2002 to June 2019.

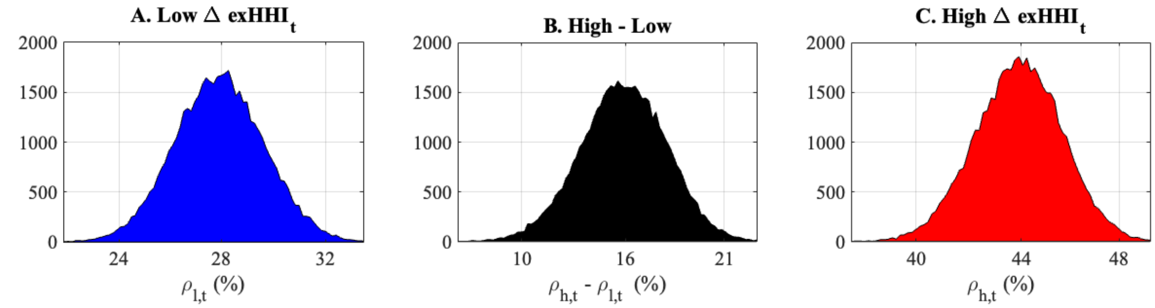

Figure 2.6: Conditional Distribution of Equity-Bond Correlations, ΔexHHI.

This figure plots the conditional EBC when granularity shocks are below the 30_{th} percentile (Panel A), above the 70_{th} percentile (Panel C), and the corresponding difference (Panel B). First, correlations are computed based on 90 random portfolios (each portfolio 20 firms). These correlations are then re-sampled 50,000 times and the the distribution of the bootstrapped mean of conditional correlations is plotted. Granularity shocks are measured based on changes in the excess HHI measure. Enhanced TRACE and Mergent FISD datasets are main sources of corporate bond data. Equity data is retrieved from CRSP. Data span July 2002 to June 2019.

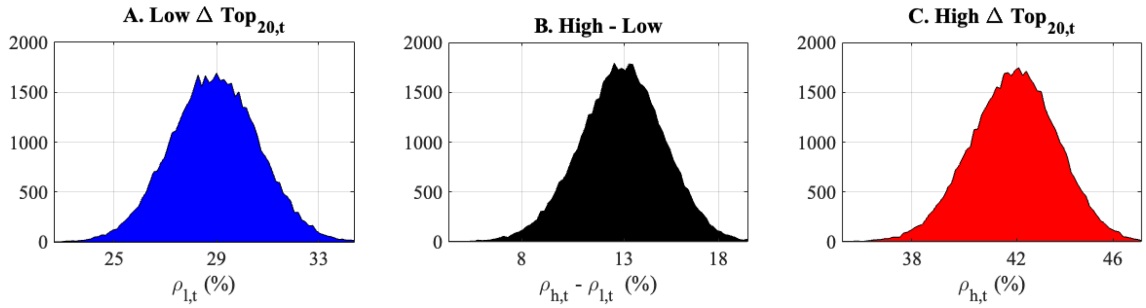

Figure 2.7: Conditional Distribution of Equity-Bond Correlations, ΔTop$_{20}$.

This figure plots the conditional EBC when granularity shocks are below the 30_{th} percentile (Panel A), above the 70_{th} percentile (Panel C), and the corresponding difference (Panel B). First, correlations are computed based on 90 random portfolios (each portfolio 20 firms). These correlations are then re-sampled 50,000 times and the the distribution of the bootstrapped mean of conditional correlations is plotted. Granularity shocks are measured based on changes in the Top$_{20}$ measure. Enhanced TRACE and Mergent FISD datasets are main sources of corporate bond data. Equity data is retrieved from CRSP. Data span July 2002 to June 2019.

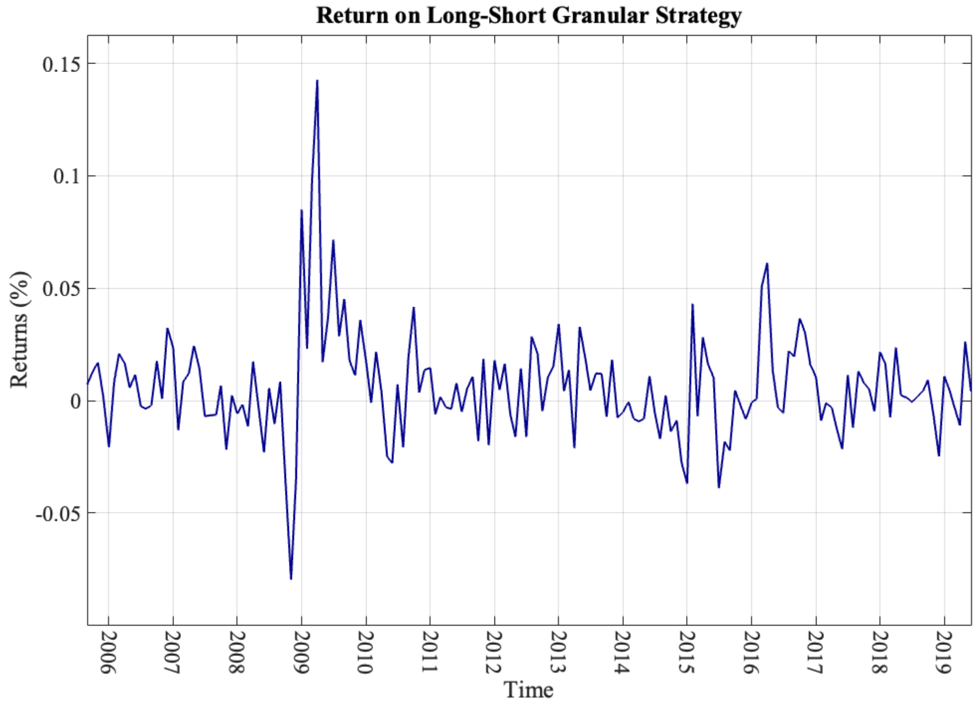

Figure 2.8: Return on Long-Short Granular Portfolios.

This figure plots return on a strategy that is long the low beta and short the high beta decile of granularity sorted portfolios (36-month rolling window). To obtain pre-formation betas I regress bond excess returns granularity shocks, market returns, and an interaction term. Granularity shocks are measured based on changes in the Top_{20} measure. Enhanced TRACE and Mergent FISD datasets are main sources of corporate bond data. Data span July 2002 to June 2019.

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General Conclusion

In this thesis we study equity and corporate bond prices when the economy is relatively dominated by a small number of firms, hence granularity. We show that a more granular economy is deemed unfavorable to risk-averse investors who care about diversification. As a result, assets with negative granularity beta are considered riskier, and yield higher expected returns. We provide ample empirical evidence from corporate bond and equity markets to support this hypothesis.

