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US House Price Risk: Searching for Heterogeneity Using Panel Quantile Regression

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Résumé

Dans l'après-guerre, le rôle des banques dans les pays développés s'est transformé d'une manière importante. Traditionnellement, l'objectif économique d'une banque est d'encourager l'initiative entrepreneuriale privée. Or, le crédit hypothécaire constitue aujourd'hui la majeure partie du bilan comptable des banques (Jordà, Schularick and Taylor, 2016a). De plus les cycles financiers sont considérablement plus amples et longs dans les pays ayant un taux élevé de propriété immobilière, comme en Espagne et au Royaume-Uni (Rünstler and Vlekke, 2016). À la lumière de la Grande Récession, il est évident que le marché immobilier constitue un élément primordial de risque international de crise bancaire structurelle.

S'inspirant de l'article de Case et Shiller (2003), nous rassemblons des données trimestrielles de facteurs « fondamentaux » et « non-fondamentaux » du marché immobilier. Ainsi, nous considérons des éléments clés de la conjoncture économique et des variables explicatives qui visent à caractériser les effets de la spéculation immobilière du public. En effet, les auteurs ont observé que les prévisions du public sont largement influencées sur les plus récentes variations de prix. Ainsi, un petit groupe d'acheteurs optimistes peut déclencher une hausse séquentielle de prix et potentiellement une bulle immobilière. Nous quantifions les effets de l'optimisme et des attentes extrapolatives des agents du marché immobilier.

Dans la présente étude, nous modélisons les fluctuations d'indices régionaux de prix immobiliers aux États-Unis à l'aide d'une régression linéaire des quantiles. Nous étudions des données de panel regroupant les 50 états et le Washington DC. Cette méthode nous permet d'estimer une structure autorégressive pour un ensemble de quantiles de la variable d'intérêt. Les coefficients estimés dans la régression varient en fonction du quantile, nous permettant d'identifier l'hétérogénéité des effets des facteurs déterminants du marché immobilier.

Mots-clés : régression quantile, bulle immobilière, stabilité financière, données de panel, effets fixes, erreur-types robustes au partitionnement des données, corrélation sérielle

Abstract

The role economists traditionally assign to the banking sector is to assist in the financing of entrepreneurial initiatives. However, during the second half of the 20th century, the share of domestic banking credit allocated to the business sector plummeted in favour of mortgage loans, such that the balance sheet compositions of banks in industrialized countries now resemble that of real estate companies. In turn, the threat of a global financial crisis appears increasingly daunting as mortgages further leverage private banks (Jordà, Schularick and Taylor, 2016b). Moreover, financial cycles are larger and longer for countries with large rates of homeownership like Spain and the United Kingdom (Rünstler and Vlekke, 2016). In light of the Great Recession, the housing market is a primary source of risk in international structural banking crises.

In line with Case and Shiller's paper (2003), we gather quarterly data of real estate fundamentals and non-fundamentals. In other words, we consider the economic drivers of housing together with public speculation. Indeed, strong serial correlation in regional house price movements suggests that market participants form expectations in an extrapolative fashion. If the public expects the latest price trends to persist, a small group of confident buyers can trigger a sense of euphoria in the market, resulting in sequential price increases and potentially a bubble. Our model quantifies the effect of consumer sentiment and extrapolative expectations on housing growth.

In this study, we estimate linear quantile regressions of changes in US state-level house price indexes in a panel setup. This method enables us to evaluate the autoregressive structure of a quantile of the outcome variable. In turn, the covariates' coefficients depend on the given quantile, allowing us to detect the heterogeneous effects of real estate market determinants.

Keywords : quantile regression, housing bubbles, financial stability, panel data, fixed effects, cluster robust standard errors, serial correlation

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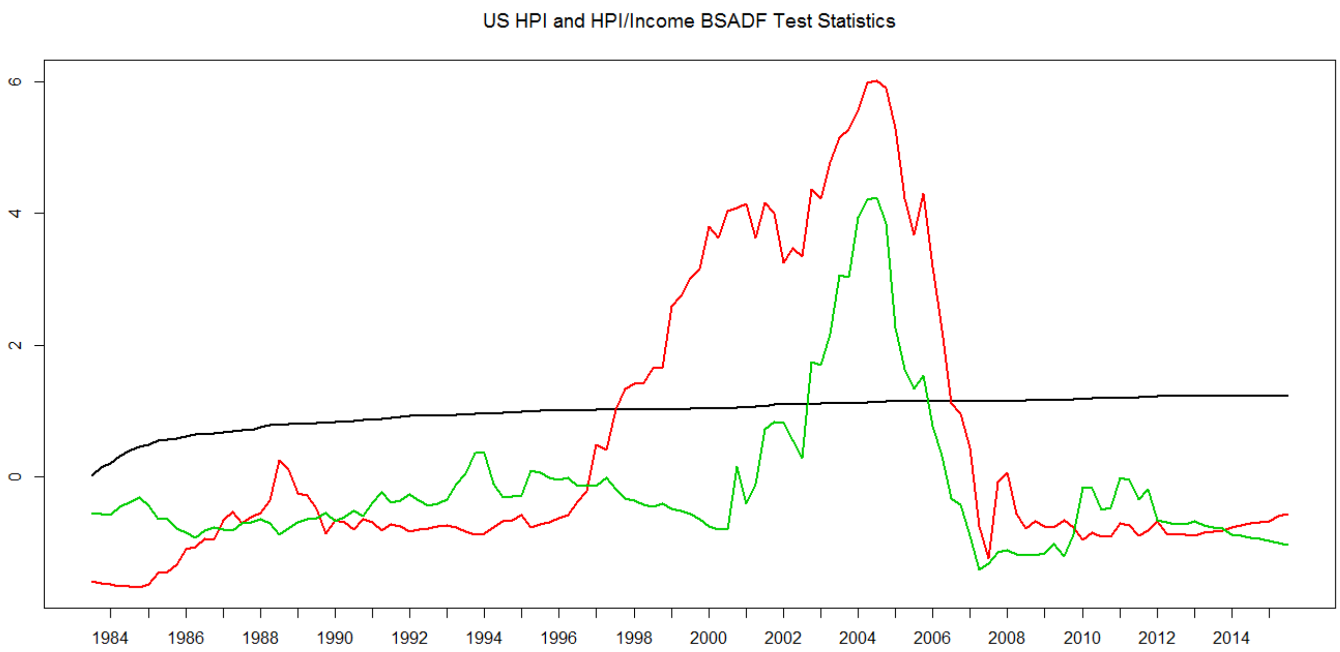

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List of Abbreviations

ADF: Augmented Dickey-Fuller

BEA: US Bureau of Economic Analysis

BLS: US Bureau of Labor Statistics

BSADF: Backward Augmented Dickey-Fuller

CCM: Clustered covariance matrix

CDF: Cumulative distribution function

DC: District of Columbia

FE: Fixed effects

FHFA: Federal Housing Finance Agency

FRB Dallas: Federal Reserve Bank of Dallas

FRED: Federal Reserve Economic Data

GDP: Gross Domestic Product

GSE: Government-sponsored enterprise

HPI: House Price Index

iid: Independent and identically distributed

QR: Quantile regression

OECD: Organisation for Economic Co-operation and Development

OFHEO: Office of Federal Housing Enterprise Oversight

OLS: Ordinary least squares

SADF: Supremum Augmented Dickey-Fuller

US: United States

VECM: Vector error correction model

WWII: World War II

Acknowledgments

I would first like to thank my teacher and research director Professor Simon van Norden for his wisdom, patience and kindness. He has been a true mentor to me since the very beginning of my graduate studies at HEC Montréal.

Je veux également remercier les membres de ma famille pour leur soutien inconditionnel et leurs encouragements.

1. Introduction

The role economists traditionally assign to the banking sector is to assist in the financing of entrepreneurial initiatives. However, during the second half of the 20th century, the share of domestic banking credit allocated to the business sector plummeted in favour of mortgage loans, such that the balance sheet compositions of banks in industrialized countries now resemble that of real estate companies. In turn, the threat of a global financial crisis appears increasingly daunting as mortgages further leverages private banks (Jordà, Schularick and Taylor, 2016b).

Jordà, Schularik, and Taylor (2016a) examined the composition of historical domestic credit of 17 industrialized countries with new long-term international aggregate credit data going back to 1870. They found that in the second half of the 20th century, advanced economies experienced an exceptional surge in non-financial credit (business and private sectors) relative to GDP, nearly doubling between 1980 and 2009. Moreover, aggregate household loans relative to GDP attained 68% in 2013, after averaging about 20% during the first half of the century.

The authors also point out that global financialization played an important role in the expansion of private credit. As a result, the banking sector's shift toward mortgage lending together with increasing global financial linkages makes housing finance a primary source of international banking risk. In hindsight of the global chaos triggered by the US housing bust, it appears that expansionist credit measures stimulated demand for houses. The resulting price increase subsequently sustained the growth of the banks' assets (Jordà, Schularick and Taylor, 2015). This stood as an incentive for financial institutions to further engage in real estate financing, at the cost of aggravating the consequences of a real estate downturn. Ultimately, the preponderant share of mortgage credit made housing finance the cornerstone of developed economies' banking system.

In the US, the Federal Housing Administration (established in 1934), Fannie Mae (established in 1938, privatized in 1968), and Freddie Mac (established in 1970) engineered and encouraged the mortgage credit expansion. The government-sponsored enterprises Fannie

Mae and Freddie Mac created a liquid secondary market of household debt, setting the stage for the notorious credit run-up.

They also played a role in standardizing mortgage lending practices across the nation, inducing important dependence in regional real estate fluctuations. Nevertheless, the financial sector grossly underestimated the risk of a simultaneous housing crash (Zimmer, 2012). Indeed, at the eve of the Great Recession, the joint distribution of regional hosue prices was estimated with Gaussian copulas, effectively assuming cross-regional independence of house prices. In his study on quarterly changes in FHFA state-level House Price Indexes, Zimmer (2012) found that even before the market crash, a simple model specification procedure could strongly reject the Gaussian mixture in favour of the Clayton, Gumbel and Student mixtures, which all identify highly dependent house prices for tail events. In this way, the Gaussian copula lead credit rating agencies to underestimate house price dependence and over-price mortgage-backed securities.

In an effort to catalogue the primary factors underlying house price movements, we first consider macroeconomic and financial fundamentals of the real estate market. Next, our analysis incorporates public sentiment variables. Since a lion's share of market participants is composed of financial amateurs, it follows that the public's flawed speculative approach — or limited cognition — can create and maintain a bubble (Case and Shiller, 2003). As a result, economical misconceptions held as truths by the public can unsustainably drive the markets. Indeed, if house prices reflect the marginal buyer's willingness to pay, a small group of optimistic buyers can influence regional house prices.

In a historical overview of housing crashes in the US, Glaeser (2013) shows that during the last two centuries, notable episodes of house price convulsions were influenced by public sentiment. For example, demand for land in Alabama skyrocketed during the 19th century because land fertility and maritime access made Alabama an early supplier of cotton. Investors from across the nation were optimistic about Alabamian land returns, but failed to anticipate that other rural US regions would eventually engage in cotton production as well. Alabama subsequently lost its grip on the cotton trade, and land returns collapsed. A more urban case would be that of Manhattan during the 1920s. In that period, builders benefited from low financing costs and a growing housing demand. Due to territorial constraints, skyscrapers naturally came as costly but effective means to realize rental revenue. Builders did not foresee that a slump in rental demand would dramatically sink their profit margins, until the Wall Street Crash of 1929. Suburban developments also procured an affordable substitute to inner-city housing, hence reducing the demand for rental spaces. Glaeser (2013) provides examples of instances when price movements are no longer supported by real economic progress, resulting in unsustainable real estate prices.

Additionally, state-level house prices empirically display significant serial correlation, possibly stemming from real estate market participants' extrapolative expectations. Indeed, real economic growth can trigger sequential house price increases that are maintained by the optimism it creates among consumers (Granziera and Kozicki, 2015). In a similar fashion, buyers become cautious when signs of a slowdown arise. Consequently, fire sales and mortgage delinquency stand as increasingly attractive options in an extrapolative mindset, posing a tangible threat of sharp price corrections (Glaeser and Nathanson, 2017). To put it another way, the public's expectations are sensitive to current market trends. In sum, the exposure of the US banking system to consumer sentiment raises a number of issues regarding the prediction of domestic credit crises.

We have seen that consumer psychology plays a role in market booms, but it is uncertain whether the effects of psychological factors are of similar importance in downturns. In this regard, the distributional response of house price changes to consumer sentiment may be asymmetric. Moreover, mortgage leveraging is not perceived as a driver of house price growth, though it helps predict the size of house price corrections (Jordà, Schularick and Taylor, 2015), again suggesting asymmetry in the distributional response of house price changes. How can we forecast a sharp downturn in the housing market, and how large might that potential correction be? More generally, what is the distributional response of house prices to real estate shocks?

Quantile regression (QR) is an adequate tool for detecting translations (location shifts) and stretches (scale shifts) in the conditional distribution of house price changes. We surmise that the effects of the key factors underlying housing market fluctuations are susceptible to display heterogeneity, meaning that the distributional response to housing factor shocks is not a pure translation of the distribution. Rather, different regions of the distribution of price changes respond differently to housing factors, resulting in a combination of location and scale shifts. QR methods allow us to approximate a finite set of conditional quantile processes for house price variations, providing a clear depiction of the distributional response to real estate shocks. In doing so, we drop OLS regression assumptions regarding symmetrical, well-behaved conditional expectations of quantiles. Moreover, QR easily adapts to various forms of error dependence and heteroscedasticity. In conclusion, the QR methodology enables one to highlight distributional asymmetry in price movements as well as persistence in response to a variety of shocks that policy-makers and investors should be mindful of.

The contribution of the present study lies in the identification of the heterogeneous effects of market determinants cited in recent housing finance literature. We employ recent advancements in QR inference due to Yoon and Galvao (2016) and Hagemann (2017) to make our estimates' standard errors robust to serial correlation, but they did not succeed in making them robust enough.

The remainder of the paper is organized as follows. In Chapter 2, we address the leading trends in the post-WWII house price dynamics literature. We classify real estate market determinants in two main categories: fundamentals and public speculation. In Chapter 3 we present the sources of the data used to construct our model. We then discuss our covariates' predicted impact on house prices in accordance to the reviewed literature. In Chapter 4, we provide the methodological details of our house price application. We first detail the computation of regional market exuberance series. We then present an introduction to QR methods and consider two novel cluster-robust QR inference techniques. Chapter 5 presents QR estimates with robust standard errors. We also analyze the finite sample performance of our robust estimators under various forms of error dependence. Chapter 6 concludes.

2. Housing market determinants

This section reviews the key factors affecting the housing market in order to construct a sensible regression model. Although, macro-financial trends do have a direct impact on access to home ownership, there is growing evidence that psychological factors play an important role in explaining house price movements. Indeed, the irrational public speculation can contribute to creating a housing bubble. This real estate pricing problem is also examined in papers by Case and Shiller (2003), Glaeser (2013), Glaeser and Nathanson (2017), Granziera and Kozicki (2015), and Pavlidis *et al.* (Pavlidis *et al.*, 2016) as well as in the books *Animal Spirits* (Akerlof and Shiller, 2010) and *This Time It's Different: Eight Centuries of Financial Folly* (Reinhart and Rogoff, 2009).

From a purely financial standpoint, one might think of a house as an asset whose price reflects the present value of the stream of housing services (or rent for non-occupying owners) it will generate. In such a model, housing market factors are those that directly affect the net value of those housing services (e.g. mortgage rates, employment, vacancy rates), and are called "fundamentals". We hereafter describe fundamentals as macroeconomic and financial trends that may impact the net value of housing services. However, most households base purchasing decisions on naïve, simplistic speculations of local housing trends rather than on a comprehensive asset-pricing rationale.

Two surveys carried out in 1988 and 2003 (both sampling 2000 American homebuyers) support the hypothesis that price changes are closely related to the public's market anticipation, especially in times of manifest optimism or pessimism (Case and Shiller, 2003). Indeed, in both surveys, over half of the respondents affirmed that their purchasing decision was influenced by word-of-mouth transmission of excitement. Indeed, price changes are closely related to the public's market anticipation, especially in times of manifest optimism or pessimism (Case and Shiller, 2003).

In our analysis we vaguely discern the effects of fundamentals and irrational economic behaviour. To that end, we classify housing determinants into two categories: fundamentals and public speculation. Fundamentals include supply and demand factors along with financial risk factors. Public speculation variables aim to capture market optimism and extrapolative expectations. The variables we incorporate in the quantile regression are largely based on the post-2007 body of work on housing business cycles and housing finance discussed below.

Fundamentals

We loosely base our list of housing fundamentals on the house price study of Case and Shiller (2003). We categorize fundamentals as follows: demand factors, supply factors, and credit risk factors. Demand factors influence the demand for existing houses, mainly via household wealth and financial factors that are meant to capture the ease with which households can finance their purchases. Supply factors affect the supply of new houses. Finally, credit risk factors impact the exposure of the banking sector to housing finance.

Demand Factors

Houses have a dual economic nature for they are simultaneously durable consumption goods and assets. This forces us to consider a number of elements that might affect the price of a house, but also the utility of housing services. We will examine factors related to housing affordability, such as unemployment, property taxes, personal income, and mortgage rates.

Some years before the Great Depression, Case and Shiller (2003) found evidence that income per capita proved to be a powerful single predictor of house prices in almost all states between 1985 and 2002. Indeed, markets with low house price-to-income per capita ratios have stable house prices that are highly correlated with per capita income trends. Conversely the few 8 states[1] with larger ratios had the largest house price variability. In these states, the model required additional predictors, such as unemployment, mortgage rates, housing starts, population, and income-to-mortgage payments ratios. For these states, unemployment had a negative impact on house prices between 1985 and 1999, although the relation is less clear over the 1985 to 2002 period (Case and Shiller, 2003). Interestingly, mortgage rates have no apparent effect on house prices between 1985 and 2002. Authors argue that simultaneity in

[1] The 8 states displaying the highest standard deviation of Home Price to Income per capita ratios, in descending order: Hawaï, Connecticut, New Hampshire, California, Rhode Island, Massachusetts, New Jersey, New York

their dataset can cause the effects of mortgage rates to be ambiguous. For example, cheap financing costs are supposed to encourage housing demand, but banks may set low mortgage rates to restore the demand when real estate prices enter a downturn.

Kahn (2009) used a regime-switching model to point out how changes in productivity levels can be a forward-looking variable income patterns. We can therefore expect household income to grow when productivity shifts into higher gear. However, long-run productivity growth forecasts prove to be unreliable (Christensen, Gillingham and Nordhaus, 2016), and to our knowledge, there are no good proxies to control for this variable.

Liquidity risk is also commonly dealt with in stock-pricing frameworks, and the real estate liquidity premium can be defined analogously to stock liquidity premiums. Based on the ratio of houses for sale to houses sold, Peterson and Zheng (2011) propose a measure of real estate liquidity risk. They found that liquidity changes can explain a portion of the variance of house prices in Canadian provinces.

One might contend that demographics are inherently connected to housing demand, and prices should change accordingly. Although this claim holds true in the long run, total population is not a leading indicator of real estate investment. Monnet and Wolf (2017) recently found that in OECF countries, demographic trends of the 20-49 age group are better predictors for house price variations than any other macro-financial correlate. However, this analysis is undependable in instances of intense migratory flows because demographic cycles are hard to assess when a region experiences migration. Since migration patterns widely differ across the US, state level demographic trends are difficult to measure and compare, so we choose to omit this predictor in the model.

Supply Factors

Since the Great Recession, a sizeable body of literature has addressed the role of demand factors, but supply factors are seldom identified as the source of the crash. We attempt to complement our analysis by including supply-side factors, namely: terms-of-trade and supply elasticity.

Corrigan (2017) quantifies the cointegrating relationship between the relative price of housing and international trade conditions — or the terms-of-trade. Indeed, domestic commodity prices are likely to fluctuate in response to global credit inclinations, financial development, monetary policy interactions, and productivity forecasts. He asserts that an increase in foreign productivity is liable to decrease the price of imports relative to domestic consumption. In accordance with the Baumol-Bowen effect[2], the price of housing would then increase relative to non-housing consumption. With a panel VECM framework of 18 developed economies[3], Corrigan (2017) identifies a strong negative long-run correlation between import and house prices over the 1994-2015 period, where persistent declines in import prices explain a substantial portion of house price variance. He finds evidence that a decline in real import prices tends to drive down domestic non-housing consumption prices, resulting in decreased inflation expectations and pressuring short-term interest rates downward. In turn, as the price of non-housing services and interest rates fall, real house prices and household debt increase. The relationship is much weaker for net exporters, however. We will use the terms-of-trade index from the US Bureau of Economic Analysis to control for international-exchange driven housing supply shocks.

Knoll, Schularick and Steger (2017) show that in a sample of 14 developed countries[4] dating back to 1870, land prices play an important role in explaining long-run house price growth. In fact, at the turn of the 20th century, housing growth was relatively stable and widely attributed to the tangible improvements made on a home, such as access to electricity and water. In the late 1900s however, land appreciation appears to be a primary factor in house price growth. In this regard, supply elasticity stands as a key element in understanding land price trends. When constructors are capable of promptly responding to a surge in demand, they

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[2] This hypothesis by Beaumol and Bowen (1966) suggests that productivity growth tends to push up the opportunity cost — and the price — of less productive sectors, such as land and housing. This pattern comes in disagreement with classical economic principles according to which long-run prices should solely reflect the production costs.

[3] 4 net commodity exporters (Australia, Canada, New-Zealand, Norway) and 14 net commodity importers (Belgium, Denmark, Finland, France, Germany, Ireland, Italy, Japan, Netherlands, Spain, Sweden, Switzerland, United Kingdom, United States)

[4] Australia, Belgium, Canada, Denmark, Finland, France, Germany, Japan, Netherlands, Norway, Sweden, Switzerland, United Kingdom, United States.

are effectively clearing the scarcity effect that would otherwise inflate prices, hence substantially impeding a home's ability to store value. However, speculation of high housing returns can lead constructors to overshoot new home starts. For example, Los Angeles's housing suppliers can adjust quickly to a surge in demand for the city is literally surrounded by unused space — the desert. In contrast, housing supply in New York is somewhat inelastic due to the limited, expensive land. During the 1980's and 2000's housing bubbles, states with highly elastic supply experienced markedly shorter booms than metropolitan areas with inelastic supply (Glaeser, Gyourko and Saiz, 2008). Following that logic, a region's elasticity can provide clues regarding the magnitude of price variations.

Credit Risk Factors

Reinhart and Rogoff (2009) put together new long-run data covering almost 140 years of credit fluctuations in 14 developed countries. One conclusion we can draw from this historical perspective is that credit growth is a powerful predictor of financial crises (Schularick and Taylor, 2009). This comes as evidence that macro-financial analysts failed to examine past episodes of financial turbulence, overlooking the pivotal role that domestic credit plays in our financial system. Later, Jordà, Schularik and Taylor (2016a) point out that real estate downturns are liable to lead to deeper recessions and slower recoveries. By observing financial cycles in the US and in the five largest European economies[5], Rünstler and Vlekke (2016) also find that empirically, GDP, housing and credit cycles are related at high frequencies. Bauer (2014) uses a panel logit model with 18 OECD countries to show that the implementation of restrictive credit policies often precedes house price corrections. In fact, deviations of the short-term interest rate from the Taylor rule are significant predictors of the likelihood of a house price downturn at forecast horizons of one quarter, one year and two years. For this reason we wish to capture the ease of mortgage finance by including national variables such as the 30-year mortgage rate and a treasury constant maturity rate spread.

[5] United Kingdom, Germany, France, Italy, Spain

Bauer (2014) also finds that total credit-to-GDP and bank credit-to-GDP ratios help forecast housing turning points. Therefore, our analysis takes into account the leverage of mortgages on the economy by including the mortgage debt-to-GDP ratio in our model. Indeed, mortgage leverage stands as a critical element in detecting systemic banking crises and their financial cost (Laeven and Valencia, 2008).

In the US, important structural changes in housing market financial intermediation (e.g. Basel I (1988)) increased interstate financial operations, which changed tail-dependence structures in regional home prices (Zimmer, 2012). In turn, credit rating agencies' assumption of independent mortgage default probabilities seriously undermined their ability to predict the Great Depression. In fact, peaks and troughs in cross-sectional real estate returns are highly correlated, and simple copula specification tests can show that there is in fact significant tail risk. As seen in the work of Zimmer (2012), and later verified in Ho, Huynh and Jacho-Chávez (2016), copula models that allow for tail dependence easily outperform the Gaussian copula and indicate that interstate house price changes exhibit larger correlations in upswings and downswings. To sum up, if house prices display strong dependence during periods of financial turmoil, the event of a simultaneous drop in realty prices stands as a looming threat of sweeping mortgage defaults that the financial sector may not be prepared to withstand.

Public Speculation

In recent history, the works of Kindleberger and Aliber (2011) and Shiller (2008) expose the potential pitfalls of assuming consumer rationality. The agents' limited cognition is particularly prevalent in the context of housing finance, where capital gains (or losses) appear to have a significant impact on consumer sentiment. Glaeser (2013) chronicles the history of US real estate convulsions, and points that the public's predictions of regional real estate growth is deeply flawed. Time and again, market participants tend to form expectations in accordance with isolated, regional stimuli, turning a blind eye to the macroeconomic drivers of real estate (Glaeser, 2013). Even among professional investors, excessive optimism can cause stock prices to depart from their so-called fundamental value. Indeed, the UBS/Gallup Investor Survey (based on a sample of 1000 US investors) indicates that stock market optimism is positively correlated with stock price-to-dividend ratios, which comes in contradiction with the conclusions of Fama and French (1988) regarding rational return expectations (Adam, Marcet and Beutel, 2017). This goes to show that human psychology is closely tied to economic decisions.

It follows that a shift in consumer sentiment can trigger abrupt local house price movements. In their influential paper, Case and Shiller (2003) argue that public expectations about the market — as biased as they may be — can sustain house price overvaluations. The authors examine two nationwide US housing market surveys carried out in 1988 and 2003. One finding is that market participants commonly perceive that real estate investments as a safe-haven for household savings, especially after the poor stock market performance of the early 2000s. Another finding is that in the broad scheme of things, a propagated feeling of optimism in the market may prompt panic buying, artificially inflating prices. Conversely, the levee breaks when signs of a slowdown become too obvious: when the public is experiencing a feeling of gloom and concern, fire sales may provoke a sharp price correction, inducing and maintaining high rates of mortgage delinquency.

In order to account for episodes of market euphoria, we construct covariates that quantify the intensity of price run-ups and downturns. Pavlidis *et al.* (2016) use the Backward Supremum Augmented Dickey-Fuller Test (BSADF) as a tool to detect housing bubbles. The BSADF test statistic can be viewed as an exuberance level in that it is designed to detect multiple episodes of explosive dynamics in a univariate time series. Using data from the International House Price Database of the Federal Reserve Bank of Dallas (FRB Dallas), the authors extract yearly country-specific BSADF test statistics for real house prices and price-to-income ratios. Moreover, the FRB Dallas maintains these updated statistics as a part of its housing database. The BSADF statistic for house prices can therefore track the explosiveness of house prices, and the BSADF statistic for the house-price-to-income ratios can highlight periods during which house price movements do not reflect changes in market fundamentals. For instance, during a bubble, prices sometimes depart significantly from income trends, seemingly driven by extrapolative expectations or sheer optimism. By observing price-to-income explosiveness, the authors found that price increases appear to be unsustainable when they depart from fundamental trends, such as income. The authors also found that episodes of exuberance for house prices and price-to-income ratios roughly coincide, but periods of exuberance are shorter for price-to-fundamentals. Note that negative exuberance levels are also observed when time-series display descending unit-root-like trajectories.

