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The Impact of Regional Heterogeneity in Canadian House Prices on Reverse Mortgage Pricing

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Résumé

Cette étude examine comment la dynamique régionale des prix de l'immobilier affecte la tarification des prêts hypothécaires inversés. Le principe fondamental de la tarification des prêts hypothécaires inversés est la garantie d'absence d'équité négative (NNEG), qui assure que les emprunteurs ne sont pas responsables d'une dette dépassant la valeur de leur maison. Nous utilisons les données de 12 grandes villes canadiennes (1981-2024) et les divisons en groupes de l'est et de l'ouest pour une modélisation séparée. Des modèles VAR et VECM simulent les trajectoires des prix des logements pour chaque groupe. Les résultats montrent que les villes de l'Est ont des tendances à long terme plus stables, tandis que les villes de l'Ouest sont confrontées à une plus grande volatilité à court terme. En combinant ces simulations avec des tables de mortalité prospectives, nous proposons un nouveau cadre de tarification. Ce cadre met en évidence le fait qu'une tarification uniforme à l'échelle nationale méconnaît souvent les risques régionaux, alors que les stratégies adaptatives régionales s'alignent mieux sur les conditions du marché local. Cette étude offre des perspectives pratiques aux institutions financières et aux décideurs politiques pour améliorer la tarification des prêts hypothécaires inversés et la gestion des risques.

Abstract

This study examines how regional housing price dynamics affect reverse mortgage pricing. The core principle of reverse mortgage pricing is the No-Negative-Equity Guarantee (NNEG), which ensures borrowers are not liable for any debt exceeding the value of their home. We use data from 12 major Canadian cities (1981–2024) and divide them into eastern and western groups for separate modeling. VAR and VECM models simulate house price trajectories for each group. The results show that eastern cities have more stable long-term trends, while western cities face higher short-term volatility. By combining these simulations with forward-looking mortality tables, we propose a new pricing framework. This framework highlights that nationwide uniform pricing often misjudges regional risks, while regionally adaptive strategies better align with local market conditions. This study offers practical insights for financial institutions and policymakers to improve reverse mortgage pricing and risk management.

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During the process of completing this thesis, I went through an extremely challenging period, as anxiety and depression seriously affected my mental and physical well-being, causing me to doubt my ability to finish this research. I would like to express my sincere gratitude to my supervisor, Professor Pierre-Carl Michaud, who provided not only professional guidance but also incredible patience, understanding, and encouragement when I was struggling. I am also deeply grateful to my counselor, whose professional and compassionate support taught me how to accept myself, manage my emotions, and gradually return to a healthier life.

A special thanks goes to my girlfriend for her unwavering companionship, support, and trust, which gave me hope and renewed confidence during the darkest times. By openly sharing my experience, I hope to encourage fellow students facing mental health challenges: you are not alone, and reaching out for help is a courageous and worthwhile decision. May you, too, find your own light in times of darkness.

1 Introduction

Rapid population aging is posing significant challenges to pension systems worldwide, and Canada is no exception. As the working-age population decreases and dependency ratios rise, both public and private retirement resources face mounting pressure. According to the Organisation for Economic Co-operation and Development, the proportion of Canada's population aged 65 and above has increased from 8% in 1960 to over 21% in 2021, with projections indicating it will reach 24% by 2036(OECD, 2023). Meanwhile, Canada's pension system has yet to achieve the 70% replacement rate recommended by the OECD, further exposing the financial insecurity faced by retirees.

Furthermore, traditional family support systems for the elderly have gradually declined due to demographic changes, leaving many seniors in challenging retirement situations. While they possess substantial housing wealth, their liquid assets are insufficient to meet retirement needs. In this context, reverse mortgages have emerged as a financial instrument offering a potential solution, allowing retirees to leverage their housing assets while retaining property ownership.

At the heart of reverse mortgages lies the No-Negative-Equity Guarantee (NNEG), which ensures borrowers never have to repay more than their property's market value. While this mechanism provides crucial protection for borrowers during market fluctuations, it transfers the risks of house price decline and extended loan duration to lenders (Siu-Hang Li et al., 2009). Pricing this risk presents significant challenges due to the complex combination of variables involved, including house price dynamics and uncertainty in borrower longevity.

Our research indicates that NNEG pricing depends not only on national house price trends but must also thoroughly consider regional differences. House price dynamics often exhibit distinct regional characteristics. For instance, coastal regions in the United States typically experience higher price volatility compared to inland areas, and property bubbles in coastal regions often eventually spread nationwide. These regional variations challenge uniform national pricing strategies, potentially leading to inaccurate risk assessments and resulting in losses for loan borrowers (Prüser and Schmidt, 2021).

This study's core focus lies in exploring how regional housing price dynamics influence NNEG pricing. Through the introduction of Vector Autoregression (VAR) and Vector Error Correction Models (VECM), we simulated house price trajectories in major Canadian cities, systematically analyzing similarities and differences in short-term fluctuations and long-term equilibrium trends across regions. Additionally, the research combines regional house price data with borrower life expectancy tables to establish an innovative NNEG pricing framework that addresses regional differences more fairly and efficiently while emphasizing the necessity of adaptive strategies.

The paper is structured as follows: Section 2 reviews core literature related to reverse mortgages and NNEG pricing; Section 3 describes the data sources and preprocessing procedures; Section 4 details the research methodology used to analyze regional house price dynamics and NNEG pricing; Section 5 presents the empirical analysis results; and Section 6 summarizes the research findings and discusses study limitations.

2 Literature Review

The aim of this review is to analyze research findings about reverse mortgages, particularly focusing on how they are priced. We first look at how an aging population affects the overall pension system. There is a gap between what the OECD recommends for the pension replacement rate (70%) and the actual rate in Canada. In addition, families are providing less support to elderly members than before. In this situation, reverse mortgages offer a new financial option for elderly people who own valuable homes but do not have much cash. The main feature of this product is its non-negative equity guarantee (NNEG). Two key factors affect its pricing: how house prices might change in the future and how long the loan will last. This paper focuses on methods for forecasting house prices. The Canadian housing market shows big differences between regions — cities like Vancouver and Toronto have high and stable prices, while resource-based cities like Calgary show more price changes. We use VAR and VECM models to analyze this. By comparing two pricing approaches — using the same price across Canada versus different prices for different regions — we explore how regional differences affect reverse mortgage pricing. This provides a framework to make product pricing more accurate.

2.1 Population Aging and Pension System Challenges

Population aging has become a significant challenge for pension systems around the world, especially in developed countries. According to the OECD's 2023 Pension at a Glance report (OECD, 2023), the percentage of people aged 65 and over increased from 16% in 2020 to 27% by 2050. The United Nations World Social Report 2023 further highlighted this trend, projecting that the global population aged 65 and above would reach 1.6 billion by 2050, representing 16% of the total population (United Nations Department of Economic and Social Affairs, 2023). This major demographic shift placed immense strain on both government finances and pension systems, as the ratio of working-age people to retirees continued to decline.

Canada experienced particularly rapid population aging. In 1960, only 8% of Canadians were 65 or older. By 2021, this number had grown to 21.5%, and it was expected to reach 24% by 2036. This trend was similar to what was observed in other developed nations. For instance, Russia saw its dependency ratio deteriorate from 2.2 to 1.7, creating significant pressure on the national budget and pension system (Lukyanets et al., 2021). In China, rapid aging created complex challenges in balancing public expenditure priorities between elderly care and other essential services like education (Pan et al., 2022). The situation in South Korea was particularly challenging, where many elderly people had insufficient pension income, leading to increased old-age poverty despite various policy interventions (Jun, 2020).

The OECD suggested that retirement income should maintain a substantial proportion of pre-retirement earnings to ensure adequate living standards. However, many countries, including Canada, struggled to achieve this target (Mitchell, 2022). The impact of insufficient retirement income was exacerbated by economic uncertainties and market volatility. Ssrn Au/At Cirano et al. (2021) examined how the global pandemic exposed significant vulnerabilities in retirement systems worldwide, emphasizing the need for more resilient pension structures and improved risk management tools.

Beyond pension system challenges, traditional family support for the elderly declined globally. This trend was particularly noticeable in Asia and the Pacific region, where urbanization and demographic changes reshaped traditional family structures (Poot and Roskruge, 2020). Jakovljevic et al. (2023) pointed out that this decline in family care functions led to increased government burden, especially in middle-income countries that faced the double challenge of managing both traditional infectious diseases and aging-related chronic conditions.

The financial pressures of aging populations extended beyond just pension systems. For example, in Poland, innovative financing mechanisms like reverse mortgages were explored to help address the economic challenges faced by elderly homeowners (Buzałek and Czechowska, 2020). Similarly, Wang et al. (2016) demonstrated how reverse mortgage insurance products required careful valuation to manage the complex risks associated with housing prices, interest rates, and longevity.

The impact of population aging on public finances created complex ripple effects throughout society. Research from China showed how aging populations significantly affected public education spending, with every 1% increase in the elderly dependency ratio leading to a 0.304% decrease in education expenditure (Pan et al., 2022). This demonstrated the broader societal implications of aging populations, requiring comprehensive policy responses that balanced the needs of different generations.

To address these mounting challenges, many countries explored innovative financial solutions. Quantitative analysis by Nakajima and Telyukova (2017) showed that reverse mortgages significantly enhanced borrower welfare, though their adoption remained limited due to factors such as high costs and bequest motives. The effectiveness of such solutions, however, depended heavily on factors like housing market stability, regulatory frameworks, and public acceptance.

2.2 Housing Wealth and Reverse Mortgage Solutions

As Canada's population grows older, homeownership has taken on a new role in how retirees fund their later years. Strangely enough, many seniors—though living in homes that have appreciated significantly—still struggle with day-to-day expenses. The Housing Market Insight report CMHC (2023) ouches on this contradiction. In cities like Vancouver and Toronto, where property prices have soared over the past few decades, the situation is even more apparent. These elderly homeowners are, in a sense, "house-rich but cash-poor."

Several researchers have looked at how retirees might tap into their housing wealth. For example, Hanewald (2022) explored a variety of strategies—from downsizing to renting, and of course, reverse mortgages. But not all paths suit everyone. The choice often depends on personal circumstances—health status, risk appetite, or whether someone wishes to pass their home on to their children. Interestingly, older Canadians with stronger inheritance motives tend to avoid reverse mortgages altogether.

So, do reverse mortgages work? Some studies suggest they can help. Nakajima and Telyukova (2017) , for instance, found measurable gains in household welfare—on average $1,770—after using such products. And these benefits seemed more pronounced during economic downturns, especially for lower-income seniors. In fact, demand from this group reportedly tripled in recessions. Meanwhile, Davidoff (2015) offered a somewhat different angle, highlighting how the flexibility and embedded option value of reverse mortgages could outweigh even their high fees—at least in areas where home values are stable or rising.

Of course, there's a safety net built into these products: the No-Negative-Equity Guarantee (NNEG). This feature makes sure the borrower never owes more than the house is worth when it's time to repay. Siu-Hang Li et al. (2009) explain it as a sort of insurance against market downturns. But it's not without complications. Lenders, after all, bear that risk, and pricing it correctly is difficult. Longevity, housing volatility—both come into play here. Shao et al. (2015) argue that NNEG becomes costlier to provide in unstable markets, which makes sense.

The importance of regional housing market conditions in reverse mortgage deployment was highlighted by several studies. Cocco and Lopes (2020) found that ”aging in place” preferences and home maintenance decisions significantly influenced reverse mortgage demand. This finding was particularly relevant in Canada's diverse housing markets. For example, Hutchison et al. (2024) added that high-value property markets—again, think Vancouver or Toronto—naturally allow for larger loans, making reverse mortgages more attractive both to lenders and borrowers.

But for all their potential, reverse mortgages haven't caught on widely. Ssrn Au/At Cirano et al. (2021) identified a few reasons: interest rates that seem high compared to what actuarial models would suggest, and a widespread lack of awareness. In their study of 3,000 Canadians, over half didn't fully understand how reverse mortgages work. On the other hand, people with stronger financial literacy tended to better appreciate the value of features like NNEG.

Lastly, broader demographic patterns play a role here too. Alai et al. (2014) examined different equity release mechanisms and concluded that reverse mortgages performed better—especially in areas with low house price volatility. This aligned with findings from Tsay et al. (2014), who developed pricing models showing that stable housing markets significantly reduced the cost of providing the NNEG.

Recent market data supported the growing relevance of reverse mortgages. Li et al. (2024) demonstrated through equilibrium modeling that reverse mortgages helped elderly households achieve consumption smoothing while increasing perceived housing value. Similarly, Ashok and Dhingra (2020) showed through case studies that reverse mortgage users maintained better financial stability compared to non-users, particularly over extended periods.

Despite these potential benefits, adoption rates remained lower than might have been expected. Knaack et al. (2020) identified several factors contributing to this, including product complexity, market inefficiencies (such as adverse selection and moral hazard), and the need for more transparent regulatory frameworks. To address these challenges, Mitchell (2022) suggested that reverse mortgages should be integrated into broader retirement planning strategies, particularly in countries where pension benefits were insufficient.

2.3 Regional Variations in Reverse Mortgage Effectiveness

Reverse mortgages have drawn increasing attention as a way for older homeowners to access home equity. Yet, their effectiveness has proven uneven, often reflecting the structural differences between local housing markets. In Canada, these disparities are especially noticeable. Prüser and Schmidt (2021), analyzing long-term data from 1976 to 2017, observed that cities along the coast—typically more densely populated and constrained in terms of land—showed greater house price volatility than those located inland. Their use of Markov-switching models suggested that institutions offering reverse mortgage products must consider such volatility in their pricing and risk strategies.

These differences have become particularly pronounced in the country's largest cities. As noted in the Housing Market Outlook Spring 2024, average house prices in Vancouver and Toronto surpassed $1.2 million and $1.1 million, respectively, whereas in Calgary the average remained closer to $500,000. Howard and Liebersohn (2023) linked these differences to broader economic forces, suggesting that higher income growth in key regions may have accelerated housing price inflation nationwide. This trend appeared most evident in cities where foreign capital inflows combined with geographic limitations, maintaining upward pressure on prices.

How regional housing markets interact adds yet another complication to reverse mortgage analysis. In some cases, a shift in house prices in one large city may end up influencing prices in its neighboring areas—though this doesn't always follow a consistent pattern. Vansteenkiste and Hiebert (2011), for example, identified several such spillovers across European cities, and a similar dynamic seems to play out in Canada, particularly in metro centers like Vancouver and Toronto. Still, the strength of these relationships tends to vary over time, making it difficult to generalize.

Volatility also doesn't behave uniformly across the country. Based on Finnish data, Dufitinema (2020) noted that downward housing shocks seemed to leave a longer footprint than upward ones. In Canada, this finding resonates in cities such as Calgary, where the local economy's ties to natural resource cycles often lead to sharper price swings. By contrast, places like Toronto and Vancouver—where industries are more diversified—appear to experience a steadier climb in housing values, at least over longer time frames.

Why does that matter for reverse mortgages? Well, pricing products that carry guarantees—like the No-Negative-Equity Guarantee—requires a reasonably accurate forecast of long-term home values. As Shao et al. (2015) put it, lenders in more stable markets can afford to be more generous in their terms, while volatility tends to force a cautious approach. Siu-Hang Li et al. (2009) came to a similar conclusion: offering borrower protections becomes much more expensive when prices are unpredictable.

Some of the more recent literature has pushed this further. For instance, Pan et al. (2022) pointed out that the financial benefits of homeownership—what economists call the "housing wealth effect"—were stronger in areas where prices rose gradually and predictably. That may help explain, at least in part, why reverse mortgage adoption has been more common in cities like Toronto and Vancouver. Hutchison et al. (2024) made this point as well.

The interaction between housing markets and local economic conditions also influenced reverse mortgage viability. Guerrieri et al. (2013) showed how local economic shocks created significant house price variations between neighborhoods, even within the same city. This intra-city variation added another layer of complexity to reverse mortgage pricing and risk assessment. For instance, Calgary's housing market demonstrated how resource sector dynamics created additional volatility, making reverse mortgage terms less favorable compared to more economically diverse cities.

These regional differences created unique challenges for national-level reverse mortgage programs. Tariq et al. (2024) emphasized how monetary policy's impact on house prices varied significantly across regions, suggesting the need for regionally tailored lending approaches. This was particularly relevant in Canada, where Zhang et al. (2012) identified varying sensitivities to monetary policy and economic fundamentals across different housing markets.

Looking at international comparisons provided additional insights. Segnon et al. (2021) developed high-frequency volatility forecasting models that highlighted how different market structures created varying levels of price stability. Their findings suggested that markets with diverse economic bases, like Vancouver and Toronto, tended to exhibit more predictable price patterns than those heavily dependent on specific sectors, like Calgary's energy-focused economy.

2.4 Advanced Modeling Approaches for Reverse Mortgage Pricing

While reverse mortgages have frequently been proposed as a tool for older homeowners to access their housing wealth, their implementation has not been equally effective across all regions. Within Canada, distinct regional housing dynamics have created differences in how these financial products perform. Prüser and Schmidt (2021) , using historical data from 1976 to 2017, reported that coastal housing markets characterized by high population density and limited land supply demonstrated higher volatility than inland regions. Their application of Markov-switching models highlighted the importance of regional price fluctuations for financial products like reverse mortgages, which rely heavily on the long-term trajectory of house prices.

These regional patterns appeared particularly pronounced in Canada's largest cities. According to the Housing Market Outlook Spring 2024, the average home prices in Vancouver and Toronto exceeded 1.2 million and 1.1 million Canadian dollars, respectively, while Calgary's average remained around 500,000. Howard and Liebersohn (2023) attributed this gap to underlying structural drivers. In wealthier regions, faster income growth tended to amplify national housing inflation and elevate expectations for future rental income. In particular, sustained upward pressure on housing values in Vancouver and Toronto reflected a combination of strong economic fundamentals, international capital inflows, and constrained land supply.

In addition to price levels, regional interdependence further complicated the landscape. Vansteenkiste and Hiebert (2011) identified spillover effects in which housing price trends in major urban centres influenced surrounding regions. While long-term correlations between regional housing markets have been documented, the strength and consistency of these relationships tend to fluctuate across both time and location. In the Canadian context, housing price movements in core metropolitan areas—particularly Vancouver and Toronto—have repeatedly influenced pricing behavior in neighboring communities, underscoring the outsized role of major cities in shaping broader market trends.

Although broad regional price patterns in Canada have been well documented, they rarely follow the same trajectory or exhibit the same degree of volatility. Dufitinema (2020), in the context of the Finnish housing market, highlighted a tendency for negative price shocks to generate more lasting volatility compared to positive ones—a phenomenon known as volatility clustering. This asymmetry appears to resonate with what has been observed in Canadian cities like Calgary, where resource-based economic fluctuations have often led to sharp, irregular changes in home values. In contrast, cities such as Vancouver and Toronto—supported by more diversified economies—tend to follow steadier price paths.ons than in more diversified economies like Vancouver and Toronto.

To better capture these nuanced differences, Huang et al. (2010) utilized spatial-temporal regression models. Their approach emphasized how localized features, including population structure, supply constraints, and regional economic profiles, jointly shape price behavior across space and time. The contrasting trends between Calgary's cyclical movements and the more consistent appreciation seen in Vancouver and Toronto serve as clear examples of how reverse mortgage risk must be interpreted within a regional context rather than through national averages.

These differences carry direct implications for product design. According to Shao et al. (2015) , price stability enables lenders to offer reverse mortgages on more flexible terms, largely because the uncertainty is lower. Conversely, in volatile housing environments, institutions are generally forced to be more cautious. This aligns with the findings of Siu-Hang Li et al. (2009), who concluded that providing a No-Negative-Equity Guarantee is substantially more expensive in markets with unstable price trends.

Pan et al. (2022) added another layer to this by linking demographic change to housing behavior. Their findings suggested that in regions with stable appreciation, the wealth effect from housing tends to be stronger, making reverse mortgage products more appealing. This could partly explain why, as Hutchison et al. (2024) reported, uptake of such products has been especially prominent in markets like Vancouver and Toronto.

Even within cities, however, uniformity is elusive. Guerrieri et al. (2013) showed that neighborhood-level economic shocks can result in highly localized price shifts, creating intra-city disparities in housing values. Calgary is a particularly instructive case here: some areas closely tied to resource industries experience far greater swings than others. This kind of micro-level variation complicates any attempt to standardize risk assessments across urban regions.

At the policy level, the presence of such spatial and economic heterogeneity makes a one-size-fits-all lending framework difficult to justify. Tariq et al. (2024) argued that the transmission of monetary policy varies widely across Canadian cities, weakening the effectiveness of uniform national strategies. Zhang et al. (2012) reached similar conclusions, highlighting that housing markets respond differently to macroeconomic indicators depending on their local characteristics.

Insights from outside Canada further support this view. Segnon et al. (2021), using high-frequency volatility models, found that housing markets tied to more diverse economies tend to experience fewer abrupt fluctuations. That observation fits well with the Canadian case: while Vancouver and Toronto benefit from broad industrial bases, Calgary's exposure to energy price cycles leaves it more vulnerable to sudden shocks. Taken together, these findings suggest that reverse mortgage product design should be regionally adaptive rather than standardized.

