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Essays on Dynamic Pricing with Nuances of Consumer Behavior and Social Influences

par
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Essays on Dynamic Pricing with Nuances of Consumer Behavior and Social Influences

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Résumé

Le comportement d'achat des consommateurs évolue de manière et rapide sur les marchés complexes actuels. Un tel comportement se caractérise, par exemple, par l'attente stratégique de meilleures opportunités d'achat, la prise en compte de l'historique des prix ou des ventes avant de faire un nouvel achat, ou le conditionnement d'une décision d'achat aux autres consommateurs. Les entreprises ont vigoureusement investi dans le développement de techniques de tarification, mais elles recherchent toujours des stratégies plus sophistiquées et sur mesure en réponse à ces comportements émergents. Cette thèse, composée de trois essais, contribue à la conception de stratégies de prix dynamiques efficaces contingentes à certains types de comportements de consommation. Ce faisant, nous mettons également en lumière un certain nombre d'aspects interdépendants tels que les prévisions de la demande, la performance de l'entreprise ou la dynamique du marché. Pour déterminer les stratégies de prix optimales et d'équilibre, nous utilisons un mélange d'approches analytiques et numériques. Cette thèse apporte une contribution substantielle et méthodologique à la littérature en générant de nouvelles perspectives et en développant de nouveaux cadres de modélisation.

Dans le premier essai, nous proposons un nouveau cadre pour la diffusion de l'innovation qui relie les comportements stratégiques au niveau individuel aux effets agrégés et aux influences sociales. Nous considérons des consommateurs hétérogènes divisés en deux segments en fonction de leurs déterminants d'adoption. Nous utilisons la théorie des jeux à champ moyen pour relier les com-

portements d'adoption basés sur l'interaction de ces consommateurs hétérogènes à la diffusion du nouveau produit et, par conséquent, aux stratégies de tarification dynamiques. Pour cela, nous définissons les conditions d'existence et d'unicité de l'équilibre de champ moyen de diffusion. De plus, nous menons des expériences numériques approfondies pour fournir plusieurs informations. Notamment, nous constatons qu'une stratégie de prix de pénétration est optimale pour les consommateurs stratégiques, mais une trajectoire de prix croissante suivie d'une baisse est recommandée lorsque les consommateurs se comportent de manière myope.

Dans les deuxième et troisième essais, nous étudions la tarification dynamique en mettant l'accent sur le comportement dépendant de la référence et les externalités sociales, chaque essai étant motivé par différentes nuances du comportement du consommateur. Dans le deuxième essai, nous considérons des consommateurs stratégiques averses aux pertes dont la décision d'adoption dépend de l'effet prix de référence et des externalités. Nous développons progressivement différents modèles imbriqués pour identifier l'impact de chaque phénomène. Ce faisant, nous considérons deux types de régimes de tarification, à savoir les stratégies de tarification préannoncées et réactives. Motivés par des preuves empiriques, nous considérons dans le troisième essai des consommateurs dont la décision d'achat qui décident dépend de deux points de référence relatifs au prix et à des externalités. Par conséquent, ils enregistrent des gains (pertes) lorsque soit le prix est inférieur (supérieur) au prix de référence, soit les externalités de *crowding* dépassent (n'excèdent pas) les attentes des consommateurs en matière de ventes. Nous proposons un modèle à points de référence multiples qui imbrique deux repères, à savoir un modèle de prix de référence uniquement et un modèle d'externalités de référence uniquement. Nous caractérisons analytiquement les stratégies de tarification dynamique, puis montrons comment leurs implications profondes peuvent remettre en question de manière critique les recommandations de tarification existantes.

Mots-clés

Tarification dynamique; Comportement du consommateur; Consommateur stratégique; Influences sociales; Bouche-à-oreille; Externalités; Théorie des jeux; Jeu à champ moyen; Marketing; Diffusion de nouveaux produits; Prix de référence; Effet d'externalités; Poits de référence multiples

Méthodes de recherche

Optimisation; Théorie des jeux; Analyse numérique

Abstract

Consumer buying behavior is evolving intricately and rapidly in the current complex markets. Such behavior is characterized, for example, by waiting strategically for better purchasing opportunities, taking into account price or sales histories before making a new purchase, or conditioning a buying decision to other consumers'. Companies have vigorously invested in developing pricing techniques, yet they still seek for more far-reaching and tailor-made pricing strategies in response to these emerging behaviors. The current thesis, elaborated in three essays, aims to fill this gap by recommending effective dynamic pricing strategies contingent on certain types of consumer behavior. In doing so, we also shed light on a number of interdependent aspects such as demand forecasts, firm's performance, or market dynamics. We take a theoretical approach to analytically characterize the pricing strategies in most cases, however, we may resort to numerical analyses in some instances. This thesis makes a substantive and methodological contribution in the literature by generating new insights and developing new modeling frameworks.

In the first essay, we propose a new framework for innovation diffusion model that connects the individual-level strategic behaviors to the aggregate effects and social influences. We consider heterogeneous consumers which are composed of two segments depending on their adoption drivers. We use mean-field games (MFGs) theory to link the interaction-based adoption behaviors of such heterogeneous consumers to the new product diffusion and consequently dynamic pricing

strategies. To this end, we delineate the conditions for the existence and uniqueness of diffusion mean-field equilibrium. Additionally, we run extensive numerical experiments to provide several insights. Notably, we find that a penetration pricing strategy is optimal for strategic consumers, but an increasing price path followed by a decreasing one is recommended when consumers behave myopically.

In the second and third essays, we study dynamic pricing with a focus on the reference-dependent behavior and social externalities, however, each essay is motivated by different nuances of consumer behavior. In the second essay, we consider strategic loss-averse consumers whose adoption decision depends on the reference-price effect and externalities. We develop progressively various nested models to identify the impact of each phenomenon. In so doing, we consider two types of pricing regimes, namely preannounced and responsive pricing strategies wherein the role of firm commitment to its strategies can be uncovered. In the third essay, and motivated by empirical evidence, we consider consumers who are reference-dependent in two dimensions, namely price and externalities. Therefore, they experience gains (losses) when either the price falls behind (exceeds) the reference price or crowding externalities exceeds (falls behind) consumers' sales expectations. We propose a multiple reference-points model that nests two benchmarks, namely only reference-price model or only reference-externalities model. We characterize dynamic pricing strategies analytically and then show how their profound implications can critically challenge existing pricing recommendations.

Keywords

Dynamic pricing; Consumer behavior; Strategic consumer; Social influences; Word-of-mouth; Externalities; Game theory; Mean-field Stackelberg game; Rational expectation equilibrium; Marketing; New product diffusion; Reference-price effect;

Reference-externalities effect; Multiple reference points

Research methods

Mathematical optimization; Game theory; Numerical analysis

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List of acronyms

WoM Word-of-mouth

MFG Mean-field game

MRPs Multiple reference effects

To my parents Fatemeh and Yaghoub,

To my brothers and sister,

for their unconditional love and support.

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This dissertation is dedicated to my parents, Fatemeh and Yaghoub.

Preface

This thesis is composed of three essays which are listed below:

- Chaab, J., Salhab, R., and Zaccour, G. (2022). Dynamic pricing and advertising in the presence of strategic consumers and social contagion: A mean-field game approach. *Omega*, 109:102606. Accepted by Omega
- Chaab, J., Zaccour, G. Dynamic pricing in the presence of social externalities and reference-price effect. Under Review in Omega.
- Chaab, J., Zaccour, G. Dynamic pricing with multiple reference effects. To be submitted.

General Introduction

Consider the following situations:

- A car company wants to determine how to price a newly launched car in the face of consumers with intricate behavior. Consumers may anticipate future prices and are patient enough to wait for better deals. Moreover, car buyers might not have the same purchasing drivers where, for instance, one group makes purchasing decision independently from other car buyers whereas the other group tends to conform to trending purchasing behavior. In nowadays markets with constantly changing prices, how the automaker can tailor its pricing strategy in accordance with such heterogeneous and intricate consumer behavior to secure a high profit?

- A theater organization wants to determine the ticket prices for reference-dependent consumers who condition their new ticket purchase on previous purchasing experiences. That is, a consumer is discouraged to book a new ticket whose price exceeds her reference-price point (price expectation) whereas she is encouraged to make the purchase if the observed attendance exceeds her expectation. She may or may not book the ticket ultimately depending on whether the benefit from higher attendance surpasses the loss from prices. In the face of such multiple reference-dependent consumers, how the theater organizer should set the prices?

In the above or similar situations, the firm needs to employ dynamic pricing to respond to or exploit such consumer behavior. Dynamic pricing is the study of varying the prices with the aim of optimizing a certain metric, usually the firm's profit. It is used as a flexible pricing adjustment strategy where the firm changes the prices on the basis of time of sales, demand information and supply oscillations (Elmaghraby and Keskinocak, 2003), or consumers and circumstances (Haws and Bearden, 2006). For example, Lexus in the US luxury car markets begins with a low introductory price followed by an increasing price path in order to initiate word-of-mouth (WoM) and attract more consumers during the mid and late stages. Apple, however, has been persistent to charge a high launch price followed by a decreasing price trend for iPhone in order to skim the early consumers with high willingness to pay and then make it affordable for those with lower ones. Dynamic pricing is innately a discriminatory-based mechanism wherein the prices are charged differently across, for example, consumers or purchasing occasions (intertemporal). The discriminatory nature of dynamic pricing can arise in many forms such as markups, coupons, clearance sales, or promotions.

While the travel industry has been pioneer in using dynamic pricing, other industries such as retailing, energy, entertainment, or manufacturing among many others, are increasingly adopting such practice. Amazon, for example, is known for its successful dynamic pricing strategies that have brought an almost unbeatable competitive advantage for this giant company. By dint of sophisticated dynamic pricing techniques, Amazon changes the prices every 10 minutes and has allegedly increased its profit by 25%(Pai, 2017; Dilmegan, 2021). In other examples, Ford Motor Co. has exceeded its target profit by $1 billion in 2003 (Sahay, 2007) while American Airlines has reportedly accrued an additional $500 million each year (Altexsoft, 2021).

Consumer behavior is at the heart of devising a successful dynamic pricing strategy. Intricate consumer behavior can emerge in the form of forward-looking behavior, reference dependency, heterogeneous buying drivers, or susceptibility to social influences among others. Early research on dynamic pricing assume that consumers are myopic (Levin et al., 2009), that is, they adopt in the first profitable purchasing occasion without anticipating the future ones. However, it is well-documented that consumers may behave in a *forward-looking*[1] fashion, (Li et al., 2014) in which they predict the future and might decide to wait strategically for better deals, say, a price markdown. Consequently, if a firm falsely assumes that forward-looking consumers are of myopic type, then it can potentially face a significant revenue loss up to 20%(Aviv and Pazgal, 2008). Moreover, consumers might behave in a backward-looking reference-dependent way in the sense that they develop expectations inherited from past experiences, known as reference point, to evaluate the appeal of current offer. For instance, the price is a popular attribute whose reference effect is widely evidenced in marketing and operations management, however, the notion of reference dependency is generalized to other attributes such as observed sales or quality. A consumer's *reference price* is a benchmark against which the appeal of current selling price can be judged. Consumers might feel gains (losses) if the reference price is higher (lower) than the selling price, where they tend to weigh losses more than the same-sized gains, what is referred to as *loss aversion* (Kahneman and Tversky, 1979). The revenue of numerous companies in the retail industry, where reference-price effect is substantially documented, can surpass $30 billion annually (Chen et al., 2017) and overlooking reference-price effect leads to suboptimal pricing strategies (Rajendran and Tellis, 1994).

Consumers heterogeneity or social influences can also create many opportunities for profit-making dynamic pricing techniques. In durable goods, for instance, early buyers are known to value the new product higher than the late buyers. Consequently, the firm is able to use, say skimming pricing strategy, in order to capitalize on such consumers heterogeneity. In a skimming pricing strategy, the firm sets a high launch price and subsequently lowers them. Social influences,

[1]Forward-looking, farsighted or strategic consumers are used interchangeably.

such as WoM communications or externalities, are also of great significance in determining the choice of a pricing scheme. Positive externalities occur when product utility increases as more consumers adopt the same or complementary products. Indeed such social influences are the key motivation behind the adoption of a penetration pricing strategy, that is, the firm starts with low prices and subsequently increases them. The advantage of such dynamic pricing strategy is to facilitate early adoption, which in turn strengthens the WoM or externalities, and ultimately leads to higher sales.

The literature on dynamic pricing with consumer behavior considerations is extant with respect to, for instance, social influences and consumer heterogeneity in the context of new product diffusion (see Nair, 2019 and Peres et al., 2010 for reviews), or reference-price dependency (see Mazumdar et al., 2005 and Arslan and Kachani, 2010 for reviews). However, several research questions remain unanswered on how firms should respond to increasingly emerging nuances of consumer behavior. In the context of new product diffusion, for instance, Peres et al. (2010) states that individual-level adoption modeling is at its early stage and should embrace behavioral findings. Tereyağoğlu et al. (2018), in the context of experience goods, calls for theoretical research on multiattribute reference-dependent behavior. To this end, the current thesis aims to fill these gaps by focusing on two streams of literature, that is, new product diffusion and revenue management. Through three essays, we make methodological and substantive contributions to literature by providing new insights into the implications of intricate consumer behavior and social influences on the firm's decisions and by proposing a new framework that links individual decision-making to aggregate influences in a behaviorally rich fashion.

The first essay focuses on the dynamic pricing and advertising of a new product (consumer durables) as well as its market penetration when consumers are forward-looking and behaviorally heterogeneous. In this product category i.e., consumer durables, the literature usually uses diffusion theory to predict the demand and accordingly determine the optimal pricing strategy (Robinson and Lakhani, 1975; Besanko and Winston, 1990; Li, 2019; Zhang and Chiang, 2020), albeit from an aggregate perspective. A number of studies have incorporated the individual-level diffusion modeling (Song and Chintagunta, 2003; Chatterjee and Eliashberg, 1990; Li, 2019; Nair, 2007), however, they either assume myopic consumers or neglect the salient role of social influences. Despite multiple calls for tying individual-level decision framework to aggregate effects (e.g., Peres et al., 2010), the literature remains quite sparse. The first essay aims to fill this gap. Our objective is to develop a new product diffusion framework for durable goods that captures nuances of consumer behavior at the individual and aggregate levels to empower the firm to better devise its pricing and advertising strategies. We make a methodological and substantive contribution by introducing *mean-field game* (MFG) in new product diffusion and providing new insights on the consumers-firm interplay.

Motivated by Bass model (Bass, 1969) and its extensions on consumer segments (e.g., see Van den Bulte and Joshi, 2007), we consider two types of consumer segments, namely individualists and conformists. The individualists adopt independently from other consumers and are driven by the price and the firm's goodwill, whereas the conformists adopt in response to social influences exerted by other consumers and also by the price. Consequently, the conformists condition their decision on others and hence predict the adoption rate in a dynamic game setting, known as MFG. MFG assume that the impact of each player (consumer) on the mass behavior, where the latter is referred to as mean-field, is negligible and hence the game of an infinite number of players can be reduced to the one between a generic player and the mean-field[2]. The firm, in return, anticipates the adoption behaviors of two consumer segments and accordingly determines the dynamic pricing and advertising strategies. We establish conditions under

[2]To give an analogy, the driving time at a given hour between points A and B depends on the traffic density, which affects the decision of a driver to use or not this road. However, if an individual decides to use the road, her impact on the average driving time is clearly negligible.

which the existence and uniqueness of mean-field equilibrium are guaranteed and further propose a numerical scheme for its computation.

The second essay studies dynamic pricing by examining a different set of consumer behavior and social influences, that is, asymmetric reference-price effect and social externalities. Moreover, it implements and compares two types of dynamic pricing strategies, namely preannounced and responsive pricing regimes. There is a large body of literature on how dynamic pricing is influenced by the externalities effect (e.g., Xie and Sirbu, 1995; Gabszewicz and Garcia, 2008), reference-price effect (e.g., Fibich et al., 2003; Popescu and Wu, 2007; Nasiry and Popescu, 2011), or both effects (Bloch and Quérou, 2013; Duan and Feng, 2021; Li and Zhang, 2020; Chen et al., 2020; Zhao et al., 2019). However, they investigate these effects either as standalone phenomena, or jointly but in a static setup. To that end, we contribute to the literature by examining the role of concurrent reference-price and social externalities effects in the face of forward-looking consumers in a dynamic fashion. Our objective is to determine the optimal dynamic pricing strategies under preannounced and responsive pricing regimes. We consider a two-period choice model wherein forward-looking consumers decide whether to make a one-time purchase. The product category is assumed to be either consumer durables or experience goods where both reference price (Neumann and Böckenholt, 2014) and externalities (Yang and Mai, 2010) are relevant. We progressively develop nested models to delineate the contribution of each aspect of consumer behavior on the firm's policies and ultimately propose the general model. In doing so, we use the notion of rational expectation equilibrium whereby the consumers and the firm's predictions coincide with realized outcomes. The results across different models, and also with respect to preannounced versus responsive pricing schemes are compared.

In the third essay, we retain the reference price and externalities effects from the second essay, however, in an entirely innovative setup. More specifically, motivated by the empirical evidence on multiattribute reference-dependent behavior

(Lattin and Bucklin, 1989; Hardie et al., 1993; Tereyağoğlu et al., 2018), we study dynamic pricing with consumers who are influenced by two reference effects, namely reference price and reference externalities. Tereyağoğlu et al. (2018) show that, in addition to reference price, consumers also develop a reference point for observed sales, what we refer to as reference externalities, where they are more (less) inclined to make a purchase if actual sales exceed (fall behind) their expectations. Reference-dependent behavior across multiple attributes was theorized by prospect theory (Tversky and Kahneman, 1991) and empirically confirmed in marketing literature with respect to price and promotions (Lattin and Bucklin, 1989), price and quality (Hardie et al., 1993) and price and filled capacity of an event (Tereyağoğlu et al., 2018). Such researches are limited to empirical works and hence the theoretical implications of multiattribute reference dependency on firm's pricing strategy are yet to be explored. While the literature on theoretical stream is abundant (Kopalle et al., 1996; Fibich et al., 2003; Popescu and Wu, 2007; Nasiry and Popescu, 2011; Hu and Nasiry, 2018; Anton et al., 2022), they are mainly concerned with single reference effect, usually only reference-price effect.

Therefore, the third essay makes a substantive contribution, beyond what is considered in the second essay, to enhance our understanding on how the monopoly should adjust the prices in response to such newly-documented behaviors. Our objective is to uncover the theoretical implications of multiattribute reference dependency from consumer side on firm's pricing policies. In line with empirical evidence (Tereyağoğlu et al., 2018), we consider an experience good where consumers make purchases repeatedly and are influenced by reference effects on both price and externalities dimensions. We develop a continuous dynamic model in which consumers are myopic, but the firm is forward-looking and aims to determine its dynamic pricing trajectory based on evolution of two interdependent reference effects. We characterize the optimal pricing strategy and also study two benchmarks where either only reference-price effect or reference-externalities effect is accounted for. We also establish monotonicity properties of reference policies with respect to price and externalities. Ultimately, we compare the firm's profit in the multiple reference-points model with the two benchmarks.

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Chapter 1

Dynamic pricing and advertising in the presence of strategic consumers and social contagion: A mean-Field game approach

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Abstract

In this paper, we introduce a framework for new product diffusion that integrates consumer heterogeneity and strategic social influences at individual level. Forward-looking consumers belong to two mutually exclusive segments: indi-

vidualists, whose adoption decision is influenced by the price and reputation of the innovation, and conformists, whose adoption decision depends on social influences exerted by other consumers and on the price. We use a mean-field game approach to translate consumer strategic interactions into aggregate social influences that affect conformists' adoption decision. The game is played à la Stackelberg, with the firm acting as leader and consumers as followers. The firm determines its pricing and advertising strategies to maximize its profit over a finite planning horizon. We provide the conditions for existence and uniqueness of equilibrium and a numerical scheme to compute it. We conduct a series of numerical simulations to analyze firm's strategy and diffusion processes for different parameter constellations. Our results suggest that the firm adopts a penetration pricing strategy in the presence of strategic consumers, whereas it increases the price first and then decreases it in face of myopic consumers. Moreover, our model asserts that diffusion of innovations is considerably shaped by the consumer heterogeneity. Indeed, our results show that as the fraction of one segment increases in the market, the consumers in the other segment have less tendency towards adoption.

1.1 Introduction

Since its inception more than half century ago by Rogers (1962) and Bass (1969), the theory of diffusion of innovations in a social system has remained constantly on the research agenda. Recent developments aimed at addressing some shortcomings of classical aggregate diffusion models by incorporating explicitly consumer behavior into the adoption process. Examining a new product adoption at individual level allows to connect a micro-founded consumer behavior to aggregate effects and firm decisions. Also, the growing impact of social networks in all areas of human activities has, at least implicitly, led to redefining the concept: "*Innovation diffusion is the process of the market penetration of new products and services,* *which is driven by social influences. Such influences include all of the interdependencies among consumers that affect various market players with or without their explicit knowledge*" (Peres et al., 2010). For instance, accounting for consumer heterogeneity and social interaction can explain why the diffusion rate does not necessarily follow a typical bell-shaped curve, as well as the presence of chasm or saddle phenomenons in sales curve (Van den Bulte and Joshi, 2007; Goldenberg et al., 2002; Song and Chintagunta, 2003).

In this paper, we study how consumer heterogeneity and interaction at individual level frame market penetration of a new product and affect firm marketing strategies. More specifically, we consider a firm that launches a new product in a market composed of a large number of strategic (forward-looking) consumers divided into two mutually exclusive groups, namely individualists and conformists. The adoption decision of an individualist depends on the product price and reputation (goodwill), which is built through investment in advertising over time. Whereas an individualist choice to buy or not the product is independent of social pressure, a conformist's decision is driven by other consumers' behavior, more precisely the percentage of adopters in the social system and by the product price. In our framework, the firm's paid-for advertising corresponds to information on the product emanating from outside the social system, while the fraction of adopters (in both consumers' groups) captures the within social system communication. A large number of early adopters can be significant for certain subgroups of consumers (e.g., teenagers) who are more prone to social pressure and seek for conformity behavior (Moretti, 2011).

When the number of consumers is large, it is intuitive to suppose that each member of a social system has a negligible impact on the mass, although the aggregate behavior of the members has an important impact on any individual's decision.[1] This assumption of weak interactions between individuals (players) is

[1]To give an analogy, the driving time at a given hour between points A and B depends on the traffic density, which affects the decision of a driver to use or not this road. However, if an

at the heart of *mean-field game* (MFG) theory, which provides a natural methodological framework to study social influences in an adoption process. We assume that the firm, as a leader, plays a Stackelberg game with consumers, who act as followers, where the conformists play a mean-field game among each other in a Nash configuration.

Based on MFG, the game among an infinite number of players can be reduced to the one between a generic player and the mean-field. Technically speaking, the mean-field, as a coupling term, can be obtained using two backward-forward equations, namely backward dynamic programming equation and forward Chapman-Kolmogorov one. The former determines the generic player's best response to the mean-field while the latter computes the mean-field under the player's best responses. These two coupled equations yield the mean-field equilibrium at which no player is better off to deviate from her best response to what she anticipates as the aggregate response i.e., mean-field in the market. Thus, we use MFG to capture the strategic interactions among large number of consumers who consider adoption of a new product. The advantages of MFG framework in context of determining dynamic pricing and advertising for a new product diffusion are threefold: *i*) it channels the individuals decisions to aggregate effects from which the diffusion of new product can be anticipated, *ii*) in relation to *i*, it retains consumers heterogeneity (consumers segmentation and their heterogeneous sensitivity to adoption drivers) from which the implications of micro-level dynamics on pricing strategies can be uncovered, *iii*) it examines the impact of strategic interactions among players, measured by the mean-field, on their individual adoption decisions, which is not accessible without MFG.

Our objective is to address the following research questions:

1. How does a new product diffuse into a market composed of forward-looking consumers whose adoption behavior is framed by individual level dynam-

individual decides to use the road, her impact on the average driving time is clearly negligible.

ics and interactions in a two-segment structure?

2. How are marketing strategies influenced by market penetration of the product and consumers heterogeneity and interactions?

3. How does the consumer's strategic or myopic behavior affect the equilibrium results?

4. What is the impact of the parameter values on the firm's equilibrium strategy?

Our contributions are as follows. We propose a new game-theoretic model of innovation diffusion that involves a two-segment market of strategic consumers with heterogeneous adoption drivers and a strategic monopolist. We use the mean-field game theory to develop tractable solutions and predict the evolution of the consumers' mass adoption behavior from the individual strategic reactions to the firm's marketing strategies and Word-of-Mouth (WoM) communications. In particular, we develop a numerical scheme to compute the firm's optimal pricing and advertising policies in face of a large population of heterogeneous consumers, and to predict the evolution of adoption rate. We generate new insights via numerical examples on how market penetration of new product is built from the strategic decisions of interacting small consumers and what are the marketing implications of such consumer heterogeneity and interactions. Our model is consistent with empirical findings in the literature and allows for different types of irregularities in diffusion curves. In particular, we find that the magnitude and rate of diffusion depend significantly on the consumer sensitivity towards mass media (external influences) and WoM (internal influences) as well as the two-segment structure of the market, and accordingly can exhibit different penetration curves. Moreover, our results generalize some findings on monopolist marketing strategies. For example, we obtain that the firm implements a penetration pricing strategy[2] to avoid that strategic consumers wait to adopt and the penetration intensity depends substantially on the consumer heterogeneity and social influences.

The rest of the paper is organized as follows: We discuss in Section 1.2 the relevant literature. The model is introduced in Section 1.3. Section 1.4 provides numerical simulations and sensitivity analyses. Section 1.5 concludes the paper and proposes some future directions. A numerical scheme is provided in the appendix section to solve the game and compute the firm's optimal strategies.

1.2 Literature Review

Our paper draws and contributes to two research streams in the marketing science literature on new product diffusion, namely individual level diffusion models and aggregate diffusion models taking either normative or two-segment structure approaches. We briefly review these two streams in this section.

Individual level diffusion modeling enables marketers to better set the marketing strategies by studying consumer behavior, various forms of consumer heterogeneity, and the relationship between micro-level dynamics and aggregate behavior. One common denominator in this literature stream is to let the consumer's utility be based on the new product performance under uncertainty (Chatterjee and Eliashberg, 1990; Roberts and Urban, 1988; Horsky, 1990; Lattin and Roberts, 1988). Following the reception over time of information from mass media or WoM communications, consumers update their uncertain perception of the innovation. A consumer adopts the product if her expected utility exceeds what she gets under non adoption. The penetration curve is next obtained by aggregating individual decisions. In a similar vein, Song and Chintagunta (2003) consider forward-looking consumers and account for consumer heterogeneity

[2]Penetration pricing involves charging low initial prices in a hope to penetrate the market faster followed by a gradual increasing trend

and price sensitivity to develop an individual diffusion-choice model. The authors neglect the role of WoM communications and assume that the price is exogenously given. Li (2020) deals with the pricing of new multi-product in which consumers are myopic and adopt in a two-stage process: First, affected by external and internal influences, consumers face some purchasing occasions, and next they select one innovation based on their individual price-dependent utility. This research finds different variants of pricing regimes for new multi-product diffusion depending on the significance of innovation versus imitation effects. We depart from this literature by considering forward-looking consumers in a two-segment structure where the conformists segment is affected by the adoption behavior of entire social system. Further, we examine how consumers heterogeneity and consumers sensitivity with respect to external and internal influences are tied to aggregate market and firm's marketing strategies and performance. The literature calls for transition from aggregate models to individual ones in order to better manage firms marketing activities in response to evolving individual level behavior (Peres et al., 2010). Individual models allow to analyze the rich effects of social networks on innovation's market performance that have recently received a growing attention (see Muller and Peres, 2019 for a review)

The second relevant stream of literature is concerned with aggregate diffusion models that either study monopolist pricing and/or advertising policies given aggregate growth model or build up new product diffusion through a two-segment market. The former line of this stream takes a normative approach to maximize the monopolist profit (see Nair, 2019 for a recent review on new product pricing). It mainly considers myopic consumers and suggests skimming pricing strategy[3] unless WoM communications are strong (Kalish, 1983, 1985; Dolan and Jeuland, 1981; Horsky, 1990). Kalish (1983), for example, considers a general diffusion framework and finds that skimming strategy is optimal if WoM is not strong for two diffusion models: price-dependent multiplicative separable

[3]Skimming pricing involves setting high initial prices followed by a gradual decreasing trend.

model and price-dependent market potential model. However, if market potential, planning horizon, or other involved aspects lead to dominance of WoM effect compared to saturation one, then the penetration pricing strategy is the optimal choice for monopolist. Kalish (1985) finds a similar strategy for pricing and further suggests monotonically decreasing trend as optimal advertising strategy. Besanko and Winston (1990) also propose skimming pricing strategy for the monopolist who encounters forward-looking consumers in order to prevents them from waiting for more appealing deals in future periods. However, the paper did not account for the salient role of WoM in the diffusion process. Including a reference price in the model, Zhang and Chiang (2020) obtain that a myopic monopolist is better off implementing a skimming pricing strategy, whereas for a strategic seller, either penetration or skimming strategies could be optimal, depending on the potential market and reference effect. Additionally, some studies consider competition across firms in context of new product diffusion and examine marketing strategies along with other aspects such as cost learning or government subsidies for new technologies (see Jørgensen, 2018 for a review). Particularly, Chenavaz et al. (2020) use a mean-field game approach to examine how a new product is priced by a large number of firms, each anticipating the mean-field price and demand affecting their individual demand.

In Rogers (1962), the distinction between the different groups (segments) of consumers is based on their timing of adoption, i.e., innovators, early adopters, early majority, late majority, and laggards. In Bass (1969) and the literature that followed, the two groups of consumers (innovators and imitators) are differentiated in terms of what drive them to adopt. Whereas innovators are influenced by paid-for mass media communications emanating from outside the social system, imitators' adoption decision depends on social pressure, measured by WoM communications. The subsequent literature, however, argues that innovators are not necessarily the first adopters as defined by Rogers (1962); see, e.g., Mahajan et al. (1990). It also questions the common assumption that all consumers homogeneously have innovative and imitative tendency towards adoption regardless of their type, and hence each potential adopter might be affected by both external and internal influences. This assumption is challenged by two-segment structure diffusion models that theoretically and empirically underpins the significance of such structure where each potential adopter is affected by corresponding adoption driver depending on her type. The notion of two-segment structure has been also adopted in other contexts such as established luxury products (e.g., Zhang et al., 2020). Steffens and Murthy (1992) divide the population into innovators and imitators where the former segment is driven by external and internal influences and the latter by internal ones. Through an aggregate perspective, their model provides a better fit compared to Bass (1969) and further shows bimodal characteristics in the penetration curve. Van den Bulte and Joshi (2007) discuss five theories that articulate how potential adopters can be unbundled into a two-segment structure based on not only independent and imitative drivers but also the way the consumers imitate within or cross segments. They find that there is a chasm between early and later stages of penetration curve for a two-segment population that compete over the social status. In our study, we name self-reliant consumers who make adoption decision independently from others and are affected by external influences as **individualists**, while those who tend to conform to social norms but also are sensitive to key external influences are termed as **conformists**. In contrast to the two-segment literature that uses aggregate approach, we introduce the individualist-conformist framework into the individual-based diffusion modeling in order to explore how heterogeneity of forward-looking consumers and their strategic interactions shape the adoption process. We use a mean-field game methodology to obtain the Nash equilibrium among conformists who collectively along with individualists act as followers in a Stackelberg game, where a profit-maximizing firm determines its pricing and advertising strategies.

1.3 Model

We consider a firm launching a new product in a market formed of N consumers, where N is a large number. Consumers are divided into two groups (or market segments): N^I individualists and N^C conformists, with $N = N^I + N^C$. Individualists are individuals whose adoption decision is based on the product's characteristics, e.g., the price and brand reputation, whereas conformists' purchasing utility depends on the behavior of other consumers in addition to the prices.

At each instant of time $t \in \{0, \ldots, T\}$, a consumer who has not yet adopted the product decides whether to purchase the product or not. The date T is interpreted as the end of the selling season, after which the product is not available. The state of individualist $j \in \{1, \ldots, N^I\}$ at time t is $S_{jt} \in \{0, 1\}$, where $S_{jt} = 1$ means that individualist j has the product at time t, while $S_{jt} = 0$ refers to the opposite case. At time t, individualist j's decision variable is $A_{jt} \in \{0, 1\}$, where $A_{jt} = 1$ and $A_{jt} = 0$ refer to adopting or not the product, respectively. We assume that each consumer buys the product at most once during the planning horizon, and consequently if a purchase is made at time t, then necessarily $A_{j\tau} = 0$ for all $\tau \neq t$. We define in a similar way the state $s_{kt} \in \{0, 1\}$ and decision variable $a_{kt} \in \{0, 1\}$ of conformist $k \in \{1, \ldots, N^C\}$. Because the product is new, we suppose that $S_{j0} = s_{k0} = 0$.

At each time period t, the firm decides the price $p_t \in [0, M_p]$ of its product and its advertising (or marketing) investment $m_t \in [0, M_m]$, where M_p and M_m are positive scalars.[4] Advertising has a positive impact on the product's reputation or goodwill $G_t \in \mathbb{R}$. The goodwill dynamics are given by

$$G_{t+1} = h(t, G_t, m_t), \quad G_0 \text{ given,} \tag{1.1}$$

where G_0 is the initial goodwill's value, and h is a continuous function. A well-known instance of (1.1) is Nerlove and Arrow's model (Nerlove and Arrow,

[4] Conceptually, we could only require to have $p_t \geq 0$ and $m_t > 0$. The upper bounds M_p and M_m, which can be arbitrary large, are needed for the existence and numerical computation of the equilibrium.

1962), where $h(t, G_t, m_t) = m_t + (1-\gamma)G_t$, and γ is consumer's forgetting rate. The Nerlove and Arrow's model has received great attention in the literature (e.g., Song et al., 2021; Chenavaz and Eynan, 2021; Crettez et al., 2021). The advertising cost is given by the increasing non-negative function $c_m(m_t)$.

Consumers are described by random utility functions.[5] That is, a consumer's utility for choosing an alternative is composed of two parts: a deterministic component encapsulating the observable consumer-alternative attributes that shape the consumer's choice; and a random component that depends on the idiosyncratic unobserved attributes representing the latent utility. In particular, at each time period t, a consumer who has not adopted the product faces a binary choice of buying it or waiting until the next purchasing occasion to make a decision. Without any loss of generality, we normalize the utility of not adopting to zero. This implies that a consumer will not adopt the product if she does not perceive a positive utility at least once during the planning horizon. Formally, at time t, an individualist j who does not yet possess the product ($S_{jt} = 0$) faces a binary choice, i.e., adopt or not. If she adopts the product ($A_{jt} = 1$), her future state S_{jt+1} switches to 1, and she enjoys the following utility:

$$U_{jt} = \rho_c^t (U^I(p_t, G_t) + \epsilon_{jt}), \tag{1.2}$$

where $0 < \rho_c \leq 1$ is a discounting rate, $U^I(p_t, G_t)$ is the deterministic component, and $\epsilon_{jt} \in [-M_\epsilon, M_\epsilon]$ is the random component. We assume that U^I is a continuous function increasing in the goodwill and decreasing in the price. If individualist j does not adopt at time t ($A_{jt} = 0$), her state at time $t+1$ stays 0 and she gains a zero per-step utility. The random components ϵ_{jt} are assumed independent and identically distributed (i.i.d.). We summarize the individualist's dynamics and utility in Figure 1.1.

[5]The theory of random utility was introduced by Thurstone (1927) and further developed by Lancaster (1966) and McFadden (1974). We refer the reader to (Corstjens and Gautschi, 1983) for an overview.

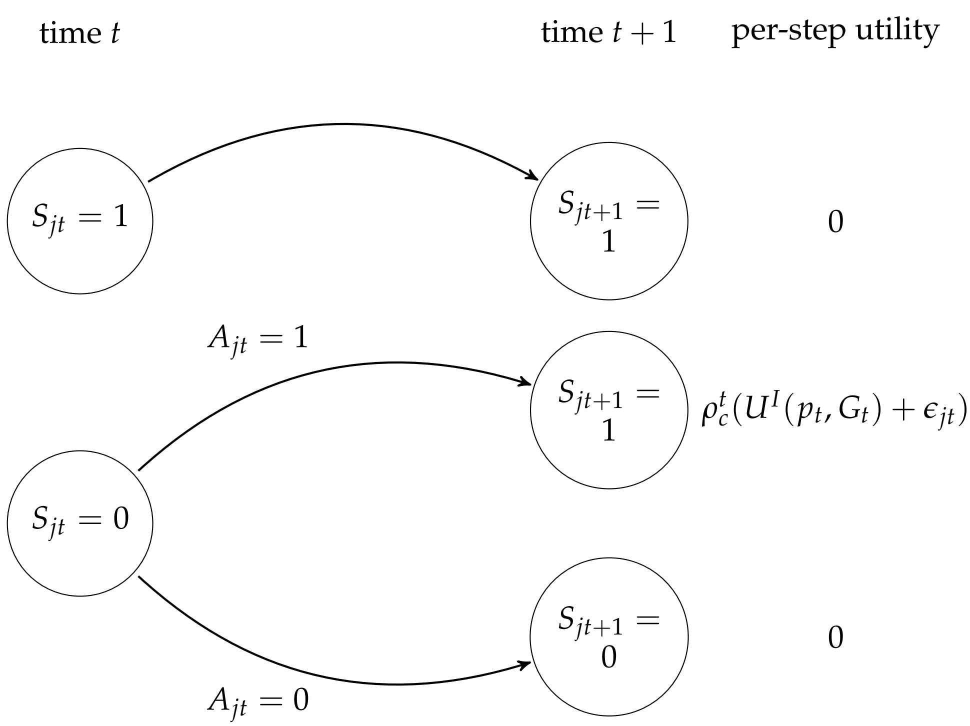


Figure 1.1: Individualist j dynamics and per-step utility.

We assume, as for individualists, that conformists' utility function depends on the product's price. Differently from individualists, conformists' adoption decision is shaped by the degree of social acceptance of the product, which is measured by the percentage of consumers who have already acquired it, i.e., $F_t := \frac{1}{N}\left(\sum_{j=1}^{N^I} S_{jt} + \sum_{k=1}^{N^C} s_{kt}\right)$, for $t \in \{0, \dots, T\}$. Consequently, we have

$$u_{kt} = \rho_c^t(u^C(p_t, F_t) + \eta_{kt}), \tag{1.3}$$

where u^C is a continuous function decreasing in p_t and increasing in F_t, and $\eta_{kt} \in [-M_\eta, M_\eta]$ are random utilities that are i.i.d. We summarize the conformist's dynamics in Figure 1.2.

In determining the timing of adoption, consumers of both types act strategically, i.e., they are forward-looking. Whereas a myopic consumer purchases the product at the first date at which her utility is positive, a strategic consumer anticipates her future payoffs and adopts at the date that yields the highest positive utility. Technically speaking, each consumer solves an intertemporal optimiza-

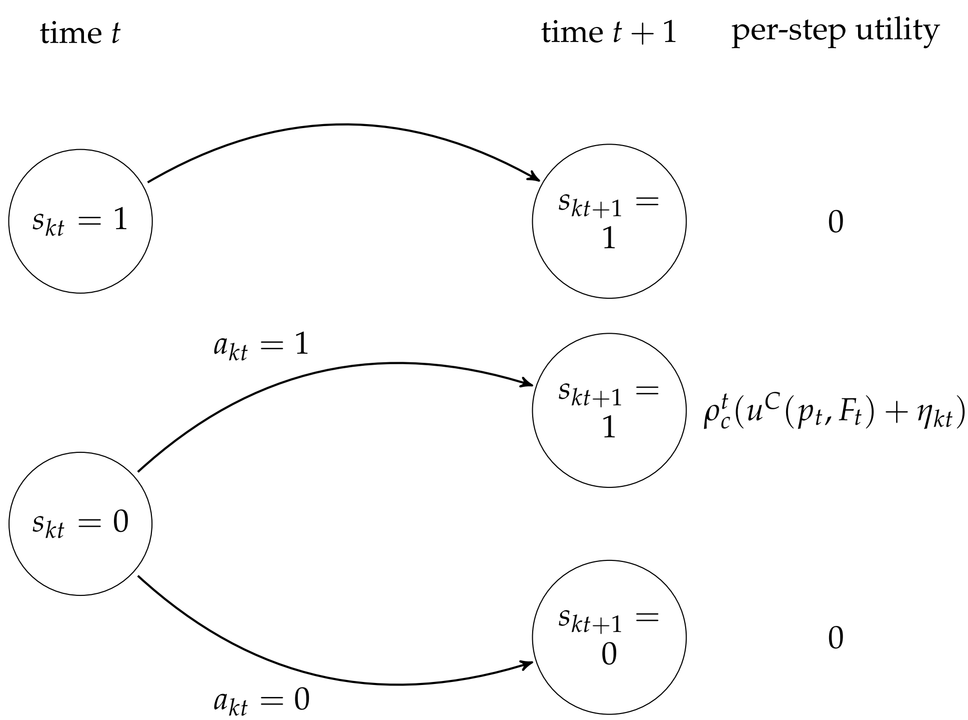


Figure 1.2: Conformist k dynamics and per-step utility.

tion problem whose output is either no adoption throughout the planning horizon, or an adoption date. Strategic consumer behavior is well-documented in marketing (Su, 2007) and in operations management (Wei and Zhang, 2018), and is significant in the context of introducing a new product (Besanko and Winston, 1990). One reason for postponing adoption can be the expectation of a drop in future prices, or to acquire more information about the product from the reviews posted by earlier adopters (Economist, 2009). These reviews are considered in the literature as a good proxy for WoM communications (Chevalier and Mayzlin, 2006).