Since granularity is an economy-wide phenomenon, an increase in state of granularity corresponds to a higher correlation between equity and corporate bond returns. This finding signifies that granularity is similarly perceived by investors in the equity and corporate bond markets.

We construct a theoretical framework that show the low-risk effect in the equity market arises naturally in a granular economy. Our model predicts that larger firms tend to have higher market betas because the market is more represented by these firms. As they are more systematic, investors require lower returns, hence the low-risk effect. On the other hand, smaller firms tend to have smaller betas as they are relatively more idiosyncratic compared to larger firms. Investors ask for compensation for exposure to less systematic risk.

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Appendix A.1

A.1.1. SDF with Cobb-Douglas Preferences

Consider an economy that consists of two independent Lucas trees. Each tree $i = \{A, B\}$ generates a stream of dividends X_i in the form of a consumption good, whose dynamics satisfies

$$\frac{dX_{i,t}}{X_{i,t}} = \mu_i dt + \sigma_i dZ_{i,t}, \tag{A.1}$$

where μ_i and σ_i reflect the expected growth and volatility, while $dZ_{i,t}$ is the incremental change of a standard Brownian motion defined on the probability space $(\Omega, \mathscr{F}, \mathbb{P})$.

The representative agent consumes a Cobb-Douglas basket of goods, denoted by Y, coming from the dividends of trees A and B:

$$Y_t = X_{A,t}^{\Phi} X_{B,t}^{(1-\Phi)}, \tag{A.2}$$

where $0 \leq \Phi \leq 1$ is the elasticity of substitution between the consumption goods.

The basket of goods Y is governed by the following stochastic process

$$\frac{dY_t}{Y_t} = \left\{ \Phi\mu_A + (1-\Phi)\mu_B - \frac{1}{2}\Phi(1-\Phi)\left[\sigma_A^2 + \sigma_B^2\right] \right\} dt + \Phi\sigma_A dZ_{A,t} + (1-\Phi)\sigma_B dZ_{B,t}, \tag{A.3}$$

which is obtained by first applying Ito's lemma:

$$\begin{aligned} dY_t &= \frac{\partial Y_t}{\partial X_{A,t}} dX_{A,t} + \frac{\partial Y_t}{\partial X_{B,t}} dX_{B,t} + \frac{1}{2}\sum_i \sum_j \frac{\partial^2 Y_t}{\partial X_{i,t} \partial X_{j,t}} d\langle X_{i,t}, X_{j,t}\rangle, \;\; i,j = \{A, B\}, \\ &= \Phi X_{A,t}^{\Phi-1} X_{B,t}^{1-\Phi} dX_{A,t} + (1-\Phi) X_{B,t}^{-\Phi} X_{A,t}^{\Phi} dX_{B,t} \\ &- \frac{1}{2}\sigma_A^2 \Phi(1-\Phi) X_{A,t}^{\Phi} X_{B,t}^{1-\Phi} dt - \frac{1}{2}\sigma_B^2 \Phi(1-\Phi) X_{A,t}^{\Phi} X_{B,t}^{1-\Phi} dt, \end{aligned}$$

and then by dividing all terms by Y_t and simplifying the terms

$$
\begin{aligned}
\frac{dY_t}{Y_t} &= \frac{\Phi X_{A,t}^{\Phi-1} X_{B,t}^{1-\Phi}}{X_{A,t}^{\Phi} X_{B,t}^{1-\Phi}} dX_{A,t} + \frac{(1-\Phi) X_{B,t}^{-\Phi} X_{A,t}^{\Phi}}{X_{A,t}^{\Phi} X_{B,t}^{1-\Phi}} dX_{B,t} \\
&\quad - \frac{1}{2}\sigma_A^2 \Phi(1-\Phi) \frac{X_{A,t}^{\Phi} X_{B,t}^{1-\Phi}}{X_{A,t}^{\Phi} X_{B,t}^{1-\Phi}} dt - \frac{1}{2}\sigma_B^2 \Phi(1-\Phi) \frac{X_{A,t}^{\Phi} X_{B,t}^{1-\Phi}}{X_{A,t}^{\Phi} X_{B,t}^{1-\Phi}} dt, \\
&= \left\{ -\frac{1}{2}\Phi(1-\Phi)\sigma_A^2 - \frac{1}{2}\Phi(1-\Phi)\sigma_B^2 \right\} dt + \Phi \frac{dX_{A,t}}{X_A} + (1-\Phi)\frac{dX_{B,t}}{X_{B,t}}, \\
&= \left\{ -\frac{1}{2}\Phi(1-\Phi)\sigma_A^2 - \frac{1}{2}\Phi(1-\Phi)\sigma_B^2 \right\} dt + \Phi \left[\mu_A dt + \sigma_A dZ_{A,t} \right] + (1-\Phi) \left[\mu_B dt + \sigma_B dZ_{B,t} \right], \\
&= \left\{ \Phi\mu_A + (1-\Phi)\mu_B - \frac{1}{2}\Phi(1-\Phi) \left[\sigma_A^2 + \sigma_B^2 \right] \right\} dt + \Phi\sigma_A dZ_{A,t} + (1-\Phi)\sigma_B dZ_{B,t}.
\end{aligned}
$$

The representative agent derives her utility of consumption from

$$
U_t = \mathbb{E}_t \left(\int_0^{\infty} e^{-\delta s} \ln(Y_{t+s}) ds \right) \tag{A.4}
$$

where Y_t is the aggregate consumption basket. This utility function implies that assets are priced by the following kernel:

$$
\pi_t = \frac{e^{\delta s}}{Y_t} \tag{A.5}
$$

with

$$
\frac{d\pi_t}{\pi_t} = -\frac{dY_t}{Y_t} + var(\frac{dY_t}{Y_t}) - \delta dt, \tag{A.6}
$$

or equivalently

$$
\begin{aligned}
\frac{d\pi_t}{\pi_t} = &\left\{ -\delta - \Phi\mu_A - (1-\Phi)\mu_B + \frac{1}{2}\Phi(1-\Phi) \left[\sigma_A^2 + \sigma_B^2 \right] + \Phi^2\sigma_A^2 + (1-\Phi)^2\sigma_B^2 \right\} dt \\
&- \Phi\sigma_A dZ_{A,t} - (1-\Phi)\sigma_B dZ_{B,t},
\end{aligned} \tag{A.7}
$$

with the equilibrium risk free rate being equal to

$$
r_t = \delta + \Phi\mu_A + (1-\Phi)\mu_B - \frac{1}{2}\Phi(1-\Phi) \left[\sigma_A^2 + \sigma_B^2 \right] - \Phi^2\sigma_A^2 - (1-\Phi)^2\sigma_B^2. \tag{A.8}
$$

The stochastic discount factor considered in Section 1.3 is thus of the form of Equation (A.7). Specifically, it corresponds to the case $\eta_A = \Phi$ and $\eta_B = 1-\Phi$.

