VaR Calculations and Backtesting for S&P 500 Index Option Straddles

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Abstract

Value-at-risk is one of the most prevalent risk-measuring approaches in finance. The models should be consistently validated to assess the quality of the VaR estimations. Backtesting is a standard method for VaR model validation. This compares actual losses to VaR calculations.

The Delta-Normal approach works for single positions. It does not work for mixed (straddle) positions. The primary goal of this thesis is to demonstrate how to resolve this problem using historical VaR. The study investigates the correctness of a VaR model used to produce VaR calculations for S&P 500 options and straddles. The different VaR calculation approaches for a two-year time period from January 2, 2020, to December 31, 2021, are employed. The thesis investigates the accuracy of value-at-Risk (VaR) calculations for S&P 500 options and straddles. It first assesses the quality of the VaR delta method (an analytical approximation less appropriate for mixed positions such as straddles). It then uses a historical VaR approach directly applied to option returns. Historical VaRs' are calculated for different batches of moneyness and time-to-maturity.

As a secondary goal, the empirical study attempts to determine which backtests are the most accurate and which tests are appropriate. The historical approach (estimation window) is evaluated through backtesting, including the unconditional coverage test and the VaR clustering test. The historical VaR approach is finally benchmarked with a parametric method that relies on the kernel density estimation (with Gaussian basis functions) of option returns.

There is no doubt that all quantiles will be lower in 2021 than they were in 2020. This is because, in part because of the COVID issue, 2020 was a more unstable year than 2021. Therefore, the study found that historical VaR performs well for mixed (straddle) options. The robustness assessment of the analysis also demonstrates that the historical distribution becomes unimodal as maturity increases. It is found that the straddles at the tails, the VaR model, and the VaR backtesting are functioning effectively.

Keywords: Value-at-Risk, VaR, Backtesting, Risk management, Risk analysis

Research methods: Analytical VaR, Historical VaR, Unconditonal Test Coverage, Violence frequency, Clustering test, Joint test, Kernel Density Estimation (KDE)

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List of abbreviations and acronyms

ATM	At The Money
C	Call
CDF	Cumulative Distribution Function
DITM	Deep In The Money
DOTM	Deep Out of The Money
ITM	In The Money
KDE	Kernel Destiny Estimation
NA	Not Applicable
OTM	Out of The Money
P	Put
VaR	Value At Risk
VDITM	Very Deep In The Money
VVDITM	Very, Very Deep In The Money
WRDS	Wharton Research Data Services

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Introduction

Value at Risk (VaR), which has become one of the most common risk measurement techniques in finance over the last decade, is becoming more and more popular. VaR seeks to capture the market risk inherent in a portfolio of assets. VaR is a measure of the highest portfolio loss that can be measured in terms of absolute time at which confidence intervals apply. The VaR method was often called unreliable in its ability to reliably estimate risks, despite its extensive use and widespread acceptance as a risk management tool (Dowd, 1998). A lot of simplifications and assumptions will still have to be made when implementing VaR systems. In addition, all the VaR models attempt to predict asset prices in terms of past market data, which does not necessarily consider present market conditions (Dowd, 1998).

The study examines several backtesting methods that are intended to evaluate the adequacy of value at Risk (VaR) measures. The characteristics of unconditional coverage, independence, and their connection to backtesting methods are described. The tests are then grouped based on whether they investigate the VAR measure's unconditional coverage feature, independence property, or both.

This thesis is organized into several chapters to emphasize each step of the research process. The first chapter describes the literature review of VaR, VaR calculations, and backtesting reviews. The second chapter concentrates on the data set and data preprocessing. The third chapter defines the methodologies used to complete the study. The fourth chapter exhibits the empirical results of VaR calculations and VaR backtesting. The results are presented in detail and analyzed thoroughly. The fifth chapter is an extension of chapters third and fourth, which demonstrate the robustness of methods for backtesting and benchmarking using Kernel Destiny Estimation (KDE). Lastly, the conclusion summarizes the contribution and findings of the study.

1. Chapter One
Literature review

A very popular method for measuring risks in the financial sector is value at risk, or VaR. Market risks related to the portfolio of assets are evaluated and quantified in value assessment reports. Risk management has evolved to such an extent in recent decades that it is regarded as a separate subfield within the theory of finance. In the 1970s, the rising volatility in financial markets was a driving force behind the growth of the risk management industry. The important events that contributed to this new paradigm for risk management were, among others, the breakdown in Bretton Woods' fixed exchange rate regime and the rapid development of a new theory named ‘Black Scholes’.

From the 1970s to the 1980s, financial institutions developed their own risk management models, but it was not until pioneering work by J.P. Morgan and its publication of Risk Metrics in 1994 that VaR became an industrywide standard. The basic idea behind VaR is straightforward since it gives a simple quantitative measure of a portfolio’s downside risk. The VaR has two major and appealing characteristics. Firstly, it introduces a common, standardized risk assessment for the various positions and types of instruments. Secondly, it should consider the correlation between various risk factors. When calculating risk data for a portfolio of more than one instrument, this property is very important (Dowd, 1998).

Therefore, the VaR models are of no use if they do not correctly predict subsequent risks. It is considered that models should be tested with appropriate statistical methods on a regular basis to verify that the results obtained from VaR calculations are both accurate and reliable. Backtesting can be defined in terms of the process by which actual earnings and losses are compared to projected profit and loss ratio estimates. These tests are appropriately referred to as reality checks (Dowd, 1998); (Jorion, 2001). Tests of independence will fail for a VaR technique that fails to adapt to the current high volatility conditions and produces clustered VaR exceptions. A model that reacts quickly to changes in the volatility and correlation of instruments can avoid exception clusters. Similar types of tests consider the independence of exceptions (Christofferssen, 1998); (Haas, 2001). The problem is presumably the most important contribution of Christoffersen's insight in 1998. The examination of the accuracy of any VaR measure may be limited to determining whether it is correct. The resulting sequence of violations and non-violations shall be subject to unconditional coverage and independence (Christofferssen, 1998).

In risk management today, testing should be at least a central element of VaR reporting. It is impossible to guarantee that the VaR system will deliver precise risk estimates unless a model has been properly validated. In the current market environment, where volatility in market prices makes investors more interested in portfolio risk data as losses accumulate, this topic is particularly important (Beder, 1995). The method for calculation of the VaR is, in general, classified into quantitative and not parametric methods. Nonparametric models are simulations or historical models that are based on the statistical characteristics of the risk factor distributions (Ammann, M. & Reich, C., 2001).

It is known that, in fact, the most common financial assets have a fat-tailed return distribution, which implies an even greater probability of extreme outcomes than normal returns. Consequently, the estimates of VaRs may be underestimated (Jorion, 2001). When instruments such as options, whose returns are highly non-normal, form part of the portfolio, these difficulties will become more severe. A first-order approximation of the returns of these instruments and then using a linear approximation for calculating VaR is one solution in this regard. The delta-normal approach makes the supposition that all asset returns have a normal distribution. The portfolio return is normally distributed since they are a linear combination of normal variables. The delta-normal approach is the method used (Jorion, 2001).

However, (Dowd, 1998) and (Britten M. and Schaefer M., 1999) have proposed quadratic Value at Risk methods, also known as delta gamma models, which use second-order approximations to go beyond first-order ones. Delta-Gamma-Normal VaR might display less accuracy than even the standard Delta VaR (Giuseppe Castellacci and Michael J. Siclari, 2003), and they demonstrated that methodologies that primarily account for non-linearity, including factors like gamma and higher moments of portfolio value, could offer substantial benefits over comprehensive Monte Carlo revaluation methods.

Identifying instruments within a portfolio shall be the first step in historical VaR and for deriving time series on these instruments over a certain defined period of history. Therefore, the weighting in the existing portfolio is used to simulate an assumed return. In this case, the estimate for VaR may be inferred from the histogram of portfolio returns. The assumption underlying this method is that the distribution of historical returns serves as a good measure of the return to be expected over the next holding period (Dowd, 1998).

Because of their simplicity, historical simulations offer a series of indisputable advantages. It does not make any assumptions about the distribution of statistics or require an estimation of volatility and correlation. The time series for portfolio returns in general is all that needs to be done (Hendricks, 1996). Most importantly, historical simulations can account for the fat tails of the return distributions (Jacquier, E., Polson, G., & Rossi, E., 2004). Virtually all types of instruments are covered by the method, which is based on full valuations (Jorion, 2001). The fact that historical VaR effectively assumes that history will repeat itself is a more serious shortcoming. In some cases, this assumption may have a major distorting effect on the VaR estimates, even if it is frequently justified (Dowd, 1998).

Determining VaRs is essentially about gauging quantiles within the distribution of portfolio returns. The two predominant methods for VaR estimation are the parametric and non-parametric approaches. It is assumed in the parametric approach that returns follow a Gaussian distribution, leading to a straightforward VaR calculation based on return volatility. This volatility can be estimated using various techniques. In contrast, the non-parametric approach involves calculating portfolio returns over a specific time frame using historical data, and the VaR is interpreted as a loss surpassed within the sample for the desired fraction of instances (El-Jahel, 1999).

For example, during times of very high volatility, which can result in severe tail losses, there may be certain risks not taken into consideration by the past data set. It is for the purposes at stake that the choice of confidence level must be made. To be able to detect as many VaR violations as possible, it is necessary to avoid high levels of confidence if validation purposes are to be achieved for the VaR model (Campbell, 2005). Risk aversion among senior management determines the degree of confidence, and risk-averse managers choose higher levels. Another consideration is whether VaR levels should be compared with estimates made from different sources (Dowd, 1998).

2. Chapter Two

Data

The delta approximation works well for single positions or options. It does not work well for mixed (straddle) positions. A historical approach would fix this problem, and the historical VaR model can be backtested to fulfill this purpose. S&P 500 is a good resource as a data source because it has underlying options data, good quality data, and many strikes are also available. The pricing of the strike has liquidation data.

The data sets are available on "Wharton Research Data Services (WRDS)" under the database "Option Metrics." The data sets contain the S&P 500 Option Index data for the 2-year period from January 2, 2020, to December 31, 2021. The datasets are as follows:

1. S&P 500 Index-Call options
2. S&P 500 Index-Put options
3. S&P 500 Index-underlying historical data

The number of records present in each dataset is shown in the table below.

S No	Data set	Number of Records
1	Call Options	4716783
2	Put Options	4716821
3	Underlying	505

Table 2.1: Number of records in each data set

2.1 S&P 500 Index-Call and put options

The data set for call and put options has the following columns:

1. secid: This column represents the security identifier, or ID, for the option contract.
2. date: The date column indicates the specific date of the option data.
3. symbol: This column represents the symbol or ticker of the underlying asset associated with the option contract. It consists of the combined information of the expiration date, option type, and strike price.
4. exdate: The exdate column refers to the expiration date of the option contract.

5. last_date: This column represents the last trading date of the option contract.
6. cp_flag: The cp_flag column could be used to distinguish between call (C) and put (P) options.
7. strike_price: This column represents the strike price of the option contract.
8. best_bid: The best_bid column indicates the highest bid price at which buyers are willing to purchase the option.
9. best_offer: The best_offer column represents the lowest asking price at which sellers are willing to sell the option.
10. impl_volatility: The impl_volatility column represents the implied volatility of the option contract.
11. delta: Delta is a measure of the sensitivity of the option's price to changes in the price of the underlying asset.
12. optionid: This column represents a unique identifier for each individual option contract.
13. expiry_indicator: The column provides additional information or a code indicating the expiration status of the option contract. It represents the frequency of the options- weekly or monthly.
14. exercise_style: The exercise_style column indicates the exercise style of the option, such as American or European.
15. issuer: This column represents the issuer or organization responsible for issuing the option contract.

2.2 S&P 500 Index-underlying data

The data set has the following columns:

1. date: This column indicates the specific date for which the data is recorded.
2. open: The open column represents the opening price of the S&P 500 Index on a given date.
3. high: The high column indicates the highest price reached by the S&P 500 Index during a particular day.
4. low: The low column represents the lowest price reached by the S&P 500 Index during a specific day.
5. close: The close column indicates the closing price of the S&P 500 Index on a given date.

2.3 Data Preprocessing in Depth

There are multiple steps performed for data cleaning and data preparation. All the records are dropped for this study where the expiration frequency is weekly or null. A unique combined key, 'Key Call' is created by joining three columns-strike price, date, and expiration date-for the data set call option. In the same way, a unique combined key, 'Key Put' is created by joining three columns-strike price, date, and expiration date-for the data set put option. Time maturity is calculated for each corresponding data record in the dataset. Both data sets, call and put, have the following columns: Key call, Key put, date, symbol, strike price, implied volatility, delta call, delta put, call premium, put premium, Date historical price, Close historical price, time maturity, and year. The 'call premium' is calculated by taking the arithmetic mean of 'best bid call' and 'best offer call'. The 'put premium' is calculated by taking the arithmetic mean of 'best bid put' and 'best offer put'. The daily implied volatility for calls and puts is calculated by a factor $\sqrt{((1 \div 252))}$ multiplied by the annual implied volatility.

To find all the possible straddle pairs, the data sets for calls and puts have been merged using the key columns 'Key call' and 'Key put'. The newly merged data set contains all the possible straddle pairs for the 2-year period from January 2, 2020, to December 31, 2021. The newly merged data set is again merged with 'trading days' dataset, and a column named 'Day ID' is assigned to each specific trading day and is used to merge these two data sets. To avoid null values, all the records have been dropped from the data set where both columns 'implied volatility call', and 'implied volatility put' are null. Then all the records have been dropped, and both columns 'delta call', and 'delta put' are null. The premium for the straddle pairs has been calculated by adding 'call premium' and 'put premium'. The records in the dataset have been sorted by the columns 'symbol call' and 'symbol put' in ascending order.

The daily implied volatility for the straddle has been calculated by taking the minimum value between the daily implied volatility for the call and the daily implied volatility for the put. The delta for the straddle pairs is calculated by adding the columns delta call and delta put. For this research study context, all the records are dropped when the delta difference between the delta call and delta put values is less than 0.94, which results in dropping non-significant records for the study. This provides a better number of straddle pairs. Then finally, this merged data set is combined with the historical price dataset using the common key, 'Date', and 'Close historical price' column is added (the closing price for each day that is coming from the historical price dataset).

To find all the possible straddle pairs for the 5-day period, it is considered that the option data should exist for the next 5th day for each pair. Therefore, when the trading days' difference for the pairs is less than five, all the corresponding records are dropped. The daily implied volatility for the five-day period straddle pairs is calculated by a factor $\sqrt{((5 \div 252))}$ multiplied by the annual implied volatility for straddle.

From Tables 2.2 and 2.3, it is concluded that 75% of call options are in the money, while on the other side, 75% of put options are out of the money, and most of them are deep out of the money. Moneyness is the ratio of the underlying asset's price to the option's strike price. In this study's purpose for straddle design, this requires that having the same moneyness increases the importance of the out of the moneyness (out of the money put).

Metric	Premium	Delta	Daily Implied Volatility	Time to Maturity	Moneyness
Mean	594.4413	0.6854	0.0202	0.4459	1.2176
St deviation	609.7315	0.2935	0.0130	0.3213	0.4479
Min	0.0250	0.0002	0.0044	0.0198	0.4864
25%	138.3000	0.4924	0.0127	0.2024	0.9958
50%	424.2000	0.7904	0.0175	0.3810	1.1061
75%	841.5000	0.9325	0.0233	0.6071	1.2701
Max	4181.9000	0.9999	0.1890	1.4881	8.6928

Table 2.2: Descriptive statistics for calls

Metric	Premium	Delta	Daily Implied Volatility	Time to Maturity	Moneyness
Mean	131.4628	-0.3093	0.0194	0.4452	1.2154
St deviation	185.3228	0.2962	0.0105	0.3204	0.4415
Min	0.0250	-0.9998	0.0025	0.0198	0.4864
25%	16.4500	-0.5036	0.0125	0.2024	0.9957
50%	65.3000	-0.2027	0.0174	0.3810	1.1059
75%	176.6000	-0.0598	0.0233	0.6071	1.2693
Max	2371.4500	0.0000	0.1770	1.4881	8.6928

Table 2.3: Descriptive statistics for puts

Call options have a higher average premium than put options. Delta represents the sensitivity of the option price to changes in the underlying asset's price. The implied volatility represents the market's expectation of future price fluctuations. Both call and put options have similar mean daily implied volatility. The time to maturity refers to the remaining time until the option's expiration. Both call and put options have a similar mean time to maturity. Both call and put options have similar mean moneyness.

Overall, the call options tend to have higher premiums and positive deltas, indicating a positive relationship with the underlying asset's price. On the other hand, put options have negative deltas, showing an inverse relationship with the underlying asset's price. The other metrics, such as daily implied volatility, time to maturity, and moneyness, show relatively similar characteristics between call and put options.

Metric	Premium	Delta	Daily Implied Volatility	Time to Maturity	Moneyness
Mean	725.7189	0.3763	0.0189	0.4460	1.2176
St deviation	546.0446	0.5897	0.0102	0.3213	0.4479
Min	29.0000	-0.9993	0.0025	0.0198	0.4864
25%	372.3500	-0.0110	0.0124	0.2024	0.9958
50%	558.8000	0.5883	0.0171	0.3810	1.1061
75%	901.3188	0.8733	0.0226	0.6071	1.2701
Max	4181.9500	0.9996	0.1858	1.4881	8.6928

Table 2.4: Descriptive statistics for straddles

From Table 2.4, the mean premium value for straddles is 725.7189. The standard deviation of the straddle premium is 546.0446, indicating significant variability in premium values. The mean delta for straddles is 0.3763. For example, 25% of the premiums are below 372.35, and on the other hand, 75% of the premiums are below 901.32. The wide range between the minimum and maximum values for each metric indicates substantial variation in straddle characteristics within the dataset. The high standard deviations for premium and delta suggest that straddles can have significantly different costs and sensitivities to underlying asset price movements. The moneyness, on average, tends to be slightly in the money, as indicated by the mean moneyness value exceeding 1.

3. Chapter Three

Methodology

Value-at-Risk is one of the most widely used risk measurement methodologies in finance. However, VaR models are only useful if they accurately predict future risks. The practitioners have extensively investigated Value-at-risk (VaR) to propose a novel risk management approach that provides valid VaR assessments for both long and short-trading positions, as well as all types of financial assets. VaR is technically defined as follows (Linsmeier, 1996): An entity's value at risk is the loss that is expected to be surpassed with a probability in the following several days.

VaR offers a straightforward mathematical estimate of a portfolio's downside risk, and its main notion is also simple. VaR has two crucial and appealing aspects. For one thing, it provides a consistent and uniform risk assessment for a wide range of scenarios. Second, it considers the link between distinct risk variables. This functionality is essential for computing risk numbers for a portfolio with various instruments (Dowd, 1998).

When evaluating VaR values, it is critical to keep the time horizon and confidence level in mind, as VaR numbers are worthless without these. Regulated investors like financial institutions utilize a one-day time horizon for market risk, but non-financial corporations sometimes prefer longer horizons. The firms should choose the holding term based on the time it takes to liquidate the portfolio.

The sample data for the year 2020 is very volatile, and it is a good representative for the year 20221. So, a hypothesis could be considered that in the year 2021, all quantiles will be lower. This is because, due in part to the COVID problem, 2020 was a more volatile year than 2021. As a result, it would be discovered during the study that historical VaR performs well for mixed (straddle) positions, and the straddles at the tails, the VaR model, and VaR backtesting would all work well.

The models should always be backtested using accepted approaches to check the quality of VaR estimations. Backtesting is a statistical technique that compares actual losses to corresponding

VaR estimates. In this study, three different confidence levels (0.1%, 1%, and 5%), respectively, are used.

In this study, the methodology has two stages:

1. VaR Calculation approaches
    - Analytical VaR (Delta Normal Approach)
    - Historical VaR
2. Backtesting
    - Unconditional coverage test
    - Testing for clustering in VaR violations
    - Joint test

During this study, both stages are applied on the whole data set and multiple batches of the data set (different chunks of data set which are extracted from the whole data set.)

3.1 VaR Calculation approaches

VaR calculation techniques are often classified as parametric or non-parametric. The statistical characteristics of the risk factor distribution serve as the foundation for parametric models. Delta-Normal is an example of a parametric model, while the historical approach is non-parametric (Ammann, M. & Reich, C., 2001).

3.1.1 Analytical VaR

The traditional analytical VaR assumes normal returns. It is only available in its closed version. Delta Normal VaR uses the delta of the position to approximate the Normal VaR of a derivative from that of its underlying (Jorion, 2001). So, the delta Var is calculated using the following formula:

$$Delta\ VaR = |\Delta \times \sigma \times \Phi^{-1}(\alpha)| \tag{3.1}$$

where, Δ is the delta of option, σ is the daily implied volatility, α is the confidence level, and $\Phi(.)$ is defined by the standard normal cumulative distribution function (cdf).

3.1.2 Historical VaR

Historical VaR is a method of calculating VaR based on historical returns. It uses historical data to estimate the potential loss that a portfolio might have experienced in the past, assuming similar market conditions. Historical VaR measures the potential loss of a portfolio by looking at past price or return data and determining how much the portfolio could have lost with a certain level of confidence over a specific time. For non-parametric approaches, historical simulation is arguably the most straightforward (Wiener, 1999). The concept is simply to calculate VaR for the present portfolio using just past market data. VaR is calculated directly from prior returns in this manner.

The initial stage of the historical approach is to identify the instruments in the portfolio and gather time series for these instruments over a specified historical period. The weights in the present portfolio are then used to model hypothetical returns that would have been realized if the current portfolio had been held during the observation period. VaR estimations may then be calculated using the portfolio return histogram. The premise underpinning this strategy is that the distribution of previous returns is a good predictor of the returns experienced over the upcoming holding period (Dowd, 1998). Most crucially, historical VaR can account for the return distributions' wide tails. This procedure is also applicable to any form of instrument and employs comprehensive values (Jorion, 2001).

A further problem is that every historical observation is assigned a weight of one if it falls within the temporal frame and zero if it falls outside of it. When large market jumps are removed from the data set, this has a negative impact on VaR estimations (Dowd, 1998) and (Wiener, 1999). The weighted historical approach, which provides lower weights to data that is further in the past, is a convenient solution to these problems (Dowd, 1998). This approach assigns weight to observed log-returns that decay exponentially as one goes back in time.

3.2 Backtesting

Value at Risk (VaR) is a widely used risk measurement technique in finance. Backtesting VaR models is an essential step in evaluating their accuracy and performance. Backtesting is a process used to evaluate the accuracy and reliability of VaR models in measuring and predicting the potential losses of a portfolio or investment. It involves comparing the VaR estimates with the actual portfolio returns over a given time period to determine whether the model is performing as expected. Backtesting is a statistical method that compares real losses with VaR-implied losses.