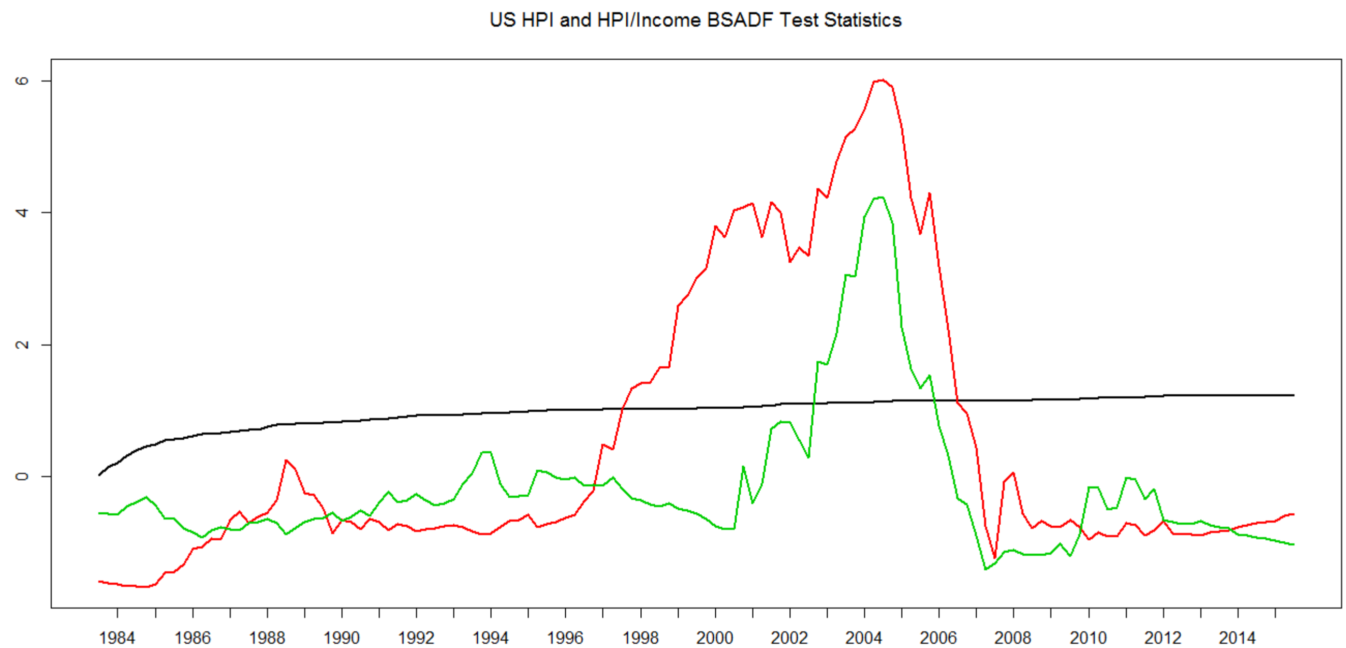

Figure 1: US univariate BSADF test statistics for house prices (red) and price-to-income ratios (green) with 95% critical values (black).
Federal Reserve Bank of Dallas: https://www.dallasfed.org/institute/houseprice#tab1

This procedure generates test statistics and critical values that allow testing for unit-root behaviour in univariate time series. In their study, periods during which the BSADF exceeds the critical value are said to show "exuberance". In an attempt to predict the likelihood of a bubble, Pavlidis *et al.* (2016) propose probit models where the outcome variable is an indicator of housing exuberance (as described above), and explanatory variables include: interest rate spreads, long-term rates, quarterly changes in the stock market, household wealth, nominal domestic credit growth to the private sector, disposable income, rents, unemployment, GDP growth, inflation, oil prices and Kilian's global indicator of real economic activity. The estimated models show that the long-term rates, private credit growth, personal disposable income growth, unemployment, and GDP growth are significant predictors of price and price-to-income exuberance.

If we look at the US house price and price-to-income BSADF statistics (Figure 1 above), we see that exuberance for US house prices started in the late 1990s during the dot-com bubble, but the price-to-income ratio only started to show explosiveness in the early 2000s (Pavlidis *et al.*, 2016). One explanation the authors propose for this is that fundamental economic growth during the tech boom caused the sequential price increases of the late 1990s and early 2000s, but later in 2003, the boom turned into a bubble. The price-to-income ratio then began to show exuberance, now indicating a rather unsustainable growth. In fact, after the dot-com bust, real economic growth came to a halt, yet real estate prices continued to grow. By the time the signs of a housing collapse became tangible in the financial and banking sectors, the run-up of house prices had already propagated around the world leading to a nearly simultaneous crash in 2006. Therefore, house price and price-to-income exuberance levels can give us clues about the underlying economic mechanism generating explosive prices. We use the BSADF statistics to construct state-specific chronological exuberance levels of house prices and house-price-to-income ratio series.

In short, house prices are influenced by macro-financial factors, but the economical misconceptions of the public may give rise to prices departing from their fundamentals. In their well-known book *Animal Spirits*, Akerloff and Shiller (2010) point out that financial and economic analysts have failed to recognize that human psychology plays a powerful role in economic decisions. We therefore complement fundamental covariates with consumer sentiment proxied by exuberance levels.

3. Data

In this section we describe the FHFA House Price Index and the data series we analyzed[6]. Our balanced panel consists of the 50 states and the District of Columbia and spans 30 years, from January 1986 to October 2015 (120 quarters).

The FHFA House Price Index

We measure fluctuations in house prices as quarterly changes in the natural logarithm of the Office of Federal Housing Enterprise Oversight's (OFHEO) House Price Index (HPI). Released for the first time in 1996, the HPI monitors the regional prices of single-family detached homes since 1975 based on Fannie May and Freddie Mac repeat-sale mortgage transaction data. The all-transactions, state-level, quarterly HPI is based on sales prices and appraisals of detached single-family homes whose mortgages are purchased by one of the twin GSEs[7] (Calhoun, 1996). Therefore, one potential caveat of using this index may be the misrepresentation of regional price trends related to vacant lands, multi-unit properties, and commercial real estate. Nevertheless, the share of total outstanding mortgages related to single-family homes historically dominated those of multifamily, commercial and farm mortgages. In fact, outstanding single-family home mortgages took about half of total outstanding mortgages in the mid-1970s. According to the last release of the Federal Reserve's Flow of Funds Report, the share of single-family home mortgages increased to over 70% since then, and has remained above that level, even throughout the dip that occurred after the crisis.

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[6] Charts of the FHFA HPI and covariates' series are included in Annex 1

[7] FHFA HPI; All-Transactions Indexes; States: https://www.fhfa.gov/DataTools/Downloads/pages/house-price-index-datasets.aspx

Fundamentals

Demand factors

In order to capture how the relative cost of housing affects demand, we consider the state-level percent change in personal income. A quarterly, current-dollar, seasonally adjusted personal income series is available via the web database[8] of the US Bureau of Economic Analysis (BEA). We expect a positive correlation between changes in personal income and HPI changes across all quantiles. Since variations in total income have a direct impact on the buyers' power to purchase, income declines are likely to be one of the most eloquent indications of an economic slowdown, especially to the economically uneducated majority of consumers. That said heterogeneity may arise if declines in income raise suspicion of market devaluation, inciting owners to sell at a discount.

We also capture the effects of property taxes by including the personal-property-taxes-to-income ratio. We use a staircase interpolation to convert annual data to quarterly observations. State-level total personal tax series are also from the BEA regional database. Personal income and personal taxes are both expressed in real terms using the BEA's quarterly GDP Implicit Price Deflator with 2009 as the base year. There can be some discrepancies between regional taxing practices. In some areas (e.g. Delaware, Hawaii, Oregon), there are no personal property taxes. For most states though, the personal-property-taxes-to-income ratio varies between 0.05% and 0.2%. We expect this covariate to have the opposite effect of household income, so property taxes should be negatively correlated with house prices.

We control for shifts in national demand by including the percent changes in the natural logarithm of real national GDP. The current-dollar quarterly GDP series is available in the BEA's regional database. Since housing growth can be associated with a surge in productivity (Kahn, 2009), we expect changes in real GDP to have a positive relationship with housing growth across all quantiles. As seen in Rünstler and Vlekke (2016), GDP and housing cycles are markedly synchronized across the 5 largest European economies and the US.

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[8] BEA Regional Data: https://www.bea.gov/iTable/index_regional.cfm

Our model takes account of the state of the local labour market with state-level unemployment rates. Seasonally adjusted monthly unemployment series are released monthly by the US Bureau of Labor Statistics (BLS)[9]. These observations are converted to quarterly frequency using end-of-period observations so as to fit the panel setup. Case and Shiller (2003) point out that high housing costs impede job creation because potential workers may opt for the job where homes are more affordable. In that aspect, unemployment and house prices suffer from simultaneity. However, rising unemployment rates may stir doubt regarding real estate growth, so we can expect unemployment to be somewhat negatively correlated with house prices.

We capture the impact of the primary mortgage market on housing demand with the national 30-year conventional mortgage rate. Based on Freddie Mac's fixed rate mortgage commitments data, this monthly, non-seasonally adjusted series was released by the US Board of Governors of the Federal Reserve System[10]. We convert this series to quarterly data using end of period observations. Since an increase in mortgage funding costs directly impact housing affordability, mortgage rates are supposed to have a negative relationship with house prices.

Supply Factors

The BEA Regional Data also includes a quarterly national terms-of-trade index, which aims to reflect the purchasing power of the US in international markets. That is, an index increase indicates a rise in the relative price of imports to exports, and conversely. Drawing from the conclusions of Corrigan (2017), we expect a negative relationship between terms-of-trade and house prices.

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[9] BLS data available via the FRED database : https://fred.stlouisfed.org/search?nasw=0&st=unemployment&t=monthly%3Bsa%3Bstate%3Bunemployment&ob=sr&od=desc

[10] Federal Reserve data available via the FRED database : https://fred.stlouisfed.org/series/MORTG

In order to capture housing stock supply trends, we consider yearly rental and home vacancy rates. Vacancy data are retrieved from the US Bureau of the Census[11]. We convert the series to quarterly data using a staircase interpolation of yearly vacancy observations. Since the FHFA HPI describe single-family home price, we consider house vacancy rates to measure house prices' response to unused housing stock. We also take rental vacancy rates into account in our model to consider both types of housing services.

If real estate property pricing methods rely on basic dividend stock pricing techniques at all, one should note that the stream of house payment and rent revenues are indicative of a property's net value. Since owning a vacant property is comparable to holding idle capital, owners are inclined to reduce the ask price when plagued by vacancy. High levels of home or rental vacancies are symptomatic excess supply, so home vacancy should be negatively correlated with single-family house prices.

The coefficient estimates for rental vacancy may capture a variety of price mechanisms, despite suffering from simultaneity. According to the substitution effect, high levels of rental vacancy encourage property owners to reduce rental costs to attract tenants. As households transition from housing to rentals, the demand and the cost of housing also fall. Therefore, the marginal effect of rental vacancy on the demand for homes should be negative, but vacancy rates may also rise when housing becomes too expensive.

Homes and rentals are also substitutes in terms of real estate investment strategies. Indeed, the two types of housing services fundamentally differ in that a house is a large investment with some real estate risk, whereas rentals cannot generate capital gains but have the benefit of being virtually devoid of risk. If we view rentals and homes as respectively safe and risky investment strategies, low rental vacancy could indicate a flight to safety as home prices enter a downfall, inducing a positive correlation between house prices and rental

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[11]: US Bureau of the Census data available via the FRED database; Rental vacancy : https://fred.stlouisfed.org/search?nasw=0&st=rental+vacancy&t=rent%3Bstate%3Bvacancy&ob=sr&od=desc ; Home vacancy : https://fred.stlouisfed.org/search?nasw=0&st=home+vacancy&t=housing%3Bstate%3Bvacancy&ob=sr&od=desc

vacancy. Conversely, high rental vacancy rates could indicate a preference for home ownership in times of growth.

Vacancy rates can also be a proxy for regional real estate supply elasticity. As seen in Glaeser, Gyourko and Saiz (2008), supply elastic regions can respond to marked real estate growth simply by boosting construction instead of inflating prices. Since overbuilding typically occurs in regions with elastic housing supply, high vacancy rates can be associated with elastic real estate supply. The authors also show that regions with elastic housing supply usually experience more moderate house price changes. If high levels of vacancy are related to supply elasticity, we can expect real estate run-ups and crashes to be briefer and of lesser magnitudes when vacancy rates are high. Following that logic, the conditional HPI changes' response to vacancy rates may be a scale shift where a high vacancy rate narrows the distribution of price changes.

Credit Risk Factors

We add the 5- to 1-year yield spread of constant maturity treasury rates to our set of covariates to control for interest rate expectations. Indeed, interest rate spreads are commonly used as a forecasting tool of financial market returns. Positive yield spreads generally indicate expectations of stable economic growth, whereas trivial or negative yield spreads signal flat or inverted yield curves. In the event of a recession, house prices usually experience sharp corrections. We therefore predict that this covariate will be positively correlated with house prices. In other words, a flat or inverted medium-to-short-term yield curve can amplify the size of housing devaluation because it signals vulnerability in economic fundamentals. 5- and 1-year constant maturity treasury rate data are retrieved from the FRED[12] and converted to quarterly series using end of period observations.

We also include the ratio of outstanding domestic mortgage debt to GDP as a real estate leverage measure. National outstanding mortgage debt data are taken from the Board of

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[12] 1-year Treasury Constant Maturity Rate: https://fred.stlouisfed.org/series/GS10 ;
5-year Treasury Constant Maturity Rate: https://fred.stlouisfed.org/series/GS5

Governors of the Federal Reserve System[13]. The series we observe contains quarterly measures of outstanding mortgages for all types of properties: one- to four-family residences, multifamily residences, farms, and non-farm/non-residential properties. Since real estate credit expansions often precede a downturn, high mortgage-to-GDP ratios are observed during a boom. For this reason, we expect an exacerbated negative effect of real estate leverage on the lower quantiles of house price changes. We also expect a credit expansion to stimulate house prices, possibly driving up the upper-quantiles. In other words, we predict that real estate leverage causes a scale shift that increases the dispersion of the distribution of house price changes.

Public Speculation

To capture the possibility of house price bubbles, we follow Pavlidis *et al.* (2016) in using the Backward Supremum Augmented Dickey-Fuller (BSADF) unit-root test developed by Phillips, Shi & Yu (2015). This method is capable of detecting multiple episodes of price explosiveness.

This procedure generates test statistics and critical values that allow testing for unit-root behaviour in univariate time series. In their study, periods during which the BSADF exceeds the critical value are said to show "exuberance". In an attempt to predict the likelihood of a bubble, Pavlidis *et al.* (2016) propose probit models where the outcome variable is an indicator of housing exuberance (as described above), and explanatory variables include: interest rate spreads, long-term rates, quarterly changes in the stock market, household wealth, nominal domestic credit growth to the private sector, disposable income, rents, unemployment, GDP growth, inflation, oil prices and Kilian's global indicator of real economic activity. The estimated models show that the long-term rates, private credit growth, personal disposable income growth, unemployment, and GDP growth are significant predictors of price and price-to-income exuberance.

[13] Board of Governors; Economic Research & Data: https://www.federalreserve.gov/econresdata/releases/mortoutstand/current.htm

We use the BSADF statistics differently in our model: instead of generating an indicator of exuberance by comparing the BSADF to critical values, we directly include the BSADF statistics as covariates. This allows us to preserve information regarding the intensity of the explosive behaviour of regional prices and price-to-income ratios. Details about the computation of the BSADF statistic appear in the next section. We argue that the price BSADF statistic reflects the public's optimism about future real estate returns, thus being positively correlated with changes is house prices. The price-to-income BSADF statistic also captures market optimism, but most importantly, it measures unsustainable housing growth relative to income patterns. On the basis of this rationale, price-to-income exuberance increases the risk of a crash. Therefore, this second exuberance level is expected to reduce the lower quantiles.

Finally, it is clear that house price changes display strong evidence of serial correlation. As we can see in Figure 2, our sample's median state-specific first-lag autocorrelation is equal to 0.59. This warrants the presence of an auto-regressive component with a 1-period lag in our model. We believe that this covariate will reflect the market participants' extrapolative short-term expectations and have a positive effect across quantiles. Some price serial correlation will not be captured by a single auto-regressive component, so serial correlation will persist in the errors. We will later see that QR estimates tolerate error autocorrelation. Moreover we use inference techniques that are robust to serially correlated residuals.

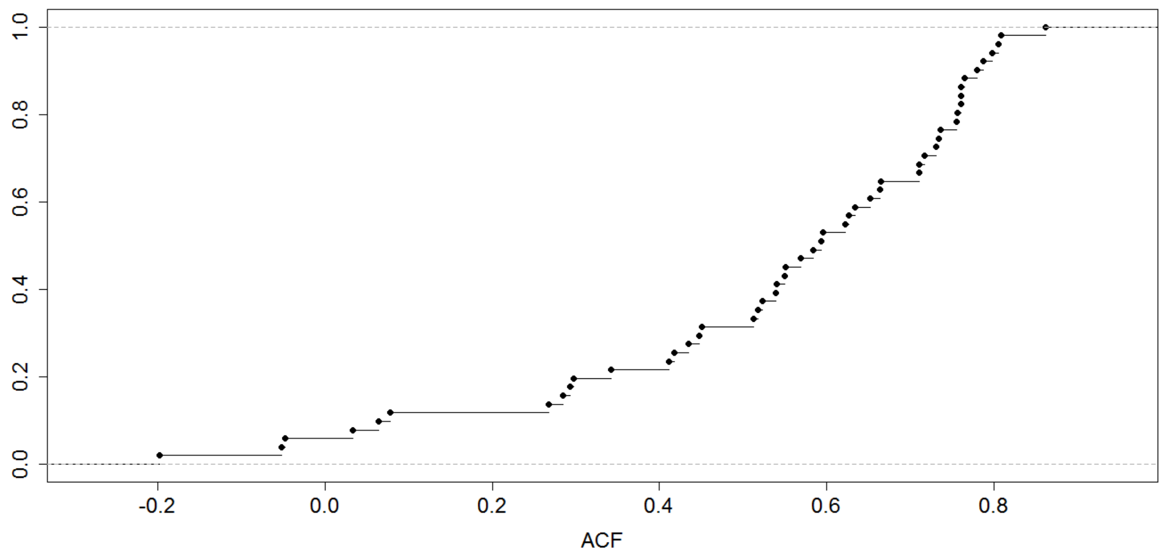

Figure 2 : Cumulative distribution of state-level first-order autocorrelations of house price changes

Model Setup

Our quantile regression is constructed upon a balanced panel of 13 covariates, 120 quarters (1986:Q1 to 2015:Q4), and 51 geographic areas. State or DC fixed effects can also be incorporated to control for regional housing market characteristics (e.g. supply elasticity). The autoregressive conditional quantile processes we estimate on a sample of T periods and N entities are of the form:

$$
\begin{aligned}
& q_\theta(\Delta log\text{HPI}_{i,t+1}) \\
&= \alpha_i + \beta_1 \cdot \Delta log\text{HPI}_{i,t} + \beta_2 \cdot \Delta\%\text{logGDP}_t + \beta_3 \cdot \%\Delta\text{Income}_{i,t} \\
&+ \beta_4 \cdot \text{Price Exuberance}_{i,t} + \beta_5 \cdot \text{Price/Income Exuberance}_{i,t} \\
&+ \beta_6 \cdot \text{Home Vacancy Rate}_{i,t} + \beta_7 \cdot \text{Rent Vacancy Rate}_{i,t} + \beta_8 \cdot \frac{\text{MDO}}{\text{GDP}}_t + \beta_9 \cdot \frac{\text{Tax}}{\text{Income}_{i,t}} \\
&+ \beta_{10} \cdot \text{TOT Index}_t + \beta_{11} \cdot \text{Rate Spread}_t + \beta_{12} \cdot \text{Mortgage Rate}_t \\
&+ \beta_{13} \cdot \text{Unemployment}_{i,t}
\end{aligned}
$$

$$i = 1, \dots, N$$

$$t = 1, \dots, T$$

where θ denotes the quantile of interest, q_{θ} denotes the conditional quantile estimate at θ, and $\beta_i \coloneqq \beta_i(\theta)$ denotes the slope of the i-th coefficient for the θ-th quantile process. α_i accounts for state or DC fixed effects[14]. The model with no fixed effects hereafter refers to a restricted model where quantile intercepts are the same across all entities ($\forall i \epsilon \{1, \dots, N\}, \alpha_i = \alpha$). In the order presented in the above, our 13 covariates include: quarterly changes in the natural logarithm of the state or DC HPI; quarterly changes in the natural logarithm of real national GDP; quarterly percentage changes in state-level personal income; price and price-to-income state-level BSADF test statistics (exuberance levels); state-level home and rental vacancy rates (expressed in percentages), the national ratio of total outstanding mortgage debt to GDP; the state-level ratio of property taxes to income; the national terms-of-trade index; the national 5- to 1-year treasury rates spread (in percentage); the national 30-year mortgage rates (in percentage); and state-level unemployment rates (in percentage).

Variable	Definition	Category	Source	Frequency	Geography type	Anticipated sign of slope
$\Delta \boldsymbol{log}$**HPI**	Previous change in logHPI	Public Speculation	FHFA	Quarterly	State or DC	+
$\Delta \boldsymbol{log}$**GDP**	Change in logGDP	Demand	BEA	Quarterly	National	+
%ΔIncome	Percent change in income	Demand	BEA	Quarterly	State or DC	+
Price Exuberance	HPI BSADF statistics	Public Speculation	FRB Dallas	Quarterly	State or DC	+
Price/Income Ex.	HPI/Income BSADF statistics	Public Speculation	FRB Dallas	Quarterly	State or DC	-
Home Vac. Rate	Vacancy rate of single-family detached houses	Supply	US Bureau of the Census	Yearly	State or DC	-

[14] Time fixed effects can also be included, but will not be considered in this paper because robust inference techniques with panel QR methods have yet to be addressed in the literature.

Variable	Definition	Category	Source	Frequency	Geography type	Anticipated sign of slope
Rent Vac. Rate	Vacancy rate of multi-unit residential buildings	Supply	US Bureau of the Census	Yearly	State or DC	±
$\mathrm{MDO}/\mathrm{GDP}$	Total mortgage debt outstanding as a fraction of GDP	Credit Risk	Board of Governors of the Federal Reserve System	Quarterly	National	-
$\mathrm{Tax}/\mathrm{Income}$	Ratio of property taxes to income	Demand	BEA	Quarterly	State or DC	-
TOT Index	National terms-of-trade index	Supply	BEA	Quarterly	National	-
Rate Spread	5- to 1-year Constant Maturity Treasury Rate Spread	Credit Risk	Board of Governors of the Federal Reserve System	Monthly	National	+
Mortgage Rate	30-year conventional mortgage rate	Demand	Board of Governors of the Federal Reserve System	Monthly	National	-
Unemployment	Unemployment rate	Demand	US Bureau of Labor Statistics	Monthly	State or DC	-

Table I: Summary of panel data

4. Methodology

In this section, we first detail the computation of the BSADF test statistic used to generate price and price-to-income exuberance levels. Second, we describe the quantile regression problem. Finally, we describe the inference issues that arise in a QR panel framework. Two robust inference procedures are implemented: bootstrapped confidence intervals and clustered covariance matrix (CCM) estimation. The former is adapted to a random effects model with clustered, serially correlated error terms. The latter allows for fixed effects and heteroscedastic, serially correlated error terms.

The BSADF Test

The Backward Supremum ADF (BSADF) test is based on the Augmented Dickey-Fuller (ADF) unit-root test, which is in turn based on the regression:

$$\Delta y_t = a_{r_1,r_2} + \beta_{r_1,r_2} y_{t-1} + \sum_{j=1}^{k} \psi_{r_1,r_2}^{j} \Delta y_{t-j} + \varepsilon_t \,,$$

where y_t is a univariate time series, k denotes the number of auto-regressive lags in the model, and ε_t is an iid, normally distributed error term with standard deviation σ_{r_1,r_2}. The interval $[r_1, r_2]$ $(r_1, r_2 \in [0,1])$ designates the portion of the sample used to calculate the ADF statistic, so with a sample with periods ranging from 0 to T, the $ADF_{r_1=n/T}^{r_2=m/T}$ statistic is based on a subset of periods ranging from n to m, inclusively $(n, m \in \{0, \dots, T\}: n < m)$. The ADF test statistic is defined as:

$$ADF_{r_1}^{r_2} = \frac{\hat{\beta}_{r_1,r_2}}{\sigma_{\hat{\beta},r_1,r_2}}.$$

The Supremum ADF (SADF) (Phillips and Yu, 2011) is then defined as:

$$SADF(r_0) = \sup_{r_2 \in [r_0,1]} ADF_0^{r_2}.$$

One can see that the SADF is calculated recursively with an expanding sample of periods with a minimum window size of r_0, while keeping the starting period fixed at $r_1 = 0$. The SADF is suited to detect a single period of unit-root behaviour in the sample (Phillips, Shi and Yu, 2015). In order to measure multiple episodes of explosiveness at a given period, we consider the BSADF test statistic :

$$BSADF_{r_2}(r_0) \sup_{r_1 \in [0, r_2 - r_0]} ADF_{r_1}^{r_2},$$

where r_0 denotes the minimal window size. This procedure can produce real-time exuberance levels by setting r_2 as the present period (the t-th period corresponds to $r_2 = {}^{t}/_{T}$), and letting the start of our estimation period r_1 vary from the beginning of our sample (0) to $r_2 - r_0$. We can then recursively create a series of BSADF statistics for both HPI and HPI-to-income (See Figure 11 for HPI BSADF and Figure 12 for HPI-to-income BSADF in Annex 1).

Quantile Regression

Here, we will discuss the key points of QR methodology. We first present QR as an alternative to OLS regression. Next, we define the QR estimator as the solution of a linear programming problem and derive its first order conditions. We then discuss finite-sample performance problems that arise in the presence of clustered errors and fixed effects. Finally, we present robust inference procedures that allow us to circumvent such issues.

Introduced by Koenker and Bassett (1978), linear quantile regression (QR) methods depart from OLS methods by directly estimating the entire quantile process instead of relying on error independence and normality assumptions to indirectly infer the quantiles from the conditional mean of the outcome variable[15]. The θ-th conditional quantile of Y is defined as the value q_θ such that $\theta = F(q_\theta | \boldsymbol{X})$, where $F(\cdot\, | \boldsymbol{X})$ is the CDF of the response conditional on $\boldsymbol{X}$.

Conditional quantiles can be estimated across a set $\boldsymbol{\theta} \in (0,1)$. By estimating QR processes at various quantiles, we can quantify the effect a given covariate has on different regions of the conditional distribution, enabling the user to uncover location and scale effects of a covariate. Here, covariates whose effects are homogeneous across the quantiles are said to induce a “location shift”, because their effect causes a parallel movement of the quantiles, resulting in a translation of the conditional distribution. Covariates that induce a “scale shift”

[15] For an introduction to QR techniques, see : Davino, Furno, Vistocco (2014); Koenker and Basset (1978).

have coefficients that vary across the quantiles. Such heterogeneous effects can in turn stretch or distort the conditional distribution of the outcome. In brief, QR methods make no assumptions regarding the distribution of the outcome variable, and therefore stand as a powerful tool to characterize asymmetrical responses in the distribution of the variable of interest.

Instead of minimizing the sum of squared residuals as in OLS, the QR linear programming problem for estimating the θ-th unconditional quantile q_θ exploits an asymmetric loss function $\rho_\theta(\cdot)$ of residuals $\boldsymbol{\varepsilon}$:

$$\rho_\theta(\varepsilon) = [\theta - \boldsymbol{I}(\varepsilon < 0)] \cdot \varepsilon,$$

where $\boldsymbol{I}(\cdot)$ is the indicator function. More generally, the loss function of the θ-th QR uses a similarly asymmetric weighting of absolute residuals (negative errors take weights of $1 - \theta$ and positive errors take weights of θ). Let $= \boldsymbol{y} - \boldsymbol{\alpha}(\theta) - \boldsymbol{x}^\top \boldsymbol{\beta}(\theta)$:

$$\begin{aligned}\rho_\theta(\varepsilon) &= [\theta - \boldsymbol{I}(\varepsilon < 0)] \cdot (\varepsilon) \\ &= [(1-\theta)\boldsymbol{I}(\varepsilon \le 0) + \theta \boldsymbol{I}(\varepsilon > 0)] \cdot |\varepsilon|.\end{aligned}$$

The conditional quantile function's coefficients $(\widehat{\boldsymbol{\alpha}}(\theta), \widehat{\boldsymbol{\beta}}(\theta))$ are the solution to the following optimization problem:

$$\begin{aligned}\big(\boldsymbol{\alpha}(\theta), \boldsymbol{\beta}(\theta)\big) &= \underset{\boldsymbol{\alpha}(\boldsymbol{\theta}), \boldsymbol{\beta}(\boldsymbol{\theta})}{argmin}\, E\big[\rho_\theta\big(\boldsymbol{y} - \boldsymbol{\alpha}(\theta) - \boldsymbol{X}\boldsymbol{\beta}(\theta)\big)\big]. \\ &= \underset{\boldsymbol{\alpha}(\boldsymbol{\theta}), \boldsymbol{\beta}(\boldsymbol{\theta})}{argmin}\left(\int_{y \in \mathbb{R}} \rho_\theta\big(\boldsymbol{y} - \boldsymbol{\alpha}(\theta) - \boldsymbol{X}\boldsymbol{\beta}(\theta)\big) \cdot dF_Y(y)\right) \\ &= \underset{\boldsymbol{\alpha}(\boldsymbol{\theta}), \boldsymbol{\beta}(\boldsymbol{\theta})}{argmin}\left((1-\theta) \int_{y < \boldsymbol{\alpha} - \boldsymbol{X}\boldsymbol{\beta}} |\boldsymbol{y} - \boldsymbol{\alpha}(\theta) - \boldsymbol{X}\boldsymbol{\beta}(\theta)|\, dF_Y(y)dy \right. \\ &\qquad \left. + \theta \int_{y > \boldsymbol{\alpha} - \boldsymbol{X}\boldsymbol{\beta}} |\boldsymbol{y} - \boldsymbol{\alpha}(\theta) - \boldsymbol{X}\boldsymbol{\beta}(\theta)|\, dF_Y(y)dy\right).\end{aligned}$$

To derive the first order condition of this minimization problem, let $\hat{q}_\theta = \boldsymbol{\alpha}(\theta) + \boldsymbol{X}\boldsymbol{\beta}(\theta)$:

$$
\begin{aligned}
0 &= \frac{d}{d\hat{q}_\theta}\, E\left[\rho_\theta\left(\boldsymbol{y} - \widehat{\boldsymbol{q}}_{\boldsymbol{\theta}}\right)\right] \\
&= \frac{d}{d\hat{q}_\theta}\left((1-\theta)\int\limits_{y<\hat{q}_\theta} |y - \hat{q}_\theta|\, dF_Y(y) + \theta \int\limits_{y>\hat{q}_\theta} |y - \hat{q}_\theta|\, dF_Y(y)\right) \\
&= \frac{d}{d\hat{q}_\theta}\left((1-\theta)\int\limits_{-\infty}^{\hat{q}_\theta} (\hat{q}_\theta - y)\, dF_Y(y) + \theta \int\limits_{\hat{q}_\theta}^{+\infty} (y - \hat{q}_\theta)\, dF_Y(y)\right) \\
&= (1-\theta)\int\limits_{-\infty}^{\hat{q}_\theta} dF_Y(y) - \theta \int\limits_{\hat{q}_\theta}^{+\infty} dF_Y(y) \\
&= (1-\theta)F(\hat{q}_\theta) - \theta\left(1 - F(\hat{q}_\theta)\right) \\
\Leftrightarrow F(\hat{q}_\theta) &= \theta
\end{aligned}
$$

The first order condition leads to $F\left(\boldsymbol{\alpha}(\theta) + \boldsymbol{X}\boldsymbol{\beta}(\theta)\right) = \theta$ where F is the CDF of the response variable y. It follows that the expected value of the θ-th quantile of the error ε is 0.

