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Mean-variance analysis of barbell strategies

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Sciences de la gestion

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Résumé

Dans notre étude empirique, nous mettons en œuvre le modèle de taux d'intérêt à facteur unique Vasicek pour effectuer des stratégies d'immunisation en utilisant un portefeuille en barbelé sur les données de la courbe de rendement fédérale dans un cadre moyenne-variance. Les statistiques de performance des stratégies traditionnelles basées sur la durée qui incluent la durée de Macaulay et la durée stochastique n'ont pas surpassé les performances de la stratégie de variance minimale utilisant le portefeuille en barbelé. Cependant, la performance de la durée de Macaulay était plus proche de la stratégie à variance minimale que la stratégie basée sur la durée stochastique. Les résultats obtenus à partir de l'analyse hors échantillon et des contrôles robustes ont réitéré une conclusion similaire.

Mots clés: Analyse empirique, stratégie d'immunisation, analyse de moyenne-variance, modèle Vasicek, stratégie en barbelé, stratégie d'immunisation à variance minimale, stratégies basées sur la durée

Méthodes de recherche: Étude empirique

Abstract

In our empirical study, we implement the Vasicek single-factor interest rate model to perform immunization strategies using a barbell portfolio on the Federal yield curve data under the mean-variance framework. The performance statistics of traditional duration-based immunization strategies which include the Macaulay's duration and stochastic duration have failed to outperform the performance of minimum-variance strategy using the barbell portfolio. However, the performance of Macaulay's duration was closer to the minimum-variance immunization barbell strategy compared to the stochastic duration-based strategy. The results obtained from the out-of-sample analysis and the robust checks have reiterated a similar conclusion.

Keywords: Empirical analysis, immunization strategy, mean-variance analysis, Vasicek model, barbell strategy, minimum-variance immunization strategy, duration-based strategies

Research methods: Empirical study

The analyses contained in this thesis are based on the research ideas developed by Professors Pascal François and Franck Moraux. I acknowledge the right for them to use these analyses for producing research articles in which I will not take part.

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1. Introduction

Immunization essentially means protection of one's financial position from the impact of interest rate fluctuations in the dynamic environment. Immunization strategies are implemented tactically with the help of active and passive investment strategies to exploit the interest rate movements and use it for one's advantage to earn profits rather than let the interest rates impact negatively. Short-horizon and long-horizon assets can be traded when interest rates are on the rise or fall to lock in the position regardless of the interest rates. Hence, immunization strategies are based on the trade-off between resale price risk and reinvestment risk.

Immunization strategies are a common way where multiple stakeholders such as banks and non-banking financial companies (NBFCs) hedge their financial position against the interest rate movements. Banks use immunization strategy to safeguard their net worth and protect their clients. On the other hand, NBFCs such as pension funds use immunization since they guarantee their clients to return the money after a certain period. An entity can make use of an immunization strategy to their advantage and make profits but if implemented incorrectly, it can substantially degrade their portfolio. In theory, a 100% immunized position implies there is no impact on the portfolio, irrespective of the future interest rates.

In the context of this paper, we use immunization for a bond portfolio. The most common method is the duration-based immunization. Duration is the weighted average of the bond's cash flow dates over its life. It is a better measure to capture the bond's volatility compared to just taking

the maturity of the bond. Thus, duration-based immunization strategies essentially immunize the bond portfolio such that the duration of the bond portfolio should match with the investment planning horizon. For instance, zero-coupon bonds with a similar maturity to the bond portfolio can help hedge fixed-income portfolios.

In this paper, we will particularly study duration-based immunization strategies using a barbell strategy. A barbell strategy is hedging the portfolio with the help of short-term maturity bonds and a long-term maturity bonds with no investment in the intermediate horizon bonds. The whole idea behind implementing a barbell strategy is to get the advantage of investing in short-term structures with the help of rolling horizon bond (i.e. we can reinvest frequently) till the planning horizon and the benefit of long-term structures since it compensates the investor with high yields due to the greater risk that it inherits compared to the short-term bonds. This strategy is an active form of portfolio management which helps with risk diversification, liquidity and flexibility but at the same time allows earning higher yields by investing in the long-term bonds.

François and Moraux (2008) have performed mean-variance analysis of immunization strategy under a two-factor model where they implement static and dynamic immunization strategies. Under the static immunization strategy, the result states that traditional and stochastic, both fail to be on the efficient frontier set of horizon. However, under dynamic immunization strategy, traditional and stochastic perform on the efficient frontier set of horizon. Hence, they conclude that the effective immunization performance is influenced by the type of strategies and not based on duration.

Bond portfolio optimization and duration-constrained mean-variance optimization is developed using dynamic factor models by Caldeira et al. (2016) in their research. They benchmark their results against different bond portfolio strategies such as bullet, barbell and ladder that are popular among bond desks. Their approach consists in implementing a dynamic portfolio selection rule by picking the policy with the highest probability of outperforming other policies, one month in advance. The proposed strategy outperformed the traditional strategies that are used in the fixed-income market in the space of Sharpe ratio and mean excess return.

Agca (2005) performs Monte Carlo simulations on a term structure that evolves to a Heath-Jarrow-Morton (1992). Agca obtains that implementing the correct term structure model and corresponding interest rate risk measure is not an effective way compared to using the right immunization strategy and portfolio formation when the objective is immunization. She further states that when the parameters of HJM model need to be estimated then traditional risk measures outperform HJM risk measures.

Fooladi and Roberts (1992) uses Canadian bond data over a period of approximately 20 years and concludes in their research that duration-matching strategies are better at immunizing interest rate risk compared to maturity matching strategies of the bond to the planning horizon. The results hold even in the absence of the term structure fitting. The various types of strategies implemented in the research paper are barbell, bullet and ladder. They further conclude that the duration-matching strategy outperforms partly due to the portfolio design.

Soto (2004) studies various duration-based strategies and portfolio strategies to immunize the short-term horizon Spanish government bonds over a period of 7 years. The empirical study portrayed that duration-based strategies outperforms partly due to number of risk factors compared to the selected model. Furthermore, traditional immunization strategies perform better if realistic strategies are added. Also, three-factor immunization strategy i.e. only duration-matching bullet and barbell portfolios including a maturity bond combined with risk factor with the common factor model can be used for effective interest rate risk management.

Mann and Ramanlal (1997) examine the relative performance of yield curve strategies that includes barbell, bullet and ladder strategies when the yield curve changes in level, slope and curvature. They find out that for downward shift of the yield curve, short-term bullet portfolio performs better compared to short-term barbell portfolios. The opposite is true for long-term portfolios. This result holds irrespective of the amount of shift, shape and coupon level in the yield curve.

Gultekin and Rogalski (1984) test the implications of alternative duration measures in terms of explaining the return variance of U.S government bonds. They find that none of the duration measures analyzed are superior to the others. Babble (1983) finds that simple duration matching strategies perform as well as the proposed alternatives. Ilmanen (1992) finds that simple duration measures explain 80% to 90% of the return variance of U.S. government bonds. Rutkowski (1999) and Andersson and Lageras, (2013) have also studied properties of the strategy consisting in rolling over one dollar in zero coupon bonds during a given time period. The user of such a

strategy typically faces a reinvestment risk. Thus, we go a step further by adding the holding of a bond for the same time period.

The prior work has mostly measured immunization performance ex-post using bond data over a sample period. In this paper, we delve deep into measuring the immunization performance ex-ante. We analyze immunization-based strategy returns using barbell strategy within the mean-variance framework. This strategy captures the fundamental issue of trading off reinvestment risk and resale price risk of the bond coupons.

First, in section 2, we state all the model assumptions and limitations that our mean-variance study of immunization strategies is based on. Subsequently, in section 3 we perform an in-sample analysis with the US data yield curves for zero-coupon bonds for immunization-based strategies using a barbell portfolio under the Vasicek term structure model. The mean, variance and Sharpe ratios of the strategy returns provide a base for comparison between the different strategies across multiple investment planning horizons. For the in-sample performance analysis, we require some base case parameters of the Vasicek model fitted to the data. Here, we rely on the approximate linear Kalman filtering applied to exponential-affine term structure models as estimated by François and Moraux (2008). Furthermore, for better visualization and intuition, we showcase the mean-variance of the immunization strategies on the efficient frontier curve. The result that we obtain from the in-sample performance is that Macaulay's duration-based strategy and stochastic duration-based strategy are not the most mean-variance efficient strategies. We also highlight the most mean-variance efficient strategy that provides maximized returns for a given level of risk that we state as minimum-variance immunization strategy.

Further, in section 4 we conduct out-of-sample performance analysis of immunization-based strategies using a barbell portfolio. However, in this case, we do not assume any parameters, but we estimate the parameters historically from one part of the data and implement this on the other part of the data yearly. The idea behind out-of-sample analysis is that any trader can implement these strategies in the real world with historical estimation.

Additionally, in section 5 we compute robustness checks using yield curves for zero-coupon bonds from Bank of Canada. The motivation behind the robustness check is to make sure that the immunization strategies using a barbell portfolio behaves in an unbiased fashion on different data that has different characteristics. The robustness performance is conducted similarly as the in-sample analysis, by assuming the same base case parameters. Finally, in the last section, we end with some concluding remarks.

2. Model assumptions and limitations

Immunization implies protection of one's financial position from the impact of interest rate fluctuations in the market. Additionally, a barbell strategy is hedging the position with the help of short-term maturity bonds and a long-term maturity bonds with no investment in the intermediate horizon bonds. Hence, our goal of immunization is to exploit this advantage of investing in short-term structures with the help of rolling horizon bond (i.e. we can reinvest frequently) till the planning horizon and the benefit of long-term structures since it compensates the investor with high yields due to the riskiness that it inherits compared to the short-term bonds to safeguard a bond portfolio.

2.1. Stylized strategy

François and Moraux (2008), in their research paper introduced a stylized strategy that captures the trade-off between reinvestment risk and resale price risk. The strategy immunizes an investor who has invested in a bond with a face value of $1 and a continuous coupon payment (c). The horizon in the strategy is θ which lies between 0 and T. They then implement the strategy by holding the coupon bond from 0 to θ, re-investing the coupon in the discount bond $P(t, \theta - t)$ every time and ending the strategy at time θ.

2.2. Assumptions

In this study, the investor invests in a bond portfolio that contains only two different maturities to be able to implement the immunization strategy. Models are a way of implementing the theory so that one can see the results in a close to a practical environment. Hence, models are supposed to be a simplified version of the real world. Our model assumes that there are assets with two

maturities only that make the investors' portfolio. A short-term zero-coupon bond with maturity T_1 and a long-term zero-coupon bond with maturity T_2. Hence, in this way, the investor can balance their exposure to reinvestment risk through exposure in the short-term bond and resale price risk through exposure in the long-term bond. This is how a barbell strategy can be implemented. We assume that the mean-variance framework captures the investor preferences.

2.3. Mean-variance analysis

For the sake of immunization strategy, we assume that an investor has an initial investment of $1 at the start of the period. The planning horizon is θ which is in the range from T_1 to T_2, and it is implemented at time 0.

In this study of the immunization strategy, the investor invests $\$x$ (one part) amount of money in a rolling horizon bond which has a maturity of time period of T_1 until planning horizon period (θ). This essentially means that the investor invests $\$ x$ which is the weight induced on the rolling zero-coupon bond that rolls from time 0 until the planning horizon period θ. This part of the strategy helps in balancing the reinvestment risk.

Additionally, the remaining weight which is $\$(1 - x)$ amount of money is invested in another zero-coupon bond with maturity period T_2 until the planning horizon period θ. This part of the leg is a buy and hold strategy that helps in balancing the exposure towards resale price risk.

P (t, τ) indicates the price of a zero-coupon bond at time *t* with time to maturity *τ* with a face value of $1. $R_\theta(\tau)$ is the value of the rolling horizon bond at time θ for $1 of initial investment invested at time 0. The terminal value of an investor's immunization strategy with time horizon θ is π_θ.

$$\pi_\theta = x * R_\theta(T_1) + \frac{1-x}{P(0,T_2)} * P(\theta, T_2 - \theta)$$

The investor invests *x* amount of money in the short-term bond (R_θ) that has a maturity T_1 (in our case one year) and the remaining amount of money (1 – *x*) is invested in the long-term bond that

has a maturity T_2. The terminal value of the long-term bond is the price of a zero-coupon bond with planning horizon θ and maturity of $T_2 - \theta$. Hence, the terminal value of an investor's immunization strategy (π_θ) with time horizon θ will be the addition of the short-term leg and the long-term leg with their respective weights.

The expected mean of the random terminal value of an investor's immunization strategy ($E(\pi_\theta)$) with time horizon θ is:

$$E(\pi_\theta) = x * E(R_\theta(T_1)) + \frac{1-x}{P(0,T_2)} * E(P(\theta, T_2 - \theta))$$

The expected variance of the random terminal value of an investor's immunization strategy ($var(\pi_\theta)$) with time horizon θ is:

$$var(\pi_\theta) = x^2 V_1 + (1-x)^2 V_2 + 2x(1-x)V_{12}$$

Where:

$V_1 = var\ [R_\theta(T_1)]$

$V_2 = \ var[P(\theta, T_2 - \theta)/\ P(0, T_2)]$

$V_{12} = cov[R_\theta(T_1), P(\theta, T_2 - \theta)/\ P(0, T_2)]$

V_1 is the variance of the rolling horizon zero-coupon bond leg that the investor invests a part of the money to balance the reinvestment risk. V_2 is the variance of the long-term zero-coupon bond leg

that the investor invests the remaining amount of money to balance the resale price risk. V_{12} is the covariance between the rolling horizon zero-coupon bond and long-term zero-coupon bond.

The following postulations will characterize the mean-variance efficiency for the barbell portfolio strategy which is free of any term-structure model. Firstly, in the mean-variance space, the terminal value of the immunization strategy forms a convex set which results in a minimum variance at $x = x^*_{mvi}$. The following results are shown in François and Moraux (2008):

$$x^*_{mvi} = \frac{V_2 - V_{12}}{V_1 + V_2 - 2V_{12}}$$

Secondly, the minimum variance of the immunization strategy is obtained by plugging the x^*_{mvi} in $var(\pi_\theta)$

$$\frac{V_1 V_2 - {V_{12}}^2}{V_1 + V_2 - 2V_{12}}$$

The immunization strategy is mean-variance efficient if the short-term bond has lesser than or equal weight (x) compared to x^*_{mvi} when,

$$(E(R_\theta(T_1)) < E(P(\theta, T_2 - \theta))$$

Or the short-term bond has greater than or equal weight (x) compared to x^*_{mvi} when,

$$(E(R_\theta(T_1)) > E(P(\theta, T_2 - \theta))$$

According to x^*_{mvi}, the new duration-based approach consists of giving weight to both the legs of the barbell strategy i.e. the short-term and the long-term to be able to minimize the variance of the

barbell portfolio. This barbell strategy can be called as minimum variance immunization. However, the minimum variance immunization still has some minimum variance and 100% of the risk is not immunized. Hence, if the expected return of the short-term bond is greater (lower) than the long-term bond in the portfolio, then any barbell portfolio with a weight (x) more than x^*_{mvi} (weight (x) less than x^*_{mvi}) will have a mean and variance higher than the minimum variance strategy.

