HEC MONTRÉAL

École affiliée à l'Université de Montréal

Essays on Macroeconomic Risk and Asset Pricing

par

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Essays on Macroeconomic Risk and Asset Pricing

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Résumé

Le risque macroéconomique apporte des éléments essentiels qui permettent une description plus précise du prix des actifs, comme suggérées dans certaines études, telles que celles Bansal et Yaron (2004), Bhamra, Kuehn et Strebulaev (2010a, b), et Chen (2010). Ce risque est caractérisé par des conditions économiques fluctuantes dans le temps qui, combinées à des préférences récursives, permettent d'évaluer les primes de risque qui lui sont associées. Ainsi, nous pouvons comprendre les impacts du risque macroeconomique dans plusieurs domaines tels que, les décisions des entreprises, celles des pays, l'évaluation des actions, des obligations d'entreprises ou les obligations souveraines.

Cette thèse propose deux articles sur les impacts des conditions macroéconomiques sur l'évaluation:
(1) des actions et des obligations d'entreprises et (2) des obligations souveraines.

Le premier article, intitulé «What Drives Corporate Asset Prices: Short- or Long-run Risk?», étudie l'impact du risque macroéconomique sur les prix des actifs des entreprises. Cet article propose un modèle de tarification des actifs basé sur la consommation qui permet de comparer les compensations pour le risque macroéconomique, dit risque à long terme et celui obtenu à partir du modèle classique d'évaluation d'Actif lié consommation, également appelé risque à court terme. Le risque à court terme provient de la corrélation positive entre les revenus des entreprises et la consommation. Le risque à long terme vient du basculement aléatoire entre les périodes d'expansion et de récession économiques et la persistance de chacun de ces périodes. Les modèles d'évaluation des actifs basés sur la consommation postulent que le premier type de risque est le principal moteur de la prime de risque sur les actions, mais des études plus récentes indiquent que le second type aide à expliquer plusieurs énigmes en finance. La principale conclusion est que le risque à long terme represente plus des deux tiers de la primes de risque, tant pour les actions que pour les obligations. De plus, le rôle du risque à long terme dans la prime de risque est amplifié pendant les périodes de récessions pour les actions, mais reste stable dans le temps en ce qui concerne les primes de risque sur les obligations. L'importance relative des risque à court et à long terme varie également entre firmes. Une analyse empirique sur la période 1952-2016 permet de corroborer les principales prédictions du modèle.

Le deuxième article, intitulé «Macroeconomic Risk, Investors Preferences, and Sovereign Credit Spreads», examine l'impact de la conjoncture macroéconomique mondiale et les préférences des investisseurs sur les écarts de crédit souverains. Ce article rejoint une littérature abondante prouvant que les écarts de crédit souverains varient beaucoup en fonction des conditions financières et économiques mondiales et peu selon les caractéristiques économiques des pays, comme le montrent, par exemple, Jeanneret (2015) et Augustin et Tédongap (2015). Ce présent article propose un modèle structurel d'évaluation de la dette souveraine dans lequel la variable décisionnel est la consommation, cela avec prise en compte du cycle économique global, dans une économie peuplée d'investisseurs de type Epstein et Zin. Augustin et Tédongap (2015) soulignent l'importance du risque macroéconomique pour mieux comprendre la dynamique des écarts de crédit souverains, mais leur modèle réduit n'indique pas comment les décisions optimales de défaut et d'endettement d'un pays varient en fonction de ce risque; ceci est un aspect crucial de notre article. Borri et Verdelhan (2012) analysent uniquement le prix du risque associé au risque à court terme, alors que nous nous concentrons sur le risque associé aux conditions macroéconomiques. Notre modèle prédit que 30% du niveau des écarts de crédit est dû à l'exposition au risque macroéconomique et que ce risque augmente la probabilité de défaut à 5 ans de 3,7% à 9,1%. En outre, nous montrons que les gouvernements choisissent un niveau d'endettement plus élevé et préfèrent faire défaut plus tôt lorsque la performance économique du pays est plus sensible au cycle économique mondial.

Mots-clés: Tarification des actifs, risque macroéconomique, série transversale des rendements, préférences récursives, dette souveraine, risque de crédit.

Méthode de recherche: Modélisation mathématique, analyse numérique, économetrie.

Abstract

Macroeconomic risk brings critical elements that allow more precise descriptions of asset prices movements, as shown by several studies (see Bansal and Yaron (2004), Bhamra, Kuehn, and Strebulaev (2010a, b) and Chen (2010)). This risk is characterized by time-varying economic conditions which combined with recursive preferences allows to price its risk premia. Hence, we can study its impacts in various domains such as, firm or country decisions and on the pricing of stocks, corporate or sovereign bonds.

This thesis proposes two articles on the impacts of macroeconomic conditions on the pricing of: (1) equity and corporate bond and (2) sovereign bond.

The first article, named "What Drives Corporate Asset Prices: Short- or Long-run Risk?", investigates the impact of macroeconomic risk on corporate asset prices. This paper proposes a consumption-based asset pricing model that allows comparing the compensations for macroeconomic risk, also labeled long-run risk and the one obtained from the classical consumption CAPM, also labeled short-run risk. Short-run risk originates from the positive correlation between firm cash flows and aggregate consumption. Long-run risk comes from the random switch between good and bad economic conditions, and the persistence of each of these state. Consumption-based asset pricing models postulate that the first type of risk is the main driver of equity risk premium, while more recent influential studies document that the second type helps explain several puzzles in finance. The key finding is that long-run risk commands more than two third of risk premia, for both equities and bonds. Second, the role of long-run risk in the equity risk premium is amplified in recessions, but remains stable over the business cycle for credit spreads. The relative importance of short- vs. long-run risk also varies at the cross-section. An empirical analysis over the period 1952-2016 provides support for the main predictions of the model.

The second article, entitled "Macroeconomic Risk, Investors Preferences, and Sovereign Credit Spreads", examines the impact of global macroeconomic conditions and investor preferences on sovereign credit spreads. This paper is related to a large literature providing evidence that sovereign

credit spreads vary with global financial and economic conditions, as documented, among others,

by Jeanneret (2015) and Augustin and Tédongap (2015). We propose a structural model for

sovereign debt valuation embedded in a consumption-based environment with a global business

cycle, in an economy populated by Epstein and Zin type of investors. Augustin and Tédongap

(2015) highlight the importance of macroeconomic risk to better understand the dynamics of

sovereign credit spreads, but their reduced-form model does not offer insights on how a sovereign's

optimal default and indebtedness decisions vary with such risk, which is a key aspect of our paper.

Borri and Verdelhan (2012) exclusively analyze the price of risk associated with short-run risk,

whereas we focus on the risk associated with macroeconomic conditions. Our model predicts that

30% of the credit spread level is due to exposure to macroeconomic risk and that this risk increases

the 5-year default probability from 3.7% to 9.1%. Moreover, we show that governments choose a

higher indebtedness level and prefer to default earlier when a country's economic performance is

more sensitive to the global business cycle.

Keywords: asset pricing, macroeconomic risk, cross-section of returns, recursive preferences,

Sovereign debt, credit risk.

Research methods: Mathematical modeling, numerical analysis, econometrics.

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Part I

What Drives Corporate Asset Prices: Short- or Long-Run Risk?

Abstract ¹

This paper investigates the relative impact of various types of systematic risk on corporate asset

prices. Equity risk premium and credit spreads are priced in a consumption-based corporate finance

model with time-varying macroeconomic conditions. We decompose the risk premia into different

sources of systematic risk compensation and show that long-run risk commands most of the risk

premium (about 70%), for both equities and bonds. The role of long-run risk in the equity risk

premium is amplified in recessions, but remains stable over the business cycle for credit spreads.

The relative importance of short- vs. long-run risk also varies at the cross-section. An empirical

analysis over the period 1952-2016 provides support for the main predictions of the model.

JEL Codes: G12, G17, E44

Keywords: Asset pricing, long-run risk, cross-section of returns, recursive preferences.

¹This article is co-authored with C. Dorion and A. Jeanneret.

1 Introduction

The nature of risk premia is central for the understanding of asset prices. It is now widely accepted that there exist different sources of systematic risk, which carry separate risk premium. First, firm cash flow shocks are partly systematic as they correlate positively with aggregate consumption. Because these shocks are short-lived, the associated premium can be viewed as a compensation for *short-run* risk. Second, the expected cash flow growth rate is exposed to aggregate economic conditions. That is, firms tend to grow less rapidly in recessions when expected consumption growth is also low. Because expected growth rates are persistent, as they vary slowly over the business cycle, the corresponding risk premium is a compensation for *long-run* risk. Consumption based asset pricing models postulate that the first type of systematic risk is the main driver of equity risk premium.² More recent studies document that the second type helps explain some asset pricing moments.³ It remains important, however, to understand the role of each type of risk for the equity risk premium and corporate credit spreads.

This paper contributes to the literature in various dimensions. First, it decomposes and quantifies the level of systematic risk and the associated risk premium. Second, it compares the role of systematic risk across asset classes, equity versus debt. And third, it separates the risk premia into the different sources of systematic risk to analyze their relative importance, both over the business cycle and at the cross-section.

We propose a consumption-based asset pricing model that allows analyzing both types of systematic risk, individually or in tandem. The environment is characterized by time-varying macroe-conomic conditions, which determine the expected growth rates of firm cash flows and aggregate consumption, as in Bansal and Yaron (2004), Bhamra, Kuehn and Strebulaev (2010a; 2010b), and Chen (2010). The representative agent is of an Epstein-Zin type. That is, this agent is not only averse to short-run risk, but also has preference for early resolution of uncertainty, i.e. averse to long-run risk.⁴ The approach to derive corporate asset prices follows the structural corporate finance

²See Rubinstein (1976), Lucas (1978), Breeden and Litzenberger (1978), Breeden (1979), Hansen and Singleton (1983), Mehra and Prescott (1985), and Hansen and Jagannathan (1991)

³Bansal and Yaron (2004) and Bhamra, Kuehn and Strebulaev (2010b) show that time-varying expected growth rates (and volatilities) generate high levels of the equity risk premium, while Bhamra, Kuehn and Strebulaev (2010a) and Chen (2010) finds that such types of risk can explain the credit-spread puzzle.

⁴The coefficient of elasticity of intertemporal substitution (EIS) is greater than the inverse of the coefficient of

models developed by Fischer, Heinkel and Zechner (1989), Leland (1994), and Hackbarth, Miao and Morellec (2006).⁵ The firm's optimal capital structure and default decisions are endogenously determined.

The contribution is to separate the quantity from the price of risk, and to disentangle the asset pricing implications of each source of systematic risk. We derive the credit spread in the full model and isolate the quantity of risk component by assuming that the agent is risk neutral, i.e. she does not command any risk premium. This is the pure compensation for holding default-risky corporate bonds. The credit spread premium is defined by the difference between the total credit spread and its default-risk compensation. Then, we separate the risk premia associated with each type of systematic risk. The credit spread premium component that relates to short-run risk is obtained by assuming that the agent has CRRA preferences.⁶ The long-run risk premium is obtained by subtracting the short-run risk premium from the total credit spread premium. The same methodology applies for the equity risk premium.

The model is calibrated to real consumption and aggregate firm profits data over the period 1952Q1–2016Q4. We respectively use U.S. real non-durables goods plus service consumption expenditures and U.S. corporate profits before tax, both from the Bureau of Economic Analysis. Short-run risk is determined by the correlation between consumption and corporate profits. Aggregate economic conditions are considered to be time-varying and to switch between two states. That is, the economy is either in expansion or in recession. The conditional expected growth rates of consumption and firm cash flows are computed using the NBER classification. The random transition from one state to another is modeled by a two-states Markov-regime switching model, following Hamilton (1989)'s approach. The transition matrix reveals that the states of the economy tend to be persistent.

The main theoretical predictions are as follow. First, systematic risk affects the expected excess return differently for equities versus bonds. For corporate bonds, the quantity of risk represents around 63% of the total credit spread, while the corresponding risk premium captures 37%. This

RRA.

⁵See also Goldstein, Ju and Leland (2001) and Strebulaev (2007).

⁶A CRRA agent has no preference for early resolution of the uncertainty, because the coefficient of elasticity of intertemporal substitution is equal to the inverse of the coefficient of relative risk aversion.

magnitude is similar to what is observed empirically.⁷ In contrast, the compensation for systematic risk represents 100% of the equity risk premium, by definition.

Second, long-run risk commands a greater risk premium than short-run risk, when the agent has a preference for early resolution of the uncertainty. This lends support to Bansal and Yaron (2004), Tedongap (2014) and Bansal, Kiku, Shaliastovich and Yaron (2014), who show that long-run risk can help explain some stylized facts in asset pricing. We find that the long-run risk component represents 72.8% of the total risk premium embedded into equity prices. The remaining 27.2% represents a compensation for short-run risk, which arises when the investor is risk averse to cash flow shocks that positively correlate with aggregate consumption shocks. In contrast, the long-run risk component of the credit spread premium represents 68.5%. These results show that the impact of aversion to long-run risk, when the agent has recursive preferences, is significantly higher than the aversion to short-run risk.

Third, the model shows how these risk premia vary over the business cycle. Long-run risk represents 89.9% of equity risk premium in periods of economic downturns, but only 60.6% during periods of economic expansion. Hence, the relative importance of long-run risk appears to be strongly countercyclical for equity valuation. In contrast, the proportion of long-run risk in the credit spread premium is stable over time, i.e. around 69%. Hence, stockholders price more their preference for early resolution of the uncertainty during recessions than bondholders do.

Fourth, we find that the relative importance of these risk premia differs at the cross-section, as it varies with firm characteristics. When idiosyncratic volatility increases, the proportion of the credit spread premium due to long-run risk decreases. However, the model predicts that the allocation of long-run vs. short-run risk in the equity risk premium is almost insensitive to the level of idiosyncratic risk. When firms perform well and financial leverage decreases, the proportion of the long-run risk premium decreases for equities but increases for bonds. These are novel testable predictions. This paper also shows how preferences affect optimal debt and default decisions. When the investor cares about long-run risk, the firm chooses to issue less debt and a lower default barrier.

⁷Elton, Gruber, Agrawal and Mann (2001) report that systematic risk may explain up to 46% of the credit spread for 10-year corporates spread. Huang and Huang (2012) estimate that in a model with counter-cyclical market risk premium the proportion of spread due to default risk is 33% for Baa and 63% for Ba, for 10-year bonds. Longstaff, Mithal and Neis (2005) show that the default component represents 56% for A-rated bonds, 71% for BBB-rated bonds, and 83% for BB-rated bonds.

This means that default probability is expected to be lower when corporate assets are valued by an agent with recursive preferences. Hence, preferences affect the firms' optimal decisions.

Finally, we consider an extension of the model in which cash flow volatility varies over the business cycle. This creates a second source of long-run risk. We show that the conditional equity risk premium is more countercyclical with time-varying cash flow volatility, as reported in Bansal and Yaron (2004). In presence of this additional source of risk, the proportion of the long-run risk component in the equity risk premium decreases to 76.1% in recession (from 89.9% without time-varying volatility) and increases to 67.6% in expansion (from 60.6% without time-varying volatility). In contrast, the conditional credit spread premium increases in both states.

An empirical analysis, using CRSP data over the period from 1952 to 2016, confirms that long-run risk is priced in U.S. firms, particularly in bad times. The compensation for long-run risk represents 73.7% of the excess return in a strategy that is long the high exposure portfolio and short the low exposure portfolio. The results also provide evidence that long-run risk better explains cross-sectional variation in equity risk premium than short-run risk (CCAPM) and that the effect is countercyclical, as the model predicts.

Overall, this paper helps us better understand the role of investor preferences and systematic risk in corporate asset valuation. The results show that the compensation for long-run risk appears to be the main component of the equity risk premium and corporate credit spreads. This paper thus highlights the importance of the risk premium associated with an expected cash flow growth rate that is time-varying and exposed to aggregate economic conditions. Yet the classical risk premium associated with the consumption beta is not negligible, as it accounts for one third of the total risk premium.

The remainder of the article is organized as follows. Section 2 reviews the literature. Section 3 describes the economy, details the sources of the systematic risks embedded into the model, and the pricing of claims, Sections 4 and 5 present respectively the data and the methodology proposed to measure the risk premiums, Section 6 studies the model's implications, Section 7 presents the empirical analysis while the Section 8 concludes. Proofs and others additional materials are contained in the Appendices.

2 Literature review

Asset pricing models like the capital asset pricing model (CAPM) of Markowitz (1952), Sharpe (1964) and Lintner (1965) (and subsequent models as in Black, Jensen and Scholes (1972); Fama and MacBeth (1973)) have postulated that the sensitivity to market or systematic volatility is the only risk that is needed to describe average returns.

However, since the 1970's, several papers (see Basu (1977); Banz (1981); Shanken (1985); Fama and French (1992; 1993)) proposed new approaches to improve the pricing performances and provide answers to some of the inconsistencies of the CAPM, shown for example by empirical analysis of cross-sectional asset data. In particular, the empirical asset pricing model proposed by Fama and French (1992; 1993) is perhaps the most important. They demonstrate that CAPM has virtually no explanatory power to explain the cross-section of average returns on assets of portfolios sorted by size and book-to-market equity ratios, among others characteristics.

A second important trend of the literature has developed theoretical models to improve the pricing performances of the CAPM via a consumption-based approach: the Consumption Capital Asset Pricing Model (CCAPM). The main innovation of the CCAPM models lied on the introduction of the macroeconomic risk into asset pricing. According to these first models, the risk premia should be proportion to the consumption beta (correlation between the firm's cash flow and the consumption).

One line of this literature built on the market-based CAPM of Sharpe (1964) and Lintner (1965), and on the Intertemporal CAPM developed by Merton (1973). The very first models of CCAPM were issued by Rubinstein (1976), Lucas (1978), Breeden and Litzenberger (1978), and Breeden (1979). However, the tests conducted on this line of the CCAPM models were not concluding.⁸ Others authors have also argued that there exist some issues regarding the accuracy of the consumption data due to the way they are recorded.⁹

Among the CCAPM models, another set of papers has introduced new features in the hope to enhance the pricing performances.¹⁰ In particular, Epstein and Zin (1989) and Weil (1989)

⁸See Hansen and Singleton (1983), Mehra and Prescott (1985), Chen, Roll and Ross (1986) and, Hansen and Jagannathan (1991).

⁹For example, Grossman, Melino and Shiller (1987) and Breeden, Gibbons and Litzenberger (1989)

¹⁰See Pye (1972) and Greenig (1986) with time-multiplicative utility functions. See also Sundaresan (1989),

have developed preferences, which allow for the separation between the intratemporal relative risk aversion (CRRA) from the elasticity of intertemporal substitution (EIS). By introducing this separability, it is now feasible to isolate, the aversion to future economic uncertainty from the aversion to the current correlation between consumption and firm's cash flows. The later type of aversion is measured through the consumption beta.

Alongside these papers, many empirical works have explored conditional versions of the consumption CAPM as for example Jagannathan and Wang (2007) and Lettau and Ludvigson (2001a; 2001b). In particular, Lettau and Ludvigson (2001a; 2001b) explore a conditional version of the consumption CAPM or CCAPM, by expressing the stochastic discount factor not in an unconditional linear model setting, as in traditional derivations of the CCAPM, but as a conditional factor model. Empirically, their model performs as good as the Fama-French three-factor model in explaining the cross-section of average returns of portfolios sorted by size and book-to-market value. Lettau and Ludvigson's conditional model captures the countercyclical risk premium and improves the performance of the CCAPM. The reason is that the correlation between stocks and the consumption growth increases more in bad times, when risk aversion is high, compare to the good times when the risk aversion is low. According to their findings, this conditionality on risk premia is missed by unconditional CCAPM models because they produce constant risk premia over time.

This indicates that the most suitable assets pricing models should incorporate not only investors preferences but also conditional pricing. Bansal and Yaron (2004) and recent papers by Bhamra, Kuehn and Strebulaev (2010a; 2010b) and Chen (2010), have successfully developed consumption-based models with a representative agent with Epstein-Zin-Weil type of preference and in a time-varying macroeconomic environment. This type of agent is not only risk-averse but also dislikes the uncertainty about the future macroeconomic conditions. These models allow to disentangle the impact of these two types of preferences on equity and debt and help resolve the credit spread puzzle and generate reasonable levels of equity risk premium. However, this paper emphasizes on quantifying the proportion of pure long-run risk into risk premia.

Constantinides (1990), Abel (1990), and Campbell and Cochrane (1999) with habit formation.

¹¹Predecessor works that have studies conditional versions of the CCAPM are Harvey (1991), Ferson and Harvey (1991), Jagannathan and Wang (1996), and Ferson and Harvey (1999).

3 The model setup

The economy consists of a representative agent and a number of firms. The agent provides capital to firms by buying equity and bond. There is no friction in the economy.

3.1 Environment

The representative agent has an exogenous stream of consumption C_t , which evolves as follows:

$$\frac{dC_t}{C_t} = \theta_{s_t} dt + \sigma dB_t, \quad s_t = \{R, E\}, \tag{1}$$

where θ_{s_t} is the consumption growth rate and σ is the consumption volatility. The standard Brownian motion under the physical measure B_t represents the continuous shocks to consumption.

Two states govern the economy - expansion, $s_t = E$ and, recession, $s_t = R$. The state-dependent consumption growth rate, θ_{s_t} , is procyclical so that $\theta_E > \theta_R$. The state of the economy, s_t , switches through a two-states Markov process. This random change in the state of the economy happens infrequently, and each state tends to be persistent.

The economy is populated by several firms. Consider that firm i has a stream of cash flows, denoted by $X_{t,i}$, given by the stochastic process:

$$\frac{dX_{t,i}}{X_{t,i}} = \mu_{s_t} dt + \sigma^f dB_{t,i}^f + \sigma^g dB_t^g, \quad s_t = \{R, E\}.$$
 (2)

The expected growth rate of the firm cash flow μ_{s_t} varies with global conditions. In particular, we assume that μ_{s_t} is procyclical so that $\mu_E > \mu_R$. Firms are more profitable in expansions than in recessions. Cash flows are subject to both firm-specific and global shocks, dB_t^f and dB_t^g , which are independent, by definition. Total volatility of cash flows equals $\sigma_X = \sqrt{(\sigma^f)^2 + (\sigma^g)^2}$. Consumption and global cash-flow shocks, B_t and B_t^g , are positively correlated such that $dB_t dB_t^g = \rho dt > 0$.

3.2 Short-run and long-run risk

The representative agent has Epstein-Zin-Weil preference.¹² This preference separates the impacts of risk aversion, γ , from the elasticity of intertemporal substitution, defined by the EIS coefficient, ψ . In equilibrium, the state-price density dynamic follows (see Appendix 8.2):

$$\frac{d\pi_t}{\pi_t} = -r_{s_t}dt + \frac{dM_t}{M_t} \tag{3}$$

$$= -r_{s_t}dt - \Theta^B dB_t - \Theta^P_{s_t}dN_{s_t,t}, \quad s_t = \{R, E\}$$
 (4)

where M_t is a martingale under the physical measure, $N_{s_t,t}$ a Poisson process which jumps upward by one whenever the state of the economy switches from the state s_t to the state $\overline{s_t} \neq s_t$, $\Theta^B = \gamma \sigma$ is the market price of risk due to Brownian shocks in the state s_t , $\Theta^P_{s_t} = 1 - \Delta_{s_t}$ is the market price of risk due to random changes of the state of economy from $s_t = \{R, E\}$, where Δ_{s_t} , is the change in the state-price density π_t at the transition time from state $s_t = \{R, E\}$. The stochastic discount factor prices two types of risk: short and long-run risk. We define them below.

3.2.1 Short-run risk

Firm cash flows are affected by global shocks that positively correlate with the agent's consumption. The agent is averse to investing in a firm that performs badly when her marginal utility of consumption is high, i.e. when consumption experiences bad shocks. Because these shocks are short-lived, we refer to this risk as to short-run risk. The agent requires a price for this risk when pricing financial claims and does so by evaluating firm assets as if a firm was less profitable than in reality. In this regard, the agent considers a risk-neutral measure of cash flows growth rate, $\hat{\mu}_{st}$, that reduces the physical rate μ_{st} by $\gamma\rho\sigma^g\sigma$. This adjustement directly depends on the constant coefficient of relative risk aversion γ , as in the standard consumption CAPM. With Epstein-Zin preferences, this adjustment is independent from the EIS.

¹²This type of utility function, developed by Kreps and Porteus (1978), Epstein and Zin (1989), Duffie and Epstein (1992), and Weil (1989).

3.2.2 Long-run risk

Time-varying consumption and cash flow expected growth rates introduces macroeconomic risk into the model. More precisely, the expected growth rate of consumption and cash flows jointly depend on the state of the economy. Hence, firm profitability decreases when the economy is in recession, in times when the agent sees a slowdown in her consumption growth. The agent dislikes recessions and demand a compensation for holding assets that perform poorly during such times. Further, the Epstein-Zin agent prefers to know when the next recessions will arrive, since she dislikes uncertainty about the future when her preferences satisfy $\psi > \frac{1}{\gamma}$. Yet future states of the economy arrive randomy and cannot be anticipated. Changes in the state of the economy are persistent and long-lived, and we thus refer to this risk as to *long-run risk*.

The agent accounts for such risk by pricing assets as if recessions last longer than in reality and so expansions last shorter than in reality. Call, λ_{s_t} , the probability per unit of time of leaving state s_t . Hence, the quantity $^1/\lambda_{s_t}$ is the expected duration of state s_t . However, recessions are shorter than expansions, so that $^1/\lambda_R < ^1/\lambda_E$. Thus, the physical probabilities λ_R and λ_E are converted to their risk-neutral counterparts $\hat{\lambda}_R$ and $\hat{\lambda}_E$ through the factor Δ_E , which is the change in the state-price density π_t at the transition time from expansion to recession. The risk-neutral probabilities per unit of time of changing state are then given by

$$\hat{\lambda}_E = \Delta_E \lambda_E \text{ and } \hat{\lambda}_R = \Delta_R \lambda_R = \frac{1}{\Delta_E} \lambda_R.$$
 (5)

When $\psi>\frac{1}{\gamma}$, the agent has preference for earlier resolution of the uncertainty, i.e. $\Delta_E>1$. In contrast, the agent is indifferent to uncertainty about the future states of the economy if $\Delta_E=\Delta_R=1$, i.e. when $\psi=\frac{1}{\gamma}$. This is the case of a CRRA agent who uses the actual transition probabilities to price assets, i.e. $\hat{\lambda}_E=\lambda_E$ and $\hat{\lambda}_R=\lambda_R$.

3.3 Asset prices

We now determine the price of corporate assets. Equity and debt are issued at initial time $s_0 = \{R, E\}$. Their values depend on the financing states s_0 as well as on the current $s_t = \{R, E\}$. We drop the firm i's subscript for convenience.

3.3.1 Bond price

The present debt value, $B_{s_0s_t}$, is the discounted coupon stream c_{s_0} before default plus the present value of the recovered firm asset liquidation value at default $(\phi_{s_D}A_{s_D})$, where ϕ_{s_t} is the state-dependent asset recovery rate and A_{s_t} is the firm asset liquidation value. The value of debt is equal to (see Appendix 8.6)

$$B_{s_0 s_t} = E_t \left[\int_t^{t_D} c_{s_0} \frac{\pi_u}{\pi_t} du \mid s_t \right] + E_t \left[\frac{\pi_u}{\pi_t} \phi_{t_D} A_{t_D} du \mid s_t \right], \quad s_t = \{R, E\}.$$
 (6)

The credit spread, $CS_{s_0s_t}$, for the present state $s_t = \{R, E\}$ is defined by

$$CS_{s_0s_t} = \frac{c_{s_0}}{B_{s_0s_t}} - r_{B,s_t}, \quad s_t = \{R, E\}$$
 (7)

where r_{B,s_t} is the perpetual risk-free discount.

3.3.2 Stock price

The stock value, $S_{s_0s_t}$, is the after-tax discounted value of future cash flows, X_t less coupon payments, c_{s_0} before bankruptcy is declared by the stockholders.

$$S_{s_0 s_t} = (1 - \tau) E_t \left[\int_t^{t_D} \frac{\pi_u}{\pi_t} (X_t - c_{s_0}) du \mid s_t \right], \quad s_t = \{R, E\}$$
 (8)

where t_D is the random default time. The absolute priority rule holds so that at default equity value is worthless.

The levered equity risk premium, $RP_{s_0s_t}$, for the current state $s_t = \{R, E\}$ is

$$RP_{s_0s_t} = \rho \Theta^B \sigma_{s_t}^B + \lambda_{s_t} \Theta^P \sigma_{s_t}^P, \quad s_t = \{R, E\}$$
(9)

with $\sigma^B_{s_t} = \frac{X_t}{S_{s_0s_t}} \frac{\partial S_{s_0s_t}}{\partial X_t} \sigma^g$ is the systematic volatility of stock returns caused by global cash flow shocks, where $S_{s_0s_t}$ represents the equity value and $\sigma^P_{s_t} = \frac{S_{s_0j}}{S_{s_0s_t}} - 1, \ s_t \neq j = \{R, E\}$ the volatility of stock returns caused by the change of state of the economy. The first term, $\rho\Theta^B\sigma^B_{s_t}$, corresponds to the compensation asked by investors to bear the short-run risk and the second term, $\lambda_{s_t}\Theta^P_{s_t}\sigma^P_{s_t}$, to the price associated to the long-run risk, where Δ_{s_t} , is the change in the state-price density π_t at the transition time from state $s_t = \{R, E\}$.

3.4 Firm decisions

The coupon value is chosen by shareholders at the time debt is issued to maximize the firm value $c_{s_0} = argmax(D_{s_0} + S_{s_0})$, where $s_0 = \{R, E\}$ is the financing state. The shareholders also determine the ex-ante default boundaries, X_{D,s_t} , corresponding to each state of the economy with the objective to optimize the equity value under the smooth-pasting condition, so that:

$$\frac{\partial S_{s_0 s_t}}{\partial X_t} \mid_{X_t = X_{D, s_t}} = 0, \quad s_t = \{R, E\}.$$
 (10)

4 Data and model calibration

This section presents the calibration of the model. Table 1 summarizes the parameter values. The model is calibrated to match the salient aspects of the market.

NBER dates are used to characterize the U.S. business cycle. The state of the economy can be either expansion (E) or recession (R). The switch from one state to another, which occurs randomly, is modeled by a Markov chain. The actual probability of transition from one state to

another, λ_{st} , the actual long-run probability of being in each state, f_{st} , and the actual rate of news arrival, denoted by p, are estimated using a two-state Markov-regime switching model on NBER recession dates over the period 1952Q1-2016Q4.¹³ The estimation approach is based on Hamilton (1989) and detailed in Appendix F. Real non-durables goods plus service consumption expenditures obtain from the Bureau of Economic Analysis is used as proxy of the aggregate consumption. The estimates of the actual probabilities of being in a expansion and in recession are respectively $f_E = 85.16\%$ and $f_R = 14.84\%$. When calibrating the conditional moments of consumption growth to the NBER recession dates, we obtain a U.S. consumption growth rate of $\theta_L = 0.65\%$ in recession and $\theta_E = 2.20\%$ in expansion, while its unconditional systematic volatility is $\sigma = 0.86\%$.

The cash flows data are the quarterly corporate profits (without inventory valuation and capital consumption adjustment) in billions of dollar before tax from the US Bureau of Economic Analysis. We use information over the period 1952Q1-2016Q4 to compute the moments of the representative firm cash flows growth. The conditional growth rate is thus equal to $\mu_R = -13.47\%$ in recession and $\mu_E = 5.62\%$ in expansion while its unconditional standard deviation $\sigma^g = 11.82\%$. The debt recovery rate is set to $\alpha_R = 40\%$ in recession and $\alpha_E = 70\%$ in expansion. Chen (2010) estimates that mean bond recovery rate is 41.8%. Longstaff, Mithal and Neis (2005) use a recovery rate of 50% and Duffee (1999), 44% using Moody's data. The corporate tax rate τ is set at 15%.

Regarding the representative agent's preferences, we consider a coefficient of risk aversion $\gamma=10$, a coefficient of elasticity intertemporal substitution (EIS) $\psi=1.5$, and an annual discount rate equal to $\beta=3.0\%$.

