HEC MONTRÉAL École affiliée à l'Université de Montréal

SECURITIZATION AND OPTIMAL RETENTION UNDER MORAL HAZARD

Par

Sara Malekan

Thèse présentée en vue de l'obtention du grade de Ph. D. en administration

(option Finance)

Avril 2015

© Sara Malekan, 2015

HEC MONTRÉAL École affiliée à l'Université de Montréal

Cette thèse intitulée :

SECURITIZATION AND OPTIMAL RETENTION UNDER MORAL HAZARD

Présentée par :

Sara Malekan

a été évaluée par un jury composé des personnes suivantes :

Pascal François HEC Montréal Président-rapporteur

Georges Dionne HEC Montréal Directeur de recherche

> Michèle Breton HEC Montréal Membre du jury

Marie-Claude Beaulieu Université Laval Examinateur externe

Fabien Chauny HEC Montréal Représentant du directeur de HEC Montréal

RÉSUMÉ

La titrisation est l'une des innovations les plus importantes sur les marchés financiers. C'est le processus de mise en commun des actifs financiers (prêts), de les arranger, de les convertir en actifs financiers structurés et de les vendre sous forme de tranches correspondant à des investisseurs ayant des appétits différents pour le risque. La structure des tranches correspond à des risques de défaut différents et à des rendements différents. Elle ressemble à une structure de capital donnant un ordre de priorité dans l'allocation des défauts.

En dépit de tous les avantages de la titrisation, elle est souvent soupçonnée d'être l'une des causes principales de la récente crise financière en raison du problème d'aléa moral qui réduit la motivation des banques dans le dépistage des prêteurs et dans la surveillance des prêts. Cette forme de risque moral est maximale lorsque les banques transfèrent tous leurs prêts aux marchés financiers. La rétention d'une quantité de prêts par la banque semble être une sage solution pour diminuer le problème d'aléa moral associé à la titrisation. Elle est fréquemment utilisée pour revitaliser les marchés de titrisation depuis la crise financière.

Méthodes de recherche : Notre objectif est de trouver la forme de rétention optimale pour les prêts sous l'aléa moral. Nous analyserons la titrisation optimale sous l'aléa moral en appliquant la méthodologie de la détermination endogène des contrats pour ces produits afin d'obtenir le partage optimal du risque entre les vendeurs et les acheteurs de ces produits.

Mots-clés: titrisation, rétention optimale, aléa moral, modèle principal-agent, tranching, rehaussement de crédit, le risque systémique.

ABSTRACT

Securitization is one of the most important innovations in financial markets. It is the process of pooling financial assets, packaging and converting them into securities and then selling them in the form of prioritized capital structure of claims to dispersed investors.

In spite of all the advantages of securitization, it is often suspected to be one of the main causes of the recent financial crisis due to the moral hazard problem in lender screening and monitoring. Tranche retention seems to be an optimal solution to solve the moral hazard problem of securitization and is used frequently to revitalise securitization markets.

Research method: Our objective is to find the optimal amount and form of retention for securitized products under moral hazard. We analyze the incentive contracting of securitized products under moral hazard and apply the security design methodology to these products in order to obtain the optimal risk sharing between sellers and buyers of these products.

Keywords: Securitization, optimal retention, moral hazard, principal-agent model, tranching, credit enhancement, systemic risk.

Table of Contents

RÉSUMÉ	iv
ABSTRACT	v
List of figures	viii
List of abreviations	ix
Dedication	x
Acknowledgment	xi
Chapter 1:Literature Review of Securitization in the Banking Industry	1
Securitization Introduction	2
1.1 "Originate-to-distribute" revolution	2
1.2 Problems of securitization	6
1. 2. 1 Adverse selection	11
1. 2. 2 Moral Hazard	14
1.3 Solutions	
1. 3. 1 Solutions for adverse selection	21
1. 3. 2 Solutions to moral hazard	
1.4 Design of research on moral hazard	
1. 4. 1 Best contract for principal-agent model	
Chapter 2:One Tranche Without Credit Enhancement Procedure	
Introduction	
2.1 Motivation	
2.2 Contributions	
2.3 The model	
2.3.1 Investors' objective function	44
2.3.2 Participation constraint	45
2.3.3 Incentive compatibility constraint	

2.3.4 Technology constraint	
2.3.5 Analyzing the result	
2.4 Conclusion	54
Chapter 3:Structured Asset-backed Securitization without Systemic Risk	56
Introduction	57
3.1 Extension of the model	59
3.1.1 Investors' objective function	61
3.1.2 Participation constraint	63
3.1.3 Incentive compatibility constraint	65
3.1.4 Technology constraint	65
3.1.5 Analyzing the result	69
3.2 Conditional distribution of loss	73
3.2.1 Analyzing the result	78
3.3 Conclusion	
Chapter 4: Structured Asset-backed Securitization with Systemic Risk	
Introduction	
4.1 Derivation of the model	
4.1.1 Investors' objective function	90
4.1.2 Participation constraint	91
4.1.3 Incentive compatibility constraint	
4.1.4 Technology constraint	93
4.1.5 Analyzing the result	97
4.2 Conclusion	
Appendix	104
References	114

List of figures

Figure 1: Simplified Overview of the Securitization Process	.38
Figure 2: Optimal Securitization Contract under Ex Ante Moral Hazard	.54
Figure 3: Optimal Securitization Contract under credit enhancement procedure	.72
Figure 4: Optimal Securitization Contract with conditional distribution of loss and credit enhancement	82
Figure 5: Optimal Securitization Contract with credit enhancement and systemic risk	101

List of abreviations

Originate-to-distribute	OTD
Credit risk transfer	CRT
Special purpose vehicle	SPV
Loan-to-value	LTV
European Union	EU
Collateralized loan obligation	CLO
Monotone likelihood ratio property	MLRP

Dedication

This thesis is dedicated to my father, who inspired me to pursue doctoral studies,

to my mother and sister, who helped me with my baby,

to my daughter, Sophia, for her patience, and

to my husband who was always there for me.

Acknowledgment

First and foremost, I am grateful to my PhD supervisor Prof. Georges Dionne for his valuable advice and guidance throughout the duration of my studies. His support and ideas were instrumental in improving the quality of the work.

I would like to thank the administrative staff at HEC, including Ms. Lise Cloutier, Ms. Claire Boisvert, and Ms. Julie Bilodeau, and Ms. Nathalie Bilodeau.

I am also thankful to all my friends, namely, Ms. Kiana Basiri, Mr. Xiaozhou Zhou, and Mr. Cédric Okou, who made my time at HEC more enjoyable and more memorable.

Chapter 1:

Literature Review of Securitization in the Banking Industry

Securitization Introduction

Securitization is one of the most important innovations in financial markets. It is a process of converting illiquid loans that cannot be sold readily to third-party investors into liquid securities and selling them to dispersed investors.

Because of the attractiveness of securitization for financial market participants, its application has widely been extended in recent years. In April 2011, the volume of outstanding securitized assets reached \$11 trillion, which is substantially greater than the overall outstanding marketable US Treasury securities of \$8 trillion (US Department of the Treasury, 2011).

In spite of all its advantages and widespread application, securitization is often suspected of being one of the main reasons for the recent financial crisis. That is why in the following pages we survey the literature on the revolutions that securitization brings to the market. We focus on major problems of securitization which are related to the recent financial crisis as well as previously proposed solutions in order to make our contribution to the literature more clear.

1.1 "Originate-to-distribute" revolution

The dramatic increase in the application of securitization during the last few decades has changed the traditional role of financial intermediaries from "originating and holding" to "originating and selling." This transfer to the "originate-to-distribute" model has significant implications for all market participants, including the originating banks, the borrowing firms, the individuals, the investors, and the regulators. "Originate-to-distribute" (OTD) model can be "socially desirable" according to the advantages that it may create. This financial innovation enhances the accessibility of credits and standardizes origination of the loans. Overall it improves the efficiency of credit market performance by creating more complete markets and facilitating the liquidity transformation that was the fundamental role traditionally performed by financial intermediaries (Diamond and Dybvig (1983)).

As a result, "Originating and selling" has increased liquidity in capital markets and provided the originating banks with additional sources of financing by allowing originators to remove the issued loans from their balance sheet and use the proceeds for other purposes or even to originate new loans (Coval, Jurek and Stafford (2009)).

Accordingly it is no more required for the banks to hold as many illiquid assets on balance sheet and since they can offer new loans by selling old loans they are not supposed to just relying on issuing new liabilities. These additional sources of financing increase the capacity of the banks' loan supply which depends upon business cycle conditions and banks' risk position. These features possibly lead to an increase in lending and promoting economic growth. Pavel and Phillis (1987) mention all the above features as issuers' incentives for securitization. They find that issuers want to have high risk or leverage as well as to mitigate or diversify risks. All these developments together help banks to extend credit reaction to the cost of funds of external shocks (Coval, Jurek and Stafford (2009); Loutskina (2011)).

For example, by using a large sample of European banks, Altunbas, Gambacorta and Marques-Ibanez (2009) demonstrate that the application of securitization could protect the bank's loan supply from the consequences of monetary policy shocks. With securitization banks are less dependent on deposits (traditional funding sources) and protected against interest-rate shocks, whether or not these shocks are derived by monetary policy (Altunbas, Gambacorta and Marques-Ibanez (2009); Gambacorta and Marques-Ibanez (2011); Goswami, Jobst and Long (2009)).

This evolution of "originate-to-distribute" model has some positive consequences in borrowers' relationship by increasing the access to financial markets for all the participants even those who had no access to the market before. It also alleviates borrower's financial constraints and provides additional financing (Gande and Saunders (2012)) as well as increasing access to debt capital for borrowers (Drucker and Puri (2009)).

As a result borrowers have access to a wide range of loans with better terms and conditions, because, in the presence of securitization, the risk of borrowing spreads among a dispersed group of investors that can bear more risk than individual banks. It reduces borrower's cost of capital as a result of valuable risk-sharing benefits from sale of loans to other investors in the secondary loan market (Gupta, Singh and Zebedee (2008); Parlour and Winton (2013)).

For the above reasons securitization is considered as one of the most efficient approaches to dispense credit risk since it improves risk sharing and decreases the cost of capital for lenders by dropping off the regulated capital. Issuers can also lower their cost of capital by securitizing low risk assets which protects them against bankruptcy (Ambrose, LaCour-Little and Sanders (2003); Ayotte and Gaon (2005); Gorton and Souleles (2007); Greenbaum and Thakor (1987); Minton, Sanders and Strahan (2004)).

Improving risk sharing and reducing the bank's cost of capital are extensively mentioned as benefits of securitization in the literature. In contrast Cheng, Dhaliwal and Neamtiu (2011) indicate that asset securitization increases cost of capital as a result of higher information uncertainty which can be seen in higher bid-ask spreads and dispersion. It is because of emergence of credit derivative markets with loan securitization activity that have transformed credit risk management by banks.

Overall "Originate-to-distribute" model provides this opportunity for the banks to diversify their asset portfolio, reach parts of the credit spectrum and transfer credit risk from their balance sheets to other economic agents. Securitization allows an originating bank to earn their fees and then transfer the interest rate and credit risks to outside investors. A potential advantage is that the banking system can reduce its exposure to risk that threatens its stability by transferring it to those most able to bear it (Brunnermeier (2008); Pennacchi (1988)). Therefore banks benefit the healthy spread due to securitization. Another advantage is that investors' desire to access a high yield on rated investments is satisfied.

Credit risk transfer (CRT), however, can have a blurred outcome on the fragility of the financial system (Allen and Carletti (2006)). On one hand it is helpful specially if the financial system is fragile and the credit risk is transferred to the non-bank sectors; it allows banks to better diversify their risks which improve financial stability (Wagner and Marsh (2006)). On the other hand, higher portfolio diversification caused by credit risk transfer can reduce financial stability and in this manner confirms the ambiguous implications for total welfare (Wagner (2005)).

Credit risk transfer can lead to contagion across different sectors in the financial system and thereby to a reduction in total welfare (Kiff and Kisser (2010)). It can also lead to an unprecedented credit expansion that helped feed recent financial crisis (Brunnermeier (2008)).

Wagner (2007) shows that although credit risk transfer improves asset liquidity for banks it also encourages them to take on higher risks which then offset the positive impact of higher liquidity.

So far some advantages and disadvantages of securitization have been discussed. These have also been considered in numerous other articles (Dell'Ariccia, Igan and Laeven (2012); Demyanyk and Van Hemert (2011); Gorton (2009); Gorton and Metrick (2012); Kashyap and Stein (2000); Keys, Mukherjee, Seru and Vig (2008); Keys, Mukherjee, Seru and Vig (2009); Kothari, Loutskina and Nikolaev (2006); Loutskina (2006); Loutskina and Strahan (2009); Mian and Sufi (2008); Morrison (2005); Parlour and Plantin (2008); Rajan, Seru and Vig (2010)).

The disadvantages of securitization are the result of some problems which are discussed next.

1.2 Problems of securitization

The above evidences, taken together, suggest that the secondary loan market has significantly transformed the nature of the banking activities and the borrower and lender relationship. Traditionally, banks held the issuing loans until they are repaid; they produced information on the nature of borrowers (screening) and monitored the borrowers of originated loans. The emergence of an active secondary market for bank loans in which loans are pooled, tranched, and then resolved through securitization, possibly diluted screening and monitoring activities carried out by banks (Diamond (1984); Holmstrom and Tirole (1997)) even if it allows for additional loans to be made.

In this way, although banks uphold a central role in originating loan and evaluating credit risk, they lose their importance as primary holders of illiquid loans (Altunbas, Gambacorta and Marques-Ibanez (2009); Gupta, Singh and Zebedee (2008)).

This transformation has some negative consequences like increasing complexity and reducing transparency in loan origination that leads to breakdown of lending relationships. Furthermore, the securitization process reduces the market participants' incentive to learn about underlying collateral information (Park (2013)).

Reduction in monitoring, in addition to breakdown of lending relationship, leads to suboptimal investment decision and harsher covenants (Drucker and Puri (2009)), as well as creating difficulties in debt renegotiation (Carey, Prowse, Rea and Udell (1994)). The participating investors who buy the loans without having a lending relationship with the borrowers are then expected to be at an information disadvantage when buying a loan originated by a bank. Selling the loans in the secondary market (securitization) could result in moral hazard and adverse selection problems (Gorton and Pennacchi (1995); Pennacchi (1988)). Because of securitization only low quality loans are securitized (adverse selection) and loans that can be sold are not initially screened, or not subsequently monitored (moral hazard).

Keys *et al.* (2008), Demyanyk & Van Hemert (2011) and Dell'Ariccia *et al.* (2012) investigate the reduction in lending standards due to securitization. Gorton (2010) goes one step further by considering structure of MBS in order to find out why this reduction happens. Parlour & Plantin (2008) established a theoretical model to show evidence of some of the above outlined effects. Nevertheless, from an empirical standpoint, it is not obvious which of these effects dominate.

The emergence of secondary loan market deteriorated the role of the monetary policy. Securitization provides banks with an additional funding sources and may weaken the impact of central banks on the lending channel by reducing the efficiency of the traditional interest-rate policy (Kuttner (2000)). Estrella (2002) suggests that central banks should rely on mechanisms

beyond the interest rate, in order to increase the impact of monetary policy. Stein (2011) explains how monetary policy can affect bank lending and real activity to achieve financial stability.

Another problem within the secondary loan market that is related to securitization process is poor credit quality of structured products. One reason for this problem is attractiveness of structured products for rating agencies due to higher collected fees that they bring out for rating agencies which leads to superior ratings for structured products compared to other bonds and securities (Brunnermeier (2008)). These credit rating agencies' incentives to issue higher rating was also considered as a possible cause of recent financial crisis.

Another reason for poor quality of structured products is that rating agencies were not diligent in assessing credit risk associated with off-balance sheet securitization activities. Barth, Ormazabal and Taylor (2011) examines the association source of credit risk with asset securitization and find that securitizing firms' credit risk is not only associated with retained portion of the securitized assets but is also associated with the non-retained portion of the securitized assets which is not considered in assessment of rating agencies. On the other hand rating agencies perceive that firms' credit risk exposure is only associated with the contractual retained interest in the securitized assets and not associated with the non-retained portion of the securitized assets. Rating agencies' assessment of the firm's credit risk is based on private information about asset securitization that can be presented by credit ratings. Credit ratings can be affected by factors other than the credit risk of the bank, such as rating agency incentives for issuing particular ratings. Credit risk can also be measured by bond spreads which is based on publicly available information about asset securitization. Bond spread measurement might be incomplete, but is not likely affected by incentives for assessing a particular level of risk. As a result credit risk assessments by bond market can be more reliable than those by rating agencies. Bond market assesses that the firm's credit risk

is associated with both the retained and non-retained portions of securitized assets which indicate the deficiency of rating agencies in assessing credit risk.

Higher ratings by rating agencies provide firms with opportunity to obtain additional financing at lower cost which can be used to fund further asset securitization (Morgenson and Story (2010); Rosenkranz (2009); Sorkin (2009)). On the other hand these higher credit rating, give the banks a wrong sense of confidence that they are far away from any trouble and encourage them to take on tail risks through issuing more short term claims rather than long term claims. This tendency increases the possibility that banks become illiquid and incapable of rolling over financing (Diamond and Rajan (2009)).

The above situation can become worse by increasing the incentives of levered institutions to become more illiquid with the expectation that future interest rates would be low (Diamond and Rajan (2009)). Now it becomes obvious why banks were willing to take illiquidity risk in case of a sharp downturn, especially when there was a great possibility of increasing the liquidity and cutting the interest rates by Federal Reserve (the so-called "Greenspan Put").

In general the nature of modern banking is unstable and risky. Shleifer and Vishny (2010) present a model in the context of securitization and leverage and show that a levered bank is naturally volatile. Gennaioli, Shleifer and Vishny (2012) show that even without leverage, financial intermediaries can be volatile and fragile, because investors neglect certain unlikely risks.

In summary, the securitized products are affected by deficiencies in the following aspects:

- Complexity,
- Transparency,
- Contracting,
- Rating,

- Pricing, and
- Regulation

These could lead to major problems, such as adverse selection and moral hazard.

1. 2. 1 Adverse selection

The complexity due to the securitization process, which bundles the loans, tranches them into different risk categories and then sells diverse packages of loans is that it decreases the transparency of loans' quality to participating investors. This reduction in transparency together with bank's superior information about the borrowers' quality give rise to information asymmetry between lenders and investors which leads to an adverse selection problem.

Adverse selection problem results in decreasing the quality of the loans in the securitized pool. Elul (2011) applies a regression approach to identify the relationship between securitization and loan performance and he found that securitized mortgages perform worse than portfolio loans. He attributes this result to adverse selection of poorer loans into securitized pools.

Downing, Jaffee and Wallace (2009) offer a sharp-test to evaluate the adverse selection problem in the context of federally guaranteed mortgages. Their findings provide strong empirical support to verify that the quality of securitized assets is lower ("lemons") compared to assets that are not sold through special purpose vehicles (SPVs).

There is vast uncertainty over how these securities should be valued which leads to considerable fear of information asymmetries about the quality of the underlying assets and banks' exposures to these securities. The issue of adverse selection raises the question for participating investors if they can trust the banks, in the sense that banks are not selling lemons. Are banks selling loans of borrowers about whom they have negative private information that is unobservable by outside investors or are they selling off the loans due to appropriate motives such as capital relief, risk diversification, improving balance-sheet liquidity, and reducing financing frictions and their cost of capital?

Akerlof (1970) shows that, in the context of incomplete markets, informational asymmetries may lead to "lemon problem" that cause the breakdown of used-car market. Hart and Holmstrom (2008) and Hellwig (2009) pay attention to the same "lemon problem," in the secondary loan market after the onset of financial crisis. They demonstrate that during financial crisis, there were a lot of concerns about the quality of the loans due to rise in information asymmetry about the quality of trading assets. Market participants performed as they expected to do in the perfect market. Investors pulled out their money and demanded a huge discount on any unknown quality assets. Uninformed investors did not fear that the sellers were trying to get rid of low quality assets while keeping the good ones. As a result liquidity in the market decreased and the price declined as the quantity sold increased (Firla-Cuchra and Jenkinson (2005)).

Decline in the pricing system aggravated the difficulties in the financial market (see Hellwig 2009). One of the main reasons behind this turndown in the asset valuation of banks was not only the loss on the underlying assets of securities, but also the general reaction of the market participants to the lemon problem. As a result of this awareness about the lemon problem and the involving risk spread among all market participants, all such assets lost their value and this process extended from a small sector of the market through the rest. Great part of banks' holding assets suddenly lost much of their value. The pricing system undermined as expectations changed (Colander, Goldberg, Haas, Juselius, Kirman, Lux and Sloth (2009)).

Colander, Goldberg, Haas, Juselius, Kirman, Lux and Sloth (2009) and Gorton and Souleles (2007) reinvestigate the pricing of asset backed securities and informational role of financial prices and financial markets in the presence of the lemon problem. They show that before securitization, asset trading could transmit information, while in the presence of securitization this information transmission seems to have broken down. They interpret this as securitization has to somewhat

bring about a loss of information by anonymous intermediation (often multiple) between borrowers and lenders. In his way, the buyer of structured financial products would not have spent any time and effort to gather information regarding his far away counterparts, and the informational gathering has been outsourced to rating agencies. This centralized information processing by rating agencies, rather than the dispersed one in traditional credit relationships, initiated a severe loss of information. As a result, standard loan default model have fallen short considerably in recent years (Rajan, Seru and Vig (2010)).

Information asymmetry can also play a crucial role in the existence of systemic risk through its impact on the expectations of market participants. The assessment of various systemic events highlights the importance of distribution of financial contracts information among market participants. Revelation of accurate information can prevent the constitution of wrong expectation among market participants which may leads to individually rational but not socially optimal systemic event (De Bandt and Hartmann (2000)). In the following we show how asymmetric information and expectations may leads to systemic events.

Assume that a bank has started selling lemons in the market. If the information about these loans and related bank were released in full, it is individually rational for investors to withdraw their money and force this bank to liquidate. But information asymmetry aggravates the negative external present of one bank's failure on the health of other banks. As a result of this information asymmetry, investors lose their trust in the banks (wrong expectation), in the sense that they are selling loans of borrowers about whom they have negative private information. Because of this awareness about the lemon problem, investors pulled out their money and all such assets lose their value and this process extended from a small sector of the market through the rest. This systemic event might still have been individually rational but it is not socially optimal because it leads to capital losses due to decline in asset valuation below the normal market level.

The lemon problem can also lead to a volatile financial market because the levered banks which are depending significantly on investors' sentiment have to liquidate a fraction of this assets on balance sheet in order to keep a haircut level (the margin between the actual market value of a security and the value assessed by investors' sentiment) even though the actual quality of these assets is high (Shleifer and Vishny (2010)). The lemon problem decreases the market value of collateral securities which makes the investor to evaluate these securities as being worth less than they actually are to give itself a cushion.

Alternatively originate-to-distribute model of credit may persuade banks to originate bad loans to increase their fee income and then sell them off in the active secondary market which may lead to underperformance of borrowers in long run along with valuation loss of their total assets in comparison with their peers (Berndt and Gupta (2009)). Additional detailed discussions of adverse selection problem can be found in (Ashcraft and Schuermann (2008); Gorton (2009); Rajan, Seru and Vig (2010)).

1. 2. 2 Moral Hazard

Due to the increase of securitization, credit quality and lending standard have declined. This is because other financial institutions, rather than the banks, bear the major part of the securitized risk. Banks basically face only the pipeline risk of holding a loan for some months until the risks are passed on to investors (Angelides and Thomas (2011)). If banks are not exposed to the default risk of the originated loans they would not have enough incentive to be vigilant in approving loan applications and monitoring loans in order to control and uphold the loans' quality. Taking off the loans from the banks' book, reduces their incentives to be engaged in costly screening and monitoring of the borrowers (Berndt and Gupta (2009)).

One concern that is frequently raised in the literature is that securitization leads to moral hazard in lender screening and monitoring (Berndt and Gupta (2009); Brunnermeier (2008); Donnelly and Embrechts (2010); Fender and Mitchell (2009); Gorton and Pennacchi (1995); Keys, Mukherjee, Seru and Vig (2008); Mian and Sufi (2008); Parlour and Plantin (2008); Rajan, Seru and Vig (2010); Selody and Woodman (2009))

Keys, Mukherjee, Seru and Vig (2008) suggest empirical evidence that increases in securitization lead to moral hazard in lender screening and considerably deteriorates the credit quality of underlying loans. They examine this issue empirically by using data on securitized subprime mortgage loan contracts in the United States. They investigate that the probability of being securitized is higher for the loans with FICO credit score (threshold) above 620. Conditional on being securitized, the ease of securitization is greater for this portfolio of the loans. At the same time they showed that in a portfolio with greater ease of securitization, default rate increases by around 10% to 25% more than in a similar risk profile group with a lesser ease of securitization. On the other hand loans with FICO credit score 620 or higher are more likely to be securitized, which leads to more delinquencies in these loans in comparison with loans with credit score less than 620 which have less prospect of being securitized. After controlling for other variables which do not show any discontinuity around this threshold (FICO credit score 620), they conclude that securitization is the main reason for the increase in default rates through decreasing the lenders' incentive to screen borrowers above this score.

After accepting the term of the loan contract by borrowers and signing it between the lender and borrower, the loan can be sold as part of the securitized pool to investors. When investors buy these loans as part of a securitized pool, they only notice the "hard information" about the borrowers (e.g. FICO score) and the contractual terms such as loan-to-value (LTV) ratio and interest rates (Keys, Mukherjee, Seru and Vig (2008)). "Hard information" is defined as information that is easy to measure, transmit and contract upon. To rate tranches of the securitized pool, rating agencies use the same information about the borrowers and loan terms as investors use to buy the loans. Since securitization increases the distance between the originators and investors, and "soft information" (e.g., a measure of future income stability, provided years of documentation and joint income status) about the borrowers. "Soft information", by definition, is something that is not easy to contract upon and transmit, while the lender has to exert an unobservable effort to collect soft information (Stein (2002)).

Accordingly, even though there is compensation for lenders to gather "hard information" about the borrowers, their incentives to collect "soft information" depend on the extent to which they are supposed to bear the risk of originated loans (Gorton and Pennacchi (1995); Parlour and Plantin (2008); Rajan, Seru and Vig (2010)).

Keys, Mukherjee, Seru and Vig (2008) proposed that the only way that may persuade lenders to incur the cost of obtaining "soft information" is that the signal provided by the borrowers' "hard information" is not as much necessary while there is enough chance that lenders would retain the loan on its balance sheet.

Keys, Mukherjee, Seru and Vig (2008) claim that lenders are less likely to expend effort to process "soft information" as the ease of securitization increases. Alternatively any residual differences in default rates around the threshold, should only be due to the decrease in the lenders' screening effort on the soft information dimension because of increase in the ease of securitization. Rajan, Seru and Vig (2010) provide evidence that securitization reduces the incentives to collect "soft information" which led to origination of loans with low quality and more delinquencies.

Parlour and Plantin (2008) show that this potential defeat in the monitoring incentives reduced the traditional bank specialness aspect. They use a theoretical model to demonstrate the reduction in the banks' incentives to monitor borrowers in the presence of secondary market loan trading. Liquid loan market enables the banks to detach balance sheet management from borrower relationship management, which leads to moral hazard in terms of destruction in the monitoring activities of banks. This has a negative effect on the borrowers. Borrowers might suffer from value destruction in the long run due to weaker relationship with banks and reduction in monitoring discipline (Berndt and Gupta (2009)).

Loss in monitoring incentives could potentially lead to a transfer of wealth from the bondholders to the shareholders of the borrowing firm due to the increased risk shifting by the shareholders of the borrowing firm at the expense of borrowing firm's bondholders (Gande and Saunders (2012)). From a longer-term perspective, there are some studies proving that any reduction in monitoring incentives is beneficiary for borrower's shareholders due to the favourable impact of new bank loan announcements on borrowers' stock returns (Best and Zhang (1993); Billett, Flannery and Garfinkel (2006); Lummer and McConnell (1989)).

On the contrary, Billett, Flannery and Garfinkel (2006) show that the bank loans announcement resulted in the negative abnormal returns in the long-run for the borrowing firm. From a short-term perspective, most studies have shown positive abnormal returns for borrower's shareholders in contrast to the announcement of other forms of corporate financing such as common stock, preferred stock, straight debt and convertible debt (Berndt and Gupta (2009)). In general the literature, on the effects of loan sales on the borrower's stock price is rather controversial. While Dahiya, Puri and Saunders (2003) recognized a negative effect of lending bank announcement of the sale of a borrower's loans, Gande and Saunders (2012) documented the reverse (positive) announcement effect.

1.3 Solutions

At present, several policy questions arise from the above discussions for regulatory authorities. The first question is that, in spite of all these benefits and weaknesses mentioned earlier, does the shift to originate-to-distribute model create value in the financial system or not? Is it socially desirable? If so, should these authorities put any restriction on these activities of the banks in order to skip the costs and take more advantage from them that leads to additional value creation? Should the regulatory authorities put into effect the enhancement of disclosure of the information about banks' activities in the loan sales market? What will be the long run effect of these regulations on the borrowing firms and individuals, the banking system and, overall, the market?

Empirical evidence alone is not sufficient to answer all of these questions, as there are both positive and negative effects of originate-to-distribute model. Ultimately, these issues must also be considered theoretically which the focus of our research is.

In spite of all the advantages that securitization brings out to the economy and the financial system, the financial crisis that began in August 2007 drew attention to this fact that advantageous financial innovation such as securitization can become a source of financial instability if industry practice and regulation do not keep pace with innovation (Selody and Woodman (2009)).

It is becoming important for regulators and market participants to understand the costs and benefits of securitization so that they can appropriately improve the incentives and scope of securitization to mitigate this information cost. Berndt and Gupta (2009) believe that the highly deregulated nature of the secondary loan market is perhaps one of the main reasons for the occurrence of the moral hazard and adverse selection problems. Stein (2011) explains the consequence of unregulated private money creation on establishment of unstable market which make it necessary to put in force supplementary policy together with open-market operation.

In summary we can mention the general solution for previously mentioned deficiencies:

- Complexity: reducing the complexity of securitized products
- Transparency: enhancing the availability and quality of information, making
- securitization transparent and based on standards
- Contracting: improve contractual part of securitization by carefully setting out the criteria for loan eligibility for a pool

19

- Ratings: improving the reliability and use of ratings by imposing new regulations on the rating agencies, given the great responsibility generally attributed to their perceived failures to mitigate rating agencies conflicts of interest.
- Pricing: improve market functioning, take into consideration the risk premia and liquidity premia for the pricing of structured products and impose new regulation to oblige banks to have sufficient funds to satisfy their solvency.
- Regulation: propose specific rules addressing both traditional financial institution and other components of the "shadow banking system" focusing on the reforms to how securitization should be done

Reviving securitization markets and bringing back investors' confidence call for a coordinating effort on all industry participants, investors, and regulators. There must be explicit or implicit contractual design features to mitigate these obvious problems. Improving the design of securitized products can result in significant reductions in the uncertainty surrounding credit quality and a reduced need for monitoring (Selody and Woodman (2009)).

