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The One-Warehouse Multi-Retailer Problem with Four Types of Emission Constraints

by
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A Thesis Submitted
In Partial Fulfilment of the Requirements
for A Master of Science in Global Supply Chain

December 2019

Declaration by student
Ethical requirements:
Research on human subjects

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Résumé

Dans ce mémoire, nous abordons le problème de production et transport entre une usine et plusieurs détaillants (« one-warehouse multi-retailer problem – OWMR »). Nous considérons une usine qui fabrique un seul type de produit et réapprovisionne multiples détaillants qui possèdent une demande dynamique sur un horizon de planification discret et fini. Nous généralisons le problème OWMR avec quatre types de contraintes d'émission: globale, cumulative, d'horizon glissant et périodique. Nous modélisons le problème en utilisant la formulation de base et la formulation multicommodité. Outre l'optimisation des coûts, nous limitons les émissions engendrées par les activités de production et de stockage dans ce réseau de distribution à deux échelons sur les horizons de temps différents. Nous utilisons deux méthodes pour établir les plafonds d'émissions. La première méthode génère les plafonds d'émission équivalents qui garantissent la faisabilité des problèmes et de qui nous permet de comparer équitablement les quatre types de contraintes d'émission. La deuxième méthode détermine le niveau d'émission minimum pour chaque type de plafond d'émission. En augmentant progressivement les plafonds d'émission, nous analysons les courbes de compromis entre les coûts et les émissions. L'analyse montre que le plafond d'émission le plus serré induit le coût total plus élevé avec tous ces quatre types de contraintes, mais plus fortement avec les contraintes périodiques et cumulatives. Ce mémoire ajoute à la littérature sur les modèles de lot-sizing dynamique multi-échelon avec les contraintes d'émission et donne un aperçu des performances des formulations et des modèles.

Mots-clés: OWMR, lot-sizing, contraintes d'émission carbone, multi-échelon

Méthode de recherche: modélisation

Abstract

In this thesis, we address the one-warehouse-multi-retailer (OWMR) problem. We consider a plant that produces a single type of product and replenishes multiple retailers with dynamic demand over a discrete and finite time horizon. We generalize the OWMR problem with four types of emission constraint: global, cumulative, rolling horizon and periodic. We model the problem using the basic formulation and the multicommodity formulation. Besides cost optimization, we limit the emissions incurred in the production and stock activities in the two-echelon supply chain network on different time horizons. We use two methods to establish the emission caps. The first method generates equivalent emission caps that ensure the feasibility of the problems, allowing us to fairly compare the four types of emission constraints. The second method determines the minimum emission level for each type of emission cap. By gradually increasing the emission caps, we analyze the cost-emission trade-off curves. The analysis shows that the tighter emission cap induces higher total costs in all four types of constraint, but most strongly in the periodic and cumulative constraints. This thesis complements the literature on the multi-echelon dynamic lot-sizing model with emission constraints and provides insights in the behaviors of different formulations and models.

Keywords: OWMR, lot sizing, carbon emission constraints, multi-echelon

Research methods: modelling

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Chapter 1
Introduction

In the past few decades, there has been growing concern about climate change, along with a rapid increase in environmental regulations globally. In order to comply with legislation and respond to social pressures, companies strive to mitigate their environmental impact.

There is a consensus that anthropogenic emissions are the main cause of global warming. Therefore, the reduction of carbon emissions plays a vital role in sustainable management. In 2016, 175 legislators signed the Paris Agreement, aiming to limit global warming this century to 2℃, or even less, to 1.5℃above pre-industrial levels (United Nations Framework Convention on Climate Change, 2015). This objective signifies an objective of a 2.7% annual emissions reduction rate in the coming decade (United Nations Environment Programme, 2019). In addition to international cooperation, countries implement carbon regulations such as carbon taxes or cap-and-trade policies, urging companies to manage their carbon footprints.

There is a growing body of literature mapping the possible sustainable strategies for companies. Dekker, Bloemhof and Mallidis (2012) suggest assessing the environmental impact along the supply chain, emphasizing on strategies such as supply chain network design and planning and control. They also shed light on tactical decisions, such as the fuel consumption in transportation. However, only a few options at the operational level have been mentioned. It is often believed that cost-optimization and environmental endeavours are incompatible. Contrary to this misconception, however, Benjaafar, Li and Daskin (2013) point out that companies can curb emissions effectively by operational adjustments without significant investment. One such measure is green lot sizing. Traditionally, a lot-sizing model determines the right amount and time to for production, commonly with an objective of minimizing total cost. A green lot-sizing model also considers the emissions induced during the production and storage activities. In this thesis, we will focus on a specific multi-echelon dynamic lot-sizing problem, namely, the one-warehouse-multi-retailer (OWMR) problem with emission constraints.

As the name suggests, the OWMR problem tackles a single plant (a distribution centre) that replenishes multiple retailers with dynamic demand over a discrete and finite time horizon. To be more specific, the dynamics of the problem are as follows. At the beginning of each period, each retailer can put an order to the production site; the plant collects the demands of all the retailers, then produces and delivers the quantity ordered to each retailer (Levi *et al.*, 2008). In this thesis, we assume that the warehouse produces, stores and replenishes the retailers; therefore, the terms "warehouse," "production site" and "plant" are used interchangeably, referring to the same node. Three types of cost are incurred: setup costs (fixed production setup costs in the production site and fixed delivery setup costs subject to the retailers), unit production and delivery costs, and holding costs at the plant and retailers. The objective is to minimize the total cost at all facilities. As discussed earlier, in addition to the cost optimization, we also limit the emissions induced in all the activities in a classical OWMR problem.

In this thesis, we model the OWMR problem with two formulations, namely the basic formulation (BF) and the multicommodity formulation (MC). Moreover, we propose an extension for the two formulations to incorporate the four types of emission constraints proposed by Absi *et al.* (2013), namely the global, cumulative, rolling horizon and periodic emission constraints, respectively. We conduct three sets of computational experiments. The first set of experiments validates the two formulations without emission constraints. The second set of experiments allows us to compare the four types of emission constraints. And in the third set of experiments, we establish the Pareto curves to analyze the cost-emission trade-offs for each type of the emission constraints separately.

In the second set of experiments, in order to have a fair comparison, we need to establish equivalent emission caps for the four different types of emission constraints. Taking an instance with 10 periods for example, if the global emission cap is 100, the corresponding periodic cap is equal to 10. If the rolling horizon window comprises 3 periods, then the equivalent cap for the rolling horizon constraints would be 30. The equivalent caps for the cumulative constraints would be as follows: 10 for period 1, 20 for period 2, 30 for period 3, ..., 100 for period 10. Having the equivalent caps allows us to directly compare the impact of different types of emission constraints. Besides, we need to make sure that the equivalent caps lead to feasible instances. Since the model with periodic constraints is the strictest, we first determine the emission caps for this model, and then we calculate the equivalent caps for the other types of emission constraints. If we can guarantee the feasibility for the cap in the periodic model, we can also ensure the feasibility of the equivalent caps in the other models. We will propose an optimization model to determine the minimum level of the period cap that still provides a feasible solution.

In the third set of computation experiments, we determine the minimum emission level of each type of caps using a series of optimization models. Again, these minimum emission levels ensure the feasibility of the instances. Moreover, using the minimum emission cap level, the emission constraints have an impact on total cost. By gradually increasing the emission caps, we establish Pareto curves to analyze the trade-offs between total cost and emissions for the global, rolling horizon, periodic and cumulative constraints, respectively.

All the computational experiments are conducted using CPLEX 12.9.0.0. The instances consist of the demand and cost parameters from previous studies of Solyali and Süral (2012), the emission factors generated using the approach in Zhong (2014), and the emissions caps generated as discussed above.

This thesis makes a fourfold contribution. First, it validates two formulations of the OWMR problem and compares the computational results with those of previous studies. Second, the thesis extends the OWMR problem by adding four types of emission constraint. It fills a gap in the literature by incorporating cumulative, rolling horizon and periodic emission constraints into multi-echelon dynamic lot-sizing models. Third, it introduces two methods to determine the emission caps that can be used in the computational experiments. Fourth, it analyzes the emission-cost trade-offs for each type of emission constraint. The analysis provides practical implications for the carbon footprint management of companies. In summary, the thesis complements the existing literature in the field of green lot-sizing models by studying the specific OWMR problem with four types of emission constraints.

The remainder of this thesis is organized as follows. Chapter 2 reviews the existing literature in modelling green supply chain problems, with a focus on dynamic lot-sizing models with emission constraints. Chapter 3 explores the formulations of the OWMR problems and proposes formulations with emission constraints. Chapter 4 reports the computational results of the OWMR.

Chapter 5 describes the computational experiments on OWMR problem with emission constraints (OWMR-E) using equivalent emission caps, then compares the four types of emission constraint. Chapter 6 provides the emission-cost trade-offs analysis and a brief analysis of the trade-offs between the total emission and the emission caps. The last chapter draws the conclusion with some final remarks.

Chapter 2
Literature Review

This section provides an overview of recent developments in modelling the green supply chain before briefly reviewing the lot-sizing models, specifically, the OWMR problem. The final section discusses sustainable lot-sizing models with a focus on different types of emission constraints and explores extensions to various carbon-pricing schemes.

2.1 Supply Chain Models with Environmental Factors

With the growing attention to environmental issues, there has been a significant increase in the literature regarding green supply chains. Modelling is the dominant approach for supply chain sustainability studies (Brandenburg and Rebs, 2015). This section investigates the methodology of modelling a supply chain problem with environmental considerations.

Scopes of Sustainable Supply Chain Models

The most common method for incorporating environmental factors into models is the Life Cycle Assessment (LCA). This approach gives a general framework for assessing environmental impact in the different stages of a supply chain (Lake *et al.*, 2015). The LCA has three scopes, namely, cradle-to-grave, cradle-to-gate and gate-to-gate. The cradle-to-grave scope assesses sustainability through the whole supply chain, from the extraction of raw materials to production, distribution, use, repair, disposal and recycling. The cradle-to-gate scope tracks the processes from raw material extraction to the production site. The gate-to-gate scope mainly monitors environmental performance at the production and distribution levels (Eskandarpour *et al.*, 2015). Some research focuses on the most polluting sectors in the supply chain (Banasik *et al.*, 2018), namely, the transport and manufacturing aspects (Brandenburg and Rebs, 2015), which represent 14% and 21% of the global carbon emissions, respectively (IPCC, 2015).

Objectives of Sustainable Supply Chain Models

In terms of their objectives, sustainable supply chain models can comprise three dimensions—the economic, environmental and social—of which the economic is most often addressed (Seuring, 2013). Models of the economic dimension prioritize cost optimization while presenting the environmental concerns in terms of constraints. Models of the environmental dimension tend to set multiple objectives, most commonly one objective on total costs and the other on environmental impacts (Banasik *et al.*, 2018). For instance, Wang, Lai and Shi (2011) propose a multi-objective green supply chain network model to analyze the trade-offs between total costs and total emission amounts. The first objective function optimizes the fixed setup costs, total transportation costs, handling costs and investment in sustainability. The second objective function measures total carbon emissions through the network. The authors employ the normalized normal constraint method to generate the Pareto frontier, then test the model using a case study. A similar yet more comprehensive multi-objective model appears in Chaabane, Ramudhin and Paquet (2012), in which the authors also consider the costs of reverse logistics and the costs and/or revenue of emission permits.

Key Environmental Indicators

According to Eskandarpour *et al.* (2015) , there are four main indicators for measuring environmental impact: (1) Greenhouse Gas (GHG) emissions, (2) amount of waste, (3) energy consumption and (4) amount of recycling. GHG emissions are composed of CO_2, CH_4, N_2O and fluorinated gases, of which carbon emissions constitute the most commonly used indicator in supply chain models. Paksoy, Bektaş and Özceylan (2011) use the amount of fuel consumption to estimate transport emissions. Similarly, Li and Hai (2019) calculate transport emissions in terms of vehicle type, fuel consumption, distance, weight of load and fuel emission rate, with warehousing emissions assumed to comprise electricity and other energy costs. Sundarakani *et al.* (2010) propose a more holistic model to measure the carbon footprint in a closed-loop end-to-end supply chain, using the long-range Lagrangian and the Eulerian transport methods, a common approach in pollution studies. They examine emissions in two categories: emission from stationary processes (material processing, production and warehousing) and from non-stationary processes (transportation). Note that in all the literature mentioned above, the researchers refer explicitly to the difficulty of collecting emissions data and calculating carbon prints. Thus, green supply chain models, all based on estimated emissions data, are more effective as a tool for strategic decision-making than tactical ones.

Modelling Emissions under Different Policies

In 2019, 57 carbon-pricing initiatives are in force (or scheduled) in 48 national and 28 subnational jurisdictions (World Bank Group, 2019). These carbon-pricing schemes include Emissions Trading Systems (ETSs), carbon taxes, strict caps and carbon offsets. An ETS, also called a cap-and-trade system, caps the emissions of firms and allows entities to trade emission allowances. A carbon tax directly sets a tax rate for the GHGs emitted. A cap policy sets a strict carbon emissions limit, whereas a carbon offset system allows firms to compensate for their excess emissions with previous unused emissions credits. The supply chain models are applicable to various schemes. Zakeri *et al.* (2015) compare two schemes in a dual-objective model, with the first objective function being to optimize total costs, and the second being to minimize carbon costs. Under carbon-tax, the second objective function minimizes the charges of the total emissions amount. Under cap-and-trade, on the other hand, the objective function minimizes the total costs of emissions allowances in the case of over-emitting or minimizes the gap between allowance costs and income from selling unused emissions credits. The authors show that cap-and-trade results in carbon reduction with fewer costs but the risks of an unpredictable carbon market can be significant.

We have reviewed the main methodology to incorporate emissions into general supply chain models. In the rest of this literature review, we will focus on green lot-sizing models.

