HEC MONTRÉAL

Options in Asset Liability Management:

Profit-Seeking and Hedging

By

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Résumé

Le but de cette recherche est d'analyser une application d'options dans la gestion d'actifs/passifs dans différentes conditions de marché. Nous allons comparer la performance de deux stratégies, une avec des produits *traditionnels* (titres à revenus fixes) et une autre avec des produits dérivés. Celles-ci seront jugées sur la base de leur solvabilité, leur coût, et leur facilité d'implémentation. Il reste à voir si la perception des options comme étant des outils très risqués reste valide ou bien s'il y a des opportunités sous la forme de stratégies efficientes de gestions de risques.

Nous prenons les données historiques du fond de pension des Forces Armées Canadiennes pour simuler des flux monétaires de passifs sur des périodes de 1 et 3 ans. Ensuite, nous comparons un portefeuille d'obligations à zéro coupon contre une stratégie d'options *collar*, qui consiste d'un achat d'un actif risqué, un achat de put et une vente de call.

Nos observations nous mènent à la conclusion que pour les fonds qui manquent de sophistication (habileté) ou de complexité (besoin) pour des approches dynamique comme le *stochastic optimal control*, il y a peut-être des raisons d'explorer des stratégies d'options. Cette stratégie nous permet gérer notre risque baissier comme avec les zéro coupon tout en profitant du rendement supérieur d'un actif risqué.

Les résultats de notre analyse montrent que les avantages de cette stratégie varient avec certaines conditions et différents horizons. Dans tous les scénarios, notre portefeuille d'options est plus cher que notre benchmark à l'initiation (entre 0.4% et 0.01% de la valeur du passif), mais permets d'obtenir des rendements supérieurs (jusqu'à 0.7425% de la valeur du passif) dans l'évènement que notre call est atteint tout en garantissant notre flux monétaire requis à maturité. En d'autres mots, ce n'est pas une stratégie plus risquée que notre benchmark malgré l'utilisation de produits dérivés.

Mots-clés: gestion actif-passif, gestion de risque, produits dérivés, options, réplication …

Abstract

The goal of this paper is to analyze the application of options for the purpose of asset liability management in pension funds. We compare 2 strategies, one composed of more traditional fixed income products and another including risky assets hedged with options. Their performance will primarily be judged on return, solvency, cost and ease of implementation. We will attempt to determine if the perception of options being risky rings true or if there is money being left on the table in the form of efficient strategies of cash flow matching.

We take historical data of a pension fund, responsible for the Canadian Armed Forces, and project liabilities over 1 and 3 year periods. We then compare a portfolio of zero coupon bonds against a collar strategy, a portfolio composed of a growth asset, a long put and a short call.

We found that, for pension funds which lack either the sophistication (ability) or complexity (need) to approach stochastic optimal control strategies there may still be reason to explore the use of derivatives. The collar provides the same floor payoff as zero coupon bonds but with a higher potential upside.

Our results show the advantages of this strategy lie in the higher upside and that it's effect varies over the different time horizons. In all scenarios our option portfolio is costlier than our benchmark at initiation (between 0.4% and 0.01% of the value of the liability) but it allows to generate higher returns (up to 0.7425% of the liability) in the event that our call is assigned all while guaranteed our required cash flow at maturity. In other words, this is not a riskier strategy than our benchmark despite the use of derivatives.

Keywords: asset-liability management, liability driven investments, derivatives, options, hedging, cash-flow matching …

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Introduction

The inherent uncertainty of cash flows, cost of funds and return on investments has caused investors to seek out greater efficiency in the management of their assets and liabilities (Kusy & Ziemba, 1986). There are consistently new opportunities to achieve this efficiency due to changing regulations, risk preferences and market conditions. Many authors agree that there is a need for pension products which can offer greater returns than guaranteed products while still protecting from downside movements (Dempster, Germano, Medova & Villaverde 2003). Indeed, there are many parties who could benefit of greater control over their liabilities. Pension funds and insurance companies have been engaging in some sort of explicit liability management since their inception but jointly managing the risks of assets and liabilities arises in many other industries and departments of financial institutions (Romanyuk 2010). One example of adjusting how firms take on risk, brought on by changing market conditions, is employers moving towards defined contribution rather than defined benefit plans.