With the intention of achieving the above goal, it is necessary to change the structure of securitized products in the way that they become less complex and opaque while at the same time they ensure the alignment of incentives among various participants in the intermediation chain (Fender and Mitchell (2009a); Paligorova (2009))

1. 3. 1 Solutions for adverse selection

The adverse selection issue has been studied extensively in the corporate finance and insurance literatures. Different solutions can be extended for securitization to convince investors that there are no incentive problems and reduce the agency problem. For example, DeMarzo and Duffie (1999) develop a "hidden knowledge" model, in which the issuer earns the knowledge after signing the contract. In this models, retaining a larger fraction of the issue is viewed as a signal of high project value for privately informed issuer (DeMarzo and Duffie (1999); Leland and Pyle (1977)). Implicit contract features such as the part of the loan retention or implicit guarantees against default may make loan sales possible and in that way reduce the adverse selection problem (Gorton and Pennacchi (1995)).

Recently some important models of securitization with the focus on asymmetric information issue were proposed. Glaeser and Kallal (1997) and Riddiough (1997) address the adverse selection problem in which an informed issuer optimally designs risky asset-backed securities with asymmetric information and liquidation motives and then sells them in the market. This requires the creation of low-risk and low information-sensitivity securities.

Dang, Gorton and Holmstrom (2009) support this idea that the issuance of information-insensitive securities can resolve adverse selection problem. They approve that the attractiveness of security design come from the fact that it reduces the incentives to privately acquire information. In this case the value of these securities is independent of the information known only by the informed issuer. Information-insensitive securities are liquid and can be traded easily in the financial market. Park (2013) empirically demonstrates that in reality the main reason of applying securitization is to generate information-insensitive securities. In this case the credit risk of the underlying

collateral can be revealed by credit enhancement procedures, including tranching. Tranching technique can also reduce the overall adverse selection problem of banks engaging in loan sales and maximize financial intermediaries' proceeds (Gorton and Pennacchi (1995)).

On the contrary, Axelson (2007) and Plantin (2004) propose optimal security designs while assuming that investors have private information and originators are uninformed. In this case it is optimal for the originator to issue a security that is information-sensitive, such as the equity. Gorton (2008) link the summer 2007 financial crisis to the traditional bank run due to transformation of information insensitive securities to information sensitive securities which is the result of aggregate shock from the declining values in the U.S housing market.

Equity tranche retention can be viewed as a signalling device of unobservable quality of securitized portfolio. Drucker and Puri (2009) suggest that in order to overcome the effects of asymmetric information and mitigate lemon problems, banks may choose to securitize loans about which they have a relatively low amount of soft information. They also could contribute in holding the risk of the securitized portfolio by retaining the riskiest part of the portfolio (equity tranche) as a signalling device (Albertazzi, Eramo, Gambacorta and Salleo (2011)). DeMarzo (2005) suggest that in presence of pooling and tranching, banks can signal the quality of the sold loan portfolio by retaining interest in the equity tranche which is confirming the optimality of a standard debt contract.

The primary U.S. bank regulators state that implicit recourse may provide banking organization with an incentive to avoid adverse selection. Issuers have incentives to commit to provide implicit recourse if they are involved in repeated-play game (Gorton and Souleles (2007)). Revolving credit card and loan securitizations have distinct features that provide issuers with greater motivations

and ability to provide implicit recourse than in other securitizations (Calomiris and Mason (2004); Higgins and Mason (2004)). Chen, Liu and Ryan (2008) believe that implicit recourse is unlikely to have any significant effect on mortgages and commercial loans securitizations.

Reputational concerns could prevent banks from selling lemons, because they deal with investors on a continuing basis and securitization process is not a once in a lifetime process (Fender and Mitchell (2009a)). On the other hand if the present value of the future profits from securitization would be above the cost of on balance sheet financing, reputational concerns can provide incentive for the banks to mitigate adverse selection problem by determining the likelihood of loan default and select which loan to put into the SPV (Gorton and Souleles (2007)). Banks might even choose to securitize loans of better-than-average (although unobservable) quality, when trying to improve their reputation (Albertazzi, Eramo, Gambacorta and Salleo (2011)). Gande, Puri, Saunders and Walter (1997) and Kroszner and Rajan (1994) emphasize the same dynamics for banks who underwrite the securities that are issued by their borrowing firms.

Albertazzi, Eramo, Gambacorta and Salleo (2011) investigate adverse selection problem of securitization from the empirical contract theory view point and its alleviation by analyzing the rich dataset on securitization in Italy. Overall, they suggest the following solutions to alleviate the adverse selection problem: selling less opaque loans, using signalling devices (i.e. retaining a share of the equity tranche) and building up a reputation for not undermining the lending standards.

Pooling of assets have an information-destruction effect that may augment adverse selection problem, but at the same time it can play a role in overcoming this problem faced by uninformed investors (Firla-Cuchra and Jenkinson (2005)). DeMarzo and Duffie (1999) and DeMarzo (2005) develop models to demonstrate the trade-offs between adverse selection issue of pooling assets

(because it eliminates the advantage of asset-specific private information "information destruction effect") against "risk-diversification effect" of pooling (because it creates a potentially large lowrisk pool, and associated securities, that are less sensitive to the seller's private information) for informed issuer. They prove how tranching can be optimal together with pooling in presence of asymmetric information, for large enough pools of assets. The intuition is as follow: as the size of asset pool increases, the information destruction effect which leads to illiquidity can be outweigh with the risk diversification effect of pooling.

Axelson (2007), Boot and Thakor (1993), Plantin (2004) and Riddiough (1997) also add some explanations to the value creation effect of pooling and tranching under asymmetric information by proposing theoretical models in the presence of various investors with different level of private information. The basic intuition of their models is that there is a separating equilibrium by creating an essentially riskless senior tranche which attracts unsophisticated investors who have low ability to screen the underlying assets. Investors with more private information are attracted to more junior tranches which allow the banks to re-cycle their capital and to raise the return to their private information ratio.

On the other hand, the optimal security design allows banks to restructure the lemon pools into tranched securities and overcome the adverse selection problem between informed investors who buy the riskier junior tranches and uninformed investors who buy the senior tranches. As Riddiough (1997) suggest even if asymmetric information steer the creation of a senior tranche, several junior tranches might be formed to serve particular tastes of different investors with the aim to facilitate the placement of the information-sensitive tranches in the market.

Agency conflicts can also play an important role in amplifying the adverse selection problem. Drucker and Mayer (2008) point at the exploitation of inside information by underwriters due to their advantage in secondary loan markets. In association with this finding, Drucker and Puri (2009), Gorton and Pennacchi (1995) and Sufi (2007) look into the ways to mitigate these agency conflicts through appropriate contract terms and conditions.

Regulatory supervision in the secondary loan market together with additional disclosure requirement on all market participants with the aim of better transparency could, to some extent, reduce agency conflicts and as a result resolve the adverse selection problem (Berndt and Gupta (2009)).

1. 3. 2 Solutions to moral hazard

Regulatory oversight alone cannot resolve moral hazard problem which is the result of information asymmetries in the OTD market on the lenders side. Keys, Mukherjee, Seru and Vig (2009) support this idea by examining the consequences of existing regulations on the quality of mortgage loans originations in the OTD market with the purpose of mitigating moral hazard problems. They find that the quality of loan origination varies inversely with the amount of regulation; more regulated lenders originate loans of worse quality. On the other hand the overall default rates for less regulated banks were lower than high regulated banks.

As an alternative stronger risk management departments with greater bargaining power inside the bank may have the power to resolve the moral hazard problem. By measuring the share of risk manager's compensation from the total compensation which is given to the five highest-paid executives in the institution, Keys, Mukherjee, Seru and Vig (2009) show that brokers with a powerful risk management department have lower default rates on the originated mortgages. Ellul

and Yerramilli (2013) also provide some evidence that financial institution with a weak risk management department are more prone to take excessive risk that brought about the financial crisis.

Having more lenders inside a mortgage pool which is associated to more competition leads to having a portfolio of loans with higher quality. Higher diversity reduces default rates and provides this opportunity for the issuer of the pools to benchmark the quality of the loans against each other. Keys, Mukherjee, Seru and Vig (2009) indicate that more competition among lenders result in better performance evaluation and consequently to some extent can mitigate the moral hazard problem. This relative performance evaluation could somewhat mitigate the moral hazard problem (Gibbons and Murphy (1991)).

Reputational concerns play an important role in mitigating moral hazard problem. Keys, Mukherjee, Seru and Vig (2009) provide some evidence that recommended banks with higher reputation tend to be more conservative and try to keep their quality by, for example, keeping more deposits, or more liquid assets which results in origination of more carefully screened and higher quality loans in the OTD market. Overall, their evidence suggests that using market forces as an internal device to align lenders' incentives with that of the investors (like the one discussed in our research), is more efficient in mitigating moral hazard in the OTD market than external policies that impose stricter lender regulations which fail to align lenders' incentives.

One possible solution for moral hazard problem, as an internal device to align lenders' incentives, is to impose restriction on the originated banks by requiring them to keep at least a certain percentage of those loans on their books or to have skin in the game. This can be interpreted as the fragility of lightly regulated originators' capital structure and support the Dodd-Frank Law

approach intended to mitigate moral hazard by requiring a minimum level of risk retention by originators.

Selody and Woodman (2009) propose that retaining a portion of an issue of a new debt with intention of sharing the default risk of loans can result in the improvement of incentives alignment. This may prevent banks from originating bad loans and give them more incentive to monitor borrowers (Berndt and Gupta (2009)).

If participation of banks in risk sharing is sufficient, they would have enough motivation to perform appropriate due diligence on loan origination, continuously monitor the behaviour of borrowers, and, perhaps, represent warranties on the quality of loans and the underwriting process (Selody and Woodman (2009)).

Donnelly and Embrechts (2010) go one step further by proposing that the banks should hold onto the riskiest part of the loan pool in order to be exposed to the risk of loan defaults and have enough incentives to control and preserve the quality of the originated loans. In practice, this did not always take place, and as a result misalignment of incentives may have played a role in reducing the lending standards and distorting the quality of loans originated in the OTD market (Brunnermeier (2008); Dell'Ariccia, Igan and Laeven (2012); Keys, Mukherjee, Seru and Vig (2008); Mian and Sufi (2008); Rajan (2006)).

Overall, tranche retention is considered as one of the most effective ways to align incentives between originators and investors. There exist different retention mechanisms with considerably different impact on the originators' efforts and incentives in screening and monitoring the borrowers. Three general types of tranche retention are as follows:

- Equity tranche retention
- Mezzanine tranche retention
- Vertical slice retention

Vertical slice retention is referred to retaining a percentage share of each of the tranches. Different retention mechanisms have different impact on the screening level, due to different sensitivities to a systematic risk factor which plays an important role in the determination of borrowers, default probabilities and asset values (Fender and Mitchell (2009a); Fender and Mitchell (2009b)). Showing different sensitivities implies that the effectiveness of tranche retention in aligning incentives will be a function of tranche thickness, the return of assets, the size of the retained interest, the economy's position in the cycle (the state of the macro economy) and most importantly how it is configured.

Vertical slice retention might be suboptimal in aligning incentives to monitor borrowers. Equity tranche is more sensitive to the realisation of systemic risk than the entire portfolio. Because the equity tranche will be exhausted when there is a large probability of a systemic risk it could decrease the originator's incentive to make a screening effort. In this case it would be better to hold also mezzanine tranche (Fender and Mitchell (2009b)). Their focus is on associating different screening effort across different retention mechanism rather than deriving an optimal profit maximization contract for the originator.

Chen, Liu and Ryan (2008) investigate the determinants of the size of the equity tranche retention by estimating the association between banks' equity risk and the characteristics of the securitized loans. They find that banks must retain larger equity tranche when the pool is riskier or the credit risk is less externally verifiable to outside investors. In other words the structure of retention is not independent of the risks of the pool (underlying portfolio). Their result is consistent with Dionne and Harchaoui (2008), Haensel and Krahnen (2007) and Niu and Richardson (2006) who investigate the relation between the measures of firms' equity risk to their off-balance sheet positions arising from securitizations. These papers conclude that the amount of equity risk which is endured by firms is associated with both the retained and non-retained portions of the securitized assets. Chen, Liu and Ryan (2008) show that the amount of equity risk associated with asset securitization varies with the type of asset securitized. Moody's Investor Service (2002) also investigates the link of ultimate amount of risk transfer to specific structure of the transaction. These findings are related to the literature on the equity risk-relevance of other off-balance sheet positions and suggest that the association of firms' equity risk with asset securitization can depend on different characteristic of securitized assets.

Selecting an appropriate type of retention scheme is really critical, because wrong retention scheme could cause unintended costs and consequently slow down efforts to restart sustainable securitization markets. It is also really critical to determine the amount of retention requirement wisely and based on precise calculation which is the focus of our research. When retention requirements are too low, screening incentives may not be sufficiently high, but if requirements are too high, securitization may no longer be an economical form of finance (Selody and Woodman (2009)). The most capital constrained banks securitize the least (Minton, Sanders and Strahan (2004)).

United States government and the European Union (EU) parliament required 5% of uniform mandatory risk retention, in the form of vertical slice with a fixed ratio (IMF, 2009). This retention scheme was criticized in recent papers. Most criticisms focus on the vertical slice retention; because it is not optimal in aligning incentives of financial intermediaries to monitor borrowers

(Fender and Mitchell (2009b); Kiff and Kisser (2010)). Jeon and Nishihara (2012) also point out the weakness of risk retention requirement from other aspect of the current regulation. They pay attention to the impact of fixed ratio on risk retention efficiency which is uniformly applied to every financial institution, without considering features of individual intermediary or business cycle. Wu and Guo (2010) show the deficiency of flat ratio of risk retention and recommend that the information disclosure requirement is more efficient than risk retention in this sense. Dugan (2010) claim that fixed risk retention will worsen the situation and suggest a minimum underwriting standard requirement instead. Levitin (2013) state that risk retention cannot resolve the moral hazard problem of credit card securitization but implicit recourse can resolve this issue. Batty (2011) addresses the current regulation may shrink the issuance of collateralized loan obligations (CLOs) which most of them are actively managed by third party managers.

Optimal security design also plays an important role in resolving the moral hazard problem which resulted in reviving the securitization market and assuring the optimality of securitization process as there is theoretical argument supporting this claim (Albertazzi, Eramo, Gambacorta and Salleo (2011)). DeMarzo and Duffie (1999) investigate optimal security design focusing on the trade-off between liquidity costs and retention costs. Hartman-Glaser, Piskorski and Tchistyi (2012) present an optimal mortgage-backed security design in a continuous basis model.

Finally, if we could find an applicable solution based on appropriate retention scheme to provide enough incentives for screening and monitoring the borrowers, as it is the focus of our research, we could guarantee the optimality of securitization process from an optimal security design perspective.

1.4 Design of research on moral hazard

We are among the first ones who investigate the optimal security design of structured products by analyzing the explicit incentive contracting under moral hazard. Our goal is to address moral hazard problem using a principal-agent model where investor is the principal and lender is the agent. Moral hazard is often defined broadly as the conflict of interest between two parties who are interacting with each other while it is not possible to determine the detail of information interaction. If investors could design a complete contract that specifies how originators should screen borrowers and what kind of information they should gather, there would be no moral hazard problem. Since in the real life screening and monitoring activities are not observed by investors or they cannot specify these behaviours of originators, then the screening behaviour taken before or after signing the contract will be distorted by selling loans to investors and removing them from the lenders' book, because originator is already paid for the securitized loans, and the amount of this payment does not depend on the screening and monitoring effort of the originator.

Securitization operates similar to partial risk sharing. Without securitization, all the costs and benefits of screening and monitoring are internal to the lender and incentive to screen and monitor is optimal. With securitization, originators sell some parts of their risk to a third party and do not bear the full cost of loan default. The more the investors contribute in covering the default cost, the less the incentive of lenders is to carefully screen and monitor borrowers. We show how we can mitigate moral hazard in screening and monitoring by specifying the optimal amount of the loan pool that should be kept by lender to maintain its incentive under ex-ante moral hazard.

In order to proceed in developing our model, it would be helpful to be familiar with the banking system and to know about the basics of principal-agent model as well as the best form of contract

for this kind of model. For this purpose we considered the book, "Micro economics of Banking" (Freixas, Xavier, and Jean-Charles Rochet (1997)) and the papers that will come in the following.

1. 4. 1 Best contract for principal-agent model

In classical principal-agent theory, moral hazard problem discussed in situations like insurance, labour contracting, and delegation the responsibility to make decision, when principal and agent engage in risk sharing circumstance however the action of agent which affect the probability distribution of the outcome is privately taken and unobservable to principal. The source of this moral hazard or incentive problem is an asymmetry of information between both parties. In such a situation Pareto-optimal risk sharing is unachievable due to the incentive distortion for taking correct action under moral hazard. The natural solution to this problem is to monitor the action of the agent and contract upon it. However in real life it is impossible because it is too costly. In such situations the best contract would be the second best solution which trades off some of the risk-sharing benefits for provision of incentives (Hölmstrom (1979)).

Hölmstrom (1979) introduces traditional moral hazard agency model in which the principal is risk neutral and the agent is risk averse. He suggests that when there is uncertainty in the evaluation of the manager performance (e.g. talents, exerted effort, and consumption on the job), then riskaverse manager will always choose to share part of these uncertainty in the measurement of manager's actions with the firm's risk bearers. He shows the optimal form of contract in this situation. It is determined through the distribution function that maps the manager actions (e.g. the importance of his decision and his ability to pay for the cash flow ownership upfront) into his riskaversion. The contract will be more convex when the manager is less risk-averse and the distribution function is more skewed. Innes (1990) investigates the principal-agent problem when effort is not contractible and observable. He considers a risk neutral entrepreneur with limited liability and access to an investment project who makes an unobservable effort choice influencing the probability of investment project' success and an outside investor who provides the required financial support for the project. He proves that in this situation, debt financing is the optimal contract and a limited liability restriction can introduce convexity. Based on this classical principal-agent theory while effort choice is unobservable, non-contactable and costly (screening and monitoring), to make the manager's residual claim more sensitive to performance, we should assign the senior claims to investors. In other words, retention of the securities that are most sensitive to the seller's private information is optimal for incentive purposes as a skin in the game. We apply his classical principal-agent theory in the development of our model for security design analysis to compare implied screening and monitoring effort across different retention mechanisms.

Gale and Hellwig (1985) show that in the context of information asymmetries, the optimal form of the contract is a standard debt contract. Diamond (1984) shows that standard debt contract is the optimal contract of financial intermediation based on delegated monitoring. DeMarzo (2005) confirms optimality of standard debt contract by showing that banks can signal the quality of the sold loan by retaining interest in equity tranche.

Morrison (2005) shows that the signalling value of debt can be demolished with credit derivatives in presence of credit risk transfer, which lead to disintermediation and lower welfare. Credit risk transfer with limited credit enhancements can improve loan monitoring and increase financial intermediation which may demolish the optimality of pure debt contract. Since in this case, the high return is not necessarily the outcome of monitoring but instead it is the outcome of realization of the favourable credit risk transfer (Chiesa (2008)). Even in the context of credit risk transfer it is still possible for banks to signal their types by using first-to-default contracts (Antonio and Pelizzon (2005)).

Winter (2000) points out, in the context of insurance shot, moral hazard rises because there is not a deterministic relation between contractible and non-contractible variables which leads to not achieving the first best by means of a non-linear contract. He also shows that the optimal form of the insurance contract in the presence of moral hazard includes a deductible. If an insurance company covers the full loss, the incentive of insured agents for taking precautions action decreased. The use of deductibles in insurance contracts restricts the coverage to some part of the loss not all of them, thus provide incentives to take precautions. On the other hand the higher the insurance coverage, the lower the level of precaution.

There are two contributions related to our paper. DeMarzo and Duffie (1999) study the optimal security design problem under private information or adverse selection. They also obtain an optimal contract with retention (or debt like) but their design is for a better allocation of liquidity instead of an optimal risk sharing between the bank and the investors. Moreover, in their paper, the bank is passive and cannot affect the distribution of the cash flows after the securitization is made.

Osano (1999) considers an incentive problem. He analyzes the problem of security design in the presence of monitoring done by a large investor to discipline the management of a firm such as a bank. He also obtains a debt-like contract but does not derive it explicitly since the model has only two states of nature. There is partial securitization at the optimum in presence of moral hazard but the endogenous form of the optimal securitization is not derived as in this article.

The outline of our research is as follows:

In chapter 2, we propose our principal-agent model without tranching. In chapter 3 with an example we show how to extend our model for structured-asset backed securitization. In chapter 4, we extend our model to consider structured asset-backed securitization with credit enhancement procedure in the presence of the correlation between tranches in the original pool.

Chapter 2:

One Tranche Without Credit Enhancement Procedure

Introduction

Securitization is the process of pooling financial assets such as debt instruments and individual loans and then packaging and converting them into securities and selling them in the form of tranches (prioritized capital structure of claims) or other credit enhancement to third party investors (Kendall and Fishman (2000)).

In Figure 1, the simplified overview of the basics of securitization is shown. First, the financial intermediary or the bank, which is referred to as originator, originates the loans that borrowers are repaying over time. Originator explores lending opportunities either through employing lending officers or by themselves. It is the originators' responsibility to investigate the quality of borrowers. The identified pool of loans should satisfy certain features such as underwriting criteria or lending standards that make them appropriate for securitization. The pool of assets is then sold to a specialpurpose vehicle (SPV), which is a legally separate authorized entity whose management and ownership are independent from the originator (Gorton (2008); Gorton and Metrick (2012)). A SPV is an artificial capitalized entity which is not operating and is established as a trust to purchase the assets and realize the off-balance-sheet treatment for legal and accounting purposes (Paligorova (2009)). The SPV funds these assets by dividing them into different classes of securities, called tranches, and then selling them to dispersed investors in the capital market. Each tranche has its own level of risk and return which is determined through consultation with credit rating agencies who assign ratings based on the credit risk of the asset pool. Generally speaking there are three levels of tranches with different seniority and priority ordering with respect to allocation of losses: equity, mezzanine and senior. The equity tranche is the least senior tranche which is the riskiest and has the last claim on the asset cash flow with the

highest expected rate of returns, while the most senior and least risky tranche, the senior tranche, is the first to receive proceeds from the income generated by underlying assets (Paligorova (2009)). This means that the junior tranche is the first tranche affected by defaults in the loan pool.

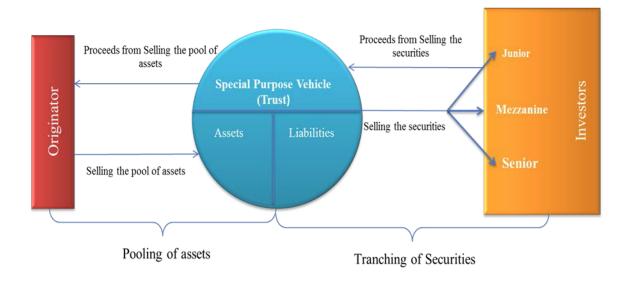


Figure 1: Simplified Overview of the Securitization Process

Securitization has a lot of advantages for market participants. Because of the attractiveness of securitization for market participants, its application has widely been extended in financial markets over the recent years. This increase in the application of securitization is an indication that market participants have better appetite to absorb risks.

Because of these advantages and the willingness of all market participants, regulators change the legislations to accommodate securitization and accelerate its process. In spite of all these advantages and widespread application of securitization, it is often suspected of being one of the main reasons for the recent financial crisis.

2.1 Motivation

There is a growing literature analyzing the causes of recent financial crisis. A key question is whether securitization played a role in the recent sharp rise in mortgage and assets defaults.

One concern that is frequently raised in the literature is that securitization leads to moral hazard in lender screening and monitoring. In the context of securitization, moral hazard refers to a decrease in the incentive of lenders to screen and monitor borrowers. In the absence of securitization they should keep all their loans in their book, therefore lenders would have more incentives to screen loan applicants, because they should bear all the cost in case of default. As a consequence of securitization, originators sell some part of their loans to the investors which lessens their incentive to screen all the loans especially those parts they will employ for securitization.

Securitization changes screening and monitoring behaviour and raises information asymmetry between lenders and investors. The resulting information asymmetry may be costly to the investors, because of the distance between borrowers and investors. The benefits of securitization are thus limited by information asymmetry between lenders and investors and in particular by the costs of this non-transparency. It is becoming important for regulators and market participants to understand the costs and benefits of securitization so that they can appropriately improve the incentives and scope of securitization to mitigate this information cost.

2.2 Contributions

Traditionally originators were supposed to retain equity tranche. Equity tranche retention is one

of the most primary methods of retention that has been applied since the early time of securitization and recently it has been considered as a way to refresh securitization market in the restorative period after the financial crisis. In the beginning of securitization, it was usual for originators to carry on equity part of the tranches, but eventually due to the further investors' appetite for absorbing risk; banks were capable to dispose of them in the emerging active market for equity tranche.

Tranche retention seems to be a wise solution to solve the moral hazard problem of securitization. It is now used frequently by the International Organization of Securities Commissions (IOSCO, 2009), the US Treasury (2009) and the European Parliament (2009) to revitalize securitization markets. To maintain incentives to screen and monitor loans, it seems logical to require lenders to retain part of securitized loans in the form of tranches so that they remain exposed to the risk.

There are also other forms of retention like percentage share retention or mezzanine tranche retention that are offered in these proposals. For example according to the accepted proposal by the European Union, originators are require to keep at least 5% of the securitized portfolio, while they have an option to choose the form of retention. This could be any of the three forms: Equity tranche retention, Mezzanine tranche retention and Vertical slice retention. Vertical slice retention is referred to retaining a percentage share of each of the tranches.

The reason behind all of these proposed solutions is that the more the fraction of loans that are sold, the less the incentive is to screen borrowers since cost of the securitization would trigger investors instead of lenders. To keep incentives to screen and monitor loans, it seems logical to require lenders to retain some part of securitized loans in order to have some skin in the game that keep their incentive in the appropriate level to screen and monitor borrowers.

All of these solutions can be appropriate in the sense that there should be retention but they are not necessarily helpful in order to find the solution to get to the bottom of this crisis. One problem is that the analysis of securitization is very general and suffers from a lack of specific security design analysis under asymmetric information. In other words, there is not yet in the literature an optimal model that endogenously specifies the exact form and amount of optimal retention to keep the lender' incentive at the optimal level.

The achievement of this research is to introduce modeling for optimal security design of structured products under moral hazard. Our research consists of three main steps.

As the first step, we investigated whether it was possible to apply the proposed model while for simplicity we assume that there is no tranching in this step. Since we assume that there is no tranching, our retention could be a fraction of the only one tranche that we assume to have (there is no senior or junior (equity) tranche in this step). We concluded that the proposed model is valid in presence of moral hazard; in the absence of moral hazard, the obtained optimal retention is equal to zero and in presence of moral hazard, it was greater than zero. As a result we introduce a platform that creates the capacity to incorporate practical complexities, e.g., tranching and credit enhancement policy.

The next step will extend this model by incorporating structured asset-backed securitization which inherently applies tranching and credit enhancement procedure.

We can take one step further by considering structured asset-backed securitization with credit enhancement procedure in the presence of the correlation between tranches in the original pool. These correlations were not considered before the recent financial crisis, which led to

41

underestimation of true default probabilities of tranches in the presence of systemic risk. This also resulted in the inaccurate rating of the financial products associated with different tranches.

2.3 The model

In this chapter, we assume that there is only one tranche (pass through securitization) and there is no credit enhancement procedure in the securitization. In this case, the special purpose vehicle issues financial assets that are not prioritized and are simply fractional claims to the payoff on the underlying portfolio. Because the expected portfolio loss is equal to the mean expected loss on the underlying securities, the average portfolio's credit rating is given by the average rating of the securities in the underlying pool (Coval, Jurek and Stafford (2009)).

We consider a risk-averse originator or bank that holds a loan pool with a value normalized to 1. The loan pool is assumed to be large enough to diversify away the idiosyncratic risk. The originator could keep these loans in the balance sheet or securitize them to risk-neutral and competitive investors. The risk-free rate of interest is normalized to zero. We assume that the average default probability of loan pool is p(e), which is a function of the screening and monitoring effort $e \in [0, \overline{e}]$. Effort *e* could be undertaken by lenders to decrease the probability of loan default. We assume p(e) is decreasing and convex in effort:

$$p'(e) < 0, p''(e) > 0, p(0) < 1 \text{ and } p(\overline{e}) > 0.$$
 (1)

Effort is costly for originators. The cost function is equal to C(e) and is increasing and convex in e:

$$C(0) = 0, C'(e) > 0, C''(e) > 0.$$
(2)

In the case of loan default there is a random conditional loss *L*, which is in the interval [0, 1]: $0 \le L \le 1$, with distribution function F(L) and density function f(L). *F* is the distribution of loss, conditional on default. In this article we assume that effort decreases the probability of default without changing the conditional distribution F(L).

The rate of return on the loan pool is R, which is assumed positive: R>0. In the presence of securitization, we assume that the originator can securitize part of the loan pool and keep a fraction α of the loans (retention). We also assume that the originator's decision to securitize or not is made at the same time as loan origination. After securitization, originators can choose their monitoring effort level. Monitoring effort cannot be observed by investors. Another interpretation would be that the average risk is evaluated by a rating agency that does not observe effort. Therefore, it cannot be contractible. Our objective is to find the optimal α .

Securitization generates liquidity for the originator prior to the loan's maturity. As mentioned previously, we assume that third-party investors are competitive. Thus the amount they are willing to pay is equal to the expected value of their payoff from the securitized loans. If the originator securitizes the entire loan pool, it will receive a return equal to 1+S. *S* is the rate of return on the loan pool that takes into account the potential default risk of the loan pool. Therefore *S* is smaller than or equal to *R*. The difference between *R* and *S* is considered as the

sum of transferred expected loss to investors plus the risk premium. The risk premium is the maximum amount that a risk-averse originator is willing to pay above the transferred expected loss to eliminate the default risk of the loan pool by transferring it to the market. If the originator keeps $a\alpha$ fraction of the loan pool, it will receive $(1-\alpha)(1+S)$ from securitization.

