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Information Acquisition in Financial Markets : A Behavioral Perspective

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Résumé

L’économie comportementale, et plus particulièrement l’étude des comportements humains sur les marchés financiers est devenue un domaine incontournable des sciences économiques en général et de la finance en particulier. Nombreuses études considérant les biais comportementaux ont permis d’expliquer des phénomènes empiriques que la finance classique ne permet pas d’élucider. Parmi les biais comportementaux les plus célèbres, nous retiendrons *l’aversion pour la perte* et l’*excès de confiance en soi*. Ils ont permis, séparément, d’apporter des explications valables à des phénomènes intrigants comme l’*énigme de la prime de risque* ou encore le *volume de transactions excessif* présents sur les marchés. Cependant, les modèles d’asymétrie d’information avec de l’aversion pour la perte ont été peu traités dans la littérature. De plus si ces deux biais cognitifs permettent d’élucider des problèmes centraux bien que différents, il n’existe pas d’étude à notre connaissance qui analyse ou modélise leur effet conjoint sur le prix des actifs, leur caractère contradictoire, leur impact marginal ou même qui tente en les incorporant conjointement dans un modèle théorique d’expliquer certaines irrégularités empiriques. Cette étude a pour objectif de combler ce manque par le biais du développement de modèles appropriés.

Le premier chapitre introduit les concepts clés de la finance comportementale relatifs à l’étude des marchés financiers. Ce chapitre analyse les modèles existants qui incorporent la théorie d’utilité non conventionnelle et les principaux biais comportementaux appliqués aux marchés financiers. Nous présentons à la fois la littérature principale en finance comportementale et à la fois les modèles théoriques impliquant l’acquisition d’information dans un cadre de préférences non-standards.

Le deuxième chapitre analyse les stratégies optimales de négociation à l'équilibre et la qualité du marché dans une économie avec de l'asymétrie d'information et dans laquelle les spéculateurs ont de l'aversion pour la perte. Dans le contexte d’asymétrie d’information et d’aversion pour la perte, la caractérisation du prix d'équilibre tractable présentée en forme fermée est obtenue

pour la première fois dans la littérature. Le modèle démêle avec succès l'effet de l'aversion pour la perte sur la stratégie optimale de négociation et le prix d'équilibre. Le modèle prédit que la profondeur du marché est non linéaire. Conformément aux observations empiriques, le modèle prédit que d'importants mouvements sur le prix peuvent se produire à la suite de petits chocs informationnels ou sur l'offre et indépendamment de la valeur de l'actif sous-jacent, et de manière asymétrique.

Enfin, le dernier chapitre propose d'explorer l'effet conjoint des deux principaux biais comportementaux qui sont dans un sens contradictoires à savoir *l'excès de confiance en soi* et l'*aversion pour la perte*. À cette fin, nous développons un modèle de différences d'opinion où les agents dits *rationnels* transigent avec des agents dits *irrationnels.* Le modèle génère une corrélation positive entre le volume et l'information agrégée, ainsi qu'une *prime de rendement sur le volume élevé*. Le modèle réussit également à concilier la variation transversale de dissymétrie au niveau de l'entreprise (la dissymétrie est négativement corrélée avec le volume des transactions, le rendement passé et la taille des entreprises) avec le fait que la dissymétrie moyenne des rendements individuels des firmes est positive.

Mots clés : spéculation, tarification des actifs, acquisition d'information, biais comportementaux, asymétrie d'information, aversion pour la perte, krachs financiers, volume de transactions, excès de confiance en soi, profondeur des marchés, coefficient de dissymétrie, divergence d'opinion, efficience des marchés.

Méthode de recherche : modélisation mathématique.

Abstract

Cognitive psychology applied to financial markets has become a key research area in finance. Behavioral finance has indeed attempted to explain several striking empirical phenomena, in particular, stock market anomalies that cannot be rationalized using the traditional finance paradigm. Loss aversion and overconfidence are arguably the most widespread cognitive biases. Over the last three decades, they have been extensively applied to problems in finance. Each of them has served to explain intriguing phenomena, such as the *equity premium puzzle* or *excessive trading*. However, models of financial markets with loss averse speculators and asymmetric information have seldom been discussed in the literature. Moreover, whereas taken separately, these two cognitive biases helped to shed light on various finance issues, there are no other studies to my knowledge that analyze their joint effect on asset prices, their contradictory nature, or their marginal impact, and no studies that attempt to incorporate them into a single equilibrium model. This study aims to fill this gap through the development of appropriate models.

The first chapter introduces behavioral concepts in finance, i.e. related to financial markets. This chapter analyzes the main existing models with non-conventional utility theories and behavioral biases applied to financial markets. We introduce both the main literature surrounding behavioral finance and the main literature surrounding theoretical models, implying information acquisition with different types of agents having non-classical preferences.

The second chapter analyses equilibrium trading strategies and market quality in an economy with information asymmetry, in which speculators display loss aversion. A closed form characterization of the equilibrium price is presented. This study introduces in the literature for the first time an analytically tractable equilibrium in an economy with information asymmetry, in which speculators display loss aversion. The model successfully disentangles the effect of loss aversion on optimal informed trading strategy and equilibrium price. The model predicts nonlinear market depth. Consistent with empirical observations, the

model finds that important price movements may occur after small shocks in the fundamentals and without any important news event. When introducing short-sale constraints, the model also supports the well-known asymmetric property of abrupt price movements.

Finally, the last chapter explores the joint effect of the two main contradictory behavioral biases, namely, overconfidence and loss aversion. We develop a model of disagreement over public signals between rational and irrational speculators, where irrational traders exhibit jointly those two biases. The model generates a positive correlation between volume and aggregate information, as well as a *high-volume return premium.* The model also succeeds in reconciling cross-sectional variation in skewness at the firm level (skewness is negatively correlated with trading volume, past return, and firm size) with the fact that, on average, skewness in individual-firm returns is positive.

Keywords: speculative trading, asset pricing, information acquisition, behavioral biases, information asymmetry, loss aversion, market crashes, trading volume, overconfidence, market depth, skewness, differences of opinion, market efficiency.

Research method: mathematical modeling.

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Liste des abréviations

CA: Centralized Auctioneer.

CAPM: Capital Asset Pricing Model.

CARA: Constant Absolute Risk Aversion.

CPT: Cumulative Prospect Theory.

EU: Expected Utility.

i.i.d: Identical and Independent Distributed.

LA: Loss Aversion.

MM: Market Maker.

PT: Prospect Theory.

REE: Rational Expectation Equilibrium.

RN: Risk Neutrality.

À Myriam

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Introduction

It is readily apparent that a number of empirical facts and anomalies relating to the aggregate stock market are difficult to explain within the traditional finance paradigm in which agents act rationally. To address this issue, several important studies depart from the classical rational framework. These studies typically introduce robust and well documented behavioral biases. Arguably two of the most robust behavioral biases that have been used to explain these anomalies are overconfidence and loss aversion. Models of financial markets with loss averse speculators and asymmetric information have not been studied extensively in the literature. Moreover, whereas overconfidence and loss aversion taken separately shed light on various empirical finance findings, there are no other studies to my knowledge that analyze their joint effect on asset prices, their contradictory nature, or their marginal impact, and no studies that attempt to incorporate them into a single equilibrium model.

Whereas models of financial markets with asymmetric information have often been applied to economies in which traders hold mistaken distributional beliefs about the payoff of the risky asset, and in particular, to economies in which traders are overconfident (Benos (1998), Caballé and Sakovics (2003), Kyle and Wang (1997), Odean (1998b), and Wang (1998)), models of financial markets with loss averse speculators and asymmetric information have seldom been discussed in the literature. Over the past three decades, we have witnessed increased interest in the study of price formation in financial markets,[1] and integration of loss aversion into the finance literature. However, only a very recent study of Pasquariello's (2014) analyzes the effect of prospect theory in general, and the effect of loss aversion in particular, on market quality. Pasquariello (2014) finds that loss averse informed traders endowed with private information have non-trivial state-dependent effects on equilibrium market liquidity, price volatility, trading volume, market efficiency, and information production. However, the nonlinear rational expectation equilibrium developed in Pasquariello's (2014) study is analytically intractable.

The first chapter of this dissertation introduces behavioral concepts in finance related to financial markets. This chapter analyses the key existing models of financial markets with non-conventional utility theories, where agents exhibit behavioral biases. We study the main literature on financial markets from a behavioral finance perspective.

In line with Grossman and Stiglitz (1980), Kyle (1985), and Vives (1995), the second chapter of this dissertation offers a noisy rational expectations equilibrium (REE) model, in which competitive price-taking speculators endowed with private information exhibit loss aversion. The proposed economy is populated with informed traders, liquidity traders, and a risk neutral market maker. Our model is inspired by Pasquariello's (2014) model. While in his original model, asset choice is based on the mean variance approach to rational investment, the informed agents

[1]Such as questions relative to the mechanism through which private information is acquired, utilized and impounded into price, optimal strategy, agents' trading incentives, equilibrium market volatility, and volume.

in our study maximize their expected utility. The proposed nonlinear equilibrium is analytically tractable and a closed-form characterization of the equilibrium price is obtained. We analyze the mechanism through which prices are formed. The model shows how asset price collapses arise endogenously from the interaction between information asymmetry and loss aversion.

In our model, speculator preferences are used to disentangle loss aversion and risk aversion. This leads to a state-dependent linear optimal trading strategy and makes the inference problem for the equilibrium price tractable. The proposed model disentangles the impact of loss aversion on optimal informed trading strategy and equilibrium price. The presence of loss averse traders who are better informed lowers the equilibrium price volatility and expected informed trading volume. Loss aversion also induces the speculators to trade less for sufficiently large signals in absolute value, and not at all for low signals. We show that since speculators' preferences successfully disentangle loss aversion and risk aversion, the trading intensity within the trading region remains unchanged in comparison to risk averse speculators only.

In our simple model, the fact that it is difficult for the market maker to assess the trading region of the informed traders can create large market price movements in the intermediate price region. Our model outlines how small trigger shocks can create market meltdowns. They occur when the absolute value of the aggregate order flow is low. In that situation, the market maker is confused about the trading status of informed traders, and price adjustment to signal and noise trading shocks becomes highly nonlinear. Large price movements following small informational shocks support the evidence reported by Culter, Porterba, and Summers (1989) according to which crashes, as well as market bubbles, appear without any preceding public news. A closed-form characterization of the market depth is provided. Asset collapses arise typically when the market depth is high.

We then introduce short-sale constraints into the model. We demonstrate that the unique interaction between loss aversion and short-sale constraints produces asymmetric price movements as reported in the literature (Pindyk (1984), French, Schwert, and Stambaugh (1987), Bekaert and Wu (2000), and Yuan (2005)), where prices are more likely to decrease than increase. In other words, we show that, as stated by Hong and Stein (2003), "markets melt down but they don't melt up."[2] Notice that while loss aversion has been shown to explain a broad variety of problems in finance, to our knowledge, this dissertation is the first attempt at establishing a connection between loss aversion and market collapses. We conjecture that this is in part because models of trading with loss aversion and information asymmetry are hardly implementable and have not been studied in the literature until very recently (Pasquariello, (2014)), due to the inherent complexity of market equilibria with asymmetric information under non-standard preferences.

Despite our initial motivation to develop a tractable equilibrium in the presence of asymmetric information and loss averse insiders, the comparative statics of the model shed light on the market crashes phenomenon. This work is not the first attempt at using a constrained information asymmetry framework to explain crashes. However, our study emphasizes the role of loss aversion and the basic

[2] Hong and Stein (2003) report that when we look directly at historical stock return data, it can be noted that nine of the ten biggest one-day movements in the S&P 500 since 1947, were declines.

framework is quite different from that of other models of market meltdowns. In Yuan (2005), and Barlevy and Verosini's work (2003), crashes are driven by uninformed investors who are uncertain about fundamentals. In contrast, Romer (1990), and Hong and Stein (2003) make the assumption that small events may reveal substantial information and that crashes are driven by the information acquisition process. In our model, asset price collapses arise because the speculators' optimal demand is partially revealing.

The third chapter proposes to explore the joint effect of overconfidence and loss aversion. Both overconfidence and loss aversion are commonly used in neoclassical finance to explain puzzling empirical evidence regarding asset prices and financial markets. However, the contradictory nature of these two behavioral biases has never been questioned. For instance, overconfidence is largely accepted as the reason behind abnormal trading volume while loss aversion is known as a potential explanation (Benartzi and Thaler (1995)) of the equity premium puzzle.[3] For this purpose, we propose a new model based on differences of opinion which can be seen as a form of overconfidence where there are two types of agents. The first type is fully rational[4] and the second type is subject to loss aversion and overconfidence. This model extends the one produced by Harris and Raviv (1993) in that the irrational group of traders not only overestimates the precision of the signal, but also exhibits loss aversion. This model produces several novel results.

[3] Given the return of stocks and bonds over the past century, an abnormally high level of risk aversion would be necessary to explain why investors are willing to hold bonds at all (Mehra and Prescott, 1985).
[4] We might assume that this type of trader represents money managers who are very sophisticated, experimented, and well trained.

Indeed, we demonstrate that aggregate information is positively correlated with trading volume. This result supports the well-accepted positive correlation between *contemporaneous* volume and return (Karpoff (1987); Stoll and Whaley (1987); Bessembinder and Seguin (1993); Bessembinder, Chan, and Seguin (1996); Chordia, Roll, and Subrahmanyam (2000); Lo and Wang (2000), and Harris and Raviv (2007)). The prediction of the model is related also to the positive correlation trading volume and future return. This result has been reported and quoted as the high-volume return premium (Gervais, Kaniel, and Mingelgrin (2001)). This chapter provides a different explanation from the visibility hypothesis introduced by Merton (1987) and generally offered to explain the *high-volume return premium*. The interaction between loss aversion and overconfidence is the main driver of the propensity for stocks experiencing high-trading volume, to generate abnormal returns. Moreover, this work supports the evidence, reported by Hong and Stein (2007), that glamour stocks tend to display higher trading volume relative to low priced value stocks. Glamour stocks tend to receive more positive coverage. In the proposed model, a greater trading volume is generated as a by-product of the increased likelihood of a positive cumulative signal. Finally, the model also succeeds in reconciling cross-sectional variation in skewness at the firm level with the fact that, on average, skewness in individual-firm returns is positive. Namely, our model generates positive skewness at the firm-level while supporting in an integrated fashion Chen, Hong, and Stein's (2001) three robust findings about conditional skewness: (1) positive skewness is more pronounced for small firms, ceteris paribus; (2) when past returns have been high, skewness is forecasted to become more negative and reciprocally, when past returns have been low, skewness is forecasted to become more positive; (3) negative skewness is greater in stocks that have experienced an increase in trading volume. Whereas Chen, Hong, and Stein (2001) have aimed to test the theory developed in Hong and Stein (2003), the size effect and influence of past return on skewness do not speak directly to the predictions of their model.

1. Deviations From Rationality and Financial Markets

The field of behavioral finance was developed in response to the increasing number of stock market anomalies that could not be explained by traditional asset pricing models (Shiller (2003)). Anomalies such as the equity premium puzzle (Mehra and Prescott (1985)), the closed-end fund puzzle (Lee, Shleifer, and Thaler, (1991)), and the forward premium puzzle (Fama (1984)), among others, have been difficult to explain using the traditional finance paradigm. While the traditional finance paradigm seeks to understand financial markets using models in which agents act rationally, behavioral finance allows economic agents to deviate from full rationality.

The standard finance topic of risk is based on quantitative measurement such as standard deviation, variance, and beta. According to Ricciardi (2008), the main foundations of the standard finance school's viewpoint of risk are the modern portfolio theory (MPT) and the capital asset pricing model (CAPM). In a survey conducted by Cooley and Heck (1981), where finance professors were asked to single out the most important contributions to the financial literature, two of the three seminal papers identified were Markowitz (1952), which is the historical basis of MPT, and Sharpe (1964), which documents the initial development of the CAPM. However, the notion of beta as a measure of risk and the CAPM has been subject to substantial criticism. There are a number of empirical facts about the *cross-section* of average return that cannot be explained by the CAPM. These so-called anomalies are, among others, the *size premium*, the *momentum effect,* and the *predictive power of scaled price ratio*. Fama and French (1992) documented these facts and concluded that beta is an inappropriate measure of risk because the CAPM did not explain the average stock returns for the 50-year period stretching from 1941 to 1990.

Behavioral finance does not rely only on quantitative measurements. It combines subjective (qualitative) aspects with objective (quantitative) elements. This distinction is, indeed, very important. The judgment process of how the individual collects information involves the assessment of consequences or outcomes and this influences the final investment decision. Therefore, based on several cognitive and psychological biases, behaviorists build models that deviate from rational expectation in order to address subjective and affective issues. The main psychological factors that influence the behavioral perception of risk are overconfidence, loss aversion, familiarity bias, representativeness heuristic, framing effect, and prospect theory.

Psychology is not the unique aspect of behavioral finance. The hypothesis that actual prices reflect fundamental values is related to the hypothesis that markets are efficient. Behavioral finance claims, however, that some characteristics of asset prices are most presumably interpreted as deviations from fundamental value. These deviations are caused by market participants who are not fully rational. De Long, Shleifer, Summers, and Waldmann (1990a) describe an economy where rational traders (arbitrageurs) bet against these irrational traders, who are often known as "noise traders." They introduce the idea of *noise trader risk*. This risk, associated with the presence of irrational traders, is the risk that the mispricing being exploited by the arbitrageur will increase over a short-term horizon.

The main literature that applied behavioral finance to information acquisition in financial markets is linked to the notion of noise trader risk and the application of overconfidence and difference in opinion on financial markets. In the following sections, we emphasize the non-conventional utility theory, the major behavioral biases from the cognitive psychology literature, and the main applications of these specific behavioral biases to financial markets.

1.1 Loss Aversion and Framing

Loss aversion and framing are two major psychological concepts that play a crucial role in behavioral finance. Loss aversion refers to individuals' tendency to be more sensitive to diminutions in their level of well-being than to increases. Loss aversion, discovered by Kahneman and Tversky (1979), has received a considerable amount of empirical attention in economics and other disciplines, such as cognitive psychology and sociology. Camerer (2000) provides an excellent summary of recent empirical work on prospect theory and shows in particular that loss aversion can explain several patterns observed in a wide variety of economic areas with a small number of modelling features. Framing in social science refers to the way individuals or groups perceive the reality. Tversky and Khaneman's (1992, p. 298) standpoint on framing is the following:

> "The rational theory of choice assumes description invariance: equivalent formulations of a choice problem should give rise to the same preference order (Arrow, 1982). Contrary to this assumption, there is much evidence that variations in the framing of options (e.g., in terms of gains or losses)

> yield systematically different preferences (Tversky and Kahneman, 1986)."

According to the analogy made by Tversky and Khaneman (1981), the effects of frames on preferences are compared to the effects of perspectives on perceptual appearance. Similar to framing, but related to economic preferences, Thaler's (1980) mental accounting describes the process whereby people code and evaluate economic outcomes. The asymmetry between gains and losses and the way individuals code them are key ingredients of a number of pioneering theoretical works in financial economics. The theoretical explanation of the equity premium puzzle is arguably one of the most important applications of these two concepts (Benartzi and Thaler (1995, 1999)).

Prospect theory (Kahneman and Tversky (1978), (1990)), disappointment aversion (Gul (1991)), and realization utility (Barberis and Xiong, (2012)) all incorporate the notions of loss aversion and framing. Among the behavioral decision models based on loss aversion and framing, these three models are of particular interest. Prospect theory refers indeed to a complete theory of choices under uncertainty. It is consistent with most choices problems, where preferences systematically violate the axioms of expected utility (EU) theory. Disappointment aversion, however, represents the most restrictive behavioral model. It includes EU theory as a special case. Finally, Realization Utility (Barberis and Xiong, 2012) refers to one of the most recent framing-based behavioral decision models, and sheds light on a variety of puzzling facts.

1.1.1 Prospect Theory

Prospect theory (Kahneman and Tversky, 1979) is built on the observation that human behavior under uncertainty is often in disagreement with the foundations of EU theory. In their seminal article, Khaneman and Tversky conduct several experiments and show that individuals tend to exhibit loss aversion, overweight small probabilities, and underweight medium and high probabilities. Based on a large number of experiments, the authors develop four specific characteristics (a four-fold pattern) of prospect theory.

1. A risk aversion for gains with medium and high probability $(p \geq 0.5)$.
2. A risk aversion for losses of low probability ($p \leq 0.1$).
3. A risk seeking for losses with medium and high probability $(p \geq 0.5)$.
4. A risk seeking for gains of low probability $(p \leq 0.1)$.

Prospect theory distinguishes two crucial steps: the edition or framing phase and the evaluation phase. Part of the edition phase is how to collect lottery results as a gain or loss, instead of final wealth. The preference order between prospects needs not to be invariant across contexts. The framing process could vary across different individuals. Therefore, the reference point, against which gains and losses are compared, is exogenous and typically differs across people. Following the framing phase, the decision maker is assumed to evaluate the lottery. In EU theory, risk aversion and risk appetite are determined only by the utility function, while in prospect theory, they are determined by the value function as well as the decision weights. Instead of making his decision relative to final wealth and objective probabilities, the decision maker uses subjective decision weights and relative performances (i.e. gains, losses, or neutral results).

The subsequently developed cumulative prospect theory (CPT), (Tversky and Kahneman (1990)), generalizes, inter alia, the original version of prospect theory by transforming the entire cumulative distribution instead of transforming each probability separately. CPT derives its name from this generalization. Given a prospect outcome x_i with probability p_i, CPT assumes that people assign the gamble the value of

$$\sum_i \pi_i v(x_i), \tag{1.1}$$

where

$$v(x) = \begin{cases} x^{\alpha} \ \text{if}\, x \geq 0 \\ -\lambda(-x)^{\beta} \ \text{if}\, x \leq 0 \end{cases} \tag{1.2}$$

and

$$\begin{aligned} \pi_i &= w(P_i) - w(P_i^*), \\ w(P) &= \frac{P^{\delta}}{(P^{\delta} + (1-P)^{\delta})^{1/\delta}}. \end{aligned} \tag{1.3}$$

P_i and P_i* refer to the probability that the gamble will yield an outcome at least as good as x_i and strictly better than x_i, respectively. The experimental parameter estimates are: $\alpha = 0.88$, $\beta = 0.88$, $\lambda = 2.25$, and $\delta = 0.65$. The median exponent of the value function was 0.88 for both gains and losses. The parameter therefore represents the coefficient of loss aversion. The value function *v* respects the four-fold patterns described above. The value function is indeed concave above the reference point and convex below, and has a kink at the origin as shown on Figure 1.1. In addition, the value function *v* is at least twice as steep for losses as it is for gains, which reflect loss aversion. Note also that the weighting function *w*(*P*) reflects the tendency for individuals to overweight small probabilities and underweight medium and high probabilities in the range of probabilities {0.05 and 0.95}.

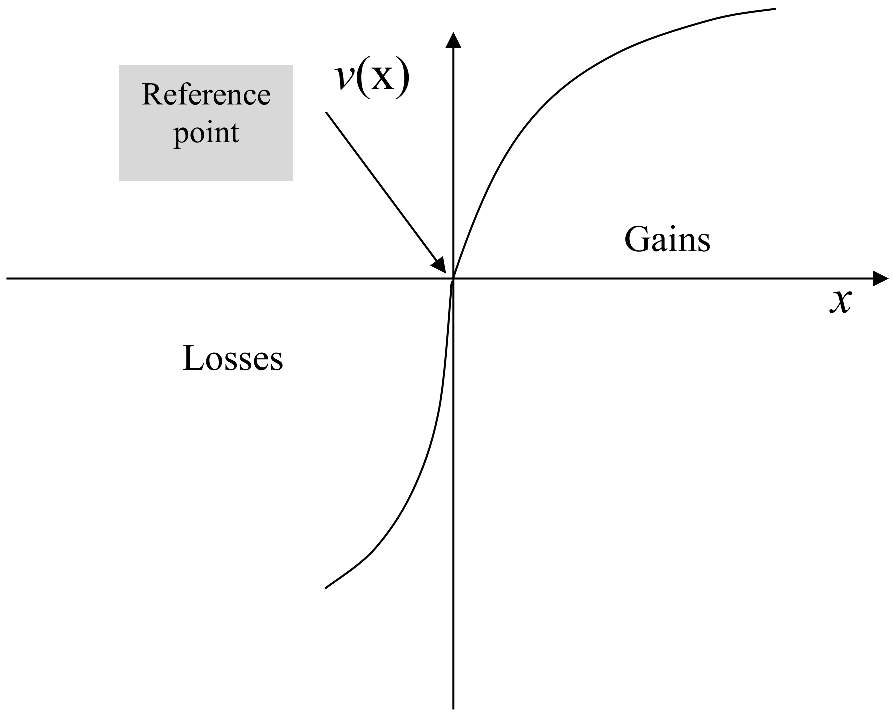

Figure 1.1. A Hypothetical PT Value Function. The value function is defined by gains and losses on deviations from a reference point, where the function is concave for gains and convex for losses. This function is steeper for losses than for gains.

1.1.2 Disappointment Aversion

Gul's (1991) disappointment aversion (DA) is based on the idea that economic agents are disappointed if the outcome of a lottery is below the certainty equivalent. Bell (1985) defines disappointment as a psychological reaction to an outcome that does not meet a decision maker's a priori expectation. DA theory is a one-parameter extension of EU theory. While including EU theory as a special case, this extension allows DA theory to be consistent with Allais Paradox (Allais

(1953)). Unlike prospect theory, which assumes that the framing phase is specific to all agents, DA theory supposes that the framing phase is endogenous to the model and that it is the same for all decision makers up to a specific parameter.

The preferences of a disappointment agent may be characterized by the pair $[u(x), \beta]$, where the function u is the traditional increasing and concave utility function of consuming x without the disappointment phenomenon, and $\beta \geq 0$, known as the disappointment rate, is a parameter that measures the level of disappointment aversion. Note that a disappointment averse agent with $\beta = 0$ refers to a risk averse agent. Let $V(\beta)$ denote the expected utility of a disappointment and let μ denote the certainty equivalent (i.e. $V(\beta) = u(\mu)$). One way to define $V(\beta)$ is

$$V(\beta) = E(u(x)) - \beta E[u(\mu) - u(x) | \mu > x], \tag{1.4}$$

where the second term in the right side of the equation (1.4) measures the average disappointment. For example, if $u(\mu)$=0, the expected utility is given by

$$V(\beta) = V(\beta)^+ + V(\beta)^- = E(u(x) | \mu < x) + E[(1+\beta)u(x) | \mu > x] \tag{1.5}$$

In that specific case, the DA utility function will be:

$$u_{DA}(x) = \begin{cases} u(x) & \text{if } x \geq \mu \\ (1+\beta)u(x) & \text{if } x < \mu \end{cases} \tag{1.6}$$

The DA utility function is concave, has a kink at $x = \mu$, and is steeper for values of x below μ than for values of x above μ, as can be seen from Figure 1.2.