2.2 Lot-sizing Models

Lot sizing determines when and how much to replenish inventory. Inventory models can be classified according to different criteria, such as the nature of the data, time scale, number of machines, number of levels and capacity constraints. In this section, we mainly review lot-sizing models with deterministic demand.

The well-known Economic Order Quantity (EOQ) model is a paradigm of deterministic models. It aims to minimize the total ordering and holding costs on the basis of a set of assumptions, including continuous time scale, deterministic and stationary demand, constant lead-time and static costs. As we all know, these assumptions are rarely achievable in the real business environment.

To address planning problems with dynamic demand, Wagner and Whitin (1958) introduce the single-item uncapacitated lot-sizing model (SI-ULSP). The objective is to optimize production and inventory costs on the basis of three critical decisions: (1) setup, (2) production size and (3) inventory level—over a discrete and finite time horizon. The SI-USLP is widely used because of its simplicity and extensibility. The simple objective function intuitively presents trade-offs of various decisions (Brahimi *et al.*, 2017; Jans and Degraeve, 2008). The model can be extended to settings that are more realistic. Pochet and Wolsey (2006), Jans and Degraeve (2008) and Brahimi *et al.* (2017) discuss extensions of the SI-ULSP. Demand-wise, the problem can include multiple items, backlogging, lost sales and stochastic demands. Time-wise, it is compatible with planning on a rolling horizon. Constraint-wise, it can comprise capacity constraints, minimum lot size and fixed batch size constraints, among others. Cost-wise, the SI-ULSP can take other elements into account, such as discounts, start-up costs, outsourcing costs and emissions costs. Structure-wise, the problem is extensible to multi-echelon distribution systems. Melo and Wolsey (2010) solve a two-echelon uncapacitated lot-sizing problem in series and reformulate it with shortest-path constraints. They also study a particular form of the two-echelon problem, namely, a one-warehouse multi-retailer multicommodity problem. Zhang, Küçükyavuz and Yaman (2012) explore a multi-echelon uncapacitated lot-sizing problem in series receiving intermediate demand. They propose an $O(T^4)$ dynamic programming algorithm, with T being the number of periods in the planning horizon. They also suggest a multicommodity formulation for the problem.

A special case of the multi-echelon lot-sizing problem is the OWMR, a two-echelon, single-sourcing multi-destination problem. There are six main formulations for the OWMR problem: (1) the basic formulation, (2) echelon stock formulation, (3) transportation formulation, (4) shortest-path formulation, (5) Wagner-Whitin based formulation and (6) multicommodity formulation. There are several other formulations that have been derived from the six main formulations above.

The basic formulation (BF) is often used to present the nature of the problem. It extends the formulation of the uncapacitated lot-sizing problem discussed by Pochet and Wolsey (2006) to a multi-echelon setting. The echelon stock formulation (ES) proposed by Federgruen and Tzur (1999) is one of the most classical formulations for the OWMR problem. ES considers the total inventory at a facility and at all its customers. In the OWMR problem, the echelon stock variable deals with the inventory level of the whole distribution system. The formulation is proven to provide stronger lower bounds using Lagrangian relaxation algorithm. Multiple reformulations are derived from ES, such as the strengthened echelon stock formulation (SES) presented in Solyali and Süral (2012). The transportation formulation (TP) introduced by Levi *et al.* (2008) tracks the location of products. The variables contain 4 timestamps: in which period the product is produced, when it is transported to the warehouse, when it is delivered to the retailers and eventually when the demand is satisfied. The shortest-path formulation (SP) proposed by Solyali and Süral (2012) derives from the transportation formulation. SP introduces new variables deciding the proportion of demand produced in period *k*, transported to the warehouse in period *t*, shipped to the retailer in period *j* to fulfill the demand in *l*. The flow of the products forms a series of arcs between the timestamps, and SP aims to choose the shortest path. Another formulation for the OWMR problem is the Wagner-Whitin based formulation (LS) introduced by Melo and Wolsey (2010). It adds the (*l, S, WW*) inequalities for certain relaxations of the problem (Cunha and Melo, 2016). Based on the it, Cunha and Melo (2016) develop the two-level lot-sizing Wagner-Whitin based formulation (2LS) and partial two-level lot-sizing Wagner-Whitin based formulation (p2LS). In addition, Cunha and Melo (2016) generalize the multicommodity formulation (MC) and two-level lot-sizing dynamic programming based formulation in Melo and Wolsey (2010) to the OWMR problem. The latest paper of Gruson *et al.* (2019) proposes eight more formulations for the OWMR problem. These formulations are based on ES and SES, combining the features of TP, SP, LS and MC.

The problem is NP-hard, since the joint replenishment problem, a special OWMR problem without warehouse inventory, is NP-hard (Arkin, Joneja and Roundy, 1989). Table 1 summarizes the work regarding the formulations for the OWMR problem. The column "New formulations" presents the formulations proposed in the paper. The column "validations" displays the formulations the author(s) have validates. The column "extension" shows the extensions of the OWMR problem introduced by the author(s). And the column "recommendations" shows the best formulation according to the authors. In the most recent two publications, MC is proven to perform better than the other formulations on the uncapacitated instances (Cunha and Melo, 2016; Gruson *et al.*, 2019). It also provides strong LP relaxation bounds in shorter solution time. Therefore, in the thesis, we focus on the MC while using BF as the baseline for comparison.

It is noteworthy that the formulations are adapted to more complex problems. Solyali and Süral (2012) and Zhong (2014) extend the formulations to include the initial inventory. Gruson *et al.* (2019) change the magnitude of the problem to a three-echelon distribution system. They validate the three-echelon formulations with balanced and unbalanced network, capacitated and uncapacitated instances. In addition to the extensions, OWMR also appears as sub-problems of complex multi-stage problems. For instance, Adulyasak, Cordeau and Jans (2014) model the OWMR as part of a production routing problem. The same approach appears in Adulyasak, Cordeau and Jans (2015).

Table 1. Formulations for OWMR

	New Formulations	**Validation**	**Extension**	**Recommendations**
Federgruen and Tzur (1999)	ES	ES	-	ES
Levi *et al.* (2008)	TP	-	-	TP
Melo and Wolsey (2010)	LS	BF, ES, LS, MC	-	MC
Solyali and Süral (2012)	SES, SP	ES, SES, TP, SP	Initial inventory	TP, SP
Zhong (2014)	-	BF, TP, SP	Initial inventory, global emission constraints	TP
Cunha and Melo (2016)	MC, LS, 2LS, p2LS, 2DP	ES, SES, MC, TP, SP, LS, 2LS, p2LS, 2DP	-	MC, p2LS
Gruson *et al.* (2019)	MCE, SES-TP, SES-LS, SES-SP	BF, ES, SP, TP, MC, ES-SP, ES-TP, ES-LS, SES, SES-TP, SES-LS, SES-LP, MCE	Three-echelon,	MC-uncapacitated, MCE-capacitated

Besides the formulations, it is also possible to find studies on developing approximation algorithms for the OWMR problem such as in Levi *et al.* (2008) and the series of papers including Stauffer (2012), Gayon *et al.* (2016), Gayon *et al.* (2017) and Stauffer (2018).

2.3 Lot-Sizing Models with Emissions

Studies on lot-sizing models with a carbon footprint limit have been growing rapidly since 2008. The EOQ and newsvendor problem were most commonly employed for green lot sizing (Bushuev *et al.*, 2015). However, we also observe a wide use of dynamic lot-sizing models with emissions.

The first main category of lot-sizing models with emissions is the EOQ-E. Authors extend the classical EOQ model with carbon costs. Hua, Cheng and Wang (2011) pioneer this stream of research. They extend the classical EOQ model to the cap-and-trade scheme by including carbon costs in the objective function and an emissions balance constraint. Moreover, they investigate the impact of the carbon price and emissions caps on operational decisions by a comparison with the classical EOQ model without emissions. Similarly, Chen, Benjaafar and Elomri (2013) formulate emissions as costs to minimize. They also examine the emissions-cost trade-off under different policies. An EOQ-E model can present the trade-offs between different cost types, hence providing insights for business and regulatory decision-makers. However, its basis in unrealistic assumptions, such as static demand, is a drawback for practitioners, who are in search of a simple yet more practical tool.

The dynamic lot-sizing model has received substantial attention, particularly as a sub-problem for a complex supply chain models. Benjaafar, Li and Daskin (2013) introduce four dominant emissions policies into a single-item lot-sizing model. Retel Helmrich *et al.* (2015) examine a similar problem with a global emissions constraint and propose multiple heuristic algorithms to solve the problem.

The above lot-sizing problems consist of a single-echelon structure. There is a paucity of literature on multi-echelon structures with carbon footprints. Bouchery *et al.* (2017) integrate the EOQ into a two-echelon multi-objective model to optimize both total costs and total emissions. Hammami, Nouira and Frein (2015) measure emissions in a multi-echelon supply chain with external suppliers, factories and warehouses. They optimize the production-inventory problem with lead-time constraints and examine the model in both cap-and-trade and carbon-tax scenarios. Furthermore, they compare aggregated emissions constraints with individual emissions caps on each node. Toptal and Çetinkaya (2017) integrate the EOQ to optimize production, inventory and emissions costs of a buyer-vendor two-echelon distribution system under two schemes, namely, cap-and-trade and carbon tax. Under each scheme, they examine two coordination strategies: joint and independent. The joint coordination strategy minimizes the total cost subject to the vendor and the buyer, and the independent strategy only considers the buyer's total cost. They show that a joint strategy can reduce costs without exceeding emission caps but that it does not mitigate the extra costs generated by emissions reductions.

The OWMR problem can also encompass emissions factors. Zhong (2014) incorporates a linear emission constraint in three formulations of the OWMR problem: the BF introduced by Solyali and Süral (2012), the four-index facility location formulation proposed by Levi *et al.* (2008) and the combined transportation and shortest-path formulation discussed by Solyali and Süral (2012). Zhong (2014) conducts experiments and solves the problem on CPLEX with different settings: (1) with or without initial inventory, (2) multiple emission datasets generated in correlation with cost parameters in the range of [-20%, 20%], [-50%, 50%] and [-100%, 100%], and (3) different levels of emission cap. Zhong (2014) also analyzes the trade-off of costs and emissions reductions, observing that the emission reduction cost increases as the reduction amount increases. In addition, he extends heuristics to solve the OWMR problem with emission constraints. Li and Hai (2019) formulate the OWMR problem with replenishing intervals and impose integer-ratio constraints. In contrast to the above, they optimize emissions as costs instead of caps. They also provide equations to calculate emissions costs in transportation and inventory management.

Lot-sizing Model with Emission Constraints

Almost all the studies above incorporate emissions as a "capacity constraint." In terms of time horizon, Absi *et al.* (2013) propose four types of carbon emission constraints: (1) periodic emission constraints, (2) cumulative emission constraints, (3) global emission constraints and (4) rolling horizon emission constraints. Figure 1 illustrates the coverage of each emission constraint type over a horizon of 6 periods.

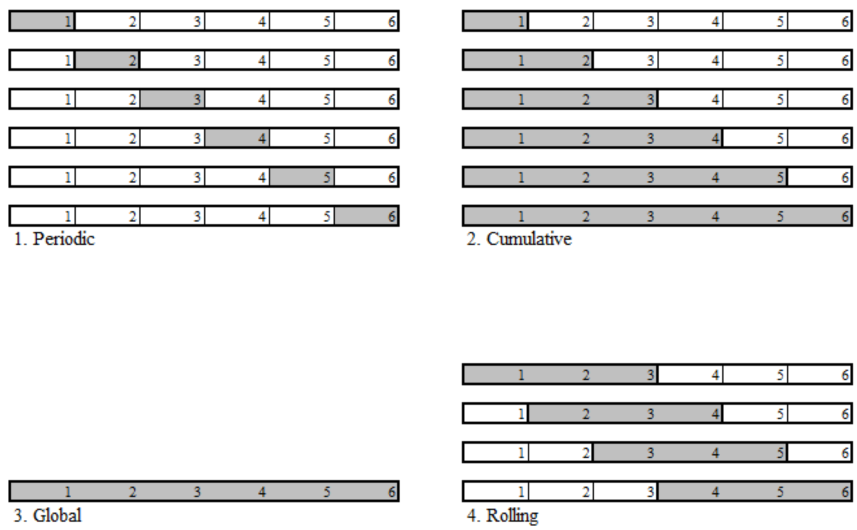

Figure 1. Four Types of Emission Constraints (Zhong, 2014)

A periodic emission constraint sets an emission cap in each period. The unused emission credits cannot be used to offset the emissions of previous periods nor transferred to the next. A cumulative emission constraint imposes a set of emission caps on all periods in a cumulative way starting from the first period. The unused emission credits can be carried onto the following periods without exceeding the cap. A global constraint limits the total emissions of all periods of the planning horizon. The rolling horizon emission constraint sets a cap on each set of *n* periods, i.e. the emissions of period *t* can be compensated for by the adjacent *n* periods. It is intuitive to see that, of the four types of constraint, the periodic constraint is the strictest and global constraint the loosest. However, Absi *et al.* (2016) do not compare the cumulative and rolling horizon emission constraints. Their study shows that the SI-ULSP with periodic emission constraints achieves the optimal solution with a polynomial-time dynamic programming algorithm but becomes NP-hard with the other three types of emission constraint. Figure 2 presents a comparison of the four types of emission constraint in terms of strictness level.