2.3.1 Investors' objective function

The objective of the model is to maximize investors' expected return under different constraints. In the non-default state, the investors' return is equal to $(1-\alpha)(1+R)$, because they have only $(1-\alpha)$ fraction of the entire loan pool. If the loan pool defaults, the α fraction kept by the originator absorbs initial losses until it is completely exhausted. Therefore, if the loss amount is small enough that it does not consume the entire α fraction, the investor loses nothing. If the loss is large enough to exhaust the α fraction, investors' expected loss is equal to $\int_{\alpha}^{1} (L-\alpha)(1+R)f(L)dL$. The investors' objective function net of $(1-\alpha)(1+S)$ is then equal to:

$$\pi_{I} \equiv (1-\alpha) (R-S) - p(e) \int_{\alpha}^{1} (L-\alpha) (1+R) f(L) dL.$$
(3)

With full securitization, when $\alpha = 0$, the investors' objective function is reduced to:

$$\pi_{I} = (R - S) - p(e) \int_{0}^{1} L(1 + R) f(L) dL.$$
(4)

We are looking for the optimal retention in the case of securitization that will keep the originator's incentive at the most favorable level. First we assume that there is no securitization and calculate the originator's first best effort. Then we assume that there is securitization, while effort is observable and there is no moral hazard. In this case, given the first best effort level, we determine the optimal amount of retention. Finally we investigate the optimal amount of retention with moral hazard. We must consider three kinds of constraints in this model:

- participation constraint
- incentive compatibility constraint
- technology constraint

2.3.2 Participation constraint

The originator's welfare in the case of securitization should be greater than or equal to its welfare in the case of no securitization. We assume the originator is risk averse, so its marginal utility of wealth is decreasing in wealth: U'(w) > 0, U''(w) < 0, where w is final wealth. This assumption is not critical to the result regarding the optimal form of contract: we obtain similar results when investors are risk averse or in a limited liability framework (Bolton and Dewatripont (2005); Winter (2000)).

In the absence of securitization, the originator holds the entire loan pool on its balance sheet and screens and monitors the loans cautiously. In this case the originator's payoff when there is no default will be U(R), and if there is default, it will be $\int_0^1 U(R - L(1+R))f(L)dL$. Its expected utility is given by:

$$\pi_{o} = \left[1 - p(e)\right] U(R) + p(e) \int_{0}^{1} U(R - L(1 + R)) f(L) \, dL - C(e).$$
(5)

From the first-order condition with respect to e, we calculate the first best effort that the originator can put forth to maximize its expected utility in the absence of securitization. This condition is equal to:

$$C'(e) = p'(e) \left[\int_0^1 U(R - L(1+R)) f(L) dL - U(R) \right].$$
(6)

The solution is e^{**} . The second-order condition is negative at the optimum. Details are in Appendix (A.1). Without securitization, the originator exerts the first best effort to maximize the expected utility that can be calculated by inserting the first best effort level in the originator's expected utility as follows:

$$\left[1 - p(e^{**})\right] U(R) + p(e^{**}) \int_0^1 U(R - L(1+R)) f(L) dL - C(e^{**}) = \overline{U}.$$
(7)

 \overline{U} is the best opportunity when considering securitization. To satisfy the participation constraint, the originator's expected utility in the presence of securitization should be at least equal to \overline{U} . In the case of securitization, we can write the originator's expected utility as follows:

$$(1-p(e)) U(R-(1-\alpha) (R-S)) +$$

$$p(e) \left[\int_0^\alpha U(R-(1-\alpha)(R-S)-L(1+R)) f(L) dL + \int_\alpha^1 U(R-(1-\alpha)(R-S)-\alpha(1+R)) f(L) dL \right] - C(e).$$
(8)

 $R - (1 - \alpha)(R - S)$ is the originator's payoff in the state of non-default. The expected return on the entire loan pool is (1+R), but because the originator keeps only the α fraction of the loan pool, it will receive the α fraction of the return: $\alpha(1+R)$. For the rest of the loan pool, the originator will receive: $(1-\alpha)(1+S)$ from securitization. Summing up these returns yields the payoff: $R - (1-\alpha)(R-S)$.

In the case of default, because the α fraction is exhausted first, the expected utility is equal to:

$$\int_{0}^{\alpha} U(R - (1 - \alpha)(R - S) - L(1 + R))f(L)dL + \int_{\alpha}^{1} U(R - (1 - \alpha)(R - S) - \alpha(1 + R))f(L)dL.$$
⁽⁹⁾

If the loss amount is less than α , the originator loses: L(1+R). When the loss amount is greater than α , the originator loses $\alpha(1+R)$. The probability of the first alternative is equal to: $F(\alpha) = 1 - \int_{\alpha}^{1} f(L) dL.$ To obtain the participation constraint, we equate the expected utility in the case of securitization to the expected utility in the case of no securitization while exerting the first best effort. We assume that this constraint is binding. We assign λ as the Lagrange multiplier to the originator's participation constraint:

$$(1-p(e))U(R-(1-\alpha)(R-S)) + (10)$$

$$p(e) \left[\int_0^{\alpha} U(R-(1-\alpha)(R-S)-L(1+R))f(L)dL + \int_{\alpha}^1 U(R-(1-\alpha)(R-S)-\alpha(1+R))f(L)dL \right] - C(e) = \overline{U}.$$

The left-hand side of this constraint must be evaluated at the optimal effort under securitization. We now compute the optimal effort by analyzing the incentive compatibility constraint under securitization.

2.3.3 Incentive compatibility constraint

This constraint ensures the effectiveness of the originator's screening and monitoring effort under moral hazard.

By differentiating the originators expected utility in the presence of securitization with respect to screening and monitoring effort (first-order condition for choice of effort), we can obtain the incentive compatibility constraint as follows:

$$p'\left(e^*\right) \begin{bmatrix} \int_0^{\alpha} U\left(R - (1 - \alpha)(R - S) - L(1 + R)\right) f\left(L\right) dL + \int_{\alpha}^{1} U\left(R - (1 - \alpha)(R - S) - \alpha(1 + R)\right) f\left(L\right) dL \\ -U\left(R - (1 - \alpha)(R - S)\right) \end{bmatrix} - C'\left(e^*\right) = 0.$$
(11)

We assign μ as a Lagrange multiplier of this constraint. When $\alpha = 0$ (full securitization), the marginal benefit of effort is equal to zero:

$$p'(e) \begin{bmatrix} \int_{0}^{\alpha} U(R - (1 - \alpha)(R - S) - L(1 + R)) f(L) dL + \int_{\alpha}^{1} U(R - (1 - \alpha)(R - S) - \alpha(1 + R)) f(L) dL \\ -U(R - (1 - \alpha)(R - S)) \end{bmatrix} = 0.$$
(12)

Because the marginal cost $C'(e^*)$ is positive, the first-order condition is strictly negative; as a result e^* is equal to zero with full securitization.

2.3.4 Technology constraint

This constraint is used to ensure that there is no over retention. It excludes $\alpha > 1$. $\delta \ge 0$ is the multiplier of this constraint.

$$\alpha \le 1 \begin{cases} \delta = 0 \text{ when } \alpha < 1 \\ \delta > 0 \text{ otherwise.} \end{cases}$$
(13)

> Optimal securitization

Optimal securitization solves the following principal-agent program:

$$\begin{aligned} &Max_{\alpha,S,e} \ \pi_{1} = (1-\alpha)(R-S) - p(e) \int_{\alpha}^{1} (L-\alpha) f(L) dL(1+R) \end{aligned} \tag{14} \\ &\mu : p'(e) \left[\int_{0}^{\alpha} U \big(R - (1-\alpha)(R-S) - L(1+R) \big) f(L) dL + \int_{\alpha}^{1} U \big(R - (1-\alpha)(R-S) - \alpha(1+R) \big) f(L) dL \right] - C'(e) = 0 \\ &\lambda : (1-p(e)) U \big(R - (1-\alpha)(R-S) \big) + \\ &p(e) \Big[\int_{0}^{\alpha} U \big(R - (1-\alpha)(R-S) - L(1+R) \big) f(L) dL + \int_{\alpha}^{1} U \big(R - (1-\alpha)(R-S) - \alpha(1+R) \big) f(L) dL \Big] - C(e) = \overline{U} \end{aligned}$$

 $\delta: \alpha \leq 1$

• No Moral Hazard: $\mu = 0$

First we assume that $\mu = 0$ when there is no moral hazard and $e = e^{**}$. The investor observes *e* and sets his level at e^{**} by using the "take it or leave it" strategy. If we differentiate (14) with respect to *S*, we have:

$$S:-(1-\alpha)+\lambda(1-p(e))U'(R-(1-\alpha)(R-S))(1-\alpha)+$$

$$\lambda p(e)\left[\int_{0}^{\alpha}U'(R-(1-\alpha)(R-S)-L(1+R))(1-\alpha)f(L)dL+\int_{\alpha}^{1}U'(R-(1-\alpha)(R-S)-\alpha(1+R))(1-\alpha)f(L)dL\right]=0.$$
(15)

By solving the above equation, we can isolate $\frac{1}{\lambda}$ as follows:

$$\frac{1}{\lambda} = (1 - p(e))U'(R - (1 - \alpha)(R - S)) +$$

$$p(e) \left[\int_{0}^{\alpha} U'(R - (1 - \alpha)(R - S) - L(1 + R))f(L)dL + \int_{\alpha}^{1} U'(R - (1 - \alpha)(R - S) - \alpha(1 + R))f(L)dL \right].$$
(16)

Differentiating (14) with respect to α yields:

$$(R-S) - p(e)(1-F(\alpha))(1+R) = \lambda(1-p(e))U'(R-(1-\alpha)(R-S))(R-S)$$

$$+\lambda p(e)\int_{0}^{\alpha}U'(R-(1-\alpha)(R-S) - L(1+R))(R-S)f(L)dL - \lambda p(e)U'(R-(1-\alpha)(R-S) - \alpha(1+R))(1+S)(1-F(\alpha)).$$
(17)

By solving this equation for $\frac{1}{\lambda}$, we obtain:

$$\frac{1}{\lambda} = \begin{cases} \frac{(1-p(e))U'(R-(1-\alpha)(R-S))(R-S)}{(R-S)-p(e)(1-F(\alpha))(1+R)} + \\ \frac{p(e)\int_{0}^{\alpha}U'(R-(1-\alpha)(R-S)-L(1+R))(R-S)f(L)dL}{(R-S)-p(e)(1-F(\alpha))(1+R)} - \frac{p(e)U'(R-(1-\alpha)(R-S)-\alpha(1+R))(1+S)(1-F(\alpha))}{(R-S)-p(e)(1-F(\alpha))(1+R)} \end{cases}$$
(18)

By equalizing the two equations (16) and (18) together we get the following equation:

$$U'(R - (1 - \alpha)(R - S) - \alpha(1 + R)) = (1 - p(e)) U'(R - (1 - \alpha)(R - S)) +$$

$$p(e) \int_{0}^{\alpha} U'(R - (1 - \alpha)(R - S) - L(1 + R)) f(L) dL + p(e)(1 - F(\alpha)) U'(R - (1 - \alpha)(R - S) - \alpha(1 + R))$$
(19)

 α =0 solves this equation. We can also verify that *R*>*S* at the optimum. In this setting, there is no retention by the bank in the absence of moral hazard.

From the participation constraint we have the following when $\alpha = 0$:

$$U(S) - C(e^{**}) = ((1 - p(e^{**}))U(R) + p(e^{**}) \left[\int_0^1 U(R - L(1 + R))f(L) dL \right] - C(e^{**}).$$
⁽²⁰⁾

By applying Taylor's expansion to both sides of equation (20), we obtain the expected loss and calculate the risk premium:

$$S = R - p(e^{**}) (1+R) \left[\int_{0}^{1} Lf(L) dL \right] + \frac{1}{2} p(e) \frac{U''(R)}{U'(R)} \int_{0}^{1} L^{2} (1+R)^{2} f(L) dL.$$
⁽²¹⁾

Details are in Appendix (A.2).

Because the risk premium is positive under risk aversion, we can verify that:

$$(R-S) > p(e^{**}) (1+R) \left[\int_0^1 Lf(L) dL \right].$$

The investor uses his power to extract the risk premium from the risk-averse bank. Under risk neutrality, R-S is equal to the expected loss of the loans.

• *Moral Hazard* : $\mu \neq 0$

Now we assume that $\mu \neq 0$ because there is potential moral hazard. Differentiating (14) with respect to *S*, we obtain after simplification:

$$\frac{1 - \lambda U'(R - (1 - \alpha)(R - S))}{\lambda p(e) + \mu p'(e)} =$$

$$\begin{bmatrix} -U'(R - (1 - \alpha)(R - S)) \\ + \int_{0}^{\alpha} U'(R - (1 - \alpha)(R - S) - L(1 + R)) f(L) dL + U'(R - (1 - \alpha)(R - S) - \alpha(1 + R))(1 - F(\alpha)) \end{bmatrix}.$$
(22)

Differentiating (14) with respect to α yields, after simplification:

$$(R-S) - p(e)(1-F(\alpha))(1+R) = \lambda U' (R - (1-\alpha)(R-S))(R-S) +$$

$$(\lambda p(e) + \mu p'(e)) \begin{bmatrix} -U' (R - (1-\alpha)(R-S))(R-S) - (1-F(\alpha))U' (R - (1-\alpha)(R-S) - \alpha(1+R))(1+S) \\ + \int_{0}^{\alpha} U' (R - (1-\alpha)(R-S) - L(1+R))(R-S)f(L)dL \end{bmatrix}.$$
(23)

The two first-order conditions can be rewritten as:

$$U'(R - (1 - \alpha)(R - S) - \alpha(1 + R)) = (\lambda U'(3) - p(e))[U'(R - (1 - \alpha)(R - S))] + p(e) \int_{0}^{\alpha} U'(R - (1 - \alpha)(R - S) - L(1 + R)) f(L) dL + p(e)(1 - F(\alpha))U'(R - (1 - \alpha)(R - S) - \alpha(1 + R))$$
(24)

2.3.5 Analyzing the result

Writing $U'(R - (1 - \alpha)(R - S) - \alpha(1 + R)) = U'(3)$ to simplify the notation, we analyze two cases.

First consider $\lambda < \frac{1}{U'(3)}$:

$$U'(R - (1 - \alpha)(R - S) - \alpha(1 + R)) < (1 - p(e))[U'(R - (1 - \alpha)(R - S))] +$$

$$p(e) \int_{0}^{\alpha} U'(R - (1 - \alpha)(R - S) - L(1 + R))f(L)dL + p(e)(1 - F(\alpha))U'(R - (1 - \alpha)(R - S) - \alpha(1 + R)).$$
(25)

Because U' is decreasing in wealth, this means that α should be smaller than or equal to zero to solve (25). Because α cannot be smaller than zero, this means that there should be full securitization. By putting $\alpha = 0$ in the above equation we get the following result:

$$U'(S) < (1 - p(e))[U'(S)] + p(e)(1 - F(\alpha))U'(S) = U'(S)$$
(26)

Which is a contradiction because F(0) = 0 and we reject this possibility.

Now considering $\lambda > \frac{1}{U'(3)}$, we obtain:

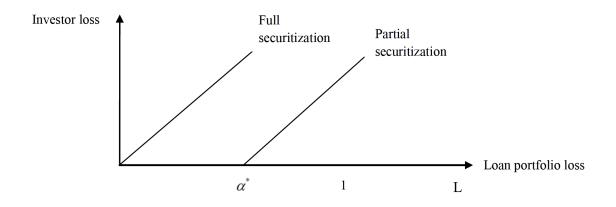
$$U'(R - (1 - \alpha)(R - S) - \alpha(1 + R)) > (1 - p(e))[U'(R - (1 - \alpha)(R - S))] +$$

$$p(e) \int_{0}^{\alpha} U'(R - (1 - \alpha)(R - S) - L(1 + R))f(L)dL + p(e)(1 - F(\alpha))U'(R - (1 - \alpha)(R - S) - \alpha(1 + R)).$$
(27)

Because U' is decreasing in wealth, this means that α must be greater than zero to solve this inequality. This implies that full securitization is not optimal and there must be retention under moral hazard. The optimal solution, $\alpha^* > 0$, means that L – Investor loss = $\alpha^* > 0$, when investor loss is greater than zero. We also verify that $\alpha^* \le 1$. Therefore, we obtain that full retention can be an optimal solution for certain parameters.

Based on the above result, we can draw the relationship between investor and loan portfolio loss in Figure 2. This contract is like a deductible contract in insurance contracting under moral hazard (Winter (2000)).





When there is full securitization, investor loss is equal to loan portfolio loss. When there is partial securitization, investor loss equals 0 when $L < \alpha^*$ and equals $L - \alpha^*$ when $L \ge \alpha^*$.

2.4 Conclusion

In this chapter, we applied the security design methodology to securitization to obtain the optimal risk sharing between sellers and buyers of these products. To the best of our knowledge, this research is the first to analyze the incentive contracting of securitized products under moral hazard.

We analyse the standard pass through securitization model. To reduce the default probability and maximize its expected utility, the originator exerts the first best effort in absence of securitization. However, the presence of securitization introduces moral hazard and reduces the effort level because it decreases the originator's incentive to carefully screen and monitor borrowers. If the originator holds nothing under moral hazard and securitizes the whole loan pool ($\alpha = 0$), he does not have any incentive to choose an appropriate effort level under moral hazard. However, when the originator holds a fraction of loan portfolio ($\alpha > 0$), he must incur a part of the default loss,

which gives him an incentive to reduce the default probability. In fact we can show that the optimal effort is strictly positive under retention by the bank.

We prove that the optimal contract must contain a partial retention clause in the presence of moral hazard. The optimal amount of retention could then be calculated using our model with different parameter values. We will take one step further by considering structured asset-backed securitization with both the absence and the presence of the correlation between assets in the original pool in the next two chapters.

Chapter 3:

Structured Asset-backed Securitization without Systemic Risk

Introduction

Structured asset-backed securitization is a kind of securitization that provides various ranges of securities with different classes of risk and return in the form of tranches to facilitate satisfying investors' demand. The Special Purpose Vehicle (SPV) offers a capital structure with prioritized claims (tranches) which are collateralized by underlying loan pool.

Tranching processes which are accompanied with credit enhancement procedure reflect the intention of achieving classes of rating higher than the pool's average rating. In order to keep the same rating mean as the underlying loan pool, other classes of rating are lower than the pool's average rating.

Generated cash flows from the underlying loan pool will be paid to investors in the prescribed sequence of tranche seniority. If cash flows are insufficient to pay all its investors, those in the lower layer (tranche) suffer losses first. The tranches are prioritized based on their losses absorbance from the underlying loan pool (Coval, Jurek and Stafford (2009)). Here is an example of tranching:

- Equity tranche which absorbs the first losses
- Mezzanine tranche which absorbs additional losses
- Senior tranche which are expected to be protected from default risk, absorbs the remaining losses

If assets of the loan pool default, the equity tranche absorbs initial losses until it is completely exhausted. So if the amount of loss is sufficiently small to not consume equity tranche, the holders of mezzanine and senior tranches lose nothing, but due to the write down of the principal amount of the equity tranche, other tranches would suffer from decline in their marked-to-market value, because the smaller equity tranche now offers less credit enhancement in comparison to the prior to default. If the amount of loss is large enough to exhaust equity tranche, the excess loss will be absorbed first by mezzanine and then senior tranche. The senior tranche only takes up losses after the junior claims (equity and mezzanine) have been exhausted. This is why the senior tranche could acquire credit ratings more than the average rating of the collateral loan pool (Coval, Jurek and Stafford (2009)). Credit rating of the more senior tranche are derived from the degree of protection provided by the junior claims, or overcollateralization which determines the maximum amount of losses that can be absorbed before the senior claims is scratched.

The three different tranches are typically designed for different investors' appetite. The equity tranche which is more risky and pays the maximum interest rate may be sold to a less risk adverse investor who is an expert in asset management and have more information about the underlying loan pool (hedge fund). The mezzanine tranche which is less risky and pays lower interest rate may be sold to investors who are looking for diversification of their credit risk while they are not interested in buying the entire underlying loan pool. The senior tranche may be sold to the more risk averse investors looking for low risk and lower return assets (pension fund). Since the equity and mezzanine tranches bear the first part of the losses of the entire loan pool, they are more sensitive to the alteration in the underlying credit spread and also more leveraged than the senior tranche or the reference loan pool as a whole.

3.1 Extension of the model

As stated before in this chapter we will consider structured asset backed securitization in our modeling. Structured asset-backed securitization is a kind of securitization that accompanied with different ranges of securities and classes of risk and return in the form of tranches to facilitate satisfying investors' demand.

Interest and principal payments are made in order of seniority. Senior tranche securities are considered as the safest securities and obtain the highest rating among other tranches and accordingly a lower yield. Junior tranches offer higher interest rates or lower prices to compensate for additional default risks. In order to understand the mechanism of interest and principal payments in the presence of having more than one tranche, it would be helpful to understand how it works based on the situation which provides the possibility of distributing credit risk through tranching. To explain the procedure, an example is shown below based on the setup of the model and considers some simplifications to the structure to make the example easy to understand.

We start with the loan pool as a debt portfolio with the value of 100\$ and 7% yield. We assume that the debt portfolio has an average rating of BBB. Suppose we securitize the portfolio as securities with different classes of risk and interest rates as follows:

- Senior tranche: 85\$, rated AAA, yields 5.8%
- Mezzanine tranche: 10\$, rated BB, yields 7.7%
- Equity tranche: 5\$, rated BBB, yield 26%

We keep equity tranche which consists of 5% of the portfolio. We obtain different ratings because of the sequence of interest and principal payments, starting with the senior tranche and ending with the equity tranche.

If there is no default, the collateral portfolio pays 7\$ (100*7%) in interest per year. 4.93\$ (85*5.8%) of this amount goes to senior tranche holder and 0.77\$ (10*7.7%) of this amount goes to mezzanine tranche holders. There is 1.3\$ left which passes through to the equity holders. The interest rate of equity tranche is equal to 26% (1.3/5) which is quite attractive.

Suppose there is a default. The story becomes different. For all investors assume that there is a 1% default in the debt portfolio. As a result, there is 99\$ in the no default portfolio with 7% yield. Now, we have 6.93\$ (99*7%) as total payment in interest instead of 7\$ in the case of no default. Like before, 5.7\$ (4.93\$+0.77\$) is consumed to pay investors (senior and mezzanine tranche holders) and the rest is 1.23\$ to pay for originator (equity tranche holders). The yield rate on equity in this case is 24.6 % (1.23/5) instead of 26%, which is still attractive. In this case the 1\$ default in the portfolio is sustained by originator since it is still smaller than 5\$ share of equity tranche. Therefore, the first capital loss in collateral and the interest loss in portfolio are sustained by equity tranche.

If the amount of loss is greater than the amount that can be absorbed by equity tranche, then other tranches in sequence of seniority would sustain rest of the loss. Now assume that there is 7% default in the collateral portfolio. Therefore we have 6.51 (93*7%) payment in interest which is still greater than the 5.7\$ that should be paid to investors. The equity tranche holders will receive 0.81\$ as interest payment which is equivalent to 16.2% (0.81/5) interest rate. In this case although investors loose nothing in interest payment, since 7\$ loss in capital is greater than 5\$ that can be sustained by equity tranche, mezzanine tranche holders lose 2\$ in capital.

The objective of this section is to show that originator must keep part of the equity tranche under moral hazard which is interpreted as retention. To simplify the analysis instead of having more than one tranche (senior and mezzanine) to securitize, we assume that there is only one tranche with the equivalent credit rating and interest rate. For example, instead of having mezzanine and senior tranches with different credit rating and interest rates, we assume that there is only one tranche with the credit rating AA and interest rate of 6%. As we will see this assumption does not make any difference in the result, because like before it is the same amount of interest and principal that should be paid to investors:

95\$ in principal and 5.7\$ (95 *6%) in interest goes to holders of securitized loans.

Based on the above example, it becomes clear that the loss on interest and capital are two different stories that should be distinguished from each other. It is quite possible that investors lose something on capital without experiencing any loss on the interest payment. That is why in our model we distinguish these two components from each other by dividing the loss into two categories: loss on interest, loss on capital. That is the main difference between this model and that in the previous chapter. Before we assumed that there is only one tranche without credit enhancement procedure. Here we assume that there is more than one tranches with credit enhancement procedure. The credit enhancement may affect the optimal retention because it creates an additional incentive.

3.1.1 Investors' objective function

The expected return of investors for a given α is equal to:

$$\pi_{1} = (1 - \alpha)(r_{1} - S) - p(e) \left[\int_{\frac{\alpha_{2}}{R}}^{1} (LR - \alpha r_{2}) f(L) dL + \int_{\alpha}^{1} (L - \alpha) f(L) dL \right]$$
(28)

 $(1-\alpha)$ is the amount of senior tranches which are securitized. r_1 is the expected rate of return of securitized loan as senior tranches. Compare to the case when there is no tranching, we noticed that the expected rate of return of securitized loan decreases from R to r_1 and, at the same time, its average rating increases to higher credit rating than the average of the loan pool because the equity tranche now offers credit enhancement by providing protection against initial losses on both capital and interest. The originator now keeps an α fraction of the loan pool as an equity tranche with a lower credit rating and higher interest rate (r_2) than the loan pool (R). We now compute the optimal α .

In the non-default state, investors' return on a securitized loan is equal to $(1-\alpha)(1+r_1)$, while investors pay $(1-\alpha)(1+S)$. The net return equals $(1-\alpha)(r_1-S)$. If the loan pool defaults, the α fraction that is kept by the originator absorbs initial losses both on interest and capital until it is completely exhausted. Consequently, the investors lose something only if the amount of loss on interest or capital or both will be large enough to exhaust the equity tranche.

Investors will lose something on interest when: $Loss * R > \alpha r_2$ or when $Loss > \frac{\alpha r_2}{R}$, and the amount they will lose is equal to $(LR - \alpha r_2)$ which is the amount that cannot be absorbed by equity tranche. Therefore investors expected loss on interest is equal to:

$$\int_{\frac{dr_2}{R}}^{l} (LR - \alpha r_2) f(L) dL$$
⁽²⁹⁾

Investors will lose something on capital when: $L_{OSS} > \alpha$ and the amount that they will lose is equal to $(L-\alpha)$. Expected loss on collateral is equal to:

$$\int_{\alpha}^{1} (L-\alpha)f(L)dL \tag{30}$$

With full securitization, when $\alpha = 0$, there is no equity tranche and investors' expected profit is equal to:

$$\pi_{I} \equiv (r_{1} - S) - p(e) \int_{0}^{1} L(1 + R) f(L) dL$$
(31)

3.1.2 Participation constraint

To satisfy the participation constraint, the expected utility of the originator in the presence of securitization should be at least equal to its expected utility in the case of no securitization, which is equal to \overline{U} . \overline{U} is the best opportunity for the bank when considering securitization.

In the absence of securitization, the originator holds the entire loan pool on its balance sheet, and screens and monitors the loans cautiously. In this case, the originator's payoff when there is no default will be U(R). When there is default, it will be $\int_{0}^{1} U(R-L(1+R))f(L) dL$. Its expected utility is given by:

$$\pi_{o} = [1 - p(e)] U(R) + p(e) \int_{0}^{1} U(R - L(1 + R)) f(L) \, dL - C(e)$$
(32)

From the first-order condition with respect to e, we calculate the first best effort that the originator will exert to maximize its expected utility in the absence of securitization. The solution is e^{**} . The second-order condition is negative at the optimum. Evaluated at e^{**} , the best alternative of the originator is equal to:

$$\left[1 - p(e^{**})\right] U(R) + p(e^{**}) \int_0^1 U(R - L(1+R)) f(L) dL - C(e^{**}) = \overline{U}$$
(33)

In case of securitization with tranching and credit enhancement, we can write the originators' expected utility as follows:

$$\pi_{O}^{S} = (1 - p(e))U(R - (1 - \alpha)(r_{1} - S)) + p(e) \left[\int_{0}^{\alpha} U(R - (1 - \alpha)(r_{1} - S) - LR - L)f(L)dL \right] + p(e) \left[\int_{\alpha}^{\frac{\alpha_{2}}{R}} U(R - (1 - \alpha)(r_{1} - S) - LR - \alpha)f(L)dL \right] + p(e) \left[\int_{\frac{\alpha_{2}}{R}}^{\frac{\alpha_{2}}{R}} U(R - (1 - \alpha)(r_{1} - S) - \alpha r_{2} - \alpha)f(L)dL \right] - C(e)$$
(34)

The expected return on the collateral loan pool is equal to 1*(1+R), which can be divided between tranches with different credit ratings and different interest rates. Therefore we have the following relationship between the return on the debt loan pool and returns on different tranches:

$$1*(1+R) = (1-\alpha)(1+r_1) + \alpha(1+r_2)$$
(35)

When there is no default, the expected payoff of the bank as equity holder is equal to $\alpha(1+r_2)+(1-\alpha)(1+S)-1$, which can be simplified to $R-(1-\alpha)(r_1-S)$ by using the above equation.

In the case of default, the equity tranche absorbs initial losses on interest and capital. If the amount of loss on capital is smaller than α , equity tranche will absorb the whole loss on capital which is equal to *L*. If the amount of loss on capital is greater than α , the equity tranche will exhaust completely and the additional loss will be absorbed by the senior tranche. Originators will lose *LR* on interest when $L < \frac{\alpha r_2}{R}$, and they will lose αr_2 when $L > \frac{\alpha r_2}{R}$.