With this calibration, the default probability is very high in recession (around 12%), whereas this probability is less than 1% in expansion. The equity risk premium is 0.88% in expansion and 3.56% in recession, showing that the equity risk premium is countercyclical. Interestingly, the unconditional credit spread obtained by computing $f_R \times CS_{ER}^{full} + f_E \times CS_{EE}^{full}$, is equal to 117 bps¹⁵ and the unconditional equity risk premium, obtained by computing $f_R \times RP_{ER}^{full} + f_E \times RP_{EE}^{full}$, is

¹³Following Boguth and Kuehn (2013) and Lettau, Ludvigson and Wachter (2008), we use postwar data. Romer (1989) has shown that data on consumption recorded at the prewar period are not reliable since they contain significant measurement errors.

¹⁴The earnings data start in 1952 to match the consumption data.

¹⁵The historical average for Baa-rated firms is around 110 bps (Chen, 2010).

equal to 1.282%. This is consistent with the admitted observation that the risk levels embedded into stocks should not be significantly higher than those carried by bonds. This unconditional equity risk premium computed by assuming rational expectation, is consistent with what similar models will predict. Bhamra, Kuehn and Strebulaev (2010a; 2010b) and Chen (2010) simulate an economy consisting of a cross-section of firms which helps increase significantly the equity risk premium, with the objective to resolve the equity premium puzzle. However, this paper addresses the question of the relative impact of the short-run risk, as in CAPM model, versus the long-run risk, as in models that incorporate macroeconomic risk. Here, we focus on modeling an individual firm which is sufficient to achieve this goal. The optimal leverage ratio is equal to 43.24% in recession and 38.45% in expansion. Fama and French (2002), Chen, Collin-Dufresne and Goldstein (2009) and Huang and Huang (2012) estimate the average leverage ratio for Baa firms to be around 38 to 44%.¹⁶

5 Theoretical predictions

This part presents and discusses the theoretical predictions of the paper. The main objective is to decompose risk premia into SRR and LRR and see how their relative weights evolve for equity vs. bond, over time and at the cross-section. Without loss of generality, it is assumed that firms finance themselves in expansion. Predictions are done for the same economy but assuming various types of agents as explained in the part 5.1. In the section 5.7, we compare different economies in which the firm can account for investor preferences while choosing its optimal policies.

5.1 Separating the short-run and long-run risk premia

We here describe the procedure to separate long- and short-run risk components in the risk premia.

The agent only cares about short-run risk when she has CRRA preferences. We can then identify the short-run risk component of the equity premium by computing the equity risk premium in the case when $\gamma=\frac{1}{\psi}$. If the agent does not care about long-run risk, she uses actual probability of being in state s_t , f_{s_t} instead of the risk neutral one \hat{f}_{s_t} to price firm assets, i.e. $\triangle_{s_s}=1$. The term

¹⁶These estimates are in line with those issued by Moody's and Standard and Poor's.

 $\lambda_{s_t}\Theta_{s_t}^P\sigma_{s_t}^P$ becomes zero and the equity risk premium due to short-run risk equals $RP_{s_0s_t}^{SRR}=\rho\Theta^B\sigma_{s_t}^B$. The long-run risk component is then given by $RP_{s_0s_t}^{LRR}=RP_{s_0s_t}^{full}-RP_{s_0s_t}^{SRR}=\lambda_{s_t}\Theta_{s_t}^P\sigma_{s_t}^P$. In this hypothetical case, the agent has no aversion to short-run risk but dislike long-run risk. That is, she is indifferent to the fact that shocks to cash flows and consumption correlate positively and thus uses actual cash flow growth rate μ_{s_t} , instead of the risk neutral one $\hat{\mu}_{s_t}$ to price corporate claims.

Similarly, we first compute the credit spread in the power utility case $(\gamma=\frac{1}{\psi})$, denoted by $CS_{s_0s_t}^{\gamma=\frac{1}{\psi}}$. We then subtract the risk-neutral compensation for default risk $CS_{s_0s_t}^{RN}$, which is computed in the power utility case and no risk aversion $(\gamma=0)$. The difference between the two reflects the short-run risk premium of the credit spread, denoted by $CS_{s_0s_t}^{SRR}$. The long-run risk premium is then obtained as the difference between the total credit spread and the one obtained with power utility: $CS_{s_0s_t}^{LRR} = CS_{s_0s_t}^{full} - CS_{s_0s_t}^{full} - CS_{s_0s_t}^{SRR} - CS_{s_0s_t}^{RN}$.

5.2 Quantity vs price of risk into credit spreads

This section presents the findings concerning default risk and risk premia embedded into credit spread. The predictions are done for the full model which we compare with the three following special cases: i) when the agent does not care about long-run risk, ii) when the investor has no aversion to the short-run risk and, finally iii) when the investor is risk-neutral. The economy is the same for all cases, i.e. coupon and barriers are kept constant to those of the full model for all cases. We used the methodology explained in the section 5.1 to produce risk premia due to the two systematic risks (short- and long-run risks) and, then, the quantity of risk, which is also obtained from the case iii) predictions. The table 2 reports the main results for the four cases (full model, case i, case ii and case iii) and the table 3 reports the quantity of risk and the risk premia related to each type of risk as well as their weights into both the equity risk and credit spread premia.

Tables 2 and 3 [about here]

The proportion of the default risk, into the credit spread, is about 63% (or 74.34 bps), the remaining represents the credit spread premium (42.7 bps). In a risk-neutral setting, Chen, Collin-Dufresne and Goldstein (2009) find an average four-year Baa credit spreads of 86.8 bps and 5.6 bps for Aaa. This proportion of default risk in bond spread (i.e. 63%) stays constant across the states

of the economy. Similar structural models are designed to capture the average spread of A-rated and B-rated bonds.¹⁷ This finding also matches those of Longstaff, Mithal and Neis (2005), who have estimated that the default component accounts for 51% of the spread for AAA-rated and 71% for BBB-rated bonds.

As expected, equity risk premium carries no default risk. Indeed, for a risk-neutral agent to both short-run ($\gamma=0$) and the long-run ($\Delta_E=\Delta_R=1$) risks, the equity risk premium yields zero, as it can be seen with the equation 9. The figure 1 summarizes these findings.

The remaining sections focus on the risk prices embedded into corporate assets.

5.3 Importance of the preference for earlier resolution of the uncertainty

Left Panel of the figure 2 shows that the unconditional equity risk premium due to long-run risk weights 72.8%. Hence, a little more than one-fourth of equity risk premium originates from the correlation between firm cash flows and consumption. The same applies for the credit spread premium. As shown in the right panel of figure 2, 68.5% of the credit risk premium comes from investors' willingness to see a quick resolution of the uncertainty regarding the future states of the economy and the remaining is due to the short-run risk. This confirms that long-run risk is the main source of uncertainty in the pricing of corporate assets and this is particularly true in an economy in which the investor has preference for early resolution of the uncertainty.

Many reasons may explain the weak impact of CRRA preferences on corporate assets prices (as compared to long-run risk).

For equity pricing, this stylized fact has been extensively reported ¹⁸. Chen, Roll and Ross (1986) performed an empirical test of classical consumption-based models, which postulate that

¹⁷Bhamra, Kuehn and Strebulaev (2010a; 2010b) restrict their analysis, as in many other papers, to BBB-rated debt. As they pointed out the spreads of top graded bonds (AAA or AA-rated) are mostly dominated by factors other than credit risk, and that structural models work well for low-grade bonds (B-rated and below). Similarly, Chen (2010) also focus mainly on Baa-rated firms (Baa in Moody's being equivalent to BBB in the S&P notation system).

¹⁸See Hansen and Singleton (1983), Mehra and Prescott (1985) and, Hansen and Jagannathan (1991).

the main factor in asset pricing should be the covariance with the aggregate consumption as in the classical CCAPM models like Rubinstein (1976) or Lucas (1978). They found that this factor is not sufficient to explain stocks price. This lends support for the Fama and French (1993) finding that the correlation with the market alone cannot explain equity premium. Bansal and Yaron (2004) provide empirical support for a model with aggregate consumption and dividend processes that contain a small persistent expected growth rate component and a conditional volatility component, in conjunction with Epstein-Zin-Weil preferences to explain many asset pricing puzzles. This underpins further the preeminent role of long-run risk in driving corporate assets prices. Here, the poor impact of short-run risk, on equity value, can easily be proved. First, the associated equity premium is measured by $\gamma \rho \sigma^B_{s_t} \sigma$, with $\sigma^B_{s_t} = \frac{X_t}{S_{s_0s_t}} \frac{\partial S_{s_0s_t}}{\partial X_t} \sigma^g = \frac{\partial ln S_{s_0s_t}}{\partial ln X_{s_t}} \sigma^g$ the systematic volatility of stock returns induced by Brownian shocks. However, the U.S. consumption growth volatility σ is around 1% (see Table 1) in the data, the corporate earnings growth volatility σ^g is around 12% and, the correlation between earnings and the consumption ρ is about 25%. Because of the fact that the term $\frac{\partial lnS_{s_0s_t}}{\partial lnX_{s_t}}$ (which is the responsiveness of stock price to a change in cash flow) is bounded, $\gamma \rho \sigma_{s_t}^B \sigma$ will stay relatively small. Second, long-run risk premium equals to $(1 - \triangle_{s_t}) \sigma_{s_t}^P \lambda_{s_t}$. In the baseline calibration, the absolute of $(1-\triangle_{s_t})$ is around 0.2. $\sigma_{s_t}^P$, which measures the jump of conditional stock price when the economy changes state, equals to 27%. And λ_{s_t} unconditional value is equal to 0.31. Hence, the long-run risk component dominates. Consequently, short-run risk will have a limited impact on stocks. A recent work by Bali and Zhou (2016) proposes a conditional intertemporal capital asset pricing model with time-varying market risk and economic uncertainty. As in the approach developed in this paper, equity risk premium consists of two separate terms; the first term compensates for the standard market risk and the second term represents additional premium for economic uncertainty. They back up their model with empirical analysis to test whether time-varying conditional covariances of equity returns with the market or economic uncertainty predicts the time-series and cross-sectional variation in stock returns. This study also concludes that exposures to economic uncertainty better predict stock returns. This finding is also supported by Lettau, Ludvigson and Wachter (2008) who document that the fall in macroeconomic risk has lead to exceptional high aggregate stock prices in the 1990 and that this phenomenon persists today.

Regarding the bond pricing, using credit default swap (CDS) spreads, Tang and Yan (2010) document that average credit spreads are decreasing in GDP growth rate, but increasing in GDP growth volatility and that, credit spreads are lower for smaller systematic jump risk. These results support the role of macroeconomic uncertainties in corporate bonds value as well.

As pointed out in Fama and French (1993), common factors seem to drive the returns on stocks and bonds. They document that stock and bond returns are related through shared variation in the bond-market factors. Besides low-grade corporates, the bond-market factors, namely maturity and default risk (but not directly the market risk) capture the common variation in bond returns. Most importantly, they have identified five factors including the market risk that may explain average returns on both stocks and bonds. The implications of these results are twofold. First, the aversion due to the correlation between consumption and cash flows (a proxy for the market risk) does not have significant repercussions on credit risk spread premium. Second, others common factors, which at least some of them are likely to vary with macroeconomic conditions, capture more of risk premia than market risk alone.

This paper provides support to these results and gives a better understanding as to why the long-run risk is dominant.

5.4 Countercyclical risk premia

The equity risk premium is 3.59% in bad times and 0.88% in good times. In particular, the compensation asked by investors to bear the risk associated with the uncertainty about future economic conditions represents 89.9% of the equity risk premium or an annual required rate of return of 3.23%, in bad times, while it worths 60.62% in recession or 0.53%.

On average, the credit spread premium is 49.7 bps in bad times and 41.5 bps in good times. However, Regardless of the state of the economy, the proportion of the credit spread premium due to the long-run risk represents 70% of the total risk premium while the remaining 30% comes from the sensitivity of the firm cash flow to the consumption.

Lettau and Ludvigson (2001a; 2001b)¹⁹ explore a conditional version of the consumption CAPM and found that their model performs as good as the Fama-French three-factor model in explaining

¹⁹Bekaert, Engstrom and Xing (2009), Bansal, Kiku, Shaliastovich and Yaron (2014) and Bali and Zhou (2016) also provide support for time-varying risk prices.

the cross-section of average returns of portfolios sorted by size and book-to-market value. They document that countercyclical risk premium help improve assets pricing.

5.5 Investors' preferences

We start by analyzing the proportion of LRR vs. SRR into risk premia to the risk aversion coefficient. Unsurprisingly, as shown in the figure 3, when the investor's risk aversion increases, both prices due to LRR and SRR go up. This means higher credit spread premium and equity risk premium when investors are more risk averse. Theoretically, the term $\gamma\rho\sigma_{s_t}^B\sigma$, that represents the risk premium due to investor's risk aversion clearly increases with the coefficient γ . The risk premium due to the uncertainty about future states of the economy is driven by the term $(1-\Delta_{s_t})\sigma_{s_t}^P\lambda_{s_t}$. However, the jump in the state-price density Δ_{s_t} decreases with respect the coefficient of risk aversion γ , increasing the premium due to the LRR increases. More importantly, the weight of the LRR in the risk premia increases when investors are more risk averse. Tang and Yan (2010) document that credit spreads are lower when investor sentiment is high, i.e. when the risk aversion is low.

Concerning the EIS, high ψ leads to high credit spread premium but the proportion of the premium due to the uncertainty about future states of the economy decreases. The equity risk premium is lower for higher EIS coefficient, however the weights of LRR and SRR remains constant.

5.6 Firm characteristics and cross-sectional asset pricing

This section attempts to identify cross-sectional assets pricing factors among firms. Investors' preferences may affect differently asset prices according to firms' characteristics. In particular, leverage, idiosyncratic volatility and firm performances are explored. Cross-sectional pricing implications of the time-varying macroeconomic conditions have studied by Boguth and Kuehn (2013), Bansal, Kiku, Shaliastovich and Yaron (2014) and Tedongap (2014), whereas this paper explores the impacts of time-varying expected growth rates.

5.6.1 Idiosyncratic volatility

The idiosyncratic volatility introduces cross-sectional heterogeneity in risks across firms.

As asset pricing theory (Merton (1973)) suggests, the equity risk premium should not change with respect to the idiosyncratic volatility. One reason is that shareholders are able to get rid of firm level idiosyncratic volatility by diversification. In fact, empirical works have found diverging results regarding the relation between firms' specific volatility and average stock returns. Fama and French (1993) model suggest that creating portfolios by sorting them on specific volatility will produce no difference in average stock returns. Hence, a firm's stock price is almost not affected by its specific volatility. Malkiel and Xu (2002) and Jones and Rhodes-Kropf (2003) argue that in a market in which investors are not able to diversify risk, they will demand a premium for holding stocks with high idiosyncratic volatility. In contrast, Ang, Hodrick, Xing and Zhang (2006) and Babenko, Boguth and Tserlukevich (2016) examine this cross-sectional relationship between idiosyncratic volatility and average stock return, where idiosyncratic volatility is defined relative to the standard Fama and French (1993) model. They find that stocks with high idiosyncratic volatility have low average returns. As in Ang, Hodrick, Xing and Zhang (2006) and Babenko, Boguth and Tserlukevich (2016), this paper's approach provides a negative relationship between idiosyncratic volatility and equity risk premium (see upper panels of Figure 7). Equity is an option on firm's asset. Hence, an increase in volatility makes it worths more, reducing risk premium. However, this reduction is mainly due to the LRR component. This paper further predicts that the weights of LRR and SRR in equity premium do not change with idiosyncratic volatility.

Regarding bond valuation, the level of the risk premium embedded into the credit spread increases with firm specific volatility. Moreover, even if the impact of LRR is higher, SRR's impact also increases as the firm specific volatility goes up. An increase in volatility from 25% to 35% leads to a nearly 80 % increase of the credit spread premium, i.e. from 42.7 to 77.5 bps. In the cross-section, Tang and Yan (2010) found out that firm-level cash flow volatility raises credit spreads. Exploring the quality of a firm's information disclosure on the term structure of its bond yield spreads, Yu (2005) documents that firms with high volatility behave differently compared to firms with low volatility. High idiosyncratic volatility makes coupon more uncertainty. Bondholders will fear any shock to firm cash flows, since the coupon payment is at risk. Therefore, as shown

in bottom left panel of Figure 7, bondholders will increase more the compensations for both SRR and LRR. In term of proportion, the impact of the LRR is reduced for firms with high level of idiosyncratic volatility, meaning that bondholders in this case are sensitive to current small shocks (see bottom right panel of Figure 7).

Figure 7 and Table 7 [about here]

5.6.2 Leverage

In this section, we explore the importance of the preferences in the cross-section of firm according to their leverage. The pricing implication of the leverage has been extensively studied in the literature. Since, the capital structure decisions are endogenized, leverage cannot be used as a parameter. Another parameter, namely the bankruptcy cost, is used after verifying that decreasing the bankruptcy cost translates into higher optimal leverage (See Leland (1994)).

An increase of 10% in bankruptcy cost translates into a reduction of about 5% in leverage. In turn, higher leverages induce higher credit spread and equity risk premium due to both investor risk aversion and preference for early resolution of the uncertainty. As shown in the Table 6, an increase of 5% in the optimal leverage, increases the equity risk premium and credit spread premium by respectively 3% and 5%. Hence, firms with high optimal leverage have also higher risk premia in level. These findings corroborate Bhandari (1988) who find empirical evidence that there exists a positive relation between leverage and expected stock returns. This characteristic is shown to have pricing implication for bond also (Yu, 2005).

Furthermore, the proportion of LRR in equity risk premium slightly increases for highly levered firms. High leverage means low equity value. So bondholders fear more the reduction of equity value as the state of the economy changes from expansion to recession. In contrast, the proportion of LRR in credit spread premium decreases since high leverage implies also high default probability, making the firm very vulnerable even to current small shocks.

Figure 6 and Table 6 [about here]

5.6.3 Firm performance

This section explores the role of the preferences in the cross-section of firms according to the cash flows level. Indeed, high cash flow ceteris paribus means better financial wealth. As such, the cash flows level is a proxy for firm performance. As it is shown in the figure 5, risk premia for equity and bond are high for firms with low cash flows level. Compared to the baseline case, both the credit spread premium and the equity risk premium increases by around 30 to 35% when the firm cash flows level is reduced by 25%. More importantly, the increase in risk premia is more pronounced for firms more distress firms. This approach predicts that equity risk premium could be as high as 15% and more. As reported in Martin (2017), the equity premium is extremely volatile and rose above 20% in the midst of the 2008 crisis. This captures the observation that firm performance is a pricing factor. It confirm the claim of Fama and French (1992) that cross-sectional irregularities are related to risk of financial distress. When market conditions deteriorate, firms are more likely to exhibit more marked cross-sectional differences.

Regarding the weights of LRR vs. SRR, the predictions are the inverse of the leverage. The idea is that firms with better performances tend to have a low leverage.

Figure 5 [about here]

5.7 Preferences and firm optimal decisions

Investors' preferences modify not only the pricing of corporate claims but also impact firm's optimal default and debt policies. In reality, managers account for investor's preference by adjusting the firms' optimal decisions accordingly. We consider the full model economy along with three special case economies (see Table 8), an economy populated by an agent who i) is risk averse (to the SRR), ii) has preference for quicker resolution of uncertainty and, iii) is risk neutral. Now, the firm endogenizes preferences while choosing its optimal policies which in turn affect the price of corporate claims.

Table 8 and Figure 8 [about here]

When investors only care about LRR, the firm is more conservative when choosing its debt level as compared to an economy in which they care only about the correlation between cash flows and

consumption. The optimal default barrier is also lower with LRR as well. Hence, the firm alters more its optimal debt and default level in presence of agent who cares more about LRR than when the agent only takes into account only SRR. The firm adjusts its policies to reduce its risk exposure, in particular by reducing its debt level. Together with these reduction, the default probability and leverage are, according to expectations, reduced. Yet, despite these adjustments, the total credit spread and equity risk premium are higher in an economy populated with a representative agent who cares about future macroeconomic conditions than in an economy with a risk averse agent. The reason is that the quantity of risk increases much that the risk premium when each source of risk is added in the economy. This is understandable since, in the case of a agent risk neutral agent, the firm will opt for a higher optimal default and debt policies than for the others economies.

5.8 Incorporating time-varying volatility

Now let consider that the volatility of consumption and cash flow growth are state-dependent. The consumption and cash flow volatilities, respectively σ_{s_t} and $\sigma_{s_t}^g$, are countercyclical in nature, implying that $\sigma_E < \sigma_R$ and $\sigma_E^g < \sigma_R^g$. Hence in the equations 2 and 1, σ and σ^g are replaced by σ_{s_t} and $\sigma_{s_t}^g$, where $\sigma_R = 1.23\%$ in recession and $\sigma_E = 0.80\%$ in expansion and $\sigma_R^g = 24.61\%$ in recession and $\sigma_E^g = 9.59\%$ in expansion.

Equity risk premium is more countercyclical with time-varying volatility of consumption and cash flow, as reported in Bansal and Yaron (2004). This feature introduces more (less) risk in recessions (expansion). With time-varying volatility, the conditional total risk premium in equity risk premium is 4.45% in recession as opposed to 3.56% without and 0.80% in expansions as compared to 0.88% without. However, because of the countercyclicality of volatility, the proportion of the LRR becomes only 76.6% (instead of 89.9% without time-varying volatility) and 67.6% (instead of 60.6% without time-varying volatility).

Credit spread premium increases in both states, respectively from 49.64 to 58.46 bps in recessions and from 41.51 to 47.21 bps in expansion when adding time-varying volatility to the model.

Table 9, Table 10 and Table 11 [about here]

This approach allows to retrieve separately from discount rate news, risk premia due to con-

sumption volatility news as in Boguth and Kuehn (2013) and Bansal, Kiku, Shaliastovich and Yaron (2014).

6 Empirical tests

In this section, we test the pricing performance of the model, in particular, the relative importance of short-run vs. long-run risk into the equity risk premium. When investors have preference for early resolution of the uncertainty, they ask, in addition to the premium for the correlation between consumption and cash flows, a compensation for cash flows exposure to the conditional moments of consumption growth. A cross-section portfolio sorting analysis shows that the exposure to LRR is priced and commands a higher compensation than the exposure to SRR.

6.1 Data

The stock sample includes all common stocks traded on the NYSE, Amex, and Nasdaq exchanges from 1952 to 2016. Data are from CRSP. The database consists of 1,048,572 firm-months data for a total of 8,600 firms.

Long-run risk is given by a firm's exposure to the conditional moments of consumption growth. Investors cannot observe the states of the economy. Hence, they estimate the conditional mean and volatility of consumption growth, which are assumed to follow independent Markov processes. Two different measures are used to proxy for the conditional moments of consumption growth. The first is the Hamilton (1989) approach and the second is the consumption forecasts of analysts.

6.2 Hamilton (1989) procedure

The perceived first $(\hat{\mu}_t)$ and second moments $(\hat{\sigma}_t)$ of consumption growth may be obtained by using Hamilton (1989) procedure as in Boguth and Kuehn (2013).

6.2.1 Step 1

To ensure that each source of risk is measured independently, we estimate the following specifications:

$$(I) R_{i,t} - R_{f,t} = \alpha_{\tau} + \beta_{c,\tau}^{i} \triangle c_{t} + \epsilon_{t}$$

$$(11)$$

$$(II) R_{i,t} - R_{f,t} = \alpha_{\tau} + \beta_{\mu,\tau}^{i} \triangle \hat{\mu}_{t} + \epsilon_{t}$$

$$(12)$$

$$(III) R_{i,t} - R_{f,t} = \alpha_{\tau} + \beta_{\sigma,\tau}^{i} \triangle \hat{\sigma}_{t} + \epsilon_{t}$$

$$(13)$$

where $t \in [\tau - 39, \tau]$, $\triangle c_t = \ln{(c_t)} - \ln{(c_{t-1})}$, $\triangle \hat{\mu}_t = \hat{\mu}_t - \hat{\mu}_{t-1}$, $\triangle \hat{\sigma}_t = \hat{\sigma}_t - \hat{\sigma}_{t-1}$, and $R_{i,t} - R_{f,t}$ is the stock i's excess returns. The risk loadings β_c^i , β_μ^i and β_σ^i are respectively obtained from timeseries regressions of individual stock returns on either log consumption growth, Δc_t , or changes in the perceived conditional wolatility, $\Delta \hat{\sigma}_t$. Ten years are needed to obtain the first risk loadings. To ensure the stability of the risk loadings estimates, firms are required to have at least 15 years of continuous observations during the sample period. The final sample covers 1,830 firms. These time-series regressions are performed using the specification (I), (II) and (III). We consider univariate regressions to avoid multicollinearity issues, as all three measures capture information related to the U.S. business cycle.

6.2.2 Step 2

We then sort the risk loading estimates to form five portfolios and compute the average excess return associated with each portfolio. This latter approach has several important advantages relative to the Fama–MacBeth regressions, as discussed in Boguth and Kuehn (2013). In particular, it allows me to allocate stocks according to their risk loadings to form portfolios, making this procedure suitable for a risk-based analysis. Panels A of Tables 12, 13 and 14 report the unconditional results. Portfolios are also sorted conditional on the state of the economy, i.e., NBER expansions vs. recessions. Panels B and C of Tables 12, 13 and 14 report the conditional results.

Table 12, Table 13 and Table 14 [about here]

6.2.3 Results

The results indicate that the only source of risk that is statistically significant is the exposure to a change in consumption growth volatility. Notably, the excess returns of the high-minus-low portfolio are particularly large in bad times.

A strategy that consists in buying high exposure portfolios and shorting low exposure portfolios yields on average 0.093% monthly (compensation for SRR) plus 0.078% + 0.141% (compensations for the two components of LRR), meaning that the proportion of the LRR is 73.73%, close to the theoretical estimate (74.6%). The average excess return is countercyclical for both types of risk. The average compensation for SRR is 0.165% monthly in recession and 0.078% in expansion. The average compensation for LRR is 0.288% (0.033%) in recession (expansion) for the exposure to the mean of consumption growth, while it is 0.566% (0.050%) for the exposure to consumption growth volatility. Therefore, the total LRR component of the equity risk premium is greater than the compensation for short-run risk, particularly during NBER recessions. This confirms the prediction that the LRR impact on equity premium is countercyclical. The proportion of LRR in equity premium is 83% in recession, the theoretical estimate being around 77 to 90% and is 52% in expansion, as compared to a theoretical estimate of 60 to 67%.

6.3 Survey of professional forecasters

Analysts' economic forecasts about future consumption can also be viewed as proxy for ex-ante measure of macroeconomic conditions. In this case, mean and dispersion of forecasts represent respectively the perceived first and second moments of consumption growth. Survey of professional forecasts on consumption are available since 1981Q3 and provided by the Federal Reserve Bank of Philadelphia.

6.3.1 Step 1

The following specifications is tested:

$$(IV) R_{i,t} - R_{f,t} = \alpha_{\tau} + \beta_{c,\tau}^{i} \triangle c_t + \beta_{\mu^{SPF},\tau}^{i} \mu_t^{SFP} + \beta_{\sigma^{SPF},\tau}^{i} \sigma_t^{SFP} + \epsilon_t$$

$$(14)$$

where $t \in [\tau - 39, \tau]$, $\triangle c_t = \ln{(c_t)} - \ln{(c_{t-1})}$ and $R_{i,t} - R_{f,t}$ is the stock i's excess returns. The risk loadings β_c^i , $\beta_{\mu^{SPF}}^i$ and $\beta_{\sigma^{SPF}}^i$ are respectively obtained from five years rolling regressions of excess returns on log consumption growth, $\triangle c_t$, or/and mean in consumption growth forecasts, μ_t^{SFP} , or/and dispersion in consumption growth forecasts, σ_t^{SFP} . The final sample consists of 4,054 firms.

6.3.2 Step 2

We then sort the stocks into equally-weighted quintile portfolios by their risk loading estimates and compute the average excess return associated with each portfolio. Panel A of Table 15 reports the t-statistics obtained from the high-minus-low portfolios for the unconditional results. Portfolios are also sorted conditional on the state of the economy. Panel B of Table 15 reports the t-statistic obtained from the high-minus-low portfolios of the conditional results. Table 16 shows average returns and t-statistics when regressing stocks excess returns on all three factors as in specification (IV).

Table 15 and Table 16 [about here]

6.3.3 Results

In the case of unconditional pricing, high-minus-low portfolios returns are statistically significant for consumption growth mean forecasts, under the assumption that only one factor is priced on the cross-section of individual stocks (see the first row of Panel A in Table 15). The *t*-statistic related to the high-minus-low returns when using expected consumption growth, as pricing factor, is about 2 whereas its dispersion is not statistically significant nor is consumption growth (*t*-statistic of 1.28). Combining two factors among the three or all three factors yields no statistically significant results.

Assuming that asset are priced conditional on the state of the economy allows to obtain better results. The main finding is that expected growth rate tends to have a good predictive power in good times (even if excess returns are economically significant in bad times, see next paragraph) whereas dispersion in expected consumption growth is consistently priced in bad times.

Again a strategy, that consists in investing adequately into high-minus-low portfolios, produces on average 0.218% monthly return (compensation for SRR) plus 0.263% + 0.047% (compensations for the two components of LRR), meaning that the proportion of the LRR is about 60%. The average excess return is countercyclical for both types of risk. The average compensation for SRR is 1.331% monthly in recession and 0.308% in expansion. It is worthwhile to observe that the *t*-statistic of medium and high betas portfolio are not significant and their returns seem too low, meaning that the compensation for the SRR in recession measured here, is presumably too

high. The average compensation for LRR is 0.717% (0.468%) in recession (expansion) for the exposure to the mean of consumption growth, while it is 0.770% (0.014%) for the exposure to consumption growth volatility. Therefore, the total LRR component of the equity risk premium is greater than the compensation for SRR. The proportion of LRR is 64% in expansion, as compared to a theoretical estimate of 60 to 67%. Notably, besides the observation that the medium portfolio obtained from the exposure to expected consumption growth (which *t*-statistic is clearly below the four others), the two components of LRR are priced in recessions.

Overall, these findings provide evidences that long-run risk better explains cross-sectional variations in the equity risk premium than short-run risk (CCAPM) and that the effect is countercyclical, as the model predicts.

7 Concluding remarks

Investor's preferences influence firm decisions, hence affecting the valuation of the firm's claims. Thus, a better assessment of the systematic risks that firms face is a good way to apprehend the risk premium embedded into corporate securities.

This paper proposes a new approach, constructed in a consumption based asset pricing environment, to understand the impact of investor preferences for the pricing of stocks and corporate bonds. This pricing of the firm's assets is done after considering two sources of systematic risks that can affect expected cash flows, i.e. the time-varying macroeconomic conditions and the instantaneous correlation between consumption and firm's cash flows. Firms' react to investor's preferences regarding these systematic risks by adjusting their default and debt decisions, in order to reduce the impacts of these preferences. The present approach allows putting the emphasis on the levels of the risk premia associated to each of these systematic risks and in various situations. This approach provides evidence that the preference for the earlier resolution of the uncertainty is preponderant into the equity risk premium, which we back with empirical tests and credit spread. This study also shows that equity and bond react differently to the investors' preferences and predicts the proportion of short-run versus long-run risks vary according to firm characteristics.