A sample of *m* daily VaRs computed in the past and their corresponding realized log-returns, a series $\{I_{(t+1)}\}_{t=1}^{m}$of VaR violations as

$$I_{(t+1)} = \begin{cases} 1, & if\ r_{(t+1)} < -VaR_{t+1}^{\alpha} \\ 0, & if\ r_{(t+1)} \geq -VaR_{t+1}^{\alpha} \end{cases} \tag{3.2}$$

where $r_{(t+1)}$ is given by log-return at day t+*1 and* VaR_{t+1}^{α} is defined as Value at Risk at given confidence level at day t+1

If the VaR methodology is correct, the frequency of VaR violations should be close to α, which represents

$$\frac{\sum I_{(t+1)}}{m} \approx \alpha \tag{3.3}$$

If $\sum I_{(t+1)} \gg m\alpha$, the VaR approach underestimates the real risk exposure and if $\sum I_{(t+1)} \ll m\alpha$, the VaR approach is too conservative.

3.2.1 Unconditional coverage test

The most typical way to evaluate a VaR model is to tally the number of VaR violations, which are days (or other holding periods) when portfolio losses exceed VaR projections. The Unconditional Coverage Test is a commonly used statistical test for backtesting the Value at Risk (VaR) model. It evaluates the ability of the VaR model to accurately capture the number of actual losses that exceed the VaR estimate. The test determines whether the number of violations observed in the historical data is consistent with the VaR level chosen.

The unconditional coverage test evaluates the likelihood of observing the number of exceptions given the confidence level. If the number of exceptions is too high compared to the expected number, it suggests that the VaR model is not accurately capturing the risk.

- If the VaR methodology is correct, the VaR violations $I_{(t+1)}$ should be Bernoulli variables equal to 1 with probability α and 0 otherwise. So, the following approach is applied in the study:

Total number of violations:

$$m_1 = \sum I_{(t+1)} \tag{3.4}$$

Total number of non-violations: $$m_0 = m - m_1 \quad (3.5)$$

- The likelihood of a Bernoulli sequence with probability α is given by:

$$L_1 = (1-\alpha)^{m_0} \times (\alpha)^{m_1} \quad (3.6)$$

- The maximum likelihood estimation from the sample is given by:

$$L_2 = (1 - \frac{m_1}{m})^{m_0} \times (\frac{m_1}{m})^{m_1} \quad (3.7)$$

Where *m* is given by the arithmetic sum of $m_1 and\ m_0$

- So, the likelihood ratio test is given by:

$$-2 \ln \frac{L_1}{L_2} \sim \gamma_1^2 \quad (3.8)$$

Where γ_1^2 is a random variable following a chi-square distribution with one degree of freedom.

3.2.2 Testing for clustering in VaR violations

Testing for clustering in Value at Risk (VaR) violations is an essential step in assessing the adequacy of risk measures and ensuring that they accurately capture the underlying risk in a portfolio. Clustering in VaR violations refers to a phenomenon where extreme losses occur more frequently and in clusters than would be expected under a well-calibrated risk model. This analysis helps identify if the VaR model adequately captures the tail risks and if there are periods of increased vulnerability to significant losses.

One common approach to testing for clustering in VaR violations is the Runs Test. The test involves counting the number of runs in a binary sequence, where "1" represents a VaR violation (an actual loss exceeding the VaR estimate) and "0" represents no violation (the actual loss is within the VaR estimate). A run is a sequence of consecutive occurrences of the same number in the binary sequence. In this case, a run represents a period of two consecutive VaR violations or two consecutive days without violations.

Count the number of occurrences in the binary sequence. Therefore, m_{ij} represents the total number of observations with a *j* following an *i*, with the same convention: 1 is a VaR violation and 0 is not.

- The following formula defines the likelihood function:

$$L = (1 - \pi_{01})^{m_{00}} \pi_{01}^{m_{01}} (1 - \pi_{11})^{m_{10}}\ \pi_{11}^{m_{11}} \tag{3.9}$$

- Under the independence assumption, it is assumed that π_{01} is equal to π_{11} and both are equal to the ratio of $\frac{m_1}{m}$.
- Therefore, the likelihood function can be defined as:

$$L_3 = \left(1 - \frac{m_1}{m}\right)^{m_{00}} \left(\frac{m_1}{m}\right)^{m_{01}} \left(1 - \frac{m_1}{m}\right)^{m_{10}} \left(\frac{m_1}{m}\right)^{m_{11}} \tag{3.10}$$

- So, the probability π_{01} is defined by the ratio of $\frac{m_{01}}{m_{00}+m_{01}}$ and the probability $(1 - \pi_{01})$ is defined by the ratio of $\frac{m_{00}}{m_{00}+m_{01}}$.
- In the same manner, the maximum likelihood of the sample is defined by the following formula:

$$L_4 = \left(\frac{m_{00}}{m_{00} + m_{01}}\right)^{m_{00}} \left(\frac{m_{01}}{m_{00} + m_{01}}\right)^{m_{01}} \left(\frac{m_{10}}{m_{10} + m_{11}}\right)^{m_{10}} \left(\frac{m_{11}}{m_{10} + m_{11}}\right)^{m_{11}} \tag{3.11}$$

- So, the likelihood ratio test is given by:

$$-2 \ln \frac{L_3}{L_4} \sim \gamma_1^2 \tag{3.12}$$

Where γ_1^2 is a random variable following a chi-square distribution with one degree of freedom.

3.2.3 Joint test

A joint test for VaR violations involves testing whether the occurrence of VaR violations is jointly related to certain factors or conditions in the financial market. This test is used to examine whether the VaR model's performance is influenced by specific market conditions or whether it remains consistent across different market environments. The joint test typically involves specifying a statistical model where the binary variable indicating VaR violations is the dependent variable, and the factors or conditions of interest are the independent variables. The goal is to determine whether these independent variables have a significant impact on the probability of VaR violations.

Therefore, a joint test for the average number of violations and their independence is defined by the following formula:

$$-2\ln\frac{L_1}{L_2} - 2\ln\frac{L_3}{L_4} \sim\gamma_2^2 \tag{3.13}$$

Where γ_2^2 is a random variable following a chi-square distribution with two degrees of freedom.

3.3 Data set distribution approach for study

In this study, the methodology for all option-type cases is applied by using the full data and creating multiple batches, which are defined with the combination of moneyness *M* and time to maturity *T*.

- **Moneyness** M is defined as the ratio of the underlying price to the strike price.

$$M = \frac{historical\ close\ price}{Strike\ price}$$

So moneyness is divided into the seven intervals to analyze the VaR analysis in the different aspects of moneyness, such as in the money, out of the money, and at the money. For better convenience in this study, the classification and batch labels are defined as follows:

	Batch Label	Moneyness interval
1	DOTM (Deep Out of the Money)	$M < 0.9$
2	OTM (Out of the Money)	$0.9 \leq M < 0.95$
3	ATM (At the Money)	$0.95 \leq M < 1.05$
4	ITM (In the Money)	$1.05 \leq M < 1.1$
5	DITM (Deep in the Money)	$1.1 \leq M < 1.5$
6	VDITM (Very deep in the Money)	$1.5 \leq M < 2.0$
7	VVDITM (Very very deep in the Money)	$M \geq 2.0$

Table 3.1: Moneyness classification

- **Time to maturity** *T* is defined as the ratio of the number of trading days to expiry to the total number of trading days in the year.

$$\text{Time to maturity } T = \frac{\text{number of days to expire option}}{252}$$

The time to maturity T is divided into the three intervals for VaR analysis in different time horizons, and the categorization and batch labels are defined as follows for this study's convenience:

	Batch Label	**Time to Maturity interval (in years)**
1	Short	$T < 0.25$
2	Medium	$0.25 \leq T \leq 0.5$
3	Long	$T > 0.5$

Table 3.2: Time to maturity classification

- **Batches classification:**

  The batch labels for the moneyness and time to maturity is defined earlier. By using the combination of seven batches of moneyness and three batches of time to maturity, 21 batches are created for the full dataset. The batch labels are defined as follows:

Batch No	**Batch Label**	**Time Batch Description**
Batch 1	DOTM/short	(M<0.9) and (T<0.25)
Batch 2	DOTM/medium	(M<0.9) and (0.25≤T≤0.5)
Batch 3	DOTM/long	(M<0.9) and (T>0.5)
Batch 4	OTM/short	(0.9≤M<0.95) and (T<0.25)
Batch 5	OTM/medium	(0.9≤M<0.95) and (0.25≤T≤0.5)
Batch 6	OTM/long	(0.9≤M<0.95) and (T>0.5)
Batch 7	ATM/short	(0.95≤M<1.05) and (T<0.25)
Batch 8	ATM/medium	(0.95≤M<1.05) and (0.25≤T≤0.5)
Batch 9	ATM/long	(0.95≤M<1.05) and (T>0.5)
Batch 10	ITM/short	(1.05≤M<1.1) and (T<0.25)
Batch 11	ITM/medium	(1.05≤M<1.1) and (0.25≤T≤0.5)
Batch 12	ITM/long	(1.05≤M<1.1) and (T>0.5)
Batch 13	DITM/short	(1.1≤M<1.5) and (T<0.25)
Batch 14	DITM/medium	(1.1≤M<1.5) and (0.25≤T≤0.5)
Batch 15	DITM/long	(1.1≤M<1.5) and (T>0.5)
Batch 16	VDITM/short	(1.5≤M<2.0) and (T<0.25)
Batch 17	VDITM/medium	(1.5≤M<2.0) and (0.25≤T≤0.5)
Batch 18	VDITM/long	(1.5≤M<2.0) and (T>0.5)
Batch 19	VVDITM/short	(M≥2.0) and (T<0.25)
Batch 20	VVDITM/medium	(M≥2.0) and (0.25≤T≤0.5)
Batch 21	VVDITM/long	(M≥2.0) and (T>0.5)

Table 3.3: Batch classification

4. Chapter Four

Empirical Results

In this chapter, all the empirical results are presented based on the methodology described in the previous chapter. All the VaR calculations are done for the three different confidence level (alpha) values:

- Alpha1= 0.001 or (0.1%)
- Alpha2= 0.01 or (1%)
- Alpha3= 0.05 or (5%)

Within this segment, the analysis and outcomes are showcased for a specific set of nine batches. This selection is predicated on the classification's foundation in moneyness categories: OTM, ATM, and ITM. The nine batches are as follows:

- OTM/short
- OTM/medium
- OTM/long
- ATM/short
- ATM/medium
- ATM/long
- ITM/short
- ITM/medium
- ITM/long

4.1 Quantiles of actual log returns for years 2020 and 2021

In the realm of finance and investment, understanding the historical performance and volatility of an asset or portfolio is of paramount importance. Log returns are used to measure the percentage change in the price of an asset over a period. Log returns are preferred over simple percentage returns because they have certain statistical properties that make them more suitable for financial analysis and modeling. Log returns, or logarithmic returns, are a key measure used to analyze financial data, assess risk, and make informed investment decisions.

Log returns are a mathematical transformation of simple returns, designed to convert multiplicative returns into additive returns. The log-return for a particular period is calculated as the natural logarithm of the ratio of the ending value of an asset to its beginning value. Mathematically, the log return (LR) for a period t is represented as: $\ln(t) = \ln(\frac{P_t}{P_{t-1}})$,Where P_t is the ending price of the option at time t, and P_{t-1} is its price at time t-1. The natural logarithm (ln) is used to transform the difference in prices into a percentage change.

The appendix section contains descriptive information for a total of 21 batches for this section.

A. Call Options

The log return for the call option is calculated by taking the natural logarithm of [call premium (today)/call premium (yesterday)]. The descriptive details of nine batches and their log returns' quantiles for the years 2020 and 2021 are shown below in Tables 4.1 and 4.2, respectively.

	Quantiles for year 2020		
	0.1%	**1%**	**5%**
OTM/short	-3.6557	-2.4908	-1.2391
OTM/medium	-0.9348	-0.6927	-0.4287
OTM/long	-0.7773	-0.4841	-0.2803
ATM/short	-1.9961	-1.0986	-0.5502
ATM/medium	-0.6748	-0.4416	-0.2593
ATM/long	-0.5158	-0.326	-0.1859
ITM/short	-0.4742	-0.3321	-0.1896
ITM/medium	-0.385	-0.2292	-0.1319
ITM/long	-0.369	-0.1853	-0.1026

Table 4.1: Descriptive statistics for call log returns for 2020

	Quantiles for year 2021		
	0.1%	**1%**	**5%**
OTM/short	-3.4752	-2.3535	-1.1632
OTM/medium	-1.0296	-0.5027	-0.3046
OTM/long	-0.4467	-0.2823	-0.1837
ATM/short	-2.0869	-0.9749	-0.4442
ATM/medium	-0.4527	-0.2665	-0.1832
ATM/long	-0.259	-0.1717	-0.1161
ITM/short	-0.2377	-0.1903	-0.1197
ITM/medium	-0.1734	-0.1394	-0.0911
ITM/long	-0.1263	-0.1083	-0.067

Table 4.2: Descriptive statistics for call log returns for 2021

This is particularly noticeable in the batches OTM/short and OTM/medium, where the log returns have experienced a downward shift. While the batch OTM/long also shows a decrease in log returns, the decline is less pronounced. Similarly, OTM batches and ATM batches also exhibit a decrease in log returns across all quantiles. The ‘short' and 'medium' categories have experienced the most significant decrease in log returns. Overall, it's evident that all the quantiles are lower in 2021 than their corresponding quantiles in 2020. This is because 2020 was a more volatile year than 2021, in part because of the COVID crisis.

B. Put Options

The log return for the put option is calculated by taking the natural logarithm of [put premium (today)/put premium (yesterday)]. Meanwhile, Tables 4.3 and 4.4 display the descriptive characteristics of nine batches, along with their corresponding log returns’ quantiles for the years 2020 and 2021.

	Quantiles for year 2020		
	0.1%	**1%**	**5%**
OTM/short	-0.4596	-0.3313	-0.22
OTM/medium	-0.3797	-0.2385	-0.1561
OTM/long	-0.3601	-0.1923	-0.1197
ATM/short	-1.5731	-0.8091	-0.4747
ATM/medium	-0.4789	-0.3345	-0.2028
ATM/long	-0.4285	-0.2528	-0.1446
ITM/short	-2.9174	-1.5599	-0.6576
ITM/medium	-0.508	-0.3437	-0.2217
ITM/long	-0.4723	-0.2447	-0.1467

Table 4.3: Descriptive statistics for put log returns for 2020

	Quantiles for year 2021		
	0.1%	**1%**	**5%**
OTM/short	-0.3725	-0.2788	-0.2107
OTM/medium	-0.2325	-0.1965	-0.1487
OTM/long	-0.1802	-0.148	-0.1077
ATM/short	-1.344	-0.8898	-0.5003
ATM/medium	-0.3485	-0.2816	-0.212
ATM/long	-0.2306	-0.1889	-0.1345
ITM/short	-2.322	-1.2613	-0.6931
ITM/medium	-0.5188	-0.3259	-0.2464
ITM/long	-0.2843	-0.2092	-0.145

Table 4.4: Descriptive statistics for put log returns for 2021

The decrease is more pronounced in the OTM/short scenario, with the largest difference observed in the 0.1% quantile. Overall, there is a consistent pattern of decreasing log return values in both ATM and ITM scenarios across different durations and quantiles. The magnitude of the decrease is most pronounced in the 0.1% quantile. Overall, the comparison between the two years' log returns suggests shifts in the distribution of returns for different batches. These changes have resulted from changing market circumstances stemming from the COVID-19 crisis.

C. One day straddles

The log return for the straddles is calculated by taking the natural logarithm of [straddle premium (today)/straddle premium (yesterday)]. In addition, Tables 4.5 and 4.6 exhibit descriptive attributes and corresponding log returns' quantiles for nine batches during the years 2020 and 2021.

	Quantiles for year 2020		
	0.1%	**1%**	**5%**
OTM/short	-0.4158	-0.3068	-0.2108
OTM/medium	-0.3301	-0.2393	-0.1221
OTM/long	-0.3169	-0.1796	-0.0767
ATM/short	-0.6182	-0.3269	-0.1844
ATM/medium	-0.3224	-0.1594	-0.0729
ATM/long	-0.2986	-0.124	-0.0445
ITM/short	-0.5898	-0.3256	-0.1502
ITM/medium	-0.3822	-0.2113	-0.0772
ITM/long	-0.3686	-0.1318	-0.046

Table 4.5: Descriptive statistics for 1-day straddle log returns for 2020

	Quantiles for year 2021		
	0.1%	**1%**	**5%**
OTM/short	-0.3686	-0.2762	-0.2049
OTM/medium	-0.1968	-0.1686	-0.1273
OTM/long	-0.1403	-0.1121	-0.0786
ATM/short	-0.4743	-0.3247	-0.1758
ATM/medium	-0.1696	-0.1222	-0.074
ATM/long	-0.1126	-0.0697	-0.0383
ITM/short	-0.2324	-0.1621	-0.0963
ITM/medium	-0.1712	-0.0707	-0.0491
ITM/long	-0.1105	-0.0484	-0.0308

Table 4.6: Descriptive statistics for 1-day straddle log returns for 2021

There are instances where the values in Table 4.6 are notably lower than those in Table 4.5. For instance, in the OTM/short batch, the values in all quantiles (0.1%, 1%, and 5%) have decreased, with the largest decrease observed in the 0.1% column (-0.3686 in Table 4.6 compared to -0.4158 in Table 4.5). This pattern of decrease is also visible in other batches like OTM/ medium and OTM/long. On the other hand, notably in the ITM/short category, the values have increased in the 1% and 5% columns. These findings collectively suggest that the quantiles for straddles are less extreme than the quantiles for options because the gain (loss) on the call partially offsets the loss (gain) on the put.

D. Five-day straddles

Tables 4.7 and 4.8 display descriptive characteristics and the corresponding logarithmic returns for nine batches spanning the years 2020 and 2021.

	Quantiles for year 2020		
	0.1%	**1%**	**5%**
OTM/short	-0.629	-0.5426	-0.4031
OTM/medium	-0.4104	-0.3467	-0.2345
OTM/long	-0.3188	-0.2557	-0.1436
ATM/short	-0.8738	-0.6314	-0.4279
ATM/medium	-0.4555	-0.3059	-0.1718
ATM/long	-0.406	-0.1993	-0.0972
ITM/short	-0.6549	-0.476	-0.306
ITM/medium	-0.3741	-0.2681	-0.1516
ITM/long	-0.2793	-0.1879	-0.0838

Table 4.7: Descriptive statistics for 5-day straddle log returns for 2020

	Quantiles for year 2021		
	0.1%	**1%**	**5%**
OTM/short	-0.5213	-0.4502	-0.3488
OTM/medium	-0.418	-0.3224	-0.2449
OTM/long	-0.2703	-0.2111	-0.1549
ATM/short	-1.0492	-0.5761	-0.3799
ATM/medium	-0.3036	-0.2229	-0.1488
ATM/long	-0.176	-0.123	-0.0818
ITM/short	-0.3444	-0.2552	-0.1712
ITM/medium	-0.2154	-0.1387	-0.0992
ITM/long	-0.1482	-0.0965	-0.0688

Table 4.8: Descriptive statistics for 5-day straddle log returns for 2021

In summary, the comparison of Tables 4.7 and 4.8 reveals dynamic shifts in logarithmic returns. Quantiles for straddles exhibit milder extremes compared to those for options, primarily due to the mitigating effect of gains on calls partially counteracting losses on puts. It is important to note that when considering 5-day returns versus 1-day returns, the former displays a greater degree of tail risk.

4.2 Analytical VaR

A. Call Options

Table 4.9 presents descriptive statistics for Delta Value at Risk (Delta VaR) values for call options in the years 2020 and 2021. Delta VaR is computed at different confidence levels, including 0.1%, 1%, and 5%. The table provides insights into the distribution of Delta VaR values for different risk levels.

The minimum Delta VaR values were all very close to 0.00001, indicating extremely low risk at these levels. Overall, this table offers a comprehensive overview of the distribution of Delta VaR values for call options, highlighting changes in risk levels between 2020 and 2021 across different confidence levels.

	Delta VaR for year 2020			**Delta VaR for year 2021**		
	0.1%	**1%**	**5%**	**0.1%**	**1%**	**5%**
Mean	0.05188	0.03906	0.02762	0.04681	0.03524	0.02491
Min	0.00001	0.00001	0.00001	0.00001	0.00001	0.00001
25%	0.02064	0.01554	0.01099	0.01820	0.01370	0.00969
50%	0.04486	0.03377	0.02388	0.03960	0.02981	0.02108
75%	0.06836	0.05147	0.03639	0.06116	0.04604	0.03255
Max	0.58214	0.43824	0.30986	0.58178	0.43797	0.30967

Table 4.9: Descriptive statistics of Delta VaR for calls

B. Put Options

Table 4.10 presents the descriptive statistics for the Delta Value at Risk (VaR) of put options for the years 2020 and 2021. The Delta VaR is calculated at different confidence levels, including 0.1%, 1%, and 5%. The minimum Delta VaR across all confidence levels was 0.00001, indicating the lowest potential loss.

Both years exhibited maximum Delta VaR values that indicated the highest potential loss. This table provides an overview of the distribution of Delta VaR for put options across different confidence levels for the years 2020 and 2021.

	Delta VaR for year 2020			Delta VaR for year 2021		
	0.1%	**1%**	**5%**	**0.1%**	**1%**	**5%**
Mean	0.01703	0.01282	0.00907	0.01035	0.00779	0.00551
Min	0.00001	0.00001	0.00001	0.00001	0.00001	0.00000
25%	0.00413	0.00311	0.00220	0.00364	0.00274	0.00194
50%	0.01224	0.00921	0.00652	0.00906	0.00682	0.00482
75%	0.02356	0.01774	0.01254	0.01618	0.01218	0.00861
Max	0.40432	0.30437	0.21521	0.22610	0.17021	0.12035

Table 4.10: Descriptive statistics of Delta VaR for puts

C. One-day straddles

Table 4.11 provides a comprehensive overview of the Delta Value at Risk (VaR) for 1-day straddles for the years 2020 and 2021. It highlights various statistical measures, shedding light on the distribution of the Delta VaR values. For 1-day straddles, there is a noticeable decrease in Delta VaR values from 2020 to 2021. The mean values are again higher in 2020, signifying a reduction in risk exposure.

	Delta VaR for year 2020			Delta VaR for year 2021		
	0.1%	**1%**	**5%**	**0.1%**	**1%**	**5%**
Mean	0.04490275	0.03380310	0.02390061	0.03928477	0.02957384	0.02091030
Min	0.00000008	0.00000006	0.00000004	0.00000004	0.00000003	0.00000002
25%	0.01758426	0.01323755	0.00935966	0.01372518	0.01033241	0.00730557
50%	0.03676949	0.02768032	0.01957148	0.02978726	0.02242405	0.01585502
75%	0.06194451	0.04663225	0.03297152	0.05488951	0.04132120	0.02921632
Max	0.54665270	0.41152390	0.29096960	0.57381220	0.43196970	0.30542590

Table 4.11: Descriptive statistics of Delta VaR for 1-day straddles

D. Five-day straddles

Table 4.12 provides a comprehensive overview of the distribution of Delta VaR values for different confidence levels, highlighting the potential losses associated with 5-day straddles for the years 2020 and 2021. 5-day straddles exhibit a decrease in Delta VaR values from 2020 to 2021. As with the other categories, the mean Delta VaR values are higher in 2020 compared to 2021, reflecting a decline in risk.

