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INTERACTIONS AMONG COSTS IN DIFFERENT LOT-SIZING PROBLEMS WITH SUBSTITUTION

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I'd like to say thank you to my supervisor, Professor Raf Jans for his time and support. I'd like to say thank you to my family and all my best friends for your love to me so that I could finish this thesis on time.

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ABSTRACT

The aim of this thesis is to understand more how different costs interact in different lot-sizing problems with substitution. It is a direct extension of the paper "On the interaction between demand substitution and production changeovers" by Dawande et al. (2010). This paper considers a small bucket model, one-way downward substitution with two products, where product 1 could substitute for product 2 but not vice versa. The purpose of this paper is to see how changes in cost parameters impact the rate of substitution and changeovers. There are three main analyses in this paper. In analysis 1, a relative ratio is proposed to see how substitution and changeover interact. Analysis 2 looks at whether a reduction in the changeover cost or a reduction in the substitution cost will be more beneficial to the total cost reduction. Finally, the purpose of analysis 3 is to find out how changes in holding cost for product 2 influence the rate of substitution and the number of changeover.

This thesis will do similar analyses, but with three more models: one-way downward substitution and big bucket model, two-way substitution and small bucket model, and two-way substitution and big bucket model. Two testbeds will be used in this thesis: the first one is similar to the one in the original paper, and the second one with a significant gap in the demand of the two products to verify the results of analysis 1 and analysis 2.

There are some important results observed from the test experiments with the two testbeds. First, substitution proves to be more important for small-bucket models than for big-bucket models. Substitution rate could reach 100% in small bucket models while this figure is only 40% in big bucket models. Second, the product with a higher demand tends to substitute for the product with a lower demand in small-bucket models, and there is not this tendency in big - bucket models. Third, variance in demand has a high impact on the rate of substitution and the number of changeovers in small-bucket models. Finally, two-way substitution proves to be more beneficial than one-way substitution: the total cost for models with two-way substitution is always lower than the total cost for models with one-way substitution.

THESIS OUTLINE

CHAPTER 1: INTRODUCTION

1.1 General discussion about substitution

1.1.1) Introduction about substitution

Product substitution could be understood simply as when a certain product is not available, customers are willing to switch to purchase other products or manufacturers decide to replace the unavailable products with products of equal or better quality and function.

Product substitution has been applied in many real-life contexts, such as retailing, airline or hotel industry, manufacturing context (cable, steel, semiconductor...). In the airline industry for example, customers in the economy class could be upgraded into business class when the actual number of passengers showing up exceeds the capacity of the economy class, while there are still empty places in the business class. In this case it is clearly beneficial for the airline, since no overbooking cost will be incurred while the lost revenue due to empty seats in the business class is reduced. Another example is in the optical cable industry: a range of products with no significant gap in function could be interchanged with each other, although the cables of higher quality might be somewhat more expensive than the cables of lower quality.

For more information on applications of substitution, please refer to Lang (2010).

1.1.2) The benefits of substitution in production planning

Why should product substitution be considered in production planning? It has many benefits. On the manufacturing side it allows companies to have more flexibility in arranging production plans, lower safety inventory levels (due to aggregation of demand) (Chopra et al. 2012), ensure high level of product availability and therefore, reduce customer dissatisfaction. On the customer side it offers customers other choices of products when the initial desired product is not available. The examples below demonstrate some of the benefits of the product substitution in production planning:

- Windshield interlayer production planning: Lang and Shen (2011) explain why it is beneficial to consider substitution in the windshield interlayer production. Several types of interlayer are produced on the same production line, and among these some certain types could substitute for the others. In order to produce these interlayers a setup is required for each type, and the setup takes some time. Instead of producing all these different types of interlayer, it is possible to produce certain types only and use these types to substitute for the others. Total setup time could be reduced and there would be more capacity (in time units) available for the production line.
- Steel manufacturing: Balakrishnan and Geunes (2003) discuss a case in steel manufacturing where customers allow the steel plates to vary within a specified range. The steel plates are cut from slabs. The price of the plate sold to customers depend on its weight: the heavier it is, the more expensive it is. If the manufacturer chooses to produce only the heaviest plates, there might be remaining pieces of the slab which could not be utilized to produce any kind of plate. This in turn results in some lost revenue. Given the fact that customers accept some allowance in product dimensions, the plates should be cut in appropriate sizes so that no surplus piece of slab is left. This would help the steel manufacturer to maximize revenue.
- In a practical case that I know in the cable manufacturing industry, customers are mainly constructors and their demand highly depends on their winning bids. Demand is very hard to predict, although it does vary somehow according to seasons. In addition, the competition among different cable suppliers is quite fierce, and one of the main criteria on which customers choose a certain supplier is the ability to ship the deliveries on time. Product substitution is very important in this case: when a required product is unavailable, the manufacturer could immediately substitute it by other types of products. This is even more important for the products whose production time are very long and when customers have urgent projects.

1.1.3) Types of substitution

Lang (2010) lists quite in detail the different substitution characteristics. For example, the transitivity of substitution options (product 1 could substitute for product 2, product 2 could substitute for product 3 but product 1 could not substitute directly for product 3), or the substitution triangle inequality (the total cost to convert product 1 to product 2 and successively convert product 2 to 3 is the same or higher than the cost to convert directly product 1 to product 3). However, here only the types of substitution which are frequently found in different papers incorporating the substitution element are mentioned: the substitution structure, the substitution ratio and the number of substitution conversion steps.

- The substitution structure:
  - Downward: only the products of higher quality could substitute for the products of lower quality but not vice versa (see figure 1.1).
  - General substitution: a subset of products could substitute for other subsets of products (see figure 1.2).

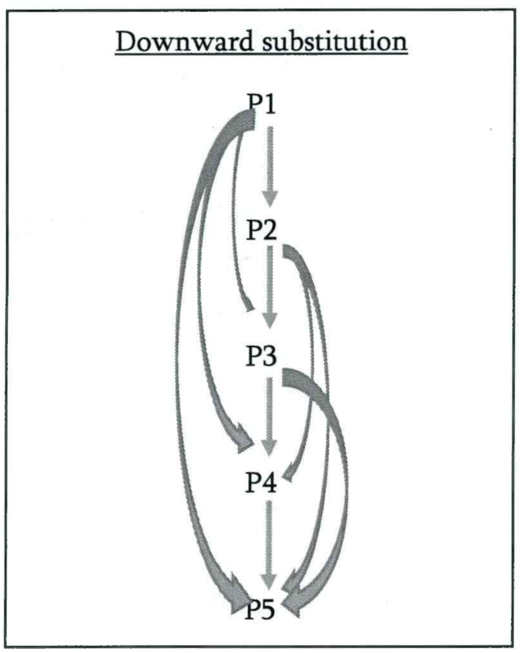


Figure 1.1: Downward substitution

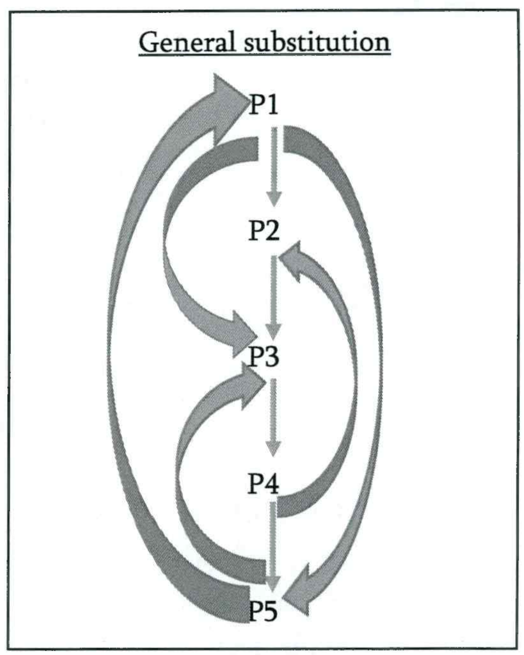


Figure 1.2: General substitution

- The substitution ratio:
  - 1-1: one unit of product A could substitute for one unit of product B.
  - 1-M: one unit of product A could substitute for M units of products B. This is often been found in the retail industry. For example, a big package of biscuits could be converted to smaller packages of biscuits. Since one big package is equivalent to several units of smaller packages, in this case one product A (big package) substitutes for M product B (small package).
  - 1-partial: one unit of product A could substitute partially for one unit of product B. For example, in fiber optics industry, product A contains connector, product B contains housing and product C contains both connector and housing. Product A and product B in this case are considered to substitute partially for product C.

- The substitution conversion steps

  Certain products could replace for other products directly without any transformation. In the retail industry for instance, if a customer would like to purchase tissue brand A but it is not available, the seller could use tissue brand B to replace for tissue brand A, without any further steps to convert B to A. However, for some other industries, some products need some transformation before they could be used to substitute for other products.
  - No conversion step: product A could substitute for product B directly without any transformation.
  - Single conversion step: in order for product A to substitute for product B, there is only one step required to convert product A to product B.
  - Multiple conversion steps: in order for product A to substitute for product B, there are several steps required to convert product A to product B.

1.2) Problem definition

This thesis is a direct extension of the paper "On the interaction between demand substitution and production changeovers" by Dawande et al. (2010) (from now on this paper will be called the original paper for short). Therefore, first we will explain in detail the analyses that were conducted in this paper and next, we will explain how this thesis extends this paper.

1.2.1) Main research questions and analysis of the original paper

1.2.1.1) Purpose of the paper

The purpose of the paper is to see *how different cost parameters impact production planning with substitution*. It considers an uncapacitated production planning context with deterministic demand, multiple periods, and a single planning level. There are two products, product 1 could substitute for product 2 but not vice versa. The substitution ratio is 1-1. Also, the model is small bucket: in one period it is only possible to produce product 1 or product 2, but not both.

The demand for product 1 and product 2 could be satisfied by the options followed:

- Demand is satisfied directly from production. There will be a changeover cost if production is switched from product 1 to product 2 and vice versa. However, there is not setup cost in this model. The difference between a setup and a changeover will be formally discussed in the section III.
- Demand for product 2 could also be satisfied by substituting product 2 by product 1. This will entail a substitution cost.
- Demand could also be satisfied by inventory from the previous period. This will entail a holding cost.

All these three costs are time-invariant over the periods as well as the same for both product. The objective is to minimize total cost, and to see which of the three options will be utilized more when there are changes in cost parameters and when there are changes in demand for product 2.

1.2.1.2) Model formulation

Basic notations used to formulate and analyze the problem

	Parameters
T	The number of period;
d_{it}	Demand of product i in period t, i= 1,2; t= 1,2,... T;
h_{it}	Per-unit holding cost of product i in period t, i= 1,2; t= 1,2,... T;
K	Changeover cost when production shifts from one product to another;
w	Per-unit substitution cost.
	Variables
x_{it}	Quantity of product i produced in period t, i= 1,2; t= 1,2,... T;
I_{it}	Inventory of product i at the end of period t, i= 1,2; t= 1,2,... T;
y_t	Quantity of product 1 used to substitute for product 2 in period t, t= 1,2,... T;
s_{it}	Setup variable: =1 if setup for product i in period t, = 0 otherwise. i= 1,2; t= 1,2,... T;

Let $e_1 = s_{11}$. Define $e_t = \begin{cases} e_{t-1}, & if\ s_{1t} + s_{2t} = 0; \\ s_{1t}, & otherwise. \end{cases}$

Thus, $\delta_{t-1,t} = |e_t - e_{t-1}|$=1 if production switches from product 1 to product 2 or vice versa.

The objective function of this model is to minimize the total changeover cost, holding cost, and substitution cost.

Minimize

$$\sum_{t=2}^{T} K * \delta_{t-1,t} + \sum_{t=1}^{T}\sum_{i=1}^{2} h_{it} * I_{it} + w * \sum_{t=1}^{T} y_t$$

Constraints

$$I_{1t} = I_{1,t-1} + x_{1t} - y_t - d_{1t}\ (t = 1,2,\ldots T) \quad (1.1),$$

$$I_{2t} = I_{2,t-1} + x_{2t} + y_t - d_{2t}\ (t = 1,2,\ldots T) \quad (1.2),$$

$$I_{i0}, I_{iT} = 0\ (i=1,2) \quad (1.3),$$

$$x_{1t} \leq (d_{2t} + \textstyle\sum_{l=t}^{T} d_{1l}) * s_{1t}\ (t = 1,2,\ldots T) \quad (1.4),$$

$$x_{2t} \leq (\textstyle\sum_{k=t}^{T} d_{2k}) * s_{2t}\ (t = 1,2,\ldots T) \quad (1.5),$$

$$s_{1t} + s_{2t} \leq 1\ (t = 1,2,\ldots T) \quad (1.6),$$

$$I_{it} \geq 0, x_{it} \geq 0, s_{it}\ binary\ (i = 1, 2; t = 1,2,\ldots T) \quad (1.7),$$

$$y_t \geq 0\ (t=1, 2,\ldots T) \quad (1.8).$$

Constraints (1.1), (1.2), and (1.3) represent the inventory balance. Since there is no limit on capacity, constraints (1.4) and (1.5) define the setup forcing. For product 1, it is allowed to produce in period t the total demand of product 1 from period t to period T, plus the demand of product 2 in period t. For product 2, it is allowed to produce in period t the total demand of product 2 from period t to period T. Note the restriction that product 1 could only be produced and substitute for product 2 in the same period (constraint (1.4)). This restriction follows from a property which is formally proven in the paper. Constraint (1.6) is for small bucket model, only 1 type of product is produced in one period.

An important contribution of the paper is the development of a polynomial time algorithm for solving this problem. Since this is not relevant for this thesis, we will not further focus on this algorithm.

1.2.1.3) Sensitivity analyses

There are three analyses related to the following three questions when there are changes in cost parameters:

- How do substitution and changeover interact? When does substitution happen more frequently than changeover and vice versa?
- If it is possible to reduce the changeover cost or the substitution cost to lower the total cost, then when is it more interesting to reduce the changeover cost or the substitution cost?
- How do changes in holding cost for product 2 impact the substitution and changeover?

The testbed used in this paper is similar to the ones used by Chand and Morton (1986) and Dawande et al. (2007). Holding cost takes one value at 1 for both products (however, in the third analysis holding cost for product 2 varies from 0.2 to 0.8). Substitution cost takes 4 values 2, 4, 6 and 8. Changeover cost takes 10 values 10, 15, 20, 30, 50, 75, 100, 150, 200 and 300. Mean demand for product 1 is 20, and mean demand for product 2 takes the values 10, 15, 20 and 40. The number of periods is 20.

The three analyses regarding changes in cost parameters are presented next:

- **Analysis 1**:
  - Issue raised: How do substitution and changeover interact? When does substitution happen more frequently than changeover and vice versa?
  - Analysis method:

    In this analysis the holding cost is fixed, only the substitution cost and the changeover cost are varied so that it is easier to see how substitution and changeover interact. A relative ratio with the following definition is suggested:

$$relative\ ratio = \frac{Changeover\ cost}{Substitution\ cost\ x\ Mean\ demand\ of\ product\ 2}$$

    The idea behind this ratio is that choosing the best production option depends on which one has the lower cost:

    + When this ratio is high, it means that the cost for changeover is relatively higher than the cost for substitution. The optimal solution could then choose to produce only product 1, substitute product 2 by product 1, and limit the switch of production to limit the cost for changeover. The higher this ratio is, the higher the rate of substitution and the lower number of changeover.

+ However, when this ratio is low, the cost for changeover is now relatively less costly than the cost for substitution. The optimal solution will then limit the rate of substitution and choose to switch production more frequently between product 1 and product 2.

Note also that the cost for substitution depends on two elements: the substitution cost and the demand for product 2, since in this model one must pay the substitution cost for each unit of demand for product 2.

- Result:

The number of changeover and the rate of substitution is computed for each instance. Overall the result is the same as hypothesized.

Figures 1.3 shows the number of changeover: X-axis shows the relative ratio, and Y-axis shows the number of changeover.

Figures 1.4 shows the rate of substitution: X-axis shows the relative ratio, and Y-axis shows the rate of substitution.

+ According to figure 1.3 and figure 1.4, when the *relative ratio is* close to 0 on the X-axis, the number changeover is almost at maximum while the rate of substitution is almost at 0. The cost for changeover is much cheaper than the cost for substitution, therefore the optimal solution limits the rate of substitution and utilizes much more often the changeover option.

+ Vice versa, when the *relative ratio* is more than 2, the number of changeover is almost at 0 while the rate of substitution almost reaches 100%. The cost for changeover is now much higher than the cost for substitution, therefore the substitution option is more often used.

+ Substitution and changeover are both used when the relative ratio is between 1 and 1.5. In this case the cost for substitution and the cost for changeover are almost equal, so the optimal solution utilizes both options in a balanced way.

Overall, the higher the ratio, the higher the rate of substitution and the lower the number of changeover. Vice versa, the lower the ratio, the lower the rate of substitution and the higher the number of changeover

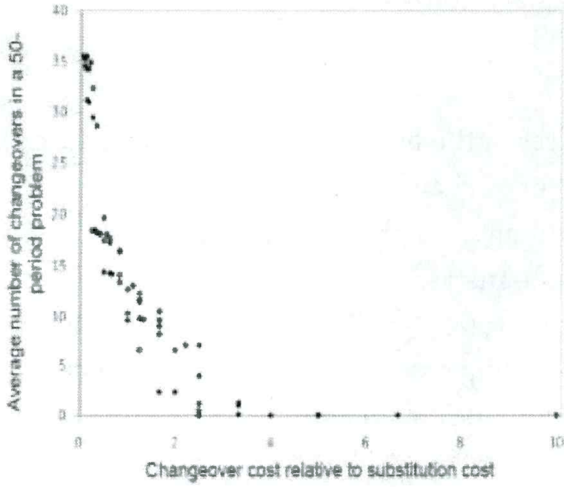


Figure 1.3: Changeover changes with relate to the relative ratio

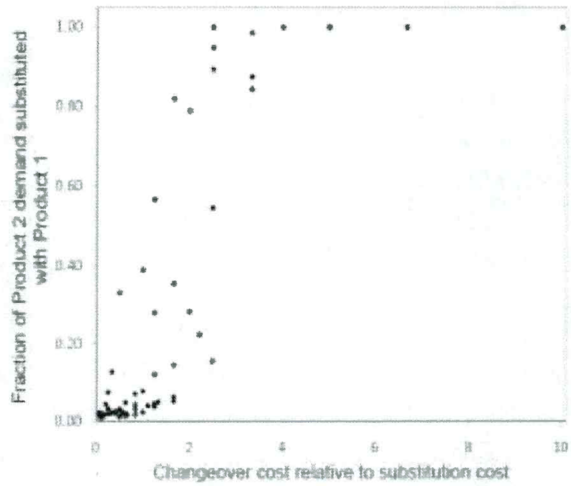


Figure 1.4: Substitution changes with relate to the relative ratio

Source: Dawande et al. (2010), "On the Interaction between demand substitution and production changeovers", Manufacturing & Service Operations Management, Vol 12(4), 682-691.

- <u>**Analysis 2**</u>:
    - Issue raised: The authors consider it is possible to reduce the changeover cost or the substitution cost to lower the total cost. Then how the impact of substitution cost reduction and the impact of changeover cost reduction would change when demand 2 increases from 10 to 40?
    - Analysis method: First the total cost when both the substitution cost and the changeover cost are at the highest value (8 and 200, respectively) is computed. Then the total cost reduction due to 50% reduction in the substitution cost and due to 50% reduction in the changeover cost are calculated separately. The cost reduction is computed averaged over 4 base demand 2 (10, 15, 20 and 25). The investment costs to reduce the substitution cost and the changeover cost to half are assumed to be the same.
    - Result:

      When the demand for product 2 increases from 10 to 25, the impact of substitution cost reduction decreases from 35.8% to 16.2%, and the impact of changeover cost reduction increases from 10.9% to 20.1%.

- <u>**Analysis 3**</u>:
  - Issue raised: how do changes in the holding cost for product 2 impact the rate of substitution and the number of changeover?
  - Analysis method: the holding cost of product 2 is varied from 0.2 to 0.8 in increments of 0.2. Two sub-analyses are performed. First the total number of changeover and rate of substitution is calculated over each instance. Second, to further see the impact of holding cost for product 2 on production planning, the analysis is based on separating the data according to high or low changeover cost and high or low substitution cost.
  - Result:

    + First, when the holding cost for product 2 increase from 0.2 to 0.8, both changeover and substitution increase: the fraction of periods with changeovers increases from 0.186 to 0.295 (or 58%), and the rate of substitution increases from 0.048 to 0.227 (or 366%).

    + Second, the changes in changeover and substitution is more clearly illustrated through figures 1.5 and 1.6 as below. Figure 1.5 shows the changes in changeover: X-axis shows the holding cost for product 2 varying from 0.2 to 0.8, and Y-axis is the percentage change in changeover. Similarly, figure 1.6 shows the changes in changeover: X-axis shows the holding cost for product 2 varying from 0.2 to 0.8, and Y-axis is the percentage change in substitution.