References

- Abel, A. B. (1990), Asset prices under habit formation and catching up with the joneses, *American Economic Review* 80(2), 38–42.
- Ang, A., Hodrick, R. J., Xing, Y. and Zhang, X. (2006), The cross-section of volatility and expected returns, *Journal of Finance* 61(1), 259–299.
- Babenko, I., Boguth, O. and Tserlukevich, Y. (2016), Idiosyncratic cash flows and systematic risk, Journal of Finance 71(1), 425–456.
- Bali, T. and Zhou, H. (2016), Risk, uncertainty, and expected returns, *Journal of Financial and Quantitative Analysis* 51(3), 707–735.
- Bansal, R., Kiku, D., Shaliastovich, I. and Yaron, A. (2014), Volatility, the macroeconomy, and asset prices, *The Journal of Finance* 69(6), 2471–2511.
- Bansal, R. and Yaron, A. (2004), Risks for the long run: A potential resolution of asset pricing puzzles, *Journal of Finance* 59(4), 1481–1509.
- Banz, R. W. (1981), The relationship between return and market value of common stocks, *Journal of Financial Economics* 9(1), 3–18.
- Basu, S. (1977), The investment performance of common stocks in relation to their price-earnings ratios: A test of the efficient market hypothesis, *Journal of Finance* 32(3), 663–82.
- Bekaert, G., Engstrom, E. and Xing, Y. (2009), Risk, uncertainty, and asset prices, *Journal of Financial Economics* 91(1), 59–82.
- Bhamra, H. S., Kuehn, L.-A. and Strebulaev, I. A. (2010a), The aggregate dynamics of capital structure and macroeconomic risk, *Review of Financial Studies* 23(12), 4187–4241.
- Bhamra, H. S., Kuehn, L.-A. and Strebulaev, I. A. (2010b), The levered equity risk premium and credit spreads: A unified framework, *Review of Financial Studies* 23(2), 645–703.
- Bhandari, L. C. (1988), Debt/equity ratio and expected common stock returns: Empirical evidence, Journal of Finance 43(2), 507–528.
- Black, F., Jensen, M. and Scholes, M. (1972), *The Capital Asset Pricing Model: Some Empirical Tests*, Studies in the Theory of Capital Markets.
- Boguth, O. and Kuehn, L.-A. (2013), Consumption volatility risk, *Journal of Finance* 68(6), 2589–2615.
- Breeden, D., Gibbons, M. R. and Litzenberger, R. (1989), Empirical tests of the consumption-oriented capm, *Journal of Finance* 44(2), 231–262.
- Breeden, D. T. (1979), An intertemporal asset pricing model with stochastic consumption and investment opportunities, *Journal of Financial Economics* 7(3), 265–96.

- Breeden, D. T. and Litzenberger, R. H. (1978), Prices of state-contingent claims implicit in option prices, *Journal of Business* 51(4), 621–651.
- Campbell, J. and Cochrane, J. (1999), By force of habit: A consumption-based explanation of aggregate stock market behavior, *Journal of political Economy* 107(2), 205–251.
- Chen, H. (2010), Macroeconomic conditions and the puzzles of credit spreads and capital structure, *Journal of Finance* 65(6), 2171–2212.
- Chen, L., Collin-Dufresne, P. and Goldstein, R. S. (2009), On the relation between the credit spread puzzle and the equity premium puzzle, *Review of Financial Studies* 22(9), 3367–3409.
- Chen, N., Roll, R. and Ross, S. A. (1986), Economic forces and the stock market, *Journal of Business* 59(3), 383–403.
- Constantinides, G. M. (1990), Habit formation: A resolution of the equity premium puzzle, *Journal of Political Economy* 98(3), 519–543.
- Duffee, G. R. (1999), Estimating the price of default risk, *Review of Financial Studies* 12(1), 197–226.
- Duffie, D. and Epstein, L. G. (1992), Stochastic differential utility, Econometrica 60(2), 353-94.
- Elton, E. J., Gruber, M. J., Agrawal, D. and Mann, C. (2001), Explaining the rate spread on corporate bonds, *Journal of Finance* 56(1), 247–277.
- Epstein, L. G. and Zin, S. E. (1989), Substitution, risk aversion, and the temporal behavior of consumption and asset returns: A theoretical framework, *Econometrica* 57(4), 937–69.
- Fama, E. F. and French, K. R. (1992), The cross-section of expected stock returns, *Journal of Finance* 47(2), 427–65.
- Fama, E. F. and French, K. R. (1993), Common risk factors in the returns on stocks and bonds, Journal of Financial Economics 33(1), 3–56.
- Fama, E. F. and MacBeth, J. D. (1973), Risk, return, and equilibrium: Empirical tests, *Journal of Political Economy* 81(3), 607–36.
- Fama, E. and French, K. (2002), Testing trade-off and pecking oder predictions about dividends and debt, *Review of Financial Studies* 15(1), 1–33.
- Ferson, W. E. and Harvey, C. R. (1991), The variation of economic risk premiums, *Journal of Political Economy* 99(2), 385–415.
- Ferson, W. E. and Harvey, C. R. (1999), Conditioning variables and the cross section of stock returns, *Journal of Finance* 54(4), 1325–1360.
- Fischer, E. O., Heinkel, R. and Zechner, J. (1989), Dynamic capital structure choice: Theory and tests, *Journal of Finance* 44(1), 19–40.

- Goldstein, R., Ju, N. and Leland, H. (2001), An ebit-based model of dynamic capital structure, *Journal of Business* 74(4), 483–512.
- Greenig, D. (1986), Non-separable preferences, stochastic returns and intertemporal substitution in consumption. Princeton University dissertation.
- Grossman, S. J., Melino, A. and Shiller, R. J. (1987), Estimating the continuous-time consumption-based asset-pricing model, *Journal of Business and Economic Statistics* 5(3), 315–327.
- Hackbarth, D., Miao, J. and Morellec, E. (2006), Capital structure, credit risk, and macroeconomic conditions, *Journal of Financial Economics* 82(3), 519–550.
- Hamilton, J. (1989), A new approach to the economic analysis of nonstationary time series and the business cycle, *Econometrica* 57(2), 357–84.
- Hansen, L. P. and Jagannathan, R. (1991), Implications of security market data for models of dynamic economies, *Journal of Political Economy* 99(2), 225–262.
- Hansen, L. P. and Singleton, K. J. (1983), Stochastic consumption, risk aversion, and the temporal behavior of asset returns, *Journal of Political Economy* 91(2), 249–265.
- Harvey, C. R. (1991), The world price of covariance risk, Journal of Finance 46(1), 111-157.
- Huang, J.-Z. and Huang, M. (2012), How much of the corporate-treasury yield spread is due to credit risk?, *Review of Asset Pricing Studies* 2(2), 153–202.
- Jagannathan, R. and Wang, Z. (1996), The conditional capm and the cross-section of expected returns, *Journal of Finance* 51(1), 3–53.
- Jagannathan, R. and Wang, Z. (2007), Lazy investors, discretionary consumption, and the cross-section of stock returns, *Journal of Finance* 62(4), 1623–1661.
- Jones, C. M. and Rhodes-Kropf, M. (2003), The price of diversifiable risk in venture capital and private equity, *Unpublished working paper*, *Columbia University*.
- Kreps, D. M. and Porteus, E. L. (1978), Temporal resolution of uncertainty and dynamic choice theory, *Econometrica* 46(1), 185–200.
- Leland, H. E. (1994), Corporate debt value, bond covenants, and optimal capital structure., *Journal of Finance* 49(4), 1213–52.
- Lettau, M. and Ludvigson, S. (2001a), Consumption, aggregate wealth, and expected stock returns, Journal of Finance 56(3), 815–849.
- Lettau, M. and Ludvigson, S. (2001b), Resurrecting the (c)capm: A cross-sectional test when risk premia are time-varying, *Journal of Political Economy* 109(6), 1238–87.
- Lettau, M., Ludvigson, S. and Wachter, J. (2008), The declining equity premium: What role does macroeconomic risk play?,, *Review of Financial Studies* 21(4), 1653–87.

- Lintner, J. (1965), Security prices, risk, and maximal gains from diversification, *Journal of Finance* 20(4), 587–615.
- Longstaff, F. A., Mithal, S. and Neis, E. (2005), Corporate yield spreads: Default risk or liquidity? new evidence from the credit default swap market, *Journal of Finance* 60(5), 2213–2253.
- Lucas, R. E. (1978), Asset prices in an exchange economy, Econometrica 46(6), 1429-1445.
- Malkiel, B. G. and Xu, Y. (2002), Idiosyncratic risk and security returns. Working paper, University of Texas at Dallas.
- Markowitz, H. M. (1952), Portfolio selection, Journal of Finance 7(1), 77-91.
- Martin, I. (2017), What is the expected return on the market?, *Quarterly Journal of Economics* 132(1), 367–433.
- Mehra, R. and Prescott, E. C. (1985), The equity premium: A puzzle, *Journal of Monetary Economics* 15(2), 145–161.
- Merton, R. C. (1973), The theory of rational option pricing, *Bell Journal of Economics* 4(1), 141–183.
- Pye, G. (1972), Lifetime Portfolio Selection with age Dependent Risk Aversion, Mathematical methods in investment and finance, Amsterdam: NorthHolland Publishing Co., 49-64.
- Romer, C. D. (1989), The pre-war business cycle reconsidered: New estimates of gross national product 1869-1908, *Journal of Political Economy* 97(1), 1–37.
- Rubinstein, M. (1976), The valuation of uncertain income streams and the pricing of options, *Bell Journal of Economics and Management Science* 7(2), 407–25.
- Shanken, J. (1985), Multivariate tests of the zero-beta capm, *Journal of Financial Economics* 14(3), 327–48.
- Sharpe, W. F. (1964), Capital asset prices: A theory of market equilibrium under conditions of risk, *Journal of Finance* 19(3), 425–42.
- Strebulaev, I. (2007), Do tests of capital structure theory mean what they say?, *Journal of Finance* 62(4), 1747–1787.
- Sundaresan, S. M. (1989), Intertemporally dependent preferences and the volatility of consumption and wealth, *Review of financial Studies* 2(1), 73–89.
- Tang, D. Y. and Yan, H. (2010), Market conditions, default risk and credit spreads, *Journal of Banking & Finance* 34(4), 743–753.
- Tedongap, R. (2014), Consumption volatility and the cross-section of stock returns, *Review of Finance* 19(1), 367–405.

- Weil, P. (1989), The equity premium puzzle and the risk free rate puzzle, *Journal of Monetary Economics* 24(3), 401–21.
- Yu, F. (2005), Accounting transparency and the term structure of credit spreads, *Journal of Financial Economics* 75(1), 53–84.

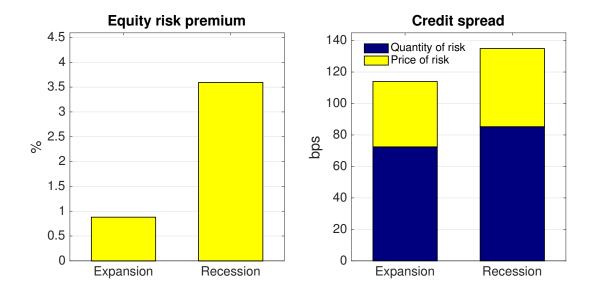


Figure 1: Quantity vs price of risk. This figure compares the quantity and the price of risk embedded into the credit spread. The right panel display predictions on conditional credit spread, the left panel on equity risk premium. The quantity and prices of systematic risk are computed using the procedure explained in the section 5.1. We use the parameters of the baseline calibration (see Table 1).

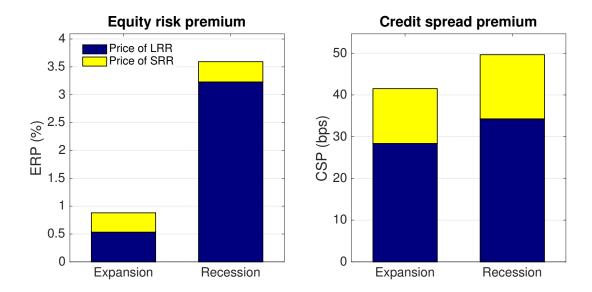


Figure 2: **Prices of long- vs short-run risk.** This figure compares the prices of the long- and short-run risk embedded into equity risk premium and credit spread premium. The right panel display predictions on the conditional risk premium in the credit spread premium and the left panel in the equity risk premium. The prices of systematic risk are computed using the procedure explained in the section 5.1. We use the parameters of the baseline calibration (see Table 1).

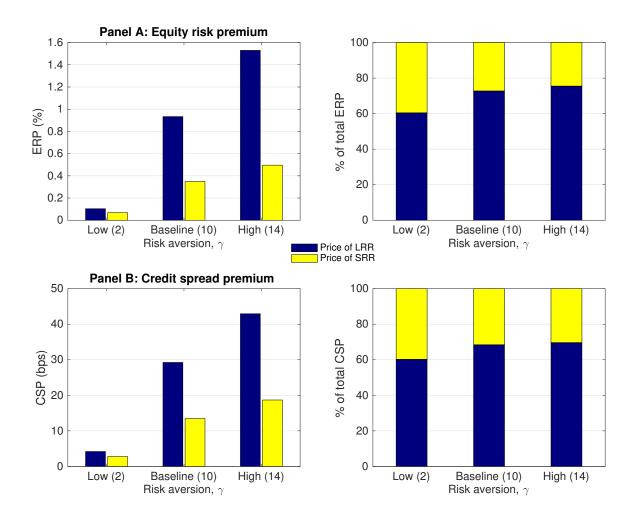


Figure 3: Price of risk and level of risk aversion. This figure shows the impact of investors' risk aversion on the equity risk premium and credit spread premium for different level of risk aversion, high, $\gamma=14$ and low, $\gamma=2$ that we compare with the baseline predictions. The upper panels show the prices of risk due to short- and long-run risk in the equity risk premium in level (top-left panel) and in proportion of the total equity risk premium (top-right panel), while the lower panels show the prices of risk due to short- and long-run risk in the credit spread premium in level (down-left panel) and in proportion of the total credit spread (down-right panel). The coupon and default boundaries are fixed to those of the full model, i.e. baseline case. Unless otherwise specified, we use the parameters of the baseline calibration (see Table 1).

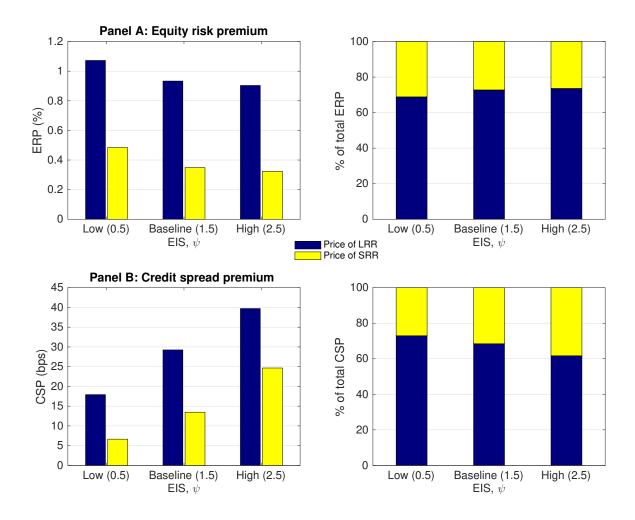


Figure 4: Price of risk and level of EIS coefficient. This figure shows the impact of EIS coefficient on the equity risk premium and credit spread premium for different level of risk aversion, high, $\psi=2.5$ and low, $\psi=0.5$ that we compare with the baseline predictions. The upper panels show the prices of risk due to short- and long-run risk in the equity risk premium in level (top-left panel) and in proportion of the total equity risk premium (top-right panel), while the lower panels show the prices of risk due to short- and long-run risk in the credit spread premium in level (downleft panel) and in proportion of the total credit spread (down-right panel). The coupon and default boundaries are fixed to those of the full model, i.e. baseline case. Unless otherwise specified, We use the parameters of the baseline calibration (see Table 1).

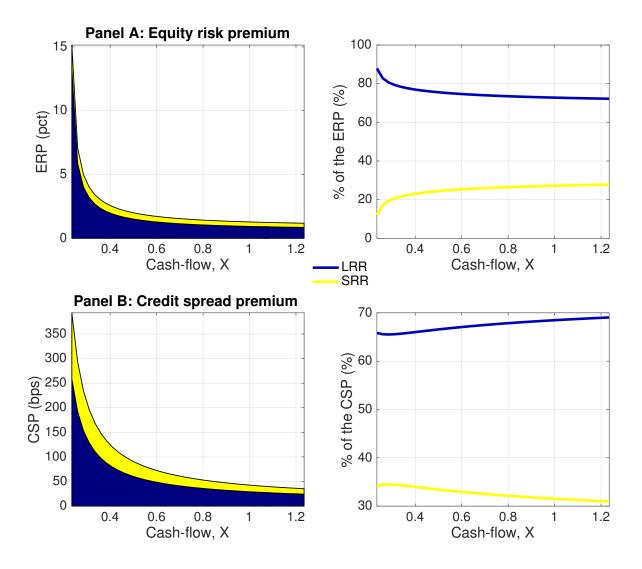


Figure 5: Cash flow, equity risk premium and credit spread premium. This graph shows how investors preferences influence the equity risk premium and credit spread premium for different cash flows level. The upper panels show the prices of risk due to short- and long-run risk in the equity risk premium in level (top-left panel) and in proportion of the total equity risk premium (top-right panel), while the lower panels show the prices of risk due to short- and long-run risk in the credit spread premium in level (down-left panel) and in proportion of the total credit spread (down-right panel). The coupon and default boundaries are fixed to those of the full model, i.e. baseline case. Unless otherwise specified, the parameters are those of the baseline calibration (see Table 1).

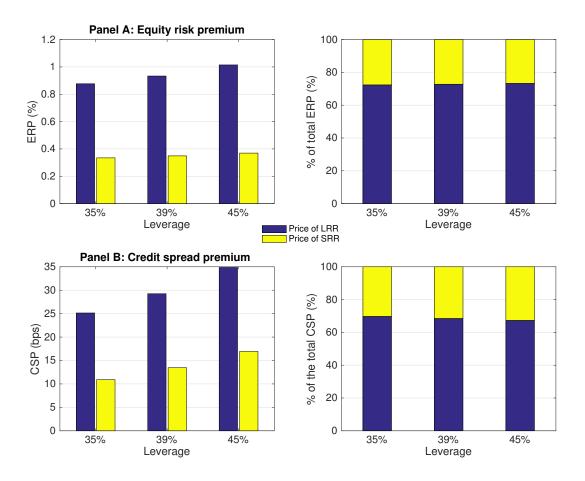


Figure 6: Leverage, equity risk premium and credit spread premium. This graph shows how investors preferences influence the equity risk premium and credit spread premium for different leverage level. The upper panels show the prices of risk due to short- and long-run risks in the equity risk premium in level (top-left panel) and in proportion of the total equity risk premium (top-right panel), while the lower panels show the prices of risk due to short- and long-run risks in the credit spread premium in level (down-left panel) and in proportion of the total credit spread (down-right panel). The coupon and default boundaries are fixed to those of the full model, i.e. baseline case. Unless otherwise specified, the parameters of those of the baseline calibration (see Table 1).

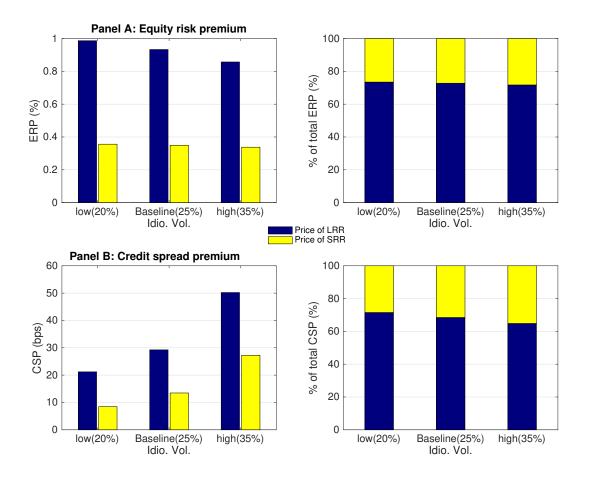


Figure 7: Idiosyncratic volatility, equity risk premium and credit spread premium. This graph shows how investors preferences influence the equity risk premium and credit spread premium for different idiosyncratic volatility level. The upper panels show the prices of risk due to short-and long-run risk in the equity risk premium in level (top-left panel) and in proportion of the total equity risk premium (top-right panel), while the lower panels show the prices of risk due to short-and long-run risk in the credit spread in level (down-left panel) and in proportion of the total credit spread (down-right panel). The coupon and default boundaries are fixed to those of the full model, i.e. baseline case. Unless otherwise specified, the parameters of those of the baseline calibration (see Table 1).

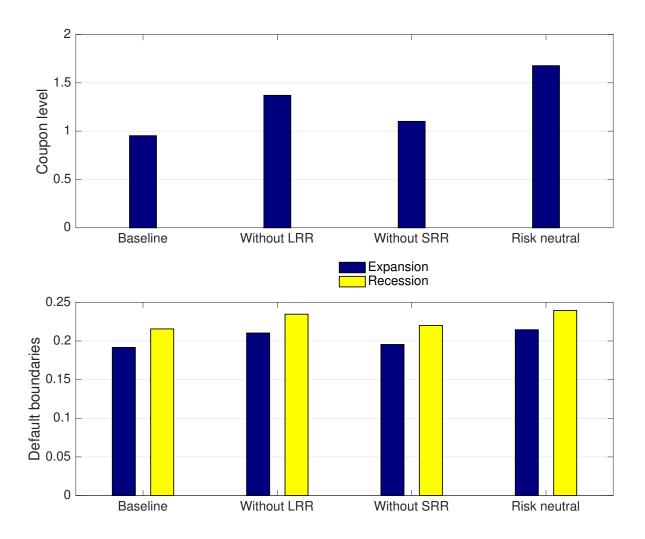


Figure 8: Conditional coupon level and default boundaries. This figure displays the coupon level and the conditional default boundaries for the full model which are compared to those of three special cases. We consider first a model in which the investor does not care about the uncertainty regarding the timing of the resolution of the uncertainty (without long-run risk - LRR), i.e. $\hat{\lambda}_{s_t} = \lambda_{s_t}$; then, a model in which the investor is not averse to the correlation between cash flows and consumption (without short-run risk - SRR), i.e. $\hat{\mu}_{s_t} = \mu_{s_t}$ and finally a model in which the investor is risk-neutral. The upper panel displays the coupon levels and the lower panel the default boundaries. The results are reported for the case when financing occurs in expansion $(s_0 = H)$. We use the parameters of the baseline calibration (see Table 1).

Table 1: Model calibration.

This table reports the parameter values used for the calibration of the model. The state of the economy is determined by the NBER recession dates in the U.S. over the period 1952Q1-2015Q4. The state $s_t=E$ refers to an expansion, whereas the state $s_t=R$ corresponds to a recession. The frequency of the data is quarterly and the values are annualized, when applicable.

Variable	Notation	Va	lue	Source
Panel A: Economic environment and agent pre	eferences			
		Recession	Expansio	n
State of the economy	s_t	R	Ε	
Consumption growth rate (%)	$ heta_{s_t}$	0.65	2.20	Bureau of Economic
Consumption growth volatility (%)	σ	0.86	0.86	Analysis,1952Q1-2015Q4
Actual long-run probability (%)	f_{s_t}	14.84	85.16	NBER recessions dates
Consumer time preference	β	0.03	0.03	
Risk aversion coefficient	γ	10.0	10.0	
Elasticity of intertemporal substitution	ψ	1.5	1.5	
Panel B: Firm characteristics				
Cash flows growth rate (%)	μ_{s_t}	-13.47	5.62	Bureau of Economic
Systematic cash flows growth volatility (%)	σ^g	11.82	11.82	Analysis,1952Q1-2015Q4
Idiosyncratic cash flows growth volatility (%)	σ^f	25	25	
Correlation	ho	0.2496	0.2496	
Tax rate (%)	au	15	15	
Bankruptcy costs (%)	ϕ_{s_t}	60	30	

Table 2: The impact of investors preferences on corporate assets.

This table reports theoretical predictions for the full model, which we compare with three special cases. We consider first the situation in which the investor does not care about the uncertainty regarding the timing of the resolution of the uncertainty, $\hat{\lambda}_{s_t} = \lambda_{s_t}$ (Column 2). Then, we consider the situation in which the investor is not averse to the correlation between cash flows and consumption, $\hat{\mu}_{s_t} = \mu_{s_t}$ (Column 3) and finally the one in which the investor is risk-neutral (Column 4). The coupon and default boundaries are those of the full model so as to keep the same economy, that of the full model, in all three cases. The results are reported for the case when financing occurred in expansion. The equity and debt values are normalized by the full model value. Panel A contains the predictions for the case when the economy is currently in recession and Panel B in expansion. Unless otherwise specified, we use the parameters of the baseline calibration (see Table 1).

	Full model	When agent does	When agent is not	Risk neutrality
		not care about LRR	averse to SRR	
Panel A : Conditional on current	t state being recess	sion		
Coupon	0.9525	0.9525	0.9525	0.9525
Default boundary	0.2156	0.2156	0.2156	0.2156
Equity risk premium (%)	3.59	0.36	3.02	0.00
Credit spread (bps)	134.90	100.63	122.30	85.26
Equity value (normalized)	100.00	159.23	123.58	207.25
Debt value (normalized)	100.00	105.67	102.33	108.87
Leverage (%)	43.24	33.58	38.68	28.58
Default probability - 5y (%)	12.15	12.15	12.15	12.15
Panel B : Conditional on current	t state being expan	sion		
Coupon	0.9525	0.9525	0.9525	0.9525
Default boundary	0.1916	0.1916	0.1916	0.1916
Equity risk premium (%)	0.88	0.35	0.48	0.00
Credit spread (bps)	113.95	85.58	103.14	72.44
Equity value (normalized)	100.0	153.33	122.24	197.85
Debt value (normalized)	100.0	104.71	102.06	107.47
Leverage (%)	38.45	29.90	34.27	25.33
Default probability - 5y (%)	0.21	0.21	0.21	0.21

Table 3: Contribution of each type of systematic risk.

This table reports theoretical predictions for the full model, the price of risk associated to the long- and the short-run risk (Columns 2 and 3) and finally the quantity of risk (Column 5). The coupon and default boundaries are those of the full model so as to keep the same economy, that of the full model, in all three cases. The results are reported for the case when financing occurred in expansion. Panel A contains the predictions for the case when the economy is currently in recession and Panel B in expansion. Unless otherwise specified, we use the parameters of the baseline calibration (see Table 1).

		Price	of the		
	Full model	LRR	SRR	Total risk premium	Total quantity of risk
	Panel A: Condit	tional on curren	t state being re	cession	
Equity risk premium					
In percentage	3.59	3.23	0.36	3.59	0.00
% of the total ERP		89.9	10.11		
Credit spreads					
In bps	134.90	34.27	15.37	49.64	85.26
% of the CSP		69.03	30.97		
% of the total CS		26.31	11.33	36.80	63.20
	Panel B: <i>Condit</i>	ional on current	t state being ex	pansion	
Equity risk premium					
In percentage	0.88	0.53	0.35	0.88	0.00
% of the ERP		60.62	39.38		
Credit spreads					
In bps	113.95	28.37	13.14	41.51	72.44
% of total CSP		68.36	31.64		
% of the total CS		24.90	11.53	36.43	63.57

Table 4: Risk aversion, equity risk premium and credit spread.

This table reports theoretical predictions for the full model, which we compare with three special cases. We consider first the case in which the investor does not care about the uncertainty regarding the timing of the resolution of the uncertainty, i.e. $\hat{\lambda}_{s_t} = \lambda_{s_t}$ (Column 2). Then, we consider a model in which the investor is not averse to the correlation between cash flows and consumption, i.e. $\hat{\mu}_{s_t} = \mu_{s_t}$ (Column 3) and finally a model in which the investor is risk-neutral (Column 4). The coupon and default boundaries are those of the full model so as to keep the same economy, that of the full model, in all three cases. Panel A report the credit spread (Panel A1) and equity risk premium (panel A2) obtained for two different levels of risk aversion, high, $\gamma=14$ and low, $\gamma=2$ that we compare with the baseline predictions and the Panel B the contribution of each type of systematic risk. The results are reported for the case when financing occurs in expansion $(s_0=E)$. Equity risk premiums and credit spreads are weighted average, where the weights are given by the actual long-run distribution, f_{s_t} . Unless otherwise specified, we use the parameters of the baseline calibration (see Table 1).

	Full model (A)	Do not care about LRR	Without aversion to SRR	Risk neutrality (D)
	(A)	(B)	(C)	(D)
Panel A1: Equity risk pren	nium (pct)	, ,	, ,	
Low CRRA	0.171	0.068	0.102	0.00
Baseline	1.282	0.349	0.855	0.00
High CRRA	2.026	0.496	1.351	0.00
Panel A2: Credit spread (I	bps)			
Low CRRA	82.81	78.63	80.17	75.87
Baseline	117.06	87.81	105.98	74.34
High CRRA	135.14	92.21	120.28	73.53
Panel B : Decomposit	Price of LRR	Price of SRR	Total	
Daniel Bl. Frank mielane	(A) - (B)	(A) - (C)	(A) - (D)	
Panel B1: Equity risk pren	піит (рст)			
	. ,	0.060	0.171	
Low CRRA	0.104	0.068	0.171	
Low CRRA Baseline	0.104 0.933	0.349	1.282	
Low CRRA	0.104			
Low CRRA Baseline	0.104 0.933 1.530	0.349	1.282	
Low CRRA Baseline High CRRA	0.104 0.933 1.530	0.349	1.282	
Low CRRA Baseline High CRRA Panel B2: Credit spread p	0.104 0.933 1.530 remium (bps)	0.349 0.496	1.282 2.026	

Table 5: EIS coefficient, equity risk premium and credit spread.

This table reports theoretical predictions for the full model, which we compare with three special cases. We consider first the case in which the investor does not care about the uncertainty regarding the timing of the resolution of the uncertainty, i.e. $\hat{\lambda}_{s_t} = \lambda_{s_t}$ (Column 2). Then, we consider a model in which the investor is not averse to the correlation between cash flows and consumption, i.e. $\hat{\mu}_{s_t} = \mu_{s_t}$ (Column 3) and finally a model in which the investor is risk-neutral (Column 4). The coupon and default boundaries are those of the full model so as to keep the same economy, that of the full model, in all three cases. Panel A report the credit spread (Panel A1) and equity risk premium (panel A2) obtained for two different levels of EIS coefficient, high, $\psi=2.5$ and low, $\psi=0.5$ that we compare with the baseline predictions and the Panel B the contribution of each type of systematic risk. The results are reported for the case when financing occurs in expansion $(s_0=E)$. Equity risk premiums and credit spreads are weighted average, where the weights are given by the actual long-run distribution, f_{s_t} . Unless otherwise specified, we use the parameters of the baseline calibration (see Table 1).

	Full model	Do not care about	Without aversion	Risk neutrality
	(A)	LRR	to SRR	(D)
	(, ,)	(B)	(C)	(-)
Panel A1: Equity risk prei	mium (pct)	()	()	
Low EIS	1.557	0.484	0.975	0.00
Baseline	1.282	0.349	0.855	0.00
High EIS	1.228	0.324	0.829	0.00
Panel A2: Credit spread (ĺbps)			
Low EIS	123.45	105.52	116.39	98.88
Baseline	117.06	87.81	105.98	74.34
High EIS	109.04	69.31	94.51	44.65
Panel B : Decomposit	tion of the price of r	isk		
Panel B : Decomposi	Price of LRR	Price of SRR	Total	
·	Price of LRR (A) - (B)		Total (A) - (D)	
Panel B1: Equity risk prei	Price of LRR (A) - (B) mium (pct)	Price of SRR (A) - (C)	(A) - (D)	
Panel B : Decomposit Panel B1: Equity risk pred Low EIS	Price of LRR (A) - (B) mium (pct) 1.073	Price of SRR (A) - (C) 0.484	(A) - (D) 1.557	
Panel B1: Equity risk prei Low EIS Baseline	Price of LRR (A) - (B) mium (pct) 1.073 0.933	Price of SRR (A) - (C) 0.484 0.349	(A) - (D) 1.557 1.282	
Panel B1: Equity risk prei Low EIS	Price of LRR (A) - (B) mium (pct) 1.073	Price of SRR (A) - (C) 0.484	(A) - (D) 1.557	
Panel B1: Equity risk prei Low EIS Baseline	Price of LRR (A) - (B) mium (pct) 1.073 0.933 0.904	Price of SRR (A) - (C) 0.484 0.349	(A) - (D) 1.557 1.282	
Panel B1: Equity risk prei Low EIS Baseline High EIS	Price of LRR (A) - (B) mium (pct) 1.073 0.933 0.904	Price of SRR (A) - (C) 0.484 0.349	(A) - (D) 1.557 1.282	
Panel B1: Equity risk prei Low EIS Baseline High EIS Panel B2: Credit spread p	Price of LRR (A) - (B) mium (pct) 1.073 0.933 0.904 premium (bps)	Price of SRR (A) - (C) 0.484 0.349 0.324	(A) - (D) 1.557 1.282 1.228	

Table 6 : Leverage, equity risk premium and credit spread.