This reduction in risk is reflected in lower mean Delta VaR values in the latter year for all types of financial instruments studied, including calls, puts, 1-day straddles, and 5-day straddles, at various confidence levels.

	Delta VaR for year 2020			Delta VaR for year 2021		
	0.1%	**1%**	**5%**	**0.1%**	**1%**	**5%**
Mean	0.09881513	0.07438870	0.05259683	0.09073512	0.06830602	0.04829604
Min	0.00000008	0.00000006	0.00000005	0.00000026	0.00000020	0.00000014
25%	0.03781124	0.02846456	0.02012598	0.03172378	0.02388188	0.01688578
50%	0.07907147	0.05952554	0.04208777	0.07166225	0.05394783	0.03814403
75%	0.13652280	0.10277530	0.07266770	0.12757990	0.09604301	0.06790760
Max	1.28308300	0.96591360	0.68295310	1.20360200	0.90608010	0.64064760

Table 4.12: Descriptive statistics of Delta VaR for 5-day straddles

4.3 Historical VaR

A. Call Options

According to the data in Tables 4.1 and 4.13, it is evident that the values for all quantiles in 2020 are equal to or very close to the values of the historical VaRs. These VaR figures indicate the potential losses that could be incurred by holding the respective call options at different confidence levels based on historical data from the year 2020.

	Historical VaR for year 2020		
	0.1%	**1%**	**5%**
OTM/short	-3.6889	-2.5029	-1.2392
OTM/medium	-0.9350	-0.6932	-0.4289
OTM/long	-0.7829	-0.4841	-0.2804
ATM/short	-1.9963	-1.0986	-0.5504
ATM/medium	-0.6754	-0.4417	-0.2593
ATM/long	-0.5202	-0.3260	-0.1859
ITM/short	-0.4749	-0.3322	-0.1896
ITM/medium	-0.3920	-0.2303	-0.1319
ITM/long	-0.3697	-0.1856	-0.1026

Table 4.13: Historical VaR for calls

B. Put Options

Based on the information provided in Tables 4.3 and 4.14, it is clear that the values for all quantiles during the year 2020 either match or closely resemble the historical Value at Risk (VaR) values. The VaR values represent the potential loss in percentage terms based on historical data, considering the strike price and time to maturity of the put options. The table highlights the potential risks associated with different types of put options at various confidence levels during the year 2020. The potential losses associated with short OTM puts were more significant under extreme market conditions in 2020. There was a higher likelihood of substantial losses in short ATM puts during extreme market events.

	Historical VaR for year 2020		
	0.1%	**1%**	**5%**
OTM/short	-0.4636	-0.3316	-0.2200
OTM/medium	-0.3800	-0.2389	-0.1561
OTM/long	-0.3639	-0.1924	-0.1197
ATM/short	-1.5736	-0.8092	-0.4748
ATM/medium	-0.4795	-0.3346	-0.2028
ATM/long	-0.4298	-0.2531	-0.1446
ITM/short	-2.9267	-1.5603	-0.6581
ITM/medium	-0.5229	-0.3440	-0.2217
ITM/long	-0.4726	-0.2449	-0.1467

Table 4.14: Historical VaR for puts

C. One day straddles

According to the data presented in Tables 4.5 and 4.15, it is evident that the quantile values for the entirety of 2020 either align with or closely resemble the historical Value at Risk (VaR) values. The quantiles pertaining to straddles exhibit comparatively milder extremes compared to those of options. This discrepancy arises from the fact that the profit (or loss) incurred on the call option partially counterbalances the loss (or gain) experienced on the put option.

	Historical VaR for year 2020		
	0.1%	**1%**	**5%**
OTM/short	-0.4218	-0.3068	-0.2108
OTM/medium	-0.3302	-0.2394	-0.1221
OTM/long	-0.3174	-0.1797	-0.0767
ATM/short	-0.6186	-0.3269	-0.1845
ATM/medium	-0.3225	-0.1594	-0.0729
ATM/long	-0.2986	-0.1248	-0.0445
ITM/short	-0.5943	-0.3264	-0.1502
ITM/medium	-0.3841	-0.2113	-0.0772
ITM/long	-0.3708	-0.1320	-0.0460

Table 4.15: Historical VaR for 1-day straddles

D. Five-day straddles

According to the data presented in Tables 4.7 and 4.16, it is evident that the quantile values for the entire year of 2020 either align with or bear a strong resemblance to the historical Value at Risk

(VaR) values. It would be prudent to note that when considering 5-day returns as opposed to 1-day returns, there is a heightened manifestation of tail risk.

	Historical VaR for year 2020		
	0.1%	**1%**	**5%**
OTM/short	-0.6338	-0.5428	-0.4031
OTM/medium	-0.4105	-0.3467	-0.2348
OTM/long	-0.3195	-0.2564	-0.1438
ATM/short	-0.8753	-0.6314	-0.4280
ATM/medium	-0.4557	-0.3064	-0.1718
ATM/long	-0.4060	-0.1997	-0.0972
ITM/short	-0.6560	-0.4766	-0.3060
ITM/medium	-0.3761	-0.2687	-0.1516
ITM/long	-0.2837	-0.1881	-0.0838

Table 4.16: Historical VaR for 5-day straddles

4.4 Backtesting-Analytical VaR violation frequencies

A. Call Options

Backtesting the Analytical VaR calculation for call options reveals that the ratio is not even close to their corresponding alpha values at any confidence level.

	Alpha1=0.001	**Alpha2=0.01**	**Alpha3=0.05**
Ratio	0.2276	0.2520	0.2817

Table 4.17: Violation frequency ratio for calls-Analytical VaR

B. Put Options

The results of evaluating the Analytical VaR calculation for put options through backtesting indicate a significant disparity between the ratio and their respective alpha values across all confidence levels.

	Alpha1=0.001	**Alpha2=0.01**	**Alpha3=0.05**
Ratio	0.5731	0.5842	0.5943

Table 4.18: Violation frequency ratio for puts-Analytical VaR

C. One day straddles and Five-day straddles

The outcomes obtained from backtesting the Analytical VaR calculation for one-day straddles and five-day straddles reveal a notable difference between the ratio and the corresponding alpha values across various confidence levels.

	Alpha1=0.001	**Alpha2=0.01**	**Alpha3=0.05**
Ratio	0.2367	0.2707	0.3091

Table 4.19: Violation frequency ratio for 1-day straddles-Analytical VaR

	Alpha1=0.001	Alpha2=0.01	Alpha3=0.05
Ratio	0.2429	0.2724	0.3074

Table 4.20: Violation frequency ratio for 5-day straddles-Analytical VaR

4.5 Backtesting-Historical VaR violation frequencies

A. Call Options

The violation frequency ratio, which ideally should align closely with alpha, signifies the efficacy of the option pricing model in predicting option prices as compared to actual market prices. The batches OTM/medium and ATM/medium exhibit violation frequency ratios close to 0.001, implying a strong performance of the model. For batches OTM/medium and ATM/medium, the violation frequency ratio is close to 0.001. While for batch OTM/short, the violation frequency ratio is close to 0.05 (0.0442).

	Batch OTM			Batch ATM			Batch ITM		
	0.1%	**1%**	**5%**	**0.1%**	**1%**	**5%**	**0.1%**	**1%**	**5%**
short	0.0002	0.0094	0.0442	0.0011	0.0077	0.0313	0.0000	0.0000	0.0102
medium	0.0011	0.0030	0.0147	0.0003	0.0011	0.0113	0.0000	0.0000	0.0142
long	0.0001	0.0009	0.0104	0.0000	0.0004	0.0066	0.0000	0.0000	0.0144

Table 4.21: Violation frequency ratio for calls-Historical VaR

B. Put Options

The batches OTM/short, OTM/medium, and OTM/long exhibit a ratio close to 0.05. Similarly, batches ATM/short and OTM/medium have ratios very close to 0.001. Moreover, the majority of batches OTM, ATM, and OTM across all time-to-maturity segments (short, medium, and long) closely align with a ratio of approximately 0.05. This indicates that the model performs well in these cases.

	Batch OTM			Batch ATM			Batch ITM		
	0.1%	**1%**	**5%**	**0.1%**	**1%**	**5%**	**0.1%**	**1%**	**5%**
short	0.0000	0.0034	0.0428	0.0003	0.0155	0.0552	0.0000	0.0038	0.0590
medium	0.0000	0.0008	0.0416	0.0000	0.0017	0.0575	0.0011	0.0084	0.0644
long	0.0000	0.0002	0.0323	0.0000	0.0002	0.0375	0.0000	0.0028	0.0488

Table 4.22: Violation frequency ratio for puts- Historical VaR

C. One day straddles

For all time-to-maturity categories-short, medium, and long-the majority of OTM and ATM batches' ratio values are in close proximity to 0.05. This suggests that the model performs well in these scenarios. On the other hand, this is not true for the batch ITM due to the fact that the ratio values are not close in any scenario.

	Batch OTM			Batch ATM			Batch ITM		
	0.1%	**1%**	**5%**	**0.1%**	**1%**	**5%**	**0.1%**	**1%**	**5%**
short	0.0002	0.0049	0.0444	0.0002	0.0097	0.0441	0.0000	0.0001	0.0118
medium	0.0000	0.0003	0.0585	0.0000	0.0017	0.0516	0.0000	0.0006	0.0090
long	0.0000	0.0002	0.0544	0.0000	0.0006	0.0359	0.0000	0.0006	0.0131

Table 4.23: Violation frequency ratio for 1-day straddles- Historical VaR

D. Five-day straddles

The ratio value for batches OTM/medium and ATM/short is close to alpha 0.01. On the other side, ratio values for batches OTM/medium and OTM/long are close to alpha value 0.05.

	Batch OTM			Batch ATM			Batch ITM		
	0.1%	**1%**	**5%**	**0.1%**	**1%**	**5%**	**0.1%**	**1%**	**5%**
short	0.0000	0.0007	0.0267	0.0028	0.0075	0.0325	0.0000	0.0000	0.0032
medium	0.0020	0.0064	0.0585	0.0000	0.0009	0.0302	0.0000	0.0000	0.0063
long	0.0000	0.0024	0.0632	0.0000	0.0004	0.0278	0.0000	0.0000	0.0207

Table 4.24: Violation frequency ratio for 5-day straddles- Historical VaR

4.6 Backtesting-Unconditional coverage tests

This test examines the effectiveness of VaR models by gauging their performance. It quantifies the instances of VaR violations when the observed loss surpasses the predicted VaR threshold. The test assesses whether the proportion of VaR violations aligns with the expected level. The tables in this section demonstrate the chi-square statistics corresponding to distinct quantiles for a single degree of freedom. Our focus here lies on Type-2 error, wherein an erroneous model is accepted. Within the financial context, the primary concern revolves around the acceptance of an incorrect model, making the decision to reject a valid model a reasonable choice. The test is considered "Not applicable" for batches where the VaR violation numbers are zero.

A. Call Options

A substantial portion of these values converges towards zero, indicative of a favorable outcome. Consequently, the model can be accepted. The batch ATM/long (2.2610) implies that at a 10% confidence level, there is insufficient evidence to dismiss the model.

	Batch OTM			Batch ATM			Batch ITM		
	0.1%	**1%**	**5%**	**0.1%**	**1%**	**5%**	**0.1%**	**1%**	**5%**
short	0.0072	0.0002	0.0047	0.0003	0.0213	0.3099	NA	0.0000	0.7888
medium	0.0002	0.0864	0.4500	0.0219	0.4960	1.7165	NA	0.0000	0.5998
long	0.0230	0.2159	0.7318	NA	0.6136	2.2610	NA	0.0000	0.5711

Table 4.25: Unconditional coverage test results for calls

B. Put Options

Most of the values are very close to zero, which is a good representation of the result. So, we can accept the model. As the statistics are closer to zero, the result is better. It means we cannot reject the model at a 10% confidence level.

	Batch OTM			Batch ATM			Batch ITM		
	0.1%	**1%**	**5%**	**0.1%**	**1%**	**5%**	**0.1%**	**1%**	**5%**
short	NA	0.0382	0.0074	0.0251	0.0971	0.0200	NA	0.0811	0.0262
medium	NA	0.1804	0.0195	NA	0.4044	0.0428	0.0002	0.0047	0.0650
long	NA	0.2765	0.1142	NA	0.6636	0.1319	NA	0.1129	0.0005

Table 4.26: Unconditional coverage test results for puts

C. One day straddles

The majority of the values are extremely near zero, which effectively depicts the outcome. Therefore, the model can be embraced. Given that the statistics closely approach zero, it is advantageous for the outcome. This indicates that at a 10% confidence level, the model cannot be dismissed.

	Batch OTM			Batch ATM			Batch ITM		
	0.1%	**1%**	**5%**	**0.1%**	**1%**	**5%**	**0.1%**	**1%**	**5%**
short	0.0072	0.0205	0.0045	0.0400	0.0003	0.0279	NA	0.3033	0.7080
medium	NA	0.2155	0.0182	NA	0.4008	0.0021	NA	0.2491	0.8564
long	NA	0.2765	0.0059	NA	0.5595	0.1688	NA	0.2434	0.6242

Table 4.27: Unconditional coverage test results for 1-day straddles

D. Five-day straddles

The majority of the values are in close proximity to zero, which accurately reflects the outcome. Consequently, the model is deemed acceptable. Given that the statistics exhibit proximity to zero, they enhance the validity of the results. This implies that, at a 10% confidence level, there are no grounds for rejecting the model.

	Batch OTM			Batch ATM			Batch ITM		
	0.1%	**1%**	**5%**	**0.1%**	**1%**	**5%**	**0.1%**	**1%**	**5%**
short	NA	0.0630	0.0576	0.0710	0.0228	0.2329	NA	NA	1.0306
medium	0.0079	0.0145	0.0144	NA	0.4555	0.3127	NA	NA	0.8381
long	NA	0.1085	0.0439	NA	0.5502	0.3980	NA	NA	0.3021

Table 4.28: Unconditional coverage test results for 5-day straddles

4.7 Backtesting-Clustering tests

When assessing the performance of a VaR model, one common concern is whether VaR violations (instances where the actual losses exceed the predicted VaR) are clustered or not. Clustering of VaR violations can indicate potential deficiencies in the VaR model, as it may suggest that the model is underestimating the true risk during certain time periods or market conditions. The tables within this segment display the chi-square statistics associated with different quantiles for one degree of freedom. The emphasis in this section is on Type-2 error, which involves the acceptance of a flawed model. Specifically, within the financial context, the main worry is centered on embracing an inaccurate model, which makes opting to reject a valid model a justifiable decision. The test is "Not applicable" for batches in which the clustering sequences '01', '10', or '11' have values equal to zero.

A. Call Options

Most of the values in Table 4.29 are very close to zero, suggesting a positive portrayal of the result. As a result, we can accept the model, indicating that it remains valid even with a confidence level of 10%.

	Batch OTM			Batch ATM			Batch ITM		
	0.1%	**1%**	**5%**	**0.1%**	**1%**	**5%**	**0.1%**	**1%**	**5%**
short	NA	0.0022	0.0040	NA	0.0152	0.2535	NA	NA	NA
medium	NA	NA	0.0183	NA	0.0044	0.0046	NA	NA	0.0023
long	NA	NA	NA	NA	NA	NA	NA	NA	0.0022

Table 4.29: Clustering test results for calls

B. Put Options

The majority of the values in Table 4.30 are near to zero, indicating that we can validate the model. This implies that, at a 10% confidence level, we do not have grounds to discard the model.

	Batch OTM			Batch ATM			Batch ITM		
	0.1%	**1%**	**5%**	**0.1%**	**1%**	**5%**	**0.1%**	**1%**	**5%**
short	NA	NA	0.0133	NA	0.3116	0.7120	NA	0.0051	0.1474
medium	NA	NA	0.0169	NA	0.0080	0.0041	NA	0.0006	0.0145
long	NA	NA	0.0051	NA	NA	0.0372	NA	NA	0.0123

Table 4.30: Clustering test results for puts

C. One day straddles

The majority of the values are near zero in Table 4.31, which indicates that we can validate the model. In other words, at a 10% confidence level, we fail to reject the model.

	Batch OTM			Batch ATM			Batch ITM		
	0.1%	**1%**	**5%**	**0.1%**	**1%**	**5%**	**0.1%**	**1%**	**5%**
short	NA	NA	0.0115	NA	0.0850	0.2935	NA	NA	NA
medium	NA	NA	0.0040	NA	NA	0.1377	NA	NA	0.0037
long	NA	NA	0.0559	NA	0.0268	0.1679	NA	NA	0.0006

Table 4.31: Clustering test results for 1-day straddles

D. Five-day straddles

Most of the values are very close to zero, which is a good representation of the result. So, we can accept the model. It means we cannot reject the model at a 10% confidence level.

	Batch OTM			Batch ATM			Batch ITM		
	0.1%	**1%**	**5%**	**0.1%**	**1%**	**5%**	**0.1%**	**1%**	**5%**
short	NA	0.0011	0.0109	0.1846	0.6372	1.9922	NA	NA	NA
medium	NA	NA	0.6359	NA	NA	1.6246	NA	NA	0.2703
long	NA	NA	0.9866	NA	0.0652	1.3738	NA	NA	0.4196

Table 4.32: Clustering test results for 5-day straddles

4.8 Backtesting-Joint tests

A joint test for the average number of violations and their independence is given by the arithmetic sum of clustering test values and conditional coverage test values. The statistical distribution of the various quantiles follows a chi-square distribution with two degrees of freedom. The test is deemed "Not Applicable" for the batches within this section in cases where either the unconditional coverage test cannot be applied or the clustering test holds no significance, as demonstrated in the preceding sections.

Most of the values within Tables 4.33, 4.34, 4.35, and 4.36 exhibit extreme proximity to zero, implying a favorable depiction of the outcome. Consequently, we are able to embrace the model, affirming its continued validity even with a confidence level of 10%. All the results are represented below:

A. Call Options

	Batch OTM			Batch ATM			Batch ITM		
	0.1%	**1%**	**5%**	**0.1%**	**1%**	**5%**	**0.1%**	**1%**	**5%**
short	NA	0.0024	0.0088	NA	0.0365	0.5634	NA	NA	NA
medium	NA	NA	0.4684	NA	0.5004	1.7211	NA	NA	0.6021
long	NA	NA	NA	NA	NA	NA	NA	NA	0.5733

Table 4.33: Joint test results for calls

B. Put Options

	Batch OTM			Batch ATM			Batch ITM		
	0.1%	**1%**	**5%**	**0.1%**	**1%**	**5%**	**0.1%**	**1%**	**5%**
short	NA	NA	0.0207	NA	0.4086	0.7320	NA	0.0862	0.1736
medium	NA	NA	0.0364	NA	0.4124	0.0469	NA	0.0053	0.0795
long	NA	NA	0.1193	NA	NA	0.1690	NA	NA	0.0128

Table 4.34: Joint test results for puts

C. One day straddles

	Batch OTM			Batch ATM			Batch ITM		
	0.1%	**1%**	**5%**	**0.1%**	**1%**	**5%**	**0.1%**	**1%**	**5%**
short	NA	NA	0.0160	NA	0.0853	0.3214	NA	NA	NA
medium	NA	NA	0.0222	NA	NA	0.1398	NA	NA	0.8601
long	NA	NA	0.0619	NA	0.5864	0.3367	NA	NA	0.6248

Table 4.35: Joint test results for 1-day straddles

D. Five-day straddles

	Batch OTM			Batch ATM			Batch ITM		
	0.1%	**1%**	**5%**	**0.1%**	**1%**	**5%**	**0.1%**	**1%**	**5%**
short	NA	0.0741	0.0685	0.2556	0.6601	2.2251	NA	NA	NA
medium	NA	NA	0.6504	NA	NA	1.9373	NA	NA	1.1084
long	NA	NA	1.0305	NA	0.6154	1.7718	NA	NA	0.7217

Table 4.36: Joint test results for 5-day straddles

5. Chapter Five

Robustness

Another type of data classification is used to perform the study for the period of year 2020.

1. First six months of 2020 (from January 1 to June 30, 2020)
2. Last six months of 2020 (from July 1 to December 31, 2020)

There was a big event, COVID-19 occurred in the first half of 2020, so the market volatility was too high. So, for the robustness check, another approach is followed for the study: to consider the last six months of the year 2020 historical data and repeat all the methodologies as we discussed earlier in Chapter 3.

5.1 Actual log return quantiles for the last six months of 2020 and 2021

The appendix section contains descriptive information for a total of 21 batches for this section.

A. Call Options

From Tables 5.1 and 5.2, it is evident that batches OTM/medium and ATM/short, show larger differences in log returns between the two years, while others, like ITM/long, exhibit relatively smaller variations.

	Quantiles for year 2020		
Batch Label	**0.1%**	**1%**	**5%**
OTM/short	-3.8393	-2.976	-1.0986
OTM/medium	-0.9368	-0.5138	-0.3273
OTM/long	-0.662	-0.3587	-0.1954
ATM/short	-1.953	-0.9556	-0.4607
ATM/medium	-0.6381	-0.3217	-0.1915
ATM/long	-0.3798	-0.2453	-0.1314
ITM/short	-0.3809	-0.2733	-0.1492
ITM/medium	-0.3005	-0.1969	-0.1054
ITM/long	-0.2486	-0.1533	-0.0829

Table 5.1: Descriptive statistics for call log returns for the last six months of 2020

	Quantiles for year 2021		
Batch Label	**0.1%**	**1%**	**5%**
OTM/short	-3.3132	-1.9755	-1.0992
OTM/medium	-1.0705	-0.5209	-0.3201
OTM/long	-0.4973	-0.2978	-0.1897
ATM/short	-2.1047	-1.047	-0.452
ATM/medium	-0.4544	-0.2881	-0.1891
ATM/long	-0.2648	-0.182	-0.1172
ITM/short	-0.2384	-0.191	-0.1182
ITM/medium	-0.1825	-0.1409	-0.0923
ITM/long	-0.1323	-0.1082	-0.0664

Table 5.2: Descriptive statistics for call log returns for the last six months of 2021

B. Put Options

Comparing the two Tables 5.3 and 5.4, it's notable that ATM/short, show larger differences in log returns between the two years, while others exhibit relatively smaller variations.

