HEC MONTRÉAL
École affiliée à l'Université de Montréal

Three Essays on Dynamic Games and Sustainable Fishery Management

par
Ilyass Dahmouni

Thèse présentée en vue de l'obtention du grade de Ph. D. en administration
(option Méthodes Quantitatives de Gestion)

Novembre 2019

HEC MONTRÉAL
École affiliée à l'Université de Montréal

Cette thèse intitulée :

Three Essays on Dynamic Games and Sustainable Fishery Management

Présentée par :

Ilyass Dahmouni

a été évaluée par un jury composé des personnes suivantes :

Mr. Pierre-Olivier Pineau
HEC Montréal
Président-rapporteur

Mr. Georges Zaccour
HEC Montréal
Directeur de recherche

Mme. Michèle Breton
HEC Montréal
Membre du jury

Mr. Fabio Lamantia
University of Calabria
Examinateur externe

Mr. André Tchokogue
HEC Montréal
Représentant du directeur de HEC Montréal

Résumé

Cette thèse porte sur des applications de la théorie des jeux dynamiques à l'analyse de problèmes de gestion de la pêche. Ce cadre méthodologique permet de prendre à la fois les externalités intertemporelles et stratégiques. La thèse est composée de trois essais.

Dans le premier essai, nous examinons le problème de la gestion efficace d'une pêcherie où des externalités de pollution sont présentes. Le modèle bionomique de pêche en libre accès est analysé dans le cadre d'un jeu différentiel à *n* joueurs avec deux variables d'état, à savoir, le stock de poissons et le stock de pollution. Nous caractérisons la solution coopérative et l'équilibre de Nash en rétroaction, et définissons une règle de partage égalitaire pour allouer le gain coopératif total sur un horizon temporel infini, et montrons que cette règle est cohérente dans le temps.

Dans le deuxième essai, nous considérons un modèle de pêcherie à deux espèces qui interagissent biologiquement de façon compétitive, symbiotique ou selon une relation proie-prédateur. Chaque espèce est pêchée par un groupe de pêcheurs différent. Nous caractérisons et comparons les stratégies de pêche à l'équilibre, les stocks de la ressource à l'état stationnaire et la somme des utilités actualisées sous différents modes de jeu, à savoir, la non-coopération dans les deux groupes, la coopération dans chacun des groupes et coopération dans un seul groupe de pêcheurs. Nos principaux résultats sont les suivants: (i) Dans tous les scénarios, la stratégie d'équilibre d'un joueur pêchant l'une des deux espèces consiste à pêcher, à chaque période, une proportion du stock disponible. (ii) Les dividendes de la coopération dans un groupe donné augmentent avec le nombre de joueurs dans ce même groupe. (iii) La coordination entre les acteurs d'un groupe donné peut être nuisible (biologiquement et économiquement) aux autres espèces.

Finalement, dans le troisième essai, nous considérons un modèle à deux périodes d'une pêcherie exploitée par deux monopoles vendant leurs prises sur deux marchés séparés. En supposant que le coût de la pêche est une information privée et qu'il existe une autorité capable de réguler le marché, nous montrons que le régulateur est en mesure de faire le design d'un contrat qui permet à la fois de résoudre le problème d'asymétrie d'information et d'amener les firmes à pêcher au niveau collectivement optimal.

Mots-clés

Pêcherie; Pollution; Jeux dynamiques; Partage égalitaire; Cohérence dans le temps; Pêcherie à deux espèces; Interactions biologiques; Strategies noncooperatives et cooperatives, Monopoles; Competition par comparaison; Régulation; Asymmetrie d'information.

Abstract

This thesis is composed of three essays concerning fisheries management problems. To account for both intertemporal and strategic interactions between agents, we adopt a dynamic game theory framework.

In the first essay, we consider the problem of efficiently managing a fishery where pollution externalities are present. The open-access bionomic model is analyzed in an n-player differential game framework with two-state variables, that is, the fish stock and the pollution stock. We characterize the noncooperative feedback-Nash equilibrium and cooperative solution. We define an egalitarian sharing rule to allocate the joint welfare maximizing payoff over an infinite time horizon and show that this rule is time consistent.

In the second essay, we consider a two-species fishery model where the species can have different biological interactions, namely, competitive, symbiotic or prey-predator relationships. Each species is harvested by a group of fisherpersons. We characterize and compare equilibrium harvesting strategies, steady-state stocks and total discounted utilities under different modes of play, that is, noncooperation in both groups, cooperation in each of the groups and cooperation in only one group of fisherpersons. Our main results are as follows: (i) In all scenarios, the equilibrium strategy of an agent fishing either species consists of harvesting, in each period, a proportion of the available stock. (ii) The dividend of cooperation in a given group is increasing with the number of members in that group. (iii) Coordination between agents fishing a given species may be detrimental (biologically and economically) to the other species.

Finally, in the third essay, we consider a two-period model of a fishery exploited by two firms selling their harvests in separate local markets. The harvesting cost is private information. Assuming there is an agency that can regulate the market, we propose a contract in the realm of yardstick competition framework that allows the regulator to solve the information asymmetry problem and achieve cooperation.

Keywords

Fisheries; Pollution; Dynamic Games; Fair Sharing; Time Consistency; Two-species Fisheries; Biological interactions; Noncooperative and cooperative strategies; Monopoly; Yardstick Competition; Regulation; Asymmetric Information.

Contents

List of Tables

List of Figures

To mama Fouzia & baba Abdenbi & Youssef
the purest LOVE one can have..

Acknowledgements

Quoi de plus spirituel que d'achever cette thèse, ici, au bord du fleuve sacré du Gange.

Ce projet ne put être réalisé que grâce aux soutien, aide et encouragement de personnes magnifiques qui ont marqué mon esprit et mon inspiration pour toujours.

Le plus gros crédit est accordé à mes chers parents qui m'ont permis d'atteindre ce rêve. Ma chère maman, pour qui, le sacrifice envers ses deux enfants représente une vocation de vie et en fait d'elle une femme extraordinaire que je prendrai toujours pour exemple. Mon feu papa qui m'a toujours appris que Savoir, Apprentissage, Persévérance devront être la devise de chacun jusqu'au "*TERMINUS*", il aurait été si fier, il l'est certainement. J'aimerais aussi remercier mon petit frère Youssef qui ne cesse d'exprimer son souhait de me voir réussir dans tout ce que j'entreprends.

Outre mes parents biologiques, j'aimerais reconnaître l'aide immense de mes *parents académiques*, pour qui, il n'existerait sans doute pas un mot au dictionnaire pouvant décrire leur impact sur ma vie, mon professeur et superviseur de thèse Georges Zaccour qui a pu faire preuve tout au long de ces années d'une patience hors norme grâce à son dévouement magique aussi bien pour moi même que pour l'ensemble de son corps estudiantin, ainsi, je témoigne fort du fait d'avoir cru en mes idées restera gravé sur mon cœur à tout jamais. Mon professeur Michèle Breton de qui j'ai énormément appris, sa façon de donner sans compter fait d'elle une académicienne extraordinaire. J'aimerais aussi saluer en Michèle et Georges l'humain sympathique avec des valeurs éthiques et sociales exquises, et sûrement ma vie n'est plus la même après vous avoir connu. Je reviendrai toujours vous revoir.

Merci aux membres du jury pour leur temps investit pour l'évaluation de cette thèse, leurs présences et commentaires respectives le jour de la soutenance orale. Je nommerais le professeur Pierre-Olivier Pineau, président-rapporteur, le professeur Fabio Lamantia, examinateur externe, et le professeur André Tchokogue, représentant du directeur de HEC Montréal.

Et pour le soutien financier pendant de longues années, je voudrais remercier également le CRSNG et le CRSH, le programme de doctorat de HEC Montréal, la chaire de la théorie des jeux et gestion, et le gouvernement du Québec.

Un remerciement honorifique est pour mes anciens professeurs tout au long de mon cursus, très spécialement, le professeur Jocelyn Donze l'auteur de mon premier cour de théorie des jeux, le professeur Ujjayant Chakravorty pour m'avoir enseigné mon premier modèle de pêcherie, et le professeur Sylvain Landry que je prendrais comme modèle d'enseignant optimal.

Une reconnaissance spéciale est pour mes meilleurs amis, Elnaz Kanani et Faouzi Moueffek, ils ont fait toute la différence aux moments clés.

Merci à tous mes amis/collègues, à savoir, Baris, Vishua, Sébastien, Mustapha, Hussain, Utsav, Nahid, Jaafar, Mahsa, Safae, Amir, Aichouche, Tarik, Pablo*, Mona, Claire, Nadia, Hossein, Samar, Can Baris Cetin, Nikos, Fabien, Lee, Roberta, Michel.

Un grand merci à tous mes proches pour leurs soutiens, notamment Mr Abderrafia Bennani et Mme Atika Demnati.

Je ne manquerais pas d'exprimer une pensée forte et profonde à Saad, Hassan, Fatima-Zahra Riad, Fatima, Mariam, Mouad, Hicham Ouahabi, Mehdi, Hicham Faraja, Ismail, Younes, Zine el Abidine, Mounia. Et à ma team de feu: Jérôme, Hélène, Marietta, Philippe, Jessica, Mathieu, Sabrina et Marianne.

Et pour finir* ..

"*La constancia vence lo que la dicha no alcanza*"..

General Introduction

Achieving a sustainable exploitation of a renewable natural resource requires a wise arbitrage between the current value of the resource, and its discounted value of future use. For example, in fisheries management, harvesting a higher level today results in limited access to the stocks for future generations, whereas, a lower level of harvest reduces the amount of fish available on the market while eliminating a considerable portion of the profit of the fishermen.

According to the Food and Agriculture Organization of the United Nations (FAO), the decline in fish stocks is a worrying and potentially dangerous threat to life in the ocean. Pollution and overfishing are considerably endangering the marine biodiversity. In the absence of successful actions to protect this resource, most of the species may be subject to a dramatic extinction in the near future. FAO reports claim that half of the world's available fish is fully exploited, with at least 76 species being an over-exploitation activity. From FAO's 2018 report *The state of world fisheries and aquaculture*, we quote its director general's foreword: *"(...) Fish production in 2016 reached an all-time high of 171 million tonnes, of which 88 percent was utilized for direct human consumption. (...) This production resulted in a record-high per capita consumption of 20.3 kg in 2016. Since 1961 the annual global growth in fish consumption has been twice as high as population growth. (...) These and other challenges engendered FAO's Blue Growth Initiative, an innovative, integrated and multisectoral approach to the management of aquatic resources aimed at maximizing the ecosystem goods and services obtained from the use of oceans, inland waters and wetlands, while also providing social and economic benefits. "*

Moreover, when the resource is exploited by more than one agent, the need to consider strategic externalities, and a game-theoretic approach, on the top of the intertemporal ones. A seminal paper in the fisheries literature is Hardin's tragedy of the commons, which stipulates that open access, or competition, leads to harvesting at levels that are higher than the resource's rate of reproduction, which may cause its depletion (Hardin, 1968). This result highlighted the need for further studies to prevent such a tragedy. A rich literature developed over the last five decades. Certainly, Munro (1979), Clark (1980), and Levhari and Mirman (1980) represent the contributions that have drawn the guidelines of this game theoretical approach for analyzing the exploitation of renewable resources, fisheries in particular; for more details, we refer the reader to the surveys in Bailey, Sumaila, and Lindroos (2010), Gronbaek, Lindroos, Munro, and Pintassilgo (2018), Hannesson (2011), Long (2011, 2018), and Sumaila (2013). In its three essays, this thesis contributes to the dynamic game theory literature in fisheries. Using dynamic games as a methodological framework is the only way to account for both the intertemporal and strategic interactions mentioned above.

In the first essay, we consider the problem of efficiently managing a fishery where pollution externalities are present. We study the open-access bionomic model in a multiplayer differential game framework with two-state variables, that is, the fish stock and the pollution stock. The reason for this essay stems from the fact that most of the contributions focused on harvesting quantity under different institutional settings, mode of play by the agents, the species and their biological interactions, leaving often aside the quality of the marine environment. For instance, Matsuoka et. al (2005) advocates that nearly 212,000 to 505,000 octopus are killed per year due to marine pollution within the area's fishing grounds in southern Japan. Few contributions shed a light on this environmental issue, e.g., Tahvonen (1991), Xepapadeas (1995), and Wirl (2004). In fact, the main reason that prevents adopting a game model, where both (nonlinear) resource dynamics and environmental dynamics are interacting, is the mathematical complexity behind the model solving, that is, the nonlinearity of the fish growth rate, renders the analytical characterization of the equilibrium a very complicated task. This chapter entitled *"A fair and time-consistent sharing of the joint exploitation payoff of a fishery"*[1] in which we consider a fishery exploited by numerous strategic agents, where harvesting activities damage the environment and accumulated pollution has a negative impact on the growth rate of the resource. For this purpose, we adopt a modified version of the dynamic game model in Xepapadeas (1995) and determine the feedback-Nash equilibrium and cooperative harvesting strategies and outcomes. Furthermore, we assume that the community of fisherpersons agree on a fair sharing of the dividend of cooperation, that is, each player gets her open-access outcome plus an equal share of the dividend of cooperation. Next, we define a sustainable sharing over time, in such a way that no player will be tempted to choose competition over cooperation. This work contributes to the fisheries literature at two levels. First, at a mathematical level, the model considers the resource-environment interactions, which should lead to better predictions and normative implications. To avoid the above-mentioned tractability difficulties, in a two-state variables framework we approximate the nonlinear resource dynamics by an inverted-V shaped function as in Benchekroun (2003). This simplifies the characterization of the equilibrium strategies and payoffs. Second, we propose a scheme to enforce the sustainability of cooperation over time, which gives larger collective and individual outcomes, as well as higher biological benefits, that is, a larger stock of the biomass than under noncooperation. The closest work to this chapter is Mazalov and Rettieva (2010), where also a time-consistent scheme is adopted to enforce cooperation over time. However, their model does not include any environmental considerations. Given the mounting evidence that fish stocks are overexploited and that fishing activities affect the marine environment, a model accounting for pollution is clearly relevant and timely.

The next chapter of this thesis is entitled *"Equilibria in a two-species fishery"*.[2] Here, we consider a two-species fishery model where the species biologically interact, according to a competitive, symbiotic or prey-predator relationship. Each species is harvested

---

[1] This chapter was published as: Dahmouni I, Vardar B, Zaccour G. A fair and time-consistent sharing of the joint exploitation payoff of a fishery. Natural Resource Modeling. 2019; e12216.

[2] This chapter is published as : M. Breton, I. Dahmouni, G. Zaccour, “Equilibria in a two-species fishery”, Mathematical Biosciences, 309, 78-91, 2019.

by a group of fisherpersons. We characterize and compare equilibrium harvesting strategies, steady-state stocks and total discounted utilities under different modes of play, that is, noncooperation in both groups, cooperation in each of the groups and cooperation in only one group of fisherpersons. This chapter is motivated by the simple idea that fisheries are typically populated by more than one species, which brings the following question on the table: how the results obtained in one-species models generalize to multiple species? Furthermore, in addition to the dynamic (intertemporal) externalities, the analysis must account for the biological interactions between the species. Fischer and Mirman (1992, 1996) studied a fishery with two species, each harvested by one player. In this paper, we extend their analysis by assuming that each species is harvested by more than one agent, more specifically, we add a competitive dimension to the fishing activities of each species. That is, the characterization of equilibrium harvesting strategies and outcomes under various mode of plays, as well as various biological relationships between the species. Our investigation mainly focus on separating the effects of three sources of externalities, namely, the intertemporal, biological and behavioral externalities. The contribution of this chapter can be summarized as follows: (i) In all setups, the equilibrium strategy of an agent fishing either species consists of harvesting, in each period, is a fixed proportion of the available stock. (ii) The dividend of cooperation in a given group is increasing with the number of agents in that group. (iii) Coordination among a group of agents fishing a given species may be detrimental (biologically and economically) to the other species.

From the political economy point of view, the last chapter is an occasion to discuss the important role of regulations and law enforcement in sustaining an environmental friendly fish stock while allowing the fishing business to stay profitable. This chapter is entitled *"A Note on Yardstick Regulation of Fishery Monopolies"*. In a principal-agent framework we consider a fishery exploited by two firms, acting as monopolies in two distinct local markets. We assume the harvesting cost to be a private information to each firm. This realistic assumption is due to the fact that the fishing vessels face different and variable costs everyday, especially in high seas. The principal in this story is a central planner that has the power of regulating fishing activities. The objective of this note is to design a mechanism that would induce the firms to truthfully report their costs and to adopt harvesting levels that are at the same time beneficial to consumer by leading to a competitive price, and to the resource by having a higher stock. Cooperation, or central management, can be a necessary condition to sustain a healthy stock level of the resource. However, when cooperation amounts to collusion between the agents, the consumer may have to spend more for less. Following Fisher and Mirman (1992, 1996), we consider a two-period game model, where the available stock grows with birth and decreases with the catch.

For this purpose, a series of mechanisms can be used, with each adopting a specific metric to benchmark the outcome or the harvesting effort exerted by the firm, e.g., the rate of return on investment, quotas, price cap, marginal cost, etc. The common denominator to these mechanisms is to bring the market to its competitive level. The Implementation of any designed mechanism becomes feasible when the fishing parameters are common knowledge to all. However, when the information is asymmetric, the regulator must incentivize the firms to reveal their private information. In this chapter, we consider the so-called yardstick competition introduced by Shleifer (1985) as a regulatory tool to encourage competitive behavior by natural monopolies.

Finally, to illustrate the kind of insight that can be obtained from our models, for the two first essays, we provide a numerical example and conduct a sensitivity analysis to check the impact of varying key parameter values on the results.

To summarize, this thesis attempts to answer the following questions:

1. While accounting for negative pollution externalities, what are the harvesting policies under noncooperative and cooperative modes of play? How to share the total dividend of cooperation among the players? How to insure that the cooperative agreement, established at the beginning of the game, remains in force as time goes by?

2. Assuming biological interaction between fish species, what are the equilibrium har-

vesting strategies corresponding to various modes of play? How do the results vary with the type of biological interaction and with parameter values? How do the various outcomes compare in terms of environmental and economical considerations? Under what conditions is it better for agents in one group to act cooperatively?

3. As a benevolent regulator pursuing the public interest, how to design a mechanism capable to simultaneously sustaining a higher fish stock and a lower price on the market? How to overcome the information asymmetry by implementing yardstick competition mechanism?

Chapter 1

A Fair and Time-Consistent Sharing of the Joint Exploitation Payoff of a Fishery

Abstract

We consider the problem of efficiently managing a fishery where pollution externalities are present. The open-access bionomic model is analyzed in an n-player differential game framework with two-state-variables, i.e., the fish stock and the pollution stock. We characterize the non-cooperative feedback-Nash equilibrium and cooperative solution, and define an egalitarian sharing rule to allocate the joint welfare maximizing payoff over an infinite time horizon, and show that this rule is time consistent.[1]

1.1 Introduction

The exploitation of a renewable resource, e.g., fishery, involves a fundamental trade-off between the current consumption value of fish, and the discounted value of future con-

---

[1] This chapter was published as: Dahmouni I, Vardar B, Zaccour G. A fair and time-consistent sharing of the joint exploitation payoff of a fishery. Natural Resource Modeling. 2019; e12216.

sumption. A large (low) harvest level today leaves less (more) for the future. Further, if the resource is exploited by few agents, then one needs to account for both the intertemporal and strategic externalities. Following the early contributions by Munro (1979), Clark (1980), and Lehvari and Mirman (1980), dynamic game theory has became a natural methodology for analyzing the exploitation of renewable resources, particularly fisheries, when these two externalities are present; see the surveys in Bailey et al. (2010), Grønbæk et al. (2018), Hannesson (2011), Long (2011, 2018), and Sumaila (2013).

The fisheries literature has repeatedly highlighted Hardin's tragedy of the commons, which stipulates that open access, or competition, leads to harvesting at levels that are higher than the resource's rate of reproduction, which may cause its depletion (Hardin (1968)). This observation clearly signals the need for mechanisms to prevent such a tragedy. One possibility is central management, or cooperation. The role of cooperation in fisheries has been investigated in, e.g., Sumaila (2002), Kaitala and Lindroos (2004), Lindroos et al. (2005), Trisak (2005), Munro (2006), Sumaila and Armstrong (2006), Kronbak and Lindroos (2007), Pintassilgo and Lindroos (2008), Mazalov and Rettieva (2010) and Breton et al. (2019). A main issue here is how to build and sustain over time a cooperative agreement.

The above-cited contributions focused on harvesting *quantity* under different institutional settings (regulated or unregulated fishery), mode of play by the agents (cooperation, competition), the number of species and their biological interactions, leaving often aside the *quality* of the marine environment. Fishing debris such as nets, buoys, and lines, accounts for a majority of plastic debris found in the oceans, and this pollution causes the so-called ghost fishing, i.e., the netting debris that entangle and capture fish. To illustrate, a study in southern Japan estimated that 212,000 to 505,000 octopus are killed per year within the area's fishing grounds (Matsuoka et al. (2005)). Few contributions accounted for the interrelationship between harvesting activities and the marine environment while also taking into account the strategic behavior by the agents; see, e.g., Tahvonen (1991), Xepapadeas (1995) and Wirl (2004). [2] [3] In fact, the main reason that prevents adopting a game model, where both (nonlinear) resource dynamics and environmental dynamics are interacting, is probably a purely technical one. Since the rate of growth is nonlinear in stock, the analytical characterization of the equilibrium becomes out of reach.

In this paper, we consider a fishery exploited by n strategic agents, where harvesting activities damage the environment and accumulated pollution has a negative impact on the growth rate of the resource. Assuming that each player aims at maximizing her welfare over an infinite planning horizon, and having in mind that cooperation yields, at least in the long term, a higher stock of the resource than noncooperation extraction, our research questions are as follows:

1. What are the harvesting policies under a noncooperative and cooperative mode of play?
2. How to share the total dividend of cooperation among the players?
3. How to insure that the cooperative agreement, established at the game's initial date, remains in force as time goes by?

To answer these questions, we adopt a tractable version of the dynamic game model in Xepapadeas (1995) and determine the feedback-Nash equilibrium and cooperative harvesting strategies and outcomes. Next, we assume that the community of fisherpersons agree on a fair sharing of the dividend of cooperation, that is, each player gets her noncooperative outcome plus $1/n$ of the dividend of cooperation. (Of course, other criteria could be used to share the dividend of cooperation, but fairness seems a natural choice in the absence of a specific context.) Finally, we build a sustainable sharing over time, in such a way that no player will be tempted to deviate from cooperation.

---

[2] Of course, many studies have considered the impact of climate change on the fishing industry—see, e.g., Miller and Munro (2004) and Cheung et al. (2009) for illustrative examples—but not in a strategic-interactions framework.

[3] There is also a (generally non-strategic) literature accounting for the impact of the quality of the habitat on the stock of the biomass. The early contributions are Barbier and Strand (1998), Barbier (2000), Fluharty (2000), Kaiser and de Groot (2000).

Our work contributes to the fisheries literature at two levels. First, at the modeling level, our model accounts for the resource-environment interactions, which should lead to better predictions and normative implications. To avoid the above-mentioned tractability difficulties, we follow the method—first proposed in Benchekroun (2003, 2008) for one state variable and extended to two state variables in Vardar and Zaccour (2018)—of approximating the nonlinear resource dynamics by an inverted-V shaped function. This simplifies the characterization of the equilibrium strategies and payoffs. Second, we propose a scheme to enforce the sustainability of cooperation over time, which gives larger collective and individual outcomes, as well as higher biological benefits, i.e., a larger stock of the biomass than under noncooperation. Here, the closest paper to ours is Mazalov and Rettieva (2010), which also adopts a time-consistent scheme to enforce cooperation over time. However, their model does not include any environmental considerations.[4] Given the mounting evidence that fish stocks are overexploited and that fishing activities affect the marine environment, which in turn has an impact on the biomass, we believe that analyzing these issues, albeit at an abstract level, is a useful addition to the literature. We mention here that our approach follows a cooperative game framework, that is, the grand coalition optimizes its total payoff and then allocates it to its members. An alternative approach is to build a coalition of cooperators adopting noncooperative thinking; here, a player would join the coalition if her payoff as a member is higher than it would be outside the coalition.

The rest of the paper is organized as follows. Section 2 introduces the model, and Section 3 the noncooperative and cooperative solutions. In Section 4, we determine a fair and time-consistent sharing of the dividend of cooperation. Section 5 provides a few numerical illustrations, and Section 6 briefly concludes.

---

[4] There is a sizeable literature in applied cooperative dynamic games on time consistency, a concept that was initially proposed in Petrosjan (1977). The concept and its ramifications are covered in the books by Yeung and Petrosjan (2005, 2012) and in the recent contributions by Yeung and Petrosjan (2018) and Petrosjan and Zaccour (2018). For tutorials on time consistency in differential games and in games played over event trees, see Zaccour (2008, 2017).

1.2 The model

We consider a fishery exploited by n symmetric players over an infinite planning horizon. We denote by $e_i(t)$ player i's fishing effort (number of boats, crew members, equipment, etc.) at time $t \in [0,\infty)$, and assume that the resulting harvest is given by a linear production function, i.e., $q_i(e_i(t)) = ae_i(t)$, where a is a positive parameter measuring productivity. The cost of the effort is given by the convex increasing function $g_i(e_i) = be_i^2$, with $b > 0$.

Denote by $S(t)$ and $Z(t)$ the resource and pollution stock at time t, respectively. Following Xepapadeas (1995), we assume that the growth rate of the resource is affected by the pollution stock. The evolution of the resource over time is described by the following differential equation:

$$\dot{S}(t) = F(S(t), Z(t)) - \sum_{i=1}^{n} q_i(e_i(t)), \quad S(0) = S^0 > 0, \tag{1.1}$$

where $F(S(t), Z(t))$ is a function describing the growth rate of the resource.

Harvesting (production) pollutes the shared environment. As we have a monotone (linear-increasing) relationship between fishing effort and production, we can directly express pollution emissions $E_i(t)$ in terms of effort. We assume that this relationship is linear, i.e., $E_i(t) = \omega e_i(t)$, where $\omega > 0$. The stock evolves according to the following ordinary differential equation:

$$\dot{Z}(t) = \sum_{i=1}^{n} \omega e_i(t) - kZ(t), \quad Z(0) = Z^0 \geq 0, \tag{1.2}$$

where $k > 0$ is the natural absorption rate of pollution. The resulting environmental damage cost is captured by the following quadratic function:

$$D_i(Z) = \frac{\phi}{2} Z^2, \tag{1.3}$$

where ϕ is a strictly positive parameter.

With few exceptions, the literature has typically assumed that the growth function $F(\cdot)$ in (1.1) depends only on the stock S. The assumption is that $F(S)$ is a concave function that reaches its maximum at $S = \frac{\bar{S}}{2}$, where $\bar{S}$ is the habitat's carrying capacity beyond which the resource growth rate is negative and $F(\frac{\bar{S}}{2})$ is the maximum sustainable yield of the resource. Benchekroun (2003) proposes the following approximation of this function:

$$F(S) = \begin{cases} \delta S, & \text{for } S \leq \frac{\bar{S}}{2}, \\ \delta\left(\bar{S} - S\right), & \text{for } S > \frac{\bar{S}}{2}, \end{cases} \tag{1.4}$$

where δ is a positive constant. This approximation allows to have (piecewise) linear dynamics, which simplifies the characterization of the noncooperative equilibrium and the optimal solution.

In this paper, we extend the above formulation by adding the impact of pollution on the stock of the resource. We suppose that this influence is linear and let $F(S(t), Z(t))$ be defined as follows:

$$F(S,Z) = \begin{cases} \delta S - \lambda Z, & \text{for } S \leq \frac{\bar{S}}{2}, \\ \delta\left(\bar{S} - S\right) - \lambda Z, & \text{for } S > \frac{\bar{S}}{2}, \end{cases} \tag{1.5}$$

where λ is a positive parameter capturing the negative externality of pollution stock on the rate of growth of the resource.