This table reports theoretical predictions for the full model, which we compare with three special cases. We consider first the case in which the investor does not care about the uncertainty regarding the timing of the resolution of the uncertainty, i.e. $\hat{\lambda}_{s_t} = \lambda_{s_t}$ (Column 2). Then, we consider a model in which the investor is not averse to the correlation between cash flows and consumption, i.e. $\hat{\mu}_{s_t} = \mu_{s_t}$ (Column 3) and finally a model in which the investor is risk-neutral (Column 4). The coupon and default boundaries are those of the full model so as to keep the same economy, that of the full model, in all three cases. Panel A report the credit spread (Panel A1) and equity risk premium (panel A2) obtained for two different levels of leverage, high, 45% and low, 35% that we compare with the baseline predictions (39%) and the Panel B the contribution of each type of systematic risk. The results are reported for the case when financing occurs in expansion ($s_0 = E$). Equity risk premiums and credit spreads are weighted average, where the weights are given by the actual long-run distribution, f_{s_t} . Unless otherwise specified, we use the parameters of the baseline calibration (see Table 1).

	Full model	Do not care about	Without aversion	Risk neutrality
	(A)	LRR	to SRR	(D)
	. ,	(B)	(C)	,
Panel A1: Equity risk prem	nium (pct)			
Low leverage	1.211	0.334	0.812	0.00
Baseline	1.282	0.349	0.855	0.00
High leverage	1.384	0.369	0.917	0.00
Panel A2: Credit spread (b	pps)			
Low leverage	107.54	82.40	98.20	71.48
Baseline	117.06	87.81	105.98	74.34
High leverage	129.77	94.91	116.31	77.99
			110.01	11.33
	on of the price of r	isk		11.55
Panel B : Decompositi	on of the price of r	isk Price of SRR	Total	
Panel B : Decompositi	on of the price of r Price of LRR (A) - (B)	isk		
Panel B : Decompositi	on of the price of r Price of LRR (A) - (B)	isk Price of SRR	Total	11.55
Panel B : Decompositi Panel B1: Equity risk prem	Price of LRR (A) - (B) sium (pct)	Price of SRR (A) - (C)	Total (A) - (D)	
Panel B : Decompositi Panel B1: Equity risk prem Low leverage	Price of LRR (A) - (B) nium (pct) 0.876	Price of SRR (A) - (C) 0.334	Total (A) - (D) 1.211	
Panel B : Decompositi Panel B1: Equity risk prem Low leverage Baseline	Price of LRR (A) - (B) nium (pct) 0.876 0.933 1.015	Price of SRR (A) - (C) 0.334 0.349	Total (A) - (D) 1.211 1.282	11.33
Panel B : Decompositi Panel B1: Equity risk prem Low leverage Baseline High leverage	Price of LRR (A) - (B) nium (pct) 0.876 0.933 1.015	Price of SRR (A) - (C) 0.334 0.349	Total (A) - (D) 1.211 1.282	
Panel B : Decompositi Panel B1: Equity risk prem Low leverage Baseline High leverage Panel B2: Credit spread present in the spr	Price of LRR (A) - (B) nium (pct) 0.876 0.933 1.015 remium (bps)	Price of SRR (A) - (C) 0.334 0.349 0.369	Total (A) - (D) 1.211 1.282 1.384	

Table 7: Idiosyncratic volatility, equity risk premium and credit spread.

This table reports theoretical predictions for the full model, which we compare with three special cases. We consider first a model in which the investor does not care about the uncertainty regarding the timing of the resolution of the uncertainty, i.e. $\hat{\lambda}_{s_t} = \lambda_{s_t}$ (Column 2). Then, we consider a model in which the investor is not averse to the correlation between cash flows and consumption, i.e. $\hat{\mu}_{s_t} = \mu_{s_t}$ (Column 3) and finally a model in which the investor is risk-neutral (Column 4). The coupon and default boundaries are fixed to those of the full model in the baseline case. Panel A report the credit spread (Panel A1) and equity risk premium (panel A2) obtained for two different levels of idiosyncratic volatility, high, $\sigma^{id}=35\%$ and low, $\sigma^{id}=20\%$ that we compare with the baseline predictions and the Panel B the contribution of each type of systematic risk. The results are reported for the case when financing occurs in expansion ($s_0=E$). Equity risk premiums and credit spreads are weighted average, where the weights are given by the actual long-run distribution, s_t . Unless otherwise specified, we use the parameters of the baseline calibration (see Table 1).

Panel A: Equity risk	premium and cred	it spread		
	Full model (A)	Do not care about LRR (B)	Without aversion to SRR (C)	Risk neutrality (D)
Panel A1: Equity risk prem	nium (pct)	(5)	(=)	
Low id. volatility	1.343	0.356	0.886	0.00
Baseline	1.282	0.349	0.855	0.00
High id. volatility	1.195	0.338	0.810	0.00
Panel A2: Credit spreads ((bps)			
Low id. volatility	73.64	52.43	65.79	43.98
Baseline	117.06	87.81	105.98	74.34
High id. volatility	224.59	174.38	204.94	147.14
Panel B : Decompositi	ion of the price of ris	sk		
	Price of LR Risk	Price of SR Risk	Total	
	(A) - (B)	(A) - (C)	(A) - (D)	
Panel B1: Equity risk prem	nium (pct)			
Low id. volatility	0.987	0.356	1.343	
Baseline	0.933	0.349	1.282	
High id. volatility	0.857	0.338	1.195	
Panel B2: Credit spread pr	remium (bps)			
Low id. volatility	21.21	8.45	29.66	
Baseline	29.25	13.47	42.71	
High id. volatility	50.22	27.24	77.45	

Table 8: The impact of investors preferences on corporate assets - Endogenous policy.

This table reports theoretical predictions for the full model, which we compare with three special models. We consider first a model in which the investor does not care about the uncertainty regarding the timing of the resolution of the uncertainty, $\hat{\lambda}_{s_t} = \lambda_{s_t}$ (Column 2). Then, we consider a model in which the investor is not averse to the correlation between cash flows and consumption, $\hat{\mu}_{s_t} = \mu_{s_t}$ (Column 3) and finally a model in which the investor is risk-neutral (Column 4). The coupon and default boundaries are endogenous, i.e., shareholders now consider investors preferences in their decision making. The results are reported for the case when financing occurred in expansion. The equity and debt values are normalized by the full model value. Panel A contains the predictions for the case when the economy is currently in recession and Panel B in expansion. Unless otherwise specified, we use the parameters of the baseline calibration (see Table 1).

	Full model	When agent does	When agent is not	Risk neutrality	
Danal A . Conditional on augus	t atata baina waasa	not care about LRR averse to SRR		•	
Panel A : Conditional on current			1 1000	1 6771	
Coupon	0.9525	1.3702	1.1009	1.6771	
Default boundary	0.2156	0.2346	0.2201	0.2395	
Equity risk premium (%)	3.59	0.42	3.22	0.00	
Credit spread (bps)	134.90	125.24	131.03	121.57	
Equity value (normalized)	100.00	132.39	114.22	160.02	
Debt value (normalized)	100.00	145.81	116.40	178.89	
Leverage (%)	43.24	45.52	43.70	45.99	
Equity return volatility (%)	51.46	52.17	51.88	52.62	
Default probability - 5y (%)	12.15	15.38	12.90	16.26	
D 10 0 11 1					
Panel B: Conditional on current	t state being expan	sion			
	t state being expan 0.9525	1.3702	1.1009	1.6771	
Coupon			1.1009 0.1954	1.6771 0.2145	
Coupon Default boundary	0.9525	1.3702			
Panel B: Conditional on current Coupon Default boundary Equity risk premium (%) Credit spread (bps)	0.9525 0.1916	1.3702 0.2103	0.1954	0.2145	
Coupon Default boundary Equity risk premium (%)	0.9525 0.1916 0.88	1.3702 0.2103 0.39	0.1954 0.49	0.2145 0.00	
Coupon Default boundary Equity risk premium (%) Credit spread (bps)	0.9525 0.1916 0.88 113.95	1.3702 0.2103 0.39 105.74	0.1954 0.49 110.22	0.2145 0.00 102.20	
Coupon Default boundary Equity risk premium (%) Credit spread (bps) Equity value (normalized)	0.9525 0.1916 0.88 113.95 100.0	1.3702 0.2103 0.39 105.74 131.12	0.1954 0.49 110.22 114.46	0.2145 0.00 102.20 158.82	
Coupon Default boundary Equity risk premium (%) Credit spread (bps) Equity value (normalized) Debt value (normalized)	0.9525 0.1916 0.88 113.95 100.0 100.0	1.3702 0.2103 0.39 105.74 131.12 144.91	0.1954 0.49 110.22 114.46 116.39	0.2145 0.00 102.20 158.82 178.56	

Table 9: The impact of investors preferences on corporate assets - Time-varying volatility. This table reports theoretical predictions for the full model, which we compare with three special cases. We consider first the situation in which the investor does not care about the uncertainty regarding the timing of the resolution of the uncertainty, $\hat{\lambda}_{s_t} = \lambda_{s_t}$ (Column 2). Then, we consider the situation in which the investor is not averse to the correlation between cash flows and consumption, $\hat{\mu}_{s_t} = \mu_{s_t}$ (Column 3) and finally the one in which the investor is risk-neutral (Column 4). The coupon and default boundaries are those of the full model so as to keep the same economy, that of the full model, in all three cases. The results are reported for the case when financing occurred in expansion. The equity and debt values are normalized by the full model value. Panel A contains the predictions for the case when the economy is currently in recession and Panel B in expansion. Unless otherwise specified, we use the parameters of the baseline calibration (see Table 1).

	Full model	When agent does	When agent is not	Risk neutrality
		not care to LRR	averse to SRR	
Panel A : Conditional on current	t state being recess	sion		
Coupon	0.9363	0.9363	0.9363	0.9363
Default boundary	0.1990	0.1990	0.1990	0.1990
Equity risk premium (%)	4.45	1.06	3.01	0.00
Credit spread (bps)	151.66	110.51	136.08	93.19
Equity value (normalized)	100.00	161.15	125.16	210.54
Debt value (normalized)	100.00	106.78	102.82	110.37
Leverage (%)	42.27	32.67	37.56	27.74
Default probability - 5y (%)	17.57	17.57	17.57	17.57
Panel B : Conditional on current	t state being expan	sion		
Coupon	0.9363	0.9363	0.9363	0.9363
Default boundary	0.1901	0.1901	0.1901	0.1901
Equity risk premium (%)	0.80	0.26	0.48	0.00
Credit spread (bps)	123.74	90.48	111.28	76.53
Equity value (normalized)	100.0	155.45	123.14	200.47
Debt value (normalized)	100.0	105.53	102.34	108.47
Leverage (%)	37.79	29.20	33.55	24.74
		0.14	0.14	0.14

Table 10: Contribution of each type of systematic risk - Time-varying volatility.

This table reports theoretical predictions for the full model, the price of risk associated to the long- and the short-run risk (Columns 2 and 3) and finally the quantity of risk (Column 5). The coupon and default boundaries are those of the full model so as to keep the same economy, that of the full model, in all three cases. The results are reported for the case when financing occurred in expansion. Panel A contains the predictions for the case when the economy is currently in recession and Panel B in expansion. Unless otherwise specified, we use the parameters of the baseline calibration (see Table 1).

	Price	of the		
Full model	LRR	SRR	Total risk premium	Total quantity of risk
Panel A: Condit	tional on curren	t state being re	cession	
4.45	3.39	1.06	4.45	0.00
	76.09	23.91		
151.66	41.15	17.31	58.46	93.19
	70.38	29.62		
	27.13	11.42	38.55	61.45
Panel B: Condit	ional on current	t state being ex	pansion	
0.80	0.54	0.26	0.80	0.00
	67.62	32.38		
123.74	33.26	13.95	47.21	76.53
	70.45	29.55		
	26.88	11.27	38.15	61.85
	Panel A: <i>Condit</i> 4.45 151.66 Panel B: <i>Condit</i> 0.80	Full model LRR Panel A: Conditional on current 4.45 3.39 76.09 151.66 41.15 70.38 27.13 Panel B: Conditional on current 0.80 0.54 67.62	Panel A: Conditional on current state being red 4.45 3.39 1.06 76.09 23.91 151.66 41.15 17.31 70.38 29.62 27.13 11.42 Panel B: Conditional on current state being exp 0.80 0.54 0.26 67.62 32.38 123.74 33.26 13.95 70.45 29.55	Full model LRR SRR Total risk premium Panel A: Conditional on current state being recession 4.45 3.39 1.06 4.45 76.09 23.91 4.45 151.66 41.15 17.31 58.46 70.38 29.62 27.13 11.42 38.55 Panel B: Conditional on current state being expansion 0.80 0.54 0.26 0.80 67.62 32.38 47.21 70.45 29.55 47.21

Table 11: Impact of the time-varying volatility in risk premia.

This table reports theoretical predictions for the full model, the price of risk associated to the long- and the short-run risk (Columns 2 and 3) and finally the quantity of risk (Column 5). The coupon and default boundaries are those of the full model so as to keep the same economy, that of the full model, in all three cases. The results are reported for the case when financing occurred in expansion. Panel A contains the predictions for the case when the economy is currently in recession and Panel B in expansion. Unless otherwise specified, we use the parameters of the baseline calibration (see Table 1).

	In re	ecession	In expansion	
	Total	LRR impact	Total	LRR impact
Equity risk premium	%	Proportion	%	Proportion
T-V growth and volatility	4.45	76.1%	0.80	67.6%
T-V growth only	3.59	89.9%	0.88	60.6%
Credit spreads premium	bps	Proportion	%	Proportion
T-V growth and volatility	58.46	70.4%	47.21	70.5%
T-V growth only	49.64	69.1%	41.51	68.4%

Table 12: Portfolio sorted on exposure to consumption growth.

This table reports average equally weighted monthly returns in percent of portfolios based unconditional and conditional risk loadings. Time-varying unconditional and conditional risk loadings, $\hat{\beta}^i_{c,t}$, $\hat{\beta}^i_{c,E}$, and $\hat{\beta}^i_{c,R}$ are obtained from 10-year rolling time-series regressions of firm excess returns on log consumption growth using quarterly data. Five portfolios are formed based on the estimated $\hat{\beta}^i_{c,t}$, $\hat{\beta}^i_{c,E}$, and $\hat{\beta}^i_{c,R}$ and held for 1 year. The column "High–Low" shows returns of a zero investment portfolio that is long in the high exposure portfolio and short in the low exposure portfolio. t-statistics are reported in parentheses and are based on Newey and West (1987) adjusted standard errors using 12 lags. The sample period is 1964 to 2016.

	Low		Med		High	High-Low
Mean	-0.106	-0.040	-0.014	0.010	0.070	0.176
(std)	(0.044)	(0.024)	(0.020)	(0.023)	(0.048)	(0.070)
Returns	1.111	1.075	0.972	1.075	1.204	0.093
(t-stat)	(5.79)	(5.76)	(6.80)	(5.86)	(6.97)	(0.97)
	Pa	nel B : Univ	variate Sort	Based on β	$\hat{c}_{c,E}^{i}$	
	Low		Med		High	High-Low
Mean	-0.108	-0.040	-0.013	0.012	0.070	0.183
(std)	(0.047)	(0.026)	(0.020)	(0.023)	(0.049)	(0.071)
Returns	1.203	1.184	1.048	1.173	1.281	0.078
(t-stat)	(6.28)	(6.02)	(6.61)	(5.92)	(7.08)	(0.73)
	Pa	nel C: Univ	ariate Sort	Based on \hat{eta}	$i \atop c,R$	
	Low		Med		High	High-Low
Mean	-0.095	-0.043	-0.021	-0.001	0.049	0.144
(std)	(0.027)	(0.014)	(0.014)	(0.017)	(0.036)	(0.049)
Returns	0.675	0.558	0.614	0.612	0.840	0.165
(t-stat)	(2.62)	(2.87)	(3.74)	(4.93)	(3.49)	(1.87)

Table 13: Portfolio sorted on exposure to change in consumption growth mean.

This table reports average equally weighted monthly returns in percent of portfolios based on unconditional and conditional risk loadings. Time-varying unconditional and conditional risk loadings, $\hat{\beta}^i_{\mu,t}$, $\hat{\beta}^i_{\mu,E}$, and $\hat{\beta}^i_{\mu,R}$ are obtained from 10-year rolling time-series regressions of firm excess returns on changes in the perceived consumption mean, using quarterly data. Five portfolios are formed based on the estimated $\hat{\beta}^i_{\mu,t}$, $\hat{\beta}^i_{\mu,E}$, and $\hat{\beta}^i_{\mu,R}$ and held for 1 year. The column "High-Low" shows returns of a zero investment portfolio that is long in the high exposure portfolio and short in the low exposure portfolio. t-statistics are reported in parentheses and are based on Newey and West (1987) adjusted standard errors using 12 lags. The sample period is 1964 to 2016.

	Pa	nel A : Uni	variate Sort	Based on /	$\hat{\beta}_{\mu,t}^i$		
	Low		Med		High	High-Low	
Mean	-0.375	-0.094	0.029	0.161	0.472	0.847	
(std)	(0.158)	(0.097)	(0.090)	(0.100)	(0.048)	(0.251)	
Returns	1.123	1.024	0.980	1.109	1.201	0.078	
(t-stat)	(5.84)	(6.28)	(6.78)	(6.63)	(5.50)	(0.68)	
Panel B : Univariate Sort Based on $\hat{eta}^i_{\mu,E}$							
	Low		Med		High	High-Low	
Mean	-0.380	-0.090	0.033	0.166	0.489	0.869	
(std)	(0.165)	(0.097)	(0.091)	(0.103)	(0.150)	(0.253)	
Returns	1.242	1.132	1.061	1.180	1.275	0.033	
(t-stat)	(6.60)	(6.26)	(6.73)	(6.41)	(5.69)	(0.29)	
	Pa	nel C: Univ	ariate Sort	Based on \hat{eta}	$i \atop u,R$		
	Low		Med		High	High-Low	
Mean	-0.350	-0.090	0.033	0.166	0.489	0.869	
(std)	(0.119)	(0.091)	(0.077)	(0.082)	(0.138)	(0.210)	
Returns	0.564	0.515	0.595	0.773	0.853	0.288	
(t-stat)	(3.12)	(3.62)	(3.30)	(4.72)	(2.65)	(1.31)	

Table 14 : Portfolio sorted on exposure to change in consumption growth volatility. This table reports average equally weighted monthly returns in percent of portfolios based unconditional and conditional risk loadings. Time-varying unconditional and conditional risk loadings, $\hat{\beta}^i_{\sigma,t}$, $\hat{\beta}^i_{\sigma,E}$, and $\hat{\beta}^i_{\sigma,R}$ are obtained from 10-year rolling time-series regressions of firm excess returns on changes in perceived consumption volatility, using quarterly data. Five portfolios are formed based on the estimated $\hat{\beta}^i_{\sigma,t}$, $\hat{\beta}^i_{\sigma,E}$, and $\hat{\beta}^i_{\sigma,R}$ and held for 1 year. The column "High–Low" shows returns of a zero investment portfolio that is long in the high exposure portfolio and short in the low exposure portfolio. t-statistics are reported in parentheses and are based on Newey and West (1987) adjusted standard errors using 12 lags. The sample period is 1964 to 2016.

	Pa	nel A : Uni	variate Sort	Based on /	$\beta_{\sigma,t}^i$	
	Low		Med		High	High-Low
Mean	-1.822	-0.273	0.400	1.079	2.650	4.472
(std)	(1.018)	(0.574)	(0.556)	(0.531)	(0.640)	(1.316)
Returns	1.293	1.054	0.885	1.049	1.152	-0.141
(t-stat)	(6.54)	(6.49)	(6.30)	(6.17)	(5.07)	(-1.07)
	Pa	nel B : Univ	variate Sort	Based on $\hat{\beta}$	σ_{r}^{i}	
	Low		Med		High	High-Low
Mean	-1.919	-0.336	0.340	1.024	2.630	4.550
(std)	(1.042)	(0.576)	(0.552)	(0.521)	(0.629)	(1.344)
Returns	1.359	1.131	0.934	1.153	1.309	-0.050
(t-stat)	(6.64)	(6.50)	(5.88)	(6.18)	(5.74)	(-0.34)
	Pa	nel C: Univ	ariate Sort	Based on \hat{eta}	$i \\ \sigma, R$	
	Low		Med		High	High-Low
Mean	-1.363	0.022	0.680	1.342	2.743	4.106
(std)	(0.740)	(0.460)	(0.484)	(0.498)	(0.680)	(1.102)
Returns	0.979	0.688	0.656	0.557	0.412	-0.566
(t-stat)	(5.49)	(3.65)	(3.20)	(2.72)	(1.97)	(-5.32)

Table 15: t-statistic of high-minus-low portfolios when using analysts forecasts.

This table reports t-statistic of average equally weighted monthly returns of high-minus-low portfolios based on unconditional (Panel A) and conditional (Panel B) risk loadings. Time-varying risk loadings are obtained from 5-year rolling time-series regressions of firm excess returns on change in log consumption growth $\triangle c_t$, one semester ahead of expected consumption growth, μ_t^{SFP} and volatility, σ_t^{SFP} from the survey of professional forecasts. t-statistics are based on Newey and West (1987) adjusted standard errors using 12 lags. The sample period is 1964 to 2016.

Panel A: Unconditional					
		$\triangle c_t$	μ_t^{SFP}	σ_t^{SFP}	
		1.28			
One factor			2.01		
				NO(-)	
Two factors		1.04	1.32		
TWO Tactors		NO(+)		NO(-)	
Three factors		1.35	1.32	NO(+)	
Panel B : Condit	ional on	the state of th	ne economy		
	Е	1.69			
One factor	R	1.89			
One factor	Е		2.86		
	R		1.06		
	Е			NO(-)	
	R			-6.95	
	Е	1.47	2.17		
Two factors	R	2.93	-1.36		
	Е	2.04		NO(-)	
	R	2.90		-5.90	
Three factors	Е	1.98	2.32	NO(-)	
Three factors	R	2.57	-1.47	-4.3	

Table 16: Portfolio sorted on exposure to consumption growth, expected consumption growth and dispersion.

This table reports average equally weighted monthly returns in percent of portfolios based on unconditional and conditional risk loadings. Time-varying unconditional and conditional risk loadings are obtained from 5-year rolling time-series regressions of firm excess returns on log consumption growth, expected consumption growth and dispersion using quarterly data. Five portfolios are formed based on the estimated risk loadings and held for 1 year. The column "High-Low" shows returns of a zero investment portfolio that is long in the high exposure portfolio and short in the low exposure portfolio. *t*-statistics are reported in parentheses and are based on Newey and West (1987) adjusted standard errors using 12 lags. The sample period is 1964 to 2016.

	Low		Med		High	High-Low			
Panel A: Univariate Sort - Consumption Growth									
\hat{eta}^i_c	1.012	1.068	0.983	0.712	1.230	0.218			
(t-stat)	(3.36)	(3.54)	(3.26)	(2.51)	(3.74)	(1.35)			
$\hat{eta}^i_{c,E}$	1.289	1.175	1.133	1.042	1.598	0.308			
(t-stat)	(6.35)	(5.29)	(4.76)	(3.67)	(6.28)	(1.98)			
$\hat{eta}^i_{c,R}$	-1.444	-0.363	-0.143	-0.351	-0.113	1.331			
(t-stat)	(-3.37)	(-2.59)	(-1.31)	(-1.63)	(-1.08)	(2.57)			
Panel B: Univariate Sort - Mean Consumption Growth forecasts									
$\beta^i_{\mu^{SPF},t}$	0.920	0.767	1.112	1.013	1.183	0.263			
(t-stat)	(4.02)	(2.32)	(3.72)	(3.36)	(3.32)	(1.32)			
$eta^i_{\mu^{SPF},E}$	1.090	0.980	1.365	1.244	1.558	0.468			
(t-stat)	(5.79)	(5.76)	(6.80)	(5.86)	(6.97)	(2.32)			
$\beta^i_{\mu^{SPF},R}$	-0.275	-0.305	-0.133	-0.828	-0.991	-0.717			
(t-stat)	(-2.54)	(-1.94)	(-0.35)	(-11.16)	(-2.51)	(-1.47)			
Panel C: Univariate Sort - Mean Consumption Growth forecasts dispersion									
$\beta^i_{\sigma^{SPF},t}$	1.042	0.913	1.087	0.866	1.089	0.047			
(t-stat)	(3.43)	(3.00)	(3.61)	(2.98)	(3.22)	(0.24)			
$\beta^i_{\sigma^{SPF}.E}$	1.364	1.136	1.176	1.212	1.350	-0.014			
(t-stat)	(4.77)	(3.74)	(5.02)	(4.91)	(6.04)	(-0.06)			
$eta^i_{\sigma^{SPF},R}$	-0.121	-0.363	-0.463	-0.704	-0.891	-0.770			
(t-stat)	(-0.75)	(-1.70)	(-7.30)	(-3.55)	(-3.17)	(-4.3)			

8 Appendix

We present the main point of the model which is a static version of the model of Bhamra, Kuehn and Strebulaev(2010a; 2010b). The model uses a state-dependent approach to derive endogenously each state optimal coupon and default boundaries which in turn are used to compute the firms claims, i.e. equity and debt. The Arrow-Debreu default claims which measure the present value of the jump in the state-price density from one state to another allow to introduce the state-dependency in the pricing.

8.1 Consumption and cash flows dynamics

The economy consists of a representative agent and a number of firms. The agent provides capital to firms by buying equity and bond. There is no friction in the economy. All variables are in real terms.

8.1.1 Consumption

Let C_t denote the perpetual stream of consumption in the economy. The output level follows the process

$$\frac{dC_t}{C_t} = \theta_{s_t} dt + \sigma_{s_t} dB_t, \quad s_t = \{R, E\},$$
(15)

where θ_{s_t} and σ_{s_t} are the drift and volatility of output, and B_t is a standard Brownian motion under the physical measure.

This economy is characterized by the long-run risk, which creates variation in the business cycle. We assume that the economy is governed by two states such that the first and second moments of output growth, θ_{s_t} and σ_{s_t} , are state-dependent. The state of the economy at time t is determined by s_t , which is equal to R in recession and to E in expansion. The first moment is procyclical, while the second one is countercyclical such that $\theta_E > \theta_R$ and $\sigma_E < \sigma_R$. The evolution of s_t is given by a 2-state Markov chain. All agents can observe the current state of the economy.

8.1.2 Firm cash flows

The firm i has a stream of cash flows, denoted by $X_{i,t}$, which is given by

$$\frac{dX_{t,i}}{X_{t,i}} = \mu_{s_t} dt + \sigma^f dB_{t,i}^f + \sigma_{s_t}^g dB_t^g, \quad s_t = \{R, E\}$$
(16)

where μ_{s_t} is the conditional growth rate of the firm's cash flows, while σ^f and $\sigma^g_{s_t}$ capture respectively the idiosyncratic and systematic volatility of the firm's earning growth rate. The standard Brownian motion $B^f_{t,i}$ is the firm-specific shock, which is uncorrelated to the shock to consumption B_t . The firm's total earning volatility is equal $\sigma_{X,s_t} = \sqrt{(\sigma^f)^2 + (\sigma^g_{s_t})^2}$. μ_{s_t} is procyclical so that $\mu_E > \mu_R$ and σ_{X,s_t} is countercyclical so that $\sigma_{X,R} > \sigma_{X,E}$.

The firms' cash flows depend on the macroeconomic environment in two ways. First, all firms share common shocks coming from the country level economic conditions. As consequence, firm cash flows shocks are correlated with shocks to aggregate consumption, so that: $dB_t dB_t^g = \rho dt$, where ρ is the constant coefficient of correlation between cash flows and consumption. Second, the firms cash flows follow the business cycles, which is determined by the state s_t .

8.2 State-price density and equilibrium risk-free rate

In this section, we provide the formula of the state-price density and the equilibrium risk-free rate.²⁰ The state-price density is initially derived by Duffie and Skiadas (1994) for the general class of stochastic differential utility function proposed by Duffie and Epstein (1992). This type of utility function incorporates not only the agent's risk aversion but also the aversion for intertemporal resolution of the uncertainty in the economy.

The representative agent's state-price density π_t , in the case $\psi \neq 1$, is given by

$$\pi_t = \left(\beta e^{-\beta t}\right)^{\frac{1-\gamma}{1-\frac{1}{\psi}}} C_t^{-\gamma} \left(p_{C,s_t} e^{\int_0^t p_{C,s_u}^{-1} du}\right)^{-\frac{\gamma - \frac{1}{\psi}}{1-\frac{1}{\psi}}}, \tag{17}$$

²⁰For additional details and complete derivation, we refer the reader to the Appendix of Bhamra, Kuehn, and Strebulaev (2010b) in the case of two states, and to the Appendix of Chen (2010) for N states.

where p_{C,s_t} is the price-consumption ratio that satisfies the following implicit non-linear equation:

$$p_{C,s_{t}}^{-1} = \overline{r}_{s_{t}} - \theta_{s_{t}} + \gamma \sigma_{s_{t}}^{2} - \left(1 - \frac{1}{\psi}\right) \lambda_{s_{t}} \left(\frac{\left(\frac{p_{C,\bar{s}_{t}}}{p_{C,s_{t}}}\right)^{\frac{1-\gamma}{1-\frac{1}{\psi}}} - 1}{1 - \gamma}\right), \ s_{t}, \bar{s}_{t} \in \{R, E\}, \bar{s}_{t} \neq s_{t}$$

$$(18)$$

with

$$\overline{r}_{s_t} = \beta + \frac{1}{\psi} \theta_{s_t} - \frac{1}{2} \gamma \left(1 + \frac{1}{\psi} \right) \sigma_{s_t}^2. \tag{19}$$

The dynamics of the state-price density π_t follow the following stochastic differential equation

$$\frac{d\pi_t}{\pi_t} = -r_{s_t}dt + \frac{dM_t}{M_t} \tag{20}$$

$$= -r_{s_t}dt - \Theta_{s_t}^B dB_t - \Theta_{s_t}^P dN_{s_t,t}, \tag{21}$$

where M is a martingale under the physical measure, $N_{s_t,t}$ a Poisson process which jumps upward by one whenever the state of the economy switches from s_t to $\overline{s_t} \neq s_t$, $\Theta^B_{s_t} = \gamma \sigma_{s_t}$ is the market price of risk due to Brownian shocks in state s_t , and $\Theta^P_{s_t} = 1 - \Delta_{s_t}$ is the market price of risk due to Poisson shocks when the economy switches out of state $s_t = \{R, E\}$.

Finally, r_{s_t} represents the equilibrium real risk-free rate, which is given by

$$r_{s_t} = \begin{cases} \overline{r}_L + \lambda_L \left[\frac{\gamma - \frac{1}{\psi}}{1 - \gamma} \left(\Delta^{-\frac{\gamma - 1}{\gamma - \frac{1}{\psi}}} - 1 \right) - (\Delta^{-1} - 1) \right] &, s_t = R \\ \overline{r}_H + \lambda_H \left[\frac{\gamma - \frac{1}{\psi}}{1 - \gamma} \left(\Delta^{\frac{\gamma - 1}{\gamma - \frac{1}{\psi}}} - 1 \right) - (\Delta - 1) \right] &, s_t = E \end{cases}$$

$$(22)$$

with $\Delta_H=\Delta_L^{-1}=\Delta$, where Δ is the solution of $G(\Delta)=0$ from

$$G(x) = x^{-\frac{1-\frac{1}{\psi}}{\gamma-\frac{1}{\psi}}} - \frac{\overline{r}_H + \gamma \sigma_H^2 - \theta_H + \lambda_H \frac{1-\frac{1}{\psi}}{\gamma-1} \left(x^{\frac{\gamma-1}{\gamma-\frac{1}{\psi}}} - 1 \right)}{\overline{r}_L + \gamma \sigma_L^2 - \theta_L + \lambda_L \frac{1-\frac{1}{\psi}}{\gamma-1} \left(x^{-\frac{\gamma-1}{\gamma-\frac{1}{\psi}}} - 1 \right)}, \quad \psi \neq 1$$
(23)

The agent has preference for earlier resolution of uncertainty in the case when $\gamma>\frac{1}{\psi}$ and thus

cares about the rate of news arrival, denoted by p. When p is small, the speed at which information arrives is low, thereby increasing the risk of the intertemporal substitution for an agent averse to such risk. The rate at which the distribution for the state of the economy converges to its steady state is given by $p = \lambda_R + \lambda_E$, where λ_{s_t} is the probability per unit of time of leaving state s_t . The quantity $1/\lambda_{s_t}$ is the expected duration of state s_t . Recessions are shorter than expansions, such that $1/\lambda_R < 1/\lambda_E$.