2.4. Macaulay and stochastic durations

To observe the performance of the minimum variance immunization strategy in comparison with other strategies, we compare it with Macaulay's duration-based strategy and stochastic duration-based strategy. The duration-based strategies allocate bond portfolio by matching the investment planning horizon.

The immunization strategy using Macaulay's duration implements:

$$x(T_1) + (1 - x)T_2 = \theta$$

Which results in Macaulay's duration-based bond portfolio allocation to be

$$x^*_{mac} = \frac{T_2 - \theta}{T_2 - T_1}$$

Macaulay's duration is a model-free term structure which is simply a function of the rolling horizon period, planning horizon period and the long-term horizon period of the bond portfolio.

On the other hand, the Cox-Ingersoll-Ross stochastic duration is not a model-free term structure. According to Cox, Ingersoll and Ross (1979), define stochastic duration as the time to maturity of a discount bond with the same basis risk as to the initial instrument.

For one-factor models, the portfolio duration is given by θ.

$$\theta = b^{-1}\big(xb(T_1) + (1 - x)b(T_2)\big)$$

Where,

$b(t) = -\frac{1}{P(0)}\frac{\partial P(0,t)}{\partial t}$ and $r =$ Instantaneous risk-free rate

The immunization strategy using Cox-Ingersoll-Ross stochastic duration-based results in the following bond portfolio allocation:

$$x_{cir}^{*} = \frac{b(T_2) - b(\theta)}{b(T_2) - b(T_1)}$$

x_{cir}^{*} is a non-linear transformation of the Macaulay's duration-based immunization strategy.

2.5. Immunization performance in the Vasicek Model

We observe the performance of duration-based immunization strategies which includes Macaulay's duration and stochastic duration in the mean-variance framework under the Vasicek term structure model.

The instantaneous risk-free rate (r_t) follows a Gaussian mean-reverting process under the physical probability measure.

$$dr_t = \alpha(\beta - r_t)dt + \eta\, dZ_t$$

where,

α = speed reversion, β = long-term mean of r_t, Z_t = Brownian motion and η = volatility

The value of a default-free discount bond with face value $1 at time t and time to maturity τ is:

$$P(t,\tau) = a(\tau)\exp\left(-b\,(\tau)\,r_t\right)$$

Where,

$$a(\tau) = \exp\left((b(\tau)-\tau)(\beta + \frac{\eta\lambda}{\alpha} - \frac{\eta^2}{2\alpha^2}) - \frac{\eta^2 b^2(\tau)}{4\alpha}\right)$$

$$b(\tau) = \frac{1-\exp\left(-\alpha\tau\right)}{\alpha}$$

λ = risk premium parameter

In the mean-variance framework under the Vasicek term structure model, the mean ($E(\pi_\theta)$) and variance ($var(\pi_\theta)$) of the immunization strategy for any arbitrary portfolio allocation (x) can be computed.

The bond portfolio allocations for all the different immunization strategies using a barbell portfolio are as follows:

$$x^*_{mac} = \frac{T_2 - \theta}{T_2 - T_1}$$

$$x^*_{cir} = \frac{b(T_2) - b(\theta)}{b(T_2) - b(T_1)}$$

$$x^*_{mvi} = \frac{V_2 - V_{12}}{V_1 + V_2 - 2V_{12}}$$

Firstly, Macaulay's duration-based strategy is a model-free term structure. Secondly, the stochastic duration-based strategy is a transformation of the Macaulay's strategy but is not model-free, since it is dependent on b which is dependent on the α parameter i.e. the mean reversion parameter. Lastly, the mean-variance immunization-based strategy is affected by interest rate dynamics across all term structure model parameters. François and Moraux (2008) derive x^*_{mvi} in a closed-form solution in the Vasicek model.

2.6. Immunization strategy implementation

For any given starting date t_0, the value at time θ of the continuous investment in the rolling horizon is proxied by a daily rollover of

$$R_\theta(T_1) \approx \frac{P(t_0+\theta,T_1)}{P(t_0,T_1)} \prod_{j=0}^{n-1} \left(1 + \Delta f_{t_j}(T_1)\right)$$

Where,

Δ is the daily time step

n is the number of days within the investment planning horizon ($n = \theta / \Delta$)

$f_{t_j}(T_1)$ is the instantaneous forward rate at day t_j with time to maturity T_1

The second component of the immunization strategy is the ratio of two zero-coupon bond prices:

$$\frac{P(\theta, T_2 - \theta)}{P(0, T_2)}$$

All the zero-coupon bond prices are obtained as

$$P(t,\tau) = \exp\left(-z_t\,(\tau)\tau\right)$$

Where $z_t\,(\tau)$ is the zero-coupon spot rate at time t with time to maturity τ.

We analyze the mean-variance performance of three different immunization strategies that differ in the portfolio allocation according to their respective strategies. In Macaulay's duration-based

strategy, the portfolio Macaulay duration should match with the investment planning horizon ($x = x^*_{mac}$). In the stochastic duration-based strategy, the portfolio stochastic duration should match with the investment planning horizon ($x = x^*_{cir}$). Lastly, in the minimum variance strategy, the variance of the portfolio should be minimized at the investment planning horizon ($x = x^*_{mvi}$). The first strategy is a purely model-free strategy. The second strategy uses the estimated alpha parameter to compute x^*_{cir}. The last strategy uses all the estimated parameters along with *r* which is observed one-year zero-coupon rate to compute x^*_{mvi}.

3. In-sample analysis

3.1. Data and estimation

For the in-sample analysis, we model theoretical and empirical immunization strategies to understand how our models perform in the theoretical space compared to the real data on actual U.S. term structures in the mean-variance space.

We extracted zero-coupon yield curve rates and instantaneous forward rates from the U.S Treasury yield curve of the Federal Reserve Board. The zero-coupon spot rates are computed daily for maturities of 1, 2, 5, 10, 15 and 20 years. The instantaneous forward rate is also computed daily for 1-year maturity. Our sample period starts on July 2, 1981 and ends on April 30, 2019, which gives us a total of 9,437 daily observations over a period of approximately 38 years.

In our study of the immunization strategies using barbell portfolio, we use one of the simplest interest rate models which is the Vasicek model. We can derive the x^*_{mvi} explicitly from this model, thus, making it less complex to perform in-sample analysis. First, we implement the Vasicek model to understand the theoretical performance of Macaulay's duration-based strategy, the Cox-Ingersoll-Ross duration-based strategy and the minimum-variance immunization strategy in the mean-variance space.

The Vasicek model requires some base case parameters that we rely on the approximate linear Kalman filtering applied to exponential-affine term structure models as estimated by François and Moraux (2008) in their paper. Thus, we assume the base case parameters as $\alpha = 0.0329$, $\beta = 0.0257$, $\lambda = 0.4284$, $\eta = 0.0124$ that apply to the whole period of our Federal data from July 2, 1981 to April 30, 2019. However, for the theoretical immunization we do not implement our strategies on the real U.S. data. The investment planning horizons (θ) of 5, 10 and 15 years, the bond maturities of $T_1 = 1$ year and $T_2 = 20$ years are all considered for the calibration of the theoretical model. On numerically verifying the relationship between *r* (maturity rate) and x^*_{mvi}, we obtain that x^*_{mvi} is not dependent on *r*. However, the theoretical proof of this relationship is beyond the scope of this paper. Here, the base case parameters are estimated only once over the entire sample data.

3.2. Results

3.2.1. Theoretical model immunization

Figures 1, 2 and 3 plot the theoretical mean and standard deviation of the immunization strategy using the Vasicek model for all possible bond portfolio allocations (x). For the immunization in the mean-variance space, we particularly highlight five cases for the bond portfolio allocations. Firstly, when the initial investment ($1) is wholly invested in the rolling horizon bond (x = 1). Secondly, when the initial investment ($1) is wholly invested in the buy and hold part of the portfolio (x = 0). Thirdly, the bond portfolio allocation is determined by matching the Macaulay's duration with the investment planning horizon ($x = x^*_{mac}$). Fourthly, the portfolio allocation is determined by matching the stochastic duration of the portfolio with the investment planning horizon ($x = x^*_{cir}$). Lastly, the portfolio allocation is determined by minimizing the variance of the portfolio with the investment planning horizon ($x = x^*_{mvi}$).

Figure 1: In-sample theoretical immunization in the mean-variance space (θ = 5)

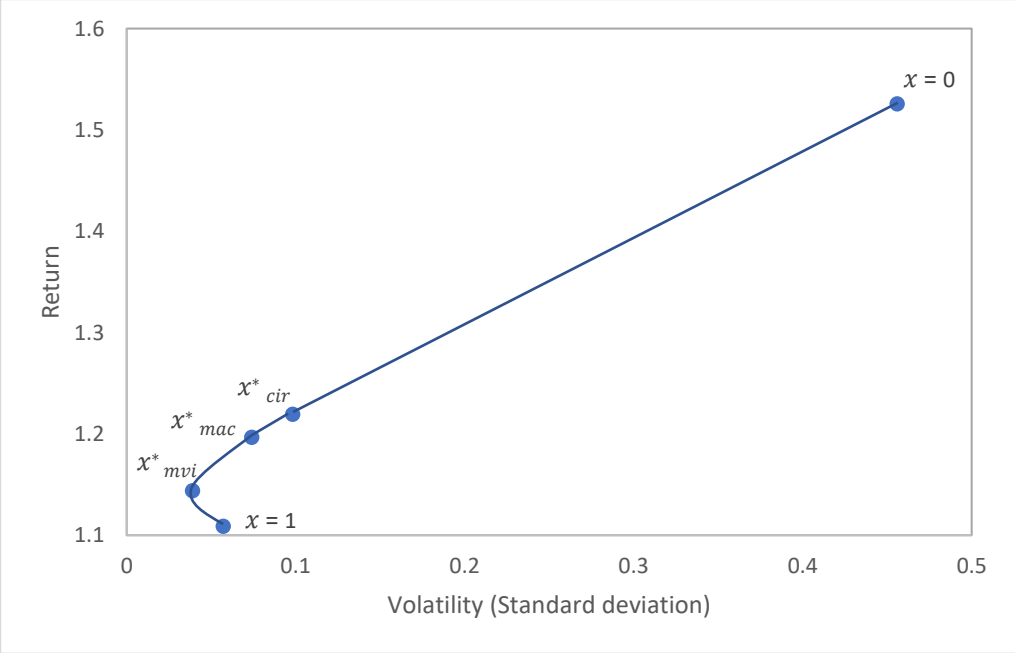


Figure 2: In-sample theoretical immunization in the mean-variance space (θ = 10)

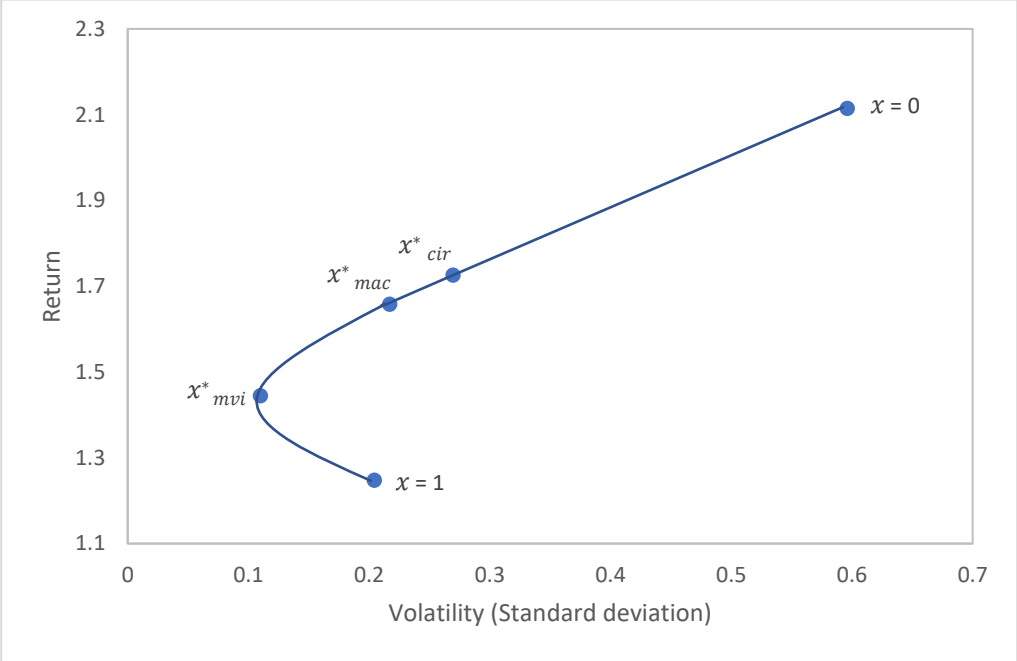
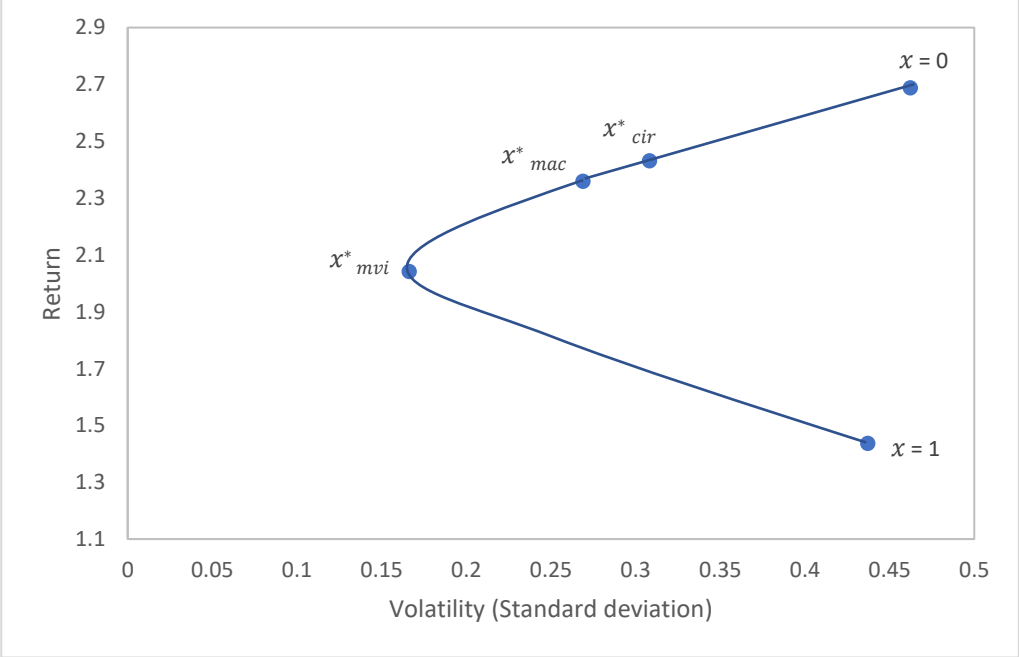


Figure 3: In-sample theoretical immunization in the mean-variance space (θ = 15)



Hence, the theoretical immunization in the mean-variance space depicts the outcome of the different bond portfolio allocations under the Vasicek model. For instance, if we use all the information on the parameter values at one point in time then we can compute the different bond portfolio allocations such as x^*_{mvi}, x^*_{mac} and x^*_{cir}. Therefore, an investor can use these bond portfolio allocations today for a planning horizon of θ years and expect an outcome with the respective return and volatility of the strategy on an average. In Figures 1, 2 and 3, we can see that the x^*_{mvi} has the minimum variance with a weight lesser than 1.