    According to figure 1.5, when the holding cost for product 2 increases, the fraction of periods with changeover increase more significantly for low changeover cost or high substitution cost. However, as shown in figure 1.6, for low substitution cost or high changeover cost, the rate of substitution increases more significantly.

    It is then concluded that when the holding cost for product 2 increases, it is better to increase changeovers when the changeover cost is low or substitution cost is high, and increase substitution when substitution cost is low or changeover cost is high.

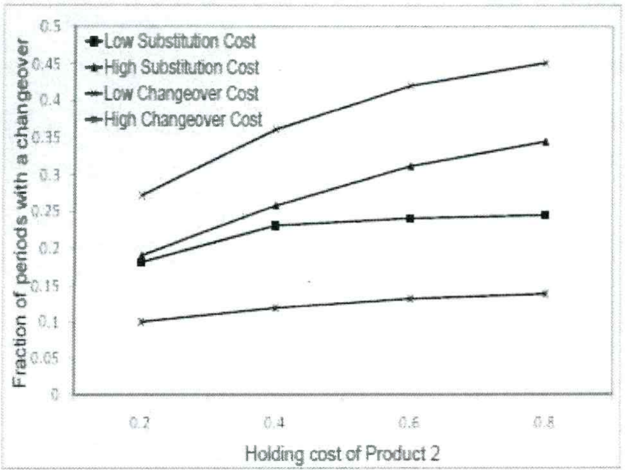


Figure 1.5: Changeover changes

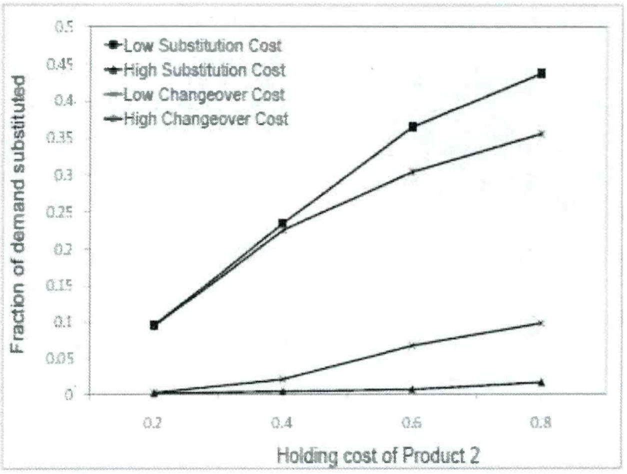


Figure 1.6: Substitution changes

Source: Dawande et al. (2010), "On the Interaction between demand substitution and production changeovers", Manufacturing & Service Operations Management, Vol 12(4), 682-691

1.2.2) Comment about the relative ratio proposed in the original paper

+ First, this ratio should be understood not in the absolute term, but in the relative term. That is, a higher *relative ratio* does not always mean a higher percentage of substitution and vice versa, a lower *relative ratio* does not in all the cases means a higher number of changeover. Or that the same *relative ratio* does not mean the optimal solution results in the same number of changeover and the same rate of substitution.

For example, two sets of instance:

Instance 1: D1= 50, D2= 25, substitution cost = 8, changeover cost = 200
Instance 2: D1= 50, D2= 25, substitution cost = 2, changeover cost = 50
These two instances have the same ratio, but both the number of changeover and the rate of substitution are lower in the first instance than in the second instance. The reason is that in the first instance, both the changeover cost and the substitution cost are high while the holding cost for both products stay unchanged at the value of 1. Therefore, instead of substituting product 1 for product 2 or switching the setup sequentially between product 1 and product 2, the optimal solution increases inventory for both products. Using this way, it is possible to exploit the low value of holding cost when both the changeover cost and the substitution cost are high.

+ Second, this ratio only considers the changeover cost and the substitution cost but ignores the holding cost. In addition, the demand for product 1 is not included. For these reasons there are several cases which might not have the results as expected, for example when the demand for product 1 is much higher than the demand for product 2.

Example (see figure 1.7):

Mean D1= 10,000, mean D2= 10

Substitution cost = 8, changeover cost = 10, holding cost = 1.

The number of period: 10

For simplification, we assume that the actual demand is the same as mean demand in each period.

The relative ratio is (changeover cost / (substitution cost x mean D2))
= (10 / (8*10)) = 0.125. *This ratio is close to 0, so it is expected that there are lots of periods with changeovers.*

Below are the three possible options substitution and changeover:

Option 1- 100% substitution: produce product 1 in all 10 periods and substitute 100% for product 2. Total cost is equivalent to total substitution cost only:
Substitution cost x mean D2 x 10 periods = 8 x 10 x 10 = **800**

Option 2- Produce product 1 in the 1st period, switch to produce product 2 in the 2nd period and switch back to produce product 1 in the 3rd period. Continue to produce product 1 from period 4 – 10. Demand for product 2 for period 3 -10 is satisfied by the inventory from period 2.
Total cost = Substituting product 2 by product 1 in period 1 + 2 changeovers + holding cost for product 1 in period 2 + holding cost product 2 from period 3 -9
= 8*10 + 10*2 + 1*10000 + 1*10*(7+6+5+4+3+2+1) = **10,380**

Option 3- 100% changeover: Produce product 1 in the 1st period, switch to produce product 2 in period 2, then produce product 1 in period 3. Continue to produce alternately between product 1 and product 2 to the end of planning.

Total cost = Substituting product 2 by product 1 in period 1 + (9 x changeover cost) + (5 periods inventory for product 1) + 4 periods inventory for product 2) = (8*10) + (9*10) + (5 * 1 * 10,000) + (4*1*10) = **50,210**

➔ We could see that among the 3 options, option 1 has the lowest cost. For option 2 and 3, holding cost constitutes a big portion of the total cost. The optimal solution therefore might be option 1, and this is against the expected result that when the *relative ratio* is close to 0, the number of changeover will almost reach maximum.

This example proves that ignoring the holding cost as well as ignoring the demand for product 1 might make this ratio not relevant when there is a big variance in demand. Further tests for instances with big difference in demand for different products will be shown in chapter 4.

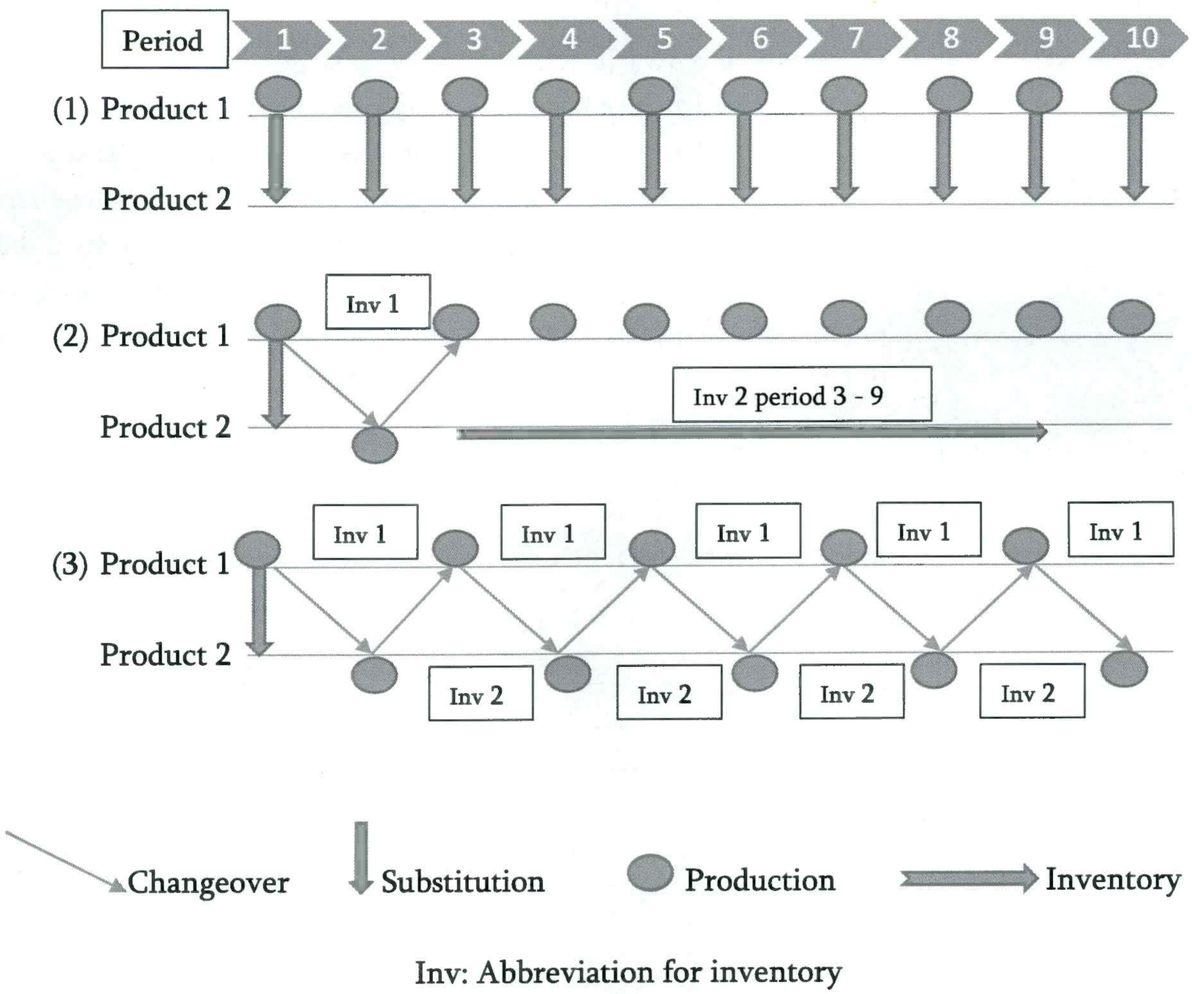


Figure 1.7: Illustration of three different production plans

1.2.2) <u>Thesis research questions</u>

First, with the purpose of verifying whether the conclusions in the original paper hold true in other lot-sizing problems, this thesis will investigate four lot-sizing models, perform calculation tests and compare the results with the original paper.

Below are the four models which will be tested in this thesis:

- Model 1: Uncapacitated, small bucket, one-way downward substitution.

  Note that this is the lot-sizing model with substitution in the original paper.

- Model 2: Uncapacitated, big bucket, one-way downward substitution

- Model 3: Uncapacitated, small bucket, two-way substitution

- Model 4: Uncapacitated, big bucket, two-way substitution

These models are different from each other mainly in two points:

- One-way downward substitution versus general substitution: for one-way downward substitution models, only product 1 could substitute for product 2. For two-way substitution, both products 1 & 2 could substitute each other.
- Small bucket versus big bucket: in a big bucket model, more than one type of product could be produced in one time period, while in a small bucket model, at most one type of product is produced in one time period. A big bucket model entails only setup cost, but a small bucket model entails changeover cost. The difference between setup and changeover will be formally defined by the models in chapter 3.

For more information on different lot-sizing problems, please see Jans and Degraeve (2008).

Table 1.1 summarises the different characteristics of the four models:

Model	Changeover cost	Setup cost	Product 1→ substitute product 2	Product 2→ substitute product 1
1	x		x	
2		x	x	
3	x		x	X
4		x	x	X

Table 1.1: List of differences among 4 models

Next, the conclusions in the original paper will be validated to see whether they continue to be true in the case where the difference in demand between the two products is large. They are validated upon a data set with limited variability: the difference between demand for product 1 and demand for product 2 is not significant (base demand for product 1 is at 20, and base demand for product 2 takes four values: 10, 15, 20 and 25). If there is big gap between demand for product 1 and that for product 2 (for example, demand for product 1 is at 2000 but demand for product 2 is at 20), the results in analysis 1 and 2 of the original paper might be no longer relevant.

1.2.3) Methodology

The test bed utilized in this thesis is almost the same as the one used in the original paper, although it is somewhat adjusted and the number of instances is reduced (more details are given in section 4.1). The models are written in OPL language and solved in CPLEX 12.6 using the MIP solver. They are solved on a 3.07 GHz computer with an Intel Xeon X5667 processor and with 12GB of RAM. The new instances are generated in Excel to test the problems.
Different lot-sizing problems are tested to have better insights on how changes in parameters impact the production planning with substitution. There are two testbeds: one is almost identical to the testbed used in the original paper, and the other with a significant gap in demand of different products.
In addition, the models in this thesis are reformulated using the facility location reformulation for lot-sizing problems. This reformulation is based on the correlation of some elements between the production planning problem and the facility location problem. A setup in one period and the fixed setup cost for a product in the production planning are similar to the setup and fixed opening cost of a new warehouse. Fulfilment of demand for a demand class and the fulfilment cost in the production planning are considered similar to the transportation quantity and transportation cost from a warehouse to customer.
The reformulation was first proposed by Krarup and Bilde (Krarup & Bilde 1977). Later, Geunes (2003), Lang and Domschke (2010) in their experiments showed that the solution time for lot-sizing problems could be significantly reduced when they are reformulated as facility location problems. For more information on the reformulation, see Pochet & Wolsey (2006).

1.2.4) Thesis outline

The outline of this thesis is as follows. The current chapter 1- Introduction gives an overview of substitution, the benefits of applying substitution and its applications in different industries. Papers researching substitution will be briefly reviewed in chapter 2 - Literature review, with the focus on manufacturer-driven substitution. Chapter 3 and chapter 4 are the main parts of this thesis. Chapter 3 presents in details the models of the different lot-sizing problems with substitution. Chapter 4 presents the test and the results to see whether the conclusions in the original paper hold true in different lot-sizing problems, as well as with data sets having significant variance in demand for different products. Limitations and future outlook will be presented in chapter 5 and finally, chapter 6 will be the conclusions.

CHAPTER 2:

LITERATURE REVIEW

In this section two streams of research with substitution will be presented: customer-driven substitution and manufacturer-driven substitution. The relation with this thesis is also mentioned after.

2.1) Literature review

There are various streams of research regarding substitution in the literature: in a manufacturing context, in a retailer context, or in revenue management (for airlines for examples). However, within the limitation of this thesis, only two streams are mentioned: customer-driven substitution and manufacturer-driven substitution. The former is mainly found in a retail context and the latter is mainly found in a manufacturing context. The two streams differ with respect to the person who decides on the substitution. For customer-driven substitution, customers decide whether they will substitute one product by other types when the desired products are unavailable. Therefore the retailers must ***predict the probability*** that the customers will substitute unavailable products with other products. For manufacturer-driven substitution, manufacturers could control the substitution since the substitution is at their discretion. When a customer orders a certain kind of product but it is unavailable, they could decide whether they should replace this product with other types of products.

2.1.1) Customer-driven substitution

As mentioned above, research in this stream mainly focuses on the retailers and aims at finding the optimal inventory levels so that the total cost is minimized or the total profit is maximized.

Yucel et al. (2008) investigate a retail context with deterministic demand, general substitution (a product could be substituted by several other types of products), limited space for storing products, and list of suppliers who could supply more than one type of product but one product is only supplied by one supplier. The purpose is to find, under space constraints and with substitution considered, the optimal assortment of products (which means which products to buy and sell) so that the profit is maximized.

Experiments in this paper show that total profit will decrease when substitution, supplier selection and space constraints are ignored. Regarding substitution, the result shows that when the substitution cost increases, the rate of substitution decreases. This is because when the substitution cost increases, the optimal solution will increase the number of products in the assortment and the number of supplier chosen, thereby reducing the substitution rate. Nagarajan and Rajagopalan (2008) study the system with two products whose demand are negatively correlated and a fixed proportion of customers will switch to the other product when their desired product is unavailable. Consistent with prior papers in the literature, this paper finds that "as items become more substitutable, the retailer needs to carry lower inventories". Another interesting paper by Stavrulaki (2011) focuses on the inventory decisions for a system of two products when sales are impacted by two elements: substitution and demand stimulation. For substitution, product 1 and product 2 could substitute each other. For demand stimulation, higher inventory could boost the demand for a product and thereby boosting the sales of products (this point is quite different from other papers, which usually focus on how to minimize inventory). Also, product 2 has a higher profit margin than product 1. The experiment shows two noted results. When the percentage of customers who substitute product 1 with product 2 increases, the optimal solution will increase the inventory for product 2, thereby stimulating both demand for product 2 and increasing the total profit without lost sales for product 1. In this case substitution is important and a significant loss will occur if substitution is not considered. Conversely, when the percentage of customers who substitute product 2 with product 1 increases, if the optimal solution increases the inventory for product 1, it could stimulate more demand for product 1. This might lower the total profit and this is the undesired result since product 1 has a lower profit margin than product 2. In this case it is difficult to find an optimal solution capturing both demand stimulation and substitution. In the same manner as the papers mentioned above, Tan and Karabati (2013) also prove that incorporating substitution could improve profit. The model in this paper is a fixed-review period and order-up to level system. In addition, for each product, it is required to satisfy the minimum service level by direct sales (direct sales means the demand for each product must be satisfied directly by that product, not by substituting this product with other products).

2.1.2) Manufacturer-driven substitution

Different papers about production planning with substitution could be found in the literature, each paper considers different combinations of production planning characteristics and substitution types. Some different characteristics of production planning settings are single-level versus multi-levels, single item versus multi-items, single period versus multi-periods, capacitated versus un-capacitated, big bucket versus small bucket, setup carry-over versus no setup carry-over, with scheduling versus no scheduling, with capacity constraint versus no capacity constraint.

Bassok et al. (1999) build a profit maximization, single period, multiproduct (although the experiment is restricted to two products), stochastic demand and downward substitution model with two decisions: first the ordering decision before the demand realization and second, the allocation of products to demand classes. Through various experiments (the method is varying one parameter while fixing the other parameters), the results show that incorporating substitution improves the total profit, especially when substitution cost is low. Similarly, Rao et al. (2004) build a model with two-stage decisions, minimizing cost, a single period, multiple product, stochastic demand and downward substitution with setup cost. Two important results are shown regarding substitution in this paper. First, the total cost when substitution is considered in both two stages are compared with two scenarios: 1) when substitution is completely not considered and 2) when substitution is only considered in the second stage (allocation of products to demand after actual demand is known). The total cost when substitution is allowed in both two stages is lower than the total cost in each of the two scenarios. Second, this paper also finds that savings are more significant when substitution cost is low. This result is consistent with the result by Bassok et al. (1999). Balakrishnan and Geunes (2000) focus on the complex interaction between setup cost and substitution cost as well as the benefit of substitution for a production planning problem with the following characteristics. In their uncapacitated, big bucket and two component model, component C2 could substitute for C1 and the objective is minimizing the total cost. The total demand for both products are the same over the planning periods, but there are two demand scenario: first, the demand for component 1 is increasing over the planning periods while the demand for component 2 is decreasing over the planning periods; second, both components have seasonal demand, with the peak demand for this component corresponding to the low demand for the other. Their analysis is as follows: substitution cost and holding cost for component 1 is fixed at 1; holding cost for component 2 takes 3 values, and the setup cost for both product varies from 0 to 1000. The results show that overall the substitution rate and the percentage of savings due to substitution increase when the setup cost increases over both demand scenarios, although the substitution rate and the percentage of savings are not the same for the three values of holding cost for component 2. The other interesting result is that both the substitution rate and the percentage of savings in the first demand scenario is higher than in the second demand scenario. This result is attributed to "the greater disparity in the magnitudes of the two product's demands over an extended period of time. That is, substitution depends not only on the magnitude of difference in product demands, but also on the duration of time during which this difference in volume continues".

Some other papers focus more on the efficiency of the solution methods rather than the analysis of costs as the three papers mentioned above in this section. Balakrishnan and Geunes (2003) incorporate the flexibilities of customer demand in the maximizing profit production planning problem for the steel manufacturing: in certain cases customer requirements for the weight of plates could vary within a certain range. Therefore, exploiting this flexibility could allow to further increase total profit. They suggest a composite solution including branch and bound, Lagrangian relaxation and valid inequalities to run this model.

Hsu et al. (2005) analyze two models with one-way downward substitution, one with conversion and the other with no conversion. The two models differ in two points. The first difference is, in the first model, a product needs to be converted before it could substitute other products while in the second model, a product could substitute other products directly without any conversion. Secondly, since the holding cost for different products is time-varying, in order to exploit the lower holding cost, it is possible to convert a product before it is used to replace other products in the first model. In the second model however, a product is always kept in its own inventory before it is used to meet the demand for other products. Hsu et al. also build a heuristic to run these models and compare the efficiency of this heuristic with a Wagner-Whitin algorithm over seven sets of instances and over six parameters: conversion rate, holding cost rate, demand variability over time, production cost variability over time, demand variability across products and production cost variability across time.

Yaman (2009) has a model quite similar to Dawande et al. (2010) except that the substitution cost is zero and the context is big-bucket. The solution suggested in this paper is LP relaxation.