	Quantiles for year 2020		
Batch Label	**0.1%**	**1%**	**5%**
OTM/short	-0.294	-0.2491	-0.1987
OTM/medium	-0.1935	-0.1684	-0.1381
OTM/long	-0.1507	-0.1294	-0.1042
ATM/short	-1.7099	-0.7761	-0.4312
ATM/medium	-0.2553	-0.2182	-0.176
ATM/long	-0.1901	-0.1576	-0.1272
ITM/short	-3.0011	-1.6083	-0.6647
ITM/medium	-0.4592	-0.2614	-0.1936
ITM/long	-0.2901	-0.1813	-0.1337

Table 5.3: Descriptive statistics for put log returns for the last six months of 2020

	Quantiles for year 2021		
Batch Label	**0.1%**	**1%**	**5%**
OTM/short	-0.3733	-0.2856	-0.219
OTM/medium	-0.2395	-0.2005	-0.149
OTM/long	-0.1848	-0.1518	-0.1062
ATM/short	-1.2523	-0.9234	-0.5222
ATM/medium	-0.3508	-0.2904	-0.2228
ATM/long	-0.2368	-0.1909	-0.1327
ITM/short	-2.5336	-1.3177	-0.7669
ITM/medium	-0.527	-0.3677	-0.2534
ITM/long	-0.2842	-0.2081	-0.1443

Table 5.4: Descriptive statistics for put log returns for the last six months of 2021

C. One day straddles

The mean log returns for OTM batches decrease as the quantile range increases (e.g., -0.2957 for 0.1% to -0.1786 for 5%). ATM batches generally have more negative mean log returns compared to OTM and ITM batches across different quantile ranges.

	Quantiles for year 2020		
Batch Label	**0.1%**	**1%**	**5%**
OTM/short	-0.2957	-0.2385	-0.1786
OTM/medium	-0.1489	-0.1229	-0.0982
OTM/long	-0.1056	-0.0816	-0.0593
ATM/short	-0.5975	-0.2852	-0.1627
ATM/medium	-0.3007	-0.0932	-0.0575
ATM/long	-0.195	-0.0652	-0.0368
ITM/short	-0.5141	-0.2321	-0.1253
ITM/medium	-0.3333	-0.1117	-0.0595
ITM/long	-0.251	-0.0812	-0.0382

Table 5.5: Descriptive statistics for 1-day straddle log returns for the last six months of 2020

	Quantiles for year 2021		
Batch Label	**0.1%**	**1%**	**5%**
OTM/short	-0.3691	-0.2834	-0.215
OTM/medium	-0.1968	-0.1744	-0.131
OTM/long	-0.1406	-0.1168	-0.0795
ATM/short	-0.4742	-0.3656	-0.1825
ATM/medium	-0.1763	-0.1272	-0.0749
ATM/long	-0.115	-0.0664	-0.0355
ITM/short	-0.273	-0.1763	-0.0978
ITM/medium	-0.2138	-0.0837	-0.049
ITM/long	-0.1312	-0.0484	-0.0312

Table 5.6: Descriptive statistics for 1-day straddle log returns for the last six months of 2021

D. Five-day straddles

ATM batches generally have more negative log returns compared to OTM and ITM batches.

	Quantiles for year 2020		
Batch Label	**0.1%**	**1%**	**5%**
OTM/short	-0.6089	-0.4567	-0.3479
OTM/medium	-0.3441	-0.2848	-0.1938
OTM/long	-0.2411	-0.1676	-0.1138
ATM/short	-0.8869	-0.5736	-0.3974
ATM/medium	-0.3856	-0.2125	-0.1428
ATM/long	-0.2844	-0.1561	-0.0829
ITM/short	-0.5479	-0.4745	-0.2957
ITM/medium	-0.3685	-0.2606	-0.1424
ITM/long	-0.2887	-0.1907	-0.0837

Table 5.7: Descriptive statistics for 5-day straddle log returns for the last six months of 2020

	Quantiles for year 2021		
Batch Label	**0.1%**	**1%**	**5%**
OTM/short	-0.4998	-0.4501	-0.3499
OTM/medium	-0.4648	-0.3393	-0.2594
OTM/long	-0.2742	-0.2229	-0.1713
ATM/short	-1.125	-0.6742	-0.3955
ATM/medium	-0.3137	-0.2438	-0.1526
ATM/long	-0.185	-0.1263	-0.0761
ITM/short	-0.355	-0.2732	-0.1784
ITM/medium	-0.2308	-0.1441	-0.1004
ITM/long	-0.1598	-0.0927	-0.0665

Table 5.8: Descriptive statistics for 5-day straddle log returns for the last six months of 2021

5.2 Historical VaR for the last six months of year 2020

A. Call Options

Based on the information presented in Tables 5.1 and 5.9, it becomes apparent that the values corresponding to all quantiles in the year 2020 closely match or are nearly identical to the historical VaR values.

Batch Label	0.1%	1%	5%
OTM/short	-3.8607	-3.0068	-1.0986
OTM/medium	-0.9417	-0.5138	-0.3273
OTM/long	-0.6842	-0.3683	-0.1954
ATM/short	-1.9561	-0.9589	-0.4608
ATM/medium	-0.6420	-0.3217	-0.1915
ATM/long	-0.3872	-0.2454	-0.1314
ITM/short	-0.3841	-0.2737	-0.1493
ITM/medium	-0.3012	-0.1985	-0.1054
ITM/long	-0.2490	-0.1541	-0.0829

Table 5.9: Historical VaR for calls for the last six months of 2020

B. Put Options

From Tables 5.3 and 5.10, it is shown that the values for all quantiles in the year 2020 either match or closely resemble the historical VaR values.

Batch Label	0.1%	1%	5%
OTM/short	-0.2959	-0.2492	-0.1988
OTM/medium	-0.1939	-0.1684	-0.1382
OTM/long	-0.1508	-0.1294	-0.1043
ATM/short	-1.7525	-0.7786	-0.4313
ATM/medium	-0.2556	-0.2182	-0.1760
ATM/long	-0.1903	-0.1576	-0.1272
ITM/short	-3.0020	-1.6094	-0.6678
ITM/medium	-0.4595	-0.2686	-0.1936
ITM/long	-0.2903	-0.1816	-0.1337

Table 5.10: Historical VaR for puts for the last six months of 2020

C. One day straddles

The data in Tables 5.5 and 5.11 illustrate that the quantile values in the year 2020 either align closely with or bear a strong resemblance to the historical VaR values.

Batch Label	0.1%	1%	5%
OTM/short	-0.3067	-0.2386	-0.1786
OTM/medium	-0.1496	-0.1229	-0.0982
OTM/long	-0.1070	-0.0818	-0.0593
ATM/short	-0.5993	-0.2854	-0.1627
ATM/medium	-0.3007	-0.0932	-0.0575
ATM/long	-0.2027	-0.0653	-0.0369
ITM/short	-0.5170	-0.2321	-0.1255
ITM/medium	-0.3337	-0.1122	-0.0595
ITM/long	-0.2512	-0.0813	-0.0382

Table 5.11: Historical VaR for 1-day straddles for the last six months of 2020

D. Five-day straddles

Tables 5.7 and 5.12 demonstrate that the quantile values for the year 2020 align closely with, or are similar to, the historical VaR values.

Batch Label	0.1%	1%	5%
OTM/short	-0.6157	-0.4570	-0.3479
OTM/medium	-0.3455	-0.2852	-0.1938
OTM/long	-0.2412	-0.1677	-0.1139
ATM/short	-0.8889	-0.5736	-0.3974
ATM/medium	-0.3856	-0.2125	-0.1428
ATM/long	-0.2867	-0.1566	-0.0829
ITM/short	-0.5703	-0.4747	-0.2957
ITM/medium	-0.3719	-0.2610	-0.1424
ITM/long	-0.2901	-0.1908	-0.0837

Table 5.12: Historical VaR for 5-day straddles for the last six months of 2020

5.2.1 Backtesting-VaR violation frequencies

The violation frequency ratio, which ideally should align closely with alpha.

A. Call Options

After comparing the results presented in Table 5.13 with Table 4.21, the violation frequency ratio in Table 5.13 for batch ATM/medium improves and is close to alpha 0.001. It improves for batches OTM/medium, ATM/short, and ATM/medium and is close to alpha 0.01. Similarly, it performs better for OTM and ATM for all durations and is close to alpha 0.05.

	Batch OTM			**Batch ATM**			**Batch ITM**		
	0.1%	**1%**	**5%**	**0.1%**	**1%**	**5%**	**0.1%**	**1%**	**5%**
short	0.0000	0.0041	0.0500	0.0012	0.0116	0.0473	0.0000	0.0000	0.0278
medium	0.0016	0.0108	0.0477	0.0005	0.0064	0.0483	0.0000	0.0000	0.0380
long	0.0003	0.0022	0.0480	0.0004	0.0017	0.0353	0.0000	0.0000	0.0304

Table 5.13: Violation frequency ratio for calls for last six months

B. Put Options

From the results of Table 5.14 and Table 4.22, the violation frequency ratio improves for all alpha values for most batches (OTM,ATM, and ITM) and is very close to the corresponding alpha values.

	Batch OTM			**Batch ATM**			**Batch ITM**		
	0.1%	**1%**	**5%**	**0.1%**	**1%**	**5%**	**0.1%**	**1%**	**5%**
short	0.0079	0.0287	0.0663	0.0000	0.0210	0.0682	0.0000	0.0052	0.0606
medium	0.0134	0.0308	0.0601	0.0250	0.0527	0.0927	0.0020	0.0444	0.0979
long	0.0108	0.0287	0.0518	0.0103	0.0296	0.0545	0.0007	0.0178	0.0566

Table 5.14: Violation frequency ratio for puts for last six months

C. One day straddles

It shows from Tables 5.15 and 4.23 that for the batch ATM/long, the ratio improves for both alphas (0.01 and 0.05) and close to alphas, respectively.

	Batch OTM			**Batch ATM**			**Batch ITM**		
	0.1%	**1%**	**5%**	**0.1%**	**1%**	**5%**	**0.1%**	**1%**	**5%**
short	0.0069	0.0311	0.0819	0.0002	0.0182	0.0635	0.0000	0.0019	0.0222
medium	0.0317	0.0612	0.1073	0.0000	0.0312	0.0781	0.0000	0.0037	0.0250
long	0.0194	0.0460	0.1158	0.0000	0.0108	0.0463	0.0000	0.0032	0.0262

Table 5.15: Violation frequency ratio for 1-day straddles for last six months

D. Five-day straddles

It shows from Tables 5.16 and 4.24 that for the batch ATM for all durations, the ratio improves for alpha 0.05 and closes to alpha, respectively.

	Batch OTM			Batch ATM			Batch ITM		
	0.1%	**1%**	**5%**	**0.1%**	**1%**	**5%**	**0.1%**	**1%**	**5%**
short	0.0000	0.0077	0.0519	0.0045	0.0144	0.0492	0.0000	0.0000	0.0067
medium	0.0092	0.0298	0.1231	0.0000	0.0187	0.0584	0.0000	0.0001	0.0102
long	0.0039	0.0556	0.1167	0.0000	0.0036	0.0398	0.0000	0.0000	0.0189

Table 5.16: Violation frequency ratio for 5-day straddles for last six months

5.2.2 Backtesting-Unconditional coverage tests

The test is deemed "inapplicable" for batches with zero VaR violation counts.

A. Call Options

From Tables 4.25 and 5.17, it is concluded that in Table 5.17, the test is not applicable for the Batch ITM for all durations. While all the values approach close to zero. This implies that we do not have sufficient evidence to reject the model at a 10% confidence level.

	Batch OTM			Batch ATM			Batch ITM		
	0.1%	**1%**	**5%**	**0.1%**	**1%**	**5%**	**0.1%**	**1%**	**5%**
short	NA	0.0191	0.0000	0.0008	0.0046	0.0032	NA	NA	0.1076
medium	0.0026	0.0005	0.0009	0.0052	0.0311	0.0013	NA	NA	0.0285
long	0.0057	0.0839	0.0008	0.0104	0.2084	0.0971	NA	NA	0.0766

Table 5.17: Unconditional coverage test results for calls for last six months

B. Put Options

From Tables 4.26 and 5.18, the value of chi square for batch ATM/medium is maximum (2.3161) in Table 5.18. On the other hand, this value is still less than the threshold value (2.70554) for a confidence level of 90%. So, in summary, we cannot reject the model at the 10% confidence level.

	Batch OTM			Batch ATM			Batch ITM		
	0.1%	**1%**	**5%**	**0.1%**	**1%**	**5%**	**0.1%**	**1%**	**5%**
short	0.0790	0.0983	0.0213	NA	0.1848	0.1250	NA	0.0251	0.0192
medium	0.3624	0.2270	0.0161	2.3161	1.8697	0.6328	0.0063	0.5597	0.3302
long	0.2961	0.2172	0.0006	0.5708	0.4923	0.0082	0.0006	0.0411	0.0073

Table 5.18: Unconditional coverage test results for puts for last six months

C. One day straddles

From Tables 4.27 and 5.19, the value of chi square for batch OTM/medium is maximum (1.2766) in Table 5.19. On the other hand, the test is not applicable for ITM (0.1% and 1%). The value is less than the threshold value (2.70554) for confidence level 90%. This indicates that at a 10% confidence level, the model cannot be dismissed.

	Batch OTM			Batch ATM			Batch ITM		
	0.1%	**1%**	**5%**	**0.1%**	**1%**	**5%**	**0.1%**	**1%**	**5%**
short	0.0629	0.0034	0.0755	0.0225	0.1647	0.0702	NA	NA	0.1768
medium	1.2766	0.0119	0.4248	NA	0.0504	0.2917	NA	NA	0.1390
long	0.7254	0.0022	0.6235	NA	0.0002	0.0057	NA	NA	0.1169

Table 5.19: Unconditional coverage test results for 1-day straddles for last six months

D. Five-day straddles

From Tables 4.28 and 5.20, the values of chi square for any batch are less than the threshold value (2.70554) for confidence level 90%. This suggests that the model retains significance at a confidence level of 10%.

	Batch OTM			Batch ATM			Batch ITM		
	0.1%	**1%**	**5%**	**0.1%**	**1%**	**5%**	**0.1%**	**1%**	**5%**
short	NA	0.0016	0.0002	0.1131	0.0297	0.0003	NA	NA	0.4355
medium	0.1599	0.1683	0.5303	NA	0.1079	0.0249	NA	0.1311	0.3435
long	0.0367	0.7909	0.5375	NA	0.0924	0.0402	NA	NA	0.1813

Table 5.20: Unconditional coverage test results for 5-day straddles for last six months

5.2.3 Backtesting-Clustering tests

The test does not apply to batches where the values of the clustering sequences '01', '10', or '11' are zero.

A. Call Options

It is clear from Tables 4.29 and 5.21 that in this case the test is applicable to more batches than Table 4.29. The majority of the values in Table 5.21 are near zero, indicating that we can validate the model.

	Batch OTM			Batch ATM			Batch ITM		
	0.1%	**1%**	**5%**	**0.1%**	**1%**	**5%**	**0.1%**	**1%**	**5%**
short	NA	NA	0.0051	NA	0.0123	0.2167	NA	NA	NA
medium	NA	0.0091	0.0025	NA	NA	0.0012	NA	NA	NA
long	NA	NA	0.0001	NA	NA	0.0008	NA	NA	NA

Table 5.21: Clustering test results for calls-last six months

B. Put Options

The test is applicable for the batches OTM/short (1%) and ATM/medium (0.1%). Most of the values in Table 5.22 are clustered around zero, suggesting that the model can be validated.

	Batch OTM			Batch ATM			Batch ITM		
	0.1%	**1%**	**5%**	**0.1%**	**1%**	**5%**	**0.1%**	**1%**	**5%**
short	NA	0.0025	0.0151	NA	0.4153	0.5009	NA	0.0052	0.1180
medium	NA	NA	0.0044	0.0085	0.0105	0.0250	NA	0.0154	0.0364
long	NA	NA	0.0000	NA	NA	0.0089	NA	NA	0.0278

Table 5.22: Clustering test results for puts-last six months

C. One day straddles

The test is applicable for batches OTM, ATM, and all durations (1%). Most of the values in Table 5.23 cluster around zero, suggesting that the model's validation is feasible.

	Batch OTM			Batch ATM			Batch ITM		
	0.1%	**1%**	**5%**	**0.1%**	**1%**	**5%**	**0.1%**	**1%**	**5%**
short	NA	0.0034	0.0038	NA	0.1647	0.2104	NA	NA	0.0058
medium	NA	0.0119	0.0173	NA	0.0504	0.1524	NA	NA	0.0056
long	NA	0.0022	0.0699	NA	0.0002	0.1060	NA	NA	0.0016

Table 5.23: Clustering test results for 1-day straddles-last six months

D. Five-day straddles

As the statistics are closer to zero, the result is better. It means we cannot reject the model at 10% confidence level.

	Batch OTM			Batch ATM			Batch ITM		
	0.1%	**1%**	**5%**	**0.1%**	**1%**	**5%**	**0.1%**	**1%**	**5%**
short	NA	NA	0.0370	0.1430	0.7024	1.4742	NA	NA	NA
medium	NA	0.1725	0.8299	NA	0.5393	1.8111	NA	NA	0.2420
long	NA	0.5132	1.1209	NA	0.1866	1.0491	NA	NA	0.2473

Table 5.24: Clustering test results for 5-day straddles-last six months

5.2.4 Backtesting-Joint tests

The test is considered "Not Applicable" for the batches in this segment if either the unconditional coverage test cannot be executed or the clustering test doesn't carry any significance, as shown in the previous sections. While comparing the test result of this section to Section 4.8, it is concluded that the test is applicable to more batches in the case of options and straddles.

Most values in all cases tend to zero, which is better than the results that are shown in Section 4.8. As a result, we can adopt the model, confirming its validity even with a confidence level of 10%. The outcomes are detailed below:

A. Call Options

	Batch OTM			Batch ATM			Batch ITM		
	0.1%	**1%**	**5%**	**0.1%**	**1%**	**5%**	**0.1%**	**1%**	**5%**
short	NA	NA	0.0051	NA	0.0169	0.2198	NA	NA	NA
medium	NA	0.0096	0.0033	NA	NA	0.0025	NA	NA	NA
long	NA	NA	0.0008	NA	NA	0.0978	NA	NA	NA

Table 5.25: Joint test results for calls-last six months

B. Call Options

	Batch OTM			Batch ATM			Batch ITM		
	0.1%	**1%**	**5%**	**0.1%**	**1%**	**5%**	**0.1%**	**1%**	**5%**
short	NA	0.1008	0.0364	NA	0.6001	0.6259	NA	0.0303	0.1372
medium	NA	NA	0.0205	2.3246	1.8802	0.6577	NA	0.5752	0.3666
long	NA	NA	0.0007	NA	NA	0.0171	NA	NA	0.0351

Table 5.26: Joint test results for puts-last six months

C. Call Options

	Batch OTM			Batch ATM			Batch ITM		
	0.1%	**1%**	**5%**	**0.1%**	**1%**	**5%**	**0.1%**	**1%**	**5%**
short	NA	0.1240	0.0793	NA	0.2744	0.2806	NA	NA	0.1827
medium	NA	0.9926	0.4420	NA	0.6446	0.4441	NA	NA	0.1446
long	NA	0.6446	0.6934	NA	0.0014	0.1117	NA	NA	0.1184

Table 5.27: Joint test results for 1-day straddles-last six months

D. Call Options

	Batch OTM			Batch ATM			Batch ITM		
	0.1%	**1%**	**5%**	**0.1%**	**1%**	**5%**	**0.1%**	**1%**	**5%**
short	NA	NA	0.0372	0.2561	0.7321	1.4744	NA	NA	NA
medium	NA	0.3407	1.3602	NA	0.6472	1.8360	NA	NA	0.5856
long	NA	1.3041	1.6583	NA	0.2791	1.0893	NA	NA	0.4286

Table 5.28: Joint test results for 5-day straddles-last six months

5.3 Kernel Density Estimation (KDE)

Kernel Density Estimation (KDE) is a powerful statistical technique used for estimating the underlying probability density function of a continuous random variable from a set of observations. It approximates the density function using a set of data points and a smoothing parameter known as the kernel. The kernel acts as a window function centered at each data point, and its shape determines how neighboring points influence the estimation at a specific location. A key aspect of KDE is the selection of an appropriate kernel function. Gaussian (normal) kernel is used here in the study to find the density estimation. Each data point is spread using a kernel and a parameter, and the bandwidth.

The percentile, or estimate, of VaR is computed using the Gaussian kernel density estimator. The bandwidth (also known as the smoothing parameter) plays a crucial role in KDE. It controls the trade-off between bias and variance in the density estimation process. In this study, Silverman's rule of thumb is used to find the appropriate bandwidth. The scope of the histogram is expanded using the kernel density estimator. The KDE generates a smooth and continuous representation of the underlying distribution.

The historical VaR approach is finally benchmarked with a parametric method that relies on the kernel density estimation (with Gaussian basis functions) of option returns. Therefore, the kernel density estimation value of the log returns for each quantile would be very close to the log return calculated under the historical approach for the year 2020. This validation is shown for each option in the next sections below.

A. Call Options

Figure 5.1 shows the histograms for the different batches of call options. For the batch OTM with all the duration, there are multiple peaks, which represent that historical distribution is slightly multimodal. On the other hand, for the batches ATM/short, ATM/medium, and ITM/short the historical log return distribution is bimodal. While the batches ATM/long, ITM/medium, and ITM/long show nearly the unimodal distribution for the year 2020. It depicts that the distribution becomes unimodal as maturity increases.

When the kernel density estimation curves from Figure 5.2 are compared with the histograms, most of the left -or right-tail kernel density quantiles are very similar to the historical ones. The KDE curves are continuous and smooth for the batches ATM/long, ITM/medium, and ITM/long.