3.2.2. Empirical model immunization

Now, we compute ex-post in-sample analysis using the Federal data i.e. had the investor known the exact dynamics of the interest rate parameters then one would implement the immunization strategy and obtain the results according to the Table 1.

We compute the terminal value of $1 invested in barbell immunization strategies that use the T_1 = 1-year bonds and the T_2 = 20-year bonds daily from July 2, 1981 until the end of our data (April 30th, 2019). For our investment planning horizons (θ) of 5, 10, and 15 years, we have a total of 8,187, 6,937 and 5,687 immunization strategies computed daily, respectively. In this study, we have short-term and long-term strategies that have strong constraints in implementing them in the current time frame where our data is restricted to the end of April 2019. Thus, our methodological standpoint is to take advantage of the maximum number of points with the limited data we have over 38 years approximately.

The output indicates that if the investor had known about the dynamics in advance then on average the strategies would perform according to Table 1.

Table 1: In-sample performance statistics for immunization strategies

	Minimum Variance Immunization	Duration-based Immunization	
		Macaulay duration	Stochastic duration
Panel A: θ = 5 (8,187 strategies)			
Mean	1.2924	1.3523	1.3875
Variance	0.0661	0.0867	0.1010
Sharpe Ratio	0.0431	0.1286	0.1686
Panel B: θ = 10 (6,937 strategies)			
Mean	2.0611	2.2312	2.3564
Variance	0.5617	0.7048	0.8222
Sharpe Ratio	0.1955	0.2386	0.2646
Panel C: θ = 15 (5,687 strategies)			
Mean	3.8151	3.9311	4.1037
Variance	2.9947	3.2107	3.5469
Sharpe Ratio	0.2792	0.2864	0.2961

**Values in the table are rounded up to 4 decimal places. The Sharpe ratio is computed as $\left(\frac{E(\pi_\theta)-1}{\theta} - \bar{y}_\theta\right) / \left(\frac{\sqrt{var(\pi_\theta)}}{\sqrt{\theta}}\right)$ where $\bar{y}_\theta$ is the average yield of the planning horizon.

Figures 4, 5 and 6 in the appendix shows the histograms of outcomes for each of the strategies for θ is 5, 10 and 15 years respectively when we repeat the strategy over a thousand times. The outcome implies to the terminal value of the $1 invested at time 0. The histogram depicts that on repeating each strategy's outcomes daily, we get a highly skewed and a long right-tailed distribution. The distribution of all the histograms also indicates that the Sharpe ratio is solely an indicative performance metric and should not be used as an ultimate performance measure. This could give rise to a further discussion when using other metrics of performance. The goal in each of our immunization strategies is to try to make a trade-off on the shocks of the short end and the long end of the yield curve. Despite that, the strategy outcomes have a significant dispersion and variance because of the global volatility of the yield curve.

Subsequently, Figures 7, 8 and 9 replicates the in-sample analysis in Table 1 in a mean-variance space, by plotting the empirical mean and standard deviation of the immunization strategy using Federal data for all possible bond portfolio allocations (x) in the mean-variance space. Figures 7, 8 and 9 are the outcome of the in-sample analysis where we repeat ex-post many experiments. Hence, it is an empirical estimate that represents the Federal data that we have tested. The respective positions of weights induced by the Macaulay duration-based, the Cox-Ingersoll-Ross duration-based strategy as well as the weight induced by the minimum-variance immunization is highlighted. The investment planning horizons (θ) are 5, 10 and 15 years. The bond maturities are T_1 = 1 year and T_2 = 20 years. Additionally, the base case parameters used are α = 0.0329, β = 0.0257, λ = 0.4284, η = 0.0124.

Figure 7: In-sample empirical immunization in the mean-variance space (θ = 5)

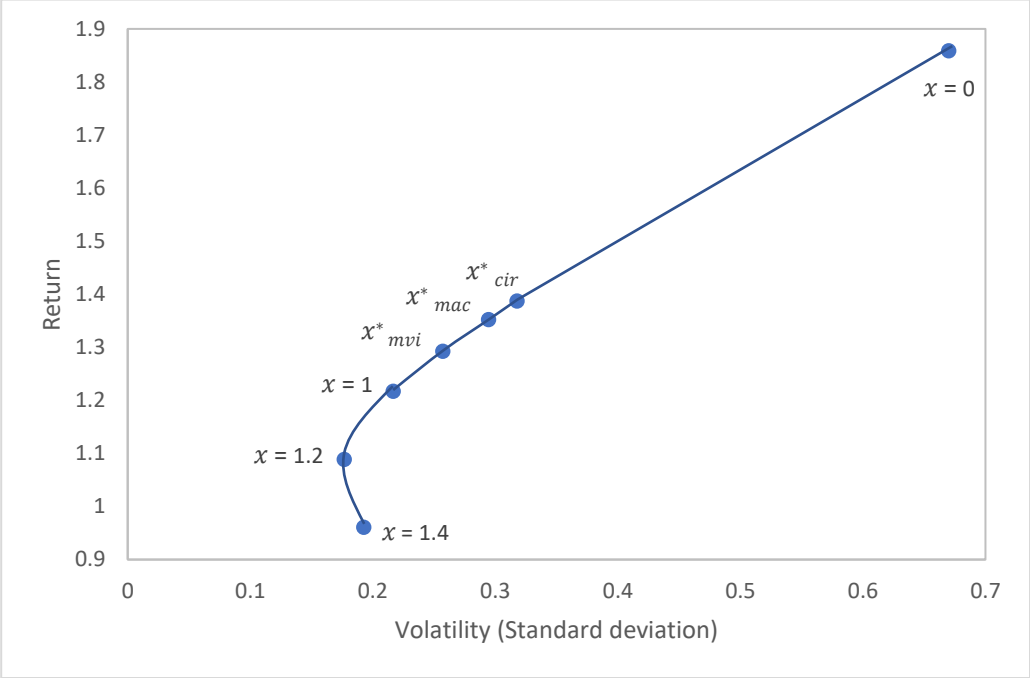


Figure 8: In-sample empirical immunization in the mean-variance space (θ = 10)

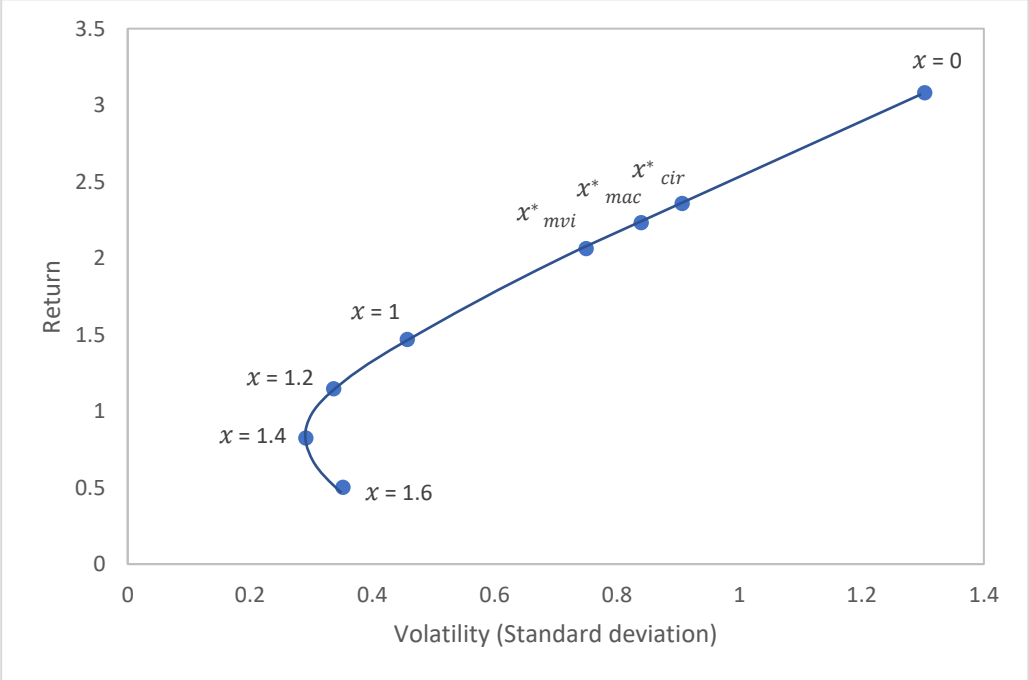
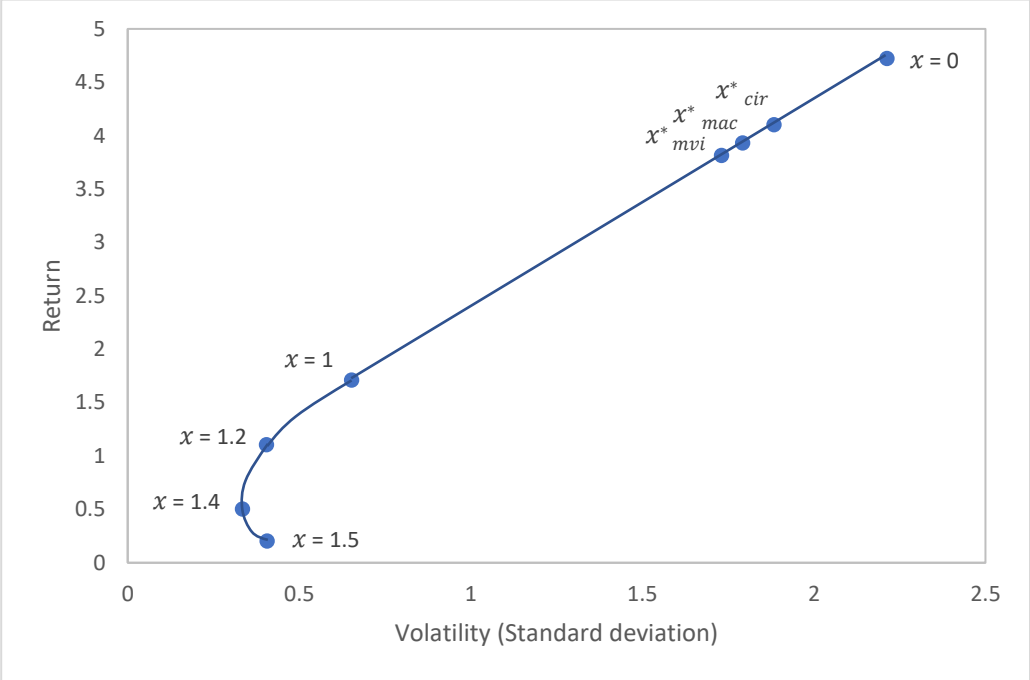


Figure 9: In-sample empirical immunization in the mean-variance space (θ = 15)



Furthermore, the portfolio allocation (x) is theoretically determined to match the duration in case of x^*_{mac} and x^*_{cir}. In the case of x^*_{mvi}, the portfolio allocation is determined to minimize the variance of the terminal value. The barbell strategy is implemented such that the short-term bond is rolled over, and the long-term bond is held until the investment planning horizon (θ).

The point of minimum variance depends on the characteristics of rolling horizon bond and long-term bond as well as their mean and variance. Theoretically speaking, a rolling horizon bond is generally more volatile as it is dependent on dynamic short-term rates. Hence, the x^*_{mvi} is relatively low in a theoretical calibration as it is a minimum-variance barbell strategy. On the contrary, when we move to empirical calibration using the real data (Federal data), the x^*_{mvi} is not as low as compared to theoretical estimate. Since we use fully overlapping strategies by

repeating thousands of experiments daily that differ only by a day, the resulting outcome of all the daily rolling horizon bonds is highly correlated with each other.

Additionally, the long-term bond relies on the price of a bond with T_2 maturity and T_2 - θ maturity. Hence, it depends on two points of the curve that are relatively less volatile and as a reason their experiments are relatively less correlated. Thus, due to the highly correlated experiments combined partly with model risk leads to an overall underestimated variance for the rolling bond. In this way, the minimum variance for θ = 5 is at a point x = 1.2, for θ = 10 and 15 is at point x = 1.4. Intuitively, an x greater than 1 means short selling the long-term bond and over-investing in the short-term bond.

Therefore, in Figures 7, 8 and 9 we can observe that the x^*_{mac} and x^*_{cir} are above the x^*_{mvi} on the efficient frontier curve. However, the minimum variance point is when x is 1.2, 1.4 and 1.4 respectively. This is the minimum point when the returns are maximized at a given level of volatility for the respective planning horizon.

Moreover, there are multiple implications for the investor while using the real data. Firstly, all the three strategies here yield mean-variance efficient results but the more the strategy is away from the minimum-variance point the riskier it is. Hence, if an investor aims to minimize the risk then the results of those strategies would be disappointing. On the other hand, if the investor is more aggressive and is a risk lover then he is probably happy with the outcome. Secondly, the Sharpe

ratios of all the strategies increase with the increase in the horizon. Thirdly, the differences in the performance between the three strategies decreases as the horizon increases. Finally, these investor implications for the in-sample analysis also applies for the out-of-sample analysis, which makes our conclusions robust.