Lang and Shen (2011) combine scheduling and substitution in one model, instead of separating the two aspects in different models. They consider the case where setups are sequence-dependent. In a context where it is required to schedule the production of different products in different periods and where it is possible for certain products to substitute the others, it is better to combine these aspects in one model to find the optimal solution rather than sub-optimal solutions for each problem. The benefit would be reducing setup times, changeover times and thereby increasing available capacity for production. The solution methods in this paper are Relax&Fix and Fix&Optimize heuristics.

Geunes (2003) and Lang and Domschke (2010) prove that a reformulation of the production planning problems using the facility location problem helps the models run faster. Geunes (2003) reformulates the model by Balakrishnan and Geunes (2000) using this technique. Their results show that, over three main experiments, the running time of reformulation using the facility location problem is by far lower than the running time of the shortest path reformulation. For exactly the same set of instances, Geunes (2003) solves all the instances of this set in less than 1 second while Balakrishnan and Geunes (2000) solve them on average in 9 minutes. Next a larger set of instances is tested, and the solving time is also within 1 second. Finally the reformulation could also solve a much larger set of instances within 3 seconds. Similarly, Lang et Domschke (2010) have comparisons among different formulations and the results show that the reformulation using the facility location problem is most of the time superior to other formulations (including original formulation, user cut and valid inequalities) in five experiments. There are two models proposed in this paper, one with no additional resource and the other with some additional resources, including lost sales, over-time and some capacity. The difference between this paper and other papers is that it includes initial inventory and in addition, it deals with a general substitution structure. In experiment 1 and 2 for model 1 (no resource), the reformulation is superior both in the lowest running time and in the ability to solve 100% instances in the set time limit. The result is quite similar in experiments 3 – 5, although in some certain instances of experiments 5, the reformulation does not prove to be superior.

2.2) Relation with this thesis

So far in the manufacturer-driven literature there are very few papers focusing on the analysis of costs and the benefit of general (or two-way) substitution. In addition, in the papers which do analyze the costs, the conclusions are quite general. For example, substitution happens more frequently when substitution cost is low, or that the total savings is higher when substitution cost is low and setup cost is high. Usually experiments are based on sets of data with very low variance in demand of different products. This might be the reason leading to these general conclusions, which might not be true in all contexts. Finally, there is almost no paper analyzing how costs interact in different lot-sizing problems. In most of the cases there is only one type of lot-sizing problem in these papers. This thesis, therefore, hopes to have a better look at the missing points which have not been mentioned in the literature.

CHAPTER 3:

MODELS & FORMULATION

This chapter presents the formulation details of the four models mentioned in chapter 2: first the formulations for the standard lot-sizing problems and next the reformulations for the lot-sizing problems using the facility location problem.

The models presented in this thesis have the following characteristics:

- Deterministic demand
- Two products
- Multiple periods, a single planning level
- No backlog
- No setup carry over
- 1-1 substitution ratio
- Independent demand
- No initial inventory and no inventory left at the end of the planning horizon.

3.1) Standard formulation

Notation	Meaning
Sets:	
P	Set of products, = {1…M=2};
T	Set of periods, = {1…N};
Parameters:	
d_{it}	Demand of product i in period *t;*
h_{it}	Per-unit holding cost of product i in period *t;*
w	Per-unit substitution cost;
K	Changeover cost;
S_{it}	Setup cost for product i in period *t;*
Decision variables:	
x_{it}	Quantity of product i produced in period *t;*
I_{it}	Inventory of product i at the end of period *t;*
y_{ijt}	Quantity of product i used to substitute for product j in period *t;*
s_{it}	Setup for product i in period *t*: =1 if setup for product i in period *t*, = 0 otherwise;
z_{it}	Changeover for product i in period *t*: =1 if product i is produced in period *t* but it was not produced in period *t-1*, = 0 otherwise.

3.1.1) Model from Dawande et al. (2010): one-way substitution, small-bucket

Though the model in the original paper is explicitly written for two products, here a model for the multi-product case is presented.

Minimize

$$\sum_{t=1}^{N}\sum_{i=1}^{M} K * z_{it} + \sum_{t=1}^{N}\sum_{i=1}^{M} h_{it} * I_{it} + w * \sum_{t=1}^{N}\sum_{i=1}^{M}\sum_{j=i+1}^{M} y_{ijt}$$

Subject to

$$I_{it} = I_{i,t-1} + x_{it} - \textstyle\sum_{j=i+1}^{M} y_{ijt} + \sum_{k=1}^{i-1} y_{kit} - d_{it}, for\ 1 \le t \le N, 1 \le i \le \mathrm{M} \tag{3.1}$$

$$x_{it} \le (\textstyle\sum_{j=i+1}^{M} d_{jt} + \sum_{l=t}^{N} d_{il}) * s_{it}, for\ 1 \le t \le N, 1 \le i \le \mathrm{M} \tag{3.2}$$

$$z_{it} \ge s_{it} - s_{i,t-1}, for\ 1 \le t \le N, 1 \le i \le \mathrm{M} \tag{3.3}$$

$$\textstyle\sum_{i=1}^{M} s_{it} \le 1, for\ 1 \le t \le N \tag{3.4}$$

$$z_{it}\ binary \tag{3.5}$$

$$s_{it}\ binary \tag{3.6}$$

$$I_{it} \geq 0, x_{it} \geq 0, y_{ijt} \geq 0 \tag{3.7}$$

Constraint (3.1) calculates the remaining inventory for product i at the end of period t. Three sources increase the inventory for product i in period t: the remaining inventory at the end of period *t-1* carried on to period t, the quantity of product i produced in period t and the total quantity of the products substituting for product i. Two sources decrease the inventory for product i in period t: the consumption of product i in period t and the quantity of product i allocated to substitute for other products. The remaining inventory for product i in period t is the balance between the total quantity that increases the inventory and the total quantity that decreases the inventory.

This model is uncapacitated, therefore constraint (3.2) allows the quantity of product i produced in period t to its maximum: total demand for product i from period t to the end of planning horizon (period N) plus the total quantity of product that product i could substitute for in the same period t. This property was formally proven in Dawande et al. (2010) for two products. The purpose of this constraint is to be consistent with constraint (1.4) in the original paper:

$$x_{1t} \leq (d_{2t} + \sum_{l=t}^{T} d_{1l}) * s_{1t}\ (t = 1,2,\ldots T)$$

For constraint (3.3), there are three scenarios as below:

If $(s_{it} - s_{i,t-1}) = 0$, there are two cases: or product i is produced in period *t-1* and t, or product i is not produced both in period t and *t-1*. In both these two cases they all mean that there is no changeover from period *t-1* to t for product i. z_{it} could be both 0 or 1, but since the objective function is minimize, z_{it} will take the value 0, which corresponds to the case that there is no changeover from period *t-1* to t for product i.

If $(s_{it} - s_{i,t-1}) = 1$, it means product i is produced in period t but not produced in period *t-1*, *i.e* there is a changeover from period *t-1* to t for product i. In this case z_{it} could only take the value 1, which corresponds to the case that there is a changeover from period *t-1* to t for product i.

The last scenario is $(s_{it} - s_{i,t-1}) = -1$. This equals to the case that product i is produced in period *t-1* but not produced in period *t*. There is no changeover in this case, and z_{it} will take the value 0 since the objective function is minimize.

Constraint (3.4) imposes that only one type of product is produced in one period.

The changeover decision variable and setup decision variable are defined as binary variables in constraints (3.5) and (3.6), correspondingly. Constraints (3.7) are the non-negativity constraints.

3.1.2) Model 2: one-way downward, big bucket and uncapacitated.

In this model it is allowed to produce more than one type of product in each period, therefore there is no changeover decision variable z_{it}. The model is similar to model 1, except that the variable z_{it} is replaced by the variable s_{it} in the objective function and it is now associated to a setup cost S_{it}. Also, constraints (3.3), (3.4) and (3.5) are removed in this model.

Minimize

$$\sum_{t=1}^{N}\sum_{i=1}^{M} S_{it} * s_{it} + \sum_{t=1}^{N}\sum_{i=1}^{M} h_{it} * I_{it} + w * \sum_{t=1}^{N}\sum_{i=1}^{M}\sum_{j=i+1}^{M} y_{ijt}$$

Subject to constraints (3.1), (3.2), (3.6) & (3.7).

Note that this model continues to be subject to constraint (3.2) of model 1: the maximum production amount that product i is produced is limited to the sum of the remaining demand until the end of planning horizon plus the quantity of product that product i could substitute for in period *t*. Therefore this model is somewhat limited and would be applied in some certain contexts.

3.1.3) Model 3: General substitution, small bucket, uncapacitated

In this model it is necessary to separate two set of products: P_i^+ is the set of products that product i could substitute for, and P_i^- is the set of products that could substitute for product i.

Minimize

$$\sum_{t=1}^{N}\sum_{i=1}^{M} K * z_{it} + \sum_{t=1}^{N}\sum_{i=1}^{M} h_{it} * I_{it} + w * \sum_{t=1}^{N}\sum_{i=1}^{M}\sum_{j \epsilon P_i^+} y_{ijt}$$

Subject to

$$I_{it} = I_{i,t-1} + x_{it} - \sum_{j \in P_i^+} y_{ijt} + \sum_{k \in P_i^-} y_{kit} - d_{it}, for\ 1 \le t \le N, 1 \le i \le M \quad (3.8)$$

$$x_{it} \le (\sum_{j \in P_i^+} d_{jt} + \sum_{l=t}^{N} d_{il}) * s_{it,}\ for\ 1 \le t \le N, 1 \le i \le M \quad (3.9)$$

$$z_{it} \ge s_{it} - s_{i,t-1}, for\ 1 \le t \le N, 1 \le i \le M \quad (3.10)$$

$$\sum_{i=1}^{M} s_{it} \le 1, for\ 1 \le t \le N \quad (3.11)$$

$$z_{it}\ binary \quad (3.12)$$

$$s_{it}\ binary \quad (3.13)$$

$$I_{it} \ge 0, x_{it} \ge 0, y_{ijt} \ge 0 \quad (3.14)$$

Constraints (3.8) – (3.14) are similar to constraints (3.1) – (3.7).

3.1.4) Model 4: Two-way, big bucket, uncapacitated

In this model it is allowed to produce more than one type of product in each period, therefore there is no changeover decision variable z_{it}. The model is similar to model 3, except that the variable z_{it} is replaced by the variable s_{it} in the objective function and it is now associated to a setup cost S_{it}. Also, constraints (3.10), (3.11) and (3.12) are removed.

Minimize

$$\sum_{t=1}^{N}\sum_{i=1}^{M} S_{it} * s_{it} + \sum_{t=1}^{N}\sum_{i=1}^{M} h_{it} * I_{it} + w * \sum_{t=1}^{N}\sum_{i=1}^{M}\sum_{j \epsilon P_i^+} y_{ijt}$$

Subject to constraints (3.8), (3.9), (3.13) & (3.14).

Note that this model continues to be subject to constraint (3.9) of model 3 with respect to the maximum production amount.

3.2) <u>Reformulation</u>

Pochet & Wolsey (2006) reformulate the uncapacitated lot-sizing problem using the facility location problem as follows. The setup variable s_{it} and setup cost S_{it} are kept unchanged, but the production variable x_{it} and inventory I_{it} are replaced by the variable

w_{iut}, with $u \leq t$. This variable represents the amount of product i produced in period u used to satisfy the demand of product i in period t. The cost to produce w_{iut} is p_{iut}, which represents the combination of production cost in period u plus the holding cost to keep w_{iut} from period u to period t.

The objective function is now to minimize the total setup cost and total cost to produce w_{iut} in all periods in the planning horizon. For the standard multi-products, uncapacitated lot-sizing problem, the reformulation is as follows:

Mimimize

$$\sum_{u=1}^{N}\sum_{t=u}^{N}\sum_{i=1}^{M} p_{iut} * w_{iut} \quad + \sum_{t=1}^{N}\sum_{i=1}^{M} S_{it} * s_{it}$$

Subject to

$$\sum_{u=1}^{t} w_{iut} = d_{it}, for\ 1 \leq \mathrm{t} \leq N, 1 \leq \mathrm{i} \leq \mathrm{M}$$

$$w_{iut} \leq d_{it} * s_{iu},\ for\ 1 \leq u \leq t \leq T, 1 \leq \mathrm{i} \leq \mathrm{M}$$

$$s_{it}\ binary,\ \ w_{iut} \geq 0$$

Next, we reformulate the previous models using this facility location problem. x_{it} and I_{it} are replaced by w_{iut} . In this thesis there is no production cost, therefore p_{iut} is the holding cost to keep w_{iut} from period u to period t.

3.2.1) Model 1: One-way substitution, small bucket and uncapacitated.

Minimize

$$\sum_{u=1}^{N}\sum_{i=1}^{M} K * z_{iu} + \sum_{t=1}^{N}\sum_{u=1}^{t}\sum_{i=1}^{M} p_{iut} * w_{iut} \quad + w * \sum_{t=1}^{N}\sum_{i=1}^{M}\sum_{j=i+1}^{M} y_{iju}$$

Subject to

$$\textstyle\sum_{u=1}^{t} w_{iut} + \sum_{k=1}^{i-1} y_{kit} = d_{it,} for\ 1 \leq t \leq N, 1 \leq i \leq M \qquad (3.15)$$

$$\sum_{t=u}^{N} w_{iut} \leq (\sum_{l=u}^{N} d_{il}) * s_{iu} , for\ 1 \leq u \leq N, 1 \leq i \leq M \quad (3.16)$$

$$\sum_{j=i}^{M} y_{iju} \leq (\sum_{j=i}^{M} d_{ju}) * s_{iu}\ for\ 1 \leq u \leq N, 1 \leq i \leq M \quad (3.17)$$

$$z_{iu} \geq s_{iu} - s_{i,u-1}, for\ 1 \leq u \leq N \quad (3.18)$$

$$\sum_{i=1}^{M} s_{iu} \leq 1, for\ 1 \leq u \leq N \quad (3.19)$$

$$z_{iu}\ binary \quad (3.20)$$

$$s_{iu}\ binary \quad (3.21)$$

$$w_{iut} \geq 0, y_{ijt} \geq 0 \quad (3.22)$$

Constraint (3.15) states that demand for product i in period t is satisfied by two ways: 1) the quantity of product i produced in period u to satisfy the demand of product i in period t and 2) the quantity of products that substitute for product i in period t. Constraints (3.16) - (3.22) are similar to constraints (3.2) – (3.7).

3.2.2) Model 2: one-way downward, big bucket and un-capacitated.

In this model it is allowed to produce more than one type of product in one period, therefore there is no changeover decision variable z_{it}. The model is similar to model 1, except that the variable z_{it} is replaced by the variable s_{it} in the objective function and is now associated with the setup cost S_{it}. Also, constraints (3.18), (3.19) and (3.20) are removed.

Minimize

$$\sum_{u=1}^{N}\sum_{i=1}^{M} S_{iu} * s_{iu} + \sum_{t=1}^{N}\sum_{u=1}^{t}\sum_{i=1}^{M} p_{iut} * w_{iut} + w * \sum_{t=1}^{N}\sum_{i=1}^{M}\sum_{j=i+1}^{M} y_{iju}$$

Subject to constraints (3.15), (3.16), (3.17), (3.21) & (3.22).

3.2.3) Model 3: Two-way substitution, small bucket and uncapacitated

In this model it is necessary to separate two set of products: P_i^+ is set of products that product i could substitute for, and P_i^- is the set of products that could substitute for product i.

Minimize

$$\sum_{u=1}^{N}\sum_{i=1}^{M} K * z_{iu} + \sum_{t=1}^{N}\sum_{u=1}^{t}\sum_{i=1}^{M} p_{iut} * w_{iut} \quad + w * \sum_{t=1}^{N}\sum_{i=1}^{M}\sum_{j \epsilon P_i^+} y_{iju}$$

Subject to

$$\sum_{u=1}^{t} w_{iut} + \sum_{k \epsilon P_i^-} y_{kit} = d_{it,}\, for\ 1 \le \mathrm{t} \le N, 1 \le \mathrm{i} \le \mathrm{M} \qquad (3.23)$$

$$\sum_{u=t}^{N} w_{iut} \le (\sum_{l=u}^{N} d_{il}) * s_{iu}\ , for\ 1 \le u \le N, 1 \le i \le \mathrm{M} \qquad (3.24)$$

$$\sum_{j \epsilon P_i^+} y_{iju} \le (\sum_{j \epsilon P_i^+} d_{ju}) * s_{iu}, for\ 1 \le u \le N, 1 \le i \le \mathrm{M} \qquad (3.25)$$

$$z_{iu} \ge s_{iu} - s_{i,u-1}, for\ 1 \le \mathrm{u} \le N \qquad (3.26)$$

$$\sum_{i=1}^{M} s_{iu} \le 1, for\ 1 \le \mathrm{u} \le N \qquad (3.27)$$

$$z_{iu}\ binary \qquad (3.28)$$

$$s_{iu}\ binary \qquad (3.29)$$

$$w_{iut} \ge 0, y_{ijt} \ge 0 \qquad (3.30)$$

Constraints (3.23) – (3.30) are similar to constraints (3.15) – (3.22).

3.2.4) Model 4: Two-way, big bucket, uncapacitated

In this model it is allowed to produce more than one type of product in one period, therefore there is no changeover decision variable z_{it}. The model is similar to model 5, except that the variable z_{it} is replaced by the variable s_{it} in the objective function and it is now associated with a setup cost S_{it}. Also, constraints (3.26), (3.27) and (3.28) are removed in this model.

Minimize

$$\sum_{t=1}^{N}\sum_{i=1}^{M} S_{it} * s_{it} + \sum_{t=1}^{N}\sum_{u=1}^{t-1}\sum_{i=1}^{M} p_{iut} * w_{iut} \quad + w * \sum_{t=1}^{N}\sum_{i=1}^{M}\sum_{j \epsilon P_i^+} y_{ijt}$$

Subject to constraints (3.23), (3.24), (3.25), (3.29) & (3.30).

CHAPTER 4:

TEST EXPERIMENTS

In this section two testbeds will be considered. The results of the four models with the two testbeds will be presented, and the differences in the results of the four models will also be mentioned.

4.1) Testbeds

Testbed 1 is similar to the testbeds tested in the original paper, and the purpose is to verify whether the conclusions in the original paper are valid in different lot-sizing problems. Testbed 2 is to verify whether the conclusions still hold true when there is a significant gap in the demand of different products. This testbed is new.

The two testbeds respect the order: holding cost < substitution cost < changeover cost as in the original paper. The purpose of this order is to ensure the three options of satisfying demand (inventory left over, product 1 substituting for product 2, and producing for product 1 and product 2) are not artificially excluded in the optimal solution.

For more information we refer to the original paper.

For example:

If substitution cost = changeover cost = 10, holding cost =1.

Demand 1 = demand 2 = 10, the number of periods = 10.

Option 1- 100% substitution: produce product 1 in all 10 periods and substitute 100% for product 2. Total cost is equivalent to total substitution cost only: Substitution cost x mean D2 x 10 periods = 10 x 10 x 10 = **1,000**

Option 2- 100% changeover: produce product 1 in the 1st period, then production is switched alternately between product 1 and product 2 in each period. Total cost = Substituting product 2 by product 1 in period 1 + (9 x changeover cost) + (5 periods inventory for product 1) + 4 periods inventory for product 2) = (10*10) + (9*10) + (5 * 1 * 10) + (4*1*10) = **280.**
When substitution cost = changeover cost = 10, clearly option 1 has higher cost than option 2, and this means there will be no substitution but only changeover. If substitution cost is less than changeover cost, for instance at 2, total cost for option 1 will be 200. The optimal solution would probably utilize both options substitution and changeover to satisfy demand.

+ **Testbed 1:** Base demand D_{i0} for product 1 is 20 and base demand D_{i0} for product 2 takes three value: 10, 20 and 40. For each period, mean demand is generated by the function $d_i = D_{i0}$ for each product and actual demand is generated by the function $d_{it} = \max(1, D_{i0} + SD_{i0}\xi_i)$, where i represents the product and ξ_i represents the standard normal variate. S takes three values 0.15, 0.5 and 1.15. Holding cost, substitution cost and changeover cost are time-invariant and the same for both products. For the first and second analysis, holding cost is fixed at 1 for both products. But for the third analysis, holding cost for product 2 takes three values 0.2, 0.8 and 1 while holding cost for product is still fixed at 1. Substitution cost takes two values, 2 and 8, and changeover cost takes four values, 10, 75, 150 and 300. (Notice that in big bucket lot-sizing problems, changeover cost is setup cost). The total combination of different parameter settings is 216 and for each combination, 10 instances are generated.

+ **Testbed 2:** In order to see how variance in demand impacts the results, testbed 2 is further classified into three sub-types. The first one, when the demand for product 1 is substantially higher than the demand for product 2. The second one, when the demand of the two products are equal. And finally the third one, when the demand for product 1 is much lower than the demand for product 2.