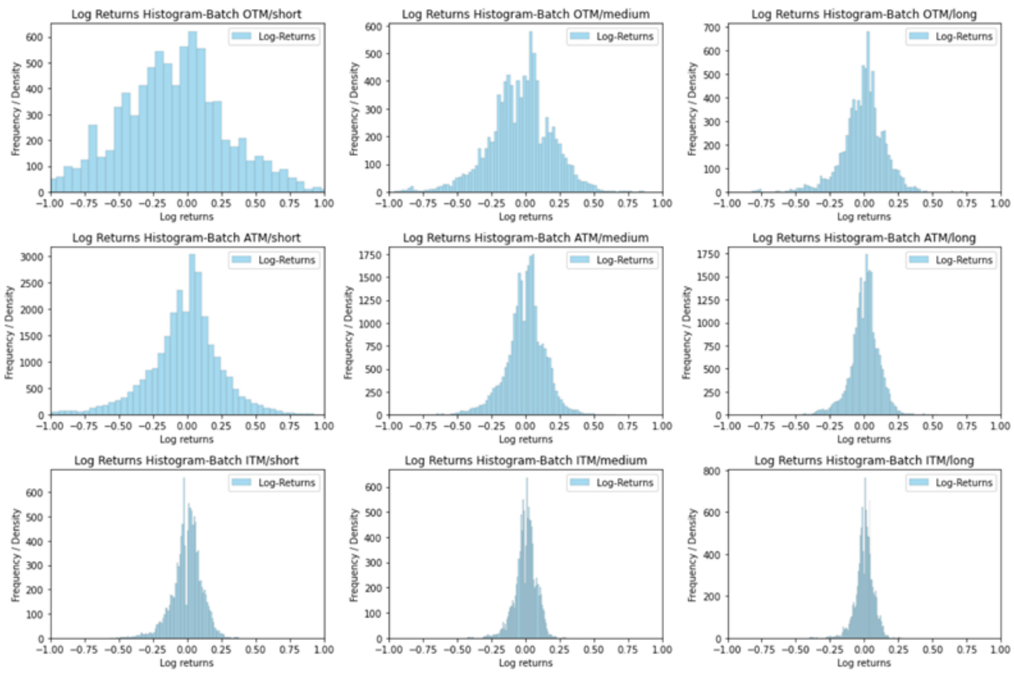

Figure 5.1: Histograms of calls

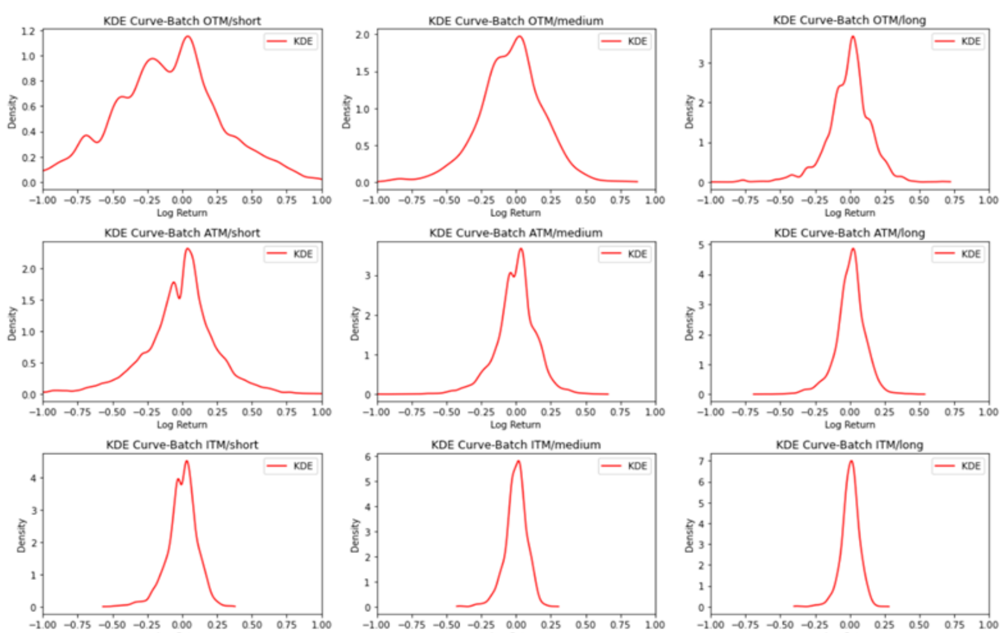

Figure 5.2: KDE curves of calls

From Table 5.29, the log return value and the KDE value are very close to each other For the 5% quantile. It means the KDE yields a smooth density. Therefore, the historical VaR is benchmarked for a 5% quantile.

	Log return	**KDE**
	5%	**5%**
OTM/short	-1.2391	-1.2231
OTM/medium	-0.4287	-0.4361
OTM/long	-0.2803	-0.2813
ATM/short	-0.5502	-0.5490
ATM/medium	-0.2593	-0.2613
ATM/long	-0.1859	-0.1870
ITM/short	-0.1896	-0.1907
ITM/medium	-0.1319	-0.1370
ITM/long	-0.1026	-0.1076

Table 5.29: Log returns and KDE for calls of 2020 for 5%

	Log return	**KDE**	**Log return**	**KDE**
	0.1%	**0.1%**	**1%**	**1%**
OTM/short	-3.6557	-3.6570	-2.4908	-2.4917
OTM/medium	-0.9348	-0.9357	-0.6927	-0.6871
OTM/long	-0.7773	-0.7887	-0.4841	-0.4870
ATM/short	-1.9961	-1.9977	-1.0986	-1.1030
ATM/medium	-0.6748	-0.6788	-0.4416	-0.4402
ATM/long	-0.5158	-0.5141	-0.326	-0.3284
ITM/short	-0.4742	-0.4754	-0.3321	-0.3330
ITM/medium	-0.385	-0.3593	-0.2292	-0.2306
ITM/long	-0.369	-0.3454	-0.1853	-0.1853

Table 5.30: Log returns and KDE for calls of 2020

From Table 5.30, it is evident that KDE is nearly the same or very close to historical values for all quantiles. The historical VaR is benchmarked, indicating acceptance of the VaR calculation process and backtesting approaches defined in the previous sections.

B. Put Options

Figure 5.3 shows the histograms for the different batches of put options. For the batch OTM/short, and all ATM batches, the historical log return distribution is bimodal. While for all the batches, ITM with all the duration distribution of log-returns for the year 2020 is nearly unimodal. It shows that the distribution becomes unimodal as maturity increases.

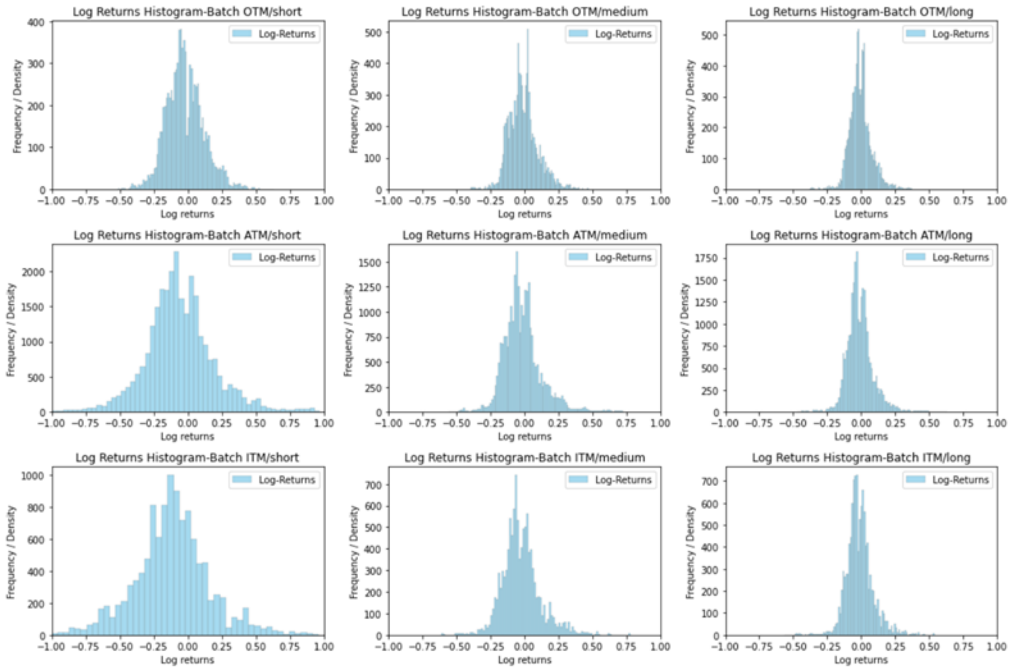

Figure 5.3: Histograms of puts

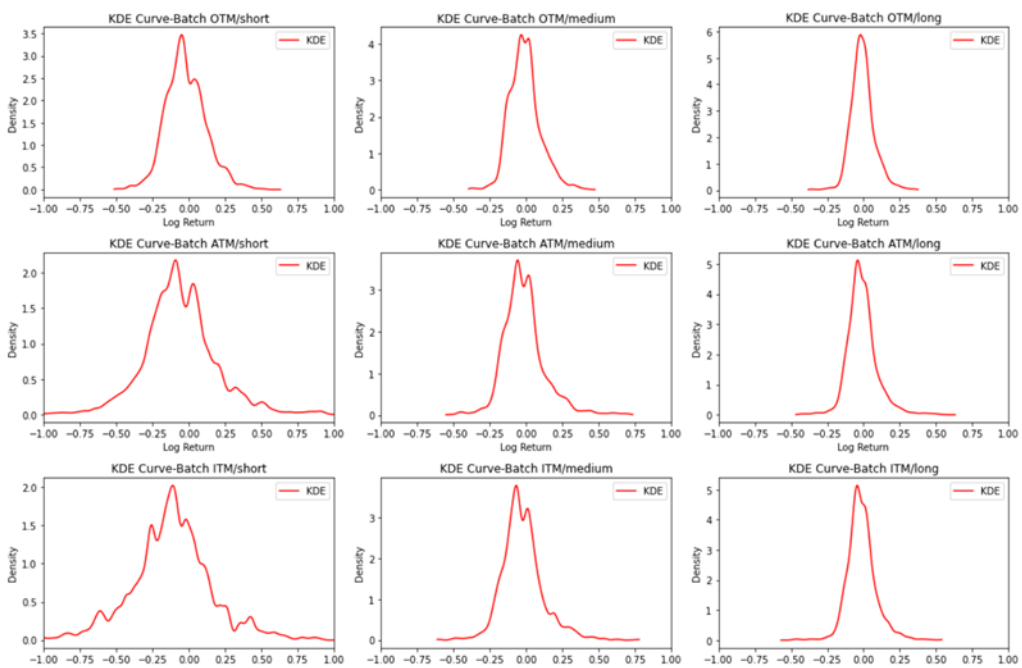

Figure 5.4: KDE curves of puts

When the kernel density estimate curves in Figure 5.4 are compared to the histograms, the majority of the left- or right-tail kernel density quantiles are extremely comparable to the historical ones. For the batches ATM/long, ITM/medium, and ITM/long, the KDE curves are continuous and smooth.

	Log return	**KDE**
	5%	**5%**
OTM/short	-0.22	-0.22532
OTM/medium	-0.1561	-0.16243
OTM/long	-0.1197	-0.12496
ATM/short	-0.4747	-0.47493
ATM/medium	-0.2028	-0.20785
ATM/long	-0.1446	-0.15029
ITM/short	-0.6576	-0.66035
ITM/medium	-0.2217	-0.2253
ITM/long	-0.1467	-0.15427

Table 5.31: Log returns and KDE for puts of 2020 for 5%

For the 5% quantile, the log return value and the KDE value are quite close to each other in Table 5.31. It signifies that the KDE produces a smooth density. As a result, the historical VaR is calibrated for the 5% quantile.

	Log return	**KDE**	**Log return**	**KDE**
	0.1%	**0.1%**	**1%**	**1%**
OTM / short	-0.4596	-0.45816	-0.3313	-0.33658
OTM / medium	-0.3797	-0.37756	-0.2385	-0.23995
OTM / long	-0.3601	-0.35898	-0.1923	-0.19312
ATM / short	-1.5731	-1.57094	-0.8091	-0.80943
ATM / medium	-0.4789	-0.48083	-0.3345	-0.33662
ATM / long	-0.4285	-0.42799	-0.2528	-0.25676
ITM / short	-2.9174	-2.92006	-1.5599	-1.55571
ITM / medium	-0.508	-0.51071	-0.3437	-0.34782
ITM / long	-0.4723	-0.47137	-0.2447	-0.24408

Table 5.32: Log returns and KDE for puts of 2020

According to Table 5.32, KDE is around the same or very similar to historical levels for all quantiles. The historical VaR is benchmarked, suggesting that the VaR calculation procedure and backtesting approaches specified in the preceding sections have been accepted.

C. One day straddles

Figure 5.5 shows the histograms for the different batches of one day straddles. For most batches, the historical log return distribution looks unimodal for the year 2020 with respect to options.

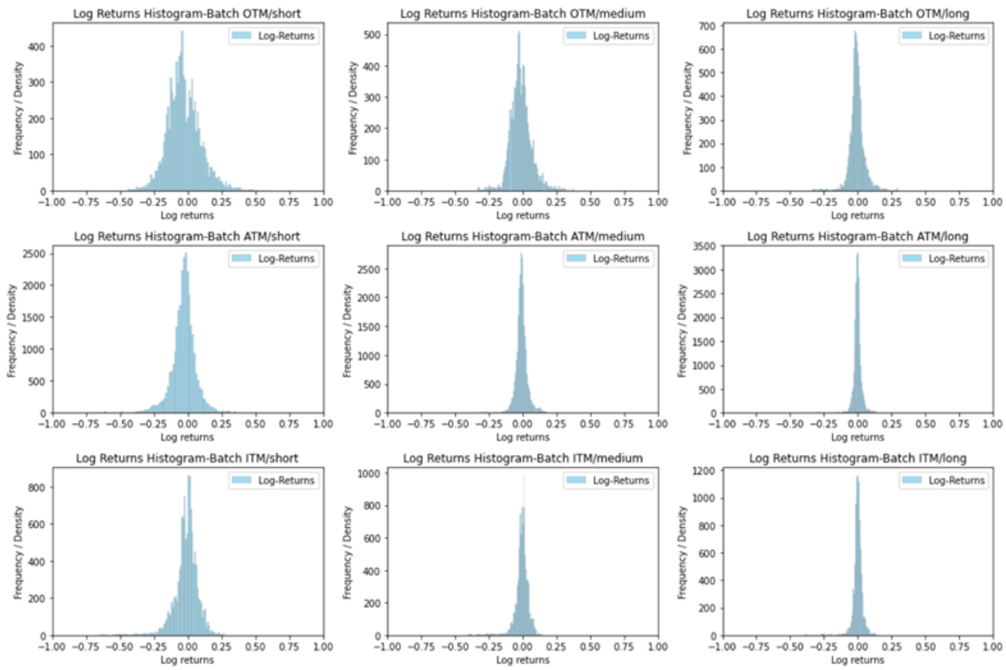

Figure 5.5: Histograms of 1-day straddles

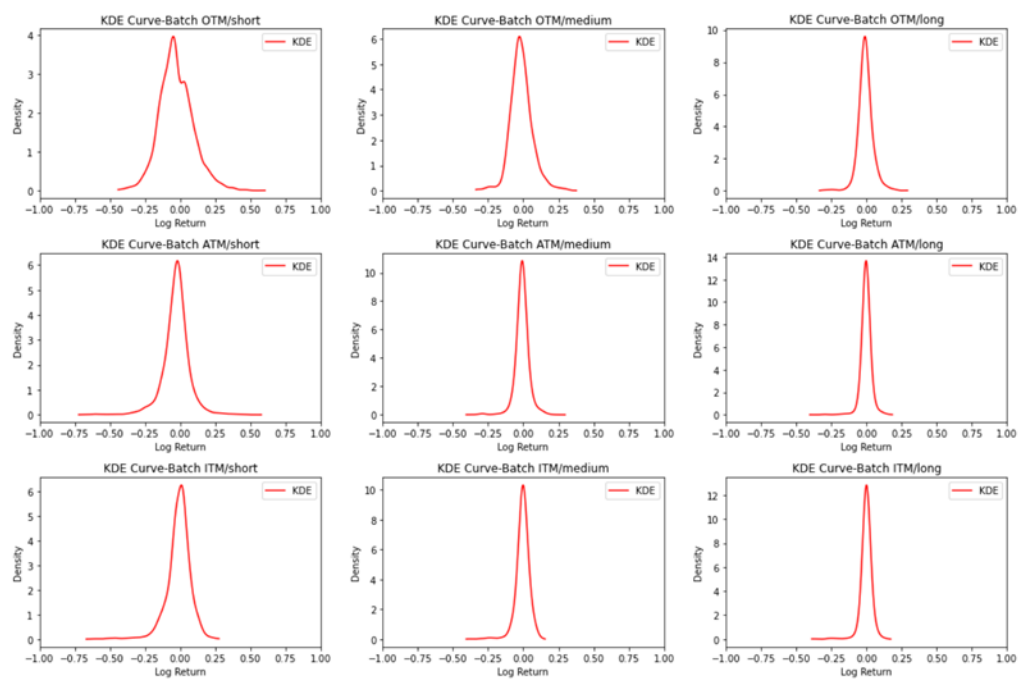

Figure 5.6: KDE curves of 1-day straddles

When the kernel density estimate curves in Figure 5.6 are compared to the histograms, most of the kernel density quantiles are extremely comparable to the historical ones. Except batch OTM/short, for all rest batches, the KDE curves are very continuous and smooth.

	Log return	KDE
	5%	**5%**
OTM/short	-0.2108	-0.21312
OTM/medium	-0.1221	-0.12769
OTM/long	-0.0767	-0.08568
ATM/short	-0.1844	-0.18679
ATM/medium	-0.0729	-0.08032
ATM/long	-0.0445	-0.05618
ITM/short	-0.1502	-0.15457
ITM/medium	-0.0772	-0.08205
ITM/long	-0.046	-0.05726

Table 5.33: Log returns and KDE for 1-day straddles of 2020 for 5%

In Table 5.33, the log return value and the KDE value for the 5% quantile are quite similar. It denotes that the KDE generates a smooth density. As a result, the historical VaR is set to the 5% quantile.

	Log return	KDE	Log return	KDE
	0.1%	**0.1%**	**1%**	**1%**
OTM/short	-0.4158	-0.40884	-0.3068	-0.30836
OTM/medium	-0.3301	-0.32008	-0.2393	-0.23354
OTM/long	-0.3169	-0.3113	-0.1796	-0.17744
ATM/short	-0.6182	-0.62365	-0.3269	-0.33241
ATM/medium	-0.3224	-0.33399	-0.1594	-0.16417
ATM/long	-0.2986	-0.31014	-0.124	-0.12496
ITM/short	-0.5898	-0.58333	-0.3256	-0.32928
ITM/medium	-0.3822	-0.35958	-0.2113	-0.20565
ITM/long	-0.3686	-0.31747	-0.1318	-0.13481

Table 5.34: Log returns and KDE for 1-day straddles of 2020

Table 5.34 shows that KDE is around the same or very comparable to historical values for all quantiles. The historical VaR is benchmarked, implying that the VaR calculation technique and backtesting approaches described in the preceding sections have been used.

D. Five-day straddles

Figure 5.7 depicts the histograms for the various batches of five-day straddles. In terms of options, the historical log return distribution appears unimodal for most batches in 2020, similar to one-day straddles.

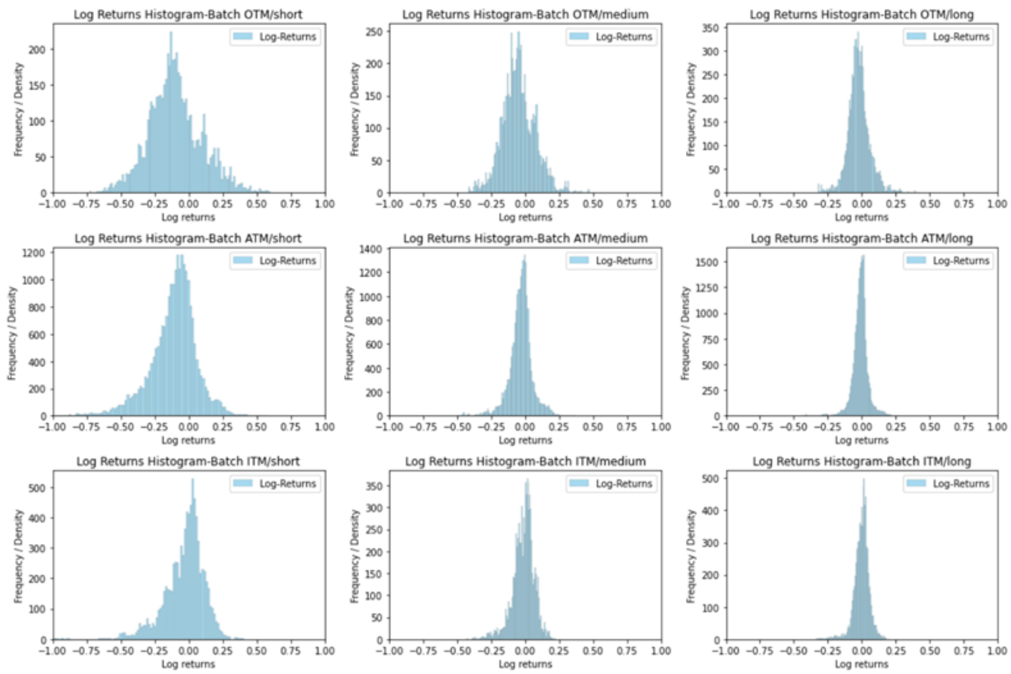

Figure 5.7: Histograms of 5-day straddles

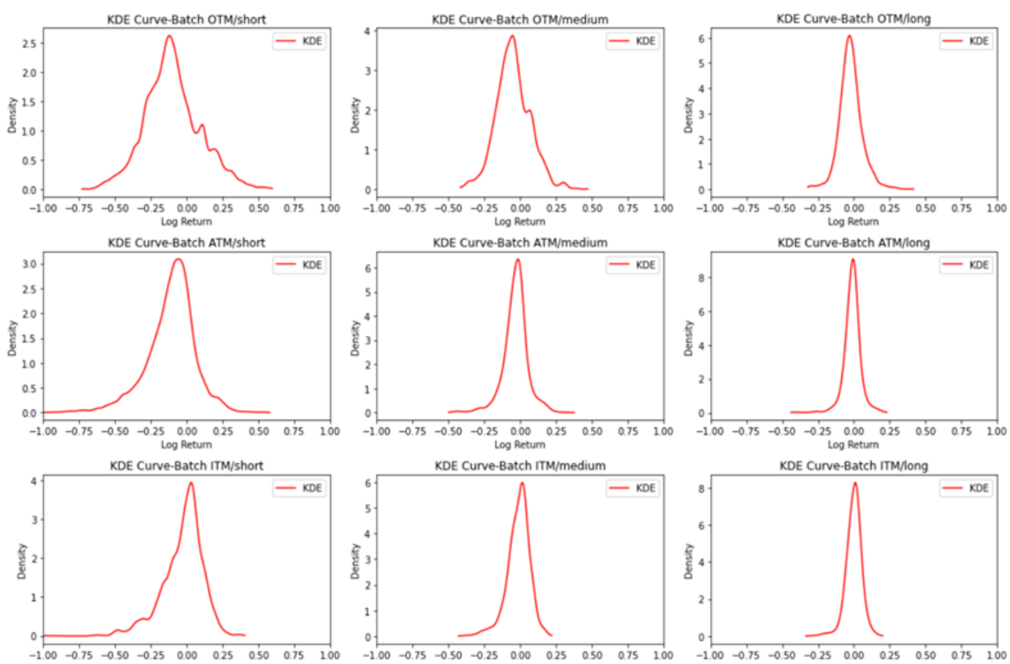

Figure 5.8: KDE curves of 5-day straddles

When the kernel density estimate curves in Figure 5.8 are compared to the histograms, most of the kernel density quantiles are very similar. The KDE curves for most batches are highly continuous and smooth.