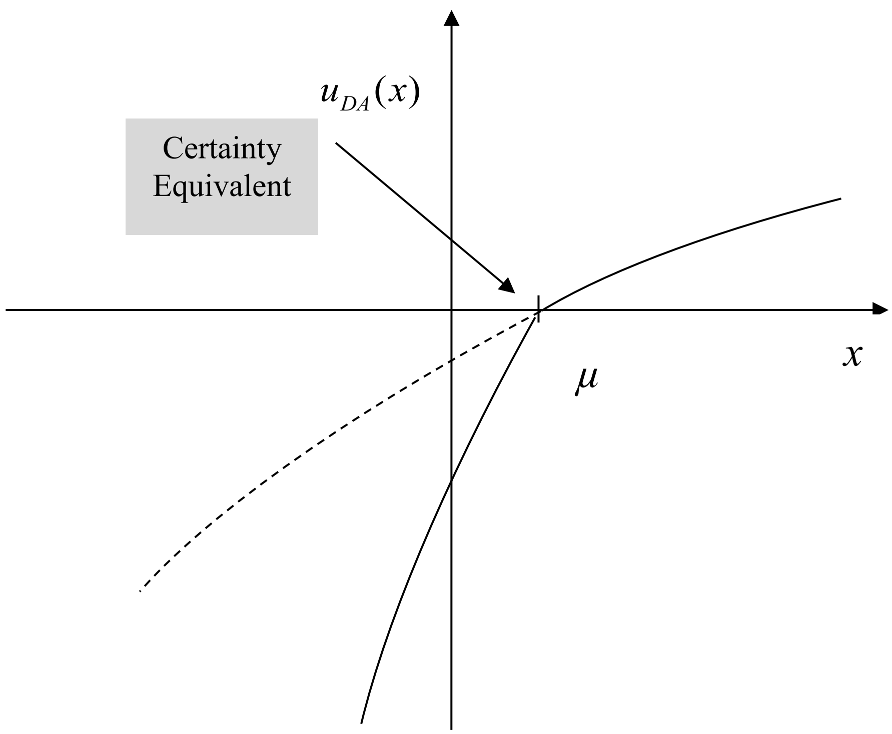

Figure 1.2 An Hypothetical DA Utility Function. The value function is defined by gains and losses on deviations from the certainty equivalent, where the function is concave for gains as well as for losses

Good outcomes above the certainty equivalent are downweighted relative to bad outcomes. The certainty equivalent represents the reference point against which gains and losses are compared. It is endogenous and depends on β. Unlike in prospect theory, the reference point is not arbitrary. Thus, the notion of loss aversion relative to the certainty equivalent can be in fact interpreted as a disappointment aversion. According to DA theory, the framing phase is identical for all individuals sharing the same degree of disappointment aversion. Since DA utility function is globally concave; it provides solvable portfolio allocation whereas with prospect theory preferences, optimal finite portfolio allocation may not exist.

1.1.3 Realization Utility

According to Thaler (1999), a *realized* loss is more painful than a paper loss. Individuals might indeed derive utility from *realized* gains and losses, rather than from terminal wealth. This idea first appears in Shefrin and Statman (1985). They combine prospect theory, mental accounting, tax consideration, regret aversion, and self-control in order to explain the tendency to sell winning stocks too early and hold losing stocks too long. They were the first to call this famous phenomenon the "disposition effect."

Barberis and Xiong (2009) investigate whether prospect theory can predict a disposition effect. They find that framing gains and losses in terms of realizing gains and losses predicts a disposition effect, while framing them in terms of annual gains and losses fails to predict a disposition effect. They isolate and emphasize the central role of *realized transactions* on individual investor perception. Barberis and Xiong (2012) subsequently develop a more comprehensive analysis on this subject, which they define as realization utility. This model assumes that the investment process is seen by investors as a series of distinct episodes during each of which they either made or lost money. Investors indeed receive a burst of pleasure upon selling an asset at a gain, and a burst of pain when they sell an asset at a loss relative to purchase price. Frydman, Barberis, Camerer, Bossaerts, and Rangel (2014) report evidence of behaviors supporting the realization utility hypothesis using neural data.

1.2 Overconfidence

Overconfidence refers to individuals' tendency to overestimate or exaggerate their ability to successfully perform a particular task. Debondt and Thaler (1995) argue that perhaps the most robust finding in the psychology of judgment is that people are overconfident. According to Plous (1993), overconfidence has been called the most "pervasive and potentially catastrophic" of all the cognitive biases to which human beings fall victim. It has been blamed for lawsuits, strikes, wars, and stock market bubbles and crashes.

The two principal aspects of overconfidence are miscalibration of subjective probability and the better-than-average effect. Calibration measures the validity of probability assessments. Calibration problems can be categorized as either discrete-type problems or continuous-type problems. The first type is calibration for events for which the outcome is discrete. These include probabilities assigned to statements like "I am the smartest among my colleagues" or "It will rain tomorrow." The second class of tasks is calibration for probabilities assigned to uncertain continuous quantities. For example, what is the length of the Congo River? Or, how long will it take to finish this project? Lichtenstein, Fischhoff, and Phillips (1982) perform a comprehensive review of the research literature on calibration. Their main findings, associated with the discrete-type calibration, are that individuals are generally poorly calibrated, since people act as though they can make much finer distinctions in their degree of uncertainty than is actually the case. They note that overconfidence is found in most tasks, that is, assessors tend to overestimate how much they know, and that the degree of overconfidence depends of the difficulty of the task. The more difficult the task, the greater the overconfidence. Moreover, they also demonstrate that training can improve calibration only to a limited extent. Lichtenstein, Fischhoff, and Phillips (1982) also report similar results in regards to the continuous-type of calibration. Individuals think they know more about uncertain quantities than they actually do, while only a few continuous-type studies have indicated that with practice, people can learn to become somewhat better calibrated.

The second facet of overconfidence is the *better-than-average-effect*. Taylor and Brown (1988) document that people have unrealistically positive views of themselves. People indeed judge themselves to be better than others concerning skills or positive character traits. For example, Sevenson (1981) documents a widely cited experiment on student respondents. In this experiment, students were asked to compare themselves with the drivers they encounter on the road. Eighty-two percent of respondents rank themselves among the top 30 percent of safe drivers.

1.3 Heuristics in Decision Making

In standard economic theory, when faced with a decision task, individuals often lack the ability to carry out the complex optimization expected of them. Instead, the decision maker may rely on simple cognitive strategies or intuitive heuristics (mental shortcuts) in order to eliminate some dimensions of the decision task. Although these strategies facilitate decision-making, they also induce decisions that deviate from outcomes prescribed by economic theory. Representativeness, anchoring, contamination effect, and familiarity bias are among the most important heuristics.

One of the main heuristics is representativeness. It can be conceptualized as the tendency to assess the similarity of outcomes based on stereotypes and then to use these assessments of similarity as a basis for judgment. Psychologists observe that individuals often try to interpret random events as the result of a thoughtful and predictable series of events. For instance, society frequently tries to assign blame to some individual when a major accident occurs.

The terms *apophenia* and *pareidolia*, which Powers (2012) defines as a phenomenon whereby people believe that systematic patterns exist in entirely random data, are related to the interpretation of random events. Some contend that the Rorschach inkblot test, where psychologists analyze a subject's interpretation of inkblots using an interpretation grid, falls within the realm of apophenia. According to Chapman and Chapman (1982), the Rorschach inkblot test was once the most widely used test for revealing hidden emotions. The “science” behind such tests is that humans have attitudes and motivations that are hidden from conscious awareness. Hume (1757, p. 11) recognized this human tendency more than 250 years ago when he stated:

> "There is a universal tendency among mankind to conceive all beings like themselves, and to transfer to every object those qualities with which they are familiarly acquainted, and of which they are intimately conscious. We find human faces in the moon, armies in the clouds; and by a natural propensity, if not corrected by experience and reflection, ascribe malice and good will to everything that hurts or pleases us"

The human tendency of trying to find a pattern in randomness is a normal human bias. McFadden (1999, p. 93) writes:

> "Tune (1964) and Kahneman and Tversky (1972) document experimentally that individuals intuitively reject randomness when they see recognizable patterns or streaks, systematically underestimating the probability that these can occur by chance. These biases reinforce the influence of random coincidences on beliefs and behavior."

Sterman (1994) and McFadden (1999) call this search for confirmation of current beliefs a quest for "emotional and spiritual sustenance." This confirmation bias serves as one explanation for the hot-hand fallacy as described by Gilovich, Tversky, and Vallone (1985). They find that people often assume the existence of a "hot hand" in basketball. Although no empirical evidence shows that a basketball player has a higher chance of making a successful shot if the previous shots were successful, many people still assume a positive correlation between events. This phenomenon occurs when people perceive outcomes as depending on abilities. In the case of random events such as flipping a coin, individuals will often assume a negative correlation; this is known as the *gambler's fallacy* (Tversky and Kahneman (1971)). Assuming a fair coin, the probability of throwing heads after a sequence of tails is still one half, but people often overestimate this probability.

By way of explanation, Tversky and Kahneman propose the so-called *law of small numbers* whereby people expect the large sample properties of a random event to be reflected in a small sample. In other words, because the number of heads should equal the number of tails in an infinite sample of coin tosses, both outcomes are assumed to occur equally often in a sample of say 10 tosses. Both fallacies can be observed in a financial decision setting, where individuals exhibit greater trust in human performance when predicting a random sequence, than they do when betting randomly (Huber, Kirchler, and Stockl (2010)).

Another example of apophenia is the quest for predictability of business cycles. Within insurance markets, the property and liability insurance market was in upheaval in the 1970s and 1980s because of the rapid rise of insurance premiums. The cyclicality of insurer profitability was a hot topic and generated a wide body of literature (Boyer, Jacquier, and Van Norden (2012)). Despite the lack of accepted econometric evidence of such cycles, the fascination with assuming their existence, and therefore predictability, remains.

1.4 Applications to financial markets

This section tries to link psychological factors described in the last sections with puzzling empirical evidence and anomalies pertaining to capital markets. These so-called market anomalies typically appear when agents' behaviors deviate from the prediction of efficient market hypothesis and subjective expected utility theory. This section also summarizes very briefly the major risks faced by financial agents from a behavioral standpoint and sheds light on some striking features of trading behavior and asset pricing.

1.4.1 The Familiarity Bias

People dislike ambiguity and tend to invest in what they know. This phenomenon is important for behavioral finance because manifesting a bias toward the familiar suggests a potential lack of diversification. According to French and Poterba (1991), a lack of diversification appears to be the result of investor choices rather than institutional constraints. Familiarity could refer either to a *local bias*, when investors display a preference for local assets, or to a *home bias*, when an individual's portfolio is heavily biased toward domestic equity.

Both institutional and behavioral explanations exist for familiarity bias. Perhaps the most popular rational explanation is asymmetric information. A number of studies contend that the local bias may be a rational response to better information about familiar assets (Ivković and Weisbenner (2005), Massa and Simonov, (2006)). Economic and cultural distance, which represents a barrier to the flow of information, could explain home bias (Hau (2001)). However, rational explanations for familiarity can only account for part of the home bias. Among popular behavioral explanations, researchers have analyzed the role of overconfidence, regret, patriotism, and social identification. This relatively new and growing literature finds that investors exhibit overconfidence in predicting returns on familiar assets. They then may prefer to invest in local assets to avoid regret or because of social identification and patriotism (Fouad (2010)).

Similar to familiarity, the *availability bias* is a cognitive heuristic whereby a decision maker will rely upon knowledge that is readily available rather than examine other alternatives or procedures. Availability can be viewed as the tendency to disproportionally recall very recent events or events in which the person is emotionally involved. The more salient an event, the more likely it is to drive an individual's investment decision.

1.4.2 Noise Trader Risk

Does the presence of mispricing predict possible profit? In the traditional framework, economic agents are rational, while markets are frictionless, and hence efficient. The *efficient market hypothesis* implies that observed asset prices reflect fundamental values, and that deviations only occur in the short run. Behavioral finance, however, claims that some characteristics of asset prices are most presumably interpreted as persistent deviations from fundamental value. The main argument against persistent mispricing is that it creates arbitrage opportunities so that rational traders will immediately take advantage of these price deviations, thereby correcting the mispricing (Shleifer and Summers, 1990). By contrast, De Long, Shleifer, Summers, and Waldmann (1990a) and Shleifer (2000) maintain that correcting the mispricing is not straightforward and can be very risky. This means that some rational individuals may prefer to behave irrationally.

Black (1986) was the first to introduce the notion of noise traders, who trade on noise as opposed to information. Shleifer and Vishny (1997) further analyze this phenomenon and describe an economy in which rational traders (arbitrageurs) bet against irrational traders—another name for "noise traders." They associate noise trader risk with the risk of mispricing being exploited by arbitrageurs. Shleifer and Vishny (1997) show that under certain conditions, arbitrageurs face difficulties in exploiting profitably market inefficiencies even if implementation costs, such as transactions costs and a short-sell constraint, are low. De Long, Shleifer, Summers, and Waldmann (1990a) show that if noise trader risk is systematic and if arbitrageurs are risk averse and have a short planning horizon, arbitrage will be limited even without implementation costs. In a related study, De Long, Shleifer, Summers, and Waldmann (1990b) describe an economy where noise traders follow positive-feedback strategies (i.e., buy when prices rise and sell when prices fall). For this type of economy, the authors show that arbitrageurs may prefer to trade in the same direction as noise traders. Instead of correcting the mispricing, arbitrageurs will therefore exacerbate it.

In theory, any evidence of persistent mispricing will be sufficient to assert that there are limits to an informed trader's ability to benefit from arbitrage opportunities. The presence of mispricing is not testable because any mispricing is inevitably tested jointly with some equilibrium asset-pricing model (Fama, (1970)). Despite this obvious problem, Barberis and Thaler (2005) note several cases where researchers report financial market anomalies that almost certainly indicate persistent mispricing.

1.4.3 Myopic Loss Aversion and the Equity Premium Puzzle

The equity premium puzzle is arguably one of the most emblematic enigmas of finance. Given the return of stocks and bonds over the past century, an abnormally high level of risk aversion would be necessary to explain why investors are willing to hold bonds at all (Mehra and Prescott, 1985). Mehra and Prescott (1985) believe that in order to explain the historic level of the equity premium, investor risk aversion should exceed 30, whereas theoretical arguments suggest that it should rather be around 1. Benartzi and Thaler (1995) combine loss aversion and mental accounting to provide a theoretical basis for the observed equity premium puzzle.

The mental accounting aspect considered by the authors encompasses the dynamic aggregation rules that people follow. An investor can have an evaluation period of six month with a 30-year horizon. The investment horizon is not part of the framing process. However, the evaluation period is a crucial component since it drives investor preferences. From the investors' point of view, they will be more inclined to take risks if they evaluate the portfolio performance infrequently.

The authors have found that after an evaluation period of about one year, investors with prospect theory preferences are left indifferent as regards holding all their assets in stocks or bonds. This evaluation period coincides with practical evidence. Loss aversion is the main driver of their finding. Replacing weighting functions by actual probabilities or using the simple piecewise linear function given by

$$u(x) = \begin{cases} x & \text{if } x \geq 0 \\ 2.5x & \text{if } x < 0 \end{cases}, \tag{1.7}$$

instead of the value function of equation (2.12), yields close results. Moreover, given an evaluation period of one year, they find that a portfolio containing between about 30 percent and 55 percent stocks maximizes prospective utility.

This result is roughly consistent with observed behavior, where most frequent allocation is equally distributed between stocks and bonds. According to Benartzi and Thaler's (1995) theory, the equity premium is produced by a combination of loss aversion and frequent evaluation (over a one-year period or so). They show that if the evaluation period increases, stocks become more attractive under prospect theory/loss aversion preferences. They demonstrate that if individual investors, in aggregate, evaluate their portfolios every five, ten or twenty years, then the equity premium would fall to 3 percent, 2 percent and 1.4 percent respectively. This result is important from a behavioral risk perspective. There is in fact, a particular behavioral risk associated with the psychic cost of evaluating the portfolio frequently. Benartzi and Thaler (1995, p. 86) illustrate this concept and state:

> One way to think about this result is that someone with a twenty year investment horizon, the psychic cost of evaluating the annually are 5.1 percent per year! That is, someone with a twenty-year horizon would be indifferent between stocks and bonds if the equity premium were only 1.4 percent, and the remaining 5.1 percent is potential rents payable to those who are able to resist to the temptation to count their money often. In a sense, 5.1 percent is the price of excessive vigilance.

1.4.4 Overconfidence and Asymmetric Information: Momentum, Excessive Trading and Market Underreaction

Overconfidence may be considered as a key aspect of behavioral finance. While rational investors try to maximize returns and to minimize risk, overconfident investors misinterpret the level of risk they take. Overconfident investors have a tendency to purchase high risk stocks and to underdiversify their portfolio. In regards to overconfidence among investors, Nofsinger (2007, p. 10) writes:

> "People can be overconfident. Psychologists have determined that overconfidence causes people to overestimate their knowledge, underestimate risks, and exaggerate their ability to control events. Does overconfidence occur in investment decision making? Security selection is a difficult task. It is precisely this type of task at which people exhibit the greatest overconfidence."

Overconfident investors are generally modeled as individuals who overestimate (underestimate) the precision (variance) of the private signal they receive. The main prediction of most overconfident investor models is the high trading volume. This market characteristic refers to the excessive trading phenomenon. Besides the excessive trading prediction, Odean (1998b) reports a positive correlation between the presence of overconfident traders and the volatility of asset prices. He demonstrates also that overconfident traders have lower expected utility than rational traders and hold undiversified portfolios. Daniel, Hirshleifer, and Subrahmanyam (1998) show that overreaction and self-attribution bias can reconcile short-run positive autocorrelation with long-run negative autocorrelation. They provide evidence on short-term momentum with long-term mean reversal, where momentum refers to the tendency for winning (loser) stocks over 3 to 12 months to remain winners (losers) in the subsequent period. Daniel, Hirshleifer, and Subrahmanyam (1998) theory also provides an explanation for the pattern that stock price reactions, following public events, are of the same sign as post-event long-run abnormal returns. This phenomenon has sometimes been interpreted as market underreaction. Daniel and Titman (1999) find also that momentum is stronger for growth stocks. Lee and Swaminathan (2000), and Glaser and Weber (2003) demonstrate that momentum is stronger among high-turnover stocks, where turnover, defined as a measure of trading volume, is the number of shares traded divided by the number of shares outstanding. From a behavioral risk perspective, in Daniel, Hirshleifer, and Subrahmanyam's (1998) model, on average, overconfident informed traders lose money. However, the notion of noise trader risk defined earlier (De Long, Shleifer, Summers, and Waldmann (1990a)), combined with risk averse overconfident traders developed by Daniel, Kent, Hirshleifer, and Subrahmanyam (2001), offer a different view on overconfident trades. In fact, in Daniel, Kent, Hirshleifer, and Subrahmanyam's (2001) model, overconfidence allows risk averse traders to exploit information more effectively and thus, to increase their profits.

1.4.5 Realization Utility: Trading Behavior and Asset Pricing

According to Barberis and Xiong's (2012) partial equilibrium model, investors may derive utility from realizing gains and losses on assets they own. Their model assumes first a linear functional form, and subsequently a piecewise-linear functional form for the realization utility. They incorporate realization utility both into a model of trading behavior and into a model of asset pricing, where they derive an analytical solution for the investor's optimal strategy. One of the main implications of that model is that even if the functional form of realization utility is linear and concave, the investor can be risk seeking. Indeed, a highly volatile stock may imply a large gain in the future that investors can enjoy realizing.

Besides the disposition effect (Shefrin and Statman, 1985 and Odean, 1998) reported earlier, realization utility sheds light on a number of puzzling phenomena in trading behavior. Barber and Odean (2000), report that after transaction costs, the average return of the individual investors in their sample is below the return of multiple benchmarks. This phenomenon is known as excessive trading behavior, since their trading hurts their performance. Barberis and Xiong's (2012) model offers an explanation for this behavior. They argue that after stocks have risen in value relative to the purchase price, the investors of Barber and Odean's (2000) sample are tempted to trade (sell the stock) in order to receive a burst of positive utility during the transaction and subsequently reinvest in new stocks. The transaction costs incurring, lead the investors to underperform the benchmarks. Realization utility also offers an explanation for another related phenomenon which is the underperformance before transaction cost (Barber, Lee, Liu, and Odean (2009)). In fact, the model predicts that investors are often willing to buy a stock with a low expected return as long as the stock's volatility is sufficiently high. Another anomaly concerns the phenomenon whereby rising markets exhibit higher trading volume than falling markets. This anomaly was first introduced by Stein (1995). Barberis and Xiong's (2012) model predicts that individual investors sell assets either when the price reaches the liquidation point that is above the purchase price, presumably in a rising market, or because of a liquidity shock. The effect of historic highs on the propensity to sell is another puzzling phenomenon for which the model offers a similar explanation. Finally, the last trading behavior that the authors report is the individual preference for volatile stock (Kumar, 2009). As noted previously, investors driven by realization utility might strongly prefer volatile stocks.

Barberis and Xiong's (2012) model also provides insights on some asset pricing patterns. Furthermore, they argue that realization utility can explain the low average return of volatile stocks (Ang, Hodrick, Xing, and Zhang (2006)). This finding goes against the conventional notion of risk since the principal tenet of the traditional risk theory is that riskier stocks should have higher average returns. According to Barberis and Xiong's (2012) model, as previously motioned, investors may be risk seeking and exert buying pressure on volatile stocks. This behavior may destabilize prices, and highly volatile stocks may then become overpriced. The second asset pricing pattern that the authors describe is the heavy trading of highly valued assets (Hong and Stein (2007)). The authors propose similar arguments about the attractiveness of highly volatile stocks for individual investors. They argue however that the coincidence of high prices and heavy trading will occur specifically for assets whose value is especially uncertain. Finally, realization utility may partially explain momentum, since as mentioned earlier, a related realization utility concept in combination with prospect theory may explain momentum (Grinblatt and Han (2005)).

1.5 Concluding Remarks

Irrational behavior applied to financial markets has become a hot topic in financial research. The efficient market hypothesis is no longer free from attacks from either the theoretical or empirical perspectives. This chapter highlights some biases in investor behavior, theories explaining some apparently irrational market behavior, and fields other than finance where behavioral biases are present. After introducing "traditional" approaches in finance, the chapter focuses on modifications to the neo-classical expected utility paradigm such as loss aversion, framing, realization theory, and prospect theory. These modifications introduce biases in the way that individuals treat gains and losses.

The chapter also covers heuristics in decision-making including representativeness, anchoring, and contamination. Additionally, the chapter examines overconfidence because of its economic impact on financial markets. The risk that the different biases, heuristics, and apparent sub-optimal behaviors discussed in this chapter affect investment decisions is an important concern for financial markets, financial regulators, and public policy makers.

2. Loss Aversion, Asymmetric Information, and Equilibrium Asset Prices

This chapter analyses equilibrium trading strategies and market quality in an economy with information asymmetry, in addition to which speculators display loss aversion. A closed-form characterization of the equilibrium price is presented. The model successfully disentangles the effect of loss aversion on optimal informed trading strategy and equilibrium price. We study the impact of loss aversion on asset prices, market depth, informed trading volume and price volatility. The model predicts nonlinear market depth. Consistent with empirical observations, the model finds that important price movements may occur following small shocks in the intermediate price region, regardless of the value of the underlying asset.

2.1 Introduction

Whereas models of financial markets with asymmetric information have often been applied to economies in which traders hold mistaken distributional beliefs about the payoff of the risky asset, and in particular, to economies in which traders are overconfident (Benos (1998), Caballé and Sakovics (2003), Kyle and Wang (1997), Odean (1998), and Wang (1998)), models of financial markets with loss averse speculators and asymmetric information have seldom been discussed in the literature. Over the past three decades, interest in the study of price formation in financial markets has grown, in particular in regards to the mechanism through which private information is acquired, utilized and impounded into price, optimal strategy, agents' trading incentives, and equilibrium market volatility and volume. We have also seen a widespread successful application of loss aversion to problems in finance. However, only a very recent study by Pasquariello (2014) analyzes the effect of prospect theory in general, and that of loss aversion in particular, on market quality. Pasquariello (2014) finds that loss averse informed traders endowed with private information have non-trivial state-dependent effects on equilibrium market liquidity, price volatility, trading volume, market efficiency, and information production. Unfortunately, the nonlinear rational expectation equilibrium developed in Pasquariello's (2014) study is analytically intractable.

In line with Grossman and Stiglitz (1980), Kyle (1985), and Vives (1995), we propose a noisy rational expectation equilibrium (REE) model in which competitive price-taking speculators endowed with private information exhibit loss aversion. The proposed economy is populated with informed traders, liquidity traders, and a risk neutral market maker (MM). Our model is a modified version of Pasquariello's (2014) model. While in his original model, asset choice is based on the mean variance approach to rational investment, the informed agents in our study maximize their expected utility. The proposed nonlinear equilibrium is analytically tractable, and a closed-form characterization of the equilibrium price is obtained. We analyze the mechanism through which prices are formed. The model shows how asset price collapses arise endogenously from the interaction between information asymmetry and loss aversion.

The speculators' preferences in our model disentangle loss aversion and risk aversion. This leads to a state dependant linear optimal trading strategy and makes the inference problem for the equilibrium price tractable. The proposed model disentangles the impact of loss aversion on optimal informed trading strategy and equilibrium price. The presence of loss averse better informed traders lowers the equilibrium price volatility and expected informed trading volume. Loss aversion induces also the speculator to trade less for sufficiently large signals in absolute value and not at all for very low signals. We show that since speculators' preferences successfully disentangle loss aversion and risk aversion, the trading intensity within the trading region remains unchanged in comparison to risk averse speculators only.