The utility function of player i from consumption is given by

$$U_i(e_i(t)) = q_i(e_i(t)) - g(e_i(t)) = ae_i(t) - be_i^2(t), \tag{1.6}$$

where a and b are strictly positive parameters, and her instantaneous welfare function is given by

$$J_i(t) = U_i(e_i(t)) - D_i(Z(t)) = ae_i(t) - be_i^2(t) - \frac{\phi}{2}Z^2(t), \tag{1.7}$$

that is, the difference between the resource-based utility and the environmental damage cost.

Assuming welfare-optimizing behavior over the infinite planning horizon, then player i's objective function is given by

$$J_i = \int_0^{\infty} e^{-\rho t}\left(ae_i(t) - be_i^2(t) - \frac{\phi}{2}Z^2(t)\right)dt, \tag{1.8}$$

where $\rho > 0$ is the common discounting rate.

Remark 1. *One should normally add the constraint that total production cannot exceed the available stock. We deal with this constraint by seeking a positive steady-state value of the resource.*

Remark 2. *If the fisherpersons do not internalize the pollution damage, then it suffices to set* $\phi = 0$ *in the results to follow.*

To wrap up, by (1.8), (1.1), and (1.2), we have defined an n-player differential game, with one control variable for each player ($e_i(t) \geq 0$) and two state variables ($S(t)$ and $Z(t)$). To keep the focus on designing a sustainable agreement, we assumed that the players are symmetric. We discuss in the conclusion the impact of relaxing this assumption.

At the modeling level, the closest model to ours is the one in Vardar and Zaccour (2018). In both contributions, the model involves a renewable resource and pollution externalities. However, we differ from Vardar and Zaccour in at least three respects. First, in our model, each player's utility function depends only on her control, whereas in Vardar and Zaccour, it depends on all the players' controls. Put differently, here, the players are linked through the scarcity of the resource and the pollution stock, whereas in Vardar and Zaccour, they are also linked through a market price, i.e., they compete à la Cournot. In this sense, the model is simpler here, as are the characterizations of the noncooperative equilibrium and the optimal solution. Second, Vardar and Zaccour assumes one-to-one relationship between effort and the resulting harvest, but here, the productivity of effort is also taken into account in the model as a parameter, which changes the fish stock dynamics. Third, and more importantly, in Vardar and Zaccour, cooperation and its sustainability over time are not at all on the menu, whereas here, they are the main focus.

In the next section, we characterize the noncooperative and cooperative solutions. In the noncooperative game, which can be seen as a benchmark, we seek a stationary feedback-Nash equilibrium, which is subgame perfect. By stationary, we mean that the strategies depend on the state variables and not (explicitly) on time, i.e., we look for strategies of the form $e_i(S,Z)$. In the cooperative game, we suppose that the players maximize their joint payoff.

1.3 Solutions

We shall superscript the equilibrium strategies and outcomes with N (for Nash equilibrium), and the jointly optimal solution with C (for cooperation). Denote by $V_i^j(S,Z)$ player i's value function in game $j \in \{N,C\}$. Recall that the value function $V_i^j(S,Z)$ is a mapping from the state space into R, and gives the payoff-to-go of player i from any position (S,Z) in game $j \in \{N,C\}$.

Although we shall focus in the sequel on interior solutions, we mention the existence of the following three regions in the two-state-variables space:

Region of no economic activity $\mathscr{R}_0^j$: In this region, we have

$$e_i^j(S,Z) = 0 \text{ and } a \leq a\frac{\partial V_i^j(S,Z)}{\partial S} - \omega\frac{\partial V_i^j(S,Z)}{\partial Z}.$$

Region of scarce resource $\mathscr{R}_S^j$: In this region, we have

$$e_i^j(S,Z) > 0, \quad a > a\frac{\partial V_i^j(S,Z)}{\partial S} - \omega\frac{\partial V_i^j(S,Z)}{\partial Z} \text{ and } \frac{\partial V_i^j(S,Z)}{\partial S} > 0.$$

Region of abundant resource $\mathscr{R}_A^j$: In this region, we have

$$e_i^j(S,Z) > 0, \quad a > a\frac{\partial V_i^j(S,Z)}{\partial S} - \omega\frac{\partial V_i^j(S,Z)}{\partial Z} \text{ and } \frac{\partial V_i^j(S,Z)}{\partial S} = 0.$$

In these definitions, the term $a\frac{\partial V_i^j(S,Z)}{\partial S} - \omega\frac{\partial V_i^j(S,Z)}{\partial Z}$ is the opportunity cost of a unit of harvesting effort of player i in game $j \in \{N,C\}$. Accordingly, in region $\mathscr{R}_0^j$, the marginal utility of the initial unit of effort (denoted by $U_i^\prime(0) = a$) is less than its opportunity cost for the given position in the game, and thus, harvesting the resource is not profitable because productivity a is too low. In the two other regions, the fishing effort is positive. The regions $\mathscr{R}_S^j$ and $\mathscr{R}_A^j$ differ in the magnitude of the marginal value of the resource; in $\mathscr{R}_S^j$, this value is positive, whereas it is zero in region $\mathscr{R}_A^j$.

Remark 3. *In this work, we only focus on the behavior in the scarcity region ($\mathscr{R}_S^j$), where players' strategy depends on both the resource stock and the pollution stock. We refrain from characterizing the regions $\mathscr{R}_0^j$ and $\mathscr{R}_A^j$, and refer interested readers to Vardar and Zaccour (2018) for their characterization method. (A full characterization involves a lot of technicalities, which would take us away from our main objective of building a sustainable cooperative agreement between the players.)*

The next subsection studies the noncooperative game. Then in Subsection 1.3.2, we will determine the optimal behavior of agents under cooperation.

1.3.1 Noncooperative solution

The following proposition characterizes the unique feedback-Nash equilibrium strategies in region $\mathscr{R}_S^N$.

Proposition 1. *For $i = 1, \ldots, n$, the unique symmetric feedback-Nash equilibrium harvesting-effort is given by*

$$e^N(S,Z) = \frac{1}{2b}\left(a + \theta^N + \eta^N S + \zeta^N Z\right), \tag{1.9}$$

where the constants θ^N, η^N, and ζ^N are given by

$$\begin{aligned}
\theta^N &= \frac{\left(an + (2n-1)\,\theta^N\right)}{2b}\left(\frac{\omega}{(k+\rho)}\left(\zeta^N - \frac{2b\lambda\eta^N}{(\rho-\delta)}\right) - \frac{a\eta^N}{(\rho-\delta)}\right), && (1.10)\\
\eta^N &= \frac{-1+\bar{q}\zeta^N}{2\left(\bar{s}\bar{r}^2 + \bar{q}^2\bar{v} - \bar{u}\bar{s}\bar{q}\right)}, && (1.11)\\
\zeta^N &= \frac{\bar{r}^2 - \bar{u}\bar{r} + 2\bar{q}\bar{v} \pm \sqrt{\left(\bar{u}\bar{r} - \bar{r}^2 - 2\bar{q}\bar{v}\right)^2 - 4\left(\bar{v} - \bar{t}\bar{r}^2\right)\left(\bar{s}\bar{r}^2 + \bar{q}^2\bar{v} - \bar{u}\bar{r}\bar{q}\right)}}{2\left(\bar{s}\bar{r}^2 + \bar{q}^2\bar{v} - \bar{u}\bar{r}\bar{q}\right)}, && (1.12)
\end{aligned}$$

where $\{\bar{q}, \bar{r}, \bar{s}, \bar{t}, \bar{u}, \bar{v}\}$ are constants given in the Appendix.

Proof. See Subsection 1.7.1 in the Appendix. ☐

The above proposition shows that the harvesting effort is linear in both state variables in region $\mathscr{R}_S^N$. It is shown in Subsection 1.7.1 of the Appendix, that player i's value function in the noncooperative game is quadratic in the state variables and given by

$$V_i^N(S,Z) = V^N(S,Z) = \mathscr{A}^N + \frac{\mathscr{B}^N}{2}S^2 + \mathscr{C}^N S + \frac{\mathscr{D}^N}{2}Z^2 + \mathscr{E}^N Z + \mathscr{F}^N SZ, \tag{1.13}$$

where $\mathscr{A}^N, \dots, \mathscr{F}^N$ are constants given in the Appendix. This value will play an important role in the sharing of the cooperative outcome and its sustainability over time. Given the complexity of the expressions, it is unfortunately out of reach to get much qualitative insight into the equilibrium strategies and outcome.

Denote by

$$x_1^N = \frac{na\left(a+\theta^N\right)}{2b\delta - na\eta^N}, \quad x_2^N = \frac{2b\lambda + na\zeta^N}{2b\delta - na\eta^N}, \quad x_3^N = \frac{n\omega\left(a+\theta^N\right)}{2bk - n\omega\zeta^N}, \quad x_4^N = \frac{n\omega\eta^N}{2bk - n\omega\zeta^N}$$

Proposition 2. *The feedback-Nash equilibrium steady-state values are given by*

$$S_\infty^N = \frac{x_1^N + x_2^N x_3^N}{1 - x_2^N x_4^N}, \tag{1.14}$$

$$Z_\infty^N = \frac{x_3^N + x_1^N x_4^N}{1 - x_2^N x_4^N}. \tag{1.15}$$

Proof. Substituting the equilibrium strategies $e^N(S,Z)$ given in Proposition 1 into (1.1)–(1.2), then solving the system $\dot{S}(t) = 0$ and $\dot{Z}(t) = 0$ for S and Z. We then use the solutions given in (1.10) to (1.12) and rearrange the terms to obtain the result. □

Again, the expressions of the steady-state values are by no means amenable to a qualitative analysis. We shall conduct a numerical sensitivity analysis in Section 1.5.

1.3.2 Cooperative solution

The following proposition characterizes the optimal solution when the players jointly maximize their welfare:

Proposition 3. *The unique optimal harvesting-effort policy $e^C(S,Z), \forall i$ is given by*

$$e^C(S,Z) = \frac{1}{2b}(a + \theta^C + \eta^C S + \zeta^C Z), \tag{1.16}$$

where the constants θ^C, η^C, *and* ζ^C *are given by*

$$\begin{aligned}
\theta^C &= \left(a+\theta^C\right) n \left(\frac{\omega}{2b(\rho+k)}\left(\zeta^C - \frac{\lambda \eta^C}{\rho-\delta}\right) - \frac{a\eta^C}{2b(\rho-\delta)}\right), && (1.17) \\
\eta^C &= \frac{-1+\bar{g}\zeta^C}{2\left(\bar{\imath}\bar{h}^2+\bar{g}^2\bar{l}-\bar{k}\bar{h}\bar{g}\right)} && (1.18) \\
\zeta^C &= \frac{\bar{h}^2-\bar{k}\bar{h}+2\bar{g}\bar{l} \pm \sqrt{\left(\bar{k}\bar{h}-\bar{h}^2-2\bar{g}\bar{l}\right)^2 - 4\left(\bar{l}-\bar{j}\bar{h}^2\right)\left(\bar{\imath}\bar{h}^2+\bar{g}^2\bar{l}-\bar{k}\bar{h}\bar{g}\right)}}{2\left(\bar{\imath}\bar{h}^2+\bar{g}^2\bar{l}-\bar{k}\bar{h}\bar{g}\right)}, && (1.19)
\end{aligned}$$

where $\{\bar{g}, \bar{h}, \bar{\imath}, \bar{j}, \bar{k}, \bar{l}\}$ *are constants given in the Appendix.*

Proof. See Subsection 1.7.2 in the Appendix. □

As in the noncooperative equilibrium, the harvesting policy is linear in both state variables. It is shown in Subsection 1.7.2 of the Appendix that the value function in the cooperative solution is quadratic in the state variables and given by

$$V^C(S,Z) = \mathscr{A}^C + \frac{\mathscr{B}^C}{2}S^2 + \mathscr{C}^C S + \frac{\mathscr{D}^C}{2}Z^2 + \mathscr{E}^C Z + \mathscr{F}^C SZ, \tag{1.20}$$

where $\mathscr{A}^C, \ldots, \mathscr{F}^C$ are constants also given in the Appendix.

Let us denote by

$$x_1^C = \frac{na\left(a+\theta^C\right)}{2b\delta - na\eta^C},\quad x_2^C = \frac{2b\lambda + na\zeta^C}{2b\delta - na\eta^C},\quad x_3^C = \frac{n\omega\left(a+\theta^C\right)}{2bk - n\omega\zeta^C},\quad x_4^C = \frac{n\omega\eta^C}{2bk - n\omega\zeta^C}$$

Proposition 4. *The steady-state values in the optimal solution are given by*

$$\begin{aligned}
S_\infty^C &= \frac{x_1^C + x_2^C x_3^C}{1 - x_2^C x_4^C}, && (1.21) \\
Z_\infty^C &= \frac{x_3^C + x_1^C x_4^C}{1 - x_2^C x_4^C}. && (1.22)
\end{aligned}$$

Proof. We substitute $e^C(S,Z)$ into (1.1)–(1.2), and then solve for $\dot{S}(t) = 0$ and $\dot{Z}(t) = 0$. It suffices to use the values given in (1.17) to (1.19) to get the results. □

To implement a time-consistent sharing, we need to compute the trajectories of the two state variables. We show in the Appendix that they are given by

$$\begin{aligned}
S^j(t) &= e^{\Gamma_1 t}c_2^j + e^{\Gamma_2 t}c_1^j + \frac{\bar{b}^j\bar{d}^j + \bar{l}^j\bar{c}^j}{\bar{l}^j\bar{a}^j - \bar{b}^j\bar{k}^j}, \\
Z^j(t) &= \frac{-\Gamma_1^j e^{\Gamma_1 t}c_2^j}{\bar{b}^j} - \frac{\Gamma_2 e^{\Gamma_2 t}c_1^j}{\bar{b}^j} + \frac{\bar{a}^j\left(e^{\Gamma_1 t}c_2^j + e^{\Gamma_2 t}c_1^j + \frac{\bar{b}\bar{d}+\bar{l}\bar{c}}{\bar{l}\bar{a}-\bar{b}\bar{k}}\right) - \bar{c}^j}{\bar{b}^j},
\end{aligned}$$

for $j \in \{N, C\}$. All the constants involved in the above expressions are defined in the Appendix. Writing them in full yields very large expressions with no apparent qualitative insight.

1.4 A fair and time-consistent allocation

In this section, we define a time-consistent decomposition over time of each player's total welfare, based on an egalitarian sharing rule. The objective is, as stated in the introduction, to have a sustainable agreement. This is achieved by implementing the following four-step algorithm:

Step 1: Compute the total payoff to be shared by the players.

Step 2: Determine the individual payoffs in the absence of an agreement.

Step 3: Share the total cooperative payoff.

Step 4: Define a time-consistent solution.

The total payoff to be shared is the jointly maximized discounted welfare over the infinite planning horizon, that is, J^C. In the parlance of cooperative game theory, this collective payoff corresponds to the characteristic-function value of the grand coalition. It is also the optimal cooperative value function evaluated at initial state, that is,

$$J^C = V^C\left(S^0, Z^0\right) = \mathscr{A}^C + \frac{\mathscr{B}^C}{2}\left(S^0\right)^2 + \mathscr{C}^C S^0 + \frac{\mathscr{D}^C}{2}\left(Z^0\right)^2 + \mathscr{E}^C Z^0 + \mathscr{F}^C S^0 Z^0. \quad (1.23)$$

In the absence of an agreement, the game is played noncooperatively and each player gets her feedback-Nash equilibrium outcome J_i^N, that is,

$$J_i^N = V^N\left(S^0, Z^0\right) = \mathscr{A}^N + \frac{\mathscr{B}^N}{2}\left(S^0\right)^2 + \mathscr{C}^N S^0 + \frac{\mathscr{D}^N}{2}\left(Z^0\right)^2 + \mathscr{E}^N Z^0 + \mathscr{F}^N S^0 Z^0, \quad (1.24)$$

which is obtained by evaluating (1.13) at $\left(S^0, Z^0\right)$.

Cooperative game theory offers a series of solutions to share the grand coalition's outcome in (1.23), e.g., core, Shapley value, nucleolus, Nash bargaining solution. Here, we require that the solution satisfy the following three properties, which are all embedded in the Nash bargaining solution:

Individual rationality: Globally, no player would accept a form of sharing that left her with less than what she could achieve in the noncooperative game. Formally, denote by $(J_1^*, \ldots, J_n^*)$ an allocation of the total cooperative payoff. Then, we require that this allocation satisfy the following inequalities:

$$\text{Individual rationality} : J_i^* \geq J_i^N, \quad i = 1, \ldots, n. \tag{1.25}$$

Collective rationality: The total payoff obtained under joint maximization must be fully allocated, that is,

$$\text{Collective rationality} : J^C = \sum_{i=1}^{n} J_i^*. \tag{1.26}$$

Any vector $(J_1^*, \ldots, J_n^*)$ satisfying (1.25) and (1.26) is called an imputation. (Collective rationality means that the players do not waste resources, nor expect a subsidy from an outside entity.)

Egalitarian sharing: The total dividend of cooperation D, which is measured by

$$D = J^C - \sum_{i=1}^{n} J_i^N = V^C\left(S^0, Z^0\right) - \sum_{i=1}^{n} V_i^N\left(S^0, Z^0\right),$$

that is, the total cooperative payoff minus the sum of individual noncooperative outcomes must be shared equally. This is achieved by allocating to each player her outcome in the noncooperative game plus $1/n$ of the dividend of cooperation. Formally, player i obtains

$$J_i^* = J_i^N + \frac{D}{n}, \quad i = 1, \ldots, n. \tag{1.27}$$

The last step is to design a time-consistent mechanism of J_i^*. Time consistency means that, at any intermediate instant of time $\tau \in (0, \infty)$, no player finds it individually rational to abandon the agreement and switch to her noncooperative strategy. A formal definition follows.

The fair allocation in (1.27) is time consistent at initial state $\left(S^0, Z^0\right)$ if, at any $\left(S^C(\tau), Z^C(\tau)\right)$ and all $\tau \in (0, \infty)$, it holds that

$$J_i^*\left(S^C(\tau), Z^C(\tau)\right) \geq J_i^N\left(S^C(\tau), Z^C(\tau)\right), \quad i = 1, \ldots, n, \tag{1.28}$$

where $\left(S^C(\tau), Z^C(\tau)\right)$ denotes the joint maximization state trajectory.

The condition in (1.28) states that the cooperative payoff-to-go of player i, $i = 1, \ldots, n$, must dominate, at least weakly, her noncooperative payoff-to-go under noncooperation. Note that this dominance test is carried out along the cooperative state trajectory $\left(S^C(\tau), Z^C(\tau)\right)$, meaning that, at any comparison time τ, the assumption is that the players have cooperated from the initial instant of time till τ.

Since the condition in (1.28) must hold at any instant of time, we need to compute the players' cooperative and noncooperative payoffs-to-go for all t, not only at the initial time. These values are simply obtained by evaluating J^C and J_i^N at $\left(S^C(\tau), Z^C(\tau)\right)$, that is,

$$\begin{aligned} J^C\left(S^C(\tau), Z^C(\tau)\right) &= V^C\left(S^C(\tau), Z^C(\tau)\right) = \mathscr{A}^C + \frac{\mathscr{B}^C}{2}\left(S^C(\tau)\right)^2 + \mathscr{C}^C S^C(\tau) \\ &\quad + \frac{\mathscr{D}^C}{2}\left(Z^C(\tau)\right)^2 + \mathscr{E}^C Z^C(\tau) + \mathscr{F}^C S^C(\tau) Z^C(\tau), \\ J_i^N\left(S^C(\tau), Z^C(\tau)\right) &= V^N\left(S^C(\tau), Z^C(\tau)\right) = \mathscr{A}^N + \frac{\mathscr{B}^N}{2}\left(S^C(\tau)\right)^2 + \mathscr{C}^N S^C(\tau) \\ &\quad + \frac{\mathscr{D}^N}{2}\left(Z^C(\tau)\right)^2 + \mathscr{E}^N Z^C(\tau) + \mathscr{F}^N S^C(\tau) Z^C(\tau). \end{aligned}$$

To determine the allocation that player i gets in the subgame starting at time τ, it suffices to use the above formulas and (1.27) to compute $J_i^*\left(S^C(\tau), Z^C(\tau)\right)$.

Observe that, at any $\tau \in (0, \infty)$, $J_i^*\left(S^N(\tau), Z^N(\tau)\right)$ and $J_i^*\left(S^C(\tau), Z^C(\tau)\right)$ are (generally) not equal. The reason is that $\left(S^N(\tau), Z^N(\tau)\right) \neq \left(S^C(\tau), Z^C(\tau)\right)$.

To implement a time-consistent solution, we introduce the concept of an imputation distribution procedure (IDP) of the game (Petrosjan (1977)).

A vector $\beta(t) = (\beta_1(t), \ldots, \beta_n(t))$ of time functions is an imputation distribution procedure if, for all $i = 1, \ldots, n$, it satisfies

$$J_i^*\left(S^0, Z^0\right) = \int_0^\infty e^{-\rho t} \beta_i(t) dt. \tag{1.29}$$

An IDP simply decomposes over time the imputation that player i is entitled to receive under the agreement. Clearly, there is an infinite number of time functions that satisfy the equality in (1.29). We select the time functions that possess the additional property of being time consistent.

The IDP $\beta(t)$ is time consistent, if

$$J_i^*\left(S^0,Z^0\right) = \int_0^\tau e^{-\rho t}\beta_i(t)dt + e^{-\rho\tau}J_i^*\left(S^C(\tau),Z^C(\tau)\right), \tag{1.30}$$

where $J_i^*\left(S^C(\tau),Z^C(\tau)\right)$ is player i's payoff-to-go in the subgame starting at time τ, along the cooperative state trajectory.

The interpretation of the condition in (1.30) is as follows: suppose that the players wish to renegotiate the agreement at (any) intermediate instant of time $\tau \in (0,\infty)$. At that moment, the state of the system is $\left(S^C(\tau),Z^C(\tau)\right)$, meaning that cooperation has prevailed from the initial time until τ, and that each player i would have been allocated the stream of monetary amounts given in the first right-hand-side term. Now, if the subgame starting with initial condition $(S(\tau),Z(\tau)) = \left(S^C(\tau),Z^C(\tau)\right)$ is played cooperatively, then player i will get her fair share in this game, which is given by the second right-hand-side term of (1.30). If what she has received on $[0,\tau]$ and what she will get from τ onward add up to her payoff under the original agreement, i.e., $J_i^*\left(S^0,Z^0\right)$, then a renegotiation would leave the original agreement unaltered. If one can find an IDP $\beta(t) = (\beta_1(t),\cdots,\beta_n(t))$ such that (1.30) holds true, then this IDP is time consistent.

Proposition 5. *Let $\beta_i(t), i = 1,\ldots,n$ be given by*

$$\beta_i(t) = \rho J_i^*\left(S^C(t),Z^C(t)\right) - \frac{d}{dt}J_i^*\left(S^C(t),Z^C(t)\right). \tag{1.31}$$

Then, $\beta(t) = (\beta_1(t),\cdots,\beta_n(t))$ is a time-consistent IDP.

Proof. First, we show that it satisfies (1.29):

$$\begin{aligned}\int_0^\infty e^{-\rho t}\beta_i(t)dt &= \int_0^\infty e^{-\rho t}\left(\rho J_i^*\left(S^C(t),Z^C(t)\right) - \frac{d}{dt}J_i^*\left(S^C(t),Z^C(t)\right)\right)dt,\\ &= -e^{-\rho t}J_i^*\left(S^C(t),Z^C(t)\right)\Big|_0^\infty = J_i^*\left(S^C(0),Z^C(0)\right) = J_i^*\left(S^0,Z^0\right).\end{aligned}$$

Second, we show that it satisfies (1.30):

$$
\begin{aligned}
& \int_0^\tau e^{-\rho t}\beta_i(t)dt + e^{-\rho\tau}J_i^*\left(S^C(\tau),Z^C(\tau)\right) \\
= & \int_0^\tau e^{-\rho t}\left(\rho J_i^*\left(S^C(t),Z^C(t)\right) - \frac{d}{dt}J_i^*\left(S^C(t),Z^C(t)\right)\right)dt + e^{-\rho\tau}J_i^*\left(S^C(\tau),Z^C(\tau)\right) \\
= & -e^{-\rho t}J_i^*\left(S^C(t),Z^C(t)\right)\Big|_0^\tau + e^{-\rho\tau}J_i^*\left(S^C(\tau),Z^C(\tau)\right) \\
= & -e^{-\rho\tau}J_i^*\left(S^C(\tau),Z^C(\tau)\right) + J_i^*\left(S^C(0),Z^C(0)\right) + e^{-\rho\tau}J_i^*\left(S^C(\tau),Z^C(\tau)\right) \\
= & J_i^*\left(S^C(0),Z^C(0)\right).
\end{aligned}
$$

□

Two comments can be made regarding this result. First, the time-consistent IDP defined in (1.31) holds true for any functional form of the payoff functions. Second, it has a nice interpretation. Indeed, the formula in (1.31) allocates at instant of time t to player i a reward corresponding to the interest payment (interest rate times her cooperative payoff-to-go) minus the variation over time of this payoff-to-go.

Replacing the terms appearing in (1.31) by their values, we get

$$\beta_i(t) = \frac{1}{4b}\left(a^2 - 2b\phi\left(Z^C(t)\right)^2 - \left(\theta^C + \eta^C S^C(t) + \zeta^C Z^C(t)\right)^2\right),$$

which can be rewritten as follows:

$$\beta_i(t) = e^C(S^C(t),Z^C(t))\left(a - be^C(S^C(t),Z^C(t))\right) - \frac{\phi\left(Z^C(t)\right)^2}{2}.$$

Therefore, the IDP is expressed in terms of the optimal harvesting effort and the damage cost. Note that the IDP is equivalent to the instantaneous welfare function evaluated at the optimal harvesting effort, i.e.,

$$\beta_i(t) = U_i(e^C(S^C(t),Z^C(t))) - D_i(Z^C(t)).$$

To wrap up, it is collectively in the best interest of the fisher community to coordinate harvesting strategies by jointly maximizing their payoff. The next step is to share the resulting outcome, which can be done according to principles acceptable to the players.

Assuming that fairness is the criterion adopted by the fisherpersons, we equally divided the dividend of cooperation among them. Now, it is well known that a cooperative solution is not, in general, an equilibrium, implying that it is not self-supported. Consequently, it may be rational for a player to deviate at some subsequent date to her noncooperative strategy. A time-consistent solution gives a sufficient incentive to abide by the agreement as time goes by. Note that this solution is not an equilibrium; it is a necessary, not sufficient, condition for the sustainability of cooperation. In deriving such mechanism, the assumption is that either the players will continue playing the game cooperatively or the agreement will fall apart. Put differently, if one player deviates, the whole agreement is canceled, and we do not seek a new agreement with $n-1$ players.

1.5 Numerical illustration

Although all our results are analytical, the complexities of all the derived expressions (e.g., steady-state values, harvesting effort trajectories, etc.) do not allow for an analytical comparison between the different cases under consideration. To illustrate the kind of insight that can obtained from our model, we provide a numerical example and compare the outcomes of cooperative and noncooperative solutions. Also, we conduct a sensitivity analysis to check the impact of varying key parameter values on the results. Since the analytical derivations focused on the behavior in $\mathscr{R}_S^j$, we set a constellation of parameters such that the initial state and the trajectories of the state variables remain in region $\mathscr{R}_S^j$ for both solutions ($j \in \{N, C\}$).

As a benchmark scenario, the following parameter values are chosen:

$$
\begin{aligned}
\text{Welfare function parameters} &: \; a = 4, \;\; b = 10, \;\; \phi = 0.1, \\
\text{Resource dynamics parameters} &: \; S^0 = 10, \;\; \delta = 0.1, \;\; \lambda = 0.1, \\
\text{Pollution dynamics parameters} &: \; Z^0 = 0.1, \;\; \omega = 0.5, \;\; k = 0.12, \\
\text{Other parameters} &: \; \rho = 0.002, \;\; n = 2.
\end{aligned}
$$

To approximate the infinite horizon, we run the model for $t \in [0; 15.000]$ time interval. In the figures, we plot the first $t \in [0;150]$ periods of the game. The steady-state values for this parameter constellation are:

$$S_\infty^N = 10.2135, \;\; S_\infty^C = 11.9889, \;\; Z_\infty^N = 1.0655, \;\; Z_\infty^C = 1.2528,$$

that is, larger resource and pollution stocks under cooperation than in the Nash equilibrium. The result that central management (cooperation or joint optimization) yields higher resource stock in the long term is expected.