	Log return	KDE
	5%	**5%**
OTM/short	-0.4031	-0.40481
OTM/medium	-0.2345	-0.23953
OTM/long	-0.1436	-0.14558
ATM/short	-0.4279	-0.42775
ATM/medium	-0.1718	-0.17571
ATM/long	-0.0972	-0.10343
ITM/short	-0.306	-0.30796
ITM/medium	-0.1516	-0.15664
ITM/long	-0.0838	-0.09101

Table 5.35: Log returns and KDE for 5-day straddles of 2020 for 5%

The log return value and the KDE value for the 5% quantile in Table 5.35 are very comparable. It implies that a smooth density is produced by the KDE. The historical VaR is therefore set to the 5% quantile.

	Log return	KDE	Log return	KDE
	0.1%	**0.1%**	**1%**	**1%**
OTM/short	-0.629	-0.63717	-0.5426	-0.54422
OTM/medium	-0.4104	-0.4023	-0.3467	-0.34538
OTM/long	-0.3188	-0.31213	-0.2557	-0.2458
ATM/short	-0.8738	-0.87221	-0.6314	-0.63569
ATM/medium	-0.4555	-0.46155	-0.3059	-0.30858
ATM/long	-0.406	-0.40125	-0.1993	-0.20181
ITM/short	-0.6549	-0.65965	-0.476	-0.47255
ITM/medium	-0.3741	-0.37433	-0.2681	-0.2694
ITM/long	-0.2793	-0.27642	-0.1879	-0.18639

Table 5.36: Log returns and KDE for 5-day straddles of 2020

Table 5.36 demonstrates that for all quantiles, KDE is roughly the same as or very comparable to historical values. The historical VaR is benchmarked, indicating that the backtesting methods and VaR calculation methods stated in the earlier sections have been applied.

E. Call options for last six months of 2020

The histograms for the various batches of call options for the last six months of 2020 are displayed in Figure 5.9. The historical distribution for the batch OTM with all the durations is slightly multimodal. While for the year 2020, the batches ATM/long, ITM/medium, and ITM/long virtually exhibit a unimodal distribution, as maturity rises, and it shows, the distribution becomes unimodal.

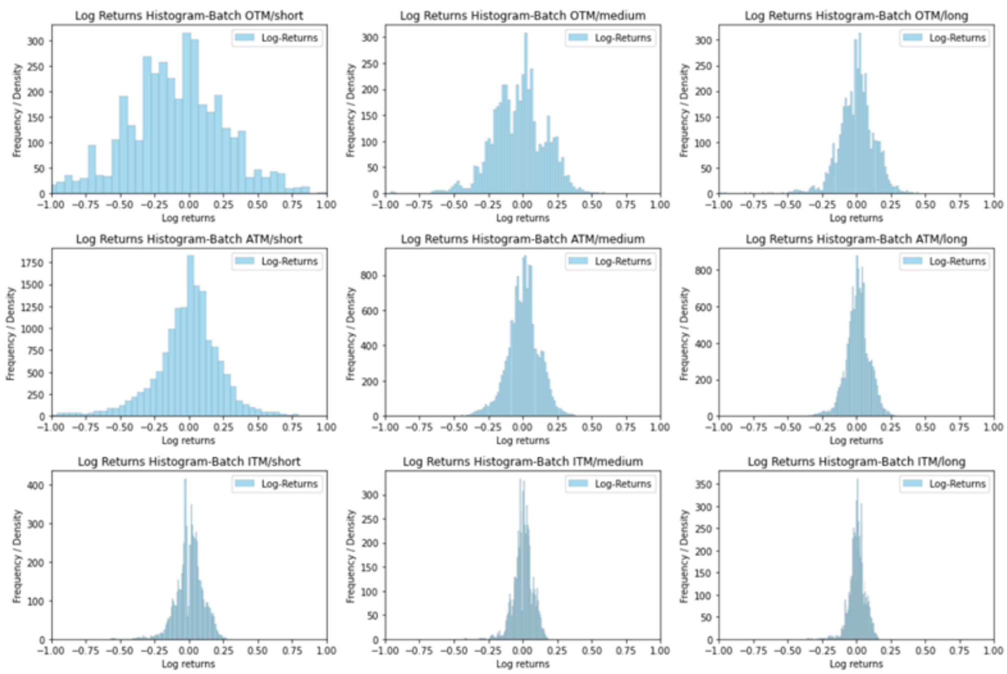

Figure 5.9: Histograms of calls for last six months of 2020

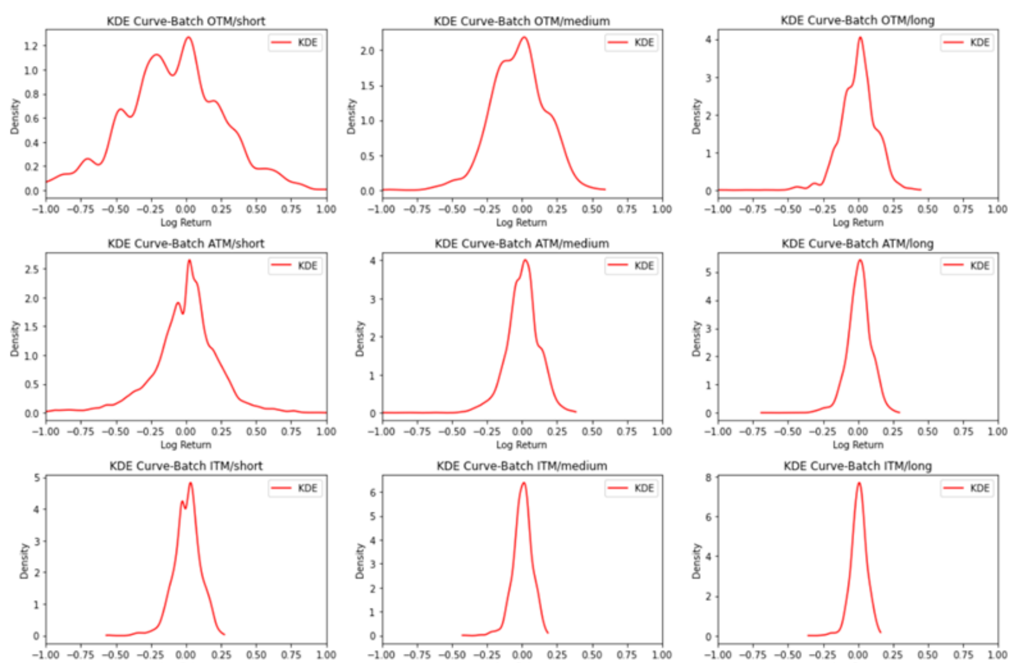

Figure 5.10: KDE curves of calls for last six months of 2020

Most of the left-tail kernel density quantiles are extremely comparable to the historical ones when the histograms are compared to the kernel density estimate curves from Figure 5.10. For the batches ATM/long, ITM/medium, and ITM/long, the KDE curves are continuous and smooth.

	Quantiles for year 2020	
	Log return	**KDE**
	5%	**5%**
OTM/short	-1.0986	-1.1091
OTM/medium	-0.3273	-0.3359
OTM/long	-0.1954	-0.1990
ATM/short	-0.4607	-0.4597
ATM/medium	-0.1915	-0.1939
ATM/long	-0.1314	-0.1354
ITM/short	-0.1492	-0.1511
ITM/medium	-0.1054	-0.1108
ITM/long	-0.0829	-0.0883

Table 5.37: Log returns and KDE for calls for last six months of 2020 for 5%

	Log return	KDE	Log return	KDE
	0.1%	**0.1%**	**1%**	**1%**
OTM/short	-3.8393	-3.8401	-2.976	-2.9835
OTM/medium	-0.9368	-0.9272	-0.5138	-0.5178
OTM/long	-0.662	-0.6627	-0.3587	-0.366
ATM/short	-1.953	-1.9534	-0.9556	-0.9636
ATM/medium	-0.6381	-0.6421	-0.3217	-0.3238
ATM/long	-0.3798	-0.3753	-0.2453	-0.2469
ITM/short	-0.3809	-0.3897	-0.2733	-0.2726
ITM/medium	-0.3005	-0.3006	-0.1969	-0.1981
ITM/long	-0.2486	-0.2527	-0.1533	-0.1592

Table 5.38: Log returns and KDE for calls for last six months of 2020

The log return value and the KDE value for the 5% quantile are shown in Table 5.37 to be fairly similar. In other words, the KDE produces a smooth density. The historical VaR is therefore compared to a 5% quantile.

Table 5.38 makes it clear that KDE is quite similar to or almost identical to historical values for all quantiles. The historical VaR is benchmarked, demonstrating acceptance of the backtesting methods and VaR calculation methods described in the earlier sections.

F. Put options for last six months of 2020

The histograms for the various batches of put options for the final six months of the year 2020 are shown in Figure 5.11. For the year 2020, the duration distribution of log-returns for all batches using ITM is almost unimodal. It demonstrates how, as maturity rises, the distribution becomes unimodal.

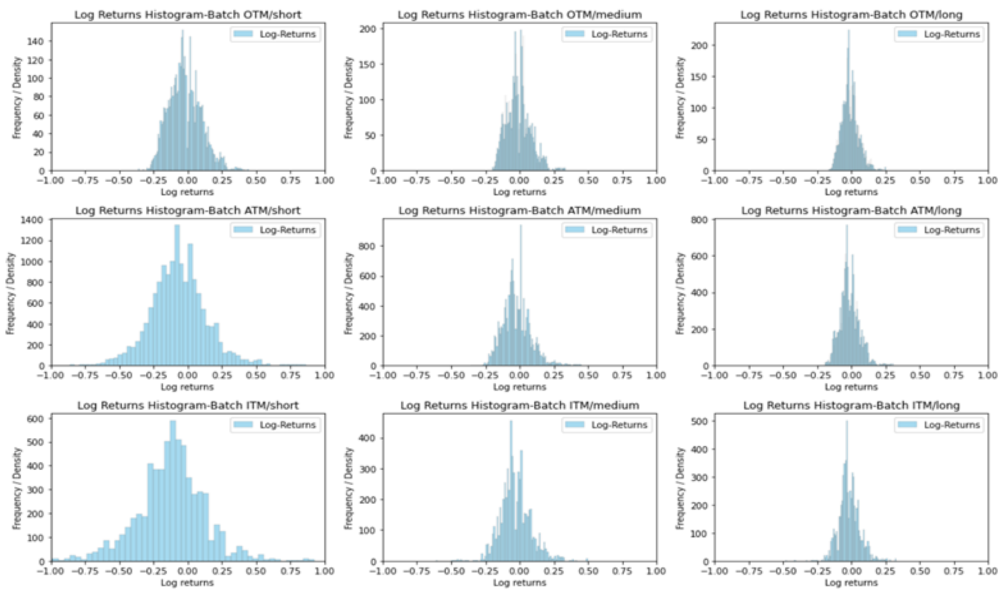

Figure 5.11: Histograms of puts for last six months of 2020

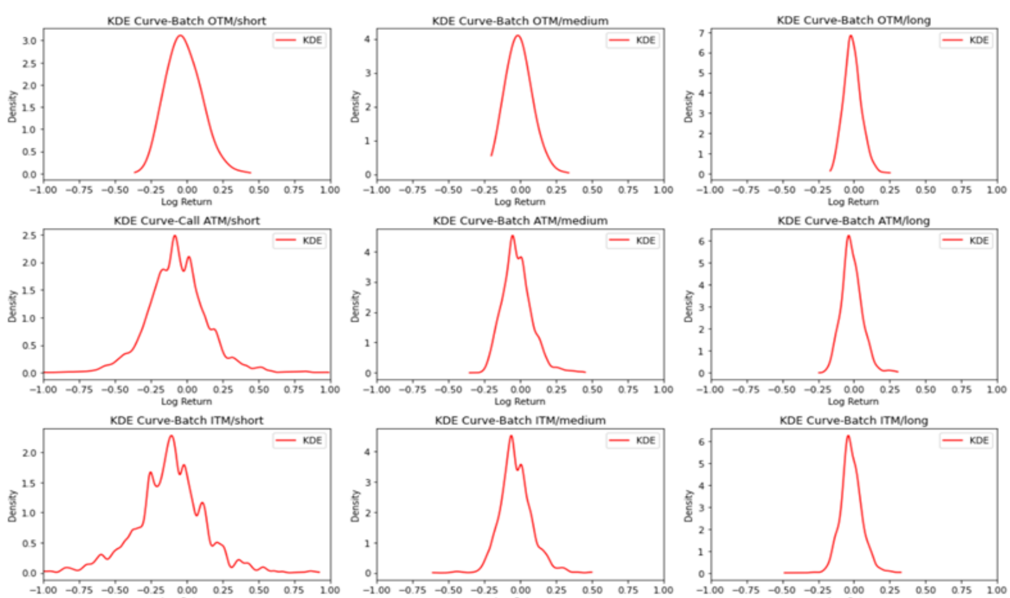

Figure 5.12: KDE curves of puts for last six months of 2020

The majority of the left- or right-tail kernel density quantiles are very similar to the historical ones when the histograms are compared to the kernel density estimate curves in Figure 5.12. The KDE curves are smooth and continuous for the batches ATM/long, ITM/medium, and ITM/long.

	Log return	**KDE**
	5%	**5%**
OTM/short	-0.1987	-0.218
OTM/medium	-0.1381	-0.153
OTM/long	-0.1042	-0.107
ATM/short	-0.4312	-0.4303
ATM/medium	-0.176	-0.1783
ATM/long	-0.1272	-0.1297
ITM/short	-0.6647	-0.6629
ITM/medium	-0.1936	-0.1997
ITM/long	-0.1337	-0.1380

Table 5.39: Log returns and KDE for puts for last six months of 2020 for 5%

	Log return	**KDE**	**Log return**	**KDE**
	0.1%	**0.1%**	**1%**	**1%**
OTM/short	-0.294	-0.305	-0.2491	-0.2536
OTM/medium	-0.1935	-0.1969	-0.1684	-0.1753
OTM/long	-0.1507	-0.1515	-0.1294	-0.1390
ATM/short	-1.7099	-1.6899	-0.7761	-0.7779
ATM/medium	-0.2553	-0.2661	-0.2182	-0.2234
ATM/long	-0.1901	-0.2001	-0.1576	-0.1622
ITM/short	-3.0011	-2.9992	-1.6083	-1.607
ITM/medium	-0.4592	-0.4617	-0.2614	-0.263
ITM/long	-0.2901	-0.2930	-0.1813	-0.181

Table 5.40: Log returns and KDE for puts for last six months of 2020

In Table 5.39, the log return value and the KDE value for the 5% quantile are relatively similar to one another. It indicates that a smooth density is produced by the KDE. The historical VaR is therefore adjusted for the 5% quantile.

Table 5.40 shows that for all quantiles, KDE is roughly the same as or extremely similar to historical levels. The historical VaR is benchmarked, indicating that the backtesting techniques and VaR calculation method described in the preceding sections have gained acceptance.

G. One day straddles for last six months of 2020

The histograms for the several batches of one-day straddles for the last six months of the year

2020 are displayed in Figure 5.13. The historical log return distribution for 2020 appears to be unimodal regarding options for most batches.

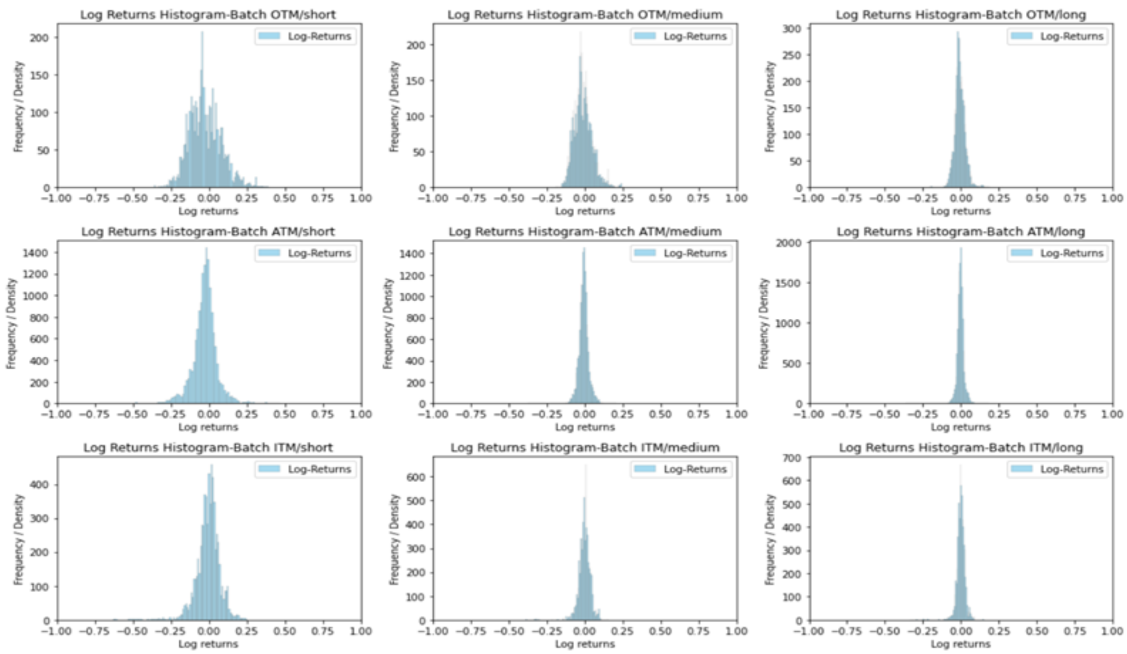

Figure 5.13: Histograms of 1-day straddles for last six months of 2020

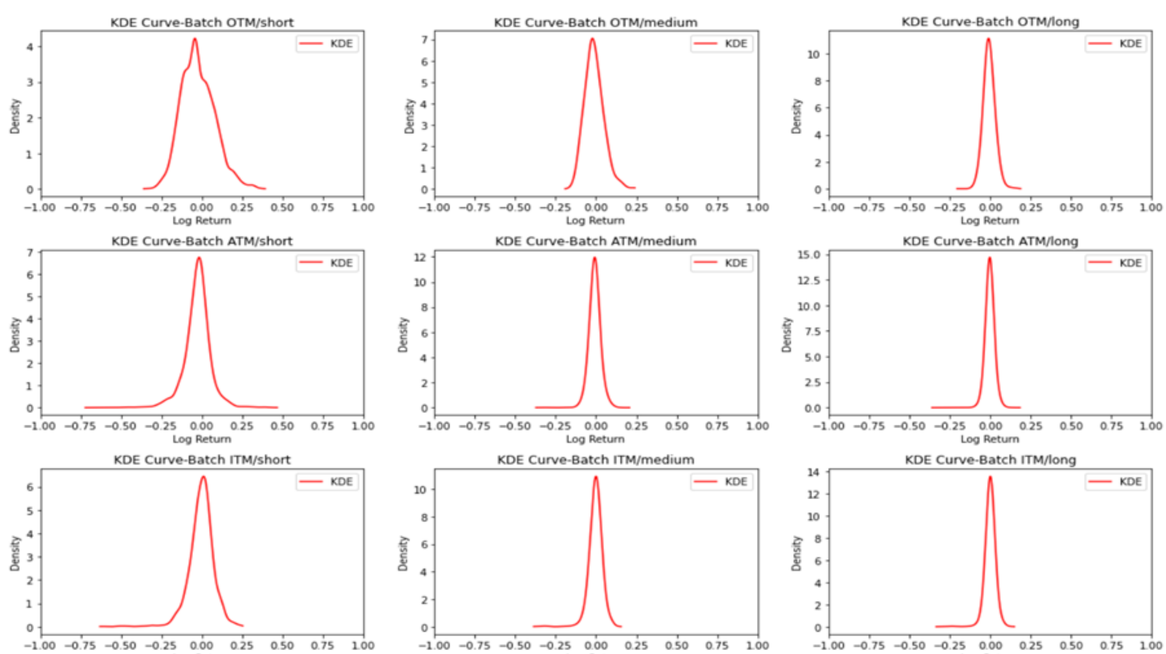

Figure 5.14: KDE curves of 1-day straddles for last six months of 2020

Most of the kernel density quantiles are very similar to the historical ones when the histograms are contrasted with the kernel density estimate curves in Figure 5.14. The KDE curves for all other batches, aside from batch OTM/short, are exceedingly smooth and continuous.

	Log return	KDE
	5%	**5%**
OTM/short	-0.1786	-0.1786
OTM/medium	-0.0982	-0.1008
OTM/long	-0.0593	-0.0616
ATM/short	-0.1627	-0.1656
ATM/medium	-0.0575	-0.0585
ATM/long	-0.0368	-0.03711
ITM/short	-0.1253	-0.128
ITM/medium	-0.0595	-0.0607
ITM/long	-0.0382	-0.0391

Table 5.41: Log returns and KDE for 1-day straddles for last six months of 2020 for 5%

	Log return	KDE	Log return	KDE
	0.1%	**0.1%**	**1%**	**1%**
OTM/short	-0.2957	-0.2982	-0.2385	-0.2403
OTM/medium	-0.1489	-0.154	-0.1229	-0.1276
OTM/long	-0.1056	-0.1116	-0.0816	-0.0852
ATM/short	-0.5975	-0.5939	-0.2852	-0.2882
ATM/medium	-0.3007	-0.2992	-0.0932	-0.095
ATM/long	-0.195	-0.1911	-0.0652	-0.0687
ITM/short	-0.5141	-0.513	-0.2321	-0.2342
ITM/medium	-0.3333	-0.3323	-0.1117	-0.1136
ITM/long	-0.251	-0.255	-0.0812	-0.082

Table 5.42: Log returns and KDE for 1-day straddles for last six months of 2020

The log return value and the KDE value for the 5% quantile in Table 5.41 are very comparable. It concludes that a smooth density is produced by the KDE. The historical VaR is therefore set to the 5% quantile.

Table 5.42 demonstrates that for all quantiles, KDE is roughly the same as or very comparable to historical values. The historical VaR is benchmarked, indicating that the backtesting methods and VaR calculation methods stated in the earlier sections have been applied.

H. Five-day straddles for last six months of 2020

The histograms for the various batches of five-day straddles for the final six months of 2020 are shown in Figure 5.15. The historical log return distribution appears unimodal for most batches in 2020 in terms of options.