In our simple model, the market maker's problem of assessing the trading region of the informed traders can create large market price movements in the intermediate price region. Our model outlines how small trigger shocks can create market meltdowns. They appear when the absolute value of the aggregate order flow is low. In that situation, the market maker is confused about the trading status of informed traders, and price adjustment to signal and noise trading shocks becomes highly non-linear. Large price movements following small informational shocks are consistent with the evidence reported by Culter, Porterba, and Summers (1989), according to which crashes, as well as market bubbles, appear without any preceding public news. A closed-form characterization of the market depth is provided. Asset collapses arise typically when the market depth is high.

We then introduce short-sale constraints into the model. We demonstrate that the unique interaction between loss aversion and short-sale constraints produces asymmetric price movements similar to those reported in the literature (Pindyk (1984), French, Schwert, and Stambaugh (1987), Bekaert and Wu (2000), and Yuan (2005)). Specifically, price changes are more likely to decrease rather than increase. In other words, we show that, as Hong and Stein (2003) state, "markets melt down but they don't melt up." Hong and Stein (2003) report that when we look directly at historical stock return data, it can be noted that nine of the ten biggest one-day movements in the S&P 500 since 1947 were declines. Note that as mentioned previously, while loss aversion has successfully explained a broad variety of problems in finance, this work represents the first attempt in our knowledge to establish a connection between loss aversion and market collapses. We conjecture that this is in part because models of trading with loss aversion and information asymmetry are difficult to implement and have not been studied in the literature until very recently (Pasquariello, (2014)), due to the inherent complexity of market equilibriums with asymmetric information under non-standard CARA preferences.

Despite the initial motivation of the chapter to develop a tractable equilibrium in presence of asymmetric information and loss averse insiders, the model's comparative statics allow us to study market crashes. We are not the first to address a constrained information asymmetry framework in order to explain crashes. However, this study largely emphasizes the role of loss aversion and the basic framework is hardly different from other works related to market meltdown.

Yuan (2005), and Barlevy and Verosini (2003) have found that crashes are driven by uninformed, uncertain about fundamentals, investors. In contrast, Romer (1990), and Hong and Stein (2003) make the assumption that small events may reveal substantial information and that the crashes are driven by the information acquisition process. Conversely, in our model, asset price collapses arise predominantly because of the partly non-revealing aspect of speculators' optimal demand.

The analysis presented in this chapter is also related to research produced by Ozsoylev and Werner (2011), Condie and Ganguli (2011), and Mele and Sangiorgi (2015). Using a REE framework, they study to what extent ambiguity (Knightian uncertainty, Knight, 1921)) about fundamentals affects asset prices. Those papers typically start from the premise of the standard Grossman and Stiglitz model (1980), and make the assumption that markets are subject to ambiguity and that the risky payoff cannot be quantified probabilistically. Like loss aversion, ambiguity aversion represents a robust and well documented behavioral bias. Similarly to this study, Ozsoylev and Werner (2011), and Mele and Sangiorgi (2015) incorporate a noisy supply in their model and induce partial revelation of information. While in our model, important price movements occur following small supply or informational shocks, Ozsoylev and Werner (2011), Condie and Ganguli (2011), and Mele and Sangiorgi (2015) typically show how large price swings occur after a small change in the uncertainty parameters.

2.2 Model of Trading with Loss Aversion

Since the introduction of the Allais paradox (1953), several violations of the basic expected utility theory have been documented. According to Starmer's (2000) review of the literature, one specifically persistent empirical finding in experiments is a greater sensitivity of losses than to gains of similar size. This idea that people are loss averse with respect to changes in wealth is a central feature of prospect theory (Khaneman and Tversky (1979)). Recently, van Gaudecker, van Soest, and Wengstrom (2011) have analyzed risk preferences using an experiment with real incentives in a representative sample of 1,422 respondents. They find that utility curvature and loss aversion are the key determinants of individuals' choices under risk. We adopt the utility specification of von Gaudecker, van Soest, and Wengstrom (2011) to model speculators' preferences. We describe a noisy rational expectation equilibrium model of sequential trading in the presence of better informed, loss averse speculators. In the spirit of Grossman and Stiglitz (1980), Diamond and Verrecchia (1981), and Vives (1984), we assume that speculators are competitive and submit limit orders instead of market orders. Allowing for perfect competition, informed limit orders, and loss aversion, our model is similar to the model of Pasquariello (2014) who studies market quality with prospect theory driven preference speculators. However, while in Pasquariello (2014) equilibrium quantities are approximate using a numerical approach via OLS, the equilibrium developed in this study is analytically tractable.

2.2.1 The basic economy

We describe a noisy equilibrium model of sequential trading in the presence of better informed speculators, who are competitive and submit limit orders. The economy is populated with informed traders, liquidity ("noise") traders whose demand is exogenous and who trade for idiosyncratic life-cycle or liquidity reasons, and a risk-neutral competitive market maker. Informed traders are competitive and form a continuum with measure one. The model has two dates, time 0 and time 1. At time 0, investors trade competitively in the market based on their private information. At time 1, payoffs from the assets are realized and consumption occurs.

There is one risk-free asset and one risky asset. The risk-free asset is a claim to one unit of terminal-period wealth, and the risky asset pays $\tilde{v}$ units of the single consumption good. While taking the risk-free asset to be the numeraire, we let P be the price for the risky asset. Prior to trading, informed investors receive private information related to the payoff of the risky asset. The signal s is a noisy signal of the asset final payoff v, given as $s = v + \varepsilon$. We assume that all informed investors receive the same private signal s and possess identical preference. Unfortunately, a model with diverse signals and/or diverse preferences does not allow us to derive tractable equilibrium. The random variables v and ε are assumed to be mutually independent and normally distributed with mean zero and variance σ_v^2 and σ_ε^2. In order to save on notation, we assume that the mean of v is zero. However, for general value of $E[v]$ the derivation remains the same for $v - E[v]$. Liquidity ("noise") traders produce a random, normally distributed demand z with mean zero and variance σ_z^2. Moving first, liquidity traders submit market orders and speculators submit demand schedules or generalized limit orders contingent on their information to the market maker, before the equilibrium price P has been set. Market clearing proceeds through a simultaneous placement of orders with a centralized auctioneer (CA) (see also Yuan, (2005), and Ozsoylev and Werner, (2011)). When speculators optimize their demand, they take into consideration the relationship between equilibrium functional price and the random variables in the economy. Then a competitive risk neutral market maker sets the price efficiently given the observed aggregate order flow. It is well known that in large markets, competitive noisy rational equilibriums are implementable, thus allowing agents to use demand schedule as strategies. We denote a speculator demand schedule by $x_i(s,.)$; thus when the price is P, the desired position of the informed trader is $x_i(s,P)$. We assume that the speculator perceives the investment of all his or her wealth in the risk free asset as the *reference point*, and any other outcomes as changes or profits with respect to this reference point. So the profits from speculator i are given by $\pi_i = x_i(v-P)$.

2.2.2 Loss Averse Speculators

The CARA-normal model is popular in the study of financial markets with asymmetric information. For various settings it admits linear equilibria. Hellwig (1980), Grossman and Stiglitz (1980), Diamond and Verrecchia (1981), Admati (1985) and Vives (1995) all analyze competitive rational expectation models with asymmetric information with constant absolute risk aversion (CARA) and normally distributed random variables.

In the standard CARA-Normal framework, the speculator maximizes $E_0\left[-\exp(-\gamma\tilde{W}_{1,i})\right]=E\left[-\exp(-\gamma\tilde{W}_{1,i})\middle|s\right]$ over the final wealth, where γ is the coefficient of absolute risk aversion. E_0 refers to the expectation operator, conditional on investor information at time 0, and $\tilde{W}_{1,i}$ is the final wealth of speculator *i*. It is well known that the optimization result does not depend on initial wealth, and it is equivalent to maximize $E_0\left[-\exp(-\gamma\pi_i)\right]$ over the speculator's profits.

We extend the CARA-Normal model and we add loss aversion into speculator preferences. Since the risk-free rate is equal to zero, speculator *i* perceives the reference point as $W_{i,0}$. We suppose that preferences are continuous and display a kink at a reference point. Relative to the reference point, losses hurt more individuals than comparable gains, and thus the slope of the utility function is steeper for losses than for gains. We assume that all speculators have the same utility function $U(\pi,\gamma,\lambda)$ and so we drop the subscripts *i*. Moreover, in line with the specification of von Gaudecker, van Soest, and Wengstrom (2011), we assume that the utility function is given by

$$U(\pi,\lambda,\gamma)=\begin{cases}-\dfrac{1}{\gamma}e^{-\gamma\pi} & \text{for } \pi\geq 0\\ \dfrac{\lambda-1}{\gamma}-\dfrac{\lambda}{\gamma}e^{-\gamma\pi} & \text{for } \pi<0\end{cases} \tag{2.1}$$

where λ represents the degree of loss aversion [5]

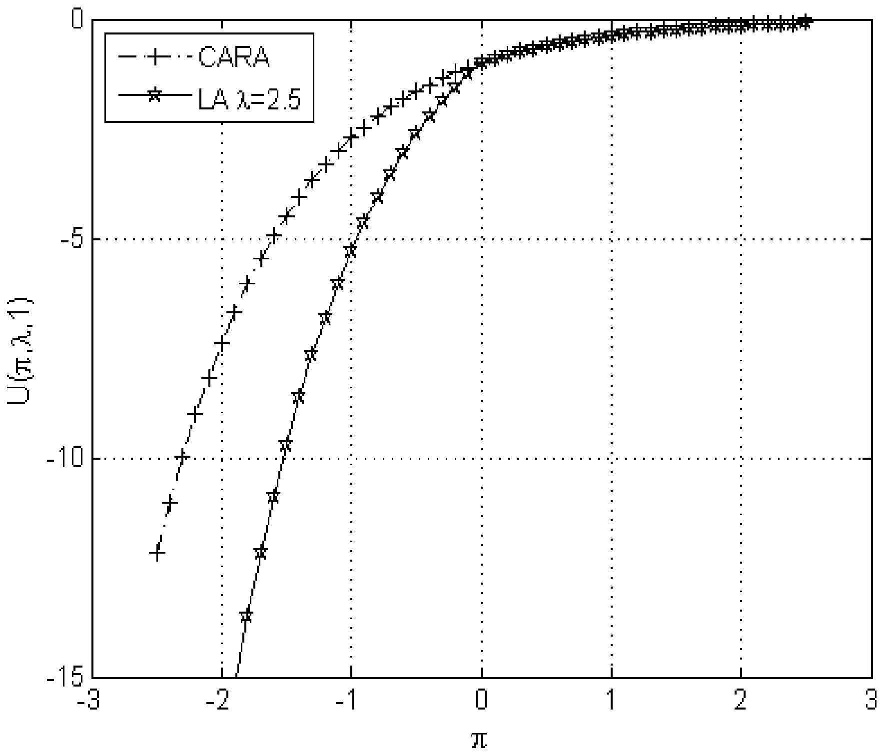

Figure 2.1. Utility function. This function in line with loss aversion exhibits a kink at the origin. The starred line is for a loss aversion parameter of $\lambda = 2.5$ and the crossed line represents a particular case where $\lambda = 1$, (CARA preferences). The risk aversion parameter is $\gamma = 1$.

Von Gaudecker, van Soest, and Wengstrom (2011) compare in their web appendix the proposed specification with prospect theory specification and report slightly larger values of λ for prospect theory preferences, which they attribute as a mechanical consequence of the different assumptions on the shape of the utility function on the negative domain. They report for high-incentive treatment a

[5]Recent empirical evidence challenges prospect theory's original utility function for mixed gamble (Baltussen, Post, and Pim van Vielt (2006)). Moreover, von Gaudecker, Martin, van Soest, and Wengstrom (2011) show in their study that changing the assumption curvature to prospect theory-type preferences does not substantially affect their main estimates.

median estimate parameter of loss aversion of 2.38, in line with previous estimates[6].

2.3 The Optimal Demand of Informed Traders

Let $x(s,.)$ represent the demand schedule for the risky asset of an informed trader given private signal s. When the price realization is P, the demand function is then $x(s,P)$. The only information available to the informed trader at time 0 is the noisy signal s. According to Vives (1995) and Pasquariello (2014), speculators neither learn from market prices nor internalize the impact on their trades on market prices. Thus, the demand of the informed trader submitted at time 0 is given by the maximization of the expected utility

$$E\left(U(\pi,\gamma,\lambda)\middle|s\right)=\begin{cases}-\frac{1}{\gamma}\left[\begin{array}{l}e^{-\gamma x(\mu_{v/s}-P)+\frac{\gamma^2x^2\sigma_{v/s}}{2}}\left[1+(\lambda-1)\Phi(\frac{x\gamma\sigma_{v/s}^2\text{-}(\mu_{v/s}-P)}{\sigma_{v/s}})\right]\\ \text{-}(\lambda-1)\Phi(\frac{\mu_{v/s}-P}{\sigma_{v/s}})\end{array}\right] & \text{for } x\le 0\\ -\frac{1}{\gamma}\left[\begin{array}{l}e^{-\gamma x(\mu_{v/s}-P)+\frac{\gamma^2x^2\sigma_{v/s}}{2}}\left[1+(\lambda-1)\Phi(\text{-}\frac{x\gamma\sigma_{v/s}^2\text{-}(\mu_{v/s}-P)}{\sigma_{v/s}})\right]\\ \text{-}(\lambda-1)\Phi(\text{-}\frac{\mu_{v/s}-P}{\sigma_{v/s}})\end{array}\right] & \text{for } x>0\end{cases} \quad (2.2)$$

[6] The authors demonstrate that despite the fact that Köbberling and Wakker's (2005) definition of loss aversion is model independent, the measurement parameters depend on the complete structure of the utility function. It is thus not possible to directly compare the parameters across models. We analysis in the present study the effect of loss aversion on the price formation process and Coval and Shumway (2005) reported in their empirical study, that the degree of loss aversion might vary across investors depending on prior gains and losses. Thus, since the model is static, one might chose the appropriate degree of loss aversion depending on market conditions.

where $\mu_{v/s} = \rho^2 \tilde{s}$ and $\sigma_{v/s}^2 = \sigma_v \sqrt{1-\rho^2}$ is the conditional mean and variance of the random risky payoff v given the private signal received by each speculator and where $\rho = \dfrac{\sigma_v}{\sqrt{\sigma_v^2 + \sigma_\varepsilon^2}}$, and $\Phi(.)$ refers to the cumulative distribution function of the standard normal distribution. The derivation of (2.2) and the conditional mean and variance are presented in Appendix A.

Equation (2.2) admits only one bounded maximum value for each region since, as we will see in the ensuing analysis, the first order condition of equation (2.2) is solved for at most one value in each region. For unbounded values of x in each region, the objective function is equal to minus infinity.

Taking the first order condition of equation (2.2) with respect to x, yields for $x > 0$

$$e^{-\gamma x(\mu_{v/s}-P)+\frac{\gamma^2 x^2 \sigma_{v/s}^2}{2}} \left[\begin{array}{l} \dfrac{(\lambda-1)\gamma\sigma_{v/s}}{\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x\gamma\sigma_{v/s}^2-(\mu_{v/s}-P)}{\sigma_{v/s}}\right)^2} \\ +\left(1+(\lambda-1)\Phi(\dfrac{x\gamma\sigma_{v/s}^2-(\mu_{v/s}-P)}{\sigma_{v/s}})\right)\left(x\gamma^2\sigma_{v/s}^2-\gamma(\mu_{v/s}-P)\right) \end{array} \right] = 0 \qquad (2.3a)$$

and for $x \leq 0$

$$e^{-\gamma x_i(\mu_{v/s}-P)+\frac{\gamma^2 x^2 \sigma_{v/s}^2}{2}} \left[\begin{array}{l} -\dfrac{(\lambda-1)\gamma\sigma_{v/s}}{\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x\gamma\sigma_{v/s}^2-(\mu_{v/s}-P)}{\sigma_{v/s}}\right)^2} \\ +\left(1+(\lambda-1)\Phi(-\dfrac{x\gamma\sigma_{v/s}^2-(\mu_{v/s}-P)}{\sigma_{v/s}})\right)\left(x\gamma^2\sigma_{v/s}^2-\gamma(\mu_{v/s}-P)\right) \end{array} \right] = 0 \qquad (2.3b)$$

The term in the bracket of equation (2.3a) and equation (2.3b) should be equal to zero since the exponential function $e^{-\gamma x_i(\mu_{v/s}-P)+\frac{\gamma^2 x^2 \sigma_{v/s}^2}{2}}$ is bounded below by a positive number. Dividing both sides of equation (2.3a) and equation (2.3b) by $\gamma\sigma_{v/s}$ and defining $\Lambda = x\gamma\sigma_{v/s} - \frac{\mu_{v/s}}{\sigma_{v/s}} - \frac{P}{\sigma_{v/s}}$ we get for $x > 0$

$$\Lambda\left(1+(\lambda-1)\Phi(\Lambda)\right)+\frac{(\lambda-1)}{\sqrt{2\pi}}e^{-\frac{1}{2}\Lambda^2}=0 \tag{2.4a}$$

and equivalently for $x \le 0$

$$\Lambda\left(1+(\lambda-1)\Phi(\text{-}\Lambda)\right)-\frac{\left(\lambda-1\right)}{\sqrt{2\pi}}e^{-\frac{1}{2}\Lambda^2}=0 \tag{2.4b}$$

For any degree of loss aversion $\left(\lambda \ge 1\right)$ one can solve equations (2.4a) and (2.4b). numerically. Estimates of loss aversion from previous studies are typically in the neighborhood of 2.5. For example, if we set $\lambda = 2$ we find that $\Lambda = -0.276$ for positive value of *x* and $\Lambda = 0.276$ for negative value of *x*. Thus, the optimal positive demand is $x = \frac{(\mu_{v/s} - P)}{\gamma\sigma^2_{v/s}} - \frac{0.276}{\gamma\sigma_{v/s}}$ and the optimal negative demand is $x = \frac{(\mu_{v/s} - P)}{\gamma\sigma^2_{v/s}} + \frac{0.276}{\gamma\sigma_{v/s}}$. We notice however that a positive or negative demand will depend on the magnitude and the precision of the private signal. In order to push demand to positive ranges, the signal should be relatively high, i.e. $s \ge \frac{P + 0.276\sigma_{v/s}}{\rho^2}$, and inversely to push the demand to the negative range the signal should be relatively low, i.e. $s \le \frac{P - 0.276\sigma_{v/s}}{\rho^2}$. Outside this range, in the interval $\left|\mu_{v/s} - P\right| \le 0.276\sigma_{v/s}$ we note that neither the objective function for positive value of *x* nor for negative value of *x* admits local minimum in their respective ranges. Thus $x = 0$ maximizes the expected utility in the range $\left|\mu_{v/s} - P\right| \le 0.276\sigma_{v/s}$.

We can generalize to any value of λ greater o equal to one. Thus the solution of equation (2.4a) and (2.4b) may be expressed as a function of λ, $\Lambda = \Lambda(\lambda)$.

Result A1: *The optimal demand for the informed trader is given by*

$$x_{LA}^{*} = \begin{cases} \dfrac{\rho^2 s - P}{\gamma\sigma_v^2\left(1-\rho^2\right)} - \dfrac{\Lambda(\lambda)}{\gamma\sigma_v\sqrt{1-\rho^2}} & \text{for } s > \dfrac{P + \Lambda(\lambda)\sigma_v\sqrt{1-\rho^2}}{\rho^2} \\ 0 & \text{elsewhere} \\ \dfrac{\rho^2 s - P}{\gamma\sigma_v^2\left(1-\rho^2\right)} + \dfrac{\Lambda(\lambda)}{\gamma\sigma_v\sqrt{1-\rho^2}} & \text{for } s \le \dfrac{P - \Lambda(\lambda)\sigma_v\sqrt{1-\rho^2}}{\rho^2} \end{cases} \tag{2.5}$$

where $\Lambda = \Lambda(\lambda)$ solves $\Lambda\left(1+(\lambda-1)\Phi(-\Lambda)\right) - \dfrac{(\lambda-1)}{\sqrt{2\pi}}e^{-\frac{1}{2}\Lambda^2} = 0$.

Figure 2.2 plots the functional form $\Lambda(\lambda)$ for loss aversion parameters in the range [1, 10]. From figure 2.2 we see that $\Lambda(\lambda)$ is concave and increases with λ. We notice that for $\lambda = 1$, $\Lambda(\lambda) = 0$ and the optimal demand reduces to the optimal generalized limit order under the regular CARA-Normal model with negative exponential utility ((Vives (1995) and Grossman and Stiglitz (1980)).

$$x_{MV} = \frac{\mu_{v/s} - P}{\gamma\sigma_{v/s}^2} = \frac{\rho^2 s - P}{\gamma\sigma_v^2\left(1-\rho^2\right)} \tag{2.6}$$

We find that loss aversion has additional effects on speculator trading strategies. As for the standard CARA-Normal setting, the proposed model predicts that informed traders submit cautious limit orders. The optimal demand is a state-dependant linear function of the private signal and the equilibrium price. Increasing loss aversion or increasing risk aversion increases the cautiousness of the trade. The losses induced by trading, which obviously hurt more in proportion to the speculator's degree of loss aversion, are reflected in a reduction of optimal trading activity compared to risk averse speculators only. According to the intensity of the private signal, losses are reflected in less trading or no trading at all.

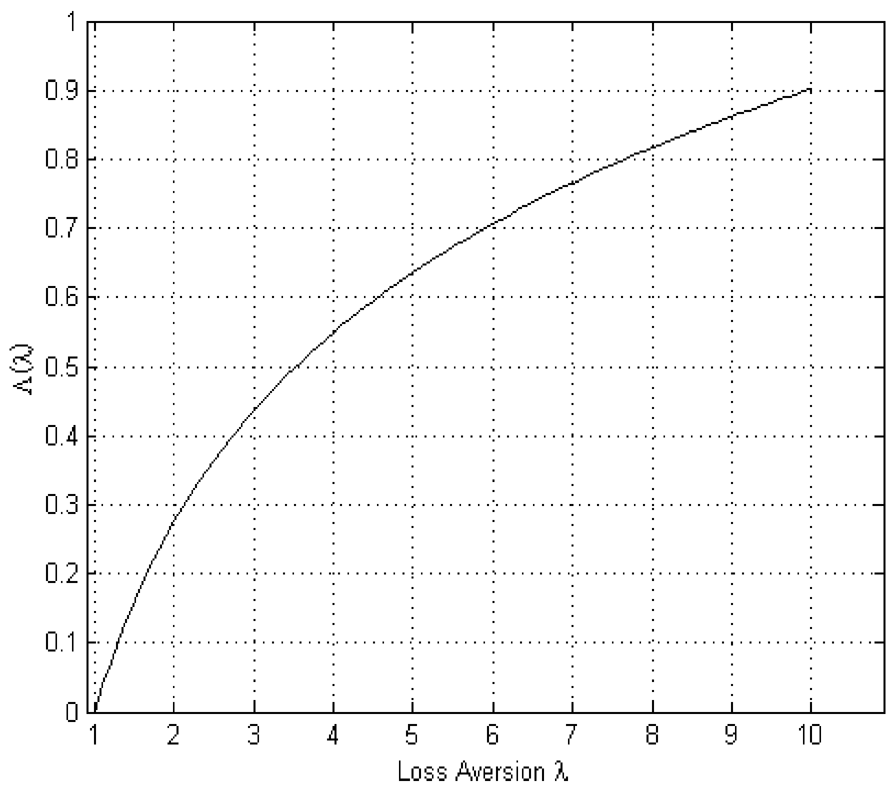

Figure 2.2. The effect of loss aversion on optimal demand: $\Lambda(\lambda)$ **.** The graph plots the solution of equation $\Lambda\left(1+(\lambda-1)\Phi(-\Lambda)\right)-\frac{(\lambda-1)}{\sqrt{2\pi}}e^{-\frac{1}{2}\Lambda^2}=0$ as a function of λ.

Trading intensity (Vives (1995)) is defined by the sensitivity of speculators' demand function to information shocks $\zeta = \frac{\partial x}{\partial s}$. In our model the trading intensity is

$$\zeta = \begin{cases} 0 & \text{for } \left|s - \Delta\right| \le \frac{P}{\rho^2} \\ \frac{1}{\gamma \sigma_\varepsilon^2} & \text{for } \left|s - \Delta\right| \le \frac{P}{\rho^2} \end{cases} . \tag{2.7}$$

where $\Delta(\lambda, \sigma_v, \sigma_\varepsilon) = \frac{\Lambda(\lambda)\sigma_v \sqrt{(1-\rho^2)}}{\rho^2}$. Figure 2.3 depicts an example optimal demand for a given private signal precision and noise trading precision, and a correlation coefficient.

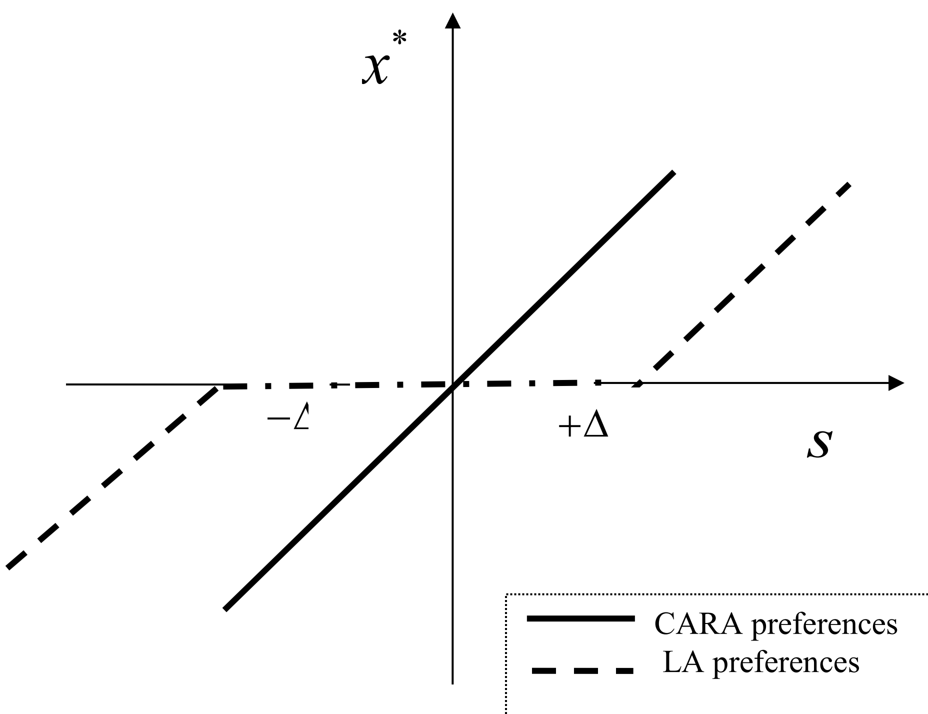

Figure 2.3. An example of optimal demand. For given economy parameters, the figure represents the optimal demand x^* for the informed traders as a function of private signal intensity. The solid line refers to the case with risk averse insiders and the dashed line refers to loss averse speculators.