Finally the participation constraint is obtained as follows. We assign λ as the Lagrange multiplier to the participation constraint:

$$\lambda : (1 - p(e))U(R - (1 - \alpha)(r_1 - S)) + p(e)\int_0^{\alpha} U(R - (1 - \alpha)(r_1 - S) - LR - L)f(L)dL + p(e)\int_{\alpha}^{\frac{\alpha r_2}{R}} U(R - (1 - \alpha)(r_1 - S) - LR - \alpha)f(L)dL + p(e)\int_{\alpha}^{\frac{1}{R}} U(R - (1 - \alpha)(r_1 - S) - LR - \alpha)f(L)dL + \overline{U}$$
(36)

3.1.3 Incentive compatibility constraint

This constraint ensures the effectiveness of originator's effort for screening taking into account the moral hazard problem. By differentiating the originator's expected utility in the presence of securitization with respect to screening and monitoring effort (first order condition for choice of effort), we can calculate the incentive compatibility constraint as follows. We assign μ as the Lagrange multiplier of this constraint. This constraint is the derivation of the participation constraint with respect to effort.

$$\mu : -p'(e)U(R - (1 - \alpha)(r_1 - S)) + p'(e)\int_0^{\alpha} U(R - (1 - \alpha)(r_1 - S) - LR - L)f(L)dL + p'(e)\int_{\alpha}^{\frac{\alpha r_2}{R}} U(R - (1 - \alpha)(r_1 - S) - LR - \alpha)f(L)dL + p'(e)\int_{\frac{\alpha r_2}{R}}^{\frac{\alpha r_2}{R}} U(R - (1 - \alpha)(r_1 - S) - \alpha r_2 - \alpha)f(L)dL = C'(e)$$
(37)

3.1.4 Technology constraint

This constraint can be applied to ensure that there is no over retention. It excludes $\alpha > 1$.

$$\delta : \alpha \le 1 \begin{cases} \frac{\delta = 0 \longrightarrow \alpha < 1}{\delta > 0 \longrightarrow \alpha = 1} \end{cases}$$
(38)

> Optimal securitization contract model

The optimal structured securitization contract with tranching and credit enhancement is attained by solving the following maximization program:

$$Max \ \pi_{1} = (1 - \alpha)(r_{1} - S) - p(e) \left[\int_{\frac{dr_{2}}{R}}^{1} (LR - \alpha r_{2}) f(L) dL + \int_{\alpha}^{1} (L - \alpha) f(L) dL \right]$$
(39)

$$\lambda : (1 - p(e))U(R - (1 - \alpha)(r_1 - S)) + p(e) \int_0^{\alpha} U(R - (1 - \alpha)(r_1 - S) - LR - L)f(L)dL + p(e) \int_{\alpha}^{\alpha r_2} U(R - (1 - \alpha)(r_1 - S) - LR - \alpha)f(L)dL + p(e) \int_{\alpha}^{\alpha r_2} U(R - (1 - \alpha)(r_1 - S) - \alpha r_2 - \alpha)f(L)dL - C(e) = \overline{U}$$

$$\mu :-p'(e)U(R - (1 - \alpha)(r_1 - S)) + p'(e)\int_0^{\alpha} U(R - (1 - \alpha)(r_1 - S) - LR - L)f(L)dL + p'(e)\int_{\alpha}^{\frac{\alpha r_2}{R}} U(R - (1 - \alpha)(r_1 - S) - LR - \alpha)f(L)dL + p'(e)\int_{\frac{\alpha r_2}{R}}^1 U(R - (1 - \alpha)(r_1 - S) - \alpha r_2 - \alpha)f(L)dL = C'(e)$$

 $\delta : \alpha \leq 1$

• No Moral Hazard: $\mu = 0$

First we assume that $\mu = 0$ when there is no moral hazard and $e = e^{**}$. The investor observes *e* and sets its level at the full information level e^{**} by using the "take it or leave it" strategy. If we differentiate the design model in (39) with respect to *S*, we have:

$$S: - (1 - \alpha) + \lambda (1 - p(e)) U'(R - (1 - \alpha)(r_1 - S))(1 - \alpha) + \lambda p(e) \int_{0}^{\alpha} U'(R - (1 - \alpha)(r_1 - S) - LR - L)(1 - \alpha)f(L)dL + \lambda p(e) \int_{\alpha}^{\alpha} U'(R - (1 - \alpha)(r_1 - S) - LR - \alpha)(1 - \alpha)f(L)dL +$$
(40)
$$\lambda p(e) \int_{R}^{1} U'(R - (1 - \alpha)(r_1 - S) - \alpha r_2 - \alpha)(1 - \alpha)f(L)dL = 0.$$

By solving the above equation, we can isolate $1\!\!/_\lambda$ as follows:

$$\frac{1}{\lambda} = (1 - p(e)) U'(R - (1 - \alpha)(r_1 - S)) +$$

$$p(e) \int_0^{\alpha} U'(R - (1 - \alpha)(r_1 - S) - LR - L) f(L) dL + p(e) \int_{\alpha}^{\frac{\alpha r_2}{R}} U'(R - (1 - \alpha)(r_1 - S) - LR - \alpha) f(L) dL +$$

$$p(e) \int_{\frac{\alpha r_2}{R}}^{\frac{\alpha r_2}{R}} U'(R - (1 - \alpha)(r_1 - S) - \alpha r_2 - \alpha) f(L) dL$$
(41)

Differentiating the model in (39) with respect to α when $\mu = 0$ yields:

$$-(r_{1}-S) + p(e) r_{2} \left(1 - F\left(\alpha \frac{r_{2}}{R}\right)\right) + p(e)(1 - F(\alpha)) + \lambda (1 - p(e)) U'(R - (1 - \alpha)(r_{1} - S)) (r_{1} - S) + \lambda p(e) \int_{0}^{\alpha} U'(R - (1 - \alpha)(r_{1} - S) - LR - L) (r_{1} - S) f(L)dL + \lambda p(e) \int_{\alpha}^{\frac{\alpha r_{2}}{R}} U'(R - (1 - \alpha)(r_{1} - S) - LR - \alpha)(r_{1} - S - 1) f(L)dL + \lambda p(e) \int_{\alpha}^{\frac{1}{R}} U'(R - (1 - \alpha)(r_{1} - S) - \alpha r_{2} - \alpha)(r_{1} - S - r_{2} - 1) f(L)dL = 0.$$

$$(42)$$

By solving this equation for $\frac{1}{\lambda}$, we obtain:

$$\frac{1}{\lambda} = \begin{cases}
\frac{(1-p(e)) U'(R-(1-\alpha)(r_{1}-S)) (r_{1}-S)}{(r_{1}-S) - p(e) r_{2}\left(1-F\left(\alpha \frac{r_{2}}{R}\right)\right) - p(e)(1-F(\alpha))} + \frac{p(e)\int_{\alpha}^{\frac{\sigma_{2}}{R}} U'(R-(1-\alpha)(r_{1}-S) - LR - L) (r_{1}-S) f(L)dL}{(r_{1}-S) - p(e) r_{2}\left(1-F\left(\alpha \frac{r_{2}}{R}\right)\right) - p(e)(1-F(\alpha))} + \frac{p(e)\int_{\alpha}^{\frac{\sigma_{2}}{R}} U'(R-(1-\alpha)(r_{1}-S) - LR - \alpha)(r_{1}-S - 1)f(L)dL}{(r_{1}-S) - p(e) r_{2}\left(1-F\left(\alpha \frac{r_{2}}{R}\right)\right) - p(e)(1-F(\alpha))} + \frac{p(e)\int_{\alpha}^{\frac{\sigma_{2}}{R}} U'(R-(1-\alpha)(r_{1}-S) - LR - \alpha)(r_{1}-S - 1)f(L)dL}{(r_{1}-S) - p(e) r_{2}\left(1-F\left(\alpha \frac{r_{2}}{R}\right)\right) - p(e)(1-F(\alpha))} + \frac{p(e)\int_{\alpha}^{\frac{\sigma_{2}}{R}} U'(R-(1-\alpha)(r_{1}-S) - \alpha r_{2} - \alpha)(r_{1}-S - r_{2} - 1)f(L)dL}{(r_{1}-S) - p(e) r_{2}\left(1-F\left(\alpha \frac{r_{2}}{R}\right)\right) - p(e)(1-F(\alpha))} + \frac{p(e)\int_{\alpha}^{\frac{\sigma_{2}}{R}} U'(R-(1-\alpha)(r_{1}-S) - \alpha r_{2} - \alpha)(r_{1}-S - r_{2} - 1)f(L)dL}{(r_{1}-S) - p(e) r_{2}\left(1-F\left(\alpha \frac{r_{2}}{R}\right)\right) - p(e)(1-F(\alpha))} + \frac{p(e)\int_{\alpha}^{\frac{\sigma_{2}}{R}} U'(R-(1-\alpha)(r_{1}-S) - \alpha r_{2} - \alpha)(r_{1}-S - r_{2} - 1)f(L)dL}{(r_{1}-S) - p(e) r_{2}\left(1-F\left(\alpha \frac{r_{2}}{R}\right)\right) - p(e)(1-F(\alpha))} + \frac{p(e)\int_{\alpha}^{\frac{\sigma_{2}}{R}} U'(R-(1-\alpha)(r_{1}-S) - \alpha r_{2} - \alpha)(r_{1}-S - r_{2} - 1)f(L)dL}{(r_{1}-S) - p(e) r_{2}\left(1-F\left(\alpha \frac{r_{2}}{R}\right)\right) - p(e)(1-F(\alpha))} + \frac{p(e)\int_{\alpha}^{\frac{\sigma_{2}}{R}} U'(R-(1-\alpha)(r_{1}-S) - \alpha r_{2} - \alpha)(r_{1}-S - r_{2} - 1)f(L)dL}{(r_{1}-S) - p(e) r_{2}\left(1-F\left(\alpha \frac{r_{2}}{R}\right)\right) - p(e)(1-F(\alpha))} + \frac{p(e)\int_{\alpha}^{\frac{\sigma_{2}}{R}} U'(R-(1-\alpha)(r_{1}-S) - \alpha r_{2} - \alpha)(r_{1}-S - r_{2} - 1)f(L)dL}{(r_{1}-S) - p(e) r_{2}\left(1-F\left(\alpha \frac{r_{2}}{R}\right)\right) - p(e)(1-F(\alpha))} + \frac{p(e)\int_{\alpha}^{\frac{\sigma_{2}}{R}} U'(R-(1-\alpha)(r_{1}-S) - \alpha r_{2} - \alpha)(r_{1}-S - r_{2} - 1)f(L)dL}{(r_{1}-S) - p(e) r_{2}\left(1-F\left(\alpha \frac{r_{2}}{R}\right)\right) - p(e)(1-F(\alpha))} + \frac{p(e)\int_{\alpha}^{\frac{\sigma_{2}}{R}} U'(R-(1-\alpha)(r_{1}-S) - \alpha r_{2} - \alpha)(r_{1}-S - r_{2} - 1)f(L)dL}{(r_{1}-S) - p(e) r_{2}\left(1-F\left(\alpha \frac{r_{2}}{R}\right)\right) - \frac{p(e)}{(r_{1}-F\left(\alpha \frac{r_{2}}{R}\right)} - \frac{p(e)}{(r_{1}-F\left(\alpha \frac{r_{2}}{R}\right)} + \frac{p(e)}{(r_{1}-S} - \frac{p(e)}{R} + \frac{p(e)}{(r_{1}-S} - \frac{p(e)}{R} + \frac{p(e)}{(r_{1}-S} - \frac{p(e)}{R}\right)} + \frac{p(e)}{(r_{1}-S} - \frac{p(e)}{(r_{1}-S} - \frac{p(e)}{R} + \frac{p(e)}{(r_{1}-S} - \frac{p(e)}{R} + \frac{p(e)}{(r_{1}-S} - \frac{p($$

By equalizing the two equations (41) and (43) together we get the following result:

$$p(e)\int_{\alpha}^{\frac{\alpha_{2}}{R}}U'(R-(1-\alpha)(r_{1}-S)-LR-\alpha)(-r_{2})f(L)dL + p(e)\int_{\frac{\alpha_{2}}{R}}^{1}U'(R-(1-\alpha)(r_{1}-S)-\alpha r_{2}-\alpha)(-r_{2}-1)f(L)dL =$$

$$-p(e)\left\{r_{2}\left(1-F\left(\alpha\frac{r_{2}}{R}\right)\right) + (1-F(\alpha))\right\} \begin{cases} (1-p(e))U'(R-(1-\alpha)(r_{1}-S)) + p(e)\int_{0}^{\alpha}U'(R-(1-\alpha)(r_{1}-S)-LR-L)f(L)dL + \\ p(e)\int_{\alpha}^{\frac{\alpha_{2}}{R}}U'(R-(1-\alpha)(r_{1}-S)-LR-\alpha)f(L)dL + p(e)\int_{\frac{\alpha_{2}}{R}}^{1}U'(R-(1-\alpha)(r_{1}-S)-\alpha r_{2}-\alpha)f(L)dL \end{cases}$$

$$(44)$$

 $\alpha = 0$ solves this equation. We can also verify that $r_1 > S$ at the optimum and calculate the risk premium (like what it is exactly done in the previous chapter with a modified notation):

$$S = r_1 - p(e^{**})(1+R) \int_0^1 Lf(L) dL - risk \ premium$$
(45)

 $p(e^{**})(1+R)\int_{0}^{1} Lf(L)dL$ is the expected loss and the risk premium is equal to (Details are in Appendix (*B.1*)):

$$risk \ premium = -\frac{1}{2} p(e) \frac{U''(R)}{U'(R)} \int_{0}^{1} L^{2} (1+R)^{2} f(L) dL$$
(46)

• Moral Hazard : $\mu \neq 0$

Now we assume that $\mu \neq 0$ and there is potential moral hazard. If we differentiate the model in (39) with respect to *S*, when $\mu > 0$, we obtain:

$$S:-(1-\alpha)+\lambda(1-p(e))U'(R-(1-\alpha)(r_{1}-S))(1-\alpha)+$$

$$\lambda p(e) \begin{cases} \int_{0}^{\alpha} U'(R-(1-\alpha)(r_{1}-S)-LR-L)(1-\alpha)f(L)dL + \int_{\alpha}^{\frac{\alpha r_{2}}{R}} U'(R-(1-\alpha)(r_{1}-S)-LR-\alpha)(1-\alpha)f(L)dL + \\ \int_{\alpha}^{1} \frac{1}{m_{2}}U'(R-(1-\alpha)(r_{1}-S)-\alpha r_{2}-\alpha)(1-\alpha)f(L)dL \\ -\mu p'(e)U'(R-(1-\alpha)(r_{1}-S))(1-\alpha)+ \\ \mu p'(e) \begin{cases} \int_{0}^{\alpha} U'(R-(1-\alpha)(r_{1}-S)-LR-L)(1-\alpha)f(L)dL + \int_{\alpha}^{\frac{\alpha r_{2}}{R}} U'(R-(1-\alpha)(r_{1}-S)-LR-\alpha)(1-\alpha)f(L)dL + \\ \int_{\alpha}^{\frac{1}{m_{2}}} U'(R-(1-\alpha)(r_{1}-S)-LR-L)(1-\alpha)f(L)dL + \int_{\alpha}^{\frac{\alpha r_{2}}{R}} U'(R-(1-\alpha)(r_{1}-S)-LR-\alpha)(1-\alpha)f(L)dL + \\ \end{bmatrix} = 0$$

By simplifying the above equation, we find:

$$\frac{-U'(R-(1-\alpha)(r_{1}-S))+\int_{0}^{\alpha}U'(R-(1-\alpha)(r_{1}-S)-LR-L)f(L)dL+\int_{\alpha}^{\frac{\alpha r_{2}}{R}}U'(R-(1-\alpha)(r_{1}-S)-LR-\alpha)f(L)dL+}{\frac{\int_{\alpha}^{\frac{\alpha r_{2}}{R}}U'(R-(1-\alpha)(r_{1}-S)-\alpha r_{2}-\alpha)f(L)dL}{1-\lambda \ U'(R-(1-\alpha)(r_{1}-S))}}$$
(48)

If we differentiate the model in (39) with respect to α we have:

$$-(r_{1}-S) + p(e) r_{2}\left(1 - F\left(\alpha \frac{r_{2}}{R}\right)\right) + p(e)(1 - F(\alpha)) + \lambda (1 - p(e)) U'(R - (1 - \alpha)(r_{1} - S)) (r_{1} - S) +$$

$$\lambda p(e) \begin{cases} \int_{a}^{a} U'(R - (1 - \alpha)(r_{1} - S) - LR - L) (r_{1} - S) f(L)dL + \\ \int_{a}^{a} \frac{r_{2}}{R} U'(R - (1 - \alpha)(r_{1} - S) - LR - \alpha)(r_{1} - S - 1)f(L)dL + \\ \int_{a}^{1} \frac{r_{2}}{R} U'(R - (1 - \alpha)(r_{1} - S) - \alpha r_{2} - \alpha)(r_{1} - S - r_{2} - 1)f(L)dL \end{cases} - \mu p'(e) U'(R - (1 - \alpha)(r_{1} - S)) (r_{1} - S) +$$

$$\mu p'(e) \begin{cases} \int_{a}^{a} U'(R - (1 - \alpha)(r_{1} - S) - LR - \alpha)(r_{1} - S - r_{2} - 1)f(L)dL \\ \int_{a}^{a} \frac{r_{2}}{R} U'(R - (1 - \alpha)(r_{1} - S) - LR - \alpha)(r_{1} - S - 1)f(L)dL + \\ \int_{a}^{a} \frac{r_{2}}{R} U'(R - (1 - \alpha)(r_{1} - S) - LR - \alpha)(r_{1} - S - 1)f(L)dL + \\ \int_{a}^{a} \frac{r_{2}}{R} U'(R - (1 - \alpha)(r_{1} - S) - LR - \alpha)(r_{1} - S - 1)f(L)dL + \\ \int_{a}^{a} \frac{r_{2}}{R} U'(R - (1 - \alpha)(r_{1} - S) - LR - \alpha)(r_{1} - S - 1)f(L)dL \\ \end{cases} = 0.$$

This can be simplified as follow:

$$\frac{1}{\lambda p(e) + \mu p'(e)} = \frac{\int_{a}^{l} U'(R - (1 - \alpha)(r_1 - S))(r_1 - S) + \int_{0}^{\alpha} U'(R - (1 - \alpha)(r_1 - S) - LR - L)(r_1 - S)f(L)dL + \int_{\alpha}^{\frac{\alpha_2}{R}} U'(R - (1 - \alpha)(r_1 - S) - LR - \alpha)(r_1 - S - 1)f(L)dL + \int_{\alpha}^{\frac{\alpha_2}{R}} U'(R - (1 - \alpha)(r_1 - S) - LR - \alpha)(r_1 - S - 1)f(L)dL + \int_{\alpha}^{\frac{\alpha_2}{R}} U'(R - (1 - \alpha)(r_1 - S) - LR - \alpha)(r_1 - S - 1)f(L)dL + \int_{\alpha}^{\frac{\alpha_2}{R}} U'(R - (1 - \alpha)(r_1 - S) - LR - \alpha)(r_1 - S - 1)f(L)dL + \int_{\alpha}^{\frac{\alpha_2}{R}} U'(R - (1 - \alpha)(r_1 - S) - LR - \alpha)(r_1 - S - 1)f(L)dL + \int_{\alpha}^{\frac{\alpha_2}{R}} U'(R - (1 - \alpha)(r_1 - S) - LR - \alpha)(r_1 - S - 1)f(L)dL + \int_{\alpha}^{\frac{\alpha_2}{R}} U'(R - (1 - \alpha)(r_1 - S) - LR - \alpha)(r_1 - S - 1)f(L)dL + \int_{\alpha}^{\frac{\alpha_2}{R}} U'(R - (1 - \alpha)(r_1 - S) - LR - \alpha)(r_1 - S - 1)f(L)dL + \int_{\alpha}^{\frac{\alpha_2}{R}} U'(R - (1 - \alpha)(r_1 - S) - LR - \alpha)(r_1 - S - 1)f(L)dL + \int_{\alpha}^{\frac{\alpha_2}{R}} U'(R - (1 - \alpha)(r_1 - S) - LR - \alpha)(r_1 - S - 1)f(L)dL + \int_{\alpha}^{\frac{\alpha_2}{R}} U'(R - (1 - \alpha)(r_1 - S) - LR - \alpha)(r_1 - S - 1)f(L)dL + \int_{\alpha}^{\frac{\alpha_2}{R}} U'(R - (1 - \alpha)(r_1 - S) - LR - \alpha)(r_1 - S - 1)f(L)dL + \int_{\alpha}^{\frac{\alpha_2}{R}} U'(R - (1 - \alpha)(r_1 - S) - LR - \alpha)(r_1 - S - 1)f(L)dL + \int_{\alpha}^{\frac{\alpha_2}{R}} U'(R - (1 - \alpha)(r_1 - S) - LR - \alpha)(r_1 - S - 1)f(L)dL + \int_{\alpha}^{\frac{\alpha_2}{R}} U'(R - (1 - \alpha)(r_1 - S) - LR - \alpha)(r_1 - S - 1)f(L)dL + \int_{\alpha}^{\frac{\alpha_2}{R}} U'(R - (1 - \alpha)(r_1 - S) - LR - \alpha)(r_1 - S - 1)f(L)dL + \int_{\alpha}^{\frac{\alpha_2}{R}} U'(R - (1 - \alpha)(r_1 - S) - LR - \alpha)(r_1 - S - 1)f(L)dL + \int_{\alpha}^{\frac{\alpha_2}{R}} U'(R - (1 - \alpha)(r_1 - S) - LR - \alpha)(r_1 - S - 1)f(L)dL + \int_{\alpha}^{\frac{\alpha_2}{R}} U'(R - (1 - \alpha)(r_1 - S) - LR - \alpha)(r_1 - S - 1)f(L)dL + \int_{\alpha}^{\frac{\alpha_2}{R}} U'(R - (1 - \alpha)(r_1 - S) - LR - \alpha)(r_1 - S - 1)f(L)dL + \int_{\alpha}^{\frac{\alpha_2}{R}} U'(R - (1 - \alpha)(r_1 - S) - LR - \alpha)(r_1 - S - 1)f(L)dL + \int_{\alpha}^{\frac{\alpha_2}{R}} U'(R - (1 - \alpha)(r_1 - S) - LR - \alpha)(r_1 - S - 1)f(L)dL + \int_{\alpha}^{\frac{\alpha_2}{R}} U'(R - (1 - \alpha)(r_1 - S) - LR - \alpha)(r_1 - S - 1)f(L)dL + \int_{\alpha}^{\frac{\alpha_2}{R}} U'(R - 1)f(L)dL + \int_{\alpha}^{\frac{\alpha_2}{R$$

By equalizing the two equations together we get the following result:

$$p(e)\left\{r_{2}\left(1-F\left(\alpha\frac{r_{2}}{R}\right)\right) + (1-F(\alpha))\right\}\left\{\int_{\alpha}^{\frac{\sigma_{2}}{R}}U'(R-(1-\alpha)(r_{1}-S)-LR-\alpha)f(L)dL + \int_{\alpha}^{\frac{\sigma_{2}}{R}}U'(R-(1-\alpha)(r_{1}-S)-LR-\alpha)f(L)dL + \int_{\alpha}^{\frac{1}{2}}U'(R-(1-\alpha)(r_{1}-S)-\alpha r_{2}-\alpha)f(L)dL + \int_{\alpha}^{\frac{1}{2}}U'(R-(1-\alpha)(r_{1}-S)-\alpha r_{2}-\alpha)f(L)dL + \int_{\alpha}^{\frac{1}{2}}U'(R-(1-\alpha)(r_{1}-S)-LR-\alpha)f(L)dL + \int_{\alpha}^{\frac{1}{2}}U'(R-(1-\alpha)(r_{1}-S)-\alpha)f(L)dL + \int_{\alpha}^{\frac{1}{2}}U'(R-(1-\alpha)(r_{1}$$

3.1.5 Analyzing the result

When there is no moral hazard we have the following particular case:

$$(1-p(e)) \ U'(R-(1-\alpha)(r_{1}-S)) = \frac{\left\{ \int_{\alpha}^{\frac{dr_{2}}{R}} U'(R-(1-\alpha)(r_{1}-S)-LR-\alpha)f(L)dL + \\ \left(\int_{\alpha}^{\frac{dr_{2}}{R}} U'(R-(1-\alpha)(r_{1}-S)-\alpha r_{2}-\alpha)(r_{2}+1)f(L)dL \right) - p(e) \int_{0}^{\alpha} U'(R-(1-\alpha)(r_{1}-S)-LR-L) \ f(L)dL - \\ \left\{ r_{2} \left(1-F\left(\alpha \frac{r_{2}}{R}\right) \right) + (1-F(\alpha)) \right\} - p(e) \int_{0}^{\alpha} U'(R-(1-\alpha)(r_{1}-S)-LR-L) \ f(L)dL - \\ p(e) \int_{\alpha}^{\frac{dr_{2}}{R}} U'(R-(1-\alpha)(r_{1}-S)-LR-\alpha)f(L)dL - p(e) \int_{\frac{dr_{2}}{R}}^{1} U'(R-(1-\alpha)(r_{1}-S)-\alpha r_{2}-\alpha)f(L)dL - \\ p(e) \int_{\alpha}^{\frac{dr_{2}}{R}} U'(R-(1-\alpha)(r_{1}-S)-LR-\alpha)f(L)dL - p(e) \int_{\frac{dr_{2}}{R}}^{1} U'(R-(1-\alpha)(r_{1}-S)-\alpha r_{2}-\alpha)f(L)dL - \\ p(e) \int_{\alpha}^{\frac{dr_{2}}{R}} U'(R-(1-\alpha)(r_{1}-S)-LR-\alpha)f(L)dL - p(e) \int_{\frac{dr_{2}}{R}}^{1} U'(R-(1-\alpha)(r_{1}-S)-\alpha r_{2}-\alpha)f(L)dL - \\ p(e) \int_{\alpha}^{\frac{dr_{2}}{R}} U'(R-(1-\alpha)(r_{1}-S)-LR-\alpha)f(L)dL - \\ p(e) \int_{\frac{dr_{2}}{R}}^{\frac{dr_{2}}{R}} U'(R-(1-\alpha)(r_{1}-S)-\alpha r_{2}-\alpha)f(L)dL - \\ p(e) \int_{\frac{dr_{2}}{R}}^{\frac{dr_{2}}{R}} U'(R-(1-\alpha)(r_{2$$

We use the following notation to make the above equation simpler:

$$x = \frac{\begin{cases} \int_{\alpha}^{\frac{\alpha r_{2}}{R}} U'(R - (1 - \alpha)(r_{1} - S) - LR - \alpha)f(L)dL + \\ \frac{\int_{\alpha}^{\frac{1}{R}} U'(R - (1 - \alpha)(r_{1} - S) - \alpha r_{2} - \alpha)(r_{2} + 1)f(L)dL \\ \end{cases}}{\left\{ r_{2} \left(1 - F\left(\alpha \frac{r_{2}}{R}\right) \right) + (1 - F(\alpha)) \right\}}$$
(53)

and

$$y = p(e) \int_{0}^{\alpha} U'(R - (1 - \alpha)(r_{1} - S) - LR - L) f(L)dL + p(e) \int_{\alpha}^{\frac{dr_{2}}{R}} U'(R - (1 - \alpha)(r_{1} - S) - LR - \alpha)f(L)dL + p(e) \int_{\frac{dr_{2}}{R}}^{1} U'(R - (1 - \alpha)(r_{1} - S) - \alpha r_{2} - \alpha)f(L)dL$$
(54)

Equation (54) becomes:

$$(1-p(e)) \ U'(R-(1-\alpha)(r_1-S)) = x - y \longrightarrow U'(R-(1-\alpha)(r_1-S)) = \frac{x-y}{(1-p(e))}$$
(55)

With moral hazard we have the following equation using the above notations:

$$(\lambda x - p(e)) \ U'(R - (1 - \alpha)(r_1 - S)) = x - y \longrightarrow U'(R - (1 - \alpha)(r_1 - S)) = \frac{x - y}{(\lambda x - p(e))}$$
(56)

To compare (56) with the no moral hazard case, we can write:

$$\lambda = \frac{1}{x}$$
, which makes (55) and (56) equivalent.

We now suppose that $\lambda < \frac{1}{x}$. It indicates that $U'(R - (1 - \alpha)(r_1 - S))$ is higher in (56) than in (55):

$$U'(R - (1 - \alpha)(r_1 - S)) > \frac{x - y}{(1 - p(e))}$$
(57)

Given that U' is decreasing in wealth, and, as shown before, $r_1 > S$, this means that α should be smaller than or equal to zero to satisfy the inequality. Because α cannot be smaller than zero, this means that there should be full securitization. By putting $\alpha = 0$ in the above inequality, we get the following result:

$$U'(R - r_1 - S) > U'(R - r_1 - S)$$
(58)

which is a contradiction and we reject this possibility. Now considering $\lambda > \frac{1}{x}$, we obtain:

$$U'(R - (1 - \alpha)(r_1 - S)) < \frac{x - y}{(1 - p(e))}$$
⁽⁵⁹⁾

Because U' is decreasing in wealth and $r_1 > S$, this means that α should be greater than zero to solve this inequality. This implies that full securitization is not optimal and there must be retention under moral hazard with tranching and credit enhancing.

In fact, we may suspect that the optimal positive α from (51) is lower than the optimal positive α from (24) because credit enhancement introduces an additional incentive for the originator by relating losses to returns of the equity tranche. In Appendix *B.2*, we show this is the case under risk neutrality and uniform distribution for *L*.

Since $\frac{dx}{de} = 0$, the optimal form of contract like before is a deductible which indicates retaining the constant amount of loan portfolio as equity tranche under moral hazard. The only difference is that the amount of optimal α^* is smaller with credit enhancement comparing to the previous one without credit enhancement. Based on the above result, we can draw the relationship between investor and loan portfolio loss in Figure 3. To compare the result with the previous case without tranching and credit enhancement procedure, we can write the optimal α^* as α_1^* and α_2^* corresponding to optimal amount of retention without and with credit enhancement procedure.

With full securitization, investor loss is equal to loan portfolio loss (L(1+R)) which can be shown as l(R) = L(1+R) and the slope of the 45° line is equal to 1 (Figure 3): *investor loss* = l(R).

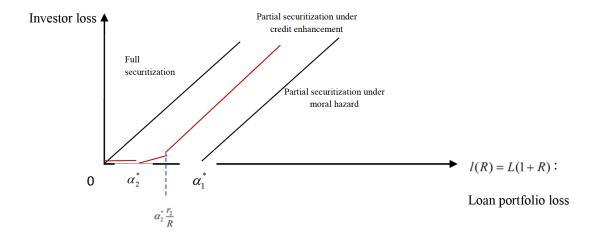
With partial securitization under credit enhancement, we have the following:

When $L < \alpha_2^*$, investor will lose nothing, not on interest and collateral: *investor* loss = 0.

When $\alpha_2^* < L < \alpha_2^* \frac{r_2}{R}$, investor will start losing something on collateral, but nothing on interest. In this case: *investor loss* = $L - \alpha_2^*$, which can be rewritten as *investor loss* = $\frac{l(R)}{(1+R)} - \alpha_2^*$, compare to the full securitization, the slope of the line decreased from 1 to $\frac{1}{(1+R)}$.

When $L > \alpha_2^* \frac{r_2}{R}$, investor will lose something both on collateral and interest. In this case: investor loss = $l(R) - \alpha_2^*(1+r_2)$.

Figure 3: Optimal Securitization Contract under credit enhancement procedure



Investor loss equals 0 when $L < \alpha_2^*$, equals $(L - \alpha_2^*)$ when $\alpha_2^* < L < \alpha_2^* \frac{r_2}{R}$ and equals $(L - \alpha_2^*) + LR - \alpha_2^* r_2$ when $L > \alpha_2^* \frac{r_2}{R}$

 α_2^* (Optimal α with credit enhancement) is smaller than α_1^* (Optimal α without credit enhancement)(see Appendix *B.2*) While

$$\alpha_2^* \frac{r_2}{R}$$
 Could be smaller than or greater than α_1^* .

3.2 Conditional distribution of loss

In this section, we assume that the originator can affect the distribution function F(L|e) and density function f(L|e) of the loss with screening and monitoring effort. In the case of loans default there is a random conditional loss L which is in the interval $[0, 1]: 0 \le L \le 1$.