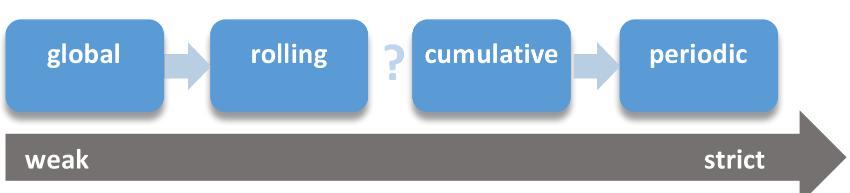

Figure 2. Comparison of Four Types of Emission Constraints

Velázquez Martínez *et al.* (2014) set a global emission constraint on a single-item lot-sizing problem. They aggregate the transportation carbon emissions in the dynamic lot-sizing model on six different levels, and analyze which aggregation method is best. To date, most of the research has studied the global emission constraint, and few studies can be found on the other three types of emission constraint. Absi *et al.* (2016) introduce a single-item lot-sizing model with periodic emission constraints, assuming that a mode will emit a fixed amount of CO_2 if it is selected. The authors consider average emissions per product. This can be relevant for practitioners who want to display the carbon details of their products, contributing to a sustainable brand image. Phouratsamay and Cheng (2019) impose a periodic emission constraint on a two-mode single-item lot-sizing problem with inventory bounds. The authors also propose an extension on the cap-and-trade policy by adding the carbon trade costs and/or revenue to the objective function and the amount of traded emission permits to the constraint. Wu *et al.* (2018) investigate the capacitated multi-item lot-sizing problem with non-identical parallel machines and periodic emission constraints. Yu *et al.* , in contrast to the literature mentioned above, consider cumulative emission constraints in a single-item lot-sizing model. To the best of our knowledge, no literature deals with lot-sizing models with a rolling horizon emission constraint.

Extension on Various Regulations

As discussed in the previous section, more and more countries are implementing carbon-pricing policies. Consequently, studies of lot-sizing models under various carbon schemes are emerging.

The regulations induce two extensive parts, in addition to a classical lot-sizing model: the carbon cost and the carbon cap. He *et al.* (2015) analyze the trade-offs between holding, ordering and carbon costs under cap-and-trade and carbon-tax approaches. Akbalik and Rapine (2013) model cap-and-trade into an uncapacitated lot-sizing problem by adding the carbon trade cost to the objective function and a global emissions cap to the constraints. In addition, two scenarios are tested to analyze the complexity of the problem, namely, with and without budget limits on emission transactions. The authors conclude that the uncapacitated lot-sizing problem with cap-and-trade without budget constraints can be solved in polynomial time, and that the same model with global budget constraints is NP-hard.

In conclusion, the integration of lot-sizing models with carbon footprint is an important topic. To review the current literature in this field systematically, we adopt the framework of Bushuev *et al.* (2015) to classify lot-sizing models with carbon footprint, as shown in Table 2. We first categorize the lot-sizing models by the demand type, then by the number of products and the size of the distribution system. Besides, the table displays the type of emission constraints used in the lot-sizing model and the carbon pricing initiatives the model takes into account.

Table 2. Summary of Literature on Lot-sizing Models with Emissions

	Emission constraint type	Static demand				Dynamic demand				Regulation
		Single-item		Multi-item		Single-item		Multi-item		
		Single echelon	Multi echelon	Single echelon	Multi echelon	Single echelon	Multi echelon	Single echelon	Multi echelon	
Hua, Cheng and Wang (2011)		✓								ETS
Chen, Benjaafar and Elomri (2013)	G	✓								ETS, CT, CO
He *et al.* (2015)		✓								ETS, CT
Bouchery *et al.* (2017)			✓							
Toptal and Çetinkaya (2017)					✓					ETS, CT
Benjaafar, Li and Daskin (2013)	G					✓				ETS, CT, CO
Retel Helmrich *et al.* (2015)	G					✓				
Absi *et al.* (2013)	G, R, C, P					✓				
Absi *et al.* (2016)	P					✓				
Wu *et al.* (2018)	P							✓		
Phouratsamay and Cheng (2019)	P					✓				
Yu *et al.* (2013)	C					✓				
Velázquez Martínez *et al.* (2014)	G					✓				
Akbalik and Rapine	G					✓				ETS
Hammami, Nouira and Frein (2015)	G						✓			ETS, CT
Zhong (2014)	G						✓			
Li and Hai (2019)			✓							CT

Note: G=global, R=rolling horizon, C=cumulative, P=periodic, ETS=cap-and-trade, CT=carbon tax, CO=carbon offset

Chapter 3
OWMR and OWMR-E Formulations

In this chapter, we describe two formulations of the one-warehouse multi-retailer problem: the basic formulation introduced by Pochet and Wolsey (2006) and the multicommodity formulation proposed by Cunha and Melo (2015). We then generalize the OWMR problem with four types of emission constraints, namely the global, cumulative, rolling horizon and periodic emission constraints proposed by Absi *et al.* (2013).

In this thesis, we assume that (1) all demands must be satisfied, (2) no backorder is allowed, (3) the demands of each retailer can only be fulfilled by the production site, (4) the production and delivery time is 0, and there is neither production nor delivery loss, and (5) no capacity constraint.

3.1 Basic Formulation (BF)

The decision variables and parameters are denoted as follows:

Parameters

N	Number of retailers
T	Number of periods of the planning horizon
FC_t^0	Production setup cost at the plant in period *t*
FC_t^r	Transportation setup cost from the plant to retailer *r* in period *t*
HC_t^0	Unit holding cost at the plant in period *t*
HC_t^r	Unit holding cost at the retailer *r* in period *t*
PC_t^0	Unit production cost at the plant in period *t*
PC_t^r	Unit transportation cost from the plant to retailer *r* in period *t*

D_t^0 Demand at the plant in period t, equal to the aggregated demands the retailers receive in period *t*

D_t^r Demand at retailer *r* in period *t*

D_{tk}^0 Total demands at the plant from period *t* to period *k*

D_{tk}^r Total demands at retailer *r* from period *t* to period *k*

s_0^0 Initial inventory at the plant

s_0^r Initial inventory at retailer *r*

Decision variables

y_t^0 Production setup variable, equal to 1 if production occurs at the plant in period *t*, and 0 otherwise

y_t^r Transportation setup variable, equal to 1 if transportation occurs from the plant to the retailer *r* in period *t*, and 0 otherwise

s_t^0 Inventory at the plant at the end of period *t*

s_t^r Inventory at the retailer *r* at the end of period *t*

q_t^0 Quantity produced at the plant in period *t*

q_t^r Quantity delivered from the plant to retailer *r* in period *t*

The formulation of the problem BF is as follows:

$$\text{Min } \sum_{t=1}^{T}(FC_t^0 y_t^0 + PC_t^0 q_t^0 + HC_t^0 s_t^0) + \sum_{r=1}^{N}\sum_{t=1}^{T}(FC_t^r y_t^r + PC_t^r q_t^r + HC_t^r s_t^r) \quad (1)$$

$$s.t.\ s_{t-1}^0 + q_t^0 = \sum_{r=1}^{N} q_t^r + s_t^0 \qquad \forall t \in \{1, 2, \dots, T\} \quad (2)$$

$$s_{t-1}^r + q_t^r = d_t^r + s_t^r \qquad \forall r \in \{1, 2, \dots, N\}, \forall t \in \{1, 2, \dots, T\} \quad (3)$$

$$q_t^0 \leq D_{tT}^0 y_t^0 \qquad \forall \mathrm{t} \in \{1, 2, \dots, T\} \qquad (4)$$

$$q_t^r \leq D_{tT}^r y_t^r \qquad \forall \mathrm{r} \in \{1, 2, \dots, N\}, \forall \mathrm{t} \in \{1, 2, \dots, T\} \qquad (5)$$

$$y_t^0 \in \{0, 1\} \qquad \forall \mathrm{t} \in \{1, 2, \dots, T\} \qquad (6a)$$

$$y_t^r \in \{0, 1\} \qquad \forall \mathrm{r} \in \{1, 2, \dots, N\}, \forall \mathrm{t} \in \{1, 2, \dots, T\} \qquad (6b)$$

$$q_t^0, s_t^0 \geq 0 \qquad \forall \mathrm{t} \in \{1, 2, \dots, T\} \qquad (7a)$$

$$q_t^r, s_t^r \geq 0 \qquad \forall \mathrm{r} \in \{1, 2, \dots, N\}, \forall \mathrm{t} \in \{1, 2, \dots, T\} \qquad (7b)$$

The objective function (1) minimizes the total cost including the production setup costs, variable production costs, inventory holding costs at the plant, transportation setup costs, variable delivery costs and holding costs at the retailers over a finite time horizon. Constraints (2) and (3) are the inventory balance constraints for the plant and retailers, respectively. Constraints (4) and (5) set the binary variables to one if a production or delivery setup occurs. Constraints (6) and (7) are the binary and non-negativity constraints on the variables.

As we assume that all demands must be satisfied and no backorder is allowed in this problem, the total quantity of products manufactured and delivered is fixed. $\sum_{t=1}^{T} q_t^0$ is equal to the difference between the total demand and the initial stock (at the plant and at the retailers). Similarly, $\sum_{r=1}^{N} \sum_{t=1}^{T} q_t^r$ is equal to the difference between the total demand and the initial stock at the retailer. Furthermore, we assume that the unit production cost PC_{t}^0 and the unit transportation cost PC_t^r are time invariant. Thus, the total production and delivery costs are constant. To facilitate the computation, we exclude the two constant costs mentioned above from the objective function, and replace the objective function (1) with the function (8). The basic formulation is therefore as follows.

$$(BF) \ \mathrm{Min} \sum_{\mathrm{t}=1}^{\mathrm{T}} (FC_t^0 y_t^0 + HC_t^0 s_t^0) + \sum_{r=1}^{N} \sum_{t=1}^{T} (FC_t^r y_t^r + HC_t^r s_t^r) \qquad (8)$$

s.t. constraints (2) – (7)

3.2 Multicommodity Formulation (MC)

The multicommodity formulation recognizes each pair of retailer-period. It considers each demand D_t^r as a distinct product (Cunha and Melo, 2016). In this formulation, we define new continuous multicomodity variables. We also introduce the new parameters.

New parameters

δ_{kt} Kronecker delta, δ_{kt} equals to one if k = t, and zero otherwise

w_{0t}^{0r} Initial inventory in the production site to satisfy the demand of retailer *r* in period *t*

w_{0t}^{1r} Initial inventory at retailer r to satisfy its demand in period t

Decision variables

p_{kt}^{0r} Quantity produced in the production site in period *k* to satisfy the demand of retailer *r* in period *t*

p_{kt}^{1r} Quantity transported from the production site to retailer *r* in period *k* to satisfy the retailer's demand in period *t*

w_{kt}^{0r} Quantity stocked in the production site in period *k* to satisfy the demand of retailer *r* in period *t*

w_{kt}^{1r} Quantity stocked locally at retailer *r* in period *k* to satisfy its demand in period *t*

The formulation MC is as follows:

$$(MC)\ \text{Min} \sum_{k=1}^{T} FC_k^0 y_k^0 + \sum_{r=1}^{N}\sum_{k=1}^{T} FC_k^r y_k^r + \sum_{r=1}^{N}\sum_{k=1}^{T}\sum_{t=k}^{T} HC_k^0 w_{kt}^{0r} + \sum_{r=1}^{N}\sum_{k=1}^{T}\sum_{t=k}^{T} HC_k^r w_{kt}^{1r} \quad (9)$$

$$\text{s.t. } w_{\text{k}-1,t}^{0r} + p_{kt}^{0r} = w_{kt}^{0r} + p_{kt}^{1r} \quad \forall \text{r} \in \{1,2,\dots,N\}, \forall k \in \{1,2,\dots,T\}, \forall t \in \{k,k+1,\dots,T\} \quad (10)$$

$$w_{k-1,t}^{1r} + p_{kt}^{1r} = \delta_{kt} d_t^r + (1-\delta_{kt}) w_{kt}^{1r} \quad \forall \text{r} \in \{1,2,\dots,N\}, \forall k \in \{1,2,\dots,T\}, \forall t \in \{k,k+1,\dots,T\} \quad (11)$$

$$p_{kt}^{0r} \le D_t^r y_k^0 \quad \forall \text{r} \in \{1,2,\dots,N\}, \forall k \in \{1,2,\dots,T\}, \forall t \in \{k,k+1,\dots,T\} \quad (12)$$

$$p_{kt}^{1r} \le D_t^r y_k^r \quad \forall \text{r} \in \{1,2,\dots,N\}, \forall k \in \{1,2,\dots,T\}, \forall t \in \{k,k+1,\dots,T\} \quad (13)$$

$$y_k^0 \in \{0, 1\} \qquad \forall k \in \{1, 2, \dots, T\} \tag{14a}$$

$$y_k^{\mathrm{r}} \in \{0, 1\} \qquad \forall \mathrm{r} \in \{1, 2, \dots, N\}, \forall k \in \{1, 2, \dots, T\} \tag{14b}$$

$$w_{kt}^{0r}, p_{kt}^{0r} \geq 0 \qquad \forall \mathrm{r} \in \{1, 2, \dots, N\}, \forall k \in \{1, 2, \dots, T\}, \forall t \in \{k, k+1, \dots, T\} \tag{15a}$$

$$w_{kt}^{1r}, p_{kt}^{1r} \geq 0 \qquad \forall \mathrm{r} \in \{1, 2, \dots, N\}, \forall k \in \{1, 2, \dots, T\}, \forall t \in \{k, k+1, \dots, T\} \tag{15b}$$

The objective function (9) minimizes the production setup cost, the transportation setup cost, the holding cost at the warehouse and at the retailers, respectively. Constraints (10) are the balance constraints for each commodity at the plant. Constraints (11) ensure the inventory balance for each commodity at the retailers. Constraints (12) and (13) enforce the setups if an amount of product is produced or shipped. Constraints (14) and (15) are the binary constraints and non-negative constraints, respectively.