The coefficient estimates are the solution to the following optimization problem:

$$
\left(\widehat{\boldsymbol{\alpha}}(\theta), \widehat{\boldsymbol{\beta}}(\theta)\right) = \underset{\boldsymbol{\alpha}(\boldsymbol{\theta}),\boldsymbol{\beta}(\boldsymbol{\theta})}{argmin} \left\{(1-\theta)\sum_{y\leq 0}\left|y_i - \widehat{\boldsymbol{\alpha}}(\theta) - x_i^{\top}\boldsymbol{\beta}(\theta)\right| \cdot f(y) \right.
$$

$$
\left. + \ \theta \sum_{y>0}\left|y_i - \widehat{\boldsymbol{\alpha}}(\theta) - x_i^{\top}\boldsymbol{\beta}(\theta)\right| \cdot f(y)\right\},
$$

Which, for a given θ, is a convex minimization problem that can effectively be solved by the iterative simplex method proposed by Dantzig (1963). This well-known computing algorithm is easily implemented and works well with medium-sized samples.[16]

[16] The simplex method stands as a time-costly option for large problems of over 100,000 observations. The QR version of the Frisch-Newton interior-point algorithm introduced by Portnoy and Koenker (1997) is better suited to accelerate the procedure with samples of this size (Davino, Furno and Vistocco, 2014).

Moreover, QR is "distribution-free", referring to the absence of parametric distributional assumptions. In fact, QR tolerates error terms with distributional asymmetries and skewness distortions, heteroscedasticity, and serial correlation. $\widehat{\boldsymbol{\alpha}}(\theta)$ and $\widehat{\boldsymbol{\beta}}(\theta)$ are also known to be asymptotically normal in general cases of non-identically distributed or dependent errors (Davino, Furno and Vistocco, 2014). In finite samples, slope estimates become skewed to the left for the lower quantiles and skewed to the right for higher quantiles as we move away from the median quantile estimate. Hypothesis testing performance with skewed coefficients can therefore lead to over-rejection rates at extreme quantiles, but simulation evidence shows that the estimators remain unbiased (Davino, Furno and Vistocco, 2014).

Since its inception, QR has found a number of applications. With panel data applications becoming particularly popular in recent years, clustered data and fixed-effects are also growing topics of interest in QR modelling.

Robust Inference with Clustered Data

We refer to clustered data when error terms in the model are not identically distributed across the panel's entities. Errors within an entity form a *cluster* of errors that may be serially correlated and/or heteroscedastic. In the presence of within-cluster autocorrelation and heteroscedasticity, the QR coefficient estimates are consistent and remain asymptotically normal, but traditional QR standard errors yield unreliable confidence intervals because residuals are assumed to be independent and identically distributed (iid) (Koenker and Bassett, 1978). This assumption is bound to lead to invalid covariance estimation in our application because the dependence between regional economic covariates and the response typically leads to heteroscedastic, serially correlated errors.

For instance, in our house price panel application, the high regional autocorrelation of house prices persists in the error clusters. As we can see in Figure 3, first order serial correlations of the state-specific residuals vary between -0.6 to 0.6. At the three quartiles, the median first-order autocorrelation across states is below 0.14. A large range of autocorrelation levels justifies the use of cluster-robust inference methods that allow for cluster-specific calibration of the error autocorrelation structures. This also implies that a model where all entities have a common correlation structure may be mispecified.

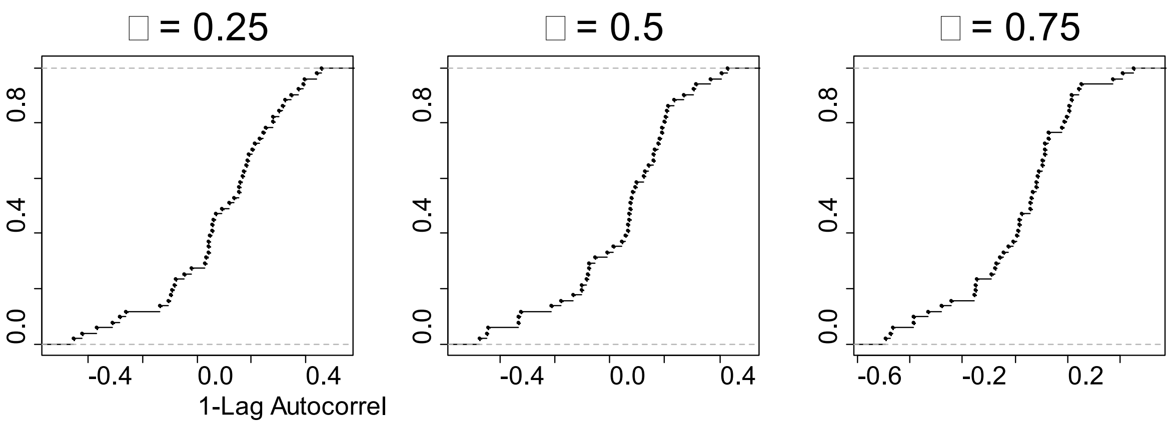

Figure 3: Cumulative distributions of state-level first-order autocorrelation of the residuals in the fixed effects QR model

Distributions of the residuals are reported for QRs with θ in $\{0.25, 0.5, 0.75\}$. The distributions of state-specific first-order serial correlations are similar to the model without fixed effects.

As a result, traditional standard errors fail to take into account that the densities of the errors changes through time and across states, but robust standard errors estimate entity-specific conditional error densities that allow consistent measuring of the QR covariance matrix.

Bootstrapped Confidence Intervals

The *wild gradient bootstrap* (Hagemann, 2017), extended from the *wild bootstrap* of Chen, Wei, and Parzen (2003), enables us to compute valid standard errors with arbitrary forms of cluster autocorrelation. This method is not robust to heteroscedasticity, however[17]. Standard errors are computed by resampling the QR estimator's subgradient — or first order conditions.

[17] Alternatively, Parente and Santos Silva (2016) present an analytical approach to cluster-robust covariance matrix estimation under serially correlated errors, but assume homoscedasticity. Machado and Santos Silva (2013) present a heteroscedasticity-robust covariance matrix estimator, but not in a clustering context.

The QR estimator $\left(\widehat{\boldsymbol{\alpha}}(\theta), \widehat{\boldsymbol{\beta}}(\theta)\right)$ in panel data solves the following condition:

$$\frac{1}{\sqrt{N}}\sum_{i=1}^{N}\sum_{j=1}^{T}\left[\theta - \boldsymbol{I}\left(y_{ij} - \alpha_i(\theta) - \boldsymbol{x}_{ij}{}^{\top}\boldsymbol{\beta}(\theta) < 0\right)\right]\boldsymbol{x}_{ij} = 0.$$

The wild gradient bootstrap process $W_N(\boldsymbol{\alpha}(\theta), \boldsymbol{\beta}(\theta), \theta)$ introduces state-level perturbations w_i such that:

$$W_N(\boldsymbol{\alpha}(\theta), \boldsymbol{\beta}(\theta), \theta) = \frac{1}{\sqrt{N}}\sum_{i=1}^{N} w_i \sum_{j=1}^{T}\left[\theta - \boldsymbol{I}\left(y_{ij} - \alpha_i(\theta) - \boldsymbol{x}_{ij}{}^{\top}\boldsymbol{\beta}(\theta) < 0\right)\right]\boldsymbol{x}_{ij},$$

where w_i's are drawn from an iid random variable[18] with $E[w_i] = 0$ and $E[|w_i|^q] < \infty$ for $q > 2$.

We now denote the QR objective function $M_N(\boldsymbol{\alpha}(\theta), \boldsymbol{\beta}(\theta), \theta)$, which can be expressed as:

$$\frac{1}{\sqrt{N}}\sum_{i=1}^{N}\sum_{j=1}^{T}\rho_\theta\left(y_{ij} - \alpha_i(\theta) - \boldsymbol{x}_{ij}{}^{\top}\boldsymbol{\beta}(\theta)\right),$$

Each replication of the wild gradient bootstrap will then solve for $\left(\widehat{\boldsymbol{\alpha}}^*(\theta), \widehat{\boldsymbol{\beta}}^*(\theta)\right)$ that minimizes a new objective function M_N^*, expressed as:

$$M_N^*(\boldsymbol{\alpha}(\theta), \boldsymbol{\beta}(\theta), \theta) = M_N(\boldsymbol{\alpha}(\theta), \boldsymbol{\beta}(\theta), \theta) + W_N(\boldsymbol{\alpha}(\theta), \boldsymbol{\beta}(\theta), \theta) \cdot \frac{\left(\boldsymbol{\alpha}(\theta), \boldsymbol{\beta}(\theta)\right)}{\sqrt{N}}.$$

The QR estimator's standard errors are subsequently estimated using the empirical distribution of the bootstrapped parameters.

Hagemann (2017) presents Monte Carlo evidence that empirical critical values of standard errors offer a well-sized testing procedure for numerous clusters at varying levels of within-cluster correlation. One drawback of this method is that it does not support fixed effects because of incidental parameter difficulties. This implies that for a given θ, all entities in the model share the same intercept.

[18] Hagemann (2017) suggest that w_i should be drawn from the Mammen (2012) 2-point distribution.

Robust Inference with State-Specific Fixed Effects

In a panel QR with clusters, we may also want to include entity fixed effects to capture unobserved, time-invariant regional factors, but the methods above are not adapted to a fixed effects model. Indeed, estimation in the presence of fixed effects proves to be more cumbersome for QR than OLS.

When estimating an OLS regression, fixed effects and slope coefficients can be estimated in two separate steps. First, entity-specific means are subtracted from each variable. Next, the slope coefficients are estimated based on the correlation structure of the demeaned data. Solving the OLS problem with entity-demeaned variables is adequate because we estimate the relationship between explanatory variables and the conditional *expectation* of the outcome, a linear operator. In contrast, QR estimates the relationship between explanatory variables and the conditional *quantile* of the outcome. As seen above, the first order condition of the QR problem requires that $F\left(\boldsymbol{\alpha}(\theta) + \boldsymbol{X\beta}(\theta)\right) = \theta$. Since the conditional quantile is not a linear operator, QR slopes cannot be estimated separately from fixed effects, which greatly increases the dimension of the optimization problem (Powell, 2016). The QR fixed effects literature also cautions that in panel data, a large number of individuals relative to the number of observations may induce data multicollinearity (see Hagemann (2017); Davino, Furno and Vistocco (2014)).

Clustered Covariance Matrices

Yoon and Galvao (2016) propose an analytical covariance estimator that is robust to both serial correlation and various forms heteroscedasticity in a fixed effects model. We will reproduce their methodology in a fixed effects model with our panel of house price data.

They present 2 tests in their paper: the Score Test and the Wald Test. One advantage of the Wald test is that it tolerates errors with autocorrelation and heteroscedasticity. The Score test also permits serially correlated errors, but restricts the model to a homoscedastic error mixture. The main drawback of the Wald test is that it requires estimation of the conditional density of the error terms, making it less reliable in smaller samples when the density is difficult to estimate. The Score test has slightly better size properties than the Wald test, but does not produce clustered standard errors. This is because the Score test is not based on the clustered covariance matrix (CCM) of the coefficients. Rather, the Score test estimates the CCM of the coefficients' sub-gradient[19].

For both tests, statistics are asymptotically distributed as chi-squared distributions with a number of degrees of freedom equal to the number of tested parameters.

The CCM is of the following form[20]:

$$\boldsymbol{\Sigma} = \boldsymbol{\Lambda}^{-1}\boldsymbol{V}\boldsymbol{\Lambda}^{-1}$$

In a panel with N entities, T periods, and p covariates, let $\boldsymbol{X}_{\cdot t}$ be a $N \times p$ matrix of observations at period t in $\{1, \dots, T\}$ and $\boldsymbol{c}_i = \frac{E[f_{it}(0|\boldsymbol{X}_{it})\boldsymbol{X}_{it}]}{E[f_{it}(0|\boldsymbol{X}_{it})]}$, where $f_{it}(\cdot)$ is the density function[21] of the residuals. The $\boldsymbol{\Lambda}$ term captures heteroscedasticity and is defined as:

$$\boldsymbol{\Lambda} \coloneqq \underset{N\to\infty}{plim} \frac{1}{N}\sum_{i=1}^{N} E[f_{it}(0|\boldsymbol{X}_{it})\boldsymbol{X}_{it}(\boldsymbol{X}_{it} - \boldsymbol{c}_i)'].$$

Additionally, let:

$$\varrho_{its} \coloneqq E[(\mathbf{1}(e_{it}(\theta) \leq 0) - \theta)(\mathbf{1}(e_{it}(\theta) > 0) - \theta)|\boldsymbol{X}_{i1}, \dots, \boldsymbol{X}_{iT}, \alpha_i].$$

$\boldsymbol{V}$ is then defined as:

$$\mathbf{V} \coloneqq \underset{N\to\infty}{plim} \frac{1}{N}\sum_{i=1}^{N} V_{N,i}^0 + V_{N,i}^1,$$

where

$$V_{N,i}^0 = \frac{1}{T}\sum_{t=1}^{T} \theta(1-\theta)E[(\boldsymbol{X}_{it} - \boldsymbol{c}_i)(\boldsymbol{X}_{it} - \boldsymbol{c}_i)'],$$

$$V_{N,i}^1 = \frac{1}{T}\sum_{s=1}^{T-1}\sum_{t=s+1}^{T} E\left[\varrho_{it,t-s}(\boldsymbol{X}_{it} - \boldsymbol{c}_i)(\boldsymbol{X}_{it-s} - \boldsymbol{c}_i)'\right] + E\left[\varrho_{it-s,t}(\boldsymbol{X}_{it-s} - \boldsymbol{c}_i)(\boldsymbol{X}_{it} - \boldsymbol{c}_i)'\right].$$

[19] See Yoon and Galvao (2016), pp. 13-15

[20] See Yoon and Galvao (2016), p. 10

[21] Kernel density estimation details will follow shortly.

The $\boldsymbol{V}$ matrix captures serial correlation through $V_{N,i}^{1}$. In practice, the autocorrelation structure is not always computed up to $T-1$ lags. Rather, we consider a truncation parameter $r_T \leq T-1$ that determines how many lags $V_{N,i}^{1}$ will account for. Theoretical support for the selection of r_T is a problem that has yet to be addressed in time-series literature (Yoon and Galvao, 2016). Indeed, the authors caution about the truncation lag choice, as the $\widehat{\boldsymbol{V}}$ estimate can be sensitive to r_T. Again, we follow the methodology suggested by the authors. They found that the rule $r_T = \max\left(2, \left\lfloor 1.2 \cdot T^{1/3} \right\rfloor\right)$ worked well for panel data of dimension similar to ours. This means that r_T is equal to 5 in our application. Hence, our $\widehat{\boldsymbol{V}}$ estimate considers time-dependence in errors that can be detected within 5 consecutive quarters.

The authors also present the $\widehat{\boldsymbol{V}}^{\boldsymbol{m}}$ estimator, a modified version of $\widehat{\boldsymbol{V}}$ that restricts serial dependence structures to be identical across entities ($\varrho_{its} = \varrho_{ts}$)[22]. They claim that the modified variance estimator is robust to the truncation parameter choice. We follow them and set r_T equal to $T-1$ so as to cover the entire time dependence structure in our sample.

Kernel Density Estimation

The CCM procedure requires state-specific error density estimates $\hat{f}_i(\cdot)$, which we estimate with the expression:

$$\hat{f}_i(x) = \frac{1}{Th}\sum_{t=1}^{T} K\left(\frac{\hat{e}_{it} - x}{h}\right),$$

where T is the number of periods in our sample. h and $K(\cdot)$ are the user-chosen bandwidth and kernel. We will use the Gaussian kernel, such that $K(\cdot)$ is the normal density function. Bandwidth validation is an issue that has not been resolved in the QR literature. The Silverman Rule of Thumb bandwidth, although *ad hoc*[23], is still widely used as a consistent

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[22] See Yoon and Galvao (2016), p. 12

[23] This method has been widely used since it was introduced by Silverman (1986). In fact, we will use the Silverman bandwidth in some initial steps of the computation of the Wald statistic. In cases where the error distribution is multimodal or asymmetric, the Silverman's rule of thumb is said to over-smooth the mode of the distribution causing a noisier estimate in the tails (Davino, Furno and Vistocco, 2014).

bandwidth selection. For a given set of QR residuals across the panel $\hat{\boldsymbol{e}}_{it}$, we can compute the state-level Silverman bandwidths h_{1i} as:

$$h_{1i} = \left(\frac{4\hat{\sigma}_e^5}{3n}\right),$$

where σ_e denotes the sample standard deviation of errors. This bandwidth selection issue is of less concern for our purposes because the calculation of the Wald test does not require the whole density to be estimated. Rather, we estimate the density at 0 with $\hat{f}_h(0)$. As suggested in Yoon and Galvao (2016), density estimates used for the calculation of the $\boldsymbol{\Lambda}$ term of the CCM estimator are conducted with h_2:

$$h_{2i} = \min\left(\hat{\sigma}_e\,; \frac{IQR}{1.34}\right),$$

where $\hat{\sigma}_e$ and IQR denote the QR residuals standard error and their inter-quartile range. This bandwidth is usually larger than the Silverman bandwidth, which leads to smoother densities.

In summary, the Wald test with the CCM estimator is a very flexible tool because it implements significance tests that are robust to state-specific heteroscedasticity, serial correlation and non-standard error-term distributions. Note that the $\widehat{\boldsymbol{V}}^m$ poses the restriction that all entities in the panel are subject to the same autocorrelation structure, but produces estimates more robust to the choice of lag truncation.

Monte Carlo Study

Simulation evidence shows that in a fixed effect QR model, testing procedures with no cluster correction perform poorly in the presence of serial correlation. In a panel with dimension $(N, T) = (50{,}100)$, Yoon and Galvao (2016) find that the CCM estimators effectively tackle the size-distortion issues that arise when using fixed effects QR[24].

[24] Simulation evidence shows that for error correlation of $\rho = 0.4$, the uncorrected Score test has a size of 0.136 while the corrected Score test has a size of 0.046. For error correlation $\rho = 0.8$, the uncorrected Score test has a size of 0.422 and the corrected Score test has a size of 0.062. These empirical test sizes are reported for data

In fact, our panel dimension is similar to the one presented by the authors with $(N, T) = (51{,}120)$. We investigate the finite sample performance of the CCM estimators by measuring empirical test sizes in a fixed effects QR model[25]. To that end, we calculate Wald and Score test statistics for a randomly generated spurious variable. We consider 2 simulation procedures.

The first simulation generates a process with fixed effects and homogeneous, serially correlated error terms as presented in Yoon and Galvao (2016) :

$$y_{it} = \alpha_i + x_{it}\beta_{it} + z_{it}\gamma_{it} + e_{it}\,,$$
$$\alpha_i \overset{iid}{\sim} Unif(0{,}1)\,,$$
$$x_{it} = 0.3\alpha_i + \varepsilon_{it}, \quad \varepsilon_{it} \overset{iid}{\sim} \chi^2_3\,,$$
$$e_{it} = \rho\, e_{it-1} + \nu_{it}, \quad \nu_{it} \overset{iid}{\sim} N(0{,}1-\rho^2)\,.$$

In this case all entities have the same error dependence structure, but with different innovations. We then incorporate a spurious treatment variable[26] on which we will carry out the Wald and Score tests. We generate the treatment variable as follows. We first randomly pick half of the entities in the panel and select a random period t in $(\lceil 0.1T \rceil, \dots, \lfloor 0.9T \rfloor)$. The spurious variable's observations that occur after period t in the randomly selected entities are then considered to be treated. In this way, we can measure and compare empirical test sizes by observing the distributions of Score and Wald statistics simulated under the null. We report empirical test sizes with first-order serial correlation ρ in $\{0, 0.4, 0.8\}$. At a nominal significance level of α, the cluster-robust significance test for the spurious variable should have a rejection rate of α.

We can see in Table IV of Annex 2 that the Wald statistic has much worse test sizes when the data are correlated, whereas the Score test and the modified Score test only

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simulated under a mixture with homoscedastic correlated error terms. The nominal rate is 5%; See Yoon and Galvao (2016)'s Annex : Table I: Sizes of Tests. Model 1

[25] We do not include empirical test sizes for the wild gradient bootstrap method in a no-fixed effects model due to computational costs. Test sizes are reported in Hagemann (2017); pp. 15-22.

[26] This simulation design is proposed in Bertrand, Duflo and Mullainathan (2004).

experience mild size distortions, even for data generated with serial correlation $\rho = 0.8$. The modified Wald test statistic seems to be slightly better-sized than the Score test in the case of no autocorrelation, though less decisively for medium correlation ($\rho = 0.4$). For highly correlated data, the modified Wald test's rejection rates can be over twice as much as the nominal rate, reaching 14.2% at the 5% level and 20.9% at the 10% level.

The second simulation aims to assess the size distortions that we encounter when we test the effects of a spurious random variable in the context of our full pane. In this case, we observe empirical test sizes for the same spurious treatment variable, but instead of including one other covariate in the model by generating N independent processes, the second simulation uses our panel of 13 covariates. This leads to severe size distortions for all CCM testing procedures[27]. Note that the CCM estimator rules out cross-sectional error mixtures, only allowing for within-state dependence. A weakness of our study therefore lies in the house price dependence between states. Indeed, Zimmer (2012) finds convincing copula-based evidence of dependence across regional housing markets.

We circumvent this by using size-corrected p-values. Size-corrected p-values are obtained by simulating the distributions of the test statistics for a spurious treatment variable in our whole panel. With 5000 replications, we estimate the cumulative distribution of the statistic under the null hypothesis so as to extract adjusted p-values for our 13 covariates[28]. This provides an *ad hoc* statistical tool to conduct well-sized tests when the fixed effects QR model presented in Yoon and Galvao (2016) is misspecified.

[27] See Table IV in Annex 2 for empirical test sizes in the second simulation

[28] See in Table V in Annex 2 for size-corrected p-values

5. Results and Analysis

In this section, we examine the estimated quantile effects of our covariates. In the tables below, we report coefficient estimates for the 13 covariates at the 3 quartiles with standard errors and p-values. Table II shows coefficient estimates in a model with no fixed effects with standard errors and p-values inferred from the wild gradient bootstrap procedure. Table III shows the coefficient estimates in a fixed effects model, with standard errors produced by the Wald and modified Wald tests. Since these tests tend to over-reject in the presence of serial correlation and cross-cluster dependence, we report p-values obtained from the better-sized Score test.

QR Estimates — Model without Fixed Effects

Covariates	Parameter	θ		
		0.25	**0.50**	**0.75**
HPI % change	$\hat{\beta}(\theta)$	0.41823***	0.43022***	0.46840***
	$\hat{\sigma}_b$	0.03257	0.03310	0.04868
Real GDP % change	$\hat{\beta}(\theta)$	0.05988***	0.02178*	0.02328*
	$\hat{\sigma}_b$	0.00733	0.00973	0.01125
Real personal income % change	$\hat{\beta}(\theta)$	0.00006	-0.00022	-0.00001
	$\hat{\sigma}_b$	0.00018	0.00017	0.00014
HPI exuberance (BSADF)	$\hat{\beta}(\theta)$	0.00124***	0.00095***	0.00081***
	$\hat{\sigma}_b$	0.00010	0.00007	0.00013
HPI / income exuberance (BSADF)	$\hat{\beta}(\theta)$	-0.00013	0.00018	0.00056*
	$\hat{\sigma}_b$	0.00017	0.00014	0.00024
Housing vacancy	$\hat{\beta}(\theta)$	-0.00299***	-0.00190***	-0.00130*
	$\hat{\sigma}_b$	0.00048	0.00044	0.00053
Rental vacancy	$\hat{\beta}(\theta)$	0.00000	-0.00015	-0.00027**
	$\hat{\sigma}_b$	0.00008	0.00008	0.00010
Total outstanding mortgage debt / GDP	$\hat{\beta}(\theta)$	-0.02875***	-0.01589***	-0.00600*
	$\hat{\sigma}_b$	0.00285	0.00251	0.00305
Property tax / personal income	$\hat{\beta}(\theta)$	-0.07945	0.01333	0.00677
	$\hat{\sigma}_b$	0.36475	0.28108	0.56047
Terms-of-trade index	$\hat{\beta}(\theta)$	0.00010	0.00020**	-0.00001
	$\hat{\sigma}_b$	0.00007	0.00006	0.00009
5- to 1-year treasury rate spread	$\hat{\beta}(\theta)$	0.00099***	0.00044*	0.00081**
	$\hat{\sigma}_b$	0.00022	0.00019	0.00028
30-year mortgage rate	$\hat{\beta}(\theta)$	0.00004	0.00034**	0.00059***
	$\hat{\sigma}_b$	0.00012	0.00011	0.00015
Unemployment	$\hat{\beta}(\theta)$	-0.00028	0.00006	0.00044
	$\hat{\sigma}_b$	0.00015	0.00011	0.00024

Table II: Coefficients estimates and bootstrapped standard with a QR model no fixed effects
***: p<0.001, **: p<0.01, *: p<0.05 ; 300 bootstrap replications

QR Estimates — Model with Individual Fixed Effects

Covariates	Parameter	θ		
		0.25	**0.50**	**0.75**
HPI % change	$\hat{\beta}(\theta)$	0.38158**	0.40900**	0.41888**
	$\hat{\sigma}_W$	0.02475	0.03379	0.07053
	$\hat{\sigma}_{W^m}$	0.02224	0.03285	0.02707
Real GDP % change	$\hat{\beta}(\theta)$	0.05299*	0.03130	0.03757
	$\hat{\sigma}_W$	0.00603	0.00815	0.00472
	$\hat{\sigma}_{W^m}$	0.00880	0.00879	0.01444
Real personal income % change	$\hat{\beta}(\theta)$	-0.00014	-0.00034	-0.00027
	$\hat{\sigma}_W$	0.00015	0.00022	0.00023
	$\hat{\sigma}_{W^m}$	0.00015	0.00019	0.00018
HPI exuberance (BSADF)	$\hat{\beta}(\theta)$	0.00107**	0.00093**	0.00092**
	$\hat{\sigma}_W$	0.00010	0.00025	0.00009
	$\hat{\sigma}_{W^m}$	0.00013	0.00026	0.00015
HPI / income exuberance (BSADF)	$\hat{\beta}(\theta)$	0.00008	0.00022	0.00055
	$\hat{\sigma}_W$	0.00015	0.00049	0.00019
	$\hat{\sigma}_{W^m}$	0.00024	0.00061	0.00028
Housing vacancy	$\hat{\beta}(\theta)$	-0.00403	-0.00299	-0.00199
	$\hat{\sigma}_W$	0.00056	0.00059	0.00065
	$\hat{\sigma}_{W^m}$	0.00045	0.00057	0.00054
Rental vacancy	$\hat{\beta}(\theta)$	-0.00042	-0.00030*	-0.00030**
	$\hat{\sigma}_W$	0.00010	0.00026	0.00016
	$\hat{\sigma}_{W^m}$	0.00011	0.00032	0.00015
Total outstanding mortgage debt / GDP	$\hat{\beta}(\theta)$	-0.02194*	-0.01367**	-0.00423**
	$\hat{\sigma}_W$	0.00344	0.01913	0.00740
	$\hat{\sigma}_{W^m}$	0.00502	0.02239	0.01029
Property tax / personal income	$\hat{\beta}(\theta)$	0.37880	-0.08077	-1.29721
	$\hat{\sigma}_W$	0.94069	0.86360	0.15573
	$\hat{\sigma}_{W^m}$	0.95796	1.3099	1.3680

Covariates	Parameter	$\boldsymbol{\theta}$		
		0.25	**0.50**	**0.75**
Terms-of-trade index	$\widehat{\boldsymbol{\beta}}(\boldsymbol{\theta})$	0.00022	0.00020	0.00005
	$\widehat{\boldsymbol{\sigma}}_{\boldsymbol{W}}$	0.00016	0.00119	0.00060
	$\widehat{\boldsymbol{\sigma}}_{\boldsymbol{W}^{\boldsymbol{m}}}$	0.00022	0.00141	0.00081
5- to 1-year treasury rate spread	$\widehat{\boldsymbol{\beta}}(\boldsymbol{\theta})$	0.00108*	0.00039*	0.00054*
	$\widehat{\boldsymbol{\sigma}}_{\boldsymbol{W}}$	0.00012	0.00052	0.00038
	$\widehat{\boldsymbol{\sigma}}_{\boldsymbol{W}^{\boldsymbol{m}}}$	0.00027	0.00057	0.00041
30-year mortgage rate	$\widehat{\boldsymbol{\beta}}(\boldsymbol{\theta})$	-0.00032	0.00013	0.00056
	$\widehat{\boldsymbol{\sigma}}_{\boldsymbol{W}}$	0.00011	0.00056	0.00026
	$\widehat{\boldsymbol{\sigma}}_{\boldsymbol{W}^{\boldsymbol{m}}}$	0.00017	0.00063	0.00037
Unemployment	$\widehat{\boldsymbol{\beta}}(\boldsymbol{\theta})$	-0.00077	-0.00006	0.00051
	$\widehat{\boldsymbol{\sigma}}_{\boldsymbol{W}}$	0.00015	0.00068	0.00028
	$\widehat{\boldsymbol{\sigma}}_{\boldsymbol{W}^{\boldsymbol{m}}}$	0.00023	0.00079	0.00033

Table III: Coefficient estimates and robust standard errors in a fixed effects QR model

***: p<0.001, **: p<0.01, *: p<0.05 ; We report Score test significance levels and Wald standard errors ; The superscript m indicates the use of the modified CCM estimator

In order to appreciate the richness of QR models, we also estimated the coefficient estimates at all the deciles. Annex 3 includes plots of the coefficients against the 9 deciles along with 95% confidence intervals computed with the Wald and modified Wald standard errors. At a glance, coefficient plots allow us to visualize how a covariate's effect changes in different regions of the distribution of house price growth. In some cases, it can also be relevant to compare how the QR process at a specific quantile may differ from the OLS coefficient values. In the remainder of this chapter, we heuristically[29] assess coefficient heterogeneity by observing the patterns in the coefficient plots.