A.1.2. Alternative Granularity Measures

Table A1 displays the correlation structure among alternative measures for granularity. The Herfindahl-Hirschman Index (HHI) is a measure of market concentration. It is defined as

$$HHI = \sqrt{\sum_{1}^{N} w_i^2}, \tag{A.9}$$

where w_i is the market value of firm i as fraction of total market capitalization. N is the number of firms in the market. Gabaix and Koijen (2019) introduce a modified HHI measure, called the excess HHI measure, defined as

$$exHHI = \sqrt{-\frac{1}{N} + \sum_{1}^{N} w_i^2}. \tag{A.10}$$

A higher $exHHI$ means that the market is more granular. When all the firms have the same size, $exHHI = 0$.

We also use the skewness of the cross section of firm values (*skw*) as well as the dispersion of the log of total market capitalization (σ_{logmkt}) as other potential measures. By Panel A, our measure, Top100, is highly positively correlated with HHI and exHHI. It is positively correlated with skewness but negatively related to σ_{logmkt}. When we consider the change in granularity (Panel B), all correlation coefficients are both positive and larger than corresponding coefficients in Panel A. This table suggests that our results are not driven by the specificity of a measure for granularity.

A.1.3. Alternative Measures and Slope of the SML

Table A2 shows estimation results for the regression analysis developed in the paper. The slope of the SML is similarly related to granularity when it is proxied by the alternative measures introduced in Section 2.8. Except for skewness, *skw*, other measures explain the slope of the SML significantly, and as expected, with a negative sign. T-statistics are reported in parentheses.

Table A1
Correlation Structure: Granularity Measures

This table presents the correlation matrix among alternative measures for granularity.

Panel A: Granularity measures								
Measure	Top1%	Top20%	Top50	Top100	HHI	exHHI	skw	σ_{logmkt}
Top1%	1	0.77	0.75	0.83	0.31	0.34	-0.10	0.08
Top20%		1	0.46	0.58	0.02	0.04	-0.30	0.47
Top50			1	0.98	0.80	0.82	0.32	-0.20
Top100				1	0.72	0.74	0.22	-0.12
HHI					1	0.99	0.76	-0.46
exHHI						1	0.76	-0.45
skw							1	-0.50
σ_{logmkt}								1

Panel B: Change in granularity measures								
Measure	Top1%	Top20%	Top50	Top100	HHI	exHHI	skw	σ_{logmkt}
Top1%	1	0.76	0.96	0.96	0.77	0.80	0.36	0.13
Top20%		1	0.73	0.80	0.53	0.56	0.18	0.49
Top50			1	0.97	0.80	0.84	0.37	0.14
Top100				1	0.75	0.78	0.30	0.18
HHI					1	0.99	0.75	0.08
exHHI						1	0.76	0.09
skw							1	0.01
σ_{logmkt}								1

Table A2
Slope of the SML and alternative Granularity Measures

The dependent variable is the conditional slope of the SML, obtained from regressing equal weighted monthly returns of 25 beta portfolios at time t on the market betas of the same portfolios at time $t-1$. Conditional betas are estimated by rolling a 60-month window. Explanatory variables include granularity, G_t, measured by the alternative definitions introduced in Section 2.8, and control variables; investor sentiment, inflation, TED Spread, and excess market return. Details about the control variables are provided in Table 1.1 in the paper. Robust Newey-West t-statistics with 12 lags are reported in parentheses. Monthly data span January 1973 to December 2018, obtained from CRSP. All variables are normalized. *, **, and *** indicate statistical significance at the 10%, 5%, and 1% level, respectively.

	HHI	exHHI	skw	σ_{logmkt}
ΔG_t	-0.22**	-0.20***	-0.01	-0.21***
	(-2.41)	(-2.73)	(-0.20)	(-4.14)
Inf_{t-1}	-0.00	-0.00	-0.01	0.03
	(-0.09)	(-0.10)	(-0.13)	(0.69)
TED_t	0.10***	0.10***	0.09***	0.13***
	(3.60)	(3.64)	(3.17)	(4.65)
$R_{m,t}$ - R_f	0.78***	0.78***	0.81***	0.85***
	(12.27)	(12.18)	(14.06)	(14.30)
Adj. R-Squared	0.66	0.66	0.64	0.69
Observations	387	387	387	387

Appendix A.2

A.2.1. Data Cleaning Procedure Applied to Enhanced TRACE Transactions

To clean the Enhanced TRACE transaction data I first apply the procedure offered by Nielsen (2014), where 35% of the raw transaction data is deleted due to various problematic issues. Then, I use the following criterion, à la Bail, Bali, and Wen (2019), to filter the corporate bond dataset.

- Remove bonds that are not listed or traded in the US public market, which include bonds issued through private placement, bonds issued under the 144A rule, bonds that do not trade in US dollars, and bond issuers not in the jurisdiction of the United States.
- Remove bonds that are structured notes, mortgage backed or asset backed, agency backed, or equity linked.
- Remove convertible bonds since this option feature dis- torts the return calculation and makes it impossible to compare the returns of convertible and nonconvertible bonds
- Remove bonds that trade under \$5 or above \$1,000.
- Remove bonds that have a floating coupon rate, which means the sample comprises only bonds with a fixed or zero coupon.

- Remove bonds that have less than one year to maturity.
- Eliminate bond transactions that are labeled as when-issued, locked-in, or have special sales conditions, and that have more than a two- day settlement.
- Remove transaction records that are canceled and ad- just records that are subsequently corrected or re- versed.
- Remove transaction records that have trading volume less than $10,000.

A.2.2. Cross-sectional Regressions with Alternative Granularity Measures

This table presents Fama-MacBeth cross sectional regressions when the granularity alternates across columns. As you can observe, the negative and economically significant effect of granularity remains robust to measuring granularity.

Table A3

Fama-Macbeth Regressions, Cross Section of Individual Bond Returns and Characteristics, Different Granularity Measures (2002 - 2019)

In Table A3 shows estimation results of Fama-Macbeth cross-sectional regressions where the dependent variable is individual bond excess returns. Columns correspond to estimation results with different granularity measures. In columns 1-4, respectively, granularity is the total market value of equity and debt for the {5,10,15,20} largest firms as fraction of total market value of equity and debt of all firms. In the last column, granularity is the exHHI index. Other explanatory variables include innovations to the relative market value of smallest 50 firms among the 200 largest firms, bond market return, default and term spreads, the three factors of **?**, downside risk (DRF), credit risk (CRF) and liquidity risk (LRF), and measures of financial (Unc_f) and economic (Unc_e) uncertainty of **?**. Columns 1-4 augment the list of risk factors with firm level characteristics including reversal (Rev), Size, credit ratings (Rating), and time to maturity (TTM), respectively. The last column reports results for the full specification. Bond data are from Enhanced TRACE and Mergent FISD datasets. Data span July 2002 to June 2019. *, **, and *** indicate statistical significance at the 10%, 5%, and 1% level, respectively.