3.3 Basic Formulation with Emission Constraints (BF-E)

We follow the classical assumption of a fixed-plus-linear emission structure (Retel Helmrich *et al.*, 2015) to formulate the emission constraints. The constraints account for the carbon footprint related to the production setup, transportation setup, production quantities, delivery quantities and inventory level.

Similar to the cost function, we assume that (1) the demand is deterministic and (2) the production and transportation emission parameters are time invariant. Hence, the total emission related to the production and transportation quantities becomes constant. We remove the constant. The emission function comprises the emissions induced in production setups, transportation setups and inventory at the warehouse and at the retailers. We introduce a set of emission parameters, corresponding to the emission activities above.

FE_t^0 Production setup emission in period *t*

FE_t^r Transportation setup emission at retailer *r* in period *t*

HE_t^0 Emission to hold one unit of product at the plant in period *t*

HE_t^r Emission to hold one unit of product at the retailer *r* in period *t*

PE_t^0	Emission to produce one unit of commodity at the production site in period *t*
PE_t^r	Emission to deliver one unit of product from the production site to the retailer *r* in period *t*
EG^{max}	Global emission cap
ER^{max}	Rolling horizon emission cap
EP^{max}	Periodic emission cap
EC_t^{max}	Cumulative emission cap

The global emission constraint sets a cap on total emission of all facilities in all periods over the planning horizon. Structurally it resembles the objective function, which minimizes the total cost. As discussed above, we remove the constant production and transport emissions, i.e. $\sum_{t=1}^{T} PE_t^0 q_t^0 + \sum_{r=1}^{N} \sum_{t=1}^{T} PE_t^r q_t^r$. The basic formulation with global constraint (BF-G) is as follows:

Objective function (8)

s.t. constraints (2) – (7)

$$\sum_{t=1}^{T} \left(FE_t^0 y_t^0 + HE_t^0 s_t^0 \right) + \sum_{r=1}^{N} \sum_{t=1}^{T} \left(FE_t^r y_t^r + HE_t^r s_t^r \right) \leq EG^{max} \tag{16}$$

The first part in constraint (16) represents the total emission incurred in the plant, and the second part is emission incurred at the retailers. The total emission of the distribution system cannot exceed the global cap.

The rolling horizon constraints limit the total emission of any *u* consecutive periods over the planning horizon. For instance, the total emission of periods 1-5, 2-6, 3-7, … 11-15, for *u* = 5 and *T* = 15. The basic formulation with rolling horizon emission constraint (BF-R) can be expressed as follows:

Objective function (8)

s.t. constraints (2) – (7)

$$\sum_{l=t-u+1}^{t} \left(FE_l^0 y_l^0 + HE_l^0 s_l^0\right) + \sum_{r=1}^{N} \sum_{l=t-u+1}^{t} \left(FE_l^r y_l^r + HE_l^r s_l^r\right) \leq ER^{max} \quad \forall t \in \{u, u+1, \dots, T\} \quad (17)$$

Constraints (17) deal with the total emission in two echelons in any consecutive *u* periods. Note that we can consider the global and periodic constraints as variations of the rolling horizon constraint. When the rolling horizon consists of one period, the rolling horizon constraints are equal to the periodic constraints; when the length of the rolling horizon *u* is equal to *T*, the rolling horizon constraint becomes equal to the global emission constraint.

Alternatively, we can formulate the periodic constraint (BF-P) as follows:

$$\left(FE_t^0 y_t^0 + HE_t^0 s_t^0\right) + \sum_{r=1}^{N} \left(FE_t^r y_t^r + HE_t^r s_t^r\right) \leq EP^{max} \qquad \forall t \in \{1, 2, \dots, T\} \qquad (18)$$

The constraints (18) limit the setup and holding emissions at the warehouse and the retailers by the periodic cap. The cumulative emission constraints distinguish from the other three types. As the name suggests, each constraint cumulates the total emission from period 1 to period *t*. Instead of one static right-hand-side value, the cumulative constraints require an incremental set of *T* caps. We express the basic formulation with cumulative emission constraints (BF-C) as follows:

$$\sum_{l=1}^{t} \left(FE_l^0 y_l^0 + HE_l^0 s_l^0\right) + \sum_{r=1}^{N} \sum_{l=1}^{t} \left(FE_l^r y_l^r + HE_l^r s_l^r\right) \leq EC_t^{max} \qquad \forall t \in \{1, 2, \dots, T\} \qquad (19)$$

Constraints (19) ensure that the sum of setup and holding emissions from period 1 to any period *t* does not exceed the cumulative cap EC_t^{max}.

3.4 Multicommodity Formulation with Emission Constraints (MC-E)

The emission constraints for the multicommodity formulation have the same structure as BF-E. We replace the inventory variables *s* in BF with the multicommodity variables *w*. The multicommodity formulation with global emission constraints (MC-G) is as follows.

Objective function (9)

s.t. (10) – (15)

$$\sum_{k=1}^{T} FE_k^0 y_k^0 + \sum_{r=1}^{N}\sum_{k=1}^{T}\sum_{t=k}^{T} HE_k^0 w_{kt}^{0r} + \sum_{r=1}^{N}\sum_{k=1}^{T} FE_k^r y_k^r + \sum_{r=1}^{N}\sum_{k=1}^{T}\sum_{t=k}^{T} HE_k^r w_{kt}^{1r} \leq EG^{max} \qquad (20)$$

Constraints (20) limit the production setup emissions, the holding emissions at the plant, the transportation emissions and the storage emissions at the retailers over the total time horizon of the problem. The rolling horizon constraints limiting the total emission of any *u* consecutive periods over the planning horizon, are formulated as follows.

$$\sum_{l=k-u+1}^{k} FE_l^0 y_l^0 + \sum_{r=1}^{N}\sum_{l=k-u+1}^{k}\sum_{t=l}^{T} HE_l^0 w_{lt}^{0r} + \sum_{r=1}^{N}\sum_{l=k-u+1}^{k} FE_l^r y_l^r + \sum_{r=1}^{N}\sum_{l=k-u+1}^{k}\sum_{t=l}^{T} HE_l^r w_{lt}^{1r} \leq ER^{max}$$

$$\forall k \in \{u, u+1, \dots, T\} \qquad (21)$$

Constraints (21) ensure that the total setup and holding emissions of any *u* periods do not exceed the rolling horizon cap. Similarly, the periodic emission constraints are expressed as follows.

$$FE_k^0 y_k^0 + \sum_{r=1}^{N}\sum_{t=k}^{T} HE_k^0 w_{kt}^{0r} + \sum_{r=1}^{N} FE_k^r y_k^r + \sum_{r=1}^{N}\sum_{t=k}^{T} HE_k^r w_{kt}^{1r} \leq EP^{max} \qquad \forall k \in \{1, 2, \dots, T\} \quad (22)$$

Constraints (22) set a cap on the emissions of any period k over the planning horizon. The cumulative constraints (23) limit the cumulative emissions from period 1 to any period *k* at the warehouse and the retailers.

$$\sum_{l=1}^{k} FE_l^0 y_l^0 + \sum_{r=1}^{N}\sum_{l=1}^{k}\sum_{t=l}^{T} HE_l^0 w_{lt}^{0r} + \sum_{r=1}^{N}\sum_{l=1}^{k} FE_l^r y_l^r + \sum_{r=1}^{N}\sum_{l=1}^{k}\sum_{t=l}^{T} HE_l^r w_{lt}^{1r} \leq EC_k^{max} \; \forall k \in \{1, 2, \dots, T\} \qquad (23)$$

We introduce four types of emission constraints with different time horizon. The use of the constraints is not limited to emissions. They can be generalized to any other capacity or cost limitations over certain time horizons. For instance, the constraints can apply to the energy consumption in the setups and stocks over rolling horizon in a company.

Chapter 4
Computational Experiments on OWMR

In this section, we report the results of the computational experiments. We validate the standard formulations of BF and MC in the experiments. The basic formulation serves as a comparison baseline, whereas the MC formulation represents the strongest formulation for the OWMR problem according to Cunha and Melo (2016) and Gruson *et al.* (2019). All formulations are coded in Python and solved using CPLEX 12.9.0.0 on an Intel(R) Core i7-7800X 3.5GHz machine with 128GB. The optimality tolerance is set to 10^{-9} and MIP relative gap tolerance ((best bound – best integer) / (1^{-10} + best integer) to 0.0 (IBM, 2019). The other parameters remain as default. Each instance has a time limit of 7200 seconds.

4.1 Computational Environment Settings and Instances

We first perform experiments on the basic OWMR to validate the basic code and to check the performances of the two formulations. We carry out the computational experiments with the instances provided in Solyali and Süral (2012). The instances consider a distribution system of 50, 100 and 150 retailers, and a planning horizon of 15 and 30 periods. Therefore, we have six combinations of retailer-horizon. For each combination, we have four groups of instances considering static or dynamic demand and cost patterns. We run two separate experiments with and without initial inventory. A standard single-item dynamic lot-sizing problem often assumes zero initial inventory at all facilities. However, that is rarely the case in reality. To adapt to the real business setting and to compare the performance of the formulations, we run the same instances with and without initial inventory at the warehouse. Each group consists of 10 instances. We solve 480 instances in total.

The demand and cost consider two patterns: time variant and time invariant. The demand of retailers is randomly generated within the range of [5, 100]. In the scenario without initial inventory, we set the initial stock to zero in all the instances. In the experiments with initial inventory, we set the warehouse's initial inventory to $\sum_{r=1}^{N} D_1^r$, i.e. the aggregated demands of all retailers in the first period if the demands are time invariant. If the demands are dynamic, the warehouse estimates the required initial inventory to be $\frac{100+5}{2} \cdot N$. In terms of costs, the fixed cost at the warehouse is set in the range of [1500, 4500]; the setup cost at the retailer has the value of [5, 100]. Note that all the parameters mentioned above are integral. The holding costs, on the other hand, are decimal and time invariant. The holding cost at the warehouse is fixed at 0.5 per unit per period, and the cost to hold a product at the retailers ranges from 0.5 to 1.0. Table 3 shows the details of the parameters.

Table 3. Characteristics of the Basic Parameters

Parameter	Value	Nature of data
Number of retailers	{15, 30, 45}	
Number of periods	{15, 30}	
Demand at retailers	U[5, 100]	static & dynamic
Initial inventory	$\{\sum_{r=1}^{N} D_1^r, \frac{100+5}{2} \times N, 0\}$	
Fixed cost at warehouse	U[1500, 4500]	static & dynamic
Fixed cost at retailer	U[5, 100]	dynamic
Holding cost at warehouse	0.5	static
Holding cost at retailers	U[0.5, 1.0]	static

4.2 Computational Results of OWMR

We compare the two formulations based on the following criteria: (1) CPU time to solve the MILP problem, (2) CPU time to solve the LP relaxation, and (3) the gaps between the MILP and LP objective value. Table 4 presents the results of both formulations with and without initial inventory, denoted by II > 0 and II = 0, respectively. To be specific, the column MILP time refers to the average elapsed time (in seconds) required to solve the problem. The column LP time shows the mean time to solve the formulation using continuous variables instead of the Boolean setup decisions. The column LP gap% reflects the average LP relaxation gap, i.e. gap = (MIP solution - LP solution)/MIP solution. The column optimal% describes the proportion of instances that are solved to optimality, i.e. with a relative MIP gap of 0.0. The last column compares the MIP CPU time of the formulations (BF MIP time/MC MIP time). Each result is the average of 40 instances.

Table 4. Computational Results of BF and MC

II = 0		BF				MC				BF/MC
N	T	MIP time	LP time	LP gap%	optimal%	MIP time	LP time	LP gap%	optimal%	MIP time
50	15	13.6	0.001	67.9	100.0	0.4	0.6	0.004	100.0	34.5
100	15	133.7	0.006	71.0	100.0	1.1	1.7	0.018	100.0	118.5
150	15	304.8	0.010	72.2	100.0	2.0	3.1	0.015	100.0	152.6
50	30	1702.6	0.008	80.3	87.5	2.5	6.2	0.019	100.0	675.8
100	30	-	-	-	-	8.4	19.2	0.025	100.0	-
150	30	-	-	-	-	16.0	33.5	0.020	100.0	-
Average		**-**	**-**	**-**	**-**	**5.1**	**10.7**	**0.017**	**100.0**	**-**

II > 0		BF				MC				BF/MC
N	T	MIP time	LP time	gap%	optimal%	MIP time	LP time	gap%	optimal%	MIP time
50	15	14.0	0.016	67.9	100.0	0.9	0.7	1.627	100.0	15.7
100	15	188.4	0.011	70.9	100.0	2.9	1.9	1.104	100.0	64.6
150	15	354.7	0.010	72.2	100.0	5.3	3.3	1.007	100.0	67.4
50	30	1713.5	0.009	80.3	90.0	5.8	5.9	0.888	100.0	296.3
100	30	-	-	-	-	29.7	35.4	0.653	100.0	-
150	30	-	-	-	-	47.9	63.6	0.572	100.0	-
Average		**-**	**-**	**-**	**-**	**15.4**	**18.5**	**0.975**	**100.0**	**-**

Our first observation aligns with the conclusions of Cunha and Melo (2016) and Gruson *et al.* (2019): the multicommodity formulation performs better than the basic formulation. BF requires substantially higher solution time. On the smallest instance set, i.e. the 50-retailer-15-period instances, BF spends 34.5 times more time to solve the MIP problem with zero initial inventory and 15.7 times more time on the MIP problem with inventory. Note that the difference of the solution time becomes larger as the instances grow larger. In addition, BF cannot solve most of the larger instances (100 or 150 retailers, 30 periods) within the 7,200 seconds' time limit. Furthermore, MC formulation provides better linear relaxation bounds. It is noteworthy that MC provides very good bounds: the mean LP gap is below 0.02% in the non-initial-inventory scenario and below 1% with initial stock. The multicommodity formulation provides a better performance with respect to the solution time and linear relaxation bounds.