Holding cost is fixed at 1 for both products. The substitution cost and the changeover cost in this testbed only fall into two extremes: 8 & 10 and 2 & 300 for each of the three sub-types. Usually when the substitution cost is at the lowest value and the changeover cost is at the highest value, the rate of substitution will be high while the number of changeovers will be low. The result is reversed when the substitution cost is at the lowest value and the changeover cost is at the highest value. Therefore, the purpose of this analysis is to see if the conclusions in the original paper are still valid for this testbed.

The set of combinations is listed in table 4.1. The actual demand is generated in the same way as in testbed 1.

Combination	D1	D2	Substitution cost	Changeover cost
Testbed 2a) D1 > D2				
1	1000	10	8	10
2	1000	10	2	300
3	2000	20	8	10
4	2000	20	2	300
5	4000	40	8	10
6	4000	40	2	300
Testbed 2b) D1 = D2				
1	1000	1000	8	10
2	1000	1000	2	300
3	2000	2000	8	10
4	2000	2000	2	300
5	4000	4000	8	10
6	4000	4000	2	300
Testbed 2c) D1 < D2				
1	10	1000	8	10
2	10	1000	2	300
3	20	2000	8	10
4	20	2000	2	300
5	40	4000	8	10
6	40	4000	2	300

Table 4.1: Set of combinations for testbed 2

4.2) Results with test bed 1

All instances of this testbed are solved to optimality and the solution time is typically less than 10 seconds.

4.2.1) Results for analysis 1

In this part, first the results of the four models will be reviewed to see if they are consistent with the analysis 1 of the original paper. Next the results of the four models are compared to have more insights on the differences between small bucket models and big bucket model, between one-way downward substitution and two way substitution.

Notice that for model 1 and model 2, with one-way downward substitution, the *relative ratio* is exactly the same as in the original paper: $relative\ ratio = \frac{changover\ cost}{substitution\ cost\ x\ Demand\ 2}$.

However, for models 3 and 4, with two-way substitution, since both product 1 and product 2 could substitute each other, two *relative ratios* will be used:

$relative\ ratio\ 1 = \frac{changover\ cost}{substitution\ cost\ x\ \mathbf{Demand\ 2}}$ and $relative\ ratio\ 2 = \frac{changover\ cost}{substitution\ cost\ x\mathbf{Demand\ 1}}$

Besides, in order to see how variance in demand impacts the rate of substitution, for model 3 & 4 there will be three different substitution graphs corresponding to three cases: 1) demand 1 > demand 2, 2) demand 1< demand 2, and 3) demand 1 = demand 2. The relative ratio 1 and 2 will be used for case 1 and case 2 respectively. For case 3, it is possible to use either of the two ratios since the two ratios are equivalent in this case. Also, the rate that product 1 substituted by product 2 and the rate that product 2 substituted by product 1 are also shown separately for each case.

<u>Observation for each model</u>:

+ Model 1: result is consistent with the analysis 1 of the original paper: the higher the relative ratio, the higher the rate of substitution and the lower the number of changeover (figures 3.1 & 3.2).

+ Model 2:

- The setup behaviour is similar to the analysis 1: the higher the relative ratio is, the lower the number of setups (figure 3.4)

- However, the substitution behaviour is somewhat different: the substitution rate is almost 0% when the relative ratio is less than 1, it reaches its peak when the relative ratio is in the range around 3- 5, and decreases to around 10 – 20% when the relative ratio exceeds 5 (figure 3.3)

It will be easier to be explained in the following example:

- Instance 1: D1= 19, D2=10, Substitution cost = 2, setup cost = 75. Relative ratio is 3.75 and substitution rate is 40%. The number of setup for product 1 is 8 and the number of setup for product 2 is 4.
- Instance 2: D1= D2=8, Substitution cost = 2, setup cost = 300. Relative ratio is 18.75 and substitution rate is 10%. The number of setup for each product is 2.

In these models, it is required that if product i substitutes for product k a quantity q in period t, this quantity q must also be produced in period t and not in $t-1$. For the instance number 2, the number of setups for product 1 is 2 period, therefore product 1 could only substitute for product 2 maximum 2 periods. For the instance number 1, product 1 could substitute for product 2 in 8 periods. The rate of substitution, therefore, is higher for the instance 1 than the instance 2.

+ Model 3:

- The number of changeovers is consistent with the conclusion 1 of the original paper: the higher the relative ratio is, the lower the number of changeover (figures 3.7, 3.10 & 3.13)
- Similarly, the rate of substitution is also consistent with the conclusion 1 of the original paper: the higher the relative ratio is, the higher the rate of substitution. Besides, there is the tendency that the product with a higher demand will substitute for the product with a lower demand:
  - When demand 1 > demand 2 (figures 3.5 & 3.6): the rate that product 1 is substituted by product 2 is 0% while the rate that product 2 is substituted by product 1 could reach 100%. Also, the higher the *relative ratio 1* is, the higher the rate that product 2 is substituted by product 1.
  - When demand 1 < demand 2 (figures 3.8 & 3.9): the rate that product 1 is substituted by product 2 could reach 100% while the rate that product 2 is substituted by product 1 is 0%. Also, the higher the *relative ratio 2* is, the higher the rate that product 1 is substituted by product 2.
  - When demand 1 = demand 2 (figures 3.11 & 3.12): the rate that product 1 is substituted by product 2 and the rate that product 2 is substituted by product 1 are almost equal.

+ Model 4:

- The number of setups is consistent with the conclusion 1 of the original paper: the higher the relative ratio is, the lower the number of setup (figures 3.16, 3.19 & 3.22)
- However, substitution behaviour is somewhat different: the substitution rate is almost 0% when the relative ratio is less than 1, it reaches its peak when the relative ratio is in the range around 3- 5, and decreases to around 10 – 20% when the relative ratio exceeds 5. This is similar to model 2. Also, for the three cases when demand 1 > demand 2 (figures 3.14 & 3.15), when demand 1 < demand 2 (figures 3.8 & 3.9), and when demand 1 = demand 2 (figures 3.11 & 3.12): variance in demand seems to have little effect on the substitution rate. For example, when demand 1 > demand 2, there are instances of which product 2 is substituted by product 1 and at the same time product 1 is also substituted by product 2.

Comparison of the four models:

+ The substitution behaviour is not the same for small bucket models and big bucket models. The rate of substitution in small bucket models (model 1& 3, figures 3.1, 3.5, 3.6, 3.8, 3.9, 3.11 & 3.12) could reach 100% while in big bucket models (model 2 & 4, figures 3.3, 3.14, 3.15, 3.17, 3.18, 3.20 & 3.21) this figure reaches maximum only 40%. There are two reasons for this result. First, in these big bucket models there is a setup cost for each period of production, while for small bucket models, there is only a changeover cost when production is switched from one product to another. Second, there is the limitation that if product i substitutes for product k a quantity q in period t, this quantity q must also be produced in period t. Therefore if there is a limit on the number of setups, this would also lead to a limited substitution.

+ For small-bucket & two-way substitution model, there is a tendency that the product with a higher demand substitutes for the product with a lower demand. There is no setup cost associated with production for small bucket model, but only substitution cost and changeover cost. Therefore this tendency is understandable, since producing the product with a higher demand and substituting for the product with a lower demand will lower the cost for substitution compared to producing the product with a lower demand and substituting the product with a higher demand.

However, there is not this tendency for big-bucket & two-way substitution model, since there is a setup cost associated with production for big bucket model. The higher the setup cost, the more the optimal solution will restrict the number of setup. The rates of substitution of the two products now depend largely on the number of setups.

For example:

Instance 1: Demand 1= 20 > demand 2 = 10. Substitution cost =2, setup cost = 75.

Instance 2: Demand 1= 20 > demand 2 = 10. Substitution cost =2, setup cost = 150.

The number of period is 20.

For the instance 1, the number of setups for product 1 is 8, and the number of setup for product 2 is 4. Product 1 substitutes 25% the demand of product 2.

For the instance 2, compare the two solutions:

1) The setup cost is much higher, so the number of setup for each product is now 4. Product 2 substitutes 20% the demand for product 1. Product 2 is setup in periods 1, 6, 11 & 16 and substitute the demand for product 1 in these periods. For product 1, the setups are in periods 2, 7, 12 and 17. Total cost is then 2240.

2) If still let product 1 substitute for product 2: setup for product 1 in periods 1, 5, 9, 13 & 17. Product 2 will be substituted by product 1 in these periods. About product 2, setup will be in periods 2, 8 and 14. Total cost for this solution is 2260, which is higher than the first solution (2240).

Therefore when the number of setup is equal between the two products, the product with a lower demand will tend to substitute for the product with a higher demand in big bucket model.

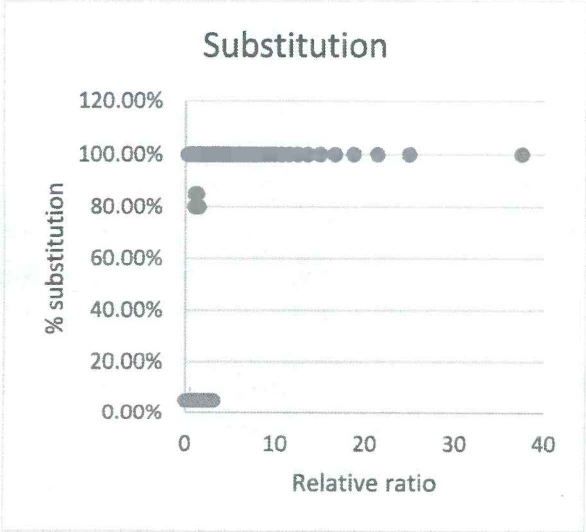


Figure 3.1 (Model 1- Small bucket, one-way substitution)

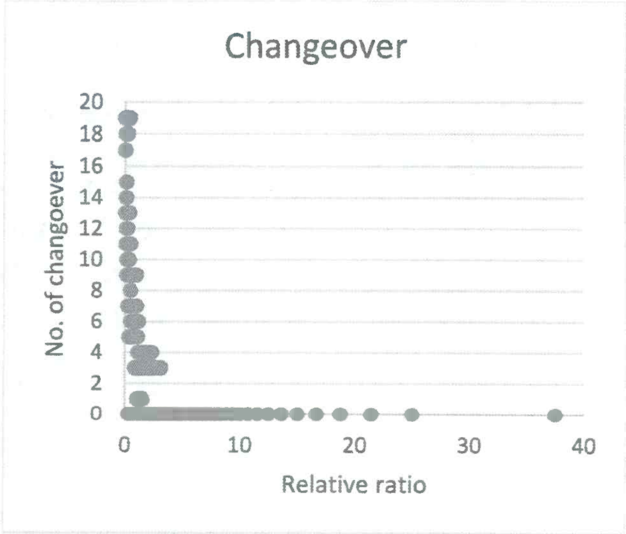


Figure 3.2 (Model 1- Small bucket, one-way substitution)

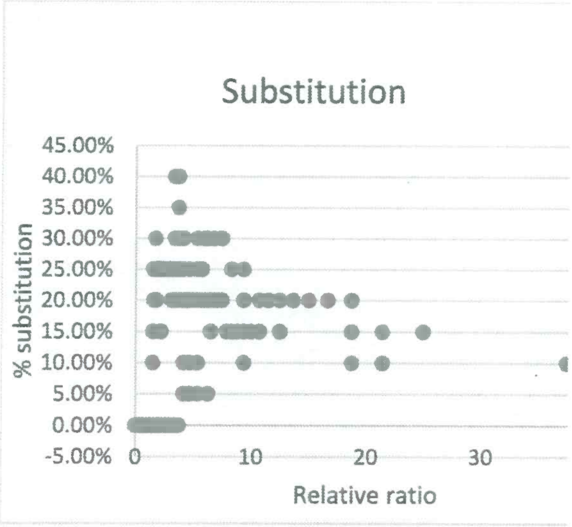


Figure 3.3 (Model 2- big bucket, one-way downward substitution)

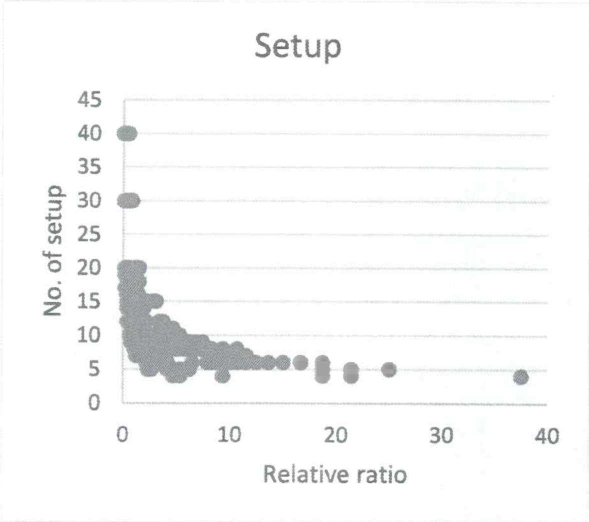
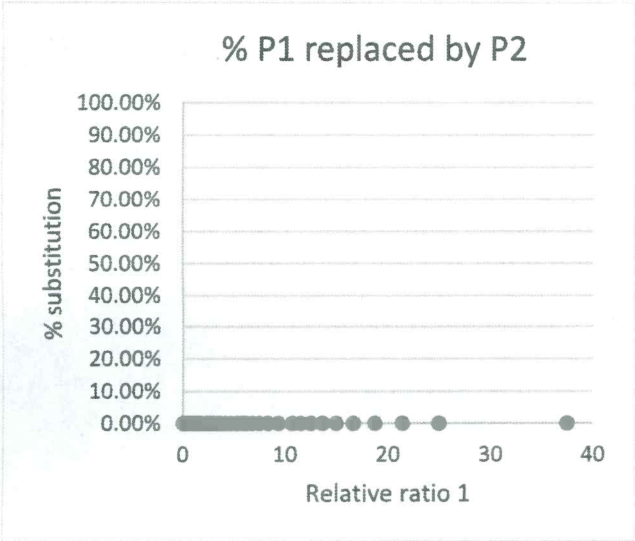


Figure 3.4 (Model 2- big bucket, one-way downward substitution)



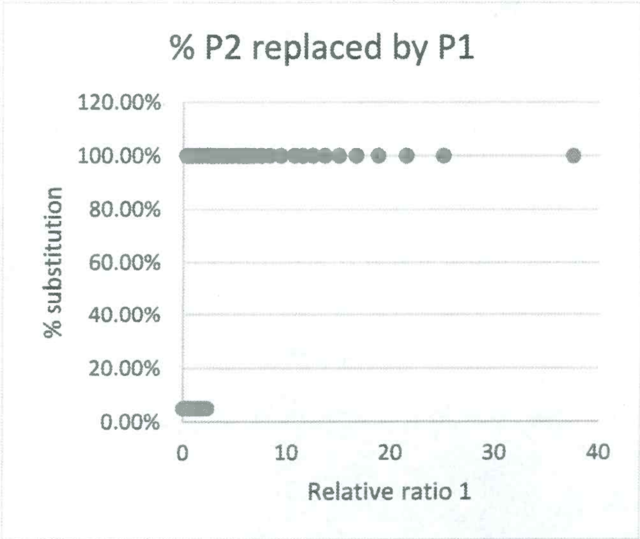


Figure 3.5 (Model 3- small bucket, two-ways substitution): Demand P1 > Demand P2

Figure 3.6 (Model 3- small bucket, two-ways substitution): Demand P1 > Demand P2

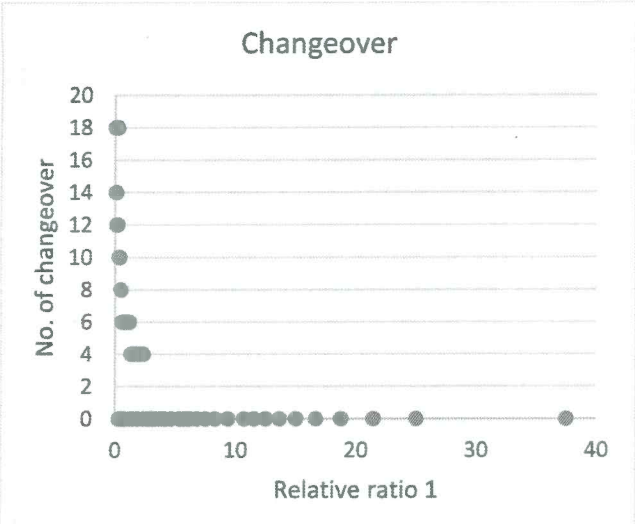


Figure 3.7 (Model 3- small bucket, two-ways substitution): Demand P1 > Demand P2

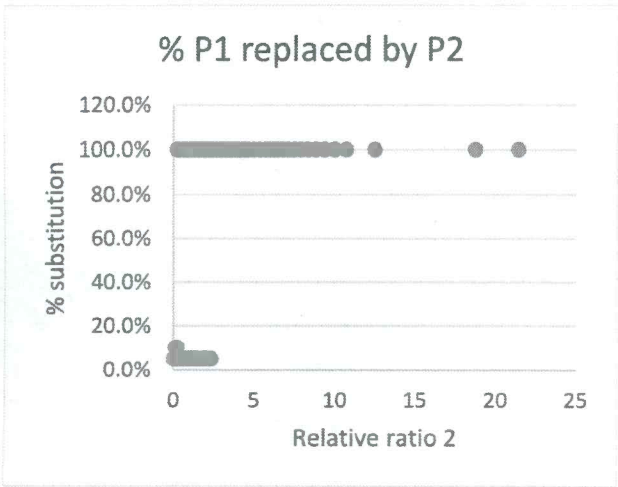


Figure 3.8 (Model 3- small bucket, two-ways substitution): Demand P1 < Demand P2

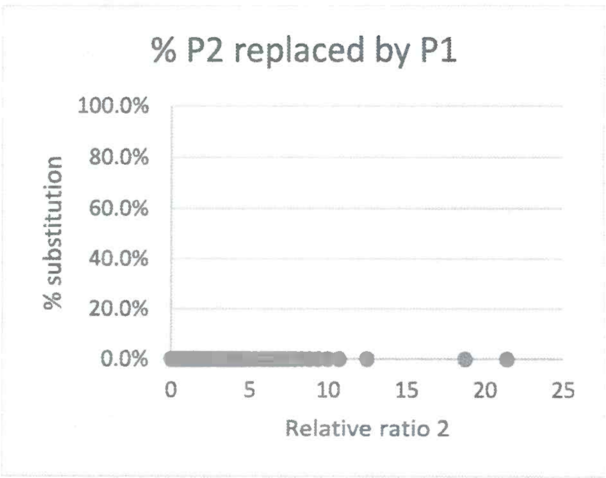


Figure 3.9 (Model 3- small bucket, two-ways substitution): Demand P1 < Demand P2

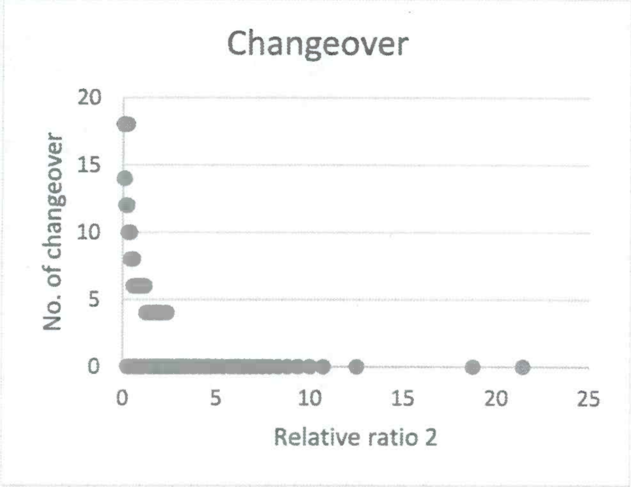


Figure 3.10 (Model 3- small bucket, two-ways substitution): Demand P1 < Demand P2

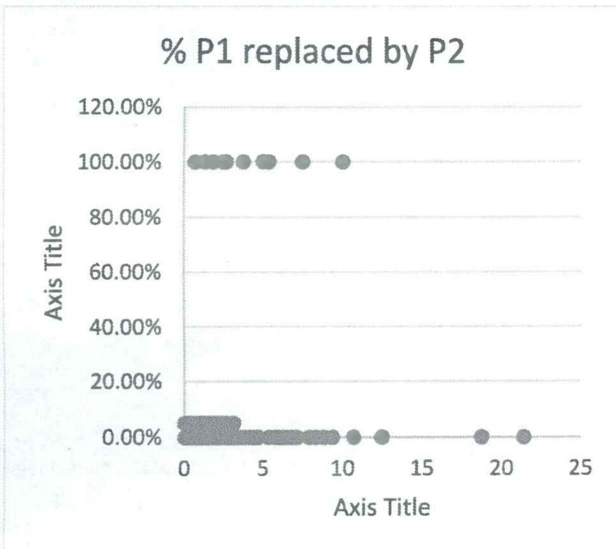


Figure 3.11 (Model 3- small bucket, two-ways substitution): Demand P1 = Demand P2