	(1)	(2)	(3)	(4)	(5)
Constant	1.026^{***}	1.026^{***}	1.018^{***}	1.003^{***}	1.010^{***}
	(3.44)	(3.52)	(3.52)	(3.58)	(3.60)
ΔG_t	-0.117^{***}	-0.165^{***}	-0.185^{***}	-0.223^{***}	-0.007^{***}
	(−3.62)	(−3.35)	(−2.86)	(−2.74)	(−2.88)
R_b - R_f	0.207^{**}	0.200^{**}	0.199^{**}	0.205^{**}	0.208^{**}
	(2.51)	(2.43)	(2.45)	(2.49)	(2.49)
Term	0.043	0.038	0.024	0.023	0.034
	(0.16)	(0.14)	(0.09)	(0.09)	(0.13)
Def	0.229	0.202	0.205	0.211	0.212

Continued on next page

Table A3 – *Continued from previous page*

	(1)	(2)	(3)	(4)	(5)
	(1.51)	(1.30)	(1.31)	(1.36)	(1.39)
DRF	0.368***	0.344***	0.359***	0.370***	0.357***
	(2.81)	(2.69)	(2.86)	(2.93)	(2.84)
CRF	0.146	0.123	0.134	0.143	0.141
	(1.26)	(1.03)	(1.12)	(1.20)	(1.19)
LRF	0.157**	0.148**	0.152**	0.154**	0.154**
	(2.36)	(2.21)	(2.18)	(2.18)	(2.29)
Unc_{econ}	−0.648**	−0.571**	−0.565**	−0.617**	−0.666**
	(−2.21)	(−2.08)	(−2.14)	(−2.18)	(−2.26)
Unc_{fin}	−0.513***	−0.504**	−0.495**	−0.519**	−0.524**
	(−2.60)	(−2.53)	(−2.51)	(−2.53)	(−2.54)
ΔVIX	−48.522	−47.233	−48.673	−50.194	−49.961
	(−1.51)	(−1.46)	(−1.49)	(−1.53)	(−1.55)
Rev	−0.175***	−0.175***	−0.175***	−0.175***	−0.174***
	(−9.58)	(−9.54)	(−9.57)	(−9.57)	(−9.57)
Size	−0.015	−0.015	−0.013	−0.013	−0.015
	(−1.07)	(−1.07)	(−0.99)	(−0.98)	(−1.06)
TTM	0.000	0.000	0.000	0.000	0.000
	(0.83)	(0.93)	(1.05)	(1.02)	(0.94)
Mom	−1.068	−1.203	−1.339	−1.351	−1.074
	(−1.00)	(−1.10)	(−1.23)	(−1.26)	(−1.00)
Illiq	3.695***	3.640***	3.661***	3.630***	3.691***
	(2.98)	(2.95)	(2.96)	(2.95)	(2.99)
Rate	−0.048**	−0.048**	−0.048**	−0.047**	−0.048**
	(−2.16)	(−2.15)	(−2.13)	(−2.14)	(−2.17)
Volume	−0.000	−0.000	−0.000	−0.000	−0.000
	(−0.09)	(−0.14)	(−0.08)	(−0.05)	(−0.12)
adj-R^2	33.17	33.25	33.31	33.24	33.18
Obs.	137.00	137.00	137.00	137.00	137.00

A.2.3. Sketch of a Toy Model

This Appendix sketches a theoretical framework for how granularity affects equity and bond returns. The results in this Appendix are still preliminary and incomplete.

Setting

There are N firms in the economy, represented by a cashflow dynamics that is governed, under the $\mathbb{P}$ measure, by

$$\frac{dX_i}{X_i} = \mu_i dt + \sigma_i dB_i, \tag{A.11}$$

where μ_i and σ_i are constant parameters, and dB_i is the incremental change of a standard Brownian motion defined on the probability space $(\Omega, \mathscr{F}, \mathbb{P})$. The parameters of the model are common knowledge. The cashflow of the grains is less volatile than other firms in the economy[2], $\sigma_g < \sigma_i$. The firm cashflow process is composed of a market-wide systematic as well as an orthogonal firm-specific component,

$$\frac{dX_i}{X_i} = \mu_i dt + \sigma_i \big(\rho_i dZ_M + \sqrt{1-\rho_i^2} dZ_i\big), \tag{A.12}$$

where $\rho_i = Corr(dB_i, dZ_M)$, and $dZ_M \perp dZ_i$ for all i. The typical investor is concerned about shocks to the earnings process of the firms. Therefore, the SDF that prices assets in this economy looks like

$$\begin{aligned} \frac{d\pi_t}{\pi_t} &= \sum_{i=1}^{N} w_i \frac{dX_i}{X_i} \\ &= \sum_{i=1}^{N} w_i \Big[\mu_i dt + \sigma_i \big(\rho_i dZ_M + \sqrt{1-\rho_i^2} dZ_i\big)\Big], \\ &= \mu_M dt + \sigma_M dZ_M + \sum_{i=1}^{N} w_i \sigma_i \sqrt{1-\rho_i^2} dZ_i, \end{aligned} \tag{A.13}$$

I further assume that all firms are similarly productive, that is for any firm i, $\mu_i = \mu$. So far this setting does not differ from standard pricing models we already know of.

[2] **?** argue that the volatility of earnings of small firms has grown more volatile since the 1980s. I also provide evidence in the empirical section that the equity return volatility is negatively related to firm size.

However, the second term in Equation (A.13) reveals a plausible interpretation. Granularity boils down to how idiosyncratic shocks are weighted by relative size of firms in the economy. When size is normally distributed, by Law of Large Numbers, diversification is proportionate to $\frac{1}{N}$, and with a large enough number of firms $\sum_{i=1}^{N} w_i \sigma_i \sqrt{1-\rho_i^2} dZ_i$ is eventually compressed. But we know that firms size is power-law distributed (Gabaix, 2011), the LLN does not hold, and $\sum_{i=1}^{N} w_i \sigma_i \sqrt{1-\rho_i^2} dZ_i$ remains a significant component regardless of the number of firms in the economy.

From the perspective of an investor, idiosyncratic innovations stemming from firms that dominate the portfolio are too large to ignore.

Unlevered Risk Premia

Cochrane, Longstaff, and Santa Clara (2008) solve a similar consumption based model in a representative agent framework with two firms in an endowment economy. In such setting w_i is a stochastic process, affecting the valuation ratios. Martin (2013) solves for the price dividend ratio in a three-firm economy model and provides semi-closed form solutions.

Such partial equilibrium model leads to an endogenous stochastic discount factor. On one hand, the representative agent cares about the firms in her consumption portfolio, on the other hand she needs to be compensated for the risk arising from the stochasticity of the weights. She wonders if the more volatile firm ends up dominating her basket or vice versa. Solving this model endogenously is quite complex. Cochrane, Longstaff, and Santa Clara (2008) propose closed form solutions for a two-tree economy. Martin (2013) derives semi-analytical solutions for a three-firm economy. In both these papers the firms do not undertake leverage.