There is a multitude of variables which contribute to the complexity of this entire problem. The main variable studied in ALM due to the nature of the assets is interest rates. Rates are continuously fluctuating and will affect our asset returns and how the present value of the cash flows matches with our liabilities and their cash flows. Indeed, the main asset class dealt with in these scenarios is typically fixed income as their value is directly related to interest rates and deal with relatively long investment horizons. Changing interest rates will affect how we discount our future liabilities to determine their present value. Derivative products, on the other hand, tend to be shorter maturities and investors in these products place more importance on the sensitivities to other variables such as the underlying’s price movements or to volatility. However the longer we extend the maturities of these products the more we expose them to interest rate risks. The longer term derivative products are therefore more so affected by interest rates than their shorter term counterparts, and coincidentally are gaining popularity in research and application.

We are interested in observing the effect of interest rates on options and what types of strategies can be used, and how, to still create reliable payoffs for liability management with additional return potential. The verdict on options is still split as there are parties who use these instruments for refined hedging strategies and others who engage in highly speculative behavior. Although there is room for both, and having multiple types of agents in fact improves the liquidity of the market as a whole, we believe the reputation of these assets contributes to their underuse in the pension industry.

This paper will focus on the possibility of using options as a substitute for fixed income securities within ALM allocations for pension funds. We believe that the use of derivatives strategies, in particular collars, can lead to building more effective ALM strategies by reducing costs and assisting in reaching solvency targets without employing heavily sophisticated techniques such as stochastic optimal control, or by making more passive some heavily dynamic techniques. The strategies we explore are an attempt to blend liability hedging portfolio with profit-seeking portfolios, usual components of an organization who engages in asset liability management. In addition to this, we hope that the customizability and specificity of such options can perhaps lead to advancements in terms of non-parallel shifts in the yield curve or new types of pension products if explored further.

To achieve this, we will generate liabilities, and compare the performance and costs of the benchmark zero coupon portfolio and the test portfolios, which will vary in their composition. The liabilities will be based on member data found in the Canadian Armed Forces (CAF) actuarial reports. The available assets will include a universe of Canadian indexes to eliminate the aspect of stock picking and to have highly liquid growth assets. Because we are interested in a relatively long period and such options do not exist, we will then price options on these growth assets for maturities for 1 and 3 years using the Heston-Nandi GARCH approximation for reasons discussed later.

We will be observing the test portfolios at 3 different strike pairs, 3 interest rate environments and 2 investment horizons. The strike pairs will always be some percentage higher than the underlying risky asset at time of purchase, meaning our collar strategy is dealing with long in the money puts and short out of the money calls. The following ratios show the relationship of our strike price to our underlying starting price for the 1 year horizon and will be discussed further in the methodology.

$$Puts : 1.05 \leq \frac{K}{X_t} \leq 1.15 \qquad Calls : 0.86 \leq \frac{X_t}{K} \leq 0.94$$

The interest rates will be represented as a low rate environment, a mid rate environment and a high rate environment associated with different parts of the economic cycle and are 0.5%, 1.5% and 2.5% respectively. The horizons studied are 1 and 3 years. The 3 year maturity in particular was selected to observe possible legal solvency requirements and how this type of strategy would behave under these regulations. The 1 year maturity was selected to determine the potential for rebalancing on this strategy and to reflect a more traditional long term horizon for options, which are typically shorter term instruments. In this analysis we hope to find some market dynamics which demonstrate the potential of options in long term liability management particularly in the case of increasing the solvency of a pension fund. We are comparing the option portfolio to the benchmark on many levels, our attention primarily be focused on overall return, risk or expected cash flows, initial setup costs, ease of implementation or rebalancing, customizability and more.

Chapter 1 will cover existing literature on the topics of ALM, option pricing (in particular for long maturities and sensitivity to interest rates) and the combination of the two. Chapter 2 will pertain to the data and methodologies used for our research including the option pricing model and option strategies used. Chapter 3 consists of the results and our analysis. Finally our conclusions are presented in Chapter 4.

Literature Review

The literature discussed in the following paper will touch on the slightly broader and separate fields of ALM, option pricing and hedging in addition to the use of options in ALM. Throughout the following chapter, we will look at various types of entities ranging from pension funds (Dempster, Germano, Medova & Villaverde 2003, Abourashchi, Clacher, Hillier, Freeman, Kemp & Zhang 2013) to insurance firms (Meer & Smink 1993) to banks (Romanyuk 2010, Kusy & Ziemba 1986). We will observe mostly ALM models along with some references on liability driven investments (LDI) (Bragt & Kort, 2010), although these terms are sometimes used interchangeably. Then we will cover literature about options plus static and dynamic hedging models (Chen 2011, Stulz 2004 and more).