The mean LP gaps obtained in the experiments are slightly smaller than the results reported in Solyali and Süral (2012), Zhong (2014) and Cunha and Melo (2016). The difference stems from the MIP gap tolerance setting of the solver. Instead of CPLEX' default value of 1^{-4}, we set the relative MIP gap tolerance to 0.0. In the computational experiments conducted, MC solves all the instances to optimality (relative MIP gap = 0.01. BF solves all the instances with 15 retailers to optimality. For the instances with 50 retailers and 15 periods, BF cannot solve 4 instances with initial inventory and 5 instances with initial inventory to optimality, leading to an average optimality gap of 4.5% and 0.3%, respectively. Hence, MC also has a better performance with respect to MIP solution quality.

The number of retailers and periods have a great impact on the solution time. Given the instances over the same horizon (T=15), the solution time of MC increases considerably by 1.87 and BF by 8.82 as the number of periods doubles. Given the same number of retailers, MC spends at least 5.39 time more CPU time to solve the instances of 30 periods than 15 periods, and BF 125 times more. Based on the comparison above, we have the following observations: (1) both the number of retailers and number of periods lead to a substantial increase in CPU time. The length of the planning horizon has greater impact on the solution time than the number of retailers for both formulations; (2) BF is more sensitive to the magnitude of the problem.

In addition, we compare the two scenarios, with and without initial inventory. Both formulations use more computational time with the initial inventory. BF spends 69% more time and MC 126% to solve the instances with initial inventory than without initial inventory.

In brief, the multicommodity formulation has a better performance than the basic formulation in the numeric experiments we conducted. It obtains stronger linear relaxation bounds and solves the MIP with much less computational time.

Chapter 5
Computational Experiments on OWMR-E: Comparison of Four Types of Emission Constraints

In this chapter, we introduce the emission parameters, particularly the method to determine the equivalent emission cap. Then we report the computational results of OWMR-E and compare the impact of the four types of emission constraints.

5.1 Emission Parameters

To solve the formulations with emission constraints, we introduce a new set of emission parameters for the instances generated by Solyali and Süral (2012). The emission parameters comprise the setup emissions, the holding emissions and emission caps. First, we generate the setup emissions and holding emissions applying the method in Zhong (2014). We assume that the emissions are correlated with the costs. The correlation is conceptually quantified as a 50% positive or negative deviation. Thus, an emission parameter can be generated using uniform distribution within the range of 50% - 150% of the corresponding cost parameter. For instance, given the transportation setup cost at a retailer *r* of 100$, a fixed amount of [50, 150] units of CO_2 are emitted.

Second, we determine the five types of emission caps for each instance, namely a global cap, a 10-period rolling horizon cap, a 5-period rolling horizon cap, a periodic cap and a string of T cumulative caps. We generate the caps in two steps. The first step ensures the feasibility of the instances, and the second creates connections between different types of emission constraints, which allow us to make fair comparison. The infeasibility occurs when an emission cap is too tight. It is most probable to occur with the periodic constraints since they are the strictest. Therefore, in the first step, we determine the minimum periodic cap level that guarantees the feasibility for the problem with periodic constraints. To find this minimum periodic cap level, we use a model that contains all the constraints of the OWMR problem, to make sure that all demand is satisfied. Besides, we define a new variable z, which is greater than the total emission in each period. We minimize z so that we find exactly the highest value of the total emissions in each period. Using this value as our emission cap guarantees a feasible solution for the problem with periodic constraints.

$$\text{Min } z \tag{24}$$

s.t. constraints (2) - (7)

$$z \geq (FE_t^0 y_t^0 + HE_t^0 s_t^0) + \sum_{r=1}^{N} (FE_t^r y_t^r + HE_t^r s_t^r) \qquad \forall t \in \{1, 2, \ldots, T\} \tag{25}$$

$$z \geq 0 \tag{26}$$

Objective function (24) minimizes the maximum amount of emissions permitted per period. Constraints (25) ensure that the emission level of each period is lower than the cap. Constraint (26) ensures the non-negativity. We solve the model above using all the instances without time limit in order to obtain the optimal solutions. We can set EP^{max} equal to the optimal $z*$.

In the second step, we calculate the global, rolling horizon and cumulative caps based on the calculated minimum level for the periodic cap. Under the periodic policy, a firm can emit a maximum amount of z^* units of CO_2 each period. Thereby, the firm's total emission equals to or is less than $T \cdot z^*$ over T periods, and $u \cdot z^*$ over u periods of time. Similarly, at the end of each period k, the firm cumulates a maximum emission amount of $k \cdot z^*$. Hence, we set the emission caps as follows:

Periodic emission cap	$EP^{max} = z^*$
Global emission cap	$EG^{max} = T \cdot z^*$
Rolling horizon emission cap	$ER^{max} = u \cdot z^*$
Cumulative emission cap	$[z^*, 2z^*, 3z^*, \ldots, T \cdot z^*]$

We set the length of rolling horizon to 5 and 10 periods, respectively. Using the two steps above, we incorporate the tailored emission cap parameters to each instance. We ensure the feasibility of the OWMR-E problem with each type of emission constraints. Knowing that the periodic emission constraints are binding, we further investigate the financial impact of the other three types of emission constraints and compare their performance.

5.2 Computational Results of OWMR-E

We examine the two formulations for the OWMR problem with four types of emission constraints on each of the 480 instances (240 with initial inventory and 240 without initial inventory). The environment of the computational experiments is identical to the experiments on OWMR as described in Chapter 4. Note that we discuss the performance of OWMR-E problem with initial inventory in this section. The results of the experiments without initial inventory can be found in the appendix.

An OWMR-E problem contains from 1 to 30 more constraints than the OWMR problem. Thus, intuitively the OWMR-E problem may require higher CPU time. Table 5 compares the mean CPU time to solve the OWMR problem and the OWMR-E problem ((CPU time of OWMR-E – CPU time of OWMR)/CPU time of OWMR) with two formulations using the same instances. The CPU time increase significantly with the emission constraints. The CPU time of MC-E is 0.43 times to 3290 times higher than MC; the solution time of BF-E is 0.4% to 209 times higher than the BF. Note that BF-E has a less significant increase in CPU time because it only solves the smaller instances. The solution time increase is determined by the nature of the formulation, the scale of the problem (*N* and *T*), and most importantly the type of emission constraints.

Table 5. Comparison of CPU Time between OWMR and OWMR-E

		BF-E/BF%					MC-E/MC%				
N	**T**	**Global**	**Rolling 10**	**Rolling 5**	**Cumulative**	**Periodic**	**Global**	**Rolling 10**	**Rolling 5**	**Cumulative**	**Periodic**
50	15	1.4	4.4	6.8	1.5	20913.6	79.0	104.7	271.4	732.5	328980.3
100	15	1.9	106.1	143.7	0.8	-	132.9	441.4	9982.3	2971.1	-
150	15	66.1	49.1	123.3	9.0	-	159.0	768.2	9701.0	7820.1	-
50	30	0.2	0.4	0.4	0.1	-	110.8	224.4	3883.4	600.7	-
100	30	-	-	-	-	-	132.0	1667.7	8806.3	3299.7	-
150	30	-	-	-	-	-	43.3	2824.0	7446.0	4001.5	-
average%		-	-	-	-	-	**109.5**	**1005.1**	**6681.7**	**3237.6**	-

Table 6 reports the results of MC-E and BF-E using four types of constraints. MC-E shows better performance with the global and the rolling horizon constraints, whereas BF-E performs better with periodic constraints. MC-E is better than BF-E on smaller instances but gives poorer results on instances of 150 retailers with cumulative constraints. Both formulations cannot solve the larger instances with periodic constraints within 7200 seconds. BF-E cannot solve the instances of 100 and 150 retailers and 30 periods within the time limit. Comparing the results of the 15-period instances, the CPU time of BF-G is 35 times higher than MC-G; the CPU time of BF-R10 is 14 times higher than MC-R10; BF-R5 is 1.6 times of MC-R5; and BF-C is 1.2 times of MC-C. BF-P, on the other hand, requires 40% less solution time than MC-P, corresponding to 1,185 seconds' extra solution time on each instance. In terms of solution quality, MC-E performs better with global, rolling horizon and cumulative constrains. Comparing the results for all the instances with 15 periods and the instances with 50 retailers and 15 periods, MC-E solves all but 5 instances to optimality with an average optimality gap of 0.01%. BF-E has 15 suboptimal solutions; the average optimality gap of these instances is 0.03%. BF-E provides stronger LP bounds with periodic constraints. It solves 90% of the instances with periodic constraints to optimality. The non-optimal instances have a mean optimality gap of 0.077%. MC-E only gets optimal solutions with 72.5% of the instances using periodic emission constraints, leading to the mean optimality gap of 0.085%.

Comparing the two formulations, we observe that BF is less sensitive to emission constraints. Given the smallest instances in Table 5, the global constraints cause a 1.4% increase of CPU time with BF, however 79% with MC. The periodic constraints increase the CPU time dramatically by 209 times with BF and 329 times with MC compared with the original formulation without emission constraints.

Similar to the formulations without emission constraints, the scale of the problem has great impact on the solution time. The solution time can increase up to approximately 25 times as the number of the retailer doubles, and 70 times as the time scale doubles. The length of the rolling horizon plays an equally important role. A shorter rolling horizon of 5 periods makes the emission constraints much tighter. The mean solution time is approximately quadruple of the constraints compared with the 10-period-rolling-horizon.

Table 6. Results of BF-E and MC-E Using Equivalent Emission Caps

MC-E		global			rolling u =10			rolling u=5			cumulative			periodic		
N	T	CPU time	Optimal%	Gap%	CPU time	Optimal%	Gap%	CPU time	Optimal%	Gap%	CPU time	Optimal%	Gap%	CPU time	Optimal%	Gap%
50	15	1.6	100.0	0.0	1.8	100.0	0.00	3.3	100.0	0.00	7.4	100.0	0.000	2934.9	72.5	0.09
100	15	6.8	100.0	0.0	15.8	100.0	0.00	294.1	100.0	0.00	89.6	100.0	0.000	-	-	-
150	15	13.6	100.0	0.0	45.7	97.5	0.01	515.6	92.5	0.02	416.7	97.5	0.008	-	-	-
50	30	12.2	100.0	0.0	18.8	100.0	0.00	230.4	100.0	0.00	40.5	100.0	0.000	-	-	-
100	30	68.9	100.0	0.0	525.1	95.0	0.00	2645.7	77.5	0.01	1009.9	97.5	0.02	-	-	-
150	30	68.7	100.0	0.0	1401.5	87.5	0.04	3616.8	60.0	0.02	1965.9	85.0	0.01	-	-	-
average		**28.6**	**100.0**	**0.0**	**334.8**	**96.7**	**0.01**	**1217.6**	**88.3**	**0.01**	**588.3**	**96.7**	**0.01**	-	-	-

BF-E		global			rolling u =10			rolling u=5			cumulative			periodic		
N	T	CPU time	Optimal%	Gap%	CPU time	Optimal%	Gap%	CPU time	Optimal%	Gap%	CPU time	Optimal%	Gap%	CPU time	Optimal%	Gap%
50	15	14.2	100.0	0.00	14.6	100.0	0.00	14.9	100.0	0.00	14.2	100.0	0.000	1749.1	90.0	0.08
100	15	192.1	100.0	0.00	388.4	97.5	0.01	459.1	97.5	0.06	189.9	100.0	0.000	-	-	-
150	15	589.1	100.0	0.00	528.8	97.5	0.04	792.1	95.0	0.04	386.7	100.0	0.000	-	-	-
50	30	2023.3	92.5	0.03	2346.0	90.0	0.04	2352.3	95.0	0.05	1970.5	97.5	0.015	-	-	-
100	30	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
150	30	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
average		-	-	-	-	-	-	-	-	-	-	-	-	-	-	-

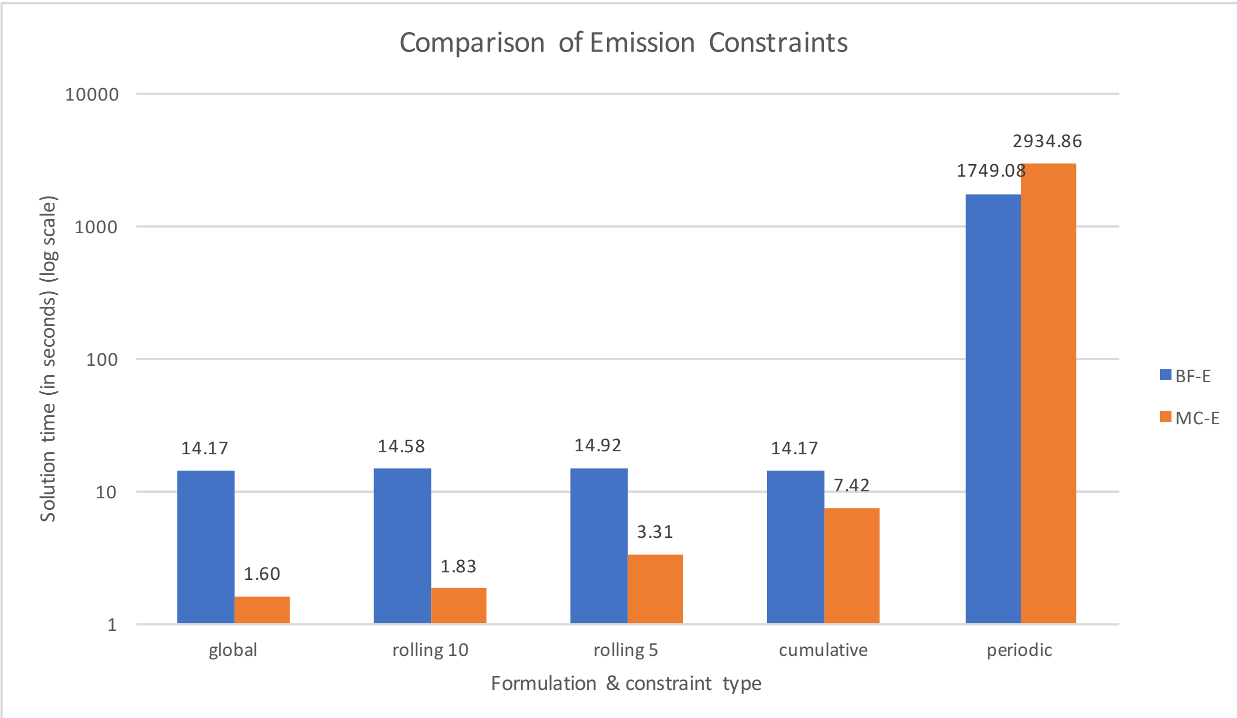

Figure 3. Comparison of CPU Time between Different Types of Emission Constraints (40 instances, N=50, T=15)

Among all types of emission constraints, the global constraints require the least solution time. This can be partly explained by the number of constraints. It is also due to how restrictive these constraints are. There is only one emission constraint in the model with global emission constraint, T-u+1 constraints in the scenario of rolling horizon, and T constraints in the cases of cumulative and periodic restrictions. The global emission constraint is the least restrictive. According to the experiments, the solution time of the emission constraints may rank as follows: global < rolling horizon (10 periods) < rolling horizon (5 periods) < cumulative and periodic. On the other hand, the caps have a great impact on the solution time. The periodic constraints are binding with the minimum periodic caps. Nevertheless, the other types of caps may not be tight in the experiments we conduct. We further investigate the impact of emission caps in next chapter.