The optimal amount and form of the structured asset-backed securitization contract is obtained by solving the following maximization program:

$$Max \ \pi_{1} = (1 - \alpha)(r_{1} - S) - p(e) \left[\int_{\frac{dm_{2}}{R}}^{1} (LR - \alpha r_{2}) f(L|e) dL + \int_{\alpha}^{1} (L - \alpha) f(L|e) dL \right]$$
(60)

$$\begin{aligned} \lambda : (1 - p(e))U(R - (1 - \alpha)(r_1 - S)) + \\ p(e) \int_0^{\alpha} U(R - (1 - \alpha)(r_1 - S) - LR - L)f(L|e)dL + p(e) \int_{\alpha}^{\frac{\alpha r_2}{R}} U(R - (1 - \alpha)(r_1 - S) - LR - \alpha)f(L|e)dL + \\ p(e) \int_{\frac{\alpha r_2}{R}}^{1} U(R - (1 - \alpha)(r_1 - S) - \alpha r_2 - \alpha)f(L|e)dL - C(e) = \overline{U} \end{aligned}$$

$$\begin{split} & \mu : -p'(e)U(R - (1 - \alpha)(r_1 - S)) + \\ & p'(e)\int_0^{\alpha} U(R - (1 - \alpha)(r_1 - S) - LR - L)f(L|e)dL + \\ & p'(e)\int_{\alpha}^{\frac{\alpha r_2}{R}} U(R - (1 - \alpha)(r_1 - S) - LR - \alpha)f(L|e)dL + p'(e)\int_{\frac{\alpha r_2}{R}}^{1} U(R - (1 - \alpha)(r_1 - S) - \alpha r_2 - \alpha)f(L|e)dL + \\ & p(e)\int_0^{\alpha} U(R - (1 - \alpha)(r_1 - S) - LR - L)f_e(L|e)dL + p(e)\int_{\alpha}^{\frac{\alpha r_2}{R}} U(R - (1 - \alpha)(r_1 - S) - LR - \alpha)f_e(L|e)dL + \\ & p(e)\int_{\frac{\alpha r_2}{R}}^{1} U(R - (1 - \alpha)(r_1 - S) - \alpha r_2 - \alpha)f_e(L|e)dL = C'(e) \end{split}$$

 $\delta : \alpha \leq 1$

• No Moral Hazard: $\mu = 0$

First we assume that $\mu = 0$ when there is no moral hazard and $e = e^{**}$. The investor observes *e* and sets its level at the full information level e^{**} by using the "take it or leave it" strategy. If we differentiate with respect to *S*, we have:

$$S: - (1 - \alpha) + \lambda (1 - p(e)) U'(R - (1 - \alpha)(r_1 - S))(1 - \alpha) +$$

$$\lambda p(e) \int_0^{\alpha} U'(R - (1 - \alpha)(r_1 - S) - LR - L)(1 - \alpha) f(L|e) dL + \lambda p(e) \int_{\alpha}^{\frac{\alpha r_2}{R}} U'(R - (1 - \alpha)(r_1 - S) - LR - \alpha)(1 - \alpha) f(L|e) dL +$$

$$\lambda p(e) \int_{\frac{\alpha r_2}{R}}^{1} U'(R - (1 - \alpha)(r_1 - S) - \alpha r_2 - \alpha)(1 - \alpha) f(L|e) dL = 0.$$
(61)

By solving the above equation, we can isolate $\frac{1}{\lambda}$ as follows:

$$\frac{1}{\lambda} = (1 - p(e)) U'(R - (1 - \alpha)(r_1 - S)) +$$

$$p(e) \int_0^{\alpha} U'(R - (1 - \alpha)(r_1 - S) - LR - L) f(L|e) dL + p(e) \int_{\alpha}^{\frac{\alpha r_2}{R}} U'(R - (1 - \alpha)(r_1 - S) - LR - \alpha)(1 - \alpha) f(L|e) dL +$$

$$p(e) \int_{\frac{\alpha r_2}{R}}^{\frac{\alpha r_2}{R}} U'(R - (1 - \alpha)(r_1 - S) - \alpha r_2 - \alpha) f(L|e) dL$$
(62)

Differentiating with respect to α yields:

$$-(r_{1} - S) + p(e) r_{2}\left(1 - F\left(\alpha \frac{r_{2}}{R}\right)\right) + p(e)(1 - F(\alpha)) + \lambda (1 - p(e)) U'(R - (1 - \alpha)(r_{1} - S)) (r_{1} - S) + \lambda p(e) \int_{0}^{\alpha} U'(R - (1 - \alpha)(r_{1} - S) - LR - L) (r_{1} - S) f(L|e)dL + \lambda p(e) \int_{\alpha}^{\frac{\alpha_{2}}{R}} U'(R - (1 - \alpha)(r_{1} - S) - LR - \alpha)(r_{1} - S - 1) f(L|e)dL + \lambda p(e) \int_{\alpha}^{1} U'(R - (1 - \alpha)(r_{1} - S) - \alpha r_{2} - \alpha)(r_{1} - S - r_{2} - 1) f(L|e)dL = 0.$$
(63)

By solving this equation for $\frac{1}{\lambda}$, we obtain:

$$\frac{1}{\lambda} = \begin{cases}
\frac{(1-p(e)) U'(R-(1-\alpha)(r_{1}-S)) (r_{1}-S)}{(r_{1}-S) - p(e) r_{2} \left(1-F\left(\alpha \frac{r_{2}}{R}\right)\right) - p(e)(1-F(\alpha))} + \frac{p(e)\int_{\alpha}^{\alpha} U'(R-(1-\alpha)(r_{1}-S) - LR - L) (r_{1}-S) f(L|e)dL}{(r_{1}-S) - p(e) r_{2} \left(1-F\left(\alpha \frac{r_{2}}{R}\right)\right) - p(e)(1-F(\alpha))} + \frac{p(e)\int_{\alpha}^{\frac{\alpha}{R}} U'(R-(1-\alpha)(r_{1}-S) - LR - \alpha)(r_{1}-S - 1)f(L|e)dL}{(r_{1}-S) - p(e) r_{2} \left(1-F\left(\alpha \frac{r_{2}}{R}\right)\right) - p(e)(1-F(\alpha))} + \begin{cases}
\frac{\lambda p(e)\int_{\alpha}^{\frac{1}{2}} U'(R-(1-\alpha)(r_{1}-S) - \alpha r_{2} - \alpha)(r_{1}-S - r_{2} - 1)f(L|e)dL}{(r_{1}-S) - p(e) r_{2} \left(1-F\left(\alpha \frac{r_{2}}{R}\right)\right) - p(e)(1-F(\alpha))} \end{cases} + \begin{cases}
\frac{\lambda p(e)\int_{\alpha}^{\frac{1}{2}} U'(R-(1-\alpha)(r_{1}-S) - \alpha r_{2} - \alpha)(r_{1}-S - r_{2} - 1)f(L|e)dL}{(r_{1}-S) - p(e) r_{2} \left(1-F\left(\alpha \frac{r_{2}}{R}\right)\right) - p(e)(1-F(\alpha))} \end{cases} + \begin{cases}
\frac{\lambda p(e)\int_{\alpha}^{\frac{1}{2}} U'(R-(1-\alpha)(r_{1}-S) - \alpha r_{2} - \alpha)(r_{1}-S - r_{2} - 1)f(L|e)dL}{(r_{1}-S) - p(e) r_{2} \left(1-F\left(\alpha \frac{r_{2}}{R}\right)\right) - p(e)(1-F(\alpha))} \end{cases} + \begin{cases}
\frac{\lambda p(e)\int_{\alpha}^{\frac{1}{2}} U'(R-(1-\alpha)(r_{1}-S) - \alpha r_{2} - \alpha)(r_{1}-S - r_{2} - 1)f(L|e)dL}{(r_{1}-S) - p(e) r_{2} \left(1-F\left(\alpha \frac{r_{2}}{R}\right)\right) - p(e)(1-F(\alpha))} \end{cases} + \begin{cases}
\frac{\lambda p(e)\int_{\alpha}^{\frac{1}{2}} U'(R-(1-\alpha)(r_{1}-S) - \alpha r_{2} - \alpha)(r_{1}-S - r_{2} - 1)f(L|e)dL}{(r_{1}-S) - p(e) r_{2} \left(1-F\left(\alpha \frac{r_{2}}{R}\right)\right) - p(e)(1-F(\alpha))} \end{cases}$$

By equalizing the two equations together we get the following result:

$$p(e)\int_{\alpha}^{\frac{\alpha r_{2}}{R}} U'(R-(1-\alpha)(r_{1}-S)-LR-\alpha)(-1)f(L|e)dL + p(e)\int_{\frac{\alpha r_{2}}{R}}^{1} U'(R-(1-\alpha)(r_{1}-S)-\alpha r_{2}-\alpha)(-r_{2}-1)f(L|e)dL =$$

$$-p(e)\left\{r_{2}\left(1-F\left(\alpha\frac{r_{2}}{R}\right)\right) + (1-F(\alpha))\right\} \begin{cases} (1-p(e)) U'(R-(1-\alpha)(r_{1}-S)) + p(e)\int_{0}^{\alpha} U'(R-(1-\alpha)(r_{1}-S)-LR-L) f(L|e)dL + \\ p(e)\int_{\alpha}^{\frac{\alpha r_{2}}{R}} U'(R-(1-\alpha)(r_{1}-S)-LR-\alpha)f(L|e)dL + p(e)\int_{\frac{\alpha r_{2}}{R}}^{1} U'(R-(1-\alpha)(r_{1}-S)-\alpha r_{2}-\alpha)f(L|e)dL \end{cases}$$

$$(65)$$

 α =0 solves this equation.

• *Moral Hazard* : $\mu \neq 0$

Now we assume that $\mu \neq 0$ and there is potential moral hazard. If we differentiate the model with respect to *S*, we have:

$$S: - (1-\alpha) + \lambda (1-p(e)) U'(R-(1-\alpha)(r_{1}-S))(1-\alpha) +$$

$$\begin{pmatrix} \int_{0}^{\alpha} U'(R-(1-\alpha)(r_{1}-S) - LR - L)(1-\alpha)f(L|e)dL + \int_{\alpha}^{\frac{\sigma_{2}}{R}} U'(R-(1-\alpha)(r_{1}-S) - LR - \alpha)(1-\alpha)f(L|e)dL + \\ \int_{\frac{\sigma_{2}}{R}}^{\alpha} U'(R-(1-\alpha)(r_{1}-S) - \alpha r_{2} - \alpha)(1-\alpha)f(L|e)dL + \\ \mu p'(e) \int_{0}^{\alpha} U'(R-(1-\alpha)(r_{1}-S) - LR - L)(1-\alpha)f(L|e)dL + \int_{\alpha}^{\frac{\sigma_{2}}{R}} U'(R-(1-\alpha)(r_{1}-S) - LR - \alpha)(1-\alpha)f(L|e)dL + \\ \int_{\frac{\sigma_{2}}{R}}^{\frac{\sigma_{2}}{R}} U'(R-(1-\alpha)(r_{1}-S) - LR - L)(1-\alpha)f(L|e)dL + \\ \int_{\alpha}^{\frac{\sigma_{2}}{R}} U'(R-(1-\alpha)(r_{1}-S) - LR$$

By simplifying the above equation, we find.

$$\frac{1}{\lambda p(e) + \mu p'(e)} = \frac{\int_{a}^{1} U'(R - (1 - \alpha)(r_1 - S)) + \int_{a}^{\alpha} U'(R - (1 - \alpha)(r_1 - S) - LR - L)f(L|e)dL + \int_{\alpha}^{\frac{\alpha_2}{R}} U'(R - (1 - \alpha)(r_1 - S) - LR - \alpha)f(L|e)dL + \int_{\alpha}^{1} U'(R - (1 - \alpha)(r_1 - S) - LR - \alpha)f(L|e)dL + \int_{\alpha}^{1} U'(R - (1 - \alpha)(r_1 - S) - LR - \alpha)f(L|e)dL + \int_{\alpha}^{1} U'(R - (1 - \alpha)(r_1 - S) - LR - \alpha)f(L|e)dL + \int_{\alpha}^{1} U'(R - (1 - \alpha)(r_1 - S) - LR - \alpha)f(L|e)dL + \int_{\alpha}^{1} U'(R - (1 - \alpha)(r_1 - S) - LR - \alpha)f(L|e)dL + \int_{\alpha}^{1} U'(R - (1 - \alpha)(r_1 - S) - LR - \alpha)f(L|e)dL + \int_{\alpha}^{1} U'(R - (1 - \alpha)(r_1 - S) - LR - \alpha)f(L|e)dL + \int_{\alpha}^{1} U'(R - (1 - \alpha)(r_1 - S) - LR - \alpha)f(L|e)dL + \int_{\alpha}^{1} U'(R - (1 - \alpha)(r_1 - S) - LR - \alpha)f(L|e)dL + \int_{\alpha}^{1} U'(R - (1 - \alpha)(r_1 - S) - LR - \alpha)f(L|e)dL + \int_{\alpha}^{1} U'(R - (1 - \alpha)(r_1 - S) - LR - \alpha)f(L|e)dL + \int_{\alpha}^{1} U'(R - (1 - \alpha)(r_1 - S) - LR - \alpha)f(L|e)dL + \int_{\alpha}^{1} U'(R - (1 - \alpha)(r_1 - S) - LR - \alpha)f(L|e)dL + \int_{\alpha}^{1} U'(R - (1 - \alpha)(r_1 - S) - LR - \alpha)f(L|e)dL + \int_{\alpha}^{1} U'(R - (1 - \alpha)(r_1 - S) - LR - \alpha)f(L|e)dL + \int_{\alpha}^{1} U'(R - (1 - \alpha)(r_1 - S) - LR - \alpha)f(L|e)dL + \int_{\alpha}^{1} U'(R - (1 - \alpha)(r_1 - S) - LR - \alpha)f(L|e)dL + \int_{\alpha}^{1} U'(R - (1 - \alpha)(r_1 - S) - LR - \alpha)f(L|e)dL + \int_{\alpha}^{1} U'(R - (1 - \alpha)(r_1 - S) - LR - \alpha)f(L|e)dL + \int_{\alpha}^{1} U'(R - (1 - \alpha)(r_1 - S) - LR - \alpha)f(L|e)dL + \int_{\alpha}^{1} U'(R - (1 - \alpha)(r_1 - S) - LR - \alpha)f(L|e)dL + \int_{\alpha}^{1} U'(R - (1 - \alpha)(r_1 - S) - LR - \alpha)f(L|e)dL + \int_{\alpha}^{1} U'(R - (1 - \alpha)(r_1 - S) - LR - \alpha)f(L|e)dL + \int_{\alpha}^{1} U'(R - (1 - \alpha)(r_1 - S) - LR - \alpha)f(L|e)dL + \int_{\alpha}^{1} U'(R - (1 - \alpha)(r_1 - S) - LR - \alpha)f(L|e)dL + \int_{\alpha}^{1} U'(R - (1 - \alpha)(r_1 - S) - LR - \alpha)f(L|e)dL + \int_{\alpha}^{1} U'(R - (1 - \alpha)(r_1 - S) - LR - \alpha)f(L|e)dL + \int_{\alpha}^{1} U'(R - (1 - \alpha)(r_1 - S) - LR - \alpha)f(L|e)dL + \int_{\alpha}^{1} U'(R - (1 - \alpha)(r_1 - S) - LR - \alpha)f(L|e)dL + \int_{\alpha}^{1} U'(R - (1 - \alpha)(r_1 - S) - LR - \alpha)f(L|e)dL + \int_{\alpha}^{1} U'(R - (1 - \alpha)(r_1 - S) - LR - \alpha)f(L|e)dL + \int_{\alpha}^{1} U'(R - (1 - \alpha)(r_1 - S) - LR - \alpha)f(L|e)dL + \int_{\alpha}^{1} U'(R - (1 - \alpha)(r_1 - S) - LR - \alpha)f(L|e)dL + \int_{\alpha}^{1} U'(R - (1 - \alpha)(r_1 - S) - LR - \alpha)f(L|e)dL + \int_{\alpha}^{1} U'(R - (1 - \alpha)(r_1$$

If we differentiate with respect to α we have:

$$-(r_{1}-S) + p(e) r_{2}\left(1 - F\left(\alpha \frac{r_{2}}{R}\right)\right) + p(e)(1 - F(\alpha)) + \lambda (1 - p(e)) U'(R - (1 - \alpha)(r_{1} - S)) (r_{1} - S) +$$

$$k p(e) \left\{ \int_{0}^{\alpha} U'(R - (1 - \alpha)(r_{1} - S) - LR - \alpha)(r_{1} - S) f(L|e)dL + \int_{\alpha}^{\frac{\sigma_{2}}{R}} U'(R - (1 - \alpha)(r_{1} - S) - LR - \alpha)(r_{1} - S - 1)f(L|e)dL + \int_{\alpha}^{\frac{\sigma_{2}}{R}} U'(R - (1 - \alpha)(r_{1} - S) - LR - \alpha)(r_{1} - S - 1)f(L|e)dL + \int_{\alpha}^{\frac{\sigma_{2}}{R}} U'(R - (1 - \alpha)(r_{1} - S) - LR - \alpha)(r_{1} - S - r_{2} - 1)f(L|e)dL + \int_{\alpha}^{\frac{\sigma_{2}}{R}} U'(R - (1 - \alpha)(r_{1} - S) - LR - \alpha)(r_{1} - S - r_{2} - 1)f(L|e)dL + \int_{\alpha}^{\frac{\sigma_{2}}{R}} U'(R - (1 - \alpha)(r_{1} - S) - LR - \alpha)(r_{1} - S - 1)f(L|e)dL + \int_{\alpha}^{\frac{\sigma_{2}}{R}} U'(R - (1 - \alpha)(r_{1} - S) - LR - \alpha)(r_{1} - S - 1)f(L|e)dL + \int_{\alpha}^{\frac{\sigma_{2}}{R}} U'(R - (1 - \alpha)(r_{1} - S) - LR - \alpha)(r_{1} - S - 1)f(L|e)dL + \int_{\alpha}^{\frac{\sigma_{2}}{R}} U'(R - (1 - \alpha)(r_{1} - S) - LR - \alpha)(r_{1} - S - 1)f(L|e)dL + \int_{\alpha}^{\frac{\sigma_{2}}{R}} U'(R - (1 - \alpha)(r_{1} - S) - LR - \alpha)(r_{1} - S - 1)f(L|e)dL + \int_{\alpha}^{\frac{\sigma_{2}}{R}} U'(R - (1 - \alpha)(r_{1} - S) - LR - \alpha)(r_{1} - S - r_{2} - 1)f(L|e)dL + \int_{\alpha}^{\frac{\sigma_{2}}{R}} U'(R - (1 - \alpha)(r_{1} - S) - LR - \alpha)(r_{1} - S - r_{2} - 1)f_{e}(L|e)dL + \int_{\alpha}^{\frac{\sigma_{2}}{R}} U'(R - (1 - \alpha)(r_{1} - S) - LR - \alpha)(r_{1} - S - r_{2} - 1)f_{e}(L|e)dL + \int_{\alpha}^{\frac{\sigma_{2}}{R}} U'(R - (1 - \alpha)(r_{1} - S) - \alpha r_{2} - \alpha)(r_{1} - S - r_{2} - 1)f_{e}(L|e)dL + \int_{\alpha}^{\frac{\sigma_{2}}{R}} U'(R - (1 - \alpha)(r_{1} - S) - \alpha r_{2} - \alpha)(r_{1} - S - r_{2} - 1)f_{e}(L|e)dL + \int_{\alpha}^{\frac{\sigma_{2}}{R}} U'(R - (1 - \alpha)(r_{1} - S) - \alpha r_{2} - \alpha)(r_{1} - S - r_{2} - 1)f_{e}(L|e)dL + \int_{\alpha}^{\frac{\sigma_{2}}{R}} U'(R - (1 - \alpha)(r_{1} - S) - \alpha r_{2} - \alpha)(r_{1} - S - r_{2} - 1)f_{e}(L|e)dL + \int_{\alpha}^{\frac{\sigma_{2}}{R}} U'(R - (1 - \alpha)(r_{1} - S) - \alpha r_{2} - \alpha)(r_{1} - S - r_{2} - 1)f_{e}(L|e)dL + \int_{\alpha}^{\frac{\sigma_{2}}{R}} U'(R - (1 - \alpha)(r_{1} - S) - \alpha r_{2} - \alpha)(r_{1} - S - r_{2} - 1)f_{e}(L|e)dL + \int_{\alpha}^{\frac{\sigma_{2}}{R}} U'(R - (1 - \alpha)(r_{1} - S) - \alpha r_{2} - \alpha)(r_{1} - S - r_{2} - 1)f_{e}(L|e)dL + \int_{\alpha}^{\frac{\sigma_{2}}{R}} U'(R - (1 - \alpha)(r_{1} - S) - \alpha r_{2} - \alpha)(r_{1} - S - r_{2} - 1)f_{e}(L|e)dL + \int_{\alpha}^{\frac{\sigma_{2}}{R}} U'(R - (1 - \alpha)$$

This can be simplified as follow:

$$\frac{1}{\lambda p(e) + \mu p'(e)} = \frac{\int_{a}^{a} U'(R - (1 - \alpha)(r_1 - S))(r_1 - S) + \int_{0}^{a} U'(R - (1 - \alpha)(r_1 - S) - LR - L)(r_1 - S)f(L|e)dL + \int_{a}^{\frac{\omega_1}{R}} U'(R - (1 - \alpha)(r_1 - S) - LR - \alpha)(r_1 - S - 1)f(L|e)dL + (r_1 - \alpha)(r_1 - S) - \alpha r_2 - \alpha)(r_1 - S - r_2 - 1)f(L|e)dL}{(r_1 - S) - \lambda U'(R - (1 - \alpha)(r_1 - S))(r_1 - S) - p(e)\left\{r_2\left(1 - F\left(\alpha \frac{r_2}{R}\right)\right) + (1 - F(\alpha))\right\}} - \mu p(e)\left\{\int_{0}^{a} U'(R - (1 - \alpha)(r_1 - S) - LR - L)(r_1 - S)f_e(L|e)dL + \int_{a}^{\frac{\omega_1}{R}} U'(R - (1 - \alpha)(r_1 - S) - LR - \alpha)(r_1 - S - 1)f_e(L|e)dL + \int_{a}^{\frac{\omega_1}{R}} U'(R - (1 - \alpha)(r_1 - S) - LR - \alpha)(r_1 - S - 1)f_e(L|e)dL + \int_{a}^{\frac{\omega_1}{R}} U'(R - (1 - \alpha)(r_1 - S) - \mu r_2 - \alpha)(r_1 - S - r_2 - 1)f(L|e)dL + \int_{a}^{\frac{\omega_1}{R}} U'(R - (1 - \alpha)(r_1 - S) - LR - \mu r_2 - \alpha)(r_1 - S - 1)f_e(L|e)dL + \int_{a}^{\frac{\omega_1}{R}} U'(R - (1 - \alpha)(r_1 - S) - LR - \mu r_2 - \alpha)(r_1 - S - 1)f_e(L|e)dL + \int_{a}^{\frac{\omega_1}{R}} U'(R - (1 - \alpha)(r_1 - S) - LR - \mu r_2 - \alpha)(r_1 - S - 1)f_e(L|e)dL + \int_{a}^{\frac{\omega_1}{R}} U'(R - (1 - \alpha)(r_1 - S) - \mu r_2 - \alpha)(r_1 - S - r_2 - 1)f_e(L|e)dL + \int_{a}^{\frac{\omega_1}{R}} U'(R - (1 - \alpha)(r_1 - S) - \mu r_2 - \alpha)(r_1 - S - r_2 - 1)f_e(L|e)dL + \int_{a}^{\frac{\omega_1}{R}} U'(R - (1 - \alpha)(r_1 - S) - \mu r_2 - \alpha)(r_1 - S - r_2 - 1)f_e(L|e)dL + \int_{a}^{\frac{\omega_1}{R}} U'(R - (1 - \alpha)(r_1 - S) - \mu r_2 - \alpha)(r_1 - S - r_2 - 1)f_e(L|e)dL + \int_{a}^{\frac{\omega_1}{R}} U'(R - (1 - \alpha)(r_1 - S) - \mu r_2 - \alpha)(r_1 - S - r_2 - 1)f_e(L|e)dL + \int_{a}^{\frac{\omega_1}{R}} U'(R - (1 - \alpha)(r_1 - S) - \mu r_2 - \alpha)(r_1 - S - r_2 - 1)f_e(L|e)dL + \int_{a}^{\frac{\omega_1}{R}} U'(R - (1 - \alpha)(r_1 - S) - \mu r_2 - \alpha)(r_1 - S - r_2 - 1)f_e(L|e)dL + \int_{a}^{\frac{\omega_1}{R}} U'(R - (1 - \alpha)(r_1 - S) - \mu r_2 - \alpha)(r_1 - S - r_2 - 1)f_e(L|e)dL + \int_{a}^{\frac{\omega_1}{R}} U'(R - (1 - \alpha)(r_1 - S) - \mu r_2 - \alpha)(r_1 - S - r_2 - 1)f_e(L|e)dL + \int_{a}^{\frac{\omega_1}{R}} U'(R - (1 - \alpha)(r_1 - S) - \mu r_2 - \alpha)(r_1 - S - r_2 - 1)f_e(L|e)dL + \int_{a}^{\frac{\omega_1}{R}} U'(R - (1 - \alpha)(r_1 - S) - \mu r_2 - \alpha)(r_1 - S - r_2 - 1)f_e(L|e)dL + \int_{a}^{\frac{\omega_1}{R}} U'(R - (1 - \alpha)(r_1 - S) - \mu r_2 - \alpha)(r_1 - S - r_2 - 1)f_e(L|e)dL + \int_{a}^{\frac{\omega_1}{R}} U'(R - (1 - \alpha)(r_1 - S) - \mu r_2 - \alpha)(r_1 - S - r_2 - 1)$$

By equalizing the two equations together we get the following result:

$$p(e)\left\{r_{2}\left(1-F\left(\alpha\frac{r_{2}}{R}\right)\right)+(1-F(\alpha))\right\}\left\{\begin{matrix}-U'(R-(1-\alpha)(r_{1}-S))+\int_{0}^{\alpha}U'(R-(1-\alpha)(r_{1}-S)-LR-L)f(L|e)dL+\\\int_{\alpha}^{\frac{\sigma_{2}}{R}}U'(R-(1-\alpha)(r_{1}-S)-LR-\alpha)f(L|e)dL+\\\int_{\alpha}^{\frac{1}{M}}U'(R-(1-\alpha)(r_{1}-S)-\alpha r_{2}-\alpha)f(L|e)dL\end{matrix}\right\}$$

$$=$$

$$\mu \ p(e)\left\{\int_{\alpha}^{\frac{\sigma_{2}}{R}}U'(R-(1-\alpha)(r_{1}-S)-LR-\alpha)f_{e}(L|e)dL+\int_{\frac{1}{M}}^{\frac{\sigma_{2}}{R}}U'(R-(1-\alpha)(r_{1}-S)-\alpha r_{2}-\alpha)(r_{2}+1)f_{e}(L|e)dL\right\}$$

$$+\left\{1-\lambda U'(R-(1-\alpha)(r_{1}-S))\right\}\left[\int_{\alpha}^{\frac{\sigma_{2}}{R}}U'(R-(1-\alpha)(r_{1}-S)-LR-\alpha)f(L|e)dL+\\\int_{\alpha}^{\frac{1}{M}}U'(R-(1-\alpha)(r_{1}-S)-LR-\alpha)f(L|e)dL+\\\int_{\alpha}^{\frac{1}{M}}U'(R-(1-\alpha)(r_{1}-S)-LR-\alpha)f(L|e)dL+\\\int_{\alpha}^{\frac{1}{M}}U'(R-(1-\alpha)(r_{1}-S)-LR-\alpha)f(L|e)dL+\\\int_{\alpha}^{\frac{1}{M}}U'(R-(1-\alpha)(r_{1}-S)-LR-\alpha)f(L|e)dL+\\\int_{\alpha}^{\frac{1}{M}}U'(R-(1-\alpha)(r_{1}-S)-LR-\alpha)f(L|e)dL+\\\int_{\alpha}^{\frac{1}{M}}U'(R-(1-\alpha)(r_{1}-S)-LR-\alpha)f(L|e)dL+\\\int_{\alpha}^{\frac{1}{M}}U'(R-(1-\alpha)(r_{1}-S)-LR-\alpha)f(L|e)dL+\\\int_{\alpha}^{\frac{1}{M}}U'(R-(1-\alpha)(r_{1}-S)-LR-\alpha)f(L|e)dL+\\\int_{\alpha}^{\frac{1}{M}}U'(R-(1-\alpha)(r_{1}-S)-LR-\alpha)f(L|e)dL+\\\int_{\alpha}^{\frac{1}{M}}U'(R-(1-\alpha)(r_{1}-S)-LR-\alpha)f(L|e)dL+\\\int_{\alpha}^{\frac{1}{M}}U'(R-(1-\alpha)(r_{1}-S)-LR-\alpha)f(L|e)dL+\\\int_{\alpha}^{\frac{1}{M}}U'(R-(1-\alpha)(r_{1}-S)-LR-\alpha)f(L|e)dL+\\\int_{\alpha}^{\frac{1}{M}}U'(R-(1-\alpha)(r_{1}-S)-LR-\alpha)f(L|e)dL+\\\int_{\alpha}^{\frac{1}{M}}U'(R-(1-\alpha)(r_{1}-S)-LR-\alpha)f(L|e)dL+\\\int_{\alpha}^{\frac{1}{M}}U'(R-(1-\alpha)(r_{1}-S)-LR-\alpha)f(L|e)dL+\\\int_{\alpha}^{\frac{1}{M}}U'(R-(1-\alpha)(r_{1}-S)-LR-\alpha)f(L|e)dL+\\\int_{\alpha}^{\frac{1}{M}}U'(R-(1-\alpha)(r_{1}-S)-LR-\alpha)f(L|e)dL+\\\int_{\alpha}^{\frac{1}{M}}U'(R-(1-\alpha)(r_{1}-S)-LR-\alpha)f(L|e)dL+\\\int_{\alpha}^{\frac{1}{M}}U'(R-(1-\alpha)(r_{1}-S)-LR-\alpha)f(L|e)dL+\\\int_{\alpha}^{\frac{1}{M}}U'(R-(1-\alpha)(r_{1}-S)-LR-\alpha)f(L|e)dL+\\\int_{\alpha}^{\frac{1}{M}}U'(R-(1-\alpha)(r_{1}-S)-LR-\alpha)f(L|e)dL+\\\int_{\alpha}^{\frac{1}{M}}U'(R-(1-\alpha)(r_{1}-S)-LR-\alpha)f(L|e)dL+\\\int_{\alpha}^{\frac{1}{M}}U'(R-(1-\alpha)(r_{1}-S)-LR-\alpha)f(L|e)dL+\\\int_{\alpha}^{\frac{1}{M}}U'(R-(1-\alpha)(r_{1}-S)-LR-\alpha)f(L|e)dL+\\\int_{\alpha}^{\frac{1}{M}}U'(R-(1-\alpha)(r_{1}-S)-LR-\alpha)f(L|e)dL+\\\int_{\alpha}^{\frac{1}{M}}U'(R-(1-\alpha)(r_{1}-S)-LR-\alpha)f(L|e)dL+\\\int_{\alpha}^{\frac{1}{M}}U'(R-(1-\alpha)(r_{1}-S)-LR-\alpha)f(L|e)dL+\\\int_{\alpha}^{\frac{1}{M}}U'(R-(1-\alpha)(r_{1}-S)-LR-\alpha)f(L|e)dL+\\\int_{\alpha}^{\frac{1}{M}}U'(R-(1-\alpha)(r_{1}-S)-LR-\alpha)f(L|e)dL+\\\int_{\alpha}^{\frac{1}{M}}U'(R-(1-\alpha)(r_{1}-S)-LR-\alpha)f(L|e)dL+\\\int_{\alpha}^{\frac{1}{M}}U'(R-(1-\alpha)(r_{1}-S)-LR-\alpha)f(L|e)dL+\\\int_{\alpha}^{\frac{1}{M}}U'(R-(1-\alpha)(r_{1$$