Hence, we can conclude that in the mean-variance space, the minimum-variance immunization strategy as a benchmark against Macaulay's duration and stochastic duration, has clearly outperformed the traditional duration-based strategies.

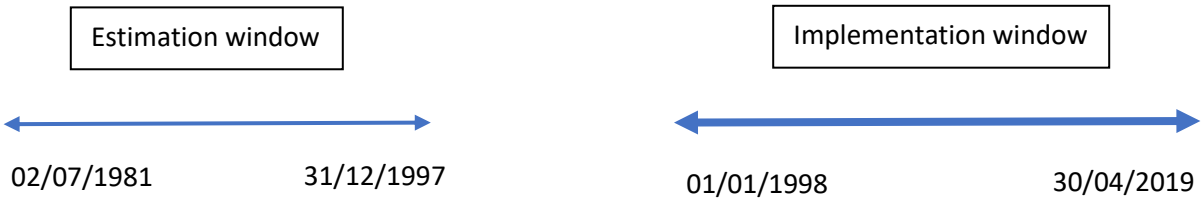
4. Out-of-sample Analysis

4.1. Data and estimation

For the out-of-sample analysis, we implement a historical approach on actual U.S. term structures to understand how our theoretical models work without any assumptions. Out-of-sample analysis indicates that how in real-time one can implement the strategies and considers the model risk exposure that the investor faces using the barbell strategy on stochastic duration-based and minimum-variance immunization strategies. Macaulay's duration-based strategy is not affected as it is model-free. Thus, it portrays how a market participant would have invested in a bond portfolio using the following strategies with the knowledge of the historical data. In the real world, an investor doesn't anticipate how the evolution of the rates will flow. Hence, the output of the out-of-sample analysis is what performance (in terms of expected returns and volatility) an investor can expect on an average from implementing the three different strategies.

In case of conducting out-of-sample analysis, we don't assume the base case parameters (α = 0.0329, β = 0.0257, λ = 0.4284, η = 0.0124) unlike the in-sample analysis. Our historical estimation window is from the years July 2nd, 1981 to December 31st, 1997 of the Federal data and our implementation window is from January 1st, 1998 to the end of our data. We compute the terminal value of \$1 invested in barbell immunization strategies that use the T_1= 1-year, T_2 = 20-year bonds and the re-estimated parameters. We repeat it yearly (every 250 trading days) from January 1st, 1998 until the end of available data in our sample (ends on April 30, 2019) for investment planning horizons of 5, 10, and 15 years employing a rolling window.

Figure 10: Historical approach for out-of-sample analysis



The window from 1981 to 1997 is to get acquainted with the dynamics in those periods and compute parameters from the historical estimation window. The knowledge grasped and the parameters estimated from the first part of the window are then applied to the second part of the window by applying all the strategies to the data starting from January 1st, 1998 to April 30th, 2019, annually. In this way, our parameters are computed on one part of the dataset and then implemented to the rest of the data on a rolling basis.

Macaulay's duration-based strategy is not dependent on any external base case parameters. Thus, we compute the Macaulay's immunization strategy from January 1st, 1998 to the end of the sample by employing the x^*_{mac} from the rolling window. Hence, we get various strategy outcomes for each planning horizon but a constant x^*_{mac}. Subsequently, the stochastic duration-based strategy is a non-linear transformation of the Macaulay's strategy. Therefore, it requires an additional parameter α, which we recompute using the historical estimation window. Our α parameter is recomputed for every rolling window that we slide over by 250 trading days over the historical period. As a result, we recompute our bond portfolio allocation for the stochastic duration-based strategy (x^*_{cir}). Hence, we use different weights computed over the historical

estimation window to recompute the outcome of the new strategy using the implementation window.

$$\alpha = - \frac{cov(r_t, \Delta r_t)}{var(r_t)}$$

where,

$r_{t=}$ One-year zero-coupon rate

Δr_t = First difference of the one-year zero-coupon rate

Similarly, the minimum-variance immunization strategy requires the variance and covariance of the rolling horizon bond and the long-term bond. Thus, these statistics are recomputed for every rolling window that we keep sliding over our historical estimation window to recalibrate our bond portfolio allocation for minimum-variance strategy (x^*_{mvi}).

$$x^*_{mvi} = \frac{V_2 - V_{12}}{V_1 + V_2 - 2V_{12}}$$

Where,

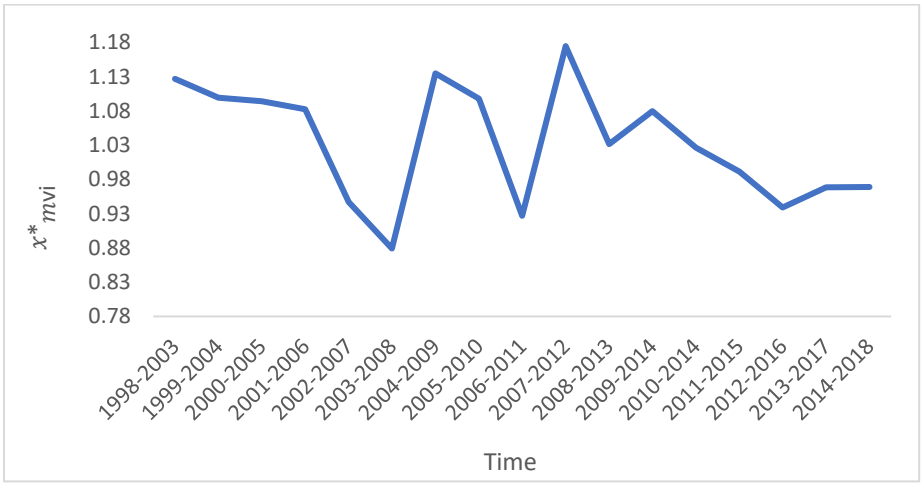
V_1 = Variance of the rolling horizon bond

V_2 = Variance of the long-term bond

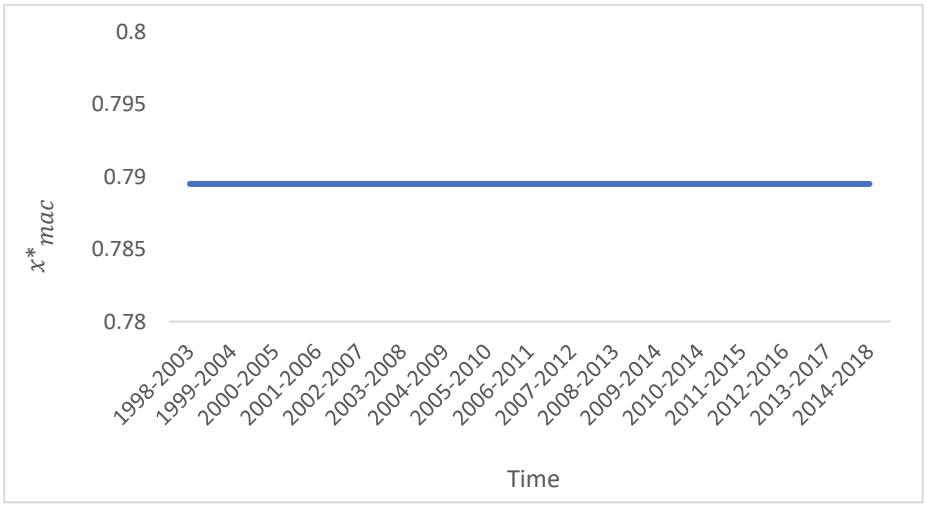
V_{12} = Covariance between the rolling horizon bond and the long-term bond

Figure 11: Bond portfolio allocations for the empirical immunization model (θ = 5)

a.



b.



c.

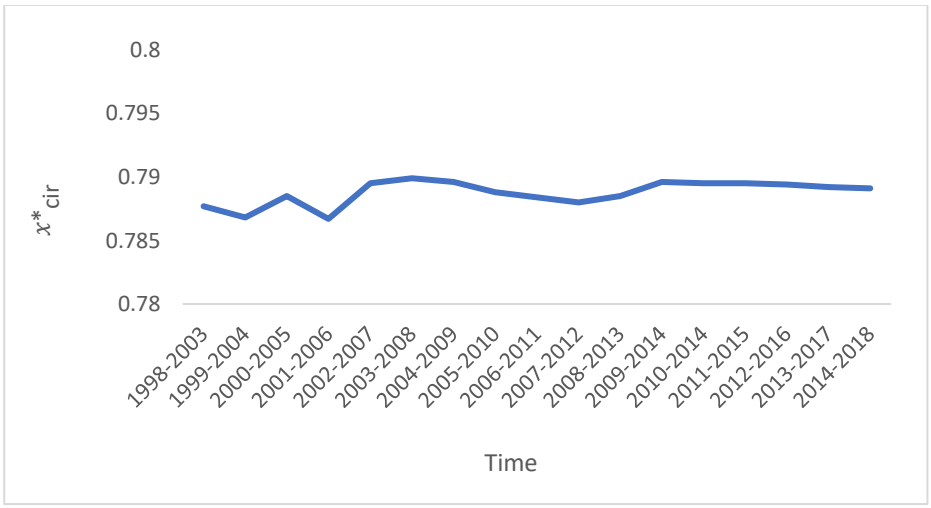
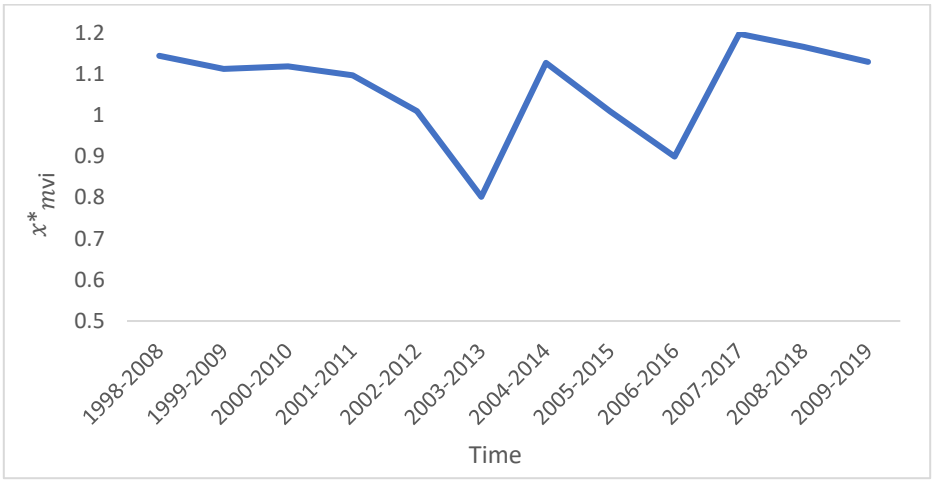
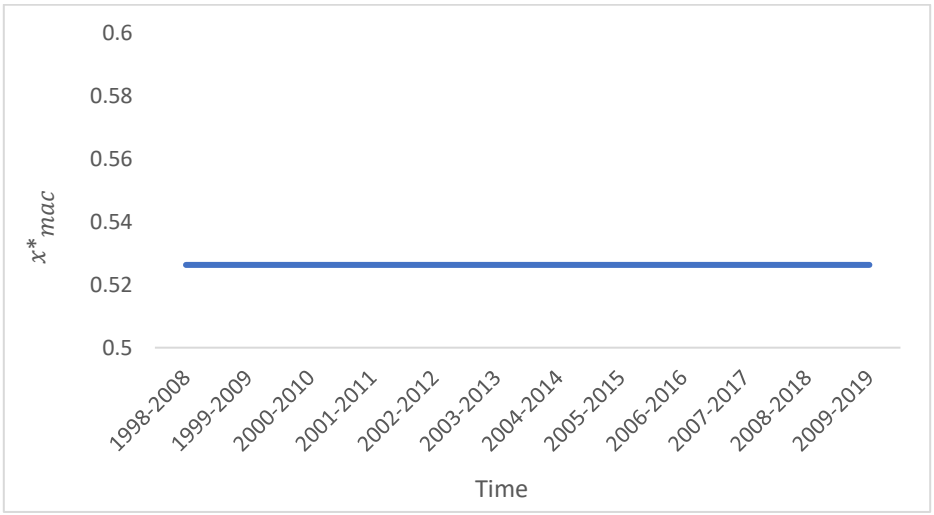


Figure 12: Bond portfolio allocations for the empirical immunization model (θ = 10)

a.



b.



c.