Figure 3 shows the average CPU time of both formulations with different types of emission constraints using the instances of 50 retailers and 15 periods. The CPU time of MC-E rises significantly as the number of constraints increases. Taking the CPU time of MC-G as benchmark, the solution time increase by 0.14, 1.06, 3,78 and 1938 times in the experiments with MC-R10, MC-R5, MC-C and MC-P, respectively. BF-E, on the other hand, shows a more gradual CPU increase. The difference of solution time is smaller than 3% using BF-G, BF-R and BF-C. However, the mean solution time of BF-C including larger instances is in line with our general observation that BF-C has a larger CPU time than BF-G. average BF-C solution time resides between the elapsed time of BF-R5 and BF-P.

Certain errors have occurred in the experiments and adjustments have been taken. In the experiment of MC-P, CPLEX stops due to "out of memory" error on two instances. We modify CPLEX' working memory parameter and node file storage parameter to prevent the error, and re-solve the problem with the two instances. The parameter may increase the solution time, however, both instances hit the 7200-second time limit with and without the same parameter, yielding the same optimality gap. Therefore, we keep the results. Another error is the "no solution exists" due to rounding issues. We relax the caps by 0.01 and re-conduct the experiment on 6 instances having the error. The adjustments generate a difference smaller than 0.0003% on the periodic emission cap and 0.004% on the global emission cap, which we consider tolerable.

To summarize, we have compared the performance of (1) OWMR with and without emission constraints, (2) the two formulations BF-E and MC-E, (3) the four types of emission constraints. The results indicate that OWMR-E takes more solution time than OWMR. MC-E demonstrates better performance than BF-E in general. However, as there are more emission capacity constraints, MC-E provides a poorer performance. To be specific, the best formulations are MC-G, MC-R5, MC-R10 and BF-P. MC-C performs better on instances of 50 and 100 retailers and BF-C is a better option for instances of 150 retailers. The CPU time of the four types of emission constraints ranks as follows: periodic > cumulative > 5-period rolling horizon > 10-period rolling horizon > global constraints.

5.3 Comparison of Four Types of Emission Constraints

In the previous section, we have compared the performance of the four types of emission constraints. In this section, we analyze the financial and environmental impacts of the four types of emission constraints.

We first compare the financial impact of each type of constraint. For each instance, we compute the minimum total cost (min TC) by solving a standard OWMR problem without emission constraints. We then solve the OWMR-E problems for the total cost with each type of emission constraints (TC-E). If the instance is solved to optimality and TC-E is equal to TC, it suggests that the emission constraints have no impact on the costs. In order to have a fair comparison, we only consider the instances that are solved to optimality with all types of emission constraints. Table 7 depicts the proportion of instances that have binding emission constraints using MC-E.

Table 7. Proportion of Binding Emission Constraints Using Equivalent Emission Caps

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		% Binding				
N	**T**	**Global**	**Rolling 10**	**Rolling 5**	**Cumulative**	**Periodic**
50	15	0.0	2.5	22.5	90.0	100.0
100	15	2.5	25.0	87.5	100.0	-
150	15	0.0	43.6	91.9	100.0	-
50	30	0.0	10.0	37.5	92.5	-
100	30	3.2	36.8	77.4	100.0	-
150	30	0.0	34.3	79.2	100.0	-
		1.0	**25.4**	**66.0**	**97.1**	**-**

All the periodic emission constraints have an impact on total cost. 97% of the cumulative constraints are binding, and the proportion grows to 100% in larger instance groups. 66% of the 5-period rolling horizon constraints have an impact on costs, however, only 25% of the 10-period rolling horizon constraints and 0.95% global constraints are binding. Recall that the MC-P deals with the minimum emission level for the periodic cap, whereas the caps of the other emission constraints may not be tight.

Table 8. Financial Impact of Four Types of Emission Constraints – Equivalent Emission Caps

		% Cost increase				
N	**T**	**Global**	**Rolling 10**	**Rolling 5**	**Cumulative**	**Periodic**
50	15	0.000	1.22	1.2	6.90	30.8
100	15	0.001	0.76	1.9	6.64	-
150	15	0.000	0.96	1.2	0.04	-
50	30	0.000	0.04	0.7	2.84	-
100	30	0.676	0.44	1.4	3.50	-
150	30	0.000	0.64	1.0	2.09	-
		0.113	**0.68**	**1.2**	**3.67**	**-**

Knowing the proportion of instances with binding emission constraints, we evaluate their financial impact explicitly. We calculate the average cost increase rate (TC-E – TC)/TC of each instance group using each type of emission constraints. The global emission constraints lead to a very small increase of 0.113%. The rolling horizon also result in a small impact. Despite the fact that 66% of the 5-period rolling horizon constraints cause extra costs, the costs only increase by 1.23% on average. Similarly, 97% cumulative constraints have financial impact, but the average cost increase is less than 5%. Notably, the periodic constraints cause a significant cost increase of 31%.

Based on the analysis, we have the following observations. **Observation 1**: The different types of emission constraints can have very different impacts on costs. Two different types of constraints can incur up to 31% difference in total cost. **Observation 2**: the periodic emission constraints have greatest impact on costs. On the contrary, the global constraints show little influence on total cost. **Observation 3**: the financial impact of rolling horizon constraints is associated with the length of the rolling horizon. The shorter the rolling horizon is, the higher costs the constraints induce.

Chapter 6
Analysis of Emission Trade-offs

In this chapter, we explore the trade-offs between the total cost and the emission of each of the four types of emission constraints separately. The global, rolling horizon and cumulative emission constraints in Section 5.1 do not necessarily have any impact on the total cost because the caps may not be binding. Therefore, in order to analyze the trade-offs, we need to establish again the minimum emission level for each type of the emission caps. We introduce the optimization models to determine the minimum emission level for each type of the emission constraints.

6.1 Analysis of Cost-emission Trade-offs

It is easy to determine the minimum emission level for the global caps. We can simply minimize the total emission without any cost constraint. However, it becomes more complicated to determine the minimum emission level for the other types of caps. As explained in Section 5.1, to determine the minimum periodic cap level, we define a new variable z, which is larger than the total emission in any period t. Let's define E_t as the total emission in period t. We then minimize z so that we can find the highest value of the total emission in each period. Using this value as the periodic emission cap, we can guarantee a feasible solution for the OWMR-P problem. To determine the minimum emission level for the global and rolling horizon constraints, we now define the variable z as follows:

Global cap $\quad z \geq E_1 + E_2 + E_3 + \dots + E_T$

Rolling horizon cap (u=10) $\quad z \geq E_1 + E_2 + E_3 + \dots + E_{10}$

$$z \geq E_2 + E_3 + E_4 + \cdots + E_{11}$$

$$\dots$$

$$z \geq E_{T-9} + E_{T-8} + E_{T-7} + \dots + E_T$$

Rolling horizon cap (u=5) $z \geq E_1 + E_2 + E_3 + E_4 + E_5$

$$z \geq E_2 + E_3 + E_4 + E_5 + E_6$$

$$\ldots$$

$$z \geq E_{T-4} + E_{T-3} + E_{T-2} + E_{T-1} + E_T$$

To determine the minimum rolling horizon emission level, we first construct a model which comprises all the constraints in the standard OWMR problem, i.e. constraints (2) – (7). This step guarantees that all demand is satisfied. We then set the variable *z* larger than the total emission of any *u* consecutive periods, as shown in constraints (27). Constraint (26) guarantees the non-negativity of *z*. The objective function (24) minimizes *z*, so that we can find the highest value of the total emission during any consecutive *u* periods.

Objective function (24)

s.t. constraints (2) – (7)

$$z \geq \sum_{l=t-u+1}^{t} (FE_l^0 y_l^0 + HE_l^0 s_l^0) + \sum_{r=1}^{N} \sum_{l=t-u+1}^{t} (FE_l^r y_l^r + HE_l^r s_l^r) \qquad \forall t \in \{u, u+1, \ldots T\} \quad (27)$$

constraint (26)

As discussed, we can consider the periodic emission constraints and global emission constraints as variations of the rolling horizon constraints. When *u* equals to 1, constraints (27) are equal to periodic emission constraints; when *u* equals to *T*, constraint (27) becomes equal to the global constraint. We solve the model using all the instances with the value of *u* to be 5, 10, T, respectively. Thus, we capture the tightest rolling horizon and global emission caps for each instance.

However, this method cannot be applied to the cumulative emission caps. The cumulative constraints require an incremental set of *T* different caps instead of one fixed right-hand-side constant for multiple constraints. Each cap corresponds to a specific period. Table 9 lists the emission constraints for a OWMR-E problem with 10 periods as an example. In the example, the global emission cap is 10, periodic cap is 1, and rolling horizon cap is 5. On the other hand, the cumulative caps are the tuple (1, 2, 3, …, 10).

Table 9. Example: Different Types of Emission Caps

Global	Rolling horizon 5	Periodic	Cumulative
$E_1 + E_2 + E_3 + \cdots + E_{10} \leq 10$	$E_1 + E_2 + E_3 + E_4 + E_5 \leq 5$	$E_1 \leq 1$	$E_1 \leq \mathbf{1}$
	$E_2 + E_3 + E_4 + E_5 + E_6 \leq 5$	$E_2 \leq 1$	$E_1 + E_2 \leq \mathbf{2}$
	$E_3 + E_4 + E_5 + E_6 + E_7 \leq 5$	$E_3 \leq 1$	$E_1 + E_2 + E_3 \leq \mathbf{3}$
	…	…	…
	$E_6 + E_7 + E_8 + E_9 + E_{10} \leq 5$	$E_{10} \leq 1$	$E_1 + E_2 + E_3 + \cdots + E_{10} \leq \mathbf{10}$

Recall that the cumulative caps must satisfy the following conditions in order to lead to a feasible instance:

$$cap\ 1 \geq E_1$$

$$cap\ 2 \geq E_1 + E_2$$

$$cap\ 3 \geq E_1 + E_2 + E_3$$

…

$$cap\ T \ \geq E_1 + E_2 + E_3 + \cdots + E_T$$

To generate the set of cumulative caps, we construct a sequence of *T* models and solve them iteratively. Each model provides the minimum level of a cumulative cap in a specific period. The model is based on the decisions made in the previous round. Figure 4 illustrates the dynamics of the problem. Model 1 seeks the minimum amount the company can emit in period 1. By solving the model, we obtain the minimum emission cap in period 1 and a set of decisions made, such as whether a production setup occurs in period 1. These decisions are fixed in model 2, which continues to search for the minimum cumulative emission allowed for period 1 and 2. Likewise, we solve model 2, and then fix its outputs in model 3. The process iterates until model *T*. Thus, we ensure the connection of the models and the coherence of the decisions. At the end of the experiments, we obtain the minimum emission level of all the *T* cumulative caps.