The physical probabilities λ_R and λ_E are converted to their risk-neutral counterparts $\hat{\lambda}_R$ and $\hat{\lambda}_E$ through a risk distortion factor Δ_E , which is defined as the change in the state-price density π_t at the transition time from expansion to recession. The risk-neutral probabilities per unit of time of changing state are then given by

$$\hat{\lambda}_E = \Delta_E \lambda_E$$
 and $\hat{\lambda}_R = \frac{1}{\Delta_E} \lambda_R$. (24)

The agent prefers earlier resolution of the uncertainty, which implies that $\Delta_E>1$. Hence, this agent prices securities as if recessions last longer $(\lambda_R>\hat{\lambda}_R)$ and expansions shorter $(\lambda_E<\hat{\lambda}_E)$ than in reality. The risk-neutral rate of news arrival is $\hat{p}=\hat{\lambda}_R+\hat{\lambda}_E$, which implies that the long-run risk-neutral distribution is determined by $(\hat{f}_R,\hat{f}_E)=(\frac{\hat{\lambda}_E}{\hat{p}},\frac{\hat{\lambda}_R}{\hat{p}})$.

The equilibrium risk-free rate prevailing in equilibrium in state s_t is given by (see Appendix A)

$$r_{s_{t}} = \overline{r}_{s_{t}} - \left(\frac{\gamma - \frac{1}{\psi}}{\gamma - 1}\right) \lambda_{s_{t}} \left(1 - \Delta_{s_{t}}^{\frac{\gamma - 1}{\gamma - \frac{1}{\psi}}}\right) + \lambda_{s_{t}} \left(1 - \Delta_{s_{t}}\right), \quad \psi \neq 1, \ s_{t} = \{R, E\}, \quad (25)$$

where

$$\overline{r}_{s_t} = \beta + \frac{1}{\psi} \theta_{s_t} - \frac{1}{2} \gamma \left(1 + \frac{1}{\psi} \right) \sigma_{s_t}^2. \tag{26}$$

Higher uncertainty ($\sigma_E < \sigma_R$) and lower economic growth ($\theta_E > \theta_R$) in recession induce greater demand for the risk-free bond, thereby reducing the equilibrium interest rate ($r_E > r_R$). The risk-free interest rate is therefore procyclical.

8.3 Arrow-Debreu default claims

This Appendix derives two kinds of Arrow-Debreu claims that are used to discount cash flows. The first kind captures the default triggered by the firm's earning falling below the default boundary, whereas the second kind additionally accounts for the default related to a change in the state of the economy. In the second case, default can occur instantaneously because of a change in state although the firm's earning remains unchanged. This situation can occur when the economy is in good economic state ($s_t = E$) and switches to the bad state ($s_D = R$), and the firm's earning is above the good state's default boundary, but below the bad state's default boundary. The reason is that the default boundary is countercyclical ($X_{D,R} > X_{D,E}$), as shown in Bhamra, Kuehn, and Strebulaev (2010a, p.4238). The first kind of the Arrow-Debreu claims is defined as

$$q_{s_t s_D} = E_t \left[\frac{\pi_{t_D}}{\pi_t} Prob \left(s_D \mid s_t \right) \mid s_t \right], \tag{27}$$

while the second kind corresponds to

$$q'_{s_t s_D} = E_t \left[\frac{\pi_{t_D}}{\pi_t} \frac{X_{t_D}}{X_{s_t}} Prob\left(s_D \mid s_t\right) \mid s_t \right]. \tag{28}$$

8.4 First kind

The Arrow-Debreu default security $q_{s_ts_D}$ is the present time t value of a security that pays one unit of consumption at the moment of default t_D , where s_t represents the present state of the economy, and s_D the state at the default time. The time of default is the first time that the earning level of the firm falls to the boundary X_{D,s_D} . By definition, this Arrow-Debreu claim is given by

$$q_{s_t s_D} = E_t \left[\frac{\pi_{t_D}}{\pi_t} Prob\left(s_D \mid s_t\right) \mid s_t \right], \tag{29}$$

which is solution of the two ordinary differential equations (ODE)

$$\frac{1}{2}\sigma_{X,s_t}^2 X^2 \frac{d^2 q_{s_t s_D}}{dX^2} + \mu_{s_t} X \frac{d q_{D,s_t s_D}}{dX} + \hat{\lambda}_{s_t} \left(q_{j s_D} - q_{s_t s_D} \right) - r_{s_t} q_{s_t s_D} = 0, \ s_t = \{R, E\}, \ (30)$$

where $\sigma_{X,s_t} = \sqrt{(\sigma^f)^2 + (\sigma_{s_t}^g)^2}$ denotes the total volatility of cash-flows in state s_t .

The above ODEs are obtained by applying Ito's Lemma to the classical non-arbitrage condition

$$E_t^Q \left[dq_{s_t s_D} - r_{s_t} q_{s_t s_D} \right] = 0, (31)$$

The Arrow-Debreu claim payoffs are such that:

$$q_{s_t s_D}(X) = \begin{cases} 1, & s_t = s_D, & X \le X_{D, s_t} \\ 0, & s_t \ne s_D, & X \le X_{D, s_t}. \end{cases}, s_t, s_D = \{R, E\}$$
(32)

Therefore, each state of the economy is characterized by a specific default boundary. The default barriers are higher in recession and lower in expansion, that is $X_{D,E} \leq X_{D,R}$. Each of the four Arrow-Debreu claims is then determined over three separate intervals: $X \geq X_{D,R}$, $X_{D,R} \geq X \geq X_{D,E}$, and $X \leq X_{D,E}$.

From the payoff equations, we can infer the values of the four Arrow-Debreu claims in the interval $X \leq X_{D,E}$. For the interval $X \geq X_{D,R}$, we are looking for a solution of the following general form:

$$q_{s_t s_D}(X) = h_{s_t s_D} X^k, (33)$$

which implies that k must be a root of the quartic equation

$$\left[\frac{1}{2}\sigma_{X,E}^{2}k(k-1) + \mu_{R}k + \left(-\hat{\lambda}_{R} - r_{R}\right)\right] \left[\frac{1}{2}\sigma_{X,E}^{2}k(k-1) + \mu_{E}k + \left(-\hat{\lambda}_{E} - r_{E}\right)\right] - \hat{\lambda}_{R}\hat{\lambda}_{E} = 0.$$
(34)

The Arrow-debreu claims can be written as

$$q_{s_t s_D}(X) = \sum_{m=1}^{4} h_{s_t s_D m} X^{k_m}$$
(35)

with $k_1,k_2<0$ and $k_3,k_4>0$. However, when X goes to infinity the Arrow-Debreu claims must

be zero, which indicates that we should have $h_{s_t s_D,3} = h_{s_t s_D,4} = 0$. We then obtain

$$q_{Rs_D}(Y) = \sum_{m=1}^{2} h_{Rs_D,m} X^{k_m}$$
(36)

$$q_{Es_D}(Y) = \sum_{m=1}^{2} h_{Es_D,m} \varepsilon(k_m) X^{k_m}, \qquad (37)$$

where

$$\varepsilon(k_{m}) = -\frac{\hat{\lambda}_{H}}{\frac{1}{2}\sigma_{X,H}^{2}k(k-1) + \mu_{E}k - (\hat{\lambda}_{E} + r_{E})} = -\frac{\frac{1}{2}\sigma_{X,R}^{2}k(k-1) + \mu_{R}k - (\hat{\lambda}_{R} + r_{R})}{\hat{\lambda}_{R}}.$$
(38)

Finally, over the interval $X_{D,R} \ge X \ge X_{D,E}$, both $q_{D,RR}$ and $q_{D,RE}$ are known from the payoffs equations and are respectively equal to 1 and 0. Then,

$$q_{ER}(X) = \frac{\hat{\lambda}_E}{r_E + \hat{\lambda}_E} + \sum_{m=1}^2 s_{R,m} X^{j_m}$$
 (39)

$$q_{EE}(X) = \sum_{m=1}^{2} s_{E,m} X^{j_m}, \tag{40}$$

where

$$\frac{1}{2}\sigma_{X,E}^{2}j(j-1) + \mu_{R}j - (\hat{\lambda}_{E} + r_{E}) = 0$$
(41)

with $j_1 < j_2$.

To summarize, the four Arrow-Debreu claims can be written as follows

$$q_{RR} = \begin{cases} \sum_{m=1}^{2} h_{RR,m} X^{k_m}, & X \ge X_{D,R} \\ 1, & X_{D,R} \ge X \ge X_{D,E} \\ 1, & X \le X_{D,E} \end{cases}$$

$$q_{RE} = \begin{cases} \sum_{m=1}^{2} h_{RE,m} X^{k_m}, & X \ge X_{D,E} \\ 0, & X_{D,R} \ge X \ge X_{D,E} \\ 0, & X \le X_{D,E} \end{cases}$$

$$q_{ER} = \begin{cases} \sum_{m=1}^{2} h_{RR,m} \varepsilon (k_m) X^{k_m}, & X \ge X_{D,R} \\ \frac{\lambda_E}{r_E + \lambda_E} + \sum_{m=1}^{2} s_{R,m} X^{j_m}, & X_{D,R} \ge X \ge X_{D,E} \\ 0, & X \le X_{D,E} \end{cases}$$

$$q_{EE} = \begin{cases} \sum_{m=1}^{2} h_{RE,m} \varepsilon (k_m) X^{k_m}, & X \ge X_{D,R} \\ \sum_{m=1}^{2} s_{E,m} X^{j_m}, & X_{D,R} \ge X \ge X_{D,E} \\ 1, & X \le X_{D,E} \end{cases}$$

$$q_{EE} = \begin{cases} \sum_{m=1}^{2} h_{RE,m} \varepsilon (k_m) X^{k_m}, & X \ge X_{D,R} \\ \sum_{m=1}^{2} s_{E,m} X^{j_m}, & X_{D,R} \ge X \ge X_{D,E} \\ 1, & X \le X_{D,E} \end{cases}$$

$$q_{EE} = \begin{cases} \sum_{m=1}^{2} h_{RE,m} \varepsilon (k_m) X^{k_m}, & X \ge X_{D,R} \\ \sum_{m=1}^{2} s_{E,m} X^{j_m}, & X_{D,R} \ge X \ge X_{D,E} \\ 1, & X \le X_{D,E} \end{cases}$$

$$q_{EE} = \begin{cases} \sum_{m=1}^{2} h_{RE,m} \varepsilon (k_m) X^{k_m}, & X \ge X_{D,R} \\ \sum_{m=1}^{2} s_{E,m} X^{j_m}, & X_{D,R} \ge X \ge X_{D,E} \\ 1, & X \le X_{D,E} \end{cases}$$

$$q_{EE} = \begin{cases} \sum_{m=1}^{2} h_{RE,m} \varepsilon (k_m) X^{k_m}, & X \ge X_{D,R} \\ \sum_{m=1}^{2} s_{E,m} X^{j_m}, & X_{D,R} \ge X \ge X_{D,E} \\ 1, & X \le X_{D,E} \end{cases}$$

$$q_{EE} = \begin{cases} \sum_{m=1}^{2} h_{RE,m} \varepsilon (k_m) X^{k_m}, & X \ge X_{D,R} \\ \sum_{m=1}^{2} s_{E,m} X^{j_m}, & X_{D,R} \ge X \ge X_{D,E} \end{cases}$$

$$q_{RE} = \begin{cases} \sum_{m=1}^{2} h_{RE,m} X^{k_m}, & X \ge X_{D,E} \\ 0, & X_{D,R} \ge X \ge X_{D,E} \\ 0, & X \le X_{D,E} \end{cases}$$
(43)

$$q_{ER} = \begin{cases} \sum_{m=1}^{2} h_{RR,m} \varepsilon(k_m) X^{k_m}, & X \ge X_{D,R} \\ \frac{\hat{\lambda}_E}{r_E + \hat{\lambda}_E} + \sum_{m=1}^{2} s_{R,m} X^{j_m}, & X_{D,R} \ge X \ge X_{D,E} \\ 0, & X \le X_{D,E} \end{cases}$$
(44)

$$q_{EE} = \begin{cases} \sum_{m=1}^{2} h_{RE,m} \varepsilon(k_m) X^{k_m}, & X \ge X_{D,R} \\ \sum_{m=1}^{2} s_{E,m} X^{j_m}, & X_{D,R} \ge X \ge X_{D,E} \end{cases}$$

$$1, & X \le X_{D,E}.$$

$$(45)$$

The eight constants are determined by eight boundary conditions, which are

$$\lim_{X \to X_{D,R}} q_{EE} = 1, \quad \lim_{X \to X_{D,R}} q_{RE} = 0 \tag{46}$$

$$\lim_{X \to X_{D,R}^{+}} q_{ER} = \lim_{X \to X_{D,R}^{-}} q_{ER}, \quad \lim_{X \to X_{D,R}^{+}} q_{EE} = \lim_{X \to X_{D,R}^{-}} q_{EE}$$

$$\lim_{X \to X_{D,R}^{+}} \dot{q}_{ER} = \lim_{X \to X_{D,R}^{-}} \dot{q}_{ER}, \quad \lim_{X \to X_{D,R}^{+}} \dot{q}_{EE} = \lim_{X \to X_{D,R}^{-}} \dot{q}_{EE}$$

$$(47)$$

$$\lim_{X \to X_{D,R}^+} \dot{q}_{ER} = \lim_{X \to X_{D,R}^-} \dot{q}_{ER}, \quad \lim_{X \to X_{D,R}^+} \dot{q}_{EE} = \lim_{X \to X_{D,R}^-} \dot{q}_{EE} \tag{48}$$

$$\lim_{X \to X_{D,E}} q_{ER} = 0, \quad \lim_{X \to X_{D,E}} q_{EE} = 1. \tag{49}$$

8.5 Second kind

We use the same approach to derive the second kind of Arrow-Debreu default claims, which account for the possibility that a default can when the state of the economy changes. The only claim that is different from that of the first kind is q_{HL} , whose expression is now given by

$$q_{ER}' = \begin{cases} \sum_{m=1}^{2} h_{RR,m} \varepsilon \left(k_{m} \right) X^{k_{m}}, & X \ge X_{D,R} \\ \frac{\hat{\lambda}_{E}}{r_{E} + \hat{\lambda}_{E} - \mu_{E}} \frac{X}{X_{D,R}} + \sum_{m=1}^{2} s_{R,m} X^{j_{m}}, & X_{D,R} \ge X \ge X_{D,E} \\ 0, & X \le X_{D,E}. \end{cases}$$
(50)

8.6 Corporate debt

The present debt value, $B_{s_0s_t}$, is the discounted coupon stream c_{s_0} , before default plus the present value of the recovered firm asset liquidation value at default $(\phi_{s_D}A_{s_D})$, where ϕ_{s_t} is the state-dependent asset recovery rate and A_{s_t} is the firm asset liquidation value. Hence, the debt value is:

$$D_{s_0 s_t} = E_t \left[\int_t^{t_D} c_{s_0} \frac{\pi_u}{\pi_t} du \mid s_t \right] + E_t \left[\frac{\pi_u}{\pi_t} \phi_{t_D} A_{t_D} du \mid s_t \right]$$
 (51)

$$= \frac{c_{s_0}}{r_{B,s_t}} - \sum_{s_D} \left(\frac{c_{s_0}}{r_{B,s_D}} - \phi_{t_D} A_{t_D} \right) q_{s_t s_D}, \quad s_t, s_D = \{R, E\}$$
 (52)

where $s_0=\{R,E\}$ is financing state, ϕ_{s_t} is the state-dependent asset recovery rate, $A_{s_t}=(1-\tau)\frac{X}{r_{A,s_t}}$ is the firm asset liquidation value, $r_{B,s_t}=r_{s_t}+\frac{r_j-r_{s_t}}{\hat{p}+r_j}\hat{p}\hat{f}_j, \quad j\neq s_t$ is the bond discount rate and $r_{A,s_t}=r_{s_t}-\hat{\mu}_{s_t}+\frac{(r_j-\hat{\mu}_j)-\left(r_{s_t}-\hat{\mu}_{s_t}\right)}{\hat{p}+r_j-\hat{\mu}_j}\hat{p}\hat{f}_j, \quad j\neq s_t,$ is the risky discount rate with $\hat{\mu}_{s_t}=\mu_{s_t}-\gamma\rho\sigma_{s_t}^g\sigma_{s_t}.$

8.7 Equity value

The equity value, $S_{s_0s_t}$, is the after-tax discounted value of future cash flows (i.e. EBIT) less coupon payments before bankruptcy is declared by the stockholders

$$S_{s_0 s_t} = (1 - \tau) E_t \left[\int_t^{t_D} \frac{\pi_u}{\pi_t} (X_t - c_{s_0}) du \mid s_t \right], \quad s_t = \{R, E\},$$

hence

$$S_{s_0 s_t} = A_{s_t} - (1 - \tau) \frac{c_{s_0}}{r_{B, s_t}} + \sum_{s_D} \left((1 - \tau) \frac{c_{s_0}}{r_{B, s_D}} - A_{t_D} \right) q_{s_t s_D}, \quad s_t, s_D = \{R, E\} \quad (53)$$

where A_{t_D} is the asset recovery rate at the time the shareholders decide to declare bankruptcy (see section 8.6).

8.8 Estimation of the transition probabilities

This section describes the estimation of the transition probabilities considered in the paper. We estimate a Markov regime-switching model with two regimes using the NBER recession dates over the period 1952Q1-2015Q4. The transition probability matrix, is obtained by maximum likelihood using the Hamilton (1989)'s approach. There are some issues with the estimation when using consumption date between of the period spanning the 2007-09 financial crisis. This originates from the fact that the consumption growth slowdown was so pronounced during this period that it is recognized as representing by itself one more regime. To overcome that, we have replaced the consumption data by the NBER dates and put 1 or 0 when the economy is respectively in recession or in expansion. The estimation gives the following transition matrix:

$$T = \begin{bmatrix} T_{RR} & T_{RE} \\ T_{ER} & T_{EE} \end{bmatrix} = \begin{bmatrix} 0.9603 & 0.0397 \\ 0.2275 & 0.7725 \end{bmatrix}$$
 (54)

where T_{ij} denotes the probability of a switch from state i to state j.

The actual long-run probability f_{s_t} to be in the state $s_t \in \{R, E\}$ is determined by $f_R = \left(1 + \frac{T_{RE}}{T_{ER}}\right)^{-1}$ and $f_E = 1 - f_R$. The probability λ_{s_t} that the economy leaves the state $s_t \in \{R, E\}$ is then given by $\lambda_R = pf_E$ and $\lambda_E = pf_R$, with $p = -4ln\left(1 - \frac{T_{RE}}{1 - f_R}\right)$.

Part II

Macroeconomic Risk, Investor Preferences, and Sovereign Credit Spreads

Abstract¹

This paper examines the impact of global macroeconomic conditions and investor preferences on

sovereign credit spreads. We propose a structural model for sovereign debt valuation embedded

in a consumption-based environment with a global business cycle. Governments choose a higher

indebtedness level and prefer to default earlier when a country's economic performance is more

sensitive to the global business cycle. Moreover, investors demand a compensation for intertemporal

risk, as they dislike countercyclical default risk and uncertainty about future global conditions.

Our model predicts that a country's exposure to macroeconomic risk increases default probability,

particularly in bad times, and the price of risk, but particularly in good times.

JEL Codes: F34, G12, G13, G15, G32

Keywords: Sovereign debt, credit risk, asset pricing, macroeconomic conditions.

¹This article is co-authored with A. Jeanneret.

1 Introduction

There is strong evidence that a country's default risk varies with global economic and financial conditions (see, e.g., Longstaff, Pan, Pedersen, and Singleton, 2011). One reason is that most countries are exposed to the global business cycle and tend to perform poorly during global economic slowdowns. This exposure to world-wide macroeconomic fundamentals should have important implications for a government's policies, the pricing of default risk, and eventually the level of sovereign credit spreads. Yet the existing literature thus far lacks a theoretical framework that helps us understand the magnitude of and the mechanism behind such effects.

This paper examines the role of time-varying global macroeconomic conditions, and their interaction with investor preferences, for the explanation of sovereign credit risk. We develop a model that provides insights on how sovereign credit spreads depend on a country's economic exposure to the global business cycle, in particular when the government can adjust its debt and default policies. Importantly, the model allows disentangling the impact of macroeconomic risk into the compensation for default risk and the price of this risk, thereby offering new predictions regarding the contribution of investors' preferences to sovereign spreads.

We propose a model that embeds the structural approach for valuing sovereign credit risk in a consumption-based environment with global macroeconomic risk. The economic environment is characterized by a global business cycle; the state of the global economy can be in expansion or in recession and switches slowly via a Markov chain.² The economic performance of a country depends on this business cycle, such that the expected growth rate is lower and volatility higher during global recessions. Yet output shocks are independent across countries. Hence, the source of systematic risk within the model is the exposure to low-frequency global macroeconomic conditions, rather than some high-frequency systematic shocks (e.g., Borri and Verdelhan, 2012). Each country has a government that decides the optimal level of debt to issue and when to default.³ A representative

²Our model builds on the works of Bhamra, Kuehn, and Strebulaev (2010a,b) and Chen (2010), who show that time-varying macroeconomic conditions in the U.S. help better explain the credit spread puzzle and corporate financing decisions.

³The optimal indebtedness level is based on the trade-off between fostering public investments and the increase in default risk, whereas the optimal default policy is determined by the trade-off between the benefits of a debt reduction (i.e., haircut upon restructuring) and the economic costs of default (i.e., reduced output). These features follow the model of Jeanneret (2015), which helps explain a large fraction of the time-variation in sovereign credit spreads.

global agent with recursive preferences prices financial securities, thereby determining the yield for default-risky government debt and the equilibrium risk-free interest rate.⁴ The difference between the two consists of the sovereign credit spread.

The presence of macroeconomic risk matters through various complementary channels. As recessions correspond to times of economic slowdowns and high uncertainty, default probabilities become countercyclical. Global investors particularly dislike the increased default risk during recessions, as it coincides with a rise in their marginal utility of consumption. Moreover, they prefer uncertainty about the future state of the global economy to be resolved sooner than later and thus demand a premium for intertemporal risk. Sovereign bonds are priced accordingly and governments adjust both their default policy and debt issuance in anticipation of the future business cycle. This model is thus particularly insightful to study the interaction between time-varying global economic conditions, governments' optimal default and debt decisions, and the pricing of sovereign credit risk. We calibrate the model using NBER dates determine the recession/expansion periods. We consider aggregate consumption in the U.S. to calibrate the moments in global consumption and real GDP data for a large set of emerging and developed economies to determine the conditional moments of a representative sovereign country.

The first contribution of the paper is to provide new predictions regarding the influence of global macroeconomic risk on a country's sovereign creditworthiness. The model suggests that countries that are more exposed to the global business cycle risk have wider sovereign credit spreads, not only in recessions, but also unconditionally. The effect arises from a combination of two channels. First, ecomic volatility increases and expected growth rate decreases during recessions, thus directly increasing the likelihood of default. This is the case even though governments have less incentives to default during a global recession which implies a cyclical default policy. Second, governments tend to optimally increase their indebtedness in presence of macroeconomic risk, thereby amplifying the increase in the default probability. The reason is that the equilibrium risk-free rate is procyclical, being lower in recessions for precautionary saving motive. Investors dislike

⁴We consider Epstein-Zin-Weil preferences such that the agent is able to disentangle between risk aversion and aversion about the timing of the resolution of uncertainty. This type of utility function, developed by Kreps and Porteus (1978), Epstein and Zin (1989), Duffie and Epstein (1992a,b), and Weil (1990), has been recently used to resolve some important asset pricing problems related to the equity premium, credit spread, and forward-premium puzzles, for example. Significant contributions include Bansal and Yaron (2004), Bhamra, Kuehn, and Strebulaev (2010a,b), Chen (2010), and Colacito and Croce (2013), among others.

macroeconomic uncertainty and thus price government bonds with a discount rate that overweights the low risk-free rate in recession and underweights the high risk-free rate in expansion. As a consequence, governments respond to these favorable financing conditions (i.e., the discount rate being unconditionally lower than the risk-free rate) by issuing a greater amount of debt.

The model predicts that 30% of the credit spread level is due to exposure to macroeconomic risk, and that this risk increases the 5-year default probability from 3.7% to 9.1%. The presence of a global business cycle, which determines the conditional moments of a country's output growth, thus exerts an economically strong influence on a country's default risk and thus on its borrowing costs. As a result, countries whose economic growth and volatility are more sensitive to global macroeconomic conditions should exhibit greater sovereign credit spreads. Notably, we find that this effect is particularly pronounced for less-performing and riskier countries (i.e., lower output, higher indebtedness, or more economic uncertainty).

Our second contribution is to highlight the role of investor preferences for the pricing of sovereign credit risk. Credit spreads are expected to reflect not only a fair compensation for default risk but also the price of macroeconomic risk. Agents are more sensitive to a rise in default risk during global recessions, when marginal utility of consumption is high, which makes sovereign default risk partially systematic in nature. In addition, global macroeconomic risk introduces uncertainty about future bad states of the economy. Agents prefer earlier resolution of the uncertainty as to when recessions will arrive and price financial assets as if recessions had a higher duration than in reality. Investors thus require a premium for macroeconomic risk when holding risky government bonds. This price of risk represents 7% of the credit spread level, which is economically meaningful.⁵ The remaining part is the direct credit risk compensation. Our analysis thus indicates that global business cycle risk contributes not only to the level of risk but also to the risk premium component of credit spreads. Notably, the premium is greater in expansion, when investors care about the next recession, and increases in countries with better creditworthiness (i.e., greater output, less indebtedness, or lower economic volatility). That is, the compensation for default risk contributes to a relatively greater fraction of sovereign spreads when a bond faces a high default probability. In

⁵To disentangle the price from the quantity of risk, we maintain the same default and debt policies in the analysis. As discussed earlier, investor preferences can also influence a government's optimal debt level, which would then impact a country's default probability.

contrast, safer bonds reflect a much higher price of risk. Our model thus enriches our understanding regarding the magnitude of the pricing of sovereign credit risk and how it varies across sovereign bonds. This paper thus complements the findings of Borri and Verdelhan (2012) on the role of systematic shocks in sovereign spreads. While they analyze high-frequency, systematic ouput growth shocks, we rather focus on large slowly-moving shocks that influence the moments of a country's growth rate over the business cycle.

The model also sheds light on the dynamics of sovereign credit risk. A simulation of the model shows that, during global recessions, sovereign credit spreads tend to increase, to exhibit higher co-movement across countries, and sovereign defaults tend to cluster. We provide evidence that, in the absence of macroeconomic risk, the model would fail to reproduce these key conditional properties of sovereign credit risk. Hence accounting macroeconomic risk is critical to explain stylized facts about sovereign credit risk data.

Overall, our theoretical model shows that the economic risk associated with the global business cycle affects a government's debt and default policies, and helps understand how investor preferences drive sovereign credit spreads. The results suggest that a country's exposure to global macroeconomic conditions can help explain cross-country differences and over time fluctuations in both the quantity and the price of sovereign default risk. The paper thus contributes to the literature documenting the global component in sovereign credit risk (e.g., Longstaff et al, 2011; Borri and Verdelhan, 2012; Augustin and Tédongap, 2015) and provides an explanation for the stylized fact that sovereign defaults tend to occur at business-cycle frequencies (Reinhart and Rogoff, 2008).

The remainder of the paper is organized as follows. Section 2 provides a review of the related empirical and theoretical literature. Section 3 outlines a model to explore the role of macroeconomic conditions for sovereign credit risk. Section 4 analyzes the main theoretical predictions. Finally, Section 5 concludes.

2 Literature review

This paper is related to a large literature providing evidence that sovereign credit spreads vary with global financial and economic conditions. For example, Pan and Singleton (2008), Remolona,

Scatigna, and Wu (2008), Hilscher and Nosbusch (2010), Benzoni, Collin-Dufresne, Goldstein, and Helwege (2015), and Jeanneret (2015) show that sovereign credit risk varies with global uncertainty measures, such as the option-implied volatility index (VIX), which are known to increase during U.S. recessions. In addition, Uribe and Yue (2006) find that U.S. interest rates drive sovereign spreads in emerging markets, Monfort and Renne (2013) demonstrate that a global liquidity component drives European sovereign spreads, whereas Ang and Longstaff (2013) suggest that the systemic part of European spreads varies negatively with the German stock market. Turning to economic fundamentals, Augustin and Tédongap (2015) find that international co-movements in sovereign spreads strongly depend on global macroeconomic conditions, as measured with the growth and volatility of U.S. consumption. Overall, this literature confirms that global macroeconomic conditions must be key ingredients in a theoretical analysis of sovereign credit risk. We thus aim to explore how a country's exposure to such macroeconomic risk affects the level of sovereign spreads and the default probability.

Some of this empirical literature also contributes to identifying the risk premium embedded in sovereign spreads. Remolona, Scatigna, and Wu (2008) use sovereign ratings announcements to decompose sovereign credit spreads into default-risk and risk-premium components. Pan and Singleton (2008) and Longstaff et al. (2011) rather propose an affine sovereign credit model to provide this decomposition. These studies conclude that there is a significant risk premium embedded in sovereign credit spreads. In a different approach, Borri and Verdelhan (2012) exploit asset pricing tests to analyze the cross-section of sovereign bond returns. They show that sovereign bonds with greater covariances with U.S. economic conditions offer positive excess returns, which is indicative of a significant price of systematic risk. Investors thus view such bonds are particularly risky, as they correlate with their consumption, and expect to be compensated for that risk through a high return. An implication shared by these different studies is that global investors appear to play a central role in the sovereign debt market. This result highlights the importance of better understanding how investor preferences affect the pricing of sovereign bonds, in particular when governments can adjust their debt and default policies accordingly.

On the theoretical side, several papers propose models that help explain the level and the time-variation in sovereign credit spreads. Some of these models are based on a contingent claims

framework with optimal default decision, following the structural modeling developed in corporate finance (see, e.g., Fischer, Heinkel, and Zechner, 1989; Leland, 1994).⁶ This is the approach that we follow in this paper. Another strand of studies develops dynamic stochastic equilibrium models, based on the classic work of Eaton and Gersovitz (1981), to explain why countries default and borrowing costs are countercyclical (see, e.g., Aguiar and Gopinath, 2006; Arellano, 2008; Yue, 2010). As an extension, Mendoza and Yue (2012) propose a general equilibrium model of sovereign default and business cycle to explain the reduction in economic activity around defaults and the countercyclical sovereign spreads, among other stylized facts. This literature contributes to our understanding of the default-risk component in sovereign spreads, but remains silent on the global risk premia, as lenders are typically risk-neutral. Moreover, these models generally consider a single-country environment, which makes it difficult to understand why sovereign credit spreads vary with global economic conditions.

A recent literature helps to address these issues and proposes to examine sovereign credit risk in presence of global shocks and risk-averse investors. Borri and Verdelhan (2012) extend the two-country model of Aguiar and Gopinath (2006) and Arellano (2008) to introduce investors with habit preferences and study how the correlation between emerging countries and the U.S. business cycle affects the optimal debt level and the probability of default. Their key finding is that the higher the correlation with the global business cycle, the higher the average sovereign excess returns. This effect is amplified in bad times when the U.S. consumer's risk aversion increases. Augustin and Tédongap (2015) develop an equilibrium pricing model with recursive preferences to explain international co-movement in the term structure of sovereign CDS spreads observed in the data. Their model estimation requires that the conditional mean and volatility of U.S. consumption evolves according to a two-state process, as well as a long-run risk component à la Bansal and Yaron (2004).

Although these studies are conceptually close to ours, we depart in several dimensions. First, Borri and Verdelhan (2012) exclusively analyze the price of risk associated with high-frequency systematic shocks, whereas we focus exclusively on the risk associated with low-frequency global macroeconomic conditions. We show that such slowly-moving business conditions play a fundamen-

⁶See, for example, Hayri (2000), Gibson and Sundaresan (2001), Andrade (2009), François, Hübner, and Sibille (2011), or Jeanneret (2015, 2017).

tal role in the default probability and in the pricing of risk when agents have recursive preferences and thus care about intertemporal risk. In addition, our time-variation in the price of risk does not depend on time-varying preferences but rather on the state of the business cycle. Augustin and Tédongap (2015) similarly highlights the importance of macroeconomic risk to better understand the dynamics of sovereign credit spreads, but their reduced-form model does not offer insights on how a sovereign's optimal default and indebtedness decisions vary with such risk, which is a key aspect of our paper. Moreover, their analysis is geared towards explaining key properties of the term structure, whereas we focus on the impact of macroeconomic risk on the probability and the clustering of default. Moreover, we disentangle the price of risk from the quantity of credit risk and examine how their relative importance varies across sovereigns and time.