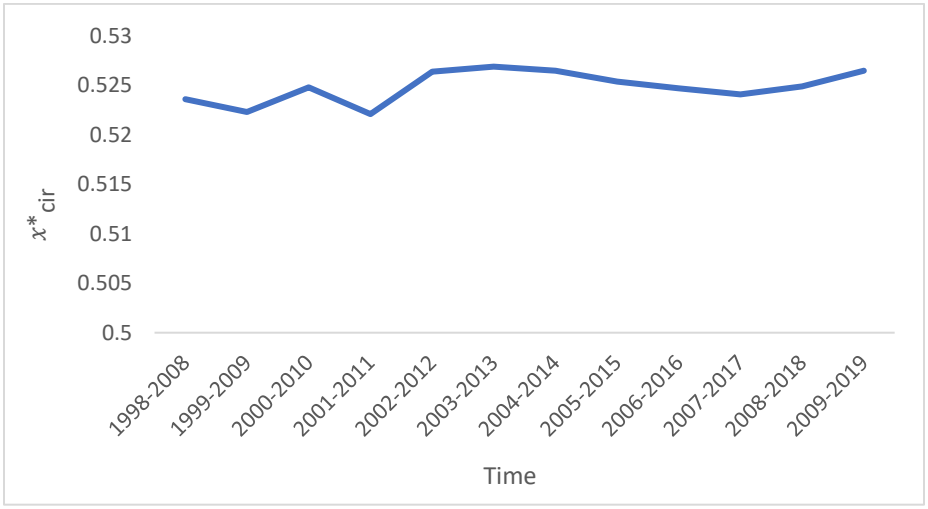
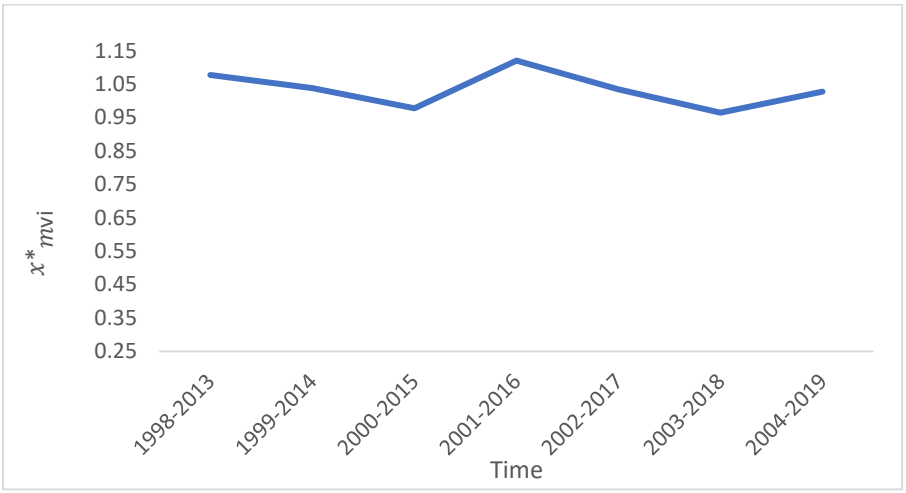
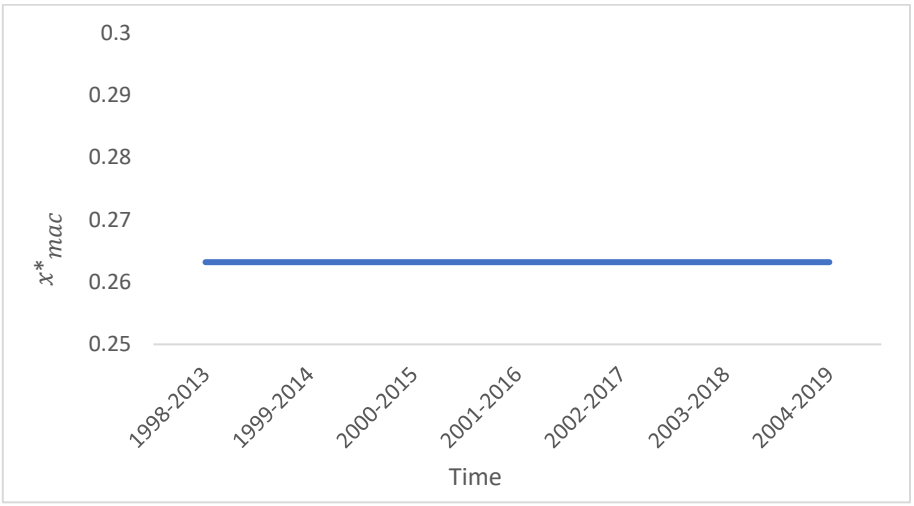


Figure 13: Bond portfolio allocations for the empirical immunization model (θ = 15)

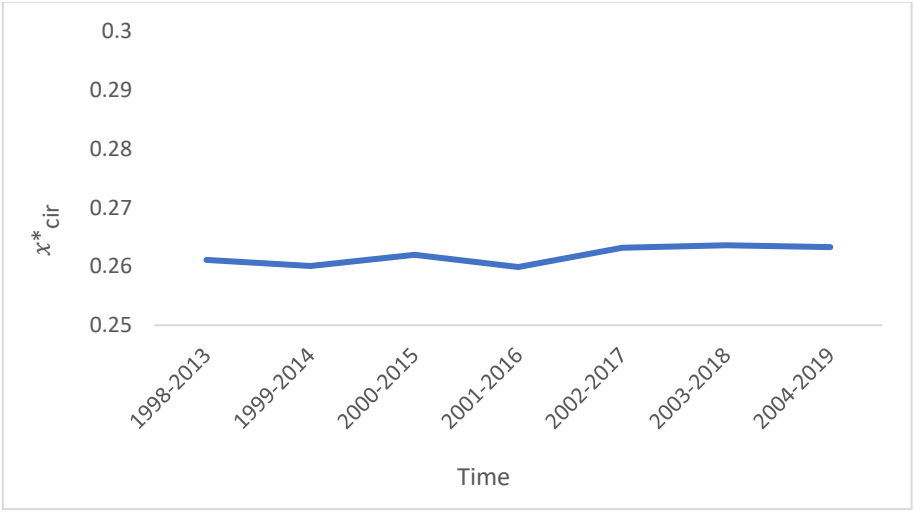
a.



b.



c.



Figures 11, 12 and 13 show the dynamic nature of x^*_{mvi} and x^*_{cir}, whereas, a constant level of x^*_{mac} for each θ. x^*_{mvi} has comparatively a greater level of variation compared to x^*_{cir}. A large goal of performing an out-of-sample analysis is to make sure that the strategies are valid in the long-run and it can be implemented by an investor without knowing the rate dynamics in a real environment. Thus, the reason behind taking the same data periods used for the in-sample analysis and then splitting it out for the out-of-sample analysis so that we can take maximum advantage of the limited data that is at our disposal. In this way, we can implement the strategies for a greater number of business cycles which gives us more reliable and true results. However, our results could be highly biased due to the high dependence and correlation. Therefore, we try to reduce our estimation bias by implementing the bond portfolio allocations in the strategy outcome on a yearly rolling basis compared to computing it daily.

In this study, we have short-term and long-term strategies that have strong constraints in implementing them in the current time frame where we are restricted to the end of April 2019 in our datasets. Thus, our methodological standpoint is to take advantage of the maximum number of points.

4.2. Results

Table 2: Out-of-sample performance statistics for immunization strategies

	Minimum Variance Immunization	Duration-based Immunization	
		Macaulay duration	Stochastic duration
Panel A: θ = 5			
Mean	1.0325	1.1520	1.1523
Variance	0.0018	0.0032	0.0032
Sharpe Ratio	-1.5448	-0.5019	-0.4994
Panel B: θ = 10			
Mean	1.0635	1.6261	1.6276
Variance	0.0164	0.0216	0.0216
Sharpe Ratio	-2.5318	1.5554	1.5823
Panel C: θ = 15			
Mean	1.0917	2.4791	2.4814
Variance	0.0065	0.0625	0.0629
Sharpe Ratio	-7.9314	4.7183	4.7091

**Values in the table are rounded up to 4 decimal places. The Sharpe ratio is computed as $\left(\frac{E(\pi_\theta)-1}{\theta} - \bar{y}_\theta\right) / \left(\frac{\sqrt{var(\pi_\theta)}}{\sqrt{\theta}}\right)$ where $\bar{y}_\theta$ is the average yield of the planning horizon.

In comparison with the results obtained from the in-sample model on the Federal data, we can see that the Table 2 has the mean and variance of all the strategies are lower in magnitude. In addition, the stochastic and Macaulay's duration-based strategies have a close mean and variance

output. The underlying reason is that, as we have mentioned before that the minimum-variance immunization strategy and stochastic duration-based strategy are affected by the historical approach whereas, the Macaulay's duration-based strategy is not affected since it is a model-free approach. Hence, the historical methodology could be a driving factor for the stochastic duration-based strategy to overlap the Macaulay's strategy.

Figures 14, 15 and 16 replicates the out-of-sample analysis in Table 3 in a mean-variance space, by plotting the mean and standard deviation of the immunization strategy using Federal data for all possible bond portfolio allocations (x). They are an outcome of the out-of-sample analysis where we repeat ex-post many experiments. Hence, it is an estimate that represents the Federal data that we have tested.

The graphs below plot the empirical mean and the standard deviation of the immunization strategy using the U.S. data. The respective positions of weights induced by the Macaulay duration-based, the Cox-Ingersoll-Ross duration-based strategy as well as the weight induced by the minimum-variance immunization is highlighted. Investment planning horizon (θ) is 5, 10 and 15 years respectively. The bond maturities are $T_1 = 1$ year and $T_2 = 20$ years.

Figure 14: Out-of-sample empirical immunization in the mean-variance space (θ = 5)

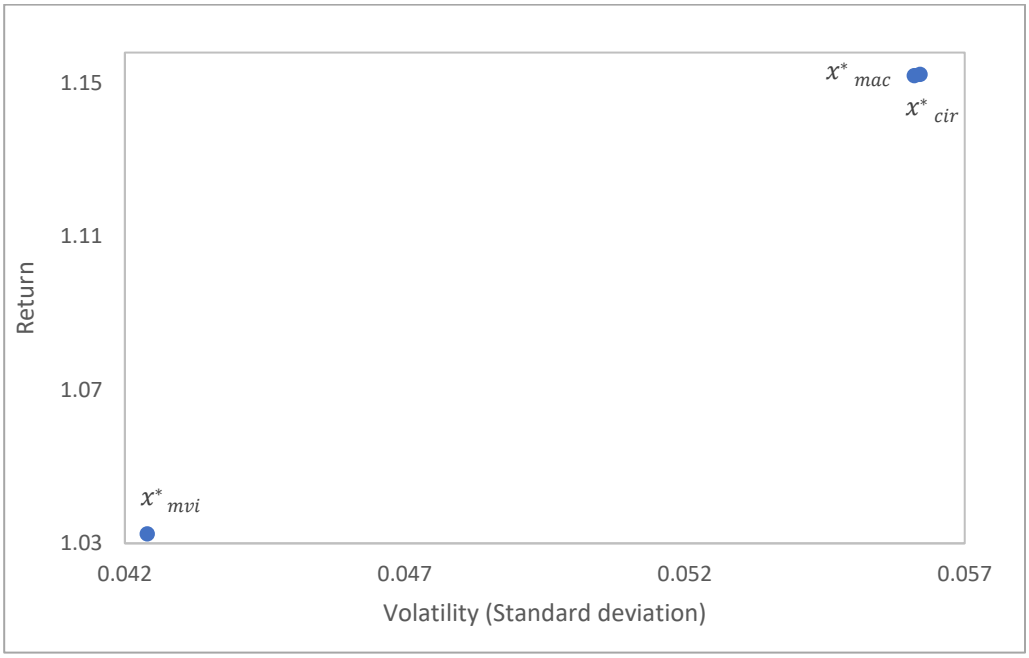


Figure 15: Out-of-sample empirical immunization in the mean-variance space (θ = 10)

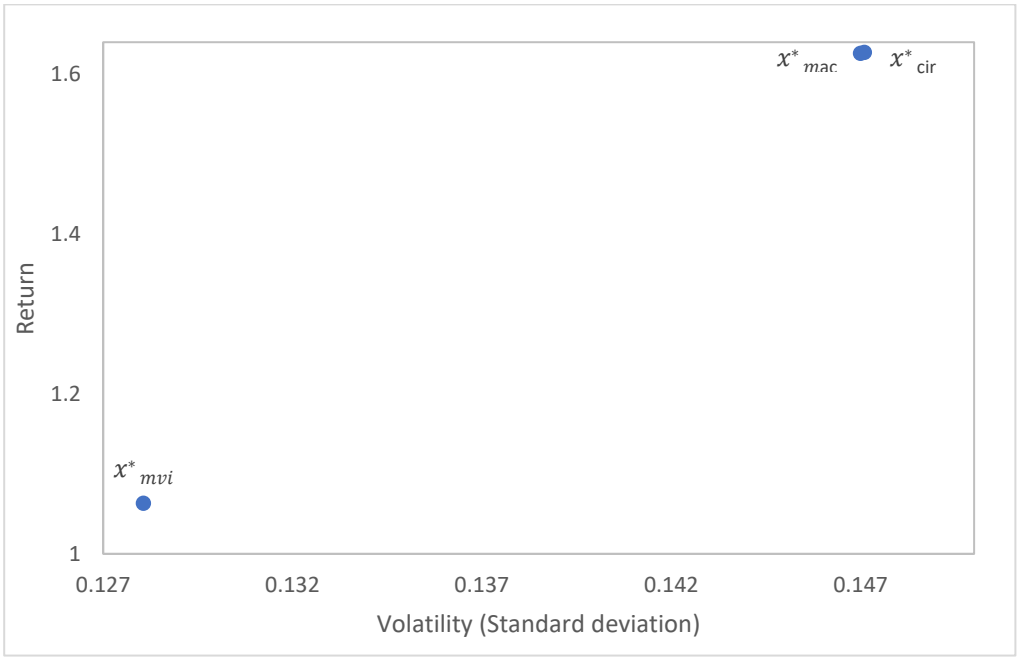
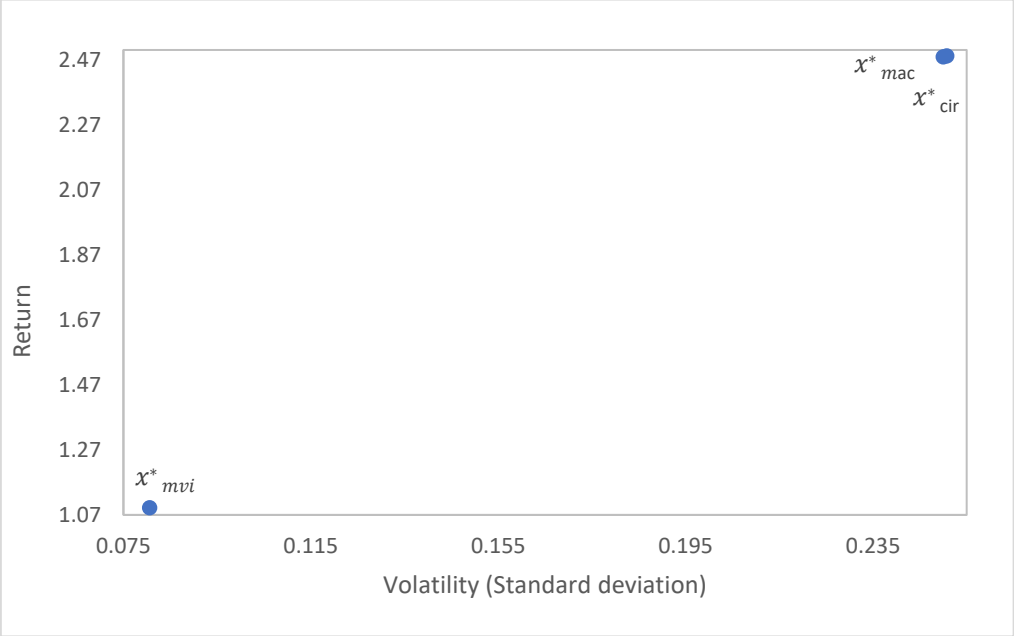


Figure 16: Out-of-sample empirical immunization in the mean-variance space (θ = 15)



In Figures 14, 15 and 16, we can observe that the x^*_{mac} and x^*_{cir} are above the x^*_{mvi}. Here, we can observe that the two traditional duration-based strategies have an almost overlapping strategy outcomes of a close mean and variance. However, the stochastic duration-based strategy is yet marginally above the Macaulay's duration-based strategy. On the other hand, the minimum-variance strategy is below both the traditional duration-based strategies. The points plotted in these figures refer to the average position of the respective strategies in our implementation sample period from January 1, 1998 to April 30, 2019.