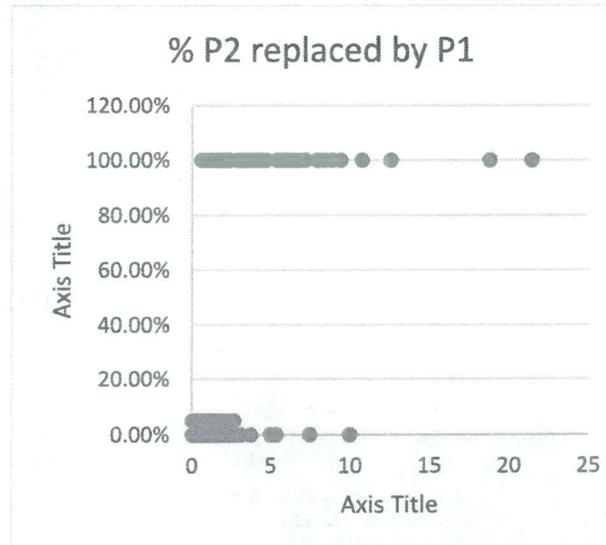


Figure 3.12 (Model 3- small bucket, two-ways substitution): Demand P1 = Demand

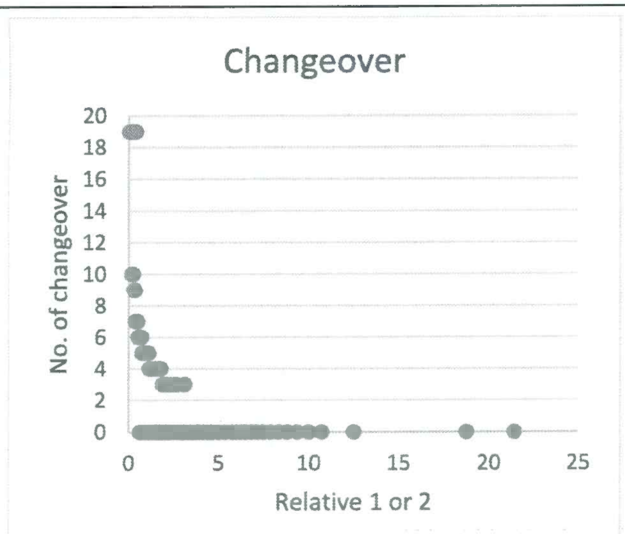


Figure 3.13 (Model 3- small bucket, two-ways substitution): Demand P1 = Demand P2

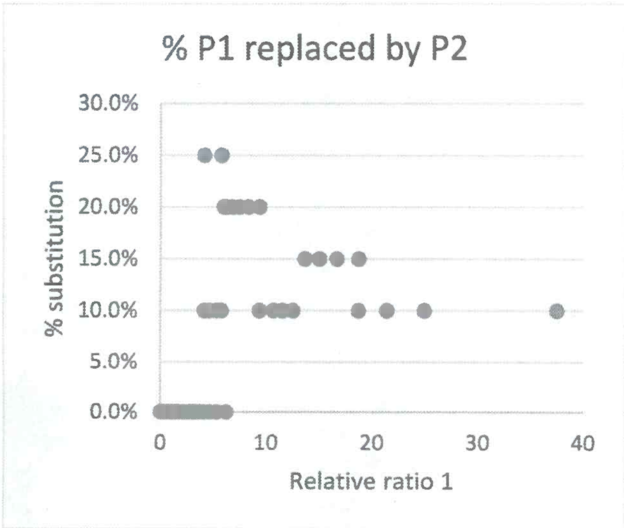


Figure 3.14 (Model 4- big bucket, two-ways substitution): Demand P1 > Demand P2

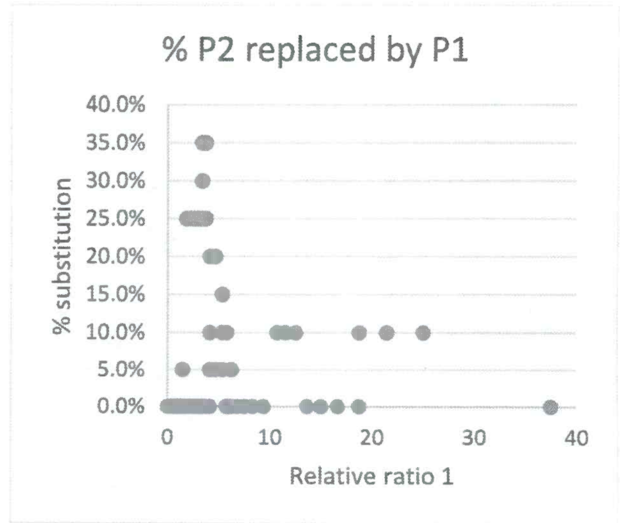


Figure 3.15 (Model 4- big bucket, two-ways substitution): Demand P1 > Demand P2

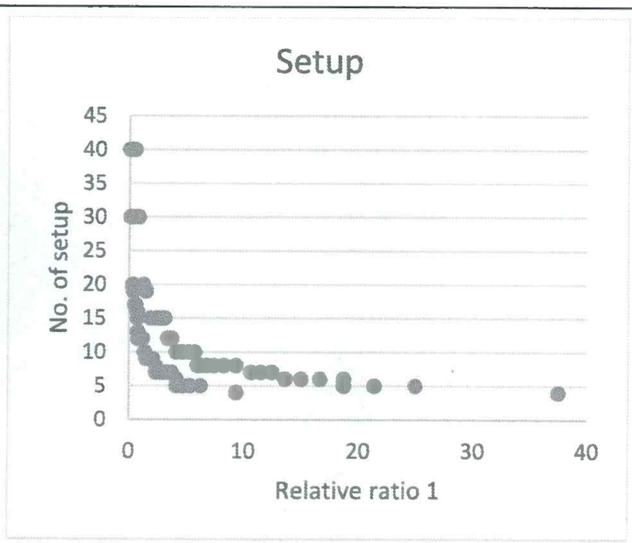


Figure 3.16 (Model 4- big bucket, two-ways substitution): Demand P1 > Demand P2

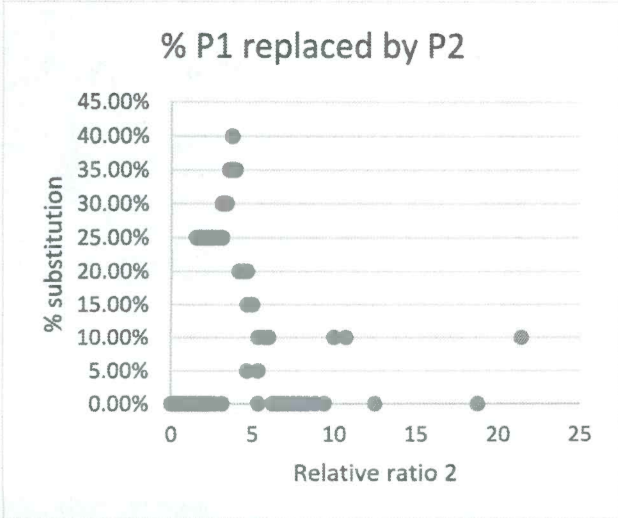


Figure 3.17 (Model 4- big bucket, two-ways substitution): Demand P1 < Demand P2

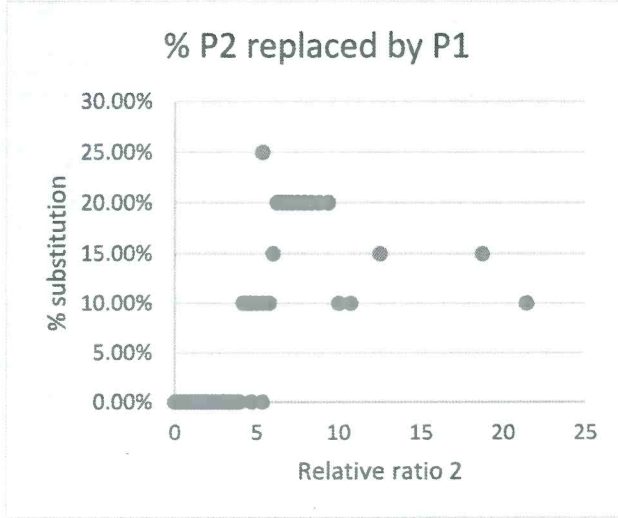


Figure 3.18 (Model 4- big bucket, two-ways substitution): Demand P1 < Demand P2

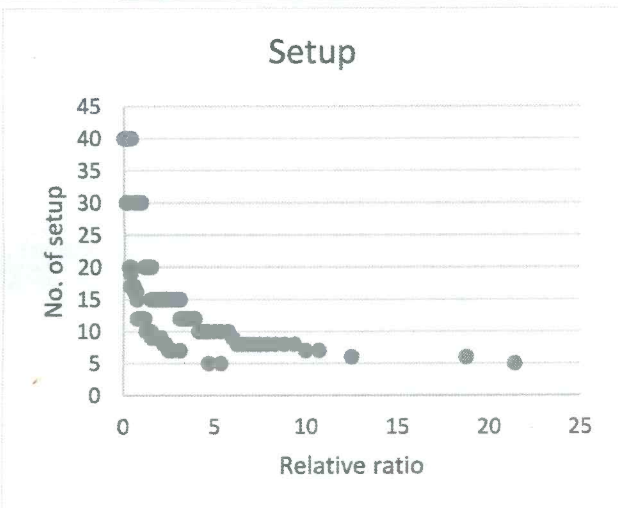


Figure 3.19 (Model 4- big bucket, two-ways substitution): Demand P1 < Demand P2

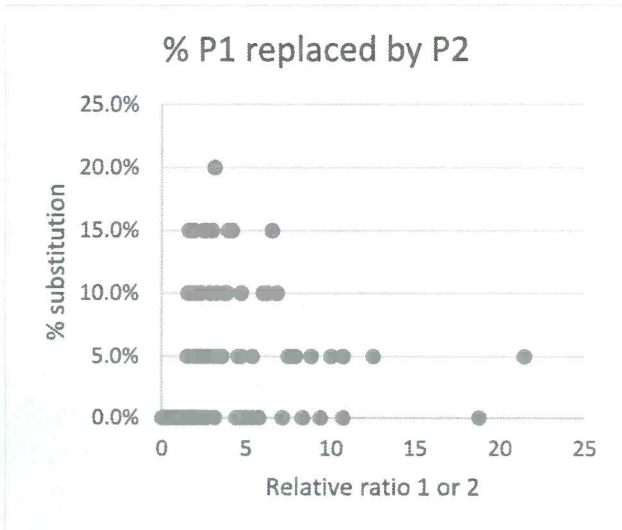


Figure 3.20 (Model 4- big bucket, two-ways substitution): Demand P1 = Demand P2

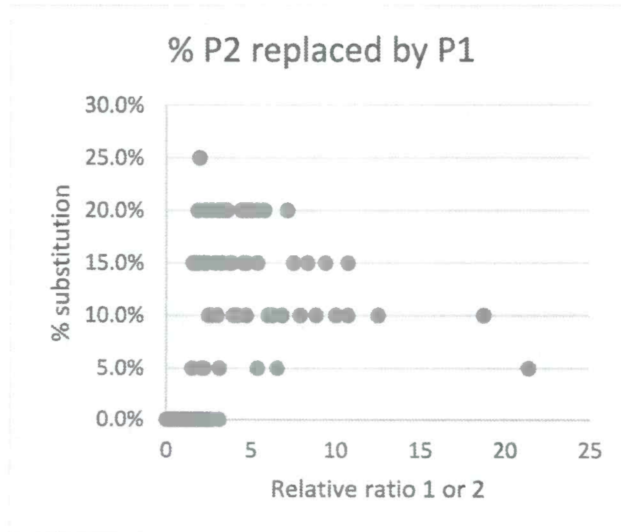


Figure 3.21 (Model 4- big bucket, two-ways substitution): Demand P1 = Demand P2

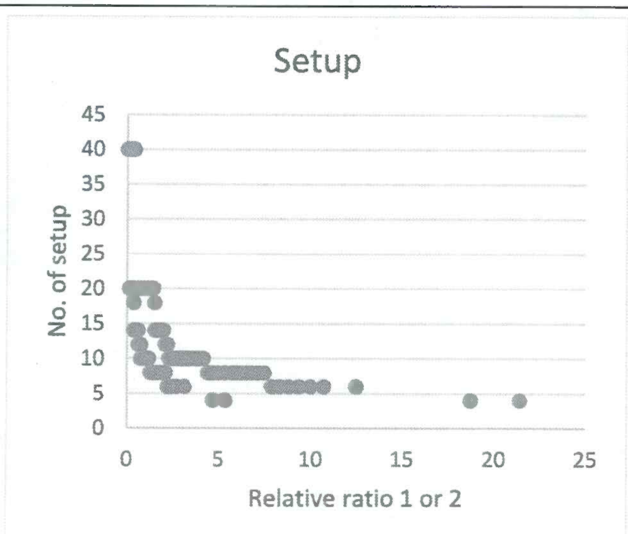


Figure 3.22 (Model 4- big bucket, two-ways substitution): Demand P1 = Demand P2

4. 2.2) Results for analysis 2

The purpose of this part is to see whether a reduction in the substitution cost or a reduction in the changeover cost will be more beneficial. The base case is when both the substitution cost and the changeover cost are at their highest value, 8 and 300. Then, first we calculate the total cost reduction due to a reduction in the substitution cost: fixing the changeover cost at 300 and decreasing the substitution cost to 2. Next the total cost reduction due to a reduction in the changeover cost is also calculated in similar manner: fixing the substitution cost at 8 and decreasing the changeover cost from 300 to 75. The total cost reduction is calculated over three base demand values of product 2 (10, 20 and 40) while the base demand for product 1 is kept at 20.

The results of the four models will be compared with the conclusion of the analysis 2. The differences among the four models will also be compared after.

Comparison with analysis 2:

Analysis 2 mentions that "When demand for product 2 increases from 10 to 25, the impact of substitution cost reduction decreases from 35.8% to 16.2%".

Model 1 (figure 3.2.3) shows a similar tendency, the benefit of substitution cost reduction decreases from around 73% to around 43% when the base demand for product 2 increases from 10 to 40.

Model 2 (figure 3.2.4) does not show the same result, the total cost reduction due to a reduction in the substitution cost increases slightly when the base demand for product 2 increases from 10 to 40.

Model 3 (figure 3.2.5) does not show the same result. The benefit of substitution cost reduction decreases from around 73% to around 65% when base demand 2 increases from 10 to 20, but it increases to 70% when demand 2 increases from 20 to 40.

Model 4 (figure 3.2.6) does not show the same result, total cost reduction due to a reduction in the substitution cost decreases slightly when demand 2 increase from 10 to 20, but it increases slightly when demand 2 increases from 20 to 40.

<u>Comparing the results of the four models</u>:

+ Small bucket models versus big bucket models

As it is already shown in part 2.1, results for analysis 1, substitution might be more important in small bucket models than in big bucket models (substitution rate reaches 100% in small bucket models while this figure is only maximum 40% in big bucket models). This observation would be again strengthened in this part. In big bucket models, figure 3.24 and figure 3.26 (model 2 & model 4) demonstrate that reducing the substitution cost does contribute to total cost reduction, yet this reduction is significantly overweighed by a reduction in setup cost. Conversely, in small bucket models, substitution cost reduction is more important: while a reduction in changeover cost helps to reduce the total cost from 35-40% in both model 1 & 3 (figures 3.23 & 3.25), a reduction in the substitution cost helps to reduce 45-70% the total cost for model 1 (figure 3.23) and 60%-75% the total cost for model 3 (figure 3.25).

Table 4.3 will help to explain more why reducing substitution cost is more important for small-bucket models and why reducing setup cost is more important for big-bucket models. The cost for substitution in model 1 and model 3 are 38.79% and 39.51% of the total cost respectively, and the cost for changeover are only 24.28% and 24.26% respectively. For model 2 and model 4, the cost for substitution is significantly lower than the cost for setup: 2.83 % versus 58.42% for model 2 and 3.16% versus 58.51 % for model 4. Since the cost of substitution constitutes a higher portion in the total cost than the cost for changeover, reducing the substitution cost will be more beneficial for small-bucket models. Similarly, since the cost for setup constitutes a higher portion in the total cost than the cost for substitution, reducing the setup cost will be more beneficial for big-bucket models.

Model	Cost for substitution (%)	Cost for changeover/setup (%)	Cost for inventory (%)	Total cost (value)
1	38.79%	24.28%	36.93%	860446
2	2.83%	58.42%	38.75%	1517370
3	39.51%	24.26%	36.23%	776885
4	3.16%	58.51%	38.33%	1513511

Table 4.2: Percentage of each cost in the total cost

+ One-way substitution models versus two-way substitution models

Comparing model 1 with model 3 (figure 3.2.3 and 3.2.5): the total cost reduction due to a reduction in the substitution cost are the same between model 1 and model 3 when demand 2 are 10 and 20. However, when demand 2 increases to 40, the total cost reduction due to a reduction in the substitution cost in model 1 is around 40%, while this figure is 70% in model 3. Recalling in the previous analysis (section 4.2.1 of this chapter), for small - bucket, two-way substitution model, there is a tendency that the product with a higher demand will substitute for the product with a lower demand. Therefore when demand 2 exceeds demand 1, two-way substitution model shows to be more beneficial than one-way substitution model since product 2 could substitute for product 1. In model 1, only product 1 could substitute for product 2, so when demand 2 begins to exceed demand 1, substitution will almost not happen, which means the substitution cost reduction will become less important.

Comparing model 2 with model 4 (figure 3.2.4 and 3.2.6): Total cost reduction due to a reduction in the substitution cost in model 4 is slightly higher than in model 2 when demand 2 is 10 and 40. The value of two-way substitution in big bucket model is also demonstrated, though it is far lower than in small bucket model.

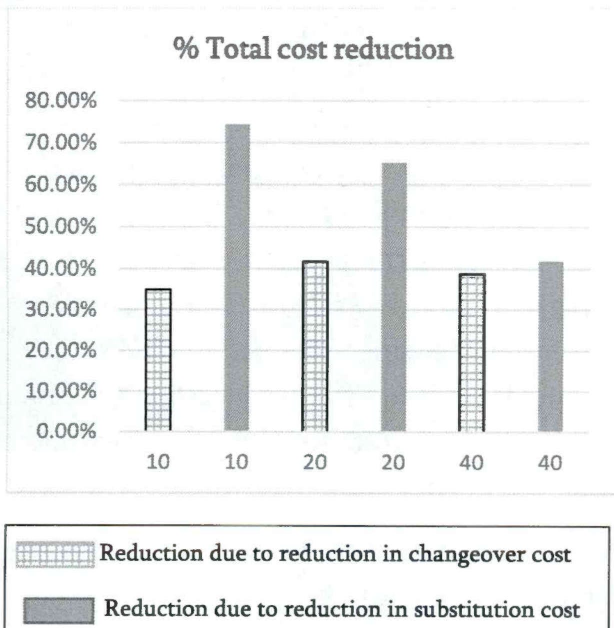


Figure 3.23- Model 1 (small bucket, one-way downward)

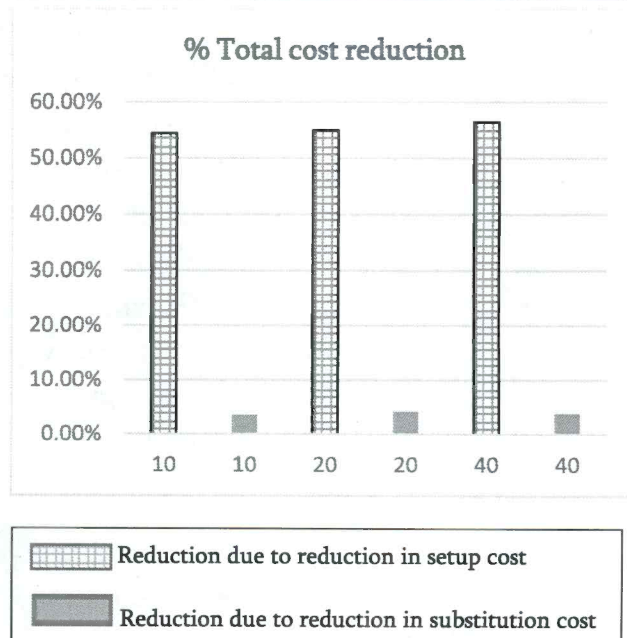


Figure 3.24- Model 2 (big bucket, one-way downward)

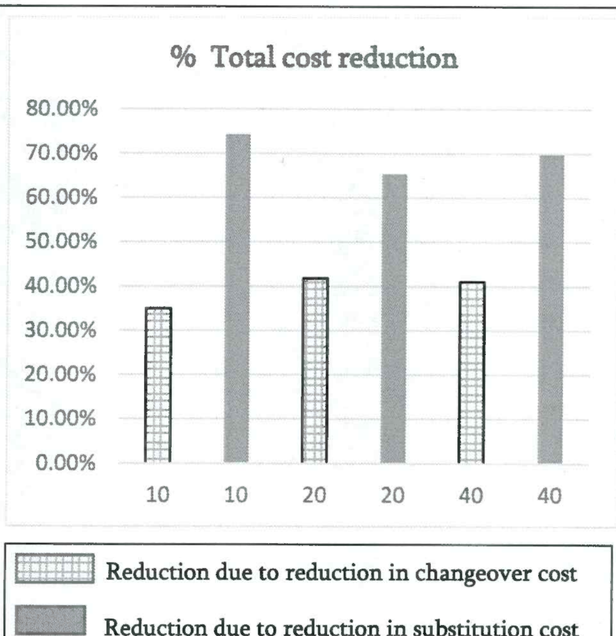


Figure 3.25- Model 3 (small bucket, two-way substitution)

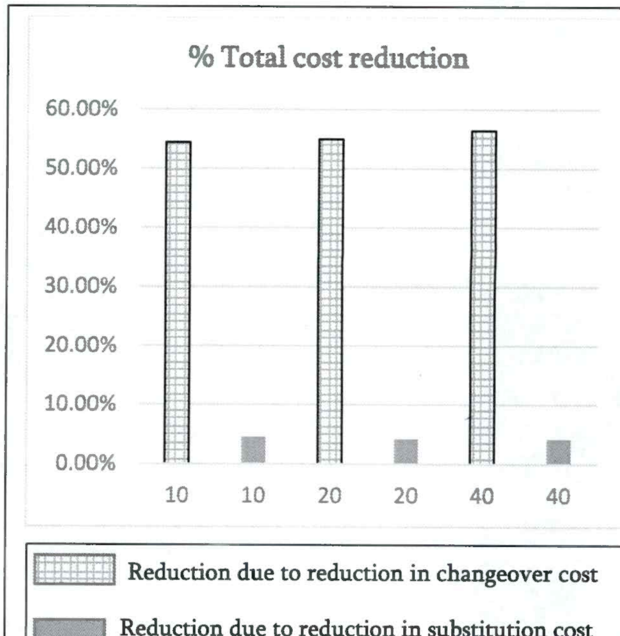


Figure 3.26- Model 4 (big bucket, two-way substitution)

4.2.3) Results for analysis 3

The results with the test experiments over four models will be compared with the two conclusions in analysis 3 of the original paper: 1) When holding cost for product 2 increases, both changeover and substitution increase and 2) When holding cost for product 2 increases, percentage increase in changeover is significantly higher than increase in substitution when changeover cost is low or substitution cost is high. The result is reversed when changeover cost is high and substitution cost is low.