Define by $f_t = \frac{1}{N}\left(\sum_{j=1}^{N^I} A_{jt} + \sum_{k=1}^{N^C} a_{kt}\right)$ the fraction of consumers who adopt the product at time t. The firm maximizes the following utility function:

$$U^f = \sum_{t=0}^{T-1} \rho_f^t \left(p_t f_t - c_m(m_t)\right) + s\left(G\left(T\right)\right), \tag{1.4}$$

where $s\left(G\left(T\right)\right)$ is an increasing salvage-value function and $0 < \rho_f \leq 1$ is a discounting rate.

Remark 1.1 *It should be noted that $A_{jt} = 1$ (resp. $A_{kt} = 1$) if and only if $S_{jt+1} - S_{jt} = 1$ (resp. $s_{kt+1} - s_{kt} = 1$). Thus,*

$$f_t = \frac{1}{N}\left(\sum_{j=1}^{N^I} S_{jt+1} + \sum_{k=1}^{N^C} s_{kt+1}\right) - \frac{1}{N}\left(\sum_{j=1}^{N^I} S_{jt} + \sum_{k=1}^{N^C} s_{kt}\right) = F_{t+1} - F_t.$$

We suppose that the firm pre-announces its price and advertising paths at the beginning of the planning horizon (Dasu and Tong, 2010). Aviv and Pazgal (2008) showed that in the presence of strategic consumers, pre-announcing prices is more profitable for the seller (up to 8.32% more profits) than is responsive pricing (see also Dasu and Tong, 2010). Examples of the implementation of pre-announced prices include Wanamaker's discount department store in Philadelphia, Pricetack.com, Tuesday Morning discount stores, Land's End Overstocks, Sam's club, Dress for Less, and TKTS ticket booths in London and New York City (Yin et al., 2009; Liu et al., 2019). The pre-announced advertising plan can be interpreted as the contract between the firm and a marketing agency defining the promotional activities to be implemented throughout the entire planning horizon.

We summarize the notation introduced above in Tables 1.1.

Table 1.1: Notation

Consumers		**Firm**	
Notation	Description	Notation	Description
N^I and N^C	Number of individualists and conformists	p_t	Price decision variable
A_{jt} and a_{kt}	Individualist and conformists adoption decision variables	m_t	Advertising decision variable
S_{jt} and s_{kt}	Individualists and conformists adoption state variables	G_t	Goodwill state variable
$\mathcal{I}^I_{jt}$ and $\mathcal{I}^C_{kt}$	Individualists and conformists information state variables	$\mathcal{I}^f_t$	Firm information state variable
U_{jt}, u_{kt}, U^I and u^C	Individualists and conformists deterministic utilities	U^f	Profit
ϵ_{jt} and η_{kt}	Individualists and conformists random utilities	h	Goodwill function
$f\epsilon$ and $f\eta$	P.d.f. of ϵ and η	M_p	Upper bound for price
$M\epsilon$ and $M\eta$	Upper bounds for ϵ and η	M_m	Upper bound for advertising
$N = N^I + N^C$	Number of consumers	c_m	Advertising cost function
F_t and f_t	Cumulative and instantaneous fraction of adopters	c	Advertising cost coefficient
Π^I and Π^C	Fraction of individualists and conformists	ρ_f	Discounting rate
π^I and π^C	Flow probability of individualists and conformists		
V^I and V^C	Value function of individualists and conformists		
L	Scalar constant		
ρ_c	Discounting rate		

1.3.1 A mean-field game

In making their adoption decision, consumers interact strategically with the firm and among themselves. We model this context as a game played à la Stackelberg with the firm acting as leader and consumers as followers.

Denote by $p_{0:T-1} = (p_0, \ldots, p_{T-1})$ and $m_{0:T-1} = (m_0, \ldots, m_{T-1})$ the firm's price and advertising strategies, respectively. We look for open-loop (global) Stackelberg solutions (Basar and Olsder, 1999), where the firm pre-announces its strategies $(p_{0:T-1}, m_{0:T-1})$ before the start of the game. The individualists react to these strategies and decide their optimal adoption time, and the conformists respond to the individualists' adoption rate while reacting to each others à la Nash. That is, the conformists play a non-cooperative game among each others, where a conformist's decision depends on the other conformists' decisions through the fraction of adopters. Therefore, the information states for the different players are as follows. Individualist j observes at time t her current state S_{jt}, the random utility ϵ_{jt}, the prices $p_{0:T-1}$ and goodwill profile $G_{0:T}$ and makes a decision. Hence, the information state of individualist j at time t is $\mathcal{I}^I_{jt} = \{S_{jt}, \epsilon_{jt}, p_{0:T-1}, G_{0:T}\}$. Conformist k observes her current state s_{kt}, the random utility η_{kt}, the prices $p_{0:T-1}$, the individualist fraction of adopters $\pi^{N^I}_{0:T} = \frac{1}{N^I}\sum_{j=1}^{N^I} S_{j0:T}$ and the current fraction of adopters F_t. Her information state is $\mathcal{I}^C_{kt} = \{s_{kt}, \eta_{kt}, p_{0:T-1}, \pi^{N^I}_{0:T}, F_t\}$. Since the firm pre-announces its strategies before the start of the game, its information state at time t is $\mathcal{I}^f_t = \{G_0\}$. As we shall see later, the MFG methodology allows the conformists to anticipate the fraction of adopters F_t, which will be dropped from their information state in the following sections.

To compute a Stackelberg equilibrium, one starts by determining the reaction functions of the followers to any announcement made by the leader. Typically in a hierarchical multistage game with many followers, one assumes that each follower's decision is affected by each of the other followers.[6] In our case, a con-

[6]To illustrate, if the leader is a regulator and the followers are firms in an industry competing in prices, then each firm would take into account the price sets by each of the competitors.

formist does not need to know who precisely are the consumers who already adopted the product; she only needs to know the fraction of adopters in the social system. Similarly, the firm only needs to know the aggregate market reaction function to its policies and not the specific reaction of each of the N consumers, where (recall) N is a very large number. Consequently, such information structure points out towards the theory of mean-field game, which provides a powerful methodology to analyze dynamic games involving a large number of weakly couple players.

The basic ideas of a MFG approach are as follows: It starts by considering an infinite number of players and capitalizes on the weak coupling and law of large numbers to anticipate the coupling term (mean-field). Indeed, the mean-field under the players' Nash strategies satisfies two coupled backward-forward equations. The first, a backward dynamic programming equation, describes a generic player's best response to the mean-field. The second, a forward Chapman-Kolmogorov equation, propagates the mean-field under the players' best responses to it. The advantage of the infinite population case is that it reduces the game between an infinite number of players to that between a generic player and the mean-field. This is to be contrasted with the finite number of players case alluded to it above, where each individual is solely able to manipulate the game, and a Nash equilibrium is characterized by a number of equations proportional to that of the players. The infinite population assumption, however, makes the equilibrium less robust in face of unilateral deviant behavior. In fact, when applied by a finite number of players, the mean-field strategies constitute an approximate Nash equilibrium. In particular, each player has a room to improve his profit by a unilateral deviation from the equilibrium. This improvement vanishes however as the number of players increases to infinity. Particularly, in a conventional Nash equilibrium, if a player changes her strategy, she either receives less payoff or maintains the same one. However, in a mean-field equilibrium, a change in the strategy might be profitable but the improvement is negligible for a large number of players. Thus, a unilateral deviation will either leads to a negligible profit or a lower payoff (see Huang et al., 2007 for further discussions). The approximate Nash equilibrium is called $\epsilon-$Nash (Huang et al., 2006, 2007) and is defined as follows:

Definition 1.1 *Let S be a set. For all $N \in \mathbb{N}$, define on the set S^N the utility function of agent k, $1 \leq k \leq N$, $J_k^{(N)}(s_1, \ldots, s_N)$. Let ϵ_N, $N \geq 0$, be a sequence of real numbers converging to 0 as $N \to \infty$. A strategy profile $\{s_i^*, i \in \mathbb{N}\}$ is called an ϵ_N-Nash equilibrium with respect to the utilities $J_k^{(N)}$, if for all N, for any $1 \leq i \leq N$ and any $s_i \in S$, we have $J_i^{(N)}(s_i, s_{-i}^*) \leq J_i^{(N)}(s_i^*, s_{-i}^*) + \epsilon_N$, where $s_{-i}^* = (s_1^*, \ldots, s_{i-1}^*, s_{i+1}^*, \ldots, s_N^*)$.*

This ϵ_N-Nash equilibrium provides the reaction function to the leader, who then solves her optimization problem. Mean-field game theory was developed in a series of papers by Huang et al. (2003, 2007, 2006), and independently by Lasry and Lions (2006a,b, 2007). It has found applications ranging from transportation, energy systems and smart grids (Kizilkale et al., 2019) to finance. The first mean-field game marketing model was introduced by Salhab et al. (2018), whereby a producer makes advertising investments to sway consumer choices in favor of its product. Later, Salhab et al. (2019) propose a dynamic marketing and pricing MFG model involving a large number of online review sensitive consumers. For a comprehensive introduction to MFG, see Caines et al. (2018).

In what follows, we first introduce our technical assumptions, which we assume to hold throughout the paper. Afterwards, we present the equilibrium results and the firm's problem.

Assumption 1.1 *The random utilities $\{\epsilon_{jt}, \eta_{kt}, 1 \leq j \leq N^I, 1 \leq k \leq N^C, 0 \leq t \leq T\}$ are independent. The probability density functions (p.d.f) of ϵ_{jt} and η_{kt} are respectively $f\epsilon$ and $f\eta$. These p.d.f's are uniformly bounded by a positive scalar M_f.*

Assumption 1.1 is standard in the discrete-choice models literature (Rust, 1996, 1987; McFadden, 1974). An interesting class of probability distributions considered in this literature are the extreme value distributions.

Assumption 1.2 *The function u^C is Lipschitz continuous in F_t, with Lipschitz constant equal to L.*

Assumption 1.3 *As the size of population N increases to infinity, the individualists' and conformists' fractions N^I/N and N^C/N converge to $\Pi^I > 0$ and $\Pi^C = 1 - \Pi^I > 0$, respectively.*

Assumption 1.4 *The parameters f_ϵ, f_η, Π^I, and Π^C are known by the firm and all the consumers.*

Assumptions 1.3, and 1.4 are standard in the MFG literature.

1.3.2 Equilibrium results

We develop in this section a solution to our diffusion game using the MFG methodology. The key feature of our model on which we rely is the form of interactions between the players. In particular, the consumers and firm interact only through the empirical distributions $\left(\pi_t^{N^I}, 1 - \pi_t^{N^I}\right)$ and $\left(\pi_t^{N^C}, 1 - \pi_t^{N^C}\right)$, where $\pi_t^{N^I} = \frac{1}{N^I}\sum_{j=1}^{N^I} S_{jt}$ and $\pi_t^{N^C} = \frac{1}{N^C}\sum_{k=1}^{N^C} s_{kt}$. Following the MFG methodology, we assume throughout this section an infinite number of consumers. It is consistent to suppose that the couplings $\pi_t^I = \lim_{N\to\infty} \pi_t^{N^I}$ and $\pi_t^C = \lim_{N\to\infty} \pi_t^{N^C}$ in the infinite population are deterministic for the following reason. If all the consumers optimally respond to deterministic π_t^I and π_t^C, then their states at any time t are independent, and the Law of Large Numbers insures that $\pi_t^{N^I}$ and $\pi_t^{N^C}$ converge respectively to the deterministic probabilities $\pi_t^I = P(S_{jt} = 1)$ and $\pi_t^C = P(s_{kt} = 1)$. As a result, in the limiting problem, the conformists interact with each others through the cumulative fraction of adopters $F_t = \lim_{N\to\infty} \frac{1}{N}\left(\sum_{j=1}^{N^I} S_{jt} + \sum_{k=1}^{N^C} s_{kt}\right) = \Pi^I\pi_t^I + \Pi^C\pi_t^C$.

The consumers, however, interact with the firm through the instantaneous fraction of adopters $f_t = F_{t+1} - F_t$.

The consumers' game

We suppose that the firm fixes and announces its strategies $(p_{0:T-1}, m_{0:T-1})$ before the start of the game. By the MFG methodology, we assume that the consumers' flows of probabilities $\pi^I_{0:T}$ and $\pi^C_{0:T}$ under Nash strategies are given for now. We show later that these flows always exist and can be computed by knowing f_ϵ, f_η, Π^I and Π^C. A generic individualist with information state $\mathcal{I}^I_t$ (we drop the index j to refer to a generic individualist in the infinite population) solves the following backward dynamic program (Bertsekas, 1995):

$$\begin{aligned} &V^I_t\left(\mathcal{I}^I_t(1,\epsilon)\right) = 0, \\ &V^I_t\left(\mathcal{I}^I_t(0,\epsilon)\right) = \max\left(U^I(p_t, G_t) + \epsilon, \rho_c \int V^I_{t+1}\left(,\mathcal{I}^I_{t+1}(0,\epsilon')\right) f_\epsilon(\epsilon') d\epsilon'\right). \end{aligned} \tag{1.5}$$

with $V^I_T = 0$. Here, $\mathcal{I}^I_t(S,\epsilon)$ refers to the realization of $\mathcal{I}^I_t$ when the vector (S_t, ϵ_t) takes the value (S,ϵ), and V^I_t is the optimal utility-to-go at time t. The individualist's optimal choice at time t is then,

$$A^*_t\left(\mathcal{I}^I_t(0,\epsilon)\right) = \begin{cases} 0, & \text{if } \epsilon \leq \epsilon^*_t\left(\mathcal{I}^I_{1t}(0)\right), \\ 1, & \text{if } \epsilon > \epsilon^*_t\left(\mathcal{I}^I_{1t}(0)\right), \end{cases} \tag{1.6}$$

where $\mathcal{I}^I_{1t} = \mathcal{I}^I_t \setminus \{\epsilon_t\}$, $\mathcal{I}^I_{1t}(S)$ refers to the realization of $\mathcal{I}^I_{1t}$ when the vector S_t takes the value S, and

$$\epsilon^*_t\left(\mathcal{I}^I_{1t}(0)\right) = -U^I(p_t, G_t) + \rho_c \int V^I_{t+1}\left(\mathcal{I}^I_{t+1}(0,\epsilon')\right) f_\epsilon(\epsilon') d\epsilon'. \tag{1.7}$$

It should be noted that once an individualist decides to adopt, she does not make any future decisions. For this reason, the optimal choice (1.6) is only defined on $\mathcal{I}^I_t(S,\epsilon) = \mathcal{I}^I_t(0,\epsilon)$. A generic conformist with information state $\mathcal{I}^C_t$ computes her best response to the cumulative fraction of adopters $F^a_{0:T}$ by solving the following backward dynamic program:

$$\begin{aligned}
&V^C_t\left(\mathcal{I}^C_t(1,\eta)\right) = 0,\\
&V^C_t\left(\mathcal{I}^C_t(0,\eta)\right) = \max\left(u^C(p_t,F_t)+\eta, \rho_c\int V^C_{t+1}\left(\mathcal{I}^C_{t+1}(0,\eta')\right) f_\eta(\eta')d\eta'\right),
\end{aligned} \tag{1.8}$$

with $V^C_t = 0$. The conformist's best response is

$$a^*_t\left(\mathcal{I}^C_t(0,\eta)\right) = \begin{cases} 0, & \text{if } \eta \le \eta^*_t\left(\mathcal{I}^C_{1t}(0)\right), \\ 1, & \text{if } \eta > \eta^*_t\left(\mathcal{I}^C_{1t}(0)\right), \end{cases} \tag{1.9}$$

where $\mathcal{I}^C_{1t} = \mathcal{I}^C_t \setminus \{\eta_t\}$, and

$$\eta^*_t\left(\mathcal{I}^C_{1t}(0)\right) = -u^C(p_t,F_t) + \rho_c\int V^C_{t+1}\left(\mathcal{I}^C_{t+1}(0,\eta')\right) f_\eta(\eta')d\eta'. \tag{1.10}$$

We now turn to the problem of finding consistent flows of probabilities $\pi^I_{0:T}$ and $\pi^C_{0:T}$, i.e., equal to the generic individualist and conformist's distributions under their best responses (1.6) and (1.9) to these flows. Since an individualist's policy (1.9) does not depend on the flow of probabilities, then a consistent π^I is the unique solution of the following forward Chapman-Kolmogorov equation (Durrett, 2010):

$$\pi^I_{t+1} = \pi^I_t + \left(1-\pi^I_t\right)\int_{\epsilon^*_t\left(\mathcal{I}^I_{1t}(0)\right)}^{\infty} f_\epsilon(\epsilon')d\epsilon', \qquad \pi^I_0 = 0. \tag{1.11}$$

This equation can be solved forward by knowing the firm's strategies. A consistent conformist's flow of probabilities $\pi^C_{0:T}$ must satisfy the following Chapman-Kolmogorov equation:

$$\pi^C_{t+1} = \pi^C_t + \left(1-\pi^C_t\right)\int_{\eta^*_t\left(\mathcal{I}^C_{1t}(0)\right)}^{\infty} f_\eta(\eta')d\eta', \qquad \pi^C_0 = 0, \tag{1.12}$$

where $\mathcal{I}^C_{1t}(0)$ depends on $\pi^C_{0:T}$ itself. Thus, a consistent flow $\pi^C_{0:T}$ is a fixed point of the map $\pi^C_{0:T} \mapsto \mathcal{L}(\pi^C_{0:T}, p_{0:T-1}, \pi^I_{0:T})$, where $\mathcal{L} = \mathcal{L}_2 \circ \mathcal{L}_1$, $\mathcal{L}_1 : [0,1]^{T+1} \times$

$[0, M_p]^T \times [0,1]^{T+1} \mapsto \mathbb{R}^T$ maps $(\pi^C_{0:T}, p_{0:T-1}, \pi^I_{0:T})$ to $\left(\eta^*_0\left(\mathcal{I}^C_{10}(0)\right), \ldots, \eta^*_{T-1}\left(\mathcal{I}^C_{1T-1}(0)\right)\right)$ defined by (1.10), and $\mathcal{L}_2 : \mathbb{R}^T \mapsto [0,1]^{T+1}$ maps $(\eta_0, \ldots, \eta_{T-1})$ to the probability flow $\pi^C_{0:T}$ generated by equation (1.12) with $\eta^*_t\left(\mathcal{I}^C_{1t}(0)\right)$ replaced by η_t, for $0 \le t \le T-1$. Unlike (1.11), equation (1.12) cannot be solved recursively since the threshold η^*_t depends on the entire trajectory $\pi^C_{0:T}$. Hence, the existence of solution is not trivial and is analyzed in Theorem 1.1 below.

Given the firm's strategy, the individualists react following a unique adoption rule given by (1.6). The fraction of adopters among the individualists is given then by the unique solution of (1.11). The conformists, however, may exhibit multiple adoption behavior given by (1.9). Each behavior corresponds to a fraction of adopters among the conformists given by a fixed point of $\mathcal{L}$. As a result, the conformists' Nash equilibria are totally determined by the fixed points of $\mathcal{L}$.

Intuitively, the monopolist pre-announces the pricing and advertising plan upon the launch of new product first. Next, the firm's mass media campaign influences the individualists who are more in touch with innovations. The individualists' adoption entices those consumers who tend to conform with social norms. The social contagion is not only exerted from the individualists rather the conformists per se also contribute in cascading pattern across the potential market. In doing so, each conformist anticipates the aggregate adoption behavior from all consumers and responds optimally which collectively yields mean-field equilibrium.

In the following, we denote by $|x_s|_s = \max_s |x_s|$, where the maximum is taken over the domain of s. We now state the main result of this section which asserts that there always exists a Nash equilibrium for an infinite number of consumers.

Theorem 1.1 *The following statements hold:*

1. *For all $\pi^C_{0:T}$ and $\bar{\pi}^C_{0:T}$ in $[0,1]^{T+1}$, we have*

$$\left|\mathcal{L}(\pi^C_{0:T}, p_{0:T-1}, \pi^I_{0:T})_t - \mathcal{L}(\bar{\pi}^C_{0:T}, p_{0:T-1}, \pi^I_{0:T})_t\right|_t \le L_{\mathcal{L}} |\pi^C_t - \bar{\pi}^C_t|_t,$$

where $L_{\mathcal{L}} := M_f L \rho_c \Pi^C \sum_{t=0}^{T-1} 2^{T-t-1} \frac{1-\rho_c^{T-t+1}}{1-\rho_c}$.

2. *There exists at least one fixed point of $\mathcal{L}$. Equivalently, the limiting conformists' game has always a Nash equilibrium given by* (1.9) *for a fixed point $\pi_{0:T}^C$ of $\mathcal{L}$.*

3. *The mean field strategies* (1.9)*, when applied by a finite number N of consumers, constitute an ϵ_N-Nash equilibrium (see Definition 1.1), where*

$$|\epsilon_N| \leq L\rho_c \left(\frac{1}{N} + \sqrt{\frac{4}{N} + \left(\frac{N^I}{N} - \Pi^I\right)^2 + \left(\frac{N^C}{N} - \Pi^C\right)^2} \right). \tag{1.13}$$

Proof. All proofs can be found in appendix section.

In the finite population, the Nash equilibrium is characterized by a set of $2N$ coupled equations. N backward dynamic program equations that describe the consumers' best responses, and N forward Chapman-Kolmogorov equations that propagate the consumer states' distributions under their best responses. When the number of consumers is large, solving for a Nash equilibrium becomes computationally intractable. The MFG methodology leverages the game's symmetries to develop simple solutions described by only 4 equations (1.5), (1.8), (1.11), and (1.12). These solutions constitute a Nash equilibrium for the infinite population. When applied by a finite number of players, they induce a loss of performance in that the consumers can profit by unilateral deviation. According to the third point of Theorem 1.1, this loss of performance becomes negligible in large populations.

The firm's problem

As discussed in the previous section, the set of potential consumers' reactions to the firm's strategy is determined by the set of fixed points of $\mathcal{L}$. In order to design an optimal strategy, the firm needs to know how the consumers select a Nash equilibrium given her prices and advertising investments. To this end, we make the following assumption.

Assumption 1.5 *The conformists select a Nash equilibrium according to a predefined deterministic mechanism, which is given by a continuous "selection" function* $S : [0, M_p]^T \times [0,1]^{T+1} \mapsto [0,1]^{T+1}$*, that maps* $(p_{0:T-1}, \pi^I_{0:T})$ *to a fixed point* $\pi^C_{0:T}$ *of* $\mathcal{L}$*.*

According to Assumption 1.5, the firm anticipates the consumers' reaction to its strategies. As a result, an optimal pricing and advertising strategy is a solution to the following optimization problem:

$$\begin{aligned}
\max_{p_{0:T-1}, m_{0:T-1}} \sum_{t=0}^{T-1} & \rho_f^t \left(p_t f_t - c_m(m_t)\right) + s\left(G\left(T\right)\right), \\
\text{s.t. } & p_{0:T-1} \in [0, M_p]^T, \\
& m_{0:T-1} \in [0, M_m]^{dT}, \\
& \pi^C_{0:T} = S(p_{0:T-1}, \pi^I_{0:T}), \\
& f_t = \Pi^I(\pi^I_{t+1} - \pi^I_t) + \Pi^C(\pi^C_{t+1} - \pi^C_t),
\end{aligned} \tag{1.14}$$

where $\pi^I_{0:T}$ satisfies (1.11). Under Assumption 1.5, the continuity of c_m, and the compactness of the firm's strategy set, problem (1.14) has at least one optimal solution. Notable, too, is the necessity of development of a numerical scheme in order to determine the firm's optimal marketing strategies. More specifically, in appendix 1.6.2, we show that there exists a continuous selection function necessary for establishing the existence of firm's optimal strategies and further provide a numerical scheme to illustrate these strategies.

1.4 Numerical simulations

In this section, we illustrate our model with some numerical examples and provide insights on the interplay between the diffusion process, consumers behavior, and firm's marketing strategies. In what follows, we specify the individual level consumer decision process as well as firms' decision problem, and start by determining the marketing strategies and penetration curves for each segment and aggregate market in a benchmark case. Next, we carry out sensitivity analysis with respect to the proportion of individualists to conformists, WoM, goodwill, and price parameters. Afterwards, we examine how the results differ in two cases. In the first case, we add a goodwill-dependent salvage value to the firm's optimization problem, and next suppose that consumers are myopic. The first case captures the impact of future benefits on the marketing strategies implemented during the current planning horizon. The second case allows us to see how the new product diffusion and marketing strategies vary with consumer's type (myopic or strategic).

1.4.1 Benchmark case

We consider a binary discrete choice model by which the consumers decide whether to buy the new product or not, and if yes, when to do so. As in Keeney and Raiffa (1993), we assume that the deterministic components in (1.2) and (1.3) of consumer utilities are additive, that is,

$$\begin{aligned} U^I(p_t, G_t) &= k_p^I U_p(p_t) + k_G U_G(G_t) \\ u^C(p_t, F_t) &= k_p^C u_p(p_t) + k_F u_F(F_t) \end{aligned} \tag{1.15}$$

where k_p^I, k_p^C, k_G and k_F are scaling coefficients that represent the significance of each uni-attribute utility function in the decision process. We consider linear price utilities, i.e., $U_p(p_t) = A^I - \beta^I p_t$ and $u_p(p_t) = A^C - \beta^C p_t$, and exponential forms utility with respect to goodwill and WoM, i.e., $U_G(G_t) = B^I - \alpha^I \exp(-r^I G_t)$ and $u_F(F_t) = B^C - \alpha^C \exp(-r^C F_t)$. Linear and exponential utilities are widely adopted in the literature (e.g., Chatterjee and Eliashberg (1990)) and capture constant risk-aversion, which was supported by empirical studies in marketing literature (e.g., Hauser and Urban (1977)). Without loss of generality, we assume that k_p^I, k_p^C, k_G and k_F are equal to 1, and A^I, A^C, B^I and B^C are equal to 0. We suppose that the random utilities ϵ_{it}^I and η_{it}^C are uniformly distributed on the unit interval $[0, 1]$. We also would like to highlight here that if the random component, which reflects the heterogeneity among adopters, is not present at all, then all individualists adopt in the same period at which the price and goodwill gives the highest utility across all purchasing occasions. All conformists will adopt one period later because the WoM effect is the highest at that period. We believe that this case lacks interest as consumers have typically heterogenous taste over performance/quality of innovation, and accordingly adoption timing. Alternatively, if the random (positive) component turned out to be larger, then the consumers receive relatively a higher utility and hence the adoption accelerates. This result is intuitive since the consumers consider the trade-off between all the components that affect their utility and if one with positive effect (random component) gets larger, then it would be less likely that consumers get a negative utility and consequently we have faster adoption rate.

The goodwill dynamics are given by a discrete time version of Nerlove-Arrow's model (Nerlove and Arrow, 1962), that is,

$$G_{t+1} = m_t + (1-\gamma)G_t, \quad G(0) = G_0, \tag{1.16}$$

where $0 < \gamma < 1$ is the decay rate of goodwill stock and G_0 the initial goodwill. The advertising cost is assumed quadratic and equal to $c_m(m_t) = \frac{1}{2}cm_t^2$, with $c > 0$. The assumption of quadratic cost function for advertising is widely adopted in the literature (e.g., Buratto et al., 2006; Farshbaf-Geranmayeh and Zaccour, 2021). The values of the different parameters in the benchmark case are given in Table 1.2. According to the literature, the percentages of individualists Π^I and conformists Π^C depend on the type of product. Based on empirical studies for 11 durable products, Mahajan et al. (1990) adopt Bass framework and report that the relative impact of external influences to internal ones can range nearly from 1% to 20%. However, Van den Bulte and Joshi (2007) argue that the proportion of individualists to conformists can vary in large intervals from zero to 1 depending on the type of innovation. For instance, for a low-risk innovation, the size of conformists in a population can be relatively small. In the benchmark case, we consider the percentage of individualists to be 30% given the salient presence of imitative behavior in social system shown in the literature (e.g., Steffens and Murthy (1992) and Mahajan et al. (1990)).

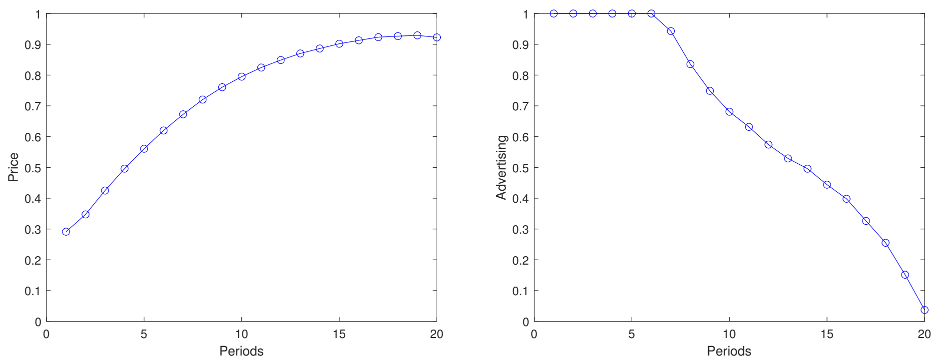
Table 1.2: Benchmark

Consumer's parameters		**Firm's parameters**	
Parameter	Description	Parameter	Description
$\alpha^I = 0.9$	Scaling constant	$c = 0.01$	Advertising cost coefficient
$\alpha^i = 0.7$	Scaling constant	$\gamma = 0.05$	Decay rate
$\beta^I = 0.6$	Individualists' price sensitivity	$G_0 = 1$	Initial goodwill
$\beta^i = 0.6$	Conformists' price sensitivity	$\rho_c = 0.9$	Consumers' discounting rate
$r^I = 0.1$	Goodwill sensitivity	$\rho_f = 0.9$	Firm's discounting rate
$r^C = 1$	WoM sensitivity		
$\Pi^I = 0.3$	Fraction of individualists		
$\Pi^C = 0.7$	Fraction of conformists		
$\pi_0^I = 0$	Initial individualist's flow probability		
$\pi_0^C = 0$	Initial conformist's flow probability		

Figures 1.3(a) and 1.3(b) report the pricing and advertising strategies in the benchmark case. Our results show that the monopolist adopts a penetration pricing strategy. This strategy is in response to consumers forward-looking behavior that can potentially cause adoption delay if future deals become more appealing. This is to be contrasted to the literature that widely finds skimming strategy for myopic consumers unless the WoM communications are strong where penetration or mixed strategies are recommended (e.g., Kalish, 1983, 1985; Horsky, 1990).[7] A few studies find that penetration pricing strategy is optimal for innovative products only when social influences are absent (Nair, 2007; Li, 2019), however, we show that the marketer maintains this strategy under presence of social influences. Indeed, the intensity of such pricing strategy depends significantly on the consumer heterogeneity and social influences, which is shown in Figures 1.5(a) and 1.9(a). Figure 1.3(b) shows that advertising expenditures monotonically decrease over time, a result often obtained in the literature (see the survey in Huang et al., 2012). One conclusion here is that the firm behaves in the same way

[7]Note, however, that we do not account for cost learning in our model, which typically also leads to a decrease in price over time.

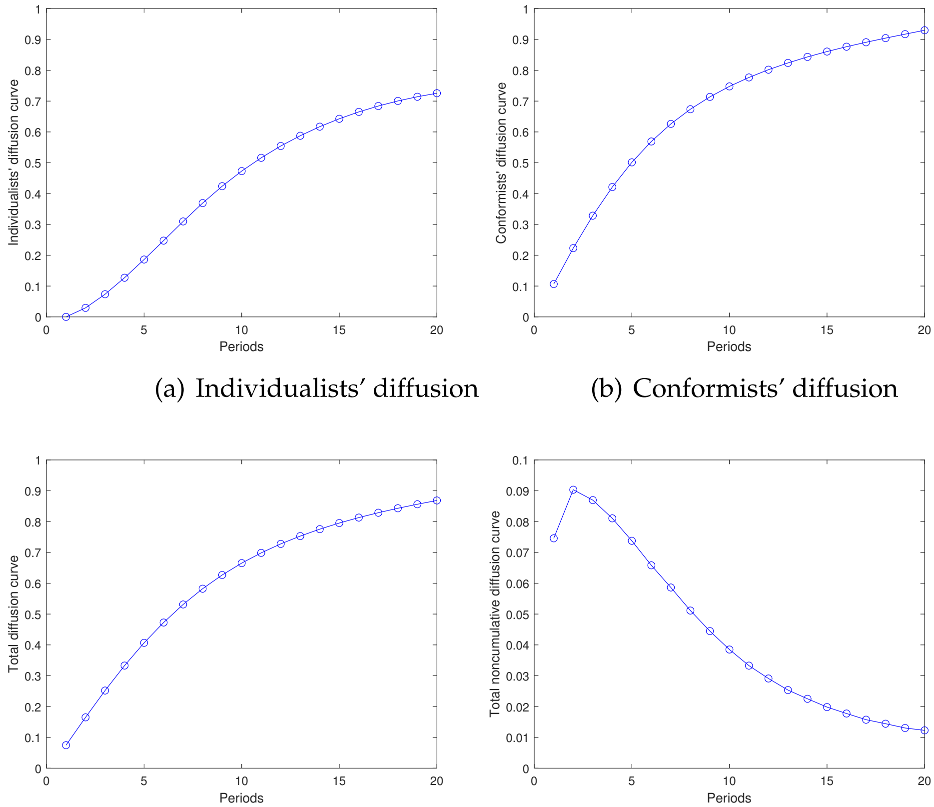
when we retain an individual-based diffusion model with strategic consumers. In fact, the rationale for advertising heavily at early stages remains the same, that is, to stimulate demand from individualists, which facilitates social contagion through WoM, and consequently incentivizes conformists to adopt the product.



(a) Pricing strategy (b) Advertising strategy

Figure 1.3: Marketing strategies in the benchmark case

Figures 1.4(a) and 1.4(b) exhibit the penetration curves in the individualists and conformists segments, respectively, and Figure 1.4(c) depicts the total penetration curve. While the general finding in the literature is an S-curve for penetration and a bell-shaped noncumulative distribution, these figures (and those to come later in the sensitivity analysis) show that different adoption curves can materialize, depending on consumer's response to external and internal influences as well as the composition of segments. In the benchmark case, we obtain a concave diffusion curve. The individualists' diffusion curve has an S-curve form due to the trade-off between goodwill effect and saturation effect over time, where the former has a function similar to WoM for the conformists (see Figure 1.4(a)). The early diffusion for this segment is slowly increasing given the low initial stock of goodwill till the point that aggressive advertising campaign accelerates the individualists' diffusion. This trend continues until the saturation effect overcomes goodwill effect where the number of individualists not having



(a) Individualists' diffusion (b) Conformists' diffusion

(c) Total diffusion (d) Noncumulative diffusion

Figure 1.4: Penetration curves in the benchmark case

adopted yet starts declining. Additionally, the individualists diffusion curve is consistent with Van den Bulte and Joshi (2007) findings, which states that the magnitude of adoption does not decrease monotonically.

The trade-off between WoM and saturation effects act differently in our framework. The forward-looking conformists anticipate the future outcomes and respond optimally to the mass adoption behavior of players. The individualists diffusion path is a priori in the conformist's intertemporal problem. Hence, the conformist would focus on how to respond to mean-field equilibrium and firm's pricing strategy. The conformists' strategic behavior favors early adoption more rapidly at early stages since it would not only serve to reinforce the social contagion and its ripple effects on diffusion process, but also to allow them to enjoy the low initial prices. This induces a concave penetration curve as shown in Figure 1.4(b). This result can explore the role of social influences in the market depending on the consumers buying motives and behaviors. Comparatively, the individualists adopt more slowly since the firm's brand image can not be built over a short period of time and requires gradual investments.

1.4.2 Sensitivity analysis

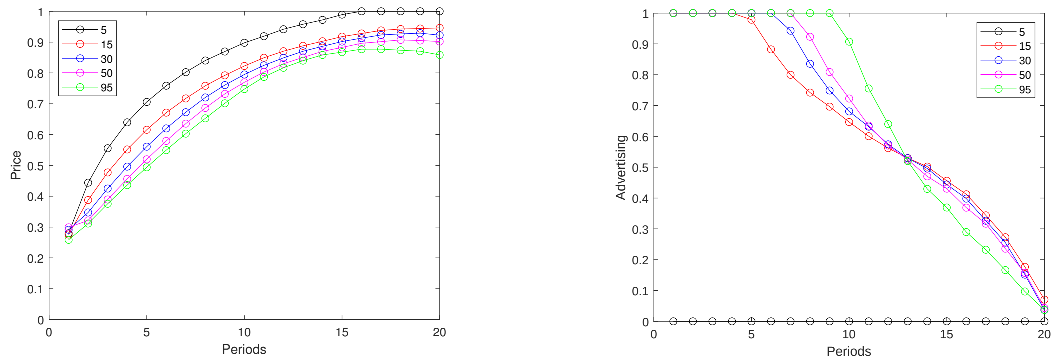
In this section, we run a series of sensitivity analysis to assess the impact of some parameter values on the results.

Type of consumers

One important feature in our model is the distinction between conformists and individualists consumers. In the benchmark case, the fraction of individualists was set equal to 30%. In Figures 1.5(a) and 1.5(b) we provide the pricing and advertising trajectories for the following fractions of individualists: 5%, 15%, 30%, 50% and 95%, that is, two below the benchmark value and two above it. For instance, in Figure 1.5(a), the black color represents the pricing regime when 5% of consumers are individualists and 95% of them are conformists.

Figure 1.5(a) shows that a penetration pricing strategy remains the optimal choice for the firm under various market configurations. However, this pricing regime is more aggressive when the role of social influences measured by fraction of individualists to conformists is more prevalent. This suggests that the firm can exploit better the penetration pricing scheme when the peer-induced adoption is salient in the social system which can accelerate the diffusion as discussed already in the benchmark case and improve the firm's discounted profit.

From Figure 1.5(b), we see that the shape of the advertising trajectory is the same for all considered configurations of the two segments, that is, the firm starts by advertising heavily in the early periods and decreases its expenditures over

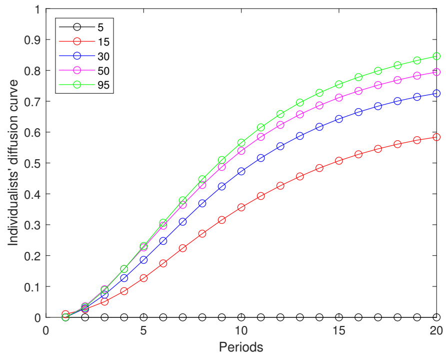


(a) Pricing strategy (b) Advertising strategy

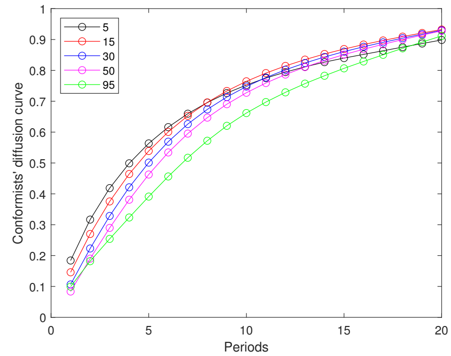
Figure 1.5: Marketing strategies for different fraction of individualists

time. The firm stops investing in the advertising when the fraction of individualists falls below a certain threshold. The reason is that advertising only targets individualists and the firm adapts its effort to the importance of their share in the society/market. Throughout the late stages, the marketer decreases the advertising faster as fraction of individualists increases given relatively more appealing prices.