In the out-of-sample analysis, we are implementing immunization strategies every consecutive year with updated parameters using historical approach in the dynamic Federal environment.

Hence, unlike the in-sample analysis our efficient frontier curve is not constant but keeps moving with the updated parameter values. However, the efficient frontier curve does exist in one point in time in the mean-variance space using a model with calibrated parameter values. Thus, we have only highlighted the average position weights (x^*_{mac}, x^*_{cir} and $x^*_{mvi)}$ for the Macaulay's duration-based strategy, stochastic duration-based strategy and minimum-variance strategy.

We are aware that our historical approach is a drawback to a certain level as the present level of interest rates are not always explained by the past behavior of interest rates. Hence, we have used a similar historical estimation window across all the strategies. The mean-variance immunization strategy and stochastic duration-based strategy are affected by the historical approach whereas the Macaulay's duration-based strategy is unaffected as it is a model-free approach. Therefore, we have used the same interest rate environment for all. Moreover, the point of our study is not to prove whether the mean-variance immunization strategy is a superior approach, but we are simply interested in benchmarking the mean-variance immunization strategy to the traditional duration-based strategy.

5. Robustness Checks

5.1. Data and estimation

We further compute robustness checks, which is essentially checking how our immunization-based strategies behave when we implement them on a different dataset. Robustness of a model is a common methodology in empirical studies to check how the model performs when it is put in a different environment compared to the data used for the in-sample analysis data. It is simply an exercise to check that our model is not biased or skewed to an outcome due to the Federal data used in our in-sample analysis. Thus, in this case, we use Canadian data instead of the U.S data that we used in the in-sample analysis. The structure of the analysis is on the same lines with the in-sample analysis.

For the robustness check analysis, we implemented immunization strategies on actual Canadian term structure data to understand how our theoretical models work on real interest rate yields.

We constructed the daily term structure of interest rates using the data from the Bank of Canada. The daily yield curves for zero-coupon bonds were generated by the Bank of Canada using the pricing data for the government of Canada bonds and Treasury bills. We extracted the zero-coupon spot rates from the Bank of Canada for maturities from 4 months to 20 years with a quarterly pattern. Our sample period starts on January 2, 1986 and ends on April 30, 2019, which gives us a total of 8,164 daily observations.

The forward rates were missing so we computed instantaneous forward rate using the following theoretical formula:

$$\left[\frac{(1+\mathrm{R(s)})^{1.25}}{(1+\mathrm{R(t)})}-1\right]\times 100$$

Where,

$\mathrm{R(t)} = \text{zero} - \text{coupon rate with maturity of 1 year}$

$\mathrm{R(s)} = \text{zero} - \text{coupon rate with maturity of 1.25 years}$

Like the in-sample analysis, to be able to determine the bond portfolio allocations, we need the Vasicek model parameters that should be in lines with our data sample. First, we implement the Vasicek model to understand the theoretical performance of Macaulay's duration-based strategy, the Cox-Ingersoll-Ross duration-based strategy and the minimum-variance immunization strategy in the mean-variance space. Thus, we assume the same base case parameters as $\alpha = 0.052$, $\beta = 0.0267$, $\lambda = 0.1329$, $\eta = 0.0286$ that we apply to the whole period of our data from January 2, 1986 to April 30, 2019. We compute ex-post robustness check analysis on Canadian data and compute what the investor on an average would receive as the output of the strategies as in the Table 3 had he known in advance about the interest rate dynamics.

5.2. Results

5.2.1. Theoretical model immunization

We compute the terminal value of $1 invested in barbell immunization strategies that use the T_1= 1-year bonds and the T_2 = 20 years bonds daily from January 2, 1986 until April 30th, 2019. Figures 17, 18 and 19 plot the theoretical mean and standard deviation of the immunization strategy using the Vasicek model for all possible bond portfolio allocations (x) for the investment planning horizons (θ) of 5, 10, and 15 years respectively. For the immunization in the mean-variance space, we particularly highlight five cases for the bond portfolio allocations), similar to the in-sample analysis.

Figure 17: Robust theoretical immunization in the mean-variance space (θ = 5)

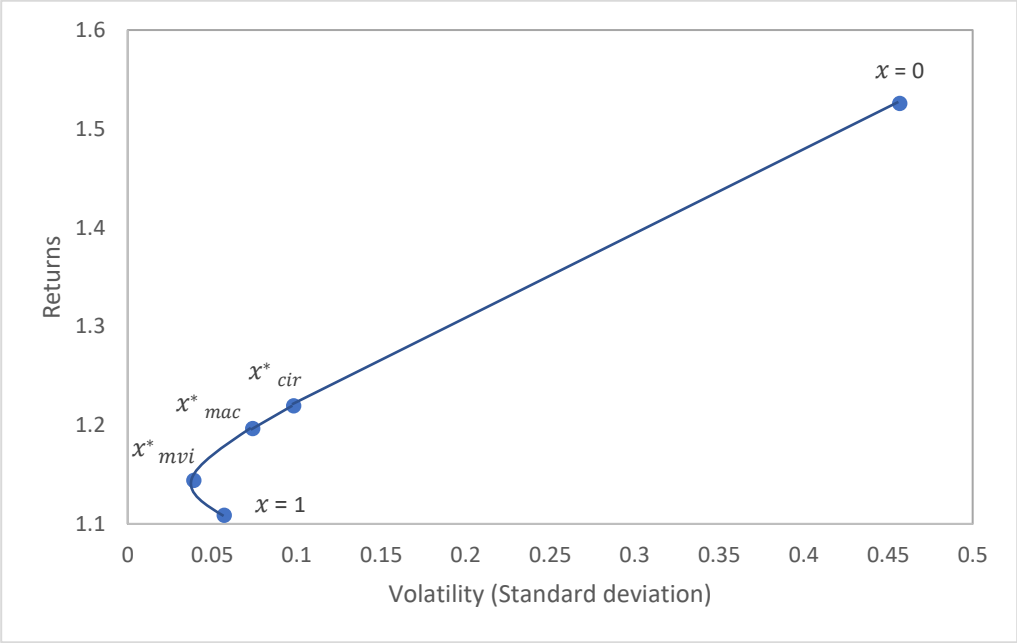


Figure 18: Robust theoretical immunization in the mean-variance space (θ = 10)

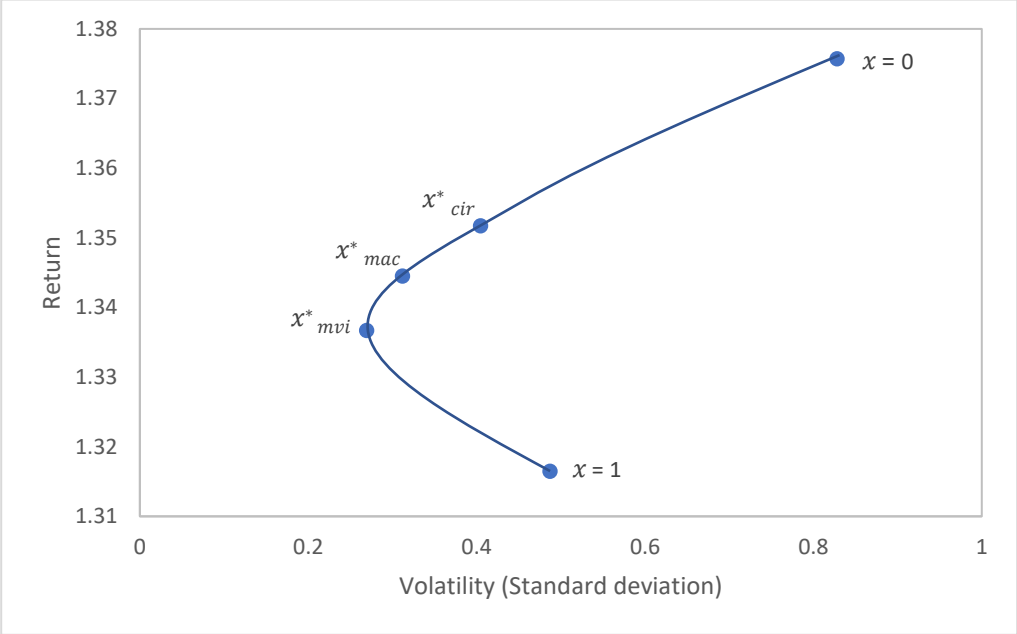
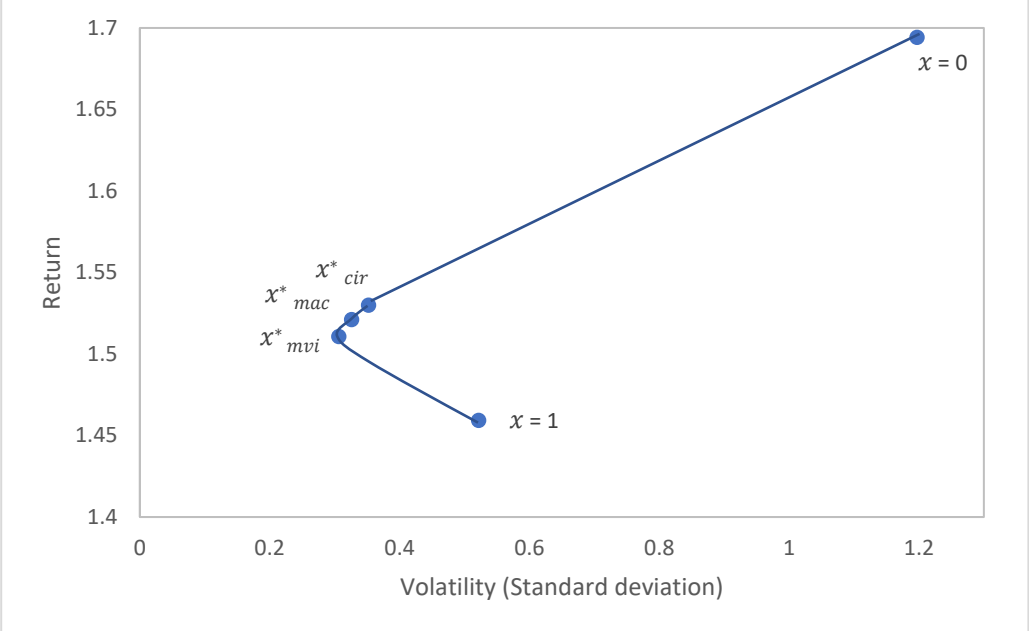


Figure 19: Robust theoretical robust immunization in the mean-variance space (θ = 15)



Figures 17, 18 and 19 shows the theoretical outcome of the bond portfolio allocations using the various immunization strategies, strictly under the Vasicek model. When $x = 1$, $x = 0$, x^*_{mac}, x^*_{mvi}, x^*_{cir} along with $x = 1.2$ and $x = 1.4$ in the theoretical immunization mean-variance space. Hence, the investor can use these bond portfolio allocations today for a planning horizon of 5, 10 and 15 years and project that the average outcome will be the respective return and volatility of the strategy. In Figures 17, 18 and 19, we can observe that the x^*_{mvi} has the minimum variance with a weight lesser than 1.

5.2.2. Empirical model immunization

Now, we compute empirically, using the Canadian data i.e. had the investor known the exact dynamics of the interest rate parameters then one would implement the immunization strategies and obtain the results as per the Table 3.

We compute the terminal value of $1 invested in barbell immunization strategies that use the T_1= 1-year bonds and the T_2 = 20-year bonds daily from January 2, 1986 until April 30, 2019. For our investment planning horizons of 5, 10, and 15 years, we have a total of 6,914, 5,664 and 4,414 immunization strategies computed daily, respectively.

Table 3: Robustness performance statistics for immunization strategies

	Minimum Variance Immunization	Duration-based Immunization	
		Macaulay duration	Stochastic duration
Panel A: θ = 5 (6,914 strategies)			
Mean	1.1367	1.2008	1.2679
Variance	0.0043	0.0095	0.0174
Sharpe Ratio	-0.7490	-0.2087	0.0734
Panel B: θ = 10 (5,664 strategies)			
Mean	1.9743	2.0069	2.2441
Variance	0.1943	0.2079	0.3211
Sharpe Ratio	0.3129	0.3250	0.3939
Panel C: θ = 15 (4,414 strategies)			
Mean	3.5578	3.6509	3.9490
Variance	1.0064	1.0785	1.3388
Sharpe Ratio	0.4380	0.4462	0.4670

**Values in the table are rounded up to 4 decimal places. The Sharpe ratio is computed as $\left(\frac{E(\pi_\theta)-1}{\theta} - \bar{y}_\theta\right) / \left(\frac{\sqrt{var(\pi_\theta)}}{\sqrt{\theta}}\right)$ where $\bar{y}_\theta$ is the average yield of the planning horizon.

In Table 3, we can observe that the robustness check produces similar output as in Table 1, where we observe that minimum-variance immunization strategy has lower mean and variance compared to the Macaulay's duration-based strategy and the stochastic duration-based strategy.

Figures 20, 21 and 22 shows the histograms of outcomes for each of the strategies when θ is 5, 10 and 15 years respectively. The outcome implies to the terminal value of the $1 invested at time 0. The histogram depicts that on repeating each strategy's outcomes daily, we get bimodal distributions. The distribution of all the histograms also indicates that the Sharpe ratio is solely an indicative performance metric and should not be used as an ultimate performance measure. The strategy outcomes have a significant volatility because of the global volatility of the yield curve in the Canadian market.

Subsequently, Figures 23, 24 and 25 replicates the robustness analysis in Table 3 in a mean-variance space, by plotting the empirical mean and standard deviation of the immunization strategy using Federal data for all possible bond portfolio allocations (x) in the mean-variance space. These figures are the outcome of the robustness analysis where we repeat ex-post many experiments. Hence, it is an empirical estimate that represents the Canadian data that we have tested. The respective positions of weights induced by the Macaulay duration-based, the Cox-Ingersoll-Ross duration-based strategy as well as the weight induced by the minimum-variance immunization is highlighted. The investment planning horizons (θ) are 5, 10 and 15 years. The bond maturities are T_1 = 1 year and T_2 = 20 years. Additionally, the base case parameters used are $\alpha = 0.052$, $\beta = 0.0267$, $\lambda = 0.1329$, $\eta = 0.0286$.