Comparison with the first conclusion:

In order to see how the holding cost of product 2 impacts the substitution rate and the number of changeovers, the holding cost for product 1 is fixed at 1, the holding cost of product 2 is varied from 0.2 to 0.8, and all the other parameters stay the same. Overall the four models have the same result: when the holding cost of product 2 increases from 0.2 to 0.8, both substitution and changeover increase. The details are shown in the following table:

Model	Holding cost	% substitution: P1 substitute P2	% substitution: P2 substitute P1	No. of changeover or setup	% change in the number of changeover or setup
1	0.2	21.32%	0.00%	2680	
1	0.8	35.57%	0.00%	3807	42.05%
2	0.2	0.27%	0.00%	9034	
2	0.8	4.72%	0.00%	11556	27.92%
3	0.2	10.05%	14.04%	2288	
3	0.8	18.81%	15.24%	3533	54.41%
4	0.2	0.13%	0.34%	8933	
4	0.8	1.24%	0.52%	11500	28.74%

Table 4.3: % change in substitution and changeover

Comparison with the second conclusion:

Next we further investigate how changeover/setup and substitution increase by separating two pairs of parameter: substitution cost and changeover cost/setup cost at 8 & 10 and 2 & 300. The holding cost for product 1 is still at 1 and the holding cost for product 2 varies from 0.2 to 0.8. Also, for each of these pairs of parameter, the increase is calculated separately over three base demand 2 values 10, 20 and 40. We provide the percentage increase in the substitution rate and the number of changeover/setup when the holding cost for product 2 varies from 0.2 to 0.8.

Models 1, 3 and 4 shows that the results are consistent with analysis 3 in the original paper: when holding cost of product 2 increase from 0.2 to 0.8, if the substitution cost is low and the changeover cost is high, % increase in substitution will be higher than % increase in changeover and vice versa.

- For figures 3.27, 3.31 & 3.33, the substitution cost is at the highest value while the changeover cost/setup cost is at the lowest value, the percentage increase in changeover/setup is significantly higher than the percentage increase in substitution.
- For figures 3.28, 3.32 & 3.34, the substitution cost is at the lowest value while the changeover cost/setup cost is at the highest value, the percentage increase in substitution is significantly higher than the percentage increase in changeover/setup.

However, for the model 2, the result is a little different. When the substitution cost is at the highest value while the setup cost is at the lowest value (figure 3.29), the percentage increase in setup is significantly higher than the percentage increase in substitution. And when the substitution cost is at the lowest value while the setup cost is at the highest value, the percentage increase in substitution is also lower than the percentage increase in setup.

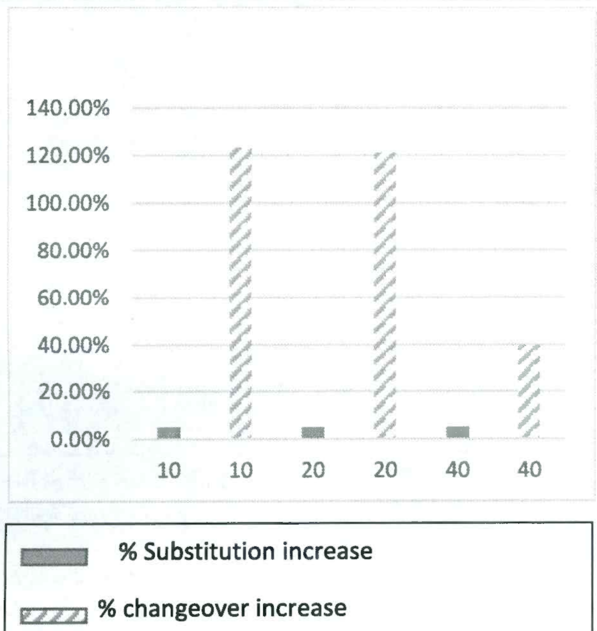


Figure 3.27- Model 1, substitution cost-changeover cost at 8-10

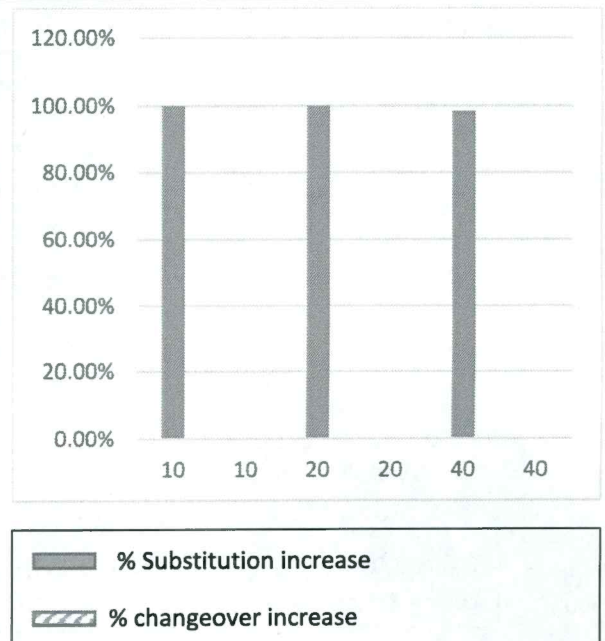


Figure 3.28- Model 1, substitution cost-changeover cost at 2-300

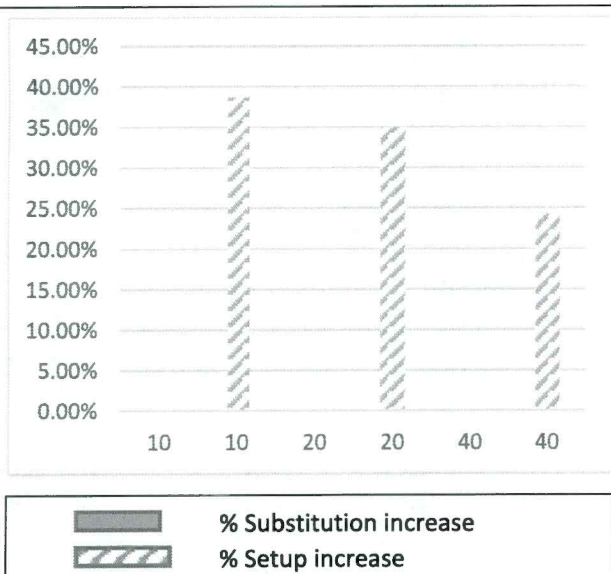


Figure 3.29- Model 2, substitution cost-setup cost at 8-10

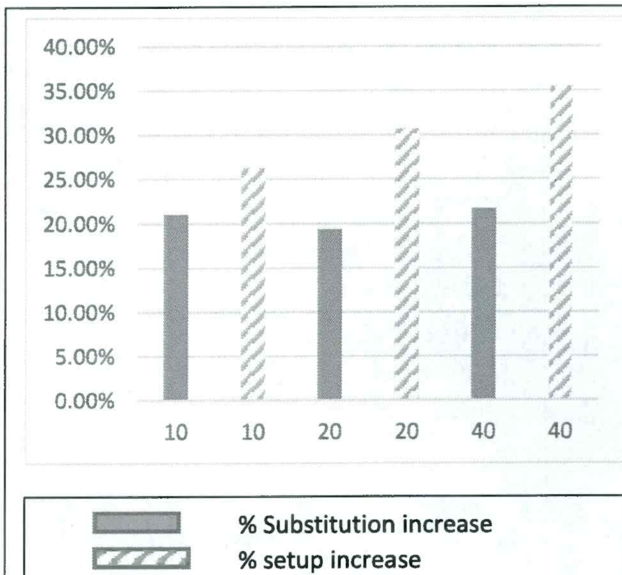


Figure 3.30- Model 2, substitution cost-setup cost at 2-300

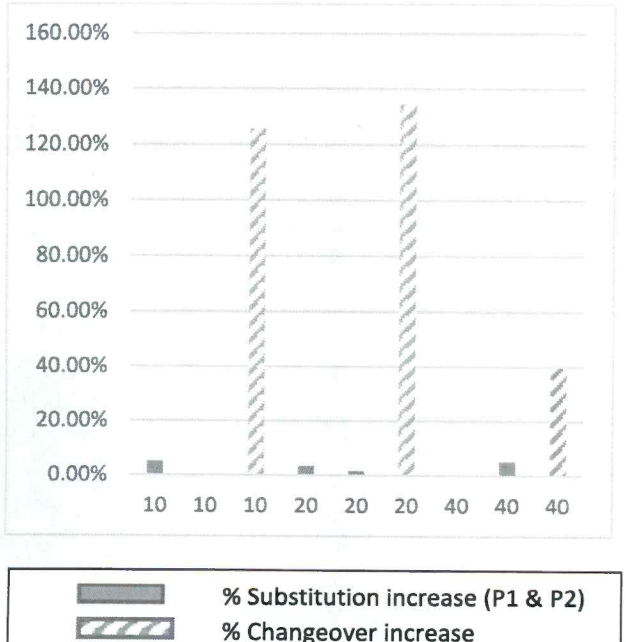


Figure 3.31- Model 3, substitution cost-changeover cost at 8-10

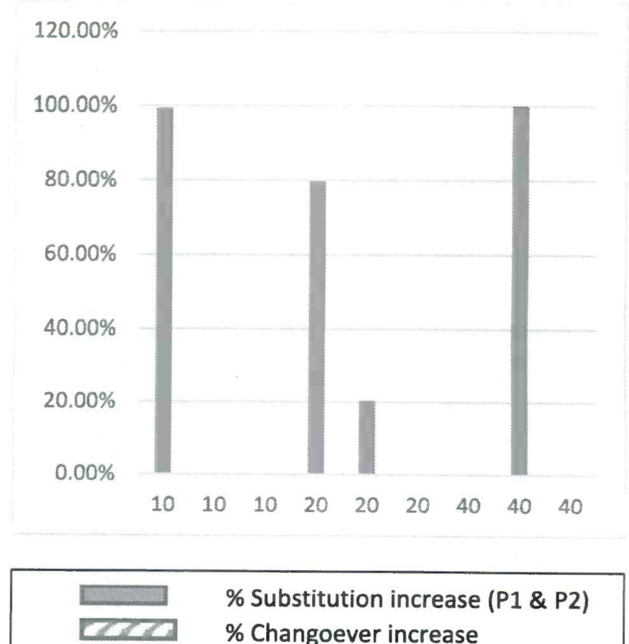


Figure 3.32- Model 3, substitution cost-changeover cost at 2-300

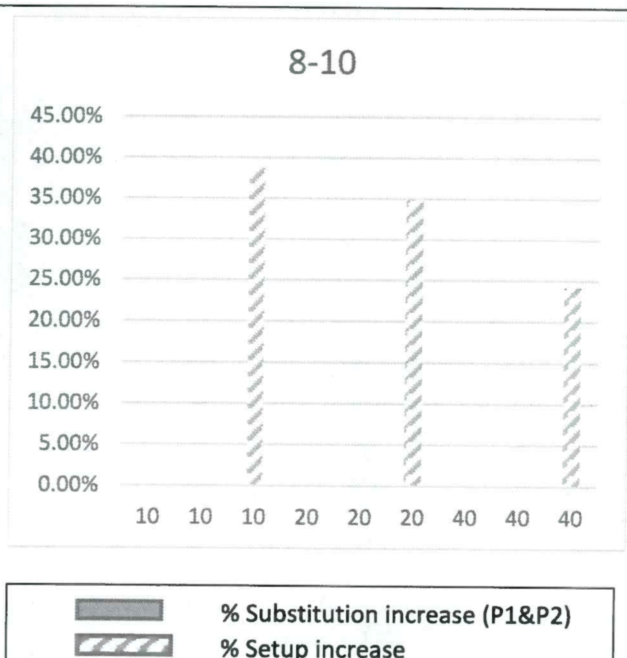


Figure 3.33- Model 4, substitution cost-setup cost at 8-10

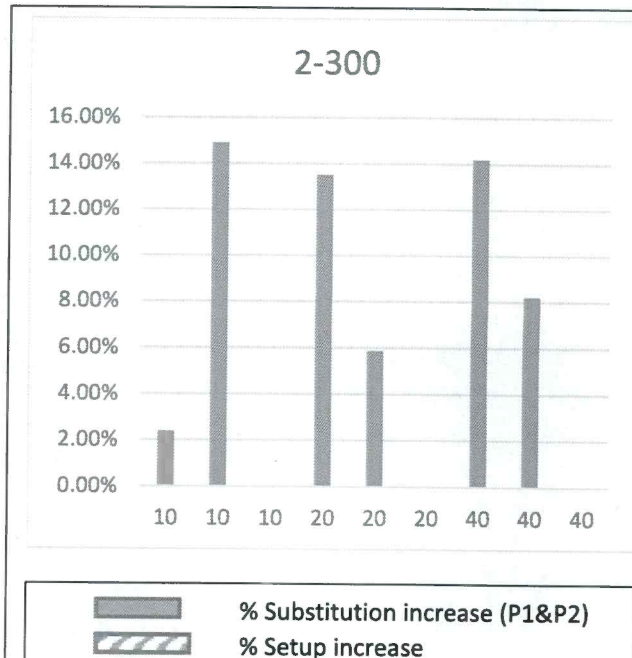


Figure 3.34- Model 4, substitution cost-s cost at 2-300

4.2.4) Comparing the total costs of the four models

In this part the benefits of two-way substitution is demonstrated through the comparison in total cost between two pairs: model 1 minus model 3 and model 2 minus model 4. As shown in table 3, certain instances of model 3 have lower cost than those of model 1. The result is similar between model 2 and model 4 (table 4).

No.	D1	D2	Scost	Ccost	Total cost Model 1	Total cost Model 3	**Cost for model 1 minus Cost for model 3**
1	20	10	2	10	12297	12297	**0**
2	20	10	8	10	16011	16011	**0**
3	20	10	2	75	12360	12360	**0**
4	20	10	8	75	31312	31307	**5**
5	20	10	2	150	12360	12360	**0**
6	20	10	8	150	40838	40838	**0**
7	20	10	2	300	12360	12360	**0**
8	20	10	8	300	48145	48145	**0**
9	20	20	2	10	18157	18157	**0**
10	20	20	8	10	22005	22005	**0**
11	20	20	2	75	24560	24560	**0**
12	20	20	8	75	41323	41323	**0**
13	20	20	2	150	24560	24560	**0**
14	20	20	8	150	53647	53647	**0**
15	20	20	2	300	24560	24560	**0**
16	20	20	8	300	70859	70859	**0**
17	20	40	2	10	24848	23588	**1260**
18	20	40	8	10	32342	27796	**4546**
19	20	40	2	75	43198	24560	**18638**
20	20	40	8	75	51750	48013	**3737**
21	20	40	2	150	48366	24560	**23806**
22	20	40	8	150	65255	62214	**3041**
23	20	40	2	300	49198	24560	**24638**
24	20	40	8	300	84593	81310	**3283**

Table 4.4- difference in total cost between model 1 and model 3

No.	D1	D2	Scost	Setup cost	Total cost Model 2	Total cost Model 4	**Cost for model 2 minus Cost for model 4**
1	20	10	2	10	11184	11184	**0**
2	20	10	8	10	11200	11200	**0**
3	20	10	2	75	44712	44625	**87**
4	20	10	8	75	45947	45947	**0**
5	20	10	2	150	66250	65668	**582**
6	20	10	8	150	68534	68534	**0**
7	20	10	2	300	97281	96215	**1066**
8	20	10	8	300	100818	100804	**14**
9	20	20	2	10	11404	11404	**0**
10	20	20	8	10	11420	11420	**0**
11	20	20	2	75	51907	51907	**0**
12	20	20	8	75	52260	52260	**0**
13	20	20	2	150	75957	75957	**0**
14	20	20	8	150	78574	78574	**0**
15	20	20	2	300	111072	111072	**0**
16	20	20	8	300	116002	116002	**0**
17	20	40	2	10	11454	11454	**0**
18	20	40	8	10	11470	11470	**0**
19	20	40	2	75	59259	58649	**610**
20	20	40	8	75	59405	59405	**0**
21	20	40	2	150	89865	88700	**1165**
22	20	40	8	150	91298	91298	**0**
23	20	40	2	300	130914	130594	**320**
24	20	40	8	300	136303	136288	**15**

Table 4.5- difference in total cost between model 2 and model 4

Comparison between model 1 & 3 (table 3): there is no difference between model 1 and model 3 when demand 2 is less than or equal to demand 1 (from cases 1- 16). However, when demand 2 is higher than demand 19 (from cases 17 – 24), the total cost in model 3 is much lower than the total cost in model 1, especially when the substitution cost is low and the changeover cost is high (cases 19, 21 and 23). This point could be interpreted as follows. When demand 1 is higher than demand 2, product 1 tends to substitute for product 2. Therefore although model 3 has the characteristics that product 2 could also substitute for product 1, this characteristics is not exploited. But when demand 2 is higher than demand 1, product 2 could substitute for product 1, and this is the case when model 3 shows to be more beneficial than model 1, the one-way substitution.

Comparison between model 2 and 4 (table 4): the total cost in model 4 is lower than the total cost in model 2 when the substitution cost is low and the setup cost is higher than 10 (cases 3, 5, 7, 19, 21, 23). This implies that substitution is mainly exploited when substitution cost is low in big bucket model.

It is noted that model 4 has a lower cost than model 2 in cases 3, 5 & 7 even when demand 1 is higher than demand 2. This is not the same for small-bucket, two-way substitution model and the reason is as follows. Model 4 is big-bucket, two-way substitution model, so the product with the higher demand will not tend to substitute for the product with a lower demand as in model 3. However, since both products 1 & 2 could substitute each other in model 4, the optimal solution will exploit this characteristics to let product 2 substitute for product 1 in certain periods (even when demand 1 is higher than demand 2), especially for instances with high setup cost. However, when demand 1 and demand 2 are equal, there is no benefit in allowing two-way substitution.

Parts 2.1, 2.2, 2.3 & 2.4 in this section all illustrate that for small bucket and one-way substitution model, the difference among the demand levels should be an important criterion to look at, not the substitution cost or the changeover cost (this is already explained in section 1.2.2, figure 1.7 and it will be explained again in section 4.3). This finding is not exactly similar to the conclusions in the original paper, which always look at the substitution cost and the changeover cost at the first place. Therefore in the next section, analysis 1 and analysis 2 are again tested with different testbeds which have significant gap between demand 1 and demand 2.