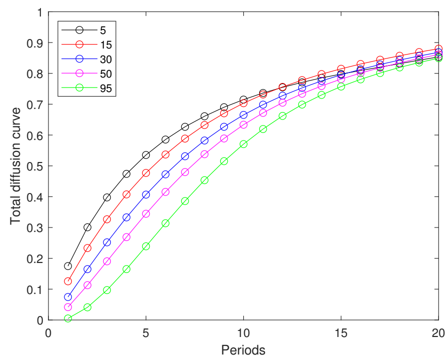
The three panels in Figure 1.6 exhibit the adoption rate over time in the individualists and conformists segments and in the whole market, respectively. We can make the following three remarks: First, depending on the segments compositions, the adoption curves can be S-shaped or concave, with the total penetration curve becoming more S-shaped when the proportion of individualists to conformists is increased. These results highlight the importance of accounting explicitly for the two segments and their different buying motives in understanding the adoption dynamics. In particular, when individualists segment is small, their social contagion role through WoM communication is of lesser importance. In fact, here conformists themselves are the main contributor to this contagion. Second, if the planning horizon is long enough, as here with 20 periods, the final



(a) Individualists' diffusion



(b) Conformists' diffusion



(c) Total diffusion

Figure 1.6: Penetration curves for different fraction of individualists

penetration percentage is almost the same for total diffusion across all configurations. However, it takes lower value in individualists diffusion curve for cases involving too low fraction of individualists. When this fraction is below a certain threshold, the final penetration rate in individualists segment drops to zero. Recall that in this case (of low fraction of individualists), the firm is investing little or no effort in advertising. Consequently, the goodwill, which is a main driver of individualists demand, is plateauing at a low level. Third, the presence of conformists decreases the penetration rate for the individualists, whereas the presence of individualists decelerates the adoption rate for the conformists. The reason is that the firm is less motivated to spend in the advertising when the

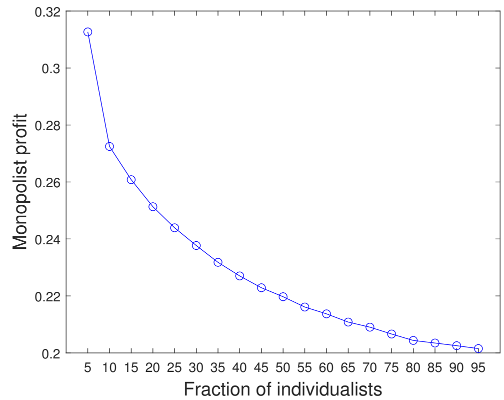
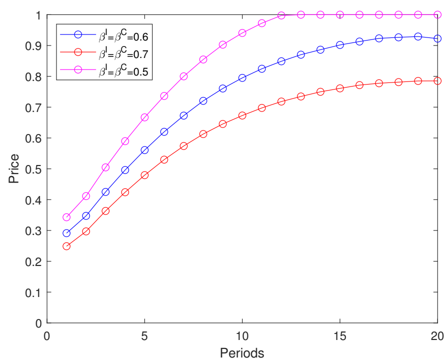
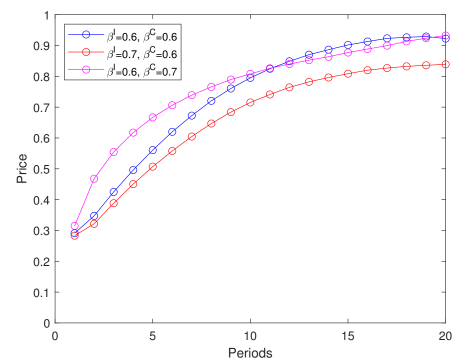


Figure 1.7: Monopolist profit for different compositions of market segments



(a) Pricing strategy

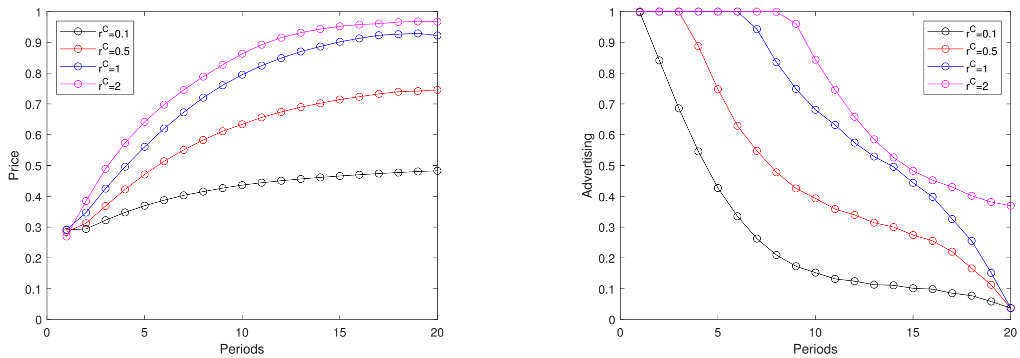


(b) Pricing strategy

Figure 1.8: Pricing strategies under different price sensitivities

conformist behavior is more common among the consumers, whereas the effect of social contagion becomes less significant when the individualists are the main contributors in spreading the innovation in the market since they adopt with a slower rate.

Finally, we look at the impact of segments composition on the firm's profit. Figure 1.7 shows that the total profit is monotonically decreasing with the fraction of individualists. This results from the prominent role of social influences in markets with large fraction of conformists and the firm adopting more aggressive penetration pricing strategy. On the other hand, when the fraction of individu-



(a) Pricing strategy (b) Advertising strategy

Figure 1.9: Marketing strategies under different WoM sensitivities

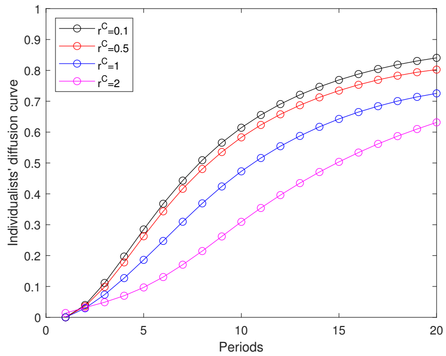
alists is sufficiently low, the firm needs to advertise less, and counts on WoM communications to stimulate the diffusion process (see, for example, the 5 case in Figure 1.6(b)) which ultimately improves its performance.

Price, goodwill and WoM coefficients

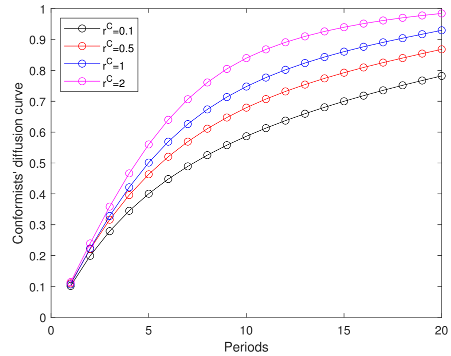
Recall that the deterministic parts of the utility functions in the individualists and conformists segments are given, respectively, by

$$U^I(p_t, G_t) = -\left(\beta^I p_t + \alpha^I e^{-r^I G_t}\right),$$
$$u^C(p_t, F_t) = -\left(\beta^C p_t + \alpha^C e^{-r^C F_t}\right).$$

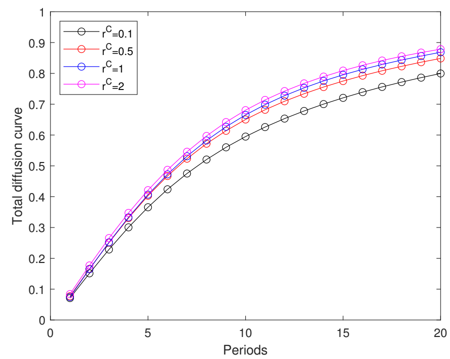
In what follows, we assess the impact of varying the price and WoM sensitivity parameters on the results. As expected, increasing consumer's price sensitivity (higher β) leads to a lower price (see Figure 1.8(a)). Note that for all considered values of β, the pricing strategy remains of the penetration type, which is reminiscent to our assumption that consumers are strategic. In what follows, we assess the impact of varying the price and WoM sensitivity parameters on the results. As expected, increasing consumer's price sensitivity (higher β) leads to a lower price (see Figure 1.8(a)).



(a) Individualists' diffusion



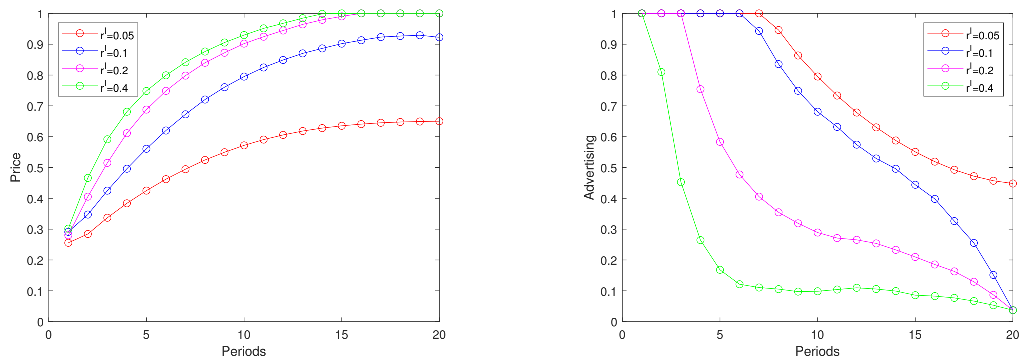
(b) Conformists' diffusion



(c) Total diffusion

Figure 1.10: Penetration curves under different WoM sensitivities

One important insight is that the intensity of penetration pricing strategy critically depends on how much the consumers are prone to the social contagion in the market. The larger r^C, the larger is the conformists' marginal utility of adoption, independently of the price level. This implies that the firm can afford to increase its price without much damage to demand. Further, to benefit from higher marginal contagion effect, the firm invests more in advertising (when r^C is higher) to increase buying by individualists who influence adoption by conformists. The impact of varying r^C on adoption over time is reported in Figures 1.10(a) and 1.10(b). A higher WoM sensitivity decreases individualists penetration curve but increases the conformists ones. The firm adopts more aggressive

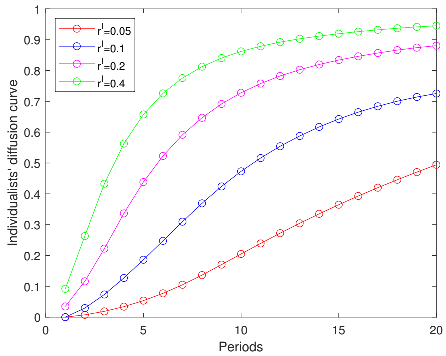


(a) Pricing strategy (b) Advertising strategy

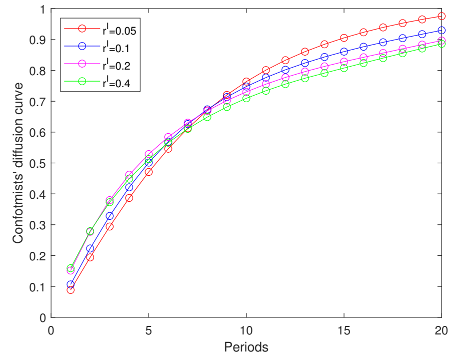
Figure 1.11: Marketing strategies under different goodwill sensitivities

pricing policy under strong social influences which decelerates the individualists diffusion curve and decreases its final penetration rate. The firm, however, mitigates the damage on individualists' demand with higher investments in the advertisement. The opposite effects of r^C on individualists and conformists penetration curves implies an almost negligible effect on total diffusion (see Figure 1.10(c)). This illustrates the significance of accounting for consumer heterogeneity and its different implications on consumers behavior.

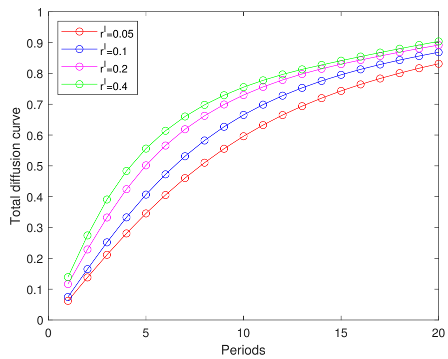
Figures 1.11(a) and 1.11(b) show marketing strategies when individualists have different sensitivities towards goodwill effect. Like for high WoM coefficient, the firm can exploit the individualists' high marginal utility of adoption and charge higher price while maintaining the demand when individualists are more sensitive to goodwill. However, it can accelerate adoption rate with lesser advertising investments by relying on impact of high goodwill sensitivity that maintains the same final penetration rate. Figures 1.12(a) and 1.12(b) show that penetration rate for individualists can significantly increase as they become more sensitivity to the firm's brand image. However, this comes at the expense of lower conformists penetration rate given the higher prices.



(a) Individualists' diffusion



(b) Conformists' diffusion

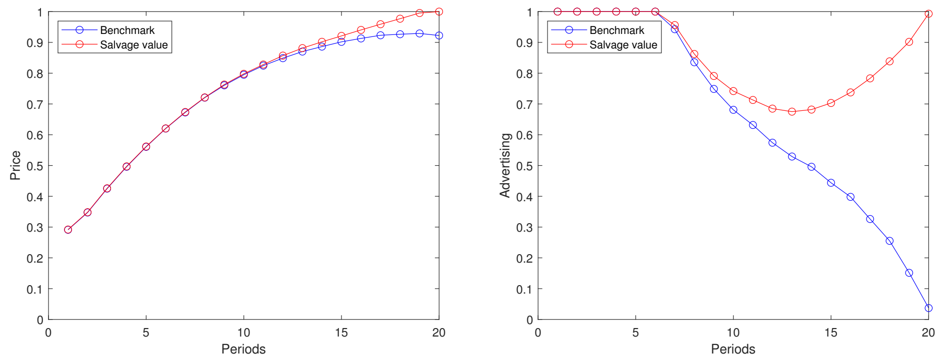


(c) Total diffusion

Figure 1.12: Penetration curves under different goodwill sensitivities

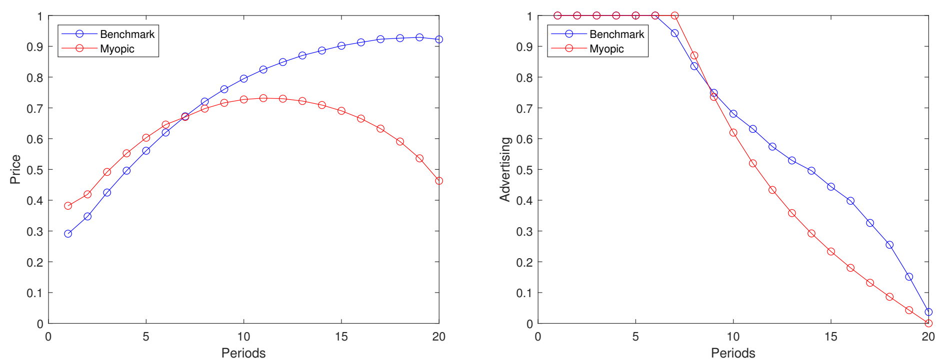
1.4.3 Salvage value

All results so far have been obtained under the assumption that the firm does not account for any potential revenue after the end of the planning horizon. One reported impact of such assumption in the dynamic advertising models literature is the monotonic decline over time of advertising spending, reaching eventually zero at the last period (see the survey in Huang et al., 2012). To verify this result in our context, we added a linear function of goodwill at terminal time to the firm's payoff function, given by $s(G(T)) = bG(T)$, where b is a positive parameter that is assumed to be equal to 0.01. As we can clearly see from Figure 1.13(a), having a positive salvage value does not have any significant impact on the pricing



(a) Pricing strategy (b) Advertising strategy

Figure 1.13: Marketing strategies with and without salvage value



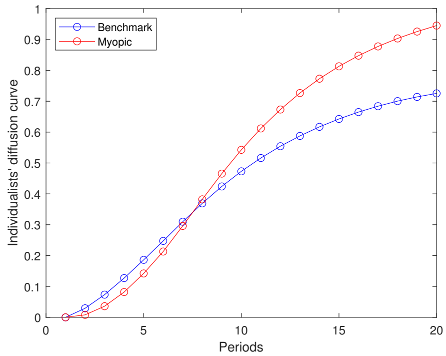
(a) Pricing strategy (b) Advertising strategy

Figure 1.14: Marketing strategies in the myopic case

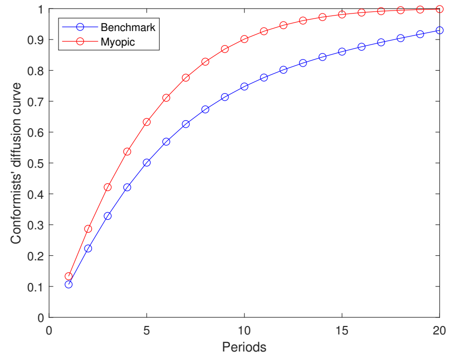
strategy. However, the advertising strategy is affected as the firm reverts towards the end of the planning horizon the decline in its advertising effort and increases expenditures sharply. In fact, the firm wants to raise its goodwill, i.e., invests in its future business. In short, the result is nothing but surprising.

1.4.4 Myopic case

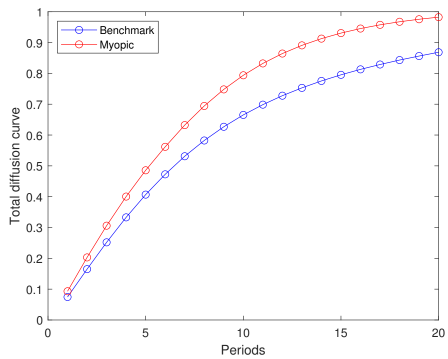
Up to now, we assumed that the consumers are strategic (or forward-looking), that is, they solve an inter-temporal optimization problem to decide whether and



(a) Individualists' diffusion



(b) Conformists' diffusion



(c) Total diffusion

Figure 1.15: Penetration curves in the myopic case

when to adopt the product. In this section, we consider the case where the consumers are myopic, which means that, at each period, they solve a static optimization problem and adopt at the first occasion the expected utility turns out to be positive. By comparing the results to the benchmark case, we can shed a light on the impact of myopia on the firm's strategies and outcome and on the adoption curves in both market segments.

The pricing strategy is not monotone anymore. The firm starts by implementing a penetration strategy followed by a skimming one (see Figure 1.14(a)). The marketer's penetration pricing strategy along with aggressive investments in advertising campaign favors WoM communications and goodwill effect till the point that combined WoM and goodwill effects dominate saturation one. The marketer then begins to cream skim the remaining untapped market. The penetration-skimming strategy is not optimal when consumers are forward-looking since they can delay their adoption for appealing future prices that can decelerate the magnitude and rate of diffusion at both segments. The prices are mainly lower in myopic case that makes a relatively lesser advertising more viable. Indeed, the marketer aims to achieve a higher penetration rate in both segments under these strategies.

A series of studies have reported that strategic consumers, who typically wait for bargains before buying, affect negatively a seller's profit. This is also the result we reach here. Whereas the firm makes a profit of 0.2377 when consumers are strategic, it realizes a profit of 0.2980 when consumers are myopic, that is an increase of 25%. The reason is clear from the discussion above, i.e., the firm exploits myopic consumers by offering better deals and therefore focusing on the attracting more consumers in the market.

1.5 Conclusion

This paper is an exploratory first step in understanding adoption dynamics in a context characterized by the presence of strategic consumers, who are either individualists and conformists, having different adoption drivers. The equilibrium among consumers is shaped by social contagion and the pricing and advertising of the firm. To the best of our knowledge, this is the first attempt to integrate in the same model forward-looking consumers, two-segment market structure, and pricing and advertising strategies in a fully dynamic framework. Also, it is the first application of mean-field game, an area witnessing an astonishing growth, to new product diffusion.

The main takeaways of this study are as follows:

1. The firm adopts a penetration pricing strategy when consumers are strategic. This result holds for all parameter constellations, however, the level of penetration depends on the consumer heterogeneity and the social influences. Interestingly, a mix of an increasing pricing strategy followed by a decreasing one materialize when consumers are myopic.

2. Advertising strategy is highly intuitive: invest heavily in early stages to build the goodwill, incentivize individualists to adopt early, and trigger a social contagion effect.

3. The penetration curves can take different forms such as S-curve and concave depending on the mixture of individualists and conformists in the social system. Moreover, the presence of each consumer segment affects negatively the adoption tendency of the other segment.

4. The firm earns higher profit when the market tends to be essentially populated by consumers with conformists behavior.

5. Individualists' and conformists' adoption processes are different, which fully justify our two-segment model.

6. The numerical results have been shown to be largely robust to variations in the parameter values.

As in any modelling work, we made some assumptions that should be relaxed in future work. Let us first consider two long-shot extensions. First, we assumed absence of competition. The new product diffusion literature is replete of monopoly models, which signals a methodological difficulty in introducing formally competition. This difficulty is huge in the context of MFG, where a theory of multi-stage equilibria are yet to be developed. Second, we assumed that the firm preannounces its strategy from the outset. It is definitely a welcome move to attempt to consider feedback (state-dependent) strategies. Again, the characterization of feedback Stackelberg equilibria is still out of reach for the moment.

Other possible avenues for future works include (i) an attempt to have both strategic and myopic consumers in the market, and (ii) estimation of the parameter distributions (see Assumptions 1.4) and segment sizes from the game's output, for example, adoption rate data. Moreover, consumers might have heterogeneous susceptibility towards WoM (Lee et al., 2021) and hence applying such concept can explore the role of consumer heterogeneity even more in this framework.

1.6 Appendix

In this appendix, we provide the proofs along with Lemma 1 in subsection 1.6.1, and selection functions and numerical scheme in subsection 1.6.2.

1.6.1 Proofs

Lemma 1 *The following statements hold:*

1. *We have for all $\epsilon \in [0, M_\epsilon]$ and $\eta \in [0, M_\eta]$,*

$$\begin{aligned} \left| V_t \left(\mathcal{I}_t^I(0,\epsilon) \right) \right| &\leq |U^I(p,G)|_{p,G} + M_\epsilon, \\ \left| V_t \left(\mathcal{I}_t^C(0,\eta) \right) \right| &\leq |u^C(p,F)|_{p,F} + M_\eta. \end{aligned} \tag{1.17}$$

2. *For all $\pi_{0:T}^C$ and $\bar{\pi}_{0:T}^C$ in $[0,1]^{T+1}$, we have,*

$$\left| V_t^C \left(\mathcal{I}_t^C(0,\eta) \right) - V_t^C \left(\bar{\mathcal{I}}_t^C(0,\eta) \right) \right|_\eta \leq L \sum_{\tau=t}^{T} \rho_c^{\tau-t} |F_\tau - \bar{F}_\tau|, \tag{1.18}$$

$$\left| \eta_t^* \left(\mathcal{I}_{1t}^C(0) \right) - \eta_t^* \left(\bar{\mathcal{I}}_{1t}^C(0) \right) \right| \leq L\rho_c \sum_{\tau=t}^{T-1} \rho_c^{\tau-t} |F_\tau - \bar{F}_\tau| \tag{1.19}$$

where F_t and $\mathcal{I}_t^C$ correspond to $\pi_{0:T}^C$, and $\bar{F}_t$ and $\bar{\mathcal{I}}_t^C$ correspond to $\bar{\pi}_{0:T}^C$.

Proof of Lemma 1. The first point is a direct consequence of (1.5) and (1.8). We prove the second point using the equality $\max(a,b) = (a+b+|a-b|/2)$. We have

$$\begin{aligned} V_t^C\left(\mathcal{I}_t^C(0,\eta)\right) = \frac{1}{2}\left(u^C(p_t,F_t)+\eta+\rho_c\int V_{t+1}^C\left(\mathcal{I}_{t+1}^C(0,\eta')\right)f_\eta(\eta')d\eta'\right) \\ +\frac{1}{2}\left|u^C(p_t,F_t)+\eta-\rho_c\int V_{t+1}^C\left(\mathcal{I}_{t+1}^C(0,\eta')\right)f_\eta(\eta')d\eta'\right|. \end{aligned} \tag{1.20}$$

Hence, for all $\pi_{0:T}^C$ and $\bar{\pi}_{0:T}^C$ in $[0,1]^{T+1}$,

$$\left|V_t^C\left(\mathcal{I}_t^C(0,\eta)\right)-V_t^C\left(\bar{\mathcal{I}}_t^C(0,\eta)\right)\right|_\eta \leq L|F_t-\bar{F}_t|+\rho_c\left|V_{t+1}^C\left(\mathcal{I}_{t+1}^C(0,\eta)\right)-V_{t+1}^C\left(\bar{\mathcal{I}}_{t+1}^C(0,\eta)\right)\right|_\eta.$$

(1.18) follows by induction, and (1.18) and (1.10) imply (1.19) ■

Proof of Theorem 1.

1. Following (1.12) and (1.19), we get

$$\begin{aligned} &\left|\mathcal{L}(\pi_{0:T}^C,p_{0:T-1},\pi_{0:T}^I)_{t+1}-\mathcal{L}(\bar{\pi}_{0:T}^C,p_{0:T-1},\pi_{0:T}^I)_{t+1}\right| \leq \\ &2\left|\mathcal{L}(\pi_{0:T}^C,p_{0:T-1},\pi_{0:T}^I)_t-\mathcal{L}(\bar{\pi}_{0:T}^C,p_{0:T-1},\pi_{0:T}^I)_t\right|+M_fL\rho_c\sum_{\tau=t}^{T}\rho_c^{\tau-t}|F_\tau-\bar{F}_\tau|. \end{aligned} \tag{1.21}$$

   This implies,

$$\left|\mathcal{L}(\pi_{0:T}^C,p_{0:T-1},\pi_{0:T}^I)_t-\mathcal{L}(\bar{\pi}_{0:T}^C,p_{0:T-1},\pi_{0:T}^I)_t\right| \leq M_fL\rho_c\textstyle\sum_{t_2=0}^{t-1}\sum_{t_1=t_2}^{T}2^{t-t_2-1}\rho_c^{t_1-t_2}|F_{t_1}-\bar{F}_{t_1}|.$$

   But, $F_{t_1}^a-\bar{F}_{t_1}=\Pi^C(\pi_{t_1}^C-\bar{\pi}_{t_1}^C)$. This implies the first point.

2. The second point follows from the continuity of $\mathcal{L}$ (first point of the Theorem) and Brouwer's fixed point theorem (Conway, 1985, Section V.9).

3. Fix $1 \leq k_0 \leq N^C$, and let $t_{k_0} \in \{0,\ldots,T-1\}$ be the adoption time of imitator k_0, and $a_{k_00:t_{k_0}}$ and $s_{k_00:T}$ the corresponding action and state, where $a_{k_00:t_k-1}=0$, $a_{k_0t_{k_0}}=1$, $s_{k_0t}=0$ if $t \leq t_{k_0}$, and $s_{k_0t}=1$ if $t > t_{k_0}$. Denote by $A_{j0:T-1}^*$ and $a_{k0:T-1}^*$, $1 \leq j \leq N^I$, $1 \leq k \leq N^C$, the individualists' and conformists' mean field strategies (1.6) and (1.9) that correspond to the unique

solution $\pi^I_{0:T}$ of (1.11) and a fixed point $\pi^C_{0:T}$ of $\mathcal{L}$. Denote by $S^*_{j0:T}$ and $s^*_{k0:T}$ the corresponding states. Define the fraction of adopters under the mean field strategies $F^N_t = \frac{1}{N}\left(\sum_{j=1}^{N^I} S^*_{jt} + \sum_{k=1}^{N^C} s^*_{kt}\right)$, and the fraction of adopter when imitator k_0 deviates from the mean field strategies

$$F^N_{-k_0,t} = \frac{1}{N}\left(\sum_{j=1}^{N^I} S^*_{jt} + \sum_{k=1,k\neq k_0}^{N^C} s^*_{kt} + s_{k_0 t}\right).$$

The utility of conformist k_0 when she deviates from the mean field strategies is

$$\begin{aligned}
J^C_{k_0}(a_{k_0 0:t_{k_0}}, F^N_{-k_0,0:T}) &= \mathbb{E}\rho_c^{t_{k_0}}\left(u^C(p_{t_{k_0}}, F^N_{-k_0,t_{k_0}}) + \eta_{k_0 t_{k_0}}\right) \\
&= J^C_{k_0}(a_{k_0 0:t_{k_0}}, F^N_{-k_0,0:T}) - J^C_{k_0}(a_{k_0 0:t_{k_0}}, F^N_{0:T}) \qquad := \xi_1 \\
&+ J^C_{k_0}(a_{k_0 0:t_{k_0}}, F^N_{0:T}) - J^C_{k_0}(a_{k_0 0:t_{k_0}}, F_{0:T}) \qquad := \xi_2 \\
&+ J^C_{k_0}(a_{k_0 0:t_{k_0}}, F_{0:T}) - J^C_{k_0}(a^*_{k_0 0:T-1}, F_{0:T}) \qquad := \xi_3,
\end{aligned}$$

where F_t is the mean field fraction of adopters, i.e $F_t = \Pi^C\pi^C_t + \Pi^I\pi^I_t$. By the definition of the mean field strategies, $\xi_3 \leq 0$. We have,

$$|\xi_1| = \left|\mathbb{E}\rho_c^{t_{k_0}}\left(u^C(p_{t_{k_0}}, F^N_{-k_0,t_{k_0}}) - u^C(p_{t_{k_0}}, F^N_{t_{k_0}})\right)\right| \leq L\rho_c\mathbb{E}\left|F^N_{-k_0,t_{k_0}} - F^N_{t_{k_0}}\right| \leq \frac{L\rho_c}{N}$$

The states S^*_{jt}, $1 \leq j \leq N^I$, are i.i.d., as well as the states s^*_{kt}, $1 \leq k \leq N^C$. Hence,

$$\begin{aligned}
|\xi_2|^2 &= \left|\rho_c^{t_{k_0}}\mathbb{E}\left(u^C(p_{t_{k_0}}, F^N_{t_{k_0}}) - u^C(p_{t_{k_0}}, F_{t_{k_0}})\right)\right|^2 \leq L^2\mathbb{E}\left|F^N_{t_{k_0}} - F_{t_{k_0}}\right|^2 \leq \\
&L^2\rho_c^2\mathbb{E}\left|F^N_{t_{k_0}} - F_{t_{k_0}}\right|^2 = L^2\rho_c^2\mathbb{E}\left(\frac{1}{N}\sum_{j=1}^{N^I}\left(S^*_{jt_{k_0}} - \mathbb{E}S^*_{jt_{k_0}}\right)\right)^2 + L^2\rho_c^2\mathbb{E} \\
&\left(\frac{1}{N}\sum_{k=1}^{N^C}\left(s^*_{kt_{k_0}} - \mathbb{E}s^*_{kt_{k_0}}\right)\right)^2 + L^2\rho_c^2\left(\frac{N^I}{N} - \Pi^I\right)^2\mathbb{E}\left(S^*_{jt_{k_0}}\right)^2 + \\
&L^2\rho_c^2\left(\frac{N^C}{N} - \Pi^C\right)^2\mathbb{E}\left(s^*_{kt_{k_0}}\right)^2 \leq L^2\rho_c^2\left(\frac{4}{N} + \left(\frac{N^I}{N} - \Pi^I\right) + \left(\frac{N^C}{N} - \Pi^C\right)^2\right).
\end{aligned}$$

This shows the result.

■

1.6.2 Selection functions and numerical scheme

The existence of a firm's optimal policy requires the existence of a continuous selection function S. Recall that S maps $(p_{0:T-1}, \pi^I_{0:T})$ to a root $\pi^C_{1:T}$ of the function

$$\Delta L(\pi^C_{0:T}, p_{0:T-1},, \pi^I_{0:T}) := \pi^C_{0:T} - \mathcal{L}(\pi^C_{0:T}, p_{0:T-1}, \pi^I_{0:T}).$$

We describe in the following a general procedure to construct an approximate continuous selection function. Let us assume that there exists an algorithm of the following form that converges to a root $\pi^C_{0:T}$,

$$(\pi^C_{0:T})^{(n+1)} = (\pi^C_{0:T})^{(n)} - \mathcal{F}\left((\pi^C_{0:T})^{(n+1)}, p_{0:T-1}, \pi^I_{0:T}\right), \tag{1.22}$$

where $\mathcal{F}$ is a continuous function. Moreover, suppose that the algorithm converges uniformly, i.e., for every $\epsilon > 0$, there exists $n_\epsilon > 0$, such that for all $n > n_\epsilon$, $\sup\limits_{p_{0:T-1}, \pi^I_{0:T}} |\Delta L((\pi^C_{0:T})^{(n)}, p_{0:T-1},, \pi^I_{0:T})| < \epsilon$. Thus, for each $\epsilon > 0$, one can define an approximate selection function $S_\epsilon(p_{0:T-1}, \pi^I_{0:T}) = (\pi^C_{0:T})^{(n_\epsilon)}$, which, following our assumptions, is a continuous function of $(p_{0:T-1}, \pi^I_{0:T})$. S_ϵ is approximate in the sense that $\sup\limits_{p_{0:T-1}, \pi^I_{0:T}} |\Delta L\left((S_\epsilon(p_{0:T-1}, \pi^I_{0:T}), p_{0:T-1}, \pi^I_{0:T}\right)| < \epsilon$, i.e. $S_\epsilon(p_{0:T-1}, \pi^I_{0:T})$ is almost a fixed point. The family of algorithms in (1.22) includes a large number of members, such as Newton's and the fixed-point iterations methods (Ortega and Rheinboldt, 1970), which we discuss in details later.

In the following, we propose an algorithm to compute an optimal solution $(m_{0:T-1}, p_{0:T-1})$ of (1.14). The algorithm includes two nested loops. The external one is the projected gradient descent method (Bertsekas, 1999) that solves for a maximizer $(m^*_{0:T-1}, p^*_{0:T-1})$ of the firm's utility, $U^f = \sum_{t=0}^{T-1}(p_t f_t - c_m(m_t))$, with $\pi^C_{0:T} = S_\epsilon(p_{0:T-1}, \pi^I_{0:T})$, and $f_t = \Pi^I(\pi^I_{t+1} - \pi^I_t) + \Pi^C(\pi^C_{t+1} - \pi^C_t)$.

The iterations of the external loop are as follows:

$$m_t^{(n+1)} = \pi_{[0,M_m]}\left(m_t^{(n)} + \xi \frac{\partial U^f}{\partial m_t}\left(m_t^{(n)}, p_t^{(n)}\right)\right), \tag{1.23}$$

$$p_t^{(n+1)} = \pi_{[0,M_p]}\left(p_t^{(n)} + \xi \frac{\partial U^f}{\partial p_t}\left(m_t^{(n)}, p_t^{(n)}\right)\right),$$

for $1 \leq t \leq T-1$, where $\xi > 0$ and π_C is the Euclidean projection on the set C. Recall that the Euclidean projection of a point x on a cube $C = [a, b]^k$ is

$$\pi_C(x_1, \ldots, x_k) = (\min(b, \max(a, x_1)), \ldots, \min(b, \max(a, x_k))).$$

The partial derivatives are computed using the finite difference formulas. This involves computing the value of U^f at different $(m_{0:T-1}, p_{0:T-1})$, and more specifically, the fraction of adopters $f_{0:T-1}$, which is computed in an internal loop according to iterations (1.22). If the algorithm converges to a global maximum $(m^*_{0:T-1}, p^*_{0:T-1})$ of U^f, then $m^*_{0:T-1}$ and $p^*_{0:T-1}$ are the optimal marketing and pricing policies of the firm, respectively . In addition to the optimal strategies, one can anticipate the evolution of adoption or the cumulative fraction of adopters $F_{0:T} = \Pi^I \pi^I_{0:T} + \Pi^C \pi^C_{0:T}$, where $\pi^C_{0:T} = S_\epsilon(m^*_{0:T-1}, \pi^I_{0:T})$, and $\pi^C_{0:T}$ is the solution of (1.11).

Newton's method

In Newton's method,

$$\mathcal{F}\left(\pi^C_{0:T}, p_{0:T-1}, \pi^I_{0:T}\right) = (J(\pi^C_{0:T}, p_{0:T-1}, \pi^I_{0:T}))^{-1} \Delta L(\pi^C_{0:T}, p_{0:T-1}, \pi^I_{0:T}),$$

where J is the finite difference approximation of the derivative of ΔL with respect to $\pi^C_{0:T}$. The main challenge here is the non-smoothness of $\mathcal{F}$, which may result from the non-smoothness of the max operator in (1.5) and (1.8), and that of the functions U^I ad u^C. A remedy would be to assume that U^I and u^C are smooth and to replace the function $\max(x, y) = \frac{1}{2}(x + y + |x - y|)$ by the smooth function $\frac{1}{2}(x + y + \sqrt{(x-y)^2 + \epsilon})$, where ϵ is a small positive number. The uniform convergence follows in this case from (i) the smoothness of $\mathcal{F}$, and (ii) the compactness of the firm's strategy set.

Fixed-point iterations method

In the fixed-point iterations method,

$$\mathcal{F}\left(\pi^C_{0:T}, p_{0:T-1}, \pi^I_{0:T}\right) = \pi^C_{0:T} - \mathcal{L}(\pi^C_{0:T}, p_{0:T-1}, \pi^I_{0:T}).$$

In this case, iterations (1.22) take the following form:

$$(\pi^C_{0:T})^{(n+1)} = \mathcal{L}\left((\pi^C_{0:T})^{(n)}, p_{0:T-1}, \pi^I_{0:T}\right). \tag{1.24}$$

The following assumption guarantees that $\mathcal{L}$ is a contraction in $\pi^C_{0:T}$, and by Banach fixed point Theorem (Berinde, 2007), the iterations (1.24) converge to the unique fixed point of $\mathcal{L}$.

Assumption 1.6 *We assume that $L_{\mathcal{L}} < 1$, where $L_{\mathcal{L}}$ is defined in Theorem 1.1.*

In this case, the selection map S sends $(p_{0:T-1}, \pi^I_{0:T})$ to the unique fixed point of $\mathcal{L}$. The following Theorem shows that S is continuous.

Theorem 1.2 *The selection function S, defined as the unique fixed point of $\mathcal{L}$, is continuous.*

Proof of Theorem 1.2. We have for all $(p_{0:T-1}, \pi^I_{0:T})$ and $(p'_{0:T-1}, (\pi^I_{0:T})')$,

$$\begin{aligned}
&\left|S(p'_{0:T-1}, (\pi^I_{0:T})')_t - S(p_{0:T-1}, \pi^I_{0:T})_t\right|_t \\
&= \left|\mathcal{L}\left(S(p'_{0:T-1}, (\pi^I_{0:T})'), p'_{0:T-1}, (\pi^I_{0:T})'\right)_t - \mathcal{L}\left(S(p_{0:T-1}, \pi^I_{0:T}), p_{0:T-1}, \pi^I_{0:T}\right)_t\right|_t \\
&\leq \left|\mathcal{L}\left(S(p'_{0:T-1}, (\pi^I_{0:T})'), p'_{0:T-1}, (\pi^I_{0:T})'\right)_t - \mathcal{L}\left(S(p_{0:T-1},, \pi^I_{0:T}), p'_{0:T-1}, (\pi^I_{0:T})'\right)_t\right|_t \\
&+ \left|\mathcal{L}\left(S(p_{0:T-1},, \pi^I_{0:T}), p'_{0:T-1}, (\pi^I_{0:T})'\right)_t - \mathcal{L}\left(S(p_{0:T-1}, \pi^I_{0:T}), p_{0:T-1}, \pi^I_{0:T}\right)_t\right|_t.
\end{aligned}$$

This implies that

$$\begin{aligned}
&\left|S(p'_{0:T-1}, (\pi^I_{0:T})')_t - S(p_{0:T-1}, \pi^I_{0:T})_t\right|_t \leq \\
&\frac{1}{1-L_{\mathcal{L}}}\left|\mathcal{L}\left(S(p_{0:T-1}, \pi^I_{0:T}), p'_{0:T-1}, (\pi^I_{0:T})'\right)_t - \mathcal{L}\left(S(p_{0:T-1}, \pi^I_{0:T}), p_{0:T-1}, \pi^I_{0:T}\right)_t\right|_t.
\end{aligned}$$

The result follows from the continuity of $\mathcal{L}$, which can be shown using (1.5), (1.8), (1.11), and (1.12). ■

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Chapter 2

Dynamic pricing in the presence of social externalities and reference-price effect

Abstract

This paper considers the pricing of a new product in the face of sophisticated consumer behaviors. At the individual level, consumers are forward-looking, whereby they may wait strategically for intertemporal arbitrage. Additionally, and in line with prospect theory, consumers might also look back to form a reference-price point with which they can compare the current price. Consumers are assumed to be loss averse where losses resonate more than gains. At the aggregate level, we account for the role of social influences in the form of externalities in consumers' adoption decision. We develop progressively different nested models to account for impact of each behavior. We utilize two types of pricing regimes, that is, preannounced and responsive pricing strategies, where the firm commits to the price path from the outset in the former while it varies the prices over time in the latter. We find that a penetration pricing strategy can be both strengthened

or weakened by forward-looking consumer behavior depending on the underlying dynamics. We show that a preannounced pricing regime does not necessarily lead to a higher profit.