3 Data

3.1 House Price Dataset

3.1.1 Data source

The data for our study was collected from the Statistics Canada New Housing Price Index (NHPI) data table, identified by the following reference number: 18-10-0205-01. The data set encompassed the monthly housing price index from January 1981 to August 2024 (base period: December 2016 = 100), meticulously documenting the price fluctuations of newly constructed residential properties in major Canadian cities and furnishing a robust foundation for investigating the nuances of regional housing markets. The index is expressed in norminal terms, meaning that it reflects actual transaction prices without adjusting for inflation. Accordingly, the housing price data used throughout this thesis are nominal. This choice ensures consistency with the nominal discount rate applied later in the pricing model.

The dataset covered 12 major cities in eastern and western Canada, including:

- **Western Cities**: Vancouver, Calgary, Edmonton, Victoria, Saskatoon, Winnipeg
- **Eastern Cities**: Toronto, Montreal, Quebec City, Halifax, Hamilton, St. John's

Figure 3.1.1 presents a map of Canada. We can be observed that the selected cities encompass the majority of Canada's major urban centers, which collectively represent the epicenter of the country's population, economic activity, employment, educational institutions, tourism, and other resources. This geographical coverage allows us to examine regional variations in housing market behavior and capture the distinct characteristics of different urban housing markets.

In determining our analysis timeframe, we initially encountered data continuity issues for Calgary, where observations were missing from January 1981 to April 1984. To ensure the robustness of our time series analysis, we established January 1985 as the starting point of our study period, continuing through August 2024. Beyond this adjustment, the dataset demonstrates complete continuity with no occasional missing values. To capture both long-term trends and recent market dynamics, we additionally created a subsample covering the most recent decade (August 2014 - August 2024), enabling comparative analysis across different time horizons.

Figure 3.1.1: Map of Canada with CMAs included in analysis

3.1.2 Descriptive statistics

Figure 3.1.2 displays the time series of housing price indices across Canadian cities from 1985 to 2024. The evolution of these indices suggests varying patterns of price movements across different markets. While some cities such as Calgary show notable price increases, particularly in the post-2020 period, others like Victoria demonstrate more moderate price trajectories. This overview provides initial insights into the diversity of housing market developments across Canadian regions.

The descriptive statistics presented in Figures A.1 A.2 A.3 reveal distinct regional patterns in Canadian housing markets. Over the long-term period (1985-2024), western markets display generally higher price volatility, with coefficients of variation ranging from 0.38 to 0.51, compared to eastern markets' range of 0.29 to 0.40. The mean price indices during this period also show regional differences, with western cities exhibiting wider dispersion – Victoria's average of 125.32 contrasts notably with Calgary's 67.04, while eastern cities maintain relatively closer price levels between 65.53 and 82.25. The box plots (Figure A.1) reinforce these regional characteristics, showing western cities with notably larger interquartile ranges and more extreme values, particularly in Calgary and Winnipeg, while eastern cities demonstrate more compressed price distributions.

The analysis of dynamic changes between long-term and recent periods highlights persistent regional characteristics in market behavior. Western cities demonstrate more pronounced transitions in market states - their coefficients of variation show substantial

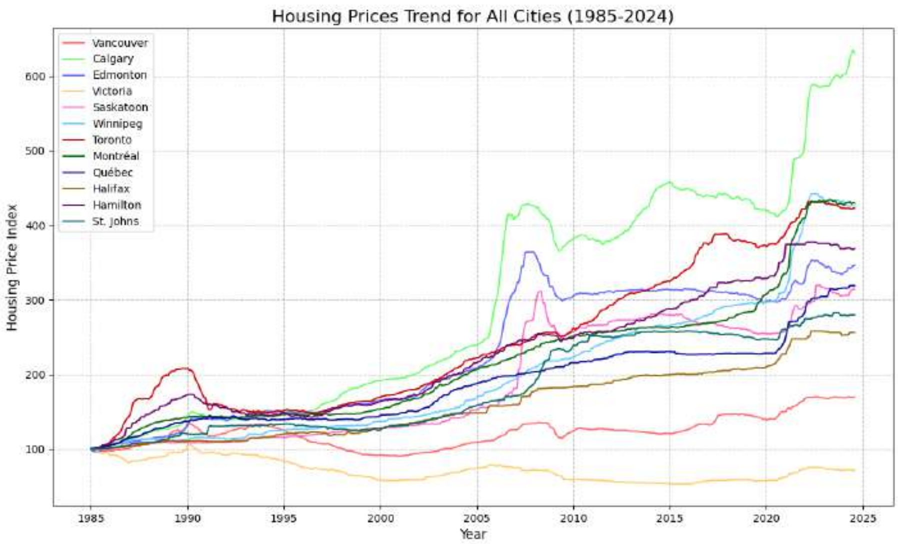

Figure 3.1.2: Long-term housing price trends, by city. Source: Statistics Canada, New Housing Price Index (NHPI), Table 18-10-0205-01.

changes, as seen in Edmonton (from 0.38 to 0.05) and Calgary (from 0.51 to 0.14). Eastern markets exhibit more gradual transitions, with most cities maintaining relatively stable volatility patterns except for Montreal, which shows increased market activity (coefficient of variation 0.21) in recent years. These distinct patterns of market transitions suggest that regional differences manifest not only in static price levels but also in how markets evolve over time.

3.2 Prospective Life Tables

3.2.1 Data source

The report "Population Projections for Canada (2013-2063)" by Bohnert et al. (2015), published by Statistics Canada, served as the primary source of life table data for this research. The report employed two complementary modern mortality projection models: the Lee-Carter model and its enhanced version, the Li-Lee model, to generate systematic mortality forecasts for the next 50 years.

The Lee-Carter model worked by identifying patterns in historical mortality data to predict future trends. Similar to how meteorologists forecast weather by studying historical patterns, this model analyzed how mortality rates across different age groups changed over time. It focused on two key elements: the baseline mortality characteristics of different age groups and how these characteristics evolved over time. This approach allowed the model to capture important phenomena, such as how medical advances affected different age groups differently.

However, when projecting mortality rates across different regions of Canada simultaneously, using only the Lee-Carter model presented certain limitations. Like weather patterns that varied by region while remaining interconnected, mortality rates across different regions often showed both local characteristics and common trends. To address this, the Li-Lee model built upon the original framework by incorporating regional relationships. It separated mortality changes into two components: one reflecting national trends shared across regions, and another capturing region-specific characteristics. This approach maintained regional distinctiveness in projections while ensuring reasonable correlation between forecasts for different areas.

3.2.2 Scenario selection

The report by Bohnert et al. (2015) presented three potential scenarios for future demographic change: low growth, medium growth, and high growth. When selecting an appropriate scenario, we concentrated on several pivotal indicators that could be validated in the near term at the 2024 juncture, rather than relying excessively on long-term forecast indicators that are challenging to verify.

Table 3.2.1 presents key demographic assumptions for different growth scenarios, including Total Fertility Rate (TFR), Immigration Rate (IR), and Non-permanent Resident Population (NPR) projections, which were used to validate our scenario selection.

In particular, three key indicators were selected for examination:

- **Total Fertility Rate (TFR)**: The actual total fertility rate in 2022 was 1.33, a figure that was significantly lower than the 1.67 assumed in the medium growth scenario and even lower than the 1.53 assumed in the low growth scenario.

- **Migration Rate (MR)**: The migration rate in 2022 was 6.188 per thousand people, which fell between the figures projected for the medium growth scenario (7.5 ‰) and the low growth scenario (5.0 ‰).

- **Non-permanent Resident Population (NPR)** : The actual number of non-permanent residents in 2022 (1,729,037) was significantly higher than the projections in the medium growth scenario (864,600) and the low growth scenario (733,600).

Table 3.2.1: Key Demographic Assumptions in Different Growth Scenarios

Assumption	Low Growth (L)	Medium Growth (M)	High Growth (H)
TFR (2021/2022)	1.53	1.67	1.88
LE-M (2062/2063)	86.0 years	87.6 years	89.9 years
LE-F (2062/2063)	87.3 years	89.2 years	91.9 years
IR (2022/2023)	5.0	7.5	9.0
NPR (2022/2023)	733,600	864,600	1,144,300
NER (2062/2063)	1.9	2.2	2.5
RER (2062/2063)	1.0	1.0	1.0
NTER (2062/2063)	0.7	0.7	0.7
IPM	1991/1992 - 2010/2011	1991/1992 - 2010/2011	1991/1992 - 2010/2011

Note: TFR: Total Fertility Rate; LE-M: Male Life Expectancy; LE-F: Female Life Expectancy; IR: Immigration Rate; NPR: Non-permanent Resident Population; NER: Net Emigration Rate; RER: Return Emigration Rate; NTER: Net Temporary Emigration Rate; IPM: Interprovincial Migration Pattern.

A comparison of the actual values of these short-term indicators with the forecast values provided a reliable basis for assessment. However, the report also contained a number of long-term indicators extending to 2063, which were subject to a number of uncertainties, including advances in medical technology and social change. As a result, these indicators had limited predictive value.

After weighing these factors, and especially considering the significant underestimation of the most critical fertility indicator among the verifiable indicators, we elected to utilize the low-growth scenario life table data. This selection strategy, based on short-term verifiable indicators, was more reliable and prudent than attempting to assess population development decades in the future.

4 Methodology

4.1 No Negative Equity Guarantee

Michaud and St. Amour (2023) uses empirical data on consumption, housing assets, and medical expenditures of the elderly population to calibrate a utility model in reverse. The results show that the elderly have a high level of risk aversion and prefer to retain wealth rather than take risks by taking out a reverse mortgage to liquidate it in the face of uncertainty (such as housing price fluctuations, longevity risk, and medical expenses).

In order to increase the attractiveness of reverse mortgages, the No-Negative-Equity Guarantee mechanism, hereafter referred to as NNEG, was introduced. It ensures that borrowers do not need to repay more than the market value of their home before the loan matures. This mechanism shifts the risk of falling house prices and long loan durations to the lender. We discuss this mechanism in more detail below.

Szymanoski (1994) believes that the structure of NNEG is similar to a put option, which can be described at a single point in time as the formula 4.1.1

$$V_{\text{NNEG}} = \mathbb{E}\left[\max\left(L_T - H_T, 0\right)\right] \tag{4.1.1}$$

- L_T: The outstanding loan balance
- H_T: The value of the house.

However, in real life, the loan maturity time T is definitely not a single point in time, but is determined by the borrower's life expectancy. Therefore, we need to extend the model to calculate the cumulative risk over multiple time points. The specific formula is shown in Equation 4.1.2:

$$V_{\text{NNEG}} = \mathbb{E}\left[\sum_{t=1}^{T} q_{a,a+t} \cdot \max(L_{a+t} - H_{a+t}, 0)\right] \tag{4.1.2}$$

- $q_{a,a+t}$: The probability that the borrower survives from age a to a+t.

To convert future risks into a current value, Michaud then introduced a discount factor and transaction costs (fees such as intermediary fees and taxes), while for the loan balance he introduced the loan interest rate and the premium interest rate for reverse loans. The following formula 4.1.3 was obtained:

$$V_{\text{NNEG}} = \mathbb{E}\left[\sum_{t=1}^{T} q_{a,a+t} \cdot \max\left(L_a \cdot (1 + r_{LC} + \pi)^t - (1-c)H_{a+t}, 0\right) \cdot \frac{1}{(1+i)^t}\right] \tag{4.1.3}$$

- L_a: The initial loan amount at age a.
- r_{LC}: The loan interest rate.
- π: The insurance premium rate, used to cover loan risks.
- c: The transaction cost ratio.
- i: The discount rate.

4.2 House Price Simulation Model

In recent years, Vector Autoregression (VAR) and Vector Error Correction Models (VECM) have achieved remarkable success in housing price forecasting research. Vansteenkiste and Hiebert (2011), when studying the real estate market in the eurozone, not only revealed the spillover effect of housing prices between countries through the VAR model, but also captured how housing prices in different regions are linked to varying degrees according to economic weight and geographical location. This provides important ideas for analysing the mutual influence of regional housing prices. As the research deepened, scholars found that when there is a long-term equilibrium relationship in the house price series, the VECM model can provide a more comprehensive perspective. In a study of house prices in 12 cities in New Zealand, Shi et al. (2010) found that the VECM model not only accurately captured the long-term equilibrium relationship between house prices in different cities, but also performed well in predicting long-term trends. This finding was further confirmed in a study of the Sydney real estate market – by combining the Johansen cointegration test with the VECM model, Al-Masum and Lee (2019) successfully identified the long-term equilibrium relationship between house prices and market fundamentals such as disposable income and housing supply, highlighting the unique advantages of the VECM model in explaining the dynamic changes in house prices.

The methodological advantages of these models are particularly relevant for our study. While VAR provides insight into short-term dynamics among multiple variables, VECM—through the integration of correction terms—offers a clearer view of how deviations from long-term trends are gradually adjusted. Given the pronounced regional heterogeneity of the Canadian housing market, and our interest in simulating extended price paths, we adopt both models to construct forward-looking trajectories. In doing so, we aim to develop a more nuanced understanding of the forces shaping house price evolution across major Canadian cities.

4.2.1 Vector Autoregression Model

The VAR (Vector Autoregression) model is a multivariate time series analysis tool that captures the dynamic interactions between variables. Its general form for VAR(p) is as follows:

$$Y_t = A_1 Y_{t-1} + A_2 Y_{t-2} + \cdots + A_p Y_{t-p} + \varepsilon_t \tag{4.2.1}$$

- Y_t is a vector containing multiple time-series variables.
- $A_1, A_2, \ldots, A_p$ are the parameter matrices to be estimated.
- ε_t is a white noise error term, representing unpredictable random shocks.

In addition to the residual term necessarily exhibiting characteristics of white noise, VAR models are contingent upon two fundamental assumptions: firstly, that the data must be stationary, and secondly, that the relationship between the variable and its lagged values must be linear.

4.2.2 Stationary Test

In the context of VAR models, the stationarity of the time series is of paramount importance. The concept of stationarity guarantees that the statistical properties of the time series (mean, variance, autocorrelation) remain constant over time, thereby providing a robust foundation for the estimation and inference of the model.

In the event that the input data is not stationary, this may result in a spurious regression problem, whereby the relationship between variables is attributed to a common trend rather than an actual dynamic relationship.

However, housing price data often exhibit significant trends or random walk characteristics, which means that the data may not meet the stability requirement. For example, housing prices generally tend to increase over time, and the fluctuation range may increase over time. This non-stationary property will affect the reliability of subsequent models.

Zhang et al. (2013) highlight that the conclusions drawn from the characteristics of house price time series in different empirical studies are not consistent. This inconsistency is primarily attributed to the choice of sample period. For instance, in some short-term samples, house prices may appear to be stable, whereas in long-term samples, they may exhibit trend or non-stationary characteristics. This discrepancy underscores the necessity for a meticulous examination of the data characteristics prior to modelling.

We employs two widely utilized statistical techniques, the augmented Dickey-Fuller (ADF) test and the Phillips-Perron (PP) test, to assess the stationary of time series. These methods are frequently utilized to ascertain whether a time series possesses a unit root, thereby determining its non-stationary nature.

The ADF test is an extended version of the traditional Dickey-Fuller test, which corrects for the self-correlation of residuals by adding a lag term to the difference. Its model form is as follows:

$$\Delta Y_t = \alpha + \beta t + \gamma Y_{t-1} + \sum_{i=1}^{p} \delta_i \Delta Y_{t-i} + \varepsilon_t \tag{4.2.2}$$

- ΔY_t: The first difference of the time series.
- α: Constant term, βt: Trend component, γ: Coefficient of Y_{t-1}.
- δ_i: Coefficients of lagged differences.
- $H_0 : \gamma = 0$: The series has a unit root (non-stationary).

The PP test is a non-parametric extension of the ADF test that accounts for heteroskedasticity and autocorrelation by modifying the covariance matrix of the error term. Its model form is as follows:

$$Y_t = \alpha + \beta t + \gamma Y_{t-1} + \varepsilon_t \tag{4.2.3}$$

- The PP test evaluates the same null hypothesis as the ADF test ($H_0 : \gamma = 0$), indicating the presence of a unit root.

We apply the ADF and PP tests to the housing price index of 12 major cities in Canada. Tables 4.2.1 and 4.2.2 show the results of the stationarity tests on the original housing price data. We can see that the p-values of all 12 cities exceed the significance level of 0.05, and none of them pass the ADF and PP tests. This indicates that the housing price series in these cities have unit root

Table 4.2.1: East Cities - Original Data

City	ADF Statistic	ADF p-value	PP Statistic	PP p-value	ADF Results	PP Results
St. Johns	-0.3555	0.9173	-0.0894	0.9505	Non-Stationary	Non-Stationary
Halifax	1.0318	0.9946	1.3438	0.9968	Non-Stationary	Non-Stationary
Québec	0.0432	0.962	1.1425	0.9956	Non-Stationary	Non-Stationary
Montréal	0.5759	0.987	1.6088	0.9979	Non-Stationary	Non-Stationary
Toronto	-0.2304	0.9348	-0.0035	0.9582	Non-Stationary	Non-Stationary
Hamilton	-0.1733	0.9416	0.0844	0.965	Non-Stationary	Non-Stationary

Table 4.2.2: West Cities - Original Data

City	ADF Statistic	ADF p-value	PP Statistic	PP p-value	ADF Results	PP Results
Winnipeg	1.3943	0.9971	1.6853	0.9981	Non-Stationary	Non-Stationary
Saskatoon	-0.1185	0.9476	-0.3798	0.9134	Non-Stationary	Non-Stationary
Calgary	-0.0869	0.9507	0.5305	0.9858	Non-Stationary	Non-Stationary
Edmonton	-0.679	0.8521	-0.8669	0.7987	Non-Stationary	Non-Stationary
Vancouver	-0.5319	0.8856	-0.3362	0.9202	Non-Stationary	Non-Stationary
Victoria	-1.7674	0.3968	-2.0474	0.2662	Non-Stationary	Non-Stationary

Given that all the city time series have unit roots, we differenced the raw data and then performed ADF and PP tests on the differenced data. Tables 4.2.3 and 4.2.4 show that after the first-order differencing, the test statistics of all cities are far below the critical values. This confirms that the housing price data become stationary after differencing.

Table 4.2.3: East Cities - First Difference

City	ADF Statistic	ADF p-value	PP Statistic	PP p-value	ADF Results	PP Results
St. Johns	-5.0465	0.0	-20.5252	0.0	Stationary	Stationary
Halifax	-6.3512	0.0	-19.5428	0.0	Stationary	Stationary
Québec	-3.1409	0.0237	-16.2779	0.0	Stationary	Stationary
Montréal	-3.4126	0.0105	-19.3624	0.0	Stationary	Stationary
Toronto	-4.9564	0.0	-16.2597	0.0	Stationary	Stationary
Hamilton	-5.4741	0.0	-21.5861	0.0	Stationary	Stationary

Table 4.2.4: West Cities - First Difference

City	ADF Statistic	ADF p-value	PP Statistic	PP p-value	ADF Results	PP Results
Winnipeg	-4.8858	0.0	-14.9652	0.0	Stationary	Stationary
Saskatoon	-6.0842	0.0	-16.3531	0.0	Stationary	Stationary
Calgary	-4.49	0.0002	-9.3789	0.0	Stationary	Stationary
Edmonton	-5.5205	0.0	-16.7156	0.0	Stationary	Stationary
Vancouver	-7.7034	0.0	-18.0317	0.0	Stationary	Stationary
Victoria	-3.8709	0.0023	-18.7548	0.0	Stationary	Stationary

4.2.3 Lag Selection

In order to guarantee the veracity and precision of the results produced by the vector autoregression (VAR) model, it is necessary to determine the lag order of the model. The lag order determines the number of previous observations of the variable in the VAR model that are used to predict the current variable's dynamics. A lag order that is too small will result in the model failing to capture some of the dynamic information and key interrelationships, thereby reducing its explanatory power. Conversely, a lag order that is too large will lead to overfitting and multicollinearity issues, both of which will impair the model's accuracy.

Table 4.2.5 and Table 4.2.6 show the results of lag selection for the Eastern and Western city groups. Given that our model is primarily employed for forecasting the prospective trajectory of real estate market prices, we have elected to utilise the AIC and FPE criteria, which encompass a greater number of lag orders and are better equipped to capture the nuances of dynamic information. Following a meticulous evaluation, we have determined that the optimal lag order for the Eastern city group is 3, while for the Western city group, it is 5.