As can be observed, outside the no-trade interval the measure of loss averse trading aggressiveness is the same as for the standard CARA-normal model and depends solely on the precision of the private signal and on risk tolerance. In our model, since the speculators' preferences disentangle risk aversion and loss aversion, loss aversion does not affect the trading intensity for sufficiently large signals. Intuitively, while trading more with a better signal involves risking more, this does not increase the likelihood of losing more in expectation, and thus the trading intensity is not affected by loss aversion.

2.4 Equilibrium

We will now characterize equilibrium prices and trading behavior in the model. We denote the aggregate order flow by $\omega = x + z$, which refers to the noisy limit-order book schedule observed by the market maker. The market maker earns zero expected profit, conditional upon the order flow. In fact, according to Vives (1995), this condition can be justified with Bertrand competition among risk neutral market makers who observe the limit order book and have symmetric information. It can also be explained by a situation where there is a continuum of risk neutral market makers who submit a limit order to a central mechanism jointly with informed traders where prices are set by a Walrasian centralized auctioneer (CA) to equate the aggregate excess demand from all the model's market participants to zero.

In this case, the equilibrium equation (2.8) is necessarily verified since otherwise market makers would like to take unbounded positions. The market clearing price P set by the market maker satisfies

$$P(\omega) = E\left[v|\omega\right]. \tag{2.8}$$

Risk neutrality and dealership competition imply the semi-strong market efficiency rule expressed by equation (2.8)[7]. From equation (2.5), this implies that the optimal demand schedule x_{LA}^{*} depends on risk aversion, loss aversion, market clearing price, and the intensity of the private signal. For a given intensity of the private signal at a given equilibrium price, speculators' optimal demand falls either within a no-trade interval or a trading interval. Thus, the market maker has to speculate as to the speculators' trading status. Following Pasquariello (2014), and in the same spirit of Yuan (2005), the risk neutral market maker inference problem can be expressed as

$$\begin{aligned} P = {} & E\left[v\left|\omega, s \geq \frac{P}{\rho^2} + \Delta\right.\right]\Pr\left[s \geq \frac{P}{\rho^2} + \Delta|\omega\right] + E\left[v\left|\omega, s \leq \frac{P}{\rho^2} - \Delta\right.\right]\Pr\left[s \leq \frac{P}{\rho^2} - \Delta|\omega\right] \\ & + E\left[v\left|\omega, |s - \Delta| \leq \frac{P}{\rho^2}\right.\right]\Pr\left[|s - \Delta| \leq \frac{P}{\rho^2}|\omega\right] \end{aligned} \tag{2.9}$$

where $\Pr\left[s \leq \frac{P}{\rho^2} - \Delta\right]$, $\Pr\left[s \geq \frac{P}{\rho^2} + \Delta\right]$ are the probability of the order flow being informative while $\Pr\left[|s - \Delta| \leq \frac{P}{\rho^2}\right]$ is the probability that the order flow is uninformative as regards the risky payoff *v*.

[7] Similar condition is found for instance in Kyle (1995), Hirshleifer, Subramanyam, and Titman (1994), Vives (1995), and Pasquariello (2014).

Since the optimal demand schedule x_{LA}^* of equation (2.5) makes ω a linear function of P and of the private signal s and since the boundaries are not functions of the received private signal s (i.e. Δ does not depend on s), the inference problem of equation (2.9) is analytically tractable and it is described in Appendix B. $\psi(x)$ refers to the probability density function of the standard normal distribution.

Result A.2: *The rational expectations equilibrium price function of the model is the unique fixed point of the implicit function*

$$P^* = \frac{\gamma\sigma_v^2\sigma_\varepsilon^2}{\sigma_v^2+\sigma_\varepsilon^2\left(1+\gamma^2\sigma_\varepsilon^2\sigma_z^2\right)}\left(\frac{s}{\gamma\sigma_\varepsilon^2}+z\right)\left[1-\Phi\left(\frac{\frac{P}{\rho^2}+\Delta}{\sqrt{\sigma_v^2+\sigma_\varepsilon^2}}\right)+\Phi\left(\frac{\frac{P}{\rho^2}-\Delta}{\sqrt{\sigma_v^2+\sigma_\varepsilon^2}}\right)\right]$$
$$+\sigma_v\rho\left(1-\sqrt{\frac{\gamma^2\sigma_z^2\sigma_\varepsilon^4}{\sigma_v^2+\sigma_\varepsilon^2\left(1+\gamma^2\sigma_\varepsilon^2\sigma_z^2\right)}}\right)\left[\psi\left(\frac{\frac{P}{\rho^2}-\Delta}{\sqrt{\sigma_v^2+\sigma_\varepsilon^2}}\right)-\psi\left(\frac{\frac{P}{\rho^2}+\Delta}{\sqrt{\sigma_v^2+\sigma_\varepsilon^2}}\right)\right] \quad (2.10)$$

Proof: Given $g(P)=f(P)-P$, where $f(P)$ represents the right side of equation (2.9), and since $\lim_{x\to\infty}\Phi(x)=0$, $\lim_{x\to-\infty}\Phi(x)=1$, and $\lim_{x\to\pm\infty}\psi(x)=0$, it is immediately clear that $\lim_{P\to\infty}g(P)<0$ and $\lim_{P\to-\infty}g(P)>0$. According to the Intermediate Value Theorem, at least one solution to $g(P)=0$ exists. As $g(P)$ is a decreasing function, the solution to $g(P)=0$ is therefore unique. Hence P exists and is unique. Q.E.D.

If speculators do not exhibit loss aversion (i.e. $\lambda=1$, $\Delta(\lambda)=0$), the rational equilibrium price function of equation (2.10) is reduced to equilibrium price when speculators have CARA preferences

$$P^*_{CARA} = \frac{\gamma\sigma_v^2\sigma_\varepsilon^2}{\sigma_v^2 + \sigma_\varepsilon^2\left(1+\gamma^2\sigma_\varepsilon^2\sigma_z^2\right)}\left(\frac{s}{\gamma\sigma_\varepsilon^2} + z\right). \tag{2.11}$$

Equation (2.11) is identical to the mean variance preferences and equilibrium price found by Pasquariello (2014) and is a special case of the linear equilibrium in Vives (2008, Proposition 1.11) when a continuum of risk averse speculators receives identical noisy signals of the asset payoff.

In equilibrium, informed agent i buys or sells according to whether s, the private estimate of v is larger than $\frac{P}{\rho^2}+\Delta$ or smaller than $\frac{P}{\rho^2}-\Delta$, and does not trade otherwise. In their trading region, informed agents trade more intensively if risk aversion (γ) is lower, and if the precision of the signal $\left(1/\sigma_\varepsilon^2\right)$ is higher. Moreover, in our model, the precision of the signal $\left(1/\sigma_\varepsilon^2\right)$ also shortens the no-trade region $\left(2\Delta\right)$, while γ has no impact on the determination of that region. As in the CARA model, trading intensity is independent of the amount of noise trading.

There is a trade in this type of model because of the presence of noise traders and because of the information advantage that informed agents hold on the market maker. The asymmetric information between speculators and market maker typically creates two opposite effects, namely the selection effect and the information (efficiency) effect. While a private signal of higher quality encourages the informed agent to trade more and more aggressively, thus more efficiently exploiting their information premium, they also typically reveal to the market maker more private information, hence increasing the precision of the equilibrium market price.

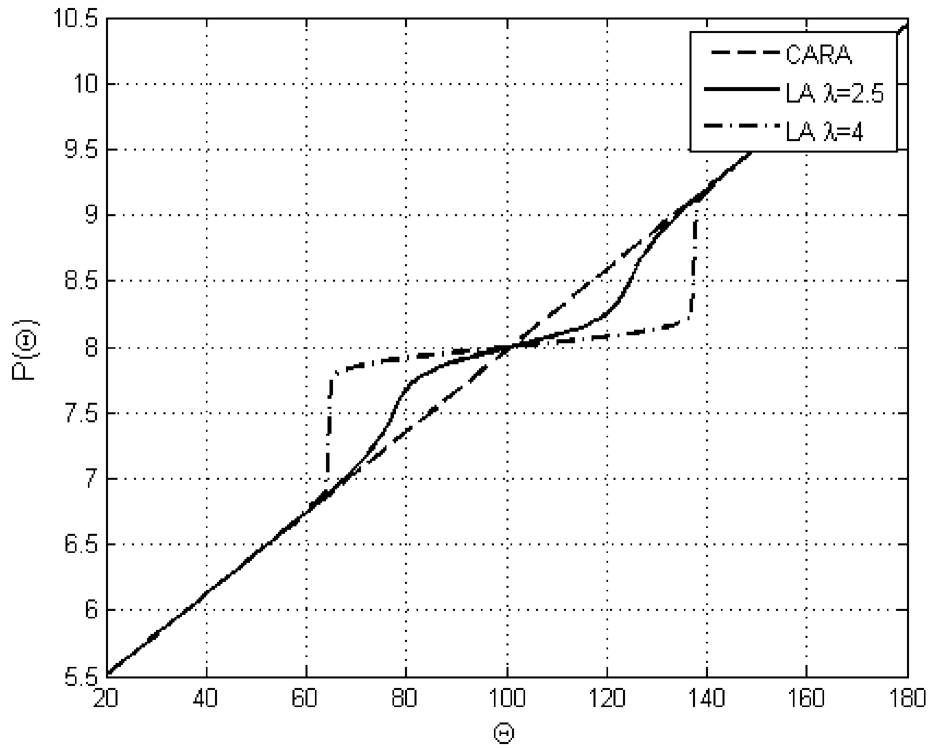

Figure 2.4. Equilibrium price. The dash line, and the solid line and the dash-dotted line, represent equilibrium price for risk averse speculators and loss averse speculators with coefficient of loss aversion of 2.5 and 4 respectively.

The insiders' information advantage still holds but it could diminish or increase depending on risk aversion, the quality of the private signal, and the noise. In that sense the camouflage which conceals informed agents' trading from the market maker varies with the parameters of the economy.

In our model, another dimension is added to the efficiency effect or equivalently to the information process by which the private information is revealed to the market maker. The degree of uncertainty regarding the informed investors' trading region indeed plays a crucial role in the inference process of the market maker. Intuitively, when the magnitude of the aggregate order flow is very low, the market maker conjectures with a relatively high probability (depending primarily on the degree of loss aversion) that the informed traders did not submit any limit order, inferring that the information advantage held by insiders is not exploited due to their loss aversion. Conversely, when the magnitude of the order flow is very high, the market maker conjectures with high probability that the insiders exploit (equation (2.5)) their information advantage. Since in the case of informative aggregate order flow, the trading intensity (ζ) outside the no trade region is similar to the CARA model, the information content of the price should be close to that given by the CARA model and thus the price should be very close as well.

To illustrate our main intuition, and to clarify the effect of loss aversion on information sharing and on the equilibrium price formation process, we will perform some numerical analyses of an economy with typical market-specific calibration where the parameters are chosen to equate the expected return on the risky asset to 6% and the standard deviation to 20%. We follow Hirshleifer, Subrahmanyam, and Titman (1994) and we set $\gamma = 2.5$, $\sigma_\varepsilon^2 = 8, \sigma_v^2 = 1$.[8] Figure 2.4 illustrates an example of equilibrium price $P_{LA}^*(\Theta)$ where $\Theta = \frac{s}{\gamma\sigma_\varepsilon^2} + z$, as a function of the noisy demand and the intensity of the private signal scaled by the risk tolerance and its precision, which refer to the statistically relevant part of the

[8] The value of the risk aversion coefficient is consistent with historical estimates of the market risk premium. Similar implications are obtained using other market specification calibrations proposed in the literature (Gennotte and Leland (1990), Leland (1992), and Yuan (2005)).

informative aggregate order flow observed by the market maker. It is important to emphasize that the equilibrium price is a non linear function of the noisy demand and of the private signal intensity, while for the CARA model, $P^{*}_{CARA}(\Theta)$ is linear in Θ. This non linearity arises because of the uncertainty regarding informed investor trading status. In the two extreme regions (when $|\Theta|$ is high) there is very little uncertainty regarding informed trader status and thus the price is melding with the linear price function of the CARA-Model. However, in the region around the expected value of the payoff *v,* the equilibrium price exhibits the smallest variation with Θ. When Θ is around zero, the market maker assigns the highest probability to the event that informed investors do not trade status. In the intermediate region a small movement in Θ can create large asset price movements.

The unique interaction between loss aversion of the speculators and adverse selection involving the informed traders and the market maker can generate steep price movements, up or down. Figure 2.5 graphs the sensitivity of equilibrium price to signal and noise trading shocks for the simplest economy with asymmetric information and a risk averse informed agent (CARA-Normal) and for an economy populated by loss averse informed traders with private information. The equilibrium price becomes sensitive to shocks in the intermediate price region when it is more difficult for the market maker to infer the quality of the private signal and to conjecture about the trading status of informed traders. The magnitude of such sensitivity decreases with the degree of precision of the private signal and increases with the level of speculators' loss aversion. Large market downturns or upturns in this model may occur regardless of the value of the underlying asset. Our model is consistent with empirical findings, reported by Culter, Poterba and Summers (1989), that important prices movement can occur without any particular news event.

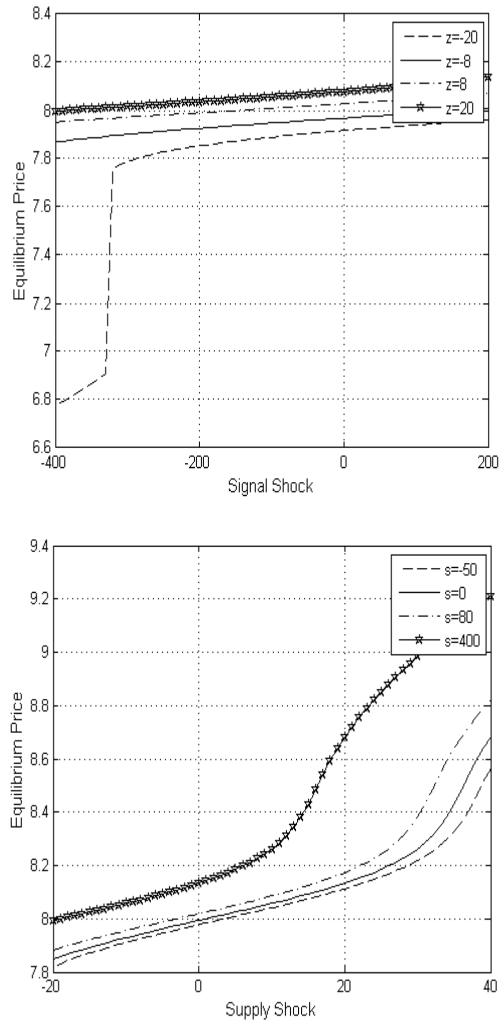

Figure 2.5. Price sensitivity to signal and supply shocks. The lines in the left graph represent equilibrium price as a function of the signal shock when the supply shock is -20,-8,20 and ,8, respectively. The lines in the right graph represent equilibrium price as a function of supply shock when the signal shock is -50,0,80, and 400.

2.5 Market Liquidity

All rational expectation equilibrium models have the particular property where equilibrium price has a dual effect: a substitution effect and an information effect. The market maker attempts to offset losses of the noise traders due to adverse selection of the speculators. As in Kyle (1985), we denote the market liquidity measure λ_{LA}^{-1} as the inverse of the *price impact* $\lambda_{LA} = \frac{\partial P}{\partial z}$ where the underscript LA refers to loss aversion preferences. With the implicit function theorem, the equilibrium market liquidity is the inverse of

$$\lambda_{LA} = \frac{A\left[1-\Phi(H)+\Phi(L)\right]}{1+\frac{A}{\sigma_v\rho}\left(\frac{s}{\gamma\sigma_\varepsilon^2}+z\right)\left(\psi(H)-\psi(L)\right)+\left(1-\sqrt{B}\right)\left(L\psi(L)-H\psi(H)\right)} \tag{2.12}$$

where $A = \frac{\gamma\sigma_v^2\sigma_\varepsilon^2}{\sigma_v^2+\sigma_\varepsilon^2\left(1+\gamma^2\sigma_\varepsilon^2\sigma_z^2\right)}$, $\quad B = \frac{\gamma^2\sigma_z^2\sigma_\varepsilon^4}{\sigma_v^2+\sigma_\varepsilon^2\left(1+\gamma^2\sigma_\varepsilon^2\sigma_z^2\right)}$, $\quad H = \frac{\frac{P}{\rho^2}+\Delta}{\sqrt{\sigma_v^2+\sigma_\varepsilon^2}}$, and

$$L = \frac{\frac{P}{\rho^2}-\Delta}{\sqrt{\sigma_v^2+\sigma_\varepsilon^2}}.$$

For $\lambda = 1, \Delta = 0$ as in Pasquariello (2014), the price impact is reduced to the equilibrium price impact of a risk averse speculator with constant absolute risk aversion

$$\lambda_{CARA} = A = \frac{\gamma\sigma_v^2\sigma_\varepsilon^2}{\sigma_v^2+\sigma_\varepsilon^2\left(1+\gamma^2\sigma_\varepsilon^2\sigma_z^2\right)}. \tag{2.14}$$

As in Kyle (1985) and Vives (1995), the equilibrium price impact for CARA speculators is positive $A > 0$ and increases in σ_v^2, highlighting the market maker's willingness to offset losses due to the speculator's adverse selection with profits to noise trading. Thus, the more uncertain the payoff, the more valuable the private information is, and hence the less liquid in equilibrium the market becomes.

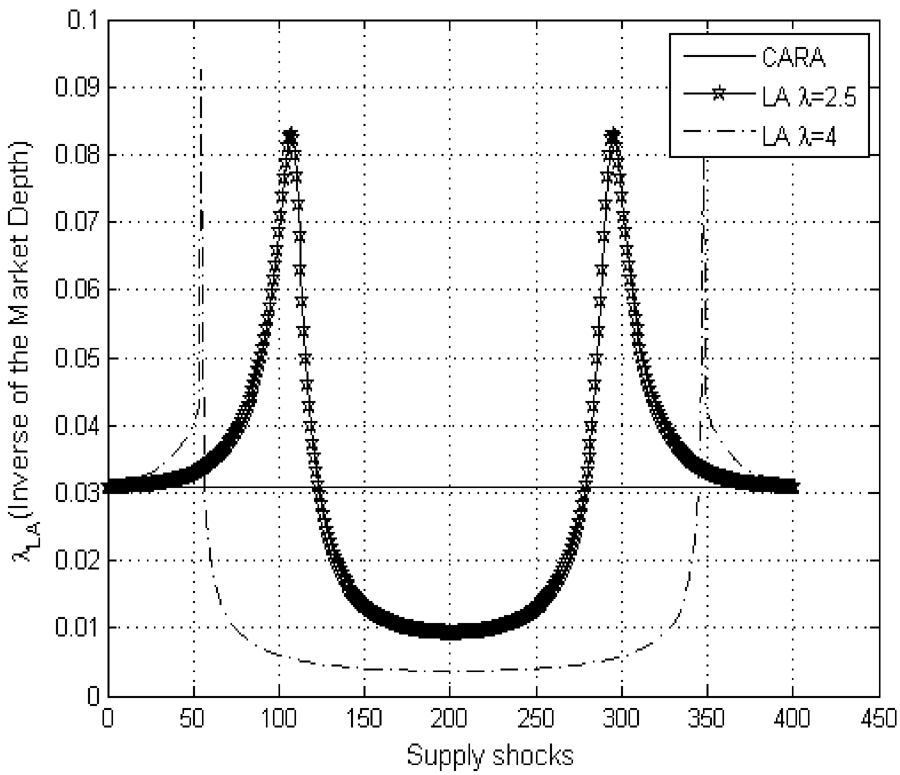

Figure 2.6. Price impact. The graph above represents the price impact (Inverse measure of liquidity) in function of noise traders' shocks where the private signal shock is 0. For risk averse speculators and loss averse speculators with loss aversion coefficients of 2.5 and 4 respectively.

However, consistent with Vives (1995) the depth of the market λ_{CARA}^{-1} is increasing in noise trading σ_z^2 and nonmonotonic in risk aversion (γ) and the precision of the signal $\left(1/\sigma_\varepsilon^2\right)$. In equilibrium, it is easy to show that if $\gamma < \frac{\sqrt{\sigma_\varepsilon^2 + \sigma_v^2}}{\sigma_\varepsilon^2 \sigma_z}$, the depth of the market increases in risk tolerance and if $\sigma_\varepsilon^2 < \frac{\sigma_v}{\gamma \sigma_z}$ the depth increases with the precision of the private signal. The reason is that although these changes tend to increase the adverse selection of speculators, it likewise increases the trading intensity and the information revealed to the market maker.

As stated above, the equilibrium price in our model is a non-linear function of both signal intensity and the noise trader's demand, z. Thus, the price impact is not a constant. However, in the extreme region of the equilibrium price, the price impact λ_{LA} of the implicit function is equal to the price impact in the presence of CARA speculators.

$$\lim_{P\to\infty}\lambda_{LA} = A = \lambda_{CARA}. \tag{2.14}$$

For a sufficiently large P value, the relation between equilibrium market liquidity and all the parameters of the model (except loss aversion) is indeed the same as for the case of risk averse informed traders.

For the intermediate price region, the market depth is highly nonlinear in noise trading demand. As for the CARA normal case, the price impact is nonnegative since the market maker attempts to offset losses due to the presumably adverse selection of the speculator with profits from noise trading. Figure 2.6 illustrates a numerical example of price impact for a given signal shock, with the specific calibration of the technology parameters discussed above, and for different degrees of loss aversion. We can separate the price impact into three distinct states corresponding to three different levels of inferred likelihoods of informed trading status by the market maker. When the price impact is close to zero, the market maker conjectures with high probability that the insider did not trade and he or she does not need to cope with the adverse selection problem. However, when the price impact is constant, the problem is reduced to the mean variance case since the trading intensity (ζ) for the trading region is equivalent to the trading intensity of the mean variance speculator. Finally, in between, market depth emphasizes the difficulty for the market maker to infer the trading status and thus a small supply shock can have a huge effect, at first not justified, on the equilibrium price while the market maker misinterprets the trading status of the informed trader. Indeed, the market maker cannot distinguish between a shock in the private signal and a shock in the noisy demand. Our model supports recent empirical evidence suggesting that the relationship between orders and price adjustment may be nonlinear. Large price fluctuations occur when the market depth is high in line with the presented comparative static analysis. It is consistent with the empirical study conducted by Pastor and Stambaugh (2003) where the authors use a related measure of price sensitivities as measures of market liquidity. They find several episodes of extremely low aggregate liquidity, including the October-1987 crash and the LTCM crisis of September 1998.

2.6 Asymmetric Price Movements

We demonstrate in earlier sections how asymmetric information, with the presence of loss aversion among informed traders, creates large price movements in financial markets. In addition, we show that big price changes may occur without being accompanied by any particularly dramatic news events. This result is consistent with the work of Roll (1984, 1988) and French and Roll (1986), where they demonstrate in various ways that it is difficult to explain asset price movements with tangible public information. Yet, the previous section fails to explain the other main evidence surrounding the literature on market crashes; namely, the fact that big price movements are more likely to be decreases than increases. Few theoretical models address this issue of asymmetric price movements (Yuan (2005), Yuan (2006), and Hong and Stein (2003)). Interestingly, Hong and Stein (2003) and Yuan (2006) introduce short-sale constraints in their model in conjunction with other market imperfections[9] to explain the pervasive tendency of markets to melt down rather than to melt up. Short-sale constraints in Yuan's (2006) theoretical model accentuate the asymmetry of large price movements. Following the same line of thought, we introduce short-sale constraints in our model. The unique interaction between asymmetric information, loss aversion and short-sale constraints supports the empirical evidence for large price movements tending to be downward rather than upward.

2.6.1 Equilibrium with Short-sale Constraints and Loss Averse Speculators

In this section, we consider a market that is identical to that of section II. Furthermore, we assume that informed traders might be subject to short-sale constraints. In particular, short-sale constraints mean that investor *i*'s position is bounded below by a non-positive number: $\theta_i \geq -b_i$ where $b_i \geq 0$. We assume that

[9] Hong an Stein (2003) developed a model based on short sales constraints and differences of opinion and Yuan (2006) proposed a generalized non-linear REE to examine the interaction between short-sale constraints, borrowing constraints and information asymmetry. Both models predict large asymmetric price movements.

$0 \leq \kappa < 1$ proportion of informed traders are subject to short-sale constraints[10], and index them by $i \in [0, \kappa)$, while the remaining, with mass $(1-\kappa)$ are unconstrained. For convenience, the short-sale constraint is assumed to be the same for all constrained speculators, so we drop the subscript i associated with constraint b. Based on the development for the optimal demand of unconstrained speculators, one can extend and find that the optimal demand schedules for constrained informed traders are $\max\left\{x_{LA}^{uc}, b\right\}$, where x_{LA}^{uc} refers to the optimal demand of equation (2.5). Following our previous development, this result is straightforward and is provided by the following.