3 The model

We develop a structural model for sovereign credit risk in a consumption-based environment with global business cycle risk. We endogenously derive a government's default and debt policies and price sovereign debt using the state-price density of a representative risk-averse agent. The model allows us to study how a country's sovereign credit risk relates to the agent's preferences and to intertemporal macroeconomic risk.

3.1 Global economic environment

The economic environment is characterized by a global stochastic consumption stream that exhibits both low and high frequency variations. There is a representative Epstein-Zin-Weil global agent who consumes this exogenous consumption stream and who prices assets internationally. All variables are in real terms.

3.1.1 Aggregate consumption

Let C_t denote the perpetual stream of global consumption, which follows the process

$$\frac{dC_t}{C_t} = \theta_{s_t} dt + \sigma_{s_t} dB_t, \quad s_t = \{L, H\},$$
(55)

where θ_{s_t} and σ_{s_t} are the drift and volatility of global consumption, and B_t is a standard Brownian motion under the physical measure.

The global economy is governed by two states such that the first and second moments of consumption growth, θ_{s_t} and σ_{s_t} , are stochastic. The dynamics of global consumption thus vary over the business cycle. More precisely, the state of the global economy at time t is determined by s_t , which is equal to L in recession and to H in expansion. The first moment is procyclical, while the second one is countercyclical such that $\theta_H > \theta_L$ and $\sigma_H < \sigma_L$. The evolution of s_t is given by a 2-state Markov chain. All agents can observe the current state of the economy.

3.1.2 State-price density

There exists a representative agent who is able to distinguish between risk aversion and aversion to intertemporal resolution of the uncertainty. The agent's state-price density, π_t , is given by (see Appendix A):

$$\pi_t = \left(\beta e^{-\beta t}\right)^{\frac{1-\gamma}{1-\frac{1}{\psi}}} C_t^{-\gamma} \left(p_{C,t} e^{\int_0^t p_{C,u}^{-1} du}\right)^{-\frac{\gamma - \frac{1}{\psi}}{1-\frac{1}{\psi}}}, \tag{56}$$

where γ is the coefficient of relative risk aversion for timeless lottery, ψ the aversion to intertemporal substitution for deterministic consumption scheme (i.e., elasticity of intertemporal consumption), β is the subjective time discount factor, and $p_{C,t} = \frac{P_t}{C_t}$ is the price-consumption ratio, which is a claim that pays the aggregate consumption per unit consumption.

We consider the case $\gamma>\frac{1}{\psi}$, such that the agent is sensitive to the intertemporal substitutability and has a preference for earlier resolution of uncertainty. This agent dislikes uncertainty about the future business cycle, which affects its consumption, and thus prefers intertemporal risk to be resolved as early as possible. The agent thus cares about the rate of news arrival, which we denote by p. When p is small, the speed at which information arrives is low, thereby increasing the risk of the intertemporal substitution for an agent averse to such risk.

⁷The price-consumption ratio claim vanishes from the stochastic discount factor when $\gamma = \frac{1}{\psi}$. In that case, the agent risk attitudes are all reflected into the risk aversion coefficient γ and one cannot distinguish the two risk preferences.

3.1.3 State transition probabilities and equilibrium risk-free rate

The rate at which the distribution for the state of the global economy converges to its steady state is given by $p = \lambda_L + \lambda_H$, where λ_{s_t} is the probability per unit of time of leaving state s_t . The quantity $1/\lambda_{s_t}$ is the expected duration of state s_t . We assume that recessions are shorter than expansions, such that $1/\lambda_L < 1/\lambda_H$.

Following Bhamra, Kuehn and Strebulaev (2010a), we convert the physical probabilities λ_L and λ_H to their risk-neutral counterparts $\hat{\lambda}_L$ and $\hat{\lambda}_H$ with a risk distortion factor Δ_H , which is defined as the change in the state-price density π_t at the transition time from expansion to recession (see Appendix A). The risk-neutral probabilities per unit of time of changing state are then given by

$$\hat{\lambda}_H = \Delta_H \lambda_H$$
 and $\hat{\lambda}_L = \frac{1}{\Delta_H} \lambda_L$. (57)

The agent prefers earlier resolution of the uncertainty, which implies that $\Delta_H > 1$. Hence, this agent prices securities as if recessions last longer $(\lambda_L > \hat{\lambda}_L)$ and expansions shorter $(\lambda_H < \hat{\lambda}_H)$ than in reality. The risk-neutral rate of news arrival is $\hat{p} = \hat{\lambda}_L + \hat{\lambda}_H$, which implies that the long-run risk-neutral distribution is determined by $(\hat{f}_L, \hat{f}_H) = (\frac{\hat{\lambda}_H}{\hat{p}}, \frac{\hat{\lambda}_L}{\hat{p}})$.

The equilibrium risk-free rate prevailing in equilibrium in state s_t is given by (see Appendix A)

$$r_{s_{t}} = \overline{r}_{s_{t}} - \left(\frac{\gamma - \frac{1}{\psi}}{\gamma - 1}\right) \lambda_{s_{t}} \left(1 - \Delta_{s_{t}}^{\frac{\gamma - 1}{\gamma - \frac{1}{\psi}}}\right) + \lambda_{s_{t}} \left(1 - \Delta_{s_{t}}\right), \quad \psi \neq 1, \ s_{t} = \{L, H\}, \quad (58)$$

where

$$\overline{r}_{s_t} = \beta + \frac{1}{\psi} \theta_{s_t} - \frac{1}{2} \gamma \left(1 + \frac{1}{\psi} \right) \sigma_{s_t}^2. \tag{59}$$

Higher uncertainty ($\sigma_H < \sigma_L$) and lower economic growth ($\theta_H > \theta_L$) in recession induce greater demand for the risk-free bond, thereby reducing the equilibrium interest rate ($r_H > r_L$). The risk-free interest rate is therefore procyclical.

3.1.4 Country's output

The world consists of several small countries that are exposed to the global business cycle. A country i has stream of output, denoted by $Y_{i,t}$, which is given by

$$\frac{dY_{i,t}}{Y_{i,t}} = \mu_{i,s_t} dt + \sigma_{i,s_t} dW_{i,t}, \quad s_t = \{L, H\},$$
(60)

where μ_{i,s_t} is the expected growth rate of the country i's output, while σ_{i,s_t} captures the volatility of this growth rate. The standard Brownian motion $W_{i,t}$ is a shock specific to country i, which is independent of the global consumption shock B_t . Hence, there is no systematic component in the shock $W_{i,t}$, which implies that output growth shocks are instantaneously uncorrelated across countries. However, the country's performance depends on the global economic environment, as its conditional moments vary with the global business cycle, which is determined by the state s_t . In particular, the procyclical and countercyclical nature of economic growth and volatility implies that $\mu_{i,H} > \mu_{i,L}$ and $\sigma_{i,H} < \sigma_{i,L}$ respectively.

3.2 Government bonds and sovereign credit risk

This section evaluates a country's sovereign debt and derives the government's state-dependent debt and default policies. For ease of notation, we drop the country's subscript i in the remainder of the paper.

3.2.1 Sovereign debt valuation

Each country has a government that issues debt with infinite maturity. Debt is characterized by a perpetual coupon c, which will be chosen endogenously. One can view such a contract as a bond with a principal that is permanently rolled-over.

In absence of default, the debt value equals $\frac{c}{r_{B,s_t}}$ when the current state is s_t , which is the present value of the continuous stream of coupons c, where the discount rate for a riskless perpetuity r_{B,s_t} equals

$$r_{B,s_t} = r_{s_t} + \frac{r_j - r_{s_t}}{\hat{p} + r_j} \hat{p} \hat{f}_j, \quad j \neq s_t.$$
 (61)

This discount rate for the riskless claims is different from the risk-free rate in each state, r_{s_t} , because of the expectation that the risk-free rate can change in the future according to the state of the global economy.

Consider now that the government can default on the bond at time t_D , which can occur in either state s_t . When the government defaults on its debt obligations, we suppose that the coupon c is reduced by a fraction $\phi \in (0,1)$ due to debt restructuring. We assume that a default can occur only once. The value of the sovereign debt D_{s_t} , conditional on the current and default states being s_t and s_D , equals (see Appendix C)

$$D_{s_{t}} = E_{t} \left[\int_{t}^{t_{D}} c \frac{\pi_{u}}{\pi_{t}} du \mid s_{t} \right] + E_{t} \left[\int_{t_{D}}^{\infty} (1 - \phi) c \frac{\pi_{u}}{\pi_{t}} du \mid s_{t} \right]$$
 (62)

$$= \frac{c}{r_{B,s_t}} \left[1 - \sum_{s_D} \phi \frac{r_{B,s_t}}{r_{B,s_D}} q_{s_t s_D} \right], \quad s_t, s_D = \{L, H\},$$
 (63)

with

$$q_{s_t s_D} = E_t \left[\frac{\pi_{t_D}}{\pi_t} Prob \left(s_D \mid s_t \right) \mid s_t \right], \tag{64}$$

where $q_{s_ts_D}$ represents the Arrow-Debreu default claim, which corresponds to the asset that pays one unit of consumption at default time t_D if the current state is s_t and the state at the moment of default is s_D (see Appendix B). The debt value is thus equal to the risk-less consol bond value $\frac{c}{r_{B,s_t}}$ minus a discount for default risk. The default risk term depends on the Arrow-Debreu default claim, the bond discount rate in both states (at the issuance date and at the default time), and on the coupon haircut. Because the representative agent cares about the state of the global economy at the time of sovereign default, she demands an additional compensation for a default that is expected to occur in recession. The increased difference between the risk-neutral and the physical probability of default raises the Arrow-Debreu default claim $q_{s_ts_D}$. This leads to a lower price of debt D_{s_t} and a higher sovereign credit spread CS_{s_t} , which is determined by (see Appendix C)

$$CS_{s_t} = \frac{c}{D_{s_t}} - r_{B,s_t}. \tag{65}$$

3.2.2 Incentive for indebtedness

Governments perceive debt as long-term liabilities, which are suitable for long-term investments. These investments create higher economic value for the country by attracting investors and increasing fiscal revenue. That is the reason why a government prefers to be indebted and thus commits to service debt. We assume that the government uses the value of debt at issuance to finance public investments, which yield a return r_g per unit of time (see Jeanneret, 2015). We denote the present value of the returns from public investments by I_{s_t} , which equals (see Appendix D.2)

$$I_{s_t} = E_t \left[\int_t^\infty r_g D_{s_0} \frac{\pi_u}{\pi_t} du \mid s_t \right] \tag{66}$$

$$= \frac{r_g}{r_{B,s_t}} D_{s_0} \tag{67}$$

Excessive debt raises the risk of a default event, which is typically costly for a country's economic performance. To model this cost, we suppose that a default reduces the country's level of output by a fraction $\alpha \in (0,1)$.⁸ Hence, there is an indebtedness level at which the economic benefit of debt issuance is offset by the increased cost of default. Similarly, the government optimally chooses the time of default, which is determined by the trade-off between the debt haircut and the economic costs of default. Before deriving these policies, let us first introduce the government's objective function.

3.2.3 Fiscal revenue

The government cares about its present and future budget balance. Hence, it is important to determine the present value of fiscal revenue, which we denote by F_{s_t} .

The government raises fiscal revenue by taxing the country's output Y_t at the tax rate τ . In the absence of default, the present value of the fiscal revenue τY_t is given by $\frac{\tau Y_t}{r_{Y,s_t}}$, when current state is s_t , with r_{Y,s_t} being the discount rate for risky flows. This discount rate is given by (see

⁸The presence of economic cost provides the government the motivation for avoiding default, as in Arellano (2008), Andrade (2009), Hatchondo and Martinez (2009), Yue (2010), and Borri and Verdelhan (2012).

Appendix D.1)

$$r_{Y,s_t} = r_{s_t} - \mu_{s_t} + \frac{(r_j - \mu_j) - (r_{s_t} - \mu_{s_t})}{\hat{p} + r_j - \mu_j} \hat{p} \hat{f}_j, \quad j \neq s_t,$$
(68)

which accounts for the risk-neutral time spent in recession and in expansion at future times. These flows are thus discounted with the risk-free rate r_{st} under the risk-neutral probability measure \mathbb{Q} . We now introduce the presence of default risk. Suppose that default occurs when the output level Y_t falls to a state-dependent default boundary Y_{D,s_D} . We can express the probability of default in terms of a new Arrow-Debreu claim q'_{sts_D} , which is the present value of one unit of consumption at the moment of default t_D , which occurs either when the output level of the country falls to Y_{D,s_D} or when the default occurs instantaneously because the global economy changes state. The Arrow-Debreu claim q'_{sts_D} is defined as follows (see Appendix B.2)

$$q'_{s_t s_D} = E_t \left[\frac{\pi_{t_D}}{\pi_t} \frac{Y_{t_D}}{Y_{s_t}} Prob(s_D \mid s_t) \mid s_t \right], \ s_t, s_D = \{L, H\}.$$
 (69)

Eventually, the present value of fiscal revenue, accounting for the reduction in output after default, when current state is s_t , is given by (see Appendix D.1)

$$F_{s_t} = E_t \left[\int_t^{t_D} \tau Y_u \frac{\pi_u}{\pi_t} du \mid s_t \right] + E_t \left[\int_{t_D}^{\infty} \tau (1 - \alpha) Y_u \frac{\pi_u}{\pi_t} du \mid s_t \right]$$
 (70)

$$= \frac{\tau Y_t}{r_{Y,s_t}} - \alpha \tau \sum_{s_D} \frac{Y_{D,s_D}}{r_{Y,s_D}} q'_{s_t s_D} (Y_t), \qquad (71)$$

where the two terms of Equation 70 respectively represent the discounted tax revenue before and after default.

⁹In the present model, the default boundary is cyclical, with $Y_{D,L} < Y_{D,H}$, such that an instantaneous default can take place when the global economy is in bad state (L) but switches to the good state (H).

3.2.4 Optimal policies

The government has control over both the default and the debt policies. The optimal level of sovereign debt arises from the trade-off between the economic benefits of indebtedness, represented by the return on the public investment r_g and the economic cost of default, captured by the fraction of output lost at default α . When choosing the debt policy, the objective of the government is to maximize the ex ante level of sovereign wealth, which we define as the present value of tax revenue plus the present value of the returns from public investments. Sovereign wealth thus equals $W_{s_t} = F_{s_t} + I_{s_t}$ when current state is s_t . The optimal state-dependent coupon c_{s_t} , at time t=0, then satisfies

$$c_{s_0}^* = \arg\max W_{s_0}, \tag{72}$$

where s_o is the state of the global economy at the moment the debt is contracted. The chosen coupon thus depends on the initial state of the economy.

In a rational expectations model, the solution of the above problem reflects the fact that the government chooses a default policy that maximizes sovereign wealth *after* debt has been issued. The government maximizes sovereign wealth by choosing the optimal state-contingent default boundaries Y_{D,s_D} , which are determined by solving the following two smooth-pasting conditions (see Appendix D.4):

$$\frac{\partial (W_{s_t}(Y_t) - D_{s_t})}{\partial Y_t} |_{Y_t = Y_{D,s_t}} = \frac{\tau (1 - \alpha)}{r_{Y_{s_t}}}, \quad s_t = \{L, H\}.$$
 (73)

The problem of the government thus consists of solving Equation 72 subject to Equation 73. A closed-form solution to this optimization problem does not exist and standard numerical procedures are used.

¹⁰In practice, governments also have control over the fiscal policy. Yet countries display strong heterogeneity in the way they set tax rates over the cycle. The fiscal policy appears to be countercyclical or acyclical among OECD countries, whereas it tends to be procyclical (i.e., low tax rate in good times) in developing countries (Kaminsky, Reinhart, and Végh, 2005). In this paper, we follow Arellano and Bai (2014) and assume that a government faces a fiscal constraint in that it cannot raise tax rates to prevent a default.

4 Data and model calibration

This section presents the calibration of the model. Table 1 summarizes the parameter values. To calibrate a representative country with sovereign default risk, we consider the information provided by seven European countries (France, Greece, Iceland, Ireland, Italy, Portugal, and Spain) and eight emerging countries (Argentina, Brazil, Chile, India, Mexico, Russia, South Africa, and Turkey). This choice of the countries allows us to benefit from long time series of economic data, which span the years between 1960 and 2014.

Table 1 [about here]

4.1 Global environment and preferences

The global economy can be in expansion or in recession. The state of the economy, which is characterized by the conditional moments of consumption growth, switches randomly across states. To calibrate the Markov chain, we obtain the probability of transition from one state to another, λ_{s_t} , and the long-run probability of being in particular state, f_{s_t} , using a two-state Markov-regime switching model on quarterly U.S. consumption data over the period 1960Q1-2014Q4. The estimation approach is based on Hamilton (1989) and details are provided in Appendix E. We proxy for global consumption using data on real non-durables goods plus service consumption expenditures from the Bureau of Economic Analysis. The estimates of the actual probabilities of being in a expansion and in recession are respectively $f_H=68.6\%$ and $f_L=31.4\%$, whereas the probability per unit of time of leaving the expansion and recession states are respectively $\lambda_H=15.2\%$ and $\lambda_L=33.3\%$. We use NBER dates of the U.S. business cycle to characterize periods of expansions and recessions. When calibrating the conditional moments of consumption growth to these periods, we obtain a growth rate of consumption of $\theta_L=0.60\%$ in recession and $\theta_H=3.17\%$ in expansion, while the conditional standard deviation is $\sigma_L=1.16\%$ in recession and $\sigma_H=0.9\%$ in expansion.

Regarding the representative agent's preferences, we consider a coefficient of risk aversion $\gamma=7.5$, a coefficient of elasticity intertemporal substitution (EIS) $\psi=1.5$, and an annual discount rate equal to $\beta=1.5\%$. The values match key moments in financial markets (see Chen,

2010) and generate realistic levels of real risk-free interest rates, which are here equal to 1.72% in recession and 3.49% in expansion.

4.2 Sovereign output and characteristics

To calibrate the characteristics of a representative country, we use quarterly real GDP data for the 15 individual countries listed in Table 2. We use information over the period 1960Q1-2014Q4 for all countries, except for Argentina, Brazil, Chile, India, and Russia, whose information is available since 1996Q1 only. Real GDP data are 2005 constant price and obtained from the OECD. We use expansion and recession periods characterized by the NBER dates to compute the conditional moments of each country's real GDP growth. The dynamics of output growth are determined by the average conditional moments obtained for each country, which are reported in Table 2. We set the unconditional output growth rate to zero such that the sovereign displays a constant indebtedness level over the long term. 11 The conditional mean output growth rate is thus equal to $\mu_L=-2.12\%$ in recession and to $\mu_H=0.97\%$ in expansion. The conditional volatility of real GDP growth rate corresponds to 3.31% in recession and to 2.64% in expansion. As Chen (2013) points out, the volatility of real GDP growth severely underestimates the volatility of government net revenue, which requires a leverage adjustement in the calibration. 12 We select a leverage factor of 7 (i.e., ratio of fiscal revenue volatility to real GDP growth volatility) to generate credit spread levels close to those observed for BBB bonds. 13 In the baseline calibration, the conditional volatility of the net fiscal revenue in the generic country is thus $\sigma_L=23.16\%$ in recession and $\sigma_H=18.50\%$ in expansion.

Table 2 [about here]

The return on public investment r_g is set 1.4%, which corresponds to the average estimate reported

¹¹Having a growth rate that is unconditionally zero is similar to assuming that the debt coupon increases at the same rate as output, which corresponds to the case of a sovereign that maintains a constant long term debt-to-output ratio.

¹²The theory implicitly models the dynamics of a government's revenue net of the component that needs to be allocated to public spending. Public spending is rather constant over time and close to the level of fiscal revenue, thus generating a strong amplification effect. Hence, it is reasonable to assume that government net revenue is substantially more volatile than gross revenue (or GDP).

¹³Augustin and Tedongap (2015) report an average CDS credit spread of 152 bps for the longest (10 year) maturity (see their Table 2).

in Jeanneret (2015), using a structural estimation of 12 countries. The tax rate is set to 34%, which equals the average government revenue (as a percentage of GDP) for our 15 countries over the period 1996-2014. The economic contraction α when the sovereign defaults is set to 5%, which is the average estimate reported in Mendoza and Yue (2012) across 23 sovereign default events for the period 1977-2009. Similarly, De Paoli, Hoggarth, and Saporta (2009) analyze sovereign crises in 35 countries over the 1970-2000 period and highlight a median estimate of output loss that is slightly over 5%. Finally, the expected fraction of debt ϕ that is lost to investors is set to 60%.

5 Model predictions

We now present and discuss the theoretical predictions. The main objective of this study is to identify the various sources of risk embedded in sovereign bond valuation and to quantify their importance. This analysis will help us improve our understanding of how a country's sovereign spreads vary with macroeconomic conditions and investor preferences

It is useful to first discuss the predictions of the model under the baseline calibration. Table 3 reports the results. The model generates an unconditional credit spread of 150.9 bps and a 5-year default probability of 9.1%. Default risk is countercyclical with respect to the global business cycle, as the difference in credit spread equals 6% between recessions and expansions, for a *given level of output* in the country. The difference is amplified as a country's output becomes lower in recessions than in expansions. The unconditional default boundary indicates that a government is expected to default when its fiscal revenue fall to 46.4% of the level observed at the time of debt issuance, while the conditional default boundary equals 45.6% during expansions and 46.8% during recessions. Finally, the level of debt to service (coupon) corresponds to 2.8% of the initial level of the country's output.

Table 3 [about here]

 $^{^{14}}$ Throughout the analysis, we compute unconditional credit spreads and default boundaries as weighted averages of the state-dependent values with weights given by the long-run distribution of the Markov chain, f_L and f_H .

5.1 Macroeconomic risk

We start by investigating the role of global macroeconomic conditions in a country's sovereign credit risk. Countries are exposed to macroeconomic risk, as their growth dynamics fluctuate over the global business cycle. To explore and quantify the effect of a country's exposure to the global business cycle on the pricing of sovereign debt, we compare the full model's predictions with a special case that shuts down business cycle risk. In the latter case, the expected growth rate and volatility of the country's output and of global consumption are fixed at their unconditional means. We first compare predictions arising from an identical debt policy across both scenarios and then relax this assumption.

5.1.1 Baseline analysis

The results presented in Table 3 (Panel A) demonstrate that, on average, 30.4% of the credit spread (i.e., from 115.8 to 150.9 bps) is due to macroeconomic risk, and that this risk increases the 5-year default probability from 3.7% to 9.1%. This finding suggests that the presence of macroeconomic risk exerts an economically strong influence on a country's creditworthiness. The mechanism is as follows. Countries experience a lower expected growth rate and a higher level of economic volatility during global recessions. The direct consequence is a greater likelihood of reaching the default boundary, which is the level of output at which defaulting on sovereign debt is an optimal choice for a government. We obtain this effect even though the default boundary is cyclical in the global economic conditions, thus suggesting that governments have relatively less incentive to default during a global recession (see Table 3, Panel B). Hence, lower expected growth rates and higher volatility increase a country's default probabilities and credit spreads, not only in recessions but also unconditionally.

5.1.2 Role of debt policy

The possibility to select an optimal debt policy amplifies the effect macroeconomic risk on sovereign creditworthiness, which arises from the procyclical risk-free rate in equilibrium.¹⁵ The central

¹⁵The time-variation in the moments of consumption growth creates procyclicality in the risk-free rate due to the greater demand for risk-free savings during bad states. This equilibrium property is in line with the evidence that

mechanism behind this effect is the prediction that recessions not only lower the risk-free interest rate but also reduce the unconditional discount rate relevant for pricing securities. The reason is that risk-averse investors dislike bad states of the global economy, thus generating a bond discount rate r_{B,s_t} that is lower than the risk-free rate r_{f,s_t} , on average (see Table 3, Panel B). Investors thus overweight the expected fall in risk-free rates during recessions.

The effect of a lower discount rate is to raise the valuation of debt and to encourage governments to increase their indebtedness level with the aim of fostering public investments at the cost of raising default risk, which can become a severe issue when a country's economic conditions deteriorate. Hence, countries issuing debt with a procyclical risk-free interest rate benefit from cheap funding but exhibit less incentives to discipline themselves regarding the size of their debt issuance. To confirm this intuition, Table 3 (Panel A) suggests that governments prefer to increase their debt issuance in presence of global business cycle risk, despite the anticipation of future bad macroeconomic shocks would amplify sovereign credit risk. As a consequence, the probability of default more than doubles when one accounts for both global macroeconomic risk and an endogenous debt policy, and sovereign spreads become meaningfully wider.

5.1.3 Conditional effect

The impact of global macroeconomic risk on a country's sovereign spreads varies over time and across countries. The model predicts that it is particularly pronounced when economic conditions deteriorate in the country (see Figure 1), when a government tends to be more indebted (i.e., higher public investment returns), when the country displays higher economic uncertainty, and when the representative agent displays greater risk aversion and dislikes intertemporal uncertainty (see Figure 2). The model thus sheds new light on the role of global business cycle risk for sovereign debt valuation. Overall, this analysis suggests that countries with higher sensitivity to the global macroeconomic conditions should be characterized by a greater time-variation in default risk and by higher sovereign credit spreads. We find that this prediction is particularly pronounced for countries

flight-to-safety episodes coincide with a decrease in real GDP growth and a rise in economic uncertainty (see Engle, Fleming, Ghysels, and Ngyuen, 2012; Baele, Bekaert, Inghelbrecht, and Wei, 2014).

¹⁶In practice, this would correspond to the case of emerging countries issuing dollar-denominated debt and of European countries issuing bonds in euro, for example. Indeed, risk-free assets in these currencies (respectively the U.S. Treasuries and German Bunds) typically act as safe-havens during flight-to-safety episodes (see Krishnamurthy and Vissing-Jorgensen, 2012).

close to financial stress.

Figures 1, 2, and 3 [about here]

5.1.4 Source of macroeconomic risk

We now deepen our analysis on the role of macroeconomic risk by decomposing its effect on each conditional moment of a country's output growth. Recessions induce a lower mean and a higher volatility of economic growth, but which moment matters more? The results presented in Figure 3 address this question. We find that the sensitivity to the global business cycle matters largely through changes in a country's economic growth, and relatively less so through variations in its uncertainty. Our model therefore suggests that a central component of a country's sovereign default risk is that its economic performance is expected to slow down in times of global economic downturns. Similarly, the results suggest that the time-variation in the mean of consumption growth play a dominant role, as variations in the volatility of consumption contribute almost insignificantly to the increase in sovereign credit spreads.

5.2 Investor preferences

Sovereign credit spreads reflect not only a country's default risk but also the market price of such a risk. The reason being that investors are averse to intertemporal uncertainty and thus require a compensation for holding risky government bonds based on their risk-neutral valuation. Consequently, investor preferences are expected to play an important role in explaining the level of sovereign credit spreads.

Investors require a premium for essentially two types of risk. First, the exposure to the global business cycle causes sovereign default risk to be partly systematic. Risk-averse agents are particularly sensitive to a rise in default risk during global recessions, when marginal utility of consumption is high, and thus demand a greater compensation for default risk. The second component comes from the exposure to global macroeconomic risk, which introduces uncertainty about the future state of the global economy. Investors dislike uncertainty about when bad states of the economy will occur and thus price financial assets as if recessions had a higher duration than in reality, using

the risk-neutral rather than the actual transition probabilities. That is, agents prefer earlier resolution of the uncertainty as to when recessions will arrive. The risk distortion factor Δ_H , which allows converting the physical transition probability λ_{s_t} to the risk-neutral counterpart $\hat{\lambda}_{s_t} = \Delta_H \lambda_{s_t}$, is equal to 1.48. We can thus expect the price of macroeconomic risk to be economically important. To gauge the magnitude of the effect, we determine the price of macroeconomic risk by comparing the predictions of the full model with those of an hypothetical identical case but in which we assume no intertemporal risk, that is when $\Delta_H = 1$ (i.e., $\hat{\lambda}_{s_t} = \lambda_{s_t}$). To be consistent in the comparison, we consider the same optimal barriers and debt coupons, along an identical risk-free rate, across both cases.

Table 4 [about here]

Table 4 indicates that agents' preferences for early resolution of uncertainty contribute to 7% in the level of sovereign credit spreads. Thus, 93% of the level of credit spreads represents a compensation for credit risk. The price of macroeconomic risk is greater in expansion than in recession, as investors dislike uncertainty about the next recession (see Figure 5, upper panels). Moreover, Figure 4 suggests that the price of risk is particularly important when a country's conditions are more favorable and can reach 25% of the credit spread level. The price of risk contributes more to credit spread levels when economic performance is favorable (high Y) and rather stable (low σ_{i,s_t}), and when there is less incentive for indebtedness (low r_g). These characteristics correspond to cases of lower sovereign credit risk. Our analysis thus suggests that the magnitude of the price of risk, and in particular the price of macroeconomic risk, is greater for relatively safer bonds. This is clear from Figure 5. In contrast, credit spreads mostly reflect the compensation for expected default losses in the case of distressed bonds. The price of macroeconomic risk is particularly sensitive to the level of default risk because investors care more about the uncertainty regarding business cycle changes when a bond is expected to default in a more distant future.

The same conclusion is obtained when we focus on the ratio between the risk-neutral and the physical default probability, computed over a 5-year horizon (see Figure 5, lower panels). The ratio varies between 1 and 1.6 depending on a country's economic conditions, which compares to Huang and Huang (2012)'s ratio ranging between 1.1 and 1.7 for corporate bonds. Further, Augustin and Tedongap (2015) show that the ratios that best explain sovereign CDS data increase with better

credit ratings, consistent with our prediction. Hence, both the magnitude and the time-variation of our model-implied ratio of risk-neutral to physical default probabilities are in line with the literature.

Figures 4 and 5 [about here]

We also investigate how investor preferences themselves drive the price of risk. Table 4 and Figure 4 present the results for different levels of risk aversion and preferences for early resolution of uncertainty. To disentangle the effect of preferences on the price of risk from the effect on the equilibrium risk-free rate, Table 4 report results under two scenarios: the risk-free rate either remains identical to that of the baseline calibration or, alternatively, varies with the agent's preferences. In both cases, the results indicate that the credit spreads increase when the pricing agent is more risk-averse (high γ) and displays higher intertemporal elasticity of substitution (high ψ). The price of risk increases in absolute and in relative terms. The prediction is further strengthened when the risk-free rate responds to a change in investor preferences. This increase in credit spreads is a combination of both a higher price of risk and a greater quantity of risk, as governments optimally choose to increase indebtedness and to default sooner under such preferences.

Overall, our model provides new insights regarding the influence of investor preferences on the pricing of debt. We show that sovereign credit spreads capture both the risk of default and the price of this risk. Notably, our analysis suggests that the price of risk is economically important and plays a relatively greater role for sovereign bonds with lower default risk.

5.3 Conditional credit spreads, default clustering, and co-movement

We now consider a simulation of the model to illustrate how macroeconomic risk helps explain countercyclicality in sovereign credit spreads, their co-movement across countries, as well as the default clustering during recessions.

Our simulation procedure consists of generating time-series of quarterly output for 50 countries over the period from 1947 to 2014. We use the mean and volatility of conditional output growth rate obtained from our calibration exercise (see Table 1). Importantly, countries only experience idiosyncratic shocks, as in the model, which implies that the instantaneous correlation in output growth across countries is zero. Yet each country's output moments vary over the global business

cycle, as determined by NBER official dates. Each quarter, the government observes its output level and the state of the global economy before making the decision to default or to continue servicing the current debt. To be realistic, we suppose that the number of countries remains constant over the sample period. To this end, we assume that defaulted countries benefit from debt restructuring so that they emerge in the next quarter as new debtors. We repeat the simulation 100 times to control for the sensitivity to macroeconomic history and average the results.

Figure 6 illustrates the model predictions and the shaded areas represent the periods when the global economy is in the bad state, which corresponds to the NBER recession periods. Panel A of Figure 6 plots the time series of average credit spreads, Panel B their co-movement, and Panel C illustrates the quarterly default rates. Our co-movement measure is the percentage of countries for which the variation in spreads between two consecutive quarters move in the same direction (i.e., same sign).¹⁷ Table 5 summarizes the results.