Figure 23: Robust empirical immunization in the mean-variance space (θ = 5)

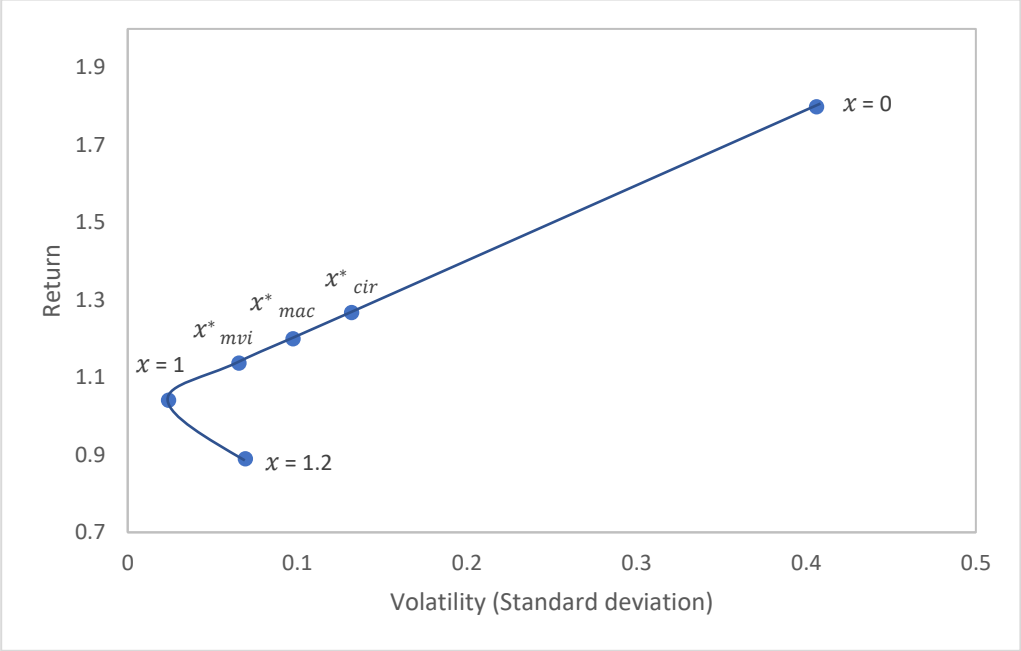


Figure 24: Robust empirical immunization in the mean-variance space (θ = 10)

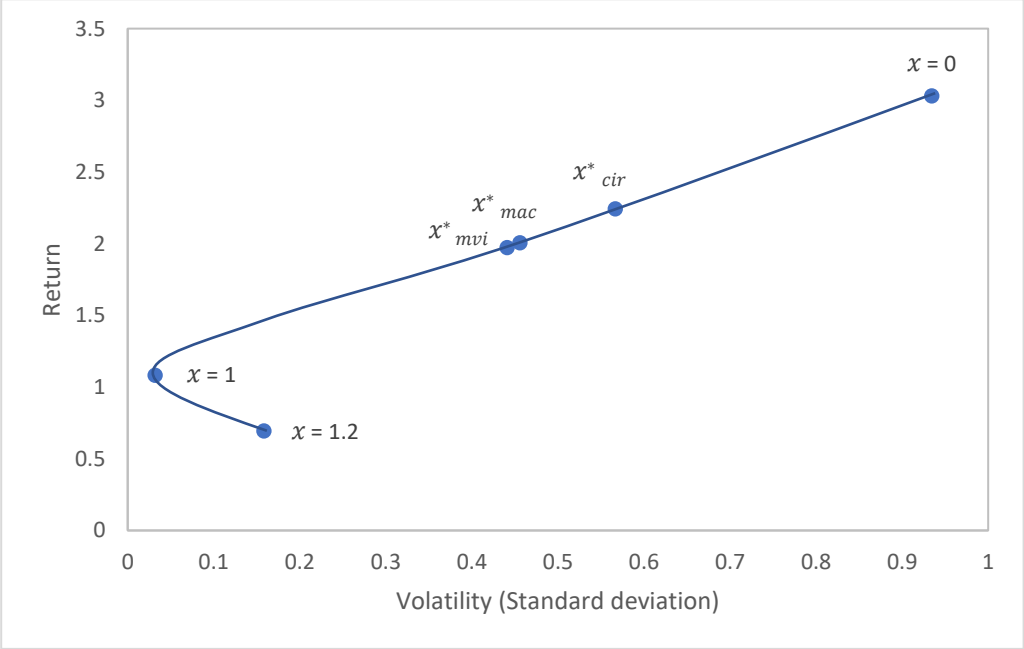
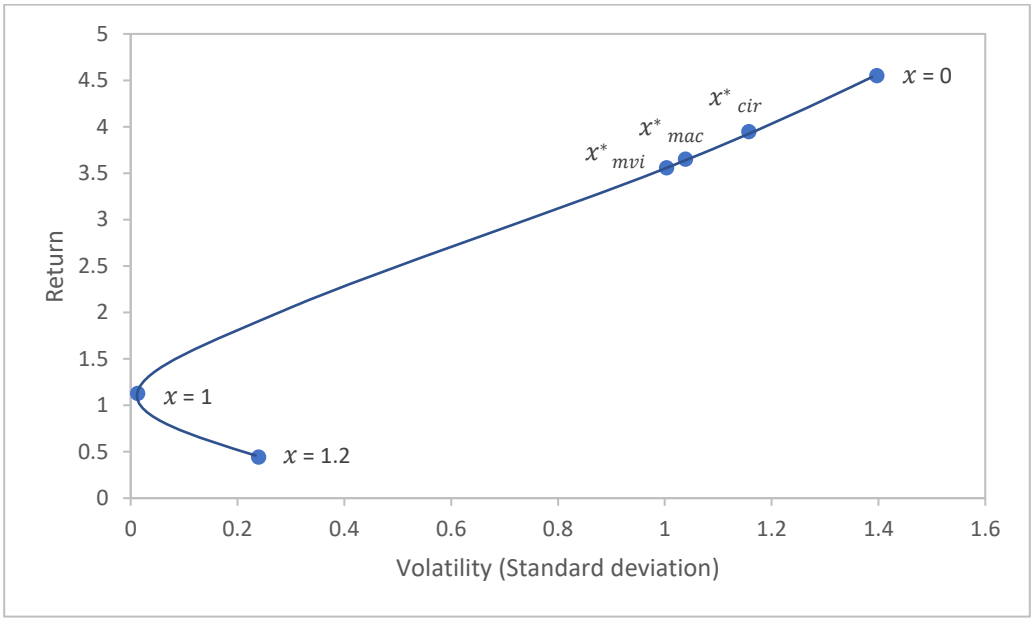


Figure 25: Robust empirical immunization in the mean-variance space (θ = 15)



In all the graphs of empirical immunization in the mean-variance space, the minimum variance point is at $x = 1$. When we compute the empirical estimate using the real data (Canadian data), the x^*_{mvi} is not as low as compared to the theoretical estimate. An x equal to 1 indicates fully investing in the short-term bond and nothing in the long-term bond. Therefore, due to the highly correlated experiments combined partly with model risk leads to an overall underestimated variance for the rolling bond in the empirical experiment.

Reiterating the implications for the investor as pointed out in the in-sample analysis. All the above immunization strategies yield mean-variance efficient results but the greater the strategy is away from the minimum-variance point the higher the risk it carries. Hence, a risk-averse investor will not be satisfied with the results of the strategies, but the risk-lover will probably be happy with the outcome. Additionally, the Sharpe ratios of all the strategies increase with the increase in the

horizon. Lastly, the differences in the performance between the three strategies decreases as the horizon increases.

6. Conclusion

In our in-sample analysis, we have presumed that we have knowledge of the model that is generating the data. Hence, we could be fitting the model if the model is poor and missing important features of the yield curve. However, in the out-of-sample analysis, we go a step further and compute the x^*_{mac}, x^*_{cir} and x^*_{mvi} by purely relying on the Federal data and no other presumed information. Therefore, we are testing the Vasicek interest rate model, its ability to capture the yield curve dynamics and check for model risk. Finally, we perform robust checks on Canadian data, to understand if the model and results were biased due to a particular data environment.

The underlying reason for our empirical study is not to prove that the minimum-variance immunization strategy is a good or bad approach in absolute terms, but we are simply interested in benchmarking the minimum-variance immunization strategy to the traditional duration-based strategies. Our in-sample analysis, out-of-sample analysis and robustness checks, all indicate that the minimum-variance immunization strategy has relatively outperformed the Macaulay's and stochastic duration-based strategy under the mean-variance framework using the Vasicek model.

Our empirical work is based on the Vasicek model as it is one of the simple interest rate models. However, further investigation and research could be performed based on a more complex model where the x^*_{mvi} could be derived numerically.

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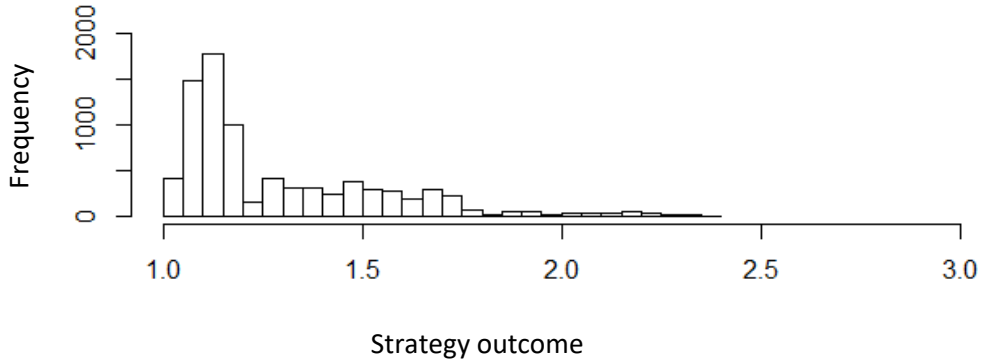
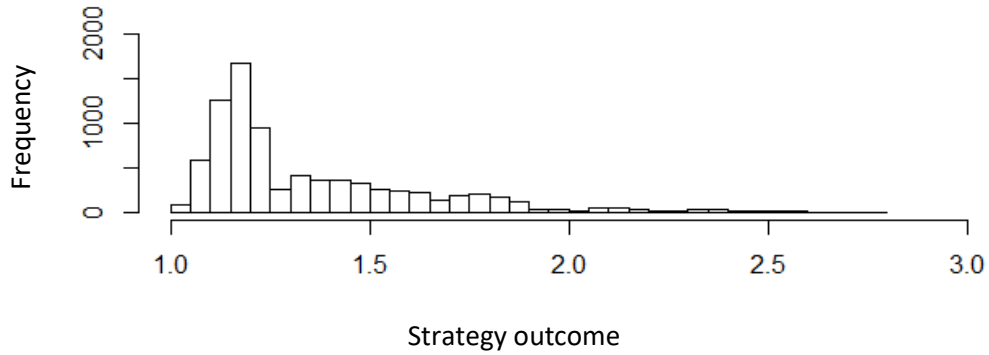
Appendix

Figure 4: In-sample histograms of strategy outcomes (θ = 5)

a. Minimum-variance duration-based



b. Macaulay's duration-based strategy



c. Stochastic duration-based strategy

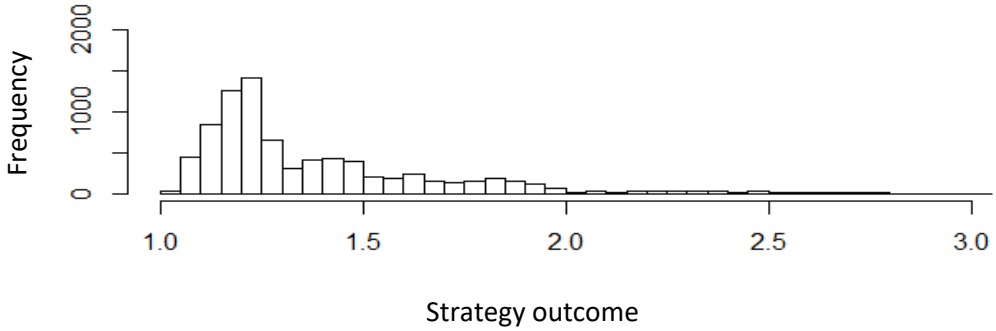
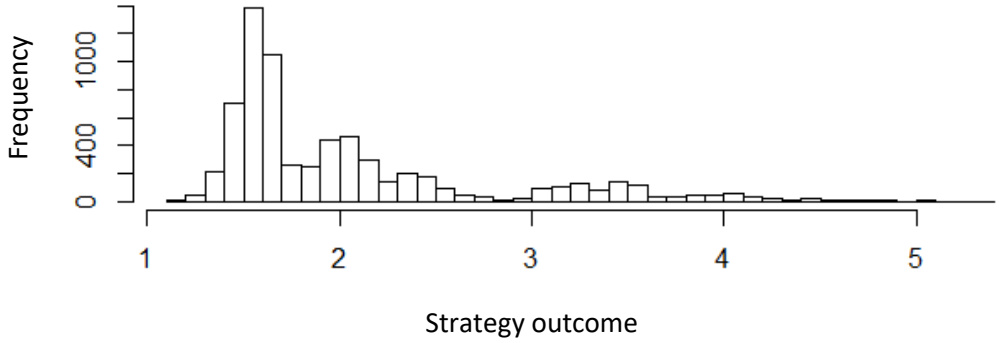
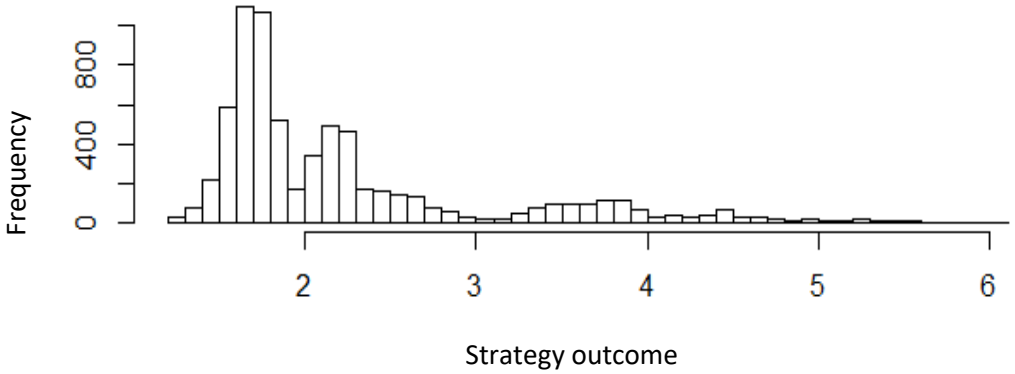