[29] A variant of the traditional Wald test for QR allows for joint statistical testing of coefficient equality quantiles and stands as a potential test for heterogeneity (Koenker and Bassett, 1982). Alternatively, one can also test for the specification of a location shift or a location-scale shift model (Koenker and Xiao, 2002).

Referring to the tables above, we find that most variables in our model are not significant. Indeed, income growth, HPI/Income BSADFs, vacancy rates, property taxes, terms-of-trade, mortgage rates, and unemployment have insignificant coefficients at most quantiles. Covariates with significant coefficients include the autoregressive component of house price changes, GDP growth, HPI BSADFs, mortgage-debt-to-GDP ratios, and treasury rate spreads. In those cases the signs of the coefficients match our predictions based on recent house price dynamics literature.

We included an autoregressive component of house price changes in the model to capture sequential price movements due to extrapolative expectations. In both models, the coefficients are highly significant and positive across all quartiles. As we can see in Figure 21, the coefficients are roughly constant across the deciles, so the distributional response to a 0.05 increase in regional HPI (such events occurred primarily in the late 1980s and early 2000s) is a +0.02 location shift of the conditional quantiles. Looking at our series of state-level house price changes (Figure 8 in Annex 1), one can see that such a location shift is not trivial. Indeed, across all states, the first and ninth deciles of house price changes are -0.0092 and +0.0253. The impact of extrapolative expectations on the conditional distribution of house price changes is therefore homogeneous and positive.

Growth in GDP stimulates house prices in both models with larger effect at the lower quartile. One can notice in Figure 22 that the coefficients at the first decile appear to be over twice that of the coefficients at the 9^{th} decile, possibly suggesting heterogeneity. Indeed, results from both models indicate that a 1% increase in GDP would drive the lower decile up by approximately 0.08, but upper quantiles would only increase by 0.02 to 0.04. In turn, when GDP increases, the conditional distribution of house prices moves slightly up and tightens as the lower quantiles are driven up further. This means that GDP growth reduces the magnitude of house price downturns in the next quarter by pushing up the lower quantiles. Conversely, a drop in GDP reduces most conditional quantiles and stretches the lower region of the distribution downward. Variations in GDP growth therefore cause a combination of location and scale shifts.

The HPI exuberance displays positive coefficients at the quartiles. In the model with fixed effects, the coefficients appear homogeneous and roughly match the OLS coefficient, but the model without fixed effects finds lower coefficients in the upper deciles. As we can see in Figure 24, the coefficient at the first decile is almost twice as much as the coefficient lower decile. Moreover, results from Table II indicate that the coefficient is 0.0012 at the first quartile and 0.0008 at the third quartile. It may seem counter-intuitive that the euphoria of the consumers drives up the lower quantiles. Indeed, one can argue that over-optimistic consumers push prices up unsustainably, which is liable to cause larger price corrections and push down lower conditional quantiles. However, following the idea of Pavlidis *et al.* (2016), we argue that unsustainable prices are instead captured by the HPI-to-income ratio exuberance. Indeed, a bubble appears when prices depart from fundamentals like income. The HPI exuberance captures market optimism and has a positive effect on the distribution.

In the model with no fixed effects, the mortgage-debt-to-GDP ratio displays heterogeneity with strongly significant negative effects in the lower region of the distribution and no significant effect in the upper region. We see in Figure 28 that the covariate has a highly significant coefficient of -0.04 at the first decile. The coefficients then steadily increase to about 0 at the 9^{th} decile. This is in line with the conclusions of the study of Jordà, Schularick and Taylor (2016a) according to which the surge of mortgage debt aggravates the risk of a financial crash in developed economies. In 2014, the mortgage-debt-outstanding-to-GDP ratio reaches its minimum in our sample at 69%, causing a response of -0.0276 at the first decile. In the 2007 to 2009 period, it peaks at above 100%, causing a response of approximately -0.04. In contrast, fluctuations of the ratio have no significant impact on the 8^{th} and 9^{th} deciles. This is indicative of a scale shift. Indeed, as mortgages further leverage the economy, house price devaluations are expected to be larger. A similar coefficient pattern is observable in the fixed effects model, although only significantly at the first quartile.

We also proxy the medium-term yield curve slope with the 5- to 1-year treasury rate spread. In both models, the treasury rate spread effects are positive as expected and significant across all the quartiles. In both models we detect a slightly enhanced effect below the median. Indeed, the coefficients at the first quartile are approximately equal to 0.001 in both models, but take values less than half of that at the median. This suggests that a steeper yield curve causes a positive location shift together with an upward scaling of the lower quantiles.

QR Fit

Here, we compare QR and OLS fitted values by representing the historical data alongside conditional quantile estimates. For the QR, we represent the paths of 5 conditional quantiles $\hat{q}_\theta$ with θ in $\{0.10, 0.25, 0.50, 0.75, 0.90\}$ alongside the actual historical data. Note that coefficients may be increasingly skewed (although unbiased) as we move away from the median. Therefore, fitted values at the extreme deciles may be less reliable than at the quartiles.

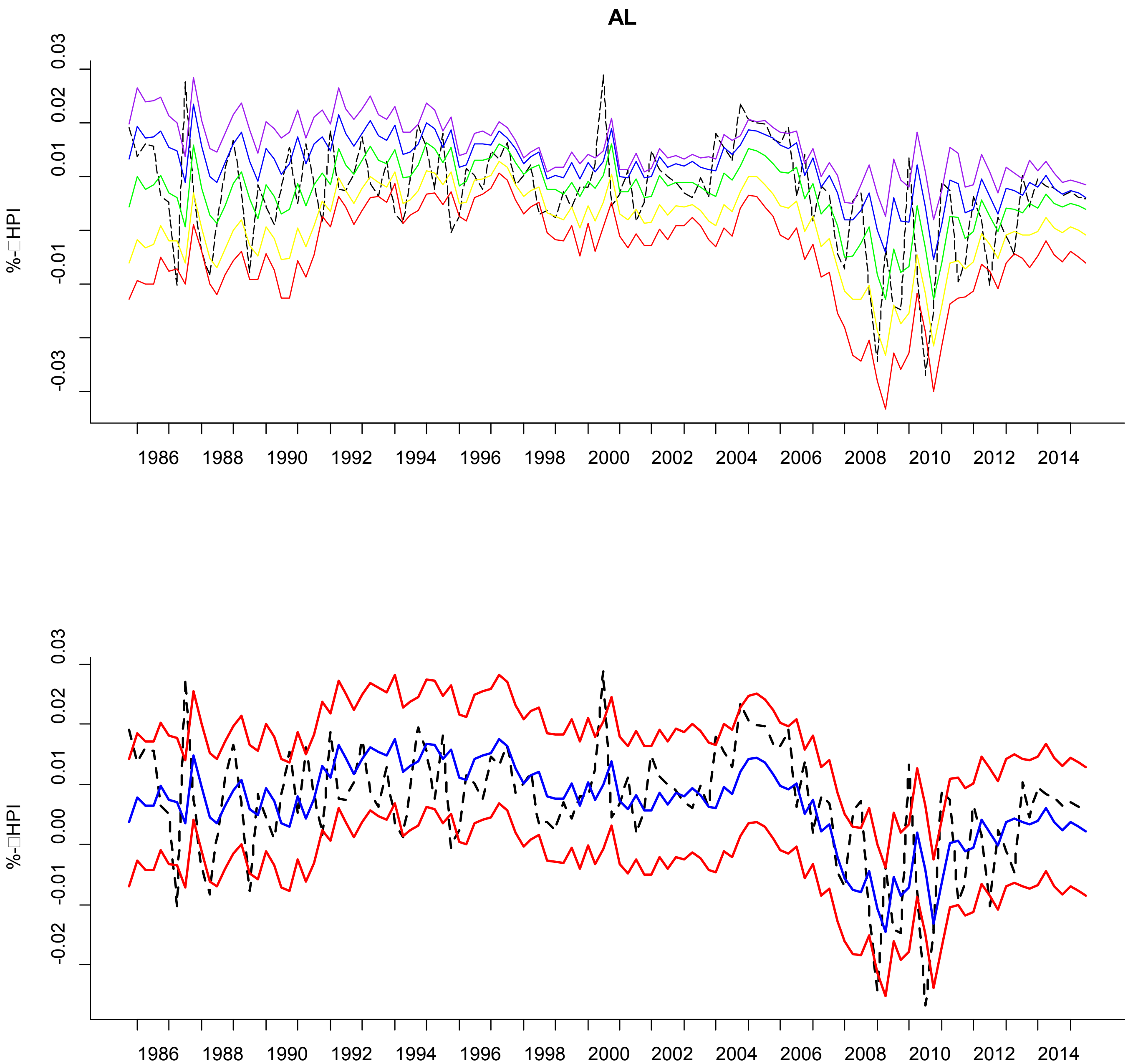
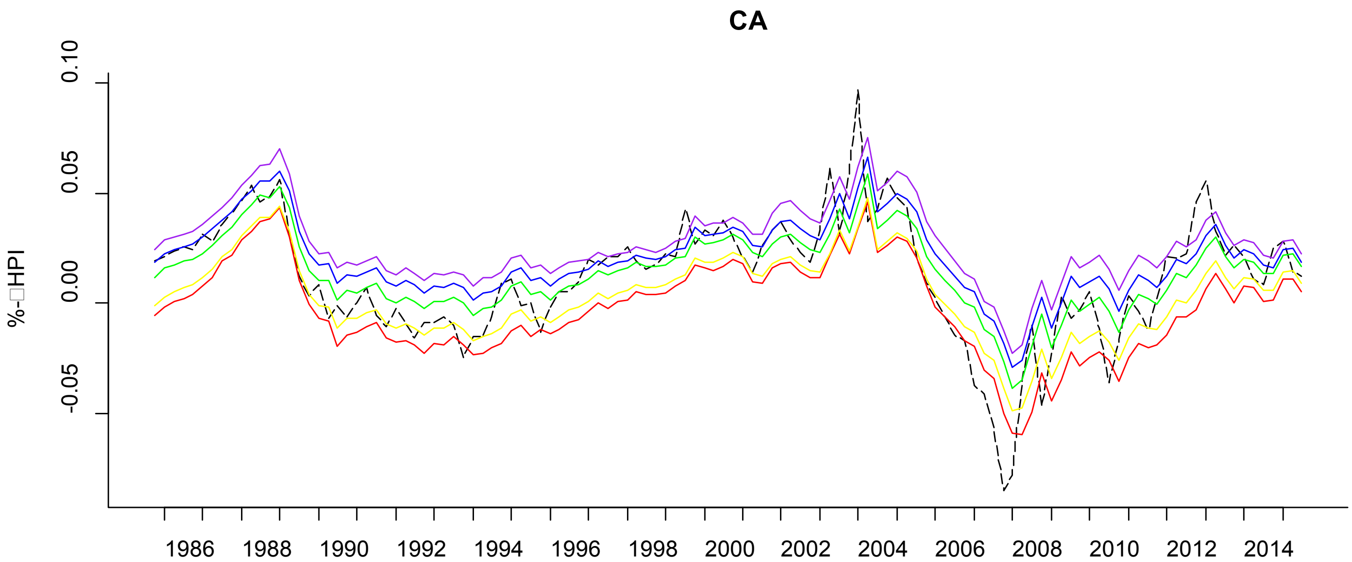


Figure 4 : Fitted quantile estimates for the state of Alabama

We present fitted values for the FE QR model (top), where fitted quantile estimates are reported for θ in {0.1,0.25,0.5,0.75,0.9}. For the FE OLS model (bottom), we present fitted conditional means (blue). The first and last deciles (red) of the fitted values are calculated according to OLS error normality conditions. Historical data is illustrated by the dashed black line.



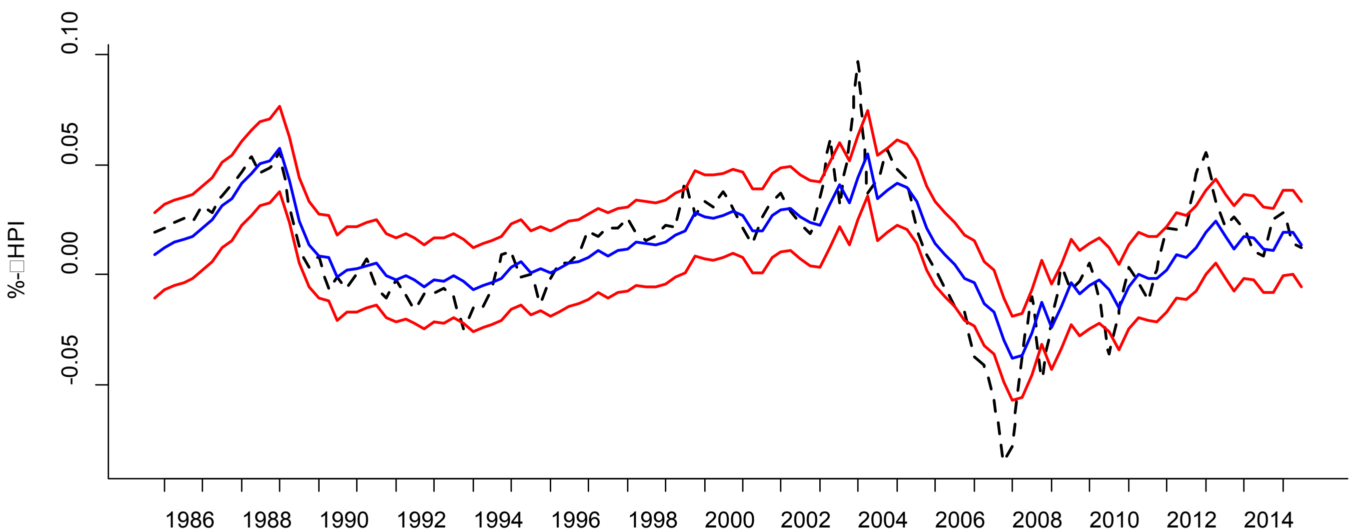


Figure 5 : Fitted quantile estimates for the state of California

We present fitted values for the FE QR model (top), where fitted quantile estimates are reported for θ in {0.1,0.25,0.5,0.75,0.9}. For the FE OLS model (bottom), we present fitted conditional means (blue). The first and last deciles (red) of the fitted values are calculated according to OLS error normality conditions. Historical data is illustrated by the dashed black line.

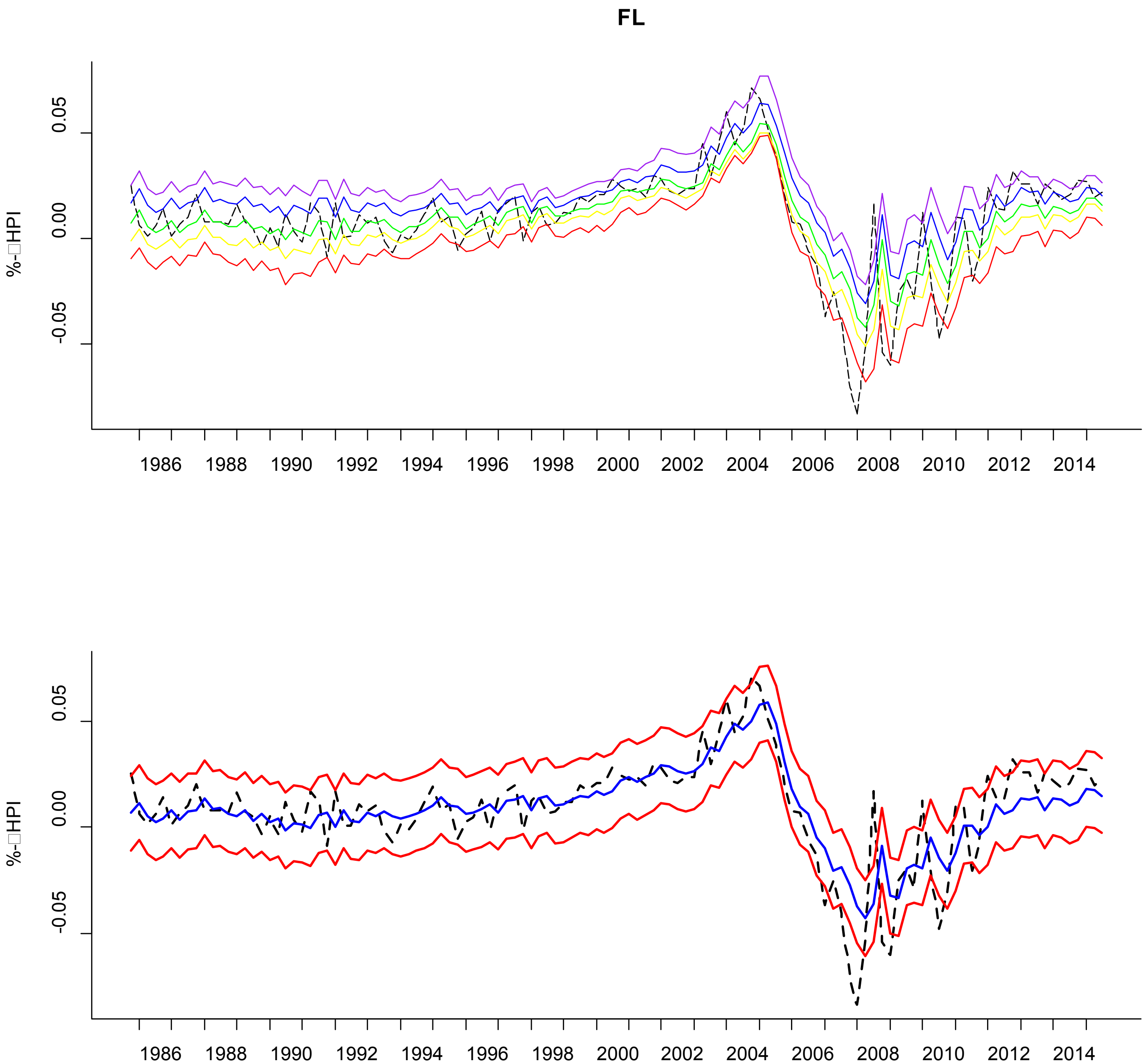


Figure 6 : Fitted quantile estimates for the state of Florida

We present fitted values for the FE QR model (top), where fitted quantile estimates are reported for θ in {0.1,0.25,0.5,0.75,0.9}. For the FE OLS model (bottom), we present fitted conditional means (blue). The first and last deciles (red) of the fitted values are calculated according to OLS error normality conditions. Historical data is illustrated by the dashed black line.

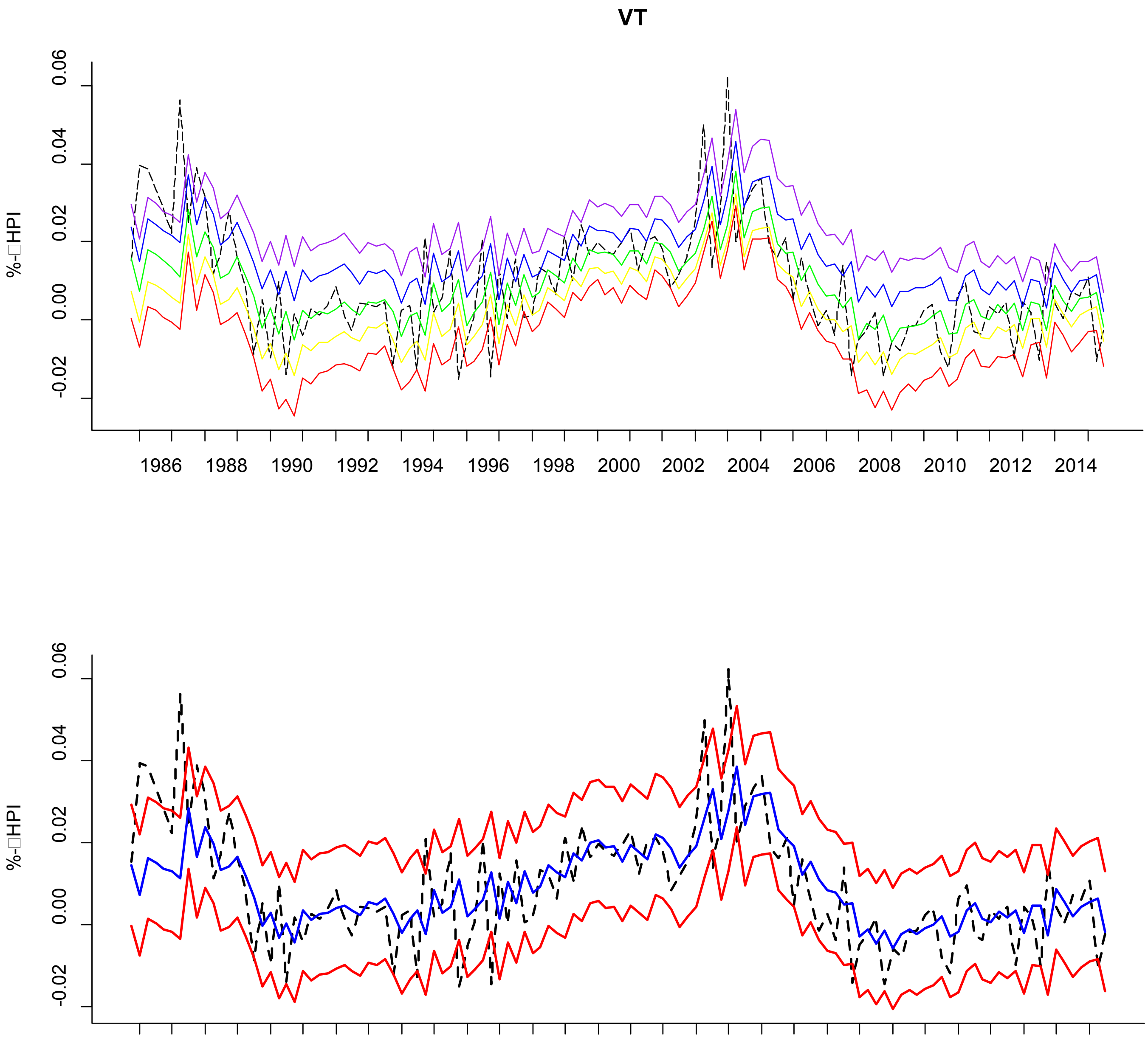


Figure 7 : Fitted quantile estimates for the state of Vermont

We present fitted values for the FE QR model (top), where fitted quantile estimates are reported for θ in $\{0.1,0.25,0.5,0.75,0.9\}$. We present fitted conditional mean (blue) for a FE OLS model. The first and last deciles (red) of the fitted values are calculated according to OLS error normality conditions. Historical data is illustrated by the dashed black line.

Note that the estimated quantile process can sometimes underestimate the magnitude of sharp changes. For instance, between late 2005 and 2008, Florida experienced a peak-to-trough shift from 0.07 to -0.08. Even after 3 years of sinking prices, the lowest point of housing returns still was a low-probability event. The QR model also seems to struggle in predicting price surges, as we can see in the case of Vermont in the first half of the 2000s with surprising HPI growth of 0.06. Also, in the aftermath of the housing bust, post-2010 median growth in California is expected to be between 0 and 0.02 and our model failed to predict the observed 0.05 increase in California's HPI.

One persistent element across all the states and DC is that fitted conditional quantiles move in parallel, implying that the covariates are largely capturing a shift in the location of the house price growth distribution. However, we do note that the dispersion of the estimated distribution of house price fluctuations changes throughout the sample. For instance, in the states of Alabama, the first and last deciles are farther apart between the mid-1980s and the mid-1990s than between the mid-1990s through the mid-2000s. Indeed, the fitted 9th decile stays relatively stable throughout the sample, but the 1st decile has much larger convulsions. By observing the isolated quantile effects of each covariate, we find that the observed scale shift in the distribution of house price growth is mainly attributable to the movements of the HPI exuberance and the mortgage rates. The surge in exuberance that started in the late 1990s can therefore explain a contraction of first and last deciles' range. We also observe that the effects of the 30-year mortgage are negative for the lower decile and positive for the upper decile. As the mortgage rates steadily declined from 10% to 4% between 1986 and 2014, the fitted quantiles were subsequently drawn closer. The heterogeneous effects of certain covariates can therefore capture asymmetric quantile movements, thereby indicating varying levels of downside house price risk.

6. Conclusion

In this study, we modelled the distribution of US state-level house price changes. Guided by recent developments in behavioural economics and housing finance literatures[30], we consider consumer speculation as an important factor of housing risk. In line with Case and Shiller (2003)'s argument, Glaeser (2013) provides historical examples that display the public's inaptitude to collectively recognize unsustainable prices, which may lead to a euphoria-driven house price bubble. That is not to say that housing asset returns are not affected by macro-financial business cycles, yet houses significantly differ from financial instruments in that the parties involved in a real estate transaction aren't professional traders. In short, real estate speculation in the US suffers from various malfunctions that may result in large housing market convulsions. We therefore considered two classes of house price drivers to construct a sensible model: macroeconomic fundamentals and public speculation variables.

A quantile regression approach allowed us to approximate a finite set of conditional quantile processes for house price variations. In doing so, we dropped distributional assumptions that otherwise appear in OLS methods. This allowed us to detect location and scale shifts in the conditional distribution of house price changes. Moreover, we use novel QR inference methods that are adapted to clustering, entity fixed effects, and heteroscedasticity.