2.1 Introduction

Farsighted or strategic consumers purchase a product during the period that yields the highest utility, that is, they consider current and future prices when making a decision. Such forward-looking behavior has been documented in many product categories, e.g., consumer durables and electronic products (McWilliams, 2004), video games (Nair, 2007), and fashion goods (Dasu and Tong, 2010; Soysal and Krishnamurthi, 2012), and this impacts, notably, a new product launch (Lobel et al., 2016) and pricing strategies (Papanastasiou and Savva, 2017). Consumers may also look backward to judge the fairness of the current price by comparing it to an anchor value, a reference price, which could be the last-period price or the price history. These two (forward- and backward-looking) behaviors are practiced by technology-savvy consumers, for example, in purchasing Apple iPhones (Zhang and Chiang, 2020; Lobel et al., 2016). Research on behavioral decision-making suggests that consumers derive various transaction values from the difference between the current and the reference price (Thaler, 1985). This comparison plays a salient role in purchasing intentions and the timing of adoption, again, in different product categories (Kalyanaram and Winer, 1995; Lowe and Alpert, 2010; Mazumdar et al., 2005). Interestingly, the impact of this difference, however, is asymmetric, in the sense that the consumer reacts more strongly to a loss than to a gain (which is known as loss aversion), and this effect is manifested more in durable than in non-druable products (Neumann and Böckenholt, 2014).

Beside these individual-based behaviors, social influences play a major role in the diffusion of a new product. Specifically, a positive externality, meaning that the utility of a product increases with the number of adopters, is widely considered as a growth driver, independently of the type of product (Peres et al., 2010; Huang et al., 2018). While some studies state that network externalities can accelerate adoption rate (e.g., Rohlfs, 2003), others suggest that it can decelerate the initial growth since consumers take a wait-and-see approach until more people adopt the product (Srinivasan et al., 2004). Consequently, the diffusion process is slow at the beginning and fast later on (Rogers, 2003). Further, it has been shown that externality can create a chilling effect on the diffusion of new product (Goldenberg et al., 2010; Mukherjee, 2014) or mitigate negative psychological aspects such as consumers' anxiety (Huang et al., 2018).

In this research, we consider a two-period choice model that captures both the individual and aggregate adoption behaviors of consumers. We assume that the consumers have heterogeneous valuations of the new product and use the concept of the rational expectation equilibrium (Stokey, 1979) to forecast future prices. Accordingly, their derived utility depends on the price and its psychological effects (considered an external influence) along with the network (social) externality (internal influence). In this setup, consumers solve an intertemporal optimization problem whereas the forward-looking monopolist uses a backward induction approach. Huang et al. (2018) provides a classification of different externality effects depending on the type of utility and their impact (see Table 2.1). Here, we consider a new product[1] where a consumer's (psychological) utility increases with the total number of adopters (upper right quadrant in Table 2.1). Our comprehensive model nests different scenarios which are progressively developed and each studies one aspect of consumer behavior.

The firm should view the pricing design through a holistic lens in the face of behaviorally sophisticated consumers. Two common approaches can be used, namely, preannounced pricing and responsive pricing. In the former, the firm commits to a predetermined pricing path, while in the latter it updates the prices in response to market conditions. Preannounced pricing has been implemented

[1] We do not specify the type of product, which can be a durable or an experience good.

Table 2.1: Four types of externality

	Functional utility	Psychological utility
Positive externality	Networked goods or complementary products	New technology products, innovations, restaurants, movies, fashion (conformity-seeking behavior)
Negative externality	Services (utilities, roads) due to congestion	Luxury products(exclusivity seeking behavior)

by, e.g., Wanamaker's discount department store in Philadelphia, Pricetack.com, Tuesday Morning discount stores, Filene Overstocks, Sam's Club, Dress for Less, and TKTS ticket booths in London and New York City (Yin et al., 2009; Liu et al., 2019). The responsive pricing is popular in the online commerce (Papanastasiou and Savva, 2017). We examine the merit of each pricing regime under various consumer behaviors.

Our research aims to answer the following questions:

- What is the optimal pricing strategy when consumers are forward-looking and are sensitive to network externality and reference-price effect?
- What are the marketing implications of preannounced and responsive pricing regimes in this context?

The main results are as follows. First, the firm may employ different pricing schemes, including skimming, constant or penetration pricing strategies, of varying intensities, depending on the strength of forward- versus backward-looking behavior, and of consumers' psychological biases. When consumers are loss averse, these conflicting forces may result in inertia, where constant prices defeats dynamic pricing strategy under certain conditions. Second, while the reference-price effect calls for skimming pricing strategy, the externality effect pushes towards a penetration pricing strategy. Interestingly, forward-looking behavior can both advocate and work against penetration pricing strategy depending on the presence and intensity of underlying effects. In particular, forward-looking behavior favors penetration pricing strategy when only externality exists, however, it might work against it when both externality and reference-price effect coexist and social. Third, the monopolist may charge a high launch price if consumers are sufficiently either forward-looking or sensitive to their price anchor. This might later favor the psychological surplus at the expense of no early adoption. Fourth, in the presence of reference-dependent behavior, the firm may earn a higher profit under responsive pricing when consumers are neither little nor too farsighted; otherwise the profit under preannounced pricing prevails. Papanastasiou and Savva (2017) also shows that, despite the popularity of preannounced pricing reported by the literature in the face of forward-looking consumers, it can be suboptimal in the presence of social learning. We, however, find that the presence of the reference-price effect can lead to such an outcome, which underscores the salience of accounting for nuances in consumer behavior.

The remainder of the paper is organized as follows: Section 2.2 reviews the relevant literature, and Section 2.3 describes the model. In Sections 2.4 and 2.5, we examine all considered scenarios when the firm adopts responsive pricing and preannounced pricing strategies, respectively. Section 2.6 compares the prices, demands, and profits of the under preannounced and responsive pricing regimes. Finally, we briefly conclude in Section 2.7.

2.2 Literature

Bass's seminal paper (Bass, 1969) initiated a large literature on the diffusion of new products and technologies.[2] The model applies to durable products and does not involve any decision variables. A number of studies have extended the framework to incorporate marketing-mix variables, especially price and advertis-

[2] In 2004, Bass (1969) was voted one of the ten most influential papers published in Management Science during the last fifty years.

ing, in both a single-firm context and a competitive setup. Relative to our area of concern, we note that the price effect has been embedded through either the consumers adoption probability (Robinson and Lakhani, 1975; Dolan and Jeuland, 1981; Bass, 1980; Kalish, 1983; Breton et al., 1997) or the market potential (Horsky, 1990; Kalish, 1985); see Nair (2019) for a recent review on new product pricing. One main recommendation to the firm is to implement a skimming pricing strategy, unless consumers are highly affected by WoM communications. Zhang and Chiang (2020) incorporate reference price in market potential and assume a fixed adoption rate. If the firm is myopic, then a skimming pricing strategy is optimal. However, either penetration or skimming pricing strategies could be optimal for a farsighted monopolist.

Xie and Sirbu (1995) consider a market for a new product where consumers benefit from consumption externality. Through a numerical simulation, they show that when the externality is strong, the pricing trajectory is increasing in a market monopoly; however, under a duopoly, it is increasing, followed by a decreasing trend. Goldenberg et al. (2010) use both agent-based model and aggregate one to show that network externality has chilling effects on new product growth. However, their model does not show how the individual consumer behavior is related to network externality. Gabszewicz and Garcia (2008) suggest zero pricing in the initial periods for a monopolist who offers network goods to myopic consumers. Li and Zhang (2020) study how cross-group externality along with the reference-price effect influence pricing decisions in a one-shot game. Their single-stage setup assumes an exogenous reference price, which does not allow to examine how pricing strategy evolves over time. Bloch and Quérou (2013) tackle a similar problem while considering a network structure, where consumers care either about the local network externality or the aspiration reference price. In the latter model, it is implicitly assumed that consumers consider only transaction utility, by comparing the price they pay to the ones paid by their neighbors. In a similar vein, Duan and Feng (2021) study a static pricing problem, however, by integrating the network externality and the aspiration-based reference price into consumers's utility. Fainmesser and Galeotti (2015) extend Bloch and Quérou (2013) by relaxing the assumption of both the firm and consumers having full information about network effects to examine the value of information and its pricing implications.

We depart from this literature in two ways. First, we consider a dynamic two-period framework where both consumers and the firm are forward-looking. Second, we consider the standard internal reference price along with its asymmetric effect on consumers' choice in the presence of social influences. Put differently, the proposed model features the situation where consumers look both forward and backward during adoption occasions.

The notion of the reference price stems from adaptation-level theory (Helson, 1964) and prospect theory (Kahneman and Tversky, 1979), and it has found empirical generalizations (Kalyanaram and Winer, 1995) and extensions to other stimuli (Lattin and Roberts, 1988). Chen et al. (2020) considers a manufacturer-retailer supply chain to examine how the reference-price effect and consumers' forward-looking behavior affect pricing strategy in a centralized and a decentralized channel. While many studies have focused mainly on nondurable goods (see Mazumdar et al., 2005, for a review), the literature calls for study of the reference price's impact on consumer adoption behavior for a new product category (Lowengart, 2002; Mazumdar et al., 2005; Kalyanaram and Winer, 1995; Biswas and Sherrell, 1993) and to tie the findings on the nuances of consumer behavior to new product diffusion, and their significance for optimal marketing strategies (Nair, 2019; Peres et al., 2010). Prospect theory also proposes that backward-looking consumers are influenced by a psychological bias, known as loss aversion. Hu and Nasiry (2018) demonstrates that loss aversion is an individual phenomenon and that the aggregate market may not replicate consumers' micro-level behavior. Our study stands out from this literature by considering a product where both internal and external influences affect the consumer adoption dynamics.

Stokey (1979) and Besanko and Winston (1990) are among the early works on preannounced and responsive pricing strategies, respectively, where the former was generalized by Gul et al. (1986). Dasu and Tong (2010) examine both pricing approaches for a perishable product, while Papanastasiou and Savva (2017) do the same for a new product launch. The latter paper incorporates the social learning effect in a two-period adoption game and proposes that the monopolist is not generally better off with preannounced pricing. Huang et al. (2018) and Zhao et al. (2019) adopt a responsive pricing strategy in a similar time frame. Zhao et al. (2019) study the reference-price effect with and without price matching; however, the focus lies on how prices and the firm's profit vary based on market dynamics such as discount factor, intensity of the reference effect, or the ratio of myopic to strategic consumers. Jing (2011) and Chen and Jiang (2021) study the role of price commitment versus other pricing schemes, in order to determine conditions under which the ex-ante commitment is beneficial for the firm. Chen et al. (2020) adopt a responsive pricing strategy and consider the joint impact of the reference-price effect and forward-looking behavior in centralized and decentralized supply chains. Following the literature, we consider both responsive and preannounced pricing strategies, however, in a different framework, to see when each pricing scheme better serves the monopolist.

Our contributions are as follows. First, we contribute to the dynamic pricing literature by examining how monopoly pricing is formed when consumers look both backward and forward. While these two consumer behaviors are examined as standalone phenomena in the literature, we unify them to capture more intricate consumer behavior in context of a new product launch. Arslan and Kachani (2011) explicitly suggest that the incorporation of forward-looking behavior in the context of the reference-price effect is useful, since consumers might be able to learn to anticipate future prices. Second, we explore the role of the firm's commitment in this context by considering both preannounced and responsive pricing strategies. Third, in line with prospect theory, since consumers have asymmetric reactions when they look backward, we contribute to this growing literature by studying how loss aversion impacts the results.

2.3 Model development

Consider a monopolist that launches a new product in a market composed of a unit-measure continuum of consumers who have a uniformly distributed private valuation $v \in [0,1]$. To capture the impact of buying time on pricing strategy, the formation of a reference price, and the effect of network externality in the most parsimonious way, we retain a two-period model. The firm's objective is to maximize its profit with respect to price. For ease of exposition, without loss of generality, we assume away discounting and production cost for the firm.

The consumer behaves strategically by choosing the adoption timing that maximizes her utility, which integrates three components: (i) an economic utility derived from consumption of the new product; (ii) a transaction utility measured by the difference between the current price and the (mental) reference price; and (iii) a network externality in the second period, measured by the first-period demand. The firm adopts either a responsive or a preannounced pricing strategy. For each pricing scheme, we study five scenarios:

B: Benchmark scenario, where only economic utility matters;

N: Network externality effect;

R: Reference price effect;

NR: Network externality and symmetric reference price effects;

NRL: Network externality and asymmetric reference price effect.

Remark 2.1 *When the firm uses responsive pricing, the results in the five scenarios will be superscripted with RB, RN, RR, RNR, and RNRL; and with PB, PN, PR, PNR, and PNRL when the firm implements preannounced pricing.*

Denote by u_t, p_t and D_t the utility, price, and demand in period $t = 1, 2$, respectively. Let w be a positive parameter measuring the impact of the first-period demand on the second-period utility. In period 2, the reference price considered by consumers is the observed price p_1 in the first period. Let $\theta \in [0, 1]$, $\gamma \in [0, 1]$, and $\lambda \in [0, 1]$ be positive parameters, used to assess the impact of the reference price on second-period utility. Note that θ is used when the impact of the reference price is considered symmetric, regardless of whether it is a gain or a loss, whereas γ and λ are used when consumers encode the impact of the reference price as a gain or a loss, respectively, however, in an asymmetric way with $\lambda > \gamma$. Denote by $\delta \in (0, 1)$ the common discount factor to all consumers. Table 2.2 defines the consumer utility in each periods of the five scenarios where $x^+ = \max(x, 0)$. We make the following comments.

1. In the first period, the only available piece of information is the price, which explains why the utility is the same in all scenarios. Whereas a myopic consumer would adopt the product in the first period if u_1 is positive, a strategic consumer compares her utilities in both periods and adopts at the period that yields the highest (positive) utility. If the utility is negative in both periods, then the consumer will not purchase the product.

2. In the second period, the utility varies across scenarios. In the benchmark scenario, the utility in period 2 depends only on the price. Network externality, which appears in 3 of the 5 scenarios, is captured by the additional term wD_1. For instance, in the network externality scenario, we see that the result of the comparison of $u_1 = \nu - p_1$ to $u_2 = \delta(\nu - p_2 + wD_1)$ depends

on the firm's pricing policy (p_1, p_2), the influence of first-period adopters, and on the degree of consumers' patience, captured by the discount rate.

3. As consumers do not have information on past prices in the first period, and in line with Nasiry and Popescu (2011), Zhao et al. (2019), and Chen et al. (2020), we assume that the reference price effect only appears in the second period. This effect is measured by the difference between p_1 and p_2 scaled by an appropriate parameter. In the third and fourth scenarios, independently of which price is higher, this impact is given by $\theta(p_1 - p_2)$, meaning that consumer reacts to gains $(p_1 > p_2)$ or losses $(p_1 < p_2)$ in the same way. In the last scenario, as suggested by prospect theory, where "losses loom larger than gains", we suppose that consumers react more strongly to losses compared to gains; hence our assumption that λ is larger than γ.

4. We assume that the utility function is additive in the three components. Such a functional form is widely adopted in the literature (e.g., Xie and Sirbu, 1995; Li and Zhang, 2020; Nasiry and Popescu, 2011).

Table 2.2: Consumer utility in each period in the five scenarios

	Period 1	Period 2
B	$u = v - p_1$	$u = \delta(v - p_2)$
N	$u = v - p_1$	$u = \delta(v - p_2 + wD_1)$
R	$u = v - p_1$	$u = \delta(v - p_2 + \theta(p_1 - p_2))$
NR	$u = v - p_1$	$u = \delta(v - p_2 + wD_1 + \gamma(p_1 - p_2))$
NRL	$u = v - p_1$	$u = \delta(v - p_2 + wD_1 + \gamma(p_1 - p_2)^+ - \lambda(p_2 - p_1)^+)$

2.4 Responsive pricing

Under responsive pricing, the sequence of events is as follows: First, the monopolist determines the price p_1 in period 1. Consumers subsequently compare their utilities across two periods and accordingly choose either to adopt in period 1, adopt in period 2, or leave the market. Since the demand D_1 and price p_2 are yet to be realized, consumers develop rational expectations on these values in order to predict their utility in period 2. In a rational expectation equilibrium, the predictions, here of D_1 and p_2, coincide with the realized ones. The demand D_1 is realized by the end of period 1. Second, the monopolist determines p_2 in the second period, and the remaining consumers choose to adopt or not, knowing the intrinsic psychological surplus and extrinsic social surplus.

To demonstrate our solution procedure, we show how a rational expectation equilibrium is obtained in the benchmark case. In this scenario, consumers adopt in period 1 if $\nu - p_1 \geq \delta(\nu - p_2)$. Suppose there exists a threshold τ such that all consumers with valuations $\nu \geq \tau$ adopt in the first period. Under the assumption that the new product valuation is uniformly distributed $\nu \in [0, 1]$, the demand in the first period would be $D_1 = (1 - \tau)$. Consequently, the remaining consumers in the second period would have valuations $\nu \in [0, \tau]$. A generic consumer in period 2 will adopt the new product if $u = \nu - p_2 > 0$, and the demand will be $\max(\tau - p_2, 0)$. The firm's optimization problem in period 2 can then be expressed as follows:

$$\max_{p_2} \pi_2 = p_2 D_2 = p_2(\tau - p_2). \tag{2.1}$$

The unique solution to this strictly concave optimization problem is $p_2^* = \frac{\tau}{2}$.

Next, we consider the firm's problem in the first period to determine p_1^*. In a rational expectation equilibrium, the consumers adopt the new product in the first period if, and only if, their utility in period 1 is nonnegative and higher than the one in period 2, that is, $\nu - p_1 \geq 0$ and $\nu - p_1 \geq \delta(\nu - p_2)$. In particular, a consumer with valuation τ is indifferent between adopting in either period. Therefore, we have

$$\tau - p_1 = \delta(\tau - p_2), \tag{2.2}$$

and using $p_2^* = \frac{\tau}{2}$, we obtain the threshold τ as a function of p_1, that is,

$$\tau(p_1) = \begin{cases} \frac{2p_1}{2-\delta}, & \text{if } p_1 \leq \frac{2-\delta}{2} \\ 1, & \text{otherwise} \end{cases} \tag{2.3}$$

If $p_1 > \frac{2-\delta}{2}$, then no consumer adopts in the first period, and demand is only positive in the second period. If $p_1 \leq \frac{(2-\delta)}{2}$, then the overall firm's optimization problem becomes

$$\max_{p_1} \pi = \pi_1 + \pi_2^* = p_1(1-\tau(p_1)) + \frac{(\tau(p_1))^2}{4}. \tag{2.4}$$

It is easy to verify that π is concave in p_1, and from the first-order optimality conditions, that the maximum is achieved at $p_1^* = \frac{(2-\delta)^2}{2(3-2\delta)}$. Clearly, we have $0 < p_1^* < \frac{2-\delta}{2} \leq 1$. Substituting for p_1^*, we get $\tau = \frac{2-\delta}{3-2\delta} < 1$, and $0 < p_1^* < \frac{2-\delta}{2} \leq 1$ and $0 < p_2^* = \frac{2-\delta}{2(3-2\delta)} \leq 1$. The other results in the benchmark as well as in all other scenarios are given in the next proposition. We introduce the following notations, which are used throughout our analysis:

$g(i) = 2(1+i)\left((3-2\delta-i(1-\delta)^2) + w(2+\delta(2+\delta i))\right) - 2w^2 > 0, \quad i \in (\theta, \gamma, \lambda),$
$h(i) = (1+i)((2-\delta)^2 + w(2+\delta(2-\delta))) - 2w^2 > 0, \quad i \in (\theta, \gamma, \lambda),$
$k(i) = (1+i)(2-\delta) + w(2+\delta(1+2i)) > 0, \quad i \in (\theta, \gamma, \lambda),$
$l(i) = 2(1-i+w) - \delta(2-i(2-\delta)) > 0, \quad i \in (\theta, \gamma, \lambda),$
$\delta_\pi^{RR} = \frac{1+\theta^2-(1+\theta)\sqrt{\theta}}{1+\theta+\theta^2} > 0,$

$\delta_\pi^{RNR} = \min\{1, \frac{1+\theta w+w+\theta^2-\sqrt{\theta^4 w-2\theta^3 w^2+3\theta^3 w+\theta^3+\theta^2 w^3+\theta^2 w+2\theta^2+\theta w^3+3\theta w^2+3\theta w+\theta}}{-\theta^2 w+\theta^2-\theta w+\theta+1}\} > 0,$

$\delta_{\tau^\gamma}^{RNRL} = \min\{1, \frac{(-1+\gamma+\sqrt{1-\gamma^2+2\gamma w})}{\gamma}\} > 0,$
$\delta_{p^\gamma}^{RNRL} = \min\{1, \frac{\sqrt{-8(\gamma+1)w^3+(8\gamma(\gamma+2)+9)w^2+2(\gamma+1)w+(\gamma+1)^2}-3\gamma+w-3}{2(\gamma+1)(w-1)}\} > 0,$
$\delta_{p^\lambda}^{RNRL} = \min\{1, \frac{\sqrt{-8(\lambda+1)w^3+(8\lambda(\lambda+2)+9)w^2+2(\lambda+1)w+(\lambda+1)^2}-3\lambda+w-3}{2(\lambda+1)(w-1)}\} > 0,$
$\delta_{\pi^u}^{RNRL} = \min\{1, \frac{2\sqrt{\gamma w^4-\gamma w^2+w^4}-\gamma w^2+\gamma-w^2-2w+1}{\gamma w^2-2\gamma w+\gamma+w^2-2w+1}\} > 0,$

$\delta_{\pi^\gamma}^{RNRL} = \min\{1, \frac{1+\gamma w+w+\gamma^2-\sqrt{\gamma^4 w-2\gamma^3 w^2+3\gamma^3 w+\gamma^3+\gamma^2 w^3+\gamma^2 w+2\gamma^2+\gamma w^3+3\gamma w^2+3\gamma w+\gamma}}{-\gamma^2 w+\gamma^2-\gamma w+\gamma+1}\} > 0,$

$$\delta_{\pi^{\lambda}}^{RNRL} = \min\{1, -\frac{\gamma(\lambda+1)(\lambda+w-1)-\sqrt{(\lambda+1)((\gamma+1)\lambda w+\gamma)(\gamma(\lambda-w+1)^2+(\lambda+w+1)^2)}+\lambda(\lambda+w+2)+w+1}{(\lambda+1)((\gamma+1)\lambda(w-1)-1)}\} > 0.$$

Proposition 2.1 *If the firm adopts responsive pricing regime, then the optimal pricing strategies and the demands and profits in the five scenarios are those given in Tables 2.3 and 2.4 , respectively.*

Proof See Appendix. ■

Table 2.3: Responsive pricing strategies

Scenarios	**Pricing strategies**
RB	$(p_1, p_2) = (\frac{(2-\delta)^2}{2(3-2\delta)}, \frac{(2-\delta)}{2(3-2\delta)})$
RN	$(p_1, p_2) = (\frac{(2-\delta)^2+w(2+\delta(2-\delta))-2w^2}{2(3-w)(1+w)-4\delta(1-w)}, \frac{(2-\delta)+w(2+\delta)}{2(3-w)(1+w)-4\delta(1-w)})$
RR	$(p_1, p_2) = \begin{cases} (1, \frac{1}{2}), & \text{if } \delta > \delta_{\pi}^{RR} \\ (\frac{(2-\delta)^2}{2(3-2\delta-\theta(1-\delta)^2)}, \frac{(2-\delta)}{2(3-2\delta-\theta(1-\delta)^2)}), & \text{if } \delta \leq \delta_{\pi}^{RR} \end{cases}$
RNR	$(p_1, p_2) = \begin{cases} (1, \frac{1}{2}) & \text{if } \delta > \delta_{\pi}^{RNR} \\ (\frac{h(\theta)}{g(\theta)}, \frac{k(\theta)}{g(\theta)}) & \text{if } \delta \leq \delta_{\pi}^{RNR} \end{cases}$
RNRL	$(p_1, p_2) = \begin{cases} \begin{cases} (\frac{h(\gamma)}{g(\gamma)}, \frac{k(\gamma)}{g(\gamma)}) & \text{if } 0 < \delta \leq \delta_{\pi^{\gamma}}^{RNRL} \\ (1, \frac{1}{2}) & \text{if } \delta_{\pi^{\gamma}}^{RNRL} < \delta < 1 \end{cases}, & \text{if } \delta_{\tau\gamma}^{RNRL} \leq \delta_{p^{\gamma}}^{RNRL} \\ \begin{cases} (\frac{h(\gamma)}{g(\gamma)}, \frac{k(\gamma)}{g(\gamma)}) & \text{if } 0 < \delta \leq \delta_{p^{\gamma}}^{RNRL} \\ (\frac{w}{1-\delta+w(1+\delta)}, \frac{w}{1-\delta+w(1+\delta)}) & \text{if } \delta_{p^{\gamma}}^{RNRL} < \delta \leq \delta_{\pi^{u}}^{RNRL} \\ (1, \frac{1}{2}) & \text{if } \delta_{\pi^{u}}^{RNRL} < \delta \leq \delta_{p^{\lambda}}^{RNRL} \\ (\frac{h(\lambda)}{g(\lambda)}, \frac{k(\lambda)}{g(\lambda)}) & \text{if } \delta_{p^{\lambda}}^{RNRL} < \delta \leq \delta_{\pi^{\lambda}}^{RNRL} \\ (1, \frac{1}{2}) & \text{if } \delta_{\pi^{\lambda}}^{RNRL} < \delta < 1 \end{cases}, & \text{if } \delta_{\tau^{\lambda}}^{RNRL} > \delta_{p^{\lambda}}^{RNRL} \end{cases}$

We note that the results in the benchmark scenario in Tables 2.3 and 2.4 are quite similar to those established in Proposition 1 in Huang et al. (2018) and Proposition 4 in Papanastasiou and Savva (2017). In these papers, the authors use the same benchmark, against which they compare the situation where consumers face anxiety or engage in social learning in their adoption decision, respectively. However, our focus is different from theirs, as we previously stated. In what follows, we highlight a series of results derived from Proposition 2.1.

Table 2.4: Demands and profit under responsive pricing

Scenarios	Demands and profit
RB	$(D_1, D_2, \pi) = (\frac{1-\delta}{3-2\delta}, \frac{2-\delta}{2(3-2\delta)}, \frac{(2-\delta)^2}{4(3-2\delta)})$
RN	$(D_1, D_2, \pi) = (\frac{1-\delta+w}{(3-w)(1+w)-2\delta(1-w)}, \frac{2-\delta+w(2+\delta)}{2(3-w)(1+w)-4\delta(1-w)}, \frac{(2-\delta)^2+4w}{4(3-w)(1+w)-8\delta(1-w)})$
RR	$(D_1, D_2, \pi) = \begin{cases} (0, \frac{1+\theta}{2}, \frac{1+\theta}{4}), & \text{if } \delta > \delta_{\pi}^{RR} \\ (\frac{2(1-\delta)-\theta(2-\delta(2-\delta))}{2(3-2\delta-\theta(1-\delta)^2)}, \frac{(2-\delta)(1+\theta)}{2(3-2\delta-\theta(1-\delta)^2)}, \frac{(2-\delta)^2}{4(3-2\delta-\theta(1-\delta)^2)}) & \text{if } \delta \leq \delta_{\pi}^{RR} \end{cases}$
RNR	$(D_1, D_2, \pi) = \begin{cases} (0, \frac{1+\theta}{2}, \frac{1+\theta}{4}) & \text{if } \delta > \delta_{\pi}^{RNR} \\ (\frac{(1+\theta)l(\theta)}{g(\theta)}, \frac{(1+\theta)k(\theta)}{g(\theta)}, \frac{((1+\theta)(2-\delta)^2+4w)}{2g(\theta)}) & \text{if } \delta \leq \delta_{\pi}^{RNR} \end{cases}$
RNRL	$(D_1, D_2, \pi) = \begin{cases} \begin{cases} (\frac{(1+\gamma)l(\gamma)}{g(\gamma)}, \frac{(1+\gamma)k(\gamma)}{g(\gamma)}, \frac{((1+\gamma)(2-\delta)^2+4w)}{2g(\gamma)}) & \text{if } 0 < \delta \leq \delta_{\pi^{\gamma}}^{RNRL} \\ (0, \frac{1+\gamma}{2}, \frac{1+\gamma}{4}) & \text{if } \delta_{\pi^{\gamma}}^{RNRL} < \delta < 1 \end{cases}, & \text{if } \delta_{\tau^{\gamma}}^{RNRL} \leq \delta_{p^{\gamma}}^{RNRL} \\ \begin{cases} (\frac{(1+\gamma)l(\gamma)}{g(\gamma)}, \frac{(1+\gamma)k(\gamma)}{g(\gamma)}, \frac{((1+\gamma)(2-\delta)^2+4w)}{2g(\gamma)}) & \text{if } 0 < \delta \leq \delta_{p^{\gamma}}^{RNRL} \\ (\frac{1-\delta}{1-\delta+w(1+\delta)}, \frac{w}{1-\delta+w(1+\delta)}, \frac{w(1-\delta+w)}{(1-\delta+w(1+\delta))^2}) & \text{if } \delta_{p^{\gamma}}^{RNRL} < \delta \leq \delta_{\pi^{u}}^{RNRL} \\ (0, \frac{1+\gamma}{2}, \frac{1+\gamma}{4}) & \text{if } \delta_{\pi^{u}}^{RNRL} < \delta \leq \delta_{p^{\lambda}}^{RNRL} \\ (\frac{(1+\lambda)l(\lambda)}{g(\lambda)}, \frac{(1+\lambda)k(\lambda)}{g(\lambda)}, \frac{((1+\lambda)(2-\delta)^2+4w)}{2g(\lambda)}) & \text{if } \delta_{p^{\lambda}}^{RNRL} < \delta \leq \delta_{\pi^{\lambda}}^{RNRL} \\ (0, \frac{1+\gamma}{2}, \frac{1+\gamma}{4}) & \text{if } \delta_{\pi^{\lambda}}^{RNRL} < \delta < 1 \end{cases}, & \text{if } \delta_{\tau^{\lambda}}^{RNRL} > \delta_{p^{\lambda}}^{RNRL} \end{cases}$

Result 2.1 *In the benchmark scenario, the firm adopts a skimming pricing strategy, and the demand is increasing over time.*

In the absence of WoM and learning-by-doing effects, it is optimal to first sell the product to consumers having a high willingness to pay, and next decrease the price to reach other market segments. This result is in line with the literature; see, e.g., Kalish (1983); Krishnan et al. (1999).

Result 2.2 *In the presence of network externality:*

1. *If*
$$w > w^{RN} = f(\delta) = \frac{\delta(1-\delta) + \sqrt{\delta^2(1-\delta)^2 + 8(1-\delta)(2-\delta)}}{4},$$
then it is optimal to implement a penetration pricing strategy; otherwise skimming pricing is optimal,

2. *The market penetration of the new product is higher than in the benchmark scenario, i.e.,* $D^{RN} = D_1^{RN} + D_2^{RN} > D^{RB} = D_1^{RB} + D_2^{RB}$*, because* $D^{RN} - D^{RB} = \frac{w(4(1+w)-\delta(7-4\delta+3w))}{2(3-2\delta)((3-w)(1+w)-2\delta(1-w))} > 0.$

Clearly, the pricing strategy depends critically on the intensity of social influences. When this intensity is strong enough, then it is optimal to start with a low price to stimulate early adoption and benefit from a high externality effect in the second period. Otherwise, it is optimal to follow a skimming pricing strategy, for the same reason as in the benchmark scenario. As compared to the benchmark, the price is lower in the first period for all admissible parameter values, which results in a higher demand. Even a small social effect is worth exploiting. Therefore, the aggregate demand would be higher than in the benchmark scenario, and the marginal difference is increasing with respect to the degree of social influences.

Remark 2.2 *The set of values that satisfy the condition* $w > f(\delta)$ *is not empty. Indeed, we have* $f(\delta) \in [0,1]$*, with* $f(0) = 1$*,* $f(1) = 0$*, and* $f'(\delta) < 0$*. If consumers are perfectly farsighted, i.e.,* $\delta = 1$ *, then the condition is always satisfied.*

Thus, Remark 2.2 suggests that forward-looking behavior paves the way for adopting penetration pricing strategy.

Result 2.3 *In the presence of the reference-price effect:*

1. *The firm adopts a skimming pricing strategy,*

2. *The market penetration of a new product is higher than in the benchmark scenario i.e.,* $D^{RR} = D_1^{RR} + D_2^{RR} > D^{RB} = D_1^{RB} + D_2^{RB}$*, because*

$$D^{RR} - D^{RB} = \begin{cases} \frac{3\theta - 1 + 12\delta\theta}{6-4\delta} > 0, & \textit{if } p_1 = 1 \textit{ (high launch price);} \\ \frac{\theta(2-\delta)^2(1-\delta)}{2(3-2\delta)(3-2\delta+\theta(1-\delta)^2)} > 0, & \textit{otherwise.} \end{cases}$$

The presence of a reference price in the second-period demand leads the firm to adopt a skimming pricing strategy. In both periods, the price is higher in this scenario than in the benchmark scenario. The firm achieves a dual benefit from this behavior. First, it allows, in period 1, to target consumers with a high utility for the product, as in the benchmark scenario. Second, it provides a psychological surplus to consumers in the second period, which is measured by $\theta(p_1 - p_2)$. Consequently, consumers adopt the product in larger numbers than in the benchmark scenario. Interestingly, if the marginal impact of the reference price is beyond a certain threshold, that is, $\theta > \frac{2(1-\delta)}{2(1-\delta)+\delta^2}$, then it is optimal to set a maximum price in period 1, that is, $p_1 = 1$, which leads to zero demand in that period. The rationale for such action is to maximize the reference-price effect on the second-period demand. We refer to this situation as *a high launch price*. Additionally, as consumers become more forward-looking, i.e., $\delta \to 1$, it would be easier to satisfy the above condition, which means that the firm would be able to charge aggressive skimming prices even when consumers are not strongly backward-looking. Besides, the firm is able to capture a higher market penetration than in the benchmark scenario.

Result 2.4 *In the presence of network externality and a symmetric reference-price effect:*

1. *If $\delta \leq \delta_{\pi}^{RNR}$ and $w > w^{RNR}$, then the firm implements a penetration pricing strategy where*

   $$w^{RNR} = \min\{1, \frac{\delta(1-\delta)+\theta(2-\delta^2)+\sqrt{(\delta(1-\delta)+\theta(2-\delta^2))^2+8(1-\delta)(2-\delta)(1+\theta)}}{4}\} > 0.$$

   otherwise, skimming pricing strategy would be the optimal choice,

2. *If $\delta \leq \delta_{\pi}^{RNR}$, then market penetration under RNR is greater than the benchmark. For $\delta > \delta_{\pi}^{RNR}$, however, it is greater if $\theta > \frac{1-\delta}{3-2\delta}$.*

Result 2.4 suggests that strong forward-looking behavior may work against adoption of penetration pricing strategy which is ironic when compared with Result 2.2. However, this can be explained when we account for the implications of reference price presence. In particular, when the reference-price effect accompanies social externalities, then strong strategic behavior can make the psychological benefits more appealing in period 2, and as a result, a skimming pricing strategy materializes. However, the firm may lose interest to skimming pricing when weak farsightedness undermines the psychological surplus in the late period. Alternatively, the firm will be better off with a penetration pricing strategy if strong externalities can compensate for the weak farsightedness.

Moreover, the above results suggest that market penetration is higher under RNR in the presence of strong forward-looking behavior. When this is not the case and when the reference-price effect is not prominent, then fewer consumers may end up adopting in the RNR scenario compared to the benchmark. Moreover, the extent of penetration pricing is less, compared to the RN scenario, because $w^{RNR} > w^{RN}$, which signals that this pricing strategy is not favorable in the presence of backward-looking behavior.

Result 2.5 *In the presence of network externality and an asymmetric reference-price effect, the pricing strategy might be skimming, constant or penetration.*

The literature reports, that depending on the magnitude of the initial reference point, the firm may adopt a contingent pricing strategy, that is, a penetration, constant, or skimming pricing strategies. Our result generalizes to forward-looking consumers the similar result obtained in the literature with reference-dependent loss-averse myopic consumers (see, for example, Popescu and Wu, 2007, Theorem 4). However, the optimal pricing strategy depends on the whole dynamics of consumer adoption behavior, including social influences and the intensity of backward-looking behavior. If we had assumed that there is an initial reference point in the launch period too, then the choice of pricing strategy would additionally depend on the initial reference price. The main takeaway is that the firm takes into account the whole dynamics of consumers' adoption behaviors in pricing decision, and not solely their initial reference point.

To summarize, the monopoly pricing and market penetration of new products depend critically on the social influences, forward-backward-looking behavior, and whether consumers are loss averse. For instance, the monopolist's pricing strategy can change from a very aggressive skimming pricing strategy in the *RR* case to constant or penetration pricing in the *RNRL* case, highlighting the impact of the interplay between the asymmetric reference-price effects and social influences.

2.5 Preannounced pricing

In this section, we examine the role of commitment in the firm's pricing strategy. The firm preannounces the full price path at the launch period, and consumers make their decisions accordingly. This pricing regime has been shown to be effective, that is, leading to higher outcomes, when consumers are forward-looking (Aviv and Pazgal, 2008).