Result A3: *The optimal demand for the short-sale constrained informed trader is*

$$x_{LA}^{*c} = \begin{cases} \dfrac{\rho^2 s - P}{\gamma\sigma_v^2\left(1-\rho^2\right)} - \dfrac{\Lambda(\lambda)}{\gamma\sigma_v\sqrt{1-\rho^2}} & \text{for } s > \dfrac{P + \Lambda(\lambda)\sigma_v\sqrt{1-\rho^2}}{\rho^2} \\ 0 & \text{elsewhere} \\ \dfrac{\rho^2 s - P}{\gamma\sigma_v^2\left(1-\rho^2\right)} + \dfrac{\Lambda(\lambda)}{\gamma\sigma_v\sqrt{1-\rho^2}} & \text{for } \dfrac{P - \Lambda(\lambda)\sigma_v\sqrt{1-\rho^2} - b\gamma\sigma_v^2(1-\rho^2)}{\rho^2} < s \leq \dfrac{P - \Lambda(\lambda)\sigma_v\sqrt{1-\rho^2}}{\rho^2} \end{cases} \tag{2.15}$$

where $\Lambda = \Lambda(\lambda)$ solves $\Lambda\left(1+(\lambda-1)\Phi(-\Lambda)\right) - \dfrac{(\lambda-1)}{\sqrt{2\pi}} e^{-\frac{1}{2}\Lambda^2} = 0$. Notice that for the present study, we keep b as any non-negative arbitrary number and we do not set it as zero. We maintain the general case where $b \neq 0$ since the interaction between loss aversion and short-sale constraint is sensitive to the choice of parameter b. For $b = \infty$, equation (2.15) is reduced to equation (2.5).

[10] Short-sale constraints are due to various restrictions on the market such as the proportion of institutional trading in the market, cost of lenders and regulatory restrictions.

The equilibrium price resulted from the presence of both unconstrained and constrained informed traders. We denote the aggregate order flow by $\omega = \kappa x^{c} + (1-\kappa)x^{uc} + z$, which refers to the noisy limit-order book schedule observed by the market maker, where x^{uc} is provided by equation (2.5) and x^{c} refers to equation (2.15). The market maker earns zero expected profit conditional on the order flow. The market clearing price P set by the market maker satisfies the semi-strong market efficiency rule expressed by equation (2.8). Aggregating the order flows coming from both unconstrained and constrained informed traders, the decision rule becomes

$$\begin{aligned} P = {} & E\left[v \middle| \omega, s \geq \frac{P}{\rho^2} + \Delta \right] \Pr\left[s \geq \frac{P}{\rho^2} + \Delta \right] + E\left[v \middle| \omega, s \leq \frac{P}{\rho^2} - \Gamma \right] \Pr\left[s \leq \frac{P}{\rho^2} - \Gamma \middle| \omega \right] \\ & + E\left[v \middle| \omega, \frac{P}{\rho^2} - \Gamma \leq s \leq \frac{P}{\rho^2} - \Delta \right] \Pr\left[\frac{P}{\rho^2} - \Gamma \leq s \leq \frac{P}{\rho^2} - \Delta \middle| \omega \right] \\ & + E\left[v \middle| \omega, \left| s - \Delta \right| \leq \frac{P}{\rho^2} \right] \Pr\left[\left| s - \Delta \right| \leq \frac{P}{\rho^2} \middle| \omega \right], \end{aligned} \tag{2.16}$$

where $\Gamma\left(\lambda, \sigma_v, \sigma_\varepsilon, b\right) = \dfrac{\Lambda\left(\lambda\right)\sigma_v\sqrt{\left(1-\rho^2\right)} + b\gamma\sigma_v^2(1-\rho^2)}{\rho^2}$. For $b = \infty$, $\Gamma = \infty$ and for $b = 0$, $\Gamma = \Delta$. For $b = \infty$, the decision rule is reduced to equation (2.9). Similarly to the previous results with unconstrained speculators only, $\Pr\left[\left| s - \Delta \right| \leq \dfrac{P}{\rho^2} \right]$ is the probability that the order flow is uninformative about the risky payoff v. and $\Pr\left[s \leq \dfrac{P}{\rho^2} - \Delta \right]$, $\Pr\left[s \geq \dfrac{P}{\rho^2} + \Delta \right]$ are the probabilities of the order flow being informative. However, the degree of informativeness differs within the different states. When $s \geq \frac{P}{\rho^2} + \Delta$, the order flow is fully informative since the short-sale constraints never bind. For $s \leq \frac{P}{\rho^2} + \Delta$, the degree of informativeness depends on the proportions of constrained informed traders. The analytical tractability remains because both unconstrained and constrained optimal demand schedules of equations (2.5) and (2.15) make ω a linear function of the private signal s and because the boundaries are not functions of the received private signal s (i.e. Δ and Γ do not depend on s). Since the inference problem uses the same mathematical properties and follows very similar steps to those described in appendix A, we skipped intermediary steps. The equilibrium price is quite cumbersome and lengthy; it is thus reported in appendix C.

Figure 2.7 graphs the sensitivity of equilibrium price to signal and noise trading shocks. The multiplier effect of short-sale constraints, information asymmetry, and loss aversion can be indeed seen in a graph in Figure 2.7. In our model, the unique interaction between loss aversion of the speculator, adverse selection between the informed traders and the market maker, and the presence of short-sale constraints among a fraction of speculators can produce market crisis[11]. The asymmetry between market meltdown and upward market price movement is highlighted. Large price drops are more severe than upward movements. Equilibrium price becomes sensitive to shocks in the intermediate price region when it is more difficult for the market maker to infer the quality of the private

[11] As in Yuan (2005), we define market crisis as a large price drop in response to a small shock to the economic environment.

signal and to make conjectures about the trading status of informed traders. The magnitude of such sensitivity decreases with the private signal's degree of the precision and increases with the level of the speculators' loss aversion.

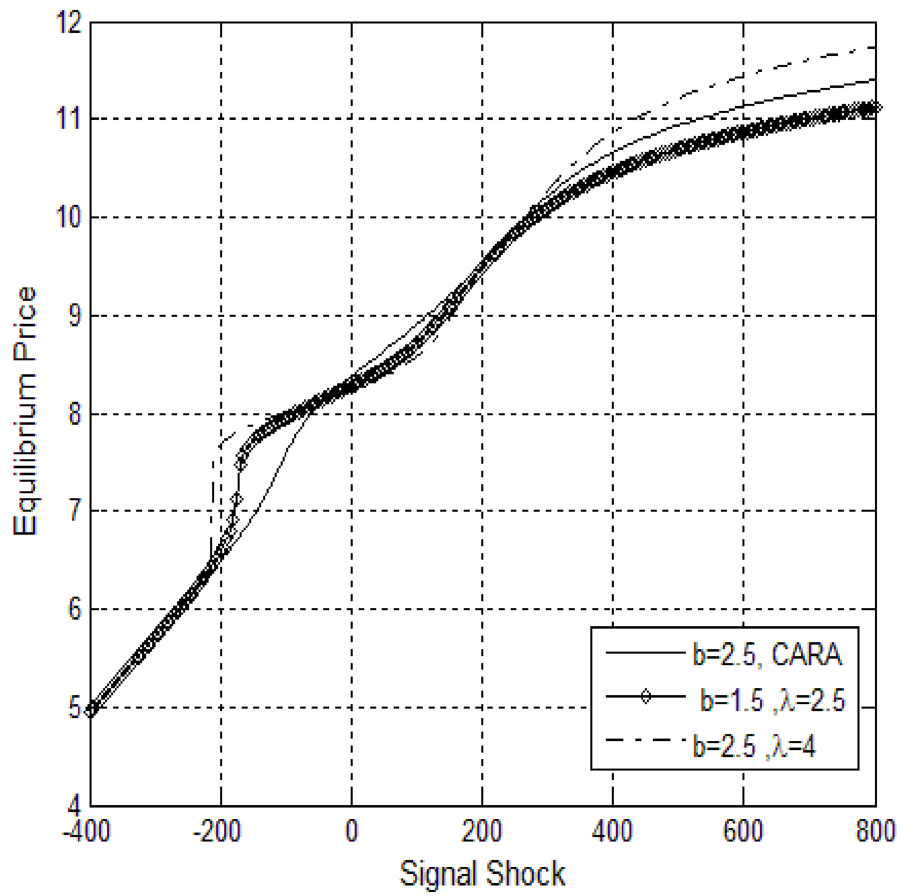

Figure 2.7. Equilibrium price in function of signal shocks. This graph represents equilibrium price as a function of the signal shock when the noise traders' intensity is 0.

Notice that asymmetry increases with the number of constrained informed traders and the level of constraint *b*. Short-sales constraints interact with loss aversion at a very fundamental level. One speculator may not trade because of either loss aversion or of short-sale constraint or of these two factors simultaneously. The more speculators are constrained, the more difficult it is for the market maker to infer a positive signal from the insider.

2.7 Model of Asymmetric Information: Concluding Remarks

This model developed in this chapter adds to the sparse asset pricing literature on information asymmetry with loss averse preferences. This study provides for the first time an analytical and tractable equilibrium solution in an economy in which loss averse speculators hold private information. Loss aversion, while affecting speculators' willingness to trade, also adds a further dimension to the market maker's inference problem about the extent of the private information from the informed trader's optimal demand schedule. When the aggregate order flow is low, the market maker cannot precisely infer the speculators' trading status, so the equilibrium price as well as the market depth becomes highly non-linear. In that situation, a small adverse shock to the fundamentals can trigger a large drop in asset value. When introducing short-sale constraints into the present model, the model provides asymmetry between upward and downward price movements. The model's predictions match historical evidence of market crashes.

3 Loss Aversion and Overconfidence: Their Joint Impact on Trading, Volume, and Skewness

This chapter analyzes equilibrium trading strategies in an economy with information asymmetry in which speculators display both loss aversion and overconfidence. Both effects have non-trivial and opposite impacts on equilibrium market quality. People are simultaneously loss averse and overconfident. This affects financial markets. This paper develops a model of disagreement over public signals between rational and irrational speculators, where irrational traders exhibit these two biases jointly. The model generates a positive correlation between volume and aggregate information, as well as a *high-volume return premium.* The model also reconciles the fact that we observe cross-sectional variation in skewness at the firm level (skewness is negatively correlated with trading volume, past return, and firm size) with the fact that on average, skewness in individual firm returns is positive.

3.1 Introduction

During Daniel Kahneman's lecture on his recent book *Thinking Fast and Slow* (Kahneman, (2011)) held at the UBS International Center of Economics in Society of the University of Zurich on April 16, 2013, a member of the audience asked a question about the seemingly contradictory nature of overconfidence and loss aversion. Professor Kahneman’s response was as follows:

> “We are loss averse independently of optimism and in general, there is a lot of evidence for both humans and other animals that we are more sensitive to loss than to gains. Therefore, loss aversion

> exists independently of risk and independently of uncertainty. We are optimistic even when we do not have control, so, those are just two facts about human nature and you can see where they might come from. Clearly, there is an obvious evolutionary advantage to loss aversion but there is probably a biological advantage to optimism too. They are contradictory in a way but they are both useful."

The purpose of this chapter was initially to translate Kahneman's remarks into a speculative trading model, and by doing so, to deepen our understanding of the joint and marginal impact of loss aversion and overconfidence on asset returns and higher moments. This initial objective has shifted slightly. During the development of the model, this initial objective morphed into a theory that aims to explain the *joint behavior of trading volume and asset prices*[12] and asymmetries in price changes. Interestingly, scholars in the behavioral finance field have put considerable effort into the development of theories that justify a variety of trading (market) anomalies, but they have remained relatively silent about: (1) patterns in prices and returns that are tightly linked to volume; (2) properties surrounding skewness in asset return. In particular, empirical evidence documents a positive (negative) correlation between volume and returns, the existence of a high-volume return premium, an average positive (negative) skewness in individual firm (market) returns, and a negative correlation between (i) trading volume and skewness; (ii) past return and skewness; (iii) firm size and skewness. We conjecture in this chapter that, to understand some of these phenomena from a

[12] Hong and Stein (2007) provide an excellent essay on disagreement models and their capacity unlike the majority of rational expectation models and traditional asset pricing theories to give a central role to volume.

behavioral finance perspective, we need to consider the fact that some investors exhibit both overconfidence and loss aversion.

Our intuition is motivated by the following observation made in Chapter 1 of the thesis: these two cognitive biases are systematically called upon to explain striking empirical evidence where traditional finance has a hard time to rationalize them. The boldest example that we can think of are the *active investing puzzle* and the *equity premium puzzle*. These two market anomalies are very often attributed to overconfidence and loss aversion respectively in behavioral finance literature.[13] One obvious question that might arise, though, is how two apparently contradicting biases succeed in explaining two key financial market puzzles. We do not aim to answer this question in the present study but instead we seek first, to isolate their respective effects, second, to understand their marginal contribution to speculative trading, and third, to shed light on the economic interplay on speculation of these two basic facts about human behavior.

The apparent contradiction between loss aversion and overconfidence is that on the one hand, overconfidence seems to encourage risk taking, while on the other hand, loss aversion seems to discourage it. Nevertheless, beyond the risk aspect, overconfidence refers to an individual's beliefs, while loss aversion relates to an individual's preferences. On the other hand, overconfidence is associated with the

[13] The active investing puzzle, a term coined by Daniel and Hirshleifer (2015) refers to the excessive trading of individual investors.It is widely accepted that it is excessively important to explain the total volume of trades in financial markets by traditional asset pricing models , and that the most prominent behavioral explanation of such excessive trading is overconfidence (Barberis and Thaler (2003)). The equity premium puzzle (Mehra and Prescott, (1985)) refers to the observation that given the return of stocks and bonds over past centuries, an abnormally high level of risk aversion would be necessary to explain why investors are willing to hold bonds at all. The myopic loss aversion concept introduced by Benartzi and Thaler (1995, 1999) arguably represents the most prominent behavioral explanation of the equity premium puzzle.

way people interpret and overweigh information. It does not bear asymmetry, unlike loss aversion, where the asymmetry relative to the *reference point* against which gains and losses are compared, represents its key feature.[14]

We extend the model of Harris and Raviv (1993) on speculative trading when agents agree to disagree and short-selling is not possible. We assume that there are two types of speculative traders. One group is risk neutral and perfectly assesses the released information, and the other group is loss averse and thinks that the signal is more informative than it really is. We model overconfidence as the belief of an agent that the information is more accurate than it is. As in Harris and Raviv (1993), we assume that traders start with common prior beliefs about the return on a particular asset. As information flows, the forecasts by agents of the two groups oscillate, since each trader updates his belief about returns using his *own* model of the relationship between the news and asset return. The two groups agree on whether a particular signal is favorable or unfavorable, but they disagree on the extent to which information is important. Speculators in the irrational group increase (decrease) their probability of high returns more upon receipt of favorable (unfavorable) information than those in the irrational group. However, unlike the Harris and Raviv (1993) model, trading does not occur necessarily when information switches from favorable to unfavorable. Loss aversion deters investors from trading unless the extent of their information is sufficiently large

[14] Relative to the reference point, losses hurt more individuals than comparable gains, and thus the slope of the utility function is steeper for losses than for gains. We will see through the model that the *reference point* is indeed manly responsible for the marginal effect of loss aversion on equilibrium price.

(above a certain positive threshold). This threshold is partly responsible for the asymmetric predictions of the models.

We derive a closed-form solution for the equilibrium price and for the expected trading volume conditional on aggregate information. We observe first, that at any time *t*, the maximum conditional volume occurs when aggregate information is positive. We then demonstrate that aggregate information is positively correlated with trading volume. This result is consistent with the well-accepted positive correlation between *contemporaneous* volume and return (Karpoff (1987); Stoll and Whaley (1987); Bessembinder and Seguin (1993); Bessembinder, Chan, and Seguin (1996); Chordia, Roll, and Subrahmanyam (2000); Lo and Wang (2000), and Harris and Raviv (2007)). It is also consistent, although more subtly, with the tendency for glamour high-priced stocks to have significantly higher turnover than low-priced value stocks reported, among others, by Harris and Raviv (2007), as well as the very strong lead-lag relationship between returns and turnover that are both market-wide and at the individual security level reported by Statman, Thorley, and Vorkink (2006). The second result indicates that the correlation between aggregate information and speculative volume increases with loss aversion. Based on assumptions we made on the nature of institutional traders and retail investors, this relation implies novel predictions about fixed-firm characteristics associated with the correlations between price levels, returns and trading volume.

We then show that the model generates a high-volume return premium (Gervais, Kaniel and Mingelgrin (2001)), the propensity for individual stocks with unusually large trading volume shocks to experience large subsequent returns. This study provides an alternative behavioral explanation to Merton's (1987) recognition hypothesis. Several testable predictions arise from the proposed theory, in line with empirical evidence reported in the work of Kaniel, Ozoguz and Starks (2012). Particularly, the theory supports the negative correlation between the levels of premium and institutional ownership.

Finally, we consider the implications of the model on skewness in returns. The model succeeds in reconciling several striking empirical facts that have been difficult to rationalize within a single theory framework. Our theory generates positive average skewness, a standard empirical feature of individual stock returns. Our theory also explains the cross-sectional variation in skewness at the firm level reported by Chen, Hong and Stein (2001), which is that skewness is negatively correlated with trading volume, past return, and firm size.

In this chapter, we contribute more generally to a growing literature that uses loss aversion and overconfidence to explain financial phenomena.

Researchers have found that loss aversion helps to explain much striking empirical evidence in finance. Benartzi and Thaler (1995, 1999) explain the equity-premium puzzle, while Barberis, Huang and Santos (2001) find that asset returns have high means, are excessively volatile and are significantly predictable in the time series. Gomes (2005) and Berkelaar et al. (2004) apply loss aversion to portfolio choice and find that loss averse investors abstain from holding stocks, unless they expect the equity premium to be quite high. Barberis and Huang (2001) explain the "value effect",[15] McQueen and Vorkink, (2004) rationalize the GARCH effect in stock returns, Dittmann, Maug, and Spalt (2010) succeed in explaining (under some assumptions) observed compensation practices, and Pasquariello (2014) explains patterns related to liquidity and price efficiency. More recently, Ouzan (2016) succeeds in explaining stylized facts about market crashes within an asymmetric information framework.

Several authors, such as Kyle and Wang (1997), Odean (1998b), Daniel, Hirshleifer, and Subrahmanyam (1998), and Caballé and Sákovics (2003) find that models of overconfidence predict a high trading volume. Besides the excessive trading prediction, Odean (1998b), shows that there is a positive correlation between the presence of overconfident traders and the volatility of asset prices. In their model, Daniel, Hirshleifer, and Subrahmanyam (1998) show that overreaction and self-attribution bias can reconcile short-run positive autocorrelation with long-run negative autocorrelation. Indeed, they provide evidence on short-term momentum with long-term mean reversal. Daniel, Hirshleifer, and Subrahmanyam's (1998) theory provides an explanation for market underreaction.[16] Daniel and Titman (1999) also find that momentum is stronger for growth stocks. Lee and Swaminathan (2000) and Glaser and Weber (2003) demonstrate that momentum is stronger among high-turnover stocks. Scheinkman and Xiong (2003) and Hong Scheinkman, and Xiong (2006) study the joint behavior of volume and overpricing. Finally, Burnside, Han, Hirshleifer,

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[15] *Value effect* or *value premium* refers to the phenomenon where stocks with low price to fundamentals ratios have a higher average return.

[16] Market underreaction refers here to the pattern where stock price reactions after public events are of the same sign as post-event long-run abnormal returns.

and Wang (2011) base their studies on investor overconfidence to explain why high-interest-rate currencies tend to appreciate relative to low-interest-rate currencies.[17] Recently, Daniel and Hirshleifer (2015) provided an excellent survey on models of overconfidence that can plausibly explain patterns that are puzzling from the perspective of fully rational models.

3.2 Model of Disagreement

The model described in this paper is based on differences of opinions. There are two groups of traders, namely group A and B. Group A consists of fully rational, risk-neutral speculators. We may consider group A as sophisticated and experienced traders, typically institutional investors. Group *B,* however, which may be identified as non-professional, retail investors, is loss averse and exhibits overconfidence toward the precision of the information released. At dates $t = 1,...,T,$ both groups of speculators trade shares of an asset that makes a single random payment $\boldsymbol{R}$ at date $T.$ There is one risk-free asset and one risky asset. We neglect discounting; the risk-free asset is thus a claim to one unit of terminal-period wealth.

We follow the model of differences of opinion of Harris and Raviv (1993) and we let the final payoff $\boldsymbol{R}$ be either high (H) or low (L) with $L \leq H$. We assume that the probabilities of high and low payoffs are equal.[18] At each date $t = 1,...,T$, new public information is revealed and all investors receive the same public signal.

[17] This market anomaly is known as the *forward premium puzzle*.
[18] For tractability, we assume that traders realize that the true prior probabilities are equal.

Following reception of public information, speculators update their beliefs and may trade at a price P_t. We may think of public signals as macroeconomic news, dividend announcements, quarterly earnings, merger announcements, acquisition announcements, political events or any release of information that influences the future prospect of the stock price. The signals are independent and identically distributed conditional on the true payoff. The rational group (group A) knows the true distribution of the public signal given the final payoff $\boldsymbol{R} \in \{H, L\}$ denoted by $\delta_{ra}(s|\boldsymbol{R})$. It is given by

$$\delta_{ra}(s|H) = \delta_{ra}(-s|L) = \begin{cases} k_{ra}a_{ra}{}^{s} & \text{for} \quad s \geq 0 \\ k_{ra}b_{ra}{}^{-s} & \text{for} \quad s < 0 \end{cases} \tag{3.1}$$

This distribution is chosen in order to allow the true posterior at date t to depend on the history of the signal only through the cumulative signal. The parameters a_{ra} and b_{ra} are strictly between 0 and 1, and k_{ra} is a constant required to make $\delta_{ra}(s|H)$ and $\delta_{ra}(s|L)$ density functions. Using Bayes' rule, after observing a history of signals $s^t = (s_1, ..., s_t)$, the true posterior probability that $\boldsymbol{R} = H$ is

$$\pi_{ra}(s^t|H) = \frac{\prod_{\tau=1}^{t} \delta_{ra}(s_t|H)}{\sum_{\kappa=\{H,L\}} \prod_{\tau=1}^{t} \delta_{ra}(s_t|\kappa)} = \left[1 + \theta_{ra}^{m}\right]^{-1} \equiv \pi_{ra}^{H}(m) \tag{3.2}$$

where $m = s_1 + ... + s_t$ is the cumulative signal, and $\theta_{ra} = b_{ra} / a_{ra}$. The independent assumption of the signals and the specific form of the likelihood function imply that signals are *additive* and the posterior depends on the signal history only through cumulative signal m.[19] Therefore, since the posterior depends on the signal history only through the cumulative signal, we substitute m for s_t and drop the subscript in the posterior. The probability that $\boldsymbol{R} = L$, given m, is $\pi^H(m) = 1 - \pi^L(m)$.

Since we assign larger values of the signal as more favorable information, we assume that the posterior probability of high outcome is increasing in m $(\theta_{ra} < 1)$. θ_{ra} can be interpreted as an inverse measure of the quality of the signal.[20] For example, if $\theta_{ra} = 0$, then any positive signal results in a posterior that assigns probability 1 to $\boldsymbol{R} = H$, and any negative signal results in a posterior that the probability 1 to $\boldsymbol{R} = L$. Conversely, for $\theta_{ra} = 1$, the posterior is independent of the signal.

After having defined the true distribution of the signal and the payoffs, we now consider the beliefs of the speculators regarding the distribution of the signal and the payoff. All speculators know the correct prior that $\boldsymbol{R}$ can be either H or L with equal probabilities. Differences of opinion are generated by assuming that speculators have different models of interpreting the signals. After observing the signal, each speculator revises his belief regarding the final payoff, using Bayes' rule and his own model (likelihood function) of the relation between signal and the payoff. The rational group, as already mentioned, has a true model and updates

[19] For instance, at $t = 2$, to infer the likelihood of the final payoff, the stream $s_1 = 1$, and $s_2 = 3$ of public information is equivalent to $s_1 = 6$, and $s_2 = -2$.

[20] We refer the true quality of the signal as a measure interpreted by group A since we assume that this group of traders is rational and therefore truly interprets the quality of the signal.

its belief regarding the true distribution. We assume that each speculator is absolutely convinced that his model is correct. Indeed, each group believes the other group is basing its decision on an incorrect model. All information, including all speculator models, is assumed to be common knowledge, so speculators do not attempt to infer the prices from the behavior of other speculators. This model, in line with other models of differences of opinion (Harris and Raviv (1993), Kandel and Pearson (1995)), suggests that even when all investors observe the same information, they may induce to trade with one another.[21] In order to generate important trades, the model combines heterogeneous priors with the assumption that the investors do not fully update their beliefs based on each other's trading decisions. Investors agree to disagree in equilibrium. One of the two aspects of overconfidence that best represents the stylized behavior of our speculators refers to the *better-than-average effect* described earlier. Although in finance literature, overconfidence tends to be more often modeled by *miscalibration* rather than the *better than average effect*, in a compelling work, Glaser and Weber (2007) test the two distinct features of overconfidence on trading volume and conclude that the *better-than-average effect/ difference of opinions* models better explain the enormous volume observed in financial markets.

The model of the irrational group (group *B*) shares the same functional form as the rational group (group *A*). Consequently, the resulting posteriors also exhibit the same functional form as the true posteriors. In particular, speculators of group

[21] Notice that, traditional rational expectation models, unlike difference of opinions models where trading volume takes its source in speculation, have a hard time rationalizing the enormous observed volume in financial markets with motives of hedging or portfolio rebalancing per se.

B realize that the signals are *i.i.d.* conditional on the final payoff, but they do not know the true density functions $\delta_{ra}(s/R)$. Instead, group B believes that the conditional density of a signal s given final payoff $\boldsymbol{R}$ is given by $\delta_{irr}(s/R)$, which represents the same functional form as the true density function but with different parameters.

Since both groups are to have models resulting in different posterior beliefs, they have different values of θ. In the present model, $\theta_{irr} < \theta_{ra}$, group B typically overestimates the precision of the signal and believes the signal is of higher quality (more informative) than group A and therefore, responds to a given signal history to a greater extent. Group B amplifies the sentiment concerning the public news, resulting in traders within this group being and more optimistic when the cumulative signal is positive and more pessimistic when m is negative. When $m = 0$, both groups revert to their prior beliefs. Namely, that the prior is high with probability 0.5. Figure 3.1 depicts the posterior beliefs of both groups. Equation (3.2) reflects that $\pi_H(m)$ is monotone increasing in m and is concave for $m > 0$ and convex for $m < 0$. These characteristics imply that larger cumulative signals indicate greater likelihood of high final payoff and that the posterior is more sensitive to changes in cumulative signals when beliefs are more diffuse (i.e. when the absolute value of m is very low).