We found that most covariates had insignificant effects across the quartiles, namely: income growth, the house-price-to-income ratio exuberance, vacancy rates, the property-taxes-to-income ratio, the terms-of-trade index, mortgage rates, and unemployment. We also found that the last observed price change, GDP growth, price exuberance, the outstanding-mortgage-debt-to-GDP ratio and the treasury rate spread are capable of capturing sharp downturns in house prices (location shifts) as well as varying dispersion in the distribution (scale shifts). Moreover, our predictions regarding the sign of the coefficients were supported by the literature and were correct for all the significant coefficients.

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[30]See *Animal Spirits* (Akerlof and Shiller, 2010)

References

Adam, Klaus, Albert Marcet and Johannes Beutel (2017). « Stock Price Booms and Expected Capital Gains », *American Economic Review*, vol. 107, no 8, p. 2352-2408.

Akerlof, George A and Robert J Shiller (2010). *Animal Spirits: How Human Psychology Drives the Economy, and Why It Matters for Global Capitalism*, Princeton, New Jersey, Princeton University Press Princeton, 248 p.

Bauer, Gregory H (2014). *International House Price Cycles, Monetary Policy and Risk Premiums*, Staff Working Paper no 2014-54, Ottawa, Ontario, Bank of Canada, 51 p. Récupéré de http://www.bankofcanada.ca/2014/12/working-paper-2014-54/

Bertrand, Marianne, Esther Duflo and Sendhil Mullainathan (2004). « How Much Should We Trust Differences-in-Differences Estimates? », *The Quarterly Journal of Economics*, vol. 119, no 1, p. 249-275.

Calhoun, Charles A (1996). *Ofheo House Price Indexes: Hpi Technical Description* Washington, DC, Federal Housing Finance Agency. Récupéré de https://www.fhfa.gov/PolicyProgramsResearch/Research/Pages/HPI-Technical-Description.aspx

Case, Karl E and Robert J Shiller (2003). « Is There a Bubble in the Housing Market? », *Brookings Papers on Economic Activity*, vol. 2003, no 2, p. 299-342.

Christensen, Peter, Kenneth Gillingham and William Nordhaus (2016). « Uncertainty in Forecasts of Long-Run Productivity Growth », communication présentée au *19th Annual Conference on Global Economic Analysis*, April 15, 2016, Washington, DC.

Corrigan, Paul (2017). *Terms-of-Trade and House Price Fluctuations: A Cross-Country Study*, Staff Working Paper no 2017-1, Ottawa, Ontario, Bank of Canada, 59 p. Récupéré de http://www.bankofcanada.ca/2017/01/staff-working-paper-2017-1/

Davino, Cristina, Marilena Furno and Domenico Vistocco (2014). *Quantile Regression: Theory and Applications*, Oxford, John Wiley & Sons, coll. Wiley Series in Probability and Statistics, 276 p.

Glaeser, Edward L (2013). « A Nation of Gamblers: Real Estate Speculation and American History », *American Economic Review*, vol. 103, no 3, p. 1-42.

Glaeser, Edward L, Joseph Gyourko and Albert Saiz (2008). « Housing Supply and Housing Bubbles », *Journal of Urban Economics*, vol. 64, no 2, p. 198-217.

Glaeser, Edward L and Charles G Nathanson (2017). « An Extrapolative Model of House Price Dynamics », *Journal of Financial Economics*.

Granziera, Eleonora and Sharon Kozicki (2015). « House Price Dynamics: Fundamentals and Expectations », *Journal of Economic Dynamics and Control*, vol. 60, p. 152-165.

Hagemann, Andreas (2017). « Cluster-Robust Bootstrap Inference in Quantile Regression Models », *Journal of the American Statistical Association*, vol. 112, no 517, p. 446-456.

Ho, Anson TY, Kim P Huynh and David T Jacho-Chávez (2016). « Flexible Estimation of Copulas: An Application to the U.S. Housing Crisis », *Journal of Applied Econometrics*, vol. 31, no 3, p. 603-610.

Jordà, Òscar, Moritz Schularick and Alan M Taylor (2015). « Betting the House », *Journal of International Economics*, vol. 96, p. S2-S18.

Jordà, Òscar, Moritz Schularick and Alan M Taylor (2016a). « The Great Mortgaging: Housing Finance, Crises and Business Cycles », *Economic Policy*, vol. 31, no 85, p. 107-152.

Jordà, Òscar, Moritz Schularick and Alan M Taylor (2016b). « Macrofinancial History and the New Business Cycle Facts », dans *Nber Macroeconomics Annual 2016, Volume 31*, University of Chicago Press.

Kahn, James A (2009). « Productivity Swings and Housing Prices », *Current Issues in Economics and Finance*, vol. 15, no July 3.

Kindleberger, Charles P and Robert Z Aliber (2011). *Manias Panics and Crashes: A History of Financial Crises*, 6e éd., United Kingdom, Palgrave Macmillan.

Knoll, Katharina, Moritz Schularick and Thomas Steger (2017). « No Price Like Home: Global House Prices, 1870–2012 », *The American Economic Review*, vol. 107, no 2, p. 331-353.

Koenker, Roger and Gilbert Bassett (1978). « Regression Quantiles », *Econometrica: Journal of the Econometric Society*, vol. 46, no 1, p. 33-50.

Koenker, Roger and Gilbert Bassett (1982). « Robust Tests for Heteroscedasticity Based on Regression Quantiles », *Econometrica: Journal of the Econometric Society*, p. 43-61.

Koenker, Roger and Zhijie Xiao (2002). « Inference on the Quantile Regression Process », *Econometrica: Journal of the Econometric Society*, vol. 70, no 4, p. 1583-1612.

Laeven, Luc and Fabian Valencia (2008). *Systemic Banking Crises: A New Database*, IMF Working Papers, International Monetary Fund. Récupéré de http://www.imf.org/en/Publications/WP/Issues/2016/12/31/Systemic-Banking-Crises-A-New-Database-22345

Machado, José AF and João Santos Silva (2013). *Quantile Regression and Heteroskedasticity*[Manuscript], University of Essex, Department of Economics.

Mammen, Enno (2012). *When Does Bootstrap Work?: Asymptotic Results and Simulations*, vol. 77, New York, Springer Science & Business Media, coll. Lecture Notes in Statistics.

Monnet, Éric and Clara Wolf (2017). « Is Demographics the Housing Cycle? », *Rue de la Banque*, no 41.

Parente, Paulo and João Santos Silva (2016). « Quantile Regression with Clustered Data », *Journal of Econometric Methods*, vol. 5, no 1, p. 1-15.

Pavlidis, Efthymios, Alisa Yusupova, Ivan Paya, David Peel, Enrique Martínez-García, Adrienne Mack*, et al.* (2016). « Episodes of Exuberance in Housing Markets: In Search of the Smoking Gun », *The Journal of Real Estate Finance and Economics*, vol. 53, no 4, p. 419-449.

Peterson, Brian and Yi Zheng (2011). « Medium-Term Fluctuations in Canadian House Prices », *Bank of Canada Review*, vol. 2011, no Winter, p. 30-42.

Phillips, Peter CB, Shuping Shi and Jun Yu (2015). « Testing for Multiple Bubbles: Historical Episodes of Exuberance and Collapse in the S&P 500 », *International Economic Review*, vol. 56, no 4, p. 1043-1078.

Phillips, Peter CB and Jun Yu (2011). « Dating the Timeline of Financial Bubbles During the Subprime Crisis », *Quantitative Economics*, vol. 2, no 3, p. 455-491.

Portnoy, Stephen and Roger Koenker (1997). « The Gaussian Hare and the Laplacian Tortoise: Computability of Squared-Error Versus Absolute-Error Estimators », *Statistical Science*, vol. 12, no 4, p. 279-300.

Powell, David (2016). *Quantile Treatment Effects in the Presence of Covariates*[Manuscript], RAND Corporation.

Reinhart, Carmen M and Kenneth S Rogoff (2009). *This Time Is Different: Eight Centuries of Financial Folly*, Princeton, New Jersey, Princeton University Press, 512 p.

Rünstler, Gerhard and Marente Vlekke (2016). *Business, Housing and Credit Cycles*, Working Paper no 1915, European Central Bank. Récupéré de https://www.ecb.europa.eu/pub/pdf/scpwps/ecbwp1915.en.pdf?b2544157a609ce7181a77ea9cd51bd45

Schularick, Moritz and Alan M Taylor (2009). « Credit Booms Gone Bust: Monetary Policy, Leverage Cycles and Financial Crises, 1870–2008 », *The American Economic Review*, vol. 102, no 2, p. 1029-1061.

Shiller, Robert J (2008). « Historic Turning Points in Real Estate », *Eastern Economic Journal*, vol. 34, no 1, p. 1-13.

Silverman, Bernard W (1986). *Density Estimation for Statistics and Data Analysis*, London, Chapman & Hall, 176 p.

Yoon, Jungmo and Antonio F Galvao (2016). *Robust Inference for Panel Quantile Regression Models with Individual Fixed Effects and Serial Correlation*[Manuscript].

Zimmer, David M (2012). « The Role of Copulas in the Housing Crisis », *Review of Economics and Statistics*, vol. 94, no 2, p. 607-620.

Annex 1 — Panel Historical Data

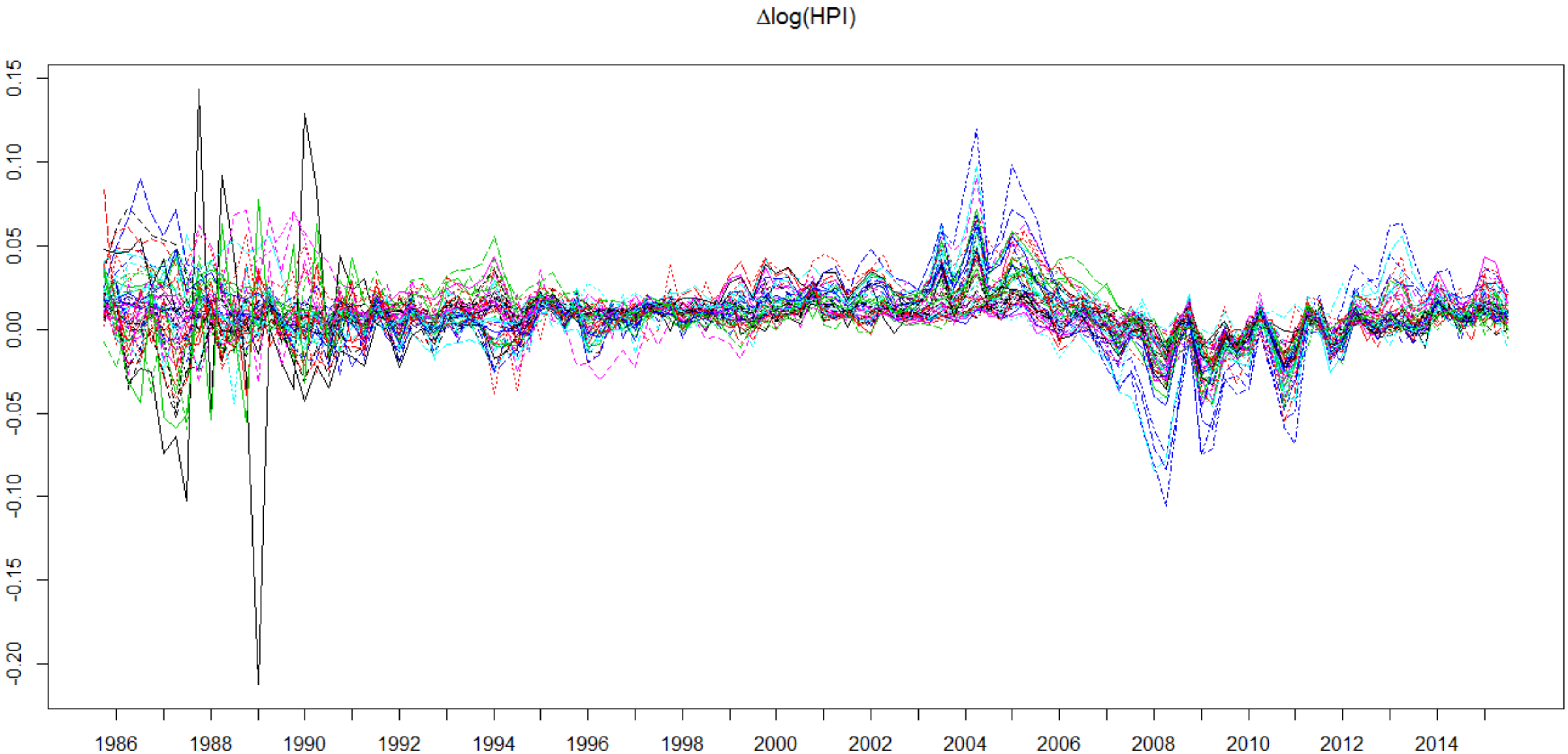


Figure 8 : Regional quarterly changes in the natural logarithm of the FHFA House Price Index

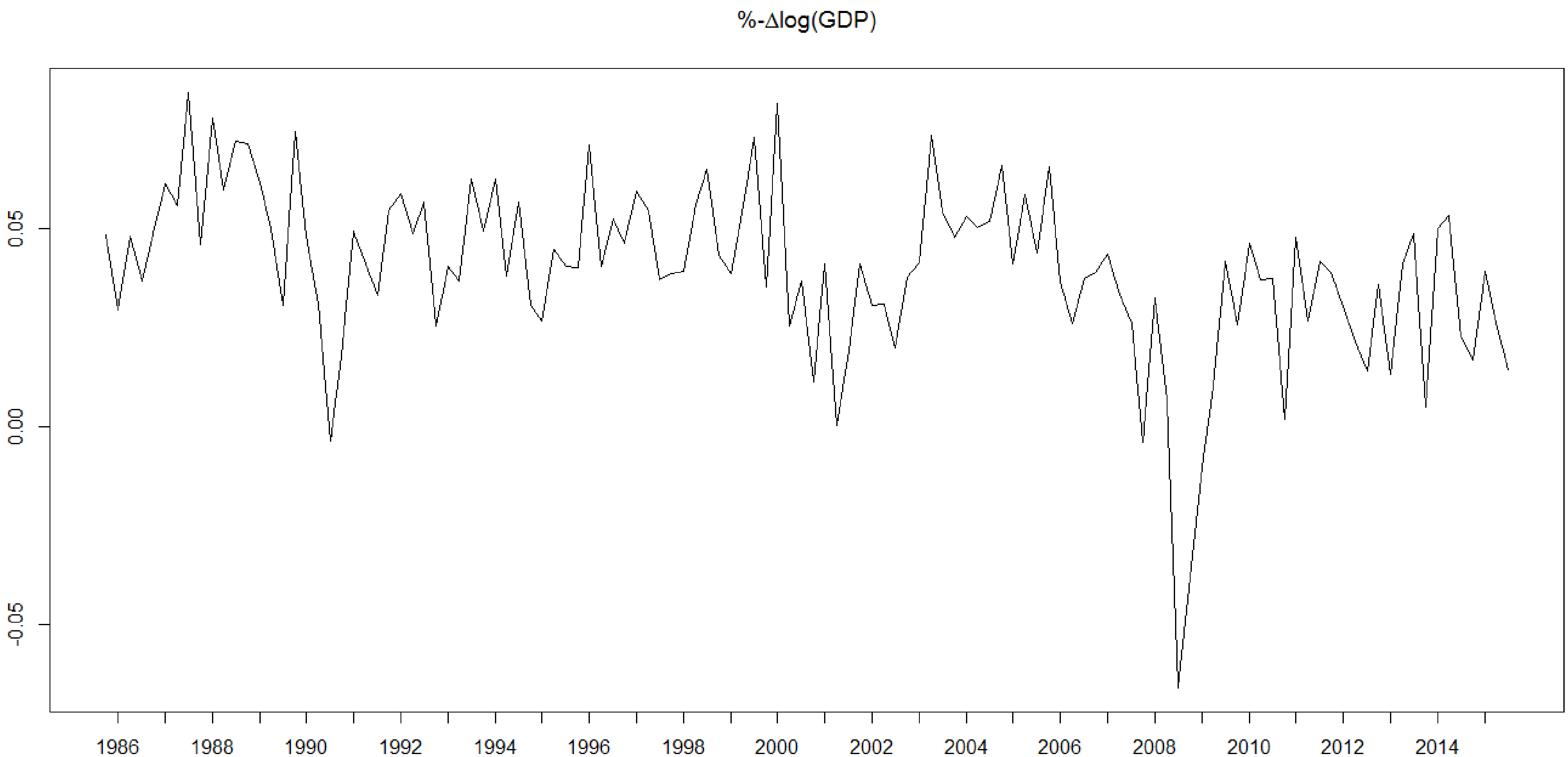


Figure 9 : Percent changes in the natural logarithm of real national GDP (expressed in %)

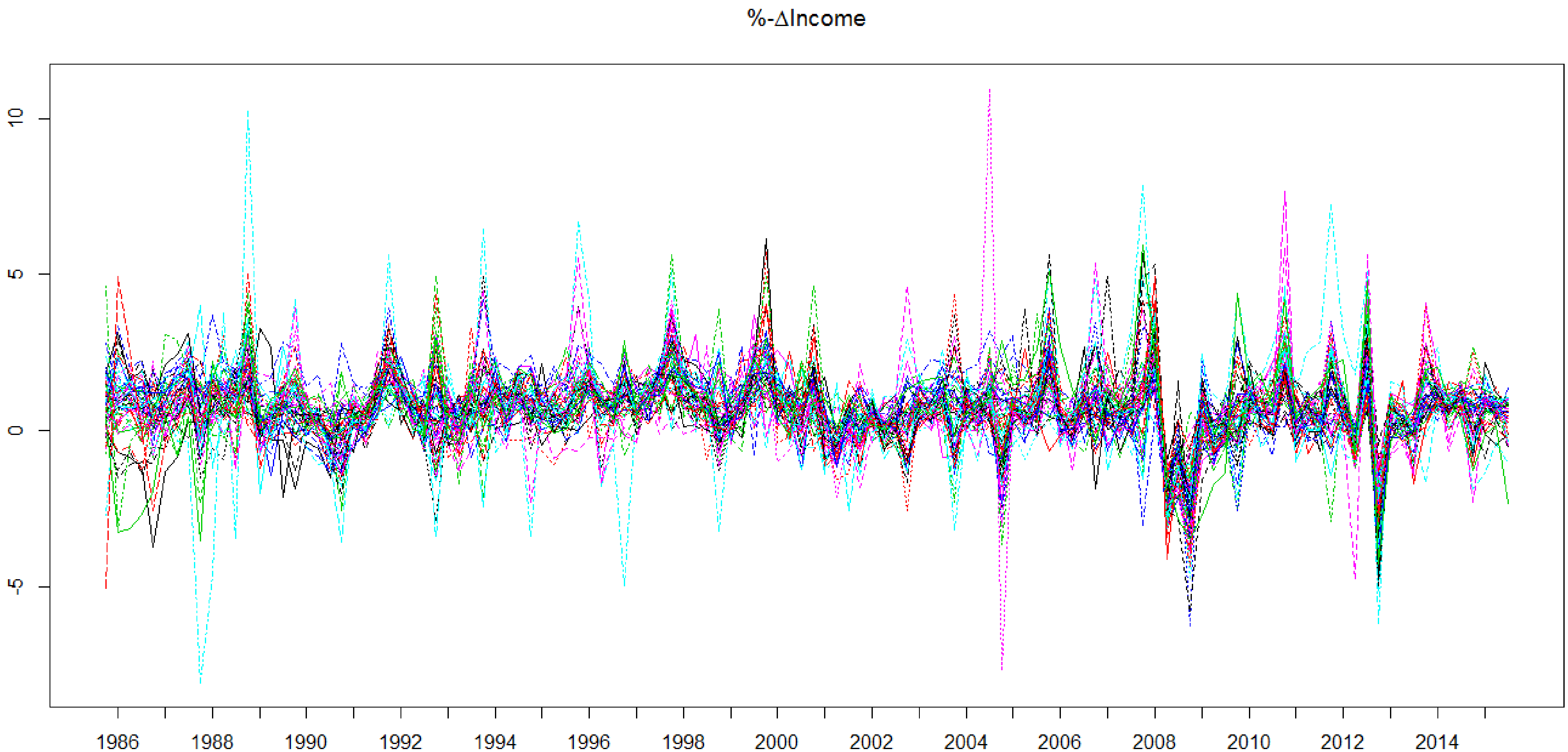


Figure 10 : Percent changes in regional real disposable income (expressed in %)

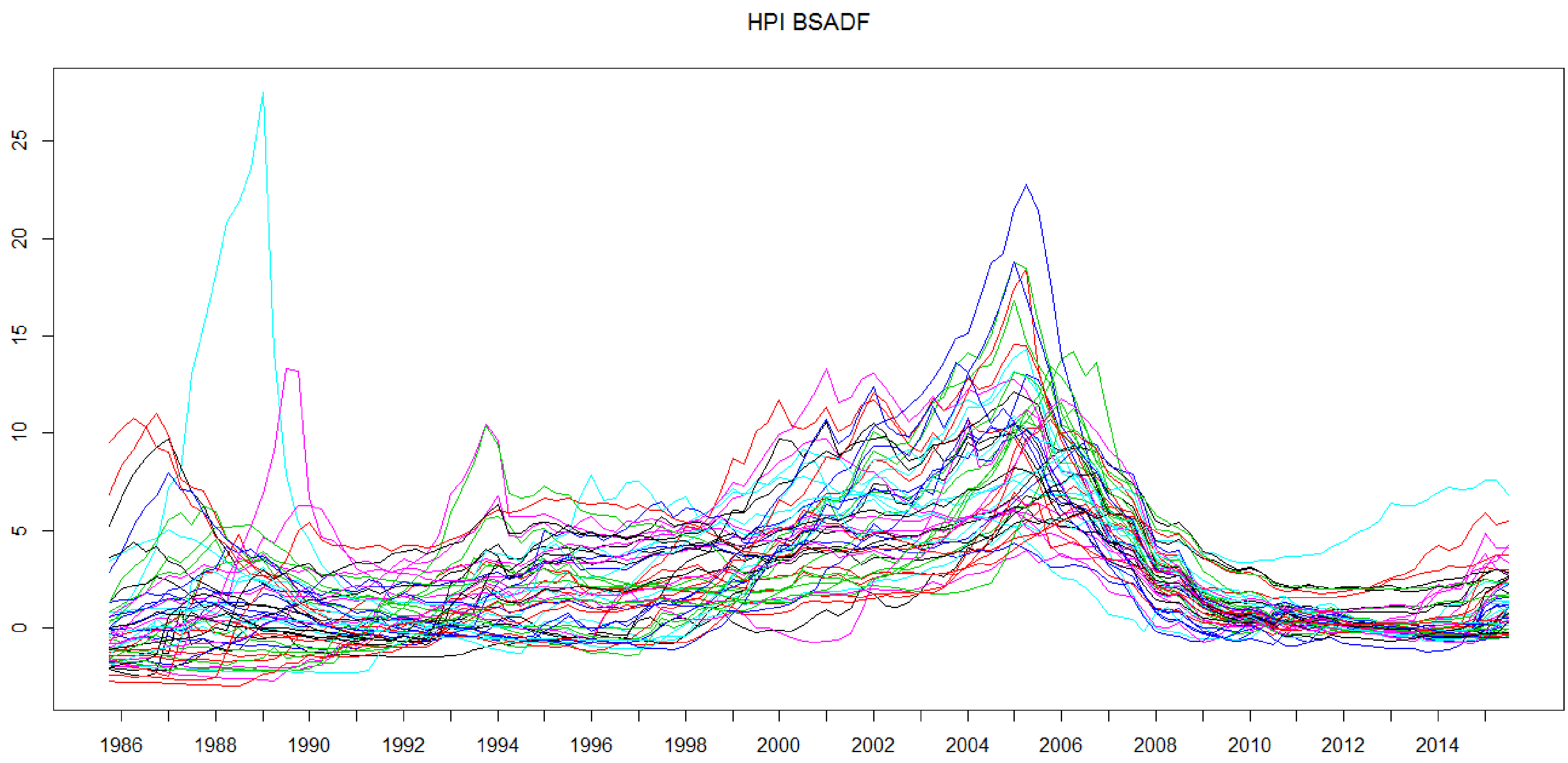


Figure 11 : Regional HPI BSADF test statistics

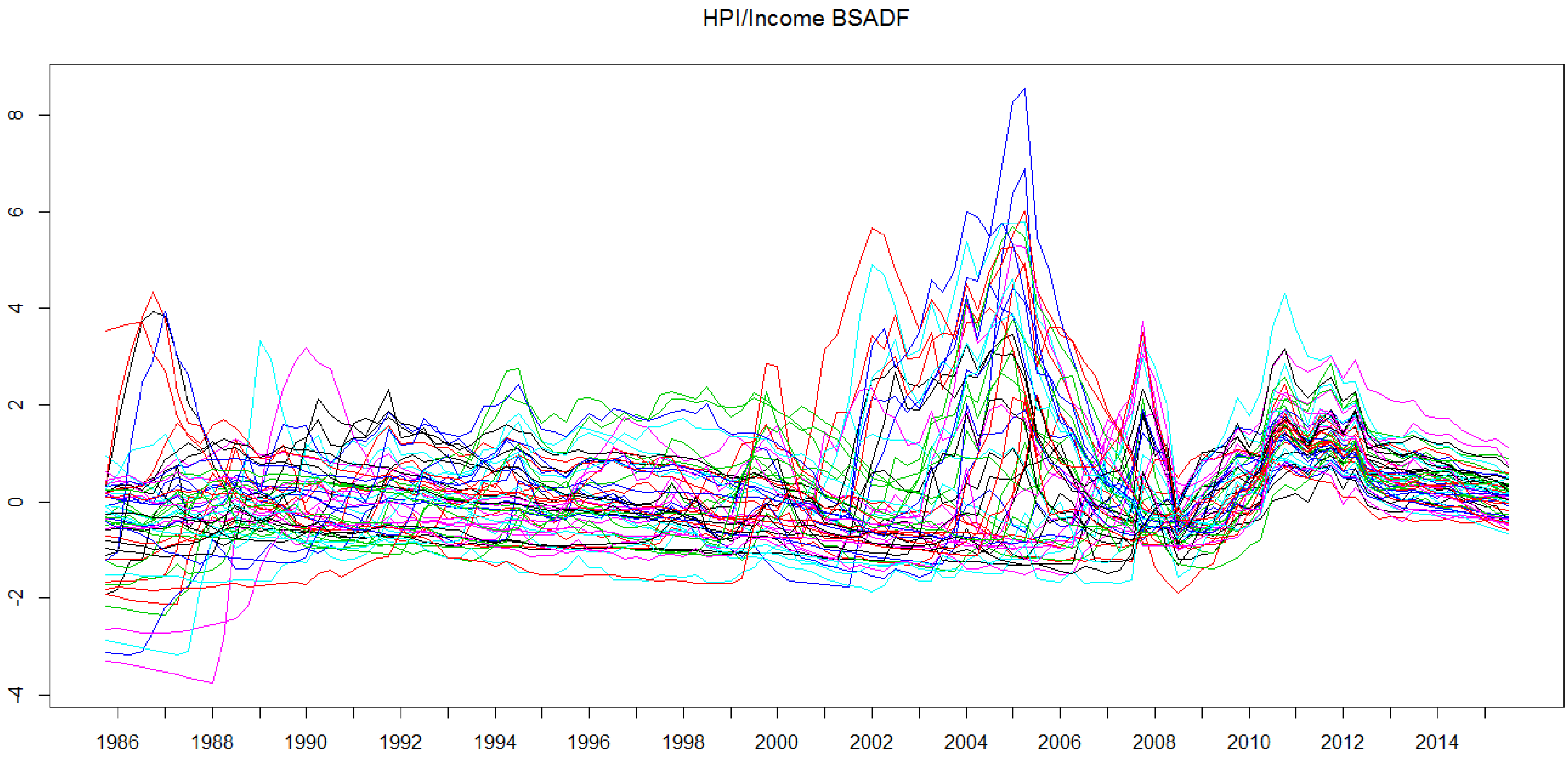


Figure 12 : Regional HPI-to-income ratio BSADF test statistics

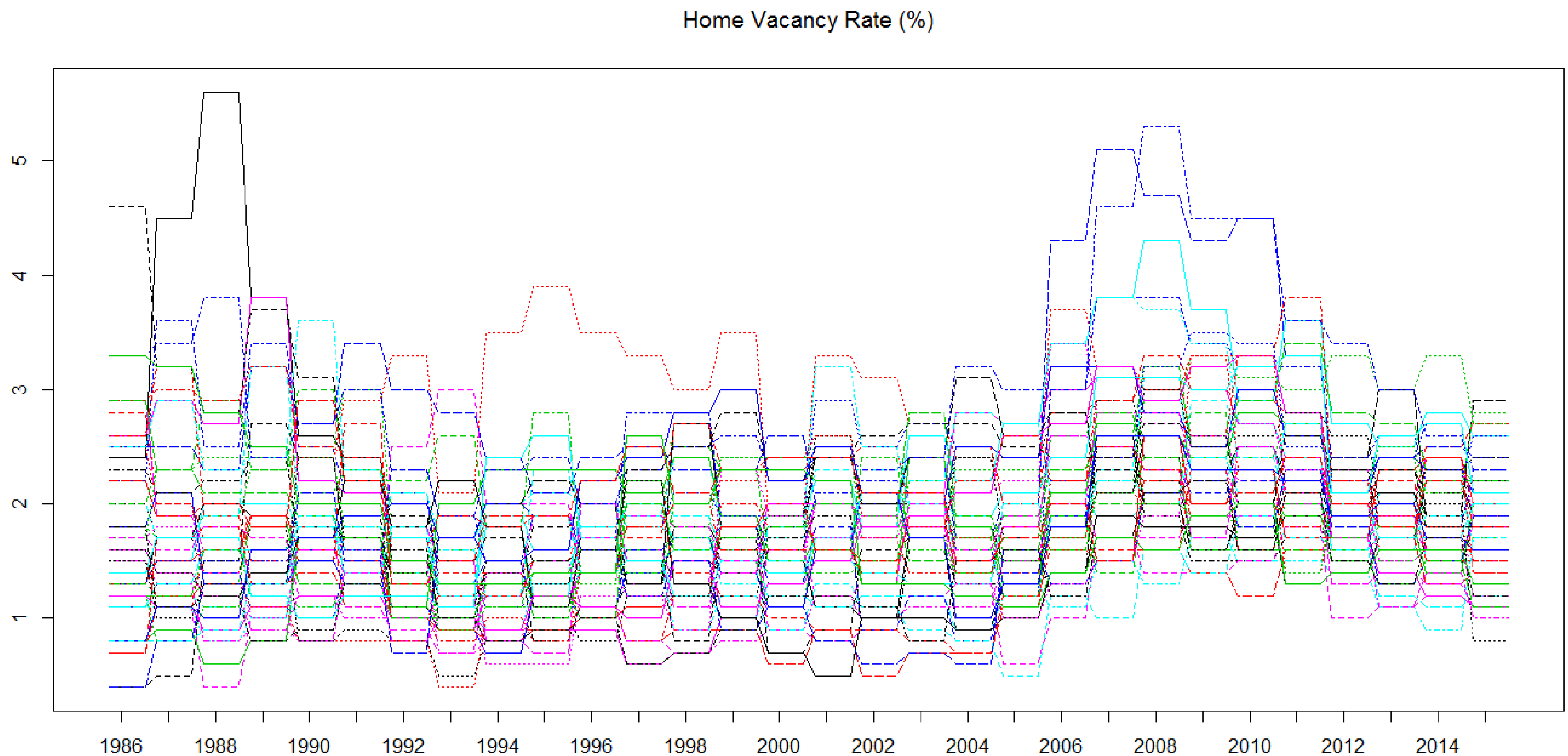


Figure 13 : Regional home vacancy rates (expressed in %)