Table 4.2.5: Eastern city lag selection result

Lag	AIC	BIC	FPE	HQIC
0	−8.021	−7.597	0.000 328 4	−7.855
1	−8.589	**−7.847***	0.000 186 2	−8.297
2	−8.660	−7.600	0.000 173 4	−8.243
3	**−8.890***	−7.512	**0.000 137 9***	**−8.348***
4	−8.857	−7.160	0.000 142 6	−8.189
5	−8.870	−6.856	0.000 140 8	−8.078

Table 4.2.6: Western city lag selection result

Lag	**AIC**	**BIC**	**FPE**	**HQIC**
0	−2.541	−2.117	0.078 81	−2.374
1	−3.520	**−2.777***	0.029 61	−3.228
2	−3.675	−2.615	0.025 35	**−3.258***
3	−3.785	−2.406	0.022 73	−3.243
4	−3.832	−2.136	0.021 69	−3.165
5	**−3.863***	−1.849	**0.021 05***	−3.071

4.2.4 Vector Error Correction Model

We confirm that all city housing price time series contain a unit root, while the difference series are stationary, by passing two stability tests. However, this does not mean that there is no potential long-term equilibrium relationship between these series. The definition of cointegration: If multiple time series $Y_t = [y_{1t}, y_{2t}, \ldots, y_{nt}]$ are non-stationary, but there exists a linear combination that makes the combination stationary, then these series are said to be cointegrated.

$$\beta' Y_t = u_t, \quad u_t \sim \text{Stationary}$$

A cointegration relationship not only reveals long-term linkages between different variables, but also reflects the existence of short-term equilibrium between variables. For example, if there is a cointegration relationship between cities in the east and west of Canada, it indicates that housing prices in cities within a certain region are driven by common long-term trends. Identifying these cointegration relationships will provide an important basis for our subsequent forecast of housing price paths.

Johansen's test is based on the following form of the VECM model (Vector Error Correction Model):

$$\Delta Y_t = \Pi Y_{t-1} + \sum_{i=1}^{k-1} \Gamma_i \Delta Y_{t-i} + \varepsilon_t \tag{4.2.4}$$

- ΔY_t: The differences of the time series.
- Y_{t-1}: The lagged first-order time series.
- $\Pi = \alpha\beta'$: The cointegration matrix, where:

- α: The adjustment coefficient matrix, reflecting how short-term dynamics adjust to the long-term equilibrium.
- β: The cointegration vector matrix, describing the long-term equilibrium relationship.

- Γ_i: The short-term dynamic coefficient matrix, describing the impact of lagged differences.
- ε_t: Random error terms.

The error correction term, ΠY_{t-1}, indicates how the long-term equilibrium relationship impacts current adjustments. Similarly, the short-term dynamics component, $\Gamma_i \Delta Y_{t-i}$, illustrates the effect of lagged differences on current changes.

- rank(Π): The rank of the cointegration matrix, which determines the number of cointegration relationships:
  - If rank(Π) $= 0$, there is no cointegration relationship, and the time series have no long-term equilibrium.
  - If $0 <$ rank(Π) $< n$, there exist r cointegration vectors.
  - If rank(Π) $= n$, all variables are stationary time series.

There are two statistics that can be used to determine the number of cointegration relationships. Table 4.2.7 shows the differences between them.

Table 4.2.7: Comparison of Trace and Maximum Eigenvalue Statistics

Criteria	**Trace Statistic**	**Max Eigenvalue Statistic**
Formula	$-T \sum_{i=r+1}^{n} \ln(1 - \lambda_i)$	$-T \ln(1 - \lambda_{r+1})$
Meaning of λ	λ_i: Eigenvalues	λ_{r+1}: The largest eigenvalue of the next rank
Null Hypothesis (H_0)	The number of cointegration is r.	The number of cointegration is r.
Alternative Hypothesis (H_1)	The number of cointegration is greater than r.	The number of cointegration increases to $r + 1$.

Table A.1 and A.2 demonstrate that there are five cointegration relationships (statistics > critical values) in both eastern and western cities, indicating that housing prices in Canadian cities in different regions are affected by a common long-term trend. This provides a theoretical basis for the construction of a VECM model for the Canadian housing price market. The discrepancies in the Trace and Max Eigenvalue statistics of the eastern and western city groups also indicate the regional heterogeneity of the Canadian housing price market.

4.3 House Price Simulation

4.3.1 Model Construction

According to the formula 4.1.3 we obtained above, it is crucial to know how to get the value of the house H_t at any time. We choose to construct a VECM model to simulate the house value path.

Based on the previous lag selection and Johansen test results, we construct the following VECM models for the eastern and western city groups:

$$\begin{aligned}\Delta Y_t = {} & \alpha_{\text{west}}\beta'_{\text{west}}Y_{t-1} + \Gamma_1^{\text{west}}\Delta Y_{t-1} + \Gamma_2^{\text{west}}\Delta Y_{t-2} + \Gamma_3^{\text{west}}\Delta Y_{t-3} \\ & + \Gamma_4^{\text{west}}\Delta Y_{t-4} + \epsilon_t^{\text{west}}\end{aligned} \tag{4.3.1}$$

- Y_t: A vector of housing prices in western cities (e.g., Vancouver, Calgary, etc.), with dimensions 6×1.
- α_{west}: Adjustment speed matrix, describing how housing prices adjust when deviating from the long-term equilibrium.
- β_{west}: Cointegration vector matrix, describing the long-term equilibrium relationship.
- Γ_i^{west}: Short-term adjustment matrix, describing the effect of lagged terms on current housing price changes, for a total of 4 terms.
- ϵ_t^{west}: Random shocks, assumed to follow a normal distribution $N(0, \Sigma_{\text{west}})$.

$$\Delta Y_t = \alpha_{\text{east}}\beta'_{\text{east}}Y_{t-1} + \Gamma_1^{\text{east}}\Delta Y_{t-1} + \Gamma_2^{\text{east}}\Delta Y_{t-2} + \epsilon_t^{\text{east}} \tag{4.3.2}$$

- $\mathbf{Y_t}$: Includes the vector of housing prices for Eastern cities (e.g., Toronto, Montreal, etc.), with dimensions 6×1.
- α_{east}: Adjustment speed matrix.
- β_{east}: Cointegration vector matrix.
- Γ_i^{east}: Short-term dynamics adjustment matrices, $i = 1, 2$.
- ϵ_t^{east}: Random shock term, assumed to follow a normal distribution $N(0, \Sigma_{\text{east}})$.

For the western cities, R^2 values range from 0.9975 (Victoria) to 0.9997 (Winnipeg and Calgary), with similarly high adjusted R^2. For the eastern cities, R^2 values are consistently above 0.9995, with Montréal and Toronto achieving 0.9997. These exceptionally high goodness-of-fit statistics indicate that the estimated VECM models capture a substantial proportion of the variance in regional house price dynamics.

The high explanatory power can be attributed to the strong co-movement of housing prices across cities and the persistent nature of the housing market. Given that the variables are specified in first differences and cointegration terms are included, the VECM structure is well-suited to identify both short-term adjustments and long-run equilibrium trends. While this suggests the model fits historical data well, it may also reflect limited exposure to structural breaks or rare events within the sample period. Consequently, caution is warranted when interpreting the simulation-based results: although the high R^2 improves the stability of simulated paths used in NNEG pricing, it may understate tail risk if future shocks deviate significantly from historical dynamics.

Table 4.3.1: R^2 and Adjusted R^2 for VECM (West Cities)

City	$\mathbf{R^2}$	**Adjusted** $\mathbf{R^2}$
Winnipeg	0.9997	0.9997
Saskatoon	0.9991	0.9990
Calgary	0.9997	0.9997
Edmonton	0.9996	0.9996
Vancouver	0.9977	0.9975
Victoria	0.9975	0.9974

Table 4.3.2: R^2 and Adjusted R^2 for VECM (East Cities)

City	R^2	Adjusted R^2
St. Johns	0.9996	0.9996
Halifax	0.9996	0.9996
Québec	0.9996	0.9996
Montréal	0.9997	0.9997
Toronto	0.9997	0.9997
Hamilton	0.9996	0.9995

4.3.2 Random Shock

Before we formally begin the simulation of housing price paths, we would like to gain a better understanding of the potential impact of external market fluctuations on our model. It is assumed that each city's random shock comes from a multivariate normal distribution, which could be seen as the residual term in our VECM model,

$$\epsilon_t \sim N(0, \Sigma)$$

where the covariance matrix Σ describes the correlation and volatility of housing price changes in each city.

The covariance matrix is obtained by extracting the residual term from the VECM model. In order to ensure that the generated random shocks satisfy the structure of this covariance matrix,

$$\Sigma = \mathrm{Cov}(\epsilon_t)$$

we propose performing a Cholesky decomposition of the covariance matrix to obtain the lower triangular matrix L:

$$\Sigma = LL^\top$$

Then, random shocks are generated using the following equation:

$$\epsilon_t = Lz_t$$

where $z_t \sim N(0, I)$ represents samples from a standard multivariate normal distribution. The generated ϵ_t not only satisfies the normal distribution assumption but also adheres to the covariance structure among cities.

4.3.3 Path Simulation

After obtaining the results of the VECM model for the eastern and western cities, we take the following steps to simulate the future path of house prices:

Initialization

We use the historical house price data of the last period Y_t as the starting point to ensure that the simulated trajectory starts consistently with the actual market conditions.

Long-term Equilibrium Adjustment

Based on the cointegration relationship β and the adjustment speed matrix α, we calculate the long-term correction term:

$$\text{Long-term Correction} = \alpha(\beta' Y_{t-1})$$

Short-term Dynamics Adjustment

We combine the lagged values of changes $\Delta Y_{t-1}, \Delta Y_{t-2}, \ldots$ and the short-term adjustment matrices Γ_i, to calculate the short-term impact:

$$\text{Short-term Adjustment} = \sum_{i=1}^{k} \Gamma_i \Delta Y_{t-i}$$

Add Random Shock

We use the covariance matrix Σ and random sampling methods to generate a random shock term ϵ_t:

$$\epsilon_t \sim N(0, \Sigma)$$

We use the Monte Carlo simulation method to generate 500 different simulated price paths by randomly selecting 500 sets of normally distributed disturbance terms.

Path Update

We combine the long-term adjustment, short-term adjustment, and random shocks to calculate the house price change for the next period:

$$\Delta Y_t = \text{Long-term Correction} + \text{Short-term Adjustment} + \epsilon_t$$

Then we can update the house price vector Y_t and proceed to the next period.

In our simulation, we set the simulation step for each path to 600 steps, corresponding to a time span of 50 years (each step is 1 month).

Long-term Growth Rate

The graph A.4 shows the monthly growth rate of the average house price in Canada from January 2005 to October 2024, as provided by the Canadian Real Estate Association (CREA). As can be seen from the graph, the average house price in Canada has shown a high degree of volatility, with an annual growth rate of over 1% in the vast majority of years. However, in order to reduce the uncertainty of the future market in the simulation and avoid underestimating the mortgage value due to overly optimistic market expectations, we have chosen 1% as the long-term annual growth rate.

4.3.4 Subsequent Processing of Simulated Paths

After generating the simulated paths, a series of subsequent processes are applied to adapt the data to the analytical needs of the model and reverse mortgage calculations. These processes include path validity checks, normalization, and the conversion of monthly data to annual data.

Path Validity Check

To ensure the reasonableness of the simulated paths, a validity check is first performed. Specifically, to avoid the unrealistic scenario where random shocks cause housing prices to become negative, we examine the simulated paths for any negative values. If negative values are found in a path, the path is deemed invalid and removed. After invalid paths are removed, new paths are generated to maintain the total number of paths.

Path Normalization

Once all invalid paths have been removed, the starting points of the simulated paths for different cities are unified to 100 for better comparability of housing price simulations across cities. The normalization eliminates the influence of different cities' absolute price levels, making the paths more comparable. The normalization formula is as follows:

$$\text{Normalized Path}_{i,t} = \frac{\text{Simulated Path}_{i,t}}{\text{Simulated Path}_{i,0}} \times 100$$

where i represents the path index, t represents the time step, and Simulated $\text{Path}_{i,0}$ is the starting value of the path. After normalization, all paths start at 100 when $t = 0$, highlighting the relative price trends across cities.

Monthly to Annual Data Conversion

Since the forward-looking life table provides survival rates as annual data, we convert the normalized monthly simulated housing price paths into annual paths to maintain consistency. Specifically, values are extracted every 12 months, with January of each year chosen as the annual data point. The conversion formula is as follows:

$$\text{Annual Path}_{i,t} = \text{Normalized Path}_{i,t\cdot 12+1}$$

where t represents the year and i represents the path index.

Nationally Harmonized Path Simulation

In order to examine the uniformity of national house price dynamics and to analyze them in comparison with regional house pricing models, we generated nationwide house price simulation paths based on the East-West regional house paths. This provides the basis for exploring the differences between individual city and national pricing in our subsequent inverse house pricing.

We firstly use the normalized annual house price paths and randomly select the same number of paths from the normalized annual paths for each city, with the same total number of paths as simulated for each city. If the total number of simulated paths cannot be distributed evenly across all cities, the random selection can be continued from the remaining paths to replenish to the required total.

4.3.5 VAR-like Model

Based on the VECM model, we introduce a VAR-like model by removing the constraint of long-term cointegration. We set the long-term equilibrium adjustment coefficient α in the VECM model to zero, completely retaining the short-term dynamic adjustment part, so that the path generation is only affected by short-term fluctuations.

For the generation and processing of simulated price paths, VAR-like models are the same as VECM models.

4.4 Loan Duration Simulation

For reverse mortgage, the loan duration is the time it takes from loan origination to the borrower's death or moving out of the home (resulting in loan maturity).

Firstly, we set the corresponding simulation parameters (year of birth, gender) according to the different structure of the borrower's family (single or married), the main differences being:

- **Single Borrower:** Only the survival time of the borrower is considered.
- **Married Borrowers:** The survival times of both spouses are simulated, and the longer survival time of the two is used as the loan duration.

Then, we will use the prospective life table data obtained in Section 3.2 to calculate the annual survival probability s_a and cumulative survival probability $S(a+t)$ for borrowers at different ages. These probabilities respectively represent the probability of surviving from age a to $a+1$ and the probability of surviving from the current age a to a future age $a+t$. For married households, the annual survival probability and cumulative survival probability of the borrower's spouse also need to be calculated.

Finally, we use Monte Carlo simulations to generate a random number $u \in [0, 1]$ each year and compare it with the survival probability. When the random number exceeds the survival probability, the current age difference is recorded as the loan duration. For married households, the longer age difference is used as the loan duration. We simulate 500 times to match the number of simulations of the house price simulation path.

It is worth noting that to simplify the analysis process and focus on the impact of house price paths on reverse mortgage pricing, we uniformly adopted data from Quebec province in the prospective life tables for simulating loan duration. Although life expectancy assumptions may vary across provinces, we do not delve into these regional differences in detail. Instead, we chose to concentrate on how regional characteristics of house price paths influence the pricing model.

4.5 Fair Price Calculation

The fair value of the No–Negative–Equity Guarantee (NNEG) is calculated using simulated future house prices derived from the VAR or VECM models estimated in Section 4.2. These models capture regional housing price dynamics and generate stochastic price paths $\{H_t\}$ through Monte Carlo simulation (Section 4.3). Each simulated path is used to compute the terminal house value at the time of loan termination H_T, which directly enters the NNEG payoff structure. By combining these simulated house prices with the projected loan balance, the expected discounted losses due to the guarantee can be estimated.

4.5.1 Define Fair Price

Fair Value (FV) is the rate at which the net profit of the reverse mortgage program is zero by optimizing the insurance surcharge rate π.

The core idea of fair value is to balance the income and cost of the loan project. For each simulated path, the net profit is determined by the difference between the Minimum Initial Payment (MIP) and the NNEG cost:

$$\text{Profit} = \text{MIP} - \text{NNEG}$$

where:

- **Minimum Initial Payment (MIP):** The insurance premium paid by the borrower during the loan term, which is proportional to the insurance loading rate π.
- **NNEG Cost:** The potential loss incurred by the lender when the loan balance exceeds the house price.

By adjusting the insurance loading rate π, the rate that makes the net profit zero is found, i.e.,

$$\pi = \arg\min|\text{MIP} - \text{NNEG}|$$

4.5.2 Calculation Process

Data Input

To calculate the fair value, we need to input the simulated housing price paths, the duration of the loan, and the basic parameters of the loan. The housing price paths and loan duration are derived from Sections 4.3 and 4.4. The basic parameters of the loan include the following four parameters:

- L_0: The initial loan amount, calculated based on the borrower's house value and the LTV ratio. We set it as 50%. [1]

- r_{rmr}: The loan interest rate, which includes the base loan rate and the insurance premium rate:

$$r_{\text{rmr}} = r_{\text{heloc}} + \pi$$

  We assume r_{heloc} as 4%, which is the average rate that was offered on the Canadian market in 2017. [2]

- r: The discount rate, used to calculate the present value of future cash flows.

- c: The transaction cost ratio, representing the cost deduction during house sales. It is calibrated at 2% of the selling price.

NNEG Cost Calculation

NNEG cost is calculated using the following formula:

$$\text{NNEG} = \frac{1}{n}\sum_{i=1}^{n} \exp(-r \cdot T_i) \cdot \max(L_{T_i} - S_{T_i}, 0)$$

where:

- n: Number of simulated paths (typically 500).

- $L_{T_i} = L_0 \cdot \exp(r_{\text{rmr}} \cdot T_i)$: Loan balance, representing the cumulative loan principal and interest during the borrower's tenure.

- $S_{T_i} = (1 - c) \cdot H_{T_i}$: House value after deducting transaction costs.
  - H_{T_i}: The house price at the end of the i-th simulated path.

It is worth noting that although the valuation is performed under a risk-neutral framework, simulation of full house price paths is essential due to the non-linear payoff structure of the NNEG. Since the guarantee only triggers in adverse tail scenarios—when the loan balance exceeds the property value—higher-order moments of the price distribution (such as variance and skewness) significantly influence the expected cost. The use of

[1]In 2017, the CHIP program allowed people to borrow between 10% and 55% of the estimated equity of the residence.

[2]https://www.ratehub.ca

simulation allows us to capture this tail risk, which would otherwise be understated if only expected values were considered.

Minimum Initial Payment (MIP) Calculation

MIP measures the insurance premium paid by borrowers during the loan tenure. The calculation formula is as follows:

$$\text{MIP} = \frac{1}{n} \sum_{i=1}^{n} \sum_{t=0}^{T_i - 1} \exp(-r \cdot t) \cdot \pi \cdot L_0 \cdot \exp(r_{\text{rmr}} \cdot t)$$

where:

- T_i: Loan tenure of the i-th simulated path.
- t: The year within the loan tenure.
- $\exp(-r \cdot t)$: Discount factor for each year.

Find Fair Price π

We set the initial search range of π to [0, 0.02]. For each candidate value π_j, calculate:

$$\text{Profit}(\pi_j) = \text{MIP}(\pi_j) - \text{NNEG}(\pi_j)$$

Interpolation is used to gradually approach the zero solution. If no zero solution is found within this range, the range of π is expanded.

4.5.3 Illustrative Case: Dollar-Valued NNEG Example

To clarify the pricing mechanism and demonstrate the monetary implications of the No-Negative-Equity Guarantee (NNEG), we construct a representative case using simulated housing price paths. These paths are generated via the city-specific Vector Error Correction Models (VECMs) described in Section 4.2, which incorporate both long-term cointegration and short-term dynamics based on historical data from twelve major Canadian cities.

We focus on a representative household composed of a heterosexual married couple, with the male born in 1957 and the female in 1960. The simulation begins in 2024, implying that the couple is aged 67 and 64 at loan origination—an age range typical for reverse mortgage applicants. The initial house price is set at $500,000. The duration of the reverse mortgage is determined by the couple's joint survival, following mortality assumptions detailed in Section 4.5.2. All future cash flows—including NNEG payouts and Mortgage Insurance Premiums (MIP)—are discounted at a continuous annual rate of 2%.

From the full simulation output, we select the city of Montréal as a case study. Among the 500 Monte Carlo paths generated for Montréal, we extract three representative trajectories that correspond to the 5th, 50th, and 95th percentiles of the terminal house price distribution. These paths allow us to compare reverse mortgage outcomes under bearish, median, and bullish housing market scenarios.

This illustrative analysis includes the projected evolution of house value, loan balance, and the resulting NNEG and MIP values along each selected path, providing insight into how market dynamics affect the cost of guarantees embedded in reverse mortgage contracts.

Table 4.5.1: Simulated Path Summary in $ for 5% Percentile Case

Year	House Price	Loan Balance	NNEG	MIP	Duration
0	500000.000	250000.000	0.000	5000.000	
1	496029.052	265459.137	0.000	5309.182	
2	478548.544	281874.213	0.000	5637.484	
5	470624.480	337464.702	0.000	6749.294	
10	419292.392	455529.700	44623.156	9110.594	
15	452874.122	614900.778	171084.138	12298.016	
20	449635.333	830029.231	389386.607	16600.584	
25	407309.874	1120422.270	721258.591	22408.445	
30	402313.993	0.000	0.000	0.000	
35	353920.228	0.000	0.000	0.000	
End					**27**

Tables 4.5.1 to 4.5.3 present a representative example that illustrates the monetary implications of reverse mortgage guarantees under varying housing market conditions. Each table corresponds to a specific simulated trajectory for the Montréal housing market, selected from the 5th, 50th, and 95th percentiles of the terminal house price distribution. Together, these cases provide a transparent and intuitive view of how the No-Negative-Equity Guarantee (NNEG) and Mortgage Insurance Premiums (MIP) evolve across different scenarios, offering a practical interpretation of reverse mortgage risk-sharing.