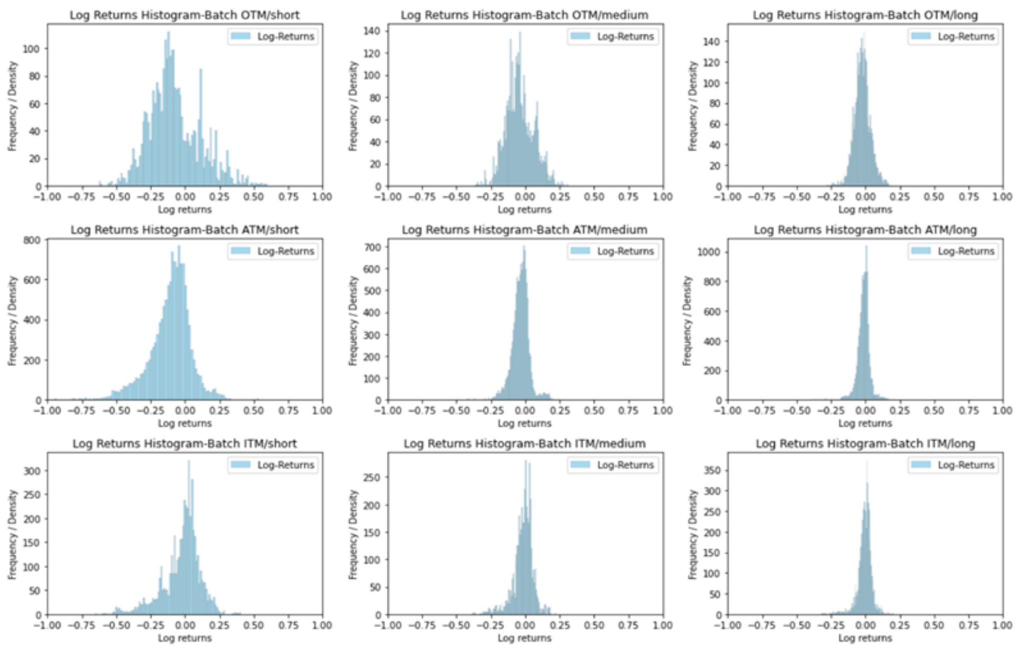

Figure 5.15: Histograms of 5-day straddles for last six months of 2020

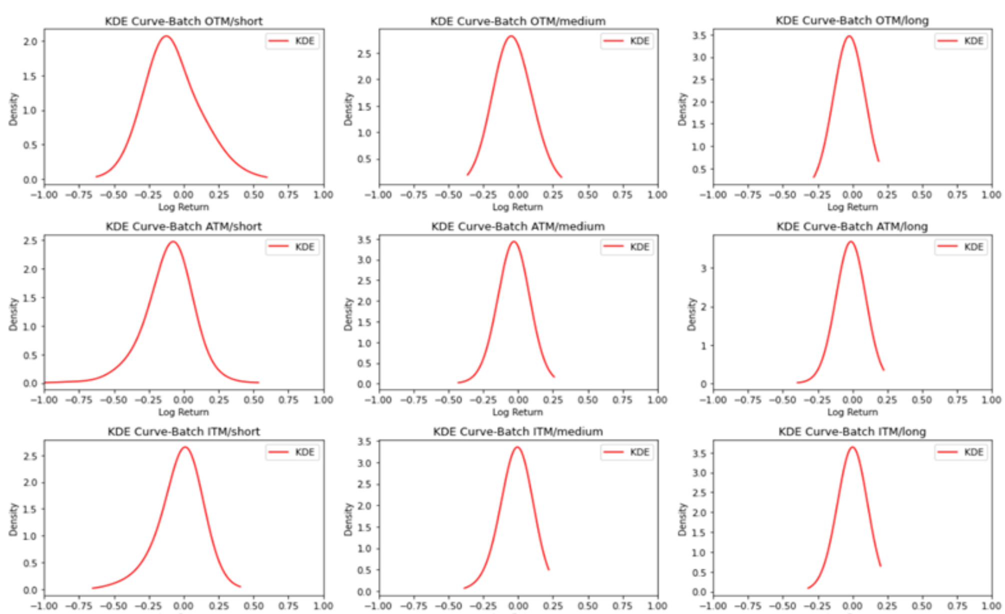

Figure 5.16: KDE curves of 5-day straddles for last six months of 2020

Most of the kernel density quantiles are extremely comparable when compared to the histograms in Figure 5.16's kernel density estimate curves. For the majority of batches, the KDE curves are extremely continuous and smooth.

	Log return	**KDE**
	5%	**5%**
OTM/short	-0.3479	-0.3917
OTM/medium	-0.1938	-0.1985
OTM/long	-0.1138	-0.1135
ATM/short	-0.3974	-0.4011
ATM/medium	-0.1428	-0.1645
ATM/long	-0.0829	-0.0951
ITM/short	-0.2957	-0.3011
ITM/medium	-0.1424	-0.1455
ITM/long	-0.0837	-0.0876

Table 5.43: Log returns and KDE for 5-day straddles for last six months of 2020 for 5%

	Log return	**KDE**	**Log return**	**KDE**
	0.1%	**0.1%**	**1%**	**1%**
OTM/short	-0.6089	-0.5837	-0.4567	-0.4553
OTM/medium	-0.3441	-0.3420	-0.2848	-0.2706
OTM/long	-0.2411	-0.2455	-0.1676	-0.1725
ATM/short	-0.8869	-0.8928	-0.5736	-0.5790
ATM/medium	-0.3856	-0.3773	-0.2125	-0.2184
ATM/long	-0.2844	-0.2735	-0.1561	-0.1591
ITM/short	-0.5479	-0.5535	-0.4745	-0.4709
ITM/medium	-0.3685	-0.3378	-0.2606	-0.2586
ITM/long	-0.2887	-0.2774	-0.1907	-0.1890

Table 5.44: Log returns and KDE for 5-day straddles for last six months of 2020

In Table 5.43, the 5% quantile's log return value and KDE value are extremely similar. It implies that the KDE creates a smooth density. Therefore, the historical VaR is set to the 5% quantile.

Table 5.44 shows that KDE is nearly the same as or very comparable to historical values for all quantiles. The historical VaR is benchmarked, demonstrating the application of the backtesting techniques and VaR calculation techniques described in the prior parts.

Conclusion

This thesis examines the accuracy of the Value at Risk and VaR computational parameters for S&P 500 options and straddles. First, it evaluates the quality of the VaR delta method by looking at an analytical approximation that is not suitable in combination with positions such as straddles. It then uses a historical VaR approach directly applicable to option returns.

Historical depreciation rates for different batches of money and the time to maturity are calculated. Various implementations of the historical approach (estimation window) are evaluated through backtesting, including the unconditional coverage test and the VaR clustering test. Finally, a parametric method based on kernel density estimation with Gaussian basis functions for option returns is used to benchmark the historical VaR approach.

So, lastly, to summarize, it's evident that all the quantiles are lower in 2021 than their corresponding quantiles in 2020. This is because 2020 was a more volatile year than 2021, in part because of the COVID crisis. The study shows during the robustness check that as maturity rises, the historical distribution becomes unimodal for the straddles. The last six months' validation approach also verifies the backtesting methodologies. The historical VaR approach performs better than the analytical VaR approach for straddles. The KDE benchmarked the historical VaR approach, and the quantiles for the log returns are very similar and close to the historical VaR. Finally, the study demonstrates that the VaR model and VaR backtesting are performing well for the straddles at the tail. Any highly volatile market event creates abrupt changes in the options and can be captured at the tails of the log return historical distribution.

The historical approach shows promising and significant results for the straddle positions. But it can be enhanced to get better results by combining the historical approach with the parametric approach. Historical VaR relies on historical price data. Parametric VaR, on the other hand, uses statistical models to estimate VaR and uses assumptions about the statistical distribution of asset returns. The same work that is described in the study can be replicated for other market options. However, there is a possibility that there might be limitations on the data. There might be fewer numbers of strikes, underlying data quality, and fewer numbers of straddle pairs.

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Appendix

1. Quantiles of actual log returns -21 batches

A. Call Options

	Quantiles for year 2020			Quantiles for year 2021		
	0.1%	**1%**	**5%**	**0.1%**	**1%**	**5%**
DOTM/short	-3.0363	-1.8146	-1.0836	-2.1112	-1.3863	-0.8983
DOTM/medium	-1.5622	-1.206	-0.8118	-1.5991	-1.4035	-0.6931
DOTM/long	-1.895	-1.269	-0.6398	-0.8775	-0.3891	-0.2253
OTM/short	-3.6557	-2.4908	-1.2391	-3.4752	-2.3535	-1.1632
OTM/medium	-0.9348	-0.6927	-0.4287	-1.0296	-0.5027	-0.3046
OTM/long	-0.7773	-0.4841	-0.2803	-0.4467	-0.2823	-0.1837
ATM/short	-1.9961	-1.0986	-0.5502	-2.0869	-0.9749	-0.4442
ATM/medium	-0.6748	-0.4416	-0.2593	-0.4527	-0.2665	-0.1832
ATM/long	-0.5158	-0.326	-0.1859	-0.259	-0.1717	-0.1161
ITM/short	-0.4742	-0.3321	-0.1896	-0.2377	-0.1903	-0.1197
ITM/medium	-0.385	-0.2292	-0.1319	-0.1734	-0.1394	-0.0911
ITM/long	-0.369	-0.1853	-0.1026	-0.1263	-0.1083	-0.067
DITM/short	-0.3917	-0.2425	-0.1386	-0.1585	-0.1142	-0.069
DITM/medium	-0.3105	-0.1982	-0.1157	-0.1314	-0.1031	-0.0614
DITM/long	-0.2865	-0.1713	-0.0935	-0.1019	-0.0796	-0.0478
VDITM/short	-0.2477	-0.1526	-0.0747	-0.0682	-0.0561	-0.0337
VDITM/medium	-0.2272	-0.1276	-0.0646	-0.0671	-0.0559	-0.0311
VDITM/long	-0.2152	-0.1117	-0.0593	-0.0594	-0.0486	-0.0266
VVDITM/short	-0.1399	-0.0841	-0.0466	-0.0524	-0.0384	-0.0233
VVDITM/medium	-0.1275	-0.0797	-0.0416	-0.0466	-0.0372	-0.0226
VVDITM/long	-0.1083	-0.0631	-0.0329	-0.0438	-0.0338	-0.0186

Table A.1: Quantiles of actual log returns-Calls

B. Put Options

	Quantiles for year 2020			Quantiles for year 2021		
	0.1%	**1%**	**5%**	**0.1%**	**1%**	**5%**
DOTM/short	-0.4126	-0.3111	-0.198	-0.2851	-0.1816	-0.1215
DOTM/medium	-0.5284	-0.3534	-0.2118	-0.3191	-0.202	-0.1235
DOTM/long	-0.6091	-0.3013	-0.1403	-0.1983	-0.1197	-0.0851
OTM/short	-0.4596	-0.3313	-0.22	-0.3725	-0.2788	-0.2107
OTM/medium	-0.3797	-0.2385	-0.1561	-0.2325	-0.1965	-0.1487
OTM/long	-0.3601	-0.1923	-0.1197	-0.1802	-0.148	-0.1077
ATM/short	-1.5731	-0.8091	-0.4747	-1.344	-0.8898	-0.5003
ATM/medium	-0.4789	-0.3345	-0.2028	-0.3485	-0.2816	-0.212
ATM/long	-0.4285	-0.2528	-0.1446	-0.2306	-0.1889	-0.1345
ITM/short	-2.9174	-1.5599	-0.6576	-2.322	-1.2613	-0.6931
ITM/medium	-0.508	-0.3437	-0.2217	-0.5188	-0.3259	-0.2464
ITM/long	-0.4723	-0.2447	-0.1467	-0.2843	-0.2092	-0.145
DITM/short	-2.9602	-1.6094	-0.8561	-2.2687	-1.3284	-0.7084
DITM/medium	-0.686	-0.4443	-0.2801	-0.4923	-0.3643	-0.2634
DITM/long	-0.5538	-0.3437	-0.1914	-0.3153	-0.2336	-0.1656
VDITM/short	-3.6336	-2.1723	-1.371	-2.3711	-1.3863	-0.8109
VDITM/medium	-0.969	-0.4982	-0.2787	-0.5248	-0.3946	-0.2627
VDITM/long	-0.847	-0.346	-0.1983	-0.4271	-0.25	-0.1635
VVDITM/short	-2.6184	-1.7918	-0.8873	-1.7931	-1.3863	-0.7885
VVDITM/medium	-1.2009	-0.6931	-0.3847	-1.0986	-0.6931	-0.3745
VVDITM/long	-0.9362	-0.3511	-0.1904	-0.6313	-0.3264	-0.1885

Table A.2: Quantiles of actual log returns-Puts

C. One day straddles

	Quantiles for year 2020			Quantiles for year 2021		
	0.1%	**1%**	**5%**	**0.1%**	**1%**	**5%**
DOTM/short	-0.404	-0.2929	-0.1925	-0.2854	-0.1815	-0.1215
DOTM/medium	-0.5222	-0.3415	-0.1971	-0.3167	-0.1986	-0.1194
DOTM/long	-0.5953	-0.2276	-0.1196	-0.1917	-0.1103	-0.0785
OTM/short	-0.4158	-0.3068	-0.2108	-0.3686	-0.2762	-0.2049
OTM/medium	-0.3301	-0.2393	-0.1221	-0.1968	-0.1686	-0.1273
OTM/long	-0.3169	-0.1796	-0.0767	-0.1403	-0.1121	-0.0786
ATM/short	-0.6182	-0.3269	-0.1844	-0.4743	-0.3247	-0.1758
ATM/medium	-0.3224	-0.1594	-0.0729	-0.1696	-0.1222	-0.074
ATM/long	-0.2986	-0.124	-0.0445	-0.1126	-0.0697	-0.0383
ITM/short	-0.5898	-0.3256	-0.1502	-0.2324	-0.1621	-0.0963
ITM/medium	-0.3822	-0.2113	-0.0772	-0.1712	-0.0707	-0.0491
ITM/long	-0.3686	-0.1318	-0.046	-0.1105	-0.0484	-0.0308
DITM/short	-0.3282	-0.2044	-0.1214	-0.1361	-0.1012	-0.0645
DITM/medium	-0.2186	-0.148	-0.0818	-0.097	-0.0708	-0.049
DITM/long	-0.2069	-0.1196	-0.0565	-0.0633	-0.0499	-0.0328
VDITM/short	-0.2349	-0.1498	-0.0739	-0.0671	-0.0551	-0.0336
VDITM/medium	-0.1999	-0.1217	-0.0621	-0.0635	-0.0531	-0.0303
VDITM/long	-0.1801	-0.1038	-0.0537	-0.0528	-0.0436	-0.0249
VVDITM/short	-0.139	-0.084	-0.0466	-0.0521	-0.0384	-0.0232
VVDITM/medium	-0.125	-0.0793	-0.0414	-0.0458	-0.0366	-0.0224
VVDITM/long	-0.1023	-0.0619	-0.0322	-0.0416	-0.0327	-0.0185

Table A.3: Quantiles of actual log returns-1-day straddles

D. Five-day straddles

	Quantiles for year 2020			Quantiles for year 2021		
	0.1%	1%	5%	0.1%	1%	5%
DOTM/short	-0.6467	-0.5069	-0.3652	-0.3586	-0.3003	-0.2157
DOTM/medium	-0.7526	-0.6122	-0.3712	-0.3443	-0.3161	-0.2334
DOTM/long	-0.6559	-0.3648	-0.2128	-0.3402	-0.2058	-0.152
OTM/short	-0.629	-0.5426	-0.4031	-0.5213	-0.4502	-0.3488
OTM/medium	-0.4104	-0.3467	-0.2345	-0.418	-0.3224	-0.2449
OTM/long	-0.3188	-0.2557	-0.1436	-0.2703	-0.2111	-0.1549
ATM/short	-0.8738	-0.6314	-0.4279	-1.0492	-0.5761	-0.3799
ATM/medium	-0.4555	-0.3059	-0.1718	-0.3036	-0.2229	-0.1488
ATM/long	-0.406	-0.1993	-0.0972	-0.176	-0.123	-0.0818
ITM/short	-0.6549	-0.476	-0.306	-0.3444	-0.2552	-0.1712
ITM/medium	-0.3741	-0.2681	-0.1516	-0.2154	-0.1387	-0.0992
ITM/long	-0.2793	-0.1879	-0.0838	-0.1482	-0.0965	-0.0688
DITM/short	-0.6288	-0.4092	-0.2203	-0.2227	-0.1606	-0.1048
DITM/medium	-0.4481	-0.2632	-0.1571	-0.1428	-0.1141	-0.0773
DITM/long	-0.3765	-0.2326	-0.1087	-0.1114	-0.0888	-0.0597
VDITM/short	-0.498	-0.3122	-0.1187	-0.1003	-0.0768	-0.0517
VDITM/medium	-0.3808	-0.2685	-0.1055	-0.0886	-0.0701	-0.0449
VDITM/long	-0.3357	-0.2248	-0.1024	-0.0821	-0.0678	-0.0415
VVDITM/short	-0.2689	-0.156	-0.0703	-0.0716	-0.0558	-0.0369
VVDITM/medium	-0.1483	-0.096	-0.0666	-0.0643	-0.052	-0.0339
VVDITM/long	-0.1942	-0.0966	-0.0601	-0.0634	-0.05	-0.0305

Table A.4: Quantiles of actual log returns-5-day straddles

2. Historical VaR-21 batches

A. Call Options

	Year 2020			Year 2021		
	0.1%	**1%**	**5%**	**0.1%**	**1%**	**5%**
DOTM/short	-3.0796	-1.8153	-1.0855	-2.1133	-1.3863	-0.9163
DOTM/medium	-1.5642	-1.2061	-0.8118	-1.5999	-1.4064	-0.6932
DOTM/long	-1.9005	-1.2691	-0.6398	-0.8795	-0.3911	-0.2253
OTM/short	-3.6889	-2.5029	-1.2392	-3.4834	-2.3582	-1.1632
OTM/medium	-0.9350	-0.6932	-0.4289	-1.0324	-0.5036	-0.3046
OTM/long	-0.7829	-0.4841	-0.2804	-0.4484	-0.2823	-0.1837
ATM/short	-1.9963	-1.0986	-0.5504	-2.0893	-0.9752	-0.4442
ATM/medium	-0.6754	-0.4417	-0.2593	-0.4559	-0.2665	-0.1832
ATM/long	-0.5202	-0.3260	-0.1859	-0.2592	-0.1717	-0.1162
ITM/short	-0.4749	-0.3322	-0.1896	-0.2379	-0.1904	-0.1197
ITM/medium	-0.3920	-0.2303	-0.1319	-0.1735	-0.1395	-0.0911
ITM/long	-0.3697	-0.1856	-0.1026	-0.1265	-0.1083	-0.0670
DITM/short	-0.3928	-0.2425	-0.1387	-0.1588	-0.1143	-0.0691
DITM/medium	-0.3106	-0.1982	-0.1158	-0.1314	-0.1031	-0.0615
DITM/long	-0.2866	-0.1713	-0.0935	-0.1019	-0.0796	-0.0478
VDITM/short	-0.2481	-0.1533	-0.0748	-0.0686	-0.0562	-0.0337
VDITM/medium	-0.2274	-0.1277	-0.0647	-0.0674	-0.0559	-0.0311
VDITM/long	-0.2152	-0.1118	-0.0593	-0.0595	-0.0487	-0.0267
VVDITM/short	-0.1438	-0.0843	-0.0467	-0.0528	-0.0385	-0.0233
VVDITM/medium	-0.1525	-0.0806	-0.0417	-0.0466	-0.0374	-0.0226
VVDITM/long	-0.1095	-0.0632	-0.0329	-0.0439	-0.0338	-0.0186

Table A.5: Historical VaR for calls

B. Put Options

	Year 2020			Year 2021		
	0.1%	**1%**	**5%**	**0.1%**	**1%**	**5%**
DOTM/short	-0.4144	-0.3113	-0.1981	-0.2879	-0.1825	-0.1216
DOTM/medium	-0.5289	-0.3534	-0.2121	-0.3193	-0.2021	-0.1239
DOTM/long	-0.6121	-0.3021	-0.1403	-0.2174	-0.1197	-0.0851
OTM/short	-0.4636	-0.3316	-0.2200	-0.3740	-0.2794	-0.2108
OTM/medium	-0.3800	-0.2389	-0.1561	-0.2330	-0.1967	-0.1487
OTM/long	-0.3639	-0.1924	-0.1197	-0.1806	-0.1480	-0.1077
ATM/short	-1.5736	-0.8092	-0.4748	-1.3455	-0.8901	-0.5003
ATM/medium	-0.4795	-0.3346	-0.2028	-0.3486	-0.2816	-0.2120
ATM/long	-0.4298	-0.2531	-0.1446	-0.2307	-0.1890	-0.1346
ITM/short	-2.9267	-1.5603	-0.6581	-2.3242	-1.2617	-0.6932
ITM/medium	-0.5229	-0.3440	-0.2217	-0.5197	-0.3265	-0.2464
ITM/long	-0.4726	-0.2449	-0.1467	-0.2844	-0.2092	-0.1450
DITM/short	-2.9618	-1.6094	-0.8561	-2.2687	-1.3291	-0.7084
DITM/medium	-0.6861	-0.4443	-0.2801	-0.4923	-0.3643	-0.2635
DITM/long	-0.5558	-0.3437	-0.1914	-0.3155	-0.2336	-0.1656
VDITM/short	-3.6481	-2.1972	-1.3789	-2.4849	-1.3863	-0.8109
VDITM/medium	-0.9995	-0.4992	-0.2789	-0.5313	-0.3947	-0.2631
VDITM/long	-0.8473	-0.3461	-0.1983	-0.4325	-0.2505	-0.1636
VVDITM/short	-2.6626	-1.7918	-0.8873	-1.9459	-1.3863	-0.7885
VVDITM/medium	-1.2528	-0.6932	-0.3852	-1.0986	-0.6932	-0.3747
VVDITM/long	-0.9631	-0.3522	-0.1907	-0.6360	-0.3267	-0.1886