As in the previous section, we illustrate the solution approach using the simplest benchmark case. First, the firm preannounces the prices p_1 and p_2. Given this information, consumers with valuations higher than a threshold τ adopt in the first period, whereas the remaining consumers may adopt in period 2. This threshold is defined by

$$\tau(p_1, p_2) = \begin{cases} p_1, & \text{if} \quad p_1 \leq p_2, \\ \frac{p_1 - \delta p_2}{1-\delta}, & \text{if} \quad p_1 > p_2 \quad \text{and} \quad p_1 - \delta p_2 \leq 1 - \delta, \\ 1, & \text{if} \quad p_1 > p_2 \quad \text{and} \quad p_1 - \delta p_2 > 1 - \delta. \end{cases} \tag{2.5}$$

The firm determines its pricing strategy by optimizing its total profit, i.e.,

$$\max_{p_1, p_2} \pi = p_1 D_1 + p_2 D_2 = p_1(1 - \tau(p_1, p_2)) + p_2(\tau(p_1, p_2) - p_2). \tag{2.6}$$

It is easy to verify that $\tau(p_1, p_2) = \frac{p_1 - \delta p_2}{1-\delta}$ maximizes the firm's profit, whereas $\tau(p_1, p_2) = p_1$ and $\tau(p_1, p_2) = 1$ yield suboptimal solutions. The following proposition shows the equilibrium solution for all cases. We use the following notations throughout our analysis.

$$
\begin{aligned}
&n(i) = 2(1-\delta)(1+i) + (1+\delta)(1+i)w - w^2 > 0, \quad i \in (\theta, \gamma, \lambda) \\
&q(i) = (1-\delta^2)(1+i) + (1+\delta+2\delta i)w > 0, \quad i \in (\theta, \gamma, \lambda) \\
&m(i) = (1-\delta)(1+i)(3+\delta-i(1-\delta)) + 2(1+\delta)(1+i)w - w^2 > 0, \quad i \in (\theta, \gamma, \lambda) \\
&t(i) = (1+i)((1-\delta)(1-i)+w) > 0, \quad i \in (\theta, \gamma, \lambda) \\
&v(i) = (1+i)((1-\delta)(1+i)+w) > 0, \quad i \in (\theta, \gamma, \lambda) \\
&y(i) = (1+i)(1+w-\delta) > 0, \quad i \in (\theta, \gamma, \lambda) \\
&s = 2(\delta(w-1)+1)(\delta(2w-1)+w+1), \\
&z = 2\delta^2(w-1) + \delta\left(2w^2 - w + 2\right) + w(w+1), \\
&\delta_{\pi}^{PR} = \tfrac{(1-\theta)^2}{(1+\theta)^2} > 0, \\
&\delta_{\pi}^{PNR} = \tfrac{1+w+w\theta+\theta^2-2\theta\sqrt{1+w+w\theta}}{(1+\theta)^2} > 0, \\
&\delta_{p^{\gamma}}^{PNRL} = 1 - \tfrac{w}{1+\gamma} > 0, \\
&\delta_{p^{\lambda}}^{PNRL} = 1 - \tfrac{w}{1+\lambda} > 0, \\
&\delta_{\pi^{\gamma}}^{PNRL} = \min\{1, \tfrac{1+w+w\gamma+\gamma^2-2\gamma\sqrt{1+w+w\gamma}}{(1+\gamma)^2}\} > 0, \\
&\delta_{\pi^{u}}^{PNRL} = \max\{0, \tfrac{w\sqrt{\gamma^2w^2-4\gamma^2w+4\gamma^2+10\gamma w^2+8\gamma+9w^2+4w+4}-\gamma w^2-2\gamma w+2\gamma-w^2-4w}{2(2\gamma w^2-3\gamma w+\gamma+2w^2-3w)}\} > 0,
\end{aligned}
$$

$$
\delta_{\pi\lambda}^{PNRL} = \min\{1, \tfrac{-2\sqrt{\gamma^2\lambda^3w+2\gamma^2\lambda^2w+\gamma^2\lambda^2+\gamma^2\lambda w+2\gamma^2\lambda+\gamma^2+\gamma\lambda^3w+2\gamma\lambda^2w+\gamma\lambda w}+\gamma w-\gamma+w+\lambda(2+w+\lambda+\gamma\lambda+\gamma w)+1}{\gamma\lambda^2+2\gamma\lambda+\gamma+\lambda^2+2\lambda+1}\} > 0.
$$

Proposition 2.2 *If the firm adopts preannounced pricing regime, then the optimal pricing strategies and the demands and profits in the five scenarios are those given in Tables 2.5 and 2.6 , respectively.*

The benchmark case in Proposition 2.2 is similar to Proposition 1 in Papanastasiou and Savva (2017). Next, we give a series of results that have some managerial implications in relation to the firm's and consumers' decisions.

Result 2.6 *In the benchmark scenario, the firm adopts a skimming pricing strategy, and the demand rate is the same in both periods.*

Result 2.6 is similar to Proposition 1 in Papanastasiou and Savva (2017), suggesting that the firm is better off with a skimming pricing strategy.

Table 2.5: Preannounced pricing strategies

Scenarios	Pricing strategies
PB	$(p_1, p_2) = (\frac{2}{3+\delta}, \frac{1+\delta}{3+\delta})$
PN	$(p_1, p_2) = (\frac{2-w}{3-w+\delta}, \frac{1+\delta}{3-w+\delta})$
PR	$(p_1, p_2) = \begin{cases} (1, \frac{1}{2}), & \text{if } \delta > \delta_{\pi}^{PR} \\ (\frac{2}{3+\delta-\theta(1-\delta)}, \frac{(1+\delta)}{3+\delta-\theta(1-\delta)}), & \text{if } \delta \leq \delta_{\pi}^{PR} \end{cases}$
PNR	$(p_1, p_2) = \begin{cases} (1, \frac{1}{2}), & \text{if } \delta > \delta_{\pi}^{PNR} \\ (\frac{2(1-\delta)(1+\theta)+(1+\delta)(1+\theta)w-w^2}{m(\theta)}, \frac{(1-\delta^2)(1+\theta)+(1+\delta+2\delta\theta)w}{m(\theta)}), & \text{if } \delta \leq \delta_{\pi}^{PNR} \end{cases}$
PNRL	$(p_1, p_2) = \begin{cases} \begin{cases} (\frac{n(\gamma)}{m(\gamma)}, \frac{q(\gamma)}{m(\gamma)}) & 0 < \delta \leq \delta_{\pi^\gamma}^{PNRL} \\ (1, \frac{1}{2}) & \delta_{\pi^\gamma}^{PNRL} < \delta < 1 \end{cases}, & \text{if } \delta_{\pi^\gamma}^{PNRL} \leq \delta_{p^\gamma}^{PNRL} \\ \begin{cases} (\frac{n(\gamma)}{m(\gamma)}, \frac{q(\gamma)}{m(\gamma)}) & 0 < \delta \leq \delta_{p\gamma}^{PNRL} \\ (\frac{w-\delta+1}{2w(1+2\delta)+2(1-\delta)}, \frac{w-\delta+1}{2w(1+2\delta)+2(1-\delta)}) & \delta_{p\gamma}^{PNRL} < \delta \leq \delta_{\pi^u}^{PNRL} \\ (1, \frac{1}{2}) & \delta_{\pi^u}^{PNRL} < \delta \leq \delta_{p^\lambda}^{PNRL} \\ (\frac{n(\lambda)}{m(\lambda)}, \frac{q(\lambda)}{m(\lambda)}) & \delta_{p^\lambda}^{PNRL} < \delta \leq \delta_{\pi^\lambda}^{PNRL} \\ (1, \frac{1}{2}) & \delta_{\pi^\lambda}^{PNRL} < \delta < 1 \end{cases}, & \text{if } \delta_{\pi^\gamma}^{PNRL} > \delta_{p^\gamma}^{PNRL} \end{cases}$

Result 2.7 *In the presence of network externality, if $w > 1 - \delta$, the firm adopts a penetration pricing strategy; otherwise, it is better off with skimming pricing strategy.*

The above result is intuitive. Indeed, if the network effect is high enough, then the firm should capitalize on it and initially offer the product at a low price and then increase it. Note that the more strategic (or farsighted) the consumer is, i.e., the higher the value of δ, then the easier it is to satisfy the inequality in the statement. In particular, if we let $\delta \to 1$, then a penetration strategy would be the only possible result.

Result 2.8 *In the presence of the reference-price effect, the firm adopts a skimming pricing strategy.*

This result is the mirror of the previous case. As the second-period utility (and demand) is increasing in the first-period price, it is in the best interest of the firm

Table 2.6: Demands and profit under preannounced pricing

Scenarios	Demands and profit
PB	$(D_1, D_2, \pi) = (\frac{1}{3+\delta}, \frac{1}{3+\delta}, \frac{1}{3+\delta})$
PN	$(D_1, D_2, \pi) = (\frac{1}{3-w+\delta}, \frac{1}{3-w+\delta}, \frac{1}{3-w+\delta})$
PR	$(D_1, D_2, \pi) = \begin{cases} (0, \frac{1+\theta}{2}, \frac{1+\theta}{4}), & \text{if } \delta > \delta_\pi^{PR} \\ (\frac{(1-\theta)}{3+\delta-\theta(1-\delta)}, \frac{(1+\theta)}{3+\delta-\theta(1-\delta)}, \frac{1}{3+\delta-\theta(1-\delta)}), & \text{if } \delta \le \delta_\pi^{PR} \end{cases}$
PNR	$(D_1, D_2, \pi) = \begin{cases} (0, \frac{1+\theta}{2}, \frac{1+\theta}{4}), & \text{if } \delta > \delta_\pi^{PNR} \\ (\frac{(1+\theta)((1-\delta)(1-\theta)+w)}{m(\theta)}, \frac{(1+\theta)((1-\delta)(1+\theta)+w)}{m(\theta)}, \frac{(1+\theta)(1-\delta+w)}{m(\theta)}), & \text{if } \delta \le \delta_\pi^{PNR} \end{cases}$
PNRL	$(D_1, D_2, \pi) = \begin{cases} \begin{cases} (\frac{t(\gamma)}{m(\gamma)}, \frac{v(\gamma)}{m(\gamma)}, \frac{y(\gamma)}{m(\gamma)}) & 0 < \delta < \delta_{\pi^\gamma}^{PNRL} \\ (0, \frac{1+\gamma}{2}, \frac{1+\gamma}{4}) & \delta_{\pi^\gamma}^{PNRL} \le \delta < 1 \end{cases}, & \text{if } \delta_{\pi^\gamma}^{PNRL} \le \delta_{p^\gamma}^{PNRL} \\ \begin{cases} (\frac{t(\gamma)}{m(\gamma)}, \frac{v(\gamma)}{m(\gamma)}, \frac{y(\gamma)}{m(\gamma)}) & 0 < \delta \le \delta_{p\gamma}^{PNRL} \\ (\frac{\delta^2(3-4w)+\delta(w-4)+w+1}{s}, \frac{z}{s}, \frac{(-\delta+w+1)^2}{2s}) & \delta_{p\gamma}^{PNRL} < \delta \le \delta_{\pi^u}^{PNRL} \\ (0, \frac{1+\gamma}{2}, \frac{1+\gamma}{4}) & \delta_{\pi^u}^{PNRL} < \delta \le \delta_{p^\lambda}^{PNRL} \\ (\frac{t(\lambda)}{m(\lambda)}, \frac{v(\lambda)}{m(\lambda)}, \frac{y(\lambda)}{m(\lambda)}) & \delta_{p^\lambda}^{PNRL} < \delta \le \delta_{\pi^\lambda}^{PNRL} \\ (0, \frac{1+\gamma}{2}, \frac{1+\gamma}{4}) & \delta_{\pi^\lambda}^{PNRL} < \delta < 1 \end{cases}, & \text{if } \delta_{\pi^\gamma}^{PNRL} > \delta_{p^\gamma}^{PNRL} \end{cases}$

to adopt a skimming pricing strategy. With such a strategy, in period 1, the firm sells at a high price to consumers having a large valuation of the product, and it attracts a higher demand in the second period with the positive effect of the reference price.

Comparing the benchmark and the reference-price scenarios, we can highlight the following features: (i) In both scenarios, the firm implements a skimming pricing strategy ($p_1^{PB} > p_2^{PB}$ and $p_1^{PR} > p_2^{PR}$). (ii) The firm charges higher prices in both periods when consumers consider a reference price ($p_1^{PR} > p_1^{PB}$ and $p_2^{PR} > p_2^{PB}$), with $\frac{p_1^{PB}}{p_2^{PB}} = \frac{p_1^{PR}}{p_2^{PR}}$, that is, the ratios of the first-period price to the second-period price are equal in both scenarios; (iii) The relative profits are equal to the relative prices, i.e., $\frac{\pi^{PB}}{\pi^{PR}} = \frac{p_1^{PB}}{p_1^{PR}} = \frac{p_2^{PB}}{p_2^{PR}}$.

Result 2.9 *In the presence of network externality and a symmetric reference-price effect, if $\delta \le \delta_\pi^{PNR}$ and $w > \min\{(1+\theta)(1-\delta), 1\}$, then the firm adopts a penetration pricing strategy; otherwise, skimming pricing is the optimal choice.*

The choice of the pricing strategy depends on the interplay between the network externality, the reference price, and the consumer's farsightedness parameters. The short interpretation is that if consumers are not too farsighted and the marginal network effect w is strong enought, then the firm is better off to adopt an increasing pricing path. However, if either the network effect is small (i.e., $w \leq \min\{(1+\theta)(1-\delta), 1\}$) or consumers are farsighted enough ($\delta > \delta_{\pi}^{PNR}$), then the skimming pricing with a high launch price is optimal to benefit from the psychological surplus in the second period. By comparing with PN scenario, we can also once again see that the presence of backward-looking behavior reduces the extent of penetration pricing strategy because $\min\{(1+\theta)(1-\delta), 1\} > (1-\delta)$.

Result 2.10 *In the presence of network externality and asymmetric reference-price effect, the firm may adopt skimming, constant or penetration pricing strategy.*

As in the previous result, the choice of a pricing strategy depends on all parameters involved in the second-period demand function. The firm may either adopt only skimming pricing strategy (if $\delta_{\pi\gamma}^{PNRL} \leq \delta_{p\gamma}^{PNRL}$) or skimming, constant and penetration pricing strategies (if $\delta_{\pi\gamma}^{PNRL} > \delta_{p\gamma}^{PNRL}$) depending on the parameters which generalize what has been reported in the literature to the case where the social influences and strategic behavior are present. constant prices can be optimal under some conditions, which was not the case before in the current section. This happens when neither externality nor the reference-price effect prevail leading to inertia wherein price variations are not optimal. Indeed given that the prices are preannounced, such pricing strategy implies that the firm is reluctant to create either a motivation for early adoption or psychological utility for consumers via dynamic pricing.

2.6 Comparison

In this section, we characterize the conditions under which the firm is better off choosing a preannounced pricing strategy (respectively, responsive pricing strategy), and check if this choice is the one preferred by consumers. The detailed results are provided in the Appendix.

Result 2.11 *For all admissible parameter values, a preannounced pricing strategy in the benchmark scenario leads to higher prices and a higher profit, and to a lower market penetration than does responsive pricing, that is,*

$$p_1^{PB} > p_1^{RB}, \quad p_2^{PB} > p_2^{RB}, \quad \pi^{PB} > \pi^{RB}, \quad D^{PB} < D^{RB}.$$

Result 2.12 *For all admissible parameter values, in the presence of network externality, a preannounced pricing strategy leads to higher prices and a higher profit, and to a lower market penetration than does responsive pricing, that is,*

$$p_1^{PN} > p_1^{RN}, \quad p_2^{PN} > p_2^{RN}, \quad \pi^{PN} > \pi^{RN}, \quad D^{PN} < D^{RN}.$$

The recommendation from Results 2.11-2.12 is clear: the firm is better off implementing a preannounced pricing strategy in both considered scenarios. This choice does not suit consumers because prices are higher in both periods and the demand is lower. In the benchmark scenario, we already obtained that, under both pricing strategies, price skimming is the optimal choice. Result 2.11 replicates what has been obtained in, e.g., Aviv and Pazgal (2008) or Papanastasiou and Savva (2017) in the absence of social learning, namely, that preannounced pricing is the right strategy when consumers are strategic. Result 2.12 is telling us that the logic remains unaltered if we add in the network externality.

Result 2.13 *For all admissible parameter values, in the presence of the reference-price effect, a preannounced pricing strategy leads to weakly higher prices. Moreover, the demand is lower and the profit is higher under preannounced pricing regime when consumers are* *either little ($0 < \delta < \delta_\pi^{PR}$) or too ($\delta_\pi^{RR} \leq \delta < 1$) farsighted. For $\delta_\pi^{PR} \leq \delta < \delta_\pi^{RR}$, the profit under responsive pricing might prevail.*

For $\delta > \delta_\pi^{RR}$, the prices, demand and profit are the same across responsive and preannounced pricing regimes whereby it is optimal to set $p_1 = 1$ (high launch price) and $p_2 = \frac{1}{2}$. For $0 < \delta \leq \delta_\pi^{PR}$, the prices and profit are higher under committed pricing. For $\delta_\pi^{PR} < \delta \leq \delta_\pi^{RR}$, it may happen that the profit under responsive pricing would be higher. This suggests that in the presence of social influences and reference-price effect, the firm might be better off with responsive pricing when consumers are neither too nor little strategic. Papanastasiou and Savva (2017) also finds that in the presence of social learning, the firm might obtain a higher profit with responsive pricing regime whereas in the literature the preannounnced pricing are deemed to be more profitable. Therefore, we show that such result can be obtained under different nuances of consumer behavior.

Result 2.14 *In the presence of network externality and a symmetric reference-price effect, a preannounced pricing strategy leads to weakly higher prices, lower demand and higher profit only when consumers are little ($0 < \delta \leq \delta_\pi^{RNR}$) or too ($\delta_\pi^{PNR} < \delta \leq 1$) farsighted. For $\delta_\pi^{RNR} \leq \delta < \delta_\pi^{PNR}$, the profit under responsive pricing might prevail.*

Result 2.15 *In the presence of network externality and a asymmetric reference-price effect, the monopoly adopts a similar pricing regimes across preannouned and responsive pricing in which either skimming, constant or penetration pricing might emerge.*

Result 2.14 is similar to 2.13 by suggesting that the preannounced pricing leads to a higher profit when consumers tend to be little or too strategic. The monopoly profit under responsive pricing might be higher, however, under certain conditions. Result 2.15 points out that the skimming, constant and penetration pricing strategies might be adopted by monopoly regardless of whether it utilizes committed or contingent pricing regimes.

2.7 Conclusion

This research is concerned with monopoly dynamic pricing for a new product when consumers look both forward and backward in the presence of social influences and loss aversion. We characterized and compared the results for two possible pricing schemes, namely, responsive and preannounced pricing. We adopt a rational expectation equilibrium between the firm and consumers to determine the prices, the demand rates, and the monopolist's performance. For both pricing strategies, we explore how the results vary with different consumer behaviors.

We find that the pricing regimes and the rate of new product demand depend heavily on the underlying consumer behaviors. When consumers are only prone to reference-price effect, the firm tend to adopt a skimming pricing strategy; however, its intensity depends on the degree to which consumers are inclined to look backward. For instance, when consumers are highly either forward-looking or sensitive to the reference price, then the firm adopts an aggressive skimming pricing strategy. When only social influences are present, the skimming pricing is optimal only when network effect is not strong; otherwise the penetration pricing strategy will be the optimal choice. In the same setting, forward-looking behavior advocates a penetration pricing strategy, so much so that such pricing strategy can be always optimal when consumers become fully strategic. Ironically, strong forward-looking behavior can work against penetration pricing strategy when both reference-price effect and social externalities are present. This is because such behavior advocates skimming pricing strategy when consumers become reference dependent, and hence only if consumers are not strongly strategic then a penetration pricing strategy might be the optimal choice when accompanied by powerful crowding externalities.

In the presence of loss aversion, the monopoly may resort to a constant pricing strategy when neither social influences nor backward-looking behavior are dominant. We also show that the monopoly might be better off with responsive pricing strategy under certain conditions, albeit the committed pricing leads to a higher profit in majority of cases. We also generalize the conditional pricing strategy composed of penetration, constant, and skimming pricing strategies in the literature to reference-dependent loss-averse forward-looking consumers. We show that the type of pricing regime depends on the whole dynamics of consumer adoption behaviors rather than just the initial reference point.

The current study can be extended by considering more sophisticated consumers. For instance, one can consider a mixed population of myopic and forward-looking behaviors and also account for other reference price mechanisms e.g. peak-end anchoring, as proposed by Nasiry and Popescu (2011). Another avenue for future research is to study a fully dynamic multi-period diffusion model to examine how the evolving reference-price effect goes hand in hand with the new product diffusion mechanism over time. This is particularly important since the literature suggests that behavioral regularities, such as loss aversion, are more prevalent in durables (Neumann and Böckenholt, 2014) and the aggregate diffusion rate does not necessarily inherit such regularities from individual-level behavior (Hu and Nasiry, 2018).

2.8 Appendix

Proof of Proposition 2.1. We demonstrate the proof of Proposition 2.1 using a backward induction for each scenario in responsive pricing settings similar to previously described benchmark.

- **Proof of RN.** The proof of *RN* is quite similar to the benchmark i.e., *RB* which was described before. Hence, the proof of *RN* is removed for the sake of brevity.
- **Proof of RR.** Using a backward induction approach, the firm's optimization

problem in period 2 is

$$\max_{p_2} \pi_2 = p_2(\tau - p_2 + \theta(p_1 - p_2)) \tag{2.7}$$

This yields $p_2 = \frac{\tau+\theta p_1}{2(1+\theta)}$ and $\pi_2 = \frac{(\tau\theta p_1)^2}{4(1+\theta)}$. Moreover, we know that the indifference equation $\tau - p_1 = \delta(\tau - p_2 + \theta(p_1 - p_2))$ leads to:

$$\tau(p_1) = \begin{cases} \frac{p_1(2+\delta\theta)}{2-\delta}, & \text{if } p_1 \leq \frac{2-\delta}{2+\delta\theta} \\ 1, & \text{otherwise} \end{cases} \tag{2.8}$$

Thus, we consider two cases where $\tau(p_1) = \frac{p_1(2+\delta\theta)}{2-\delta}$ in case *i* if $p_1 \leq \frac{2-\delta}{2+\delta\theta}$ and $\tau(p_1) = 1$ in case *ii* if $p_1 > \frac{2-\delta}{2+\delta\theta}$. Note that we might suppress the arguments of τ for ease of exposition throughout the whole appendix section.

Case *i*: $\tau(p_1) = \frac{p_1(2+\delta\theta)}{2-\delta}$ if $p_1 < \frac{2-\delta}{2+\delta\theta}$

The firm's problem over the whole planning horizon may be expressed as follows:

$$\max_{p_1} \pi = \quad p_1(1 - \tau(p_1)) + \pi_2^* = p_1(1 - \tau(p_1)) + \frac{(\tau(p_1)\theta p_1)^2}{4(1+\theta)} \tag{2.9}$$

We use the first-order conditions to obtain $p_1 = \frac{(2-\delta)^2}{2(3-2\delta-\theta(1-\delta)^2)}$. Once p_1 is obtained, the rest of decisions variables can be determined accordingly. If $\delta \leq \frac{-1+\theta+\sqrt{1-\theta}}{\theta}$, the condition $p_1 \leq \frac{2-\delta}{2+\delta\theta}$ holds and corresponding τ and π can be readily derived as follows:

$$\tau = \frac{(\delta-2)(\delta\theta+2)}{2\left((\delta-1)^2\theta + 2\delta - 3\right)}$$

$$\pi = \frac{(2-\delta)^2}{4(3-2\delta-\theta(1-\delta)^2)}$$

Case *ii*: $\tau(p_1) = 1$ if $p_1 \geq \frac{2-\delta}{2+\delta\theta}$

For this case, the firm's problem in period 2 is $\pi_2 = p_2(1 - p_2 + \theta(p_1 - p_2))$ whereby the first-order conditions give $p_2 = \frac{1+\theta p_1}{2(1+\theta)}$. Since π is an increasing linear function with respect to p_1 and $\frac{2-\delta}{2+\delta\theta} \leq p_1 \leq 1$, we can conclude that $p_1 = 1$ and hence $p_2 = \frac{1}{2}$ and $\pi = \frac{1+\theta}{4}$.

Up to now, we have obtained the equilibrium solution under each case and ensured that they are admissible. Ultimately, we need to compare the profits across two cases to determine under which conditions each case might yield a higher profit. The profit difference will be:

$$\Delta\pi = \left(\frac{(2-\delta)^2}{4(3-2\delta-\theta(1-\delta)^2)}\right) - \left(\frac{1+\theta}{4}\right) = \frac{\delta^2\left(\theta^2+\theta+1\right)-2\delta\left(\theta^2+1\right)+(\theta-1)^2}{4(3-2\delta-\theta(1-\delta)^2)}$$

Solving $\Delta\pi = 0$ in (2.8) suggests that there is a δ_π^{RR} below which the profit in case *i* is higher where $\delta_\pi^{RR} = \frac{1+\theta^2-(1+\theta)\sqrt{\theta}}{1+\theta+\theta^2}$, otherwise the profit in case *ii* prevails. It is easy to show that $\delta_\pi^{RR} < \delta_\tau^{RR}$ holds. That means for $0 < \delta \leq \delta_\pi^{RR}$, the solution of case *i* is both admissible and optimal whereas for $\delta_\pi^{RR} < \delta < 1$, the admissible solution of case *ii* leads to a higher profit. Therefore, the equilibrium outcome can be found in Proposition 2.1.

- **Proof of RNR.** We can utilize a similar approach here. The firm's optimization problem in period 2 is:

$$\max_{p_2} \pi_2 = p_2 D_2 = p_2(\tau - p_2 + \theta(p_1 - p_2) + wD_1) \tag{2.10}$$

Using the first-order conditions, one can obtain $p_2 = \frac{\tau(1-w)+\theta p_1+w}{2(\theta+2)}$ and $\pi_2 = \frac{(\tau(1-w)+\theta p_1+w)^2}{4(1+\theta)}$. The indifference equation, i.e., $\tau - p_1 = \delta(\tau - p_2 + \theta(p_1 - p_2) + w(1-\tau))$ yields

$$\tau(p_1) = \begin{cases} \frac{p_1(2+\delta\theta)+\delta w}{2-\delta+\delta w}, & \text{if } p_1 \leq \frac{2-\delta}{2+\delta\theta} \\ 1, & \text{otherwise} \end{cases} \tag{2.11}$$

Similar to previous section, we consider two cases below.

Case *i*: $\tau(p_1) = \frac{p_1(2+\delta\theta)+\delta w}{2-\delta+\delta w}$ if $p_1 \leq \frac{2-\delta}{2+\delta\theta}$

The firm's optimization problem over the two periods is as follows:

$$\max_{p_1} \pi = \quad p_1(1-\tau(p_1)) + \pi_2 = p_1(1-\tau(p_1)) + \frac{(\tau(1-w)+\theta p_1 + w)^2}{4(1+\theta)} \tag{2.12}$$

Solving the above optimization problem gives us:

$$p_1 = \frac{-((\delta-2)\delta-2)(\theta+1)w + (\delta-2)^2(\theta+1) - 2w^2}{2(\theta+1)w(\delta(\delta\theta+2)+2) - 2(\theta+1)\left((\delta-1)^2\theta+2\delta-3\right) - 2w^2}$$

For $\delta \leq \delta_{\tau}^{RNR}$, the condition $p_1 \leq \frac{2-\delta}{2+\delta\theta}$ holds and we can obtain

$$\tau = \frac{2(\theta+1)w(\delta(\delta\theta+2)+1) + (2-\delta)(\theta+1)(\delta\theta+2) - 2w^2}{2(\theta+1)w(\delta(\delta\theta+2)+2) - 2(\theta+1)\left((\delta-1)^2\theta+2\delta-3\right) - 2w^2}$$

$$\pi = \frac{(\theta+1)\left((\delta-2)^2+4w\right)}{4(\theta+1)w(\delta(\delta\theta+2)+2) - 4(\theta+1)\left((\delta-1)^2\theta+2\delta-3\right) - 4w^2}$$

Case *ii*: $\tau(p_1) = 1$ if $p_1 > \frac{2-\delta}{2+\delta\theta}$

For this case, the demand in period 1 is zero and in the absence of externalities, the results are the same as the case *ii* in *RR* scenario. Therefore, the equilibrium solution is $p_1 = 1$, $p_2 = \frac{1}{2}$ and $\pi = \frac{1+\theta}{4}$. Now we need to compare the profit across two cases. It is easy to show that for $0 < \delta \leq \delta_{\pi}^{RNR}$, the profit in case *i* is higher, otherwise, the profit in case *ii* will be optimal. One may note that $\delta_{\pi}^{RNR} < \delta_{\tau}^{RNR}$ holds. Thus, the equilibrium solution can be seen in Proposition 2.1.

- **Proof of RNRL**. A similar approach can be used for proof of *RNRL*. However, the consumers react to the reference-price effect asymmetrically, depending on whether they receive gains or losses. That means the firm's problem in period 2 can be expressed as follows:

$$\max_{p_2} \pi_2 = \quad p_2 D_2 = p_2(\tau - p_2 + \gamma(p_1-p_2)^+ + \lambda(p_1-p_2)^- + wD_1) \tag{2.13}$$

The above-mentioned firm's problem is not smooth. However, similar to Hu and Nasiry (2018), we can transform it into two smooth subproblems:

$$\max_{p_2} \pi_2^{\gamma} = \quad p_2 D_2 = p_2(\tau^{\gamma} - p_2 + \gamma(p_1 - p_2) + wD_1) \tag{2.14}$$

$$\max_{p_2} \pi_2^{\lambda} = \quad p_2 D_2 = p_2(\tau^{\lambda} - p_2 + \lambda(p_1 - p_2) + wD_1) \tag{2.15}$$

Solving these subproblems yields $p_2^{\gamma} = \frac{\tau^{\gamma}(1-w)+\gamma p_1+w}{2(\gamma+2)}$ and $\pi_2^{\gamma} = \frac{(\tau^{\gamma}(1-w)+\gamma p_1+w)^2}{4(1+\gamma)}$ and $p_2^{\lambda} = \frac{\tau^{\lambda}(1-w)+\lambda p_1+w}{2(\lambda+2)}$ and $\pi_2^{\lambda} = \frac{(\tau^{\lambda}(1-w)+\lambda p_1+w)^2}{4(1+\lambda)}$. From the indifference equation for each subproblem, we can obtain

$$\tau^{\gamma}(p_1) = \begin{cases} \frac{p_1(2+\delta\gamma)+\delta w}{2-\delta+\delta w}, & \text{if } p_1 \leq \frac{2-\delta}{2+\delta\gamma} \\ 1, & \text{otherwise} \end{cases} \tag{2.16}$$

$$\tau^{\lambda}(p_1) = \begin{cases} \frac{p_1(2+\delta\lambda)+\delta w}{2-\delta+\delta w}, & \text{if } p_1 \leq \frac{2-\delta}{2+\delta\lambda} \\ 1, & \text{otherwise} \end{cases} \tag{2.17}$$

where τ^{γ} and τ^{λ} represent the threshold policy for gain and loss subproblems, respectively. Similar to previous proofs, we consider two cases for each subproblem. We begin by solving the gain subproblem.

Case *i*: $\tau^{\gamma}(p_1) = \frac{p_1(2+\delta\gamma)+\delta w}{2-\delta+\delta w}$ if $p_1 \leq \frac{2-\delta}{2+\delta\gamma}$

The firm's optimization problem over the two periods is :

$$\max_{p_1} \pi^{\gamma} = \quad p_1(1-\tau^{\gamma}) + \frac{(\tau^{\gamma}(p_1)(1-w)+\gamma p_1 + w)^2}{4(1+\gamma)} \tag{2.18}$$

Like case *i* in RNR, the solution can be derived where

$$p_1^{\gamma} = \frac{-((\delta-2)\delta-2)(\gamma+1)w + (\delta-2)^2(\gamma+1) - 2w^2}{2(\gamma+1)w(\delta(\delta\gamma+2)+2) - 2(\gamma+1)\left((\delta-1)^2\theta + 2\delta - 3\right) - 2w^2}$$

For $\delta \leq \delta_{\tau^{\gamma}}^{RNRL}$, the condition $p_1 \leq \frac{2-\delta}{2+\delta\gamma}$ holds. Also, to ensure the gain condition i.e., $p_1 > p_2$ holds, we need to respect $\delta < \delta_{p^{\gamma}}^{RNRL}$ condition. Thus, under

$\delta \le \delta^{RNRL}_{\tau^\gamma}$ and $\delta < \delta^{RNRL}_{p^\gamma}$ conditions, τ^γ and π^γ can be expressed as follows:

$$\tau^\gamma = \frac{2(\gamma+1)w(\delta(\delta\gamma+2)+1)+(2-\delta)(\gamma+1)(\delta\theta+2)-2w^2}{2(\gamma+1)w(\delta(\delta\gamma+2)+2)-2(\gamma+1)\left((\delta-1)^2\gamma+2\delta-3\right)-2w^2}$$

$$\pi^\gamma = \frac{(\gamma+1)\left((\delta-2)^2+4w\right)}{4(\gamma+1)w(\delta(\delta\gamma+2)+2)-4(\gamma+1)\left((\delta-1)^2\gamma+2\delta-3\right)-4w^2}$$

Case *ii*: $\tau(p_1)=1$ if $p_1 > \frac{2-\delta}{2+\delta\gamma}$

Similar to case *ii* in *RR* and *RNR*, the results can be derived where $p_1 = 1$, $p_2 = \frac{1}{2}$ and $\pi^\gamma = \frac{1+\gamma}{4}$.

The comparison between the profits across two cases suggests that for $\delta < \delta^{RNRL}_{\pi^\gamma}$, the profit in case *i* is higher, otherwise, the profit in case *ii* prevails. One can also show that $\delta^{RNRL}_{\pi^\gamma} < \delta^{RNRL}_{\tau^\gamma}$ holds.

Next, we turn into solving the loss subproblem by considering two cases below.

Case *i*: $\tau^\lambda(p_1) = \frac{p_1(2+\delta\lambda)+\delta w}{2-\delta+\delta w}$ if $p_1 \le \frac{2-\delta}{2+\delta\lambda}$

The firm's optimization problem over the planning horizon is :

$$\max_{p_1} \pi^\lambda = \quad p_1(1-\tau^\lambda(p_1)) + \frac{(\tau^\lambda(1-w)+\lambda p_1 + w)^2}{4(1+\lambda)} \tag{2.19}$$

First-order optimality condition results in

$$p_1^\lambda = \frac{-((\delta-2)\delta-2)(\lambda+1)w+(\delta-2)^2(\lambda+1)-2w^2}{2(\lambda+1)w(\delta(\delta\lambda+2)+2)-2(\lambda+1)\left((\delta-1)^2\lambda+2\delta-3\right)-2w^2}$$

For $\delta \le \delta^{RNRL}_{\tau^\lambda}$, the condition $p_1 \le \frac{2-\delta}{2+\delta\lambda}$ holds. Furthermore, the loss condition i.e., $p_1 \le p_2$ holds if $\delta > \delta^{RNRL}_{p^\lambda}$. Consequently, under $\delta^{RNRL}_{p^\lambda} < \delta \le \delta^{RNRL}_{\tau^\lambda}$, we can have:

$$\tau^\lambda = \frac{2(\lambda+1)w(\delta(\delta\lambda+2)+1)+(2-\delta)(\lambda+1)(\delta\lambda+2)-2w^2}{2(\lambda+1)w(\delta(\delta\lambda+2)+2)-2(\lambda+1)\left((\delta-1)^2\lambda+2\delta-3\right)-2w^2}$$

$$\pi^\lambda = \frac{(\lambda+1)\left((\delta-2)^2+4w\right)}{4(\lambda+1)w(\delta(\delta\lambda+2)+2)-4(\lambda+1)\left((\delta-1)^2\lambda+2\delta-3\right)-4w^2}$$

Case *ii*: $\tau^{\lambda}(p_1) = 1$ if $p_1 > \frac{2-\delta}{2+\delta\lambda}$

The monopolist's optimization problem can be expressed as follows:

$$\max_{p_2} \pi_2^{\lambda} = \quad p_2 D_2 = p_2(1 - p_2 + \lambda(p_1 - p_2)) \tag{2.20}$$

where $D_1 = 0$ and hence there is no externalities effect. First-order optimality condition results in $p_2 = \frac{1+\lambda p_1}{2(1+\lambda)}$ and $\pi_2^{\lambda} = \frac{(1+\lambda p_1)^2}{4(1+\lambda)}$. We also know that $\tau^{\lambda} = 1$, $D_1 = 0$, $\pi^{\lambda} = \pi_2^{\lambda}$ and π^{λ} is an increasing function with respect to p_1. Moreover, given that $p_1 \leq p_2$ should hold for the loss subproblem, we can conclude that $p_1 = p_2 = \frac{1}{2+\lambda}$ and as a result $\pi^{\lambda} = \frac{1+\lambda}{(2+\lambda)^2}$.

Comparing the profits across case *i* and case *ii* suggests that the profit in former case is higher and hence its solution is the optimal one under the loss subproblem. Therefore, we will need to compare the monopolist's profit under loss subproblem with the one in gain subproblem under case *ii*. By so doing, we find that there is a $\delta_{\pi^{\lambda}}^{RNRL}$ such that the profit of the loss subproblem is higher for $\delta \leq \delta_{\pi^{\lambda}}^{RNRL}$. Therefore, the loss subproblem prevails for $\delta_{p^{\lambda}}^{RNRL} < \delta \leq \delta_{\pi^{\lambda}}^{RNRL}$ whereas the profit under case *ii* from gain subproblem is optimal for $\delta_{\pi^{\lambda}}^{RNRL} < \delta < 1$.

It is easy to show that $\delta_{p^{\gamma}}^{RNRL} < \delta_{p^{\lambda}}^{RNRL}$. This suggests that the firm may adopt a constant pricing strategy when neither gains nor losses situations occur i.e., $\delta_{p^{\gamma}}^{RNRL} < \delta \leq \delta_{p^{\lambda}}^{RNRL}$. Therefore, we obtain the equilibrium solution under such conditions as follows.

Under a constant pricing strategy, the firm's optimization problem in period 2 is

$$\max_{p} \pi_2^{u} = pD_2 = p(\tau - p + wD_1) \tag{2.21}$$

First-order optimality condition results in $p = \frac{\tau + w(1-\tau)}{2}$ and $\pi_2^{u} = \frac{(\tau+w(1-\tau))^2}{4}$. The indifference equation can be expressed below:

$$\tau^{u}(p) = \begin{cases} \frac{p(1-\delta)+\delta w}{1-\delta+\delta w}, & \text{if } p < 1 \\ 1, & \text{otherwise} \end{cases} \tag{2.22}$$

Assuming $p < 1$, we can solve a system of equations $\tau^u = \frac{p(1-\delta)+\delta w}{1-\delta+\delta w}$ and $p = \frac{\tau+w(1-\tau)}{2}$ that results in $p = \frac{w}{1-\delta+w(1+\delta)}$, $\tau^u = \frac{w(1+\delta)}{1-\delta+w(1+\delta)}$ and $\pi^u = \frac{w(w-\delta+1)}{(1-\delta+w(1+\delta))^2}$. Note that we did not optimize the firm's problem over two periods since we have only one unknown, however, the effects of purchasing decisions across both periods are accounted for. More specifically, τ is formed during purchasing decision in the first period and $p = \frac{\tau+w(1-\tau)}{2}$ is consumer's best response in the second period. Moreover, $p \geq 1$ and consequently $\tau = 1$ lead to $D_1 = 0$, $D_2 = 1-p$ and $\pi_2^u = p(1-p)$. First-order optimality condition results in $p = \frac{1}{2}$, however, this does not satisfy $p \geq 1$ condition and hence its solution is inadmissible. Therefore, the former solution is the admissible solution for the uniform pricing strategy case. Additionally, we need to examine whether the firm obtains a higher profit from $\pi^u = \frac{w(w-\delta+1)}{(1-\delta+w(1+\delta))^2}$ compared to case i in gain subproblem where $\pi^\gamma = \frac{(1+\gamma)}{4}$. Comparing the profits suggests that there is a $\delta_{\pi^u}^{RNRL}$ below which the profit in the constant subproblem is higher. Therefore, when $\delta_{p^\gamma}^{RNRL} < \delta \leq \delta_{p^\lambda}^{RNRL}$, the constant pricing strategy is optimal if $\delta_{p^\gamma}^{RNRL} < \delta \leq \delta_{\pi^u}^{RNRL}$ whereas the solution of case i in gain subproblem is higher if $\delta_{\pi^u}^{RNRL} < \delta \leq \delta_{p^\lambda}^{RNRL}$. One can show that $\delta_{\tau^\lambda}^{RNRL} < \delta_{\tau^\gamma}^{RNRL}$, $\delta_{p^\gamma}^{RNRL} < \delta_{p^\lambda}^{RNRL}$, $\delta_{\pi^\gamma}^{RNRL} < \delta_{\tau^\gamma}^{RNRL}$ and $\delta_{\pi^\gamma}^{RNRL} < \delta_{\tau^\lambda}^{RNRL}$ hold. We also assume that $\delta_{p^\lambda}^{RNRL} < \delta_{\pi^u}^{RNRL} < \delta_{p^\lambda}^{RNRL}$ and $\delta_{p^\lambda}^{RNRL} < \delta_{\pi^\gamma}^{RNRL}$.

Therefore, we can summarize the equilibrium solutions for $RNRL$ as follows. If $\delta_{\tau^\lambda}^{RNRL} < \delta_{\tau^\gamma}^{RNRL} < \delta_{p^\gamma}^{RNRL} < \delta_{p^\lambda}^{RNRL}$ holds, then the solution of case i from gain subproblem prevails for $0 < \delta \leq \delta_{\pi^\gamma}^{RNRL}$ whereas the solution of case ii from the same subproblem would be optimal for $\delta_{\pi^\gamma}^{RNRL} < \delta < 1$. However, if $\delta_{p^\gamma}^{RNRL} < \delta_{p^\lambda}^{RNRL} < \delta_{\tau^\lambda}^{RNRL} < \delta_{\tau^\gamma}^{RNRL}$, then the solution of case i from gain subproblem prevails for $0 < \delta \leq \delta_{p^\gamma}^{RNRL}$. The solution of the subproblem with a constant pricing strategy holds for $\delta_{p^\gamma}^{RNRL} < \delta \leq \delta_{\pi^u}^{RNRL}$ whereas the solution of case ii from gain subproblem prevails for $\delta_{\pi^u}^{RNRL} < \delta \leq \delta_{p^\lambda}^{RNRL}$. Moreover, the solution from case i in loss subproblem prevails for $\delta_{p^\lambda}^{RNRL} < \delta \leq \delta_{\pi^\gamma}^{RNRL}$ while the solution of case *ii* from gain subproblem is the optimal for $\delta^{RNRL}_{\pi^{\gamma}} < \delta < 1$.