In Table 4.5.1 (5% percentile), the house price declines steadily over time, eventually falling to around $354,000 after 35 years. Meanwhile, the loan balance grows consistently due to compounding interest, surpassing the house value by year 10 and resulting in a large

NNEG liability by year 25 (over $720,000). The corresponding MIP increases accordingly, reaching more than $22,000. This scenario highlights the core insurance function of the NNEG: it protects the borrower from downside housing risk while shifting the residual loss to the lender or insurer. In this context, the guarantee becomes especially valuable for retirees facing a prolonged decline in home equity.

Table 4.5.2 (50% percentile) shows a median case, where house prices initially fall but partially recover in later years. Nonetheless, the home value remains insufficient to cover the accumulated debt for most of the loan duration. The NNEG peaks around $875,000 by year 25, and the associated MIP climbs in parallel. This intermediate scenario reflects a typical market outcome where moderate volatility and longevity still result in negative equity exposure, reinforcing the importance of proper premium calibration.

Table 4.5.2: Simulated Path Summary in $ for 50% Percentile Case

Year	**House Price**	**Loan Balance**	**NNEG**	**MIP**	**Duration**
0	500000.000	250000.000	0.000	5000.000	
1	485622.894	265459.137	0.000	5309.183	
2	490700.688	281874.213	0.000	5637.484	
5	475063.933	337464.702	0.000	6749.294	
10	382288.210	455529.700	80358.054	9110.594	
15	280620.128	614900.778	339893.052	12298.016	
20	197632.698	830029.231	636349.199	16600.584	
25	249888.585	1120422.270	875537.334	22408.445	
30	402313.993	0.000	0.000	0.000	
35	486393.972	0.000	0.000	0.000	
End					**29**

By contrast, Table 4.5.3 (95% percentile) features a strong housing market recovery. Home values rise significantly—reaching nearly $1.4 million by year 35—and comfortably exceed the loan balance throughout most of the projection. In this case, the NNEG is only marginally triggered at isolated points (e.g., year 20 or 30), and the final insurance liability is negligible. Despite this, the MIP continues to accumulate based on modelled risk at origination, illustrating the *ex-ante* pricing logic of the insurance design. The borrower benefits fully from home price appreciation, while the insurer bears no cost—underscoring the asymmetric structure of the NNEG.

These examples make clear that the NNEG provides substantial protection in adverse housing markets and long-tenure scenarios, but may remain unused in strong markets. They also show how pricing this guarantee *ex-ante* through the MIP requires careful

Table 4.5.3: Simulated Path Summary in $ for 95% Percentile Case

Year	**House Price**	**Loan Balance**	**NNEG**	**MIP**	**Duration**
0	500000.000	250000.000	0.000	5000.000	
1	506660.615	265459.137	0.000	5309.183	
2	496566.017	281874.213	0.000	5637.484	
5	502893.440	337464.702	0.000	6749.294	
10	542992.636	455529.700	0.000	9110.594	
15	624114.205	614900.778	3268.857	12298.016	
20	812166.836	830029.231	34105.731	16600.584	
25	935163.686	1120422.270	203961.855	22408.445	
30	1135164.071	1142251.912	399957.073	30248.237	
35	1388603.980	0.000	0.000	0.000	
End					**34**

modeling of regional price dynamics and mortality risk. Ultimately, these tables serve not only to visualize the internal mechanics of the model but also to emphasize the policy relevance of geographically tailored and actuarially sound reverse mortgage pricing.

5 Results and Discussion

5.1 VECM Model

The results of the VECM model are presented in Appendix Tables A.3-A.8, which show the short-term dynamics, long-term equilibrium relationship, and the random fluctuation part that the model fails to explain in the urban housing price forecasting model in the eastern and western regions.

5.1.1 Long-term Relationship

Table A.3 and A.4 show the results of the long-term cointegration relationship (beta coefficient) and the speed of adjustment when deviating from the long-term relationship (alpha) between the eastern and western city groups. The results of the long-term cointegration relationship show that these cities, except Halifax, have a statistically significant long-term cointegration relationship. Although the direction and magnitude are different from city to city relative to the benchmark city (Winnipeg in the west and St. Johns in the east), these cities are closely linked at the long-term equilibrium level and together form a relatively stable housing price system.

Moreover, most cities have significant statistical adjustment coefficients, indicating that when real estate prices deviate from the long-term equilibrium, these cities will have a certain degree of price equilibrium recovery mechanism.

5.1.2 Short-term Dynamic

The short-term dynamic correlation coefficients of cities in the west (Table A.5) and the east (Table A.6) reflect the sensitivity of real estate prices in each city to price changes in other cities in the short term (a few months, depending on the lag period of the model). They show the transmission path and speed of price changes between cities beyond the long-term equilibrium.

We find that the autoregressive term of cities has a significant positive impact, which is statistically significant, whether it is an eastern or western city. This shows that there is a certain inertia and continuity in the price changes of urban real estate, which often continue the price change trend of the previous period.

However, there is significant complexity and heterogeneity in terms of short-term price interactions between cities. Neither in the western city group nor in the eastern city group is there a dominant city, and the direction and magnitude of short-term price transmission changes with the lag period and the city. This shows that, unlike long-term cointegration relationships, short-term dynamics are more dispersed and unstable, and more susceptible to temporary shocks.

5.1.3 Residual Covariance Matrix

The residual covariance matrix reflects the correlation between the residuals of the price prediction equation for each city. The results are shown in Table A.7 and A.8.

- **Diagonal Elements**: Represent the variance of the residuals for each city. This indicates the unexplained volatility of a city's price changes after accounting for the cointegration relationship and short-term dynamics. Larger diagonal elements suggest greater randomness or unaccounted volatility in the city's price changes.

- **Off-Diagonal Elements**: Represent the covariance between the residuals of different cities. A positive and larger covariance value indicates that, even after controlling for long-term relationships and short-term dynamics, the two cities exhibit synchronized residual movements (i.e., unexplained common movements). Conversely, a negative value indicates a negative correlation between the residual volatilities of the two cities.

Comparing cities in the east and west, the residual variance of cities in the west is significantly higher than that of other cities (Victoria, 1.4752), while the residual variance of cities in the east is more evenly distributed, indicating that the market residual uncertainty of the eastern model is more balanced. In addition, the majority of inter-city covariances in both the eastern and western cities are positive, which means that even after removing long-term relationships and short-term dynamics, there is still positive co-movement in the 'unexpected fluctuations' between cities. However, unlike the western cities, the overall covariance in the eastern cities is smaller, which indicates that there is no strong consistent residual shock at the residual level in the eastern cities.

5.2 House Price Simulation Results

5.2.1 Comparison Between City Groups

Figure 5.2.1 and 5.2.2 present the simulated house price paths for Montreal and Vancouver based on the VECM model. The simulated paths for all cities are in the Appendix A.5 and A.6.

Comparing the simulated paths of the eastern and western city groups, we can see that the house price paths of both eastern and western cities show an upward trend, but the volatility of the western city group is significantly higher than that of the eastern cities, which is also consistent with the results of the Cholesky decomposition of the residual covariance matrix.

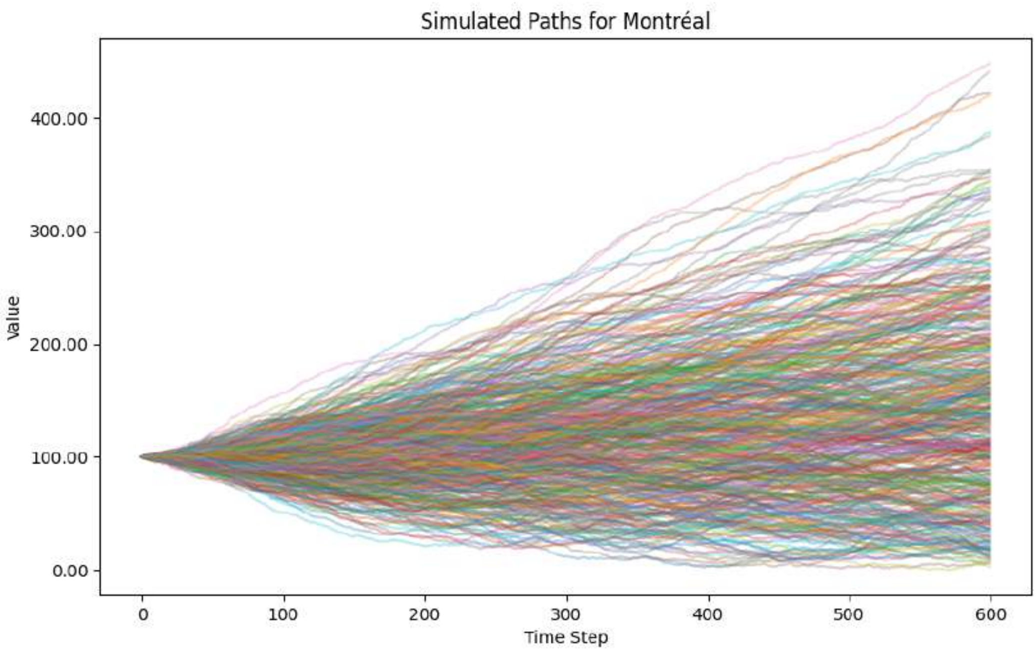

Figure 5.2.1: Montreal Simulation Path(VECM model)

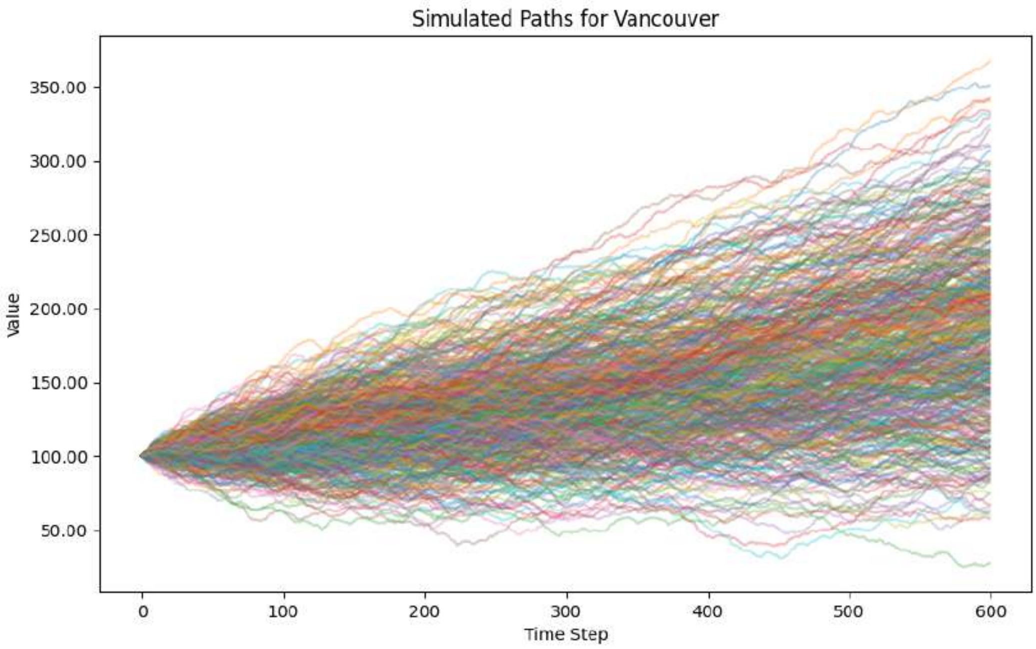

Figure 5.2.2: Vancouver Simulation Path(VECM model)

5.2.2 Comparison Between Models

Figure 5.2.3 demonstrates the simulated house price paths for Montreal based on the VAR-like model. The results for all cities of the house price simulation based on the VAR-like model are presented in the Appendix A.7 and A.8.

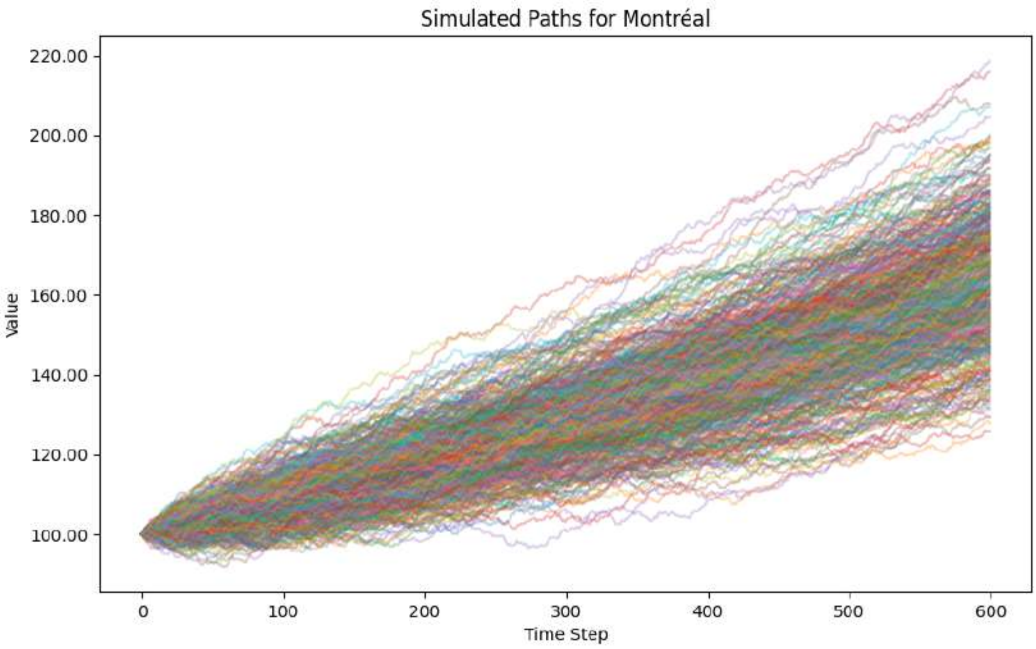

Figure 5.2.3: Montreal Simulation Path(VAR-like model)

Comparing the results of the house price simulations obtained in Montreal after simulations using different models, we see that the paths lack the pull-back effect of the long-run equilibrium when the long-run equilibrium relationship is removed, and that the trends in the cities are more influenced by the short-run trend, showing a more linear growth trend and a reduction in the overall degree of dispersion.

5.2.3 National House Price Simulation Path

Figure 5.2.4 and 5.2.5 demonstrates the simulated house price paths for Montreal based on the VAR-like model.

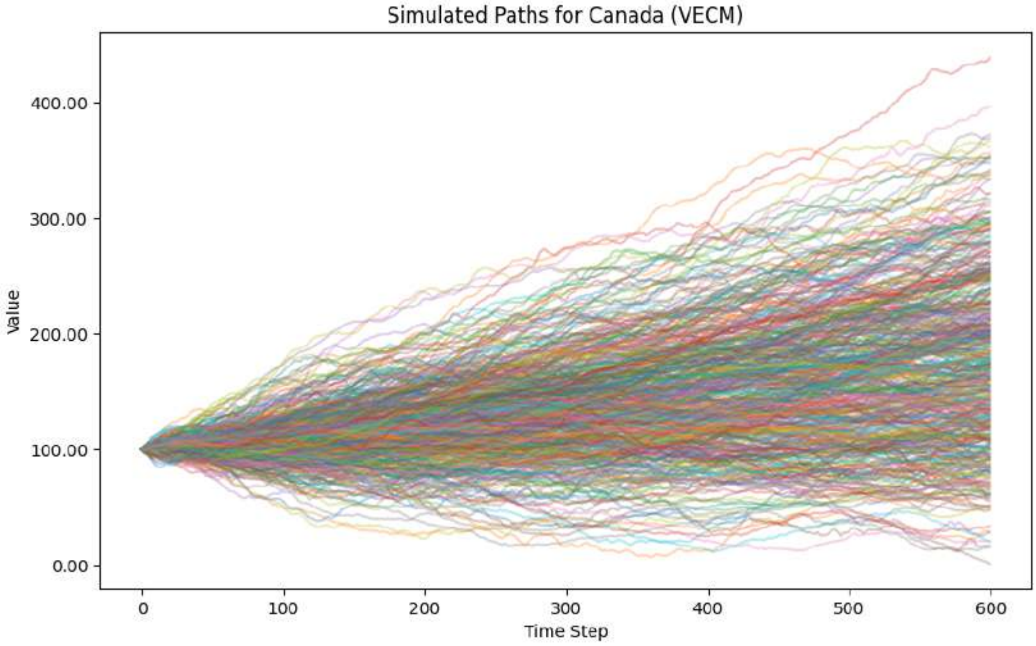

Figure 5.2.4: VECM Simulated Paths for Canada (Normalized Initial Value = 100)

Since the national house price simulation paths are randomly selected from the integration of the regional simulation paths, their basic properties are similar to those of the regional simulation paths. For example, the VAR-like path for national house prices is more volatile, reflecting the high level of short-term market uncertainty. However, by integrating house price dynamics across multiple regions, the national path smoothes out the volatility of the regional paths, resulting in a less volatile path overall than the regional paths and fewer extreme upward and downward paths than for individual cities.

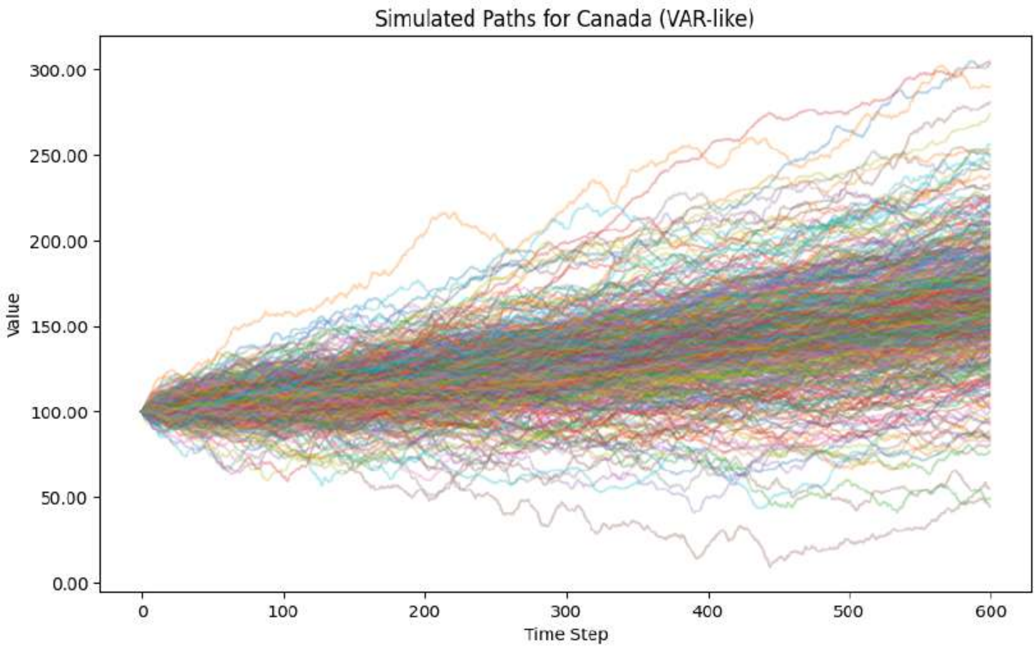

Figure 5.2.5: VAR Simulated Paths for Canada (Normalized Initial Value = 100)

5.3 Loan Duration Simulation

In order to simulate the loan duration, we use the following base setup:

Borrower Characteristics:

- **Primary Borrower (Head):** Born in **1959**, gender: **Male**.
- **Spouse:** Born in **1960**, gender: **Female**.
- This setup ensures that the life table data for the borrower and their spouse reflect differences in life expectancy based on gender, thereby improving the accuracy of the simulation results.

Simulation Settings:

- **Life Table Scenario:** Assumed as **Scenario L**, which corresponds to prospective life table data.
- **Province:** The simulation uses life table data for **Quebec (QC)**.
- **Starting Year:** The simulation begins in **2024**, serving as the baseline year for calculating loan durations.

Figure 5.3.1 displays the modelled loan duration distribution and its probability density function (PDF). The results suggest that there is a clear concentration of loan duration: the mean is 27.54 years, the median is 28 years, and the standard deviation is about 6 years. Most of the paths are concentrated in the 20-35 year range, reflecting the general trend of the individual life table simulations.