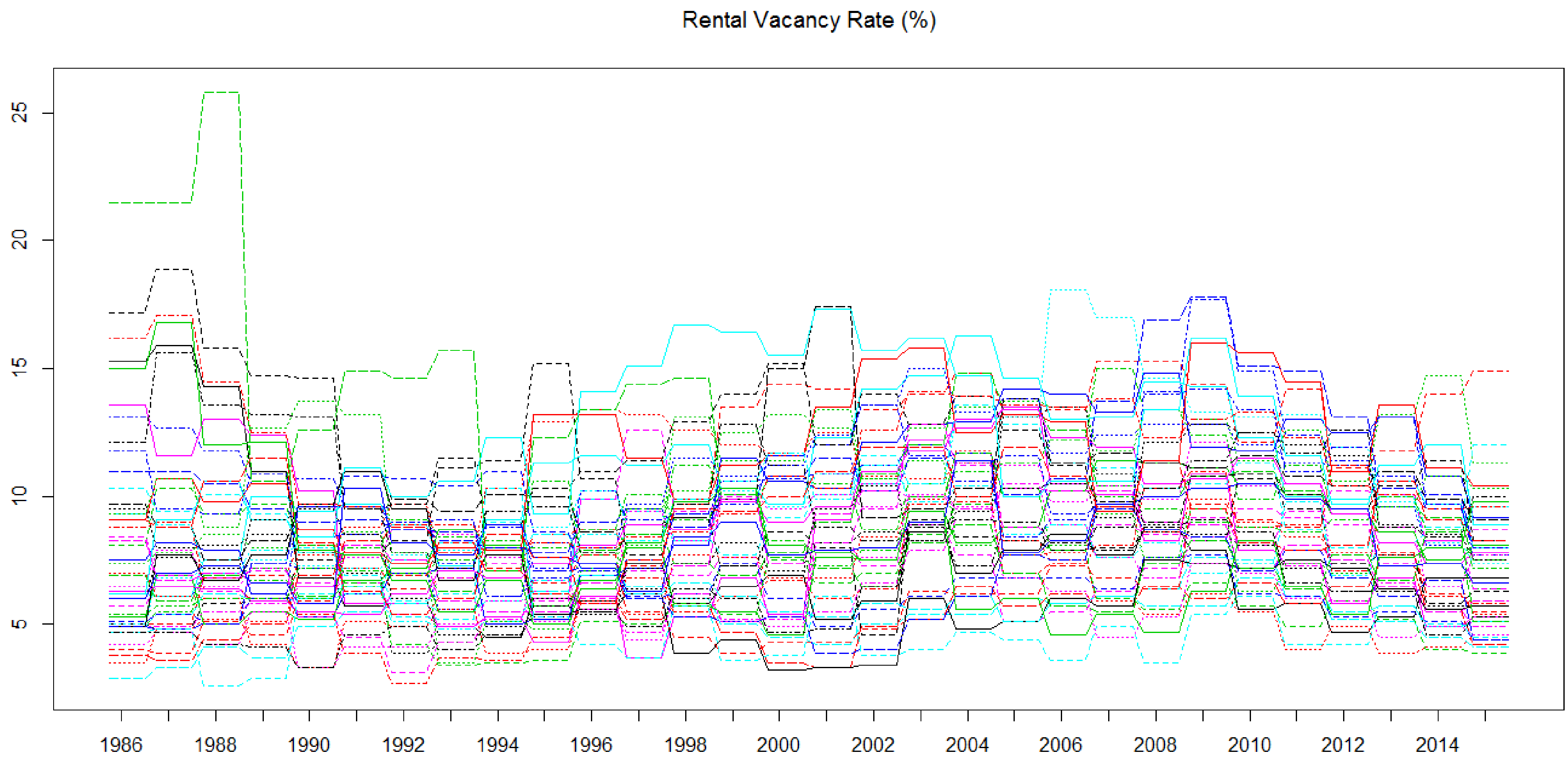


Figure 14 : Regional rental vacancy rates (expressed in %)

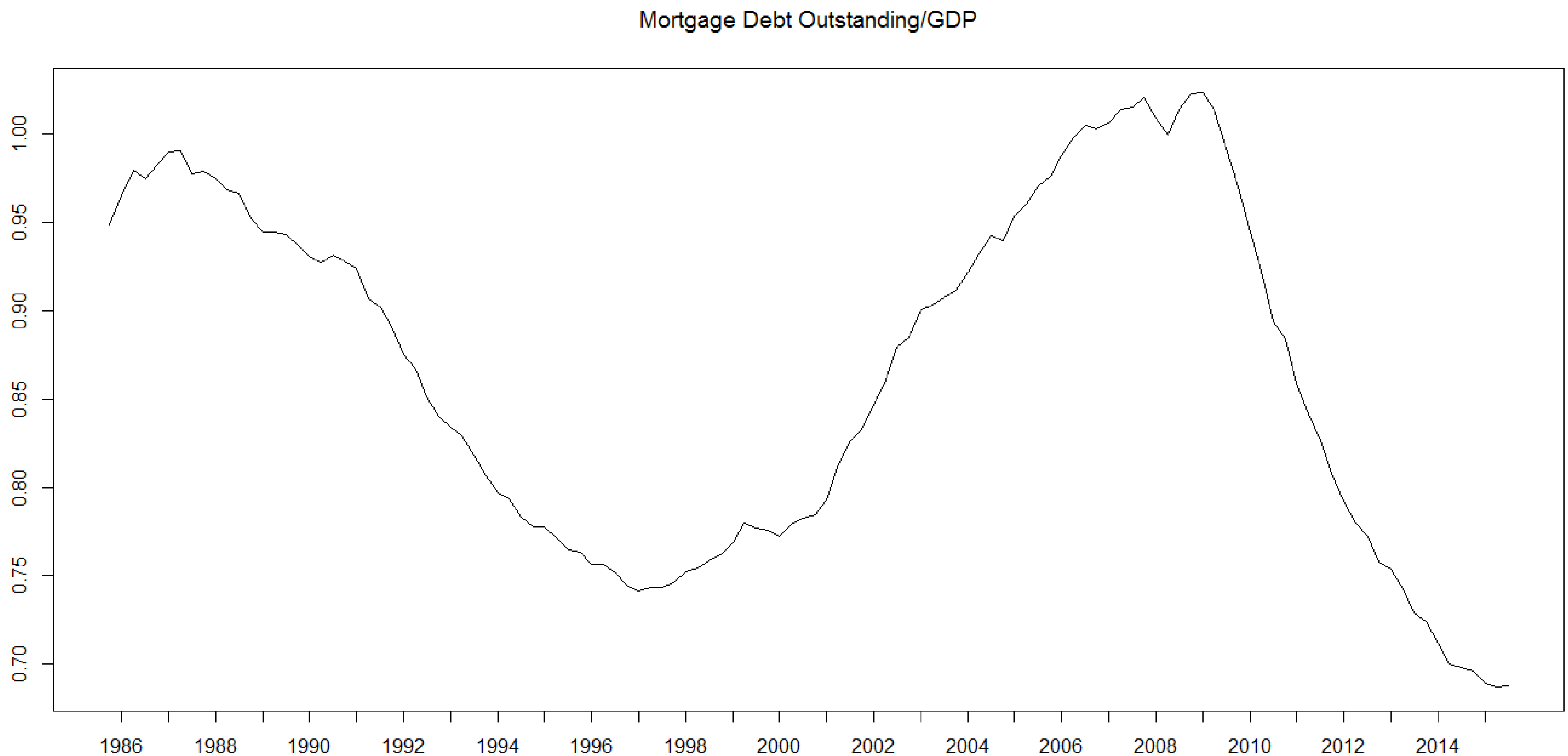


Figure 15 : National mortgage-debt-outstanding-to-GDP ratio

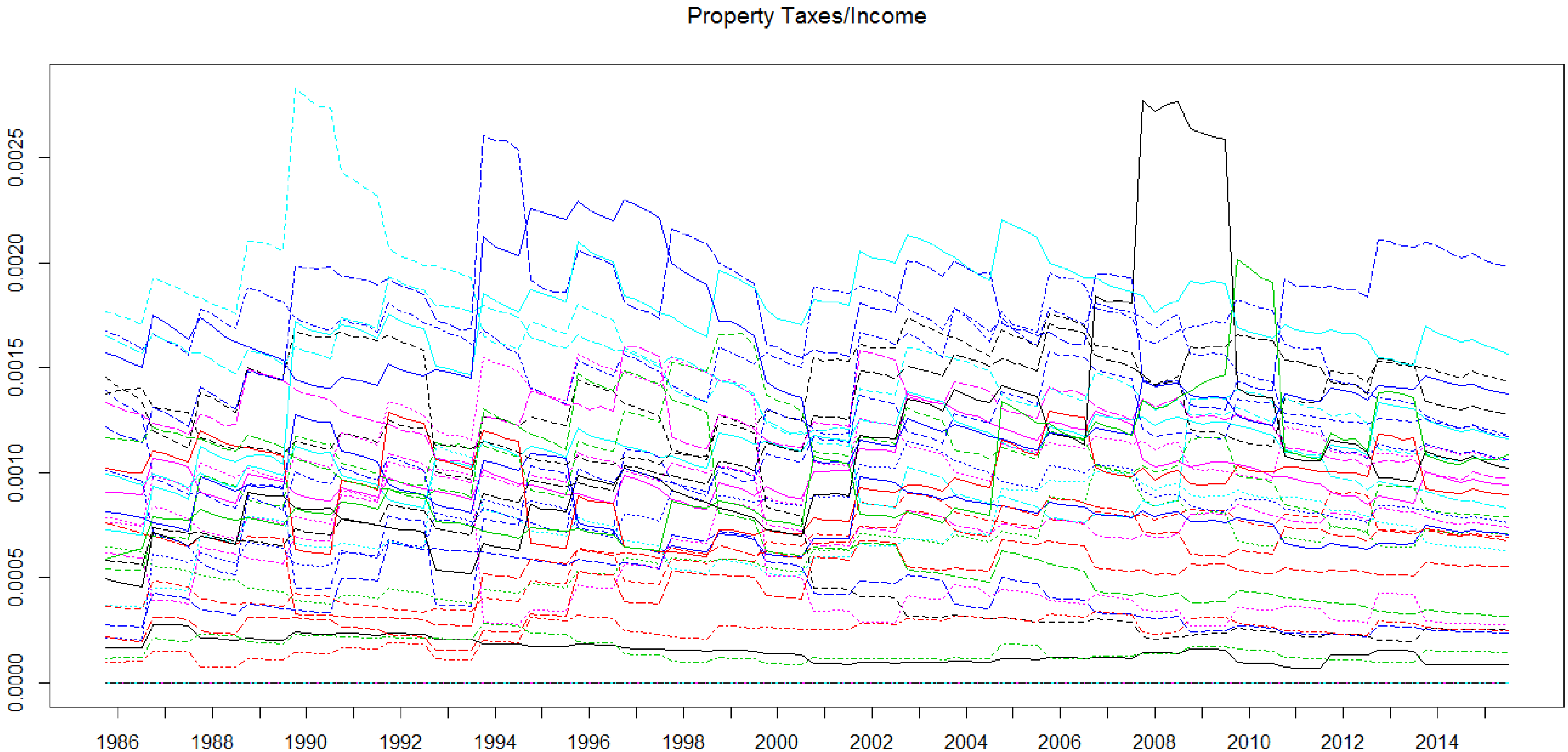


Figure 16 : Regional personal-property-taxes-to-income ratios

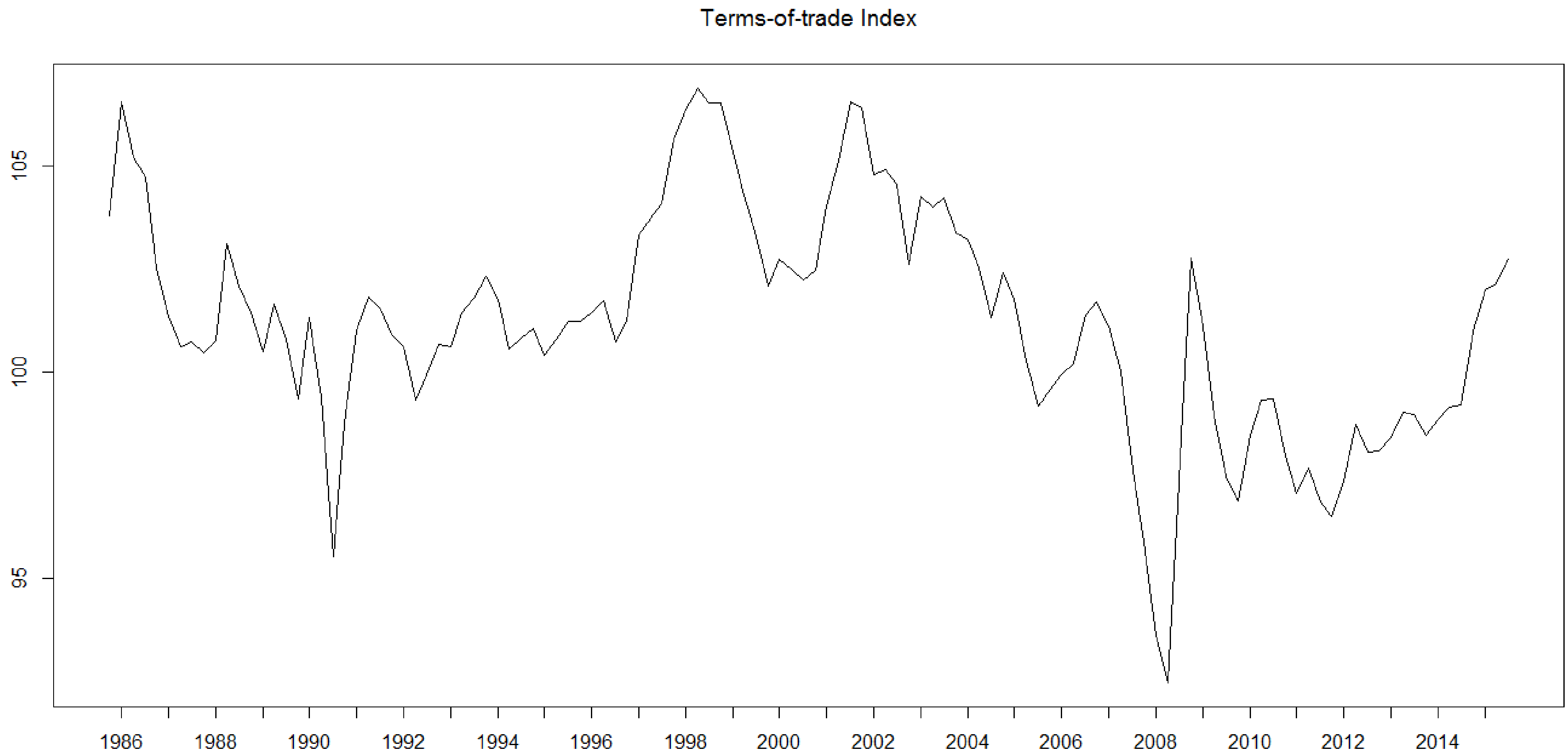


Figure 17 : National terms-of-trade index

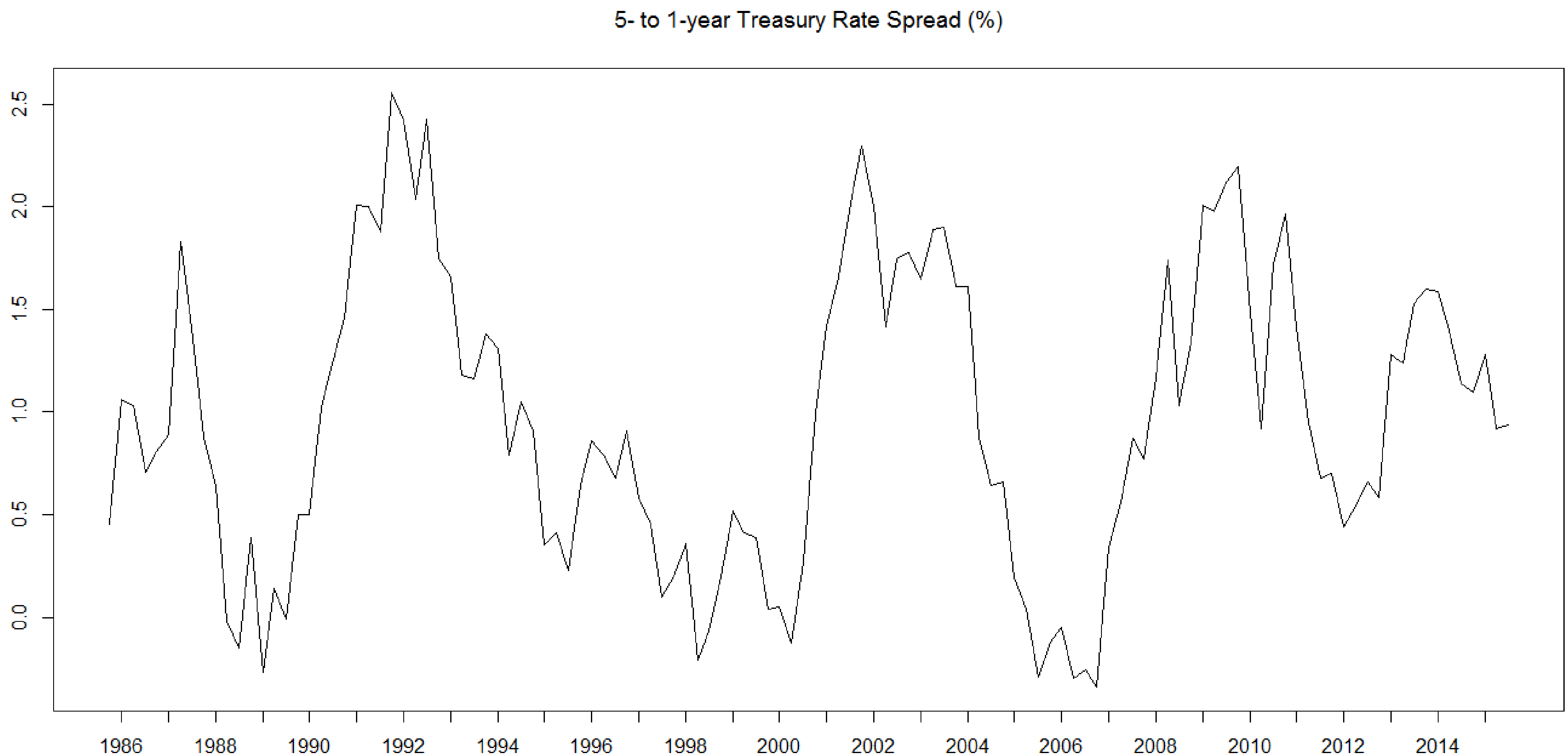


Figure 18 : 5- to 1-year Treasury Rate Spread (expressed in %)

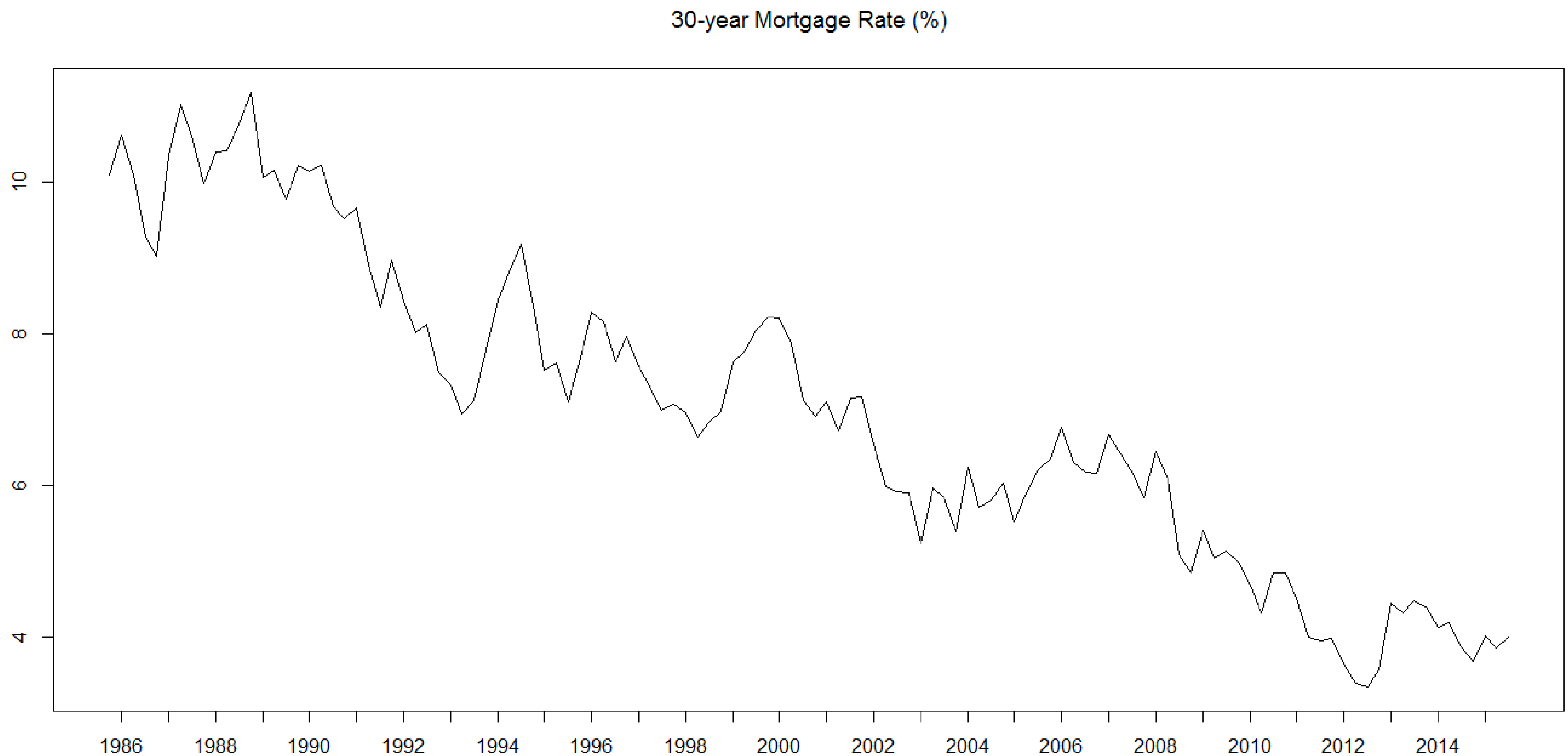


Figure 19 : 30-year mortgage rate (expressed in %)

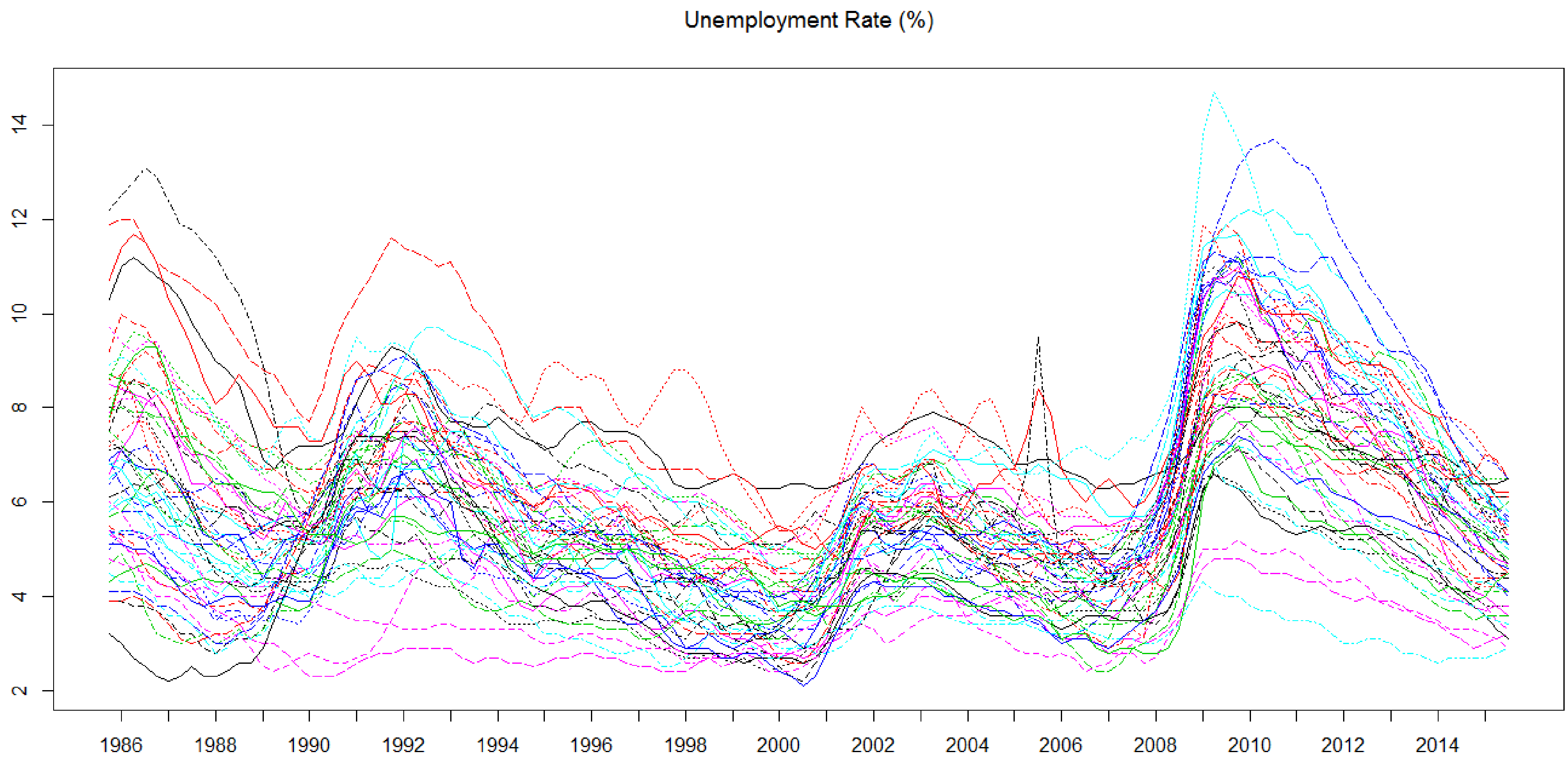


Figure 20 : Regional unemployment rates (expressed in %)

Annex 2 — Monte Carlo Study

ρ	CCM test	Empirical test size (1000 replications)					
		$\alpha = 0.05$			$\alpha = 0.10$		
		$\theta = 0.25$	$\theta = 0.5$	$\theta = 0.75$	$\theta = 0.25$	$\theta = 0.5$	$\theta = 0.75$
0	W	0.193	0.09	0.026	0.272	0.146	0.052
	W^m	0.067	0.069	0.06	0.123	0.122	0.108
	S	0.078	0.08	0.064	0.13	0.121	0.109
	S^m	0.057	0.069	0.089	0.11	0.114	0.129
0.4	W	0.264	0.097	0.014	0.349	0.147	0.034
	W^m	0.083	0.055	0.07	0.142	0.108	0.118
	S	0.073	0.066	0.064	0.12	0.121	0.103
	S^m	0.068	0.061	0.073	0.114	0.123	0.134
0.8	W	0.326	0.081	0.004	0.405	0.125	0.017
	W^m	0.142	0.107	0.139	0.209	0.179	0.214
	S	0.078	0.066	0.072	0.129	0.128	0.124
	S^m	0.078	0.076	0.08	0.127	0.144	0.145

Table IV : Empirical test sizes with the Bertrand, Duflo and Mullainathan simulation setup for Score (S) and Wald (W) tests at nominal rates equal to 5% and 10%. The superscript m indicates the use of the modified CCM estimator.