■

Proof of Proposition 2.2.

- **Proof of PN.** A similar approach to benchmark is used to determine the equilibrium solution. The threshold purchasing policy can be determined using indifference equation as follows:

$$\tau(p_1, p_2) = \begin{cases} \frac{p_1 - p_2\delta + \delta w}{1-\delta+\delta w}, & \text{if} \quad p_1 - p_2\delta \leq 1-\delta, \\ 1, & \text{if} \quad p_1 - p_2\delta > 1-\delta. \end{cases} \tag{2.23}$$

If a consumer's valuation satisfy condition $\nu > \tau$, then she adopts in the first period, otherwise the consumer waits for the purchasing occasion in period 2. To determine the equilibrium solution, we examine two cases based on τ in (2.23). We also make sure that solution of each case satisfies the corresponding admissibility conditions and then choose a solution that secures a profit for the monopolist.

Case *i*: $\tau(p_1, p_2) = \frac{p_1 - p_2\delta + \delta w}{1-\delta+\delta w}$ if $p_1 - p_2\delta \leq 1 - \delta$

Given that $\tau = \frac{p_1 - p_2\delta + \delta w}{1-\delta+\delta w}$, we can obtain $D_1 = 1 - \tau$, $D_2 = \tau - p_2 + wD_1$ and $\pi = p_1 D_2 + p_2 D_2$ accordingly. It suffices to apply the first-order optimality conditions $\frac{\partial \pi}{p_1} = 0$ and $\frac{\partial \pi}{p_2} = 0$ to obtain:

$$p_1 = \frac{2-w}{(\delta - w + 3)}; \quad p_2 = \frac{(\delta+1)}{(\delta - w + 3)}; \quad \tau = \frac{\delta - w + 2}{\delta - w + 3} < 1; \quad \pi = \frac{1}{\delta - w + 3}$$

Case *ii*: $\tau(p_1, p_2) = 1$ if $p_1 - p_2\delta > 1 - \delta$

Given that $\tau = 1$, then $D_1 = 1 - \tau = 0$, $D_2 = 1 - p_2$ and $\pi = \pi_2 = p_2 D_2$ can be determined accordingly. The first-order optimality condition $\frac{\partial \pi}{p_2} = 0$ results in $p_2 = \frac{1}{2}$ and as a result $\pi = \frac{1}{4}$.

It is easy to show that the profit under case *i* is higher and thus the equilibrium solution is the one described in case *i*.

- **Proof of PR.** As in the benchmark proof, a similar approach can be used. A consumer with a valuation $\nu \geq \tau$ may adopt in the first period where

$$\tau(p_1, p_2) = \begin{cases} p_1, & \text{if} \quad p_1 \leq p_2, \\ \frac{p_1(1+\delta\gamma) - p_2\delta(1+\gamma)}{1-\delta}, & \text{if} \quad p_1 > p_2 \quad \text{and} \quad p_1(1+\delta\gamma) - p_2\delta(1+\gamma) \leq 1-\delta, \\ 1, & \text{if} \quad p_1 > p_2 \quad \text{and} \quad p_1(1+\delta\gamma) - p_2\delta(1+\gamma) > 1-\delta. \end{cases} \tag{2.24}$$

For a consumer with $\nu < \tau$, she waits for the late purchasing occasion. To determine the equilibrium solution, we examine three cases based on $\tau(p_1, p_2)$ described in (2.23). For each case, we ensure that derived solution satisfies the admissibility conditions and then choose the one that yields the a higher profit for the monopolist.

Case *i*: $\tau(p_1, p_2) = p_1$ if $p_1 \leq p_2$

Given that $\tau = p_1$, then $D_1 = 1 - \tau$ and $D_2 = 0$ can be obtained accordingly where no consumers adopt in second period since $u_2 < u_1$ as a result of a higher price, discounting and negative reference price effect at same period. Therefore, maximizing the total profit $\pi = p_1(1 - p_1)$ leads to

$$p_1 = \frac{1}{2}; \quad \tau = p_1 = \frac{1}{2} < 1; \quad \pi = \frac{1}{4} \tag{2.25}$$

Case *ii*: $\tau(p_1, p_2) = \frac{p_1(1+\delta\gamma) - p_2\delta(1+\gamma)}{1-\delta}$ if $p_1(1+\delta\gamma) - p_2\delta(1+\gamma) \leq (1-\delta)$

Given that $\tau(p_1, p_2) = \frac{p_1(1+\delta\gamma) - p_2\delta(1+\gamma)}{1-\delta}$, one can obtain $D_1 = 1 - \tau$, $D_2 = \tau - p_2 + \theta(p_1 - p_2)$ and $\pi = p_1(1-\tau) + p_2(\tau - p_2 + \theta(p_1 - p_2))$ accordingly. It suffices to use first-order optimality conditions i.e., $\frac{\partial \pi}{p_1} = 0$ and $\frac{\partial \pi}{p_2} = 0$ to derive the equilibrium solution as follows:

$$p_1 = \frac{2}{\delta + 3}; \quad p_2 = \frac{\delta + 1}{\delta + 3}; \quad \tau = \frac{\delta + 2}{\delta + 3} < 1; \quad \pi = \frac{1}{\delta + 3} \tag{2.26}$$

It is clear that profit in case *ii* is higher than case *i*.

Case *iii*: $\tau = 1$ if $p_1(1+\delta\theta) - p_2\delta(1+\theta) > (1-\delta)$

Given that $\tau = 1$, then $D_1 = 1 - \tau = 0$, $D_2 = 1 - p_2 + \theta(p_1 - p_2)$ and $\pi = \pi_2 = p_2 D_2$. The first-order optimality condition $\frac{\partial \pi}{p_2} = 0$ results in $p_2 = \frac{\theta p_1 + 1}{2(\theta+1)}$. To determine p_1, we know from $p_1(1+\delta\theta) - p_2\delta(1+\theta) > (1-\delta)$ that

$$\frac{2-\delta}{2+\delta\theta} < p_1 \leq 1$$

we also know that $\pi = p_2(1 - p_2 + \theta(p_1 - p_2)$ is an increasing function with respect to p_1 and hence $p_1 = 1$. It is easy to show that the corner solution $p_1 = \frac{2-\delta}{2+\delta\theta}$ results in a lower profit and therefore it is not optimal under this case. Thus, the equilibrium solution in this case is:

$$p_1 = 1; \quad p_2 = \frac{1}{2}; \quad \tau = 1; \quad \pi = \frac{1+\theta}{4}$$

Since the profit in case *ii* is inferior, we need to compare the profits between case *i* and case *iii* where, as described Proposition 2.2, the former leads to a higher profit for $\delta \leq \delta_\pi^{PR}$ whereas the latter does so otherwise.

- **Proof of PNR.** Using a similar approach, the threshold purchasing policy can be described below:

$$\tau(p_1, p_2) = \begin{cases} \frac{p_1(1+\delta\gamma) - p_2\delta(1+\gamma) + \delta w}{1-\delta+\delta w}, & \text{if} \quad p_1(1+\delta\gamma) - p_2\delta(1+\gamma) \leq 1-\delta, \\ 1, & \text{if} \quad p_1(1+\delta\gamma) - p_2\delta(1+\gamma) > 1-\delta. \end{cases} \tag{2.27}$$

we examine two cases depending on τ described in (2.27) and follows a similar approach as in previous proofs.

Case *i*: $\tau(p_1, p_2) = \frac{p_1(1+\delta\theta) - p_2\delta(1+\theta) + \delta w}{1-\delta+\delta w}$ if $p_1(1+\delta\theta) - p_2\delta(1+\theta) \leq 1-\delta$
We know that $\tau = \frac{p_1(1+\delta\theta) - p_2\delta(1+\theta) + \delta w}{1-\delta+\delta w}$ and hence $D_1 = 1 - \tau$, $D_2 = \tau - p_2 + \theta(p_1 - p_2) + wD_1$, $\pi = p_1(1-\tau) + p_2(\tau - p_2 + \theta(p_1 - p_2))$ can be derived accordingly. Using the first-order optimality conditions, we can obtain:

$$p_1 = \frac{2(1-\delta)(1+\theta) + (1+\delta)(1+\theta)w - w^2}{m(\theta)}$$

$$p_2 = \frac{(1-\delta^2)(1+\theta)+(1+\delta+2\delta\theta)w}{m(\theta)}$$

$$\tau = \frac{(2\delta+1)(\theta+1)w+(1-\delta)(\theta+1)(\delta\theta+\delta+2)-w^2}{2(\delta+1)(\theta+1)w+(1-\delta)(\theta+1)((\delta-1)\theta+\delta+3)-w^2} < 1$$

$$\pi = \frac{(\gamma+1)(\delta-w-1)}{-2(\delta+1)(\gamma+1)w+(\delta-1)(\gamma+1)((\delta-1)\gamma+\delta+3)+w^2}$$

Case *ii*: $\tau(p_1, p_2) = 1$ if $\quad p_1(1+\delta\theta) - p_2\delta(1+\theta) > 1-\delta$

Given that $\tau = 1$, then $D_1 = 1 - \tau = 0$, $D_2 = 1 - p_2 + \theta(p_1 - p_2)$ and $\pi = \pi_2 = p_2 D_2$. This turns this case the same as in case *iii* in PR and hence the results are :

$$p_1 = 1; \quad p_2 = \frac{1}{2}; \quad \tau = 1; \quad \pi = \frac{1+\theta}{4}$$

By comparing the profits across cases *i* and *ii*, we can establish the conditions under which either of above cases can lead to a higher profit. We can find that there is a δ_π^{PNR} below which the profit in case *i* is higher, otherwise the profit of case *ii* prevails. The results are presented in Proposition 2.2 for PNR scenario.

- **Proof of PNRL.**

  Using a similar approach, the monopolist's problem can be expressed below:

$$\max_{p_1,p_2} \pi = \quad p_1(1-\tau) + p_2(\tau - p_2 + \gamma(p_1-p_2)^+ + \lambda(p_1-p_2)^- + w(1-\tau)) \tag{2.28}$$

  The above problem is not smooth. To tackle the non-smoothness, we can divide it into the two following subproblems:

$$\max_{p_1,p_2} \pi^\gamma = \quad p_1(1-\tau^\gamma) + p_2(\tau^\gamma - p_2 + \gamma(p_1-p_2) + w(1-\tau^\gamma)) \tag{2.29}$$

$$\max_{p_1,p_2} \pi^\lambda = \quad p_1(1-\tau^\lambda) + p_2(\tau^\lambda - p_2 + \lambda(p_1-p_2) + w(1-\tau^\lambda)) \tag{2.30}$$

  where the threshold purchasing policies can be expressed as follows:

$$\tau^{\gamma}(p_1,p_2)=\begin{cases}\frac{p_1(1+\delta\gamma)-\delta p_2(1+\gamma)+\delta w}{1-\delta+\delta w}, & \text{if } p_1(1+\delta\gamma)-\delta p_2(1+\gamma)\le 1-\delta\\ 1, & \text{otherwise}\end{cases} \tag{2.31}$$

$$\tau^{\lambda}(p_1,p_2)=\begin{cases}\frac{p_1(1+\delta\lambda)-\delta p_2(1+\lambda)+\delta w}{1-\delta+\delta w}, & \text{if } p_1(1+\delta\lambda)-\delta p_2(1+\lambda)\le 1-\delta\\ 1, & \text{otherwise}\end{cases} \tag{2.32}$$

in which τ^{γ} and τ^{λ} represent the threshold policy for gain and loss subproblems, respectively. Let us begin by solving the gain subproblem. We will have two cases, similar to PNR scenario, as follows:

Case i: $\tau^{\gamma}(p_1,p_2)=\frac{p_1(1+\delta\gamma)-\delta p_2(1+\gamma)+\delta w}{1-\delta+\delta w}$ if $p_1(1+\delta\gamma)-\delta p_2(1+\gamma)\le 1-\delta$

Recall that the solution of this case will be similar to PNR, however, we replace θ with γ since we have a gain subproblem. That is:

$$p_1=\frac{2(1-\delta)(1+\gamma)+(1+\delta)(1+\gamma)w-w^2}{m(\gamma)}$$

$$p_2=\frac{(1-\delta^2)(1+\gamma)+(1+\gamma+2\delta\gamma)w}{m(\gamma)}$$

$$\tau=\frac{(2\delta+1)(\gamma+1)w+(1-\delta)(\gamma+1)(\delta\gamma+\delta+2)-w^2}{2(\delta+1)(\gamma+1)w+(1-\delta)(\gamma+1)((\delta-1)\gamma+\delta+3)-w^2}<1$$

$$\pi=\frac{(\gamma+1)(1+w-\delta)}{2(\delta+1)(\gamma+1)w+(1-\delta)(\gamma+1)((\delta-1)\gamma+\delta+3)-w^2}$$

To ensure that the consumers experience gains in the second period, we should have $p_1>p_2$. To this end, there is a $\delta_{p^{\gamma}}^{PNRL}$ in which $p_1>p_2$ holds if $0<\delta<\delta_{p^{\gamma}}^{PNRL}$.

Case ii: $\tau^{\gamma}(p_1,p_2)=1$ if $p_1(1+\delta\gamma)-\delta p_2(1+\gamma)>1-\delta$

This is again similar to PNR in case *ii*. Thus, the solution can be expressed below:

$$p_1 = 1; \quad p_2 = \frac{1}{2}; \quad \tau = 1; \quad \pi = \frac{1+\gamma}{4}$$

Note that the above solution satisfies $p_1(1+\delta\gamma) - \delta p_2(1+\gamma) > 1-\delta$ condition. It is also clear that the skimming pricing strategy in case *ii* ensures that the consumers experience gains in this subproblem. Thus, by comparing the profits across two cases within the gain subproblem, one can show that there is a $\delta_{\pi^\gamma}^{PNRL}$ below which the profit in case *i* is higher, otherwise the profit in case *ii* would prevail.

Next, we will solve the loss subproblem that was expressed in (2.30).

Case *i*: $\tau^\lambda(p_1, p_2) = \frac{p_1(1+\delta\lambda) - \delta p_2(1+\lambda) + \delta w}{1-\delta+\delta w}$ if $p_1(1+\delta\lambda) - \delta p_2(1+\lambda) \leq 1-\delta$

This is similar to case *i* in PNR scenario. The solution can presented as follows:

$$p_1 = \frac{2(1-\delta)(1+\lambda) + (1+\delta)(1+\lambda)w - w^2}{m(\lambda)}$$

$$p_2 = \frac{(1-\delta^2)(1+\lambda) + (1+\lambda+2\delta\lambda)w}{m(\lambda)}$$

$$\tau = \frac{(2\delta+1)(\lambda+1)w + (1-\delta)(\lambda+1)(\delta\lambda+\delta+2) - w^2}{2(\delta+1)(\lambda+1)w + (1-\delta)(\lambda+1)((\delta-1)\lambda+\delta+3) - w^2} < 1$$

$$\pi = \frac{(\lambda+1)(1+w-\delta)}{2(\delta+1)(\lambda+1)w + (1-\delta)(\lambda+1)((\delta-1)\lambda+\delta+3) - w^2}$$

To ensure that the consumers experiences losses in the second period, we should have $p_1 \leq p_2$. Consequently, there is a $\delta_{p^\lambda}^{PNRL}$ above which the prices satisfy the loss condition. Next, we consider the case *ii* for the loss subproblem below.

Case *ii*: $\tau^\lambda(p_1, p_2) = 1$ if $p_1(1+\delta\lambda) - \delta p_2(1+\lambda) > 1-\delta$

Given that $\tau^\lambda = 1$, then $D_1 = 0$, $D_2 = 1 - p_2 + \lambda(p_1 - p_2)$ and $\pi = \pi_2 = p_2 D_2$. We know that the first-order optimality condition results in $p_2 = \frac{1+\lambda p_1}{2(1+\lambda)}$. Since π is an increasing function of p_1 and also $p_1 \leq p_2$ should hold to respect the loss subproblem condition, then $p_1 = p_2 = \frac{1}{2+\lambda}$ and $\pi = \frac{1+\lambda}{(2+\lambda)^2}$. It is easy to show that profit in the loss subproblem under case *i* is higher than case *ii* and hence the former case represents the equilibrium solution for this subproblem.

By comparing the profit from the loss subproblem and gain subproblem in case *ii*, we find that there is a $\delta_{\pi\lambda}^{PNRL}$ below which the profit of loss subproblem is higher, otherwise, the profit of gain subproblem under case *ii* prevails.

It is easy to show that $\delta_{p^{\gamma}}^{PNRL} < \delta_{p^{\lambda}}^{PNRL}$, which implies that the firm may adopt a constant pricing strategy. Under such setting, the problem can be solved as follows. Indeed, when $p_1 = p_2 = p$, we can obtain the threshold purchasing policy as follows:

$$\tau^u(p) = \begin{cases} \frac{p(1+\delta)+\delta w}{(1-\delta+\delta w)}, & \text{if } p(1+\delta) \leq 1-\delta \\ 1, & \text{otherwise} \end{cases} \tag{2.33}$$

Case *i*: $\tau^u(p) = \frac{p(1+\delta)+\delta w}{(1-\delta+\delta w)}$ if $p(1+\delta) \leq 1-\delta$

Given $\tau^u = \frac{p(1+\delta)+\delta w}{(1-\delta+\delta w)}$, $D_1 = 1-\tau^u$, $D_2 = \tau^u - p + w(1-\tau^u)$ and $\pi = p(D_1 + D_2)$. First-order optimality condition $\frac{\partial \pi}{\partial p} = 0$ results in:

$$p = \frac{w-\delta+1}{2w-2\delta+4\delta w+2}$$

$$\tau^u = \frac{\delta-1}{2\delta(w-1)+2} + \frac{\delta}{2\delta w-\delta+w+1} + 1$$

$$\pi^u = \frac{(-\delta+w+1)^2}{4(\delta(w-1)+1)(\delta(2w-1)+w+1)}$$

To ensure $\tau^u \leq 1$ holds, we need to assume that $0 < \delta \leq \delta_{\tau^u}^{PNRL}$.

Case *ii*: $\tau^u(p) = 1$ if $p(1+\delta) > 1-\delta$

Given that $\tau^u = 1$, then $D_1 = 0$, $D_2 = 1-p$ and $\pi = p(1-p)$. First-order condition results in $p = \frac{1}{2}$ and $\pi = \frac{1}{4}$.

Next, we need to compare the profit across these two cases, and the case *ii* from gain subproblem. Clearly, the profit of case *ii* from gain subproblem is higher than case *ii* from constant subproblem. Therefore, comparing the profit across case *i* from constant subproblem with case *ii* from gain subproblem suggests that there is a $\delta_{\pi^u}^{PNRL}$ below which the profit in case *i* from constant subproblem is higher, otherwise the profit in case *ii* from gain subproblem prevails.

Note that one can show $\delta_{p\gamma}^{PNRL} < \delta_{p\lambda}^{PNRL}$ holds. We also assume that $\delta_{p\lambda}^{PNRL} < \delta_{\pi\lambda}^{PNRL}$. Thus, we can summarize the equilibrium solutions for $PNRL$ in what follows. If $\delta_{\pi\gamma}^{PNRL} \leq \delta_{p\gamma}^{PNRL}$, then the equilibrium solution suggests a skimming pricing strategy for all admissible parameter values. More specifically, when $0 < \delta \leq \delta_{\pi\gamma}^{PNRL}$, then $p_1 > p_2$ holds and solution of case *i* from gain subproblem is the optimal one. For $\delta > \delta_{\pi\gamma}^{PNRL}$, the solution of case *ii* prevails which implies a skimming pricing strategy too.

However, if $\delta_{\pi\gamma}^{PNRL} > \delta_{p\gamma}^{PNRL}$, then for $0 < \delta \leq \delta_{p\gamma}^{PNRL}$, the solution of case *i* from gain subproblem prevails. For $\delta_{p\gamma}^{PNRL} < \delta \leq \delta_{p\lambda}^{PNRL}$, the solution of case *i* from the constant subproblem is optimal if $\delta_{p\gamma}^{PNRL} < \delta \leq \delta_{\pi^u}^{PNRL}$ whereas the solution of case *ii* from the gain subproblem prevails if $\delta_{\pi^u}^{PNRL} < \delta \leq \delta_{p\lambda}^{PNRL}$. When $\delta_{p\lambda}^{PNRL} < \delta \leq \delta_{\pi\lambda}^{PNRL}$, the solution of the loss subproblem prevails whereas for $\delta_{\pi\lambda}^{PNRL} < \delta < 1$, the solution of case *ii* from gain subproblem holds.

■

Proof of Results 2.1-2.10. The proposed properties in Results 2.1-2.10 can be directly obtained from the equilibrium outcomes in Propositions 2.1-2.2. ■

Proof of Result 2.11. We can easily obtain

$$
\begin{aligned}
p_1^{PB} - p_1^{RB} &= \frac{(1-\delta)\delta^2}{2(3+\delta)(3-2\delta)} > 0, \\
p_2^{PB} - p_2^{RB} &= \frac{3(1-\delta)\delta}{2(3+\delta)(3-2\delta)} > 0, \\
\pi^{PB} - \pi^{RB} &= \frac{(1-\delta)\delta^2}{4(3+\delta)(3-2\delta)} > 0, \\
D^{PB} - D^{RB} &= \frac{-3\delta(1-\delta)}{2(3+\delta)(3-2\delta)} < 0.
\end{aligned}
$$

■

Proof of Result 2.12. Similarly, we can obtain

$$\begin{aligned}
p_1^{PN} - p_1^{RN} &= \frac{\delta^2(1 + w(2-w) - \delta(1-w))}{2(3+\delta-w)((3-w)(1+w) - 2\delta(1-w))} > 0, \\
p_2^{PN} - p_2^{RN} &= \frac{\delta(3(1-\delta(1-w)) + w(2-w))}{2(3+\delta-w)((3-w)(1+w) - 2\delta(1-w))} > 0, \\
\pi^{PN} - \pi^{RN} &= \frac{\delta^2(1-\delta+w)}{4(3+\delta-w)((3-w)(1+w) - 2\delta(1-w))} > 0, \\
D^{PN} - D^{RN} &= \frac{-\delta(3-w)(1+w-\delta)}{2(3+\delta-w)((3-w)(1+w) - 2\delta(1-w))} < 0.
\end{aligned}$$

■

Proof of Result 2.13. One can easily show that $\delta_\pi^{RR} > \delta_\pi^{PR}$. Therefor, we compare the decision variables under three possible cases i.e., $0 < \delta \leq \delta_\pi^{PR}$, $\delta_\pi^{PR} < \delta \leq \delta_\pi^{RR}$ and $\delta_\pi^{RR} < \delta < 1$, as follows:

- If $0 < \delta \leq \delta_\pi^{PR}$:

$$\begin{aligned}
p_1^{PR} - p_1^{RR} &= \frac{\delta^2(1-\delta)(1+\theta)}{2(3+\delta-\theta(1-\delta))(3-2\delta+\theta(1-\delta)^2)} > 0, \\
p_2^{PR} - p_2^{RR} &= \frac{(1-\delta)\delta(3-\theta(1-2\delta))}{2(3+\delta-\theta(1-\delta))(3-2\delta+\theta(1-\delta)^2)} > 0, \\
D^{PR} - D^{RR} &= \frac{-\delta(1-\delta)(1+\theta)(3-\theta(1-\delta))}{2(3+\delta-\theta(1-\delta))(3-2\delta+\theta(1-\delta)^2)} < 0, \\
\pi^{PR} - \pi^{RR} &= \frac{\delta^2(1-\delta)(1+\theta)}{4(3+\delta-\theta(1-\delta))(3-2\delta+\theta(1-\delta)^2)} > 0.
\end{aligned}$$

- If $\delta_\pi^{PR} < \delta \leq \delta_\pi^{RR}$

$$\begin{aligned}
p_1^{PR} - p_1^{RR} &= 1 - \frac{(2-\delta)^2}{2\,(3-2\delta-\theta(\delta-1)^2)} > 0, \\
p_2^{PR} - p_2^{RR} &= \frac{1}{2} - \frac{\delta-2}{2\,((\delta-1)^2\gamma+2\delta-3)} > 0, \\
D^{PR} - D^{RR} &= \frac{-1-3\delta-\gamma(3+\delta)+\gamma(1-\delta)(1+\gamma)}{2(3+\delta-\gamma(1-\delta))} \lesseqgtr 0, \\
\pi^{PR} - \pi^{RR} &= \frac{-2-3\delta-\gamma(3+\delta)+\gamma(1-\delta)(1+\gamma)}{4(3+\delta-\gamma(1-\delta))} \lesseqgtr 0.
\end{aligned}$$

- If $\delta_\pi^{RR} < \delta < 1$:

$$\begin{aligned} p_1^{PR} - p_1^{RR} &= 0, \\ p_2^{PR} - p_2^{RR} &= 0, \\ D^{PR} - D^{RR} &= 0, \\ \pi^{PR} - \pi^{RR} &= 0. \end{aligned}$$

■

Proof of Result 2.14. One can show that $\delta_\pi^{RNR} < \delta_\pi^{PNR}$. Therefor, we compare the decision variables under three possible cases i.e., $0 < \delta \leq \delta_\pi^{RNR}$, $\delta_\pi^{RNR} < \delta \leq \delta_\pi^{PNR}$ and $\delta_\pi^{PNR} < \delta < 1$, as follows:

- If $0 < \delta \leq \delta_\pi^{RNR}$:

$$p_1^{PNR} - p_1^{RNR} = \frac{(\delta^2(1+\theta)((1-\delta)(1+\theta)+(1+2\theta)w)(1+\theta-\delta(1+\theta)(1-w)+(2+\theta)w-w^2))}{m(\theta)g(\theta)} > 0,$$

$$p_2^{PNR} - p_2^{RNR} = \frac{\delta((1-\delta)(1+\theta)+(1+2\theta)w)((1-\delta)(1+\theta)(3-(1-2\delta)\theta)+(1+\theta)(2+\delta(3+2\delta\theta))w-w^2)}{m(\theta)g(\theta)} > 0,$$

$$D^{PNR} - D^{RNR} = -\frac{\delta((1-\delta)(1+\theta)+(1+2\theta)w)((1-\delta)(1+\theta)(3-(1-\delta)\theta)+(2+\delta+2\theta)w-w^2)}{m(\theta)g(\theta)} < 0$$

$$\pi^{PNR} - \pi^{RNR} = \frac{(\delta^2(1+\theta)(1+\theta-\delta(1+\theta)+w+2\theta w)^2)}{2m(\theta)g(\theta)} > 0.$$

- If $\delta_\pi^{RNR} < \delta \leq \delta_\pi^{PNR}$:

$$p_1^{PNR} - p_1^{RNR} = -\frac{(\gamma+1)\left(\delta^2(\gamma+1)-\delta(2\gamma+w)+\gamma-w-1\right)}{\delta^2(\gamma+1)^2-2\delta(\gamma+1)(\gamma+w-1)+\gamma^2-2\gamma(w+1)+w^2-2w-3} < 0$$

$$p_2^{PNR} - p_2^{RNR} = -\frac{((\delta-1)(\gamma-1)+w)((\delta-1)(\gamma+1)+w)}{2(-2(\delta+1)(\gamma+1)w+(\delta-1)(\gamma+1)((\delta-1)\gamma+\delta+3)+w^2)} \lesseqgtr 0$$

$$D^{PNR} - D^{RNR} = -\frac{(\gamma+1)\left(\delta^2(\gamma+1)^2-2\delta\left(\gamma^2+\gamma w+w+1\right)+(-\gamma+w+1)^2\right)}{2(\delta^2(\gamma+1)^2-2\delta(\gamma+1)(\gamma+w-1)+\gamma^2-2\gamma(w+1)+w^2-2w-3)} \lesseqgtr 0,$$

$$\pi^{PNR} - \pi^{RNR} = \frac{(1+\theta)(\delta^2(1+\theta)^2+(1-\theta+w)^2-2\delta(1+w+\theta(\theta+w)))}{4m(\theta)} \lesseqgtr 0.$$

- If $\delta_{\pi}^{PNR} < \delta < 1$:

$$\begin{aligned} p_1^{PNR} - p_1^{RNR} &= 0, \\ p_2^{PNR} - p_2^{RNR} &= 0, \\ D^{PNR} - D^{RNR} &= 0, \\ \pi^{PNR} - \pi^{RNR} &= 0. \end{aligned}$$

■

Proof of Result 2.15. The proposed properties can be easily obtained from equilibrium results in Propositions 2.1-2.2. ■

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Chapter 3

Dynamic pricing with multiple reference effects

Abstract

This paper examines the dynamic pricing in the face of consumers who are influenced by reference effects across multiple reference points. In a market with repeated purchasing occasions, a price rise compared to the reference-price point can negatively affect the consumers' buying decision whereas a sales surge against consumers' expectations can motivate them towards adoption. To this end, we study the dynamic pricing strategies when consumers have reference dependencies towards two reference points, namely price and externalities (observed sales). Our results propose that the pricing strategies depend critically on both direction and magnitude of all reference effects and might be in stark contrast to previous pricing recommendations. We examine the monotonicity properties of reference policies and find that neither of them are necessarily monotone. Our proposed model shows that the firm's optimal profit might be overestimated or underestimated when multiple reference points are not accounted for.

3.1 Introduction

Consumers often base their buying decisions not only on current price but also on a *reference price,* an anchor price whose value results from past shopping experiences and is used as a benchmark to assess the fairness of the actual price. Consumers feel gains (losses) for prices lower (higher) than their reference price and they tend to be loss averse, that is, to weigh a loss more than a gain of the same size. Such loss aversion is observed for both frequently-purchased goods and durables (Popescu and Wu, 2007; Kalyanaram and Winer, 1995). The idea of reference-dependent loss-averse decision framework stems from prospect theory (Kahneman and Tversky, 1979) which has found widespread applications in behavioral economics and operations management. Prospect theory is further refined to accommodate for multiple reference dependencies (Tversky and Kahneman, 1991), yet the research in *multiple reference points* (MRPs) remains sparse in operations management and marketing literature. A few studies in these literature document empirical evidence on the MRPs (Lattin and Bucklin, 1989; Hardie et al., 1993; Tereyağoğlu et al., 2018), however, the theoretical implications of such framework is yet to be explored. This paper aims to bridge this gap.

Consumers consider several attributes when making purchasing decisions and they might compare appeal of each attribute against its corresponding reference point. While the reference-price effect has been extensively studied from both empirical and theoretical perspectives (Popescu and Wu, 2007; Nasiry and Popescu, 2011; Kalyanaram and Winer, 1995; Mazumdar et al., 2005), only a few contributions have incorporated MRPs. For instance, multiple reference effects have been considered for price and promotions in (Lattin and Bucklin, 1989), price and quality in (Hardie et al., 1993) and price and filled capacity of an event in (Tereyağoğlu et al., 2018). When there are multiple reference effects, one can opt for a holistic evaluation or an attribute-specific evaluation (Bleichrodt et al., 2009). In the former, one global reference point is retained (e.g., Tversky and Kahneman, 1991 and Hardie et al., 1993), whereas in the latter, the decision maker considers a reference point for each attribute (e.g., Lattin and Bucklin, 1989). Bleichrodt et al. (2009) provide preference foundation to the attribute-specific approach when utility is additive. Both Lattin and Bucklin (1989) and Tereyağoğlu et al. (2018) use attribute-specific approach to empirically validate the multiattribute reference effects.

In a revenue management context, Tereyağoğlu et al. (2018) provide strong evidence on reference dependency and loss aversion of consumers with respect to two reference points, namely reference price and reference capacity. Reference capacity is defined as the fraction of sales to capacity, with this variable carrying the same idea of a (positive) network externalities, which stipulates that consumer's utility increases with the number of other consumers buying the product. Positive externalities can be seen as a psychological utility when conformity-seeking behavior prevails, see, e.g., (Huang et al., 2018), and it can significantly impact the pricing regimes (Brynjolfsson and Kemerer, 1996). The notion of other consumers adoptions influences a consumer's purchasing decision is overwhelmingly documented as a key buying driver in the literature (e.g., Tucker and Zhang, 2011; Goldenberg et al., 2010; Peres et al., 2010), however, its psychological reference-dependent impact is not well studied. Put differently, a typical consumer's utility increases when sales exceeds her expectation (Tereyağoğlu et al., 2018) and such expectations are considered as reference points (Yang et al., 2018). Particularly, if the consumer has little exposure to the externalities (i.e., if there has been low observed sales on last purchase occasions), then the consumer expects low externalities effect from other consumers adoptions. However, if the consumer's exposure to externalities is high, then the consumer expects more crowding externalities. The difference between the consumer's expectation and observed sales influences her purchasing decision in an asymmetric way where "losses loom larger than gains". Therefore, we consider reference price and reference externalities effects in shaping the consumer's purchasing decision in order to explore their coexistent effects.

Social influence, which encompasses network externalities, has been shown to be a key driver in consumers' adoptions (e.g., Katz and Shapiro, 1985; Goldenberg et al., 2010) and an individual's choice in attending an event depends on other people's attendance and seeing their actions (e.g., Cialdini and Goldstein, 2004). It is worth mentioning that such externalities affects consumer's behavior not only for networked goods (e.g., fax, telephone) but also for non-networked goods (e.g., music plays). Becker (1991) shows that in many non-network goods cases such as restaurants, theaters, or sport events, consumer's demand depends on adoption of the same good by other consumers. Moreover, Becker (1991) not only suggest that sometimes consumers are discouraged when they encounter unfair price increases but also explain elegantly how a consumer's demand is positively related to other consumers' demands in several revenue management contexts. In this spirit, Tereyağoğlu et al. (2018) give further evidence on impact of other people's actions on the consumer's purchasing decision in line with prospect theory.

Building on the above empirical research, and in particular the compelling evidence in Tereyağoğlu et al. (2018), our objective is to explore analytically the theoretical and practical implications of MRPs on firm's pricing decision. First, we make a substantive contribution by characterizing the optimal dynamic pricing strategy in the presence of reference effects for price and demand (as an externalities), whose evolutions turn out to be interdependent. In particular, we show that accounting for only one reference point, that is, ignoring either reference price or reference externalities, leads to nonoptimal pricing trajectory when compared to the one obtained under MRPs. To this end, we show that our pricing recommendations can be in stark contrast of classic single reference-price settings. Second, we obtain surprising properties in terms of monotonicity of reference price policy and the way it evolves to steady state compared to pricing strategy. Third, we shed a light on how the firm's profit might be underestimated or overestimated when MRPs are not accounted for.

3.2 Literature

This work is related to three streams of literature in operations management, namely, behavioral dynamic pricing, MRPs, and social influences. We review the literature within these streams.

A number of studies aimed at characterizing the pricing policy and its properties analytically in face of reference-price-dependent consumers. Kopalle et al. (1996) determine the optimal pricing policy under symmetric and asymmetric reference-price effect in a discrete mode. Fibich et al. (2003) characterize the explicit pricing strategies in a continuous time for loss-neutral and loss-averse consumers. Popescu and Wu (2007) generalize Kopalle et al. (1996) and Fibich et al. (2003) by considering a general reference-dependent model when consumers are either loss neutral, loss averse, or gain seeking. Nasiry and Popescu (2011) consider a different reference price mechanism, i.e., peak-end anchoring in which reference price depends on weighted average of the lowest and most recent prices. Hu et al. (2016) extend Kopalle et al. (1996) and Popescu and Wu (2007) for gain-seeking consumers case by identifying the structure of optimal pricing strategy. In particular, they show that cyclic skimming pricing strategy is the optimal choice under certain conditions when consumers are gain seeking. Hu and Nasiry (2018) shows that the psychological biases are individual-level phenomenon and hence the aggregate demand may not inherit the individual-level biases such as the loss aversion. Our work is different from this stream of literature by considering MRPs, which leads to different findings as shown in Table 3.1 (p, p^{ss} and r_0 stand for the price, steady-state price and initial reference point, respectively).

The MRPs are studied mainly from an empirical lens in operations management literature. Lattin and Bucklin (1989) consider reference price and reference

Table 3.1: Most relevant findings on price and reference paths in literature

Papers	**Reference effect(s)**	**Pricing strategy**	**Monotonicity**	**Firm's profit**
Kopalle et al. (1996)	Price	Penetration (Skimming) if $r_0 < (>) p^{ss}$	p is monotone	Underestimated or overestimated
Fibich et al. (2003)	Price	Penetration (Skimming) if $r_0 < (>) p^{ss}$	p and r are monotone; $r \geq (\leq) p$ if $r_0 < (>) p^{ss}$	Underestimated or overestimated
Popescu and Wu (2007)	Price	Penetration (Skimming) if $r_0 < (>) p^{ss}$	p and r are monotone; $r \geq (\leq) p$ if $r_0 < (>) p^{ss}$	Underestimated or overestimated
Proposed model	Price and Externalities	$r_0 < (>) p^{ss}$ does not guarantee any pricing regime; It depends on total benefits from all reference effects	p is monotone but r is not necessarily monotone; $r \geq (\leq) p$ even if either $r_0 > p^{ss}$, or $r_0 < p^{ss}$ always holds	Optimal

promotion and Hardie et al. (1993) study reference price and reference quality, both of which provide empirical evidence on exsitence of multiattribute reference effects. The former considers attribute-specific evaluation while the latter assumes a holistic evaluation and further provide evidence for loss aversion. Kopalle and Winer (1996) built upon Hardie et al. (1993) attempt to take a normative approach in characterizing the optimal pricing strategy under asymmetric multiattribute reference effects along price and quality dimensions, however, the authors resort to numerical analyses due to problem intractability. Tereyağoğlu et al. (2018) provide empirical evidence on reference dependency as well as loss aversion of consumers with respect to price and externalities. A few research in behavioral economics follow Kőszegi and Rabin (2006)'s framework to study the MRPs along price and product variability in a newsvendor settings (Baron et al., 2015) or price and delay in service operational setting (Yang et al., 2018). Their works rely on the notion of personalized equilibrium in which the consumer's reference point is her rational expectation of outcomes along each attribute. Our work differs from these contributions in three ways. First, we take a theoretical approach to characterize the dynamic pricing strategy explicitly and identify its managerial implications. Second, we consider a well-documented and empirically-validated reference-price formation that depends on previous purchasing experiences. Third, we consider a different set of attributes, one of which is non-product-specific attribute, i.e., reference externalities, which is known to play a crucial role in consumers' adoption decision.

The literature on social influences has evidently shown the salience of other people's action on an individual choice in form of herd behavior (Banerjee, 1992), conformity (Bikhchandani et al., 1992), social learning (Moretti, 2011), network externalities (Goldenberg et al., 2010), or word-of-mouth (WoM) (Bass, 1969). Moretti (2011) empirically shows that when consumers witness positive (negative) surprise on sales compared to their expectations, the sales increases (decreases) in next purchasing occasions. It is also documented in the literature that negative social influences in the form of negative WoM looms larger than the positive one (Arndt, 1967; Mahajan et al., 1984) since it is more emotive and diffuses more quickly than cognitive-based positive WoM (Sweeney et al., 2005). Moreover, there is empirical evidence on reference dependence and loss aversion of consumers with respect to social influences (Yoon et al., 2017). Using movie industry data, Yoon et al. (2017) show that moviegoers perceive gains (losses) when positive (negative) WoM is higher than the status quo-defined as the reference point- and they react more strongly to negative WoM. Our study is different from this literature by taking a normative approach to uncover the implications of MRPs where consumers are reference dependent towards social influences measured by network externalities.

3.3 Model

We consider a market populated by consumers having heterogeneous valuations of a repeatedly-purchased product. We adopt an MRPs framework and suppose that consumers' purchasing decisions depend on a reference price and a reference demand[1]. Tereyağoğlu et al. (2018) provide empirical evidence that consumers have reference-dependent and loss-averse behaviors towards both price and observed sales. The starting point is that consumer's utility depends on the difference between filled capacity (a fraction of sold seats) and reference capacity. If this difference is positive (negative), then utility is higher (lower). Therefore, it is natural to treat the filled capacity as a proxy of network externalities and the reference capacity as reference externalities. Tereyağoğlu et al. (2018) interpret the (high) seating occupancy as crowding externalities. Here, we define reference externalities as the number of buyers (or level of demand) that the consumer expects to adopt, based on previous observed number of buyers (or demands).