Figure 6 and Table 5 [about here]

Sovereign credit spreads tend to peak and to fluctuate strongly together during global recession periods. Co-movement is particularly high at the beginning and at the end of recessions. Notably, these countercyclical properties of sovereign credit risk are obtained without introducing common shocks, which would obviously create a mechanical effect and amplify our predictions. The results are driven by time variation in the global business cycle, which influences each country's output growth moments and default decision, as discussed in Section 5.1. This is the direct effect of fluctuating macroeconomic conditions.

The model also generates susbstantial sovereign default clustering, particularly when global economic conditions detereriorate. The simulation yields lower sovereign default rates in boom times (0.552%) than during recession periods (0.758%), as reported in Table 5. Hence, the default rate is 27% higher in bad times. The phenomenon that countries tend to default simultaneously in our simulated economy arises because an unexpected deterioration in economic conditions causes a government's default incentive to suddenly increase (i.e., upward jump in the default boundary). As

¹⁷We prefer this metric over the correlation, which tends to be a biased indicator of interdependence in presence of heteroscedasticity (see Forbes and Rigobon, 2002). In addition, the sign indicator is much more convenient to analyze than having 1225 pair-wise time series of correlation.

a consequence, countries with output levels below the new default boundaries will instantaneously default, thereby generating a clustering of defaults.

Overall, this analysis shows that our model can explain a set of important sylized facts. First, sovereign credit spreads are countercyclical with respect to global economic conditions (see Arellano (2008); Neumeyer and Perri (2004); Uribe and Yue (2006)); Second, sovereign credit spreads exhibit co-movement, which is exacerbated in bad times (see Augustin and Tedongap (2015); Benzoni et al. (2015); Longstaff et al. (2010)); Third, sovereign defaults tend to be clustered around global recessions (see Reinhart and Rogoff (2008)).

One central contribution of our theory is to show that the presence of macroeconomic risk can explain these stylized facts. To provide evidence of this channel, we consider the same simulation but now turn off the impact of macroeconomic conditions. That is, each country's output growth moments are set at their unconditional values. The predictions, which are displayed in Figure 7 and reported in Table 5, greatly contrast with those of the baseline model. In absence of global macroeconomic conditions, sovereign spreads become acyclical, the degree of co-movement remains steady over time and across economic conditions, and defaults no longer cluster at the business cycle frequency.

Figure 7 [about here]

6 Concluding remarks

This paper provides new insights on the role of global macroeconomic risk and investor preferences for the explanation of sovereign credit spreads. Our theoretical analysis is based on the empirical evidence that fluctuations of global economic conditions affect most countries and exert strong influence on their sovereign credit risk. We show that macroeconomic risk fosters default risk through various complementary channels. On the one hand, sovereign countries experience lower economic growth and more volatile shocks during global recessions. On the other, under such circumstances, governments adapt their optimal policies. They choose a higher indebtedness level and to default rather sooner. Moreover, investors typically demand greater risk compensation, as they dislike the possibility of sovereign defaults during a global recession. They also prefer

uncertainty about future economic conditions to be resolved sooner than later. As a result, the exposure to global macroeconomic conditions increases the quantity and the price of sovereign default risk, both of which contribute meaningfully to the level of sovereign credit spreads.

Further, we show that time-varying global economic conditions combined with recursive preferences generate procyclicality in the risk-free rate that exerts a strong influence on the valuation of government debt. Investors price debt with a lower discount rate, as they overweight the probability of a fall in interest rate during recessions. Higher debt value then provides greater incentive for higher indebtedness, thereby worsening a country's creditworthiness. This prediction would have relevant financial and policy implications for emerging markets, which have a history of issuing sovereign bonds in U.S. dollar, mostly to benefit from better liquidity. Our analysis suggests that issuing debt in such a currency should not be viewed as a free lunch, as the procyclicality in the U.S. interest rates tend to enhance their sovereign default risk. This finding may help explain the recent development of the local-currency bond market in emerging countries, which has been of increasing size and importance over the last decade.

References

Acharya, V. V., and Rajan, R. G., 2013, Sovereign Debt, Government Myopia, and the Financial Sector, *Review of Financial Studies* 26(6), 1526–1560.

Aguiar, M., and Gopinath, G., 2006, Defaultable Debt, Interest Rates and the Current Account, *Journal of international Economics* 69(1), 64–83.

Andrade, S. C., 2009, A Model of Asset Pricing under Country Risk, *Journal of International Money and Finance* 28(4), 671–695.

Andrade, S. C., and Chhaochharia, V., 2011, How Costly is Sovereign Default? Evidence from Financial Markets, Working Paper, University of Miami.

Ang, A., and Bekaert, G., 2007, Stock Return Predictability: Is it There?, *Review of Financial Studies* 20(3), 651–707.

Ang, A., and Longstaff, F. A., 2013, Systemic Sovereign Credit Risk: Lessons from the U.S. and Europe, *Journal of Monetary Economics* 60(5), 493–510.

Arellano, C., 2008, Default Risk and Income Fluctuations in Emerging Economies, *American Economic Review* 98(3), 690–712.

Arellano, C., and Bai, Y., 2014, Fiscal Austerity during Debt Crises, Working paper, University of Rochester.

Augustin, P., and Tédongap, R., 2015, Real Economic Shocks and Sovereign Credit Risk, *Journal of Financial and Quantitative Analysis*, Forthcoming.

Baele, L., Bekaert, G., Inghelbrecht, K., and Wei, M., 2014, Flights to Safety, Working paper, Columbia University.

Bansal, R., and Yaron, A., 2004, Risks for the Long Run: A Potential Resolution of Asset Pricing Puzzles, *Journal of Finance* 59(4), 1481–1509.

Beber, A., Brandt, M. W., and Kavajecz, K. A., 2009, Flight-to-quality or Flight-to-liquidity? Evidence from the Euro-area Bond Market, *Review of Financial Studies* 22(3), 925–957.

Benzoni, L., Collin-Dufresne, P., Goldstein, R. S., and Helwege, J., 2015, Modeling Credit Contagion via the Updating of Fragile Beliefs, *Review of Financial Studies* 28(7), 1960–2008.

Bhamra, H. S., Kuehn, L.-A., and Strebulaev, I. A., 2010a, The Aggregate Dynamics of Capital Structure and Macroeconomic Risk, *Review of Financial Studies* 23(12), 4187–4241.

Bhamra, H. S., Kuehn, L.-A., and Strebulaev, I. A., 2010b, The Levered Equity Risk Premium and Credit Spreads: A Unified Framework, *Review of Financial Studies* 23(2), 645–703.

Bollerslev, T., Marrone, J., Xu, L., and Zhou, H., 2014, Stock Return Predictability and Variance Risk Premia: Statistical Inference and International Evidence, *Journal of Financial and Quantitative Analysis* 49(3), 633–661.

Borri, N., and Verdelhan, A., 2012, Sovereign Risk Premia, Working Paper, MIT.

Chen, H., 2010, Macroeconomic Conditions and the Puzzles of Credit Spreads and Capital Structure, *Journal of Finance* 65(6), 2171–2212.

Chen, H., 2013, Comment on "Systemic Sovereign Credit Risk: Lessons from the U.S. and Europe" by Ang and Longstaff, *Journal of Monetary Economics* 60(5), 511–516.

Colacito, R., and Croce, M. M., 2013, International Asset Pricing with Recursive Preferences, *Journal of Finance* 68(6), 2651–2686.

Cruces, J. J., and Trebesch, C., 2013, Sovereign Defaults: The Price of Haircuts. *American Economic Journal: Macroeconomics* 5(3), 85–117.

De Paoli, B., Hoggarth, G., and Saporta, V., 2009, Output Costs of Sovereign Crises: Some Empirical Estimates, *Bank of England Working Paper 362*.

Duffie, D., and Epstein, L. G., 1992, Stochastic Differential Utility, *Econometrica* 60(2), 353–94.

Duffie, D., and Skiadas, C., 1994, Continuous-Time Security Pricing: A Utility Gradient Approach, *Journal of Mathematical Economics* 23(2), 107–131.

Dumas, B., 1991, Super Contact and Related Optimality Conditions, *Journal of Economic Dynamics and Control* 15(4), 675–685.

Engle, R., Fleming, M., Ghysels, E., and Nguyen, G., 2012, Liquidity, Volatility, and Flights to Safety in the US Treasury Market: Evidence from a New Class of Dynamic Order Book Models, Working paper, NYU.

Epstein, L. G., and Zin, S. E., 1989, Substitution, Risk Aversion, and the Temporal Behavior of Consumption and Asset Returns: A Theoretical Framework, *Econometrica* 57(4), 937–69.

Fischer, E. O., Heinkel, R., and Zechner, J., 1989, Dynamic Capital Structure Choice: Theory and Tests, *Journal of Finance* 44, 19–40.

Forbes, K. J. and Rigobon, R, 2002, No Contagion, Only Interdependence: Measuring Stock Market Comovements, Journal of Finance 57, 2223-2261.

François, P., Hübner, G., and Sibille, J.-R., 2011, A Structural Balance Sheet Model of Sovereign Credit Risk, *Finance* 32(2), 137–165.

Gibson, R., and Sundaresan, S., 2001, A Model of Sovereign Borrowing and Sovereign Yield Spreads, Working Paper, Columbia University.

Hatchondo, J. C., and Martinez, L., 2009, Long-duration Bonds and Sovereign Defaults, *Journal of International Economics* 79(1), 117–125.

Hayri, A., 2000, Debt Relief, Journal of International Economics 52(1), 137-152.

Hilscher, J., and Nosbusch, Y., 2010, Determinants of Sovereign Risk: Macroeconomic Fundamentals and the Pricing of Sovereign Debt, *Review of Finance* 4(2), 235–262.

Jeanneret, A., 2017, Sovereign Default Risk and the U.S. Equity Market, *Journal of Financial and Quantitative Analysis*, 52(1), 305-339.

Jeanneret, A., 2015, The Dynamics of Sovereign Credit Risk, *Journal of Financial and Quantitative Analysis*, 50(5), 963–985.

Kreps, D. M., and Porteus, E. L., 1978, Temporal Resolution of Uncertainty and Dynamic Choice Theory, *Econometrica* 46, 185–200.

Krishnamurthy, A., and Vissing-Jorgensen, A., 2012, The Aggregate Demand for Treasury Debt, *Journal of Political Economy* 120(2), 233-267.

Longstaff, F. A., 2004, The Flight-to-Liquidity Premium in US Treasury Bond Prices, *Journal of Business* 77(3), 511–526.

Longstaff, F. A., Pan, J., Pedersen, L. H., and Singleton, K. J., 2011, How Sovereign Is Sovereign Credit Risk, *American Economic Journal: Macroeconomics* 3(2), 75–103.

Leland, H. E., 1994, Corporate Debt Value, Bond Covenants, and Optimal Capital Structure, *Journal of Finance* 49(4), 1213–1252.

Mendoza, E. G., and Yue, V. Z., 2012, A General Equilibrium Model of Sovereign Default and Business Cycles, *Quarterly Journal of Economics* 127(2), 889–946.

Merton, R. C., 1973, The Theory of Rational Option Pricing, *Bell Journal of Economics* 4(1), 141–183.

Monfort, A., and Renne, J. P., 2013, Decomposing Euro-Area Sovereign Spreads: Credit and Liquidity Risks, *Review of Finance* 18(6), 2103–2151.

Neumeyer, P., and Perri F., 2005, Business Cycles in Emerging Economies: The Role of Interest Rates, Journal of Monetary Economics 52(2), 345-380.

Pan, J., and Singleton, K. J., 2008, Default and Recovery Implicit in the Term Structure of Sovereign CDS Spreads, *Journal of Finance* 63, 2345–2384.

Rapach, D. E., Strauss, J. K., and Zhou, G., 2013, International Stock Return Predictability: What is the Role of the United States?, *Journal of Finance* 68(4), 1633–1662.

Reinhart, C. M., and Rogoff, K. S., 2008, This Time is Different: A Panoramic View of Eight Centuries of Financial Crises, NBER Working paper No. 13882.

Reinhart, C. M., and Rogoff, K. S., 2013, Financial and Sovereign Debt Crises: Some Lessons Learned and Those Forgotten, Working paper, Harvard University.

Remolona, E., Scatigna, M., and Wu, E., 2008, The Dynamic Pricing of Sovereign Risk in Emerging Markets: Fundamentals and Risk Aversion, *Journal of Fixed Income* 17(4), 57–71.

Uribe, M. and Yue, V. Z., 2006, Country spreads and emerging countries: Who drives whom?, Journal of International Economics 69(1), 6-36.

Weil, P., 1990, Nonexpected Utility in Macroeconomics, *Quarterly Journal of Economics* 105(1), 29–42.

Yue, V. Z., 2010, Sovereign Default and Debt Renegotiation, *Journal of International Economics* 80(2), 176–187.

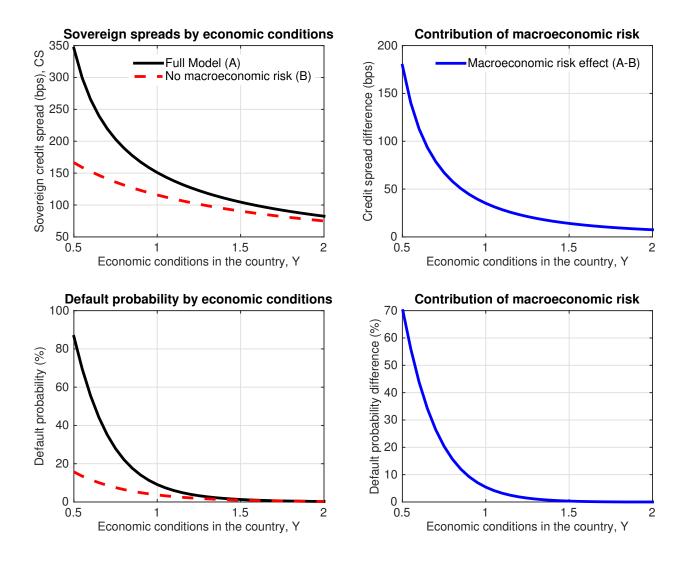


Figure 1: Macroeconomic risk and sovereign credit risk by economic conditions. This figure illustrates how macroeconomic risk influences sovereign credit risk for different levels of economic conditions in the sovereign country. The upper panels display predictions on sovereign credit spreads, whereas the lower panels show the results for the (5-year) default probability. Predictions for the full model are compared with those of the model without macroeconomic risk by switching-off all business cycles. The left panels display both predictions on sovereign credit risk, while the right panels report the marginal impact of macroeconomic risk, using the scenario without business cycle risk as the base case. Unless otherwise specified, we use the parameters of the baseline calibration (see Table 1).

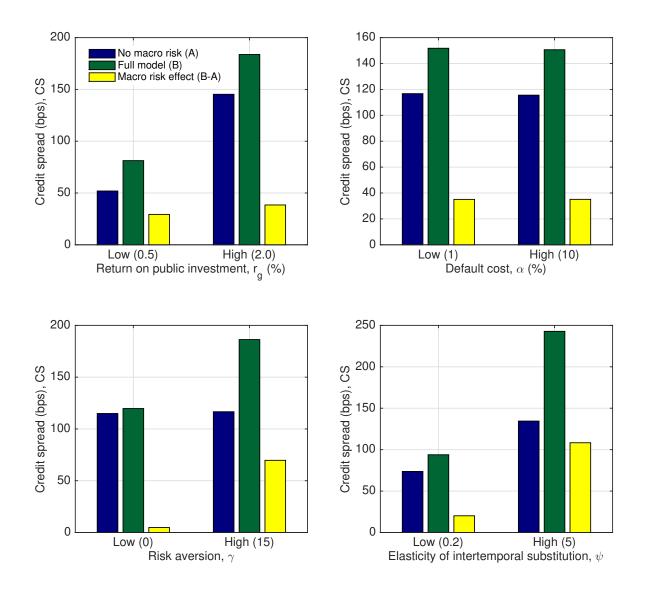


Figure 2: Comparative static analysis of the macroeconomic risk effect. This figure shows the sensitivity of the sovereign credit spread to macroeconomic risk with respect to some key model parameters. We explore the role of the government's incentive for indebtedness (top-left panel), the economic cost of default (top-right panel), and the representative agent's preferences (bottom panels). We compare the predictions of the full model with the case in which we ignore macroeconomic risk by switching-off all business cycles. Unless otherwise specified, we use the parameters of the baseline calibration (see Table 1).

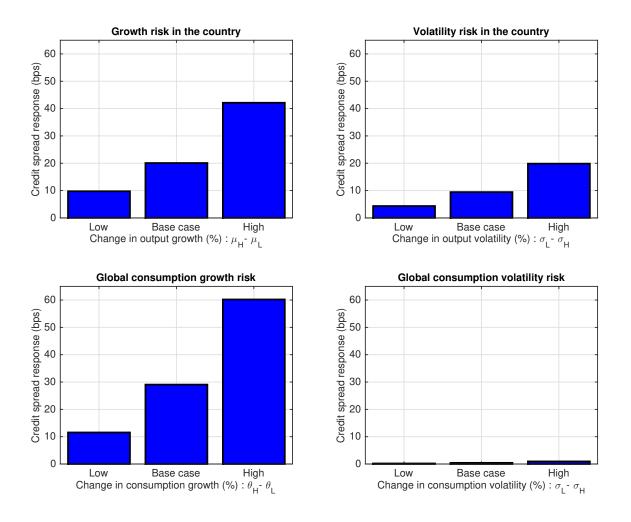


Figure 3: Source of macroeconomic risk and relative contribution to sovereign spreads. This figure illustrates how the various components of macroeconomic risk impact sovereign credit spreads. We analyze the influence of the pro(counter)cyclical variation in the mean (volatility) of the country's output growth in the top-left (right) panel and the pro(counter)cyclical variation in the mean (volatility) of global consumption growth in the bottom-left (right) panel. The figure displays the change in credit spread that is due to a time-variation in each of these parameters. The low (high) scenario represents the case in which variations in the moments are one half (twice) of those under the base case calibration. Unless otherwise specified, we use the parameters of the baseline calibration (see Table 1).

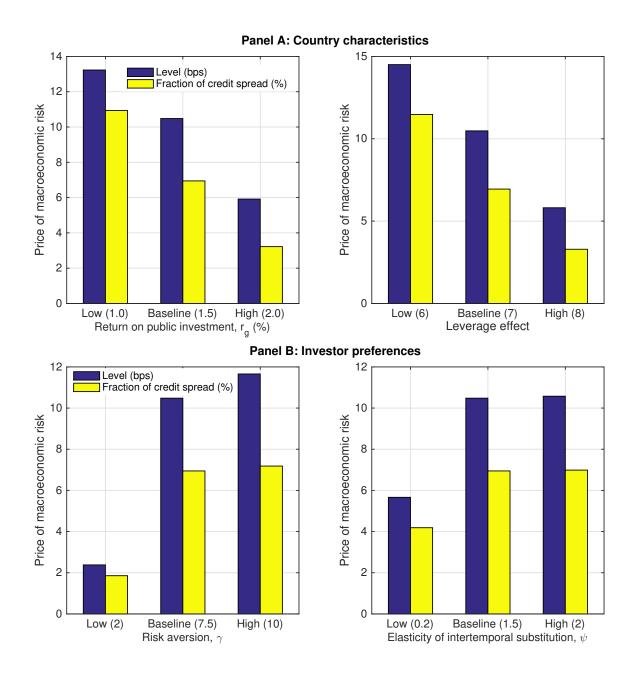


Figure 4: Price of risk by investor preferences and country characteristics. This figure presents the price of risk and its sensitivity with respect to some key parameters. We explore the role of the sovereign country's characteristics (Panel A), as determined by the government's incentive for indebtedness and the level of economic uncertainty, and the representative agent's preferences (Panel B). Unless otherwise specified, we use the parameters of the baseline calibration (see Table 1).

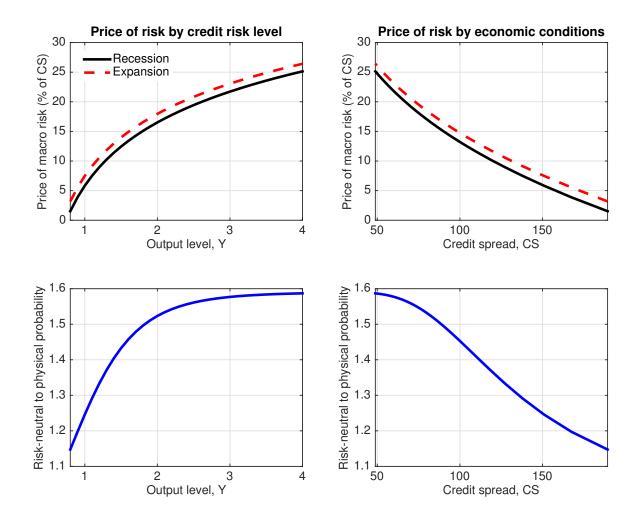


Figure 5: **Price of risk by economic conditions and default risk**. This figure presents the price of macroeconomic risk by economic conditions in the sovereign country (left Panels) and by the level of credit risk (right Panels). The upper panels display the fraction of the credit spreads that is due to the price of macroeconomic risk, whereas the lower panels show the ratio between the risk-neutral and the physical default probability. Probabilities are state-weighted and computed for a 5 year horizon. Unless otherwise specified, we use the parameters of the baseline calibration (see Table 1).

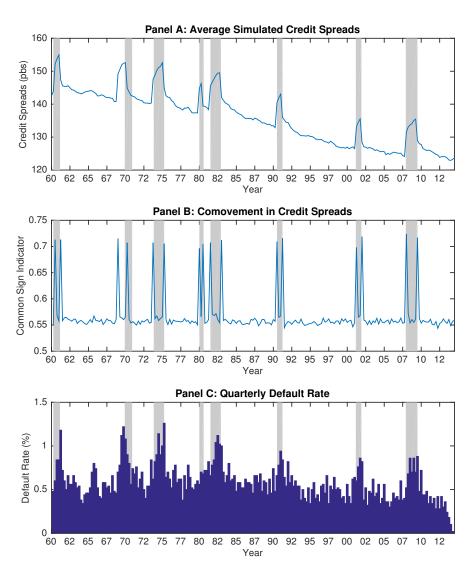


Figure 6: Sovereign Credit Spreads, Co-movement, and Default Rate – Full Model. This figure illustrates the results of a simulated economy. We use the model to simulate 50 countries and generate each country's output time-series over the 1960-2014 period with the parameters of the baseline calibration (see Table 1). Panel A presents the average sovereign credit spreads and Panel B shows the degree of co-movement, which we compute as the percentage of countries for which changes in credit spreads have the same sign during two consecutive quarters. Panel C displays the quarterly default rate. The results are obtained by averaging 100 simulations.

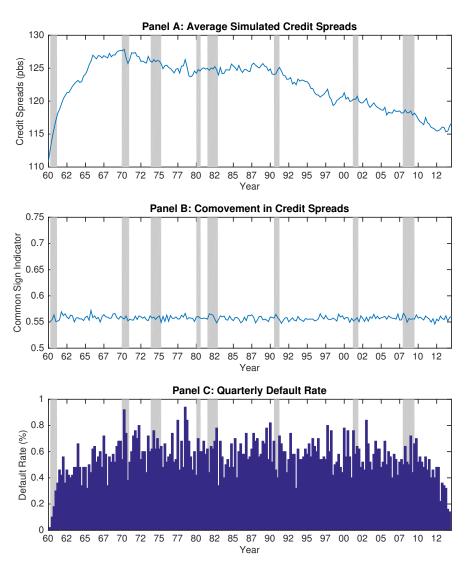


Figure 7: Sovereign Credit Spreads, Co-movement, and Default Rate — without Macrorisk. This figure illustrates the results of a simulated economy without macroeconomic risk. We use the constrained version of model to simulate 50 countries and generate each country's output time-series over the 1960-2014 period with the parameters of the baseline calibration (see Table 1). Panel A presents the average sovereign credit spreads and Panel B shows the degree of co-movement, which we compute as the percentage of countries for which changes in credit spreads have the same sign during two consecutive quarters. Panel C displays the quarterly default rate. The results are obtained by averaging 100 simulations.

Table 1: Model calibration.

This table reports the parameter values used for the calibration of the model. The state of the economy is determined by the NBER recession dates in the U.S. over the period 1960Q1-2014Q4. The state $s_t = H$ refers to an expansion, whereas the state $s_t = L$ corresponds to a recession. The frequency of the data is quarterly and the values are annualized, when applicable.

Variable	Notation	Value		Source
Panel A: Global environment and agent prefer	ences			
State of the global economy	s_t	L	Н	
Consumption growth rate (%)	$ heta_{s_t}$	0.60	3.17	D ([
Consumption growth volatility (%)	σ_{s_t}	1.16	0.90	Bureau of Economic Analysis,
Actual long-run probability (%)	f_{s_t}	31.4	68.6	1960Q1-2014Q4
Consumer time preference	eta	0.015	0.015	Chen (2010)
Risk aversion coefficient	γ	7.5	7.5	Chen (2010)
Elasticity of intertemporal substitution	ψ	1.5	1.5	Chen (2010)
Panel B: Country characteristics				
Output growth rate (%, stationarized)	μ_{s_t}	-2.12	0.97	OECD, 1960Q1-2014Q4,
Output volatility (%)	σ_{s_t}	23.16	18.50	volatility leverage factor of 7
Return on public investment	r_g	0.014	0.014	Jeanneret (2015)
Tax rate	au	0.34	0.34	Government revenue (% of GDP), 1996-2012.
Economic loss in default (fraction of output)	α	0.05	0.05	Mendoza and Yue (2012)
Debt hair cut in default	ϕ	0.6	0.6	ISDA's CDS pricing convention

Table 2 : Economic characteristics of selected sovereign countries.

This table displays the mean and volatility of output growth for a set of economies that we use to calibrate the representative country. We compute the moments of output growth using quarterly real GDP data obtained from the OECD. Data span the period 1960Q1-2014Q4 for all countries, except for Argentina, Brazil, Chile, India, and Russia, whose information is available since 1996Q1. We condition all moments by the state of the global economy, which is determined by the NBER recession dates.

	Output growth (%)		Output growth volatility (%)	
	Recession	Expansion	Recession	Expansion
European countries				
France	1.415	2.964	1.793	2.357
Greece	2.688	2.842	7.301	4.614
Iceland	1.356	3.966	4.344	3.800
Ireland	1.784	4.559	2.720	2.700
Italy	0.884	2.726	3.350	1.678
Portugal	1.062	3.520	2.865	2.254
Spain	2.780	3.530	2.615	2.000
Average	1.710	3.444	3.570	2.771
Emerging countries				
Argentina	-6.324	4.487	1.732	3.285
Brazil	-0.180	3.243	2.958	2.148
Chile	0.084	4.251	5.109	2.365
India	4.918	6.626	3.594	1.731
Mexico	1.926	4.188	3.285	2.277
Russia	-1.465	4.480	2.130	2.958
South Africa	1.939	3.274	2.148	1.978
Turkey	0.476	5.079	2.365	2.512
Average	0.172	4.453	3.125	2.532
Average (all)	0.890	3.982	3.308	2.643

Table 3: Macroeconomic risk, business cycle effect, and sovereign credit risk.

This table reports theoretical predictions for the full model, which we compare with two special cases. We consider first a model without macroeconomic risk by completely switching-off the global business cycle (Column 2, Panel A). Then, we consider a model in which the global business cycle exists (i.e., risk-free rate is procyclical) but the country is not exposed to it (Column 3, Panel A). The analysis displays the results when the debt level is either endogenous (base case) or exogenous. The Panel B presents the results of the full model for each state of the global economy. All the predictions relate to the initial state being in expansion ($s_0 = H$). We use the parameters of the baseline calibration (see Table 1).

Panel A : Predictions for different scenarios						
	Full model	Without	No exposure to			
		macroeconomic	global business			
		risk	cycle			
Endogenous debt						
Coupon	0.0279	0.0246	0.0283			
Default boundary	0.4640	0.3938	0.4266			
Credit spread (bps)	150.90	115.76	121.50			
Default probability (5y)	0.0911	0.0368	0.0564			
Exogenous debt						
Coupon	0.0246	0.0246	0.0246			
Default boundary	0.4097	0.3938	0.3715			
Credit spread (bps)	134.14	115.76	104.84			
Default probability (5y)	0.0523	0.0368	0.0266			
Panel B : State-dependency (full model)						
	Unconditional	Recession	Expansion			
Coupon	0.0279	0.0279	0.0279			
Default boundary	0.4640	0.4559	0.4677			
Credit spread (bps)	150.90	157.02	148.10			
Risk-free rate (%)	2.934	1.720	3.489			
Bond discount rate (%)	2.608	2.542	2.639			

Table 4: Price of risk by sovereign characteristics and investor preferences

This table reports the credit spreads of the full model, which we compare with a model without macroeconomic risk aversion (i.e., no preference for early resolution of uncertainty, $\hat{\lambda}_{s_t} = \lambda_{s_t}$). The fourth column reports the compensation for the price of the macroeconomic risk embedded in the sovereign credit spreads. Panel A presents results for various levels of country characteristics, as given by the levels of output and idiosyncratic volatility, as well as the return on public investment (i.e., indebtedness incentive). Panel B presents results for various levels of investor preferences, as given by the coefficient of risk aversion (RA) and the elasticity of intertemporal substitution (EIS).

	Full model	No macro risk aversion (bps)	Price of macro risk (bps)	Price of macro risk (%)
Base case	150.90	140.42	10.48	6.95
Panel A: Sovereign charact	eristics			
High output ($Y=2.0$)	82.48	68.05	14.43	17.50
Low risk $(\sigma^{id}$ low)	126.35	111.84	14.50	11.48
Low debt ($r_g=1.0\%$)	121.01	107.78	13.23	10.93
Panel B: Investor preferenc	es			
Constant interest rate				
High RA ($\gamma=10$)	162.29	150.64	11.66	7.18
High EIS $(\psi=2)$	151.38	140.81	10.58	6.99
Endogenous interest rate				
High RA ($\gamma=10$)	163.44	152.91	10.53	6.45
High EIS $(\psi=2)$	157.68	150.14	7.54	4.78

Table 5 : **Simulated Economy**.

This table reports the results of a simulated economy. We use the model to simulate 50 countries and generate each country's output time-series over the 1960-2014 period with the parameters of the baseline calibration (see Table 1). Panel A reports the average sovereign credit spreads. Panel B reports the degree of co-movement, which we compute as the percentage of countries for which changes in credit spreads have the same sign during two consecutive quarters. Panel C reports the average quarterly default rates. We compare the conditional and unconditional predictions with and without the presence of macroeconomic risk. The results are obtained by averaging 100 simulations.

Danal A . Avarage anneada							
Panel A : Average spreads							
	Recession	Expansion	Unconditional	Correlation with			
				U.S. recessions			
Baseline	142.48	134.85	135.98	0.217			
No macroeconomic risk	122.90	122.41	122.46	0.013			
Panel B : Average co-mov	Panel B : Average co-movement measure						
	Recession	Expansion	Unconditional				
Baseline	0.598	0.563	0.568				
No macroeconomic risk	0.558	0.557	0.557				
Panel C : Average default rate							
	Recession	Expansion	Unconditional				
Baseline	0.758	0.552	0.579				
No macroeconomic risk	0.554	0.557	0.551				

7 Appendix

The model uses a state-dependent approach to derive all claims and endogenous variables according to the state of the global economy, which can be in expansion or in recession.

7.1 State-price density and equilibrium risk-free rate

In this section, we provide the formula of the state-price density and the equilibrium risk-free rate, based on Bhamra, Kuehn, and Strebulaev (2010b).¹⁸ The state-price density is initially derived by Duffie and Skiadas (1994) for the general class of stochastic differential utility function proposed by Duffie and Epstein (1992). This type of utility function incorporates not only the agent's risk aversion but also the aversion for intertemporal resolution of the uncertainty in the economy.