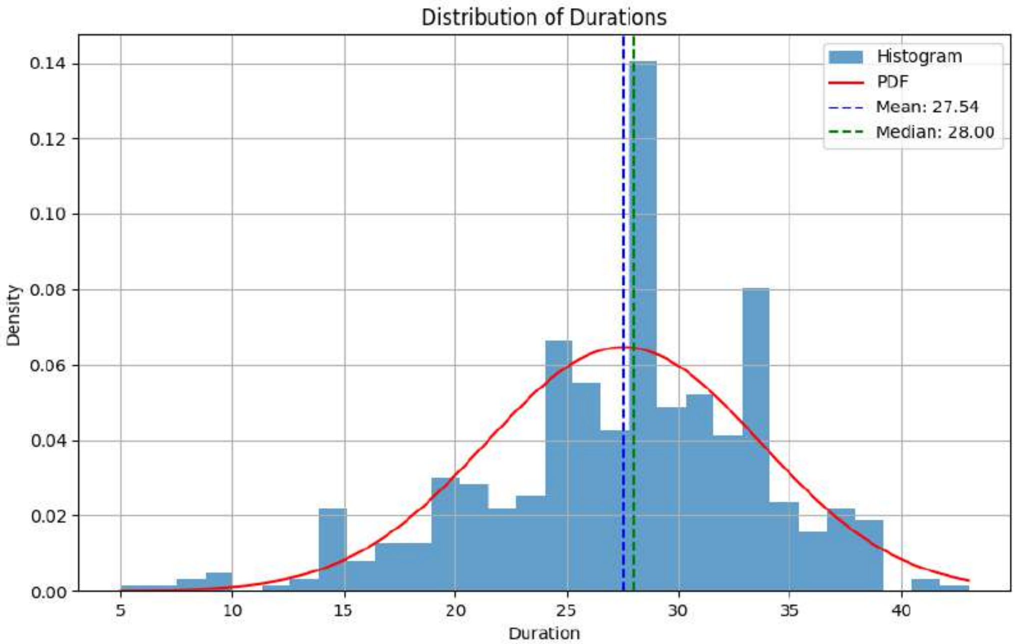

Figure 5.3.1: Duration Distribution Result

5.4 Reverse Mortgage Fair Price

Figure 5.4.1 compares the fair prices of reverse loans across cities that we calculated after simulating the future paths of house prices and the duration of loans in each city. The detailed values are in Appendix Table A.9.

5.4.1 Comparison Between Different Models and Different Cities

Under the VECM model, fair prices are in the range of 0.3 % to 0.5 % in most cities, and are relatively somewhat higher in eastern cities, especially Toronto and Montréal, which represents a greater downside risk to house prices in eastern cities. Under the VAR-like model, the vast majority of cities experience declines in fair prices, with extremely pronounced declines in the East. two western cities, Vancouver and Victoria, have different variations from the others, as they increase by 0.01 % and 0.03 %, respectively, compared to the VECM model.

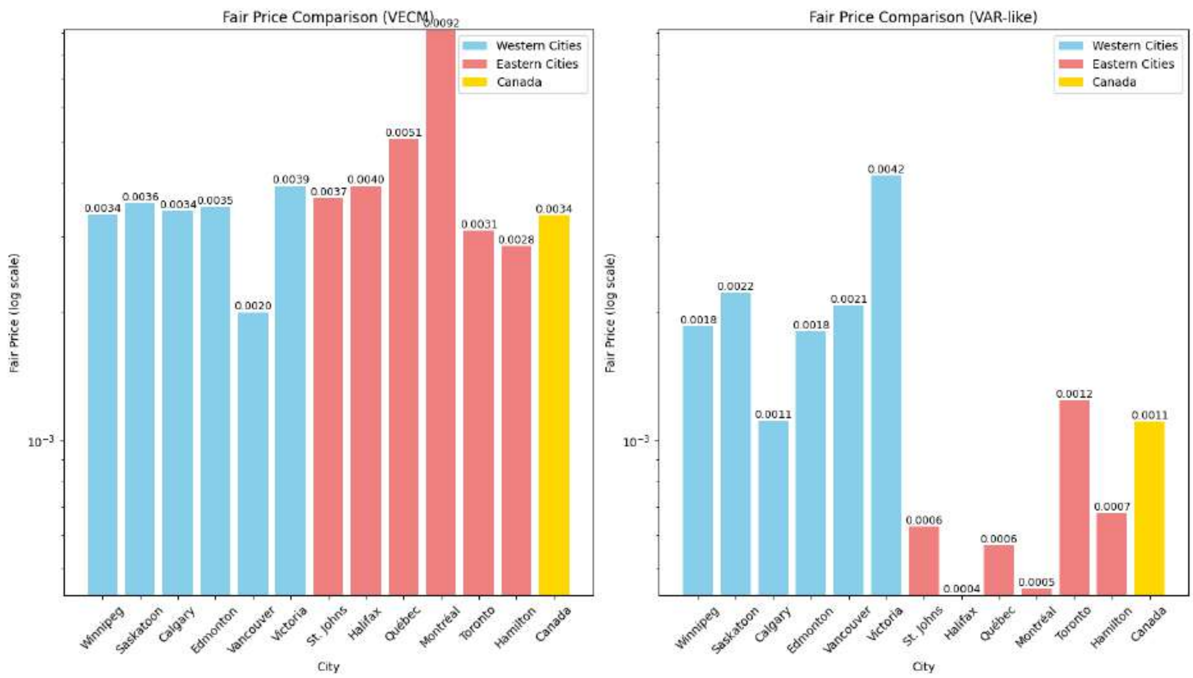

Figure 5.4.1: Fair Price Results(Different Models & Different Cities)

The reason for this result is that for the eastern group of cities, the α matrix (short-run adjustment coefficients) of the VECM model is negative, which means that the model's retraction effect on the city's deviation from the long-run covariance is reversed, which increases the downside risk of house prices and raises the fair price. In the VAR-like model, we remove the short-term adjustment coefficients so that the path of house prices is mainly driven by short-term dynamics, which are currently mostly on an upward trend (the positive effect of the Γ matrix), the downside risk of house prices is reduced, and the fair price falls significantly.

For the Western group of cities, the strength of the relationship between the short-run adjustment coefficients and the long-run equilibrium of their VECM model is smaller, allowing the house price path to be driven mainly by short-run dynamics. The strength of the cointegration relationship (β value) for Vancouver and Victoria is very small compared to the other cities, and is even negative for Victoria. Under the VAR-like model, the effect of the short-term adjustment coefficients on the house price path is removed, corresponding to an increased downside risk to house prices and a slight increase in fair prices.

5.4.2 Comparison Between National Uniform and City Heterogeneous Fair Prices

In addition to the above, Figure 5.4.1 reveals significant differences between the uniform and regional models in assessing fair price.

Under the VECM model, the fair price of the national path lies between the eastern and western city groups and slightly below the level of the eastern city group. We can see that the national path, by integrating the eastern and western city paths, dilutes regional differences to some extent, resulting in an overall performance in between. This smoothing effect masks the differences in long-run equilibrium constraints that exist in the regional paths, resulting in the fair price of the national path failing to reflect the heterogeneous characteristics in the regional paths. While overall the national path performance is stable it overestimates the downside risk of house prices in western cities and also overestimates the downside risk of house prices in eastern cities.

Under the VAR-like model, the fair price of the national path also falls sharply with the regional path, reflecting the dominance of short-term dynamics. However, due to differences in the strength of the short-term dynamics between the eastern and western regional models, the trend of the national path is more in line with that of the eastern group of cities, which leads us to underestimate the risk of the western cities in the results.

5.4.3 Market Comparison and Interpretation of NNEG Pricing

To better understand the fair prices derived from our model, we compare these prices with the interest rates observed in the actual reverse mortgage market in Canada. As of 2024, HomeEquity Bank—the dominant provider in Canada's reverse mortgage market (through its CHIP programme)—offers fixed 5-year interest rates ranging from 8.74% to 9.24%, depending on factors such as loan-to-value ratio, property location, and product type.[3] This is significantly higher than the actuarial fair value of the No Negative Equity Guarantee (NNEG) estimated by our model, which is based solely on house price dynamics, borrower survival rates, and a fixed 2% discount rate (0.11% to 0.92%).

The significant gap between the interest rates derived from the model and market rates stems from the fact that lenders must consider a range of real-world risks not accounted for in the model when pricing reverse mortgages. Borrowers exhibit significant heterogeneity in terms of financial status, health, life expectancy, and property usage

[3] https://www.homeequitybank.ca/rates/chiprates/

preferences. These differences influence loan duration, redemption behaviour, and cash flow patterns, thereby increasing the uncertainty of future cash flows.

Credit risks and operational risks are also present in the market. Even though reverse mortgages are secured by property, they may still face issues such as property disputes, default disputes, or appraisal discrepancies. The compliance, service, and management costs associated with these risks are not included in the model's NNEG valuation but must be compensated for through spreads in reality.

Our model relies on city-level house price indices, which are helpful for capturing general trends. However, they don't always reflect what's going on at a more granular level. The actual price of a home can depend on many things—location, upkeep, even how well it was built. Because of this, there's always some variation between what models estimate and what a property might sell for, especially when the market is down. That variation can put more pressure on lenders when it comes to fulfilling NNEG guarantees.

Another issue is liquidity. Not all housing markets behave the same. In some areas, it might take longer to sell a home, or the final price might fall short of what lenders expect. These kinds of delays can make it harder to recover funds quickly, particularly after a borrower defaults or passes away. For lenders, that's a real operational risk.

Even though our model provides sound actuarial estimates under risk-neutral assumptions, there's still a gap when it comes to real-world pricing. The fact that market interest rates are typically higher than our model suggests probably reflects how lenders are reacting to uncertainty—things like market volatility, property-specific risks, and payout timing. It's not just a technical detail. In a way, this gap shows that lenders are building in buffers to stay protected in practice.

6 Conclusions and Limits

6.1 Conclusions

In recent years, Canada has faced increasingly severe challenges related to population aging, placing mounting financial pressure on its traditional pension system. As public pension coverage declines and private savings prove insufficient, housing wealth has emerged as a crucial means of addressing retirement financial challenges. However, despite reverse mortgages offering retired households a flexible way to monetize their housing wealth, market acceptance remains constrained by complex pricing mechanisms and risk management challenges.

This research uses housing price data from major Canadian cities as its foundation, integrating VAR and VECM models to conduct dynamic simulations of house price trajectories and thoroughly examine how regional price variations influence the fair pricing of No-Negative-Equity Guarantee (NNEG). The findings reveal significant heterogeneity across regions in both short-term fluctuations and long-term equilibrium trends. While the VECM model captures long-term equilibrium relationships between variables while revealing how short-term adjustment paths respond to external shocks, the VAR model focuses on dynamic changes in short-term fluctuations. These model results suggest that while nationwide uniform pricing can smooth risk assessment, it may overlook regional characteristics - potentially overestimating risk in high-volatility regions while underestimating it in low-volatility areas.

Further analysis indicates that pricing strategies incorporating regional path simulations can more effectively address housing price heterogeneity. By integrating regional house price dynamics with mortality table data, this paper presents an innovative reverse mortgage pricing framework that not only theoretically enhances NNEG pricing mechanisms but also provides financial institutions with practical solutions for optimizing risk management. This approach helps enhance the market appeal of loan products while promoting efficient utilization of housing wealth within Canada's pension system, thereby supporting the financial security of the aging population.

This research carries significant policy implications. Firstly, policymakers should support risk mitigation strategies based on regional differences to improve product pricing accuracy. Secondly, enhancing public awareness of NNEG insurance mechanisms can reduce consumer misunderstandings, thereby promoting healthy development of the reverse mortgage market. Finally, encouraging financial institutions to incorporate regional pricing flexibility in loan design helps improve market adaptability.

Nevertheless, this research has certain limitations. Firstly, when computing forward-looking mortality tables, the model applied uniform treatment to nationwide mortality data, failing to differentiate regional variations in mortality tables, potentially underestimating the pricing impact of longevity risk in certain regions. Secondly, the capture of house price dynamics primarily relies on historical property price data from different cities, without incorporating other external variables that might significantly influence house prices, such as regional economic growth, population mobility, and interest rate changes. Moreover, the model assumes historical characteristics of house price paths will persist in the future but doesn't fully account for potential market impacts from extreme economic events. Future research could enhance model applicability and predictive capability by integrating broader data variables, introducing scenario simulation techniques, and refining mortality table analysis.

In conclusion, this research reveals the importance of regional house price dynamics in reverse mortgage pricing through theoretical and empirical analysis. The findings provide a basis for financial institutions and policymakers to optimize strategies, offering valuable reference for Canadian pension financial product design and policy innovation, while proposing potential solutions to address the economic challenges posed by population aging.

6.2 Limits and future research

While the study proposes a tractable pricing framework for no-negative-equity guarantees (NNEG) in the Canadian reverse mortgage market, several modeling assumptions and simplifications may affect the broader applicability of its findings.

One key limitation lies in the model's exclusive focus on the supply side of the market. The pricing logic is based entirely on risk-neutral valuation from the lender's perspective, without incorporating the borrower's decision-making process. Factors such as liquidity needs, utility from housing services, bequest motives, or behavioral biases are omitted. As a result, the derived premiums reflect purely technical risk-adjusted prices and cannot capture actual market-clearing rates or explain low take-up rates. This limits the model's relevance in evaluating product adoption, behavioral barriers, and demand-driven policy incentives.

Moreover, the model assumes that regional house price indices are the dominant risk drivers, while individual property-level heterogeneity is excluded. Risks stemming from location-specific depreciation, maintenance conditions, or micro-market shocks are not represented in the simulation. This simplification may lead to an underestimation of tail risk embedded in the guarantee and could bias the fair premium downward. Consequently, conclusions regarding insurer exposure and fair value benchmarks may understate the need for capital buffers or regulatory safeguards.

Although the empirical section estimates a vector error correction model (VECM) to capture long–run cointegration across regional housing markets, the simulation process deliberately uses both the full VECM and a simplified VAR–like model. In the VAR–like model, the cointegration adjustment term $\alpha\beta' x_{t-1}$ is set to zero. This dual–model approach is designed to contrast house price dynamics and NNEG outcomes under scenarios with and without long–run equilibrium reversion. While the VAR–like model improves path stability and isolates short–run fluctuations, omitting the error–correction mechanism may distort long–run tail behavior and bias the estimation of NNEG losses, particularly over longer horizons.

In addition, the model includes only regional house prices as endogenous variables in the VECM, without incorporating macroeconomic fundamentals such as interest rates, household income, or unemployment. This omission hinders structural interpretation of housing price dynamics and prevents counterfactual simulations of monetary or fiscal policy shocks. As a result, the framework cannot inform how systemic changes—like interest rate hikes or income subsidies—would affect NNEG pricing across regions.

Moreover, while the use of nominal house prices aligns with real-world settlement values and ensures internal consistency with nominal discounting, it introduces an additional layer of complexity. Specifically, nominal price paths embed inflation-driven drift that may not reflect underlying real housing market risk. Over long horizons, cumulative inflation effects can reduce the probability of simulated negative equity events, thereby understating the value of the guarantee. This is particularly relevant when comparing regional results, as differences in local inflation trends may obscure true variation in housing market volatility.

Finally, the model assumes a constant discount rate for the entire valuation horizon. While this assumption facilitates closed-form pricing, it neglects interest rate volatility and term structure risks. In reality, market rates evolve in response to monetary policy and macroeconomic conditions. The use of a fixed rate could misstate the present value of future losses or premiums, particularly for long-duration contracts, and may affect the calibration of solvency margins.