4.3) Result with test bed 2

As mentioned in the previous sections, the purpose of this testbed is to verify if the conclusions 1 and 2 in the original paper still hold when there is big gap between demand 1 and demand 2. Three results with three test beds will be shown from figures 3.35 – 3.64.

+ Results with the testbeds

Result with testbed 2a, D1>D2

For model 1, figure 3.35 & 3.36: the result is completely different from the analysis 1: the rate of substitution is always 100% and the number of changeovers is always 0 irrespective of the value of the relative ratio.

For model 2, figure 3.37 & 3.38: the result is still consistent with the analysis 1. The higher the relative ratio is, the lower number of setup and the higher rate of substitution.

Model 3 (figures 3.39, 3.40, & 3.41): has similar results as model 1

Model 4 (figures 3.42, 3.43 & 3.44) has similar results as model 2.

Result with testbed 2b, D1 < D2 and 2c, D1 = D2

According to the original paper, these two set of instances will have the same relative ratio, and therefore in many cases they will have similar rate of substitution and similar number of changeovers or setups. However, the results shown below are very different:

Model 1: figures 3.45 & 3.46 show a similar rate of substitution between the two set of instances, but figures 3.47 & 3.48 show very different number of changeovers. For the set D1 < D2, the number of changeovers is most of time only at 1, but for D1 = D2, the number of changeover is in the range from 10 -20.

Model 2: figures 3.49 – 3.52 show that the result still follows the conclusion according to analysis 1 of the original paper.

Model 3 (figures 3.53 – 3.58) have similar results as model 1, and model 4 (figures 3.59 – 3.64) have similar results with model 2.

+ Conclusion:

The results with these testbeds show that for small-bucket models, variance in demand is very important. When demand 1 is much higher than demand 2, substitution rate is still almost at 100% even for instances with the substitution cost at the highest value (at 8) and the changeover cost at the lowest value (at 10). When demand 1 is much lower than demand 2, changeover rarely happens even for instances with the substitution cost at the highest value (at 8) and the changeover cost at the lowest value (at 10). And when demand 1 is equal to demand 2, changeover happens frequently even for instances with the substitution cost at the lowest value (at 2) and the changeover cost at the highest value (at 300).

Noted that these three testbeds still follow the rule that holding cost < substitution cost < changeover cost.

The result is quite different for big bucket models: the results for models 2 and 4 in testbed 1 and testbed 2 are still quite consistent.

The main difference between small bucket model and big bucket model is that there is only one product produced in one period in small bucket model, while it is possible that two or more products are produced in one period in a big bucket model. This means that there is always inventory in small bucket model, or there is always the holding cost. But for big bucket model, there might or might not be inventory, or there might or might not the holding cost.

The *relative ratio* (*changeover cost /substitution cost x* ***mean demand 2***) proposed implies that, the decision to switch production between product 1 and product 2 or substitute product 2 with product 1 depends on whether this *relative ratio* is small or high: if it is small, it means that switching production between the two products is cheaper than substituting product 2 with product 1; vice versa, substitution is a better option. For big bucket models, this ratio might be still valid since there might be no holding cost. However for small bucket, one-way downward model, there is always the holding cost besides the changeover cost. Therefore if there is significant gap in demand between the two products, holding cost will account for a much larger portion in the total cost compared to changeover cost and substitution cost. For this reason the *relative ratio* suggested in the original paper might not yield correct results when there is a big gap in demand between product 1 and product 2.