A comparison of the trajectories associated to the cooperative and noncooperative solutions is presented in Figure 1.1. The results show that in the cooperative solution players harvest less of the resource in the short term and more in the long run. The long-term resource stock is higher when the players are jointly maximizing their payoffs than when they play Nash. This result holds true also for the pollution stock. In the short term, the ordering of the control and state trajectories depends on the parameter constellation. For instance, changing the value of initial resource stock to $S^0 = 15$, again shows that the harvesting level, resource and pollution stocks are indeed higher in the cooperative solution than in the Nash equilibrium in the long term, but not in the short term (see Figure 1.2). Note that for both initial resource stocks, that is, $S^0 = 10$ and $S^0 = 15$, the steady-state values are the same, which is expected in a feedback equilibrium of an infinite horizon differential game. However, whereas the convergence to the steady state is from below when $S^0 = 10$, it is from above when $S^0 = 15$ (see Figure 1.2).

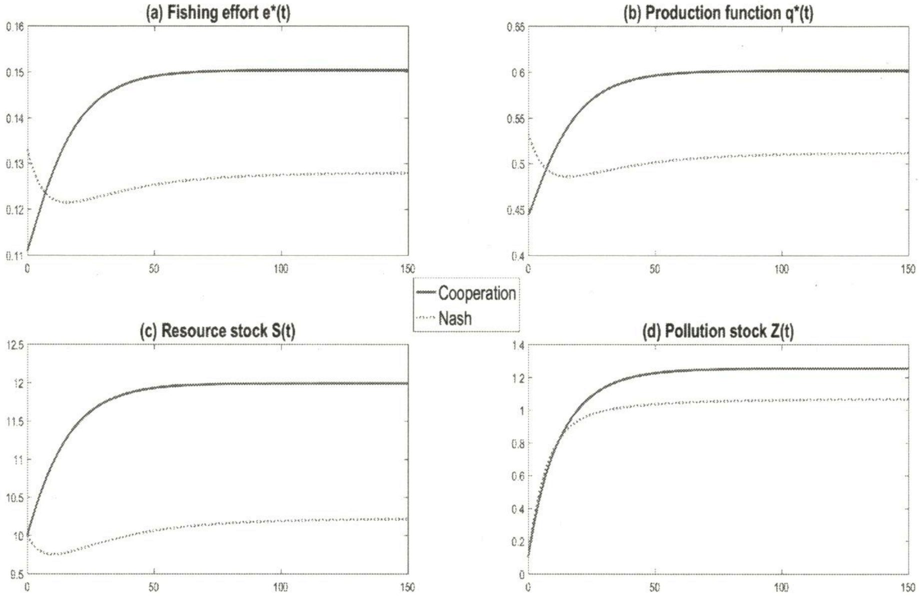

Figure 1.1 – Comparison of trajectories in the cooperative and noncooperative solutions ($S^0 = 10$)

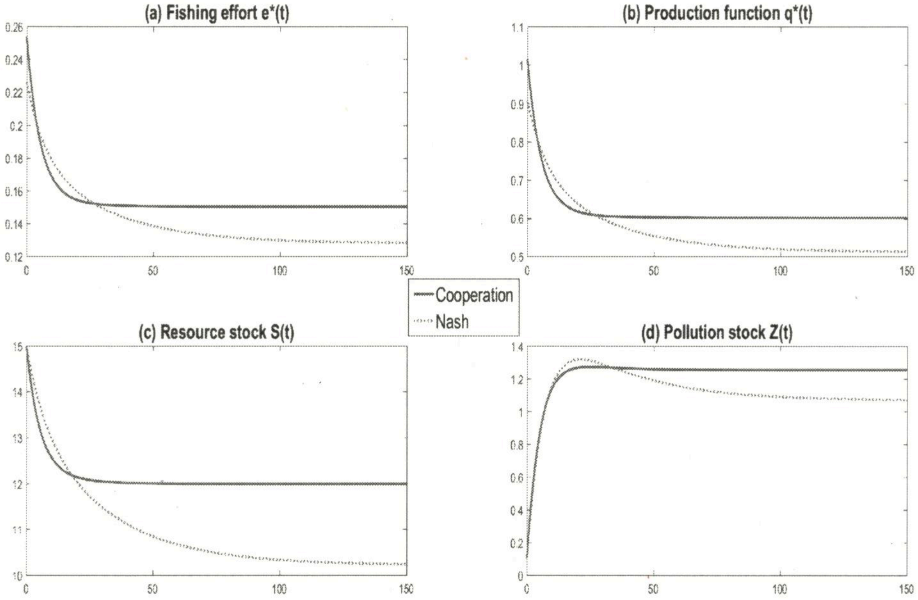

Figure 1.2 – Comparison of trajectories in the cooperative and noncooperative solutions ($S^0 = 15$)

Now, we look at the impact on the state and control trajectories of varying some parameter values, one at time, while keeping the others at their benchmark levels. We focus on the impact of the dynamic parameters of the model, namely, the discount rate (ρ), the intrinsic growth rate of the resource (δ), the decay rate of pollution (k), and pollution externalities (λ).

The effects of the discount rate (ρ) are illustrated in Figure 1.3, where we vary the value of ρ from 0.001 to 0.02. Quantitatively speaking, the changes in ρ do not have a significant effect on the long-term harvesting levels and steady-state values of pollution and resource stocks. Indeed, multiplying the discount rate by 20, leads to a variation of less than 10% in the long-term control and state variables values. Note, however, that the effect of ρ is more pronounced in the Nash equilibrium than in the cooperative solution, and also the ranking of the steady-state values is not the same in the two solutions. One possible explanation is that, in the short term, the harvesting effort is decreasing in the Nash equilibrium, whereas it is increasing in the cooperative solution.

The impact of varying the growth rate of the resource δ between 0.07 and 0.16 is shown in Figure 1.4. The two main takeaways that apply to both solutions are: (i) the higher the growth rate, the higher the harvesting effort in the short term, and so is pollution accumulation, which is intuitive; (ii) the higher the growth rate, the lower the steady-state value of the resource. This second result is counterintuitive as one would expect that a higher growth rate would lead to higher stock. However, the conservation behavior adopted in the short term when δ is low, that is, lower harvesting effort, is probably the element driving the result.

The results for varying the decay rate of pollution (k) are shown in Figure 1.5. As for δ, the ranking of trajectories is the opposite in the short and long term. When k is low (say 0.11), the players harvest at significantly higher rate in the short term than when k is large (0.14). This result holds true in both solutions and leads to higher pollution accumulation, which in turn affects negatively the growth of the resource and its steady-state value. Further, we note that the impact of varying k is much more pronounced in the Nash equilibrium than in the cooperative solution.

Figure 1.6 shows the impact of varying the pollution externality (λ) on the harvest and resource and pollution stocks. In the long term, we obtain a counterintuitive result, namely, the larger λ, the larger the long-term harvesting and resource and pollution stocks. It is counterintuitive because one expects that a higher λ means less resource and consequently lower production. In an attempt to explain this result, we look more closely at the short-term trajectories. In Figure 1.7, we provide the results for the two extreme values of λ for the first 25 time steps. In both solutions, in the short term the stock of the resource is lower when λ is higher, but it increases at a higher rate over time, which leads to a reverse situation. Interestingly, in both solutions, the trajectories of the pollution stocks are almost the same for the two values of λ. Back to Figure 1.6, we highlight the result that in fact the impact of varying λ on the two state variables is very minor. Indeed, an increase by 500% of λ $(0.05/0.01)$ leads to an increase of the resource and pollution steady-state values by at most 30%. Put differently, the results are robust to changing λ.

To illustrate the shape of the IDP, we plot in Figure 1.8 the trajectories for three initial values of the resource stock, that is, $S^0 = 4, 10$ and 15. We added here the trajectory for $S^0 = 4$ to show that the IDP can take negative values. In all cases, the convergence to a steady-state value is rather fast.

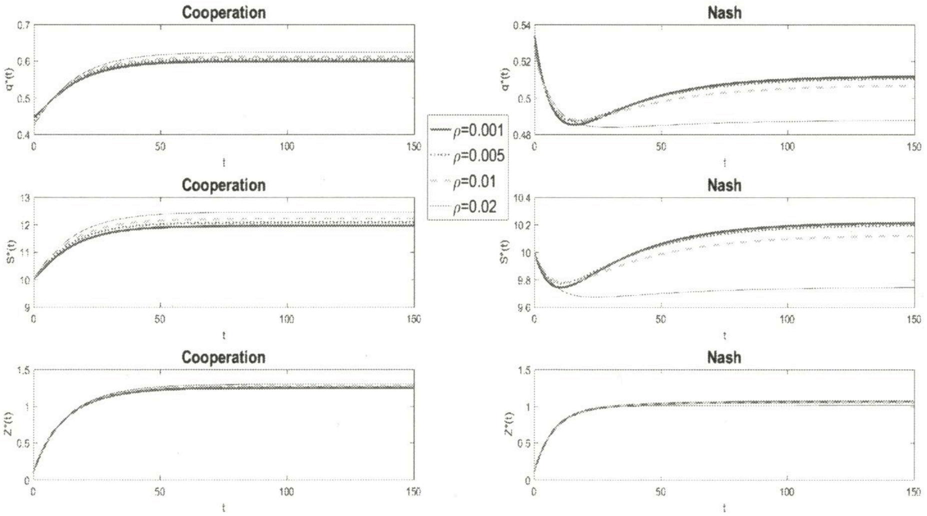

Figure 1.3 – Impact of the discount rate on harvesting and state variables

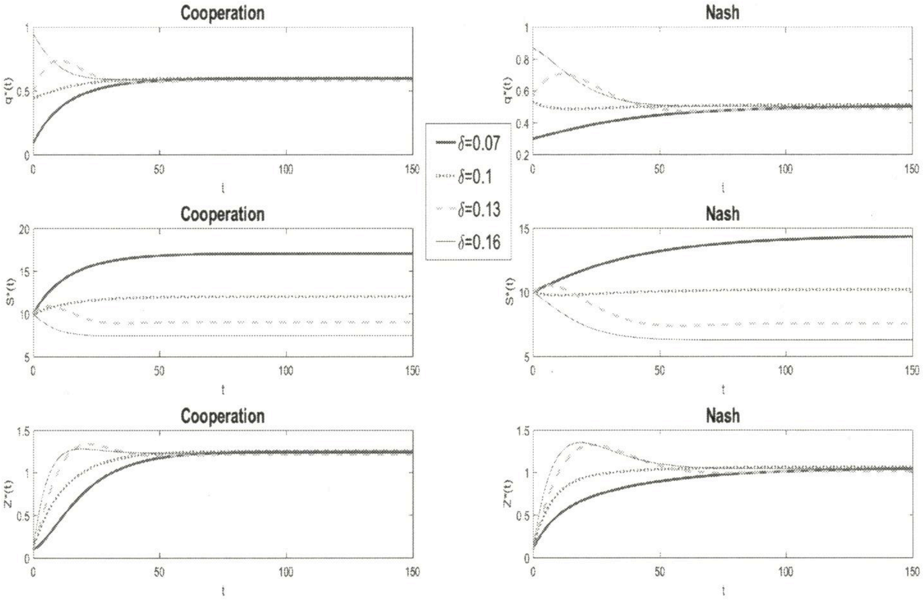

Figure 1.4 – Impact of the resource growth rate on the trajectory of production

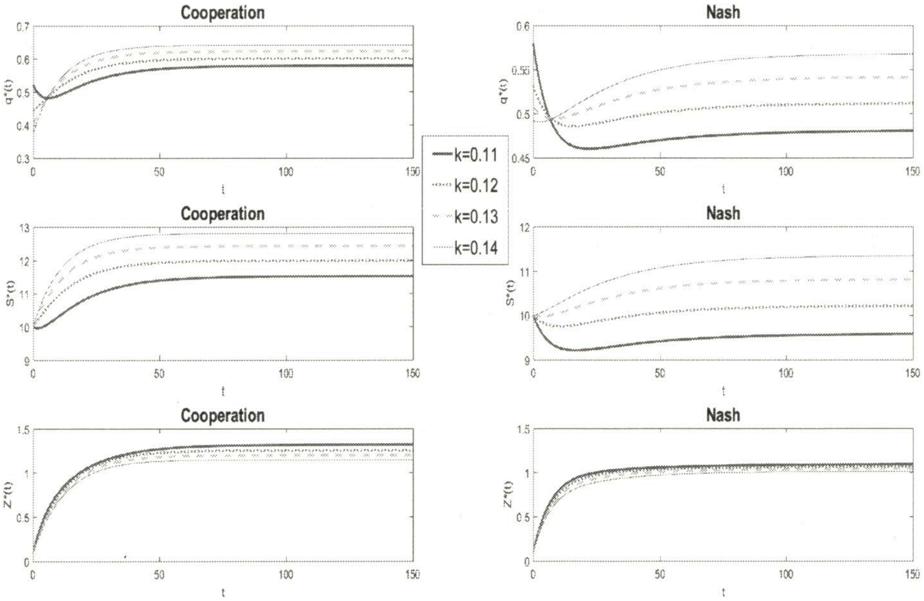

Figure 1.5 – Impact of the pollution decay on the trajectory of production

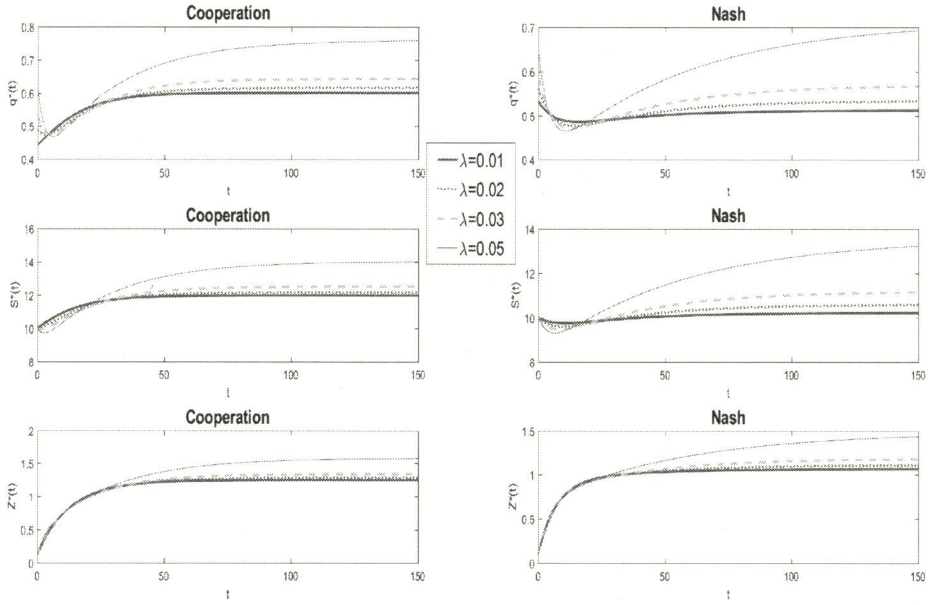

Figure 1.6 – Impact of varying λ on resource and pollution trajectories

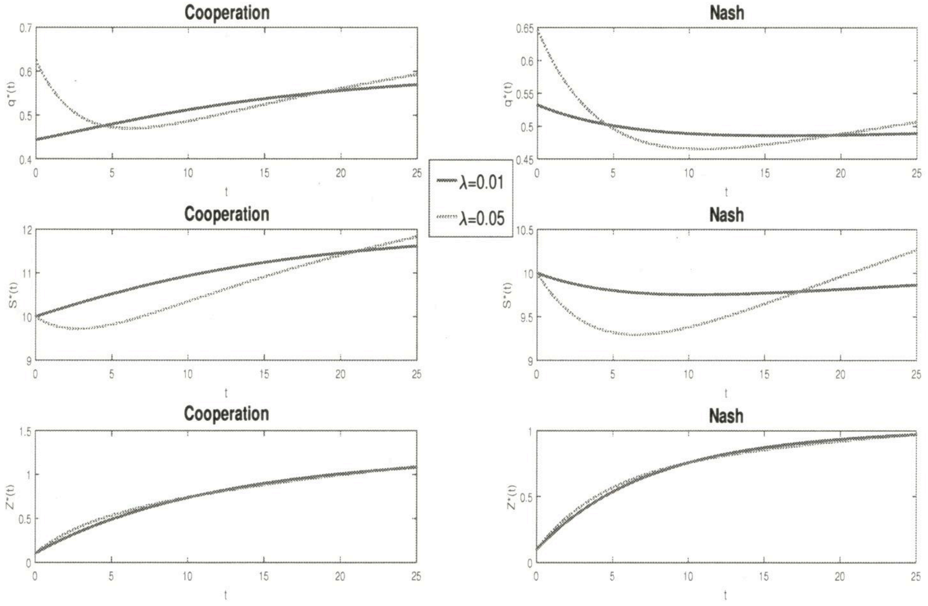

Figure 1.7 – Short-term impact of varying λ

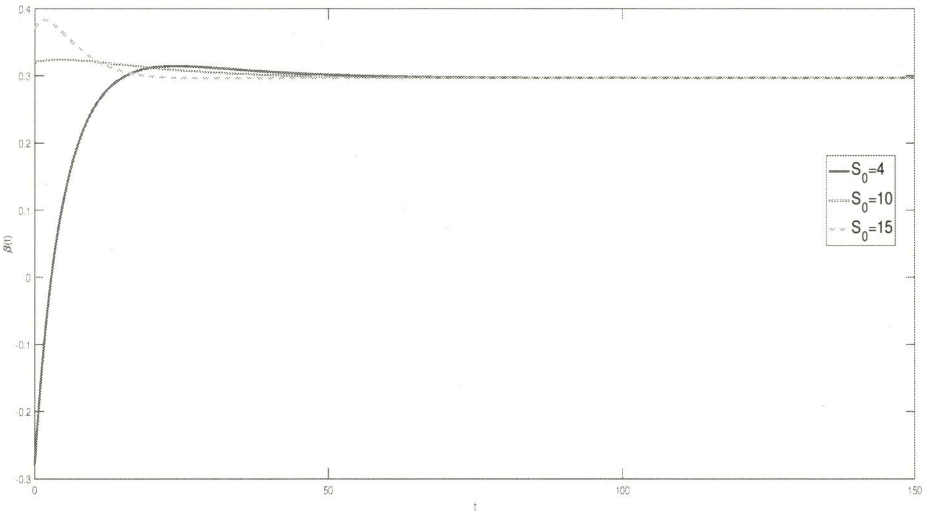

Figure 1.8 – IDP trajectories for different initial resource stock

1.6 Conclusion

Using a differential game framework, this work studies a sustainable agreement in an open-access fishery with finite number of fisherpersons where pollution externalities are present, and characterizes the symmetric noncooperative and cooperative solutions. We also define a fair and time-consistent imputation distribution procedure of the cooperative outcome. To finish, we discuss the implications of relaxing our symmetry assumption for the characterization of the equilibrium and optimal solutions, and for the derivation of the IDP.

The characterization of the noncooperative and cooperative solutions would be considerably complicated if we took into account asymmetry between the players in our two-state variable framework, with one state dynamics being approximated by a piecewise linear function. The symmetry between the players allows us to look for a single value function associated to each. Combining this property with the linear-quadratic model specification, we are able to obtain the six-dimensional equation system, then reduce it to a system of two equations, which gives us the explicit solution for the harvesting strategies. In the case of asymmetric n players, it would be necessary to find the value function associated to each player. This would mean obtaining and solving the system of $6n$ equations. Recalling that the harvesting solution can be in three different regions of the state space, the construction of trajectories and the stability analysis would become quite challenging. However, once the quantities involved in computing a fair solution are in place, asymmetry would not cause any intrinsic difficulty in computing the IDP. Recall that the formula derived in (1.31) holds for any functional forms.

A starting point to improve our understanding of how to build a sustainable agreement among asymmetric players, would be to consider a two-player game. While not easy, characterizing the feedback-Nash equilibrium is feasible, as is determining the optimal solution.

1.7 Appendix

1.7.1 Proof of Proposition 1

Using a dynamic programming approach, the Hamilton-Jacobi-Bellman (HJB) equation associated to player i's optimization problem is written as follows:

$$
\begin{aligned}
{}_i^N(S,Z) = \max_{e_i} \Bigg\{ & ae_i - be_i^2 - \frac{\phi}{2}Z^2 + \frac{\partial V_i^N(S,Z)}{\partial S}\left(F(S,Z) - \sum_{i=1}^{n} ae_i\right) \\
& + \frac{\partial V_i^N(S,Z)}{\partial Z}\left(\sum_{i=1}^{n} \omega e_i - kZ\right)\Bigg\}. \qquad (1.32)
\end{aligned}
$$

We are looking for a symmetric feedback-Nash equilibrium, which exists if there is a function $V_i^N = V^N(S,Z), \forall i$, that satisfies the above equation, and is continuously differentiable in S and Z (Haurie et al. (2012)). Maximization of the right-hand side results in the following first-order condition:

$$
a - 2be_i = a\frac{\partial V_i^N(S,Z)}{\partial S} - \omega\frac{\partial V_i^N(S,Z)}{\partial Z}. \qquad (1.33)
$$

We focus on the symmetric equilibrium in which all players harvest at the same effort level ($e_i = e^N, \forall i$), and have the same value function ($V_i^N(S,Z) = V^N(S,Z), \forall i$). Consequently, the condition above can be rewritten as follows:

$$e^N = \max\left\{0, \frac{1}{2b}\left(a - a\frac{\partial V^N(S,Z)}{\partial S} + \omega\frac{\partial V^N(S,Z)}{\partial Z}\right)\right\}. \tag{1.34}$$

We make the informed guess that, within an interior solution, the value function is a polynomial of degree 2 in two state variables. We consider the function V^N given by

$$V^N(S,Z) = \mathscr{A}^N + \frac{\mathscr{B}^N}{2}S^2 + \mathscr{C}^N S + \frac{\mathscr{D}^N}{2}Z^2 + \mathscr{E}^N Z + \mathscr{F}^N SZ. \tag{1.35}$$

Replacing $e_i = e^N$ given in (1.34) into the HJB equation and applying the undetermined coefficients method (Haurie et al. 2012) leads to the following equation system:

$$\begin{aligned}
\mathscr{A}^N =& \frac{\left(a+\omega\mathscr{E}^N - a\mathscr{C}^N\right)\left(a+(2n-1)\left(\omega\mathscr{E}^N - a\mathscr{C}^N\right)\right)}{4br},\\
\mathscr{B}^N =& \frac{(2n-1)\left(\omega\mathscr{F}^N - a\mathscr{B}^N\right)^2}{2b(\rho-2\delta)},\\
\mathscr{C}^N =& \frac{\left(\omega\mathscr{F}^N - a\mathscr{B}^N\right)\left[an+\left(\omega\mathscr{E}^N - a\mathscr{C}^N\right)(2n-1)\right]}{2b(\rho-\delta)},\\
\mathscr{D}^N =& \frac{2n-1}{2b(\rho+2k)}\\
&\left(\left(\omega\mathscr{D}^N - a\mathscr{F}^N\right)^2 - 2b\phi - \frac{2b\lambda\left(\omega\mathscr{F}^N - a\mathscr{B}^N\right)}{k+\rho-\delta}\left(\omega\mathscr{D}^N - a\mathscr{F}^N - \frac{\lambda\left(\omega\mathscr{F}^N - a\mathscr{B}^N\right)}{2b(\rho-2\delta)}\right)\right),\\
\mathscr{E}^N =& \frac{an+(2n-1)\left(\omega\mathscr{E}^N - a\mathscr{C}^N\right)}{2b(k+\rho)}\left(\omega\mathscr{D}^N - a\mathscr{F}^N - \frac{\lambda\left(\omega\mathscr{F}^N - a\mathscr{B}^N\right)}{\rho-\delta}\right),\\
\mathscr{F}^N =& \frac{(2n-1)\left(\omega\mathscr{F}^N - a\mathscr{B}^N\right)}{2b(k+\rho-\delta)}\left(\omega\mathscr{D}^N - a\mathscr{F}^N - \frac{\lambda\left(\omega\mathscr{F}^N - a\mathscr{B}^N\right)}{\rho-2\delta}\right).
\end{aligned}$$

Introduce the change of variables given by $\theta^N = \omega\mathscr{E}^N - a\mathscr{C}^N$, $\eta^N = \omega\mathscr{F}^N - a\mathscr{B}^N$, and $\zeta^N = \omega\mathscr{D}^N - a\mathscr{F}^N$. The above system becomes

$$\mathscr{A}^N = \frac{\left(a+\theta^N\right)\left(a+(2n-1)\,\theta^N\right)}{4b\rho}, \tag{1.36}$$

$$\mathscr{B}^N = \frac{(2n-1)\left(\eta^N\right)^2}{2b\,(\rho-2\delta)}, \tag{1.37}$$

$$\mathscr{C}^N = \frac{\eta^N\left(an+\theta^N\,(2n-1)\right)}{2b\,(\rho-\delta)}, \tag{1.38}$$

$$\mathscr{D}^N = \frac{(2n-1)}{2b\,(\rho+2k)}\left(\left(\zeta^N\right)^2 - 2b\phi - \frac{2b\lambda\eta^N}{k+\rho-\delta}\left(\zeta^N - \frac{\lambda\eta^N}{2b\,(\rho-2\delta)}\right)\right), \tag{1.39}$$

$$\mathscr{E}^N = \frac{an+(2n-1)\,\theta^N}{2b\,(k+\rho)}\left(\zeta^N - \frac{\lambda\eta^N}{\rho-\delta}\right), \tag{1.40}$$

$$\mathscr{F}^N = \frac{(2n-1)\,\eta^N}{2b\,(k+\rho-\delta)}\left(\zeta^N - \frac{\lambda\eta^N}{\rho-2\delta}\right). \tag{1.41}$$

It is easy to check that the equations above reduce to the following:

$$\mathscr{A}^N = \frac{\left(a+\theta^N\right)\left(a+(2n-1)\,\theta^N\right)}{4b\rho}, \tag{1.42}$$

$$\theta^N = \frac{\left(an+(2n-1)\,\theta^N\right)}{2b}\left(\frac{\omega}{(k+\rho)}\left(\zeta^N - \frac{2b\lambda\eta^N}{(\rho-\delta)}\right) - \frac{a\eta^N}{(\rho-\delta)}\right), \tag{1.43}$$

$$\eta^N = (2n-1)\,\eta^N\left(\frac{\omega}{2b\,(k+\rho-\delta)}\left(\zeta^N - \frac{\lambda\eta^N}{(k+\rho-\delta)\,(\rho-2\delta)}\right) - \frac{a\eta^N}{2b\,(\rho-2\delta)}\right), \tag{1.44}$$

$$\begin{aligned}\zeta^N =& \frac{\omega}{2b\,(\rho+2k)}\left(\frac{(2n-1)\left(\zeta^N\right)^2 - 2b\phi}{2b\,(\rho+2k)} - \frac{\lambda\,(2n-1)\,\eta^N\zeta^N}{(k+\rho-\delta)} + \frac{\lambda^2\,(2n-1)\left(\eta^N\right)^2}{2b\,(k+\rho-\delta)\,(\rho-2\delta)}\right)\\ &- \frac{a\,(2n-1)\,\eta^N}{2b\,(k+\rho-\delta)}\left(\zeta^N - \frac{\lambda\eta^N}{\rho-2\delta}\right).\end{aligned} \tag{1.45}$$

Equations (1.44)–(1.45) is a system of equations in η^N and ζ^N that has four solutions. In one pair of solutions, we have $\eta^N \neq 0$, and, in the other, $\eta^N = 0$. The solution with $\eta^N = 0$ is associated to region $\mathscr{R}_A^N$, and not characterized in this paper. The solution associated to region $\mathscr{R}_S^N$ is given by

$$\eta^N = \frac{-1+\bar{q}\zeta^N}{2\bar{s}\bar{r}^2 + 2\bar{q}^2\bar{v} - 2\bar{u}\bar{r}\bar{q}}, \tag{1.46}$$

$$\zeta^N = \frac{\bar{r}^2 - \bar{u}\bar{r} + 2\bar{q}\bar{v} \pm \sqrt{\left(\bar{u}\bar{r} - \bar{r}^2 - 2\bar{q}\bar{v}\right)^2 - 4\left(\bar{v} - \bar{t}\bar{r}^2\right)\left(\bar{s}\bar{r}^2 + \bar{q}^2\bar{v} - \bar{u}\bar{r}\bar{q}\right)}}{2\left(\bar{s}\bar{r}^2 + \bar{q}^2\bar{v} - \bar{u}\bar{r}\bar{q}\right)}, \tag{1.47}$$

where

$$\bar{q} = \frac{\omega(2n-1)}{2b(k+\rho-\delta)}, \tag{1.48}$$

$$\bar{r} = \frac{(2n-1)}{2b(\rho-2\delta)}\left(\frac{\omega\lambda}{k+\rho-\delta}+a\right), \tag{1.49}$$

$$\bar{s} = \frac{\omega(2n-1)}{2b(\rho+2k)}, \tag{1.50}$$

$$\bar{t} = \frac{2\omega b\phi}{2b(\rho+2k)}, \tag{1.51}$$

$$\bar{u} = \frac{(2n-1)}{2b(k+\rho-\delta)}\left(\frac{\omega\lambda}{\rho+2k}+a\right), \tag{1.52}$$

$$\bar{v} = \frac{\omega\lambda^2(2n-1)}{4b^2(k+\rho-\delta)(\rho+2k)(\rho-2\delta)}. \tag{1.53}$$

It can be verified that the solutions that make it possible to characterize an equilibrium are the ones with negative roots, given in Proposition 1. The terms $\mathscr{A}^N$ and θ^N are determined by these solutions, and the value function $V^N(S,Z)$ can be obtained by inserting these values into equations (1.36) to (1.41).