Figure 3.1 depicts the probability of high outcome in function of the cumulative signal m. Larger cumulative signals indicate greater likelihood of high final payoff and the posterior is more sensitive to changes in cumulative signals when beliefs are more diffuse.

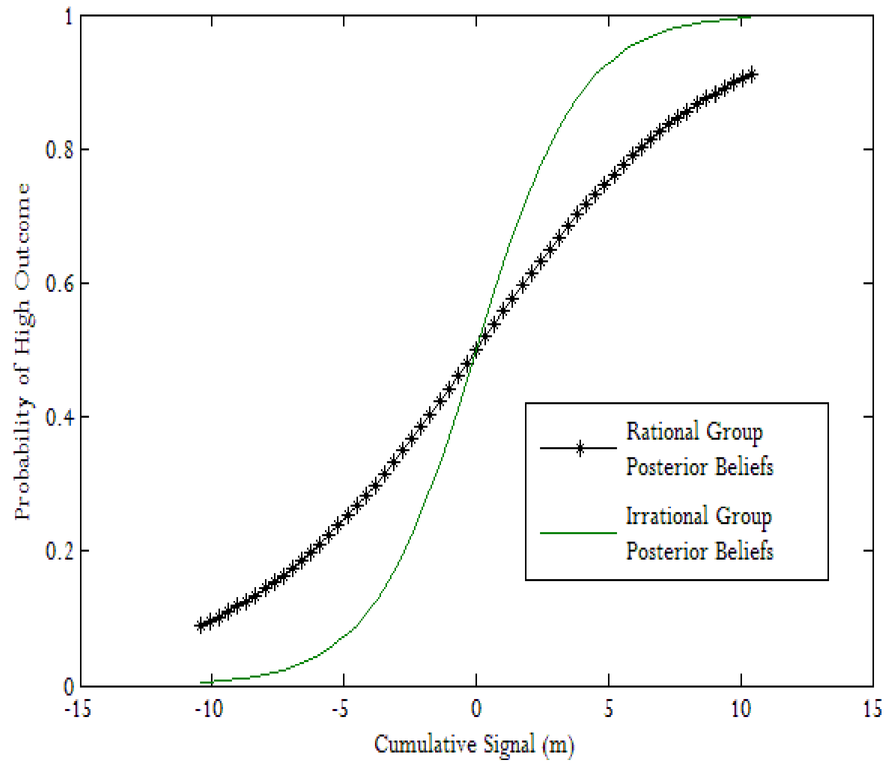

Figure 3.1. Probability of high outcome The probability of high outcome is displayed as a function of the cumulative signal *m* for both groups of speculators. The irrational group is more responsive to the signal than the rational group. For a positive (negative) signal cumulative, the irrational group values the asset more highly (little).

The two groups do not share the same preferences and utility function. The rational group is risk neutral and the irrational group is loss averse. The utility functions of both groups are depicted in Figure 3.2. The linearity of utility function is convenient for tractability reasons, and is appropriate since we are interested primarily in volume generated by speculation, as opposed to hedging for life cycle consideration. In this model, there is no trading except for speculative purposes. The only difference is that the utility function of the irrational group is a piecewise linear function and it is kinked at the origin. The irrational group's loss aversion is captured by the following utility function:

$$U(\pi,\lambda)=\begin{cases}\pi & \text{for} \quad \pi>0 \\ \lambda\pi & \text{for} \quad \pi<0\end{cases} \tag{3.3}$$

where π in our model represents the realised gain or loss and the coefficient λ stands for the degree of loss aversion. As we can see in Figure 3.2, the utility function of the irrational group has the same curvature than the rational group. Linearity of the utility functions implies that demand functions are infinitely elastic.

Therefore, any trader will seek to buy an infinite number of shares at any price below his reservation price. Following the Harris and Raviv (1993) model, to make the equilibrium well defined, we must assume that there are a fixed number of shares available. This implies that short-sales are not allowed.

Before we introduce the equilibrium, we ought to recall the concept of *mental accounting*, a term coined by Thaler (1980). It refers to the process in which people think about and evaluate their financial transactions. A question that arises in applying our analysis to the cross section at the firm level: Toward which gains and losses is the investor loss averse?

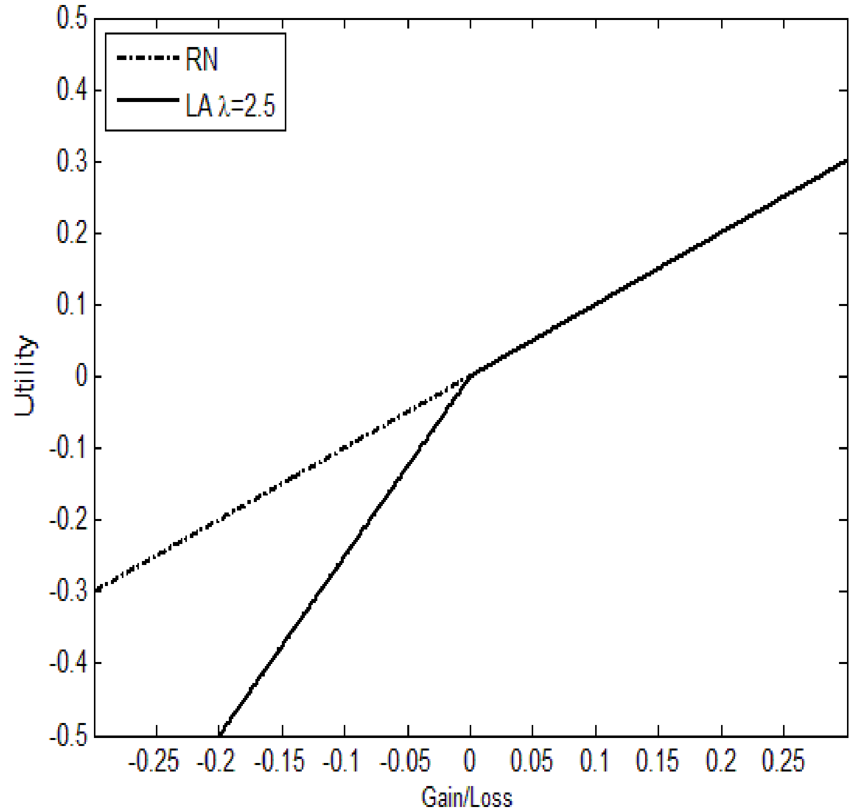

Figure 3.2. Utility of gains and losses. The dotted line represents the utility function of group A and the solid line refers to the utility function of group B with a coefficient of loss aversion of 2.5.

When investors are loss averse to changes in total wealth, we call it *broad framing*, in contrast with *narrow framing* when investors are loss averse to changes in the value of their portfolio of stocks or to changes in the value of individual stocks that they own. If we want to consider the proposed model as an equilibrium at the individual security level, we need to assume *narrow framing.* Numerous experimental studies suggest that when doing their mental accounting, people engage in narrow framing, that is, they often appear to pay attention to narrowly defined gains and losses.[22]

3.3 Equilibrium

To define the equilibrium price in this setting, we must make assumptions on how the market is organized**.** For tractability, we assume that in every period the rational group has sufficient market power to offer a price on a take-it-or-leave-it basis. This price will be equal to the price taking group's reservation price. Therefore, we restrict the price taker to be one particular group, namely the irrational group. This restriction is in line with our main assumption concerning the nature of both rational and irrational groups. Under the model assumptions, small and individual investors are viewed as irrational agents while large traders (e.g. institutional traders) are viewed as rational investors. If we follow this interpretation, market makers give preferential treatment to the large traders by filling their orders at the reservation price of the small investors.[23]

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[22] This may reflect a concern for non-consumption sources of utility, such as regret, which are often more naturally experienced over narrowly framed gains and losses. If one of an investor's many stocks performs poorly, the investor may experience a sense of regret over the specific decision to buy that stock. In other words, individual stock gains and losses can be carriers of utility in their own right, and the investor may take this into account when making decisions. Barberis and Huang (2001) study the equilibrium behavior of firm-level stock returns when investors are loss averse and exhibit narrow framing in their mental accounting. They find typical individual stock returns have a high mean and excess volatility, and there is a large "value premium."

[23] This assumption allows us to make the equilibrium tractable. Following the explanation of Harris and Raviv (1993), if we don't impose this market structure,given that the equilibrium is competitively set in each period, then the price in any period in which there is a trade will equal the reservation price of the buyer. This occurs because the buyer has an infinitely elastic demand but supply is bounded. Since the buyer's group changes from period to period, each group reservation price in any period will involve that group's expectation of the first group's expectation, and so forth which becomes quickly intractable.

The second assumption concerned the nature of preferences of the irrational group. In line with recent results of behavioral economics developed by Barberis and Xiong (2012), a number of authors have suggested that investors derive utility from realizing gains and losses on assets they own. In this work, we employ the realized utility concept and assume that the reference point from which traders derive utility is set at the end of the trading periods (at time *T*) when the payoff is realized and consumption occurs. This assumption makes the analysis much more tractable without sacrificing too much realism. Therefore, since the asset generates the payoff just at the last period and the traders know that they have the possibility to trade until the last period, the expected utility optimal demand at each period for the irrational group is

$$E\left(U(\pi,\lambda)\middle|m\right)=\begin{cases} q\left[(H-P_t)\pi_{irr}(H/m)+\lambda(L-P_t)(1-\pi_{irr}(H/m))\right] & \text{for } q\geq 0 \\ \\ q\left[\lambda(H-P_t)\pi_{irr}(H/m)+(L-P_t)(1-\pi_{irr}(H/m))\right] & \text{for } q<0 \end{cases} \tag{3.4}$$

where λ represents the coefficient of loss aversion, and q the demand for the risky asset. Maximizing the conditional expected utility gives an unbounded demand for a sufficiently large cumulative signal in absolute value

$$|m|>\frac{1}{\ln\left(1/\theta_{irr}\right)}\left[\ln(\lambda)+\frac{\ln(H-P_t)}{\ln(P_t-L)}\right], \tag{3.5}$$

and zero elsewhere.

To derive the equilibrium price, we must first define the rational group's current expectation of the payoff and it is equal to

$$E^{ra}\left[\boldsymbol{R}\middle|m\right]=H\pi_{ra}(H|m)+L\pi_{ra}(L|m)=(H-L)\pi_{ra}^{H}(m)+L. \qquad (3.6)$$

Let us assume that at time t the rational group holds the risky asset. In order to be willing and able to sell it, two conditions should be fulfilled. First, the cumulative signal should be positive. This condition arises from the differences of opinion of groups A and B. If at time $t-1$, the cumulative signal was negative and at time t, it becomes positive, the irrational group becomes optimistic while it was previously pessimistic about the information released. This arises because the groups "switch sides" in their posterior belief, and thus in their current expectation of the final payoff. The second condition implies that the cumulative signal is sufficiently large to ensure that the condition of (3.5) is met for positive value of m. This condition is driven by loss averse preferences of irrational traders.[24] Note that if $\lambda = 1$, only the first condition holds and we retrieve the condition of Harris and Raviv (1993). Since we assume that the rational group has sufficient market power to propose a price on a take-it-or-leave-it basis, the price-taking group always engages in trades that they believe have zero net present value. However, to make the equilibrium well defined, we have already assumed that there are a fixed number of shares available and short-sales are not allowed. Therefore, the reservation price of the price-taking group and the price of the risky asset at date t is the price P_t^* that solves[25]

[24] We have already assumed that the irrational group can be considered as individual non-sophisticated traders or retail investors.
[25] We solve equation (5) with equality for positive demand since short-sales are not allowed.

$$\frac{1}{\ln\left(1/\theta_{irr}\right)}\left[\ln(\lambda)+\frac{\ln(H-P_t)}{\ln(P_t-L)}\right]=m, \tag{3.7}$$

and it is equal to

$$P_t^*=\frac{H+\lambda\theta_{irr}^m L}{1+\lambda\theta_{irr}^m}. \tag{3.8}$$

The equilibrium price is smaller than the irrational group's current expectation of the final payoff unless $\lambda=1$. Loss aversion, indeed, induces the price-taking group to accept buying the risky asset at a lower price. Keeping all the other parameters fixed, the equilibrium price decreases with loss aversion. The rational group however will accept to sell the risky asset only if the cumulative signal *m* is above the threshold[26]

$$\psi=\ln(\lambda)/\ln(\theta_{ra}/\theta_{irr}). \tag{3.9}$$

As in the Harris and Raviv (1993) model, trades will occur only when the two groups "switch sides". The novelty here is that the threshold is not zero and it depends both on the precision of the public signal of both groups and on loss aversion unless $\lambda=1$. If the coefficient of loss aversion is equal to one, we retrieve the same result as in Harris and Raviv (1993). When the cumulative signal is below ψ, the risky asset is held by the rational group, while when it is above ψ the risky asset is entirely held by the irrational group of traders. The price changes

[26] The rational group would accept to sell the risky asset only if its current expectation of the payoff (equation (3.4)) is above the equilibrium price (equation (3.8)). Therefore, the threshold refers to the cumulative signal *m* that solves $(H+\lambda\theta_{irr}^m L)/(1+\lambda\theta_{irr}^m)=(H-L)\pi_H^{ra}(H)+L.$

according to the reservation price of the irrational group every time information appears. We assume that the equilibrium price is always given by equation (3.8) even in periods when there are no gains to trade.[27] Figure 3.3 illustrates the equilibrium price in function of the cumulative signal and the rational group's current expectation of final payoff. This plot emphasizes the nature of the equilibrium. Trades in this model are jointly impacted by loss aversion and differences of opinion. The rational group knows that to be able to sell the risky asset to the other group of traders, they need to set the price sufficiently low compared to the cumulative signal. This so-called "distance" between m and P_t^* highlights the very nature of loss aversion. The equilibrium price set by the market maker is always below the current expectation of group B. Therefore, on average, loss aversion featured in conjunction with short-sales constraints reduce overpricing and increase underpricing, whereas overconfidence increases mispricing with no asymmetry. The trading process and the characteristics of the equilibrium highlight how overconfidence and loss aversion are intertwined and reveal their joint impact on investors. As we can see from equation (3.8), overconfidence and loss aversion have opposite effects on the one hand; i.e., the former increases the equilibrium price and the latter decreases it.[28] On the other hand, these biases differ in the way they incite investors to react to information. Overconfidence in this model allows speculative trading and forces group B to

[27] In such periods when the cumulative signal does not cross threshold ψ, any price between P_t^* and the rational group's current expectation indeed results in no trade.

[28] This effect somewhatr reflects the contradicting aspect of these two major cognitive biases.

exploit what they consider to be mispricing,[29] whereas although loss aversion reduces their incentive to trade, it creates a no trade region in the optimal demand,[30] as it has already been emphasized in a different setting, in previous works by Pasquariello (2014) and by Ouzan (2016). In order to trade, the rational groups have to propose a sufficiently low price compared to the aggregate information.

Disagreement is not a sufficient condition for trading anymore. Traders need to disagree on the accuracy of the signal in order to trade, but they need also to consider the quality of the signal. If, for instance, the aggregate quality of the signal is judged insufficient (the so-called "distance" between aggregate information and the price is too small) the market maker cannot propose a price below its current expectation of the payoff. The market maker cannot induce an increase in the price-taker's perceived signal quality and therefore trades do not happen, although the traders' sentiment has just shifted.[31] This phenomenon is primarily responsible for the asymmetric properties on speculative volume that we further develop in the following sections.

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[29] Remember that both groups consider that the other group is falsely interpreting the signal. The more group B is overconfident, the more it views the security as being mispriced.

[30] The kink at the origin of the utility function, which characterises loss aversion as displayed in Figure 3.2 is primarily responsible for the so-called "no-trade" region that differs fundamentally and economically from simply being the mirror opposite of overconfidence.

[31] Last period rational traders were more optimistic than irrational traders while it is now the opposite.

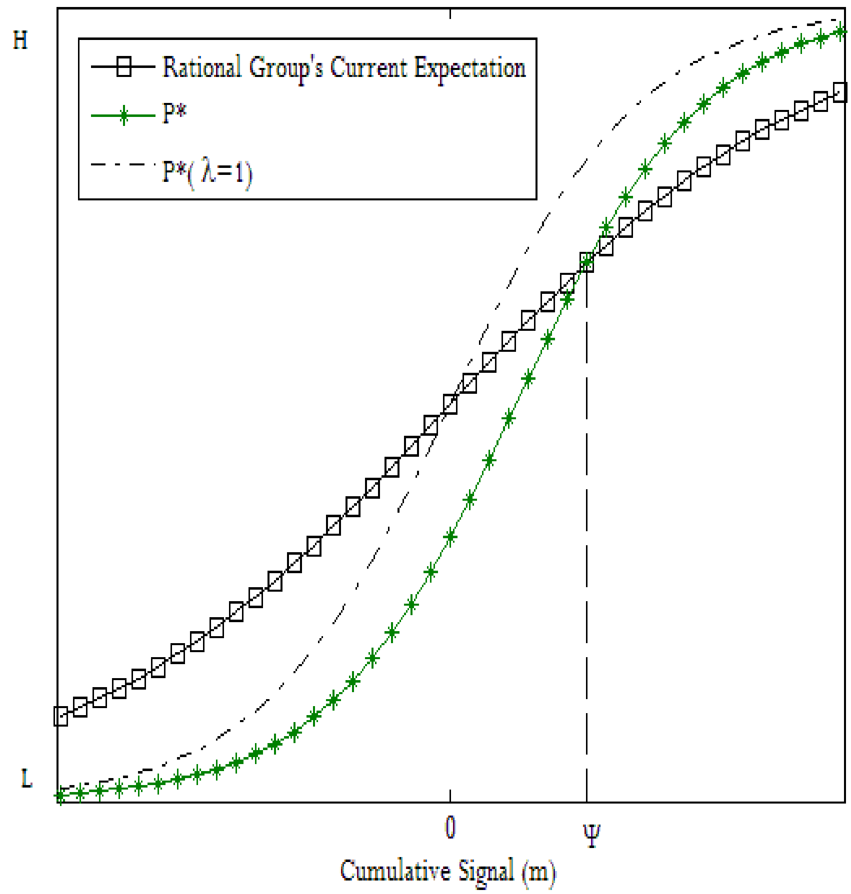

Figure 3.3. Equilibrium price. The equilibrium prices for $\lambda > 1$ and $\lambda = 1$ are displayed as a function of the cumulative signal *m*. The equilibrium price intersects the risky asset group's current expectation (reservation price) at $m = \psi$.

3.4 Speculative Volume

Having described a simple model based on heterogeneous beliefs, we now investigate the relation between volume and return. Recall that the equilibrium price in any period in our model is the reservation price of the price-taking group; namely group *B*, even if at that period no trades take place. Let $v(m,s)$ denote the volume at $t+1$ and $s_t = s$. $v(m,s)$ is equal to 0 if m and $m+s$ are both either greater or smaller than ψ, and 1 otherwise. The conditional expected volume at

$t+1$ that equals the probability of positive volume at $t+1$ given the cumulative signal $m_t = m$ at t, is[32]

$$V(m) = E^{ra}\left[v_{t+1} \middle| m_t = m\right] = \frac{\pi_{ra}^{H}(m)}{\ln(a_{ra}) + \ln(b_{ra})} \begin{cases} b_{ra}^{m}\left[b_{ra}^{-\psi}\ln(a_{ra}) + a_{ra}^{-\psi}\ln(b_{ra})\right] & m \geq \psi \\ a_{ra}^{-m}\left[b_{ra}^{\psi}\ln(a_{ra}) + a_{ra}^{\psi}\ln(b_{ra})\right] & m < \psi \end{cases}. \quad (3.10)$$

Figure 3.4 plots the expected next period volume given the current cumulative signal, for different levels of loss aversion. One can observe that the maximum expected volume occurs at $m = \psi$. Our first result shows that the speculative volume is larger on average in periods of positive aggregate information (see appendix D for formal evidence).

Result B1. *Aggregate information and speculative volume are positively correlated.*

Both practitioners and academics are aware of the tendency for higher volume to accompany higher price levels (both in time series and in the cross section). Result B1 is indeed in line with the well-accepted positive correlation between *contemporaneous* volume and return (Karpoff (1987); Stoll and Whaley (1987); Bessembinder and Seguin (1993); Bessembinder, Chan, and Seguin (1996); Chordia, Roll, and Subrahmanyam (2000); Lo and Wang (2000), and Harris and Raviv (2007)). In addition, Result B1 also supports the empirical evidence that share turnover is positively related to lagged returns (Statman, Thorley, and

[32] The derivation of the conditional volume is reported in appendix D.

Vorkink (2006)). Signals in the present model are additive. Therefore, the correlation is also valid when the positive information has been accumulated in the past, therefore implying positive past returns.

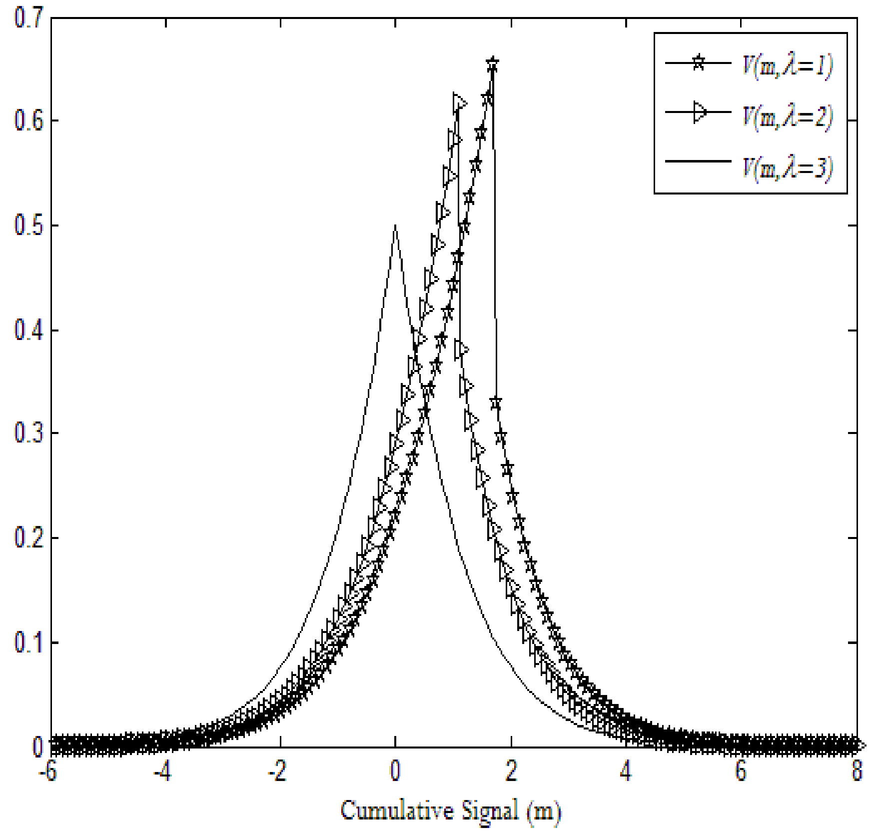

Figure 3.4. Expected volume. The expected next period volumes given the current cumulative signal are displayed for different levels of loss aversion.

This result also supports although more subtly, the work of Hong and Stein (2007). In their study, they extend the traditional link between "overtrading" and bubbles documented by Ofek and Richardson (2002) and Kindlenberg, and Aliber (2005). They show that the positive correlation between price levels relative to their fundamental values and trading volume exists not only in bubble-like situations but also in calmer times. Alternatively, to put it differently, they show that “glamour stocks tend to have higher volume than low- priced value stocks.” Result B1 demonstrates that this relation is indeed verified using aggregate information as a proxy for future growth rather than using the stock's price to fundamentals ratio.[33]

The plot of the conditional volume (Figure 3.4) exhibits a discontinuity around threshold ψ. This discontinuity of the expected conditional volume increases with loss aversion and it arises from the symmetric distribution of public information and the asymmetric nature of the equilibrium. It is straightforward to demonstrate the following result (proposition):

Result B2. *The correlation between speculative volume and aggregate information increases with loss aversion.*

Following the intuition of Barberis, Huang, and Santos (2001) on how investors frame their losses and gains,[34] Result B2 might predict that the correlation between price levels and trading volume is stronger in a bull market rather than in a bear market. After a big loss in the stock market, investors experience a sense of regret over the decision to invest in stocks and may be even more reluctant to realize further loss than usual. In Barberis, Huang and Santos’s (2001) model, to capture

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[33] Stocks deemed as glamour stocks are usually considered to have strong growth potential. We define “glamour stocks” in our setting as stocks receiving positive coverage (aggregate information). Positive coverage should represent a good proxy for high ratios of market value in terms of earnings, cash flows, or book value. The positive prospect of one share impounded into aggregate information has a direct impact on the market price while it may not be incorporated yet into book value, current cashflows, or earnings.

[34] The utility function in their asset pricing model comes from fluctuations in financial wealth.

the influence of prior outcomes, they introduce an historical benchmark level.[35] In the present work, since we haven't studied an intertemporal asset pricing model, but rather a disagreement model that captures the joint behavior of speculative volume and asset price, we simply make the assumption that irrational, less sophisticated traders are more loss averse than usual in a bear market. Thus, the first testable implication of our model refers to the average abnormal positive correlation between price level and trading volume during bear market.

In the same vein, a second testable implication arising from Result B2 is related to the number of institutional investors for a given stock. The proposed stylised model implies that all traders within a particular group exhibit the same preferences. Moreover, we assume the irrational group to be non-institutional traders. Therefore, it is fair to assume that the ratio of institutional traders for a given stock is negatively correlated to the degree of loss aversion within the model used to set a price for that particular stock. Therefore, all things being equal, our model predicts that the correlation between trading volume and price level is on average greater for stocks with a higher concentration of less sophisticated traders (retail investors). Moreover, several studies indeed confirm the conventional wisdom that institutional investment increases with firm size. According to O'Brien and Bushnan (1990), it is because firm size can be used to establish prudence on investment in legal cases. Falkenstein (1996) documents that U.S. mutual funds tilt their portfolios towards large firms, and Gompers and Metrick

[35] In their model, Barberis, Huang and Santos (2001) use historical benchmarks as fictitious secondary benchmarks to determine the magnitude of the utility received from a particular gain or loss.