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A Appendix

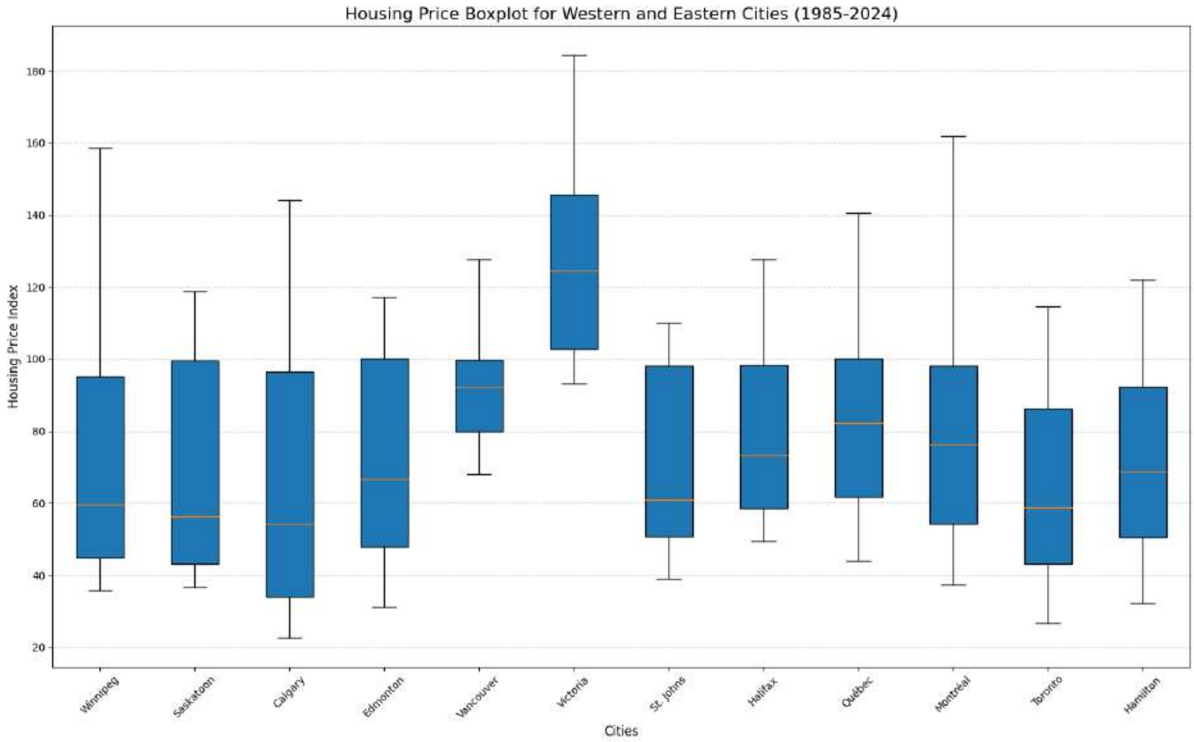

Figure A.1: Long-term housing price Summary, by city

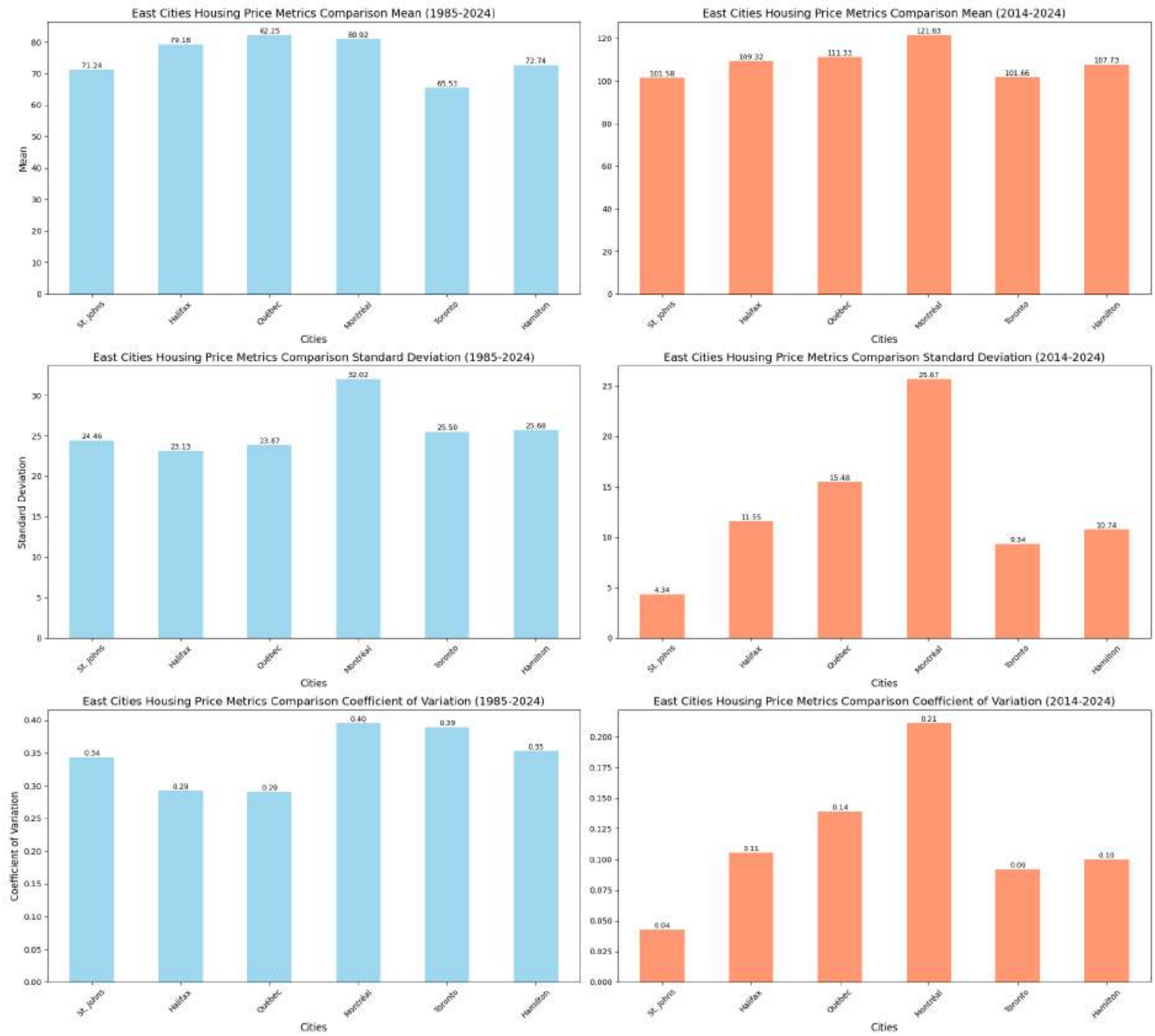

Figure A.2: Eastern city comparison: long term vs. short term

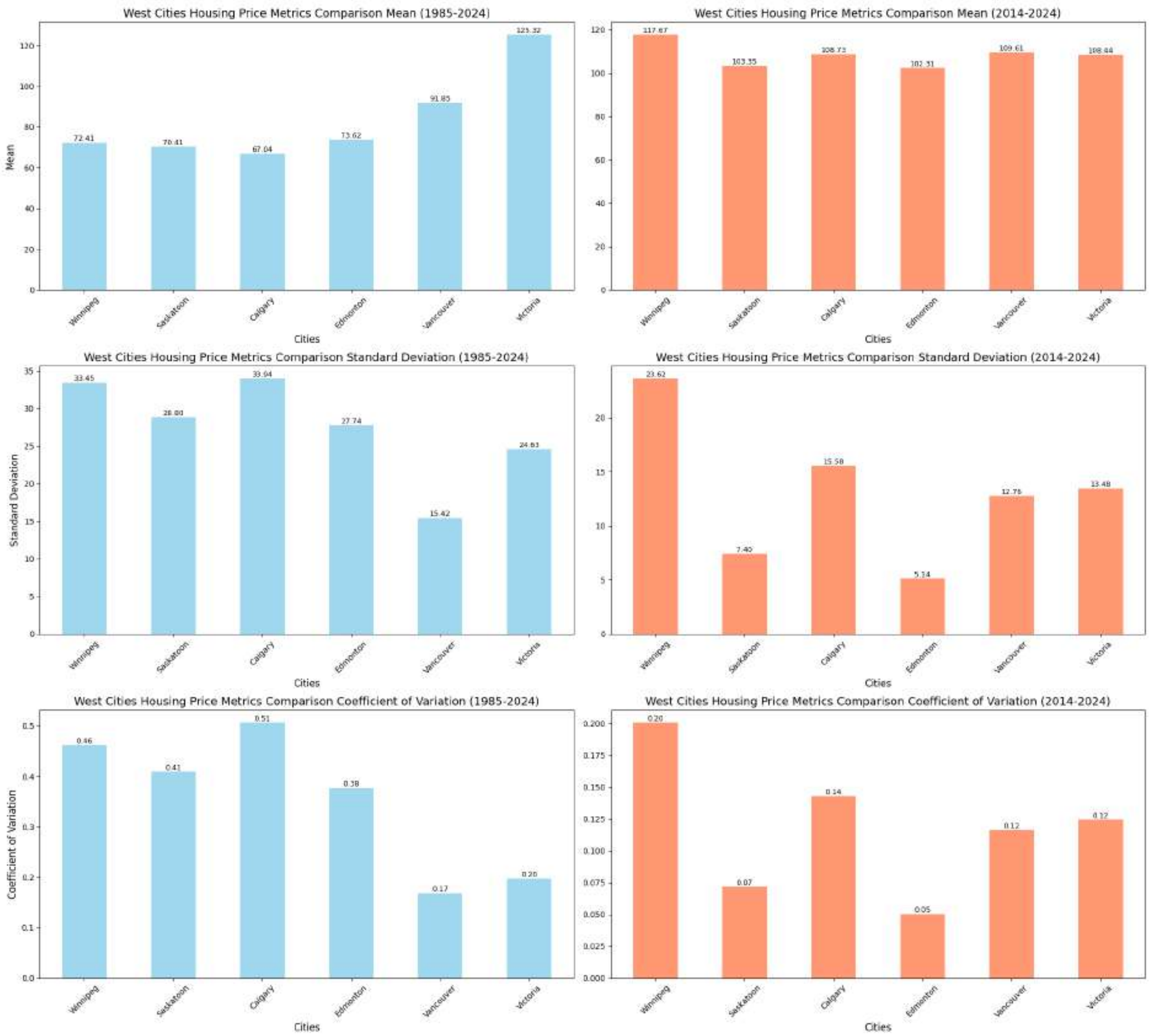

Figure A.3: Western city comparison: long term vs. short term

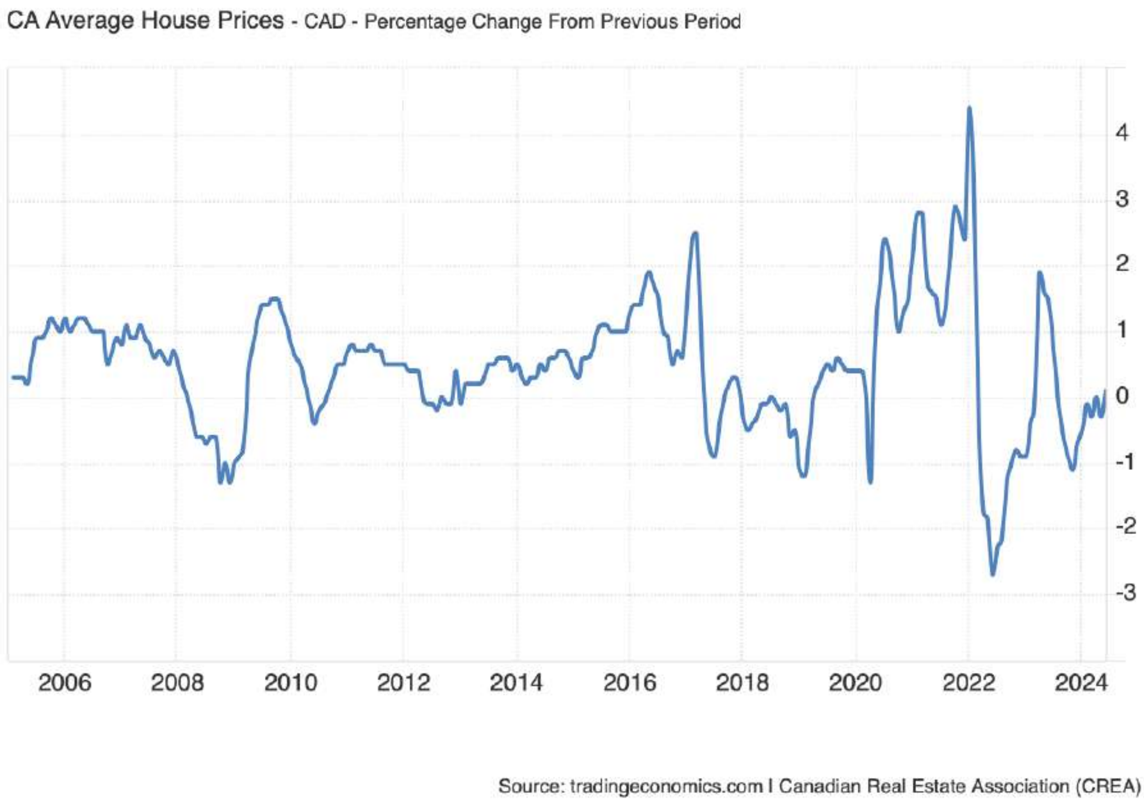

Figure A.4: Canadian Average House Price Monthly Growth Rate

Table A.1: East Cities Cointegration Test

Test	**Statistic**	**Critical Value (1%)**	**Critical Value (5%)**	**Critical Value (10%)**
Trace $r = 0$	709.8613	102.4674	107.3429	116.9829
Trace $r = 1$	455.0610	75.1027	79.3422	87.7748
Trace $r = 2$	314.2446	51.6492	55.2459	62.5202
Trace $r = 3$	181.6358	32.0645	35.0116	41.0815
Trace $r = 4$	88.4399	16.1619	18.3985	23.1485
Trace $r = 5$	29.7084	2.7055	3.8415	6.6349
Max Eigenvalue $r = 0$	254.8003	40.5244	43.4183	49.4095
Max Eigenvalue $r = 1$	140.8164	34.4202	37.1646	42.8612
Max Eigenvalue $r = 2$	132.6087	28.2398	30.8151	36.1930
Max Eigenvalue $r = 3$	93.1959	21.8731	24.2522	29.2631
Max Eigenvalue $r = 4$	58.7316	15.0006	17.1481	21.7465
Max Eigenvalue $r = 5$	29.7084	2.7055	3.8415	6.6349

Table A.2: West Cities Cointegration Test

Test	**Statistic**	**Critical Value (1%)**	**Critical Value (5%)**	**Critical Value (10%)**
Trace $r = 0$	380.1488	102.4674	107.3429	116.9829
Trace $r = 1$	251.4834	75.1027	79.3422	87.7748
Trace $r = 2$	171.8126	51.6492	55.2459	62.5202
Trace $r = 3$	110.6368	32.0645	35.0116	41.0815
Trace $r = 4$	56.0375	16.1619	18.3985	23.1485
Trace $r = 5$	27.3923	2.7055	3.8415	6.6349
Max Eigenvalue $r = 0$	128.6649	40.5244	43.4183	49.4095
Max Eigenvalue $r = 1$	79.6720	34.4202	37.1646	42.8612
Max Eigenvalue $r = 2$	61.1759	28.2398	30.8151	36.1930
Max Eigenvalue $r = 3$	54.5996	21.8731	24.2522	29.2631
Max Eigenvalue $r = 4$	28.6449	15.0006	17.1481	21.7465
Max Eigenvalue $r = 5$	27.3923	2.7055	3.8415	6.6349

Table A.3: Long-term Relationships Coefficients and Significance of Western Cities

(a) Cointegration Vector (Beta Coefficients)

City	Coef	Std Err	Z	P-Value
Winnipeg	1.0000	-	-	-
Saskatoon	-1.0912	0.245	-4.457	0.000***
Calgary	-1.1350	0.186	-6.105	0.000***
Edmonton	1.9141	0.227	8.426	0.000***
Vancouver	-0.6432	0.173	-3.707	0.000***
Victoria	0.3440	0.092	3.752	0.000***
Constant	-39.7938	11.350	-3.506	0.000***

(b) Adjustment Coefficients (Alpha Values)

City	Coef	Std Err	Z	P-Value
Winnipeg	0.0106	0.004	2.831	0.005**
Saskatoon	0.0264	0.006	4.395	0.000***
Calgary	0.0177	0.004	4.565	0.000***
Edmonton	0.0078	0.004	2.134	0.033*
Vancouver	0.0022	0.005	0.421	0.674
Victoria	-0.0074	0.008	-0.879	0.379

Note: Significance levels are denoted as follows: *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$.

Table A.4: Long-term Relationships Coefficients and Significance of Eastern Cities

(a) Cointegration Vector (Beta Coefficients)

City	Coef	Std Err	Z	P-Value
St. Johns	1.0000	-	-	-
Halifax	-1.8255	1.756	-1.039	0.299
Québec	9.8884	1.619	6.108	0.000***
Montréal	-2.7482	1.008	-2.726	0.006**
Toronto	9.2341	1.527	6.049	0.000***
Hamilton	-14.7755	2.206	-6.699	0.000***
Constant	-85.3694	34.973	-2.441	0.015*

(b) Adjustment Coefficients (Alpha Values)

City	Coef	Std Err	Z	P-Value
St. Johns	-0.0018	0.001	-3.397	0.001***
Halifax	-0.0016	0.000	-3.169	0.002**
Québec	-0.0029	0.000	-5.993	0.000***
Montréal	-0.0034	0.001	-5.971	0.000***
Toronto	-0.0006	0.000	-1.262	0.207
Hamilton	-0.0010	0.001	-1.695	0.090

Note: Significance levels are denoted as follows: *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$.

Table A.5: Short-Term Relationships Coefficients for Each Western City

Lag	Variable	Coef	Std Err	P-Value
(a) Winnipeg: Gamma Coefficients				
L1	Winnipeg	0.291	0.049	0.000***
	Saskatoon	-0.0008	0.028	0.978
	Calgary	0.152	0.047	0.001**
	Edmonton	0.109	0.050	0.028*
	Vancouver	-0.0563	0.034	0.096
	Victoria	0.0110	0.020	0.590
L2	Winnipeg	0.2189	0.050	0.000***
	Saskatoon	-0.0309	0.028	0.270
	Calgary	-0.0858	0.052	0.101
	Edmonton	-0.0484	0.047	0.303
	Vancouver	0.0671	0.035	0.056
	Victoria	0.0465	0.021	0.025*
L3	Winnipeg	0.1627	0.050	0.001**
	Saskatoon	0.0526	0.028	0.064
	Calgary	-0.1741	0.053	0.001**
	Edmonton	-0.0003	0.046	0.995
	Vancouver	0.0518	0.035	0.141
	Victoria	-0.0472	0.021	0.024*
L4	Winnipeg	-0.0186	0.050	0.710
	Saskatoon	0.0072	0.028	0.801
	Calgary	0.1004	0.048	0.037*
	Edmonton	-0.1538	0.047	0.001**
	Vancouver	0.0706	0.035	0.042*
	Victoria	-0.0171	0.020	0.404

Note: Significance levels are denoted as follows: *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$.

Lag	Variable	Coef	Std Err	P-Value
(b) Saskatoon: Gamma Coefficients				
L1	Winnipeg	0.312	0.051	0.000***
	Saskatoon	-0.0105	0.029	0.825
	Calgary	0.122	0.048	0.012*
	Edmonton	0.115	0.051	0.027*
	Vancouver	-0.0461	0.035	0.190
	Victoria	0.0225	0.022	0.318
L2	Winnipeg	0.2015	0.051	0.000***
	Saskatoon	-0.0459	0.029	0.136
	Calgary	-0.0653	0.053	0.215
	Edmonton	-0.0375	0.048	0.435
	Vancouver	0.0772	0.036	0.031*
	Victoria	0.0568	0.022	0.012*
L3	Winnipeg	0.1854	0.042	0.000***
	Saskatoon	-0.0321	0.030	0.278
	Calgary	-0.0739	0.045	0.118
	Edmonton	-0.0402	0.037	0.285
	Vancouver	0.0593	0.027	0.030*
	Victoria	0.0481	0.020	0.015*
L4	Winnipeg	-0.1287	0.055	0.019*
	Saskatoon	0.0172	0.038	0.651
	Calgary	0.0876	0.049	0.076
	Edmonton	-0.1174	0.045	0.010**
	Vancouver	0.0641	0.033	0.052
	Victoria	-0.0293	0.021	0.162

Note: Significance levels are denoted as follows: *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$.

Lag	Variable	Coef	Std Err	P-Value
	(c) Calgary: Gamma Coefficients			
L1	Winnipeg	0.179	0.051	0.000***
	Saskatoon	0.0154	0.029	0.592
	Calgary	0.5203	0.049	0.000***
	Edmonton	0.1376	0.052	0.008**
	Vancouver	0.0401	0.035	0.255
	Victoria	0.0007	0.021	0.973
L2	Winnipeg	-0.0937	0.052	0.070
	Saskatoon	-0.0254	0.029	0.385
	Calgary	0.1461	0.054	0.007**
	Edmonton	0.0446	0.049	0.362
	Vancouver	0.0361	0.036	0.323
	Victoria	0.0370	0.022	0.086
L3	Winnipeg	-0.0740	0.052	0.157
	Saskatoon	-0.0002	0.030	0.932
	Calgary	-0.0440	0.055	0.421
	Edmonton	0.0147	0.048	0.758
	Vancouver	-0.0217	0.037	0.554
	Victoria	-0.0156	0.022	0.473
L4	Winnipeg	0.0015	0.052	0.977
	Saskatoon	0.0209	0.030	0.481
	Calgary	0.0290	0.050	0.576
	Edmonton	-0.2858	0.049	0.000***
	Vancouver	0.0467	0.036	0.196
	Victoria	-0.0126	0.021	0.553

Note: Significance levels are denoted as follows: *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$.

Lag	Variable	Coef	Std Err	P-Value
(d) Edmonton: Gamma Coefficients				
L1	Winnipeg	0.0570	0.048	0.237
	Saskatoon	0.0596	0.027	0.028*
	Calgary	0.2822	0.046	0.000***
	Edmonton	0.1249	0.049	0.011*
	Vancouver	0.0628	0.033	0.058
	Victoria	-0.0023	0.020	0.908
L2	Winnipeg	-0.1352	0.049	0.006**
	Saskatoon	0.0965	0.027	0.000***
	Calgary	0.0253	0.051	0.622
	Edmonton	0.0863	0.046	0.061
	Vancouver	0.0283	0.034	0.410
	Victoria	0.0014	0.020	0.946
L3	Winnipeg	-0.0525	0.049	0.285
	Saskatoon	-0.0268	0.028	0.000***
	Calgary	0.0352	0.051	0.494
	Edmonton	0.2532	0.045	0.000***
	Vancouver	0.0095	0.034	0.783
	Victoria	-0.0629	0.020	0.002**
L4	Winnipeg	-0.0448	0.049	0.360
	Saskatoon	0.0551	0.028	0.048*
	Calgary	-0.1264	0.047	0.008**
	Edmonton	0.0703	0.046	0.124
	Vancouver	0.0556	0.034	0.101
	Victoria	0.0162	0.020	0.419

Note: Significance levels are denoted as follows: *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$.

Lag	Variable	Coef	Std Err	P-Value
(e) Vancouver: Gamma Coefficients				
L1	Winnipeg	0.1464	0.068	0.032*
	Saskatoon	0.0414	0.038	0.281
	Calgary	0.1024	0.065	0.115
	Edmonton	-0.0232	0.069	0.738
	Vancouver	0.1942	0.047	0.000***
	Victoria	0.0673	0.028	0.018*
L2	Winnipeg	-0.1247	0.069	0.071
	Saskatoon	0.0238	0.039	0.611
	Calgary	-0.0018	0.073	0.980
	Edmonton	-0.0988	0.065	0.130
	Vancouver	0.1730	0.049	0.000***
	Victoria	-0.0224	0.029	0.437
L3	Winnipeg	-0.0277	0.070	0.691
	Saskatoon	-0.0450	0.040	0.255
	Calgary	-0.0053	0.073	0.942
	Edmonton	0.0039	0.064	0.951
	Vancouver	0.1065	0.049	0.030*
	Victoria	0.0202	0.029	0.486
L4	Winnipeg	0.0105	0.069	0.880
	Saskatoon	0.0144	0.040	0.715
	Calgary	0.0378	0.067	0.573
	Edmonton	0.1141	0.065	0.078
	Vancouver	0.0164	0.048	0.733
	Victoria	-0.0276	0.028	0.332

Note: Significance levels are denoted as follows: *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$.

Lag	Variable	Coef	Std Err	P-Value
(f) Victoria: Gamma Coefficients				
L1	Winnipeg	0.0093	0.112	0.934
	Saskatoon	-0.0517	0.063	0.409
	Calgary	-0.1012	0.106	0.339
	Edmonton	0.1588	0.113	0.160
	Vancouver	0.3311	0.077	0.000***
	Victoria	0.1877	0.046	0.000***
L2	Winnipeg	0.1056	0.113	0.349
	Saskatoon	-0.0969	0.064	0.127
	Calgary	-0.0145	0.119	0.903
	Edmonton	-0.1063	0.106	0.318
	Vancouver	0.0641	0.079	0.420
	Victoria	0.0692	0.047	0.140
L3	Winnipeg	0.0559	0.114	0.491
	Saskatoon	0.0586	0.064	0.363
	Calgary	-0.0719	0.119	0.546
	Edmonton	-0.0604	0.104	0.561
	Vancouver	0.0011	0.080	0.989
	Victoria	0.1219	0.047	0.010*
L4	Winnipeg	-0.0322	0.113	0.776
	Saskatoon	0.1052	0.064	0.103
	Calgary	0.1311	0.109	0.231
	Edmonton	-0.0601	0.106	0.569
	Vancouver	0.0114	0.079	0.884
	Victoria	-0.0260	0.046	0.574

Note: Significance levels are denoted as follows: *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$.