Using these solutions, we characterize the region $\mathscr{R}_S^N$ in the noncooperative solution, written as follows:

$$\mathscr{R}_S^N = \left\{(S,Z) \mid -\frac{a+\theta^N}{\eta^N} - \frac{\zeta^N}{\eta^N}Z < S < \frac{\zeta^N(2\delta-\rho)}{\eta^N(k+\rho-\delta)}Z - \frac{(\rho-2\delta)(an+\theta^N(2n-1))}{\eta^N(2n-1)(\rho-\delta)}\right\}, \tag{1.54}$$

in which the noncooperative strategies given in Proposition 1 constitute a symmetric feedback-Nash equilibrium.

The value function in region $\mathscr{R}_0^N$ can be found by solving the following partial differential equation:

$$\rho V^N(S,Z) = -\frac{\phi}{2}Z^2 + \delta S\frac{\partial V^N(S,Z)}{\partial S} - kZ\frac{\partial V^N(S,Z)}{\partial Z}, \tag{1.55}$$

which we do not characterize in detail in this paper.

1.7.2 Proof of Proposition 3

Similarly to the noncooperative solution, we write the Hamilton-Jacobi-Bellman equation associated to the joint welfare maximization as follows:

$$\rho V^C(S,Z) = \max_{e_i} \left\{ \sum_{i=1}^{n} \left(ae_i - be_i^2 - \frac{\phi}{2} Z^2 \right) + \frac{\partial V^C(S,Z)}{\partial S} \left(F(S,Z) - \sum_{i=1}^{n} ae_i \right) \right. \\ \left. + \frac{\partial V^C(S,Z)}{\partial Z} \left(\sum_{i=1}^{n} \omega e_i - kZ \right) \right\}. \tag{1.56}$$

Maximizing the right-hand side leads to the following first-order condition:

$$a - 2be_i = a \frac{\partial V^C(S,Z)}{\partial S} - \omega \frac{\partial V^C(S,Z)}{\partial Z}. \tag{1.57}$$

Considering symmetry among the players, the above condition becomes

$$e^C = \max \left\{ 0, \frac{1}{2b} \left(a - a \frac{\partial V^C(S,Z)}{\partial S} + \omega \frac{\partial V^C(S,Z)}{\partial Z} \right) \right\}. \tag{1.58}$$

Denote the function V^C by

$$V^C(S,Z) = \mathscr{A}^C + \frac{\mathscr{B}^C}{2} S^2 + \mathscr{C}^C S + \frac{\mathscr{D}^C}{2} Z^2 + \mathscr{E}^C Z + \mathscr{F}^C SZ. \tag{1.59}$$

By substituting $e_i = e^C, \forall i$ into the HJB equation and applying the undetermined coefficients method, we obtain the following system of equations:

$$\begin{aligned}
\mathscr{A}^C &= \frac{n \left[a^2 + \left(\omega \mathscr{E}^C - a \mathscr{C}^C \right) \left(2a + \left(\omega \mathscr{E}^C - a \mathscr{C}^C \right) \right) \right]}{4br}, \\
\mathscr{B}^C &= \frac{n \left(\omega \mathscr{F}^C - a \mathscr{B}^C \right)^2}{2b(\rho - 2\delta)}, \\
\mathscr{C}^C &= \frac{n \left(\omega \mathscr{F}^C - a \mathscr{B}^C \right) \left(a + \left(\omega \mathscr{E}^C - a \mathscr{C}^C \right) \right)}{2b(\rho - \delta)}, \\
\mathscr{D}^C &= \frac{n}{2b(\rho + 2k)} \times \\
&\quad \left(\left(\omega \mathscr{D}^C - a \mathscr{F}^C \right)^2 - 2b\phi - \frac{2b\lambda \left(\omega \mathscr{F}^C - a \mathscr{B}^C \right)}{(\rho + k - \delta)} \left(\omega \mathscr{D}^C - a \mathscr{F}^C - \frac{\lambda \left(\omega \mathscr{F}^C - a \mathscr{B}^C \right)}{2b(\rho - 2\delta)} \right) \right), \\
\mathscr{E}^C &= \frac{n \left(a + \omega \mathscr{E}^C - a \mathscr{C}^C \right)}{2b(\rho + k)} \left(\omega \mathscr{D}^C - a \mathscr{F}^C - \frac{\lambda \left(\omega \mathscr{F}^C - a \mathscr{B}^C \right)}{\rho - \delta} \right), \\
\mathscr{F}^C &= \frac{n \left(\omega \mathscr{F}^C - a \mathscr{B}^C \right)}{2b(\rho + k - \delta)} \left(\omega \mathscr{D}^C - a \mathscr{F}^C - \frac{\lambda \left(\omega \mathscr{F}^C - a \mathscr{B}^C \right)}{\rho - 2\delta} \right).
\end{aligned}$$

Let $\theta^C = \omega\mathscr{E}^C - a\mathscr{C}^C$, $\eta^C = \omega\mathscr{F}^C - a\mathscr{B}^C$, and $\zeta^C = \omega\mathscr{D}^C - a\mathscr{F}^C$. The above system becomes

$$\mathscr{A}^C = \frac{n\left(a^2 + \theta^C\left(2a + \theta^C\right)\right)}{4b\rho}, \tag{1.60}$$

$$\mathscr{B}^C = \frac{n\left(\eta^C\right)^2}{2b\left(\rho - 2\delta\right)}, \tag{1.61}$$

$$\mathscr{C}^C = \frac{n\eta^C\left(a + \theta^C\right)}{2b\left(\rho - \delta\right)}, \tag{1.62}$$

$$\mathscr{D}^C = \frac{n\left(\zeta^C\right)^2 - 2nb\phi}{2b\left(\rho + 2k\right)} - \frac{2bn\lambda\zeta^C\eta^C}{\left(\rho + k - \delta\right)\left(\rho + 2k\right)} + \frac{\lambda^2 n\left(\eta^C\right)^2}{4b^2\left(\rho + k - \delta\right)\left(\rho - 2\delta\right)\left(\rho + 2k\right)}, \tag{1.63}$$

$$\mathscr{E}^C = \frac{n\zeta^C\left(a + \theta^C\right)}{2b\left(\rho + k\right)} - \frac{\lambda n\eta^C\left(a + \theta^C\right)}{\left(\rho + k\right)\left(\rho - \delta\right)}, \tag{1.64}$$

$$\mathscr{F}^C = \frac{n\zeta^C\eta^C}{2b\left(\rho + k - \delta\right)} - \frac{\lambda n\left(\eta^C\right)^2}{\left(\rho + k - \delta\right)\left(\rho - 2\delta\right)}. \tag{1.65}$$

It is easy to verify that the above system can be reduced to the following:

$$\mathscr{A}^C = \frac{n\left(a^2 + \theta^C\left(2a + \theta^C\right)\right)}{4b\rho}, \tag{1.66}$$

$$\theta^C = \left(a + \theta^C\right)n\left(\frac{\omega}{2b\left(\rho + k\right)}\left(\zeta^C - \frac{\lambda\eta^C}{\rho - \delta}\right) - \frac{a\eta^C}{2b\left(\rho - \delta\right)}\right), \tag{1.67}$$

$$\eta^C = n\eta^C\left(\frac{\omega}{2b\left(\rho + k - \delta\right)}\left(\zeta^C - \frac{\lambda\eta^C}{\rho - 2\delta}\right) - \frac{a\eta^C}{2b\left(\rho - 2\delta\right)}\right), \tag{1.68}$$

$$\begin{aligned}\zeta^C = {} & \frac{n\omega}{2b\left(\rho + 2k\right)}\left[\left(\zeta^C\right)^2 - 2b\phi - \frac{\lambda\eta^C}{\rho + k - \delta}\left(\zeta^C - \frac{\lambda\eta^C}{2b\left(\rho - 2\delta\right)}\right)\right] \\ & - \frac{an\eta^C}{2b\left(\rho + k - \delta\right)}\left[\zeta^C - \frac{\lambda\eta^C}{\rho - 2\delta}\right].\end{aligned} \tag{1.69}$$

The solution to the system given above is the pair $\left(\eta^C, \zeta^C\right)$ (with two possibilities, but, as in the noncooperative case, we do not report the solutions associated to $\mathscr{R}_A^C$):

$$\begin{aligned}
\eta^C =& \frac{-1+\bar{g}\zeta^C}{2\left(\bar{i}\bar{h}^2+\bar{g}^2\bar{l}-\bar{k}\bar{h}\bar{g}\right)} && (1.70)\\
\zeta^C =& \frac{\bar{h}^2-\bar{k}\bar{h}+2\bar{g}\bar{l}\pm\sqrt{\left(\bar{k}\bar{h}-\bar{h}^2-2\bar{g}\bar{l}\right)^2-4\left(\bar{l}-\bar{j}\bar{h}^2\right)\left(\bar{i}\bar{h}^2+\bar{g}^2\bar{l}-\bar{k}\bar{h}\bar{g}\right)}}{2\left(\bar{i}\bar{h}^2+\bar{g}^2\bar{l}-\bar{k}\bar{h}\bar{g}\right)} && (1.71)\\
\bar{g} =& \frac{\omega n}{2b\left(k+\rho-\delta\right)},\\
\bar{h} =& \frac{n}{2b\left(\rho-2\delta\right)}\left(\frac{\omega\lambda}{k+\rho-\delta}+a\right),\\
\bar{i} =& \frac{\omega n}{2b\left(\rho+2k\right)},\\
\bar{j} =& \frac{2n\omega b\phi}{2b\left(\rho+2k\right)},\\
\bar{k} =& \frac{n}{2b\left(k+\rho-\delta\right)}\left(\frac{\omega\lambda}{\rho+2k}+a\right),\\
\bar{l} =& \frac{\omega\lambda^2 n}{4b^2\left(\rho+k-\delta\right)\left(\rho-2\delta\right)\left(\rho+2k\right)}.
\end{aligned}$$

The solutions associated to the joint maximization are those with negative roots, given in Proposition 3. Lastly, region $\mathscr{R}_S^C$ is given by

$$\mathscr{R}_S^C=\left\{(S,Z)\mid -\frac{a+\theta^C}{\eta^C}-\frac{\zeta^C}{\eta^C}Z<S<\frac{\zeta^C(2\delta-\rho)}{\eta^C(k+\rho-\delta)}Z-\frac{(a+\theta^C)(\rho-2\delta)}{\eta^C(\rho-\delta)}\right\}. \quad (1.72)$$

The value function in region $\mathscr{R}_0^C$ can be obtained by solving the following partial differential equation:

$$\rho V^C(S,Z)=-\frac{\phi}{2}Z^2+\delta S\frac{\partial V^C(S,Z)}{\partial S}-kZ\frac{\partial V^C(S,Z)}{\partial Z}. \quad (1.73)$$

1.7.3 Trajectories of the state variables

In the following, we derive the analytical solutions for the trajectories of the state variables associated to solutions $j=\{N,C\}$. This includes replacing the harvesting effort $e^j\left(S,Z\right)$ in the system of ODEs in (1.1)–(1.2), then solving for $S(t)$ and $Z(t)$.

For $j = \{N, C\}$, denote by

$$\bar{a}^j = \delta - \left(\frac{na}{2b}\right)\eta^j, \qquad \bar{b}^j = \left(\lambda + \frac{na}{2b}\right)\zeta^j, \qquad \bar{c}^j = \left(\frac{na}{2b}\right)\left(a + \theta^j\right),$$
$$\bar{d}^j = \left(\frac{n\omega}{2b}\right)\left(a + \theta^j\right), \qquad \bar{k}^j = \left(\frac{n\omega}{2b}\right)\eta^j, \qquad \bar{l}^j = k - \left(\frac{n\omega}{2b}\right)\zeta^j.$$

The time representation of the stock of resource at date t is given by

$$S^j(t) = e^{\Gamma_1 t}c_2^j + e^{\Gamma_2 t}c_1^j + \frac{\bar{b}^j\bar{d}^j + \bar{l}^j\bar{c}^j}{\bar{l}^j\bar{a}^j - \bar{b}^j\bar{k}^j}, \tag{1.74}$$

and the time representation of the stock of pollution by

$$Z^j(t) = \frac{-\Gamma_1^j e^{\Gamma_1 t}c_2^j}{\bar{b}^j} - \frac{\Gamma_2 e^{\Gamma_2 t}c_1^j}{\bar{b}^j} + \frac{\bar{a}^j\left(e^{\Gamma_1 t}c_2^j + e^{\Gamma_2 t}c_1^j + \frac{\bar{b}\bar{d}+\bar{l}\bar{c}}{\bar{l}\bar{a}-\bar{b}\bar{k}}\right) - \bar{c}^j}{\bar{b}^j}, \tag{1.75}$$

The constants c_1^j and c_2^j are written as

$$c_1^j = \frac{1}{2\Theta^j}\left[2\bar{b}^j Z(0) + \left(-\bar{a}^j - \bar{l}^j + \Theta^j\right)S(0)\right] - \frac{1}{\Theta^j}\left[\Gamma_1^j\left(\frac{\bar{b}^j\bar{d}^j + \bar{l}^j\bar{c}^j}{\bar{l}^j\bar{a}^j - \bar{b}^j\bar{k}^j}\right) + 2\bar{c}^j\right], \tag{1.76}$$

$$c_2^j = \frac{1}{2\Theta^j}\left[-2\bar{b}^j Z(0) + \left(\bar{a}^j + \bar{l}^j + \Theta^j\right)S(0)\right] + \frac{1}{\Theta^j}\left[\Gamma_2\left(\frac{\bar{b}^j\bar{d}^j + \bar{l}^j\bar{c}^j}{\bar{l}^j\bar{a}^j - \bar{b}^j\bar{k}^j}\right) - 2\bar{c}^j\right], \tag{1.77}$$

and

$$\Gamma_1^j = \left(\frac{\bar{a}^j - \bar{l}^j}{2} + \frac{\Theta^j}{2}\right), \quad \Gamma_2^j = \left(\frac{\bar{a}^j - \bar{l}^j}{2} - \frac{\Theta^j}{2}\right), \quad \Theta^j = \sqrt{(\bar{a}^j)^2 + 2\bar{l}^j\bar{a}^j + (\bar{l}^j)^2 - 4\bar{b}^j\bar{k}^j}.$$

Chapter 2

Equilibria in a Two-Species Fishery

Abstract

In this paper, we consider a two-species fishery model where the species can have different biological interactions, namely, competitive, symbiotic or prey-predator relationships. Each species is harvested by a group of fisherpersons. We characterize and compare equilibrium harvesting strategies, steady-state stocks and total discounted utilities under different modes of play, that is, noncooperation in both groups, cooperation in each of the groups and cooperation in only one group of fisherpersons. Our main results are as follows: (i) In all scenarios, the equilibrium strategy of an agent fishing either species consists of harvesting, in each period, a proportion of the available stock. (ii) The dividend of cooperation in a given group is increasing with the number of members in that group. (iii) Coordination between agents fishing a given species may be detrimental (biologically and economically) to the other species.[1]

---

[1] This chapter is published as : M. Breton, I. Dahmouni, G. Zaccour, "Equilibria in a two-species fishery", Mathematical Biosciences, 309, 78-91, 2019.

2.1 Introduction

The exploitation of fisheries, and more generally of renewable resources, has been the subject of a large number of contributions over a long period of time. Two decades ago, Wilen (2000) recalled and discussed the main developments in fisheries economics and policy over the past half century. One landmark in this literature is the seminal paper by Hardin (1968) where the concept of the *tragedy of the commons* is introduced. In short, this concept specifies that open access, or competition between the agents, leads to harvesting levels above the rate of reproduction of the resource, which may cause its depletion. Levhari & Mirman (1980) were the first to model and quantify the tragedy of commons in a fishery context. A large number of contributions followed, where the Levhari & Mirman's model is modified or extended in various directions. A common denominator in this literature is the use of dynamic games to analyze the exploitation of a renewable asset over time. This choice is quite natural as a dynamic game allows to capture current and future externalities, as well as strategic interactions between agents. We refrain from reviewing this literature and refer the interested reader to the comprehensive surveys by Long (2011, 2018).[2]

As fisheries are typically populated by more than one species, a natural question is how the results obtained in one-species models generalize to multiple species. In that case, in addition to the dynamic (intertemporal) externalities, the analysis must account for the biological interactions between the species. Fischer & Mirman (1992, 1996) pioneered the domain by studying a fishery with two species, each harvested by one player. They used this fishery model, designated by *FM* in the sequel, to characterize and contrast cooperative and noncooperative strategies, taking into account various types of biological interactions between the two species.

In this paper, we extend the FM model by assuming that each species is harvested not by one, but by a group of agents, that is, we add a competitive dimension to the fishing activities of each species. Our objective is in the same vein as that of Fischer &

---

[2] For reviews of game theory applications to fisheries, see, e.g., Bailey et al. (2010), Hannesson (2011), Sumaila (2013), and Grønbæk et al. (2018).

Mirman (1992, 1996), that is, the determination and comparison of equilibrium harvesting strategies and outcomes under various behavioral assumptions about the agents exploiting the fishery, as well as various biological relationships between the species.

The papers that are most related to ours are those of Mazalov & Rettieva (2010), Rettieva (2012) and Górniewicz & Wiszniewska-Matyszkiel (2018). Mazalov & Rettieva (2010) and Rettieva (2012) extend the FM model to a case where the fishing grounds have different locations and fish can migrate. Górniewicz & Wiszniewska-Matyszkiel (2018) modify the FM model in order to account for the possibility of extinction under the so-called Allee effect.[3] The authors also offer a rigorous mathematical analysis of the equilibria in models à la FM.

Other contributions involving multi-species dynamic games use different bioeconomic models, with objectives that differ from ours.[4] For instance, Doyen et al. (2018) discuss the tragedy of open ecosystems using a model with *n* species. Wang & Ewald (2010) extend the one-fishery model of Jørgensen & Yeung (1996) to a stochastic differential game of a two-species fishery with ecological interaction and characterize the stationary feedback Nash-equilibrium. They also analyze various cases of competitive, restricted and cooperative fisheries management and their impact on the ecological system. Salenius (2018) models a three-species fishery with three players (Norway, the European Union and Iceland) as a differential game and estimates empirically open-loop equilibria under various settings related to the management of the fishery.

Our research questions are as follows:

1. What are the equilibrium harvesting strategies corresponding to various modes of play?

2. How do the results vary with the type of biological interaction and with parameter values?

---

[3] The Allee effect refers to a species starting to degenerate and soon becoming extinct when it reaches some critical level of biomass (Joosten 2016).

[4] For optimal-control two-species models, see, e.g., Hannesson (1983) and Mesterton-Gibbons (1996).

3. How do the various outcomes compare in terms of environmental and economical considerations?

4. Under what conditions is it better for agents in one group to act cooperatively?

By answering the above questions, we seek to separate the effects of three sources of externalities, namely, the intertemporal, biological and behavioral externalities. Understanding and untangling these effects are clearly of conceptual, methodological and practical values.

In a nutshell, our main results are as follows: (i) In all setups, the equilibrium strategy of an agent fishing either species consists of harvesting, in each period, a fixed proportion of the available stock. (ii) The dividend of cooperation in a given group is increasing with the number of agents in that group. (iii) Coordination among a group of agents fishing a given species may be detrimental (biologically and economically) to the other species.

The rest of the paper is organized as follows: In Section 2.2, we recall the FM model and define the scenarios we are interested in. In Section 2.3, we state some preliminary results that are valid across all scenarios. Section 2.4 is devoted to the benchmark case, where a single agent exploits the fishery. In Section 2.5, we characterize the equilibrium strategies and outcomes in various scenarios. In Section 2.6, we compare these scenarios, focusing on biological and economic outcomes. Section 2.7 briefly concludes.

2.2 Model

Consider a fishery with two species. To simplify notation, whenever a species $l \in \{1,2\}$ is considered, $m \equiv 3-l$ will represent the other species. Let x_{lt} be the stock levels of species l at date $t \in [0,\infty)$. In the absence of any human activity, the one-period growth functions of the stocks are

$$x_{l;t+1} = x_{lt}^{\alpha_l} x_{mt}^{\beta_l}, \qquad x_{l0} \text{ given}, \, l = 1,2, \tag{2.1}$$

where the parameter $\alpha_l > 0$ characterizes the regeneration capacity of species l and the parameter $\beta_l \neq 0$ characterizes the indirect effect that species m exerts on species l. Fisher

& Mirman (1992,1996) identify three possible combinations of interactions, namely: (i) *symbiotic* relationship when β_1 and β_2 are both positive; (ii) *competitive* relationship when β_1 and β_2 are both negative, which corresponds to a case where both species feed on the same resource; and, finally, (iii) *prey-predator* relationship when β_1 and β_2 have opposite signs.

We use the notation $\bar{x}_l$ to refer to the steady-state value of the stock level of species l. As in Fischer & Mirman (1992), we make the following assumption ensuring the stability of $\bar{x}_l$ in the absence of human intervention:

$$\alpha_l + |\beta_l| < 1, \quad l = 1,2.$$

Remark 4. *In the absence of human intervention, it is easy to verify that, under Assumption 1, the unique positive steady state is given by* $\left(\bar{x}_1^{nf}, \bar{x}_2^{nf}\right) = (1,1)$, *where the superscript nf stands for no fishing. The common interpretation in that case is that the saturation or natural equilibrium level of each species is normalized to 1. When the stock level* $x_{lt} < 1$, $l = 1,2$, *the impact of constant* α_l *on the fish stock is negative: the smaller is* α_l, *the higher is the regeneration capacity of the fish stock l. In the same way, the smaller is* β_l, *the higher is the (positive or negative) effect of the other species on the fish stock l.*

Suppose that two specialized fleets exploit the fishery. Denote by N_l the set of agents (fisherpersons, vessels) harvesting species l and by n_l the cardinality of N_l, $l = 1,2$. Denote by c_{ilt} the catch by agent $i \in N_l$ at date t. Consequently, when fishing activities are undertaken, the stock dynamics become

$$x_{l;t+1} = \left(x_{lt} - \sum_{i \in N_l} c_{ilt}\right)^{\alpha_l} \left(x_{mt} - \sum_{i \in N_m} c_{imt}\right)^{\beta_l}, \quad x_{l0} \text{ given}, \ l = 1,2. \tag{2.2}$$

The utility function of an agent, in both groups, is concave increasing in her catch. For simplicity, let this utility function be logarithmic, and denote by $\delta \in (0,1)$ the common periodic discount factor. Assuming that each agent maximizes her stream of utility over an infinite horizon, the optimization problem of Agent $i \in N_l$, $l = 1,2$, is then:

$$\max_{c_{ilt} \in C_{ilt}} \sum_{t=0}^{\infty} \delta^t \ln(c_{ilt}) \tag{2.3}$$

$$\text{subject to (2.2)} \tag{2.4}$$

where C_{ilt} is the set of feasible decisions for player $i \in N_l$ at date t. Clearly, feasible decisions are restricted by the level of the stock and by the decisions of other agents.[5] We suppose that the agents observe the level of the stock of the two species in each period and use a feedback (or Markovian) information structure, thus defining a discrete-time dynamic Markov game.

We consider the following scenarios:

Single owner: In this benchmark scenario (labeled S), we assume that there is only one owner of the fishery who exploits both species.

Specialized fishing: We assume, as in the FM model, that fishing the two species is specialized, that is, each species requires a specific equipment. To assess the impact of cooperation on the results, we characterize and contrast equilibrium solutions in the following setups:

- **N** The game is played fully noncooperatively. A Nash equilibrium involving $n_1 + n_2$ players is sought.
- **C** Agents in each specialized group coordinate their fishing policies, that is, they maximize their joint payoff, and play noncooperatively against the cartel formed by the other group. In this case, we solve for a Nash equilibrium involving two players.
- **Nl** Agents in the specialized group fishing species l play noncooperatively, whereas agents in the other group coordinate their strategies. We solve for a Nash equilibrium involving $n_l + 1$ players.

---

[5] A precise definition of the sets C_{ilt} is provided in Appendix 2.8.1.

As alluded to before, characterizing and contrasting the results according to these scenarios allow us to untangle the effects of three externalities, namely: dynamic interactions, biological interactions and ownership of the resource. The comparisons are made in terms of the species' steady-state stock levels, which are biological indicators, and in terms of the agents' total discounted utilities, which are economic indicators. For instance, comparing the results in Scenario Nl to those obtained in Scenario C provides a measure of the dividend of cooperation in group l, while comparing them to those obtained in Scenario N provides a measure of the dividend of cooperation in group m. Similarly, setting $n_1 = n_2 = 1$ in Scenario N and comparing the results to those according to the single owner scenario S gives an assessment of the benefit of centralization, or of having a single technology that enables fishing the two species.

2.3 Preliminary results

Before presenting the outcomes in various settings related to the number of players fishing each species and their cooperative or noncooperative behavior, we establish in this section results that are valid in all cases in order to avoid unnecessary repetitions. To save on notation, let

$$\Omega \equiv (1-\delta\alpha_1)(1-\delta\alpha_2)-\delta^2\beta_1\beta_2, \tag{2.5}$$

$$\Theta \equiv (1-\alpha_1)(1-\alpha_2)-\beta_1\beta_2, \tag{2.6}$$

$$A_l \equiv \frac{1-\delta\alpha_l}{\Omega}, \quad l=1,2, \tag{2.7}$$

$$B_l \equiv \frac{\delta\beta_l}{\Omega}, \quad l=1,2, \tag{2.8}$$

$$\rho_l = \alpha_l(1-\alpha_m)+\beta_1\beta_2, \quad l=1,2, \tag{2.9}$$

$$\xi_l = \alpha_l(1-\delta\alpha_m)+\beta_m(1+\delta\beta_l), \quad l=1,2. \tag{2.10}$$

The following two remarks establish the signs of Ω and Θ.