As I show in the empirical section, granularity is a highly persistent phenomenon with an auto-regressive coefficient of 0.97. Therefore, investors, at each point of time, have quite an accurate estimate of the level of portfolio weights. I build on this observation and assume that the agent solves the her problem knowing the vector of weights for firms in the economy.

Drawing analogy to consumption-based models, a larger w_i signifies that the consumption basket is more dominated by a specific firm. Also, by granularity, relatively (very) large firms can bring about systematic impact through their firm-specific shocks. These shocks grow incompressible because granular economies do not yield full diversification of firm-specific risk. Since granularity limits the scope of diversification of the future wealth portfolio, it is negatively related to the future opportunity set.

There is one free parameter in the model, ρ_i, that shows the systematic versus firm-specific riskiness of firms cashflow. Without loss of generality, I consider a base scenario where regardless of the composition of the consumption basket ρ_i is constant for all firms. I then discuss alternative assumptions about ρ_i in Figure A.3.

I am interested in how dominance of the grains in the investors preferences, all else equal, affects any firm i in the economy. Hence, I examine the risk premium channel,

$$
\begin{aligned}
&-\mathbb{E}\left[\frac{d\pi}{\pi}, \frac{dX_i}{X_i}\right] = \\
&-\mathbb{E}\left[\left(\sigma_M dZ_M + \sum_{i=1}^{N} w_i \sqrt{(1-\rho_i^2)}\sigma_i dZ_i\right), \left(\rho_i \sigma_i dZ_M + \sqrt{(1-\rho_i^2)}\sigma_i dZ_i\right)\right] = \\
&\rho_i \sigma_i \sigma_M + w_i (1-\rho_i^2)\sigma_i^2.
\end{aligned}
\tag{A.14}
$$

The unlevered risk premium is determined by two main components, one proportionate to systematic risk, second to firm-specific risk. Both components are functions of granularity. Notice that w_i shows up in both components. When a firm is large enough it can grow systematic such that σ_M gravitates towards the volatility of the grain. The same logic applies for the second component. The first channel affects all firms in the economy. The second channel is more prominent for firms that are more dominant in the SDF dynamics, hence with larger w_i. The overall effect determines the cross sectional behaviour of risk premia in a granular economy.

For the first component, it is straightforward to show that the volatility of the systematic component changes the share of the grains in the economy. Without loss of generality, I consider an economy with one grain, firm N. I use the subscript g to refer to the weight, volatility, or drift of the grain. Hence,

$$
\begin{aligned}
\frac{\partial \sigma_M}{\partial w_g} &= \frac{\partial \sum_{i=1}^{N} w_i \rho_i \sigma_i}{\partial w_g} = \frac{\partial \left(\rho_g w_g \sigma_g + \sum_{i=1}^{N-1} w_i \rho_i \sigma_i \right)}{\partial w_g} \\
&= \rho_g \sigma_g + \sum_{i=1}^{N-1} \frac{w_i}{w_g} \rho_i \sigma_i .
\end{aligned}
\tag{A.15}
$$

The overall effect on expected risk premia is

$$
\begin{aligned}
\frac{\partial \, \mathbb{E}\left[\frac{d\pi}{\pi}, \frac{dX_i}{X_i} \right]}{\partial w_g} &= \rho_i \sigma_i \frac{\partial \sigma_M}{\partial w_g} + \frac{\partial w_i}{\partial w_g} (1 - \rho_i^2) \sigma_i^2 \\
&= \rho_i \sigma_i \left[\rho_g \sigma_g + \sum_{i=1}^{N-1} \frac{\partial w_i}{\partial w_g} \rho_i \sigma_i \right] + \frac{\partial w_i}{\partial w_g} (1 - \rho_i^2) \sigma_i^2 \\
&= \underbrace{\rho_i \sigma_i \rho_g \sigma_g}_{>0} + \underbrace{\rho_i \sigma_i \sum_{i=1}^{N-1} \frac{\partial w_i}{\partial w_g} \rho_i \sigma_i}_{<0} + \underbrace{\frac{\partial w_i}{\partial w_g} (1 - \rho_i^2) \sigma_i^2}_{<0}
\end{aligned}
\tag{A.16}
$$

Since ρ_i, σ_i, and σ_g are constant parameters, the first out of three terms in equation (A.16) is always positively related to the risk premium. Changes in w_g in the second and third terms are negatively related to firm risk premia. To find out which effect offsets the other, I solve the model for different values for the share of the grain in the SDF dynamics. For simplicity, I consider three types of firms in the model economy ($N = 3$), grain, big, and small. Three (types of) firms capture any potential nonlinearity in behaviour of risk premiums. Yet, the result are intact for any number of firms greater than three. All else equal, I increase share of the grain in the SDF dynamics. By definition the grain is the largest among firms. Hence, when $w_g < w_i$ it means that the agent consumes $w_i C$ from a continuum of firms i.

By Figure (A.2), the market as well as small and big firm risk premia are negatively related to granularity. The risk premium of the grain increases with its importance to investors, reflected in the SDF dynamics, namely granularity. I show in the empirical section that this patter supports what I observe in the data.

To examine how equity and bond risk premia related to granularity, I need to show that this relation holds in the equity and bond dimensions too. To this end, I examine bond and equity risk premia in the next section. I solve the firm problem and show that the equity and bond market risk premia mirror those of the firm cashflows in other scenarios

Levered Risk Premia

To investigate the behaviour of risk premia for firm equity and bond, I assume firms make an optimal debt mix decision. I borrow from Leland (1994) that is a standard trade-off model in the literature where firms choose an optimal level of debt. This trade-off arises from the fact that firms have a tax advantage in issuing debt. I provide the details of the model in the Appendix. By applying Ito's lemma, equity and bond dynamics of the firm are governed by

$$\frac{dE_t}{E_t} = \mu_e dt + \rho_i \sigma_e dZ_M + \sqrt{1-\rho_i^2}\sigma_e dZ_i, \tag{A.17}$$

and

$$\frac{dB_t}{B_t} = \mu_b dt + \rho_i \sigma_b dZ_M + \sqrt{1-\rho_i^2}\sigma_b dZ_i, \tag{A.18}$$

and equity and bond risk premia are

$$-\mathbb{E}\left[\frac{d\pi_t}{\pi_t}, \frac{dE_t}{E_t}\right] = \rho_i \sigma_e \sigma_M + w_i(1-\rho_i^2)\sigma_e \sigma_i, \tag{A.19}$$

and

$$-\mathbb{E}\left[\frac{d\pi_t}{\pi_t}, \frac{dB_t}{B_t}\right] = \rho_i \sigma_b \sigma_M + w_i(1-\rho_i^2)\sigma_b \sigma_i, \tag{A.20}$$

respectively. $\sigma_e = \frac{\partial E}{\partial X}\frac{X}{E}$ is the levered equity volatility, and $\sigma_b = \frac{\partial B}{\partial X}\frac{X}{B}$ is the levered bond volatility. I solve the model for a range of cashflow values for the grain. Results are displayed in Figure (A.4). Panel A shows the equity risk premium. Similar to the patterns observed in Figure (A.2), the equity risk premium, for firms as well as the market, is negatively related to granularity. This observation is supported by empirical evidence provided in the next section.