(2001) find that American institutions invest in firms that are larger, more liquid, and have had relatively low returns in the previous year. Ferreira and Matos (2008) confirm this finding internationally, and find that all institutional investors, whatever their geographic origin, share a preference for the stock of large and widely held firms. The model suggests that a firm with less sophisticated traders would exhibit a greater correlation between trading volume and price level ceteris paribus. (This would also occur as a by-product of a smaller-size firm.) As Feng and Seasholes (2005) point out, trading experience and investor sophistication dampen reluctance to realize losses. Statman, Thorley, and Vorkink (2006) find evidence that supports our assumption both from an ownership structure and size standpoints. They demonstrate that the positive correlation between turnover and lagged returns is more pronounced in small-cap stocks and in earlier periods where individual investors hold a greater proportion of shares.

3.5 High-Volume Return Premium

Gervais, Kaniel and Mingelgrin (2001) report that individual stocks with unusually large trading volume over periods of a day or a week tend to experience large return over the subsequent month. This hypothesis is referred to as the *high-volume return premium.* The main explanation for this intriguing empirical evidence is provided by Merton’s investor recognition hypothesis (1987), also known as the visibility hypothesis. Gervais, Kaniel and Mingelgrin (2001) reinforce investors' incomplete information hypothesis. They show that the excess market-adjusted return that occurs after a stock receives substantial positive shocks is not merely a by-product of return autocorrelations, nor the result of the momentum effects that Jagadeesh and Titman (1993) document. Moreover, Kaniel, Ozoguz and Starks (2012) extend the initial work on the high-volume return premium across 41 different countries in both developed and emerging markets. The authors find that the high-volume return premium represents a pervasive phenomenon and it is present in almost all countries of their sample. Using Merton's (1987) recognition hypothesis as a guide, they find that the magnitude of the premium is associated with some country and firm characteristics. However, at the firm level, several measures of visibility do not corroborate the investor recognition hypothesis. Namely, one of their most puzzling finding is the positive correlation between analyst coverage and the high-volume return premium. Kaniel, Ozoguz and Starks' (2012) result suggests that firms that are more closely followed by analysts or S&P are more likely to experience high return following a strong volume shock. Interestingly, they show that the high-volume return premium increases with the presence of analyst coverage or S&P coverage but not in the level of analyst coverage.

The high-volume return premium can be seen in our context, before we demonstrate it formally as a gradual information flow version of Result B1.[36] To motivate our future development, the analyst coverage story may indeed to the contrary, support the proposed behavioral explanation of the premium. The S&P coverage or the existence of analyst coverage increases the likelihood of the

[36] We demonstrate that the maximum volume appears when there is positive aggregate information. Due to gradual information flow, it may take a certain time, however, for the positive signal to be incorporated into the stock price.

presence of non-specialists[37]. In the meanwhile, the number of analysts might mitigate this result. Bhushan (1989) reports that the aggregate demand for analyst services increases as more institutions hold shares in a firm or the percentage held by them increases[38].

We depart from the visibility hypothesis and demonstrate mathematically in the following equations that the high-volume return premium is required because of the presence of irrational traders that exhibit jointly overconfidence and loss aversion.

The unconditional volume v at time $t+1$ is equal to

$$\int_{-\infty}^{+\infty} E^{ra}\left[v_{t+1} \middle| m_t = m\right] g_t(m) \cdot dm, \tag{3.11}$$

where $g_t(m)$ refers to the unconditional density function of the cumulative signal m. Define the average return between $t+1$ and T, as $r = \dfrac{H + L - 2P_{t+1}^*}{2}$. Our third result shows the existence of the high-volume return premium (see the Appendix E for formal proof).

Result B3. $\dfrac{\partial r}{\partial v} > 0.$

Result B3 indicates that a positive volume shock at $t+1$ generates on average a positive future return. We demonstrate in Appendix E that the average

[37] We assume indeed that non-sophisticated and non-specialist investors are naturally a priori aware of a narrow pool of firms, so the presence of analysts should inherently increase their holding of covered stocks to a greater extent than specialists.
[38] It suggests that while analyst coverage attracts non-sophisticated traders, the level of coverage increases with institutional traders and therefore mitigates the former result, as reported in Kaniel, Ozoguz and Starks (2012).

unconditional volume increases with loss aversion, i.e. $\frac{\partial v}{\partial \psi} > 0$. The comparative static argument for the next result is quite straightforward. Let us consider two models with different degrees of loss aversion λ_1, and λ_2 with $\lambda_1 > \lambda_2$. Therefore, after a public announcement, for instance, the average volume shock will be greater for the model with λ_1, ceteris paribus. And therefore, from Result B3, we can state that the model with λ_1 will on average generate a greater premium than the model with λ_2. We state this result formally as Result B4.

Result B4. *The high-volume return premium increases with loss aversion.*

A high concentration of loss averse traders should have therefore, on average, a positive effect on the premium. A shock in loss aversion parameter due for instance, to economic downturn should also increase the average future return on the asset. Several testable implications arise from Result B4. Using the behavioral explanation as a guide, we may be able to investigate firm and country characteristics associated with the premium. The first obvious direct relation with our assumption is the negative correlation that should exist between institutional trading and the premium. Furthermore, in the time series as well we should find evidence that the high-volume return premium is more pronounced in times of market downturn.

Let us compare our theory with Merton's (1987). Indubitably, the information environment limits the investors who are aware of a firm's securities to a subset of the investing population. Therefore, the reduction in cost of capital after a volume shock associated with the visibility hypothesis can hardly be refuted. Nonetheless, some of the main implications of Merton's (1987) recognition hypothesis are in direct opposition with the behavioral motives presented in the paper. For instance, according to our model, a broader investor base increases the propensity of less sophisticated traders and therefore should increase the correlation between future return and trading volume, whereas following the visibility hypothesis it should decrease it.[39] We thus need to examine the visibility hypothesis alongside with the proposed behavioral explanation. Miller (1977) and Mayshar (1983) claim that the holders of a particular stock will on average tend to be more optimistic about its prospect. As Gervais Kaniel and Mingelgrin (2001) pointed out, it is especially true if taking a negative position is rendered difficult by institutional constraints on short-selling. Since our model imposes short-sales constraints on both types of investors, potential sellers are largely restricted to current stockholders, whereas the set of potential buyers, after any shock that attracts the attention of the investors, includes a large fraction of the market. Our model might be set nicely within the visibility hypothesis framework. Some market implications may corroborate the visibility hypothesis with the proposed behavioral explanation. For instance, at the country level, Kaniel, Ozoguz and Starks (2012) find that the premium is higher for countries with more listed companies per urban population. According to their interpretation of Merton's (1987) argument, when a country has more listed companies, the investor base for each of the companies individually would be expected to be smaller and therefore

[39] The larger the investor base, the more visible the particular stock already is.

a shock in volume should amplify the premium. Likewise, following the assumption of the number of irrational traders, the high-volume return premium should be high where there is a high concentration of listed companies per urban population. An important number of listed companies for a particular country generally implies a well-functioning financial market, where many small and heterogeneous participants are interacting[40] Kaniel, Ozoguz and Starks (2012), report also that firms with low institutional holdings have a great high volume return premium that do firms with large institutional holdings. This result supports strongly the proposed behavioral argument as well as the visibility hypothesis. Although the investor recognition hypothesis assumes that visibility is more important for individual investors, this finding represents the strongest support to our theory.

A crucial question regarding the high-volume return premium is whether there exist implementable economic trading strategies. The evidence that investors can take advantage of market anomalies in general and from the high-volume return premium in particular are mixed. Lesmond, Schill, and Zhou (2004) show that transaction costs prevent profitable strategy execution. They find that the stocks generating large momentum returns are precisely those associated with high trading costs. Transactions cost has also been found to have a detrimental effect h

[40]With respect to the work of Kaniel, Ozoguz and Starks (2012), although their results indicate that the magnitude of the high volume premium is not different in emerging countries than it is in developed countries that are not in the G-7, stocks in a G-7 country are more susceptible to the effect of extreme volume shock. This is consistent with both the Merton (1987) hypothesis and ourown. Since these markets have more individuals per capita who participate in the stock market, there may be a relatively greater proportion of retail, non-sophisticated investors, who would also be susceptible to investor visibility shock.

on possible trading profits for the high volume return premium. However, as Kaniel, Ozoguz, and Starks (2012) report that institutional investors could not profitably exploit the high volume return premium. This observation hints that it would be worthwhile to analyse economic trading strategy based on both behavioral and visibility hypotheses. After having controlled for transaction cost, stock held by an important fraction of retail investors represents a viable trading strategy, according to Kaniel, Ozoguz and Starks (2012).

3.6 Skewness

Aggregate stock market returns display negative skewness, the propensity to have market downward movement on the aggregate level with greater probability rather than upward market movement. A vast literature tends to explain this styled fact about the distribution of aggregate stock returns (e.g. Fama (1965), Christie (1982), Pyndick (1984), French, Schwert, and Stambaugh (1987), Bekaert and Wu (2000), Hong and Stein (2003), and Yuan (2005), Alburquerque (2012), Chang, Christoffersen, Jacobs (2013)). However, this evidence contrasts with the fact that firm-level returns are positively skewed. Alburquerque (2012) successfully reconciles these two counterintuitive phenomena. His model explains the positive skewness at the firm level and the generation of negative *coskewness* in the market portfolio[41].

[41] In order to generate a positive firm level skewness, Alburqueque (2012) shows that the unconditional distribution of equilibrium returns is a mixture of normal distribution. To generate a negative *coskewness*, he introduces heterogeneity in a firm's announcement events.

Our model has only one risky asset, and so depicts an incomplete picture of the market. However, since we do not introduce heterogeneity in news arrival nor multiple assets, neither correlation between assets, it is fair to say that our model aims to speak at the firm level rather than at the aggregate level. Whether it is the correlation between trading volume and high priced stocks, or the high-volume return premium, we focus in this study on the striking empirical evidence that arises from a set of particular stocks rather than from the market in general. We already conjectured that the degree of loss aversion depends on prior gains and losses and we already assumed *narrow framing*. We will see shortly that our model generates unconditional positive skewness. We are able also to speak directly about the joint behavior of return skewness and speculative volume.

Figure 3.3 suggests that risky returns are positively skewed. In our model, it is easy to show that the equilibrium price at time t is convex (concave) for $m > \ln(1/\lambda)/\ln(\theta_{irr})$ $\left(m < \ln(1/\lambda)/\ln(\theta_{irr})\right)$. Therefore, since $\frac{\ln(1/\lambda)}{\ln(\theta_{irr})} > 0$ and the distribution of the aggregate information is centered and symmetric, one can figure out that upward movements occur more often than downward movements, unless $\lambda = 1$. When irrational traders do not exhibit loss aversion, the model does not generate any skewness. We state the result formally as Result B5 (see Appendix F for a formal proof).

Result B5. *Asset returns are positively skewed unless* $\lambda = 1$.

The conjunction of loss aversion displayed by irrational traders and short-sales constraint is mainly responsible for the positive skewness of asset return.

Intuitively, loss averse traders dislike losses more than they like gains. Therefore, Group *B* will engage in trades only if they think they can realize gains more often than losses. Consequently, the market maker[42] proposes a price to the loss averse traders accordingly. This feature in addition to the short-sales constraint generate as a buy product positive skewness of asset return. It is easy to show that the positive skewness increases with loss aversion. Result B6 establishes the relation between skewness and loss aversion.

Result B6. *Skewness increases with loss aversion*

The proof of Result B6 is merely an extension of the proof of Result B5. As in Results B2 and B4, several testable implications arise from Result B6. The first implication would be the presence of more positive skewness among stocks held by a greater number of institutional traders, ceteris paribus. Furthermore, in the cross section, securities that have experienced a recent drop in their prices should exhibit more positive skewness in their subsequent returns. Chen, Hong and Stein (2001) find robust evidence about conditional skewness in line with implications generated by our model. Beyond the general evidence, it is broadly accepted that skewness is positive at the firm level and negative for the market as a whole. They come up with three robust findings. First, that positive skewness turns out to be more pronounced for small firms, ceteris paribus. Second, they find that when past returns have been high, skewness is forecasted to become more negative and reciprocally, when past returns have been low, skewness is forecasted to become

[42] Recall that the rational group, under our assumption, is indeed the market maker.

more positive. Third, that negative skewness is greater in stocks that have experienced an increase in trading volume. While Chen, Hong and Stein (2001) aim to test the theory developed in Hong and Stein (2003), the size effect and influence of past return on skewness do not speak directly to the predictions of their model.

In line with those arguments, our model may support the relation between size and skewness found in Chen, Hong and Stein (2001). Retail investors are indeed more likely to hold small firm stocks, indicating, within our context, a greater degree of loss aversion and therefore a more positive skewness in asset return. After having established a connection between loss aversion and skewness we are interested in expanding our analysis to skewness conditional on trading volume. In the same spirit as Hong and Stein (2003), we are able to show that there is a positive correlation between negative skewness and trading volume or, to put it differently, on average, an increase in trading volume generates a decrease in skewness. We state the result formally in Result B7.

Result B7. *Skewness and speculative volume are negatively correlated.*

The intuition behind Result B7 comes from the observation (in equation (10)) that the maximum conditional volume appears when $m = \psi$. At that point, the conditional skewness is negative,[43] while, as we already showed, the overall unconditional skewness is positive. Therefore, we can infer that negative skewness

[43] Notice that $\frac{\ln(1/\lambda)}{\ln(\theta_{irr})} < \psi$, therefore, the price of the risky asset, when $m = \psi$, is locally concave, suggesting negative skewness of asset return between t and $t+1$.

is more pronounced in stocks experiencing a positive shock in volume, unless $\lambda = 1$. We provide a more rigorous demonstration, of the negative correlation between skewness and trading volume using numerical (Monte Carlo simulation) techniques alongside with analytical analysis. We report it in appendix F.

Interestingly, the proposed behavioral explanation succeeds in bringing arguments in favor of the three robust findings highlighted by Chen, Hong and Stein (2001), whereas Hong and Stein's (2003) theory mainly supports just the relationship between volume and skewness. At the fundamental level, although, the proposed model differs from the model of Hong and Stein (2003), they both integrate disagreement in the way different groups of traders update their information and some restrictions for investors in taking short positions. In their model, differences of opinion and short-sales constraints are responsible for the correlation between skewness and volume but do not permit to talk about fixed-firm characteristics impacting conditional skewness. Other studies have already documented the effect of past returns and size on conditional skewness. Harvey and Siddique (2000) and Cao, Coval, and Hirshleifer (2002) show that there is negative conditional skewness after periods of positive returns. Harvey and Siddique (2000) report that skewness is more negative on average for large firms by market capitalization. Boyer, Mitton, and Vorkink (2010) find strong negative cross-sectional relation between average returns and expected skewness. They also find that other firm characteristics are also important predictors of idiosyncratic skewness, including idiosyncratic volatility, turnover, firm size, and industry designation. It is worthwhile to mention that Boyer, Mitton, and Vorkink

(2010) follow indeed the approach of Chen, Hong, and Stein (2001) to derive their results. The motivation of their study was primarily to test the prediction of recent theories that stock with high idiosyncratic skewness should have low expected returns[44] unlike Chen, Hong, and Stein (2001) where their primary motivation was to test the relation between trading volume and skewness.

There are indeed, several different theories that explain only partially empirical evidence on skewness of asset return. The proposed model hopes to encompass a broader understanding of skewness at the cross section. Hong and Stein (2001) attribute the impact of past returns on skewness to stochastic bubbles[45]. On the other hand, discretionary-disclosure is evoked to explain the effect of size. Our model represents the advantage to coming up with a means to capture the three patterns documented above in a more integrated fashion[46].

3.7 Model of disagreement: Concluding Remarks

We develop a disagreement model of speculative trading that captures for the first time the joint effect of loss aversion and overconfidence on equilibrium price, speculative volume and higher moments. Although a priori contradicting, these

[44] Mitton and Vorkink (2007), develop a model incorporating heterogeneous investor preference for skewness, predicting lower expected return for stocks with idiosyncratic skewness. Barberis and Huang (2008) show that when investors follow the cumulative prospect theory, positively skewed securities will earn lower average returns. Brunnermeier and Parker (2005) and Brunnermeier, Gollier, and Parker (2007) solve an endogenous-probabilities model that produces similar asset pricing implications for skewness.

[45] Quote "In the context of a bubble model, high past returns or a low book-to-market value imply that the bubble has been building up for a long time, so that there is a larger drop when it pops and prices fall back to fundamentals."

[46] This statement should, however, be taken with a grain of salt, since it is based on assumptions that we made regarding institutional traders and their elimination of behavioral biases through experience, as well as concerning their propensity to hold large-cap stocks.

two key cognitive biases display distinct features and do not merely cancel one another. Their unique interaction permits to generate features in line with striking empirical evidence that any of them can generate on their own. Moreover, this paper enriches also our understanding of their complex and marginal impacts on speculation.

The model sheds light indeed on different, a priori not related, cross-sectional phenomena observed in the stock market. Namely, the model generates first a positive correlation between public aggregate information and speculative volume. This feature provides an explanation for the observed correlation between high priced stocks and speculative volume (Hong and Stein (2007)). The model also generates a high-volume return premium (Gervais, Kaniel, and Mingelgrin (2001)), and fills the gap between cross-sectional evidence and fixed firm characteristic that cannot be reconciled by the widely accepted Merton's (1987) recognition investor hypothesis. For example, the proposed model may explain simultaneously why the high-volume return premium is decreasing with firm size while it is increasing with the existence of analyst coverage (Kaniel Ozoguz and Starks (2012)). Our model is consistent with the negative correlation between the concentration of institutional traders and the magnitude of the premium. Since the *visibility* hypothesis is hardly refutable and short-sales constraints are assumed in the proposed theory and amplifying Merton's (1987) recognition investor hypothesis, we should not see the proposed behavioral explanation as a substitute for Merton's hypothesis but rather as a complement to it. Using both theories as guide, our work suggests that economically implementable strategies based on the high-volume return premium should exist. The model also supports a variety of stylized facts related to the skewness of asset returns. Finally, the model succeeds to reconcile cross-sectional variation in skewness at the firm level (skewness is negatively correlated with trading volume, past return, and firm size) with the fact that on average skewness in individual firm return is positive.

It is important however to put our model's compelling supports of the impact of fixed-firm characteristics on trading volume and on conditional skewness, as described above, into some perspective. To establish the influence of firm size or past return on volume and conditional skewness, we assume: (1) *narrow framing*; (2) firm size as a proxy of institutional trading; (3) institutional traders are immune from behavioral biases (overconfidence and loss aversion in our specific case). Although, some empirical evidence corroborates these results, a rigorous empirical study should be conducted in order to strengthen the validity of the proposed theory.

Conclusion

This thesis enriches the literature on behavioral finance applied to financial markets in general and to information acquisition from a behavioral perspective in particular. We provide two important contributions to financial economics.

First, this study provides for the first time an analytical and tractable equilibrium solution in an economy in which loss averse speculators hold private information. Loss aversion, while affecting speculators' willingness to trade, adds an additional level of complexity to the difficulty for the market maker to infer the private information from the optimal demand schedule of the informed trader. We show that when the aggregate order flow is low, the market maker is confused about speculators' trading status and the equilibrium price. In this situation, where the market depth becomes highly non-linear, we demonstrate that a small adverse shock to the fundamentals can trigger a large drop in asset value. When introducing short-sales constraints into the present model, the model provides an asymmetry between upward and downward price movements. The predictions of the model match historical evidence of market crashes.

Second, we develop a disagreement model of speculative trading that captures for the first time the joint effect of loss aversion and overconfidence, on equilibrium price, speculative volume, and higher moments. Although a priori contradicting, these two key cognitive biases display distinct features and do not merely cancel one another. Their unique interaction permits features in line with striking empirical evidence that any of them can generate on their own. Moreover, this paper also enriches our understanding of their complex and marginal impacts on speculation. The model from chapter 3 can help shed light indeed on different, theoretically unrelated, cross-sectional phenomena observed in the stock market. Namely, this model generates a positive correlation between public aggregate information and speculative volume, a *high-volume return premium* (Gervais, Kaniel, and Mingelgrin (2001)), and permits to fill the gap between cross-sectional evidence and fixed-firm characteristics that fail to be rationalized using Merton's (1987) widely accepted recognition investor hypothesis. For example, this work may explain simultaneously why the high-volume return premium decreases with the firm's size while it is increases with the existence of analyst coverage (Kaniel Ozoguz and Starks (2012)). Our second model also supports the negative correlation between the concentration of institutional traders and the magnitude of the premium.

Other generalizations of the thesis could be quite interesting. Examining the impact of private information on insurance and hedging with loss aversion should allow us to study the welfare consequences of improvements in private information release with more realistic preferences. Moreover, the question of contagion in financial markets when speculators are loss averse has not, to our knowledge, been investigated in the literature

Another straightforward avenue emerges from this work. Questions of market efficiency, price-impact and survival of irrational traders in the long run are crucial to our understanding of financial markets. Several authors have been interested in these questions and have developed consumption-based asset price models where both irrational traders and rational trades are present (Hirshleifer, and Luo, (2001), Kogan, Wang, Ross, and Westerfield (2006), and Easley and Yang (2015)). Although the second model underlies the trading mechanism between rational and irrational traders, the purpose of this work is not to speak directly to questions of efficiency in financial markets, but rather to reflect the principal motivation behind the interest in the survival and price impact of irrational traders. Therefore, one could borrow the concept introduced in this thesis and develop a consumption-based asset-pricing model where irrational traders are naturally jointly loss averse[47] and overconfident (optimistic) about the prospect of the economy[48].

Finally, it will certainly be worthwhile to use similar tools to study governance and to control right choices within the context of irrational investors and managers. As Tirole (2005) stated in his remarkable book, “Despite the intensive and exciting research efforts in behavioral economics in general, behavioral corporate finance theory is still rather underdeveloped relative to its agency-based counterpart”.

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[47] Loss aversion in those studies is modeled with recursive preference representation, as in Barberis and Huang, (2009) and Easley and Yang (2015).

[48] In these models, investors who are optimistic about the state of the economy typically overestimate the rate of growth of the aggregate endowment.

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Annexes

Appendix A

Derivation of equation (2.2)

The utility function can be written as

$$U(\pi,\gamma,\lambda) = -\frac{1}{\gamma}e^{-\gamma\pi} + \frac{(\lambda-1)}{\gamma}\left(1-e^{-\gamma\pi}\right)1\{\pi<0\}\,. \tag{A1}$$

The conditional expectation is given by

$$E\left[U(\pi,\gamma,\lambda)\middle|s\right] = -\frac{1}{\gamma}E\left[e^{-\gamma\pi}\middle|s\right] + \frac{(\lambda-1)}{\gamma}\left(1-E\left[e^{-\gamma\pi}\middle|s,\ \pi<0\right]\right)\Pr\left(\pi<0\middle|s\right) \tag{A2}$$

where

$$E\left[e^{-\gamma\pi}\middle|s\right] = e^{-\gamma\left(\mu_{v/s}-P\right)x+\frac{\gamma^2x^2\sigma_{v/s}^2}{2}}. \tag{A3}$$

For $x>0$

$$\Pr\left(x(\tilde{v}-P)<0\middle|s\right) = \Phi\left(\frac{\mu_{v/s}-P}{\sigma_{v/s}}\right), \tag{A4}$$

and

$$\begin{aligned} E\left[e^{-\gamma\pi}\middle|s,\ \pi<0\right] &= e^{\gamma P}E\left[e^{-\gamma v}\middle|s,\ \tilde{v}<P\right] = \frac{\int_{-\infty}^{P}(2\pi)^{1/2}e^{-\gamma(z-P)x}e^{-\left(z-\mu_{v/s}\right)^2/2\sigma_{v/s}^2}dz}{\Phi\left(\frac{\mu_{v/s}-P}{\sigma_{v/s}}\right)} \\ &= e^{-\gamma\left(\mu_{v/s}-P\right)x+\frac{\gamma^2x^2\sigma_{v/s}}{2}}\frac{\int_{P}^{\infty}(2\pi)^{1/2}e^{-\frac{\left(z-\left(\mu_{v/s}-x\gamma\sigma_{v/s}^2\right)\right)^2}{2\sigma_{v/s}^2}}dz}{\Phi\left(-\frac{\mu_{v/s}-P}{\sigma_{v/s}}\right)} = \frac{e^{-\gamma\left(\mu_{v/s}-P\right)x+\frac{\gamma^2x^2\sigma_{v/s}}{2}}\Phi\left(\frac{\mu_{v/s}-x\gamma\sigma_{v/s}^2-P}{\sigma_{v/s}}\right)}{\Phi\left(\frac{\mu_{v/s}-P}{\sigma_{v/s}}\right)} \end{aligned} \tag{A5}$$

and for $x\le 0$,

$$\Pr\left(x(\tilde{v}-P)<0\middle|s\right) = \Phi\left(-\frac{\mu_{v/s}-P}{\sigma_{v/s}}\right), \tag{A6}$$

and

$$E\left[e^{-\gamma\pi}\left|s,\ \pi>0\right.\right]=e^{\gamma P}E\left[e^{-\gamma v}\left|s,\ \tilde{v}>P\right.\right]=\frac{e^{-\gamma\left(\mu_{v/s}-P\right)x+\frac{\gamma^2 x^2\sigma_{v/s}}{2}}\Phi\left(-\frac{\mu_{v/s}-x\gamma\sigma_{v/s}^2-P}{\sigma_{v/s}}\right)}{\Phi\left(-\frac{\mu_{v/s}-P}{\sigma_{v/s}}\right)} \quad \text{(A7)}$$

Plugging (A3), (A4), (A5), (A6) into (A7) yields equation (2.2)

Appendix B

Derivation of equation (2.9)

Each conditional expectation and probability of equation (2.10) is tractable in our setting. Since z and v are independent, for the no-trade region we have

$$E\left[v\left|\omega,\left|s-\Delta\right|\le\frac{P}{\rho^2}\right.\right]=E\left[v\left|z,\left|s-\Delta\right|\le\frac{P}{\rho^2}\right.\right]=E\left[v\left|\frac{P}{\rho^2}-\Delta\le s\le\frac{P}{\rho^2}+\Delta\right.\right] \quad \text{(B1)}$$

whereas for the informed trading region

$$E\left[v\left|\omega,s\le\frac{P}{\rho^2}-\Delta\right.\right]=E\left[v\left|\omega_1=\frac{\rho^2 s-P}{\gamma\sigma_v^2\left(1-\rho^2\right)}+\frac{\Lambda(\lambda)}{\gamma\sigma_v\sqrt{1-\rho^2}}+z,s\le\frac{P}{\rho^2}-\Delta\right.\right] \quad \text{(B2)}$$

and

$$E\left[v\left|\omega,s>\frac{P}{\rho^2}+\Delta\right.\right]=E\left[v\left|\omega_2=\frac{\rho^2 s-P}{\gamma\sigma_v^2\left(1-\rho^2\right)}-\frac{\Lambda(\lambda)}{\gamma\sigma_v\sqrt{1-\rho^2}}+z,s\ge\frac{P}{\rho^2}+\Delta\right.\right] \quad \text{(B3)}$$

We can express the conditional moments of the truncated normal variables in closed-formed (Greene (2002) (pp.781-782), and Madala (1986)).

$$E\left[v\left|\frac{P}{\rho^2}-\Delta\le s\le\frac{P}{\rho^2}+\Delta\right.\right]=\rho\sigma_v\frac{\psi\left(\dfrac{\frac{P}{\rho^2}-\Delta}{\sqrt{\sigma_v^2+\sigma_\varepsilon^2}}\right)-\psi\left(\dfrac{\frac{P}{\rho^2}+\Delta}{\sqrt{\sigma_v^2+\sigma_\varepsilon^2}}\right)}{\Phi\left(\dfrac{\frac{P}{\rho^2}+\Delta}{\sqrt{\sigma_v^2+\sigma_\varepsilon^2}}\right)-\Phi\left(\dfrac{\frac{P}{\rho^2}-\Delta}{\sqrt{\sigma_v^2+\sigma_\varepsilon^2}}\right)}. \qquad \text{(B4)}$$

While for the trading region we have

$$E\left[v\left|\omega_1,s\le\frac{P}{\rho^2}-\Delta\right.\right]=\mu_{v/\omega_1}-\frac{\rho^*\sigma_{v/\omega_1}\psi\left(\dfrac{\frac{P}{\rho^2}-\Delta}{\sqrt{\sigma_v^2+\sigma_\varepsilon^2}}\right)}{\Phi\left(\dfrac{\frac{P}{\rho^2}-\Delta}{\sqrt{\sigma_v^2+\sigma_\varepsilon^2}}\right)}, \qquad \text{(B5)}$$

and

$$E\left[v\left|\omega_2,s>\frac{P}{\rho^2}+\Delta\right.\right]=\mu_{v/\omega_2}+\frac{\rho^*\sigma_{v/\omega_2}\psi\left(\dfrac{\frac{P}{\rho^2}+\Delta}{\sqrt{\sigma_v^2+\sigma_\varepsilon^2}}\right)}{1-\Phi\left(\dfrac{\frac{P}{\rho^2}+\Delta}{\sqrt{\sigma_v^2+\sigma_\varepsilon^2}}\right)}, \qquad \text{(B6)}$$

where ρ^* refers to the correlation coefficient of the conditional bivariate normal variable $v,s|\omega$.