Remark 5. *Irrespective of the signs of* β_1 *and* β_2, Ω *is positive. Indeed, if* sign$(\beta_1) =$

$sign(\beta_2)$, then by Assumption 1, we have

$$1 - \delta\alpha_l > 1 - \alpha_l > |\beta_l| > \delta|\beta_l|, \quad l = 1,2,$$

which implies $\Omega > 0$. If $sign(\beta_1) = -sign(\beta_2)$, then Ω is trivially positive.

Remark 6. *Irrespective of the signs of β_1 and β_2, Θ is positive. If $sign(\beta_1) = sign(\beta_2)$, then by Assumption 1, we have $|\beta_l| < 1 - \alpha_l$, and consequently $\Theta > 0$. If $sign(\beta_1) = -sign(\beta_2)$, then Θ is trivially positive.*

The following proposition characterizes the steady-state values of the two fish stocks when the fishing strategy of each species is linear in the stock level of that species. We show later on that a linear strategy is part of a feedback-Nash equilibrium in a competitive setting and corresponds to the optimal policy in a cooperative setting.

Proposition 6. *For any fishing strategy of the form $c_l = \gamma_l x_l$ where $\gamma_l \in [0,1)$, $l = 1,2$, the steady-state values of the fish stocks are*

$$\bar{x}_l = (1-\gamma_l)^{\frac{\rho_l}{\Theta}} (1-\gamma_m)^{\frac{\beta_l}{\Theta}}, \; l = 1,2. \tag{2.11}$$

Proof. The steady state is obtained by solving the following system of equations:

$$x_l = (x_l(1-\gamma_l))^{\alpha_l} (x_m(1-\gamma_m))^{\beta_l}, \; l = 1,2.$$

Straightforward manipulations lead to the result. ☐

Recalling that in the absence of human intervention, the steady state is $\left(\bar{x}_1^{\mathrm{nf}}, \bar{x}_2^{\mathrm{nf}}\right) = (1,1)$, steady-state values in a given scenario can be interpreted as proportions with respect to $\bar{x}_1^{\mathrm{nf}}$ and $\bar{x}_2^{\mathrm{nf}}$.

Proposition 6 establishes the **form** that the steady-state values of the two species take in all considered scenarios, provided that the equilibrium fishing strategies be linear in the stock level. What will vary across these scenarios are the **actual values** of these steady states through the changes in γ_1 and γ_2, which depend on the setup, i.e., on the type of biological interaction, on the number of agents, and on their behavior (cooperation or not within each group). Clearly, for any γ_1 and $\gamma_2 \in [0,1)$, the steady-state values are positive, that is, a resource collapse is not possible.

The impact of the intensity of fishing on the steady-state level of the fish stock $l = 1,2$ is as follows:

$$\begin{aligned} \frac{d\bar{x}_l}{d\gamma_l} &= -\bar{x}_l \frac{\rho_l}{\Theta(1-\gamma_l)}, && (2.12) \\ \frac{d\bar{x}_l}{d\gamma_m} &= -\bar{x}_l \frac{\beta_l}{\Theta(1-\gamma_m)}. && (2.13) \end{aligned}$$

Note that $\rho_l > 0$ when $\beta_1 \beta_2 > 0$, but could be negative in the prey-predator case. We make the following additional assumption:

$$\rho_l > 0, \quad l = 1,2.$$

Under Assumption 2, the steady-state stock of a given species is decreasing in the fishing intensity of this species. In the prey-predator case, fishing the prey has a negative impact on the stock of its predator. Assumption 2 means that the direct effect of fishing the prey dominates the indirect effect due to the decrease in its predator stock. In the same way, under Assumption 2, the direct effect of fishing the predator dominates the indirect effect due to the increase in the stock of its prey.

The impact of fishing a given species on the stock of the other is positive in the competitive and predator cases, and negative in the symbiotic and prey cases. These results are fairly intuitive.

Remark 7. *Note that it is possible that the steady-state stock level of the prey species be greater than 1, if the reduction in the stock of the predator due to harvesting more than compensates for the harvesting of the prey species. Steady-state levels above the saturation level due to human intervention could also happen in the competitive case. Recall that the interpretation of the biological parameters* α *and* β *changes when the stock level is above the saturation level. For that reason, in our numerical analysis, we restrict the range of parameter values to those yielding a steady state in* $(0,1] \times (0,1]$.

The impact of parameter values on the steady-state levels for $l = 1,2$ is given by

$$\begin{aligned} \frac{\partial \bar{x}_l}{\partial \zeta} &= \bar{x}_l \left(\ln(1-\gamma_l) \frac{\partial}{\partial \zeta} \left(\frac{\rho_l}{\Theta} \right) + \ln(1-\gamma_m) \frac{\partial}{\partial \zeta} \left(\frac{\beta_l}{\Theta} \right) \right) \\ &\quad + \frac{\partial \bar{x}_l}{\partial \gamma_l} \frac{\partial \gamma_l}{\partial \zeta} + \frac{\partial \bar{x}_l}{\partial \gamma_m} \frac{\partial \gamma_m}{\partial \zeta}, \end{aligned} \tag{2.14}$$

where $\zeta \in \{\alpha_l, \alpha_m, \beta_l, \beta_m\}$.

2.4 Single owner

Suppose that a technology exists for fishing simultaneously both species, and that the fishery is owned by a single entity. This setup results in a standard infinite-horizon discrete-time dynamic optimization problem. Let $V(x)$ represent the maximal total discounted utility of the owner over an infinite horizon when the current stock is $x = (x_1, x_2) \in R^+ \times R^+$ (the *value function*). Using Bellman's principle of optimality (Bellman 1957), the optimization problem (2.3)-(2.4) has an equivalent recursive representation:

$$V(x) = \max_{\substack{c_1 \in C_1(x) \\ c_2 \in C_2(x)}} \left\{ \ln(c_1) + \ln(c_2) + \delta V\left(x_1', x_2'\right) \right\} \tag{2.15}$$

where

$$x_l' = (x_l - c_l)^{\alpha_l} (x_m - c_m)^{\beta_l}, \ l = 1,2, \tag{2.16}$$

and where $C_1(x)$ and $C_2(x)$ are the sets of feasible decisions at x. In the infinite-horizon case, some technical conditions are needed to establish the equivalence between the formulations (2.3)-(2.4) and (2.15)-(2.16) and the existence of the value function. These are discussed in Appendix 2.8.1.

In the next proposition, we show that the optimal fishing policy is linear, and that the value function is log-linear. To do so, we use the following assumption on the parameter values:

The parameter values satisfy the following restriction:

$$\xi_l > 0, \quad l = 1,2.$$

Proposition 7. *Under Assumptions 1-3, for* $(x,y) \in R^{+} \times R^{+}$*, the unique optimal policy of a single owner is to fish a proportion*

$$\gamma_l^S = \frac{1}{A_m + B_m} \in (0,1)\,,\; l = 1,2,$$

of the stock of each species, and the value function is given by

$$V^S(x_1,x_2) = \sum_{l=1}^{2} (A_m + B_m) \ln(x_l) + C^S,$$

where

$$\begin{aligned}(1-\delta)C^S &= \sum_{l=1}^{2} (A_l + B_l - 1)\ln(A_l + B_l - 1) \\ &\quad - (A_l + B_l)\ln(A_l + B_l)\,.\end{aligned}$$

Proof. See Appendix 2.8.2. □

Remark 8. *For all the scenarios analyzed in the sequel, it is straightforward to show in a similar way that the optimal response of any given player* $i \in N_l$ *to a joint strategy of the form* $\sum_{j \neq i \in N_l} c_{lj} = \gamma_l x_l$, $\sum_{j \in N_m} c_{mj} = \gamma_m x_m$, $l = 1,2$, *is a linear strategy, so that we will be looking for equilibrium in the space of linear strategies.*

From an economic point of view, the single-owner case yields the highest discounted utility. The reason is that the harvesting operations are then fully centralized and the solution is obtained by solving an optimization problem, whereas in all other scenarios we solve for an equilibrium, which at best can match the centralized (optimal) outcome.

The impact of parameter values on the optimal fishing strategies is as follows for $l = 1,2$:

$$\begin{aligned}\frac{d\gamma_l^S}{d\alpha_l} &= -\delta \frac{1-\delta\alpha_m}{1-\delta(\alpha_m - \beta_m)} < 0, \\ \frac{d\gamma_m^S}{d\alpha_l} &= -\delta^2 \frac{1-\delta(\alpha_m - \beta_m)}{(1-\delta(\alpha_l - \beta_l))^2}\beta_l, \\ \frac{d\gamma_l^S}{d\beta_l} &= -\delta^2 \frac{\beta_m}{1-\delta(\alpha_m - \beta_m)}, \\ \frac{d\gamma_m^S}{d\beta_l} &= -\delta \frac{(1-\delta\alpha_l)(1-\delta(\alpha_m - \beta_m))}{(1-\delta(\alpha_l - \beta_l))^2} < 0.\end{aligned}$$

An increase in the parameter α_l (decrease in regeneration capacity of species l) results in less fishing activity for species l and in less fishing for the other species if the interaction parameter β_l is positive (symbiotic or when l is a predator). An increase in the parameter β_l results in less fishing activity for species m. The impact of an increase in β_l on the fishing activity of species l depends on the sign of β_m, that is, on the indirect effect species l has on the other, resulting in less fishing activity when this effect is positive (symbiotic or when l is a prey).

Table 2.1 indicates the signs of the four terms of Equation (2.14) characterizing the impact of each model parameter on the steady-state stock level of species l. As indicated in this table, the impact is ambiguous for the three types of interaction. Various possibilities are illustrated in Figure 2.1.[6] Table 2.2 contains the numerical values used for the cases illustrated in Figure 2.1. Numerical experiments indicate that α_l has a negative impact on $\bar{x}_l$ in the symbiotic case and that β_l has a negative impact on $\bar{x}_l$ when $\beta_l\beta_m > 0$ and $\beta_l > 0$, and a positive impact on $\bar{x}_m$ when $\beta_m < 0$.

ζ	$-\frac{\partial}{\partial\zeta}\left(\frac{\rho_l}{\Theta}\right)$	$-\frac{\partial}{\partial\zeta}\left(\frac{\beta_l}{\Theta}\right)$	$\frac{\partial\bar{x}_l}{\partial\gamma_l}\frac{\partial\gamma_l^S}{\partial\zeta}$	$\frac{\partial\bar{x}_l}{\partial\gamma_m}\frac{\partial\gamma_m^S}{\partial\zeta}$
α_l	$-$	$-\beta_l$	$+$	$+$
α_m	$-\beta_1\beta_2$	$-\beta_l$	β_m	β_l
β_l	$-\beta_m$	$-$	β_m	β_l
β_m	$-\beta_l$	$-$	$+$	$+$

Table 2.1 – Impact of model parameters on the steady-state levels. This table contains the sign of each of the four terms of Equation (2.14) when the fishery is managed by a single owner.

2.5 Specialized fishing

Suppose that no single technology exists for harvesting simultaneously both species and that the fishery is exploited by two groups of specialized fleets, with n_l agents fishing

---

[6] Note that we restrict the range of parameter values for this numerical analysis to those satisfying Assumptions 1-3 and yielding a steady state in $(0,1] \times (0,1]$.

Cases	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10
α_l	0.5	0.5	0.5	0.3	0.5	0.9	0.36	0.4	0.3	0.6
α_m	0.5	0.5	0.6	0.2	0.5	0.2	0.97	0.9	0.8	0.6
$\lvert\beta_l\rvert$	0.1	0.05	0.005	0.5	0.1	0.01	0.4	0.13	0.1	0.1
$\lvert\beta_m\rvert$	0.1	0.1	0.05	0.5	0.05	0.5	0.015	0.01	0.1	0.1

Table 2.2 – Numerical illustrations. This table contains the parameter values used to produce the numerical illustrations in Figures 2.1 and 2.2 and in Tables 2.4 and 2.5.

species l, $l = 1,2$. Assuming that the total catch of each species is proportional to the stock of that species, the next proposition gives the form of the total discounted utility of any agent i belonging to group $l = 1,2$. Note that this form is valid for any mode of play, that is, cooperation or noncooperation within and between groups of agents.

Proposition 8. *Assume that the total catch is given by* $\gamma_1 x_1$ *and* $\gamma_2 x_2$*, where* $\gamma_l = \sum_{i\in N_l} p_{il}$*,* $l = 1,2$*. The total discounted utility over an infinite horizon of an agent* i *in group* $l \in \{1,2\}$ *corresponding to the strategy pair* $\gamma = (\gamma_1, \gamma_2)$ *is given by*

$$V_{il}(x_1, x_2; \gamma) = A_m \ln(x_l) + B_l \ln(x_m) + C_{il}(\gamma), \quad l = 1,2,$$

where

$$(1-\delta)\, C_{il}(\gamma) = \ln(p_{il}) + (A_m - 1)\ln(1-\gamma_l) + B_l \ln(1-\gamma_m).$$

Proof. See Appendix 2.8.3. □

In the specialized fishing setting, agents in a given group can coordinate their fishing strategies or not. The following sections establish the equilibrium strategies of agents in a given group according to their mode of play.

2.5.1 Non-cooperative fishing

First assume that each agent in group $l \in \{1,2\}$ unilaterally decides on her harvested quantity, given the fishing strategy of the other agents.

Proposition 9. *If agents in group $l \in \{1,2\}$ act unilaterally, given that the total catch rate in the other group is γ_m, then under Assumptions 1-3, the catch rate by agent $i \in N_l$ is*

$$p_{il}^N = \frac{1}{n_l + A_m - 1},$$

and the total catch rate by agents in group l is given by

$$\gamma_l^N = \frac{n_l}{n_l + A_m - 1} \in (0,1).$$

The total discounted utility over an infinite horizon of an agent i in group l is given by

$$V_l^N(x_1, x_2; \gamma_m) = A_m \ln(x_l) + B_l \ln(x_m) + C_l^N(\gamma_m),$$

where

$$\begin{aligned}(1-\delta)\, C_l^N(\gamma_m) &= (A_m - 1)\ln(A_m - 1) - A_m \ln(A_m + n_l - 1) \\ &\quad + B_l \ln(1 - \gamma_m).\end{aligned}$$

Proof. See Appendix 2.8.4. □

The above proposition shows that the equilibrium harvesting strategy of species $l \in \{1,2\}$ depends on the number of non-cooperating players harvesting species l and is independent of the harvesting strategy used by the players in the other group. Using (2.11), we obtain:

$$\begin{aligned}\frac{\partial \bar{x}_l}{\partial n_l} &= -\frac{\rho_l}{\Theta} \frac{\bar{x}_l}{A_m + n_l - 1} < 0, \\ \frac{\partial \bar{x}_m}{\partial n_l} &= -\frac{\beta_m}{\Theta} \frac{\bar{x}_m}{A_m + n_l - 1}.\end{aligned}$$

Increasing the number of non-cooperating players leads to a decrease in the steady-state stock of the species they are fishing. This is hardly surprising as we are simply adding fleets in an open-access fishery context. As a consequence, the impact of increasing the number of non-cooperating players on the steady state of the stock of the other species depends on the sign of the interaction parameter β_m.

2.5.2 Cooperative fishing

Now assume that the agents in group $l \in \{1,2\}$ agree to coordinate their harvesting strategies by jointly maximizing their total utility, given the fishing strategy of the agents in the other group.

Proposition 10. *If agents in group $l \in \{1,2\}$ coordinate their strategies, then under Assumptions 1-3, the catch rate by agent $i \in N_l$ is*

$$p_{il}^C = \frac{1}{n_l A_m}$$

and the total catch rate by agents in group l is given by

$$\gamma_l^C = \frac{1}{A_m} \in (0,1).$$

The total discounted utility over an infinite horizon of an agent i in group l is given by

$$V_l^C(x_1,x_2;\gamma_m) = A_m \ln(x_l) + B_l \ln(x_m) + C_l^C(\gamma_m),$$

where

$$\begin{aligned}(1-\delta)\,C_l^C(\gamma_m) &= (A_m-1)\ln(A_m-1) - A_m \ln(A_m) \\ &\quad + B_l \ln(1-\gamma_m) - \ln(n_l).\end{aligned}$$

Proof. See Appendix 2.8.5. ☐

The total catch in a cooperating group is independent of the number of agents in the group. It is equal to the optimal catch in a group containing a single agent and this total catch is distributed equally among the members of the cooperating group. As a consequence, the steady-state level of the stock is independent of the number of cooperating agents.

2.5.3 Impact of model parameters

Under specialized fishing, the impact of model parameters on the fishing strategies of cooperating and non-cooperating players is given by

$$
\begin{aligned}
\frac{d\gamma_l}{d\alpha_l} &= -\frac{\delta n (1-\delta\alpha_m)^2}{(\Omega n + \delta(\alpha_l(1-\delta\alpha_m)+\delta\beta_l\beta_m))^2} < 0, \\
\frac{d\gamma_l}{d\alpha_m} &= -\beta_l\beta_m \frac{\delta^3 n}{(\Omega n + \delta(\alpha_l(1-\delta\alpha_m)+\delta\beta_l\beta_m))^2}, \\
\frac{d\gamma_l}{d\beta_l} &= -\beta_m \frac{\delta^2 n (1-\delta\alpha_m)}{(\Omega n + \delta(\alpha_l(1-\delta\alpha_m)+\delta\beta_l\beta_m))^2}, \\
\frac{d\gamma_l}{d\beta_m} &= -\beta_l \frac{\delta^2 n (1-\delta\alpha_m)}{(\Omega n + \delta(\alpha_l(1-\delta\alpha_m)+\delta\beta_l\beta_m))^2},
\end{aligned}
$$

where $n = n_l$ when players in group l are not cooperating, and $n = 1$ when they are. As in the single owner case, an increase in α_l reduces the fishing intensity of species l; other impacts depend on the type of interaction between the two species. Table 2.3 indicates the signs of the four terms of Equation (2.14) characterizing the impact of each model parameter on the steady-state stock level of species l. As in the single owner case, the impact of all parameters is ambiguous for the three interaction possibilities.

ζ	$-\frac{\partial}{\partial\zeta}\left(\frac{\rho_l}{\Theta}\right)$	$-\frac{\partial}{\partial\zeta}\left(\frac{\beta_l}{\Theta}\right)$	$\frac{\partial\bar{x}_l}{\partial\gamma_l}\frac{\partial\gamma_l}{\partial\zeta}$	$\frac{\partial\bar{x}_l}{\partial\gamma_m}\frac{\partial\gamma_m}{\partial\zeta}$
α_l	$-$	$-\beta_l$	$+$	β_m
α_m	$-\beta_1\beta_2$	$-\beta_l$	$\beta_1\beta_2$	β_l
β_l	$-\beta_m$	$-$	β_m	$+$
β_m	$-\beta_l$	$-$	β_l	$\beta_1\beta_2$

Table 2.3 – Impact of model parameters on the steady-state values. This table contains the sign of each of the four terms of Equation (2.14) when fishing is undertaken by two independent fleets.

2.6 Comparison

In all comparisons to follow, we shall focus on the difference in the steady-state values of the two species (a biological measure), and on the difference in the individual discounted utilities (an economic measure).

2.6.1 Specialized vs. non-specialized fishing

Suppose that the sets N_l are singletons, that is, only one agent harvests each species. Obviously when $n_l = 1$, the equilibrium fishing strategies γ_l^{C} and γ_l^{N}coincide. This special case corresponds to the scenario analyzed in Fischer and Mirman (1992) and the results according to this scenario are superscipted with F. Comparing scenario F to the single-owner case (scenario S) assesses the impact of having a single technology that enables to harvest both species and the value of coordinating the harvest of two species.

The following proposition compares the steady-state values.

Proposition 11. *When the biological interaction is symbiotic, the steady-state levels with a single owner harvesting both species are higher than with two specialized agents. When one of the species preys on the other one, the steady-state level of the prey is higher with a single owner harvesting both species than with two specialized agents.*

Proof. When $n_1 = n_2 = 1$, $\gamma_l^{\mathrm{F}} = \gamma_l^{\mathrm{N}} = \gamma_l^{\mathrm{C}} = \frac{1}{A_m}$, $l = 1,2$. We have

$$\begin{aligned}\gamma_l^{\mathrm{S}} - \gamma_l^{\mathrm{F}} &= \frac{1}{A_m + B_m} - \frac{1}{A_m} \\ &= -\frac{B_m}{A_m\left(A_m + B_m\right)}, \quad l = 1,2.\end{aligned}$$

The result for the symbiotic and the prey cases follows from (2.12)-(2.13). In the two other cases, the result is ambiguous. □

Proposition 11 shows that the institutional arrangement (or alternatively the available technology), e.g., allowing one agent to exploit both species or having two players, each one harvesting one species, leads to different results depending on the relationship between the two species. In the competitive scenario and for the stock level of the predator, the differences in steady-state levels can be higher or lower when the two species are harvested by a single agent.

Using Proposition 8, the discounted utility for each player under scenario F is

$$V_l^{\mathrm{F}}(x_1,x_2) = A_m \ln(x_l) + B_l \ln(x_m) + C_l^{\mathrm{F}},\ l = 1,2,$$

where

$$(1-\delta)C_l^{\mathrm{F}} = (A_m - 1)\ln(A_m - 1) - A_m \ln(A_m) + B_l \ln\left(\frac{A_l - 1}{A_l}\right).$$

Proposition 12. *The difference in total discounted utilities between a single owner and two specialized fleets is independent of the stock levels of both species and is given by*

$$\begin{aligned} D^{SF} & \quad V^S(x_1,x_2) - V_1^F(x_1,x_2) - V_2^F(x_1,x_2), \\ &= \frac{C^S - C_1^F - C_2^F}{(1-\delta)}. \end{aligned}$$

This difference is positive for all biological interactions and all parameter values.

Proof. See Appendix 2.8.6. ☐

As alluded to before, the single owner scenario involves an optimization (centralized) problem, whereas all other specialized fishing scenarios, including scenario F, are equilibrium (decentralized) problems, leading to lower total payoffs.

Figure 2.2 illustrates the impact of the model parameters on D^{SF} by varying them around a base-case value corresponding to Case C1. These results are robust to the model parameter values.

Varying α_l has the same impact for all types of biological interactions. That is, D^{SF} is U-shaped and nearly flat for intermediate values: the difference in utilities between the single owner and the two specialized fleets is almost constant, except for extreme values of the regeneration parameter.

The impact of varying the interaction parameter β_l on D^{SF} depends on the sign of β_l; D^{SF} is increasing convex in $|\beta_l|$. Recall that the larger is $|\beta_l|$, the lower is the (positive or negative) impact of the other species on the stock of species l, and the higher are the benefits of centralization.

2.6.2 Cooperation versus competition

In this section, we assess the difference between cooperation and competition within a group given an arbitrary fishing strategy used by the agents in the other group. We start by comparing the difference in steady-state values of the two species. The following proposition shows that cooperation leads, not surprisingly, to a lower exploitation of the resource, but not necessarily to a higher steady-state level of both species.

Proposition 13. *For all biological interactions, cooperation within group* $l \in \{1,2\}$ *leads to a lower fishing intensity of species* l *than competition. Cooperation within group* l *leads to a higher steady-state level of species* l*. It leads to a higher steady-state level of species* m *when the biological interaction is symbiotic or when species* m *is a predator. When the interaction is competitive or when species* m *is a prey, cooperation within group* l *leads to a lower steady-state level of species* m*. The gap between the steady-state levels under cooperation and competition is increasing with the number of players in group* l*.*

Proof. The difference in harvesting strategies is

$$\gamma_l^{\mathrm{N}} - \gamma_l^{\mathrm{C}} = (n_l - 1)\frac{A_m - 1}{A_m\,(A_m + n_l - 1)} \geq 0,$$

where the two solutions coincide when $n_l = 1$. Recall that when agents in group l play non-cooperatively, the impact of the number of agents in group l on the steady-state stock levels is given by

$$\begin{aligned} \frac{\partial \overline{x}_l}{\partial n_l} &= -\frac{\rho_l}{\Theta}\frac{\overline{x}_l}{A_m + n_l - 1} < 0, \\ \frac{\partial \overline{x}_m}{\partial n_l} &= -\frac{\beta_m}{\Theta}\frac{\overline{x}_m}{A_m + n_l - 1}. \end{aligned}$$

Since the steady-state stock levels when agents in group l play cooperatively do not depend on n_l and coincide with the non-cooperative levels when $n_l = 1$, the result follows. □

When the game is played noncooperatively in group l, a player does not internalize the harvesting decisions of the other players when making her own, which leads to overfishing with respect to the cooperative solution. Increasing the number of players widens the gap between the cooperative and non-cooperative steady-state levels. In the case of a predator or a competing species, overfishing benefits the other species.

We now consider the economical dividend of cooperation for group $l \in \{1,2\}$, as measured by

$$\begin{aligned} D_l^{\mathrm{CN}} &= n_l\left(V_l^{\mathrm{C}}(x_1,x_2;\gamma_m) - V_l^{\mathrm{N}}(x_1,x_2;\gamma_m)\right) \\ &= n_l\left(C_l^{\mathrm{C}}(\gamma_m) - C_l^{\mathrm{N}}(\gamma_m)\right), \end{aligned}$$

where

$$\begin{aligned} (1-\delta)\left(C_l^{\mathrm{C}}(\gamma_m) - C_l^{\mathrm{N}}(\gamma_m)\right) &= -A_m \ln(A_m) - \ln(n_l) + A_m \ln(n_l + A_m - 1) \\ &= A_m \ln\left(\frac{A_m + n_l - 1}{A_m}\right) - \ln(n_l). \end{aligned}$$

Proposition 14. *Independently of the strategy used by the members of the other group, it is always beneficial for agents in group $l \in \{1,2\}$ to cooperate, and the economical benefits of cooperation are increasing with n_l. Cooperation in group l is detrimental to the agents in the other group when the biological relationship is competitive or when species l is a predator to the other species. When the biological relationship is symbiotic or when species l is a prey to the other species, cooperation in group l is beneficial to the agents of the other group. The gap between the payoffs in the other group is increasing with n_l.*

Proof. The benefit of cooperation in group l is

$$\begin{aligned} D_l^{\mathrm{CN}} &= \frac{n_l}{1-\delta}\left(A_m \ln\left(\frac{A_m + n_l - 1}{A_m}\right) - \ln(n_l)\right) \\ &= \frac{n_l}{1-\delta} f(n_l), \end{aligned}$$

and is independent of the stock of both species and of the fishing strategy used by the agents in the other group. Note that

$$f'(n_l) = \frac{1}{n_l}(A_m - 1)\frac{n_l - 1}{A_m + n_l - 1} \geq 0$$

so that

$$D_l^{\mathrm{CN}} \text{ is } \begin{cases} > 0 \text{ for } n_l > 1 \\ = 0 \text{ for } n_l = 1 \end{cases}$$

and is increasing in n_l. Using Proposition 8, the impact of cooperation in group l on the payoff of an agent i in group m is given by

$$\begin{aligned} & C_{im}(\gamma_l^{\mathrm{C}}, \gamma_m) - C_{im}(\gamma_l^{\mathrm{N}}, \gamma_m) \\ = \; & \frac{B_m}{(1-\delta)} \left(\ln\left(1-\gamma_l^{\mathrm{C}}\right) - \ln\left(1-\gamma_l^{\mathrm{N}}\right) \right) \\ = \; & \frac{B_m}{(1-\delta)} \ln\left(1 + \frac{n_l - 1}{A_m}\right). \end{aligned}$$

The gap between the payoffs in group m is positive if $\beta_m > 0$, negative otherwise. This gap is increasing with n_l:

$$\frac{d}{dn_l}\left(\ln\left(1+\frac{n_l-1}{A_m}\right)\right) = \frac{1}{A_m + n_l - 1}.$$

□

2.6.3 Global impact of cooperation

The results in the preceding section establish that it is always beneficial for agents in a given group to coordinate their fishing strategy, and that cooperation in group $l \in \{1,2\}$ has a positive impact on the steady-state fish stock level of species l. This means that, for agents in group l, Scenario C where the agents in each group coordinate their strategy is better than Scenario Nl where agents in group m coordinate their strategies while agents in group l act non-cooperatively, and results in a higher steady-state level for the species l.