CCM test	Empirical test size (1000 replications)					
	$\alpha = 0.05$			$\alpha = 0.10$		
	$\theta = 0.25$	$\theta = 0.5$	$\theta = 0.75$	$\theta = 0.25$	$\theta = 0.5$	$\theta = 0.75$
W	0.584	0.699	0.389	0.637	0.761	0.488
W^m	0.540	0.634	0.421	0.602	0.690	0.507
S	0.605	0.632	0.403	0.665	0.680	0.481
S^m	0.495	0.487	0.358	0.580	0.561	0.442

Table V : Empirical test sizes in our house price panel data setup for Score (S) and Wald (W) tests at nominal rates equal to 5% and 10%. The superscript m indicates the use of the modified CCM estimator.

Covariate	CCM test with size adjustment	p-value		
		$\theta = 0.25$	$\theta = 0.50$	$\theta = 0.75$
HPI % change	W	0.0000	0.0030	0.0034
	W^m	0.0000	0.0000	0.0000
	S	0.0054	0.0050	0.0086
	S^m	0.0000	0.0000	0.0000
Real GDP % change	W	0.0492	0.6426	0.5834
	W^m	0.1296	0.4818	0.4776
	S	0.0320	0.3032	0.4302
	S^m	0.0032	0.1732	0.2832
Real personal income % change	W	0.9820	0.8150	0.6270
	W^m	0.9766	0.7258	0.6050
	S	0.8966	0.6588	0.5188
	S^m	0.9656	0.5766	0.6190
HPI exuberance (BSADF)	W	0.0000	0.0044	0.0020
	W^m	0.0012	0.0008	0.0000
	S	0.0084	0.0044	0.0026
	S^m	0.0000	0.0000	0.000
HPI / income exuberance (BSADF)	W	0.7294	0.7170	0.0096
	W^m	0.7808	0.7992	0.1366
	S	0.7582	0.6604	0.0848
	S^m	0.8378	0.6526	0.0556
Housing vacancy	W	0.4892	0.3402	0.9428
	W^m	0.5336	0.3634	0.9340
	S	0.3010	0.2306	0.8742
	S^m	0.1750	0.1024	0.9114
Rental vacancy	W	0.1690	0.0706	0.0142
	W^m	0.1958	0.0740	0.0000
	S	0.1164	0.0100	0.0078
	S^m	0.0154	0.0000	0.0000

Covariate	CCM test with size adjustment	p-value		
		$\theta = 0.25$	$\theta = 0.50$	$\theta = 0.75$
Outstanding mortgage debt / GDP	W	0.0102	0.0334	0.0130
	W^m	0.0024	0.0152	0.0014
	S	0.0204	0.0134	0.0088
	S^m	0.0000	0.0000	0.0000
Property tax / real income	W	0.5942	0.9952	0.4824
	W^m	0.6706	0.4102	0.5268
	S	0.7796	0.4676	0.6786
	S^m	0.8324	0.3142	0.6382
Terms-of-trade index	W	0.9668	0.5074	0.3368
	W^m	0.9712	0.6412	0.1922
	S	0.8764	0.7062	0.1934
	S^m	0.9558	0.7218	0.0992
5- to 1-year treasury rate spread	W	0.0226	0.9996	0.0044
	W^m	0.0760	0.1826	0.0110
	S	0.0264	0.0240	0.0182
	S^m	0.0004	0.0030	0.0014
30-year mortgage rates	W	0.7582	0.3774	0.0330
	W^m	0.6724	0.4536	0.1794
	S	0.6992	0.7002	0.0848
	S^m	0.7830	0.6824	0.1550
Unemployment	W	0.1152	0.7410	0.0062
	W^m	0.2542	0.8538	0.3364
	S	0.3456	0.8920	0.2698
	S^m	0.1446	0.9310	0.4242

Table VI : FE QR coefficient p-values with size-corrected Wald and Score tests

The superscript m indicates the use of the modified CCM estimator.

Annex 3 — Quantile Effects

The following charts illustrate the covariates' coefficients for the QR model without fixed effects (top) and the QR model with entity fixed effects (bottom) with 95% confidence intervals. Grey bands depict the wild gradient bootstrapped confidence intervals with 300 replications. Green and blue lines respectively depict the Wald and modified Wald robust confidence intervals. OLS coefficients and confidence intervals are illustrated by the red lines. The x-axis corresponds to the 9 deciles and the y-axis corresponds to the quantile effects.

Note that for the HPI-to-Price BSADF, the property-tax-to-income ratio, and the unemployment covariates, the CCM Wald confidence intervals are not available at certain quantiles. This is due to numerical complications when estimating the Wald CCM. Indeed, we estimate non-positive semi definite covariance matrices at these quantiles. Using the modified Wald CCM avoids such complications, but restricts within-state error processes to have identical serial correlation structures. This leads us to believe that the Wald CCM struggles to tolerate entity-specific error autocorrelations.

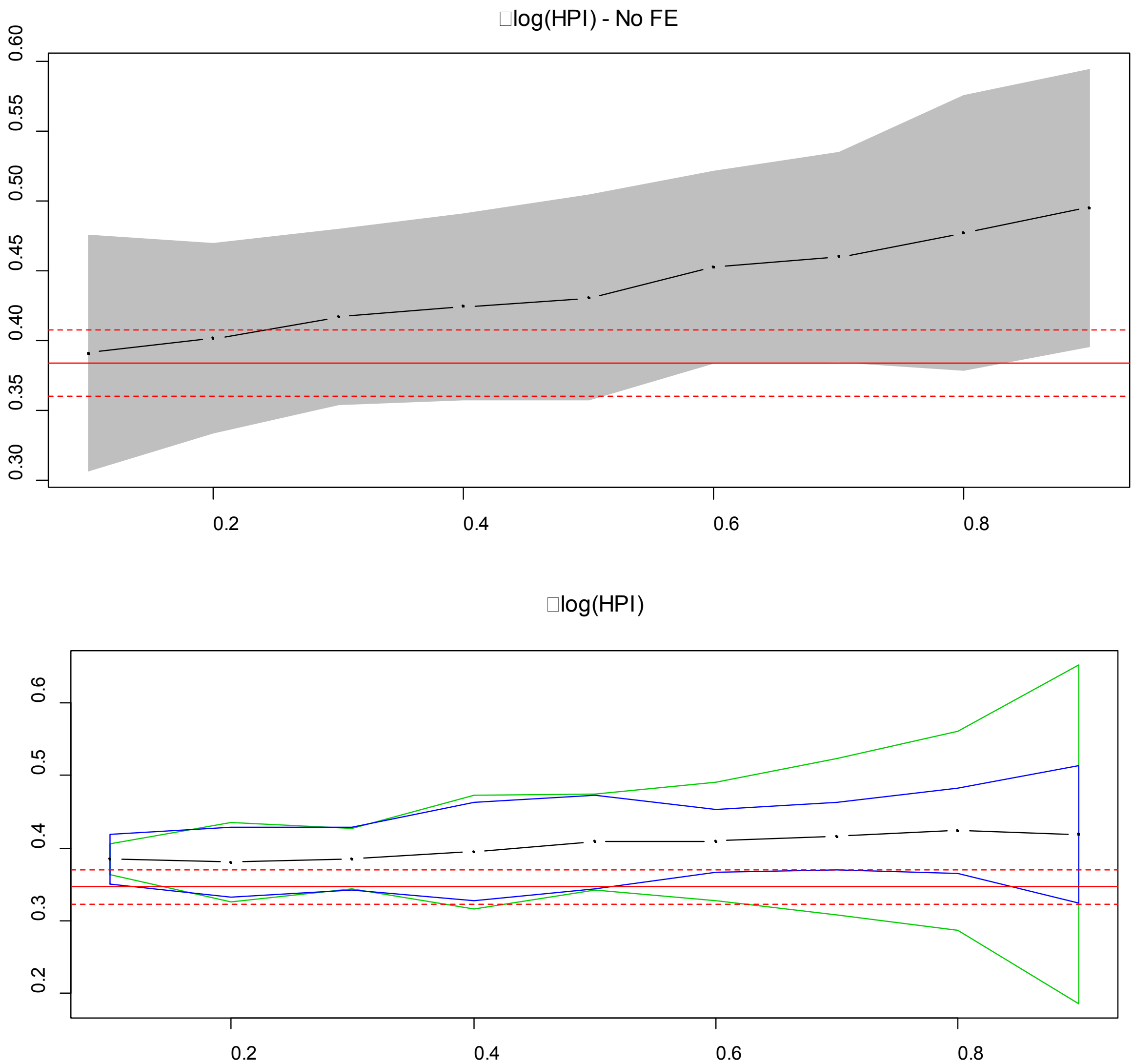


Figure 21: Sequence of QR coefficients for changes in the natural logarithm of changes in the FHFA House Price Index

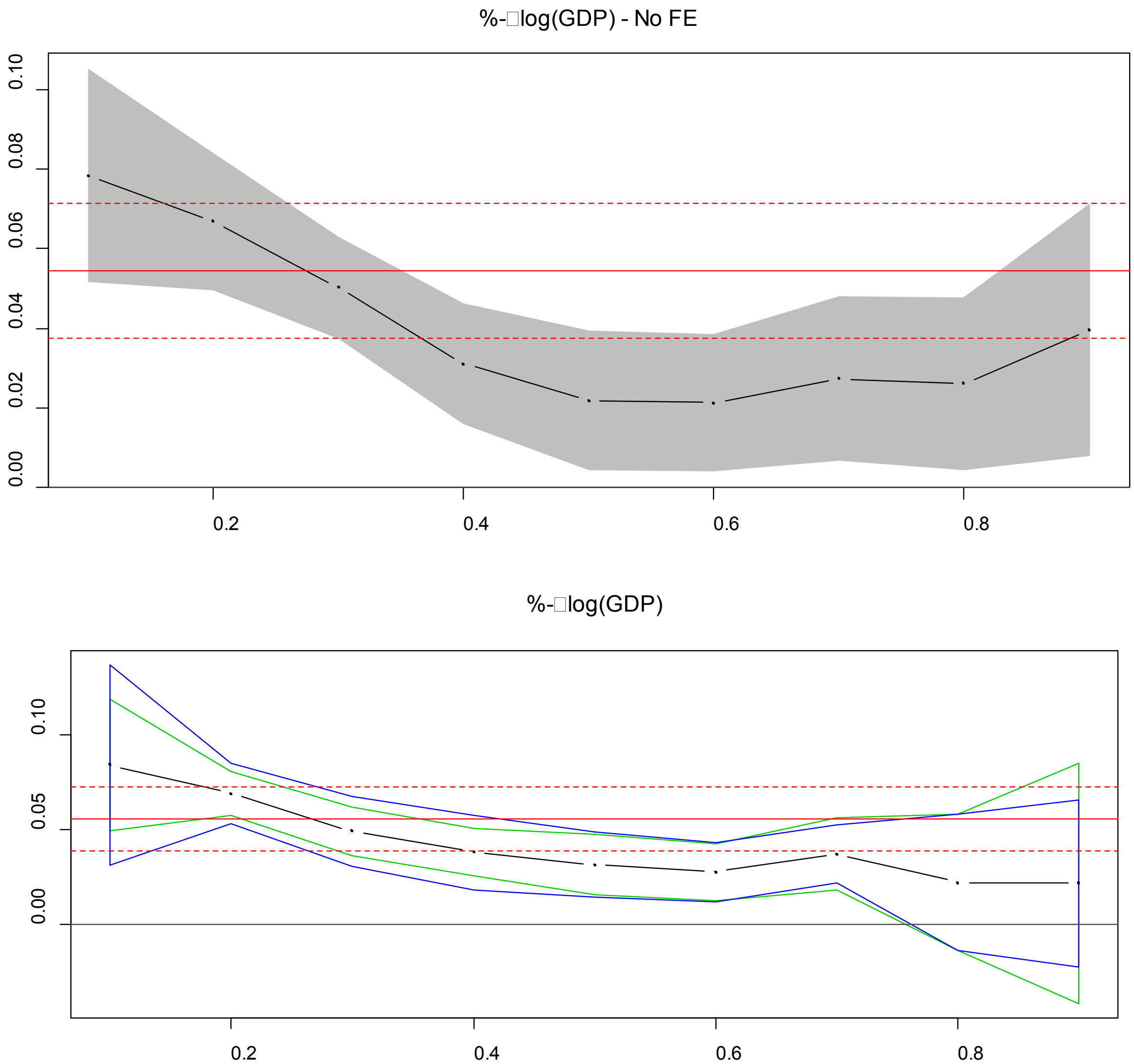
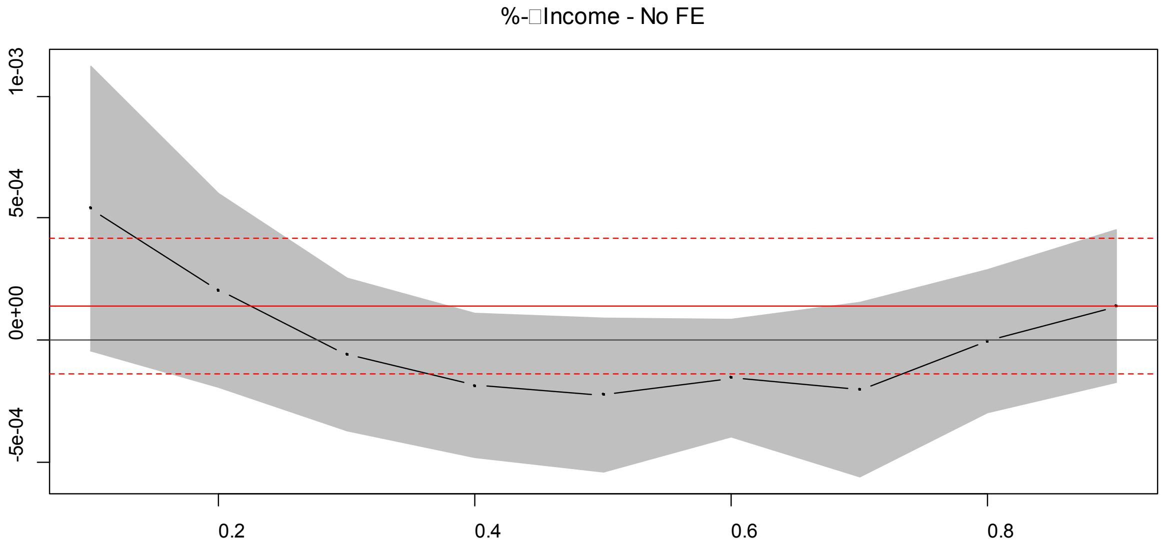


Figure 22: Sequence of QR coefficients for percent changes in the natural logarithm of real national GDP (expressed in %)



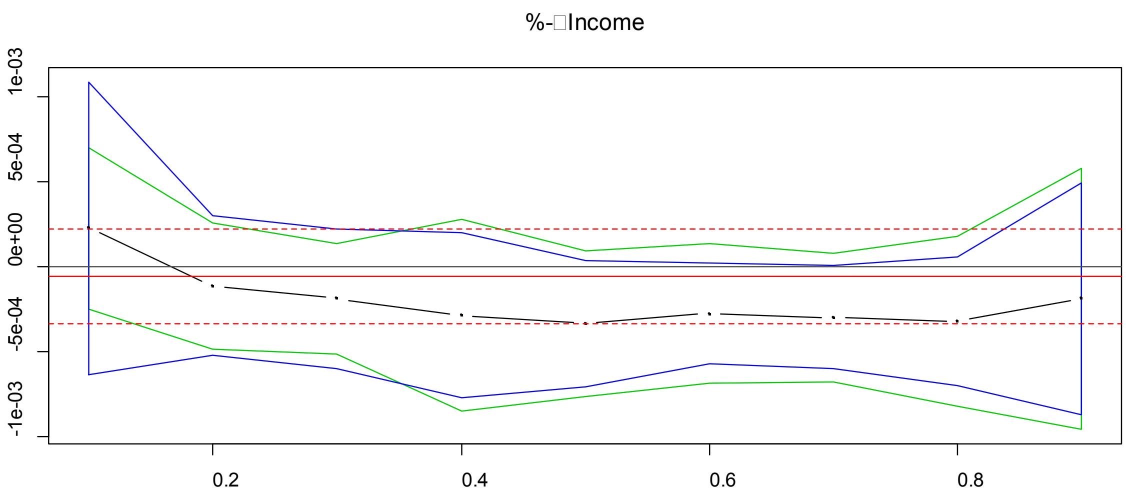


Figure 23: Sequence of QR coefficients for percent changes in regional real disposable income (expressed in %)

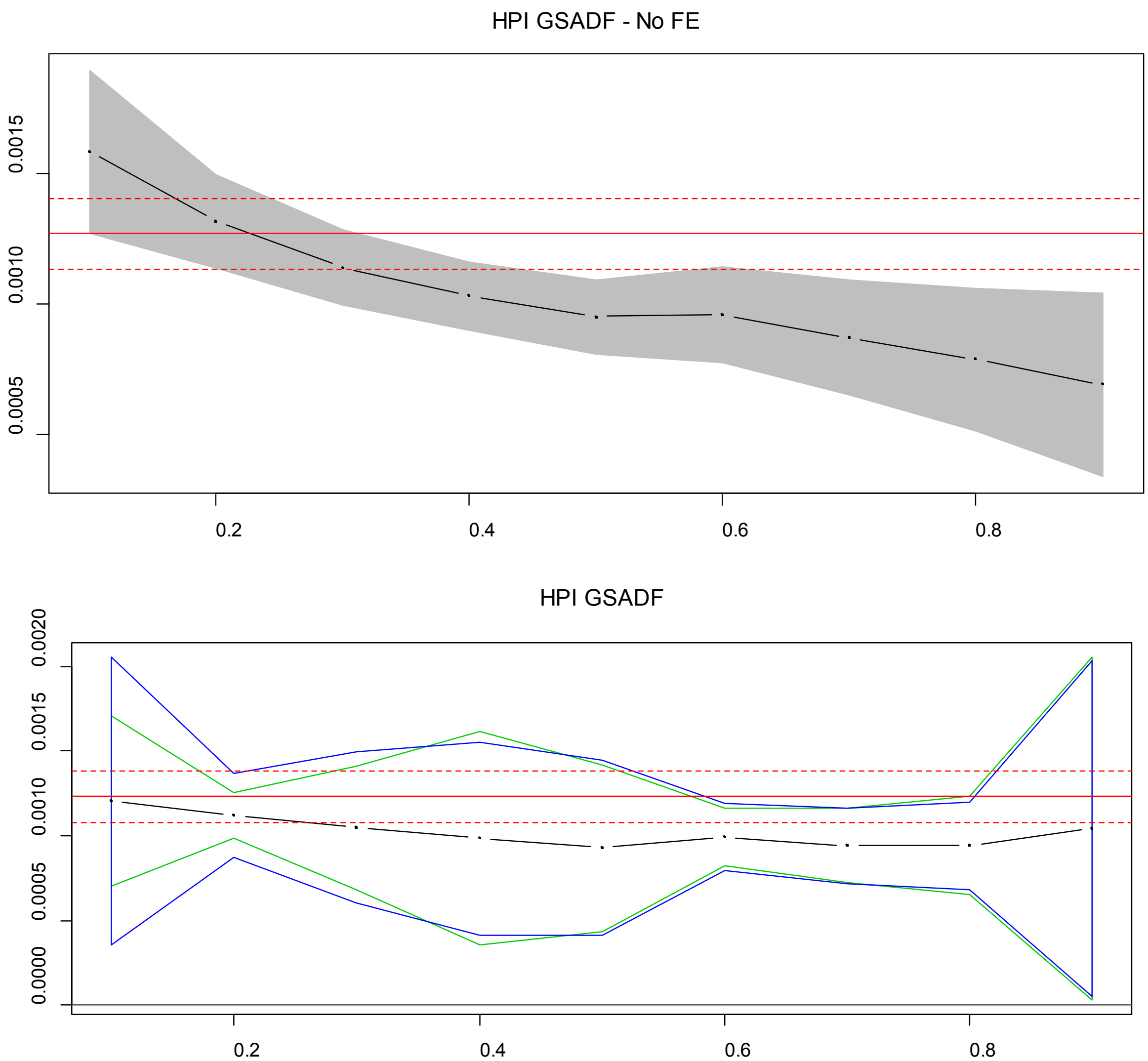


Figure 24: Sequence of QR coefficients for regional HPI BSADF test statistics

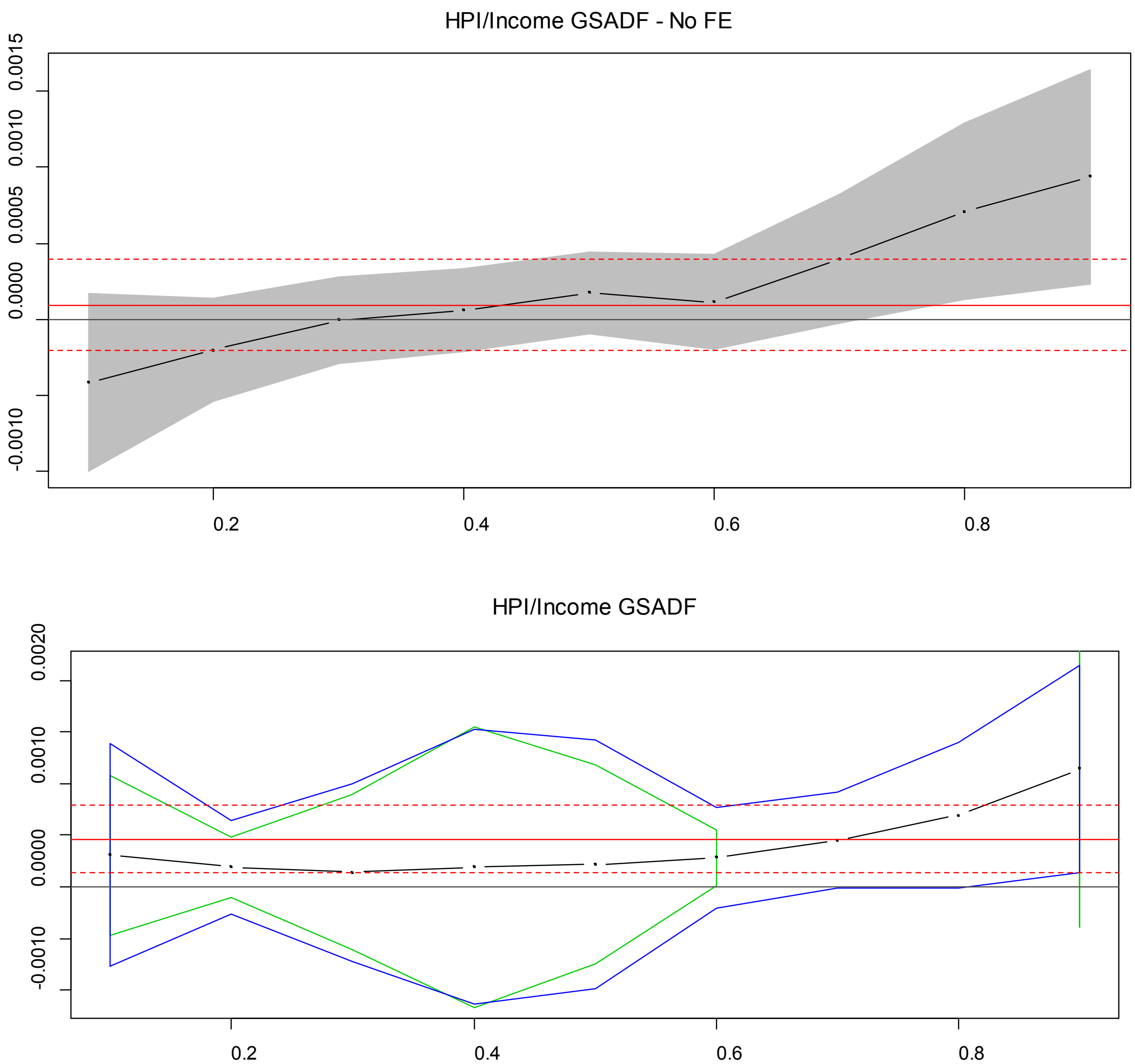


Figure 25: Sequence of QR coefficients for regional HPI-to-income BSADF test statistics. Wald confidence intervals are not available at the 7th and 8th deciles.