The representative agent's state-price density π_t , in the case $\psi \neq 1$, is given by

$$\pi_t = \left(\beta e^{-\beta t}\right)^{\frac{1-\gamma}{1-\frac{1}{\psi}}} C_t^{-\gamma} \left(p_{C,s_t} e^{\int_0^t p_{C,s_u}^{-1} du}\right)^{-\frac{\gamma - \frac{1}{\psi}}{1-\frac{1}{\psi}}}, \tag{74}$$

where p_{C,s_t} is the price-consumption ratio that satisfies the following implicit non-linear equation:

$$p_{C,s_{t}}^{-1} = \overline{r}_{s_{t}} - \theta_{s_{t}} + \gamma \sigma_{s_{t}}^{2} - \left(1 - \frac{1}{\psi}\right) \lambda_{s_{t}} \left(\frac{\left(\frac{p_{C,\bar{s}_{t}}}{p_{C,s_{t}}}\right)^{\frac{1-\gamma}{1-\frac{1}{\psi}}} - 1}{1 - \gamma}\right), \ s_{t}, \bar{s_{t}} \in \{L, H\}, \bar{s_{t}} \neq s_{t}$$
 (75)

with

$$\overline{r}_{s_t} = \beta + \frac{1}{\psi} \theta_{s_t} - \frac{1}{2} \gamma \left(1 + \frac{1}{\psi} \right) \sigma_{s_t}^2. \tag{76}$$

The dynamics of the state-price density π_t follow the following stochastic differential equation

¹⁸For additional details and complete derivation, we refer the reader to the Appendix of Bhamra, Kuehn, and Strebulaev (2010b) in the case of two states, and to the Appendix of Chen (2010) for N states.

$$\frac{d\pi_t}{\pi_t} = -r_{s_t}dt + \frac{dM_t}{M_t} \tag{77}$$

$$= -r_{s_t}dt - \Theta_{s_t}^B dB_t + \Theta_{s_t}^P dN_{s_t,t}, \tag{78}$$

where M is a martingale under the physical measure, $N_{s_t,t}$ a Poisson process which jumps upward by one whenever the state of the global economy switches from s_t to $\overline{s_t} \neq s_t$, $\Theta^B_{s_t} = \gamma \sigma^2_{s_t}$ is the market price of risk due to Brownian shocks in state s_t , and $\Theta^P_{s_t} = \Delta_{s_t} - 1$ is the market price of risk due to Poisson shocks when the economy switches out of state $s_t = \{L, H\}$.

Finally, r_{s_t} represents the equilibrium real risk-free rate, which is given by

$$r_{s_{t}} = \begin{cases} \overline{r}_{L} + \lambda_{L} \left[\frac{\gamma - \frac{1}{\psi}}{1 - \gamma} \left(\Delta^{-\frac{\gamma - 1}{\gamma - \frac{1}{\psi}}} - 1 \right) - (\Delta^{-1} - 1) \right] &, s_{t} = L \\ \overline{r}_{H} + \lambda_{H} \left[\frac{\gamma - \frac{1}{\psi}}{1 - \gamma} \left(\Delta^{\frac{\gamma - 1}{\gamma - \frac{1}{\psi}}} - 1 \right) - (\Delta - 1) \right] &, s_{t} = H \end{cases}$$

$$(79)$$

with $\Delta_H = \Delta_L^{-1} = \Delta$, where Δ is the solution of $G(\Delta) = 0$ from

$$G(x) = x^{-\frac{1-\frac{1}{\psi}}{\gamma-\frac{1}{\psi}}} - \frac{\overline{r}_H + \gamma \sigma_H^2 - \theta_H + \lambda_H \frac{1-\frac{1}{\psi}}{\gamma-1} \left(x^{\frac{\gamma-1}{\gamma-\frac{1}{\psi}}} - 1 \right)}{\overline{r}_L + \gamma \sigma_L^2 - \theta_L + \lambda_L \frac{1-\frac{1}{\psi}}{\gamma-1} \left(x^{-\frac{\gamma-1}{\gamma-\frac{1}{\psi}}} - 1 \right)}, \quad \psi \neq 1$$
(80)

7.2 Arrow-Debreu default claims

This Appendix derives two kinds of Arrow-Debreu claims that are used to discount risky cash flows. The first kind captures the default triggered by the country's output falling below the default boundary, whereas the second kind additionally accounts for the default related to a change in the state of the global economy. In the second case, default can occur instantaneously because of a change in state although the country's output remains unchanged. This situation can occur when the global economy is in good economic state ($s_t = H$) and switches to the bad state ($s_D = L$), and the country's output is above the good state's default boundary, but below the bad state's default boundary. The reason is that the default boundary is countercyclical ($Y_{D,L} > Y_{D,H}$), as shown in Bhamra, Kuehn, and Strebulaev (2010a, p.4238). The first kind of the Arrow-Debreu

claims is defined as

$$q_{s_t s_D} = E_t \left[\frac{\pi_{t_D}}{\pi_t} Prob\left(s_D \mid s_t\right) \mid s_t \right], \tag{81}$$

while the second kind corresponds to

$$q'_{s_t s_D} = E_t \left[\frac{\pi_{t_D}}{\pi_t} \frac{Y_{t_D}}{Y_{s_t}} Prob\left(s_D \mid s_t\right) \mid s_t \right]. \tag{82}$$

7.2.1 First kind

The Arrow-Debreu default security $q_{s_ts_D}$ is the present time t value of a security that pays one unit of consumption at the moment of default t_D , where s_t represents the present state of the global economy, and s_D the state at the default time. The time of default is the first time that the output level of the country falls to the boundary Y_{D,s_D} . By definition, this Arrow-Debreu claim is given by

$$q_{s_t s_D} = E_t \left[\frac{\pi_{t_D}}{\pi_t} Prob(s_D \mid s_t) \mid s_t \right], \tag{83}$$

which is solution of the two ordinary differential equations (ODE)

$$\frac{1}{2}\sigma_{i,s_t}^2 Y^2 \frac{d^2 q_{s_t s_D}}{dY^2} + \mu_{s_t} Y \frac{dq_{D,s_t s_D}}{dY} + \hat{\lambda}_{s_t} \left(q_{j s_D} - q_{s_t s_D} \right) - r_{s_t} q_{s_t s_D} = 0, \ s_t = \{L, H\}, \quad (84)$$

where σ_{i,s_t} denotes the country's output growth volatility in state $s_t.$

The above ODEs are obtained by applying Ito's Lemma to the classical non-arbitrage condition

$$E_t^Q \left[dq_{s+s_D} - r_{s_t} q_{s+s_D} \right] = 0, (85)$$

The Arrow-Debreu claim payoffs are such that:

$$q_{s_{t}s_{D}}(Y) = \begin{cases} 1, & s_{t} = s_{D}, & Y \leq Y_{D,s_{t}} \\ 0, & s_{t} \neq s_{D}, & Y \leq Y_{D,s_{t}}. \end{cases}, s_{t}, s_{D} = \{L; H\}$$
(86)

Therefore, each state of the global economy is characterized by a specific default boundary. The

Arrow-Debreu claims is derived for two distinct cases when $Y_{D,H} < Y_{D,L}$ and $Y_{D,H} > Y_{D,L}$. First, if the default barriers are higher in recession and lower in expansion, that is $Y_{D,H} < Y_{D,L}$, then each of the four Arrow-Debreu claims is determined over three separate intervals: $Y \ge Y_{D,L}$, $Y_{D,L} \ge Y \ge Y_{D,H}$, and $Y \le Y_{D,H}$.

From the payoff equations, we can infer the values of the four Arrow-Debreu claims in the interval $Y \leq Y_{D,H}$. For the interval $Y \geq Y_{D,L}$, we are looking for a solution of the following general form:

$$q_{s_t s_D}(Y) = h_{s_t s_D} Y^k, \tag{87}$$

which implies that k must be a root of the quartic equation

$$\left[\frac{1}{2}\sigma_{i,L}^{2}k(k-1) + \mu_{L}k + \left(-\hat{\lambda}_{L} - r_{L}\right)\right] \left[\frac{1}{2}\sigma_{i,H}^{2}k(k-1) + \mu_{H}k + \left(-\hat{\lambda}_{H} - r_{H}\right)\right] - \hat{\lambda}_{L}\hat{\lambda}_{H} = 0.$$
(88)

The Arrow-debreu claims can be written as

$$q_{s_t s_D}(Y) = \sum_{m=1}^{4} h_{s_t s_D m} Y^{k_m}$$
(89)

with $k_1, k_2 < 0$ and $k_3, k_4 > 0$. However, when Y goes to infinity the Arrow-Debreu claims must be null, which indicates that we should have $h_{s_t s_D, 3} = h_{s_t s_D, 4} = 0$. We then obtain

$$q_{Ls_D}(Y) = \sum_{m=1}^{2} h_{Ls_D,m} Y^{k_m}$$
(90)

$$q_{Hs_D}(Y) = \sum_{m=1}^{2} h_{Hs_D,m} \varepsilon(k_m) Y^{k_m}, \qquad (91)$$

where

$$\varepsilon(k_{m}) = -\frac{\hat{\lambda}_{H}}{\frac{1}{2}\sigma_{i,H}^{2}k(k-1) + \mu_{H}k - (\hat{\lambda}_{H} + r_{H})} = -\frac{\frac{1}{2}\sigma_{i,L}^{2}k(k-1) + \mu_{L}k - (\hat{\lambda}_{L} + r_{L})}{\hat{\lambda}_{L}}.$$
 (92)

Finally, over the interval $Y_{D,L} \ge Y \ge Y_{D,H}$, both $q_{D,LL}$ and $q_{D,LH}\left(Y\right)$ are known from the payoffs equations and are respectively equal to 1 and 0. Then,

$$q_{HL}(Y) = \frac{\hat{\lambda}_H}{r_H + \hat{\lambda}_H} + \sum_{m=1}^2 s_{L,m} Y^{j_m}$$
 (93)

$$q_{HH}(Y) = \sum_{m=1}^{2} s_{H,m} Y^{j_m},$$
 (94)

where

$$\frac{1}{2}\sigma_{i,H}^{2}j(j-1) + \mu_{H}j - (\hat{\lambda}_{H} + r_{H}) = 0$$
(95)

with $j_1 < j_2$.

To summarize, the four Arrow-Debreu claims can be written as follows

$$q_{LL} = \begin{cases} \sum_{m=1}^{2} h_{LL,m} Y^{k_m}, & Y \ge Y_{D,L} \\ 1, & Y_{D,L} \ge Y \ge Y_{D,H} \\ 1, & Y \le Y_{D,H} \end{cases}$$

$$q_{LH} = \begin{cases} \sum_{m=1}^{2} h_{LH,m} Y^{k_m}, & Y \ge Y_{D,L} \\ 0, & Y_{D,L} \ge Y \ge Y_{D,H} \\ 0, & Y \le Y_{D,H} \end{cases}$$

$$q_{HL} = \begin{cases} \sum_{m=1}^{2} h_{LL,m} \varepsilon(k_m) Y^{k_m}, & Y \ge Y_{D,L} \\ \frac{\hat{\lambda}_H}{r_H + \hat{\lambda}_H} + \sum_{m=1}^{2} s_{L,m} Y^{j_m}, & Y_{D,L} \ge Y \ge Y_{D,H} \\ 0, & Y \le Y_{D,H} \end{cases}$$

$$q_{HH} = \begin{cases} \sum_{m=1}^{2} h_{LH,m} \varepsilon(k_m) Y^{k_m}, & Y \ge Y_{D,L} \\ \sum_{m=1}^{2} s_{H,m} Y^{j_m}, & Y_{D,L} \ge Y \ge Y_{D,H} \\ 1, & Y \le Y_{D,H}. \end{cases}$$

$$(96)$$

$$q_{LH} = \begin{cases} \sum_{m=1}^{2} h_{LH,m} Y^{k_m}, & Y \ge Y_{D,L} \\ 0, & Y_{D,L} \ge Y \ge Y_{D,H} \\ 0, & Y \le Y_{D,H} \end{cases}$$
(97)

$$q_{HL} = \begin{cases} \sum_{m=1}^{2} h_{LL,m} \varepsilon(k_m) Y^{k_m}, & Y \ge Y_{D,L} \\ \frac{\hat{\lambda}_H}{r_H + \hat{\lambda}_H} + \sum_{m=1}^{2} s_{L,m} Y^{j_m}, & Y_{D,L} \ge Y \ge Y_{D,H} \\ 0, & Y \le Y_{D,H} \end{cases}$$
(98)

$$q_{HH} = \begin{cases} \sum_{m=1}^{2} h_{LH,m} \varepsilon(k_m) Y^{k_m}, & Y \ge Y_{D,L} \\ \sum_{m=1}^{2} s_{H,m} Y^{j_m}, & Y_{D,L} \ge Y \ge Y_{D,H} \end{cases}$$

$$1, \qquad Y \le Y_{D,H}.$$
(99)

The eight constants are determined by eight boundary conditions, which are

$$\lim_{Y \to Y_{D,L}} q_{LL} = 1, \quad \lim_{Y \to Y_{D,L}} q_{LH} = 0 \tag{100}$$

$$\lim_{Y \to Y_{D,L}^+} q_{HL} = \lim_{Y \to Y_{D,L}^-} q_{HL}, \quad \lim_{Y \to Y_{D,L}^+} q_{HH} = \lim_{Y \to Y_{D,L}^-} q_{HH}$$
(101)

$$\lim_{Y \to Y_{D,L}^+} \dot{q}_{HL} = \lim_{Y \to Y_{D,L}^-} \dot{q}_{HL}, \quad \lim_{Y \to Y_{D,L}^+} \dot{q}_{HH} = \lim_{Y \to Y_{D,L}^-} \dot{q}_{HH}$$
(102)

$$\lim_{Y \to Y_{DH}} q_{HL} = 0, \quad \lim_{Y \to Y_{DH}} q_{HH} = 1. \tag{103}$$

Similarly, if the default barriers are higher in expansion and lower in recession, that is $Y_{D,H}>Y_{D,L}$, then each of the four Arrow-Debreu claims is determined over three separate intervals: $Y \geq Y_{D,H}$, $Y_{D,H} \geq Y \geq Y_{D,L}$, and $Y \leq Y_{D,L}$.

By applying the same reasoning, we obtain that

$$q_{LL} = \begin{cases} \sum_{m=1}^{2} h_{LL,m} Y^{k_m}, & Y \ge Y_{D,H} \\ \sum_{m=1}^{2} s_{L,m} Y^{j_m}, & Y_{D,H} \ge Y \ge Y_{D,L} \\ 1, & Y \le Y_{D,L} \end{cases}$$

$$q_{LH} = \begin{cases} \sum_{m=1}^{2} h_{LH,m} Y^{k_m}, & Y \ge Y_{D,H} \\ \frac{\hat{\lambda}_L}{r_L + \hat{\lambda}_L} + \sum_{m=1}^{2} s_{H,m} Y^{j_m}, & Y_{D,H} \ge Y \ge Y_{D,L} \\ 0, & Y \le Y_{D,L} \end{cases}$$

$$q_{HL} = \begin{cases} \sum_{m=1}^{2} h_{LL,m} \varepsilon(k_m) Y^{k_m}, & Y \ge Y_{D,H} \\ 0, & Y_{D,H} \ge Y \ge Y_{D,L} \\ 0, & Y \le Y_{D,L} \end{cases}$$

$$q_{HH} = \begin{cases} \sum_{m=1}^{2} h_{LH,m} \varepsilon(k_m) Y^{k_m}, & Y \ge Y_{D,H} \\ 1, & Y_{D,H} \ge Y \ge Y_{D,L} \\ 1, & Y \le Y_{D,L} \end{cases}$$

$$(105)$$

$$q_{LH} = \begin{cases} \sum_{m=1}^{2} h_{LH,m} Y^{k_m}, & Y \ge Y_{D,H} \\ \frac{\hat{\lambda}_L}{r_L + \hat{\lambda}_L} + \sum_{m=1}^{2} s_{H,m} Y^{j_m}, & Y_{D,H} \ge Y \ge Y_{D,L} \\ 0, & Y \le Y_{D,L} \end{cases}$$
(105)

$$q_{HL} = \begin{cases} \sum_{m=1}^{2} h_{LL,m} \varepsilon(k_m) Y^{k_m}, & Y \ge Y_{D,H} \\ 0, & Y_{D,H} \ge Y \ge Y_{D,L} \\ 0, & Y \le Y_{D,L} \end{cases}$$
(106)

$$q_{HH} = \begin{cases} \sum_{m=1}^{2} h_{LH,m} \varepsilon(k_m) Y^{k_m}, & Y \ge Y_{D,H} \\ 1, & Y_{D,H} \ge Y \ge Y_{D,L} \\ 1, & Y \le Y_{D,L}. \end{cases}$$
(107)

The eight constants are determined by eight boundary conditions, which are

$$\lim_{Y \to Y_{D,L}} q_{LL} = 1, \quad \lim_{Y \to Y_{D,L}} q_{LH} = 0 \tag{108}$$

$$\lim_{Y \to Y_{D,H}^+} q_{LL} = \lim_{Y \to Y_{D,H}^-} q_{LL}, \quad \lim_{Y \to Y_{D,H}^+} q_{LH} = \lim_{Y \to Y_{D,H}^-} q_{LH}$$
(109)

$$\lim_{Y \to Y_{D,H}^+} \dot{q}_{LL} = \lim_{Y \to Y_{D,H}^-} \dot{q}_{LL}, \quad \lim_{Y \to Y_{D,H}^+} \dot{q}_{LH} = \lim_{Y \to Y_{D,H}^-} \dot{q}_{LH} \tag{110}$$

$$\lim_{Y \to Y_{D,H}} q_{HL} = 0, \quad \lim_{Y \to Y_{D,H}} q_{HH} = 1. \tag{111}$$

7.2.2 Second kind

We use the same approach to derive the second kind of Arrow-Debreu default claims, which account for the possibility that a default can happen when the state of the global economy changes. In the case when $Y_{D,H} < Y_{D,L}$, the only claim that is different from that of the first kind is q_{HL} , whose expression is now given by

$$q'_{HL} = \begin{cases} \sum_{m=1}^{2} h_{LL,m} \varepsilon(k_m) Y^{k_m}, & Y \ge Y_{D,L} \\ \frac{\hat{\lambda}_H}{r_H + \hat{\lambda}_H - \mu_H} \frac{Y}{Y_{D,L}} + \sum_{m=1}^{2} s_{L,m} Y^{j_m}, & Y_{D,L} \ge Y \ge Y_{D,H} \\ 0, & Y \le Y_{D,H}. \end{cases}$$
(112)

Now, in the case when $Y_{D,H} > Y_{D,L}$, that is the claim q_{LH} , whose expression is given by

$$q'_{LH} = \begin{cases} \sum_{m=1}^{2} h_{LH,m} Y^{k_m}, & Y \ge Y_{D,H} \\ \frac{\hat{\lambda}_L}{r_L + \hat{\lambda}_L - \mu_L} \frac{Y}{Y_{D,H}} + \sum_{m=1}^{2} s_{H,m} Y^{j_m}, & Y_{D,H} \ge Y \ge Y_{D,L} \\ 0, & Y \le Y_{D,L} \end{cases}$$
(113)

7.3 Sovereign debt and credit spread

This Appendix derives the value of sovereign debt in the country. The sovereign debt value, denoted by $D_{s_t}(Y_t)$ when the current state is s_t , is defined as

$$D_{s_t}(Y_t) = E_t \left[\int_t^{t_D} c \frac{\pi_u}{\pi_t} du \mid s_t \right] + E_t \left[\int_{t_D}^{\infty} (1 - \phi) c \frac{\pi_u}{\pi_t} du \mid s_t \right]$$
 (114)

$$= E_t \left[\int_t^\infty c \frac{\pi_u}{\pi_t} du \mid s_t \right] - E_t \left[\frac{\pi_{t_D}}{\pi_t} \int_{t_D}^\infty \phi c \frac{\pi_u}{\pi_{t_D}} du \mid s_t \right], \tag{115}$$

where c is the perpetual debt coupon and ϕ is the debt haircut in default.

The first term of Equation 115 represents a risk-free claim that delivers c in every period, which corresponds to pricing of a perpetual bond. Hence, we have

$$E_t \left[\int_t^\infty c \frac{\pi_u}{\pi_t} du \mid s_t \right] = \frac{c}{r_{B,s_t}}, \tag{116}$$

where r_{B,s_t} is the discount rate for a riskless perpetuity, when the current state is s_t , which is given by

$$r_{B,s_t} = r_{s_t} + \frac{r_j - r_{s_t}}{\hat{p} + r_j} \hat{p} \hat{f}_j, \quad j \neq s_t; \quad j, s_t = \{L, H\}$$
 (117)

which indicates that the discount rate $r_{B,H}$ is lower than r_H because the risk-free rate is expected to decrease in the future when the global economy enters in recession.

The second part of Equation 115 is given by

$$E_{t} \left[\frac{\pi_{t_{D}}}{\pi_{t}} \int_{t_{D}}^{\infty} \phi c \frac{\pi_{u}}{\pi_{t_{D}}} du \mid s_{t} \right] = \sum_{s_{D}} E_{t} \left[\Pr(s_{D} \mid s_{t}) \frac{\pi_{t_{D}}}{\pi_{t}} \int_{t_{D}}^{\infty} \phi c \frac{\pi_{u}}{\pi_{t_{D}}} du \mid s_{t} \right]$$

$$= \sum_{s_{D}} E_{t} \left[\Pr(s_{D} \mid s_{t}) \frac{\pi_{t_{D}}}{\pi_{t}} \mid s_{t} \right] E_{t} \left[\int_{t_{D}}^{\infty} \phi c \frac{\pi_{u}}{\pi_{t_{D}}} du \mid s_{t_{D}} \right]$$

$$= \sum_{s_{D}} \frac{\phi c}{r_{B,s_{D}}} q_{s_{t}s_{D}} (Y_{t}) .$$

$$(118)$$

The derivation is as follows. We can first separate Equation 118 into two parts, given the state-price

density is Markovian. Then, we can see that the first term of Equation 119 is

$$E_t \left[\Pr\left(s_D \mid s_t \right) \frac{\pi_{t_D}}{\pi_t} \mid s_t \right] = q_{s_t s_D} \left(Y_t \right), \tag{121}$$

which is the claim that pays one unit of consumption at the default time t_D when the initial time is t and the state is s_t , which essentially corresponds to the Arrow-Debreu claim $q_{s_ts_D}\left(Y_t\right)$. Then, we can note that $E_t\left[\int_{t_D}^{\infty}\phi c\frac{\pi_u}{\pi t_D}du\mid s_{t_D}\right]$ is the value of a claim at default time, which pays ϕc in perpetuity and whose discount rate is r_{B,s_D} . It is thus equal to $\frac{\phi c}{r_{B,s_D}}$.

Finally, combining the different parts, the sovereign debt value is equal to

$$D_{s_t}(Y_t) = \frac{c}{r_{B,s_t}} - \sum_{s_D} \frac{\phi c}{r_{B,s_D}} q_{s_t s_D}(Y_t), \ s_t, s_D = \{L, H\},$$
 (122)

where the summation over s_D indicates that a default can occur in either state, $s_D=L$ or $s_D=H$.

The sovereign credit spread that the agent requires for holding the country's government debt, when the current state is s_t , is determined as follows:

$$CS_{s_t}(Y_t) = \frac{c}{D_{s_t}(Y_t)} - r_{B,s_t}$$
 (123)

$$= \frac{1}{\left[r_{B,s_t} - \sum_{s_D} \frac{\phi}{r_{B,s_D}} q_{s_t s_D} (Y_t)\right]} - r_{B,s_t}$$
 (124)

$$= r_{B,s_t} \left[\frac{1}{1 - \sum_{s_D} \phi \frac{r_{B,s_t}}{r_{B,s_D}} q_{s_t s_D} (Y_t)} - 1 \right], \quad s_t, s_D = \{L, H\}.$$
 (125)

7.4 Government

This Appendix computes the present value of the fiscal revenue received by the country's government, the debt issuance benefits, the country's sovereign wealth, and eventually derives the optimal policies.

7.4.1 Discounted fiscal revenue

The present value of the country's fiscal revenue, denoted by $F_{s_t}(Y_t)$ when the current state is s_t , can be written as

$$F_{s_t}(Y_t) = E_t \left[\int_t^{t_D} \tau Y_u \frac{\pi_u}{\pi_t} du \mid s_t \right] + E_t \left[\int_{t_D}^{\infty} \tau (1 - \alpha) Y_u \frac{\pi_u}{\pi_t} du \mid s_t \right]$$
 (126)

$$= \tau E_t \left[\int_t^\infty Y_u \frac{\pi_u}{\pi_t} du \mid s_t \right] - \tau \alpha E_t \left[\int_{t_D}^\infty Y_u \frac{\pi_u}{\pi_t} du \mid s_t \right]. \tag{127}$$

The first term of Equation 127 is determined by

$$E_t \left[\int_t^\infty Y_u \frac{\pi_u}{\pi_t} du \mid s_t \right] = Y_t E_t \left[\int_t^\infty \frac{\pi_u}{\pi_t} \frac{Y_u}{Y_t} du \mid s_t \right]$$
 (128)

$$= Y_t \frac{1}{r_{Y,s_t}}, \tag{129}$$

where r_{Y,s_t} is the discount rate related to risky cash flows, which is given by (see Bhamra, Kuehn, and Strebulaev, 2010a, p.13)

$$r_{Y,s_t} = r_{s_t} - \mu_{s_t} + \frac{(r_j - \mu_j) - (r_{s_t} - \mu_{s_t})}{\hat{p} + r_j - \mu_j} \hat{p} \hat{f}_j, \quad j \neq s_t; \quad j, \ s_t = \{L, H\}.$$
 (130)

From the strong Markov property, we can solve for the second part of Equation 127, which yields

$$E_{t} \left[\int_{t_{D}}^{\infty} Y_{u} \frac{\pi_{u}}{\pi_{t}} du \mid s_{t} \right] = \sum_{s_{D}} q'_{s_{t}s_{D}} \left(Y_{t} \right) \frac{Y_{D,s_{D}}}{r_{Y,s_{D}}}. \tag{131}$$

Eventually, the present value of the country's fiscal revenue is given by

$$F_{s_t}(Y_t) = \frac{\tau Y_t}{r_{Y,s_t}} - \alpha \tau \sum_{s_D} \frac{Y_{D,s_D}}{r_{Y,s_D}} q'_{s_t s_D} (Y_t).$$
 (132)

7.4.2 Debt issuance benefits

The government's motivation for issuing debt is to invest in the country the amount of capital raised at time of debt issuance (t=0). Financing public investments yields a return r_g . The

government's incentives for issuing debt, denoted by $I_{s_t}(Y_t)$ when the state is s_t at time t, equal

$$I_{s_t}(Y_t) = E_t \left[\int_t^\infty r_g \frac{\pi_u}{\pi_t} du \mid s_t \right] D_{s_0}(Y_0)$$
 (133)

$$= r_g D_{s_t}(Y_t) E_t \left[\int_t^\infty \frac{\pi_u}{\pi_t} du \mid s_t \right] D_{s_0}(Y_0)$$
 (134)

$$= \frac{r_g}{r_{B,s_t}} D_{s_0}(Y_0) \tag{135}$$

7.4.3 Sovereign wealth

Sovereign wealth is defined as the present value of fiscal revenue, $F_{s_t}(Y_t)$, plus the benefits of issuing debt, $I_{s_t}(Y_t)$. From the derivation above, Sovereign wealth $W_{s_t}(Y_t)$, at time t and for current state s_t , is given by

$$W_{s_t}(Y_t) = F_{s_t}(Y_t) + I_{s_t}(Y_t)$$
(136)

$$= \frac{\tau Y_t}{r_{Y,s_t}} - \alpha \tau \sum_{s_D} \frac{Y_{D,s_D}}{r_{Y,s_D}} q'_{s_t s_D} (Y_t) + \frac{r_g}{r_{B,s_t}} D_{s_0}(Y_0)$$
 (137)

7.4.4 Smooth pasting conditions

This Appendix derives the smooth-pasting conditions that ensure continuity in the objective function at the time of default (see Merton, 1973; Dumas, 1991). For convenience, let us denote the value of sovereign wealth after debt payments have been made by $\overline{W}_{s_t}(Y_t) \equiv W_{s_t}(Y_t) - D_{s_t}(Y_t)$. Combining Equations (122) and (137), $\overline{W}_{s_t}(Y_t)$ is given by

$$\overline{W}_{s_{t}}(Y_{t}) = \frac{\tau Y_{t}}{r_{Y,s_{t}}} - \alpha \tau \sum_{s_{D}} \frac{Y_{D,s_{D}}}{r_{Y,s_{D}}} q'_{s_{t}s_{D}}(Y_{t}) + \frac{r_{g}}{r_{B,s_{t}}} D_{s_{0}}(Y_{0})
- \left[\frac{c}{r_{B,s_{t}}} - \sum_{s_{D}} \frac{c\phi}{r_{B,s_{D}}} q_{s_{t}s_{D}}(Y_{t}) \right].$$
(138)

The smooth-pasting conditions must satisfy the following equations:

$$\frac{\partial W_{s_t}(Y_t)}{\partial Y_t} \mid_{Y_t = Y_{D,s_t}} = \frac{\partial}{\partial Y_{D,s_t}} \left(\overline{W}_{s_t}(Y_t) \mid_{Y_t = Y_{D,s_t}} \right), \quad s_t = \{L, H\}.$$
 (139)

From the definition of the Arrow-Debreu claims (86), $\overline{W}_{s_t}(Y_t)$ at default time is given by

$$\overline{W}_{s_t}(Y_t) \mid_{Y_t = Y_{D,s_t}} = \tau Y_{D,s_t} \frac{1 - \alpha}{r_{Y,s_t}} + \frac{r_g}{r_{B,s_t}} D_{s_0}(Y_0) - \frac{(1 - \phi) c}{r_{B,s_t}}$$
(140)

and the right-hand side of Equation 139 is thus determined by

$$\frac{\partial}{\partial Y_{D,s_t}} \left(\overline{W}_{s_t}(Y_t) \mid_{Y_t = Y_{D,s_t}} \right) = \tau \frac{1 - \alpha}{r_{Y,s_t}}. \tag{141}$$

Hence, the smooth-pasting conditions satisfy the pair of equations given by

$$\frac{\partial \overline{W}_{s_t}(Y_t)}{\partial Y_t} \mid_{Y_t = Y_{D,s_t}} = \frac{\tau (1 - \alpha)}{r_{Y,s_t}}, \quad s_t = \{L, H\}.$$

$$(142)$$

7.5 Estimation of the transition probabilities

This Appendix describes the estimation of the transition probabilities considered in the paper. We estimate a Markov regime-switching model with two regimes on US consumption growth over the period 1960Q1-2014Q4. The transition probability matrix, which is obtained by maximum likelihood using the Hamilton (1989)'s approach, is given by

$$T = \begin{bmatrix} T_{LL} & T_{LH} \\ T_{HL} & T_{HH} \end{bmatrix} = T = \begin{bmatrix} 0.9641 & 0.0359 \\ 0.0783 & 0.9217 \end{bmatrix}$$
(143)

where T_{ij} denotes the probability of a switch from state i to state j.

Following Bhamra, Kuehn, and Strebulaev (2010a,b), the actual long-run probability f_{s_t} to be in the state $s_t \in \{L,H\}$ is determined by $f_L = \left(1+\frac{T_{LH}}{T_{HL}}\right)^{-1}$ and $f_H = 1-f_L$. The probability λ_{s_t} that the global economy leaves the state $s_t \in \{L,H\}$ is then given by $\lambda_L = pf_H$ and $\lambda_H = pf_L$, with $p = -4ln\left(1-\frac{T_{LH}}{1-f_L}\right)$.

Conclusion

Macroeconomic risk is characterized by changing economic conditions, making, countries output and firms cash flows, expected growth rates and volatility to vary overtime. In recessions, expected growth rates are lower while volatilities are higher compared to normal periods. Investors do not know with certainty the state of nature that will occur and are unwilling the current state to be a recession. They will then price financial assets accordingly. Moreover, due to their recursive preferences, they can distinguish risk aversion, as in consumption CAPM, to their aversion to macroeconomic risk. It becomes possible to measure the impacts of this latter risk on financial asset classes such equity and bonds.

This thesis quantifies the impact of macroeconomic risk and investor preferences on asset prices and economic actors (firms and governments) decision-making. The first essay predicts that two third of risk premia embedded into corporate asset arises from intertemporal uncertainty. The second essay shows that around 30% of sovereign credit spreads consists of a compensation for intertemporal uncertainty. In presence of this risk, governments choose a higher coupon level and prefer early default whereas firms opt for lower coupon and default barrier to avoid this early default.