Table A.6: Historical VaR for puts

C. One day straddles

	Year 2020			Year 2021		
	0.1%	**1%**	**5%**	**0.1%**	**1%**	**5%**
DOTM/short	-0.4056	-0.2929	-0.1926	-0.2879	-0.1820	-0.1216
DOTM/medium	-0.5226	-0.3420	-0.1972	-0.3168	-0.2004	-0.1199
DOTM/long	-0.5957	-0.2281	-0.1197	-0.2153	-0.1107	-0.0786
OTM/short	-0.4218	-0.3068	-0.2108	-0.3694	-0.2766	-0.2051
OTM/medium	-0.3302	-0.2394	-0.1221	-0.1969	-0.1687	-0.1273
OTM/long	-0.3174	-0.1797	-0.0767	-0.1404	-0.1121	-0.0786
ATM/short	-0.6186	-0.3269	-0.1845	-0.4746	-0.3248	-0.1758
ATM/medium	-0.3225	-0.1594	-0.0729	-0.1704	-0.1223	-0.0740
ATM/long	-0.2986	-0.1248	-0.0445	-0.1128	-0.0697	-0.0383
ITM/short	-0.5943	-0.3264	-0.1502	-0.2324	-0.1624	-0.0963
ITM/medium	-0.3841	-0.2113	-0.0772	-0.1719	-0.0707	-0.0491
ITM/long	-0.3708	-0.1320	-0.0460	-0.1115	-0.0484	-0.0308
DITM/short	-0.3284	-0.2044	-0.1214	-0.1361	-0.1012	-0.0645
DITM/medium	-0.2186	-0.1480	-0.0818	-0.0970	-0.0708	-0.0490
DITM/long	-0.2072	-0.1196	-0.0565	-0.0634	-0.0499	-0.0328
VDITM/short	-0.2352	-0.1500	-0.0740	-0.0672	-0.0551	-0.0336
VDITM/medium	-0.2000	-0.1222	-0.0622	-0.0636	-0.0531	-0.0303
VDITM/long	-0.1802	-0.1038	-0.0537	-0.0528	-0.0436	-0.0249
VVDITM/short	-0.1428	-0.0841	-0.0466	-0.0525	-0.0385	-0.0233
VVDITM/medium	-0.1505	-0.0803	-0.0415	-0.0458	-0.0367	-0.0224
VVDITM/long	-0.1038	-0.0621	-0.0322	-0.0418	-0.0327	-0.0185

Table A.7: Historical VaR for 1- day straddles

D. Five-day straddles

	Year 2020			Year 2021		
	0.1%	**1%**	**5%**	**0.1%**	**1%**	**5%**
DOTM/short	-0.6506	-0.5071	-0.3653	-0.4208	-0.3022	-0.2160
DOTM/medium	-0.7539	-0.6124	-0.3716	-0.3470	-0.3220	-0.2373
DOTM/long	-0.6566	-0.3652	-0.2128	-0.3554	-0.2059	-0.1521
OTM/short	-0.6338	-0.5428	-0.4031	-0.5219	-0.4505	-0.3490
OTM/medium	-0.4105	-0.3467	-0.2348	-0.4192	-0.3226	-0.2450
OTM/long	-0.3195	-0.2564	-0.1438	-0.2707	-0.2112	-0.1550
ATM/short	-0.8753	-0.6314	-0.4280	-1.0529	-0.5761	-0.3799
ATM/medium	-0.4557	-0.3064	-0.1718	-0.3039	-0.2229	-0.1488
ATM/long	-0.4060	-0.1997	-0.0972	-0.1762	-0.1230	-0.0818
ITM/short	-0.6560	-0.4766	-0.3060	-0.3448	-0.2553	-0.1712
ITM/medium	-0.3761	-0.2687	-0.1516	-0.2165	-0.1387	-0.0992
ITM/long	-0.2837	-0.1881	-0.0838	-0.1482	-0.0967	-0.0688
DITM/short	-0.6295	-0.4099	-0.2203	-0.2228	-0.1606	-0.1049
DITM/medium	-0.4486	-0.2633	-0.1572	-0.1431	-0.1141	-0.0773
DITM/long	-0.3766	-0.2327	-0.1087	-0.1115	-0.0888	-0.0597
VDITM/short	-0.4984	-0.3123	-0.1187	-0.1003	-0.0768	-0.0517
VDITM/medium	-0.3883	-0.2685	-0.1055	-0.0892	-0.0701	-0.0449
VDITM/long	-0.3363	-0.2267	-0.1025	-0.0822	-0.0678	-0.0415
VVDITM/short	-0.2899	-0.1577	-0.0704	-0.0719	-0.0559	-0.0369
VVDITM/medium	-0.1488	-0.0963	-0.0667	-0.0646	-0.0520	-0.0339
VVDITM/long	-0.1958	-0.0966	-0.0602	-0.0635	-0.0500	-0.0305

Table A.8: Historical VaR for 5-day straddles

3. **Quantiles of actual log-returns for last six months of 2020 and 2021-21 batches**

A. **Call Options**

	Year 2020			Year 2021		
	0.1%	**1%**	**5%**	**0.1%**	**1%**	**5%**
DOTM/short	-3.912	-2.3026	-1.0337	-1.8718	-1.1632	-0.6931
DOTM/medium	-1.1232	-0.6747	-0.4568	-1.6041	-1.5374	-0.6931
DOTM/long	-0.7858	-0.4943	-0.2906	-0.884	-0.4209	-0.2334
OTM/short	-3.8393	-2.976	-1.0986	-3.3132	-1.9755	-1.0992
OTM/medium	-0.9368	-0.5138	-0.3273	-1.0705	-0.5209	-0.3201
OTM/long	-0.662	-0.3587	-0.1954	-0.4973	-0.2978	-0.1897
ATM/short	-1.953	-0.9556	-0.4607	-2.1047	-1.047	-0.452
ATM/medium	-0.6381	-0.3217	-0.1915	-0.4544	-0.2881	-0.1891
ATM/long	-0.3798	-0.2453	-0.1314	-0.2648	-0.182	-0.1172
ITM/short	-0.3809	-0.2733	-0.1492	-0.2384	-0.191	-0.1182
ITM/medium	-0.3005	-0.1969	-0.1054	-0.1825	-0.1409	-0.0923
ITM/long	-0.2486	-0.1533	-0.0829	-0.1323	-0.1082	-0.0664
DITM/short	-0.2383	-0.1713	-0.088	-0.1524	-0.1105	-0.0675
DITM/medium	-0.1909	-0.149	-0.0757	-0.1284	-0.0994	-0.0606
DITM/long	-0.1641	-0.1173	-0.0596	-0.0978	-0.0775	-0.0476
VDITM/short	-0.1001	-0.0872	-0.0518	-0.0575	-0.0499	-0.0305
VDITM/medium	-0.0949	-0.0783	-0.0383	-0.0568	-0.0494	-0.03
VDITM/long	-0.0922	-0.0774	-0.0356	-0.0523	-0.0455	-0.0258
VVDITM/short	-0.069	-0.058	-0.0319	-0.0395	-0.034	-0.0219
VVDITM/medium	-0.0684	-0.0605	-0.0323	-0.0393	-0.0341	-0.0219
VVDITM/long	-0.0666	-0.0521	-0.0259	-0.0369	-0.0311	-0.018

Table A.9: Quantiles of actual log returns for last six months-Calls

B. Put Options

	Year 2020			Year 2021		
	0.1%	**1%**	**5%**	**0.1%**	**1%**	**5%**
DOTM/short	-0.2372	-0.204	-0.135	-0.1942	-0.1556	-0.1117
DOTM/medium	-0.1688	-0.1307	-0.1066	-0.213	-0.1845	-0.1117
DOTM/long	-0.2606	-0.1117	-0.0844	-0.1425	-0.1198	-0.084
OTM/short	-0.294	-0.2491	-0.1987	-0.3733	-0.2856	-0.219
OTM/medium	-0.1935	-0.1684	-0.1381	-0.2395	-0.2005	-0.149
OTM/long	-0.1507	-0.1294	-0.1042	-0.1848	-0.1518	-0.1062
ATM/short	-1.7099	-0.7761	-0.4312	-1.2523	-0.9234	-0.5222
ATM/medium	-0.2553	-0.2182	-0.176	-0.3508	-0.2904	-0.2228
ATM/long	-0.1901	-0.1576	-0.1272	-0.2368	-0.1909	-0.1327
ITM/short	-3.0011	-1.6083	-0.6647	-2.5336	-1.3177	-0.7669
ITM/medium	-0.4592	-0.2614	-0.1936	-0.527	-0.3677	-0.2534
ITM/long	-0.2901	-0.1813	-0.1337	-0.2842	-0.2081	-0.1443
DITM/short	-2.3979	-1.4663	-0.6931	-2.4849	-1.7047	-0.7676
DITM/medium	-0.4129	-0.2911	-0.2191	-0.4955	-0.3884	-0.2638
DITM/long	-0.2369	-0.2055	-0.1458	-0.3186	-0.223	-0.1661
VDITM/short	-3.7536	-1.7918	-0.8624	-2.7847	-1.6094	-0.9651
VDITM/medium	-0.4991	-0.3172	-0.2284	-0.5986	-0.4184	-0.2683
VDITM/long	-0.6767	-0.2471	-0.1614	-0.5229	-0.2494	-0.157
VVDITM/short	-2.4804	-1.3863	-0.6931	-1.9459	-1.5041	-0.9163
VVDITM/medium	-1.1708	-0.5958	-0.3365	-1.0986	-0.7833	-0.4055
VVDITM/long	-0.8883	-0.294	-0.1728	-0.6512	-0.3478	-0.1884

Table A.10: Quantiles of actual log returns for last six months-Puts

C. One day straddles

	Year 2020			Year 2021		
	0.1%	**1%**	**5%**	**0.1%**	**1%**	**5%**
DOTM/short	-0.233	-0.2037	-0.1323	-0.1943	-0.1556	-0.1116
DOTM/medium	-0.1681	-0.1213	-0.0978	-0.2116	-0.1829	-0.1117
DOTM/long	-0.2593	-0.0987	-0.07	-0.1383	-0.1118	-0.0782
OTM/short	-0.2957	-0.2385	-0.1786	-0.3691	-0.2834	-0.215
OTM/medium	-0.1489	-0.1229	-0.0982	-0.1968	-0.1744	-0.131
OTM/long	-0.1056	-0.0816	-0.0593	-0.1406	-0.1168	-0.0795
ATM/short	-0.5975	-0.2852	-0.1627	-0.4742	-0.3656	-0.1825
ATM/medium	-0.3007	-0.0932	-0.0575	-0.1763	-0.1272	-0.0749
ATM/long	-0.195	-0.0652	-0.0368	-0.115	-0.0664	-0.0355
ITM/short	-0.5141	-0.2321	-0.1253	-0.273	-0.1763	-0.0978
ITM/medium	-0.3333	-0.1117	-0.0595	-0.2138	-0.0837	-0.049
ITM/long	-0.251	-0.0812	-0.0382	-0.1312	-0.0484	-0.0312
DITM/short	-0.2369	-0.1489	-0.084	-0.1381	-0.1001	-0.0624
DITM/medium	-0.1649	-0.1076	-0.0585	-0.1012	-0.071	-0.0485
DITM/long	-0.1133	-0.0809	-0.0408	-0.0637	-0.0503	-0.0326
VDITM/short	-0.0985	-0.0859	-0.0518	-0.0562	-0.0494	-0.0302
VDITM/medium	-0.0888	-0.0761	-0.0373	-0.0555	-0.0473	-0.0289
VDITM/long	-0.0834	-0.0715	-0.0338	-0.0469	-0.041	-0.0236
VVDITM/short	-0.0682	-0.0579	-0.0319	-0.0392	-0.0339	-0.0217
VVDITM/medium	-0.0672	-0.0595	-0.0323	-0.039	-0.0337	-0.0215
VVDITM/long	-0.0641	-0.0514	-0.0257	-0.0356	-0.0306	-0.0177

Table A.11: Quantiles of actual log returns for last six months-1-day straddles

D. Five-day straddles

	Year 2020			Year 2021		
	0.1%	**1%**	**5%**	**0.1%**	**1%**	**5%**
DOTM / short	-0.4839	-0.428	-0.2895	-0.3237	-0.2699	-0.1879
DOTM / medium	-0.4181	-0.3357	-0.1934	-0.3042	-0.2449	-0.1921
DOTM / long	-0.3385	-0.2596	-0.1447	-0.2617	-0.2023	-0.1555
OTM / short	-0.6089	-0.4567	-0.3479	-0.4998	-0.4501	-0.3499
OTM / medium	-0.3441	-0.2848	-0.1938	-0.4648	-0.3393	-0.2594
OTM / long	-0.2411	-0.1676	-0.1138	-0.2742	-0.2229	-0.1713
ATM / short	-0.8869	-0.5736	-0.3974	-1.125	-0.6742	-0.3955
ATM / medium	-0.3856	-0.2125	-0.1428	-0.3137	-0.2438	-0.1526
ATM / long	-0.2844	-0.1561	-0.0829	-0.185	-0.1263	-0.0761
ITM / short	-0.5479	-0.4745	-0.2957	-0.355	-0.2732	-0.1784
ITM / medium	-0.3685	-0.2606	-0.1424	-0.2308	-0.1441	-0.1004
ITM / long	-0.2887	-0.1907	-0.0837	-0.1598	-0.0927	-0.0665
DITM / short	-0.4183	-0.2813	-0.1764	-0.2277	-0.171	-0.1108
DITM / medium	-0.3095	-0.2195	-0.1411	-0.1476	-0.1189	-0.0796
DITM / long	-0.2237	-0.1642	-0.0931	-0.1154	-0.0926	-0.0607
VDITM / short	-0.1592	-0.1351	-0.1	-0.1044	-0.087	-0.0621
VDITM / medium	-0.1466	-0.1244	-0.0825	-0.0908	-0.0696	-0.0494
VDITM / long	-0.1519	-0.1189	-0.0753	-0.0847	-0.0689	-0.0448
VVDITM / short	-0.0981	-0.0915	-0.0649	-0.0739	-0.0616	-0.0446
VVDITM / medium	-0.1034	-0.0924	-0.0659	-0.0654	-0.0549	-0.0368
VVDITM/ long	-0.1009	-0.0871	-0.0549	-0.0658	-0.0499	-0.0331

Table A.12: Quantiles of actual log returns for last six months-5-day straddles

4. Historical VaR for last six months of 2020 and 2021-21 batches

A. Call Options

	Year 2020			Year 2021		
	0.1%	**1%**	**5%**	**0.1%**	**1%**	**5%**
DOTM/short	-3.9120	-2.3026	-1.0498	-1.8718	-1.1632	-0.6932
DOTM/medium	-1.1868	-0.6834	-0.4570	-1.6094	-1.5454	-0.6932
DOTM/long	-0.7985	-0.4943	-0.2906	-0.8870	-0.4211	-0.2336
OTM/short	-3.8607	-3.0068	-1.0986	-3.3178	-2.0088	-1.1047
OTM/medium	-0.9417	-0.5138	-0.3273	-1.0709	-0.5213	-0.3202
OTM/long	-0.6842	-0.3683	-0.1954	-0.5038	-0.2979	-0.1897
ATM/short	-1.9561	-0.9589	-0.4608	-2.1476	-1.0471	-0.4520
ATM/medium	-0.6420	-0.3217	-0.1915	-0.4559	-0.2881	-0.1892
ATM/long	-0.3872	-0.2454	-0.1314	-0.2650	-0.1820	-0.1172
ITM/short	-0.3841	-0.2737	-0.1493	-0.2399	-0.1911	-0.1182
ITM/medium	-0.3012	-0.1985	-0.1054	-0.1835	-0.1410	-0.0924
ITM/long	-0.2490	-0.1541	-0.0829	-0.1325	-0.1082	-0.0664
DITM/short	-0.2390	-0.1713	-0.0880	-0.1524	-0.1105	-0.0676
DITM/medium	-0.1910	-0.1490	-0.0757	-0.1288	-0.0994	-0.0606
DITM/long	-0.1643	-0.1173	-0.0596	-0.0978	-0.0775	-0.0476
VDITM/short	-0.1004	-0.0875	-0.0519	-0.0581	-0.0502	-0.0305
VDITM/medium	-0.0951	-0.0784	-0.0384	-0.0569	-0.0494	-0.0302
VDITM/long	-0.0922	-0.0774	-0.0356	-0.0524	-0.0455	-0.0258
VVDITM/short	-0.0691	-0.0581	-0.0321	-0.0399	-0.0340	-0.0219
VVDITM/medium	-0.0685	-0.0606	-0.0323	-0.0393	-0.0341	-0.0220
VVDITM/long	-0.0667	-0.0521	-0.0260	-0.0370	-0.0312	-0.0180

Table A.13: Historical VaR for last six months-Calls

B. Put Options

	Year 2020			Year 2021		
	0.1%	**1%**	**5%**	**0.1%**	**1%**	**5%**
DOTM/short	-0.2382	-0.2058	-0.1353	-0.1942	-0.1556	-0.1117
DOTM/medium	-0.1719	-0.1307	-0.1067	-0.2170	-0.1893	-0.1117
DOTM/long	-0.2698	-0.1120	-0.0844	-0.1432	-0.1199	-0.0840
OTM/short	-0.2959	-0.2492	-0.1988	-0.3740	-0.2876	-0.2193
OTM/medium	-0.1939	-0.1684	-0.1382	-0.2395	-0.2006	-0.1490
OTM/long	-0.1508	-0.1294	-0.1043	-0.1849	-0.1519	-0.1062
ATM/short	-1.7525	-0.7786	-0.4313	-1.2632	-0.9234	-0.5222
ATM/medium	-0.2556	-0.2182	-0.1760	-0.3509	-0.2904	-0.2229
ATM/long	-0.1903	-0.1576	-0.1272	-0.2371	-0.1910	-0.1327
ITM/short	-3.0020	-1.6094	-0.6678	-2.5513	-1.3185	-0.7669
ITM/medium	-0.4595	-0.2686	-0.1936	-0.5277	-0.3682	-0.2536
ITM/long	-0.2903	-0.1816	-0.1337	-0.2843	-0.2083	-0.1443
DITM/short	-2.3979	-1.4663	-0.6932	-2.4849	-1.7048	-0.7677
DITM/medium	-0.4139	-0.2911	-0.2191	-0.4956	-0.3884	-0.2638
DITM/long	-0.2371	-0.2055	-0.1458	-0.3190	-0.2231	-0.1661
VDITM/short	-3.7992	-1.7918	-0.8650	-2.8904	-1.6094	-0.9651
VDITM/medium	-0.5978	-0.3185	-0.2284	-0.6004	-0.4231	-0.2688
VDITM/long	-0.7080	-0.2471	-0.1614	-0.5878	-0.2545	-0.1570
VVDITM/short	-2.4849	-1.3863	-0.6932	-1.9459	-1.5041	-0.9163
VVDITM/medium	-1.1750	-0.5978	-0.3365	-1.0986	-0.7850	-0.4055
VVDITM/long	-0.8950	-0.2948	-0.1728	-0.6539	-0.3483	-0.1885

Table A.14: Historical VaR for last six months-Puts

C. One day straddles

	Year 2020			Year 2021		
	0.1%	**1%**	**5%**	**0.1%**	**1%**	**5%**
DOTM/short	-0.2344	-0.2040	-0.1329	-0.1943	-0.1556	-0.1116
DOTM/medium	-0.1715	-0.1213	-0.0978	-0.2167	-0.1864	-0.1118
DOTM/long	-0.2681	-0.0991	-0.0700	-0.1400	-0.1119	-0.0782
OTM/short	-0.3067	-0.2386	-0.1786	-0.3694	-0.2839	-0.2159
OTM/medium	-0.1496	-0.1229	-0.0982	-0.1969	-0.1745	-0.1310
OTM/long	-0.1070	-0.0818	-0.0593	-0.1406	-0.1168	-0.0796
ATM/short	-0.5993	-0.2854	-0.1627	-0.4746	-0.3656	-0.1825
ATM/medium	-0.3007	-0.0932	-0.0575	-0.1769	-0.1273	-0.0750
ATM/long	-0.2027	-0.0653	-0.0369	-0.1151	-0.0664	-0.0355
ITM/short	-0.5170	-0.2321	-0.1255	-0.2790	-0.1763	-0.0978
ITM/medium	-0.3337	-0.1122	-0.0595	-0.2151	-0.0838	-0.0491
ITM/long	-0.2512	-0.0813	-0.0382	-0.1326	-0.0486	-0.0312
DITM/short	-0.2380	-0.1490	-0.0841	-0.1381	-0.1002	-0.0624
DITM/medium	-0.1651	-0.1076	-0.0585	-0.1014	-0.0710	-0.0485
DITM/long	-0.1133	-0.0809	-0.0408	-0.0637	-0.0503	-0.0326
VDITM/short	-0.0989	-0.0860	-0.0518	-0.0564	-0.0496	-0.0302
VDITM/medium	-0.0888	-0.0761	-0.0373	-0.0555	-0.0473	-0.0291
VDITM/long	-0.0836	-0.0715	-0.0338	-0.0472	-0.0410	-0.0236
VVDITM/short	-0.0682	-0.0581	-0.0321	-0.0397	-0.0339	-0.0218
VVDITM/medium	-0.0673	-0.0595	-0.0323	-0.0390	-0.0337	-0.0217
VVDITM/long	-0.0641	-0.0516	-0.0258	-0.0357	-0.0307	-0.0177

Table A.15: Historical VaR for last six months-1-day straddles

D. Five-day straddles

	Year 2020			Year 2021		
	0.1%	**1%**	**5%**	**0.1%**	**1%**	**5%**
DOTM/short	-0.4839	-0.4292	-0.2899	-0.3315	-0.2710	-0.1884
DOTM/medium	-0.4221	-0.3357	-0.1935	-0.3064	-0.2458	-0.1934
DOTM/long	-0.3403	-0.2599	-0.1448	-0.2708	-0.2024	-0.1555
OTM/short	-0.6157	-0.4570	-0.3479	-0.5058	-0.4505	-0.3502
OTM/medium	-0.3455	-0.2852	-0.1938	-0.4941	-0.3394	-0.2594
OTM/long	-0.2412	-0.1677	-0.1139	-0.2747	-0.2229	-0.1713
ATM/short	-0.8889	-0.5736	-0.3974	-1.1350	-0.6744	-0.3956
ATM/medium	-0.3856	-0.2125	-0.1428	-0.3139	-0.2440	-0.1526
ATM/long	-0.2867	-0.1566	-0.0829	-0.1851	-0.1265	-0.0761
ITM/short	-0.5703	-0.4747	-0.2957	-0.3551	-0.2732	-0.1785
ITM/medium	-0.3719	-0.2610	-0.1424	-0.2309	-0.1454	-0.1004
ITM/long	-0.2901	-0.1908	-0.0837	-0.1622	-0.0928	-0.0665
DITM/short	-0.4201	-0.2813	-0.1764	-0.2282	-0.1710	-0.1108
DITM/medium	-0.3099	-0.2196	-0.1411	-0.1477	-0.1190	-0.0796
DITM/long	-0.2239	-0.1643	-0.0931	-0.1154	-0.0926	-0.0607
VDITM/short	-0.1594	-0.1351	-0.1000	-0.1049	-0.0870	-0.0621
VDITM/medium	-0.1480	-0.1244	-0.0825	-0.0918	-0.0696	-0.0495
VDITM/long	-0.1556	-0.1189	-0.0753	-0.0848	-0.0689	-0.0448
VVDITM/short	-0.0990	-0.0919	-0.0650	-0.0750	-0.0617	-0.0447
VVDITM/medium	-0.1041	-0.0924	-0.0659	-0.0668	-0.0549	-0.0368
VVDITM/long	-0.1013	-0.0874	-0.0551	-0.0660	-0.0499	-0.0331

Table A.16: Historical VaR for last six months-5-day straddles