Panel B shows that bond risk premia behave similarly. However, the bond risk premium for the grains decreases with granularity. Since the current theoretical framework is meant to examine cross sectional effects, hence a static structural model, increasing the cashflow of the grains means that they de-lever as they grow, driving the levered bond volatility considerably low. Thus, bonds issued by grains grow less risky with granularity.

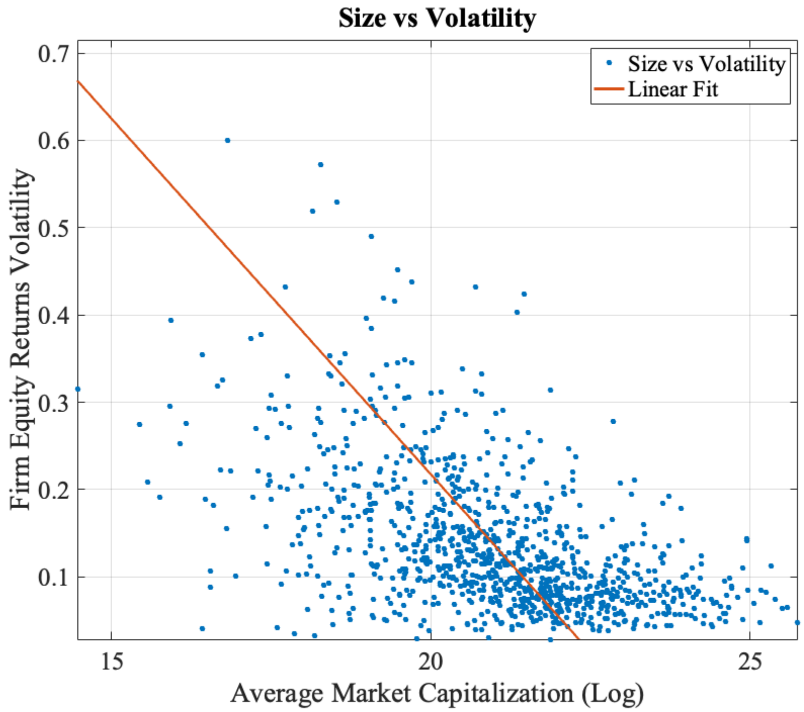

Figure A.1: Size versus Return Volatility.

This figure plots time series average of the log market capitalization of firms that issue both equity and debt versus the volatility of firms equity over the same period of time. Monthly equity data is from CRSP and it spans Jan 2002 to Dec 2019.

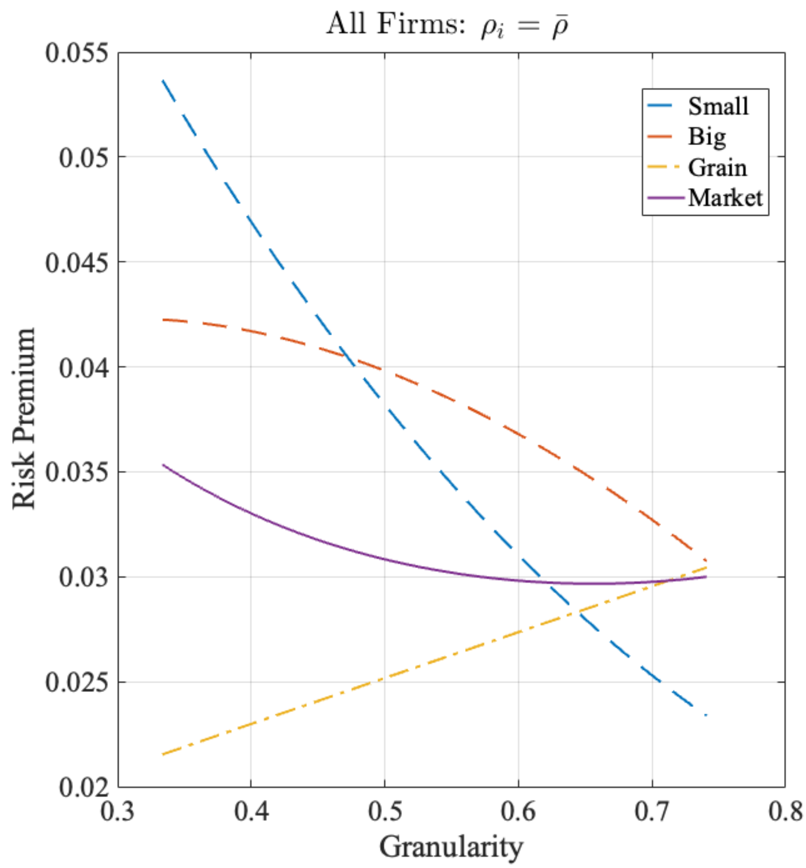

Figure A.2: Model implied Risk Premia, Firm.

This figure plots the cashflow risk premium for small, big, and grain firms in a three-firm economy. All else equal, only the level of cashflow for the grain varies. Grains are less volatile than the rest of the economy, and the systematic versus firm-specific components of firm cashflow are constant, $\rho_i = \bar{\rho}$.

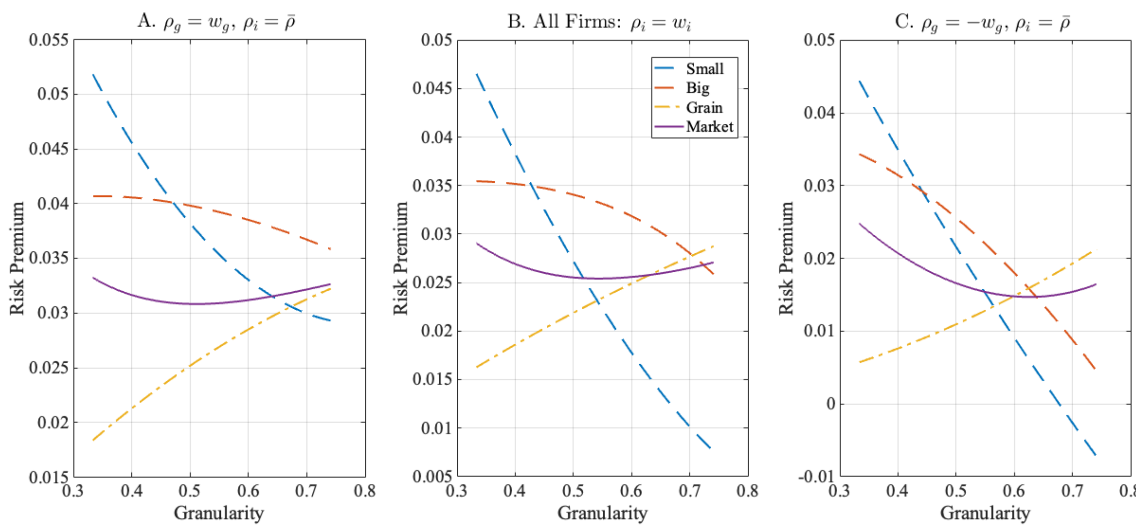

Figure A.3: Model implied Risk Premia, Firm.