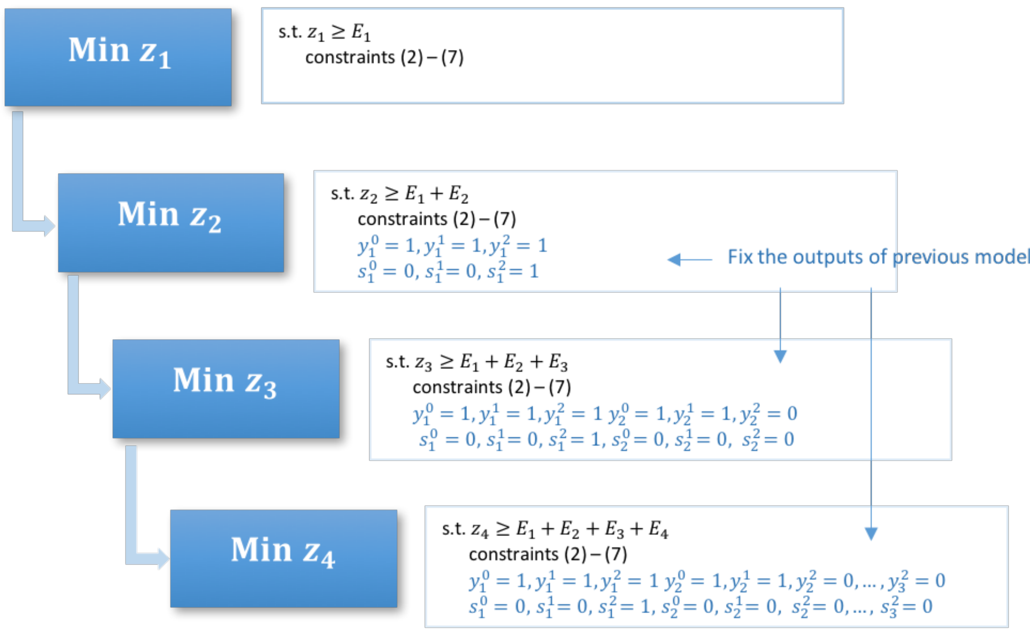

Figure 4. Process to Determine the Minimum Cumulative Emission Level

We then re-solve OWMR-E with new sets of emission caps to explore the cost-emission trade-offs. For the sake of efficiency, we select the best forming formulations in Section 3.2.2, namely MC-G, MC-R, MC-C, BF-P. Moreover, we focus on the smaller instance group of 50 retailers and 15 periods, since most of these instances can be solved within 7200 seconds. In the experiments, we first set the emission caps to their minimum level, and then gradually increase the emission caps by 5% in each experiment, thereby resolving the problems. Thus, the right-hand-side constants of the emission constraints are in the range of [minimum cap level, 200%·minimum cap level]. We run 21 experiments on each problem (MC-G, MC-R5, MC-R10, MC-C, BF-P). In the experiments of MC-G, MC-R and MC-C, we solve all the instances to optimality. In the experiments of BF-P, half of the instances cannot be solved to optimality, particularly when the emission caps are 5% - 15% higher than their minimum level. 4.3% of the solutions are suboptimal and these solutions lead to an average optimality gap of 0.04%. Since the gap is insignificant, we use the results for further analysis.

We first examine the computational results with minimum emission capacity. Table 10 demonstrates the financial impacts of the tightest emission caps with 40 instances of 50 retailers and 15 periods. The row %TC displays the additional costs induced by the emission constraints, i.e. (TC-E – min TC)/min TC.

Table 10. Financial Impact of Four Types of Emission Constraints – Minimum Emission Cap Level

	global	**rolling 10**	**rolling 5**	**cumulative**	**periodic**
%TC	2.9	8.2	8.4	49.8	30.8

Observation 2 indicates that the periodic constraints have the greatest impact on costs. However, the results in Table 10 are inconsistent with the observation. Since we now set all the emission caps to their minimum level, the cumulative emission constraints far exceed the others in cost. MC-C yields approximately 50% more costs than the cost-optimization scenario. The periodic constraints cause 31% more costs compared to minimum total cost; the rolling horizon constraints induce 8% extra costs and the global constraints incur less than 3% additional costs. With the tightest caps, two different types of emission constraints can induce a cost difference of up to 47%.

Figure 5 illustrates the trade-off curves of all types of emission constraints. Each curve synthesizes the results of 40 instances with 50 retailers and 15 periods. The curves are piecewise convex. All the curves depict that the total cost decreases as we increase the emission caps. The decrease is fastest when the emission caps are between their minimum level and the minimum level plus 5% increase. It is noteworthy that the cost increase dramatically plummets from 50% to 9% with the cumulative constraints for the tightest emission level. The global constraints lead to the lowest costs in most of the cases. Although the cumulative constraints induce the highest costs when the caps are tightest, their total cost becomes lower than the periodic constraints' when we increase both caps by 4%.

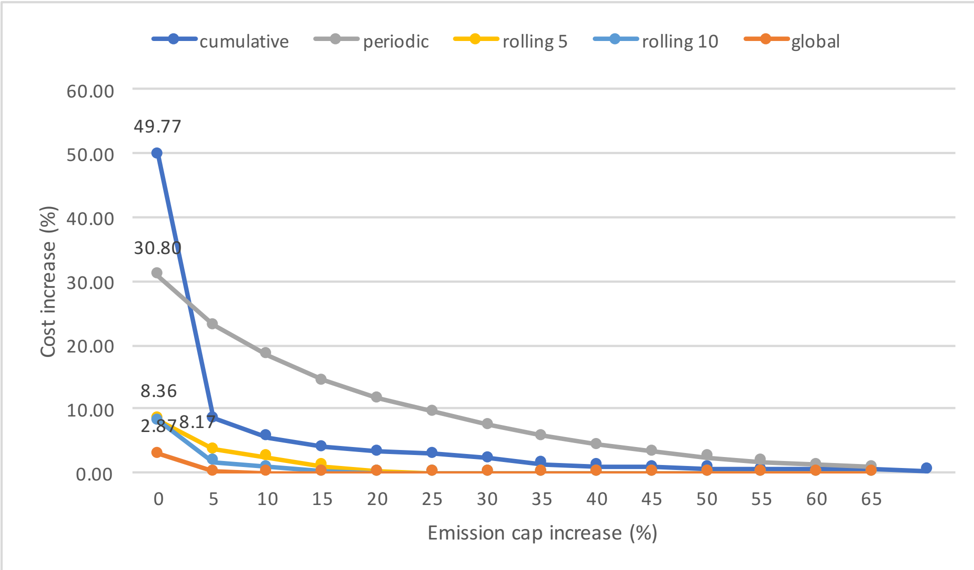

Figure 5. Cost-emission Trade-off Curves of Four Types of Emission Constraints (40 instances, N=50, T=15)

Figure 6 displays the trade-off curve of the global emission constraints. The total cost with global constraints is low compared to the other types of constraints. When we set the emission cap to its minimum level, the total cost is 2.87% higher than the minimum total cost. As we increase the cap by 10%, the total costs become equal to the minimum total cost.

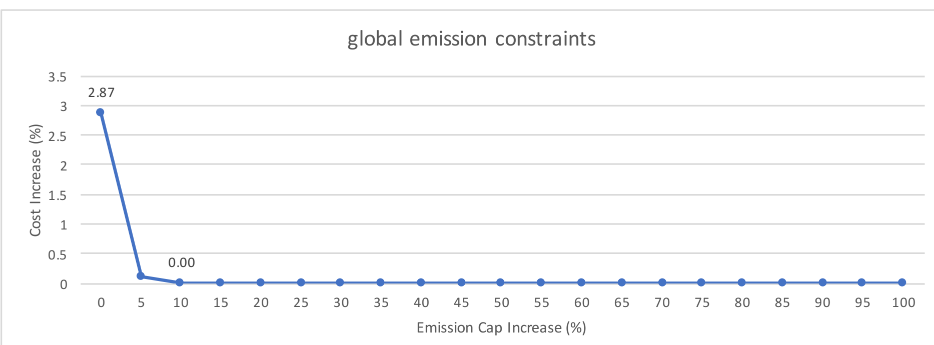

Figure 6. Cost-emission Trade-off Curve of Global Constraints (40 instances, N=50, T=15)

Figure 7 shows the trade-off curve of the rolling horizon constraints with 10 periods. The total cost plunges from 8.17% to 1.69% as the emission cap increases by 5%. And the emission constraints become not-binding when the cap increases by 25%.

The drop of total cost is less sharp in the case of 5-period rolling horizon constraints, as shown in Figure 8. The total cost declines by less than 5% and reaches 0% when the cap increases by 30%.

The decrease in costs becomes less steep as the rolling horizon window becomes longer. This can be further supported by the curve of the periodic emission constraints in Figure 9. The total cost drops from 31% to 23% in the first range (0-5% increase in caps), and it gradually declines to 0% as we increase the cap by 95%.

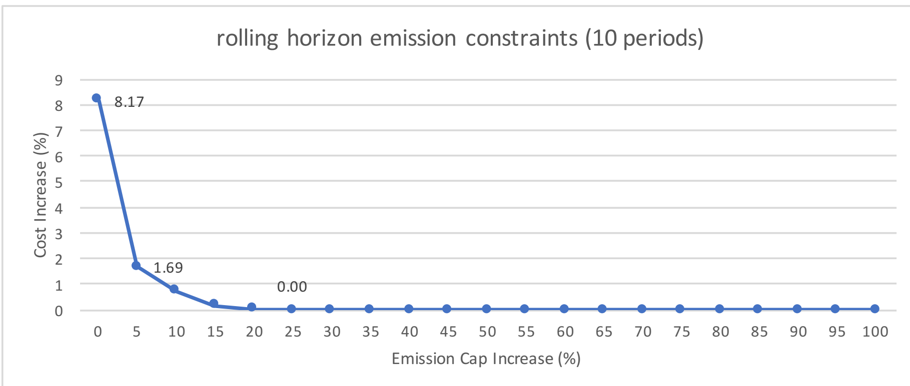

Figure 7. Cost-emission Trade-off Curve of 10-period Rolling Horizon Emission Constraints (40 instances, N=50, T=15)

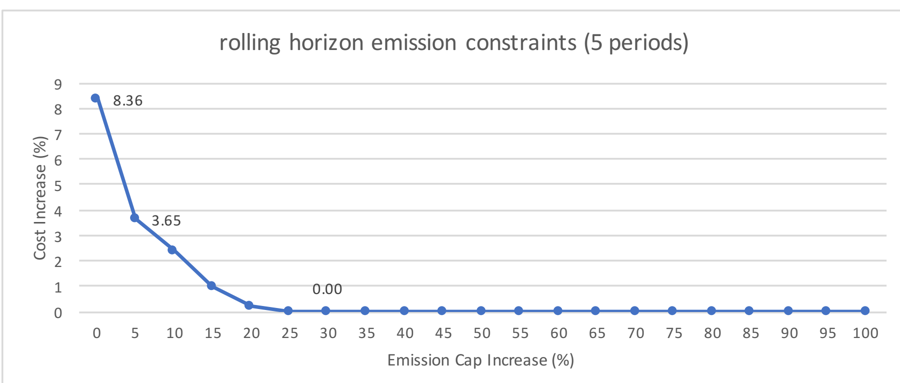

Figure 8. Cost-emission Trade-off Curve of 5-period Rolling Horizon Emission Constraints (40 instances, N=50, T=15)

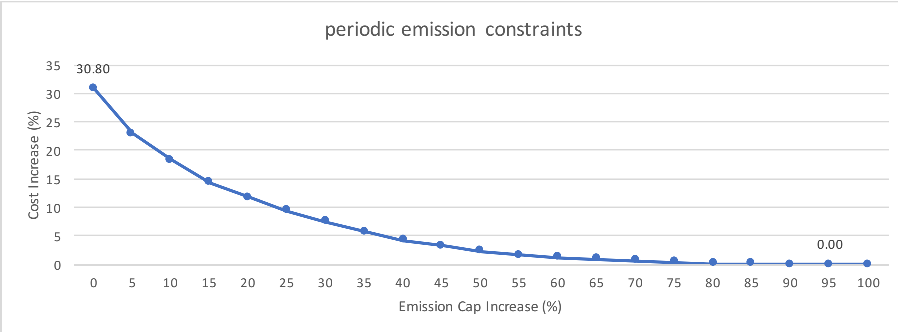

Figure 9. Cost-emission Trade-off Curve of Periodic Emission Constraints (40 instances, N=50, T=15)

Figure 10 shows that the tightest cumulative caps lead to 50% cost increase. As we increase the emission caps by 5%, the cost increase plummets to 9%. The cost increase becomes slower as we increase the emission caps. The cumulative constraints still impact the cost when we double the emission caps.

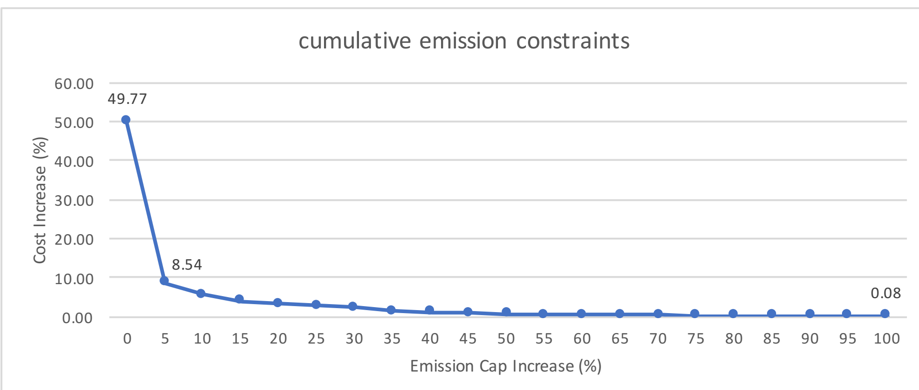

Figure 10. Cost-emission Trade-off Curve of Cumulative Constraints (40 instances, N=50, T=15)

The analysis of the trade-off curves provides the following observations. **Observation 4**: the emission caps can have significant impact on total costs. The tighter the caps are, the higher costs they induce. **Observation 5**: the total cost increases at different rate as we tighten the emission caps. The cost increases fastest when the emission caps are between their minimum level and 5% higher. Besides, we complement **observation 2** as follows: the cumulative emission constraints with tightest caps generate the highest cost. The periodic emission constraints induce higher cost than the other types of constraints within the relaxation range of 4% to 95%. On the other hand, the global constraints provide the lowest costs in most of the cases.

6.2 Analysis of Trade-offs between Total Emission and Emission Caps

We have analyzed the financial impacts of the emission constraints. In this section, we briefly analyze the trade-off between total emission and emission caps of each of the four types of emission constraints.

In the experiments in Section 6.1, we have solved the OWMR-E problem with each of the four types of emission constraints, using the minimum cap level and then gradually increasing the emission caps by 5% in each experiment. In the experiments, besides the total cost, we have also calculated the total emission induced by each type of emission constraints. We then calculate the average total emission of 40 instances with 50 retailers and 15 periods for each of the four types of emission constraints. Figure 11 illustrates the trade-off curves between the total emission and the emission caps for each type of the emission constraints.