3.2.1 Analyzing the result

We use the following notations to make the comparison possible:

$$x = \frac{\begin{cases} \int_{\alpha}^{\frac{\alpha r_{2}}{R}} U'(R - (1 - \alpha)(r_{1} - S) - LR - \alpha)f(L|e)dL + \\ \int_{\alpha}^{1} U'(R - (1 - \alpha)(r_{1} - S) - \alpha r_{2} - \alpha)(r_{2} + 1)f(L|e)dL \\ \end{cases}}{\left\{ r_{2} \left(1 - F\left(\alpha \frac{r_{2}}{R}\right) \right) + (1 - F(\alpha)) \right\}}$$

$$y = p(e) \int_{0}^{\alpha} U'(R - (1 - \alpha)(r_{1} - S) - LR - L) f(L|e)dL + \\ p(e) \int_{\alpha}^{\frac{\alpha r_{2}}{R}} U'(R - (1 - \alpha)(r_{1} - S) - LR - \alpha)f(L|e)dL + p(e) \int_{\frac{\alpha r_{2}}{R}}^{1} U'(R - (1 - \alpha)(r_{1} - S) - LR - \alpha)f(L|e)dL + p(e) \int_{\alpha}^{1} U'(R - (1 - \alpha)(r_{1} - S) - LR - \alpha)f(L|e)dL + p(e) \int_{R}^{1} U'(R - (1 - \alpha)(r_{1} - S) - LR - \alpha)f(L|e)dL + p(e) \int_{R}^{1} U'(R - (1 - \alpha)(r_{1} - S) - LR - \alpha)f(L|e)dL + p(e) \int_{R}^{1} U'(R - (1 - \alpha)(r_{1} - S) - LR - \alpha)f(L|e)dL + p(e) \int_{R}^{1} U'(R - (1 - \alpha)(r_{1} - S) - LR - \alpha)f(L|e)dL + p(e) \int_{R}^{1} U'(R - (1 - \alpha)(r_{1} - S) - LR - \alpha)f(L|e)dL + p(e) \int_{R}^{1} U'(R - (1 - \alpha)(r_{1} - S) - LR - \alpha)f(L|e)dL + p(e) \int_{R}^{1} U'(R - (1 - \alpha)(r_{1} - S) - LR - \alpha)f(L|e)dL + p(e) \int_{R}^{1} U'(R - (1 - \alpha)(r_{1} - S) - LR - \alpha)f(L|e)dL + p(e) \int_{R}^{1} U'(R - (1 - \alpha)(r_{1} - S) - LR - \alpha)f(L|e)dL + p(e) \int_{R}^{1} U'(R - (1 - \alpha)(r_{1} - S) - LR - \alpha)f(L|e)dL + p(e) \int_{R}^{1} U'(R - (1 - \alpha)(r_{1} - S) - LR - \alpha)f(L|e)dL + p(e) \int_{R}^{1} U'(R - (1 - \alpha)(r_{1} - S) - \alpha r_{2} - \alpha)f(L|e)dL$$

When there is no moral hazard we have the following equation using the above notations:

$$(1-p(e)) \ U'(R-(1-\alpha)(r_1-S)) = x - y \longrightarrow U'(R-(1-\alpha)(r_1-S)) = \frac{x-y}{(1-p(e))}$$
(72)

With moral hazard we have the following equation using above notations:

$$(\lambda x - p(e)) \ U'(R - (1 - \alpha)(r_1 - S)) = x - y + \mu p(e) x_e \longrightarrow U'(R - (1 - \alpha)(r_1 - S)) = \frac{x - y + \mu p(e) x_e}{(\lambda x - p(e))}$$
(73)

To compare the moral hazard case with the no moral hazard one (equations (73) and (72) correspondingly) and get the global maximum contract to obtain the optimal α , we impose MLRP (Monotone Likelihood Ratio Property) (see Winter (2000)). MLRP is a property of the ratio $\frac{f_e(L|e)}{f(L|e)}$ when f(L|e) is a probability density function and $f_e(L|e)$ is the derivative of f(L|e) with respect to e. We need to make an assumption about the sign of $\frac{d}{dL} \left(\frac{f_e(L|e)}{f(L|e)} \right)$ to obtain an optimal solution.

Its sign depends on L and the ratio can be >, = or < than 0. Moreover the sign of MLRP changes

as L changes. To make an assumption possible we need to analyze the sign of x_e . In the following analysis we consider 4 cases to make a correct assumption:

First we assume that
$$\lambda < \frac{1}{x}$$
 and $x_e < 0$:

In this case the comparison between equations (72) and (73) is impossible, since at the same time both numerator and the denominator of equation (73) decreases.

Second we assume that
$$\lambda < \frac{1}{x}$$
 and $x_e > 0$:

In this case since the numerator of equation (73) increases and its denominator decreases, $U'(R-(1-\alpha)(r_1-S))$ is increasing:

$$U'(R - (1 - \alpha)(r_1 - S)) > \frac{x - y}{(1 - p(e))}$$
(74)

Since U' is decreasing in wealth under risk aversion and as shown before $r_1 > S$ this means that α must be smaller than or equal to zero to satisfy the inequality. Since α cannot be smaller than zero, this means that there should be full securitization. By putting $\alpha = 0$ in the above inequality, we get the following result:

$$U'(R - r_1 - S) > U'(R - r_1 - S)$$
(75)

Which is a contradiction and we reject this possibility.

Now considering the third case, where $\lambda > \frac{1}{x}$ and $x_e > 0$:

In this case the comparison is again impossible, since at the same time both numerator and the denominator of equation (73) increase.

Finally we consider the fourth case, where $\lambda > \frac{1}{x}$ and $x_e < 0$:

In this case since the numerator of equation (73) decreases and its denominator increases, therefore $U'(R - (1 - \alpha)(r_1 - S))$ is decreasing:

$$U'(R - (1 - \alpha)(r_1 - S)) < \frac{x - y}{(1 - p(e))}$$
(76)

Because U' is decreasing in wealth and $r_1 > S$, this means that α should be greater than zero to solve this inequality. This implies that full securitization is not optimal and there must be retention under moral hazard. The optimal solution is $\alpha^* > 0$. In Appendix *B.3*, we show that under risk neutrality and exponential distribution for *L*, x_e will be negative which means that these assumptions are satisfactory to get the result.

The structure of x_e determines the structure of the contract. In the previous section when we had tranching but an unconditional distribution of loss, x_e was equal to zero:

$$x = \frac{\begin{cases} \int_{\alpha}^{\frac{\alpha_{2}}{R}} U'(R - (1 - \alpha)(r_{1} - S) - LR - \alpha)f(L)dL + \\ \int_{\alpha}^{1} U'(R - (1 - \alpha)(r_{1} - S) - \alpha r_{2} - \alpha)(r_{2} + 1)f(L)dL \\ \end{cases}}{\begin{cases} \int_{\frac{\alpha_{2}}{R}}^{\frac{\alpha_{2}}{R}} U'(R - (1 - \alpha)(r_{1} - S) - \alpha r_{2} - \alpha)(r_{2} + 1)f(L)dL \\ \end{cases}}, \frac{dx}{de} = 0 \end{cases}$$
(77)

Because MLRP is constant in that case, therefore the optimal form of the contract indicated retaining the constant amount of loan portfolio as equity tranche (retention) which was positive and could be calculated based on the model.

With tranching and conditional distribution of loss, the optimal solution corresponds to the fourth case (which is the only acceptable case). In the fourth case we have $x_e < 0$, where:

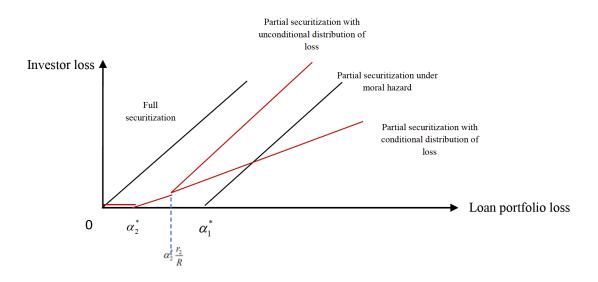
$$x_{e} = \frac{dx}{de} = \frac{\begin{cases} \int_{\alpha}^{\frac{dr_{2}}{R}} U'(R - (1 - \alpha)(r_{1} - S) - LR - \alpha)f_{e}(L|e)dL + \\ \int_{\alpha}^{\frac{1}{R}} U'(R - (1 - \alpha)(r_{1} - S) - \alpha r_{2} - \alpha)(r_{2} + 1)f_{e}(L|e)dL \\ \end{cases}}{\begin{cases} r_{2} \left(1 - F\left(\alpha \frac{r_{2}}{R}\right)\right) + (1 - F(\alpha)) \end{cases}} < 0 \end{cases}$$
(78)

The numerator is representing the loss of the senior tranche, this means that with effort, originator could decrease the loss of the senior tranche (investor). On the other hand the bank will decrease the density function by maximizing the effort level which is consistent with the investors' objective. As a result (like in Winter (2000)) we assume that $\frac{d}{dL} \left(\frac{f_e(L|e)}{f(L|e)} \right)_{<0}$, because as L increases

 $f_e(L|e)$ moves from >0 to <0. To obtain the best contract, we want that the ratio decreases when L increases. Therefore the optimal form of contract in this case, suggesting keeping a fraction of senior tranches as proportional retention (or coinsurance in insurance literature) in addition to the equity tranche retention as before (fixed amount retention as a deductible).

With conditional distribution of loss, as shown in fourth case, since $\frac{dx}{de} < 0$, the optimal form of contract is different from unconditional distribution of loss: the fixed amount of equity retention plus a fraction of higher losses as proportional retention. In Figure 4, we can draw the relationship between investor and loan portfolio loss based on the above results. As for a coinsurance contract the bank's exposure to risk increases with *L*.

Figure 4: Optimal Securitization Contract with conditional distribution of loss and credit enhancement



Investor loss equals θ when $L < \alpha_2^*$, equals $(L - \alpha_2^*)(1 + R)$ when $L > \alpha_2^*$

 α_2^* (Optimal α is the same with conditional distribution of loss) is smaller than α_1^* (Optimal α without credit enhancement)

3.3 Conclusion

In this chapter, we apply the principal-agent model together with security design methodology for structured asset-backed securitization with a credit enhancement procedure to obtain the optimal risk sharing between sellers and buyers of these products. We show that the optimal level of retention must be positive in presence of tranching and credit enhancement. Under certain conditions, the optimal amount of retention can be lower in presence of tranching and credit enhancement (compare to the case with no tranching), because losses affect the return of the equity tranche which adds an incentive scheme for the bank to screen and monitor their loans.

The optimal form of retention can be different with conditional and unconditional distribution of loss. With unconditional distribution of loss, the optimal form of contract is equity tranche retention, while with conditional distribution of loss it will be proportional to the total loss, which indicates fixed amount of equity tranche plus a fraction of higher losses as coinsurance. The optimal amount of retention in both cases is the same while the optimal form of retention would be different.

The main intuition of our results is the following¹(Malekan and Dionne (2014)): To reduce the default possibility and ensure the investor's participation incentive, the originator chooses his effort and the security price before the uncertainty is realized. Because the security price is determined in the ex-ante situation, the originator is not given any incentive to choose an appropriate effort level if it does not hold an equity tranche or vertical slice of all the tranches. However, if the originator holds an equity tranche or vertical slice of all the tranches, it must partly incur the default loss. As a result, the existence of the retention part gives the originator an incentive to reduce the default probability to avoid incurring the default loss.

However, if the equity tranche retention or proportional retention requirements are too high, securitization may no longer be an economical form of acquiring liquidity from the financial market because it could cause unintended costs and consequently impede efforts to restart sustainable securitization markets (Selody and Woodman, 2009). Therefore, it becomes critical to select an appropriate amount and form of tranche retention based on precise calculations, which was the focus of our research. The optimal amount and form of tranche retention could then be calculated using our model with different parameter values.

¹ We thank a referee for suggesting this interpretation.

A recent empirical study (Casu, Clare, Sarkisyan and Thomas (2011)) shows that securitization reduces bank credit risk taking. But the authors admit that this negative effect can be offseted by the risk arising from the securitized pool. Our contribution shows explicitly how optimal tranche retention in securitization may reduce the securitized pool risk by increasing the incentives for banks to monitor their loans. Consequently, the offsetting effect discussed by (Casu, Clare, Sarkisyan and Thomas (2011)) can be non-significant when optimal risk-sharing contracts of securitization are put in place.

For future research, we will take one step further by considering structured asset-backed securitization with a credit enhancement procedure in the presence of correlation between assets in the original pool, as observed during the recent financial crisis.

Chapter 4:

Structured Asset-backed Securitization with Systemic Risk

Introduction

In this chapter we take one step further by considering structured asset-backed securitization with credit enhancement procedure in the presence of the endogenous correlation on return between assets in the original pool. In other words, we assume that there is a systemic risk in this section.

By letting the size of the loan pool to be large enough, we could diversify away the idiosyncratic risk. But the return on the loan pool does not depend only on the loan-specific factor which constitutes idiosyncratic risks that could be diversified away by choosing a large size of the loan pool. It also depends on common factors like macroeconomic conditions that affect the entire underlying portfolio at the same time. These factors constitute the systemic risk which leads to correlation between the default risks of underlying loans and consequently different tranches. These correlations were not considered before the recent financial crisis, which led to underestimation of true default probabilities of tranches in the presence of systemic risk. This also resulted in the inaccurate rating and pricing of the financial products associated with different tranches.

Because of these correlations, this kind of risk cannot be diversified away easily and brings about more default risk than has been anticipated. That is why it becomes important to take into account the systemic risk when determining the credit risk and accordingly the rating of the entire loan pool. Systemic risk does increase the chance of many loans defaulting at the same time.

The more the correlation between defaults risks of different tranches, the more the value of the equity tranche. Thus the value of equity tranche increases with more correlation. The value of senior tranche may fall dramatically with correlation. The outcome of correlation on the value of

the mezzanine tranche is mixed; it depends on factors like the relative tranche size (Coval, Jurek and Stafford (2009)).

The objective here is to compute the optimal amount and form of retention under moral hazard and systemic risk and to compare the impact of systemic risk on the optimal amount of retention without systemic risk.

4.1 Derivation of the model

We consider a risk-averse originator or bank who detains a loan pool with a value normalized to 1. The loan pool is assumed to be large enough to diversify away the idiosyncratic risk but systemic risk which leads to correlation between the defaults risks of different loans cannot be diversified away and brings about more default risk than has been anticipated. We suppose that the average default probability of loan pool without systemic risk is p(e) which is a function of screening and monitoring effort *e*. Effort *e* could be undertaken by lenders to decrease the probability of loans default. We assume p(e) is decreasing and convex in effort:

$$p'(e) < 0, p''(e) > 0.$$
 (80)

Effort is costly for originators. The cost function is equal to C(e) and is increasing and convex in e:

$$C(0) = 0, C'(e) > 0, C''(e) > 0$$
(81)

In the case of loans default there is a random conditional loss L which is in the interval [0, 1]: $0 \le L \le 1$. F(L) is the distribution function of loss, conditional on having default with density function f(L). f(L) is not a function of effort in this section. The risk-free rate of interest is normalized to zero. Originator securitizes some part of the loan pool as senior tranches to risk-neutral and competitive investors. The amount of senior tranches which are securitized is equal to $1 - \alpha$ with higher credit rating than the average of the loan pool and a lower interest rate of r_1 . The originator can keep α fraction of the loan pool as equity tranche with lower credit rating and interest rate r_2 higher than r_1 .

Because of the systemic risk there is a correlation between default probabilities of different assets in the underlying loan pool. Systemic risk can be interpreted as domino effect in the default probability of underlying assets. In other words, if one loan defaults, the probability that other loans defaults increases because of systemic risk. We can explain the effect of systemic risk on default probabilities by an example. Let us assume a loan pool with two identical securities — call them "bonds"—both of which have a default probability of 10% and a zero recovery rate. They are securitized with two tranches as junior (*B*) and senior (*A*). The junior tranche defaults if either loan defaults, while the senior tranche defaults only if both loans default.

If we assume that the two loans are uncorrelated, the default probability of the senior tranche is $1\% (10\% \times 10\%)!$ That of the junior tranche is $19\% (10\% + 90\% \times 10\%)!$ We can use $P_N(B)$ and $P_N(A)$ to identify default probability of junior and senior tranche respectively when there is no systemic risk.

- $P_N(A) = 1\%$ (82)
- $P_N(B) = 19\%$

If both are correlated the default probability of the junior and the senior tranches are calculated as below, which can be identified by $P_s(B)$ and $P_s(A)$ respectively while there is systemic risk:

- $P_{S}(A) = 1\% + \rho 10\% (1-10\%) > P_{N}(A)$ (83)
- $P_{S}(B) = 19\% \rho 10\% (1 10\%) < P_{N}(B)$

Here ρ is the correlation coefficient. As shown in the above example, the presence of dependence increases the default probability of senior tranche $P_S(A) > P_N(A)$ while at the time decreases that of junior tranche $P_S(B) < P_N(B)$. In other words, the systemic transfer of the junior tranche risk to senior tranche is a function of correlation coefficient. The increment is a product of the correlation coefficient ρ and the probability that senior tranche defaults.

In our model, the default probability of senior and junior tranches with systemic risk can be shown as P(A) and P(B) respectively:

$$P(A) = p(e) + \rho g(P(B))$$
(84)

$$P(B) = p(e) - \rho g(P(B))$$

We assume that effort cannot affect ρ which is a natural assumption. Therefore, similar to the example, in presence of systemic risk the default probability of the senior tranche will increase as a function of the correlation coefficient (ρ) and the probability that the senior tranche defaults. The

junior tranche absorbs the first part of the loss on interest and capital. Senior tranche can participate in the default probability of the junior tranche as a function of correlation which was shown before.

As a result, the return of investors will increase correspondingly. Since they absorb the ρ fraction of the loss which was previously incurred by equity tranche α , their return increases as a ρ fraction of α .

4.1.1 Investors' objective function

The expected return of investors is equal to:

$$\pi_{1} = (1 - \alpha)(r_{1} - S) - P(A) \left[\int_{\frac{\alpha r_{2}}{R}}^{1} (LR - \alpha r_{2}) f(L) dL + \int_{\alpha}^{1} (L - \alpha) f(L) \right]$$
(85)

In the non-default state the return of investors is equal to $(1 - \alpha)(1 + r_1)$. If the loan pool defaults, the α fraction that is kept by the originator absorbs initial losses both on interest and capital. So the investors lose something if the amount of loss on interest or capital or both will be large enough to exhaust the equity tranche.

Investors will lose something on interest when: $Loss * R > \alpha r_2$ or when $Loss > \frac{\alpha r_2}{R}$, and the amount they will lose is equal to $(LR - \alpha r_2)$ which is the amount that is not absorbed by equity tranche, therefore investors' expected loss on interest is equal to:

$$\int_{\frac{\alpha r_2}{R}}^{1} (LR - \alpha r_2) f(L) dL$$
(86)

Investors will lose something on capital when: $Loss > \alpha$ and the amount that they will lose is equal to $(L - \alpha)$. The expected loss on capital is equal to:

$$\int_{\alpha}^{1} (L-\alpha)f(L) \, dL \tag{87}$$

The amount that investors are willing to pay for the securitized loan is equal to their expected value of their payoff which is equal to: $(1 - \alpha)$ (1 + S). It is the expected rate of return on the loan pool by considering the potential default risk of the loan pool. With full securitization, when $\alpha = 0$, the expected profit of investor is equal to:

$$\pi_{I} \equiv (r_{1} - S) - P(A) \int_{0}^{1} L(1 + R) f(L) dL$$
(88)

4.1.2 Participation constraint

In order to satisfy the participation constraint, the expected utility of originator in the presence of securitization should be at least equal to \overline{U} . \overline{U} is the best opportunity when considering securitization. In case of securitization, we can write the expected utility of originator as follows:

$$(1 - P(B))U(R - (1 - \alpha)(r_1 - S)) + P(B)\left[\int_0^{\alpha} U(R - (1 - \alpha)(r_1 - S) - LR - L)f(L)dL\right] + P(B)\left[\int_{\alpha}^{\frac{\alpha r_2}{R}} U(R - (1 - \alpha)(r_1 - S) - LR - \alpha)f(L)dL\right] + P(B)\left[\int_{\frac{\alpha r_2}{R}}^{1} U(R - (1 - \alpha)(r_1 - S) - \alpha r_2 - \alpha)f(L)dL\right]$$
(89)

The expected return on the collateral loan pool when there is no default is equal to: 1*(1+R) that can be divided between tranches with different credit rating and different interest rate. Therefore we have the following relationship between return on collateral loan pool and return on different tranches:

$$1*(1+R) = (1-\alpha)(1+r_1) + \alpha(1+r_2)$$
(90)

When there is no default the expected payoff of equity holder is equal to $\alpha(1+r_2) + (1-\alpha)(1+S) - 1$ that can be simplified by using the above equation to $R - (1-\alpha)(r_1 - S)$. In the case of default, since equity tranche absorbs initial losses on interest and capital, the expected utility is equal to:

$$\int_{0}^{\alpha} U(R - (1 - \alpha)(r_{1} - S) - LR - L)f(L)dL + \int_{\alpha}^{\frac{\alpha_{2}}{R}} U(R - (1 - \alpha)(r_{1} - S) - LR - \alpha)f(L)dL + \int_{\frac{\alpha_{2}}{R}}^{\frac{\alpha_{2}}{R}} U(R - (1 - \alpha)(r_{1} - S) - \alpha r_{2} - \alpha)f(L)dL$$
(91)

If the amount of loss is smaller than the expected return on equity tranche: $LR < \alpha r_2$, the equity holder loses: *LR* on interest and L on capital. If it is greater: $LR > \alpha r_2$ the equity holder loses αr_2 on interest and they will lose something on capital too, depending on the amount of loss on capital.

Finally the participation constraint is obtained as follows. We assign λ as the Lagrange multiplier to the participation constraint:

$$\lambda : (1 - P(B))U(R - (1 - \alpha)(r_1 - S)) +$$

$$P(B)\int_{0}^{\alpha} U(R - (1 - \alpha)(r_1 - S) - LR - L)f(L)dL + P(B)\int_{\alpha}^{\frac{\alpha r_2}{R}} U(R - (1 - \alpha)(r_1 - S) - LR - \alpha)f(L)dL +$$

$$P(B)\int_{\frac{\alpha r_2}{R}}^{\frac{\alpha r_2}{R}} U(R - (1 - \alpha)(r_1 - S) - \alpha r_2 - \alpha)f(L)dL = \overline{U}$$
(92)

4.1.3 Incentive compatibility constraint

This constraint ensures the effectiveness of originator's effort for screening taking into account the moral hazard problem. By differentiating the expected utility of originator in the presence of securitization with respect to screening and monitoring effort (first order condition for choice of effort), we can calculate the incentive compatibility constraint as follows. We assign μ as a Lagrange multiplier of this constraint. This constraint is the derivation of the participation constraint with respect to effort.

$$\mu : -P'(B)U(R - (1 - \alpha)(r_1 - S)) +$$

$$P'(B)\int_0^{\alpha} U(R - (1 - \alpha)(r_1 - S) - LR - L)f(L)dL +$$

$$P'(B)\int_{\alpha}^{\frac{\alpha r_2}{R}} U(R - (1 - \alpha)(r_1 - S) - LR - \alpha)f(L)dL + P'(B)\int_{\frac{\alpha r_2}{R}}^{\frac{\alpha r_2}{R}} U(R - (1 - \alpha)(r_1 - S) - \alpha r_2 - \alpha)f(L)dL = C'(e)$$
(93)

4.1.4 Technology constraint

This constraint is applied to ensure that there is no over retention excluding $\alpha > 1$.

$$\delta : \alpha \le 1 \begin{cases} \frac{\delta = 0 \longrightarrow \alpha < 1}{\delta > 0 \longrightarrow \alpha = 1} \end{cases}$$
(94)

Optimal securitization contract model

The optimal form of the structured asset-backed securitization contract is attained by solving the following maximization program:

$$Max \ \pi_{1} = (1-\alpha)(r_{1}-S) - P(A) \left[\int_{\frac{dr_{2}}{R}}^{1} (LR - \alpha r_{2})f(L)dL + \int_{\alpha}^{1} (L-\alpha)f(L)dL \right]$$

$$(95)$$

$$\lambda : (1-P(B))U(R - (1-\alpha)(r_{1}-S)) + P(B) \int_{0}^{\alpha} U(R - (1-\alpha)(r_{1}-S) - LR - L)f(L)dL + P(B) \int_{\alpha}^{\frac{dr_{2}}{R}} U(R - (1-\alpha)(r_{1}-S) - LR - \alpha)f(L)dL + P(B) \int_{\alpha}^{\frac{dr_{2}}{R}} U(R - (1-\alpha)(r_{1}-S) - LR - \alpha)f(L)dL + P(B) \int_{\alpha}^{\frac{dr_{2}}{R}} U(R - (1-\alpha)(r_{1}-S) - LR - \alpha)f(L)dL + P(B) \int_{\alpha}^{\frac{dr_{2}}{R}} U(R - (1-\alpha)(r_{1}-S) - \alpha r_{2} - \alpha)f(L)dL - C(e) = \overline{U}$$

$$\mu : -P'(B)U(R - (1-\alpha)(r_{1}-S) - LR - L)f(L)dL + P'(B) \int_{\alpha}^{\frac{dr_{2}}{R}} U(R - (1-\alpha)(r_{1}-S) - \alpha r_{2} - \alpha)f(L)dL = C'(e)$$

 $\delta : \alpha \le 1$

• No Moral Hazard: $\mu = 0$

First we assume that $\mu = 0$ when there is no moral hazard and $e = e^{**}$. The investor observes *e* and sets its level at the full information level e^{**} by using the "take it or leave it" strategy. If we differentiate the design model in (95) with respect to *S*, we have:

$$S: - (1 - \alpha) + \lambda (1 - P(B)) U'(R - (1 - \alpha)(r_1 - S))(1 - \alpha) + \lambda P(B) \int_{0}^{\alpha} U'(R - (1 - \alpha)(r_1 - S) - LR - L)(1 - \alpha)f(L)dL + \lambda P(B) \int_{\alpha}^{\alpha} U'(R - (1 - \alpha)(r_1 - S) - LR - \alpha)(1 - \alpha)f(L)dL + (96) \lambda P(B) \int_{\alpha}^{\alpha} U'(R - (1 - \alpha)(r_1 - S) - \alpha r_2 - \alpha)(1 - \alpha)f(L)dL = 0.$$

By solving the above equation, we can isolate $1/\lambda$ as follows:

$$\frac{1}{\lambda} = (1 - P(B)) U'(R - (1 - \alpha)(r_1 - S)) +$$

$$P(B) \int_0^{\alpha} U'(R - (1 - \alpha)(r_1 - S) - LR - L) f(L) dL + P(B) \int_{\alpha}^{\frac{dr_2}{R}} U'(R - (1 - \alpha)(r_1 - S) - LR - \alpha) f(L) dL +$$

$$P(B) \int_{\frac{dr_2}{R}}^{1} U'(R - (1 - \alpha)(r_1 - S) - \alpha r_2 - \alpha) f(L) dL$$
(97)

Differentiating the model in (95) with respect to α when $\mu = 0$ yields:

$$-(r_{1}-S) + P(A) r_{2}\left(1-F\left(\alpha \frac{r_{2}}{R}\right)\right) + P(A)(1-F(\alpha)) + \lambda (1-P(B)) U'(R-(1-\alpha)(r_{1}-S)) (r_{1}-S) + \lambda P(B)\int_{0}^{\alpha} U'(R-(1-\alpha)(r_{1}-S) - LR - L) (r_{1}-S) f(L)dL + \lambda P(B)\int_{\alpha}^{\frac{\alpha r_{2}}{R}} U'(R-(1-\alpha)(r_{1}-S) - LR - \alpha)(r_{1}-S - 1)f(L)dL + \lambda P(B)\int_{\frac{\alpha r_{2}}{R}}^{1} U'(R-(1-\alpha)(r_{1}-S) - \alpha r_{2} - \alpha)(r_{1}-S - r_{2} - 1)f(L)dL = 0.$$
(98)

By solving this equation for $\frac{1}{\lambda}$, we obtain:

$$\frac{1}{\lambda} = \begin{cases}
\frac{(1-P(B)) U'(R-(1-\alpha)(r_{1}-S)) (r_{1}-S)}{(r_{1}-S) - P(A) r_{2}\left(1-F\left(\alpha\frac{r_{2}}{R}\right)\right) - P(A)(1-F(\alpha))} + \\
\frac{P(B)\int_{0}^{\alpha} U'(R-(1-\alpha)(r_{1}-S) - LR - L) (r_{1}-S) f(L)dL}{(r_{1}-S) - P(A) r_{2}\left(1-F\left(\alpha\frac{r_{2}}{R}\right)\right) - P(A)(1-F(\alpha))} + \\
\frac{P(B)\int_{\alpha}^{\frac{m_{2}}{R}} U'(R-(1-\alpha)(r_{1}-S) - LR - L) (r_{1}-S) f(L)dL}{(r_{1}-S) - P(A) r_{2}\left(1-F\left(\alpha\frac{r_{2}}{R}\right)\right) - P(A)(1-F(\alpha))} + \\
\frac{P(B)\int_{\alpha}^{\frac{m_{2}}{R}} U'(R-(1-\alpha)(r_{1}-S) - \alpha r_{2} - \alpha)(r_{1}-S - r_{2} - 1)f(L)dL}{(r_{1}-S) - P(A) r_{2}\left(1-F\left(\alpha\frac{r_{2}}{R}\right)\right) - P(A)(1-F(\alpha))} + \\
\end{cases}$$
(99)

By equalizing the two equations (97) and (99) together we get the following result:

$$P(B)\int_{\alpha}^{\frac{\alpha_{2}}{R}}U'(R-(1-\alpha)(r_{1}-S)-LR-\alpha)(-1)f(L)dL+P(B)\int_{\frac{\alpha_{2}}{R}}^{1}U'(R-(1-\alpha)(r_{1}-S)-\alpha r_{2}-\alpha)(-r_{2}-1)f(L)dL =$$

$$= P(A)\left\{r_{2}\left(1-F\left(\alpha\frac{r_{2}}{R}\right)\right)+(1-F(\alpha))\right\}\left\{\left(1-P(B)\right)U'(R-(1-\alpha)(r_{1}-S))+P(B)\int_{0}^{\alpha}U'(R-(1-\alpha)(r_{1}-S)-LR-L)f(L)dL + P(B)\int_{0}^{\frac{\alpha_{2}}{R}}U'(R-(1-\alpha)(r_{1}-S)-LR-\alpha)f(L)dL + P(B)\int_{\frac{\alpha_{2}}{R}}^{1}U'(R-(1-\alpha)(r_{1}-S)-LR-\alpha)f(L)dL + P(B)\int_{\frac{\alpha_{2}}{R}}^{1}U'(R-(1-\alpha)(r_{1}-S)-\alpha r_{2}-\alpha)f(L)dL + P(B)\int_{\frac{\alpha_{2}}{R}}^{1}U'(R-(1-\alpha)(r_{1}-S)-LR-\alpha)f(L)dL + P(B)\int_{\frac{\alpha_{2}}{R}}^{1}U'(R-(1-\alpha)(r_{1}-S)-\alpha r_{2}-\alpha)f(L)dL + P(B)\int_{\frac{\alpha_{2}}{R}}^{1}U'(R-(1-\alpha)(r_{1}-S)-LR-\alpha)f(L)dL + P(B)\int_{\frac{\alpha_{2}}{R}}^{1}U'(R-(1-\alpha)(r_{1}-S)-LR-\alpha)f(L)dL + P(B)\int_{\frac{\alpha_{2}}{R}}^{1}U'(R-(1-\alpha)(r_{1}-S)-\alpha r_{2}-\alpha)f(L)dL + P(B)\int_{\frac{\alpha_{2}}{R}}^{1}U'(R-(1-\alpha)(r_{1}-S)-LR-\alpha)f(L)dL + P(B)\int_{\frac{\alpha_{2}}{R}}^{1}U'(R-(1-\alpha)(r_{1}-S)-\alpha r_{2}-\alpha)f(L)dL + P(B)\int_{\frac{\alpha_{2}}{R}}^{1}U'(R-(1-\alpha)(r_{2}-S)-\alpha r_{2}-\alpha)f(L)dL$$

 $\alpha = 0$ solves this equation only when P(A) = P(B) which is equivalent to $\rho = 0$ in our model. While $0 < \rho < 1$, full securitization is no more an optimal solution even when there is no moral hazard.