This figure plots the cashflow risk premium for small, big, and grain firms in a three-firm economy. All else equal, only the level of cashflow for the grain varies. Panel A shows a scenario where grains grow more systematic, $\rho_g = w_g$ while for other firms $\rho_i = \bar{\rho}$. Panel B displays a scenario where for all firms, $\rho_i = w_i$. Panel C shows risk premia when grains are systematically counter-cyclical, $\rho_g = -w_g$ while for non-grains $\rho_i = \bar{\rho}$.

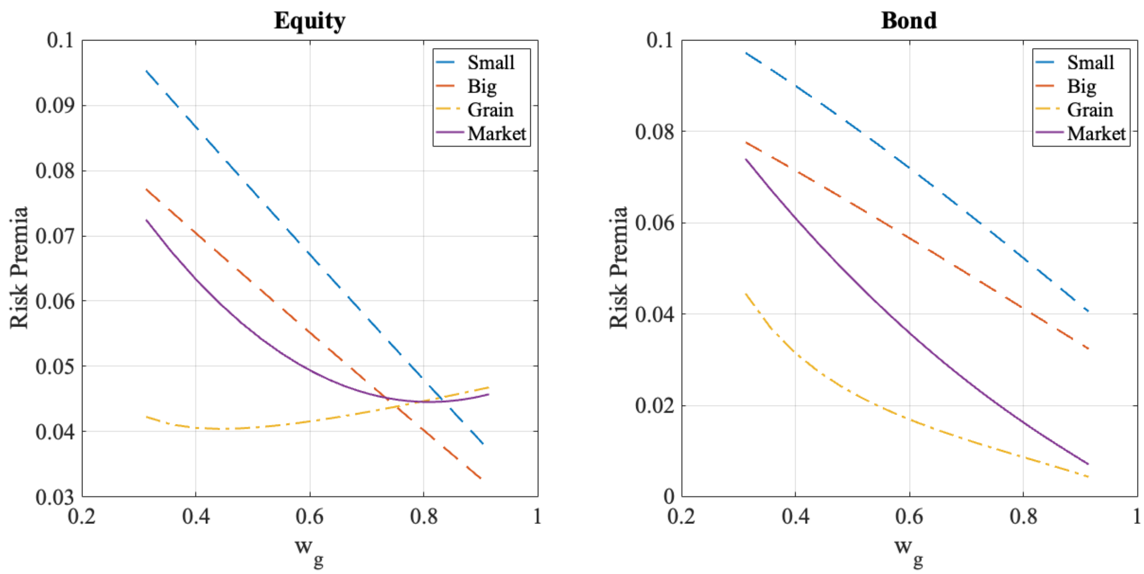

Figure A.4: Model implied Risk Premia, Equity and Bond.

This figure plots the bond and equity risk premia where the cashflow of grains and non-grains is invariatly systematic versus firm-specific, $\rho_i = \bar{\rho}$. To obtain bond and equity risk premia, I solve the firm debt-mix problem à la Leland (1994) and plot security risk premia accordingly.

A.2.4. Solving the Firm's Capital Mix Problem

Environment

There are two firms in the economy, A & B, and each owns a technology that generates a stream of cashflow ruled by

$$\frac{dX_i}{X_i} = \mu_{x,i} dt + \sigma_{x,i} dZ_i, \;\; i = \{A, B\}, \tag{A.21}$$

where $\mu_{x,i}$ and $\sigma_{x,i}$ are constant parameters and dZ_i is the incremental change of a standard Brownian motion defined on the probability space $(\Omega, \mathscr{F}, \mathbb{P})$. The cashflow processes of firms are independent and all the parameters of the model are common knowledge.

SDF

As discussed in the main text, such SDF is governed by the following dynamics,

$$\frac{d\pi_t}{\pi_t} = r_f dt + g\sigma_A dZ_A + (1-g)\sigma_B dZ_B, \tag{A.22}$$

This Appendix provides the framework explained in the paper to risk neutralize firm cashflows and solve for the optimal equity-debt mix.

Risk Neutral Dynamics of Revenue

To value firm securities, we need to write the firm revenue dynamics under the $\mathbb{Q}$ measure. Regardless of the SDF, the following adjustment presents output dynamics under the $\mathbb{Q}$ measure,

$$\frac{dX_i}{X_i} = \tilde{\mu}_{x,i} dt + \sigma_{x,i} d\tilde{Z}_i, \tag{A.23}$$

where $\tilde{\mu}_{x,i} = \mu_{x,i} + \mathbb{E}_t\left(\frac{d\pi}{\pi}\frac{dX_i}{X_i}\right)$ is the growth rate of firm i under the probability measure $\mathbb{Q}$.

Unlevered Firm Value

The unlevered firm value is a claim on the total earnings of the firm,

$$V_t^u(X) = \mathbb{E}^{\mathbb{Q}}\left[\int_t^{\infty} e^{-r(s-t)}(1-\tau)X_s ds\right] = \frac{(1-\tau)}{r-\tilde{\mu}_{x,i}}X_t. \tag{A.24}$$

Debt Value

The value of debt for firm i comprises two components, the claim on a coupon until bankruptcy and the claim on the assets at bankruptcy,

$$D_t(X) = \mathbb{E}^{\mathbb{Q}}\left[\int_t^{T_D} ce^{-r(s-t)}ds\right] + \mathbb{E}^{\mathbb{Q}}\left[\int_t^{T_D}(1-\alpha)(1-\tau)X_s ce^{-r(s-t)}ds\right] \tag{A.25}$$

in which $T_D = inf\{t \geq 0, | X_t \leq X_D\}$ is the first hitting time. We can write the first part in Equation (A.25) as

$$\mathbb{E}^{\mathbb{Q}}\left[\int_t^{T_D} ce^{-r(s-t)}ds\right] = \mathbb{E}^{\mathbb{Q}}\left[\int_t^{\infty} ce^{-r(s-t)}ds\right] - \mathbb{E}^{\mathbb{Q}}\left[\int_{T_D}^{\infty} ce^{-r(s-t)}ds\right] \tag{A.26}$$