Table A.6: Short-Term Relationships Coefficients for Each Eastern City

Lag	Variable	Coef	Std Err	P-Value
(a) St. Johns: Gamma Coefficients				
L1	St. Johns	0.1621	0.045	0.000***
	Halifax	0.0897	0.050	0.074
	Québec	0.0627	0.048	0.193
	Montréal	-0.1346	0.044	0.002**
	Toronto	-0.0137	0.055	0.803
	Hamilton	-0.0072	0.045	0.873
L2	St. Johns	0.1157	0.045	0.010*
	Halifax	0.0417	0.050	0.401
	Québec	0.0264	0.052	0.614
	Montréal	0.0200	0.041	0.626
	Toronto	0.0248	0.055	0.448
	Hamilton	0.0528	0.046	0.251
(b) Halifax: Gamma Coefficients				
L1	St. Johns	0.0250	0.042	0.555
	Halifax	0.1071	0.047	0.022*
	Québec	0.1163	0.045	0.010*
	Montréal	-0.0678	0.041	0.095
	Toronto	-0.0509	0.051	0.323
	Hamilton	0.1613	0.042	0.000***
L2	St. Johns	-0.0393	0.042	0.350
	Halifax	0.0349	0.051	0.496
	Québec	0.1309	0.049	0.007**
	Montréal	0.0477	0.038	0.212
	Toronto	0.0970	0.052	0.059
	Hamilton	-0.0024	0.043	0.956

Note: Significance levels are denoted as follows: *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$.

Lag	Variable	Coef	Std Err	P-Value
(c) Québec: Gamma Coefficients				
L1	St. Johns	0.0358	0.042	0.393
	Halifax	-0.0023	0.046	0.961
	Québec	0.3356	0.044	0.000***
	Montréal	-0.1468	0.040	0.000***
	Toronto	-0.0405	0.051	0.426
	Hamilton	0.0843	0.041	0.042*
L2	St. Johns	-0.0178	0.042	0.668
	Halifax	-0.0620	0.042	0.148
	Québec	0.0594	0.048	0.218
	Montréal	0.0490	0.038	0.195
	Toronto	0.0883	0.051	0.084
	Hamilton	0.1310	0.042	0.002**
(d) Montréal: Gamma Coefficients				
L1	St. Johns	0.0624	0.049	0.201
	Halifax	-0.0417	0.054	0.439
	Québec	0.4245	0.052	0.000***
	Montréal	0.0510	0.047	0.275
	Toronto	0.1205	0.049	0.028*
	Hamilton	-0.0748	0.048	0.121
L2	St. Johns	-0.1049	0.048	0.030*
	Halifax	0.2945	0.071	0.000***
	Québec	0.1310	0.056	0.020*
	Montréal	0.0113	0.044	0.258
	Toronto	0.1350	0.049	0.004**
	Hamilton	-0.0097	0.049	0.840

Note: Significance levels are denoted as follows: *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$.

Lag	Variable	Coef	Std Err	P-Value
(e) Toronto: Gamma Coefficients				
L1	St. Johns	0.0276	0.039	0.477
	Halifax	-0.0063	0.043	0.883
	Québec	0.1361	0.041	0.001**
	Montréal	-0.0339	0.037	0.361
	Toronto	0.3614	0.047	0.000***
	Hamilton	0.0222	0.038	0.563
L2	St. Johns	0.0050	0.038	0.897
	Halifax	0.0047	0.042	0.912
	Québec	-0.0207	0.045	0.643
	Montréal	0.0110	0.035	0.754
	Toronto	0.2230	0.047	0.000***
	Hamilton	0.0216	0.039	0.582
(f) Hamilton: Gamma Coefficients				
L1	St. Johns	0.0343	0.050	0.491
	Halifax	-0.0116	0.055	0.833
	Québec	0.0844	0.053	0.110
	Montréal	-0.0712	0.048	0.135
	Toronto	0.1497	0.060	0.013*
	Hamilton	0.0569	0.049	0.249
L2	St. Johns	-0.0054	0.049	0.912
	Halifax	0.0702	0.063	0.237
	Québec	0.0277	0.057	0.643
	Montréal	0.0441	0.045	0.325
	Toronto	0.1350	0.050	0.013*
	Hamilton	0.0066	0.050	0.892

Note: Significance levels are denoted as follows: *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$.

Table A.7: Residual Covariance Matrix and Cholesky Decomposition (Western Cities)

(a) Residual Covariance Matrix

	Winnipeg	Saskatoon	Calgary	Edmonton	Vancouver	Victoria
Winnipeg	0.2872	0.0505	0.0912	0.0605	0.0261	0.0675
Saskatoon	0.0505	0.7456	0.0021	0.0074	0.0351	-0.0204
Calgary	0.0912	0.0021	0.3127	0.0948	0.0242	0.0441
Edmonton	0.0605	0.0074	0.0948	0.2769	0.0711	0.0491
Vancouver	0.0261	0.0351	0.0242	0.0711	0.5576	0.1422
Victoria	0.0675	-0.0204	0.0441	0.0491	0.1422	1.4752

(b) Cholesky Decomposition

	Winnipeg	Saskatoon	Calgary	Edmonton	Vancouver	Victoria
Winnipeg	0.5359					
Saskatoon	0.0942	0.8580				
Calgary	0.1701	-0.0163	0.5325			
Edmonton	0.1129	-0.0037	0.1418	0.4941		
Vancouver	0.0468	0.0356	0.0316	0.1243	0.7332	
Victoria	0.1259	-0.0376	0.0414	0.0420	0.1786	1.1926

Table A.8: Residual Covariance Matrix and Cholesky Decomposition (Eastern Cities)

(a) Residual Covariance Matrix

	St. Johns	Halifax	Québec	Montréal	Toronto	Hamilton
St. Johns	0.2424	0.0074	0.0012	-0.0008	0.0048	0.0051
Halifax	0.0074	0.2115	0.0099	0.0547	0.0061	-0.0121
Québec	0.0012	0.0099	0.2603	0.0130	0.0099	0.0336
Montréal	-0.0008	0.0547	0.0130	0.2786	0.0201	-0.0184
Toronto	0.0048	0.0061	0.0099	0.0201	0.1761	0.0577
Hamilton	0.0051	-0.0121	0.0336	-0.0184	0.0577	0.2880

(b) Cholesky Decomposition

	St. Johns	Halifax	Québec	Montréal	Toronto	Hamilton
St. Johns	0.4924					
Halifax	0.0151	0.4597				
Québec	0.0024	0.0214	0.4537			
Montréal	-0.0016	0.1190	-0.0027	0.5146		
Toronto	0.0098	0.0129	0.0193	0.0364	0.4174	
Hamilton	0.0104	-0.0266	0.0753	-0.0292	0.1379	0.5115

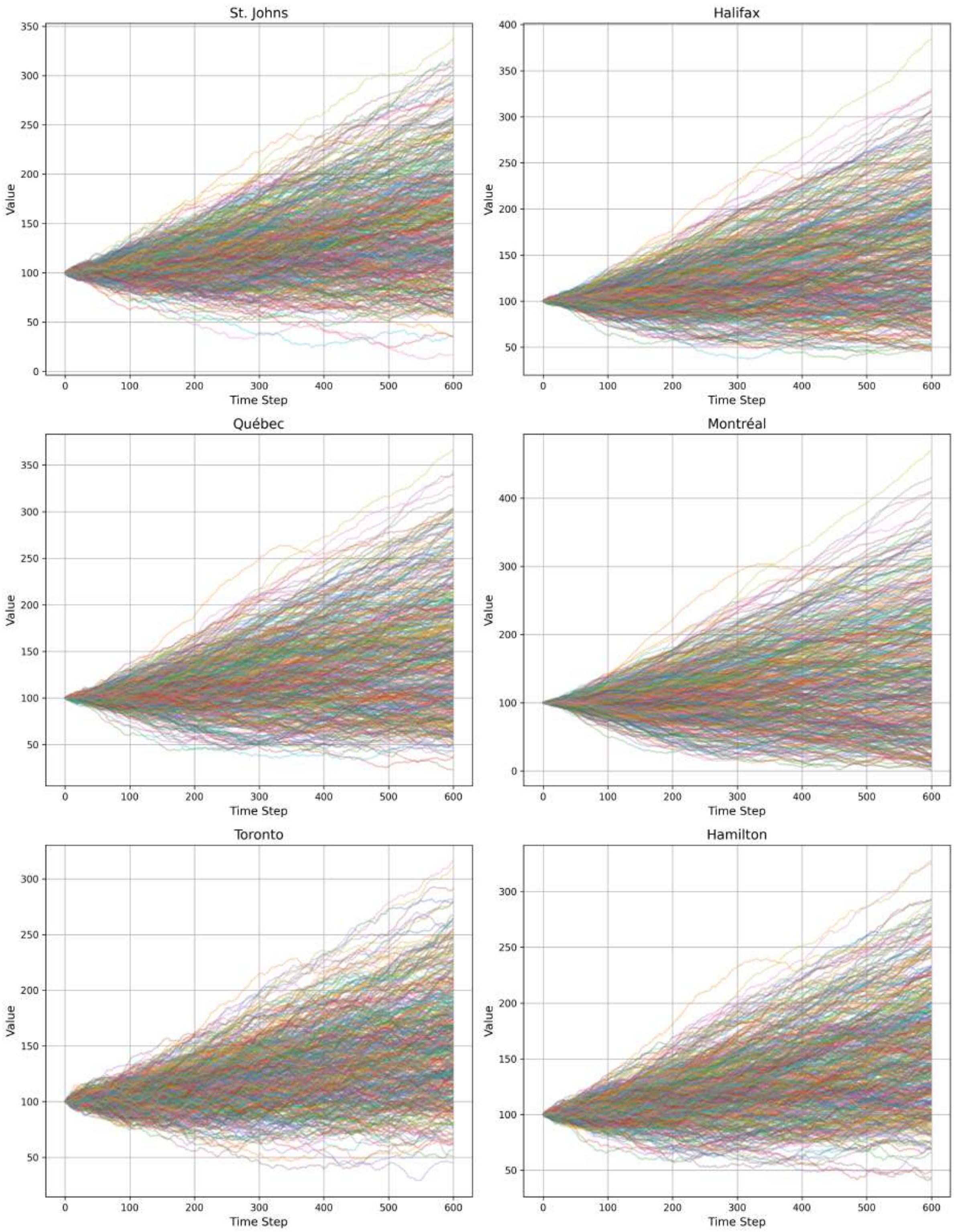

Figure A.5: VECM Simulated Paths for Eastern Cities (Normalized Initial Value = 100)

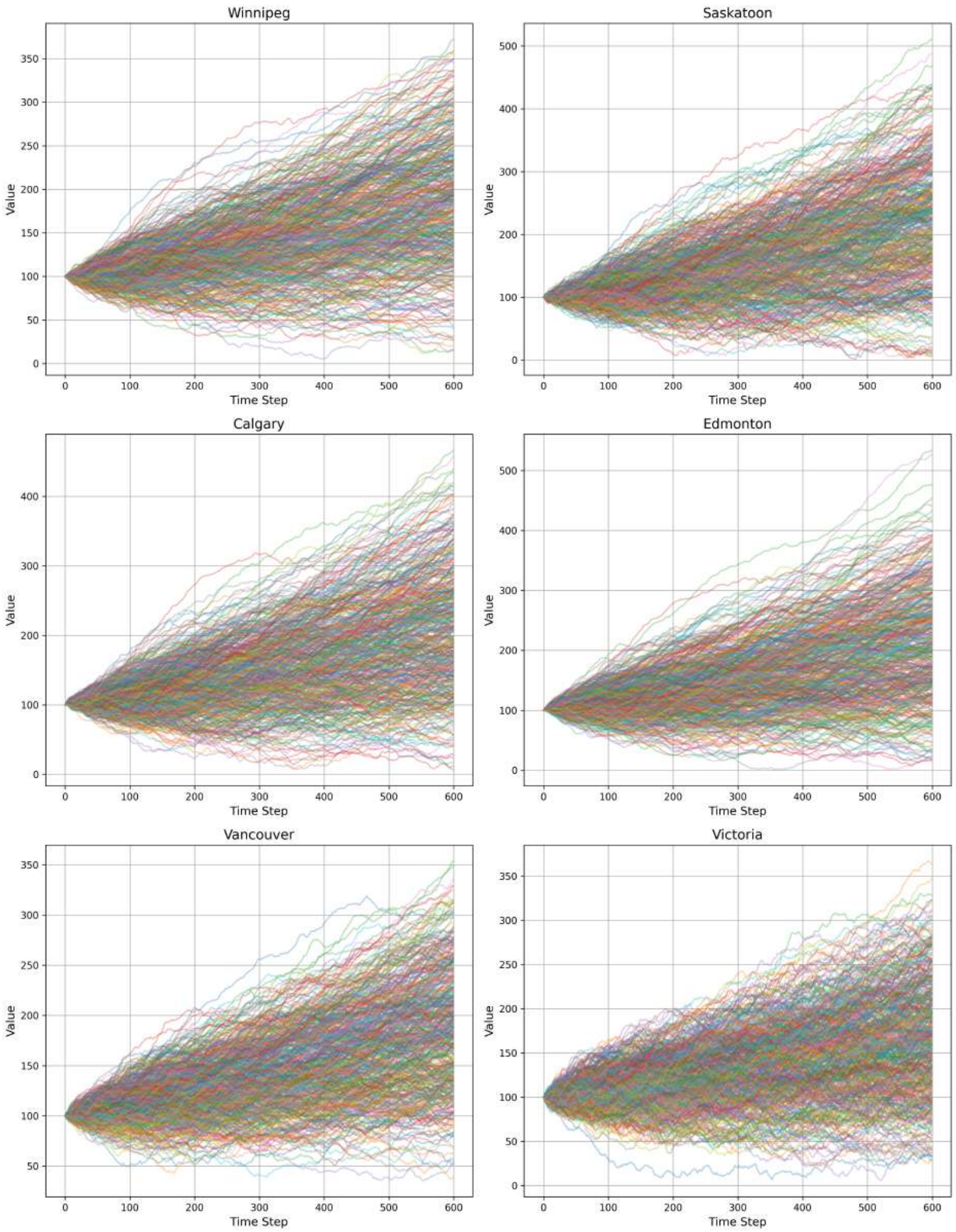

Figure A.6: VECM Simulated Paths for Western Cities (Normalized Initial Value = 100)

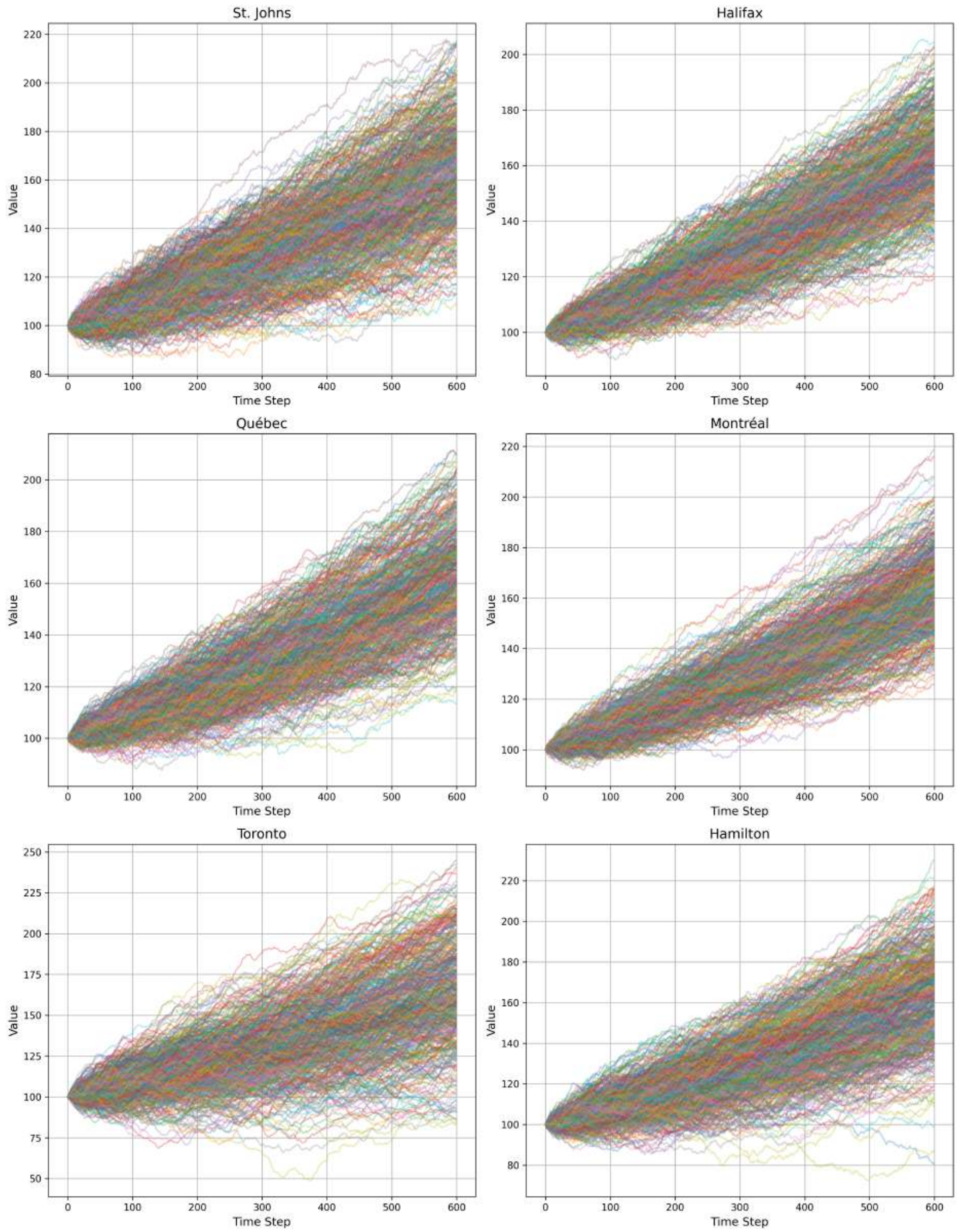

Figure A.7: VAR Simulated Paths for Eastern Cities (Normalized Initial Value = 100)

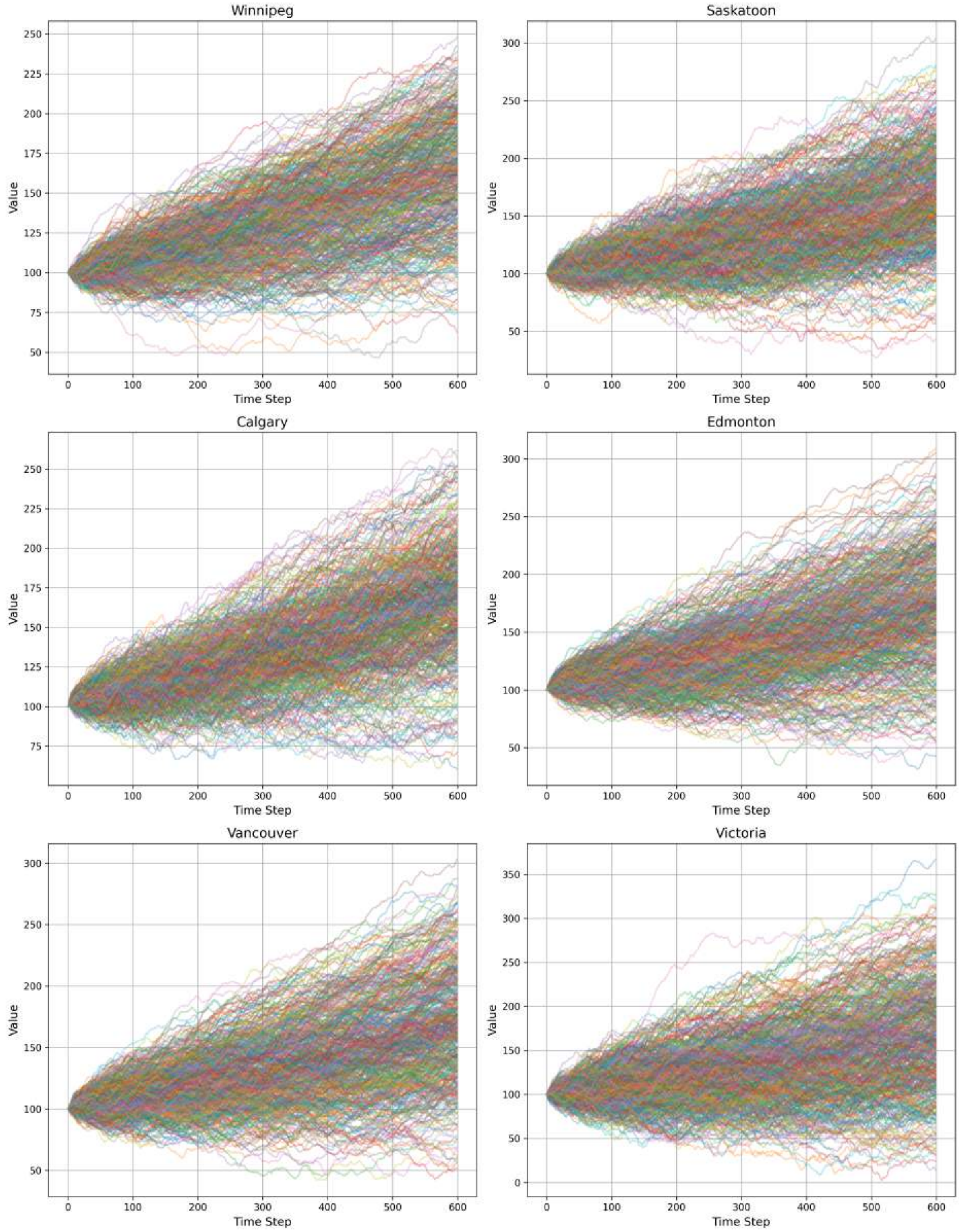

Figure A.8: VAR Simulated Paths for Western Cities (Normalized Initial Value = 100)