• Moral Hazard : $\mu \neq 0$

Now we assume that $\mu \neq 0$ and there is potential moral hazard. If we differentiate (95) with respect to *S*, when $\mu > 0$, we obtain:

$$S: - (1-\alpha) + \lambda (1-P(B)) U'(R-(1-\alpha)(r_{1}-S))(1-\alpha) +$$

$$\lambda P(B) \begin{cases} \int_{0}^{\alpha} U'(R-(1-\alpha)(r_{1}-S)-LR-L)(1-\alpha)f(L)dL + \int_{\alpha}^{\frac{\alpha r_{2}}{R}} U'(R-(1-\alpha)(r_{1}-S)-LR-\alpha)(1-\alpha)f(L)dL + \\ \int_{\alpha}^{\frac{1}{r}} U'(R-(1-\alpha)(r_{1}-S)-\alpha r_{2}-\alpha)(1-\alpha)f(L)dL \\ - \mu P'(B) U'(R-(1-\alpha)(r_{1}-S))(1-\alpha) + \\ \mu P'(B) \begin{cases} \int_{0}^{\alpha} U'(R-(1-\alpha)(r_{1}-S)-LR-L)(1-\alpha)f(L)dL + \int_{\alpha}^{\frac{\alpha r_{2}}{R}} U'(R-(1-\alpha)(r_{1}-S)-LR-\alpha)(1-\alpha)f(L)dL + \\ \int_{\alpha}^{\frac{1}{r}} U'(R-(1-\alpha)(r_{1}-S)-LR-L)(1-\alpha)f(L)dL + \int_{\alpha}^{\frac{\alpha r_{2}}{R}} U'(R-(1-\alpha)(r_{1}-S)-LR-\alpha)(1-\alpha)f(L)dL + \\ \end{bmatrix} = 0$$

By simplifying the above equation, we find:

$$\frac{1}{\lambda P(B) + \mu P'(B)} = \frac{\int_{0}^{1} U'(R - (1 - \alpha)(r_1 - S)) + \int_{0}^{\alpha} U'(R - (1 - \alpha)(r_1 - S) - LR - L)f(L)dL + \int_{\alpha}^{\frac{\alpha r_1}{R}} U'(R - (1 - \alpha)(r_1 - S) - LR - \alpha)f(L)dL + \int_{\alpha}^{\frac{\alpha r_2}{R}} U'(R - (1 - \alpha)(r_1 - S) - LR - \alpha)f(L)dL + \int_{\alpha}^{\frac{\alpha r_2}{R}} U'(R - (1 - \alpha)(r_1 - S) - LR - \alpha)f(L)dL + \int_{\alpha}^{\frac{\alpha r_2}{R}} U'(R - (1 - \alpha)(r_1 - S) - LR - \alpha)f(L)dL + \int_{\alpha}^{\frac{\alpha r_2}{R}} U'(R - (1 - \alpha)(r_1 - S) - LR - \alpha)f(L)dL + \int_{\alpha}^{\frac{\alpha r_2}{R}} U'(R - (1 - \alpha)(r_1 - S) - LR - \alpha)f(L)dL + \int_{\alpha}^{\frac{\alpha r_2}{R}} U'(R - (1 - \alpha)(r_1 - S) - LR - \alpha)f(L)dL + \int_{\alpha}^{\frac{\alpha r_2}{R}} U'(R - (1 - \alpha)(r_1 - S) - LR - \alpha)f(L)dL + \int_{\alpha}^{\frac{\alpha r_2}{R}} U'(R - (1 - \alpha)(r_1 - S) - LR - \alpha)f(L)dL + \int_{\alpha}^{\frac{\alpha r_2}{R}} U'(R - (1 - \alpha)(r_1 - S) - LR - \alpha)f(L)dL + \int_{\alpha}^{\frac{\alpha r_2}{R}} U'(R - (1 - \alpha)(r_1 - S) - LR - \alpha)f(L)dL + \int_{\alpha}^{\frac{\alpha r_2}{R}} U'(R - (1 - \alpha)(r_1 - S) - LR - \alpha)f(L)dL + \int_{\alpha}^{\frac{\alpha r_2}{R}} U'(R - (1 - \alpha)(r_1 - S) - LR - \alpha)f(L)dL + \int_{\alpha}^{\frac{\alpha r_2}{R}} U'(R - (1 - \alpha)(r_1 - S) - LR - \alpha)f(L)dL + \int_{\alpha}^{\frac{\alpha r_2}{R}} U'(R - (1 - \alpha)(r_1 - S) - \alpha r_2 - \alpha)f(L)dL + \int_{\alpha}^{\frac{\alpha r_2}{R}} U'(R - (1 - \alpha)(r_1 - S) - \alpha r_2 - \alpha)f(L)dL + \int_{\alpha}^{\frac{\alpha r_2}{R}} U'(R - (1 - \alpha)(r_1 - S) - \alpha r_2 - \alpha)f(L)dL + \int_{\alpha}^{\frac{\alpha r_2}{R}} U'(R - (1 - \alpha)(r_1 - S) - \alpha r_2 - \alpha)f(L)dL + \int_{\alpha}^{\frac{\alpha r_2}{R}} U'(R - (1 - \alpha)(r_1 - S) - \alpha r_2 - \alpha)f(L)dL + \int_{\alpha}^{\frac{\alpha r_2}{R}} U'(R - (1 - \alpha)(r_1 - S) - \alpha r_2 - \alpha)f(L)dL + \int_{\alpha}^{\frac{\alpha r_2}{R}} U'(R - (1 - \alpha)(r_1 - S) - \alpha r_2 - \alpha)f(L)dL + \int_{\alpha}^{\frac{\alpha r_2}{R}} U'(R - (1 - \alpha)(r_1 - S) - \alpha r_2 - \alpha)f(L)dL + \int_{\alpha}^{\frac{\alpha r_2}{R}} U'(R - (1 - \alpha)(r_1 - S) - \alpha r_2 - \alpha)f(L)dL + \int_{\alpha}^{\frac{\alpha r_2}{R}} U'(R - (1 - \alpha)(r_1 - S) - \alpha r_2 - \alpha)f(L)dL + \int_{\alpha}^{\frac{\alpha r_2}{R}} U'(R - (1 - \alpha)(r_1 - S) - \alpha r_2 - \alpha)f(L)dL + \int_{\alpha}^{\frac{\alpha r_2}{R}} U'(R - (1 - \alpha)(r_1 - S) - \alpha r_2 - \alpha)f(L)dL + \int_{\alpha}^{\frac{\alpha r_2}{R}} U'(R - (1 - \alpha)(r_1 - S) - \alpha r_2 - \alpha)f(L)dL + \int_{\alpha}^{\frac{\alpha r_2}{R}} U'(R - (1 - \alpha)(r_1 - S) - \alpha r_2 - \alpha)f(L)dL + \int_{\alpha}^{\frac{\alpha r_2}{R}} U'(R - (1 - \alpha)(r_1 - S) - \alpha r_2 - \alpha)f(L)dL + \int_{\alpha}^{\frac{\alpha r_2}{R}} U'(R - (1 - \alpha)(r_1 - S) - \alpha r_2 - \alpha)f(L)dL + \int_{\alpha}^{\frac{\alpha r_2}{R}}$$

If we differentiate equation (95) with respect to α we have:

$$-(r_{1}-S) + P(A) r_{2} \left(1 - F\left(\alpha \frac{r_{2}}{R}\right)\right) + P(A)(1 - F(\alpha)) + \lambda \left(1 - P(B)\right) U'(R - (1 - \alpha)(r_{1} - S)) (r_{1} - S) +$$

$$\lambda P(B) \begin{cases} \int_{0}^{\alpha} U'(R - (1 - \alpha)(r_{1} - S) - LR - L) (r_{1} - S) f(L)dL + \\ \int_{\alpha}^{\alpha \frac{r_{2}}{R}} U'(R - (1 - \alpha)(r_{1} - S) - LR - \alpha)(r_{1} - S - 1)f(L)dL + \\ \int_{\alpha}^{1} \frac{r_{2}}{r_{2}} U'(R - (1 - \alpha)(r_{1} - S) - \alpha r_{2} - \alpha)(r_{1} - S - r_{2} - 1)f(L)dL \end{cases} - \mu P'(B) U'(R - (1 - \alpha)(r_{1} - S)) (r_{1} - S) +$$

$$\mu P'(B) \begin{cases} \int_{0}^{\alpha} U'(R - (1 - \alpha)(r_{1} - S) - \alpha r_{2} - \alpha)(r_{1} - S - r_{2} - 1)f(L)dL + \\ \int_{\alpha}^{\alpha} \frac{r_{2}}{R} U'(R - (1 - \alpha)(r_{1} - S) - LR - \alpha)(r_{1} - S - 1)f(L)dL + \\ \int_{\alpha}^{\alpha} \frac{r_{2}}{R} U'(R - (1 - \alpha)(r_{1} - S) - LR - \alpha)(r_{1} - S - 1)f(L)dL + \\ \int_{\alpha}^{\alpha} \frac{r_{2}}{R} U'(R - (1 - \alpha)(r_{1} - S) - LR - \alpha)(r_{1} - S - 1)f(L)dL + \\ \int_{\alpha}^{\alpha} \frac{r_{2}}{R} U'(R - (1 - \alpha)(r_{1} - S) - \alpha r_{2} - \alpha)(r_{1} - S - r_{2} - 1)f(L)dL \\ \end{cases} = 0.$$

This can be simplified as follow:

$$\frac{1}{\lambda P(B) + \mu P'(B)} = \frac{\int_{a_{2}}^{1} U'(R - (1 - \alpha)(r_{1} - S))(r_{1} - S) + \int_{0}^{a} U'(R - (1 - \alpha)(r_{1} - S) - LR - L)(r_{1} - S)f(L)dL + \int_{a}^{\frac{m_{2}}{R}} U'(R - (1 - \alpha)(r_{1} - S) - LR - \alpha)(r_{1} - S - 1)f(L)dL + \int_{a}^{1} U'(R - (1 - \alpha)(r_{1} - S) - LR - \alpha)(r_{1} - S - 1)f(L)dL + \int_{a}^{1} U'(R - (1 - \alpha)(r_{1} - S) - LR - \alpha)(r_{1} - S - 1)f(L)dL + \int_{a}^{1} U'(R - (1 - \alpha)(r_{1} - S) - LR - \alpha)(r_{1} - S - 1)f(L)dL + \int_{a}^{1} U'(R - (1 - \alpha)(r_{1} - S) - LR - \alpha)(r_{1} - S - 1)f(L)dL + \int_{a}^{1} U'(R - (1 - \alpha)(r_{1} - S) - LR - \alpha)(r_{1} - S - 1)f(L)dL + \int_{a}^{1} U'(R - (1 - \alpha)(r_{1} - S) - LR - \alpha)(r_{1} - S - 1)f(L)dL + \int_{a}^{1} U'(R - (1 - \alpha)(r_{1} - S) - LR - \alpha)(r_{1} - S - 1)f(L)dL + \int_{a}^{1} U'(R - (1 - \alpha)(r_{1} - S) - LR - \alpha)(r_{1} - S - 1)f(L)dL + \int_{a}^{1} U'(R - (1 - \alpha)(r_{1} - S) - LR - \alpha)(r_{1} - S - 1)f(L)dL + \int_{a}^{1} U'(R - (1 - \alpha)(r_{1} - S) - LR - \alpha)(r_{1} - S - 1)f(L)dL + \int_{a}^{1} U'(R - (1 - \alpha)(r_{1} - S) - LR - \alpha)(r_{1} - S - 1)f(L)dL + \int_{a}^{1} U'(R - (1 - \alpha)(r_{1} - S) - LR - \alpha)(r_{1} - S - 1)f(L)dL + \int_{a}^{1} U'(R - (1 - \alpha)(r_{1} - S) - LR - \alpha)(r_{1} - S - 1)f(L)dL + \int_{a}^{1} U'(R - (1 - \alpha)(r_{1} - S) - LR - \alpha)(r_{1} - S - 1)f(L)dL + \int_{a}^{1} U'(R - (1 - \alpha)(r_{1} - S) - LR - \alpha)(r_{1} - S - 1)f(L)dL + \int_{a}^{1} U'(R - (1 - \alpha)(r_{1} - S) - LR - \alpha)(r_{1} - S - 1)f(L)dL + \int_{a}^{1} U'(R - (1 - \alpha)(r_{1} - S) - LR - \alpha)(r_{1} - S - 1)f(L)dL + \int_{a}^{1} U'(R - (1 - \alpha)(r_{1} - S) - LR - \alpha)(r_{1} - S - 1)f(L)dL + \int_{a}^{1} U'(R - (1 - \alpha)(r_{1} - S) - LR - \alpha)(r_{1} - S - 1)f(L)dL + \int_{a}^{1} U'(R - (1 - \alpha)(r_{1} - S) - LR - \alpha)(r_{1} - S - 1)f(L)dL + \int_{a}^{1} U'(R - (1 - \alpha)(r_{1} - S - 1)f(L)dL + \int_{a}^{1} U'(R - (1 - \alpha)(r_{1} - S - 1)f(L)dL + \int_{a}^{1} U'(R - (1 - \alpha)(r_{1} - S - 1)f(L)dL + \int_{a}^{1} U'(R - (1 - \alpha)(r_{1} - S - 1)f(L)dL + \int_{a}^{1} U'(R - (1 - \alpha)(r_{1} - S - 1)f(L)dL + \int_{a}^{1} U'(R - (1 - \alpha)(r_{1} - S - 1)f(L)dL + \int_{a}^{1} U'(R - (1 - \alpha)(r_{1} - S - 1)f(L)dL + \int_{a}^{1} U'(R - (1 - \alpha)(r_{1} - S - 1)f(L)dL + \int_{a}^{1} U'(R - (1 - \alpha)(r_{1} - S - 1)f(L)dL + \int_{a}^{$$

By equalizing the two equations together we get the following result:

$$P(A)\left\{r_{2}\left(1-F\left(\alpha\frac{r_{2}}{R}\right)\right) + (1-F(\alpha))\right\} \begin{cases} -U'(R-(1-\alpha)(r_{1}-S)) + \int_{0}^{\alpha}U'(R-(1-\alpha)(r_{1}-S)-LR-L)f(L)dL + \int_{0}^{\frac{\alpha r_{2}}{R}}U'(R-(1-\alpha)(r_{1}-S)-LR-\alpha)f(L)dL + \int_{0}^{\frac{1}{2}}\frac{U'(R-(1-\alpha)(r_{1}-S)-\alpha r_{2}-\alpha)f(L)dL + \int_{0}^{\frac{1}{2}}\frac{U'(R-(1-\alpha)(r_{1}-S)-\alpha r_{2}-\alpha)f(L)dL + \int_{0}^{\frac{\alpha r_{2}}{R}}U'(R-(1-\alpha)(r_{1}-S)-LR-\alpha)f(L)dL + \int_{0}^{\frac{\alpha r_{2}}{R}}\frac{U'(R-(1-\alpha)(r_{1}-S)-LR-\alpha)f(L)dL + \int_{0}^{\frac{\alpha r_{2}}{R}}\frac{U'(R-(1-\alpha)(r_{1}-S)-LR-\alpha)f(L)}{L}\frac{U'(R-(1-\alpha)(r_{1}-S)-LR-\alpha)f(L)}\frac{U'(R-(1-\alpha)(r_{1}-S)-LR-\alpha)f(L)}{L}\frac{U'(R-(1-\alpha)(r_{1}-S)-LR-\alpha)f(L)}L}$$

4.1.5 Analyzing the result

When there is no moral hazard we have the following particular case:

$$(1-p(B)) U'(R-(1-\alpha)(r_{1}-S)) = \frac{P(B) \begin{cases} \int_{\alpha}^{\frac{\sigma_{2}}{R}} U'(R-(1-\alpha)(r_{1}-S)-LR-\alpha)f(L)dL + \\ \int_{\alpha}^{1} U'(R-(1-\alpha)(r_{1}-S)-\alpha r_{2}-\alpha)(r_{2}+1)f(L)dL \\ \frac{\int_{\alpha}^{1}}{R} U'(R-(1-\alpha)(r_{1}-S)-LR-L)f(L)dL - P(R) \\ P(A) \begin{cases} r_{2} \left(1-F\left(\alpha \frac{r_{2}}{R}\right)\right) + (1-F(\alpha)) \\ \frac{r_{2}}{R} U'(R-(1-\alpha)(r_{1}-S)-LR-L)f(L)dL - P(R) \\ \frac{\int_{\alpha}^{1}}{R} U'(R-(1-\alpha)(r_{1}-S)-LR-\alpha)f(L)dL \\ \frac{\int_{\alpha}^{1}}{R} U'(R-($$

We use the following notation to make the above equation simpler:

$$x = \frac{P(B) \left\{ \int_{\alpha}^{\frac{dr_2}{R}} U'(R - (1 - \alpha)(r_1 - S) - LR - \alpha)f(L)dL + \int_{\frac{dr_2}{R}}^{1} U'(R - (1 - \alpha)(r_1 - S) - \alpha r_2 - \alpha)(r_2 + 1)f(L)dL \right\}}{P(A) \left\{ r_2 \left(1 - F\left(\alpha \frac{r_2}{R}\right) \right) + (1 - F(\alpha)) \right\}}$$
(107)

and

$$y = p(B) \int_{0}^{\alpha} U' (R - (1 - \alpha)(r_{1} - S) - LR - L) f(L) dL + p(B) \int_{\alpha}^{\frac{\alpha r_{2}}{R}} U' (R - (1 - \alpha)(r_{1} - S) - LR - \alpha) f(L) dL + p(B) \int_{\frac{\alpha r_{2}}{R}}^{1} U' (R - (1 - \alpha)(r_{1} - S) - \alpha r_{2} - \alpha) f(L) dL$$
(108)

Equation (108) becomes:

$$(1-p(B)) \ U'(R-(1-\alpha)(r_1-S)) = x - y \longrightarrow U'(R-(1-\alpha)(r_1-S)) = \frac{x-y}{(1-p(B))}$$
(109)

With moral hazard we have the following equation using the above notations:

$$(\lambda x - p(B)) U'(R - (1 - \alpha)(r_1 - S)) = x - y \longrightarrow U'(R - (1 - \alpha)(r_1 - S)) = \frac{x - y}{(\lambda x - p(B))}$$
(110)

To compare (110) with the no moral hazard case, we can write:

 $\lambda = \frac{1}{r}$, which makes (109) and (110) equivalent.

We now suppose that $\lambda < \frac{1}{x}$. It indicates that $U'(R - (1 - \alpha)(r_1 - S))$ is higher in (110) than in (109):

$$U'(R - (1 - \alpha)(r_1 - S)) > \frac{x - y}{(1 - p(B))}$$
(111)

Given that U' is decreasing in wealth, and, as shown before, $r_1 > S$ this means that α should be smaller than or equal to zero to satisfy the inequality. Because α cannot be smaller than zero, this means that there should be full securitization. By putting $\alpha = 0$ in the above inequality, we get the following result:

$$U'(R - r_1 - S) > U'(R - r_1 - S)$$
(112)

which is a contradiction and we reject this possibility. Now considering $\lambda > \frac{1}{x}$, we obtain:

$$U'(R - (1 - \alpha)(r_1 - S)) < \frac{x - y}{(1 - p(B))}$$
(113)

Because U' is decreasing in wealth and $r_1 > S$, this means that α should be greater than zero to solve this inequality. This implies that full securitization is not optimal and there must be retention under moral hazard with tranching and systemic risk.

In fact, we may suspect that the optimal positive α from (105) is greater than the optimal positive α from (51). Because of the systemic risk, some part of the default probability of equity tranche which is held by the bank will transfer to investors. This reduction in default probability decreases bank's exposure to the risk. Therefore we need an additional incentive for the bank to maintain its effort at a high level by increasing the amount of retention. In Appendix *C.1*, we show this is the case under risk neutrality and uniform distribution for *L*.

When $\rho = 0$, the optimal amount of retention from (105) is the same as the optimal amount of retention from (51) with no systemic risk and unconditional distribution of loss. But when $0 < \rho < 1$, the optimal amount of retention with systemic risk is greater than the optimal amount of retention with no systemic risk. Therefore an increase in optimal retention can offset the extra insecurity which is the result of systemic risk.

So far, we show that the systemic risk resulted in an increase of optimal amount of retention. But what is not obvious yet is that if this increase in the optimal amount of retention should be out of junior tranche or senior tranche. As we mentioned in previous chapter, the sign of x_e determines the structure of the contract. In the previous section when we had tranching but an unconditional distribution of loss, x_e was equal to zero:

$$x = \frac{\begin{cases} \int_{\alpha}^{\frac{dr_{2}}{R}} U'(R - (1 - \alpha)(r_{1} - S) - LR - \alpha)f(L)dL + \\ \int_{\alpha}^{1} U'(R - (1 - \alpha)(r_{1} - S) - \alpha r_{2} - \alpha)(r_{2} + 1)f(L)dL \\ \end{cases}}{\begin{cases} \int_{\frac{dr_{2}}{R}} U'(R - (1 - \alpha)(r_{1} - S) - \alpha r_{2} - \alpha)(r_{2} + 1)f(L)dL \\ \end{cases}}, \frac{dx}{de} = 0 \end{cases}$$
(114)

Because MLRP is constant in that case, therefore the optimal form of the contract indicated retaining the constant amount of loan portfolio as equity tranche (deductible) which was positive and could be calculated based on the model.

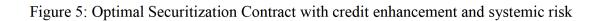
With tranching and systemic risk, we have $x_e < 0$ (details are in Appendix C.2), where:

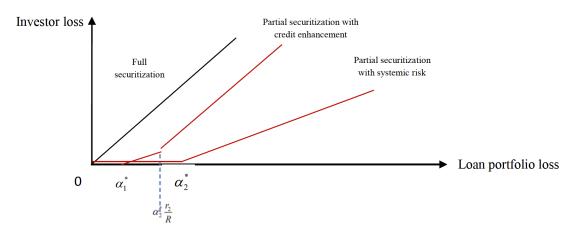
$$x_{e} = \frac{dx}{de} = \left(\frac{P_{e}(B)}{P(B)} - \frac{P_{e}(A)}{P(A)}\right) - \frac{P(B) \left\{ \int_{\alpha}^{\frac{dr_{2}}{R}} U'(R - (1 - \alpha)(r_{1} - S) - LR - \alpha)f(L)dL + \int_{\alpha}^{1} U'(R - (1 - \alpha)(r_{1} - S) - \alpha r_{2} - \alpha)(r_{2} + 1)f(L)dL \right\}}{P(A) \left\{ r_{2} \left(1 - F\left(\alpha \frac{r_{2}}{R}\right) \right) + (1 - F(\alpha)) \right\}} < 0$$

$$(115)$$

The numerator is representing the loss of the senior tranche, this means that with effort, originator could decrease the loss of the senior tranche (investor). Therefore the optimal form of contract in this case, suggesting keeping a fraction of senior tranches as proportional retention in addition to the equity tranche as before (fixed amount retention as a deductible).

With systemic risk, since $\frac{dx}{de} < 0$, the optimal form of contract is different from tranching and unconditional distribution of loss, may be it suggests to keep some part of mezzanine tranche; the fixed amount retention of equity plus a fraction of other tranches as coinsurance. In Figure 5, we can draw the relationship between investor and loan portfolio loss based on the above results.





Investor loss equals θ when $L < \alpha_2^*$, equals $(L - \alpha_2^*)(1 + R)$ when $L > \alpha_2^*$ α_2^* (Optimal α with systemic risk) is greater than α_1^* (Optimal α without systemic risk)

4.2 Conclusion

In this chapter, we take one step further by considering structured asset-backed securitization with a credit enhancement procedure in the presence of correlation between tranches in the original pool, as observed during the recent financial crisis.

These correlations were not considered by market participants before and during the recent financial crisis, which led to underestimation of the true default probabilities of all tranches in the presence of systemic risk. This also resulted in an inaccurate rating of structured financial products associated with different tranches (Dionne (2009)).

Because of extra insecurity which is the outcome of systemic risk, the optimal amount of retention should increase to the point that to offset this extra insecurity. The correlation between defaults risks of different assets in the original pool transfer some part of the default risk of junior tranche which is held by the bank to investors. It can be interpreted that in this case the default probability of the bank reduced as a function of correlation coefficient. In order to keep the incentive of the bank at the optimal level, we need to increase the amount of retention to the point that by considering this transfer, bank will exert the first best effort as previous case with no systemic risk.

The optimal form of the contract indicates that increase in optimal amount of retention should be out of senior tranches. One reason for this could be the fact that since default risk of junior tranche transfer to senior tranche, if we would like to increase the amount of retention out of junior tranche, the optimal amount of retention should increase too much. As we mentioned before too much retention can offset the advantage of securitization. This result is consistent with (Fender and Mitchell (2009b)). Because the equity tranche will be exhausted when there is a large probability of a systemic risk. It could decrease the originator's incentive to make a screening effort. In this case it would be better to hold also mezzanine tranche (Fender and Mitchell (2009b)).

A recent empirical study (Casu et al, 2011) shows that securitization reduces bank credit risktaking. However, the authors admit that this negative effect can be offset by the risk arising from the securitized pool. Our contribution explicitly shows how an optimal tranche in securitization may reduce the securitized pool risk by increasing banks' incentives to monitor their loans. Consequently, the offsetting effect discussed by Casu et al (2011) can be non-significant when optimal risk-sharing contracts of securitization are put in place.