The notation used throughout the paper is defined in Table 3.2.

Table 3.2: Notation

Parameters		**Variables**	
Notation	Description	Notation	Description
ν	Consumers' willingness to pay	$p(t)$	Price
δ	Price sensitivity	$r(t)$	Reference-price point at time t
η	Externalities sensitivity	$\rho(t)$	Reference-externalities point at time t
γ	Marginal reference-price effect	$D(t)$	Demand at time t
λ	Marginal reference-externalities effect	$\pi(t)$	Profit at time t
s	Speed of adjustment (memory parameter)		
a	Market size		
V	Upper bound for consumers' valuation		

We follow the literature by considering the reference point formations which are updated by consumers on the basis of their personal history of purchasing experiences. More specifically, we use continuous[2] exponentially decaying weighting mechanisms for both reference price and reference externalities that depend on weighted average of past prices and past sales, respectively (see, e.g., Sorger,

[1]We shall use reference demand and reference externalities interchangeably.

[2]Zhang and Chiang (2020) show that a discrete-time reference formation can be transformed to a continuous-time one.

1988; Kopalle and Winer, 1996; Winer, 1986; Fibich et al., 2003). That is, we have

$$r'(t) = e^{-st}[r'_0 + s\int_0^t e^{s\tau} p'(\tau)d\tau], \qquad r'(0) = r'_0, \tag{3.1}$$

$$\rho'(t) = e^{-st}[\rho'_0 + s\int_0^t e^{s\tau} D'(\tau)d\tau], \qquad \rho'(0) = \rho'_0, \tag{3.2}$$

where, as in Lattin and Bucklin (1989) and Tereyağoğlu et al. (2018), we have used the same memory (or speed of adjustment) parameter $s \in (0,1)$ in both reference formation mechanisms. Differentiating the above equations with respect to time, we obtain the following state equations:

$$\dot{r}'(t) = s(p'(t) - r'(t)), \qquad r'(0) = r'_0, \tag{3.3}$$

$$\dot{\rho}'(t) = s(D'(t) - \rho'(t)), \qquad \rho'(0) = \rho'_0. \tag{3.4}$$

We assume that consumers' valuation follows a uniform distribution. One may note that under the assumption of a uniform distribution, individual-based behavioral biases such as loss aversion are inherited from individual level to aggregate demand. However, Hu and Nasiry (2018) show that such biases are not inherited from individual level to aggregate demand if we consider a nonuniform probability distribution (e.g., exponential or logistic probability distributions). To control for impact of nonuniform distributions and consequently demand forms on our proposed framework, and also in line with literature (e.g., Fibich et al., 2003; Hu et al., 2016), we consider the standard settings where consumers' valuation v is distributed uniformly[3] over $[0, V]$, and denote by F the cumulative distribution. Moreover, we retain an additive linear utility function in price, externalities and their reference effects, which is a common assumption in the literature (e.g., Bleichrodt et al., 2009 and Kopalle et al., 1996). In such setup, the consumer's utility function at time $t \in [0, \infty]$ is given by

$$u(t) = \nu - \delta p'(t) + \eta D'(t) - \gamma(p'(t) - r'(t)) + \lambda(D'(t) - \rho'(t)), \tag{3.5}$$

[3]Nonetheless, it is not out of reach to examine implications of non-uniform distributions in our framework, for example, using a numerical approach.

where η measures the marginal impact of externalities on utility. The global externalities is given by $\eta D'(t)$, which implicitly implies that consumers are fully connected. A (myopic) consumer buys the product at time t only if $u(t) \geq 0$. Therefore, the aggregate demand at t is given by

$$D(t) = a(1 - F(\nu^*)) = a - a\left(\frac{\delta p'(t) - \eta D'(t) + \gamma(p'(t) - r'(t)) - \lambda(D'(t) - \rho'(t))}{V}\right).$$

Where ν^* is the threshold policy. Letting $p'(t) = p(t)\frac{V}{a}$, $r'(t) = r(t)\frac{V}{a}$, $\rho'(t) = \rho(t)\frac{V}{a}$ and $D'(t) = D(t)\frac{V}{a}$, the above equation simplifies to

$$D(t) = \frac{a - \delta p(t) - \gamma(p(t) - r(t)) - \lambda\rho(t)}{1 - \eta - \lambda}.$$

We suppose that the parameter values are such that the demand is positive. To keep the focus on multiple reference effects, we assume that $\eta = 0$, which avoids having a combined effect of the externalities itself and its reference point.[4] We note that considering $\eta > 0$ does not affect the analytical tractability of the problem, only the interpretations. With $\eta = 0$, the demand becomes

$$D(t) = \frac{a - \delta p(t) - \gamma(p(t) - r(t)) - \lambda\rho(t)}{1 - \lambda}. \tag{3.6}$$

As consumer's utility is decreasing in the reference externalities, the demand is also decreasing in $\rho(t)$. However, depending on the value of $\rho(t)$, the demand can be either increasing or decreasing in the parameter λ. Indeed, we have

$$\frac{\partial D(t)}{\partial \lambda} \lesseqgtr 0 \Leftrightarrow a - \delta p(t) - \gamma(p(t) - r(t)) \lesseqgtr \rho(t),$$

that is, $\frac{\partial D(t)}{\partial \lambda} > 0$ if the demand evaluated at $\rho(t) = 0$ (no reference externalities effect) is larger than the reference-externalities point itself.

[4]Note that if $\eta = 0$ and $\lambda = 0$, then the problem might be reduced to the one in Fibich et al. (2003), albeit not necessarily, depending on the consumers' decision model.

Assuming revenue-maximization behavior and no capacity constraint, the firm's optimization problem over the infinite planning horizon is as follows:

$$\begin{aligned} \max_{p(t)} \quad & \pi = \int_0^\infty e^{-\beta t} p(t) D(t)\, dt, \\ \text{s.t.} \quad & \dot{r}(t) = s(p(t) - r(t)), \quad r(0) = r_0, \\ & \dot{\rho}(t) = s(D(t) - \rho(t)), \quad \rho(0) = \rho_0, \end{aligned} \tag{3.7}$$

where $D(t)$ is given in (3.6).

3.4 Main Results

In this section, we first consider two benchmark cases where either only reference-price effect or reference-externalities effect is considered. We refer to the former as RP model and the latter as RE model. Next, we solve the problem (3.7) and characterize the optimal pricing strategy. Additionally, we discuss the monotonocity properties of the trajectories of optimal price, reference price, and reference externalities. Finally, we compare the firm's performance in the different considered models.

3.4.1 Reference-price effect only

Suppose that consumer's transaction utility is obtained only on the price dimension. Then, the firm's optimization problem becomes

$$\begin{aligned} \max_{p_B(t)} \quad & \pi_B = \int_0^\infty e^{-\beta t} p_B(t) D_B(t)\, dt, \\ \text{s.t.} \quad & \dot{r}_B(t) = s(p_B(t) - r_B(t)), \quad r_B(0) = r_0, \end{aligned} \tag{3.8}$$

where we have subscripted the variables with B (for benchmark). The demand is given by

$$D_B(t) = a - \delta p_B(t) - \gamma(p_B(t) - r_B(t)). \tag{3.9}$$

Let

$$\Delta = \sqrt{(2s+\beta)(\delta+\gamma)(2s\delta+\beta(\delta+\gamma))} \quad \text{and} \quad M = \frac{1}{2}\left(\frac{\Delta}{\delta+\gamma}-\beta\right) > 0.$$

Proposition 3.1 *When demand depends on reference price, then the optimal price, reference price, and demand trajectories are as follows:*

$$p_B^*(t) = p_B^{ss} + e^{-Mt}\left(1-\frac{M}{s}\right)(r_0 - p_B^{ss}), \tag{3.10}$$

$$r_B^*(t) = p_B^{ss} + e^{-Mt}(r_0 - p_B^{ss}), \tag{3.11}$$

$$D_B^*(t) = D_B^{ss} + e^{-Mt}\frac{(\Delta - (2s\delta+\beta(\delta+\gamma)))}{2s}(r_0 - p_B^{ss}), \tag{3.12}$$

with the steady-state price and demand given by

$$p_B^{ss} = r_B^{ss} = \frac{a(s+\beta)}{2\delta(s+\beta)+\beta\gamma}, \tag{3.13}$$

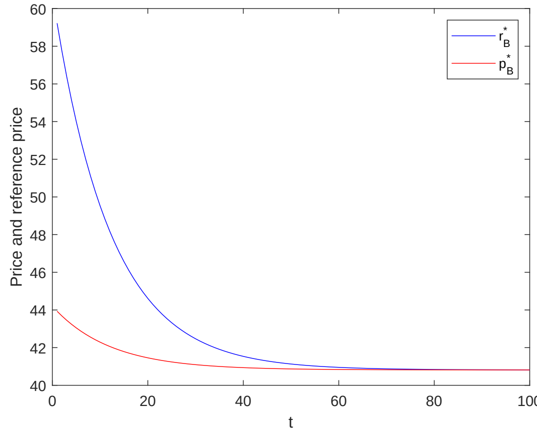
$$D_B^{ss} = a - \delta p_B^{ss}. \tag{3.14}$$

Proof See Appendix. ∎

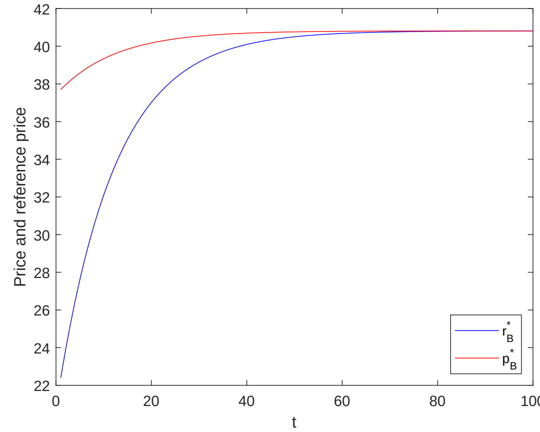
The results in Proposition 3.1 are similar to those in Proposition 1 in Fibich et al. (2003); however, ours are obtained in a revenue maximization context. Figures 3.1(a) and 3.1(b) show the pricing strategy when only reference-price effect is considered.[5] As expected, for symmetric reference effects, the price converges to a constant steady-state price since price variations over the long run is not beneficial for the firm unless consumers are gain-seeking. During the transient pricing regime, the optimal pricing depends on whether the initial-reference point is higher or lower than the steady-state price (see Eq. 3.10). The firm may adopt skimming pricing strategy if $r_0 > p_B^{ss}$ (see figure 3.1(a)), and penetration pricing strategy if $r_0 < p_B^{ss}$ (see figure 3.1(b))[6]. The intuition behind this result is as follows: when the initial reference-price point is higher than the steady state, i.e.,

[5] To draw the figures, we consider specific parameter values, i.e., $a = 100$, $\delta = 1$, $s = 0.1$, $\beta = 0.9$, $\gamma = 0.5$, $\lambda = 0.5$ and $T = 100$. However, the shape of the trajectories will be the same for any (make-sense) constellation of parameter values.

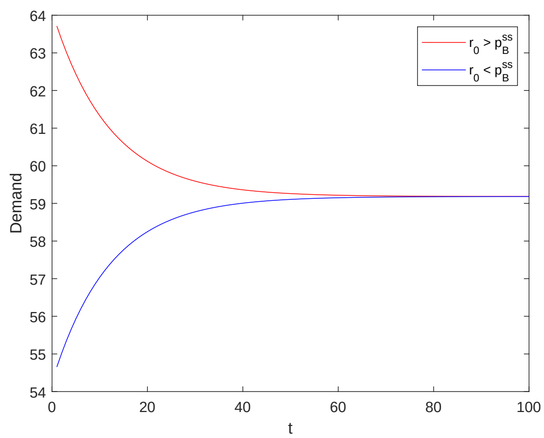
[6] It is easy to show that $1 - \frac{M}{s} > 0$.



(a) $r_0 > p_B^{ss}$



(b) $r_0 < p_B^{ss}$



(c)

Figure 3.1: Optimal solutions under RP model

$r_0 > p_B^{ss}$, the firm begins by a price higher than steady state and lower than the initial reference point, i.e., $r_0 > p_B^0 > p_B^{ss}$ [7], and decreases it gradually to boost the demand from favorable reference effect along the way to the steady-state price. Conversely, when $r_0 < p_B^{ss}$, the firm starts by charging a price between the initial reference point and the steady state, i.e., $r_0 < p_B^0 < p_B^{ss}$ to minimize the effect of

[7] p_0 is the initial price i.e., $p_B^0 = p(0)$. When $r_0 > p_B^{ss}$, it is easy to show that $r_0 > p_B^0 > p_B^{ss}$ holds, and when $r_0 < p_B^{ss}$, then $r_0 < p_B^0 < p_B^{ss}$ holds.

the unfavorable reference effects. Our results are in line with those obtained in the literature, e.g., Kopalle et al. (1996), Fibich et al. (2003) and Popescu and Wu (2007). Moreover, according to figure 3.1(c), the demand is decreasing (increasing) if $r_0 > (<) p_B^{ss}$ even though the pricing path has a decreasing (increasing) trend too. This occurs since the positive (negative) impact of reference price on demand is shrinking over time. For instance, figure 3.1(a) shows that the gap between the reference price and the price is declining over time which consequently decreases the demand despite adoption of a skimming pricing strategy since the impact of reference price on demand is higher than the impact of price per se. Thus, both demand and price trajectories have similar trends which are consistent with Eq. (3.10) and Eq. (3.12)[8].

Note that the reference price trajectory follows a similar pattern as the optimal pricing strategy. For $r_0 > p_B^{ss}$, the reference price monotonically declines and it is always greater than the price (see figure 3.1(a)), while for $r_0 < p_B^{ss}$, it monotonically increases and is always smaller than the price, until it reaches its steady-state value r_B^{ss} (see figure 3.1(b)).

3.4.2 Reference-externalities effect only

When only reference-externalities effect is present in the model, the firm's optimization problem becomes

$$\begin{aligned} \max_{p_E(t)} \quad & \int_0^\infty e^{-\beta t} p_E(t) D_E(t)\, dt, \\ \text{s.t.} \quad & \dot{\rho}_E(t) = s(D_E(t) - \rho_E(t)), \end{aligned} \tag{3.15}$$

where we have subscripted the variables with E (for demand Externality), and $D_E(t)$ is given by

$$D_E(t) = \frac{a - \delta p_E(t) - \lambda \rho_E(t)}{1 - \lambda}. \tag{3.16}$$

[8]It is easy to show that $\Delta - (2s\delta + \beta(\delta + \gamma)) > 0$.

Let

$$\Gamma = \sqrt{(2s+\beta)(1-\lambda)(2s+\beta(1-\lambda))} \quad \text{and} \quad N = \frac{1}{2}\left(\frac{\Gamma}{(1-\lambda)} - \beta\right) > 0. \tag{3.17}$$

Proposition 3.2 *When demand depends on reference externalities, then the optimal price, reference externalities, and demand trajectories are as follows:*

$$p_E^*(t) = p_E^{ss} - \frac{e^{-Nt}(2s+\beta(1-\lambda)-\Gamma)}{2\delta s}\left(\rho_0 - D_E^{ss}\right), \tag{3.18}$$

$$\rho_E^*(t) = D_E^{ss} + e^{-Nt}\left(\rho_0 - D_E^{ss}\right), \tag{3.19}$$

$$D_E^*(t) = D_E^{ss} - \frac{e^{-Nt}(\Gamma - (2s+\beta)(1-\lambda))}{2s(1-\lambda)}\left(\rho_0 - D_E^{ss}\right), \tag{3.20}$$

with the steady-state values given by

$$p_E^{ss} = \frac{a\left(s+\beta(1-\lambda)\right)}{\delta(2(s+\beta)-\beta\lambda)}, \tag{3.21}$$

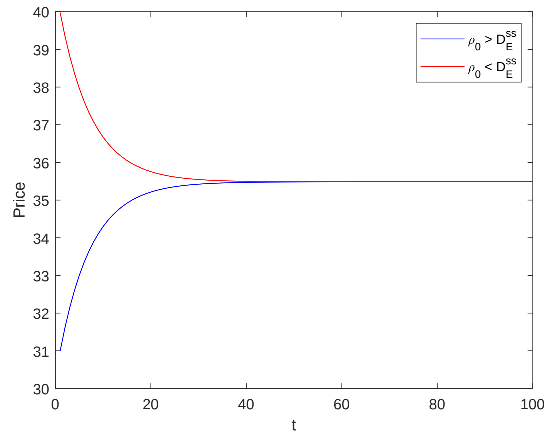
$$\rho_E^{ss} = \frac{a\left(s+\beta\right)}{2(s+\beta)-\beta\lambda}, \tag{3.22}$$

$$D_E^{ss} = \rho_E^{ss} = \frac{a - \delta p_E^{ss} - \lambda \rho_E^{ss}}{1-\lambda}. \tag{3.23}$$

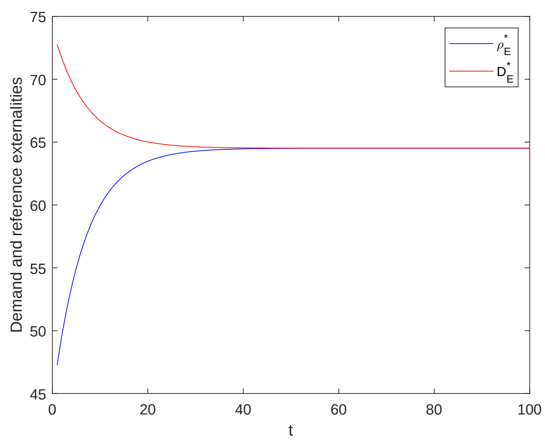
Proof See Appendix. ■

The firm's pricing trajectory is illustrated in Figure 3.2(a). For the same reason as in the RP model, the trajectory converges to the steady-state price from above, or below, depending on the values of ρ_0 and D_E^{ss} [9]. Price variations for a long term do not benefit the firm when the reference-externalities effect is symmetric. The transient pricing phase can be skimming or penetration depending on whether initial reference point is lower or higher than the steady-state value. In particular, when $\rho_0 < D_E^{ss} = \rho_E^{ss}$, the reference externalities needs to be built up to reach the optimal steady state. Recall that $D_E(t)$ is decreasing with respect to $\rho_E(t)$ (see Eq. 3.16) which implies that the demand might decrease when reference externalities increase, as shown in figure 3.2(b). The decreasing pattern of demand happens since the positive impact of reference externalities effect is shrinking over time

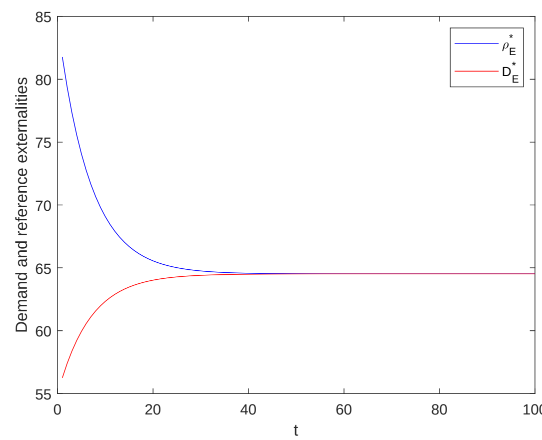
[9] It is easy to show that $2s+\beta(1-\lambda)-\Gamma > 0$ and $(\Gamma - (2s+\beta)(1-\lambda)) > 0$.



(a)



(b) $\rho_0 < D_E^{ss}$



(c) $\rho_0 > D_E^{ss}$

Figure 3.2: Optimal solutions under RE model

which consequently lowers the demands. Similar to RP model, the impact of reference externalities on demand overshadows the one from the price and hence the demand decreases despite the presence of a decreasing pricing path. The monopolist's motivation for adopting a skimming pricing strategy is to offer better deals to consumers and ultimately encourage them to buy the product as the positive reference-externalities effect fades over time. Note that according to Eq. 3.18, when $\rho_0 < D_E^{ss}$, then $p_E^0 > p_E^{ss}$.

A similar reasoning holds for the case with $\rho_0 > D_E^{ss}$. In particular, the firm adopts a penetration pricing strategy that causes a negative reference effect for consumers and hence the goal is to minimize this penalty along the way to the steady state after which the reference effect will vanish (see figure 3.2(c)).

3.4.3 Multiple reference effects

While Proposition 3.1 suggests that a skimming pricing strategy is optimal for a relatively high initial reference-price point, Proposition 3.2 states that a penetration pricing strategy is better for a high initial reference-externalities point. In this section, we characterize the optimal pricing strategy when both reference effects are present, which will allow us to shed a light on the changes induced by having MRPs instead of a single reference point. The optimization problem is defined in (3.7).

Let

$$\begin{aligned}
\Theta &= \sqrt{(\delta+\gamma)(1-\lambda)(2s+\beta(1-\lambda))(2s\delta+\beta(\delta+\gamma))} > 0, \\
\Psi &= (\delta+\gamma)(2s\delta+\beta(\delta+\gamma)) > 0, \text{ and } L = \frac{1}{2}\left(\frac{\Theta}{(\delta+\gamma)(1-\lambda)} - \beta\right) > 0.
\end{aligned}$$

Proposition 3.3 *For the multiattribute reference-dependent demand, the optimal price, reference price, reference externalities, and demand trajectories are as follows:*

$$p^*(t) = p^{ss} + \frac{e^{-Lt}\left((2s+\beta(1-\lambda))(\delta+\gamma)+\Theta\right)\left(\gamma(r_0-p^{ss})-\lambda(\rho_0-D^{ss})\right)}{2(\delta+\gamma)\left((s+\beta(1-\lambda))(\delta+\gamma)+s\delta(1-\lambda)+\Theta\right)}, \tag{3.24}$$

$$r^*(t) = r^{ss} + \frac{\left(\lambda\Psi e^{-st}-\gamma\Theta e^{-Lt}\right)(r_0-p^{ss})+\lambda\Theta\left(e^{-Lt}-e^{-st}\right)(\rho_0-D^{ss})}{\lambda\Psi-\gamma\Theta}, \tag{3.25}$$

$$\rho^*(t) = \rho^{ss} + \frac{\gamma\Psi\left(e^{-st}-e^{-Lt}\right)(r_0-p^{ss})+\left(\lambda\Psi e^{-Lt}-\gamma\Theta e^{-st}\right)(\rho_0-D^{ss})}{\lambda\Psi-\gamma\Theta}, \tag{3.26}$$

$$D^*(t) = D^{ss} + \frac{e^{-Lt}((2s\delta+\beta(\delta+\gamma))(1-\lambda)+\Theta)\left(\gamma(r_0-p^{ss})-\lambda(\rho_0-D^{ss})\right)}{2(1-\lambda)\left((s+\beta(1-\lambda))(\delta+\gamma)+s\delta(1-\lambda)+\Theta\right)}, \tag{3.27}$$

with the steady-state values given by

$$p^{ss} = r^{ss} = \frac{a\left(s + \beta(1-\lambda)\right)}{2\delta(s+\beta) + \beta(\gamma - \delta\lambda)}, \tag{3.28}$$

$$\rho^{ss} = \frac{a\left((s+\beta)\delta + \beta\gamma\right)}{2\delta(s+\beta) + \beta(\gamma - \delta\lambda)}, \tag{3.29}$$

$$D^{ss} = \rho^{ss} = \frac{a - \delta p^{ss} - \lambda\rho^{ss}}{1-\lambda}. \tag{3.30}$$

Proof See Appendix. ■

According to Proposition 3.3, the difference between the initial reference-price point and the steady-state price i.e., $r_0 - p^{ss}$ is no longer a sufficient criteria for the choice of the introductory pricing regime, which challenges the findings in the literature (Fibich et al., 2003; Popescu and Wu, 2007; Kopalle et al., 1996; Zhang et al., 2014). Instead, the choice of pricing strategies should be determined based on maximal benefits from contribution of reference effects along all attributes.

More specifically, when the initial reference-price point is lower than the steady-state price, then a penetration pricing strategy is an attempt to reach steady-state level with the minimum penalty from reference-price effect. However, if the initial reference-externalities point is lower than steady-state demand too, then the positive reference-externalities effect will shrink over time and an increasing pricing path will discourage the consumers further. Alternatively, a decreasing pricing path would temper the shrinking positive impact of reference effect on externalities dimension, however, at the cost of a higher penalty from the negative reference-price effect. These suggest that the price should follow a path that yields the maximum collective benefits from reference effects along all attributes. Such benefits depend on the difference between each initial reference point and its corresponding steady-state level weighted by respective marginal contribution, i.e., $\gamma(r_0 - p^{ss}) - \lambda(\rho_0 - D^{ss})$. Put differently, the direction and magnitude of reference-price effect measured by $\gamma(r_0 - p^{ss})$ as well as reference-externalities effect measured by $\lambda(D^{ss} - \rho_0)$ can resolve the dilemma of choice of pricing strategy under MRPs.

Table 3.3 shows eight possibilities in which different pricing strategies might emerge depending on the direction and magnitude of both reference effects. To fathom this, recall from benchmark models that when either the initial reference-price point is higher (lower) than steady-state price, or the initial reference-externalities point is lower (higher) than steady-state demand, then a skimming (penetration) pricing is proposed. Thus, in MRPs model, the monopolist adopts a skimming pricing strategy when both $r_0 > p^{ss}$ and $\rho_0 < D^{ss}$ are true (see 7 and 8 in Table 3.3) and a penetration pricing one when both $r_0 < p^{ss}$ and $\rho_0 > D^{ss}$ hold (see 5 and 6 in Table 3.3). Now if either $\rho_0 < D^{ss}$ but $r_0 < p^{ss}$ (see 1 in Table 3.3), or $r_0 > p^{ss}$ but $\rho_0 > D^{ss}$ (see 4 in Table 3.3) occur, then a skimming pricing strategy still persists if the magnitude of benefit from the reference effect that calls for a skimming pricing strategy is higher than its counterpart that advocates a penetration pricing strategy. Otherwise a penetration pricing strategy is the optimal choice (see 2 and 3 in Table 3.3). Therefore, the choice of a pricing strategy can be summarized using a single criterion, that is, skimming if $\gamma(r_0 - p^{ss}) + \lambda(D^{ss} - \rho_0) > 0$, and penetration if $\gamma(r_0 - p^{ss}) + \lambda(D^{ss} - \rho_0) < 0$. In other words, the pricing strategy should be designed in a way that favors the reference effect with more benefits. Note that if $\gamma(r_0 - p^{ss}) - \lambda(\rho_0 - D^{ss}) = 0$, then the firm resorts to a constant pricing strategy where $p^* = p^{ss}$ for the entire horizon.

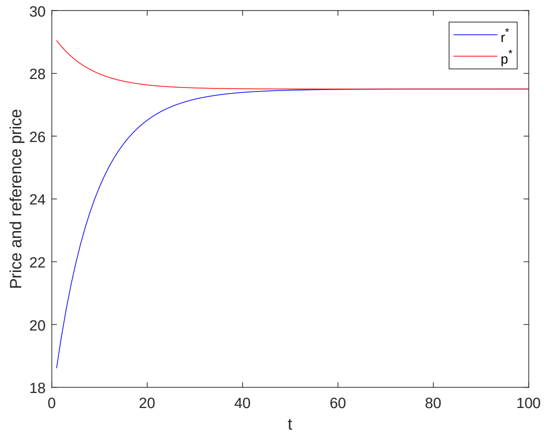
Table 3.3: Pricing strategies depending on multiattribute reference effects

No	Direction		Magnitude	Pricing strategy criterion	Optimal pricing strategy
	$r_0 - p^{ss}$	$\rho_0 - D^{ss}$	$\lvert\gamma(r_0 - p^{ss})\rvert - \lvert\lambda(\rho_0 - D^{ss})\rvert$	$\gamma(r_0 - p^{ss}) - \lambda(\rho_0 - D^{ss})$	
1	<0	<0	<0	>0	Skimming
2	<0	<0	>0	<0	Penetration
3	>0	>0	<0	<0	Penetration
4	>0	>0	>0	>0	Skimming
5	<0	>0	<0	<0	Penetration
6	<0	>0	>0	<0	Penetration
7	>0	<0	<0	>0	Skimming
8	>0	<0	>0	>0	Skimming

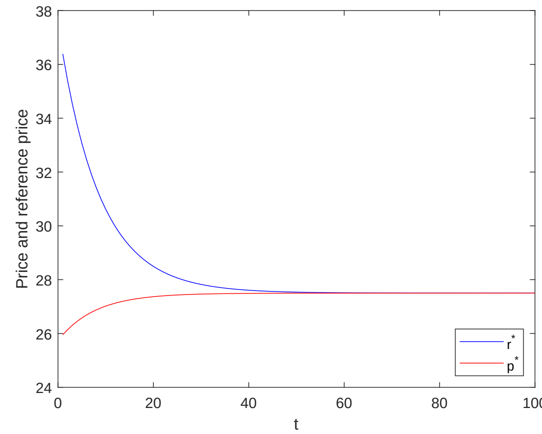
To provide intuitions, we illustrate the optimal solutions for cases 1 and 3 in Table 3.3. When $r_0 < p^{ss}$ and $\rho_0 < D^{ss}$ where the former advocates a penetration pricing strategy whereas the latter calls for a skimming pricing strategy, then the skimming pricing strategy prevails if $|\gamma(r_0 - p^{ss})| < |\lambda(\rho_0 - D^{ss})|$ (see figures 3.3(a) and 3.3(c) which correspond to case 1 in Table 3.3). Figure 3.3(c) shows that the demand is higher than reference externalities and therefore consumers can enjoy from a positive reference-dependent utility that consequently favors the aggregate demand. Since such utility on externalities dimension is relatively high at early stages but decreases over time, the firm's pricing strategy starts above the steady-state price and declines over time to mitigate the diminishing reference-externalities effect. It is clear from figure 3.3(a) that consumers and consequently the demand might suffer from negative reference-price effects, however, the firm is still better off with such pricing strategy since the positive impact of reference-externalities effect dominates the negative impact of reference-price effect under considered situation.

When $r_0 > p^{ss}$, $\rho_0 > D^{ss}$, and $|\gamma(r_0 - p^{ss})| < |\lambda(\rho_0 - D^{ss})|$ holds (case 3 in Table 3.3), then a penetration pricing is the optimal choice (see figures 3.3(b) and 3.3(d)). According to figure 3.3(d), the negative reference-externalities effect is decreasing over time which motivates the firm to start with a price lower than steady-state price and increases it over time as consumers' utility becomes more appealing. Again, the pricing strategy is aligned with the mechanics of reference-externalities effect since we have assumed in both cases that such effect has more impacts than the reference-price one. Therefore, the firm might adopt skimming (penetration) pricing strategy even though $r_0 < (>) p^{ss}$ and hence the transient pricing strategies might be in stark contrast to literature (Fibich et al., 2003; Popescu and Wu, 2007; Kopalle et al., 1996; Zhang et al., 2014) when reference-externalities effect is accounted for. Conversely, if we assume that the impact of reference effect on price dimension is higher than the externalities one, that is, $|\gamma(r_0 - p^{ss})| > |\lambda(\rho_0 - D^{ss})|$, then the pricing strategies will be consistent with the literature i.e., skimming for $r_0 > p^{ss}$, and penetration for $r_0 < p^{ss}$.

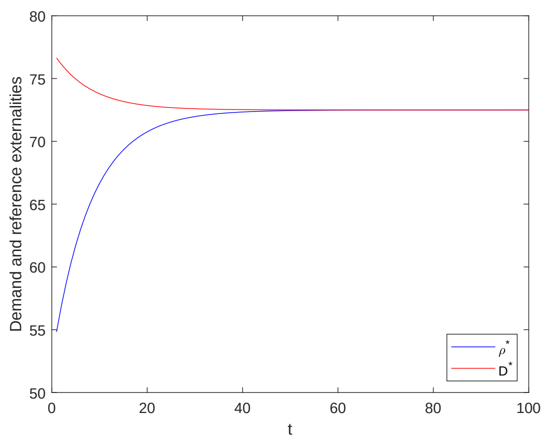
Figure 3.4 further shows how the transient pricing regime and state trajectories drastically change when consumer behavior shift from single-attribute ref-



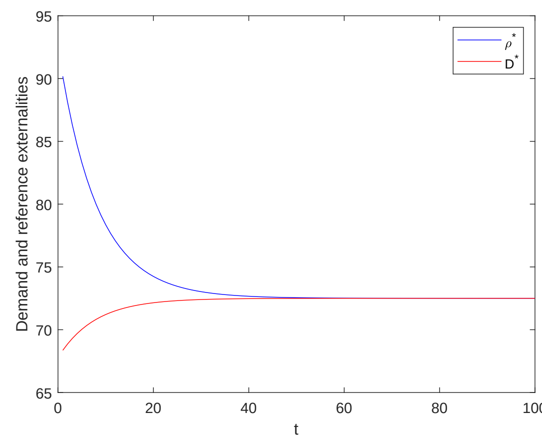
(a) $\gamma(r_0 - p^{ss}) - \lambda(\rho_0 - D^{ss}) > 0$



(b) $\gamma(r_0 - p^{ss}) - \lambda(\rho_0 - D^{ss}) < 0$



(c) $\gamma(r_0 - p^{ss}) - \lambda(\rho_0 - D^{ss}) > 0$



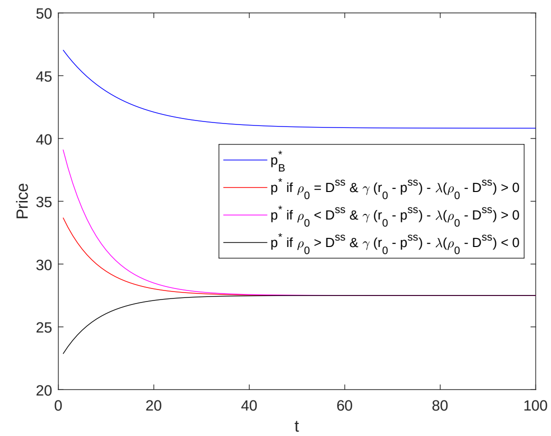
(d) $\gamma(r_0 - p^{ss}) - \lambda(\rho_0 - D^{ss}) < 0$

Figure 3.3: Optimal solutions under MRPs

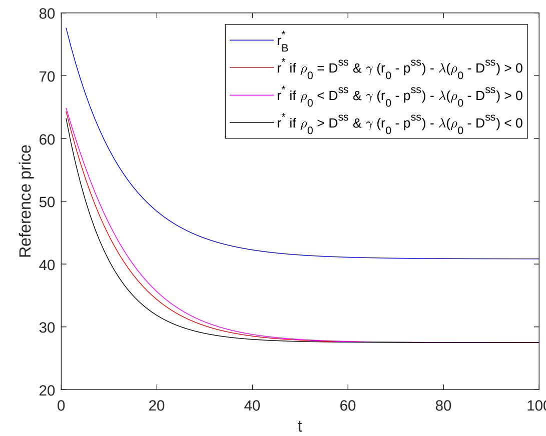
erence dependency to multiattribute one. Across these four panel, we assumed that $r_0 > p^{ss}$ [10] holds (with the same difference magnitude), to control for the role of reference-price effect[11]. According to figures 3.4(a) and 3.4(c), the skimming pricing strategy is retained when $\rho_0 = D^{ss}$, that consequently results in $\gamma(r_0 - p^{ss}) - \lambda(\rho_0 - D^{ss}) > 0$, wherein the prices are lower but the demands are higher under MRPs than their counterparts in RP model. Indeed even though

[10] We ensured that the difference between initial reference-price point and corresponding steady-state price is the same in both MRPS and RP models i.e., $r_0 - p^{ss} = r_0 - p^{ss}_B$.

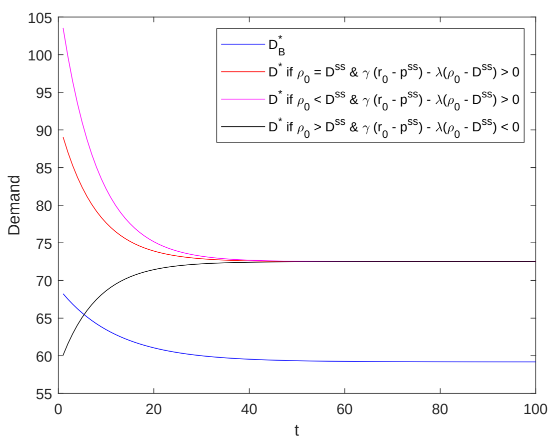
[11] Similar results can be obtained for $r_0 < p^{ss}$.



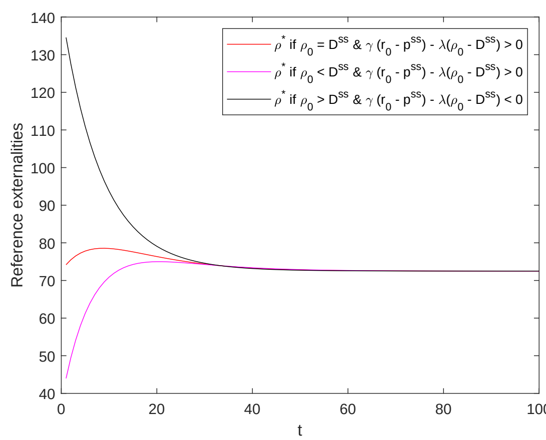
(a) Prices in MRPs vs. RP



(b) Reference prices in MRPs vs. RP



(c) Demands in MRPs vs. RP



(d) Reference externalities in MRPs

Figure 3.4: Optimal solutions in MRPs vs. RP models

$\rho_0 = D^{ss}$ holds, the consumers might enjoy from positive reference-externalities effect which allows the firm to capitalize on such effect by charging lower prices to gain a higher market share. The reference-price path follows a similar pattern like pricing strategy (see figure 3.4(b)), however, the corresponding reference-externalities trajectory follows a non-monotone path (see figure 3.4(d)), which is discussed in the next subsection.

When $\rho_0 < D^{ss}$ and $\gamma(r_0 - p^{ss}) - \lambda(\rho_0 - D^{ss}) > 0$, consumers can enjoy from positive reference effects along both attributes which enables the firm to adopt a more aggressive skimming pricing strategy with a higher demand. Finally, when $\rho_0 > D^{ss}$ and $\gamma(r_0 - p^{ss}) < \lambda(\rho_0 - D^{ss})$, then consumers experience a negative reference-externalities effect that is diminishing over time. Consequently, the firm reverse its pricing strategy to the penetration one to mitigate the negative yet declining reference effect on externalities dimension.

3.4.4 On the monotonicity of price and reference trajectories

This section examines the behavior of the control and state trajectories.

Proposition 3.4 *Problem 3.7 admits a unique steady state (p^{ss}, ρ^{ss}), where*

1. *The optimal price trajectory $p^*(t)$ monotonically converge to p^{ss};*
2. *The optimal reference-price trajectory $r^*(t)$ converges to p^{ss}, but not necessarily monotonically.*
3. *The reference-externalities trajectory $\rho^*(t)$ converges to ρ^{ss}, but not necessarily monotonically.*

Proof See Appendix. ■

Proposition 3.4 challenges the previous findings that state the reference price trajectory converges *monotonically* to p^{ss} (see Fibich et al., 2003; Popescu and Wu, 2007). To give intuitions, recall from RP model that when $r_0 > p_B^{ss}$, then $r_0 > p_B^0$ always holds, and as a result the reference price starts above the pricing path and follows its declining pattern till steady-state (see figure 3.1(a)), which ultimately leads to a monotone path . In MRPs model, however, when $r_0 > p^{ss}$, then $r_0 > p^0$ is not necessarily true and hence the reference price might starts below the prices. This might happen when the impact of reference-externalities effect dominates its counterpart which allows the monopolist to charge prices higher than initial reference-price point. Under such conditions and a skimming pricing strategy, the reference-price path might increase first because the initial high prices pull up the reference-price path until it crosses pricing path. Subsequently, as the impact of initial prices on reference-price path fades due to the decaying mechanism and prices also decrease over time, the reference-price path reverses and eventually follows the declining pricing path till the steady state (see figure 3.5(a)). Figure 3.5(a) shows the non-monotonicity of the reference-price policy under certain conditions, that is, when the pricing strategy is of a skimming type[12], $|\gamma(r_0 - p^{ss})| < |\lambda(\rho_0 - D^{ss})|$ and $r_0 < p^0$. However, according to figure 3.5(b), the reference-price policy can be monotone when either r_0 is sufficiently high so much so $r_0 > p^0$, or r_0 is sufficiently low such that the initial high prices will not be able to pull up the reference-price path enough to cross the price path. Similarly, figure 3.5(c) shows the non-monotonicity for reference externalities policy under certain conditions, that is, when the pricing strategy is of a skimming type[13], $|\gamma(r_0 - p^{ss})| > |\lambda(\rho_0 - D^{ss})|$ and $\rho_0 < D^0$. Figure 3.5(d) depicts a monotone reference externalities policy when those conditions are not satisfied.