The conditional expectation and standard deviation of normal random (Greene 26, (pp 90)) are

$$\mu_{v/\omega}=\frac{\operatorname{cov}(v,\omega)}{\operatorname{var}(\omega)}\left(\omega-E[\omega]\right) \quad \text{and} \quad \sigma_{v/\omega}=\sigma_v\left(1-\frac{E[v\omega]}{\sigma_v\sigma_\omega}\right) \qquad \text{(B7)}$$

where $w-E(\omega)=\frac{s}{\gamma\sigma_\varepsilon^2}+z$; $\operatorname{cov}(v,\omega)=\frac{\sigma_v^2}{\gamma\sigma_\varepsilon^2}$ and $\operatorname{var}(\omega)=\frac{1}{\gamma^2\sigma_\varepsilon^2}+\frac{\sigma_v^2}{\gamma^2\sigma_\varepsilon^4}+\sigma_z^2$.

Equation (B8) is the same for the lower region $\omega_1=\frac{\rho^2 s-P}{\gamma\sigma_v^2\left(1-\rho^2\right)}+\frac{\Lambda(\lambda)}{\gamma\sigma_v\sqrt{1-\rho^2}}+z$ as well as for the upper region $\omega_2=\frac{\rho^2 s-P}{\gamma\sigma_v^2\left(1-\rho^2\right)}-\frac{\Lambda(\lambda)}{\gamma\sigma_v\sqrt{1-\rho^2}}+z$.

Therefore

$$\mu_{v/\omega_1}=\mu_{v/\omega_2}=\mu_{v/\omega}=\frac{\gamma\sigma_v^2\sigma_\varepsilon^2}{\sigma_v^2+\sigma_\varepsilon^2\left(1+\gamma^2\sigma_\varepsilon^2\sigma_z^2\right)}\left(\frac{s}{\gamma\sigma_\varepsilon^2}+z\right), \tag{B8}$$

and

$$\sigma_{v/\omega}=\sigma_{v/\omega_1}=\sigma_{v/\omega 2}=\sqrt{\frac{\sigma_v^2\sigma_\varepsilon^2\left(1+\sigma_z^2\gamma^2\sigma_\varepsilon^2\right)}{\sigma_v^2+\sigma_\varepsilon^2\left(1+\sigma_z^2\gamma^2\sigma_\varepsilon^2\right)}}. \tag{B9}$$

Using well known properties of conditional multivariate normal distribution (Rencher (2002) (pp. 88)), the variance covariance matrix and correlation coefficient bivariate normal variable $v,s|\omega$ are

$$\Sigma^*=\begin{bmatrix}\frac{\sigma_v^2\sigma_\varepsilon^2\left(1+\sigma_z^2\gamma^2\sigma_\varepsilon^2\right)}{\sigma_v^2+\sigma_\varepsilon^2\left(1+\sigma_z^2\gamma^2\sigma_\varepsilon^2\right)} & \frac{\gamma^2\sigma_\varepsilon^4\sigma_z^2\sigma_v^2}{\sigma_v^2+\sigma_\varepsilon^2\left(1+\sigma_z^2\gamma^2\sigma_\varepsilon^2\right)} \\ \frac{\gamma^2\sigma_\varepsilon^4\sigma_z^2\sigma_v^2}{\sigma_v^2+\sigma_\varepsilon^2\left(1+\sigma_z^2\gamma^2\sigma_\varepsilon^2\right)} & \frac{\gamma^2\sigma_\varepsilon^4\left(\sigma_v^2+\sigma_\varepsilon^2\right)\sigma_z^2}{\sigma_v^2+\sigma_\varepsilon^2\left(1+\sigma_z^2\gamma^2\sigma_\varepsilon^2\right)}\end{bmatrix}, \tag{B10}$$

where Σ^* is the same for ω_1 as well as for ω_2. From Σ^*, we find that

$$\rho^*=\frac{\rho\gamma\sigma_z\sigma_\varepsilon}{\sqrt{1+\gamma^2\sigma_\varepsilon^2\sigma_z^2}} \tag{B11}$$

The probabilities of equation (2.10) are given by

$$\Pr\left[|s-\Delta| \leq \frac{P}{\rho^2}\right] = \Phi\left(\frac{\frac{P}{\rho^2}+\Delta}{\sqrt{\sigma_v^2+\sigma_\varepsilon^2}}\right) - \Phi\left(\frac{\frac{P}{\rho^2}-\Delta}{\sqrt{\sigma_v^2+\sigma_\varepsilon^2}}\right);\ \Pr\left[s \leq \frac{P}{\rho^2}-\Delta\right] = \Phi\left(\frac{\frac{P}{\rho^2}-\Delta}{\sqrt{\sigma_v^2+\sigma_\varepsilon^2}}\right), \tag{B12}$$

and

$$\Pr\left[s > \frac{P}{\rho^2}+\Delta\right] = 1-\Phi\left(\frac{\frac{P}{\rho^2}+\Delta}{\sqrt{\sigma_v^2+\sigma_\varepsilon^2}}\right).$$

The equation (2.10) is then obtained by replacing (B8), (B9), and (B11) into (B4) (B5), and (B4). And (B4), (B5), (B6) and (B12) into equation (2.9).

Appendix C

Equilibrium price with short-sale constraints and loss aversion

Using the same relations as in appendix B, we can develop separately all the terms of (2.16). The main difference is in the aggregation of two different types of traders, namely unconstrained and constrained traders. The first two terms are not subject to any constraints. The aggregate order flow of the first term reflects the participation of all the traders, while the second term implies the participation of noise traders only. Therefore, their developments are similar to what has been done in appendix B. The last two terms of (2.16), however, generate the short-sale constraints. The untruncated conditional expectation and standard deviation of the first term of (2.16) is given respectively by (B8) and (B9). For the second and the third term, it is equal to zero since only noise traders submit trading orders. While for the fourth term, using again well known properties (Greene (2002) (pp 90)), we find that μ_{v/ω_4} and σ_{v/ω_4} are

$$\mu_{v/\omega_4} = \frac{\gamma\sigma_v^2\sigma_\varepsilon^2(1-\kappa)}{\sigma_v^2(1-\kappa)^2(\sigma_v^2+\sigma_\varepsilon^2)+\gamma^2\sigma_\varepsilon^4\sigma_z^2}\left(\frac{(1-\kappa)s}{\gamma\sigma_\varepsilon^2}+z\right), \tag{C1}$$

$$\sigma_{v/\omega_4} = \sqrt{\frac{\sigma_v^2\sigma_\varepsilon^2\left((1-\kappa)^2+\sigma_z^2\gamma^2\sigma_\varepsilon^2\right)}{\left(\sigma_v^2+\sigma_\varepsilon^2\right)(1-\kappa)^2+\sigma_z^2\gamma^2\sigma_\varepsilon^4}}. \tag{C2}$$

The correlation coefficient of the bivariate normal variable $v,s|\omega$ of the first term of (2.16) is given in (B11). For the first and the second term it is simply ρ, and therefore, for the fourth term, $\rho^* = \dfrac{\rho\gamma\sigma_z\sigma_\varepsilon}{\sqrt{(1-\kappa)^2+\gamma^2\sigma_\varepsilon^2\sigma_z^2}}$.

Having all the parameters of the conditional truncated normal distribution of each of the four terms of (2.16), the price function for the equilibrium price can be expressed as the fixed point problem as follows:

$$P^*_{LA-SS} = \frac{\gamma\sigma_v^2\sigma_\varepsilon^2}{\sigma_v^2+\sigma_\varepsilon^2\left(1+\gamma^2\sigma_\varepsilon^2\sigma_z^2\right)}\left(\frac{s}{\gamma\sigma_\varepsilon^2}+z\right)\left[1+\Phi\left(\frac{\frac{P}{\rho^2}-\Delta}{\sqrt{\sigma_v^2+\sigma_\varepsilon^2}}\right)-\Phi\left(\frac{\frac{P}{\rho^2}+\Delta}{\sqrt{\sigma_v^2+\sigma_\varepsilon^2}}\right)-\Phi\left(\frac{\frac{P}{\rho^2}-\Gamma}{\sqrt{\sigma_v^2+\sigma_\varepsilon^2}}\right)\right]$$

$$+\frac{\gamma\sigma_v^2\sigma_\varepsilon^2(1-\kappa)}{\sigma_v^2(1-\kappa)^2(\sigma_v^2+\sigma_\varepsilon^2)+\gamma^2\sigma_\varepsilon^4\sigma_z^2}\left(\frac{(1-\kappa)s}{\gamma\sigma_\varepsilon^2}+z\right)\Phi\left(\frac{\frac{P}{\rho^2}-\Gamma}{\sqrt{\sigma_v^2+\sigma_\varepsilon^2}}\right) \tag{C3}$$

$$+\sigma_v\rho\sqrt{\frac{\gamma^2\sigma_z^2\sigma_\varepsilon^4}{\sigma_v^2+\sigma_\varepsilon^2\left(1+\gamma^2\sigma_\varepsilon^2\sigma_z^2\right)}}\left[\psi\left(\frac{\frac{P}{\rho^2}+\Delta}{\sqrt{\sigma_v^2+\sigma_\varepsilon^2}}\right)+\psi\left(\frac{\frac{P}{\rho^2}-\Delta}{\sqrt{\sigma_v^2+\sigma_\varepsilon^2}}\right)-\psi\left(\frac{\frac{P}{\rho^2}-\Gamma}{\sqrt{\sigma_v^2+\sigma_\varepsilon^2}}\right)\right]$$

$$-\sigma_v\rho\sqrt{\frac{\gamma^2\sigma_z^2\sigma_\varepsilon^4}{\left(\sigma_v^2+\sigma_\varepsilon^2\right)\left(1-\kappa\right)^2+\gamma^2\sigma_\varepsilon^4\sigma_z^2}}\;\psi\left(\frac{\frac{P}{\rho^2}-\Gamma}{\sqrt{\sigma_v^2+\sigma_\varepsilon^2}}\right)+\sigma_v\rho\left[\psi\left(\frac{\frac{P}{\rho^2}+\Delta}{\sqrt{\sigma_v^2+\sigma_\varepsilon^2}}\right)-\psi\left(\frac{\frac{P}{\rho^2}-\Delta}{\sqrt{\sigma_v^2+\sigma_\varepsilon^2}}\right)\right].$$

Appendix D

Derivation of $V(m)$

From the definition of the true posterior given by equation (3.2), it follows immediately that

$$\pi_{ra}^H(m)=\frac{a_{ra}^m}{a_{ra}^m+b_{ra}^m} \quad \text{and} \quad \pi_{ra}^L(m)=\pi_{ra}^H(-m)=\frac{b_{ra}^m}{a_{ra}^m+b_{ra}^m}. \tag{D1}$$

We get the same relation, for the irrational group, substituting the subscript *ra* with i*rr*.

Following Bayes' rule, the true density of the current signal given the cumulative signal *m* is

$$f(s|m)=\delta_{ra}(s|H)\pi_H(m)+\delta_{ra}(s|L)\pi_L(m)=\frac{\delta_{ra}(s|H)\pi_{ra}^H(m)}{\pi_{ra}^H(m+s)}. \tag{D2}$$

The expected volume when $m>\psi$ $(m\le\psi)$ refers to the probability given the cumulative signal at time *t* that the public signal at time *t+1* is in the range $s\in\left(-\infty,\psi-m\right)$ $\left(s\in\left(\psi-m,\infty\right)\right)$. Replacing k_{ra} with its value is required to make $\delta_{ra}(s|H)$ and $\delta_{ra}(s|L)$ density functions:

$$\frac{1}{k_{ra}}=\int_0^\infty(a_{ra}^s+b_{ra}^s)ds=-\frac{1}{\ln(a_{ra})}-\frac{1}{\ln(b_{ra})}. \tag{D3}$$

We thus get for $m > \psi$

$$\int_{-\infty}^{\psi - m} -\frac{b_{ra}^{-s}(a_{ra}^{m+s} + b_{ra}^{m+s})\ln(a_{ra})\ln(b_{ra})}{\left(a_{ra}^{m} + b_{ra}^{m}\right)\left[\ln(a_{ra}) + \ln(b_{ra})\right]} ds \quad \frac{a_{ra}^{m} b_{ra}^{m}\left[b_{ra}^{-\psi}\ln(a_{ra}) + a_{ra}^{-\psi}\ln(b_{ra})\right]}{\left(a_{ra}^{m} + b_{ra}^{m}\right)\left(\ln(a_{ra}) + \ln(b_{ra})\right)} \tag{D4}$$

and for $m \leq \psi$ we have

$$\int_{\psi - m}^{\infty} -\frac{a_{ra}^{s}(a_{ra}^{m+s} + b_{ra}^{m+s})\ln(a_{ra})\ln(b_{ra})}{\left(a_{ra}^{m} + b_{ra}^{m}\right)\left[\ln(a_{ra}) + \ln(b_{ra})\right]} ds = \frac{b_{ra}^{\psi}\ln(a_{ra}) + a_{ra}^{\psi}\ln(b_{ra})}{\left(a_{ra}^{m} + b_{ra}^{m}\right)\left(\ln(a_{ra}) + \ln(b_{ra})\right)} \tag{D5}$$

Replacing the posterior belief of the rational group with equation (3.2) leads to equation (3.10).

■

Let us define as $\varphi_t(m)$ the unconditional density of the cumulative signal at date *t*. We next show that $\varphi_t(m)$ is symmetric with respect to $m = 0$. The proof is by induction on *t*. For $t = 1$, $\varphi_t(m) = f(m|0) = f(-m|0)$ since $\pi_{ra}^{H}(m) = \pi_{ra}^{L}(0)$. From (A2) it follows that $f(s|-m) = f(-s|m)$. Now let us assume $\varphi_t(m) = \varphi_t(-m)$ for all $t < n$, by induction,

$$\begin{aligned}
\varphi_t(m) &= \int_{-\infty}^{\infty} \varphi_t\left(m | m_{t-1} = x\right) \varphi_{n-1}(x) dx \\
&= \int_{-\infty}^{\infty} f\left(m - x | x\right) \varphi_{n-1}(x) \, dx \\
&= \int_{-\infty}^{\infty} f\left(m + x | -x\right) \varphi_{n-1}(-x) \, dx \\
&= \int_{-\infty}^{\infty} f\left(-m - x | x\right) \varphi_{n-1}(x) \, dx \\
&= \int_{-\infty}^{\infty} f\left(-m | m_{n-1} = x\right) \varphi_{n-1}(x) \, dx = \varphi_t(-m)
\end{aligned}$$

■

Having proved that the unconditional distribution of the cumulative signal is symmetric, we are ready to demonstrate Result B1.

Result B1. $Cov_t\left[V(m),m\right]>0$.

Proof. Since the unconditional distribution is symmetric, $E\left[m\right]=0$.

$Cov_t\left[V(m),m\right]=E\left[V(m)\cdot m\right]=\int_{-\infty}^{\infty}V(m)\cdot m\cdot\varphi_t\left(m\right)\,dm$. Let us decompose this integral into the sum of three distinct integrals: I_1, I_2, and I_3, according to the brackets $m_1\in\left[-\infty,-\psi\right]$, $m_2\in\left[-\psi,\psi\right]$, and $m_3\in\left[\psi,\infty\right]$respectively. It is easy to show that $I_3+I_1>0$, since for all $m_1=-m_3$, $v(m_3)>v(m_1)$, and $m\cdot\varphi_t\left(m\right)$is an odd function.

It remains now to demonstrate that $I_2>0$. $v(m_2)$ is an increasing and positive function within the range $m_2\in\left[-\psi,\psi\right]$. It is easy to show that the integrand of I_2 is a positive (negative) function for all $m_2>0$ $(m_2<0)$. Moreover for any valueof $m_2\in\left[0,\psi\right]$, $v(m_2)>v(-m_2)$, thus $\left|v(m_2)\cdot m_2\cdot\varphi_t\left(m_2\right)\right|>\left|v(-m_2)\cdot\left(-m_2\right)\cdot\varphi_t\left(-m_2\right)\right|$. Therefore $I_2>0$.

■

Appendix E

Result B3 $\dfrac{\partial r}{\partial v}>0$.

To demonstrate Result B3 we use a comparative static argument by asking how the risky return changes with the degree of loss aversion following relation

$$\frac{\partial r}{\partial v}\frac{\partial v}{\partial \psi}\frac{\partial \psi}{\partial \lambda}=\frac{\partial r}{\partial \lambda}. \tag{E1}$$

The right hand side of equation (E1) is positive since it can be decomposed into the product of two negative terms $\frac{\partial r}{\partial P_{t+1}}\cdot\frac{\partial P_{t+1}}{\partial \lambda}$. We naturally make the straightforward assumption that equilibrium price cannot take negative value (*H* and *L* are positive). In order to prove the existence of the high-volume return premium, we just have to show that the second term of the left hand side is greater than zero, e.g. $\frac{\partial v}{\partial \psi}>0$. Using the fundamental theorem of algebra $\frac{\partial v}{\partial \psi}$ is equal to

$$\frac{\partial}{\partial \psi}\int_{-\infty}^{\psi}\frac{\left[b_{ra}^{\psi}\ln(a)+a_{ra}^{\psi}\ln(b_{ra})\right]}{(a_{ra}^{m}+b_{ra}^{m})\left[\ln(a_{ra})+\ln(b_{ra})\right]}\varphi_t(m)dm+\frac{\partial}{\partial \psi}\int_{\psi}^{\infty}\frac{a^{m}b^{m}\left[b_{ra}^{-\psi}\ln(a_{ra})+a_{ra}^{-\psi}\ln(b_{ra})\right]}{(a_{ra}^{m}+b_{ra}^{m})\left[\ln(a_{ra})+\ln(b_{ra})\right]}\varphi_t(m)dm$$

$$=\frac{\left(b_{ra}^{\psi}-a_{ra}^{\psi}\right)\left[\ln(a_{ra})-\ln(b_{ra})\right]}{(a_{ra}^{\psi}+b_{ra}^{\psi})\left[\ln(a_{ra})+\ln(b_{ra})\right]}\varphi_t(\psi)>0, \tag{E2}$$

where the last inequality follows from the fact that $1\geq a_{ra}>b_{ra}>0$, and $\varphi_t(\psi)>0$.

Appendix F

Result B5. *The unconditional skewness is positive.*

Proof. We define skewness as being the third moments of the distribution of returns between *t* and *t+1*:

$E^{ra}\left[R_{t+1}^{3}\right]=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left(P_{t+1}-P_t\right)^3 f(s/m)\ \varphi_t(m)dsdm$. From (D2) it follows that $f(s/-m)=f(-s/m)$ and $f(s/m)=f(-s/-m)$. We can decompose the unconditional skewness into the following sum of four integrals $II_1 + II_2 + II_3 + II_4 =$

$$+\int_0^{\infty}\int_0^{\infty}\left(P_{t+1}-P_t\right)^3 f(s/m)\ \varphi_t\left(m\right)ds\ dm\ +\int_{-\infty}^{0}\int_{-\infty}^{0}\left(P_{t+1}-P_t\right)^3 f(s'/m')\ \varphi_t\left(m'\right)ds'\ dm'.$$

$$+\int_{-\infty}^{0}\int_0^{\infty}\left(P_{t+1}-P_t\right)^3 f(s/m)\ \varphi_t\left(m\right)ds\ dm\ +\int_0^{\infty}\int_{-\infty}^{0}\left(P_{t+1}-P_t\right)^3 f(s'/m')\ \varphi_t\left(m'\right)ds'\ dm'$$

To demonstrate positive skewness, we show that $II_1 + II_2 > 0$ and $II_3 + II_4 > 0$.

First recall that $\varphi_t\left(m\right)=\varphi_t\left(-m\right)$ and $f(s/m)= f(-s/-m)$. Let us define $\Gamma(m,s)=P_{t+1}-P_t$. For any $s>0$ and $m>0$, we set $m'=-m$ and $s'=-s$. Therefore, we get $\varphi_t\left(m\right)\cdot f(s/m)=\ \varphi_t\left(m'\right)\cdot f(s'/m')$. Moreover $\Gamma(m,s)>0$, $\Gamma(m',s')<0$,and

$$\Gamma(m,s)+\Gamma(m',s')=\frac{(H-L)\theta_{irr}^{m}(\theta_{irr}^{2m+s}-1)(\theta_{irr}^{s}-1)(\lambda-1)\lambda(1+\lambda)}{(\theta_{irr}^{m}+\lambda)(\theta_{irr}^{m+s}+\lambda)(\lambda\theta_{irr}^{m}+1)(\lambda\theta_{irr}^{m+s}+1)}>0\,. \tag{F1}$$

Consequently, for any $s>0$ and $m>0$, $m'=-m$ and $s'=-s$, the integrand of II_1 is greater than the integrand of - II_2 . Therefore $II_1 > -II_2$. Using the same reasoning, and the fact that $f(s/-m)= f(-s/m)$, one can show that $II_3 > -II_4$.

Notice also that if $\lambda=1$, $\Gamma(m,s)+\Gamma(m',s')=0$ then $E^{ra}\left[R_{t+1}^3\right]=0$.

Result B7. $Cov_t\left[Sk(m),V(m)\right]<0.$

We define $Sk(m)$ as the conditional skewness at *t+1* given cumulative signal *m*. We already demonstrated in Result B1 that trading volume and aggregate information are positively correlated.

Thus, to demonstrate Result B7, we can show that $Cov_t\left[Sk(m),m\right]<0.$

$$Cov_t\left[Sk(m),m\right]=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}m\left(P_{t+1}-P_t\right)^3 f(s/m)\ \varphi_t\left(m\right)ds\ dm\,.$$

We divide the integral into the sum of four integrals (same intervals as in the demonstration of Result B5 and we proceed to a change of variables to permit the integration over a positive range of *m* and *s*. Since the integral of a sum of functions equals the sum of integrals, we therefore get

$$Cov_t\left[Sk(m),m\right]=\int_0^{\infty}\int_0^{\infty}\Omega(s,m)\Pi(s,m)\varphi_t\left(m\right)ds\ dm \qquad \text{(F2)}$$

where $\Omega(m,s)=\dfrac{mk_0(H-L)^s\theta^{sm}(\theta^s-1)\lambda^s a_{ra}^s}{1+\theta_{ra}^m}$,

and $\Pi(s,m)=\left(\begin{array}{l} -\dfrac{\left(1+\theta_{ra}^{m+s}\right)}{\left(\theta_{irr}^m+\lambda\right)^3\left(\theta_{irr}^{m+s}+\lambda\right)^3}+\dfrac{\left(\theta_{ra}^m+\theta_{ra}^s\right)}{\left(1+\theta_{irr}^m\lambda\right)^3\left(\theta_{irr}^s+\lambda\theta_{irr}^m\right)^3} \\ +\dfrac{\left(\theta_{ra}^m+\theta_{ra}^s\right)}{\left(\theta_{irr}^m+\lambda\right)^3\left(\theta_{irr}^m+\lambda\theta_{irr}^s\right)^3}-\dfrac{\left(1+\theta_{ra}^{m+s}\right)}{\left(1+\theta_{irr}^m\lambda\right)^3\left(1+\lambda\theta_{irr}^{m+s}\right)^3} \end{array}\right).$

We see immediately that $\Omega(m,s)<0$ for all $s>0,$and, $m>0$. Thus, one way to demonstrate the negative covariance is to show that $\Pi(s,m)>0$ for all $s>0,$and, $m>0$. Unfortunately, it is not possible to demonstrate this relation analytically. Using numerical techniques, we can show $\Pi(s,m)>0$ for all $s>0,$ and, $m>0,$ when $2.02>\lambda>1$, and $1>\theta_{irr}>\theta_{ra}+0.048>0$.