Figure 5: In-sample histograms of strategy outcomes (θ = 10)

a. Minimum-variance duration-based



b. Macaulay's duration-based strategy



c. Stochastic duration-based strategy

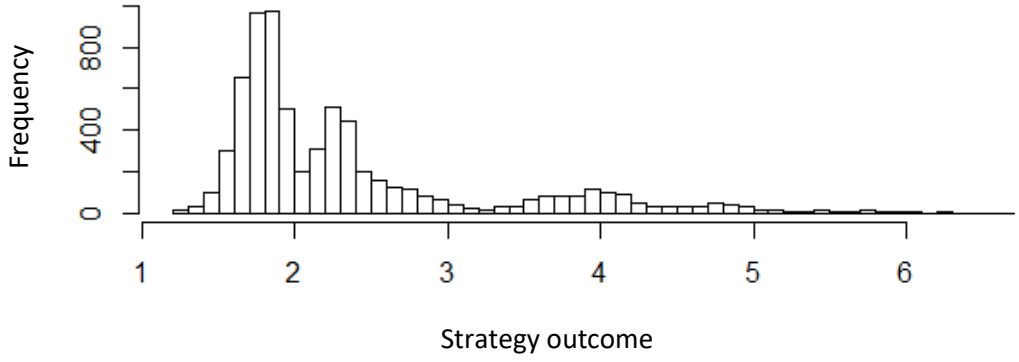
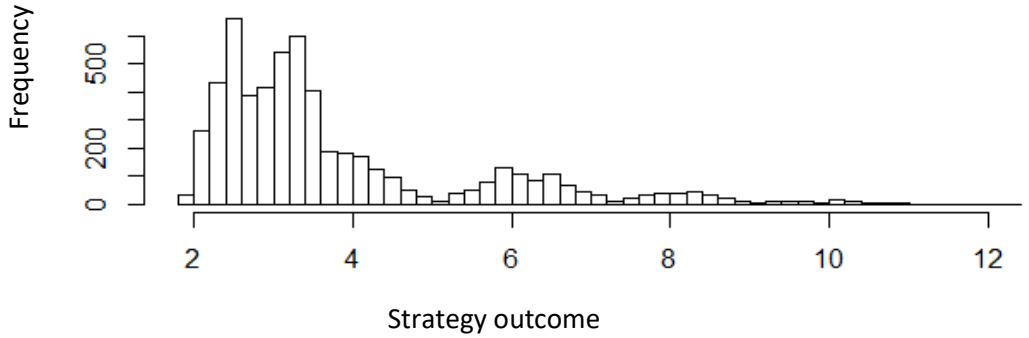
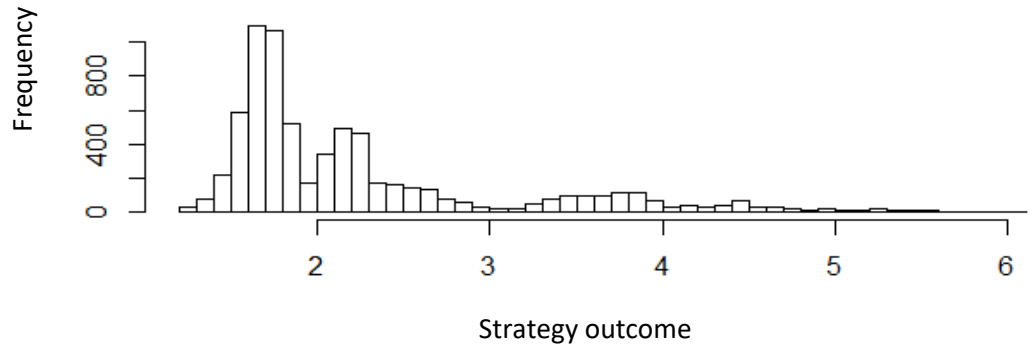


Figure 6: In-sample histograms of strategy outcomes (θ = 15)

a. Minimum-variance duration-based



c. Macaulay's duration-based strategy



b. Stochastic duration-based strategy

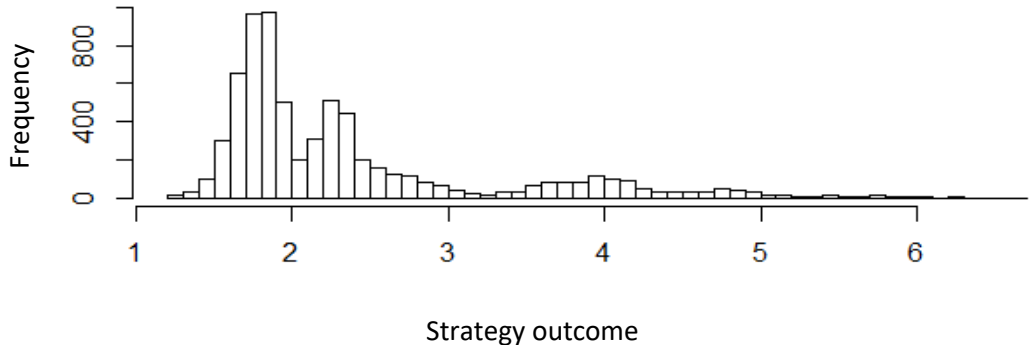
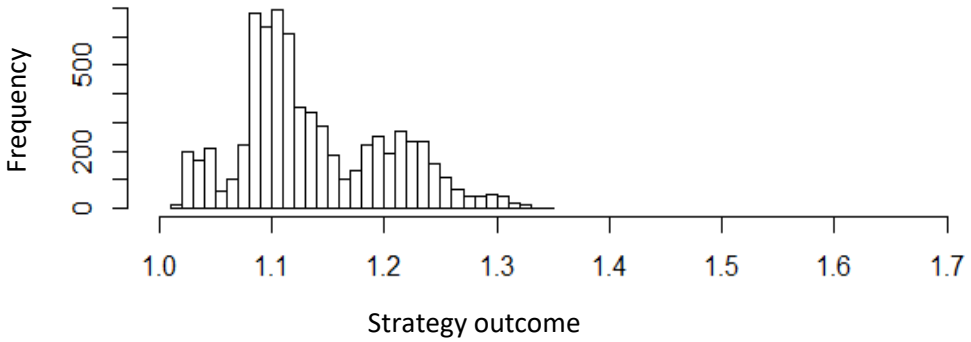
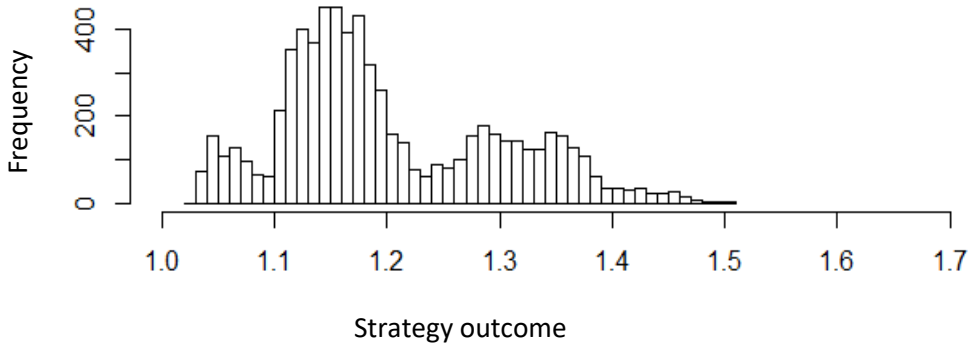


Figure 20: Robustness histograms of strategy outcomes (θ = 5)

a. Minimum-variance duration-based



b. Macaulay's duration-based strategy



c. Stochastic duration-based strategy

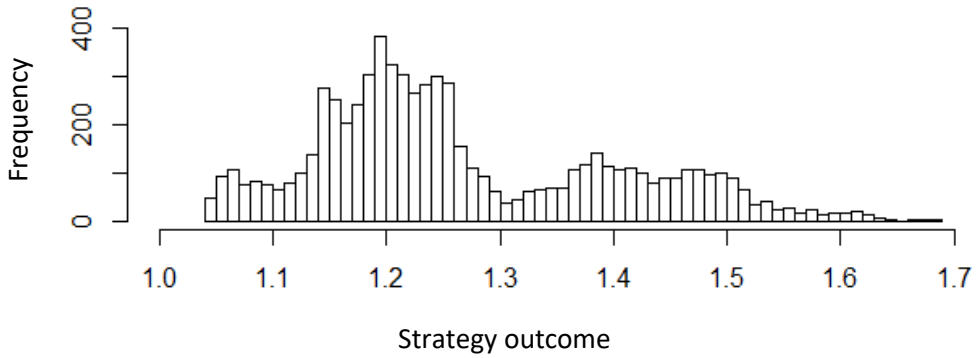
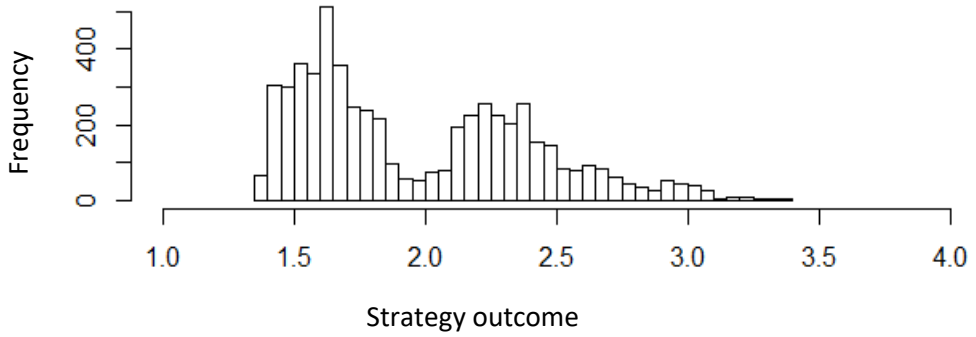
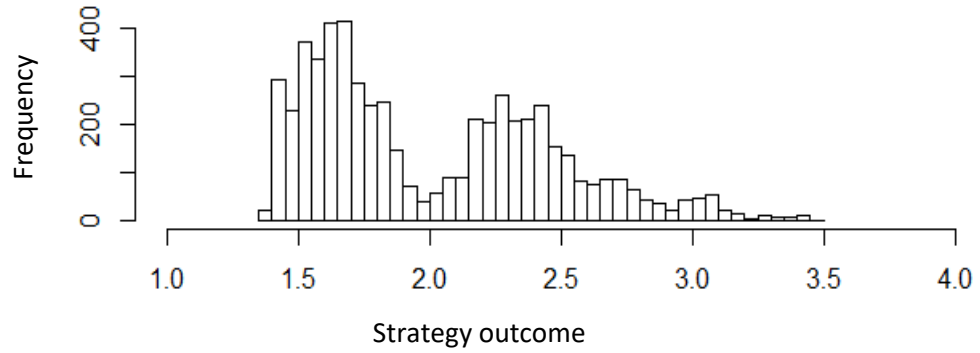


Figure 21: Robustness histograms of strategy outcomes (θ = 10)

a. Minimum-variance duration-based



b. Macaulay's duration-based strategy



c. Stochastic duration-based strategy

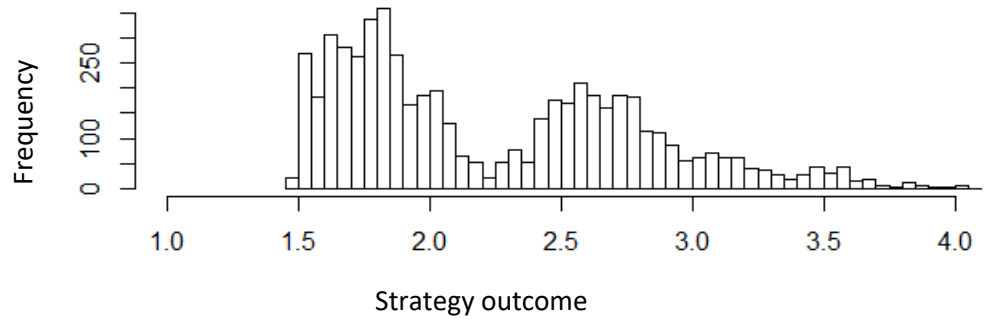
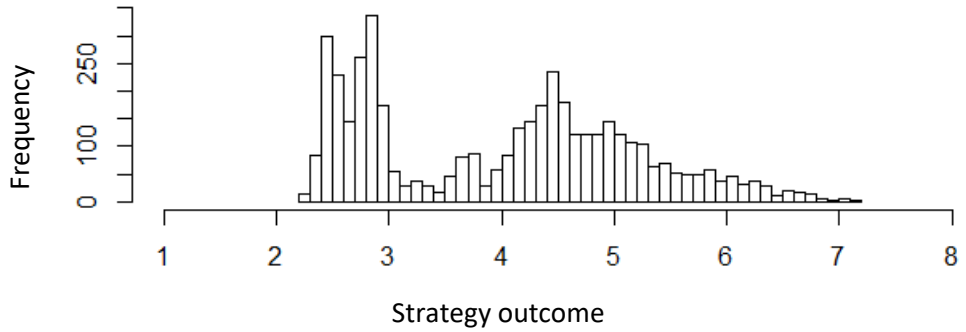
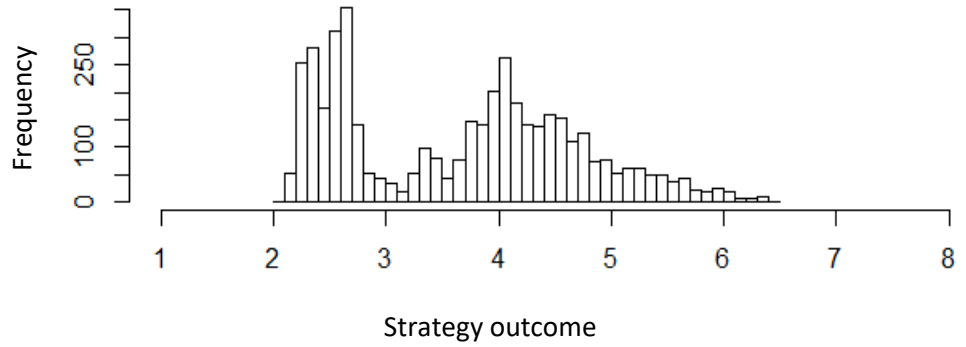


Figure 22: Robustness histograms of strategy outcomes (θ = 15)

a. Minimum-variance duration-based



b. Macaulay's duration-based strategy



c. Stochastic duration-based strategy