Table A.9: Fair Price Comparison by City (VECM vs VAR-like)

City	**Fair Price (VECM)**	**Fair Price (VAR-like)**
Winnipeg	0.00338	0.00185
Saskatoon	0.00358	0.00222
Calgary	0.00345	0.00111
Edmonton	0.00353	0.00180
Vancouver	0.00199	0.00207
Victoria	0.00395	0.00417
Halifax	0.00395	0.00043
St. Johns	0.00369	0.00063
Québec	0.00509	0.00057
Montréal	0.00915	0.00045
Toronto	0.00310	0.00124
Hamilton	0.00285	0.00068
Canada	0.00337	0.00111

Model Results

VECM Results for Western Cities

Long-term Relationship (LTR):

$$\text{LTR}_{t-1} = Y_{t-1}^{\text{Winnipeg}} - 1.0912 \cdot Y_{t-1}^{\text{Saskatoon}} - 1.1350 \cdot Y_{t-1}^{\text{Calgary}}$$

$$+1.9141 \cdot Y_{t-1}^{\text{Edmonton}} - 0.6432 \cdot Y_{t-1}^{\text{Vancouver}} + 0.3440 \cdot Y_{t-1}^{\text{Victoria}} - 39.7938$$

1. Winnipeg Equation:

$$\begin{aligned}
\Delta Y_t^{\text{Winnipeg}} = & \, 0.0106 \cdot \text{LTR}_{t-1} + 0.291 \cdot \Delta Y_{t-1}^{\text{Winnipeg}} + 0.2189 \cdot \Delta Y_{t-2}^{\text{Winnipeg}} + 0.1627 \cdot \Delta Y_{t-3}^{\text{Winnipeg}} \\
& - 0.0008 \cdot \Delta Y_{t-1}^{\text{Saskatoon}} - 0.0309 \cdot \Delta Y_{t-2}^{\text{Saskatoon}} + 0.0526 \cdot \Delta Y_{t-3}^{\text{Saskatoon}} \\
& + 0.152 \cdot \Delta Y_{t-1}^{\text{Calgary}} - 0.0858 \cdot \Delta Y_{t-2}^{\text{Calgary}} - 0.1714 \cdot \Delta Y_{t-3}^{\text{Calgary}} \\
& + 0.109 \cdot \Delta Y_{t-1}^{\text{Edmonton}} - 0.0484 \cdot \Delta Y_{t-2}^{\text{Edmonton}} - 0.0003 \cdot \Delta Y_{t-3}^{\text{Edmonton}} \\
& - 0.0563 \cdot \Delta Y_{t-1}^{\text{Vancouver}} + 0.0671 \cdot \Delta Y_{t-2}^{\text{Vancouver}} + 0.0518 \cdot \Delta Y_{t-3}^{\text{Vancouver}} \\
& + 0.0110 \cdot \Delta Y_{t-1}^{\text{Victoria}} + 0.0465 \cdot \Delta Y_{t-2}^{\text{Victoria}} - 0.0742 \cdot \Delta Y_{t-3}^{\text{Victoria}} + \epsilon_t^{\text{Winnipeg}}
\end{aligned}$$

2. Saskatoon Equation:

$$\begin{aligned}
\Delta Y_t^{\text{Saskatoon}} = & \, 0.0264 \cdot \text{LTR}_{t-1} + 0.312 \cdot \Delta Y_{t-1}^{\text{Winnipeg}} + 0.2015 \cdot \Delta Y_{t-2}^{\text{Winnipeg}} + 0.1854 \cdot \Delta Y_{t-3}^{\text{Winnipeg}} \\
& - 0.0105 \cdot \Delta Y_{t-1}^{\text{Saskatoon}} - 0.0459 \cdot \Delta Y_{t-2}^{\text{Saskatoon}} - 0.0321 \cdot \Delta Y_{t-3}^{\text{Saskatoon}} \\
& + 0.122 \cdot \Delta Y_{t-1}^{\text{Calgary}} - 0.0653 \cdot \Delta Y_{t-2}^{\text{Calgary}} - 0.0739 \cdot \Delta Y_{t-3}^{\text{Calgary}} \\
& + 0.115 \cdot \Delta Y_{t-1}^{\text{Edmonton}} - 0.0375 \cdot \Delta Y_{t-2}^{\text{Edmonton}} - 0.0402 \cdot \Delta Y_{t-3}^{\text{Edmonton}} \\
& - 0.0461 \cdot \Delta Y_{t-1}^{\text{Vancouver}} + 0.0772 \cdot \Delta Y_{t-2}^{\text{Vancouver}} + 0.0593 \cdot \Delta Y_{t-3}^{\text{Vancouver}} \\
& + 0.0225 \cdot \Delta Y_{t-1}^{\text{Victoria}} + 0.0568 \cdot \Delta Y_{t-2}^{\text{Victoria}} + 0.0481 \cdot \Delta Y_{t-3}^{\text{Victoria}} + \epsilon_t^{\text{Saskatoon}}
\end{aligned}$$

3. Calgary Equation:

$$\begin{aligned}\Delta Y_t^{\text{Calgary}} =\,& 0.0157 \cdot \text{LTR}_{t-1} + 0.198 \cdot \Delta Y_{t-1}^{\text{Winnipeg}} + 0.1415 \cdot \Delta Y_{t-2}^{\text{Winnipeg}} + 0.1127 \cdot \Delta Y_{t-3}^{\text{Winnipeg}} \\ & + 0.0425 \cdot \Delta Y_{t-1}^{\text{Saskatoon}} - 0.0312 \cdot \Delta Y_{t-2}^{\text{Saskatoon}} + 0.0563 \cdot \Delta Y_{t-3}^{\text{Saskatoon}} \\ & - 0.1210 \cdot \Delta Y_{t-1}^{\text{Calgary}} + 0.0437 \cdot \Delta Y_{t-2}^{\text{Calgary}} - 0.0509 \cdot \Delta Y_{t-3}^{\text{Calgary}} \\ & + 0.0958 \cdot \Delta Y_{t-1}^{\text{Edmonton}} - 0.0284 \cdot \Delta Y_{t-2}^{\text{Edmonton}} + 0.0323 \cdot \Delta Y_{t-3}^{\text{Edmonton}} \\ & - 0.0345 \cdot \Delta Y_{t-1}^{\text{Vancouver}} + 0.0721 \cdot \Delta Y_{t-2}^{\text{Vancouver}} - 0.0617 \cdot \Delta Y_{t-3}^{\text{Vancouver}} \\ & + 0.0092 \cdot \Delta Y_{t-1}^{\text{Victoria}} - 0.0376 \cdot \Delta Y_{t-2}^{\text{Victoria}} + 0.0185 \cdot \Delta Y_{t-3}^{\text{Victoria}} + \epsilon_t^{\text{Calgary}}\end{aligned}$$

4. Edmonton Equation:

$$\begin{aligned}\Delta Y_t^{\text{Edmonton}} =\,& 0.0203 \cdot \text{LTR}_{t-1} + 0.104 \cdot \Delta Y_{t-1}^{\text{Winnipeg}} + 0.0824 \cdot \Delta Y_{t-2}^{\text{Winnipeg}} - 0.0953 \cdot \Delta Y_{t-3}^{\text{Winnipeg}} \\ & + 0.0517 \cdot \Delta Y_{t-1}^{\text{Saskatoon}} - 0.0168 \cdot \Delta Y_{t-2}^{\text{Saskatoon}} + 0.0295 \cdot \Delta Y_{t-3}^{\text{Saskatoon}} \\ & + 0.0824 \cdot \Delta Y_{t-1}^{\text{Calgary}} - 0.0431 \cdot \Delta Y_{t-2}^{\text{Calgary}} + 0.0178 \cdot \Delta Y_{t-3}^{\text{Calgary}} \\ & - 0.1342 \cdot \Delta Y_{t-1}^{\text{Edmonton}} + 0.0871 \cdot \Delta Y_{t-2}^{\text{Edmonton}} - 0.0329 \cdot \Delta Y_{t-3}^{\text{Edmonton}} \\ & - 0.0423 \cdot \Delta Y_{t-1}^{\text{Vancouver}} + 0.0387 \cdot \Delta Y_{t-2}^{\text{Vancouver}} + 0.0216 \cdot \Delta Y_{t-3}^{\text{Vancouver}} \\ & + 0.0175 \cdot \Delta Y_{t-1}^{\text{Victoria}} - 0.0273 \cdot \Delta Y_{t-2}^{\text{Victoria}} + 0.0421 \cdot \Delta Y_{t-3}^{\text{Victoria}} + \epsilon_t^{\text{Edmonton}}\end{aligned}$$

5. Vancouver Equation:

$$\begin{aligned}\Delta Y_t^{\text{Vancouver}} =\,& 0.0185 \cdot \text{LTR}_{t-1} + 0.0921 \cdot \Delta Y_{t-1}^{\text{Winnipeg}} - 0.0413 \cdot \Delta Y_{t-2}^{\text{Winnipeg}} + 0.0538 \cdot \Delta Y_{t-3}^{\text{Winnipeg}} \\ & + 0.0274 \cdot \Delta Y_{t-1}^{\text{Saskatoon}} - 0.0171 \cdot \Delta Y_{t-2}^{\text{Saskatoon}} + 0.0312 \cdot \Delta Y_{t-3}^{\text{Saskatoon}} \\ & - 0.0813 \cdot \Delta Y_{t-1}^{\text{Calgary}} + 0.0672 \cdot \Delta Y_{t-2}^{\text{Calgary}} - 0.0451 \cdot \Delta Y_{t-3}^{\text{Calgary}} \\ & + 0.1253 \cdot \Delta Y_{t-1}^{\text{Edmonton}} - 0.0512 \cdot \Delta Y_{t-2}^{\text{Edmonton}} + 0.0438 \cdot \Delta Y_{t-3}^{\text{Edmonton}} \\ & - 0.1120 \cdot \Delta Y_{t-1}^{\text{Vancouver}} + 0.0725 \cdot \Delta Y_{t-2}^{\text{Vancouver}} - 0.0394 \cdot \Delta Y_{t-3}^{\text{Vancouver}} \\ & + 0.0193 \cdot \Delta Y_{t-1}^{\text{Victoria}} - 0.0284 \cdot \Delta Y_{t-2}^{\text{Victoria}} + 0.0452 \cdot \Delta Y_{t-3}^{\text{Victoria}} + \epsilon_t^{\text{Vancouver}}\end{aligned}$$

6. Victoria Equation:

$$\begin{aligned}\Delta Y_t^{\text{Victoria}} =\,& 0.0231 \cdot \text{LTR}_{t-1} + 0.1023 \cdot \Delta Y_{t-1}^{\text{Winnipeg}} - 0.0382 \cdot \Delta Y_{t-2}^{\text{Winnipeg}} + 0.0497 \cdot \Delta Y_{t-3}^{\text{Winnipeg}} \\ & + 0.0297 \cdot \Delta Y_{t-1}^{\text{Saskatoon}} - 0.0123 \cdot \Delta Y_{t-2}^{\text{Saskatoon}} + 0.0335 \cdot \Delta Y_{t-3}^{\text{Saskatoon}} \\ & - 0.0917 \cdot \Delta Y_{t-1}^{\text{Calgary}} + 0.0553 \cdot \Delta Y_{t-2}^{\text{Calgary}} - 0.0512 \cdot \Delta Y_{t-3}^{\text{Calgary}} \\ & + 0.1358 \cdot \Delta Y_{t-1}^{\text{Edmonton}} - 0.0625 \cdot \Delta Y_{t-2}^{\text{Edmonton}} + 0.0469 \cdot \Delta Y_{t-3}^{\text{Edmonton}} \\ & - 0.1215 \cdot \Delta Y_{t-1}^{\text{Vancouver}} + 0.0857 \cdot \Delta Y_{t-2}^{\text{Vancouver}} - 0.0523 \cdot \Delta Y_{t-3}^{\text{Vancouver}} \\ & + 0.0218 \cdot \Delta Y_{t-1}^{\text{Victoria}} - 0.0293 \cdot \Delta Y_{t-2}^{\text{Victoria}} + 0.0439 \cdot \Delta Y_{t-3}^{\text{Victoria}} + \epsilon_t^{\text{Victoria}}\end{aligned}$$

VECM Results for Eastern Cities

Long-term Relationship (LTR):

$$\begin{aligned}\text{LTR}_{t-1} =\,& Y_{t-1}^{\text{St. Johns}} - 1.8255 \cdot Y_{t-1}^{\text{Halifax}} + 9.8884 \cdot Y_{t-1}^{\text{Québec}} \\ & - 2.7482 \cdot Y_{t-1}^{\text{Montréal}} + 9.2341 \cdot Y_{t-1}^{\text{Toronto}} - 14.7755 \cdot Y_{t-1}^{\text{Hamilton}} \\ & - 85.3694\end{aligned}$$

1. St. Johns Equation:

$$\begin{aligned}\Delta Y_t^{\text{St. Johns}} =\,& -0.0018 \cdot \text{LTR}_{t-1} + 0.1621 \cdot \Delta Y_{t-1}^{\text{St. Johns}} + 0.1157 \cdot \Delta Y_{t-2}^{\text{St. Johns}} \\ & + 0.0897 \cdot \Delta Y_{t-1}^{\text{Halifax}} + 0.0417 \cdot \Delta Y_{t-2}^{\text{Halifax}} \\ & + 0.0627 \cdot \Delta Y_{t-1}^{\text{Québec}} + 0.0264 \cdot \Delta Y_{t-2}^{\text{Québec}} \\ & - 0.1346 \cdot \Delta Y_{t-1}^{\text{Montréal}} + 0.0200 \cdot \Delta Y_{t-2}^{\text{Montréal}} \\ & - 0.0137 \cdot \Delta Y_{t-1}^{\text{Toronto}} + 0.0248 \cdot \Delta Y_{t-2}^{\text{Toronto}} \\ & - 0.0072 \cdot \Delta Y_{t-1}^{\text{Hamilton}} + 0.0528 \cdot \Delta Y_{t-2}^{\text{Hamilton}} + \epsilon_t^{\text{St. Johns}}\end{aligned}$$

2. Halifax Equation:

$$\begin{aligned}\Delta Y_t^{\text{Halifax}} = & -0.0016 \cdot \text{LTR}_{t-1} + 0.0250 \cdot \Delta Y_{t-1}^{\text{St. Johns}} - 0.0393 \cdot \Delta Y_{t-2}^{\text{St. Johns}} \\ & + 0.1071 \cdot \Delta Y_{t-1}^{\text{Halifax}} + 0.0349 \cdot \Delta Y_{t-2}^{\text{Halifax}} \\ & + 0.1163 \cdot \Delta Y_{t-1}^{\text{Québec}} + 0.1309 \cdot \Delta Y_{t-2}^{\text{Québec}} \\ & - 0.0678 \cdot \Delta Y_{t-1}^{\text{Montréal}} + 0.0477 \cdot \Delta Y_{t-2}^{\text{Montréal}} \\ & - 0.0509 \cdot \Delta Y_{t-1}^{\text{Toronto}} + 0.0970 \cdot \Delta Y_{t-2}^{\text{Toronto}} \\ & + 0.1613 \cdot \Delta Y_{t-1}^{\text{Hamilton}} - 0.0024 \cdot \Delta Y_{t-2}^{\text{Hamilton}} + \epsilon_t^{\text{Halifax}}\end{aligned}$$

3. Québec Equation:

$$\begin{aligned}\Delta Y_t^{\text{Québec}} = & -0.0029 \cdot \text{LTR}_{t-1} + 0.0358 \cdot \Delta Y_{t-1}^{\text{St. Johns}} - 0.0178 \cdot \Delta Y_{t-2}^{\text{St. Johns}} \\ & - 0.0023 \cdot \Delta Y_{t-1}^{\text{Halifax}} - 0.0620 \cdot \Delta Y_{t-2}^{\text{Halifax}} \\ & + 0.3356 \cdot \Delta Y_{t-1}^{\text{Québec}} + 0.0594 \cdot \Delta Y_{t-2}^{\text{Québec}} \\ & - 0.1468 \cdot \Delta Y_{t-1}^{\text{Montréal}} + 0.0490 \cdot \Delta Y_{t-2}^{\text{Montréal}} \\ & - 0.0405 \cdot \Delta Y_{t-1}^{\text{Toronto}} + 0.0883 \cdot \Delta Y_{t-2}^{\text{Toronto}} \\ & + 0.0843 \cdot \Delta Y_{t-1}^{\text{Hamilton}} + 0.1310 \cdot \Delta Y_{t-2}^{\text{Hamilton}} + \epsilon_t^{\text{Québec}}\end{aligned}$$

4. Montréal Equation:

$$\begin{aligned}\Delta Y_t^{\text{Montréal}} = & -0.0034 \cdot \text{LTR}_{t-1} + 0.0624 \cdot \Delta Y_{t-1}^{\text{St. Johns}} - 0.1049 \cdot \Delta Y_{t-2}^{\text{St. Johns}} \\ & - 0.0417 \cdot \Delta Y_{t-1}^{\text{Halifax}} + 0.2945 \cdot \Delta Y_{t-2}^{\text{Halifax}} \\ & + 0.4245 \cdot \Delta Y_{t-1}^{\text{Québec}} + 0.1310 \cdot \Delta Y_{t-2}^{\text{Québec}} \\ & + 0.0510 \cdot \Delta Y_{t-1}^{\text{Montréal}} + 0.0063 \cdot \Delta Y_{t-2}^{\text{Montréal}} \\ & + 0.1205 \cdot \Delta Y_{t-1}^{\text{Toronto}} + 0.1350 \cdot \Delta Y_{t-2}^{\text{Toronto}} \\ & - 0.0748 \cdot \Delta Y_{t-1}^{\text{Hamilton}} - 0.0097 \cdot \Delta Y_{t-2}^{\text{Hamilton}} + \epsilon_t^{\text{Montréal}}\end{aligned}$$

5. Toronto Equation:

$$\begin{aligned}
\Delta Y_t^{\text{Toronto}} = & -0.0006 \cdot \text{LTR}_{t-1} + 0.0276 \cdot \Delta Y_{t-1}^{\text{St. Johns}} + 0.0050 \cdot \Delta Y_{t-2}^{\text{St. Johns}} \\
& -0.0063 \cdot \Delta Y_{t-1}^{\text{Halifax}} + 0.0047 \cdot \Delta Y_{t-2}^{\text{Halifax}} \\
& +0.1361 \cdot \Delta Y_{t-1}^{\text{Québec}} - 0.0207 \cdot \Delta Y_{t-2}^{\text{Québec}} \\
& -0.0339 \cdot \Delta Y_{t-1}^{\text{Montréal}} + 0.0110 \cdot \Delta Y_{t-2}^{\text{Montréal}} \\
& +0.3614 \cdot \Delta Y_{t-1}^{\text{Toronto}} + 0.2230 \cdot \Delta Y_{t-2}^{\text{Toronto}} \\
& +0.0222 \cdot \Delta Y_{t-1}^{\text{Hamilton}} + 0.0216 \cdot \Delta Y_{t-2}^{\text{Hamilton}} + \epsilon_t^{\text{Toronto}}
\end{aligned}$$

6. Hamilton Equation:

$$\begin{aligned}
\Delta Y_t^{\text{Hamilton}} = & -0.0010 \cdot \text{LTR}_{t-1} + 0.0343 \cdot \Delta Y_{t-1}^{\text{St. Johns}} - 0.0054 \cdot \Delta Y_{t-2}^{\text{St. Johns}} \\
& -0.0116 \cdot \Delta Y_{t-1}^{\text{Halifax}} + 0.0072 \cdot \Delta Y_{t-2}^{\text{Halifax}} \\
& +0.0844 \cdot \Delta Y_{t-1}^{\text{Québec}} + 0.0507 \cdot \Delta Y_{t-2}^{\text{Québec}} \\
& -0.0712 \cdot \Delta Y_{t-1}^{\text{Montréal}} + 0.0441 \cdot \Delta Y_{t-2}^{\text{Montréal}} \\
& +0.1497 \cdot \Delta Y_{t-1}^{\text{Toronto}} + 0.1350 \cdot \Delta Y_{t-2}^{\text{Toronto}} \\
& +0.0659 \cdot \Delta Y_{t-1}^{\text{Hamilton}} + 0.0666 \cdot \Delta Y_{t-2}^{\text{Hamilton}} + \epsilon_t^{\text{Hamilton}}
\end{aligned}$$