Moreover, figure 3.5(a) suggests that the price might be both higher and lower than the reference price trajectory along the way to steady state. Indeed, while Popescu and Wu (2007) argues that the pricing path is always below (above) reference price when skimming (penetration) pricing strategy is adopted, we show, in the following corollary, that this is not necessarily true.

Corollary 3.1 *Under certain conditions, the price path might exceed (fall behind) the reference-price path first when the firm adopts skimming (penetration) pricing strategy, and then fall behind (exceed) it until convergence at steady-state.*

Figure 3.6 shows that prices can be both above or below the reference price

[12]The non-monotonicity of reference price policy can be shown for a penetration pricing strategy too under similar conditions.

[13]The non-monotonicity of reference externalities policy can be shown for a penetration pricing strategy too under similar conditions.

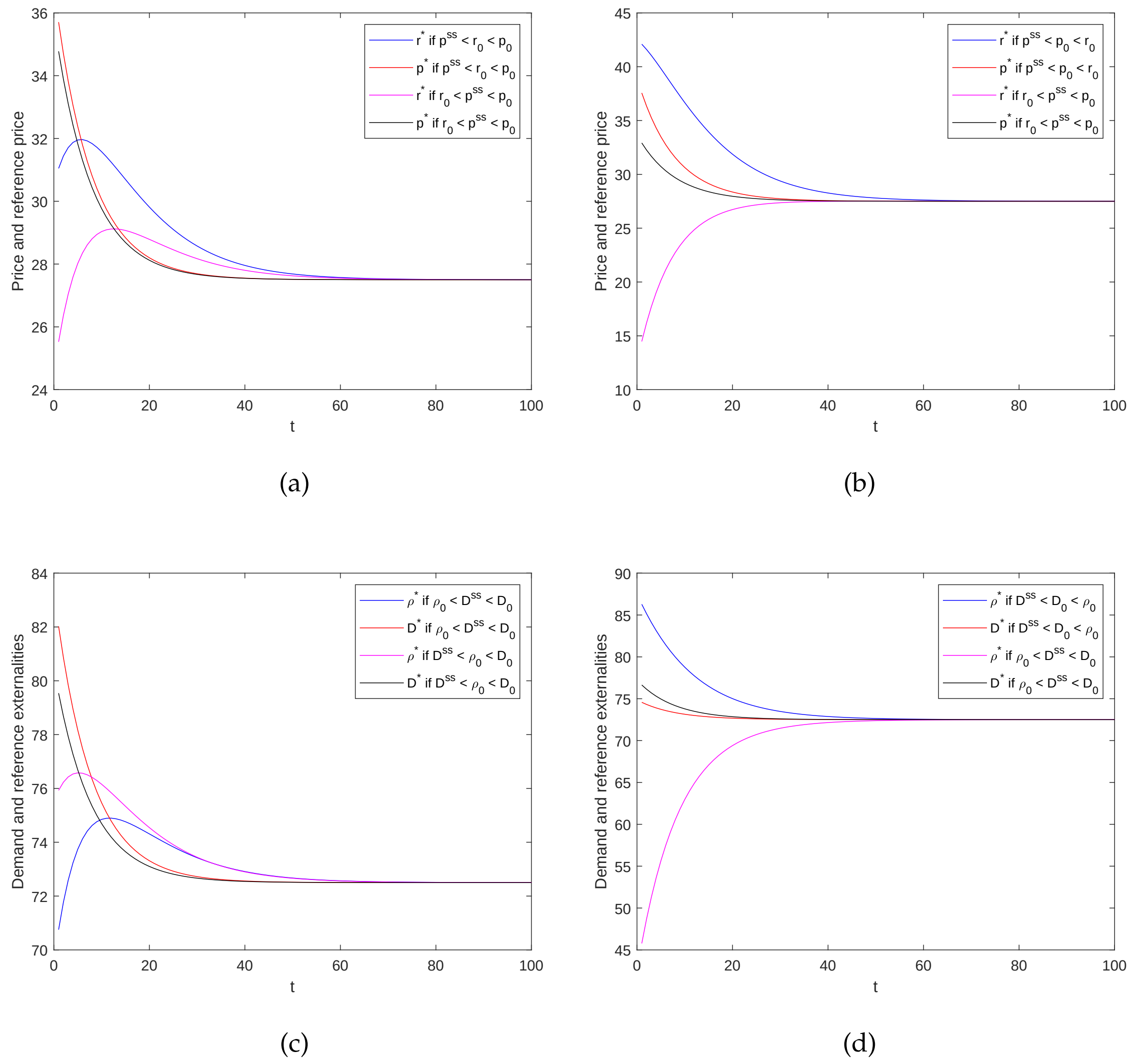


Figure 3.5: Non-monotonicity of reference policies

over time. Thus, the reference-price path might cross the price path along the way but it will converge to the same steady state. Such result has a key implication, that is, consumers may experience both positive and negative reference effects over different purchasing occasions. This can be specially important when consumers are either loss averse or gain seeking. Under such situation, the consumers might experience a shock along the way as a result of asymmetric reference effect that can ultimately influence their buying decision and accordingly firm's pricing strategies.

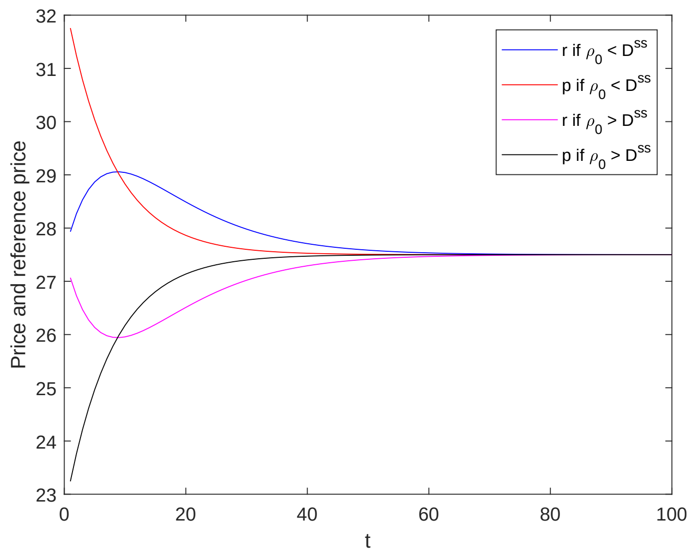


Figure 3.6: Prices and reference prices

3.4.5 Optimal versus misestimated profit

We compare the profits obtained with the MRPs model to each of the benchmark models, that is, a model with either only reference price or only reference externalities. We find that profits can be underestimated or overestimated under a partial (one-reference-point) model. For instance, if we set $r_0 = p_B^{ss} = 40.82$ and $\rho_0 = D^{ss} - 27.5 = 45$, then we have

$$\frac{\pi_B^* - \pi^*}{\pi^*} = -20.00\%$$

that is, the profit is highly underestimated with RP model. If we increase initial reference-externalities point to $\rho_0 = D^{ss} + 27.5 = 100$ while keeping $r_0 = 40.82$, then we obtain

$$\frac{\pi_B^* - \pi^*}{\pi^*} = 43.14\%$$

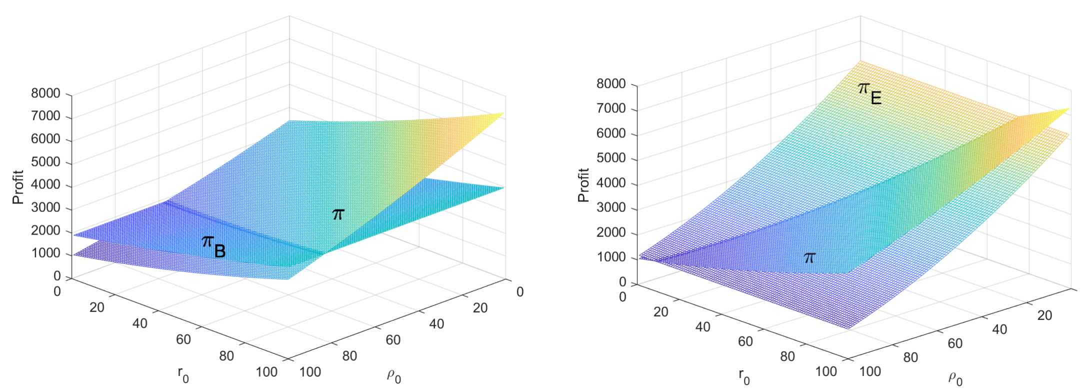
that is, the opposite result. Similarly, if we only consider reference-externalities effect, then we get the same qualitative result, namely, underestimation or overestimation of the profits. To illustrate, suppose that $r_0 = p^{ss} - 27.5 = 0$ and $\rho_0 = D_E^{ss} = 64.52$, then we get

$$\frac{\pi_E^* - \pi^*}{\pi^*} = 44.96\%$$

whereas we obtain

$$\frac{\pi_E^* - \pi^*}{\pi^*} = -20.33\%$$

if we increase the initial reference-price point to $r_0 = p^{ss} + 27.5 = 55$, while leaving $\rho_0 = 64.52$.



(a) (b)

Figure 3.7: Profit comparisons

To get a more general picture, we plot in figure 3.7 the profits for a large range of values of r_0 and ρ_0, with other parameters set as follows: $a = 100$, $\delta = 1$, $s = 0.1$, $\beta = 0.9$ and $\gamma = \lambda = 0.5$. From Figure 3.7(a), we see that the optimal profit is underestimated when initial externalities point is relatively low; otherwise, it is overestimated. The profit, for example, might be underestimated because crowding externalities for consumers with low sales expectations are not accounted for. Figure 3.7(b), which compares the profit of MRPs model to the RE one, shows that the profit is overestimated (underestimated) when both initial reference points are relatively low (high). The overestimated profit, for example, stems from the absence of reference-price effect, which can negatively affect the firm's performance when consumers have a relatively low initial reference-price point. Therefore, it is critical for the firm to account correctly for consumers' psychological behaviors to avoid misestimating its performance. Figure 3.7 also shows that the firm's profit (i.e., π) increases with r_0 and decreases with ρ_0. This is because when consumers have lower expectations from sales and tend to anticipate higher prices, then it would be easier for the firm to leverage the reference effects to secure a higher profit.

3.5 Conclusion

This study takes a normative approach of dynamic pricing in face of multiattribute reference-dependent consumers. Motivated by empirical evidence, we considered consumers who are reference-dependent along two dimensions, namely, price and externalities. We characterized the optimal dynamic pricing strategies with only reference price, only reference externalities, and with both effects. Next, we explored some monotonicity properties of reference price and reference externalities policies. We also showed that the price trajectory might cross the reference-price trajectory, which highlights the importance of the role of asymmetric reference effect. Also, we compared the optimal profits in the different setups.

We find several managerial insights on the pricing strategies and monopolist's performance most of which challenge our current understanding from the previous literature. Our results indicate that the transient pricing strategy does not necessarily depend on the direction of difference between the initial reference-price point and steady-state price, rather it depends on both direction and magnitude of all reference effects. Thus, the collective contributions of all reference effects determine the optimal choice of pricing strategy which might result in different recommendations compared to single-reference pricing strategy. We show that pricing strategy is always monotonic while reference policies might be non-monotonic depending on the dynamics of reference price and reference externalities. When non-monotonicity occurs, the price path can cross the reference price trajectory and hence the price path will not necessarily follow the same trend as reference-price policy. We also show that how the optimal profit can be overestimated or underestimated when consumer behavior is misspecified.

A number of extensions can be envisioned. First, we assumed that the reference effect is symmetric along all dimensions. The literature has extensive evidence on the asymmetric impact of reference-price effect most of which report loss aversion whereas some studies suggest gain-seeking behavior. Tereyağoğlu et al. (2018) also shows that the consumers are loss averse with respect to reference-externalities effect. Thus, it might be interesting to explore the consequence of asymmetric reference effects. This can be implemented by considering consumers who are loss averse along both attributes or even when they are gain seeking towards one attribute and loss averse towards another one. Second, and in relation to the first extension, Hu and Nasiry (2018) showed that psychological biases such as loss aversion is an individual phenomenon and hence it will not be inherited by aggregate demands. Thus, one open question is that how the loss aversion at individual level will generally be translated to aggregate demand in the presence of MRPs. This might result in more surprising result if consumers do not have similar psychological biases towards all the attributes. Third, a more technically challenging extension is to consider strategic consumers.

3.6 Appendix

Proof of Proposition 3.1 . We suppress the argument t for ease of exposition. The Hamiltonian is given by

$$H(p, r, \mu_1) = pD + \mu_1 s(p - r), \tag{3.31}$$

where μ_1 is costate variable associated with state variables r. The optimality condition i.e., $\frac{\partial H}{\partial p} = 0$, yields

$$p = \frac{a + s\mu_1 + \gamma r}{2(\delta + \gamma)}. \tag{3.32}$$

The adjoint equation along with transversality condition is:

$$\dot{\mu_1} = \beta\mu_1 - \frac{\partial H}{\partial r} = (s + \beta)\mu_1 - \gamma p, \qquad \lim_{t\to\infty} e^{-\beta t}\mu_1(t) = 0.$$

After substituting p into adjoint and state equations, we obtain the following system of non-homogeneous differential equations:

$$\dot{\mu_1} = \frac{(s + \beta)\mu_1 - \gamma(a + s\mu_1 + \gamma r)}{2(\delta + \gamma)} \tag{3.33}$$

$$\dot{r} = \frac{s(a + s\mu_1 - (2\delta + \gamma)r)}{2(\delta + \gamma)} \tag{3.34}$$

We use Matlab to solve the system of differential equations described in (3.33)-(3.34), with the initial condition $r(0) = r_0$ and transversality condition $\lim_{t\to\infty} e^{-\beta t}\mu_1(t) = 0$. Then, we obtain the optimal solution given in Proposition 1. The firm's optimal profit can be also expressed as follows:

$$\begin{aligned}\pi_B^* = \frac{p_B^{ss} D_B^{ss}}{\beta} + \frac{(r_0 - p_B^{ss})}{M + \beta}\left(\frac{\Delta - (2s\delta + \beta(\delta + \gamma))}{2s} p_B^{ss} + (1 - \frac{M}{s})D_B^{ss}\right) + \\ \frac{\Delta - (2s\delta + \beta(\delta + \gamma))}{2s(2M + \beta)}(1 - \frac{M}{s})(r_0 - p_B^{ss})^2\end{aligned} \tag{3.35}$$

Note that since the both the objective function and the state equation are concave with respect to control and state variables, we can conclude that the necessary conditions for optimality of solution are sufficient too (see e.g., Kamien and Schwartz, 2012). ■

Proof of Proposition 3.2. We utilize a same approach to show proof of 3.2. The Hamiltonian can be expressed as follows:

$$H(p, r, \mu_2) = pD + \mu_2 s(D - \rho) \tag{3.36}$$

where μ_2 is costate variable associated with state variables ρ. The optimality condition i.e., $\frac{\partial H}{\partial p} = 0$ yields:

$$p = \frac{-(\lambda\rho - a + s\delta\mu_2)}{2\delta} \tag{3.37}$$

The adjoint equation along with transversality condition is:

$$\dot{\mu_2} = \beta\mu_2 - \frac{\partial H}{\partial \rho} = \frac{(s + \beta(1-\lambda))\mu_2 + \lambda p}{1-\lambda}, \qquad \lim_{t\to\infty} e^{-\beta t}\mu_2(t) = 0.$$

After substituting p into adjoint and state equations, we obtain the following system of non-homogeneous differential equations:

$$\dot{\mu_2} = \frac{-2(s+\beta)\delta\mu_2 + \lambda(-a + (s+2\beta)\delta\mu_2 + \lambda\rho)}{2\delta(-1+\lambda)} \tag{3.38}$$

$$\dot{\rho} = -\frac{s(a + s\delta\mu_2 + (-2+\lambda)\rho)}{2(-1+\lambda)} \tag{3.39}$$

We use Matlab to solve the system of differential equations described in Eqs. (3.38-3.39) with the initial condition $\rho(0) = \rho_0$ and transversality condition $\lim_{t\to\infty} e^{-\beta t}\mu_2(t) = 0$. The solution can be found in Proposition 2.1. The optimal profit also takes the following form:

$$\begin{aligned}\pi_E^* = &\frac{p_E^{ss}\rho_E^{ss}}{\beta} + \left(\frac{(2s+\beta)(1-\lambda) - \Gamma}{2s(1-\lambda)} p_E^{ss} - \frac{2s + \beta(1-\lambda) - \Gamma}{2s\delta}\rho_E^{ss}\right)\left(\frac{\rho_0 - \rho_E^{ss}}{N+\beta}\right) - \\ &\left(\frac{(2s+\beta(1-\lambda)-\Gamma)((2s+\beta)(1-\lambda)-\Gamma)}{4s^2(1-\lambda)}\right)\frac{(\rho_0 - \rho_E^{ss})^2}{(2N+\beta)}\end{aligned} \tag{3.40}$$

Similar to Proposition 3.1, both the objective function and the state equation are concave with respect to control and state variables, and hence the necessary conditions for optimality of solution are sufficient too. ■

Proof of Proposition 3.3 . We show the proof of Proposition 3.3 using Maximum Principle. We express the Hamiltonian as follows:

$$H(p, r, \rho, \mu_1, \mu_2) = pD + \mu_1 s(p - r) + \mu_2 s(D - \rho), \tag{3.41}$$

where μ_1 and μ_2 are costate variables associated with state variables r and ρ, respectively. Differentiating the Hamiltonian with respect to p and equating to zero, we get

$$\frac{\partial H}{\partial p} = 0 \Leftrightarrow p = \frac{a - s(\mu_2(\delta + \gamma) - (1 - \lambda)\mu_1) + \gamma r - \lambda \rho}{2(\delta + \gamma)}. \tag{3.42}$$

The two adjoint equations are:

$$\begin{aligned}
\dot{\mu_1} &= \beta\mu_1 - \frac{\partial H}{\partial r} = (s + \beta)\mu_1 - s\left(\frac{\gamma\mu_2}{1 - \lambda}\right) + \frac{\gamma p}{\lambda - 1}, \\
\dot{\mu_2} &= \beta\mu_2 - \frac{\partial H}{\partial \rho} = \frac{\mu_2(s - \beta\lambda + \beta) + \lambda p}{1 - \lambda}.
\end{aligned}$$

After substituting p into adjoint and state equations, we obtain the following system of non-homogeneous differential equations:

$$\dot{\mu_1} = \frac{(1 - \lambda)(s(2\delta + \gamma) + 2\beta(\delta + \gamma))\mu_1 - \gamma s(\delta + \gamma)\mu_2 - \gamma(a + \gamma r - \lambda\rho)}{2(1 - \lambda)(\delta + \gamma)} \tag{3.43}$$

$$\dot{\mu_2} = \frac{s(1 - \lambda)\lambda\mu_1 - (\delta + \gamma)(\lambda(s + 2\beta) - 2(s + \beta))\mu_2 + \lambda(a + \gamma r - \lambda\rho)}{2(1 - \lambda)(\delta + \gamma)} \tag{3.44}$$

$$\dot{r} = \frac{s\left(s(1 - \lambda)\mu_1 - s(\delta + \gamma)\mu_2 - (2\delta + \gamma)r - \lambda\rho + a\right)}{2(\delta + \gamma)} \tag{3.45}$$

$$\dot{\rho} = \frac{s\left(-s(1 - \lambda)\mu_1 + s(\delta + \gamma)\mu_2 + \gamma r - (2 - \lambda)\rho + a\right)}{2(1 - \lambda)} \tag{3.46}$$

where the initial conditions are $r(0) = r_0$ and $\rho(0) = \rho_0$, and the transversality conditions given by

$$\lim_{t \to \infty} e^{-\beta t}\mu_1(t) = 0, \tag{3.47}$$

$$\lim_{t \to \infty} e^{-\beta t}\mu_2(t) = 0. \tag{3.48}$$

To solve such system, we first solve the homogeneous version and hence compute the corresponding eigenvalues and eigenvectors denoted by g and v, respectively, below:

$$g = \left(s+\beta, -s, \frac{1}{2}\left(\beta + \frac{\Theta}{(\delta+\gamma)(1-\lambda)}\right), \frac{1}{2}\left(\beta - \frac{\Theta}{(\delta+\gamma)(1-\lambda)}\right)\right) \tag{3.49}$$

$$v_1 = \begin{bmatrix} \frac{-(2s+\beta)}{s^2} \\ \frac{(2s+\beta)(s(2-\lambda)+\beta(1-\lambda))}{s^2(s(2\delta+\gamma)+\beta(\delta+\gamma))} \\ -\frac{s(2-\lambda)+\beta(1-\lambda)}{s(2\delta+\gamma)+\beta(\delta+\gamma)} \\ 1 \end{bmatrix}, \quad v_2 = \begin{bmatrix} 0 \\ 0 \\ \frac{\lambda}{\gamma} \\ 1 \end{bmatrix}$$

$$v_3 = \begin{bmatrix} \frac{\gamma\left(-\frac{\Theta(s(2\delta+\gamma)+\beta(\delta+\gamma))}{(\delta+\gamma)(2s\delta+\beta(\delta+\gamma))}+s(\lambda-2)+\beta(\lambda-1)\right)}{s^2(\delta\lambda+\gamma)} \\ \frac{\lambda\left(\frac{(s(2\delta+\gamma)+\beta(\delta+\gamma))(\Theta-\beta(\lambda-1)(\delta+\gamma))}{\delta+\gamma}+s(\beta(\gamma-\delta(\lambda-2))-2s\delta(\lambda-2))\right)}{s^2(\delta\lambda+\gamma)(2s\delta+\beta(\delta+\gamma))} \\ \frac{(\lambda-1)(2s-\beta\lambda+\beta)}{\Theta} \\ 1 \end{bmatrix}$$

$$v_4 = \begin{bmatrix} \frac{\gamma\left(\frac{(s(2\delta+\gamma)+\beta(\delta+\gamma))(\beta(\lambda-1)(\delta+\gamma)+\Theta)}{\delta+\gamma}-s((2-\lambda)\delta(2s+\beta)+\beta\gamma)\right)}{s^2(\delta\lambda+\gamma)(2s\delta+\beta(\delta+\gamma))} \\ \frac{\lambda\left(s((2-\lambda)\delta(2s+\beta)+\beta\gamma)-\frac{(s(2\delta+\gamma)+\beta(\delta+\gamma))(\Theta+\beta(\lambda-1)(\delta+\gamma))}{\delta+\gamma}\right)}{s^2(\delta\lambda+\gamma)(2s\delta+\beta(\delta+\gamma))} \\ \frac{(1-\lambda)(2s+\beta(1-\lambda))}{\Theta} \\ 1 \end{bmatrix}$$

where

$$\Theta = \sqrt{(1-\lambda)(\delta+\gamma)(2s+\beta(1-\lambda))(2s\delta+\beta(\delta+\gamma))}.$$

The primary solution denoted by X_c is :

$$X_c = \begin{pmatrix} \mu_1 \\ \mu_2 \\ r \\ \rho \end{pmatrix} = k_1 e^{(s+\beta)t} v_1 + k_2 e^{-st} v_2 + k_3 e^{\frac{1}{2}\left(\beta+\frac{\Theta}{(\delta+\gamma)(1-\lambda)}\right)t} v_3 + k_4 e^{\frac{1}{2}\left(\beta-\frac{\Theta}{(\delta+\gamma)(1-\lambda)}\right)t} v_4.$$

Transversality conditions imply that $k_1 = 0$ and $k_3 = 0$. Next, we derive the particular solution denoted by X_p as follows:

$$X_p = \begin{pmatrix} \frac{a\gamma}{2\delta(s+\beta)+\beta(\gamma-\delta\lambda)} \\ -\frac{a\lambda}{2\delta(s+\beta)+\beta(\gamma-\delta\lambda)} \\ \frac{a(s+\beta(1-\lambda))}{2\delta(s+\beta)+\beta(\gamma-\delta\lambda)} \\ \frac{a((s+\beta)\delta+\beta\gamma)}{2\delta(s+\beta)+\beta(\gamma-\delta\lambda)} \end{pmatrix}$$

Therefore, the solution to our system is $X = X_c + X_p$ and take the following form:

$$\mu_1(t) = k_4 \left(\frac{\gamma\left(\frac{(s(2\delta+\gamma)+\beta(\delta+\gamma))(\beta(\lambda-1)(\delta+\gamma)+\Theta)}{\delta+\gamma} - s((2-\lambda)\delta(2s+\beta)+\beta\gamma)\right)}{s^2(\delta\lambda+\gamma)(2s\delta+\beta(\delta+\gamma))} \right) e^{-Lt} + \frac{a\gamma}{2\delta(s+\beta)+\beta(\gamma-\delta\lambda)} \tag{3.50}$$

$$\mu_2(t) = k_4 \left(\frac{\lambda\left(s((2-\lambda)\delta(2s+\beta)+\beta\gamma) - \frac{(s(2\delta+\gamma)+\beta(\delta+\gamma))(\Theta+\beta(\lambda-1)(\delta+\gamma))}{\delta+\gamma}\right)}{s^2(\delta\lambda+\gamma)(2s\delta+\beta(\delta+\gamma))} \right) e^{-Lt} - \frac{a\lambda}{2\delta(s+\beta)+\beta(\gamma-\delta\lambda)} \tag{3.51}$$

$$r(t) = k_2 \frac{\lambda}{\gamma} e^{-st} + k_4 \left(\frac{(1-\lambda)(2s+\beta(1-\lambda))}{\Theta} \right) e^{-Lt} + \frac{a(s+\beta(1-\lambda))}{2\delta(s+\beta)+\beta(\gamma-\delta\lambda)} \tag{3.52}$$

$$\rho(t) = k_2 e^{-st} + k_4 e^{-Lt} + \frac{a((s+\beta)\delta+\beta\gamma)}{2\delta(s+\beta)+\beta(\gamma-\delta\lambda)} \tag{3.53}$$

where $L = \frac{1}{2}\left(\frac{\Theta}{(\delta+\gamma)(1-\lambda)} - \beta\right)$. We use initial conditions $r(0) = r_0$ and $\rho(0) = \rho_0$ to obtain coefficients k_2 and k_4 below:

$$k_2 = \frac{\gamma\left((2s\delta+\beta(\delta+\gamma))((\delta+\gamma)r_0 - r^{ss}) - \Theta(\rho_0 - \rho^{ss})\right)}{\lambda\Psi - \gamma\Theta} \tag{3.54}$$

$$k_4 = -\frac{(\delta+\gamma)(2s\delta+\beta(\delta+\gamma))\left(\gamma(r_0 - r^{ss}) - \lambda(\rho_0 - \rho^{ss})\right)}{\lambda\Psi - \gamma\Theta} \tag{3.55}$$

where $\Psi = (\delta+\gamma)(2s\delta+\beta(\delta+\gamma))$. After inserting coefficients k_2 and k_4 into Eqs. (3.52)-(3.53) and making tedious algebraic simplifications, we can obtain the optimal strategies that are given in Proposition 3.3. The optimal profit can be expressed as follows:

$$\pi^* = \frac{p^{ss}D^{ss}}{\beta} + \frac{p^{ss}\left((2s\delta+\beta(\delta+\gamma))(1-\lambda)+\Theta\right)\left(\gamma(r_0-r^{ss})-\lambda(\rho_0-\rho^{ss})\right)}{2(\beta+L)(1-\lambda)\left((s+\beta(1-\lambda))(\delta+\gamma)+s\delta(1-\lambda)+\Theta\right)} + \frac{D^{ss}\left((2s+\beta(1-\lambda))(\delta+\gamma)+\Theta\right)\left(\gamma(r_0-r^{ss})-\lambda(\rho_0-\rho^{ss})\right)}{2(\beta+L)(\delta+\gamma)\left((s+\beta(1-\lambda))(\delta+\gamma)+s\delta(1-\lambda)+\Theta\right)} + \frac{\left((2s\delta+\beta(\delta+\gamma))(1-\lambda)+\Theta\right)\left((2s+\beta(1-\lambda))(\delta+\gamma)+\Theta\right)\left(\gamma(r_0-r^{ss})-\lambda(\rho_0-\rho^{ss})\right)^2}{4(\beta+2L)(\delta+\gamma)(1-\lambda)\left((s+\beta(1-\lambda))(\delta+\gamma)+s\delta(1-\lambda)+\Theta\right)^2} \tag{3.56}$$

Note that under the assumption of $\lambda\Psi \neq \gamma\Theta$, the state variables r^* and ρ^* are well defined. Moreover, the global optimality of solution is guaranteed because both the objective function and the state equations are concave with respect to control and state variables (see, e.g., Kamien and Schwartz, 2012). ■

Proof of Proposition 3.4. For part (i), it would be sufficient to differentiate p^* with respect to t and check whether it is always an increasing or decreasing function of the time and also if it converges to p^{ss} when t tends to infinity. It is easy to show that p^* is monotonically increasing i.e., $\frac{\partial p^*}{\partial t} > 0$ if $\gamma(r_0-r^{ss})-\lambda(\rho_0-\rho^{ss}) < 0$, and p^* is monotonically decreasing i.e., $\frac{\partial p^*}{\partial t} < 0$ if $\gamma(r_0-r^{ss})-\lambda(\rho_0-\rho^{ss}) > 0$. Moreover, it is immediately clear that p^* converges to p^{ss} when $t \to \infty$.

For part (ii), if we can show that there exists certain conditions under which $\frac{dr^*}{dt}$ can be both positive and negative along its trajectory, then it can be concluded that the monotonicity does not necessarily hold. We can compute $\frac{\partial r^*}{\partial t}$ as follows:

$$\frac{dr^*}{dt} = \frac{L\Theta(\gamma(r_0-r^{ss})-\lambda(\rho_0-\rho^{ss}))e^{-Lt}}{\lambda(\delta+\gamma)(2s\delta+\beta(\delta+\gamma))-\gamma\Theta} - \frac{s\lambda e^{-st}((2s\delta+\beta(\delta+\gamma))(\delta+\gamma)(r_0-r^{ss})-\Theta(\rho_0-\rho^{ss}))}{\lambda(\delta+\gamma)(2s\delta+\beta(\delta+\gamma))-\gamma\Theta}$$

$$\frac{dr^*}{dt} = e^{-st}\left(\frac{L\Theta(\gamma(r_0-r^{ss})-\lambda(\rho_0-\rho^{ss}))e^{(-L+s)t}-\lambda s((\delta+\gamma)(2s\delta+\beta(\delta+\gamma))(r_0-r^{ss})-\Theta(\rho_0-\rho^{ss}))}{\lambda(\delta+\gamma)(2s\delta+\beta(\delta+\gamma))-\gamma\Theta}\right)$$

Let us assume that $(r_0 - r^{ss}) = 0$ and $(\rho_0 - \rho^{ss}) < 0$. Then,

$$\frac{dr^*}{dt} = \frac{\Theta\lambda(\rho^{ss}-\rho_0)e^{-st}}{\lambda(\delta+\gamma)(2s\delta+\beta(\delta+\gamma))-\gamma\Theta}\left(Le^{(-L+s)t}-s\right)$$

It is easy to show that $-L+s = \frac{\Theta-(2s+\beta)(\delta+\gamma)(1-\lambda)}{-2(\delta+\gamma)(1-\lambda)}$ can be both positive and negative depending on parameter values. If $-L+s > 0$, then $\left(Le^{(-L+s)t}-s\right)$ is negative first and then becomes positive, and vice versa. More specifically, for $-C+s>0$, $\frac{dr^*}{dt}$ will have different signs for $t<\tau$ and $t>\tau$ where $\tau=(\frac{\ln s-\ln L}{s-L})$. Thus, we have showed certain conditions under which r^* can be both increasing and decreasing along the trajectory and this complete the proof of part (ii). In addition, it is easy to show that r^* can be monotone under certain conditions. For instance, if $\rho_0=\rho^{ss}$, then r^* will be monotone.

Part (iii) can be proved similar to part (ii). To show a sketch of this proof, it would be sufficient to examine $\frac{d\rho^*}{dt}$ when $\rho_0-\rho^{ss}=0$. Consequently, ρ^* would not be necessarily monotone too. ■

Proof of Corollary 3.1. It is sufficient to show that p^*-r^* can be both positive and negative under certain conditions. Let us assume $r_0=r^{ss}$. We have:

$$p^*-r^*=-\frac{e^{-Lt}((s+\beta(1-\lambda))(\delta+\gamma)+s(\delta+\gamma)+\Theta)\lambda(\rho_0-\rho^{ss})}{2(\delta+\gamma)((s+\beta(1-\lambda))(\delta+\gamma)+s\delta(1-\lambda)+\Theta)}-\left(\frac{e^{-Lt}\lambda\Theta(\rho_0-\rho^{ss})-e^{-st}\lambda\Theta(\rho_0-\rho_{ss})}{\lambda(\delta+\gamma)(2s\delta+\beta(\delta+\gamma))-\gamma\Theta}\right)$$

$$p^*-r^*=e^{-Lt}\lambda(\rho_0-\rho^{ss})\left(\frac{e^{(-s+L)t}\Theta-\Theta}{\lambda(\delta+\gamma)(2s\delta+\beta(\delta+\gamma))-\gamma\Theta}-\frac{((s+\beta(1-\lambda))(\delta+\gamma)+s(\delta+\gamma)+\Theta)}{2(\delta+\gamma)((s+\beta(1-\lambda))(\delta+\gamma)+s\delta(1-\lambda)+\Theta)}\right) \tag{3.57}$$

If assume $-s+L>0$, then for $\rho_0-\rho^{ss}<0$, it is immediately clear that p^*-r^* is positive first and then becomes negative whereas for $\rho_0-\rho^{ss}>0$, p^*-r^* is negative first and then becomes positive. ■

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General Conclusion

This thesis, elaborated in three essays, seeks to develop dynamic pricing policies for a monopolist by featuring various nuances of consumer behavior and social influences in context of different product categories. On the consumer side, we examine strategic consumer behavior, consumers heterogeneity, social influences and reference-dependent behavior. The implications of strategic consumer behavior are studied in terms of intertemporal purchasing decisions or interacting with others whereas the consumer heterogeneity is analyzed with respect to different adoptions drivers. The social influences are addressed through WoM communications or social externalities in different setups depending on the nature of product category. Motivated by prospect theory, we also study reference-dependent behavior when consumers are influenced by either single or multiple reference effects. On the firm side, we shed light on how the optimal pricing policy and profit of the monopolist are influenced by consumer behavior, market segments or firm's commitment. On the product side, we study dynamic pricing for a new durable product or an experience good by accommodating their underlying features.

We find that consumer behavior plays a crucial role on the choice of pricing policy for the firm. Through quantitative and numerical analyses, we propose skimming, penetration, constant, or an inverted U-shaped pricing strategies depending on the underlying consumer behavior. In particular, in context of new product diffusion, we find that a penetration pricing strategy is optimal in the face of forward-looking behavior, however, its intensity depends substantially on consumers segmentation and their susceptibility to WoM and firm's goodwill. When consumers are myopic, an inverted U-shaped, that is, an increasing price path followed by a decreasing one materializes. For consumers who are prone to social externalities and reference-price effect, we find that reference-dependent behavior advocates skimming pricing strategy whereas externalities call for penetration pricing one. Under symmetric reference-price effect and social externalities, if consumers are not strongly forward-looking and social externalities are powerful, the firm adopts a penetration pricing strategy. For consumers with multiple reference-dependent behavior who consider buying experience goods repeatedly, we find that both skimming and penetration pricing strategies might emerge depending on the direction and magnitude of all reference effects, which turn to challenge our current understanding in the literature. More surprisingly, the reference price policy as well as reference-externalities policies are not necessarily monotone which has salient implications for consumers with psychological biases.

In the first essay, we study heterogeneous and forward-looking consumers who are prone to strategic interactions and social influences at individual level. In doing so, we use diffusion theory to account for heterogeneity of consumer adoption behavior and predict the new product demand. We also employ MFG theory that enables us to propose a new product diffusion framework by connecting individual-level decisions to aggregate influences and exploring the role strategic interactions among consumers. MFG methodology empowers us to exploit a rich set of consumer behavior at individual and aggregate levels that was not accessible otherwise. Thus, it can be a promising methodology in diffusion literature whose focus has shifted more towards individual-level behavior in different contexts such as social networks (see, for instance, Muller and Peres, 2019). While we do not obtain our results analytically given the complexity of the considered problem, we carry out an extensive numerical analysis to derive managerial policies. Besides dynamic pricing policies, we suggest advertising policies and forecast market penetration of the newly launched product. Overall, the first essay makes methodological and substantive contributions to literature by introducing MFG methodology and providing new managerial insights.

In the second essay, we determine dynamic pricing strategies when forward-looking consumers are reference-price dependent or prone to social influences, where the latter is measured by externalities. We develop progressively different nested models to control for role of each behavior and use the rational expectation equilibrium to obtain the results. We implement two types of pricing regimes, that is, responsive and preannounced pricing to explore the role of firm's commitment. We develop pricing strategies on the basis of different sets of consumer behavior and firm's commitment. While the second essay neither considers initial reference-price point nor the demand of second period for externalities, it retains its focus on the key ingredient of these effects consistent with the literature and exploits rational expectation equilibrium methodology to derive and assess the results analytically. In a nutshell, the second essay makes a substantive contribution to the dynamic pricing literature by developing managerial policies.

The last essay proposes dynamic pricing strategies for a monopoly in the face of consumers who make repeated purchasing decisions in a multiattribute reference-dependent fashion. Motivated by empirical evidence in marketing literature, consumers are assumed to have reference-dependent behavior across both price and externalities attributes. With the aid of optimal control methodology, we take a normative approach to characterize the optimal pricing strategies as well as reference policies. We also examine the monotonicity of reference price and reference externalities policies and further compare the firm's performance when consumers have either single-reference dependent or multiple-reference dependent behavior. Our methodology equips us to analytically obtain and assess all the results in a fully dynamic framework. The third essay contributes to behavioral dynamic pricing literature by challenging the status quo on the transient pricing recommendations and reference policies properties.

To better tailor dynamic pricing strategies to consumer behavior, future studies can incorporate more nuanced consumer behavior or develop even richer methodologies. Notably, the role of social influences on dynamic pricing, which have been a recurring theme in this thesis, can be enriched by accounting for social networks or tying it with complimentary products. The implications of referral and seeding programs or opinion leadership are usually studied within social networks where the latter is greatly tied to development of marketing strategies (Muller and Peres, 2019). Therefore, extensions of the first or second essays in this manner can be appealing for managers to strategize their prices with social networks considerations. In the context of durable complimentary products, Li (2019) proposes a skimming pricing for one product and a penetration pricing for another one to temper the impact of forward-looking behavior in an empirical setup, albeit in the absence of the social influences. A natural extension is to take a theoretical approach in presence of social influences to see how consumer behavior in context of complementary product affect pricing strategies. Behaviorally speaking, consumer may consider the future prices as their reference point, especially when the firm adopts preannouncded strategies, and hence it might be interesting, for instance in the second essay, to apply such concept. In relation to reference dependency, one avenue for research is to exploit Hu and Nasiry (2018)'s findings on non-inheritance of individual-level biases to aggregate one by considering a non-uniform distribution for consumer valuations in the third essay. Technically speaking, this can transform a linear-quadratic optimal control problem to a nonlinear one and hence it might require more methodological developments.

This thesis sought to help managers, say a car company or a theater organizer, on how to price their products/services depending on the certain consumer behavior, and what are the implications of each pricing strategy on their performance. To that end and to get back to what we started from, this thesis advises the car company to adopt a penetration pricing strategy with tailored intensities for its newly launched car, whereas it recommends to the theater organizer to employ a skimming or penetration pricing strategies for its shows' tickets depending on collective benefits from consumers reference-dependent behavior.

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