The first integral gives the value of a perpetuity,

$$\mathbb{E}^{\mathbb{Q}}\left[\int_t^{\infty} ce^{-r(s-t)}ds\right] = \frac{c}{r} \tag{A.27}$$

The second integral is over a random domain. By Karatzas & Shreve (1991)

$$\mathbb{E}^{\mathbb{Q}}\left[\int_{T_D}^{\infty} ce^{-r(s-t)}ds\right] = -\frac{c}{r}\left(\frac{X_t}{X_D}\right)^{\omega} \tag{A.28}$$

where $\left(\frac{X_t}{X_D}\right)^{\omega}$ is the Arrow-Debreu price bankruptcy and ω is the negative root of the characteristic equation $\frac{1}{2}\sigma_{x,i}^2\omega(1-\omega)+\mu_{x,i}\omega - r = 0$. Put differently, the value of a claim, v that pays a one unit of cashflow at bankruptcy and zero anywhere else, yields the risk free rate by no arbitrage,

$$\mathbb{E}^{\mathbb{Q}}[dv] = rv \tag{A.29}$$

and since v is a function of the firm earnings, it must satisfy the following PDE

$$\frac{\partial v}{\partial X}\tilde{\mu}_{x,i}X + \frac{1}{2}\frac{\partial^2 v}{\partial X^2}\tilde{\sigma}_{x,i}^2 X^2 = rv \tag{A.30}$$

The general solution to the PDE is

$$\nu(X) = a\nu^{\alpha_1} + b\nu^{\alpha_2} \tag{A.31}$$

By applying the boundary conditions the specific solution is derived,

$$\nu(X) = \left(\frac{X}{X_D}\right)^{\omega} \tag{A.32}$$

and

$$\omega = \frac{1}{2} - \frac{\mu_X}{\sigma_X^2} - \sqrt{\left(\frac{1}{2} - \frac{\mu_X}{\sigma_X^2}\right)^2 + \frac{2r}{\sigma_X^2}} \tag{A.33}$$

The Arrow-Debreu price of bankruptcy is used in other claims in a similar manner. The second part in Equation (A.25) is solved by the strong Markov property for Brownian motions by which

$$\begin{aligned} \mathbb{E}^{\mathbb{Q}}\left[\int_t^{T_D} (1-\alpha)(1-\tau)X_s c e^{-r(s-t)} ds\right] = \\ \mathbb{E}^{\mathbb{Q}}\left[(1-\alpha)(1-\tau)X_D \int_t^{\infty} e^{\sigma_{X_s} - (r-\mu_X+\frac{\sigma^2}{2})} ds\right] = \\ (1-\alpha)(1-\tau)\frac{X_D}{r-\tilde{\mu}_X}\left(\frac{X}{X_D}\right)^{\omega} \end{aligned} \tag{A.34}$$

Therefore, the value of debt is given by what we derived in Equation (A.27), (A.28) and (A.34).

$$\begin{aligned} D_t(X) =& \frac{c}{r}\left[1 - \left(\frac{X_t}{X_D}\right)^{\omega}\right] \\ &+ (1-\alpha)(1-\tau)\frac{X_D}{r-\tilde{\mu}_X}\left(\frac{X}{X_D}\right)^{\omega} \\ =& \frac{c}{r} - \left[\frac{c}{r} - (1-\alpha)(1-\tau)\frac{X_D}{r-\tilde{\mu}_{X,i}}\right]\left(\frac{X_t}{X_D}\right)^{\omega}, \end{aligned} \tag{A.35}$$

Levered Firm Value

By issuing debt, shareholders enjoy tax benefits on one hand and are exposed to costs of distress on the other hand. Tax benefits and costs of distress are both claims contingent on the earnings of the firm,

$$T_t(X) = E^{\mathbb{Q}}\left[\int_t^{T_D} \tau c e^{-r(s-t)} ds\right] \tag{A.36}$$

$$L_t(X) = E^{\mathbb{Q}}\left[\int_{T_D}^{\infty} \alpha(1-\tau)X_s e^{-r(s-T_D)} ds\right] \tag{A.37}$$

where Equations (A.38) and (A.39) are expected tax benefits and expected distress costs respectively. Following a similar procedure as in deriving debt value, the value of each claim is obtained as follows

$$T_t(X) = \frac{\tau c}{r}\left[1-\left(\frac{X_t}{X_D}\right)^{\omega}\right] \tag{A.38}$$

$$L_t(X) = \frac{\alpha(1-\tau)X_D}{r-\tilde{\mu}_{X,i}}\left(\frac{X}{X_D}\right)^{\omega} \tag{A.39}$$

$$\begin{aligned} V_t^l(X) &= V_t^u(X) + T_t(X) - L_t(X) \\ &= \frac{(1-\tau)}{r-\tilde{\mu}_{X,i}}X_t + \frac{\tau c}{r}\left[1-\left(\frac{X_t}{X_D}\right)^{\omega}\right] - \frac{\alpha(1-\tau)X_D}{r-\tilde{\mu}_{X,i}}\left(\frac{X}{X_D}\right)^{\omega} \end{aligned} \tag{A.40}$$

Equity

Shareholders have a claim on the earnings of the firm net of coupon payments until bankruptcy and the value of firm equity is then equal to

$$\begin{aligned} E_t(X) &= (1-\tau)\mathbb{E}^{\mathbb{Q}}\left[\int_t^{T_D} (X_s - c)e^{-r(s-t)} ds\right] \\ &= (1-\tau)\left[\frac{X_t}{r-\tilde{\mu}_{X,i}} - \frac{c}{r} - \left(\frac{X_D}{r-\tilde{\mu}_{X,i}} - \frac{c}{r}\right)\left(\frac{X_t}{X_D}\right)^{\omega}\right]. \end{aligned} \tag{A.41}$$

Optimal Capital Structure

The optimality of the default boundary is ensured by the smooth pasting condition

$$\frac{\partial E_t}{\partial X_t}|_{X_t = X_D} = 0, \tag{A.42}$$

that yields the optimal default boundary as

$$X_D = c\frac{\omega}{1-\omega}\frac{r-\tilde{\mu}_{X,i}}{r}. \tag{A.43}$$

By Equation (A.43), the default boundary is directly related to the value of the coupon. More leverage induces a higher default boundary, hence proximity to default.

Issuing debt is not costless, it increases the probability of default and the expected cost of distress. Thus, shareholders choose a level of leverage for which the marginal benefit of issuing and extra unit of debt is equal to the marginal cost of distress. Formally, firms solve the following problem

$$c^* = argmax\, V_0(c), \tag{A.44}$$

therefore,

$$c^* = X_0 c \frac{\omega}{1-\omega} \frac{r}{r-\tilde{\mu}_{x,i}} \left(1-\omega-\frac{\omega\alpha(1-\tau)}{\tau}\right)^{\frac{1}{\omega}}. \tag{A.45}$$