Appendix

Appendix A

A.1 Second-order condition

The second-order condition is negative at the optimum: C''(e) > 0, p''(e) > 0 and R - L(1+R) < Rimplies that $\int_0^1 U(R - L(1+R)) f(L) dL - U(R)$ is negative. As a result we verify the negativity of the *S.O.C*:

$$-C''(e^{**}) + p''(e^{**}) \left[\int_0^1 U(R - L(1+R)) f(L) dL - U(R) \right] < 0.$$
(A.1)

A.2 Risk premium when there is no moral hazard

We can verify that R>S and by calculating the difference between R and S we can calculate the risk premium. From the participation constraint we have the following:

$$\overline{U} = (1 - p(e))U(R - (1 - \alpha)(R - S)) + p(e) \left[\int_{0}^{\alpha} U(R - (1 - \alpha)(R - S) - L(1 + R))f(L)dL + \int_{\alpha}^{1} U(R - (1 - \alpha)(R - S) - \alpha(1 + R))f(L)dL \right] - C(e).$$
(A.2)

When there is no securitization ($\alpha = 0$), the originator will exert the first best effort level e^{**} , and the right-hand side of the above equation is equal to:

$$U(S) - C(e^{**}).$$
 (A.3)

The originator's expected utility without securitization while the originator exerts the first best effort is equal to:

$$(1-p(e^{**})) U(R) + p(e^{**}) \int_0^1 U(R - L(1+R)) f(L) dL - C(e^{**}).$$
(A.4)

Equalizing these two equations yields the following equality from the participation constraint:

$$U(S) - C(e^{**}) = (1 - p(e^{**})) U(R) + p(e^{**}) \int_0^1 U(R - L(1 + R)) f(L) dL - C(e^{**}).$$
(A.5)

We can calculate the risk premium by applying a Taylor expansion around R to both sides of equation (A.5). The Taylor expansion of the left side of this equation is equal to:

$$U(S) = U(R) + (S - R)U'(R).$$
 (A.6)

The Taylor expansion of the right side of equation (A.5) is equal to:

$$(1-p(e)) U(R) + p(e)U(R) - p(e) \left[\int_0^1 L(1+R) f(L) dL \right] U'(R) + p(e) \left[\int_0^1 L^2 (1+R)^2 f(L) dL \right] \frac{U''}{2}.$$
 (A.7)

Equalizing these two equations yields the following:

$$S = R - p(e)(1+R)\int_0^1 Lf(L)dL + \frac{1}{2}p(e)\frac{U''(R)}{U'(R)}\int_0^1 L^2(1+R)^2 f(L)d(L)$$
(A.8)

where the risk premium
$$= -\frac{1}{2} p(e) \frac{U''(R)}{U'(R)} \int_{0}^{1} L^{2} (1+R)^{2} f(L) dL.$$
 (A.9)

Under risk aversion, the risk premium is positive because p(e) > 0, U''(R) < 0, U'(R) > 0. Therefore, we can conclude that R-S is larger than the expected loss. The expected return of originator is then equal to:

$$1+S = \left[1-p(e)\int_{0}^{1} Lf(L)dL\right] (1+R) + \frac{1}{2}p(e)\frac{U''(R)}{U'(R)}\int_{0}^{1} L^{2}(1+R)^{2}f(L)d(L).$$
(A.10)

Appendix B

B.1 Risk premium

We can verify that $r_1 > S$ and by calculating the difference between r_1 and S we can calculate the risk premium. From the participation constraint we have the following:

$$\overline{U} = (1 - p(e))U(R - (1 - \alpha)(r_1 - S)) + (B.1)$$

$$p(e)\int_{0}^{\frac{\alpha_2}{R}} U(R - (1 - \alpha)(r_1 - S) - LR - L)f(L)dL + p(e)\int_{\frac{\alpha_2}{R}}^{\alpha} U(R - (1 - \alpha)(r_1 - S) - \alpha r_2 - L)f(L)dL + p(e)\int_{\alpha}^{1} U(R - (1 - \alpha)(r_1 - S) - \alpha r_2 - \alpha)f(L)dL - C(e)$$

When there is no securitization $\alpha = 0$, the originator will exert the first best effort level e^{**} , and the above equation is equal to:

$$\overline{U} = U(R - r_1 + S) - C(e^{**}) \tag{B.2}$$

On the other hand, $\overline{\upsilon}$ is equal to the expected utility of originator without securitization while originator exerts the first best effort:

$$\overline{U} = \left[1 - P(e^{**})\right] U(R) + p(e^{**}) \left[\int_{0}^{1} U(R - L(1 + R)) f(L) dL \right] - C(e^{**})$$
(B.3)

By putting these two equations equal we have the following equality:

$$U(R - r_1 + S) - C(e^{**}) = ((1 - p(e^{**}))U(R) + p(e^{**}) \left| \int_0^1 U(R - L(1 + R))f(L)dL \right| - C(e^{**})$$
(B.4)

Now, we have to show that:

$$S = r_1 - p(e^{**})(1+R) \int_0^1 Lf(L) dL - risk. premium$$
(B.5)

We can calculate the risk premium by applying a Taylor expansion around R to both sides of above equation. The Taylor expansion of the left side of this equation is equal to:

$$U(R - r_1 + S) = U(R) + (S - r_1)U'(R)$$
(B.6)

The Taylor expansion of the right side of equation is equal to:

$$((1 - p(e))U(R) + p(e)U(R) - p(e) \left[\int_{0}^{1} L(1 + R)f(L)dL \right] U'(R) + p(e) \left[\int_{0}^{1} L^{2}(1 + R)^{2} f(L)dL \right] \frac{U''}{2}$$
(B.7)

By putting these two equations equal we have the following:

$$S = r_1 - p(e)(1+R) \int_0^1 Lf(L) dL + \frac{1}{2} p(e) \frac{U''(R)}{U'(R)} \int_0^1 L^2 (1+R)^2 f(L) d(L)$$
(B.8)

Where the *risk premium* =
$$-\frac{1}{2}p(e)\frac{U''(R)}{U'(R)}\int_{0}^{1}L^{2}(1+R)^{2}f(L)dL$$
 (B.9)

The risk premium is positive; because: p(e) > 0, U''(R) < 0, U'(R) > 0, Therefore, we can conclude that $r_1 > S$.

B.2 Risk neutrality case

We want to show there is substitution between α and r_2 for the bank in presence of tranching and credit enhancement. For certain levels of r_2 , optimal α can be lower when there is tranching because losses reduce r_2 for the bank and creates an additional incentive for screening and monitoring the loans under moral hazard. Differentiating (39) with respect to α and S yields the following equilibrium condition, assuming risk neutrality (U'(x) = 1) and uniform distribution for L in the interval [0, 1]. The two first order conditions are as follow:

If we differentiate the model in (39) with respect to *S* we have:

$$S: -1 + \lambda \left(1 - p(e)\right) + \lambda p(e) - \mu p'(e) + \mu p'(e) = 0 \Longrightarrow \lambda = 1$$

$$(B.10)$$

If we differentiate the model in (39) with respect to α we have:

$$\alpha : -(r_1 - S) + p(e) r_2 \left(1 - \alpha \frac{r_2}{R}\right) + p(e)(1 - \alpha) + \lambda \left(1 - p(e)\right) (r_1 - S) + \lambda p(e)(r_1 - S) - \lambda p(e)(1 - \alpha) - r_2 \lambda p(e) \left(1 - \alpha \frac{r_2}{R}\right)$$

$$(B.11)$$

$$-\mu p'(e) (r_1 - S) + \mu p'(e) (r_1 - S) - \mu p'(e)(1 - \alpha) - r_2 \mu p'(e) \left(1 - \alpha \frac{r_2}{R}\right) = 0$$

This can be simplified as follow:

$$(1 - \lambda - \mu) \ p(e)(r_2 - \alpha \frac{r_2^2}{R} + 1 - \alpha) = 0$$
(B.12)

Since $\lambda = 1$ and $\mu \neq 0$, It is easy to obtain an explicit value of α :

$$\alpha = \frac{1 + r_2}{\left(1 + \frac{r_2^2}{R}\right)} > 0 \text{ under moral hazard.}$$
(B.13)

Differentiating (B.13) with respect to r_2 :

$$\frac{d\alpha}{dr_2} = \frac{(1+R) - (1+r_2)^2}{R \left(1 + \frac{r_2^2}{R}\right)^2}$$
(B.14)

Since $(1+r_2)^2 > (1+R)$ then $\frac{d\alpha}{dr_2} < 0$.

This means that a high r_2 can be a substitute to a high α , so a lower α can be sufficient to maintain the incentives in presence of credit enhancement when there is tranching.

B.3 Conditional distribution

We assume risk neutrality and negative exponential distribution for L. First, we need to calculate x_e to make sure if these are satisfactory assumptions.

Loss distribution function:

$$F(L|e) = L^{-e}$$

$$F_e(L|e) = (\ln(L))L^{-e}$$
(B.15)

Plus risk neutrality:

$$x_{e} = \frac{dx}{de} = \frac{-\alpha^{-e} \ln(\alpha) - r_{2}(\alpha \frac{r_{2}}{R})^{-e} \ln(\alpha \frac{r_{2}}{R})}{r_{2} - r_{2}(\alpha \frac{r_{2}}{R})^{-e} + 1 - \alpha^{-e}}$$
(B.16)

Since $0 < \alpha \frac{r_2}{R} < 1$, the numerator is positive but the denominator is negative, which means $x_e < 0$. Since $x_e < 0$, is the necessary condition to get the result under conditional distribution of loss, it means that these assumptions are satisfactory to get the result.

Appendix C

C.1 Risk neutrality case

We want to show that the optimal amount of retention will increase in the presence of systemic risk, because systemic risk will introduce more uncertainty about the underlying loan pool. With systemic risk the default probability of junior tranche decreases at the cost of an increase in the default probability of senior tranche. In this case we expect that the return on the equity tranche (r_2) decreases as a result of decrease in its default probability. Therefore if we can show there is a negative substitution between α and r_2 in presence of systemic risk, we can show that optimal amount of α increases (due to decrease in r_2 which is the result of systemic risk).

Differentiating (95) with respect to α and *S* yields the following equilibrium condition, assuming risk neutrality (U'(x) = 1) and uniform distribution for *L* in the interval [0,1]. The two first order conditions, are as follows:

If we differentiate the model in (95) with respect to S we have:

$$S: -1 + \lambda (1 - p(B)) + \lambda p(B) - \mu p'(B) + \mu p'(B) = 0 \Longrightarrow \lambda = 1$$
(C.1)

If we differentiate the model in (95) with respect to α we have:

$$\alpha : -(r_1 - S) + p(A) r_2 \left(1 - \alpha \frac{r_2}{R} \right) + p(A)(1 - \alpha) + \lambda \left(1 - p(B) \right) (r_1 - S) + \lambda p(B)(r_1 - S) - \lambda p(B)(1 - \alpha) - r_2 \lambda p(B) \left(1 - \alpha \frac{r_2}{R} \right)$$

$$- \mu p'(B) (r_1 - S) + \mu p'(B) (r_1 - S) - \mu p'(B)(1 - \alpha) - r_2 \mu p'(B) \left(1 - \alpha \frac{r_2}{R} \right) = 0$$

$$(C.2)$$

This can be simplified as follow:

$$(p(A) - \lambda p(B) - \mu p'(B))(r_2 - \alpha \frac{r_2^2}{R} + 1 - \alpha) = 0$$
(C.3)

In this case we can obtain an explicit value of α :

$$\alpha = \frac{1 + r_2}{\left(1 + \frac{r_2^2}{R}\right)} > 0 \text{ under moral hazard.}$$
(C.4)

Differentiating (C.2) with respect to r_2 :

$$\frac{d\alpha}{dr_2} = \frac{(1+R) - (1+r_2)^2}{R\left(1 + \frac{r_2^2}{R}\right)^2}$$
(C.5)

Since $(1+r_2)^2 > (1+R)$ then $\frac{d\alpha}{dr_2} < 0$.

This means that the reduction in r_2 due to systemic risk can be interpreted as an increase in optimal α to maintain the originator's incentives at the optimal level to take care about it.

C.2 optimal contract structure based on the sign of the x_e

We have the following equation for x with tranching and systemic risk:

$$x = \frac{P(B) \begin{cases} \int_{\alpha}^{\frac{\alpha_{2}}{R}} U'(R - (1 - \alpha)(r_{1} - S) - LR - \alpha)f(L)dL + \\ \int_{\frac{\alpha_{2}}{R}}^{1} U'(R - (1 - \alpha)(r_{1} - S) - \alpha r_{2} - \alpha)(r_{2} + 1)f(L)dL \\ \end{cases}}{P(A) \left\{ r_{2} \left(1 - F\left(\alpha \frac{r_{2}}{R}\right) \right) + (1 - F(\alpha)) \right\}}$$
(C.6)

 x_e can be calculated as below:

$$x_{e} = \frac{dx}{de} = \left(\frac{P_{e}(B)}{P(B)} - \frac{P_{e}(A)}{P(A)}\right) - \frac{P(B) \left\{ \int_{\alpha}^{\frac{dr_{2}}{R}} U'(R - (1 - \alpha)(r_{1} - S) - LR - \alpha)f(L)dL + \int_{\alpha}^{1} U'(R - (1 - \alpha)(r_{1} - S) - \alpha r_{2} - \alpha)(r_{2} + 1)f(L)dL \right\}}{P(A) \left\{ r_{2} \left(1 - F\left(\alpha \frac{r_{2}}{R}\right) \right) + (1 - F(\alpha)) \right\}} < 0$$

$$(C.7)$$

We can show that x_e is negative:

The second part of the above equation is positive, to show that x_e is negative, we need to show that the first part is negative:

$$\frac{P_e(B)}{P(B)} - \frac{P_e(A)}{P(A)} = \frac{p'(e) - \rho f'(P(B))P'(B)}{P(B)} - \frac{p'(e) + \rho f'(P(B))P'(B)}{P(A)} =$$

$$p'(e)(\frac{1}{P(B)} - \frac{1}{P(A)}) - \frac{\rho f'(P(B))P'(B)}{P(B)} - \frac{\rho f'(P(B))P'(B)}{P(A)} =$$

$$(C.8)$$

Since P(A) > P(B) then $\frac{1}{P(B)} > \frac{1}{P(A)}$. We know that p'(e) < 0 so $p'(e)(\frac{1}{P(B)} - \frac{1}{P(A)}) < 0$. As a result $x_e < 0$.

References

- Akerlof, George A, 1970, The market for" lemons": Quality uncertainty and the market mechanism, *The Quarterly Journal of Economics* 488-500.
- Albertazzi, Ugo, Ginette Eramo, Leonardo Gambacorta, and Carmelo Salleo, 2011, Securitization is not that evil after all, AFA 2012 Chicago Meetings Paper.
- Allen, Franklin, and Elena Carletti, 2006, Credit risk transfer and contagion, *Journal of Monetary Economics* 53, 89-111.
- Altunbas, Yener, Leonardo Gambacorta, and David Marques-Ibanez, 2009, Securitisation and the bank lending channel, *European Economic Review* 53, 996-1009.
- Ambrose, Brent, Michael LaCour-Little, and Anthony Sanders, 2003, Does regulatory capital arbitrage or asymmetric information drive securitization, *Unpublished Manuscript*.
- Angelides, Phil, and Bill Thomas, 2011. *The financial crisis inquiry report: Final report of the national commission on the causes of the financial and economic crisis in the united states (revised corrected copy)* (Government Printing Office).
- Antonio, Nicolo', and Loriana Pelizzon, 2005, Credit derivatives: Capital requirements and strategic contracting, (Dipartimento di Scienze Economiche" Marco Fanno").
- Ashcraft, Adam B, and Til Schuermann, 2008. Understanding the securitization of subprime mortgage credit (Now Publishers Inc).
- Axelson, Ulf, 2007, Security design with investor private information, *The Journal of Finance* 62, 2587-2632.
- Ayotte, Kenneth M, and Stav Gaon, 2005, Asset-backed securities: Costs and benefits of "bankruptcy remoteness, *Columbia Business School.*"
- Barth, Mary E, Gaizka Ormazabal, and Daniel J Taylor, 2011, Asset securitizations and credit risk, *The Accounting Review* 87, 423-448.
- Batty, David Line, 2011, Dodd-frank's requirement of skin in the game for asset-backed securities may scalp corporate loan liquidity, *NC Banking Inst.* 15, 13.
- Berndt, Antje, and Anurag Gupta, 2009, Moral hazard and adverse selection in the originate-todistribute model of bank credit, *Journal of Monetary Economics* 56, 725-743.
- Best, Ronald, and Hang Zhang, 1993, Alternative information sources and the information content of bank loans, *The Journal of Finance* 48, 1507-1522.

- Billett, Matthew T, Mark J Flannery, and Jon A Garfinkel, 2006, Are bank loans special? Evidence on the post-announcement performance of bank borrowers, *Journal of Financial and Quantitative Analysis* 41, 733-751.
- Bolton, Patrick, and Mathias Dewatripont, 2005. Contract theory (MIT press).
- Boot, Arnoud WA, and Anjan V Thakor, 1993, Security design, *The Journal of Finance* 48, 1349-1378.
- Brunnermeier, Markus K, 2008, Deciphering the liquidity and credit crunch 2007-08, (National Bureau of Economic Research).
- Calomiris, Charles W, and Joseph R Mason, 2004, Credit card securitization and regulatory arbitrage, *Journal of Financial Services Research* 26, 5-27.
- Carey, Mark, Stephen Prowse, John Rea, and Gregory Udell, 1994, Economics of the private placement market, the, *Fed. Res. Bull.* 80, 5.
- Casu, Barbara, Andrew Clare, Anna Sarkisyan, and Stephen Thomas, 2011, Does securitization reduce credit risk taking? Empirical evidence from us bank holding companies, *The European Journal of Finance* 17, 769-788.
- Chen, Weitzu, Chi-Chun Liu, and Stephen G Ryan, 2008, Characteristics of securitizations that determine issuers' retention of the risks of the securitized assets, *The Accounting Review* 83, 1181-1215.
- Cheng, Mei, Dan S Dhaliwal, and Monica Neamtiu, 2011, Asset securitization, securitization recourse, and information uncertainty, *The Accounting Review* 86, 541-568.
- Chiesa, Gabriella, 2008, Optimal credit risk transfer, monitored finance, and banks, *Journal of Financial Intermediation* 17, 464-477.
- Colander, David, Michael Goldberg, Armin Haas, Katarina Juselius, Alan Kirman, Thomas Lux, and Brigitte Sloth, 2009, The financial crisis and the systemic failure of the economics profession, *Critical Review* 21, 249-267.
- Coval, Joshua, Jakub Jurek, and Erik Stafford, 2009, The economics of structured finance, *The Journal of Economic Perspectives* 3-26.
- Dahiya, Sandeep, Manju Puri, and Anthony Saunders, 2003, Bank borrowers and loan sales: New evidence on the uniqueness of bank loans*, *The Journal of Business* 76, 563-582.
- Dang, Tri Vi, Gary Gorton, and Bengt Holmstrom, 2009, Opacity and the optimality of debt for liquidity provision, *Manuscript Yale University*.

- De Bandt, Olivier, and Philipp Hartmann, 2000, Systemic risk: A survey, *Available at SSRN* 258430.
- Dell'Ariccia, Giovanni, Deniz Igan, and Luc Laeven, 2012, Credit booms and lending standards: Evidence from the subprime mortgage market, *Journal of Money, Credit and Banking* 44, 367-384.
- DeMarzo, Peter, and Darrell Duffie, 1999, A liquidity-based model of security design, *Econometrica* 67, 65-99.
- DeMarzo, Peter M, 2005, The pooling and tranching of securities: A model of informed intermediation, *Review of Financial Studies* 18, 1-35.
- Demyanyk, Yuliya, and Otto Van Hemert, 2011, Understanding the subprime mortgage crisis, *Review of Financial Studies* 24, 1848-1880.
- Diamond, Douglas W, 1984, Financial intermediation and delegated monitoring, *The Review of Economic Studies* 51, 393-414.
- Diamond, Douglas W, and Philip H Dybvig, 1983, Bank runs, deposit insurance, and liquidity, *The economics of structured finance, The Journal of Political Economy* 401-419.
- Diamond, Douglas W, and Raghuram Rajan, 2009, The credit crisis: Conjectures about causes and remedies, (National Bureau of Economic Research).
- Diamond, Douglas W, and Raghuram G Rajan, 2009, Illiquidity and interest rate policy, (National Bureau of Economic Research).
- Dionne, Georges, 2009, Structured finance, risk management, and the recent financial crisis, *Ivey Business Journal*, 7 p.
- Dionne, Georges, and Tarek M Harchaoui, 2008, Banks' capital, securitization and credit risk: An empirical evidence for Canada, *Insurance and Risk Management* 75, 459-485.
- Donnelly, Catherine, and Paul Embrechts, 2010, The devil is in the tails: Actuarial mathematics and the subprime mortgage crisis, *Astin Bulletin* 40, 1-33.
- Downing, Chris, Dwight Jaffee, and Nancy Wallace, 2009, Is the market for mortgage-backed securities a market for lemons?, *Review of Financial Studies* 22, 2457-2494.
- Drucker, Steven, and Christopher Mayer, 2008, Inside information and market making in secondary mortgage markets, *Manuscript, Columbia University*.
- Drucker, Steven, and Manju Puri, 2009, On loan sales, loan contracting, and lending relationships, *Review of Financial Studies* 22, 2835-2872.

- Dugan, John.C., 2010, February 2, Securitization, 'skin-in-the-game' proposals, and minimum mortgage underwriting standards. *Remarks before the American Securitization Forum. Retrieved*, from http://www.occ.gov/news- issuances/speeches/2010/pub-speech-2010-13.pdf.
- Ellul, Andrew, and Vijay Yerramilli, 2013, Stronger risk controls, lower risk: Evidence from us bank holding companies, *The Journal of Finance* 68, 1757-1803.
- Elul, Ronel, 2011, Securitization and mortgage default, Available at SSRN 1484973.
- Estrella, Arturo, 2002, Securitization and the efficacy of monetary policy, *Economic Policy Review* 8, 243-255.
- Fender, Ingo, and Janet Mitchell, 2009a, The future of securitisation: How to align incentives?, *BIS Quarterly Review* 3, 25-50.
- Fender, Ingo, and Janet Mitchell, 2009b, Incentives and tranche retention in securitisation: A screening model, *National Bank of Belgium Working Paper* 177 (2009).
- Firla-Cuchra, Maciej, and Tim Jenkinson, 2005, Security design in the real world: Why are securitization issues tranched? *Working Paper, Oxford University*.
- Freixas, Xavier, and Jean-Charles Rochet. Microeconomics of banking. Vol. 2. Cambridge, MA: MIT press, 1997.
- Gale, Douglas, and Martin Hellwig, 1985, Incentive-compatible debt contracts: The one-period problem, *The Review of Economic Studies* 52, 647-663.
- Gambacorta, Leonardo, and David Marques-Ibanez, 2011, The bank lending channel: Lessons from the crisis, *Economic Policy* 26, 135-182.
- Gande, Amar, Manju Puri, Anthony Saunders, and Ingo Walter, 1997, Bank underwriting of debt securities: Modern evidence, *Review of Financial Studies* 10, 1175-1202.
- Gande, Amar, and Anthony Saunders, 2012, Are banks still special when there is a secondary market for loans?, *The Journal of Finance* 67, 1649-1684.
- Gennaioli, Nicola, Andrei Shleifer, and Robert Vishny, 2012, Neglected risks, financial innovation, and financial fragility, *Journal of Financial Economics* 104, 452-468.
- Gibbons, Robert, and Kevin J Murphy, 1991, Relative performance evaluation for chief executive officers, (National Bureau of Economic Research).
- Glaeser, Edward L, and Hedi D Kallal, 1997, Thin markets, asymmetric information, and mortgage-backed securities, *Journal of Financial Intermediation* 6, 64-86.

Gorton, Gary B, 2008, The panic of 2007, (National Bureau of Economic Research).

- Gorton, Gary B, 2009, Information, liquidity, and the (ongoing) panic of 2007, (National Bureau of Economic Research).
- Gorton, Gary B, 2010. Slapped by the invisible hand: The panic of 2007 (Oxford University Press).
- Gorton, Gary B, and George G Pennacchi, 1995, Banks and loan sales marketing nonmarketable assets, *Journal of monetary Economics* 35, 389-411.
- Gorton, Gary B, and Nicholas S Souleles, 2007, Special purpose vehicles and securitization, in *The risks of financial institutions* (University of Chicago Press).
- Gorton, Gary, and Andrew Metrick, 2012, Securitization, (National Bureau of Economic Research).
- Goswami, Mangal, Andreas A Jobst, and Xin Long, 2009, An investigation of some macrofinancial linkages of securitization, *Available at SSRN 1345576*.
- Greenbaum, Stuart I, and Anjan V Thakor, 1987, Bank funding modes: Securitization versus deposits, *Journal of Banking & Finance* 11, 379-401.
- Gupta, Anurag, Ajai K Singh, and Allan A Zebedee, 2008, Liquidity in the pricing of syndicated loans, *Journal of Financial Markets* 11, 339-376.
- Haensel, Dennis, and Jan Pieter Krahnen, 2007, Does credit securitization reduce bank risk? Evidence from the european cdo market, *Evidence from the European CDO Market* (January 29, 2007).
- Hart, Oliver, and Bengt Holmstrom, 2008, A theory of firm scope, (National Bureau of Economic Research).
- Hartman-Glaser, Barney, Tomasz Piskorski, and Alexei Tchistyi, 2012, Optimal securitization with moral hazard, *Journal of Financial Economics* 104, 186-202.
- Hellwig, Martin F, 2009, Systemic risk in the financial sector: An analysis of the subprimemortgage financial crisis, *De Economist* 157, 129-207.
- Higgins, Eric J, and Joseph R Mason, 2004, What is the value of recourse to asset-backed securities? A clinical study of credit card banks, *Journal of Banking & Finance* 28, 875-899.
- Hölmstrom, Bengt, 1979, Moral hazard and observability, The Bell Journal of Economics 74-91.
- Holmstrom, Bengt, and Jean Tirole, 1997, Financial intermediation, loanable funds, and the real sector, *the Quarterly Journal of economics* 663-691.

- Innes, Robert D, 1990, Limited liability and incentive contracting with ex-ante action choices, *Journal of Economic Theory* 52, 45-67.
- Jeon, Haejun, and Michi Nishihara, 2012, Securitization under asymmetric information and risk retention requirement, *Available at SSRN 2116770*.
- Kashyap, Anil K, and Jeremy C Stein, 2000, What do a million observations on banks say about the transmission of monetary policy?, *American Economic Review* 407-428.
- Kendall, Leon T, and Michael J Fishman, 2000. A primer on securitization (MIT press).
- Keys, Benjamin J, Tanmoy K Mukherjee, Amit Seru, and Vikrant Vig, 2008, Did securitization lead to lax screening? Evidence from subprime loans, *EFA 2008 Athens Meetings Paper*.
- Keys, Benjamin J, Tanmoy Mukherjee, Amit Seru, and Vikrant Vig, 2009, Financial regulation and securitization: Evidence from subprime loans, *Journal of Monetary Economics* 56, 700-720.
- Kiff, John, and Michael Kisser, 2010. *Asset securitization and optimal retention* (International Monetary Fund).
- Kothari, SP, Elena Loutskina, and Valeri V Nikolaev, 2006, Agency theory of overvalued equity as an explanation for the accrual anomaly, *Available at SSRN 871750*.
- Kroszner, Randall S, and Raghuram G Rajan, 1994, Is the glass-steagall act justified? A study of the us experience with universal banking before 1933, *The American Economic Review* 810-832.
- Kuttner, Kenneth, 2000, Securitization and monetary policy, *Unpublished paper, Federal Reserve Bank of New York*.
- Leland, E Hayne, and David H Pyle, 1977, Informational asymmetries, financial structure, and financial intermediation, *The journal of Finance* 32, 371-387.
- Levitin, Adam J, 2013, Skin-in-the-game: Risk retention lessons from credit card securitization, *The George Washington Law Review* 81, 813.
- Loutskina, Elena, 2006, The role of securitization in bank financial management, Federal Reserve Bank of Chicago Proceedings.
- Loutskina, Elena, 2011, The role of securitization in bank liquidity and funding management, *Journal of Financial Economics* 100, 663-684.

- Loutskina, Elena, and Philip E Strahan, 2009, Securitization and the declining impact of bank finance on loan supply: Evidence from mortgage originations, *The Journal of Finance* 64, 861-889.
- Lummer, Scott L, and John J McConnell, 1989, Further evidence on the bank lending process and the capital-market response to bank loan agreements, *Journal of Financial Economics* 25, 99-122.
- Malekan, Sara, and Georges Dionne, 2014, Securitization and optimal retention under moral hazard, *Journal of Mathematical Economics* 55, 74-85.
- Mian, Atif, and Amir Sufi, 2008, The consequences of mortgage credit expansion: Evidence from the 2007 mortgage default crisis, (National Bureau of Economic Research).
- Minton, Bernadette, Anthony Sanders, and Philip E Strahan, 2004, Securitization by banks and finance companies: Efficient financial contracting or regulatory arbitrage? *Working Paper, Ohio State University*.
- Morgenson, Gretchen, and Louise Story, 2010, Rating agency data aided wall street in deals, *New York Times*.
- Morrison, Alan D, 2005, Credit derivatives, disintermediation, and investment decisions, *The Journal of Business* 78, 621-648.
- Niu, Flora F, and Gordon D Richardson, 2006, Are securitizations in substance sales or secured borrowings? Capital-market evidence, *Contemporary Accounting Research* 23, 1105-1133.
- Osano, Hiroshi, 1999, Security design, insider monitoring, and financial market equilibrium, *European Finance Review* 2, 273-302.
- Paligorova, Teodora, 2009, Agency conflicts in the process of securitization, *Bank of Canada Review* 2009, 36-50.
- Park, Sunyoung, 2013, The design of subprime mortgage-backed securities and information insensitivity, *International Economic Journal* 27, 249-284.
- Parlour, Christine A, and Guillaume Plantin, 2008, Loan sales and relationship banking, *The Journal of Finance* 63, 1291-1314.
- Parlour, Christine A, and Andrew Winton, 2013, Laying off credit risk: Loan sales versus credit default swaps, *Journal of Financial Economics* 107, 25-45.
- Pavel, Christine, and David Phillis, 1987, Why commercial banks sell loans: An empirical analysis, *Federal Reserve Bank of Chicago Economic Perspectives* 14, 3-14.

- Pennacchi, George G, 1988, Loan sales and the cost of bank capital, *The Journal of Finance* 43, 375-396.
- Plantin, Guillaume, 2004, Tranching. London school of economics, (Working Paper April, London).
- Rajan, Raghuram G, 2006, Has finance made the world riskier?, *European Financial Management* 12, 499-533.
- Rajan, Uday, Amit Seru, and Vikrant Vig, 2010, The failure of models that predict failure: Distance, incentives and defaults, *Chicago GSB Research Paper*.
- Riddiough, Timothy J, 1997, Optimal design and governance of asset-backed securities, *Journal* of Financial Intermediation 6, 121-152.
- Rosenkranz, Robert, 2009, Let's write the rating agencies out of our law, *Wall Street Journal* (January 2).
- Selody, Jack, and Elizabeth Woodman, 2009, Reform of securitization, *Financial System Review* 47-52.
- Shleifer, Andrei, and Robert W Vishny, 2010, Unstable banking, *Journal of Financial Economics* 97, 306-318.
- Sorkin, Andrew, 2009, Too big to fail: The inside story of how wall street and washington fought to save the financial system, (Penguin Group. New York, NY: Penguin Group).
- Stein, Jeremy C, 2002, Information production and capital allocation: Decentralized versus hierarchical firms, *The Journal of Finance* 57, 1891-1921.
- Stein, Jeremy C, 2011, Monetary policy as financial-stability regulation, (National Bureau of Economic Research).
- Sufi, Amir, 2007, Information asymmetry and financing arrangements: Evidence from syndicated loans, *The Journal of Finance* 62, 629-668.
- Wagner, Wolf, 2005, Interbank diversification, liquidity shortages and banking crises, *Cambridge: Cambridge University, unpublished.*
- Wagner, Wolf, 2007, The liquidity of bank assets and banking stability, *Journal of Banking & Finance* 31, 121-139.
- Wagner, Wolf, and Ian W Marsh, 2006, Credit risk transfer and financial sector stability, *Journal* of *Financial Stability* 2, 173-193.

- Winter, Ralph A, 2000, Optimal insurance under moral hazard, in *Handbook of Insurance* (Springer).
- Wu, Ho-Mou, and Guixia Guo, 2010, Retention ratio regulation of bank asset securitization, Working paper in National School of Development, Peking University.