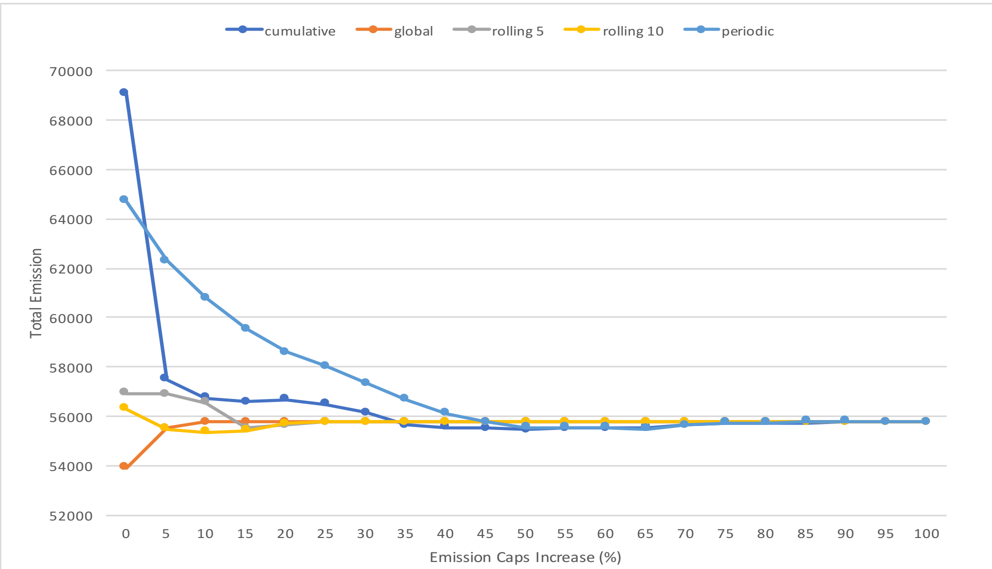

Figure 11. Trade-off Curves Between Emission Caps and Total Emissions (40 instances, N=50, T=15)

The minimum amount of emission is incurred by the global constraints when the global caps are set to their minimum level. On the contrary, the cumulative constraints with the tightest caps induce the highest total emission, 28% more than the minimum total emission. The rolling horizon, periodic and cumulative constraints always lead to a total emission higher than the minimum total emission. When we increase all the emission caps by 5-45%, the period constraints induce higher total emission than the other types of constraints. As we increase all the emission caps by more than 75%, all the four types of emission constraints lead to the similar total emission level, which is 3.4% higher than the minimum total emission.

Based on the above analysis, we provide the **observation 7**: the global emission cap can effectively reduce the total emission. However, a tighter rolling horizon, cumulative and periodic cap can paradoxically lead to higher total emission. Combining the cost-emission trade-off analysis in Section 6.1 and the analysis of the trade-offs between total emission and emission caps in this section, we have the **observation 8**: the cumulative emission constraints can lead to the highest total cost and highest total emission at the same time. The periodic constraints lead to higher total cost and total emission than all the other types of emission constraints when we increase all the emission caps by 5-45%.

Chapter 7
Conclusions

In this thesis, we consider a one-warehouse multi-retailer problem. We model the problem with two formulations: the basic formulation and the multicommodity formulation. Computational experiments with standard instances prove that the multicommodity formulation performs better. It provides stronger linear relaxation bounds and solve the MIP in shorter solution time.

To address the growing concerns about carbon footprint, we extend the formulations by adding the global, rolling horizon, cumulative and periodic emission constraints, respectively. Due to the difficulty to collect emission data, we generate the emission factors by estimation. Furthermore, we introduce two methods to obtain the emission caps for different types of emission constraints. The first method provides the equivalent emission caps for each of the four types of emission constraints, based on the minimum periodic emission level. By using these caps, we guarantee the feasibility of the OWMR-E problem. Furthermore, this allows us to fairly compare the four types of emission constraints. The computational results indicate that the model with the periodic constraints requires the longest CPU time, followed by the cumulative, the rolling horizon and the global constraints, respectively. With regard to the two different formulations, the following formulations provide better performance: BF-P, MC-C, MC-R and MC-G.

The second method determines the minimum emission level for each of the four types of emission constraints. By setting the emission caps to the minimum emission level, we ensure the feasibility of the OWMR-E problem, and we guarantee that the emission constraints have an impact on total cost. By gradually increasing the emission caps, we analyze the trade-off curves between the total cost and the emission caps for each type of the emission constraints. Furthermore, we briefly analyze the trade-off curves between the total emission and the emission caps. Certain observations emerge from the analysis. First of all, as we increase the emission caps for each type of the emission constraints, the total cost decreases. Second, the cumulative emission constraints lead to the highest total cost and total emission when the emission caps are set to the minimum level. The global emission constraints, on the other hand, induce the minimum total emission when the global caps are tightest. The periodic emission constraints induce both higher total cost and higher total emission compared to the other types of emission constraints when the emission caps are 5% to 45% higher than the corresponding minimum emission level.

The OWMR problem with four types of emission constraints could be interesting for different actors. It could help the policymakers to set more achievable reduction targets for the business entities. Note that the OWMR-E formulations can adapt to different policies by adding the emission costs in the objective function and/or additional emission permits in the capacity constraints. The models could be equally interesting for companies. Practitioners could plan the production while considering the emission caps set by the legislations using the models we have proposed.

This study is limited by the absence of emission data. Due to the difficulty to collect emission data, we assume the correlation between emission and cost parameters, which can cause bias in the analysis. Fortunately, with the enforcement of the emission regulations, more companies take their social responsibilities and invest in tracking carbon footprint. New tools are developed to measure the carbon performance. These efforts can facilitate the future studies in lot-sizing models with emission concerns. Moreover, there are other methods to determine the minimum cumulative emission level. Recall that we determine the minimum cumulative emission level by minimizing the total emission in period 1 and then solve a series of optimization models based on the decisions made in previous periods. Using the minimum cumulative emission levels obtained from these models, the cumulative emission constraints induce much higher total costs than the other types of emission constraints. As an alternative, we can simply set the minimum cumulative emission level of the last period to the minimum total emission. Using this method, the cumulative emission constraints will lead to the same results as the global emission constraints. They will induce low total cost and low total emission.

As discussed, future research might explore the adaption of regulations. The OWMR-E problem can integrate the carbon tax and cap-and-trade policies using four types of emission constraints. Another progression of this work is to model the OWMR problem with four types of emission constraints using the multicommodity echelon stock formulation, which provides better performance with capacitated instances according to Gruson *et al.* (2019).

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Appendix

Appendix 1. The Computational Results of OWMR-E without Initial Inventory

MC-E		global			rolling u =10			rolling u=5			cumulative			periodic		
N	T	CPU time	Optimal%	Gap%	CPU time	Optimal%	Gap%	CPU time	Optimal%	Gap%	CPU time	Optimal%	Gap%	CPU time	Optimal%	Gap%
50	15	1.1	100.0	0.0	1.1	100.0	0.00	2.7	100.0	0.00	19.2	100.0	0.000	3376.0	62.5	0.1
100	15	3.4	100.0	0.0	5.4	100.0	0.00	42.3	100.0	0.00	525.5	97.5	0.003	-	-	-
150	15	6.4	100.0	0.0	11.7	100.0	0.00	80.1	100.0	0.00	504.0	97.5	0.015	-	-	-
50	30	16.2	100.0	0.0	10.2	100.0	0.00	195.4	100.0	0.00	135.6	100.0	0.000	-	-	-
100	30	82.9	100.0	0.0	218.9	100.0	0.00	1358.2	100.0	0.00	1502.0	97.5	1.279	-	-	-
150	30	131.2	97.5	0.0	588.1	92.5	0.05	2706.4	82.5	0.05	2333.0	90.0	0.001	-	-	-
average		**40.2**	**99.6**	**0.0**	**139.2**	**98.8**	**0.01**	**730.9**	**97.1**	**0.01**	**836.6**	**97.1**	**0.216**	-	-	-

BF-E		global			rolling u =10			rolling u=5			cumulative			periodic		
N	T	CPU time	Optimal%	Gap%	CPU time	Optimal%	Gap%	CPU time	Optimal%	Gap%	CPU time	Optimal%	Gap%	CPU time	Optimal%	Gap%
50	15	10.0	100.0	0.00	9.4	100.0	0.00	11.6	100.0	0.00	11.5	100.0	0.00	1633.6	85.0	0.1
100	15	96.5	100.0	0.00	117.5	100.0	0.00	174.5	100.0	0.00	425.2	97.5	0.01	-	-	-
150	15	441.5	100.0	0.00	482.3	100.0	0.00	548.0	100.0	0.00	292.4	97.5	0.04	-	-	-
50	30	690.4	97.5	1.08	1474.9	95.0	4.60	1339.6	95.0	4.43	849.2	95.0	1.95	-	-	-
100	30	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
150	30	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
average		-	-	-	-	-	-	-	-	-	-	-	-	-	-	-

Appendix 2. Part of the Codes for BF

The first method constructs the basic LP model; the second method set the setup variables to binary and the inventory variables to integer, so the problem become MIP; the last method set the parameters in the objective function and the constraints.

```
class BF():
    def __init__(self, prob, N, T):
        prob.objective.set_sense(prob.objective.sense.minimize)

        ## Variables
        # create variable indices
        def varind(var, i, t):
            return var*(N+1)*T + i*T + t
        varnum = 3
        y = 0
        q = 1
        s = 2

        self.colcnt = varnum*(N+1)* T
        obj = [0] * self.colcnt
        lb = [0] * self.colcnt
        ub = [0] * self.colcnt
        name= [0]* self.colcnt

        # add variables
        for i in range(N+1):
            for t in range(T):
                obj[varind(y, i, t)] = 0
                lb[varind(y, i, t)] = 0
                ub[varind(y, i, t)] = 1
                name[varind(y, i, t)] = "y_" + str(i) + str(t+1)

                obj[varind(q, i, t)] = 0
                lb[varind(q, i, t)] = 0
                ub[varind(q, i, t)] = cplex.infinity
                name[varind(q, i, t)] = "q_" + str(i) + str(t+1)
```

```
        obj[varind(s, i, t)] = 0
        lb[varind(s, i, t)] = 0
        ub[varind(s, i, t)] = cplex.infinity
        name[varind(s, i, t)] = "s_" + str(i) + str(t+1)

prob.variables.add(obj=obj, ub=ub, lb=lb, names=name)

## Warehouse inventory balance constraint
#  If t=1: q_0_1 - s_0_1 - (q_1_1+...+q_r_1) = -s_0_0
self.cst1 = [0, "data.s_i_0[0]"]
lin = [[varind(q,r,0) for r in range(1, N+1)] + [varind(s,0,0), varind(q,0,0)], [1]*(N+1)+[-1]]
prob.linear_constraints.add(lin_expr=[lin],
                            senses="E")
#  Else: s_0_(t-1) + q_0_t - s_0_t - (q_1_t+...+q_r_t) = 0
lin = []
for t in range(1,T):
   lin += [[[varind(q,r,t) for r in range(1, N+1)] + [varind(s,0,t), varind(q,0,t), varind(s,0,t-1)], [1]*(N+1)+[-1, -1]]]
prob.linear_constraints.add(lin_expr=lin,
                            senses="".join("E"*(T-1)),
                            rhs=[0]*(T-1))

# Retailers' inventory balance constraint
# if t=0: q_r_1 - s_r_1 = D_r_1 - s_r_0
cst2 = prob.linear_constraints.get_num()
self.cst2 = []
lin = []
for r in range(1, N+1):
   lin += [[[varind(s,r,0), varind(q,r,0)], [-1, 1]]]
   self.cst2 += [[cst2, "data.D_i_t["+str(r)+'][0] - data.s_i_0['+str(r)+']']]
   cst2 += 1
prob.linear_constraints.add(lin_expr=lin,
                            senses="".join("E"*N))
# else: s_r_t-1 + q_r_t - s_r_t = D_r_t
cst3 = prob.linear_constraints.get_num()
self.cst3 = []
lin = []
for r in range(1, N+1):
   for t in range(1,T):
```

```
            lin += [[[varind(s,r,t-1), varind(q,r,t), varind(s,r,t)], [1,1,-1]]]
            self.cst3 += [[cst3, "data.D_i_t["+str(r)+']['+str(t)+']']]
            cst3 += 1
    prob.linear_constraints.add(lin_expr=lin,
                        senses="".join("E"*N*(T-1)))

    ## Forcing production/delivery constraint
    # q_i_t <= sum(D_i_tT) *  y_i_t
    cst4 = prob.linear_constraints.get_num()
    self.cst4 = []
    for i in range(N+1):
        for t in range(T):
            self.cst4 += [[cst4, [varind(q,i,t), varind(y,i,t)], [-1, "sum(data.D_i_t["+str(i)+']['+str(t)+":data.T])"]]]
            cst4 += 1
    prob.linear_constraints.add(senses="".join("G"*(N+1)*T),
                        rhs=[0]*(N+1)*T)

def mip(self, prob, N, T):
    allvars = list(range(self.colcnt))
    ct = "B" * (N+1) * T + "I" * (N+1) * T * 2
    prob.variables.set_types([(allvars[x], ct[x]) for x in range(self.colcnt)])

def parameters(self, prob, data):
    def varind(var, i, t):
        return var*(data.N+1)*data.T + i*data.T + t
    varnum = 3
    y = 0
    q = 1
    s = 2

    ## objective function
    obj = []
    for i in range(data.N+1):
        for t in range(data.T):
            obj += [[varind(y,i,t), data.FC[i][t]]]
            obj += [[varind(s,i,t), data.HC[i]]]
    prob.objective.set_linear(obj)
```

```
## constraints
# Warehouse inventory balance constraint
```

… (set the parameters to the constraints)