In the same way, for agents in group l, Scenario Nm where agents in group l coordinate their strategies while those in group m act non cooperatively is better than Scenario N, and results in a higher steady-state level for the species l.

However, depending on the biological interaction between the species, cooperation in group l may have a detrimental impact on the steady-state stock of species m and on the payoff of the agents fishing that species. This happens when the biological interaction is competitive, or when species l is a predator of species m.

Our last proposition compares the global impact of cooperation by comparing the outcome of Scenario C, where agents cooperate in both groups, with that of Scenario N, where agents do not coordinate their harvesting strategies in either group.

Proposition 15. *The steady-state stock of species* l *is larger under cooperation if* $\beta_l > 0$*. Otherwise, the steady-state stock quantity of species* l *can be higher or lower under cooperation. The economical dividend of cooperation is positive when the relationship between the species is symbiotic. In other cases, it can be positive or negative.*

Proof. See Appendix 2.8.7. □

Examples where cooperation is detrimental to the steady-state stock of one or both species and where cooperation is not economically profitable are provided in Tables 2.4 and 2.5.

Table 2.4 provides the signs of the differences in steady-state values and individual and global utilities between the fully cooperative scenario C and the fully non-cooperative scenario N, when the biological relationship is competitive. Parameter values are symmetrical and correspond to Case C1 of Table 2.2. The results in Table 2.4 show that all possible outcomes can be obtained in the competitive case, and, in particular, that cooperation may result in a decrease in the payoffs of all agents or in a decrease in the steady-state stock of one of the species.

Table 2.5 provides the signs of the differences in steady-states and individual and global utilities between the fully cooperative scenario C and the fully non-cooperative scenario N, when the biological relationship is prey/predator. As indicated in Proposition 15, the steady-state stock level of the predator is always higher under cooperation. Table 2.5 shows that the steady-state stock level of the prey can be higher or lower, and that the benefits of cooperation can be higher or lower for both types of agents. Parameter values are provided in Table 2.2.

Case C1	$n_1 = n_2 = 2$	$n_1 = n_2 = 3$	$n_1 = 2, n_2 = 10$
$\bar{x}_1^C - \bar{x}_1^N$	$+$	$+$	$-$
$\bar{x}_2^C - \bar{x}_2^N$	$+$	$+$	$+$
$C_1^C - C_1^N$	$-$	$+$	$-$
$C_2^C - C_2^N$	$-$	$+$	$+$
D^{CN}	$-$	$+$	$+$

Table 2.4 – Global impact of cooperation, competitive case. This table provides the signs of the differences in steady-states and individual and global utilities between the fully cooperative scenario C and the fully non-cooperative scenario N, when the biological relationship is competitive. Parameter values are symmetrical and correspond to Case C1 of Table 2.2.

	$n_1 = n_2 = 2$	$n_1 = n_2 = 2$	$n_1 = 2, n_2 = 3$	$n_1 = 10, n_2 = 2$	$n_1 = 2, n_2 = 10$
Case	C1	C8	C8	C9	C10
$\bar{x}_1^C - \bar{x}_1^N$	$+$	$+$	$-$	$+$	$-$
$\bar{x}_2^C - \bar{x}_2^N$	$+$	$+$	$+$	$+$	$+$
$C_1^C - C_1^N$	$-$	$-$	$-$	$+$	$-$
$C_2^C - C_2^N$	$-$	$+$	$+$	$+$	$+$
D^{CN}	$-$	$-$	$-$	$+$	$+$

Table 2.5 – Global impact of cooperation, prey/predator case. This table provides the signs of the differences in steady-states and individual and global utilities between the fully cooperative scenario C and the fully non-cooperative scenario N, when species 1 is the prey. Parameter values are provided in Table 2.2.

2.7 Concluding remarks

In this paper, we investigated the impact of biological interactions and agents' behavior on equilibrium strategies and outcomes in a two-species fishery. Our model extends the setup of Fischer & Mirman (1992, 1996) to a framework where there is a group of fisherpersons exploiting each species. Our results indicate that cooperation among two competing groups of agents is not necessarily beneficial, both from a biological and an environmental point of view. This is due to the biological interactions between the two species, and to the fact that coordination only happens among players in the same group. This is not the case when decisions are completely centralized (single owner scenario), which is shown to provide the highest overall utility. As in the FM model, two assumptions play a crucial role in the determination of equilibrium strategies and outcomes. First, the fact that both species procure the same utility, and, second, the fact that each agent harvests only one species. Relaxing either one of these assumptions would require the use of a numerical approach to obtain the resulting equilibria. This is clearly a worthy extension to this work as these two assumptions are quite restrictive.

2.8 Appendix

2.8.1 Existence of the value function

The symbiotic case

Define the set of feasible decisions by

$$C_l(x) = \{c_l : 0 < c_l < x_l\}, l = 1,2. \tag{2.17}$$

Note that in that case, the immediate reward $\ln(c_1) + \ln(c_2)$ is not bounded. When $\beta_1 > 0$ and $\beta_2 > 0$, Equation (2.2) implies that, if $x \in (0,1] \times (0,1]$, then the transition state $x' \in (0,1] \times (0,1]$ for all possible decisions, so that the state space can be restricted to $(0,1] \times (0,1]$. As a consequence, the immediate reward is non-positive for all feasible harvesting strategies and all possible states, so that the optimization problem (2.15)-(2.16) satisfies the conditions of Negative Dynamic Programming. In that case, it has been established (Strauch 1966) that the value function exists and that the optimal strategy is obtained by solving the Bellman equation. Moreover, since the discount factor $\delta < 1$, the value function is the unique solution of the Bellman equation.

The competitive and prey-predator cases

As shown in Gorniewicz & Wiszniewska-Matyszkiel (2018), when at least one of the biologic parameters is negative, the FM model needs to be slightly modified in oder to obtain the optimal strategies and the value function from the solution of the Bellman equation. Note that, when $\beta_l < 0$, Equation (2.2) implies that the stock of species m tends to infinity when the stock of species $l \in \{1,2\}$ vanishes. As the authors rightly point out, the behavior of the FM model is then unrealistic.

Define the set of feasible decisions by

$$C_l(x) = \{c_l : 0 < c_l < x_l (1-\varepsilon)\}, \tag{2.18}$$

where $0 < \varepsilon < 1$. It is then straightforward, using Equation (2.2), to obtain bounds on the value of x_1 and x_2 such that

$$0 < \eta_l^{\min} < x_l < \eta_l^{\max},\ l = 1,2.$$

By restricting the set of available decisions in that way, the immediate reward $\ln(c_1) + \ln(c_2)$ is bounded above by a constant $M = \ln\left(\eta_1^{\max}\right) + \ln\left(\eta_2^{\max}\right)$:

$$\begin{aligned} \ln(c_l) &< \ln(x_l(1-\varepsilon)) \\ &< \ln\left(\eta_l^{\max}(1-\varepsilon)\right) < \ln\left(\eta_l^{\max}\right),\ l = 1,2, \end{aligned}$$

and the value function is bounded above by $\frac{M}{1-\delta}$. It is easy to show that the optimization problem (2.15)-(2.16) is then equivalent to

$$\begin{aligned} W(x) &= \max_{\substack{c_1 \in C_1(x) \\ c_2 \in C_2(x)}} \left\{\ln(c_1) + \ln(c_2) - M + \delta W\left(x_1', x_2'\right)\right\} \\ x_l' &= (x_l - c_l)^{\alpha_l} (x_m - c_m)^{\beta_l},\ l = 1,2, \end{aligned}$$

where $V(x) = W(x) + \frac{M}{1-\delta}$ and where $W(x)$ satisfies the conditions of Negative Dynamic Programming.

Note that the positive constant ε can be selected so that the constraint on the catch is not binding, so that the solutions obtained by Fisher & Mirman (1992,1996) are also solutions of the restricted problem. As pointed out by Gorniewicz & Wiszniewska-Matyszkiel (2018), the existence of the value function in the unrestricted FM model is an open problem when at least one of the biological parameters is negative.

Assuming that there is an upper bound on the proportion of the stock that can be harvested is not necessarily unrealistic; while harvesting costs are not taken into account in the FM model, one can assume that these costs become prohibitive when the stock becomes very low, which could motivate such an assumption.

In the sequel, we will assume that the set of feasible decisions is given by Equation (2.17) in the symbiotic case and by Equation (2.18) in the two other cases, and that the constraint in Equation (2.18) is not binding.

Note that other modifications of the model can be implemented to ensure the existence of a solution, for instance by changing the dynamics so that the stock of a given species cannot be larger than the saturation level, or so that the size of the stock of a species ceases to have an impact on the evolution of the other when it falls under a given threshold.

We refer the interested reader to Gorniewicz & Wiszniewska-Matyszkiel (2018) where modifications of the dynamics, motivated by the Allee effect, are proposed, and where alternative proofs of the existence of a solution are provided for the restricted model presented above.

2.8.2 Proof of Proposition 7

Assume that, for $x_1 > 0$ and $x_2 > 0$,

$$V(x_1, x_2) = \sum_{l=1}^{2} (A_m + B_m) \ln(x_l) + C.$$

We then have

$$V(x_1, x_2) = \max_{\substack{c_1 \in C_1(x) \\ c_2 \in C_2(x)}} \left\{ G(c_1, c_2; x_1, x_2) \right\}, \tag{2.19}$$

where

$$G(c_1, c_2; x_1, x_2) = \sum_{l=1}^{2} \ln(c_l) + \delta (A_m + B_m) (\alpha_l \ln(x_l - c_l) + \beta_l \ln(x_m - c_m)) + \delta C.$$

Differenciating G with respect to the decision variables yields

$$\begin{aligned}
\frac{\partial G}{\partial c_l} &= \frac{c_l - x_l + \delta c_l (\alpha_l (A_m + B_m) + \beta_m (A_l + B_l))}{c_l (c_l - x_l)}, \\
\frac{\partial^2 G}{\partial c_l^2} &= -\frac{(c_l - x_l)^2 + \delta c_l^2 (\alpha_l (A_m + B_m) + \beta_m (A_l + B_l))}{c_l^2 (c_l - x_l)^2}, \\
\frac{\partial^2 G}{\partial c_l \partial c_m} &= 0.
\end{aligned}$$

For $l = 1,2$,

$$\begin{aligned} & \alpha_l \left(A_m + B_m\right) + \beta_m \left(A_l + B_l\right) \\ = \; & \alpha_l \left(\frac{1-\delta\alpha_m}{\Omega} + \frac{\delta\beta_m}{\Omega}\right) + \beta_m \left(\frac{1-\delta\alpha_l}{\Omega} + \frac{\delta\beta_l}{\Omega}\right) \\ = \; & \frac{\alpha_l \left(1-\delta\alpha_m\right) + \beta_m \left(1+\delta\beta_l\right)}{\Omega} \end{aligned}$$

and is strictly positive under Assumption 3. Therefore, for a given (x_1, x_2), $G(c_1, c_2; x_1, x_2)$ is a strictly concave function of (c_1, c_2). As a consequence, the first-order conditions are sufficient, and the optimal solution at (x_1, x_2) is given by

$$\begin{aligned} c_l^*(x_1, x_2) &= \frac{x_l}{1 + \delta\left(\alpha_l \left(A_m + B_m\right) + \beta_m \left(A_l + B_l\right)\right)} \\ &= \frac{x_l}{A_m + B_m}. \end{aligned}$$

Assume that $c_l^*(x)$ is interior. Replacing the optimal values for c_l, $l = 1,2$, in Equation (2.19) yields

$$\begin{aligned} V(x_1, x_2) &= \sum_{l=1}^{2} \ln\left(\frac{x_l}{A_m + B_m}\right) \\ &\quad + \delta\left(A_m + B_m\right)\left(\alpha_l \ln\left(x_l \frac{A_m + B_m - 1}{A_m + B_m}\right)\right) \\ &\quad + \delta\left(A_m + B_m\right)\left(\beta_l \ln\left(x_m \frac{A_l + B_l - 1}{A_l + B_l}\right)\right) \\ &\quad + \delta C \\ &= \sum_{l=1}^{2} \left(A_m + B_m\right) \ln\left(x_l\right) \\ &\quad + \left(A_l + B_l - 1\right) \ln\left(A_l + B_l - 1\right) \\ &\quad - \left(A_l + B_l\right) \ln\left(A_l + B_l\right) \\ &\quad + \delta C, \end{aligned}$$

which satisfies our assumption with

$$C(1-\delta) = \sum_{l=1}^{2} \left(A_l + B_l - 1\right) \ln\left(A_l + B_l - 1\right) - \left(A_l + B_l\right) \ln\left(A_l + B_l\right).$$

It remains to show that $c_l^*(x_1, x_2)$ is interior, that is

$$0 < \frac{x_l}{A_m + B_m} < x_l(1-\varepsilon), \, l = 1,2,$$

where $\varepsilon = 0$ in the symbiotic case and $\varepsilon > 0$ in the competitive and prey-predator cases. Note that

$$\begin{aligned} A_m + B_m &= \frac{1-\delta\alpha_m+\delta\beta_m}{\Omega} \\ &= 1+\delta\frac{\alpha_l(1-\delta\alpha_m)+\beta_m(1+\delta\beta_l)}{\Omega} \\ &= 1+\frac{\delta\xi_l}{\Omega} > 1. \end{aligned}$$

The solution is clearly interior in the symbiotic case. In the other cases, it suffice to set ε to any value satisfying

$$0 < \varepsilon < \min\left\{\frac{\delta\xi_1}{\Omega+\delta\xi_1}, \frac{\delta\xi_2}{\Omega+\delta\xi_2}\right\}.$$

We then have

$$\begin{aligned} \frac{1}{A_m+B_m} &= 1-\frac{\delta\xi_l}{\Omega+\delta\xi_l} \\ &< 1-\varepsilon. \end{aligned}$$

2.8.3 Proof of Proposition 8

The total discounted utility of Agent i fishing species l satisfies

$$V_{il}(x_1,x_2;\gamma) = \ln(p_{il}x_l) + \delta V_{il}(x_1-\gamma_1x_1, x_2-\gamma_2x_2;\gamma).$$

Assuming $V_{il}(x_1,x_2;\gamma) = A_m\ln(x_l) + B_l\ln(x_m) + C_{il}(\gamma)$ yields

$$\begin{aligned} V_{il}(x_1,x_2;\gamma) &= \ln(p_{il}x_l) + \delta A_m \ln\left((x_l-\gamma_lx_l)^{\alpha_l}(x_m-\gamma_mx_m)^{\beta_l}\right) \\ &\quad +\delta B_l \ln\left((x_m-\gamma_mx_m)^{\alpha_m}(x_l-\gamma_lx_l)^{\beta_m}\right) + \delta C_{il}(\gamma) \\ &= (1+\delta(\alpha_lA_m+\beta_mB_l))\ln(x_l) + \delta(\beta_lA_m+\alpha_mB_l)\ln(x_m) \\ &\quad +\delta(\alpha_lA_m+\beta_mB_l)\ln(1-\gamma_l) + \delta(\beta_lA_m+\alpha_mB_l)\ln(1-\gamma_m) \\ &\quad +\delta C_{il}(\gamma) + \ln(p_{il}), \end{aligned}$$

where

$$\begin{aligned} 1+\delta(\alpha_lA_m+\beta_mB_l) &= \frac{1-\delta\alpha_m}{\Omega} = A_m, \\ \delta(\beta_lA_m+\alpha_mB_l) &= \frac{\delta\beta_l}{\Omega} = B_l. \end{aligned}$$

It comes

$$\begin{aligned} V_{il}(x_1,x_2;\gamma) &= A_m \ln(x_l) + B_l \ln(x_m) \\ &\quad + (A_m - 1)\ln(1-\gamma_l) + B_l \ln(1-\gamma_m) + \delta C_{il}(\gamma) + \ln(p_{il}), \end{aligned}$$

which satisfies our assumption with

$$(1-\delta)C_{il}(\gamma) = \ln(p_{il}) + (A_m - 1)\ln(1-\gamma_l) + B_l \ln(1-\gamma_m).$$

2.8.4 Proof of Proposition 9

Consider Player i in group $l \in \{1,2\}$ and define P_{il} to be the total catch rate of all the other players in group l, so that $\gamma_l = p_{il} + P_{il}$.

Given γ_m and P_{il}, Player $i \in N_l$ maximizes

$$\begin{aligned} G_{il}^{\mathrm{N}}(p_{il}) &\equiv C_{il}(\gamma) \\ &= \ln(p_{il}) + (A_m - 1)\ln(1 - p_{il} - P_{il}) + B_l \ln(1-\gamma_m). \end{aligned}$$

Differentiating w.r.t. p_{il} yields

$$\begin{aligned} \frac{dG_{il}^{\mathrm{N}}}{dp_{il}} &= \frac{1}{p_{il}} \frac{P_{il} + A_m p_{il} - 1}{P_{il} + p_i - 1}, \\ \frac{d^2 G_{il}^{\mathrm{N}}}{dp_{il}^2} &= -\frac{(P_{il} + p_{il} - 1)^2 + p_{il}^2 (A_m + 1)}{p_{il}^2 (P_{il} + p_{il} - 1)^2}. \end{aligned}$$

This is a concave optimization problem and the best response of Player $i \in N_l$ to the fishing strategy of the other agents is given by

$$p_{il} = \frac{1 - P_{il}}{A_m}.$$

Simultaneously solving for $i = 1, \ldots, n_l$ yields the equilibrium fishing strategy of the players in group l, which is independent of the fishing strategy of the agents in group m:

$$\begin{aligned} p_{il}^{\mathrm{N}} &= \frac{1}{n_l + A_m - 1}, i = 1, \ldots, n_l, \\ \gamma_l^{\mathrm{N}} &= \frac{n_l}{n_l + A_m - 1}. \end{aligned}$$

Note that

$$A_l - 1 = \delta \frac{\alpha_l (1 - \delta \alpha_m) + \delta \beta_1 \beta_2}{\Omega}, \; l = 1, 2.$$

The numerator is positive if $\beta_1 \beta_2 > 0$; If $\beta_1 \beta_2 < 0$,

$$\alpha_l (1 - \delta \alpha_m) + \delta \beta_1 \beta_2 > \alpha_l (1 - \delta \alpha_m) + \beta_1 \beta_2 > 0$$

under Assumption 2. As a consequence, $A_l > 1$ for $l = 1, 2$ and the equilibrium fishing strategy $\gamma_l^{\mathrm{N}} \in (0, 1)$.

Replacing the equilibrium strategy γ_l^{N} in $C_{il}(\gamma)$ yields

$$\begin{aligned} (1 - \delta) C_l^{\mathrm{N}} (\gamma_m) &= (A_m - 1) \ln (A_m - 1) - A_m \ln (A_m + n_l - 1) \\ &\quad + B_l \ln (1 - \gamma_m). \end{aligned}$$

2.8.5 Proof of Proposition 10

Given γ_m, the agents in group l jointly maximize

$$\begin{aligned} G_l^{\mathrm{C}} (\gamma) &\equiv \sum_{i \in N_l} C_{il} (\gamma) \\ &= \sum_{i \in N_l} \ln (p_{il}) + n_l (A_m - 1) \ln \left(1 - \sum_{j \in N_l} p_{jl} \right) + n_l B_l \ln (1 - \gamma_m). \end{aligned}$$

Differentiating w.r.t. p_{il} yields

$$\begin{aligned} \frac{\partial G_l^{\mathrm{C}}}{\partial p_{il}} &= \frac{\gamma_l - 1 + n_l p_{il} (A_m - 1)}{p_{il} (\gamma_l - 1)}, \quad i = 1, \ldots, n_l, \\ \frac{\partial^2 G_1^{\mathrm{C}}}{\partial p_i^2} &= -\frac{1}{p_{il}^2} - \frac{n_l (A_m - 1)}{(1 - \gamma_l)^2}, \quad i = 1, \ldots, n_l, \\ \frac{\partial^2 G_1^{\mathrm{C}}}{\partial p_i \partial p_j} &= -n_l \frac{A_m - 1}{(1 - \gamma_l)^2}, \quad i, \, j = 1, \ldots, n_l, \; i \neq j. \end{aligned}$$

Since $A_m > 1$, this is a concave optimization problem and the best response of the group of agents fishing species l is independent of the fishing strategy of the agents fishing the other species and is given by

$$\begin{aligned} p_{il}^{\mathrm{C}} &= \frac{1}{n_l A_l}, i = 1, \ldots, n_l, \\ \gamma_l^{\mathrm{C}} &= \frac{1}{A_l} \in (0, 1). \end{aligned}$$

Replacing the equilibrium strategy γ_l^{C} in $C_{il}(\gamma)$ yields

$$\begin{aligned}(1-\delta)\,C_l^{\mathrm{C}}(\gamma_m) &= (A_m-1)\ln(A_m-1)-A_m\ln(A_m)\\ &\quad +B_l\ln(1-\gamma_m)-\ln(n_l)\,.\end{aligned}$$

2.8.6 Proof of Proposition 12

We have

$$\begin{aligned}(1-\delta)\,D^{\mathrm{SF}} &= \sum_{l=1}^{2}(A_l+B_l-1)\ln(A_l+B_l-1)-(A_l+B_l)\ln(A_l+B_l)\\ &\quad -(A_l-1)\ln(A_l-1)+A_l\ln(A_l)+B_l\ln(A_l-1)-B_l\ln(A_l)\\ &= \sum_{l=1}^{2}(A_l-1+B_l)\ln\left(1+\frac{B_l}{A_l-1}\right)-(A_l+B_l)\ln\left(1+\frac{B_l}{A_l}\right).\end{aligned}$$

Consider the function

$$\begin{aligned}f_l(A_l) &= (A_l+B_l)\ln\left(1+\frac{B_l}{A_l}\right)\\ f_l'(A_l) &= \ln\left(1+\frac{B_l}{A_l}\right)-\frac{B_l}{A_l}<0.\end{aligned}$$

Since f_l is strictly decreasing in A_l,

$$f_l(A_l-1)>f_l(A_l)$$

and

$$(1-\delta)\,D^{SF}=\sum_{l=1}^{2}f_l(A_l-1)-f_l(A_l)>0.$$

2.8.7 Proof of Proposition 15

The steady-state of species l under scenarios C (all agents coordinate their strategies) and N (all agents act individualistically) compare as follows:

$$\begin{aligned}\frac{\overline{x}_l^{\mathrm{C}}}{\overline{x}_l^{\mathrm{N}}} &= \left(\frac{1-\frac{1}{A_m}}{1-\frac{n_l}{n_l+A_m-1}}\right)^{\frac{\rho_l}{\Theta}}\left(\frac{1-\frac{1}{A_l}}{1-\frac{n_m}{n_m+A_l-1}}\right)^{\frac{\beta_l}{\Theta}}\\ &= \left(1+\frac{n_l-1}{A_m}\right)^{\frac{\rho_l}{\Theta}}\left(1+\frac{n_m-1}{A_l}\right)^{\frac{\beta_l}{\Theta}},\, l=1,2.\end{aligned}$$

When $\beta_l > 0$, this expression is larger than 1.

The global dividend of cooperation is given by

$$\begin{aligned} D^{\mathrm{CN}} &= n_l \left(V_l^{\mathrm{C}}(x_1, x_2; \gamma_m) - V_l^{\mathrm{N}}(x_1, x_2; \gamma_m) \right) \\ &= \sum_{l=1}^{2} n_l \left(C_l^{\mathrm{C}}(\gamma_m^{\mathrm{C}}) - C_l^{\mathrm{N}}(\gamma_m^{\mathrm{N}}) \right), \end{aligned}$$

where

$$\begin{aligned} (1-\delta)\, C_l^{\mathrm{N}}(\gamma_m^{\mathrm{N}}) &= (A_m - 1)\ln(A_m - 1) - A_m \ln(A_m + n_l - 1) \\ &\quad + B_l \ln\left(1 - \frac{n_m}{n_m + A_l - 1}\right), \\ (1-\delta)\, C_l^{\mathrm{C}}(\gamma_m^{\mathrm{C}}) &= (A_m - 1)\ln(A_m - 1) - A_m \ln(A_m) \\ &\quad + B_l \ln\left(1 - \frac{1}{A_l}\right) - \ln(n_l). \end{aligned}$$

It comes

$$\begin{aligned} C_l^{\mathrm{C}}(\gamma_m^{\mathrm{C}}) - C_l^{\mathrm{N}}(\gamma_m^{\mathrm{N}}) &= \frac{1}{(1-\delta)} \sum_{l=1}^{2} n_l \left(A_m \ln\left(1 + \frac{n_l - 1}{A_m}\right) - \ln(n_l) \right) \\ &\quad + n_l B_l \ln\left(1 + \frac{n_m - 1}{A_l}\right) \\ &> \frac{1}{(1-\delta)} \sum_{l=1}^{2} n_l B_l \ln\left(1 + \frac{n_m - 1}{A_l}\right). \end{aligned}$$

D^{CN} is positive when B_1 and B_2 are both positive, which is the case under symbiotic interactions.

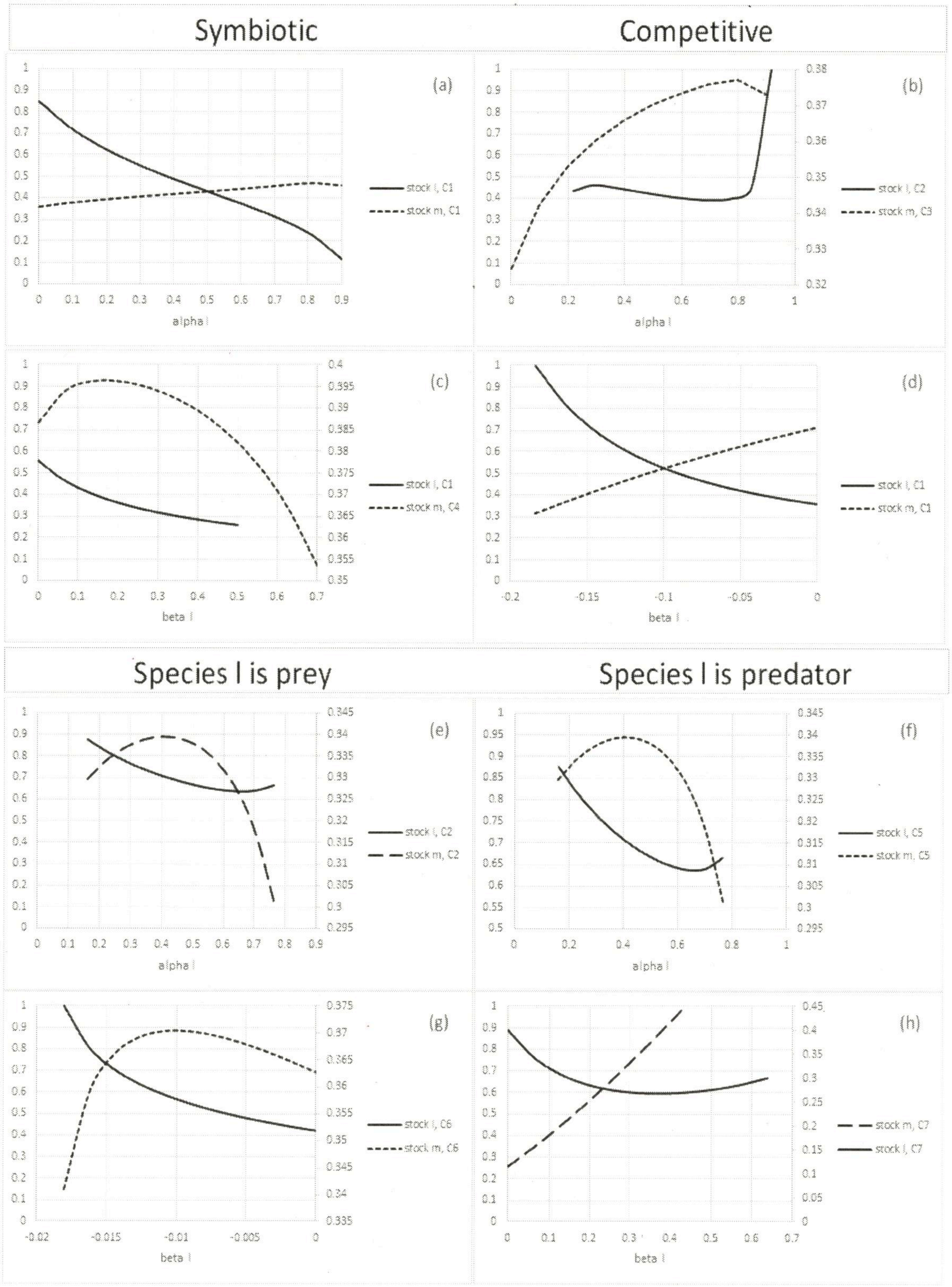

Figure 2.1 – Sensitivity of the steady-state stock levels to model parameter values for various constellations, described in Table 2.2. In all experiments, the discount factor $\delta = 0.95$.