- <u>Testbed 1a (D1>D2)</u>

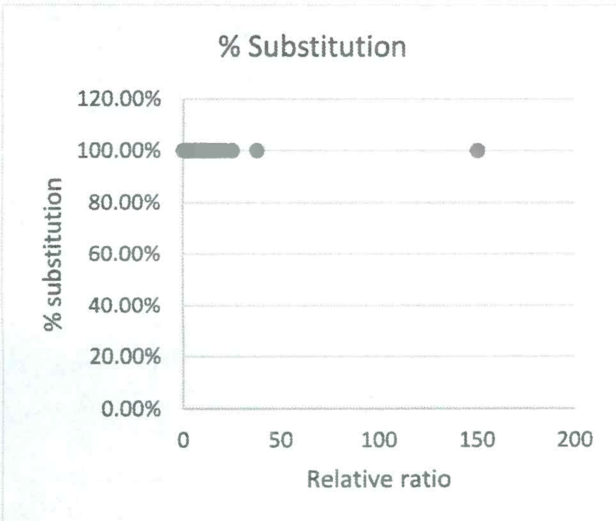


Figure 3.35- Model 1, D1>D2

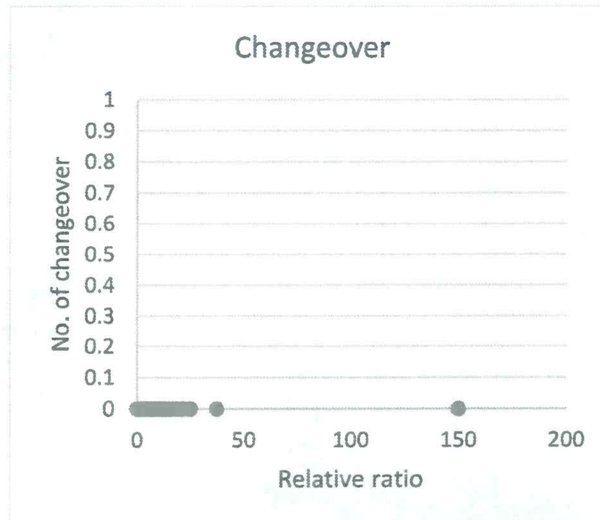


Figure 3.36- Model 1, D1>D2

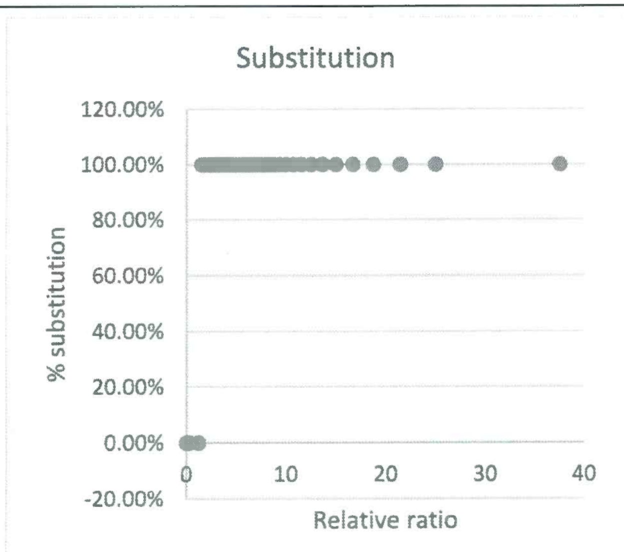


Figure 3.37- Model 2, D1>D2

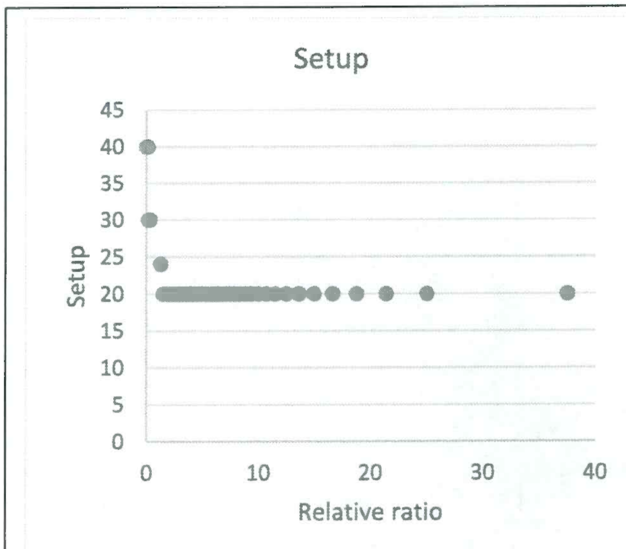


Figure 3.38- Model 2, D1>D2

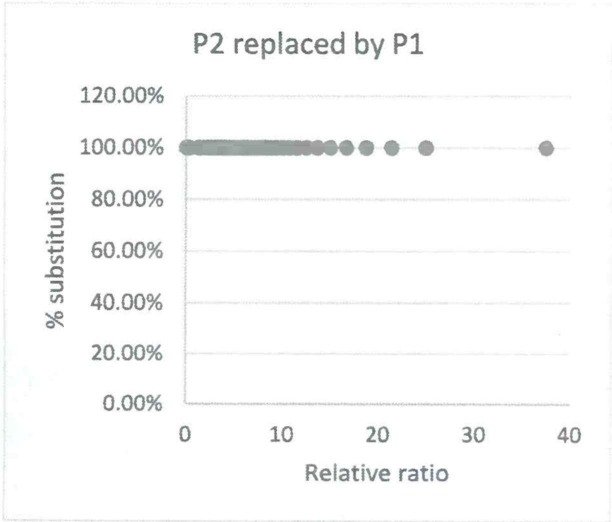


Figure 3.39- Model 3, D1>D2

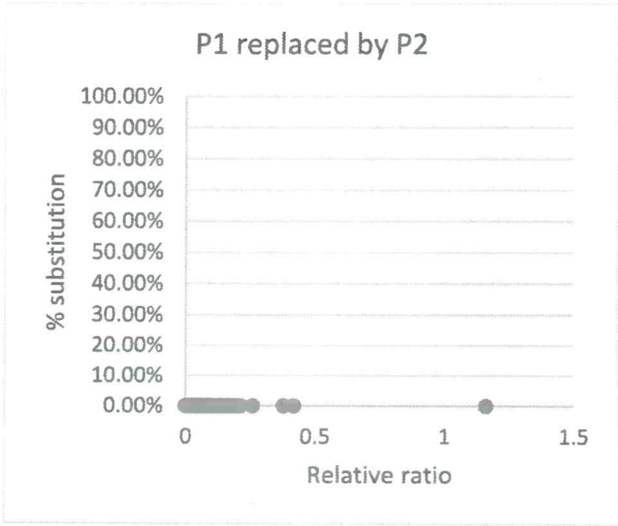


Figure 3.40- Model 3, D1>D2

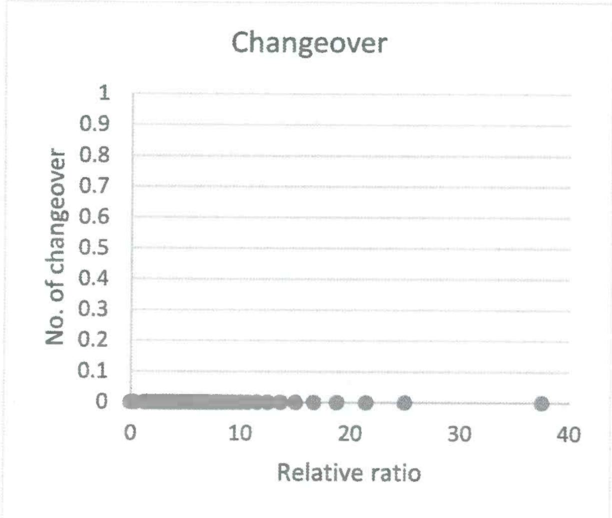


Figure 3.41- Model 3, D1>D2

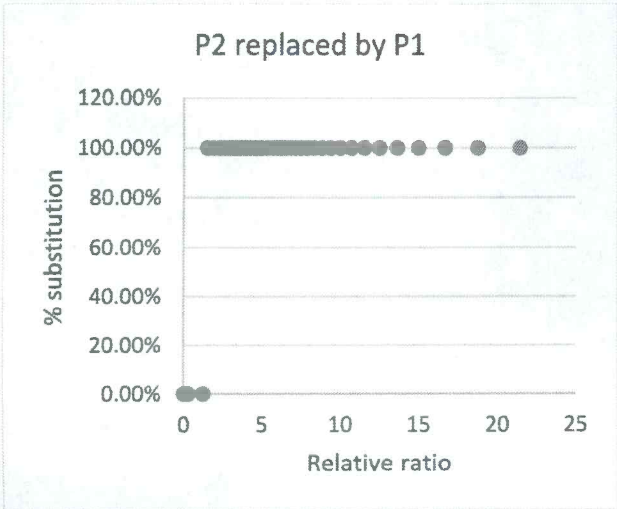


Figure 3.42- Model 4, D1>D2

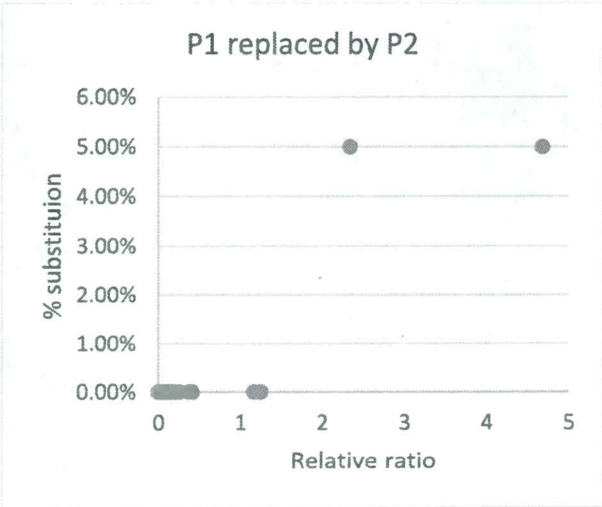


Figure 3.43- Model 4, D1>D2

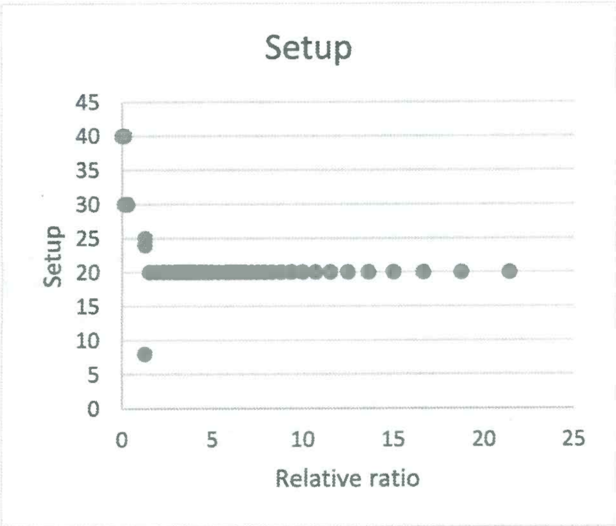


Figure 3.44- Model 4, D1>D2

- <u>D1 = D2 & D1 < D2</u>

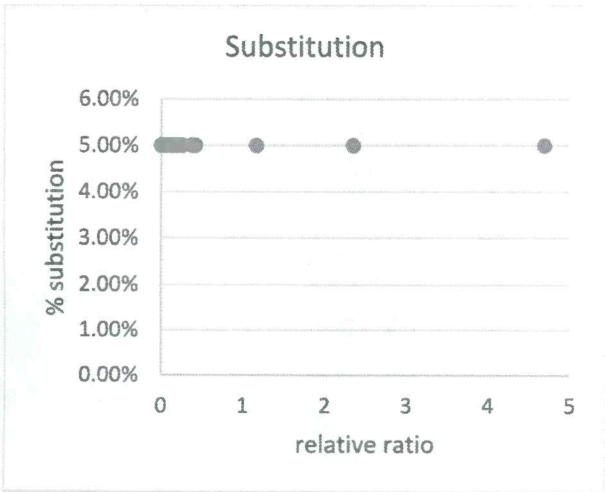


Figure 3.45- Model 1, D1<D2

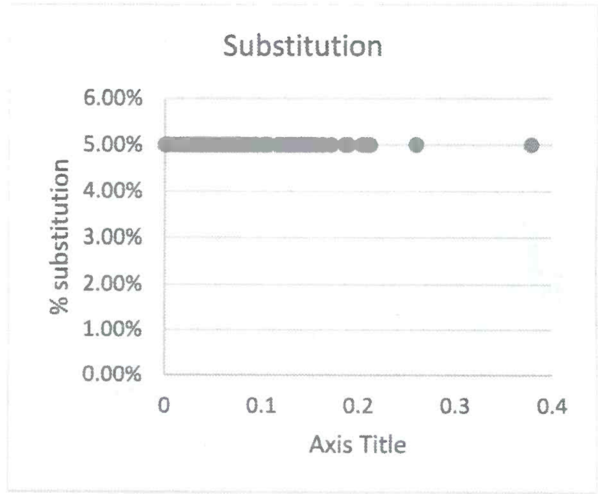


Figure 3.46- Model 1, D1=D2

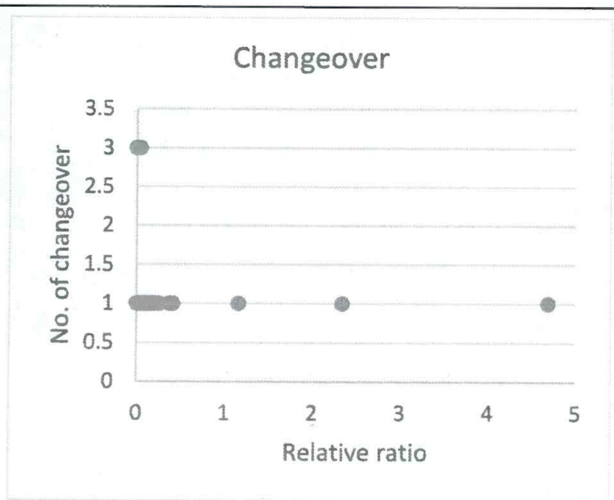


Figure 3.47- Model 1, D1<D2

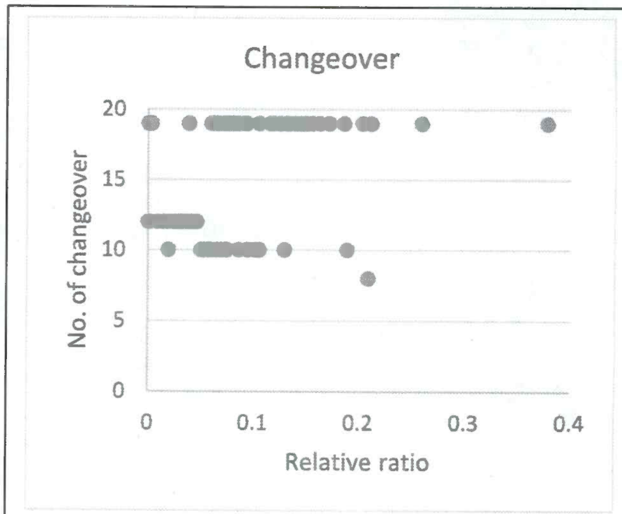


Figure 3.48- Model 1, D1=D2

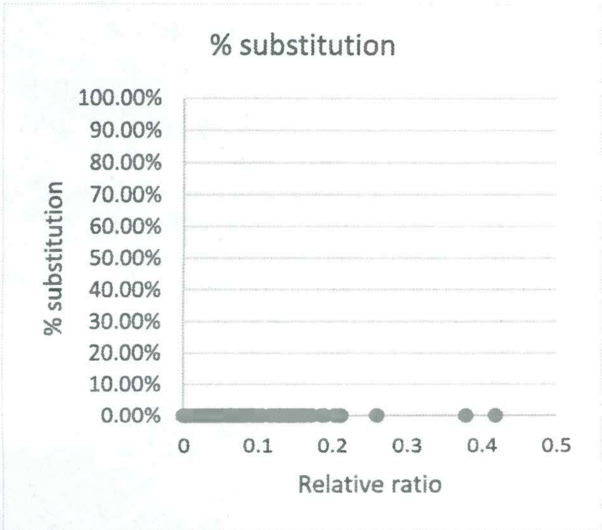


Figure 3.49- Model 2, D1<D2

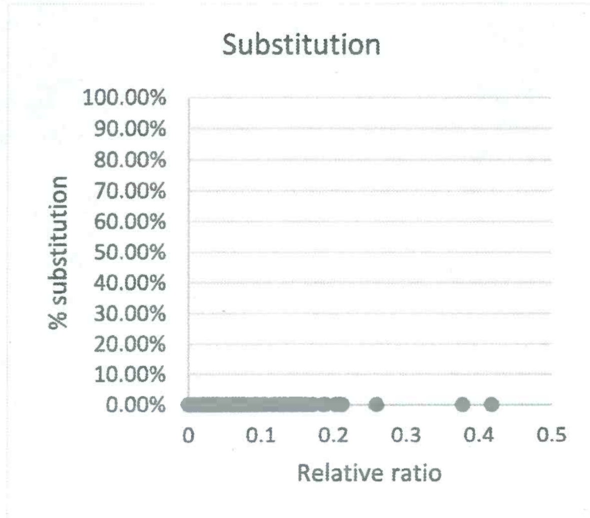


Figure 3.50- Model 2, D1=D2

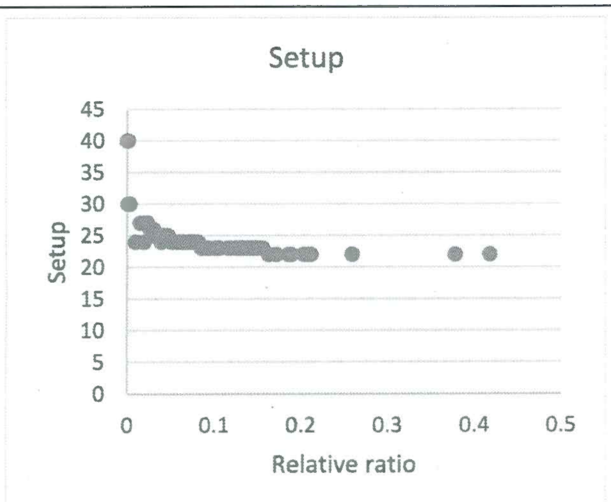


Figure 3.51- Model 2, D1<D2

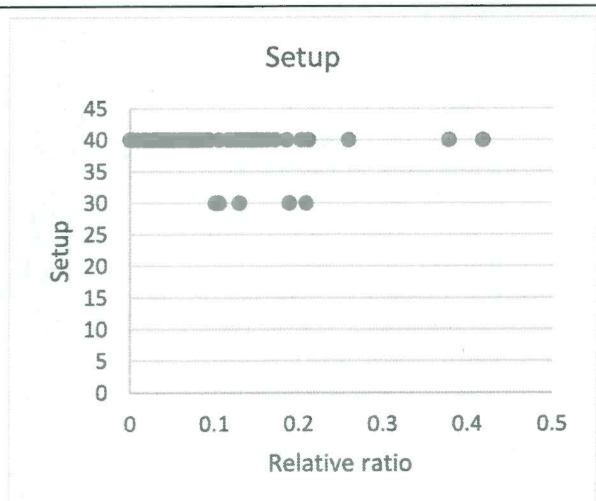


Figure 3.52- Model 2, D1=D2

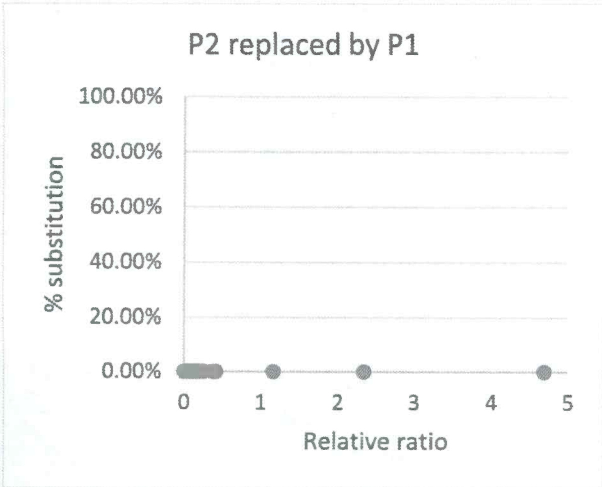


Figure 3.53- Model 3, D1<D2

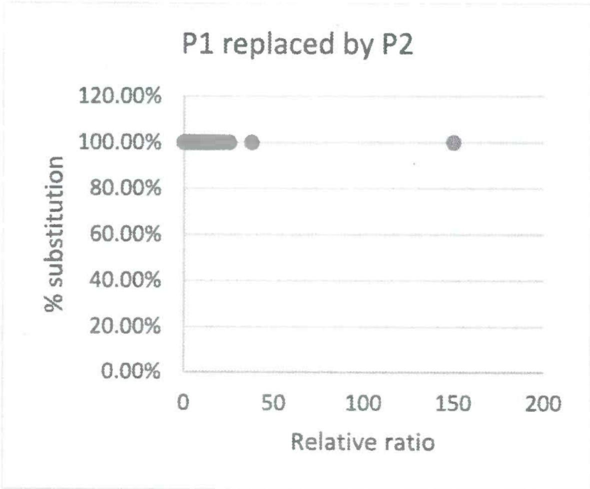


Figure 3.54- Model 3, D1<D2

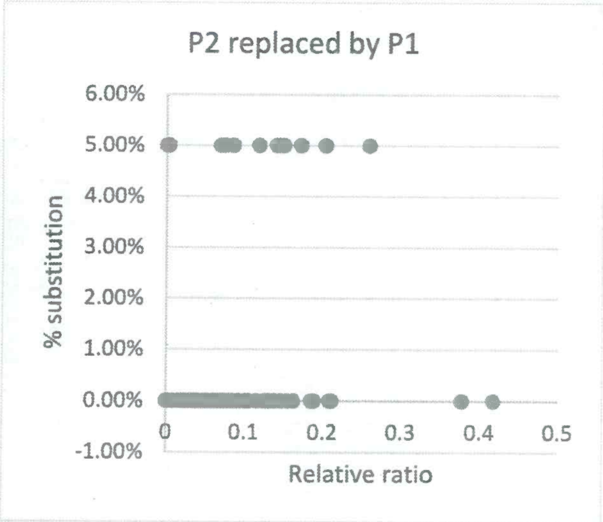


Figure 3.55- Model 3, D1=D2

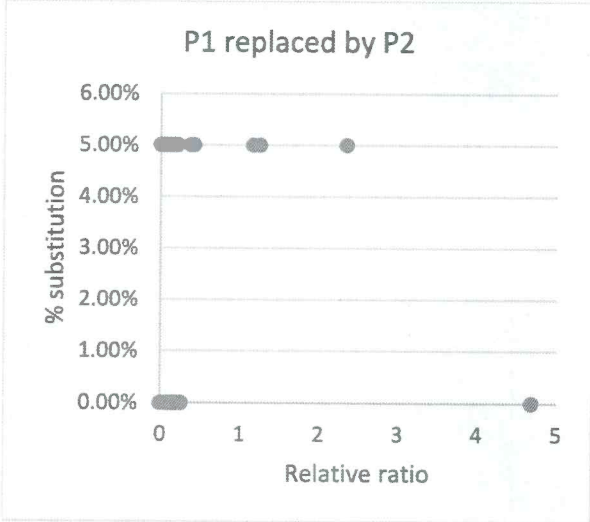


Figure 3.56- Model 3, D1=D2

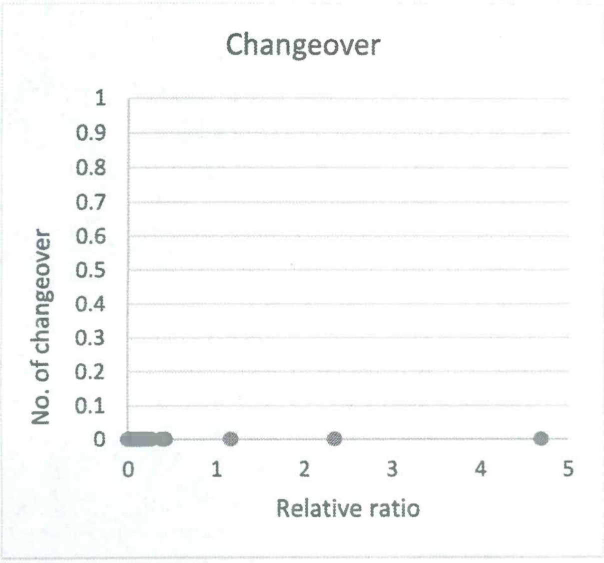


Figure 3.57- Model 3, D1<D2

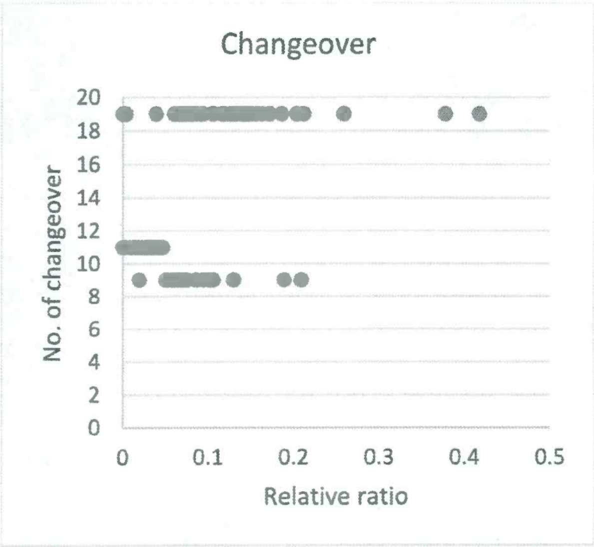


Figure 3.58- Model 3, D1=D2

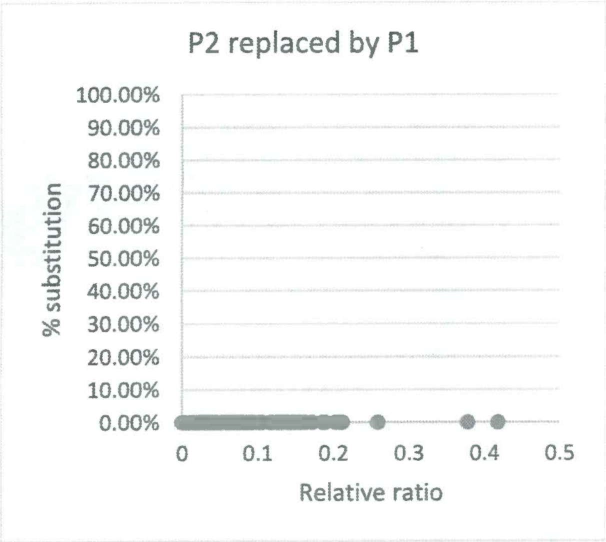


Figure 3.59- Model 4, D1<D2

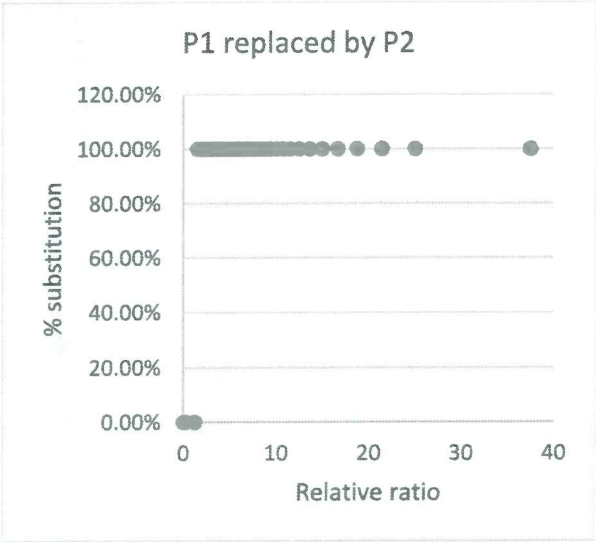


Figure 3.60- Model 4, D1<D2

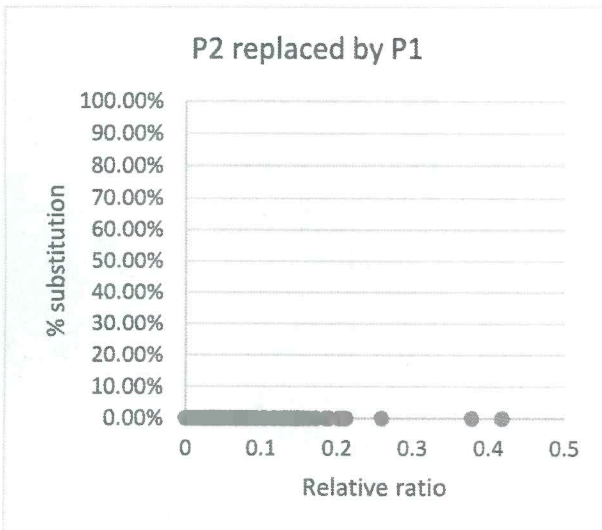


Figure 3.61- Model 4, D1=D2

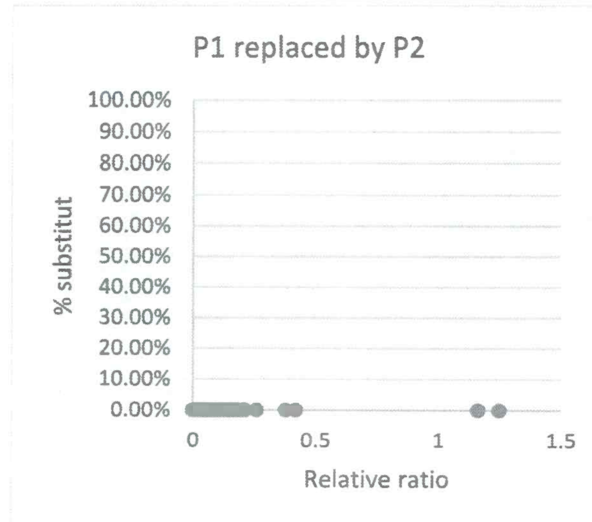


Figure 3.62- Model 4, D1=D2

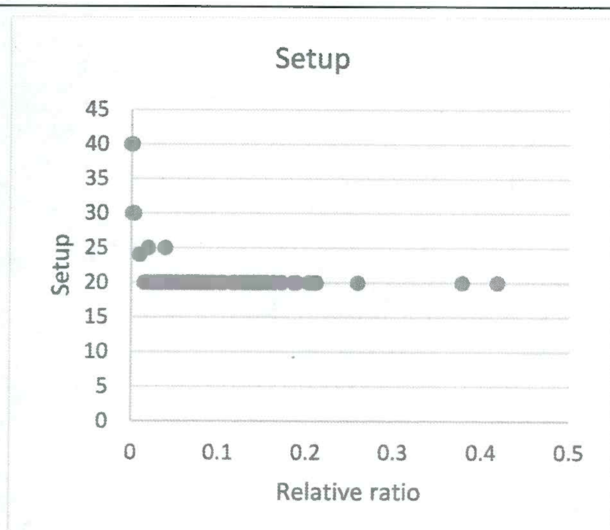


Figure 3.63- Model 4, D1<D2

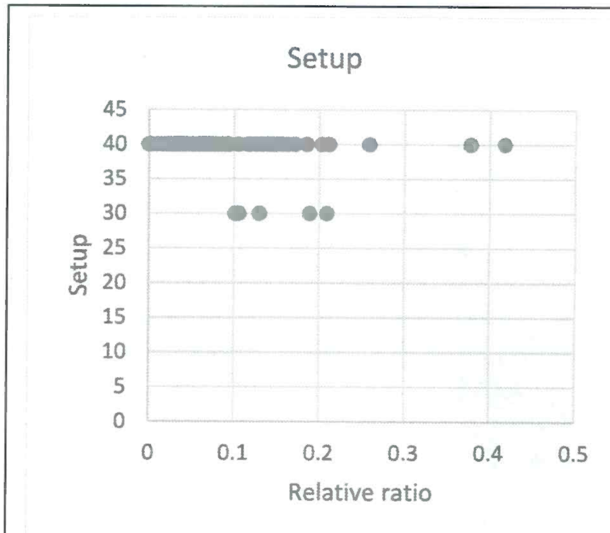


Figure 3.64- Model 4, D1=D2

CHAPTER 5: LIMITATIONS

There are some limitations in this thesis. First, due to the limitations of time and resources, this thesis analyzes four more lot-sizing problems but confines to two products as the original paper does. There will be more interesting results if there are test experiments with more than two products. Second, all the models in this thesis are uncapacitated, which will only be applicable in restricted contexts. Third, all the demand are deterministic and time-invariant over periods. Although there are still certain production planning contexts where all the demand are known in advance, the results with stochastic demand would allow them to be applied in larger circumstances. Fourth, in order to be consistent with the results in the original paper, this thesis continues to maintain the constraint that if product i substitutes for product k a quantity q in period t, this quantity q must also be produced in period t. This constraint limits the actual results for big bucket models. Finally, the *relative ratio* should be adjusted to **include the** holding cost for product 1 and holding cost for product 2. Future research could be aimed at proposing a different ratio which better explains the observed results.

CHAPTER 6: CONCLUSION

Overall, the test experiments with the four models using the testbed 1 show that the results are not the same for the four models:

- Analysis 1: Results with model 1 and model 3 are consistent with the result in the original paper: the higher the relative ratio is, the lower the number of changeover and the higher rate of substitution.
  However, the results for model 2 and model 4 are a little different. Although the setup behaviour is still similar (the higher the relative ratio is, the lower the number of setup), the substitution behaviour is different: the substitution rate reaches its peak when its value is in the range of 3 -5.
- Analysis 2: The result for model 1 is quite consistent with analysis 2 in the original paper: the benefit of substitution cost reduction decreases when demand 2 increases. However, the results for the other models are different. For model 3, although the benefit of substitution cost reduction decreases when demand 2 increases from 10 to 20, this value increases again when demand 2 increases from 20 to 40. And for models 2 & 4, there is not much impact when demand 2 increases from 10 to 40.
- Analysis 3: similar to analysis 2, the results for the four models are quite similar to the result in the original paper. When holding cost for product 2 increases, both the number of changeover and the rate of substitution increase. This increase also depends on the substitution cost and changeover cost: if substitution cost is low and changeover cost is high, the rate of substitution will increase more than the number of changeover. The result is reversed when the substitution cost is high and the changeover cost is low. However, model 2 is a bit different: % setup increase is much higher than % substitution increase when the setup cost is low and the substitution cost is high, but % setup increase is still higher than % substitution increase even when the setup cost is high and the substitution cost is low.

The test experiments using testbed 2 show completely different results for analysis 1 and analysis 2. Variance in demand has a high impact on the rate of substitution and the number of changeover for model 1 and model 3, but not for model 2 and model 4.

Some other interesting points are also observed from the test experiments. First, the substitution rate in small bucket models can reach 100%, while this figure is only maximum 40% in big bucket models. Second, there is the tendency that the product with a higher demand tends to substitute for the product with a lower demand in small bucket models. Third, variance in demand has a high impact on the substitution rate and the number of changeover for small bucket models. Finally, two-way substitution helps to lower the total cost more than one-way substitution.

LIST OF FIGURES

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REFERENCE

Bassok Y, Anupindi R, Akella R (1999). Single-Period Multiproduct Inventory Models With Substitution. *Operation Research,* Vol. 47, No. 4, 632-642

Balakrishnan A, Geunes J (2000). Requirements Planning with Substitutions: Exploiting Bill-of Materials Flexibility in Production Planning. *Manufacturing & Service Operations Management,* Vol. 2, No. 2, 166-185

Balakrishnan A, Geunes J (2003). Production planning with flexible product specifications: an application to specialty steel manufacturing. *Operations Research,* Vol. 51, No. 1, 94-112

Chopra S, Meindl P (2012). Supply Chain Mangement (5th Edition), ISBN-13: 978-0132743952

Dawande M, Gavirneni S, Mu Y, Sethi S, Sriskandarajah C (2010). On the Interaction Between Demand Substitution and Production Changeovers. *Manufacturing & Service Operations Management,* Vol. 12(4), 682-691

Geunes J (2003). Solving large-scale requirements planning problems with component substitution options. *Computers & Industrial Engineering,* Vol 44, 475-491

Hsu VN, Li CL, Xiao WQ (2005). Dynamic lot size problems with one-way product substitution. *IIE Transactions,* Vol 37, 201-215

Jans R, Degraeve Z (2008). Modeling Industrial Lot Sizing Problems: A Review. *International Journal of Production Research,* Vol. 46 (6), 1619-1643

Krarup J, Bilde O (1977). Plant location, set covering and economic lot size: an O(mn) algorithm for structural problems, *International Series of Numerical Mathematics,* Vol. 36, 155-180

Lang JC (2010). Production and Inventory Management with Substitutions. *Springer, Berlin*

Lang JC, Domschke W (2010). Efficient reformulations for dynamic lot-sizing problems with product substitution. *OR Spectrum* Vol 32(2), 263-291

Lang JC, Shen Z-J M (2011). Fix-and-optimize heuristics for capacitated lot-sizing with sequence-dependent setups and substitutions. *European Journal of Operational Research,* Vol 214, 595-605

Nagarajan M, Rajagopalan S (2008). Inventory Models for Substitutable Products: Optimal Policies and Heuristics. *Management Science,* Vol 54, No. 8, 1453-1466

Pochet Y, Wolsey LA (2006). Production Planning by Mixed Integer Programming. *Springer, Berlin*

Rao US, Swaminathan JM, Zhang J (2004). Multi-product inventory planning with downward substitution, stochastic demand and setup costs. *IIE Transactions,* Vol 36, 59-71

Stavrulaki E (2011). Inventory decisions for substitutable products with stock-dependent demand. *Int. J. Production Economics,* Vol 129, 65-78

Tan B, Karabati S (2013). Retail inventory management with stock-out based dynamic demand substitution. *Int. J. Production Economics,* Vol 145, 78-87

Yucel E, Karaesmen F, Salman FS, Turkay M (2009). Optimizing product assortment under customer-driven demand substitution. *European Journal of Operation Research,* Vol 199, 759-768

Yaman H (2009). Polyhedral analysis for the two-item uncapacitated lot-sizing problem with one-way substitution. *Discrete Applied Mathematics,* Vol 157, 3133-3151