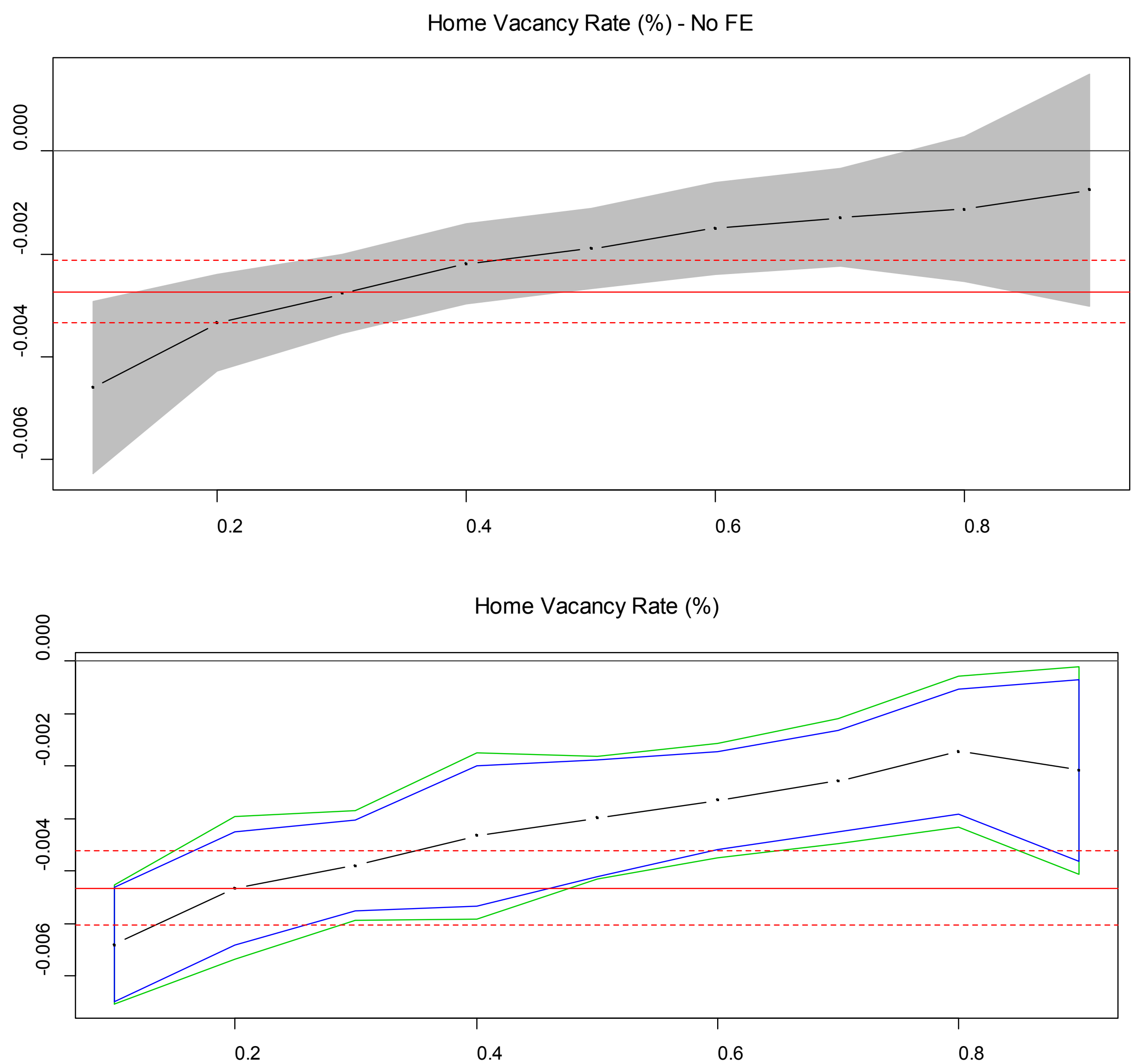


Figure 26: Sequence of QR coefficients for regional home vacancy rates (expressed in %)

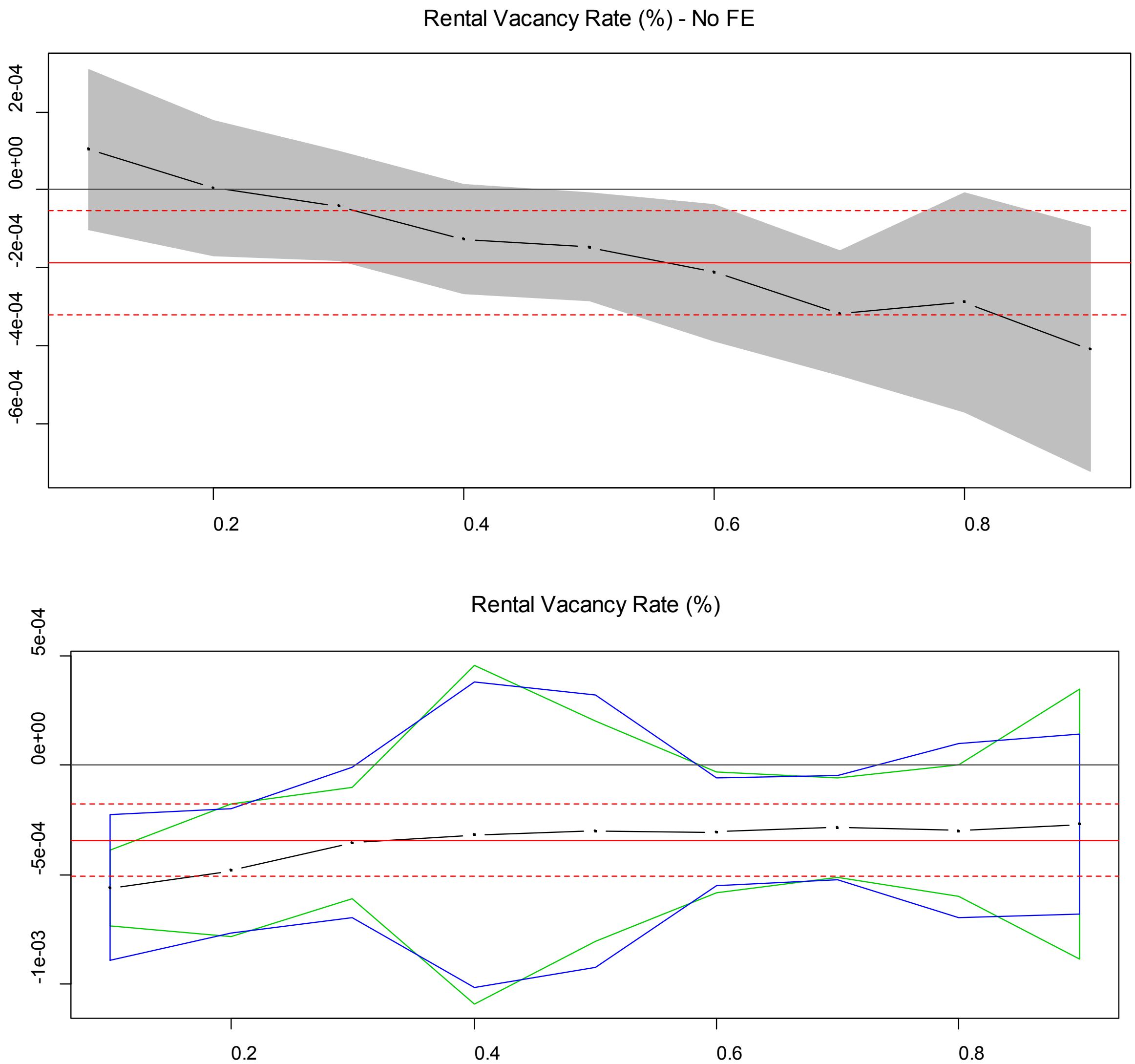


Figure 27: Sequence of QR coefficients for regional rental vacancy rates (expressed in %)

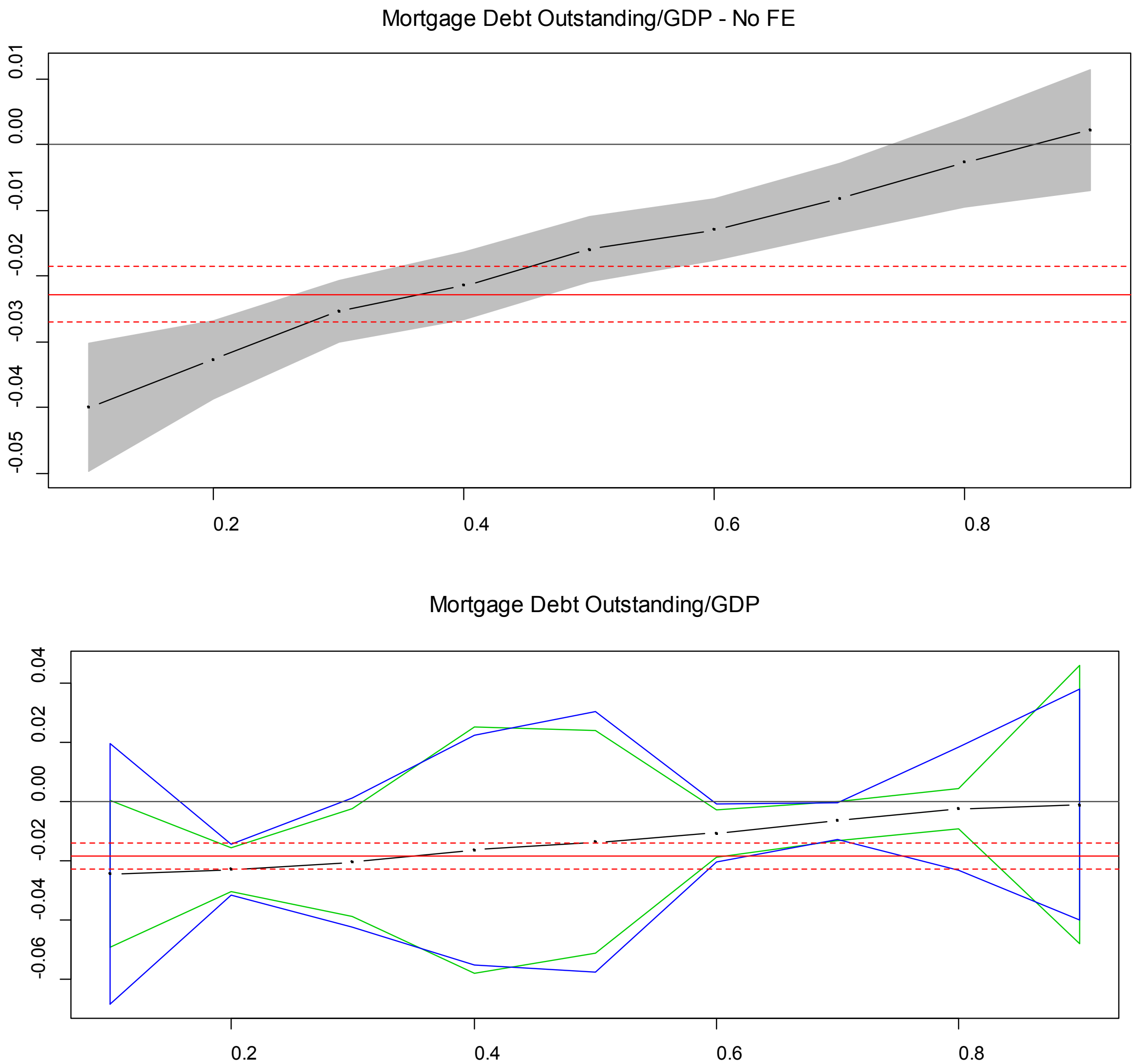


Figure 28: Sequence of QR coefficients for changes the national mortgage debt outstanding-to-GDP ratio

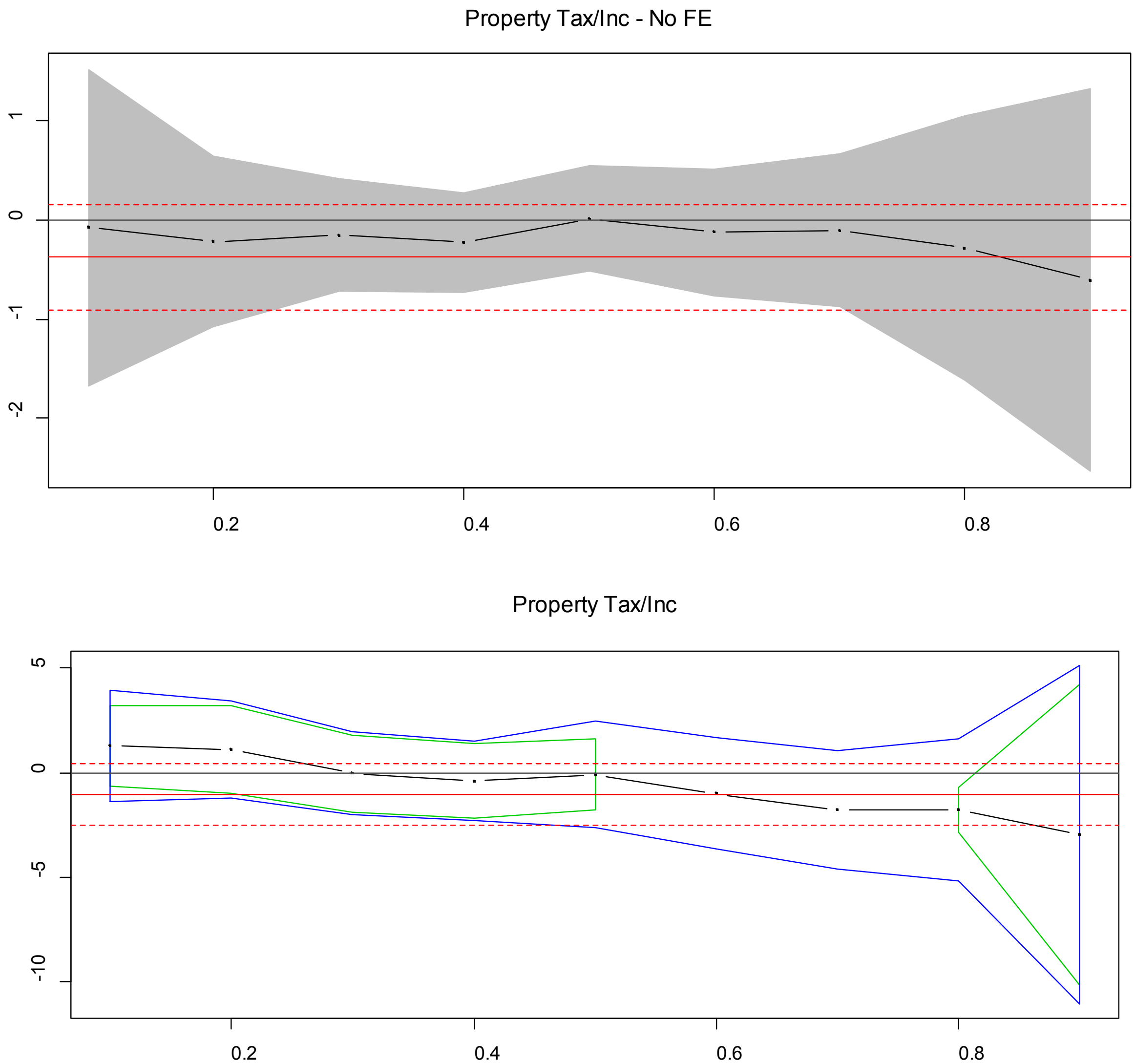


Figure 29: Sequence of QR coefficients for regional property tax-to income ratios
Wald confidence intervals are not available at the 6th and 7th deciles.

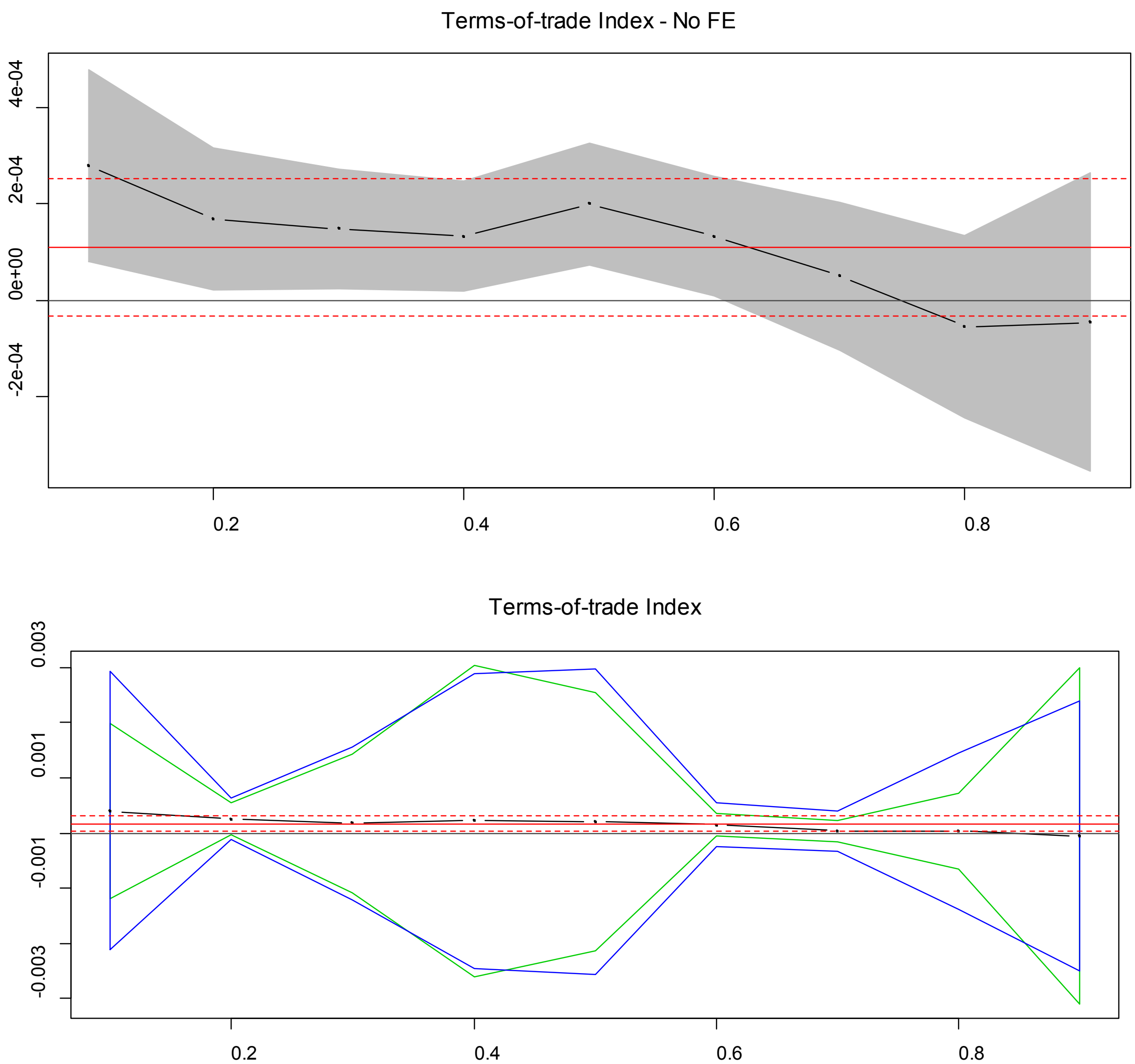


Figure 30: Sequence of QR coefficients for the national terms-of-trade index

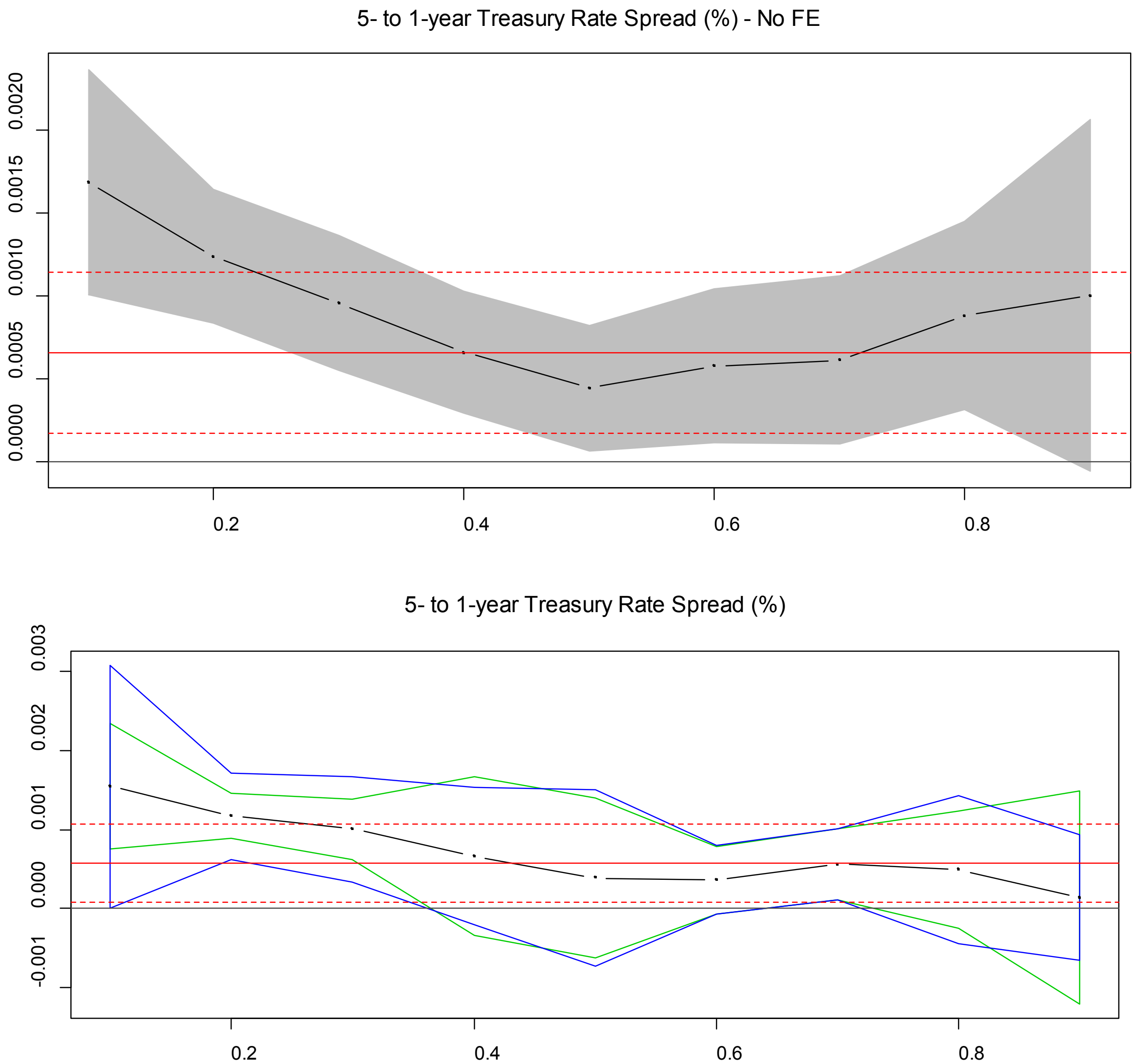


Figure 31: Sequence of QR coefficients for the 5- to 1-year Treasury Rate Spread (expressed in %)

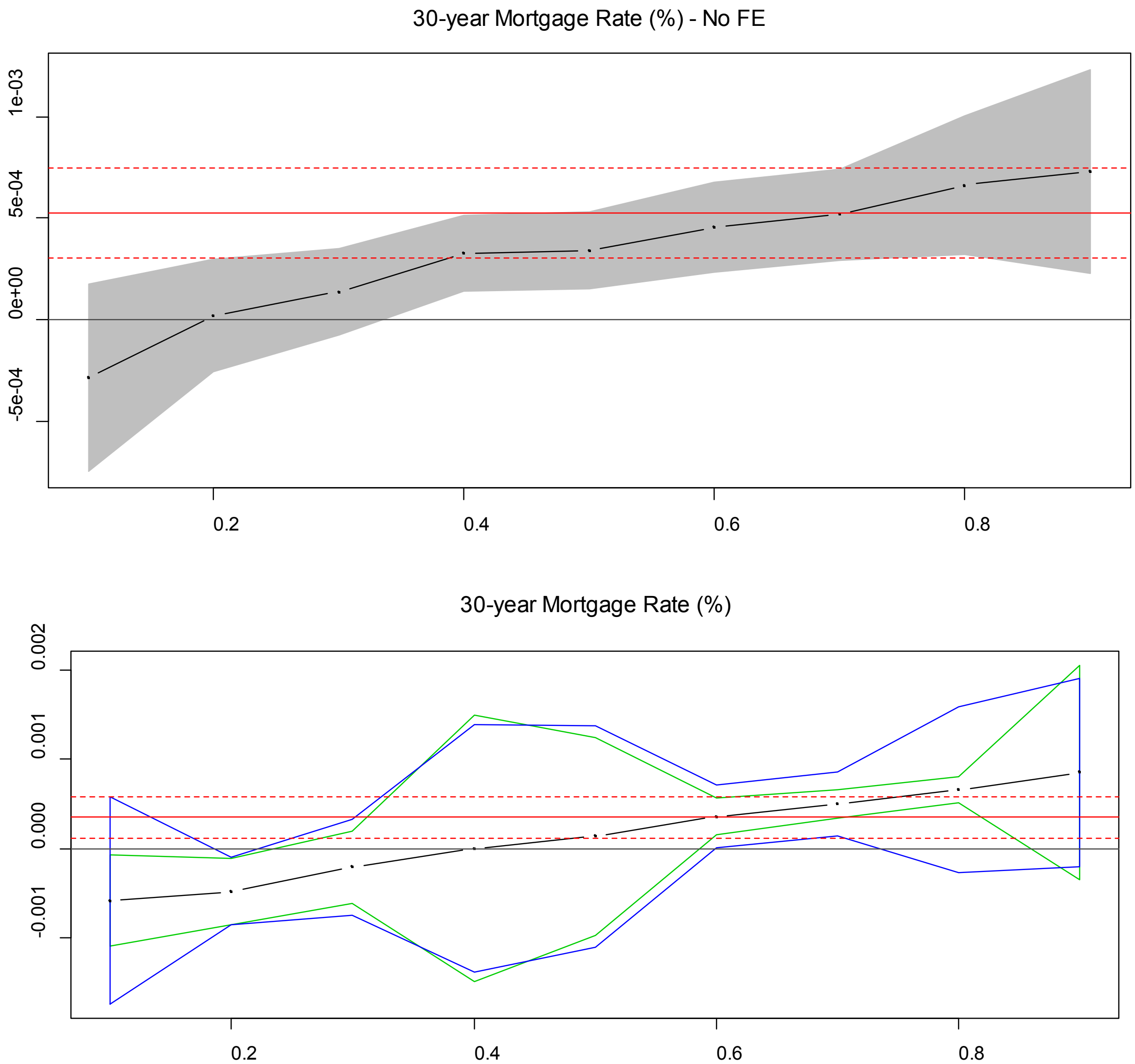


Figure 32: Sequence of QR coefficients for the 30-year mortgage rate (expressed in %)

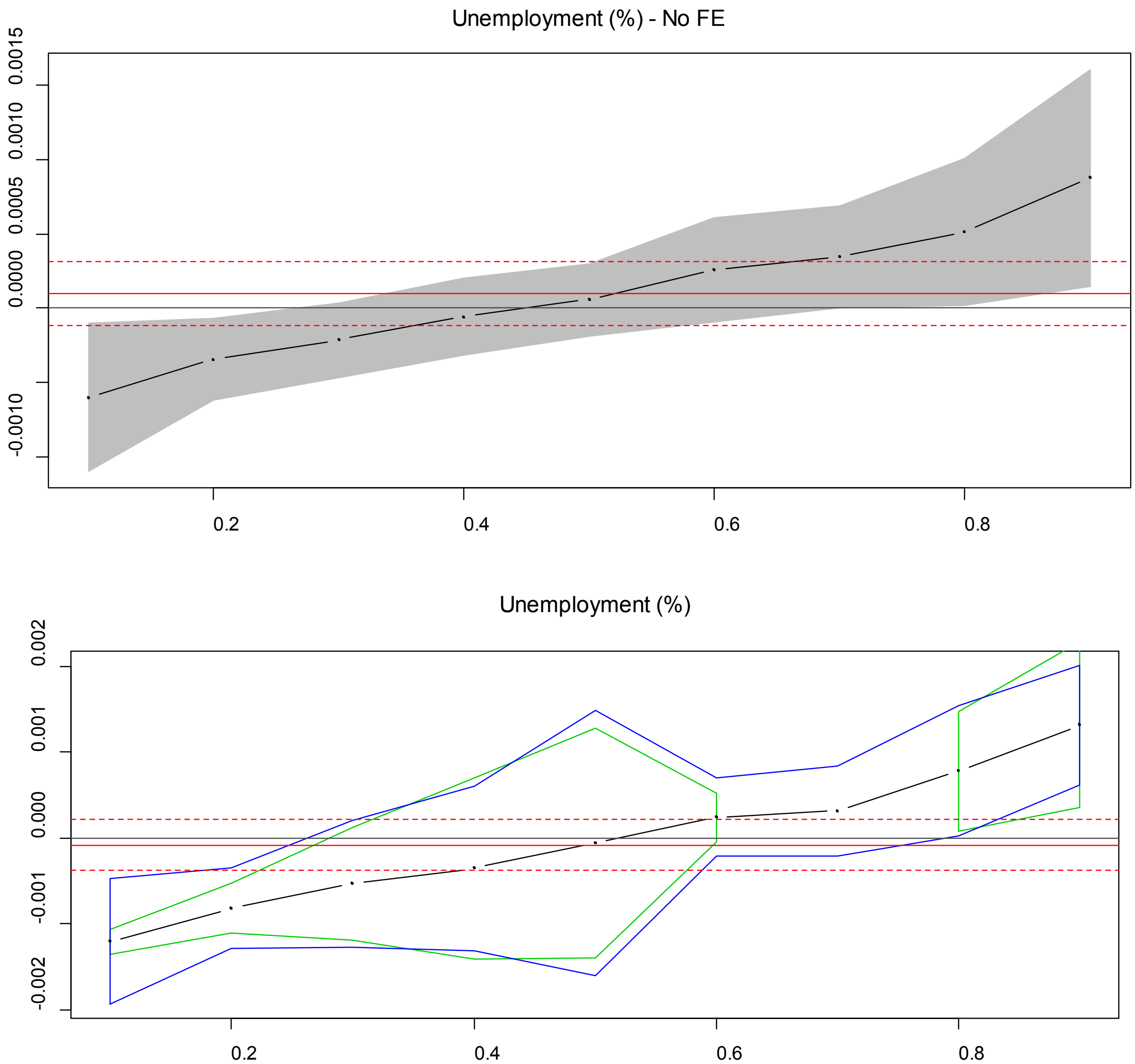


Figure 33: Sequence of QR coefficients for regional unemployment rates (expressed in %) The Wald confidence interval is not available at the 7th decile.