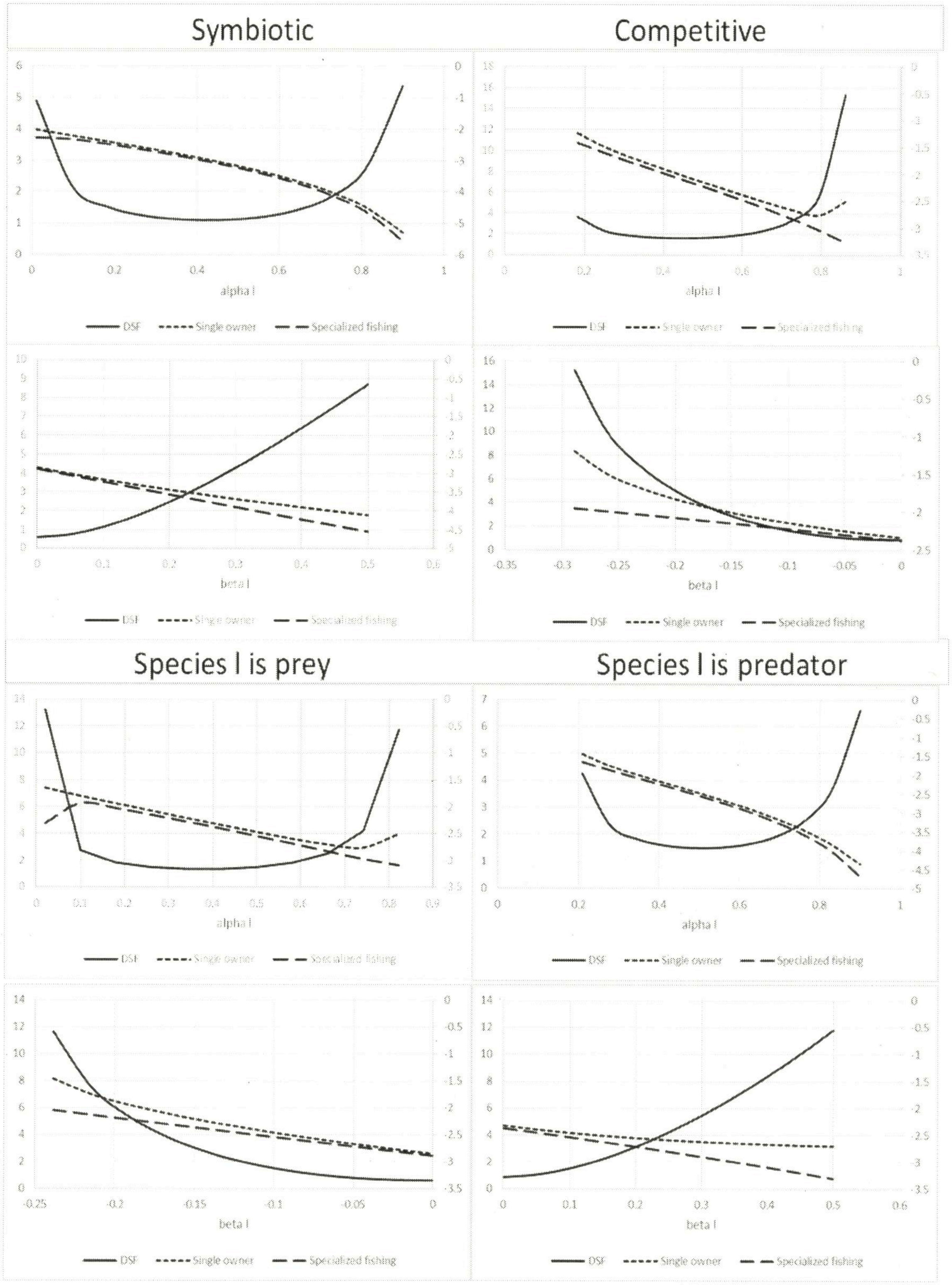

Figure 2.2 – Impact of model parameters on the benefits of centralization D^{SF} (left axis) and total discounted utilities (right axis). The base case is C1 in Table 2.2. The discount factor $\delta = 0.95$.

Chapter 3

A Note on Yardstick Regulation of Fishery Monopolies

Abstract

We consider a two-period model of a fishery exploited by two firms selling their harvests in separate local markets. The harvesting cost is private information. Assuming there is an agency that can regulate the market, we propose a contract in the realm of yardstick competition framework that allows the regulator to solve the information asymmetry problem and achieve cooperation.

3.1 Introduction

We consider a fishery exploited by two firms, selling their harvests in two separate local markets. The harvesting cost, which depends on series of items such as labour and weather conditions, is a private information to each firm. Assuming there is an authority that has the power of regulating fishing activities, the objective of this note is to design a mechanism that would induce the firms to truthfully report their costs and to adopt harvesting levels that are at the same time beneficial to consumer by leading to a competitive price, and to the resource by having a higher stock. Cooperation, or central management, can be a necessary condition to sustain a healthy stock level of the resource (see, e.g., Mazalov and Rettieva (2010) and Breton et al. (2019) for a discussion of the benefits of cooperation and its implementation under different settings). However, when cooperation amounts to collusion, consumer may suffer from higher prices. To proceed, we adopt a simple two-period game model à la Fisher and Mirman (1992, 1996), where the available stock grows with birth and decreases with harvesting.

Fisheries regulation has been an active research area for few decades (see the recent survey in Jensen et al. (2017)). There are several reasons behind the necessity of regulation, among them stock externalities, overexploitation of the resource, illegal landings, etc. Fisheries involve information uncertainties, both biological, e.g., size of the stock, and economical, e.g., prices, as well as information asymmetry about the cost, benefit and growth functions that are known to the firm but not to the regulator, which renders the regulation exercise more complicated. To highlight the importance of information asymmetry in natural resources management, we quote Hanna et al. (1996): "*Information critical for efficient management may be hard to centralize, or be asymmetric (people have different information), leading to inefficient management. Broadly viewed, natural resource problems are problems arising from incomplete and asymmetric information combined with incomplete, inconsistent, or unenforced property rights.*"

Different approaches have been proposed to regulate the industry (see, e.g., Jensen (2008) for an overview). Generally speaking, when it comes to regulating monopolies, a series of mechanisms can be used, with each adopting a specific metric to benchmark the outcome or the harvesting effort exerted by the firm, e.g., the rate of return on investment (ROR), quotas, price cap, marginal cost, etc. The common philosophy to these mechanisms is to induce the monopoly to produce/price at a (more) competitive level. Implementing any of these mechanisms is, at least in principle, easy when the benchmark is common knowledge and observable by the regulator. When the information is asymmetric, the regulator must incentivize the firms to reveal their private information, e.g., production cost. This can be done in the framework of mechanism design. For a discussion in the context of fisheries, see, e.g., Sappington (1991) and Armstrong and Sappington (2006), and for examples of principal-agent analysis in fisheries, see, e.g., Clark and Munro (1987, 1991), Jensen and Vestergaard (2002), Bailey and Sumaila (2008) and Munro et al. (2009)).

In this note, we shall use the so-called yardstick competition (YC) introduced by Shleifer (1985) as a regulatory tool to encourage competitive behavior by natural monopolies. It has been shown in Armstrong and Sappington (2006) that the regulator can ensure with a YC mechanism the full information outcome as the unique symmetric Nash equilibrium among the monopolies. The authors proposed a compensation scheme that equalizes the firm's marginal cost and the average cost-reducing expenditure of the others.

The rest of this note is organized as follows: In Section 2, we introduce the model and in Section 3, we solve for cooperative and noncooperative solutions. In section 4, we design the mechanism for the yardstick competition implementation. Finally, we briefly conclude in Section 5.

3.2 The Model

Consider a fishing zone where the resource stock is accessible to two identical firms, with each being a monopolist in its local market. To keep the model as parsimonious as possible, we retain a two-period model. Denote by h_{it} the harvest level by firm $i = 1,2$ and by $x_t \geq 0$ the stock of the resource at time $t = 0,1$. The evolution of the fish stock is described by the following difference equation:

$$x_{t+1} = x_t\left(1+\alpha\right) - h_{1t} - h_{2t}, \;\; x_0 \text{ given},$$

where $\alpha \in [0,1]$ is the fish birth rate.

The cost of fishing is given by

$$C_{it}\left(h_{it}, \theta_t, x_t\right) = \theta_t h_{it}\left(1 - c x_t\right),$$

where $c \in (0,1)$. The productivity parameter θ_t is a private information to the firm reflecting the state of nature (weather, labour, fuel price, etc.) at the beginning of each period. To keep it simple, while still being able to highlight the strategic issues involved in having private information, we assume that θ_0 is common knowledge and θ_1 can take two values, namely, high (θ_h) or low (θ_l), with $\theta_h > \theta_l$. The probability of θ_1 being equal to θ_h is p and equal to θ_l is $1-p$. The regulator cannot observe the firm's production cost in period 1, and its objective is to implement a competitive market through a mechanism design.

We make the following remarks on the cost function: First, to prevent the cost from becoming negative, we could specify the cost function as $\theta_t h_{it}(A - cx_t)$, with A being a large positive constant. To save on parameters, we set $A = 1$ and verify that in equilibrium the cost is strictly positive. Second, again for simplicity, we let θ_h and θ_l be the same for both players. Given our interpretation that θ_1 reflects exogenous state-of-nature conditions, this simplification is not unrealistic. Finally, we note that the cost is decreasing in the stock of fish, which is an intuitive common assumption in the literature.

Let the inverse-demand function in each market be given by

$$P_{it} = a - h_{it},$$

where P_{it} is the price obtained by player i and $a > 1$ is a time-invariant parameter. The linear form of the demand function can be justified on the ground that it is derivable from the maximization of a consumer's quadratic utility function. The revenues of firm i from harvesting the resource in period t are given by

$$R_{it} = h_{it}(a - h_{it}).$$

Assuming profit-maximization behavior, firm i optimizes the discounted sum of its profits over the two periods, that is,

$$\max_{h_{i0},h_{i1}} E_{\theta_1} \sum_{t=0}^{1} \delta^t (R_{it} - C_{it}) + S(x_1),$$

where $\delta \in (0,1)$ is the discount factor and $S(x_1)$ is the salvage value assumed to be linear and given by $S(x_1) = s.x_1$.

3.3 Solutions

In this section, we determine and contrast the noncooperative and collusive solutions. In the noncooperative game, we seek a feedback-Nash equilibrium, that is, harvesting strategies that are functions of the stock of the resource. When the two firms collude, they jointly maximize their payoff. Denote by $\tilde{\theta}_1$ the realization of θ_1.

Proposition 16. *Assuming an interior solution, the unique feedback-Nash equilibrium harvesting strategies are given by*

$$\{h_{10}^N(x_0,\theta_0,\tilde{\theta}_1), h_{11}^N(x_1,\theta_0,\tilde{\theta}_1)\}, \{h_{20}^N(x_0,\theta_0,\tilde{\theta}_1), h_{21}^N(x_1,\theta_0,\tilde{\theta}_1)\},$$

where,

$$h_{i0}^N(x_0,\theta_0,\tilde{\theta}_1) = \frac{1}{4\left(2-\delta c^2\left(p\theta_{1h}^2+(1-p)\theta_{1l}^2\right)\right)}\left(2cx_0\left(\theta_0 - c\delta(1+\alpha)\left(p\theta_{1h}^2+(1+p)\theta_{1l}^2\right)\right)\right.$$
$$\left.-2\left(\theta_0 - a + s\delta^2(1+\alpha)\right) + (2s\delta - a)c\delta\left((1-p)\theta_{1l}+p\theta_{1h}\right) + c\delta\left((1-p)\theta_{1l}^2+p\theta_{1h}^2\right)\right),$$

$$h_{i1}^N(x_1,\theta_0,\tilde{\theta}_1) = \frac{a-\tilde{\theta}_1\left(1-cx_1^N h_{i0}^N\right)-\delta s}{2}.$$

The equilibrium state dynamics are given by

$$x_1^N = x_0(1+\alpha) - \sum_{i=1}^{2} h_{i0}^N.$$

Proof. See Appendix. ☐

The results in the above proposition call for the following remarks. First, the strategy is linear in the fish stock, a result that is expected in view of the linear-quadratic structure of the game. Second, the harvesting strategy in period 1 is increasing in the stock. Indeed, we have

$$\frac{\partial h_{i1}^N\left(x_1,\theta_0,\tilde{\theta}_1\right)}{\partial x_1}=\frac{\tilde{\theta}_1 c h_{i0}^N}{2}>0.$$

This is an intuitive result, which however cannot be replicated in period 0. Differentiating the harvesting policy at time 0 with respect to x_0, we get

$$\begin{aligned}\frac{\partial h_{i0}^N\left(x_0,\theta_0,\tilde{\theta}_1\right)}{\partial x_0} &= \frac{c\left(\theta_0-c\delta\left(1+\alpha\right)\left(p\theta_{1h}^2+\left(1+p\right)\theta_{1l}^2\right)\right)}{2\left(2-\delta c^2\left(p\theta_{1h}^2+\left(1-p\right)\theta_{1l}^2\right)\right)} \\ & \text{is} \begin{cases} \geq 0, & \text{if } \theta_0 \geq c\delta\left(1+\alpha\right)\left(p\theta_{1h}^2+\left(1+p\right)\theta_{1l}^2\right), \\ \leq 0, & \text{otherwise.}\end{cases}\end{aligned}$$

That is, harvesting increases in the stock of resource if the productivity in period 0 exceeds the product of the expected discounted marginal cost $\left(c\delta\left(p\theta_{1h}^2+\left(1+p\right)\theta_{1l}^2\right)\right)$ times the increase in the stock $(1+\alpha)$.

The next proposition characterizes the jointly optimal solution.

Proposition 17. *Assuming an interior solution, the unique pair of strategies under collusive behavior is given by*

$$\{h_{10}^C(x_0,\theta_0,\tilde{\theta}_1),h_{11}^C\left(x_0,\theta_0,\tilde{\theta}_1\right)\},\{h_{10}^C(x_1,\theta_0,\tilde{\theta}_1),h_{11}^C\left(x_1,\theta_0,\tilde{\theta}_1\right)\},$$

where,

$$\begin{aligned}h_{i0}^C\left(x_0,\theta_0,\tilde{\theta}_1\right) &= \frac{1}{2\left(2-2\delta c^2\left(p\theta_{1h}^2+\left(1-p\right)\theta_{1l}^2\right)\right)}\left(cx_0\left(\theta_0-c\delta\left(1+\alpha\right)\left(p\theta_{1h}^2+\left(1-p\right)\theta_{1l}^2\right)\right)\right. \\ &\left.-\left(\theta_0-a+2\delta^2 s\left(1+\alpha\right)\right)+c\delta\left(\left(2\delta s-a\right)\left(p\theta_{1h}+\left(1-p\right)\theta_{1l}\right)+\left(p\theta_{1h}^2+\left(1-p\right)\theta_{1l}^2\right)\right)\right),\end{aligned}$$

$$h_{i1}^C\left(x_1,\theta_0,\tilde{\theta}_1\right)=\frac{a-\tilde{\theta}_1\left(1-cx_1^C h_{i0}^C\right)-2\delta s}{2}.$$

The state dynamics are given by

$$x_1^C=x_0\left(1+\alpha\right)-\sum_{i=1}^{2}h_{i0}^C.$$

Proof. See Appendix. □

The strategies have the same form (but not values) as in the noncooperative game. Therefore, there is no need to repeat the same comments made before.

Now, we compare the harvests obtained in the two scenarios.

Proposition 18. *Collusive and noncooperative harvesting levels compare as follows:*

1. *In period* 0*, we have*

$$h_{i0}^{C}\left(x_0,\theta_0,\tilde{\theta}_1\right)-h_{i0}^{N}\left(x_0,\theta_0,\tilde{\theta}_1\right)<0,\quad i=1,2.$$

2. *In period* 1*, if*

$$\delta s>\tilde{\theta}_1 c\left(\left[\sum_{i=1}^{2}h_{i0}^{N}\right]h_{i0}^{N}-\left[\sum_{i=1}^{2}h_{i0}^{C}\right]h_{i0}^{C}\right),\tag{3.1}$$

then

$$h_{i1}^{C}\left(x_1,\theta_0,\tilde{\theta}_1\right)-h_{i1}^{N}\left(x_1,\theta_0,\tilde{\theta}_1\right)<0,\quad i=1,2.$$

Proof. The difference in harvesting levels in period 0 is given by

$$\begin{aligned}&h_{i0}^{C}\left(x_0,\theta_0,\tilde{\theta}_1\right)-h_{i0}^{N}\left(x_0,\theta_0,\tilde{\theta}_1\right)\\&=\frac{1}{2\left(2-\delta c^2\left(p\theta_{1h}^2+(1-p)\theta_{1l}^2\right)\right)}\left(2\delta^2 s\left(c\left(p\theta_{1h}+(1-p)\theta_{1l}\right)-(1+\alpha)\right)\right.\\&\left.-\delta c\left(a\left(p\theta_{1h}+(1-p)\theta_{1l}\right)-\left(p\theta_{1h}^2+(1-p)\theta_{1l}^2\right)+cx_0(1+\alpha)\left(p\theta_{1h}^2+(1-p)\theta_{1l}^2\right)\right)\right)<0.\end{aligned}$$

Negativity follows from the facts that (i) the denominator is positive, (ii) $c\left(p\theta_{1h}+(1-p)\theta_{1l}\right)<(1+\alpha)$, and (iii) $a\left(p\theta_{1h}+(1-p)\theta_{1l}\right)>\left(p\theta_{1h}^2+(1-p)\theta_{1l}^2\right)$.

In period 1, we have

$$h_{i1}^{C}\left(x_1,\theta_0,\tilde{\theta}_1\right)-h_{i1}^{N}\left(x_1,\theta_0,\tilde{\theta}_1\right)=\frac{\tilde{\theta}_1 c\left(\left[\sum_{i=1}^{2}h_{i0}^{N}\right]h_{i0}^{N}-\left[\sum_{i=1}^{2}h_{i0}^{C}\right]h_{i0}^{C}\right)-\delta s}{2},$$

and hence the result. □

The result in period 0 recalls the established one that noncooperative behavior leads to higher production than collusion. In period 1, writing in detail the difference in harvesting levels, i.e., $h_{i1}^{C}\left(x_1,\theta_0,\tilde{\theta}_1\right)-h_{i1}^{N}\left(x_1,\theta_0,\tilde{\theta}_1\right)$ yields a very long expression whose sign cannot be unambiguously determined. However, we know that

$$\tilde{\theta}_1 c \left(\left[\sum_{i=1}^{2} h_{i0}^{N} \right] h_{i0}^{N} - \left[\sum_{i=1}^{2} h_{i0}^{C} \right] h_{i0}^{C} \right)$$

is positive. Therefore, harvesting under collusion will be lower than its noncooperative counterpart in period 1, if the discounted marginal salvage value δs, which measures future gains, is sufficiently large. In this case, collusion leaves a higher fish stock for the long term.

Remark 9. *If the players are myopic, that is, they do not account for payoffs beyond the two-period planning horizon by setting* $s = 0$*, then harvesting is higher under collusion then under noncooperative mode of play. In such case, in equilibrium the players harvest more in period* 0 *and less in period* 1 *than in the joint optimization solution.*

We assume in the rest of the paper that the assumption in (3.1) is satisfied. Consequently, we have that the price, an economic measure, and the fish stock, a biological measure, are higher under collusion than in the Nash equilibrium.

3.4 Regulation

Framework

In this section, we consider an institution having the legislative power of regulating the monopolies (federal/provincial government, international organization, local municipality commission, etc) with two objectives, (i) sustaining an environmental friendly level of the resource, (ii) eliminating the market inefficiency from dead weight loss as the result to the monopolistic pricing behaviour in each market. The regulator's objectives is in reaching the collusive resource stock level x_t^C and the Nash equilibrium price ($p_t^N = a - h_t^N$) at each period. To implement this outcome, the regulator requests that the firms harvest h_t^C and set their price at p_t^N, and reimburse them the positive amount

$$\begin{aligned}\Delta_i(\theta_0, \tilde{\theta}_1) &= \frac{1}{2}\sum_{t=0}^{1} \delta^t \left(V^C(p_t^C, h_t^C, \theta_0, \tilde{\theta}_1) - V^C(p_t^N, h_t^C, \theta_0, \tilde{\theta}_1)\right), \\ &= \frac{1}{2}\sum_{t=0}^{1} \delta^t h_t^C(\theta_0, \tilde{\theta}_1)(h_t^N(\theta_0, \tilde{\theta}_1) - h_t^C(\theta_0, \tilde{\theta}_1)) > 0,\end{aligned}$$

where V^C is value function in the joint optimization scenario. The function $\Delta_i(\theta_0, \tilde{\theta}_1)$ measures the potential loss to the firms due to the regulation.

This framework positions the regulator in an uncertain situation regarding the assessment of the harvest levels and the prices. Indeed, since $\tilde{\theta}_1$ is only known to the monopolies, the cost of harvest remains uncertain to proceed with the compensation. To obtain the necessary information, i.e., the cost of the fishing activity, we assume that the regulator uses a revelation principle approach. In each period, the monopolies are asked to report their respective total harvesting cost by revealing their value of $\tilde{\theta}_1$. Recall that this quantity is a private information to the monopolies. As cheating behavior may occur, the regulator must design a contract that would avoid it and is able to enforce a harvesting cooperative regime. At the start of the game, nature chooses the cost parameters $\{\theta_0, \tilde{\theta}_1\}$ and reveals them to the fishing monopolies at the beginning of each period. The regulator then offers a contract to the firms and commits to it. This contract will be based on the firms' reports of their respective second-period cost parameters $\{\tilde{\theta}_1\}$ under the regulation tool chosen by the regulator, e.g., a net transfer, a subsidy, a tax refund ...etc. Finally, the monopolies are allowed to start the fishing activity according to the terms of this contract. The first best contract cannot be used because there exists an incentive to deviate. Two approaches have been suggested for this situation: (i) Defining the optimal individual incentive contract to regulate the firms, (see Baron and Myerson (1982) and Laffont and Tirole (1993)); (ii) Allowing the regulator to artificially simulate competition among firms in order to overcome the asymmetry in information (see Auriol and Laffont (1992)). Yardstick competition belongs to the second stream.

Yardstick competition

In yardstick competition, the regulator offers contracts with monetary transfers, which can be either a positive (reward) or a negative (fine) amount, to incentivize the truth telling. Each monopoly $i = 1,2$ is required to reveal its second-period cost $\tilde{\theta}_i$, that is, either a high cost θ_{1h} or a low cost θ_{1l}.
If both players report the same cost $\tilde{\theta}_1 = \tilde{\theta}_2$, then the regulator offers a positive amount T to each of them.[1] If the two firms report different values, i.e., $\tilde{\theta}_1 \neq \tilde{\theta}_2$, then the monopoly with the highest cost report is the cheating player since the game is fully symmetric. Then, the regulator offers different contracts to the monopolies. The player reporting θ_{1h} must pay a fine T, which is transferred to the other firm that reported θ_{1l}.

We provide the possible rents that the players may extract from their respective reports if the regulator chooses to reimburse their costs.

- If the realized cost is θ_{1h}, then the outcomes are as follows:

		Firm 2	
		θ_{1h}	θ_{1l}
Firm 1	θ_{1h}	T , T	$-T-\Theta_1$, $T-\Theta_2$
	θ_{1l}	$T-\Theta_1$, $-T-\Theta_2$	$T-\Theta_1$, $T-\Theta_2$

---

[1] The regulator may offer a "nil" contract as the situation where he does not react and let the game as it is without any regulatory action, but in this case players will always report a lower cost as it is the unique stable Nash equilibrium for this reporting game and may harm them if their real costs are higher.

- If the realized cost is θ_{1l}, then the outcomes are given by

		Firm 2	
		θ_{1h}	θ_{1l}
Firm 1	θ_{1h}	$T+\Theta_1\ ,T+\Theta_2$	$-T,T$
	θ_{1l}	$T,-T$	$T\ ,T$

where

$$\Theta_i = \Delta_i(\theta_0,\theta_{1h}) - \Delta_i(\theta_0,\theta_{1l}), \quad i=1,2.$$

$$\Theta_i = \frac{1}{2}\sum_{t=0}^{1}\delta^t \Big(h_t^C(\theta_0,\theta_{1h}) \Big[h_t^N(\theta_0,\theta_{1h}) - h_t^C(\theta_0,\theta_{1h}) \Big] - h_t^C(\theta_0,\theta_{1l}) \Big[h_t^N(\theta_0,\theta_{1l}) - h_t^C(\theta_0,\theta_{1l})$$

Let $J_i(\tilde{\theta}_i,\tilde{\theta}_j,\theta_{1m})$ be the payoff of firm i when the report parameters of firms i and j are respectively $\tilde{\theta}_i$ and $\tilde{\theta}_j$ and the observed ex-post cost is θ_{1m}, with $m\in\{l,h\}$. The truth telling is an equilibrium only if the following inequalities are satisfied for player $i=1,2$.

$$J_i(\theta_{1h},\theta_{1h},\theta_{1h}) \geq J_i(\theta_{1l},\theta_{1h},\theta_{1h}) \tag{3.2}$$

$$J_i(\theta_{1l},\theta_{1l},\theta_{1l}) \geq J_i(\theta_{1h},\theta_{1l},\theta_{1l}) \tag{3.3}$$

$$J_i(\theta_{1h},\theta_{1l},\theta_{1h}) \geq J_i(\theta_{1l},\theta_{1l},\theta_{1h}) \tag{3.4}$$

$$J_i(\theta_{1l},\theta_{1h},\theta_{1l}) \geq J_i(\theta_{1h},\theta_{1h},\theta_{1l}) \tag{3.5}$$

Conditions (3.2) and (3.4) refer to the case where the ex-post cost is high. Here, player i is better off reporting a high cost rather than a low cost independently of the cost reported by the other player. Conditions (3.3) and (3.5) refer to the other case, namely, an ex-post low cost. In this case, player i cannot gain from reporting a high cost whatever the choice of the other player. In particular, condition (3.5) guarantees that player i will benefit from truthfully reporting the lower cost when the other player is reporting a higher cost. This condition may not bind and the mechanism requires further adjustments. The level of the transfer T is the key stone decision for this situation. Player i will prefer to truly report $\tilde{\theta}_i = \theta_{1l}$ when player 2 reports $\tilde{\theta}_j = \theta_{1h}$ if and only if this parameter is large enough. This result can be summarized in the following proposition:

Proposition 19. *The level of the reward (fine) T^k to enforce player i's true cost reporting satisfies the following condition:*

$$T^k > (\theta_{1h} - \theta_{1l})(1 - cx_1)h_{i1}^k, \quad k = \{N, C\}. \tag{3.6}$$

Proof. It is straightforward to check that this value of T satisfies the equilibrium conditions (3.2)-(3.5). □

In other words, the right-hand side of the above inequality defines the level of reward (fine) T for which player i is indifferent between reporting the true cost or not. A direct consequence of the above proposition is that the level of reward (fine) T^k will depend on the mode of play chosen by the monopolies during their fishing activities.

3.5 Conclusion

In this paper, we considered a two-period fishery model with a shared fish stock exploited by two identical firms having each a monopoly power in its local market. We showed that the regulator can overcome the information asymmetry problem by implementing a yardstick competition mechanism, which leads the monopolists to reveal their true costs and to play cooperatively.

We have made some simplifying assumptions that are worth relaxing in future work. First, the two firms could be asymmetric in their costs, which renders the model and the mechanism design problem more complicated, but more realistic. Second, it would be interesting to extend the analysis to an infinite planning horizon context.

3.6 Appendix

3.6.1 Proof of Proposition 1

To have a feedback (or subgame-perfect) equilibrium, we solve the two-period model in backward. Denote by $V_i(x)$ the value function of player i and by $V_{i1}(x)$ the value function in period 1.

Second-period equilibrium problem

The optimization problem of player i in period 1 is as follows:

$$\begin{aligned}
V_{i1}(x) &= \max_{h_{i1}} E_{\tilde{\theta}_1}\left(h_{i1}(a-bh_{i1})-\tilde{\theta}_1 h_{i1}(1-cx_1)\right)+\delta V_{i1}'(x)\},\\
&= \max_{h_{i1}} E_{\tilde{\theta}_1}\left(h_{i1}(a-bh_{i1})-\tilde{\theta}_1 h_{i1}(1-cx_1)\right)+\delta s\left(x_1(1+\alpha)-h_{i1}-h_{j1}\right)\}.
\end{aligned}$$

Assuming an interior solution, the first-order optimality conditions are as follows:

$$a-2bh_{i1}-\tilde{\theta}_1(1-cx_1)-\delta s=0,\ \ i=1,2.$$

The equilibrium harvest of player i in period 1 is given by

$$h_{i1}^N=\frac{a-\tilde{\theta}_1(1-cx_1)-\delta s}{2b}=\frac{a-\tilde{\theta}_1\left(1-c\left(x_0(1+\alpha)-(h_{i0}+h_{j0})\right)\right)-\delta s}{2b}.$$

To insure that the stock of fish is not exhausted, the following condition must hold true:

$$\left(b-\tilde{\theta}_1 c\right)x_1-a+\tilde{\theta}_1+\delta s>0 \Leftrightarrow x_1>\frac{a-\tilde{\theta}_1-\delta s}{b-\tilde{\theta}_1 c}.$$

Substituting for h_{i1}^N in $V_{i1}(x)$, we get

$$
\begin{aligned}
V_{i1}^{N} = \frac{a}{2b}&\left[a-\tilde{\theta}_1\left(1-c\left(x_0\left(1+\alpha\right)-\left(h_{i0}+h_{j0}\right)\right)\right)-\delta s\right] \\
&-\frac{1}{4b}\left[a-\tilde{\theta}_1\left(1-c\left(x_0\left(1+\alpha\right)-\left(h_{i0}+h_{j0}\right)\right)\right)-\delta s\right]^2 \\
-\frac{\tilde{\theta}_1}{2b}&\left[a-\tilde{\theta}_1\left(1-c\left(x_0\left(1+\alpha\right)-\left(h_{i0}+h_{j0}\right)\right)\right)-\delta s\right]\left(1-c\left(x_0\left(1+\alpha\right)-\left(h_{i0}+h_{j0}\right)\right)\right) \\
+\delta s&\left(\left(1+\alpha\right)\left(x_0\left(1+\alpha\right)-\left(h_{i0}+h_{j0}\right)\right)-\left(\frac{a-\tilde{\theta}_1\left(1-c\left(x_0\left(1+\alpha\right)-\left(h_{i0}+h_{j0}\right)\right)\right)-\delta s}{b}\right)\right).
\end{aligned}
$$

Overall equilibrium problem

Player i's overall optimization problem is given by

$$
V_i\left(x\right)=\max_{h_{i0}}\left\{h_{i0}\left(a-bh_{i0}\right)-\theta_0 h_{i0}\left(1-cx_0\right)+\delta\left[pV_{i1}^{N}\left(.,.,\theta_{1h}\right)+\left(1-p\right)V_{i1}^{N}\left(.,.,\theta_{1l}\right)\right]\right\}.
$$

The first-order equilibrium conditions yield

$$
\begin{aligned}
&a-2bh_{i0}-\theta_0\left(1-cx_0\right)-\left[\delta^2 s\left(1+\alpha\right)\right] \\
&\quad+\left(p\theta_{1h}^2+\left(1-p\right)\theta_{1l}^2\right)\left[\frac{\delta c\left(1-cx_1\right)}{2b}\right]+\left(p\theta_{1h}+\left(1-p\right)\theta_{1l}\right)\left[\frac{2\delta^2 sc-\delta ac}{2b}\right] \\
&\qquad=0.
\end{aligned}
$$

Define Φ^N as:

$$
\Phi^N=a-\delta^2 s\left(1+\alpha\right)+\left(p\theta_{1h}^2+\left(1-p\right)\theta_{1l}^2\right)\left(\frac{\delta c}{2b}\right)+\left(p\theta_{1h}+\left(1-p\right)\theta_{1l}\right)\left(\frac{2\delta^2 sc-\delta ac}{2b}\right).
$$

Then the above condition becomes

$$\Phi^N - \theta_0\left(1 - cx_0\right) - \left(p\theta_{1h}^2 + (1-p)\,\theta_{1l}^2\right)\left[\frac{\delta c^2 x_0\left(1+\alpha\right)}{2b}\right] + \left(p\theta_{1h}^2 + (1-p)\,\theta_{1l}^2\right)\left[\frac{\delta c^2}{b}\right] h_{i0} = 2bh_{i0},$$

which gives

$$h_{i0}^N\left(x_0, \theta_0, \tilde{\theta}_1\right) = \frac{2b\left(\Phi^N - \theta_0\left(1 - cx_0\right)\right) - \delta c^2 x_0\left(1+\alpha\right)\left(p\theta_{1h}^2 + (1-p)\,\theta_{1l}^2\right)}{4b^2 - 2\delta c^2\left(p\theta_{1h}^2 + (1-p)\,\theta_{1l}^2\right)}.$$

And therefore,

$$\begin{aligned} V_i^N &= h_{i0}^N\left(a - h_{i0}^N - \theta_0\left(1 - cx_0\right)\right) \\ &\quad + \delta E_{\tilde{\theta}_1}\left(\frac{\left(a - \tilde{\theta}_1\left(1 - cx_1^N h_{i0}^N\right) - \delta s\right)\left(a - \tilde{\theta}_1\left(1 - cx_1^N h_{i0}^N\right) - 3\delta s\right)}{4} + \delta s x_1^N h_{i0}^N\left(1+\alpha\right)\right). \end{aligned}$$

3.6.2 Proof of Proposition 2

As in the proof of Proposition 1, we solve the two-period model in backward. Denote by $V(x)$ the joint value function and by $V_1(x)$ the joint value function in period 1.

Second-period optimization problem

The joint optimization problem in period 1 is as follows:

$$V(x) = \max_{h_{11}, h_{21}} E_{\tilde{\theta}_1} \sum_{i=1}^{2}\left(h_{i1}\left(a - bh_{i1}\right) - \tilde{\theta}_1 h_{i1}\left(1 - cx_1\right) + \delta s\left(x_1\left(1+\alpha\right) - h_{i1} - h_{j1}\right)\right).$$

Assuming an interior solution, the first-order optimality conditions yield

$$h_{i1}^C = \frac{a - \tilde{\theta}_1\left(1 - cx_1\right) - 2\delta s}{2b} = \frac{a - \tilde{\theta}_1\left(1 - c\left(x_0\left(1+\alpha\right) - \left(h_{i0} + h_{j0}\right)\right)\right) - 2\delta s}{2b}, \quad i = 1, 2.$$

To insure that the stock of fish is not exhausted, the following condition must hold true:

$$\left(b-\tilde{\theta}_1 c\right)x_1 - a + \tilde{\theta}_1 + 2\delta s > 0 \Leftrightarrow x_1 > \frac{a-\tilde{\theta}_1-2\delta s}{b-\tilde{\theta}_1 c}.$$

Substituting for h_{i1}^C in $V_1(x)$, we get

$$V_1^C = \frac{a-\tilde{\theta}_1\left(1-cx_1\right)2\delta s}{2b}\left(a-\frac{a-\tilde{\theta}_1\left(1-cx_1\right)-2\delta s}{2}\right)$$
$$-\tilde{\theta}_1\left(\frac{a-\tilde{\theta}_1\left(1-cx_1\right)-\delta s}{2b}\right)\left(1-cx_1\right)+\delta s\left(x_1\left(1+\alpha\right)-\left(\frac{a-\tilde{\theta}_1\left(1-cx_1\right)-2\delta s}{b}\right)\right)$$

Giving,

$$\begin{aligned}V_1^C = {} & \frac{a}{2b}\left[a-\tilde{\theta}_1\left(1-c\left(x_0\left(1+\alpha\right)-h_{i1}-h_{j1}\right)\right)-2\delta s\right] \\ & -\frac{1}{4b}\left[a-\tilde{\theta}_1\left(1-c\left(x_0\left(1+\alpha\right)-h_{i1}-h_{j1}\right)\right)-2\delta s\right]^2 \\ & -\frac{\theta_1}{2b}\left[a-\tilde{\theta}_1\left(1-c\left(x_0\left(1+\alpha\right)-h_{i1}-h_{j1}\right)\right)-2\delta s\right]\left(1-c\left(x_0\left(1+\alpha\right)-h_{i1}-h_{j1}\right)\right) \\ & +\delta s\left(\left(1+\alpha\right)\left(x_0\left(1+\alpha\right)-h_{i1}-h_{j1}\right)-\left(\frac{a-\tilde{\theta}_1\left(1-c\left(x_0\left(1+\alpha\right)-h_{i1}-h_{j1}\right)\right)-2\delta s}{b}\right)\right).\end{aligned}$$

Overall optimization problem

The joint optimization problem is as follows:

$$V(x) = \max_{h_{10},h_{20}} \sum_{i=1}^{2}\left(h_{i0}\left(a-bh_{i0}\right)-\theta_0 h_{i0}\left(1-cx_0\right)+\delta\left(pV_{i1}^C\left(.,.,\theta_{1h}\right)+\left(1-p\right)V_{i1}^C\left(.,.,\theta_{1l}\right)\right)\right).$$

Assuming an interior solution, the first-order optimality conditions give for $i=1,2$

$$\begin{aligned}2bh_{i0} = a - \theta_0\left(1 - cx_0\right) - 2\delta^2 s\left(1 + \alpha\right) + \left(p\theta_{1h}^2 + \left(1 - p\right)\theta_{1l}^2\right)\left(\frac{\delta c}{b}\right) \\ - \left(p\theta_{1h}^2 + \left(1 - p\right)\theta_{1l}^2\right)\left(\frac{\delta c^2 x_0\left(1 + \alpha\right)}{b}\right) + 2\left(p\theta_{1h}^2 + \left(1 - p\right)\theta_{1l}^2\right)\left(\frac{\delta c^2}{b}\right)h_{i0} \\ + \left(p\theta_{1h} + \left(1 - p\right)\theta_{1l}\right)\left(\frac{\delta^2 sc - \delta ac}{b}\right)\end{aligned}$$

Define Φ^C as:

$$\Phi^C = a - 2\delta^2 s\left(1 + \alpha\right) + 2\left(p\theta_{1h}^2 + \left(1 - p\right)\theta_{1l}^2\right)\left(\frac{\delta c}{2b}\right) + 2\left(p\theta_{1h} + \left(1 - p\right)\theta_{1l}\right)\left(\frac{2\delta^2 sc - \delta ac}{2b}\right).$$

Then, the above optimality condition becomes

$$\begin{aligned}\Phi^C - \theta_0\left(1 - cx_0\right) - 2\left(p\theta_{1h}^2 + \left(1 - p\right)\theta_{1l}^2\right)\left(\frac{\delta c^2 x_0\left(1 + \alpha\right)}{2b}\right) + 2\left(p\theta_{1h}^2 + \left(1 - p\right)\theta_{1l}^2\right)\left(\frac{\delta c^2}{b}\right)h_{i0} \\ = 2bh_{i0},\end{aligned}$$

which gives

$$h_{i0}^{C*}\left(x_0, \theta_0, \tilde{\theta}_1\right) = \frac{b\left(\Phi^C - \theta_0\left(1 - cx_0\right)\right) - \delta c^2 x_0\left(1 + \alpha\right)\left(p\theta_{1h}^2 + \left(1 - p\right)\theta_{1l}^2\right)}{2b^2 - 2\delta c^2\left(p\theta_{1h}^2 + \left(1 - p\right)\theta_{1l}^2\right)}.$$

Inserting in the value function, we get

$$\begin{aligned}V_0^C &= h_{i0}\left(a - h_{i0}^C - \theta_0\left(1 - cx_0\right)\right) \\ &\quad + \delta E_{\tilde{\theta}_1}\left(\frac{\left(a - \tilde{\theta}_1\left(1 - cx_1^C h_{i0}^C\right) - \delta s\right)}{4}\left(a - \tilde{\theta}_1\left(1 - cx_1 h_{i0}^C\right) - 3\delta s\right) + \delta s x_1^C h_{i0}^C\left(1 + \alpha\right)\right).\end{aligned}$$

General Conclusion

Climate change, lack of cooperation, environmental variability, economic uncertainty and political crises are compounding threats to the sustainable exploitation of fisheries. In this thesis, we have analyzed fisheries management problems using a game theoretical approach with the objective of designing some possible solutions to overexploitation of the stock and promoting the sustainability of the resource. Given that some players may see their income decreasing in the short term, which may be the cost to pay for securing higher future revenues, the success of these solutions requires more patient-conscious behavior and stronger law enforcement for the benefit of all. The role of the central planner is crucial and is not to be neglected in a way that the existence of more powerful states (organizations) is necessary but not sufficient to solve this very complex environmental problem. The demand for this protein is continuously increasing whereas -according to the FAO- the captured amount of fish is reported to remain stable for at least during the next decade. From an economical point of view, a higher demand facing an unchanged supply results in higher prices and more profits, this situation strengthen the incentive for overexploiting the resource stocks more than ever.

We have contributed to the literature by modeling three different problems. In a dynamic game framework we have analyzed the role of designing a cooperative scheme as the corner stone solution to an environmental friendly exploitation of the world marine resource. The voluntary nature of cooperation remains a difficult problem to deal with. Using a differential game framework, we considered in the first essay an open-access fishery with a finite number of players, assuming negative externalities from endogenous pollution, and characterized noncooperative and cooperative solutions. Further, we have designed a fair and time-consistent imputation distribution procedure (IDP) of the outcome when the players are jointly maximizing their profits. Finally, we discussed the implications of relaxing our symmetry assumption on the characterization of the equilibrium and optimal solutions, and on the derivation of the IDP. A complicated, but a possible extension to this work is the relaxation of the symmetry assumption between the players in the two-state variable framework, with one-state dynamics being approximated by a piece-wise linear function. Relaxing this assumption would imply to define for each of the n players an associated value function, which means deriving and solving the system of $6n$ equations. Another challenging direction for future studies is to investigate the steady-state trends in the two other regions of the state space. The good news is that, once the quantities involved in computing a fair solution are in place, asymmetry would not cause any intrinsic difficulty in computing the IDP. Recall that the formula derived holds for any functional forms. In this essay we suggest a fair sharing rule à la Nash bargaining solution. In an n-player setting, it would be interesting to allocate the outcome among the coalition members according to different schemes, e.g., Shapley value, Nucleolus etc. and see how they compare to each others.

In the second essay, we have analyzed the impact of biological interactions and modes of play on equilibrium strategies and outcomes in a fishery with two-species, each being harvested by a group of players. Our main take-away message is that cooperation among two competing groups of agents is not necessarily beneficial, both from a biological and an environmental point of view. This counter-intuitive result is due to the biological interactions between the two species, and to the fact that coordination only happens among players in the same group. However, in the single owner scenario (centralized decision), this result does not hold true. In the same line as the FM model, relaxing the two following restrictive assumptions may lead to a worthy extension. First, the fact that both species procure the same utility, and, second, the fact that each agent harvests only one species. Relaxing either one of these assumptions would require the use of a numerical approach to obtain the resulting equilibria.

The last essay consisted of a two-period fishery model with a shared fish stock exploited by two identical firms having each a monopoly power in its local market. We showed that the regulator can overcome the information asymmetry problem by implementing a yardstick competition mechanism, which incentivizes the monopolies to truthfully report their fishing costs and adopt a cooperative behavior. One extension for this paper is to consider the firms being asymmetric in their costs in an infinite time horizon setting. Short of determining whether collusion among players will occur we can address the problem from different angles of possible strategies (Tit-for-tat, Trigger, etc.).

Finally, dealing with a sensitive topic, this thesis has probably raised more questions than responses.

Bibliography

Armstrong, M., Sappington, D.E. (2006). Regulation, competition, and liberalization. Journal of Economic Literature, 44(2), 325-366.

Auriol, E., & Laffont, J.J. (1992). Regulation by duopoly. Journal of Economics and management Strategy, 1(3): 507-533.

Bailey, M., Sumaila, U.R. (2008). Destructive fishing in Raja Ampat, Indonesia: an applied principal-agent analysis Ecological and economic analyses of marine ecosystems in the Bird's Head seascape, Papua, Indonesia. II. Fish. Centre Res. Rep. 16(1), 142-169.

Bailey, M., Sumaila, U.R., and Lindroos, M. (2010). Application of game theory to fisheries over three decades. *Fisheries Research*, 102: 1–8.

Barbier, E.B. and Strand, I. (1998). Valuing Mangrove-Fishery Linkages. A Case Study of Campeche, Mexico. *Environmental and Resource Economics*, 12(2): 151–166.

Barbier, E.B. (2000), Valuing the Environment as Input: Review of Applications to Mangrove-Fishery Linkages. *Ecological Economics*, 35: 47-61.

Baron, D. P., Myerson, R.B. (1982). Regulating a monopolist with unknown costs. Econometrica: Journal of the Econometric Society, 911-930.

Bellman, R. E. (1957) *Dynamic programming*, Princeton University Press, Princeton, N.J.

Benchekroun, H. (2003). Unilateral production restrictions in a dynamic duopoly. *Journal of Economic Theory* 111(2): 214–239.

Benchekroun, H. (2008). Comparative dynamics in a productive asset oligopoly. *Journal of Economic Theory*, 138(1): 237–261.

Breton, M., Dahmouni, I., Zaccour, G. (2019). Equilibria in a two-species fishery, Mathematical Biosciences, 309, 78-91.

Cheung, W. W., Lam, V. W., Sarmiento, J. L., Kearney, K., Watson, R., and Pauly, D. (2009). Projecting global marine biodiversity impacts under climate change scenarios. *Fish and Fisheries*, 10(3): 235–251.

Clarke, F.H., Munro, G.R. (1987). Coastal states, distant water fishing nations and extended jursdiction: a principal-agent analysis. Natural Resource Modelling, 2(1), 81-107.

Clark, C. W. (1980). *Restricted access to common-property fishery resources: a game-theoretic analysis*. In: Dynamic Optimization and Mathematical Economics, 117–132, Springer, Boston, MA.

Clarke, F.H., Munro, G.R. (1991). Coastal states and distant water fishing nations: conflicting views of the future. Natural Resource Modelling, 5(3), 345-369.

Dahmouni I, Vardar B, Zaccour G. A fair and time-consistent sharing of the joint exploitation payoff of a fishery. *Natural Resource Modeling. 2019; e12216*

Doyen, L., Cissé, A. A., Sanz, N., Blanchard, F., & Pereau, J. C. (2018). The tragedy of open ecosystems. *Dynamic Games and Applications*, 8(1), 117-140.

Fisher, R., Mirman, L. (1992). Strategic dynamic interactions: Fish wars. Journal of Economic Dynamics and Control, 16, 267-287.

Fisher, R., Mirman, L. (1996). The complete fish wars: biological and dynamic interactions. Journal of Environmental Economics and Management, 30, 34-42.

Fluharty, D. (2000), Habitat Protection, Ecological Issues, and Implementation of the Sustainable Fisheries Act. *Ecological Applications*, 10: 325–337.

Górniewicz, O., & Wiszniewska-Matyszkiel, A. (2018). Verification and refinement of a two species Fish Wars model. *Fisheries Research*, 203, 22-34.

Grønbæk, L., Lindroos, M., Munro, G., & Pintassilgo, P. (2018). Game theory and fisheries. *Fisheries Research* 203, 1-5.

Hanna, S., Folke, C., Mäler, K.-G., & Beijer Institute of Ecological Economics. (1996). Rights to nature: Ecological, economic, cultural, and political principles of institutions for the environment. Washington, D.C: Island Press.

Hannesson, R. (1983). Optimal harvesting of ecologically interdependent fish species. *Journal of Environmental Economics and Management*, 10(4), 329-345.

Hannesson, R. (2011). Game theory and fisheries. *Annual Review of Resource Economics*, 3(1), 181-202.

Hardin, G. (1968). The tragedy of the commons. *Science*, 162(3859): 1243–1248.

Haurie, A., Krawczyk, J. B., and Zaccour, G. (2012). *Games and dynamic games*. World Scientific Publishing Company.

Jensen, F., Vestergaard, N. (2002). Moral hazard problems in fisheries regulation: the case of illegal landings and discard. Resource and Energy Economics, 24(4), 281-299.

Jensen, F. (2008). Uncertainty and asymmetric information: An overview. Marine Policy, 32, 89-103.

Jensen, F., Hans, F., Abildtrup, J. (2017). Fisheries regulation: A survey of the literature on uncertainty, compliance behavior and asymmetric information. Marine Policy, 81, 167-178.

Joosten, R. (2016). Strong and Weak Rarity Value: Resource Games with Complex Price–Scarcity Relationships. *Dynamic Games and Applications*, 6(1), 97-111.

Jørgensen, S., & Yeung, D. W. K. (1996). Stochastic differential game model of a common property fishery. *Journal of Optimization Theory and Applications*, 90(2), 381-403.

Kaiser, M. J. and S.J. de Groot (2000), *Effects of Fishing on Non-Target Species and Habitats*. Blackwell Science, Oxford. 399 pages.

Kaitala, V., and Lindroos, M. (2004). When to ratify an environmental agreement: The case of high seas fisheries. *International Game Theory Review*, 6(01): 55–68.

Kronbak, L. G., and Lindroos, M. (2007). Sharing rules and stability in coalition games with externalities. *Marine Resource Economics*, 22(2): 137–154.

Laffont, J.J., Tirole, J. (1993). A theory of incentives in procurement and regulation. MIT press.

Levhari, D., and Mirman, L. J. (1980). The great fish war: An example using a dynamic Cournot-Nash solution. *The Bell Journal of Economics*, 11(1): 322–334.

Lindroos, M., Kronbak, L. G., and Kaitala, V. (2005). *Coalition games in fisheries economics*. In: Advances in Fisheries Economics: Festschrift in Honour of Professor Gordon R. Munro, 184–195.

Long, N. V. (2011). Dynamic games in the economics of natural resources: A survey. *Dynamic Games and Applications*, 1(1): 115–148.

Long, N. V. (2018). Resource Economics. In: Basar T. & Zaccour G. (eds) *Handbook of Dynamic Game Theory*, Springer, Cham.

Matsuoka, T., Nakashima, T., and Nagasawa, N. (2005). A review of ghost fishing: scientific approaches to evaluation and solutions. *Fisheries Science*, 71: 691–702.

Mazalov, V. V., & Rettieva, A. N. (2010). Fish wars and cooperation maintenance. *Ecological Modelling*, 221(12), 1545-1553.

Mesterton-Gibbons, M. (1996). A technique for finding optimal two-species harvesting policies. *Ecological Modelling*, 92(2-3), 235-244.

Miller, K. A., and Munro, G. R. (2004). Climate and cooperation: A new perspective on the management of shared fish stocks. *Marine Resource Economics*, 19(3): 367–393.

Munro, G. R. (1979). The optimal management of transboundary renewable resources. *Canadian Journal of Economics*, 12(3): 355–376.

Munro, G.R. (2003). On the management of shared fish stocks.Papers presented at the Norway-FAO expert consultation on the management of shared fish stocks. FAO Fisheries Report 695, 2-29.

Munro, G. R. (2006). *International allocation issues and the high seas: An economist's perspective*. In: Sharing the Fish Conference. Fremantle, Australia.

Petrosjan, L. (1977). Stable solutions of differential games with many participants. *Viestnik of Leningrad University*, 19: 46–52.

Petrosjan, L., and Zaccour, G. (2018). *Cooperative differential games with transferable payoffs*. In: Basar T., Zaccour G. (eds) Handbook of Dynamic Game Theory, Volume 1, 595–632, Springer.

Pintassilgo, P., and Lindroos, M. (2008). Coalition formation in straddling stock fisheries: A partition function approach. *International Game Theory Review*, 10(03): 303–317.

Rettieva, A. N. (2012). Stable coalition structure in bioresource management problem. *Ecological Modelling*, 235, 102-118..

Salenius, F. (2018). International management of North Atlantic pelagic fisheries–The role of competing species and exploiters. *Fisheries Research*, 203, 12-21.

Sappington, D.E. (1991). Incentives in principal-agent relationships. The Journal of Economic Perspectives, 5(2), 45-66.

Shleifer, A. (1985). A theory of yardstick competition. The RAND Journal of Economics, 16(3), 319-327.

Strauch, R. E. (1966). Negative dynamic programming. *The Annals of Mathematical Statistics*, 37(4), 871-890.

Sumaila, U. R. (2013). *Game theory and fisheries: Essays on the tragedy of free for all fishing*. Routledge.

Sumaila, U. R. (2002). Marine protected area performance in a model of the fishery. *Natural Resource Modeling*, 15(4): 439–451.

Sumaila, U. R. (2013). *Game theory and fisheries: Essays on the tragedy of free for all fishing*. Routledge.

Sumaila, U. R., and Armstrong, C. W. (2006). Distributional and efficiency effects of marine protected areas: A study of the Northeast Atlantic cod fishery. *Land Economics*, 82(3): 321–332.

Tahvonen, O. (1991). On the dynamics of renewable resource harvesting and pollution control. *Environmental and Resource Economics*, 1(1): 97–117.

Trisak, J. (2005). Applying game theory to analyze the influence of biological characteristics on fishers' cooperation in fisheries co-management. *Fisheries Research*, 75(1–3): 164–174.

Vardar, B. and Zaccour, G. (2018). *Exploitation of a productive asset in the presence of strategic behavior and pollution externalities*. Les Cahiers du GERAD G–2018–43.

Wang, W. K., & Ewald, C. O. (2010). A stochastic differential fishery game for a two species fish population with ecological interaction. *Journal of Economic Dynamics and Control*, 34(5), 844-857.

Wilen, J. E. (2000). Renewable resource economists and policy: What differences have we made? *Journal of Environmental Economics and Management*, 39(3), 306-327

Wirl, F. (2004). Sustainable growth, renewable resources and pollution: thresholds and cycles. *Journal of Economic Dynamics and Control*, 28(6): 1149–1157.

Xepapadeas, A. (1995). Managing the international commons: Resource use and pollution control. *Environmental and Resource Economics*, 5(4): 375–391.

Yeung, D.W.K., Petrosjan, L. (2005). *Cooperative stochastic differential games*. Springer, New York, NY.

Yeung, D.W.K., Petrosjan, L. (2012). *Subgame consistent economic optimization*. Birkhauser, New York.

Yeung, D.W.K., and Petrosjan, L. (2018). *Nontransferable utility cooperative dynamic games*. In: Basar T., Zaccour G. (eds) Handbook of Dynamic Game Theory, Volume 1, 633–670, Springer.

Zaccour, G. (2008). Time consistency in cooperative differential games: A tutorial. *INFOR*, 46(1): 81–92.

Zaccour, G. (2017). *Sustainability of cooperation in dynamic games played over event trees*. In: Melnik, R., Makarov, R. and Belair, J. (Eds.), Recent Progress and Modern Challenges in Applied Mathematics, Modeling and Computational Science, Fields Institute Communications Series, 419–437(2009). Impacts of Harvesting Rights in Canadian Pacific Fisheries Statistical and Economic Analysis Series. ISSN 1921-877X, No. 1-3. Ottawa, Fisheries and Oceans Canada.