ALM in its very simplest form refers to making investment decisions, choosing assets, based on current actual or future expected liability cash flows. In the specific case of pension funds, the goal is to hedge away client's risks, while meeting solvency requirements and make sure all benefit payments are met (Dempster, Germano, Medova & Villaverde, 2003).

According to Dempster et al., the main concerns of a pension fund manager are fourfold. They need to account for the stochastic nature of assets and liabilities. The timing and sometimes value of liability cashflows is uncertain and must be matched by the eventual asset values for which returns are unknown. The investment horizons are generally quite long (~30 years) and require portfolio rebalancing. It is extremely important to consider the risk of underfunding, i.e. failing to meet the targets of the fund without help from a guarantor. The fund must also consider possible management constraints. This includes solvency regulations as well as taxes, bid-ask spreads and other frictions in portfolio allocation.

ALM is described similarly in Choudhry (2011), but he keeps his focus mostly on what he calls the liquidity gap and the interest rate gap. These are defined as the "mismatch between the different terms of assets and liabilities across the term structure" and the "mismatch between the different interest rates that each asset or liability contract has been struck at". Both of these gaps combined form the ALM gap. This gap is likely more apparent in a bank than a pension fund due to the diversity of business lines in a bank. Each of these business lines has their own book and their own liabilities. ALM as he describes it, is a very high level and strategic endeavor and must take into account all of these various business lines.

Now for a brief look at how the literature categorizes various possible techniques. Note that for the following section the words *model, strategy* and *technique* may be used interchangeably. Firstly we can separate ALM techniques according to periodicity and stochasticity. We outline the following four possible groups as discussed in Romanyuk (2010), extrapolated from the teachings of Rosen & Zenios (2006):

- Single period static
- Single period dynamic
- Multiperiod static
- Multiperiod dynamic

Romanyuk states that the nature of assets, liabilities and risks of various industries, such as banking, insurance, pension funds and even individual households can greatly differ but generally assets and liabilities can be defined as expected cash inflows and outflows. ALM is then classified as a strategic discipline rather than a tactical one due to the, largely, long term focused outlook. It is important to note that the basis of the Romanyuk (2010) paper is to summarize existing literature on ALM for the purpose of managing a specific foreign exchange reserve fund at the Bank of Canada. Briefly, the objectives of the aforementioned fund are to maintain a high standard of liquidity, preserve capital value and maximize the return on the portfolio assets while respecting the liquidity and capital preservation objectives. For this reason some of the decisions on the performance and feasibility of certain models may differ from our interest of optimizing and improving a pension funds ALM strategies, but will still be considered; the mission of the fund is still true to the basic principles of ALM.

Dempster et al. state that most firms use static models and refer to them as "short-sighted". They consider dynamic models to be more suited to the problem faced in ALM (specifically in pension funds). By considering uncertainty in future asset returns and liability streams, dynamic (specifically stochastic programming) models are superior and lead to more robust decisions. Meer & Smink (1993) also make the distinction between static and dynamic models, but also continue by dividing dynamic strategies into return driven and value driven models. Moreover, value driven models can be split into active and passive models. Of all the strategies discussed in M&S they are evaluated on the basis of return and risk completeness, observability, model independence and data requirement. The *best* performing techniques are segmentation (static), standard (and to some degree key rate and contingent) immunization (dynamic, value-driven, passive) and spread management (dynamic, return-driven).

Standard immunization implies matching the interest sensitivities of assets and liabilities (Meer & Smink, 1993). Mathematically, we want the first order partial derivative, the duration, of the assets to be equal to that of the liabilities. Furthermore, we want the second order partial derivative, the convexity, of the asset to be equal to or greater than that of the liabilities. Such a construction means that for parallel shifts in the yield curve, both the asset and liability portfolios will change by approximately the same amount (Reitano, 1992). Creating and maintaining an asset portfolio with a larger convexity than that of the liability portfolio will ensure that any value change of the assets will not be outperformed by the value change of the liabilities and that the net value of assets minus liabilities will not decrease. M&S go on to say that this concept demonstrates the inherent weakness of immunization as a strategy, in that it assumes a flat term structure, or a single relevant interest rate resulting. This would mean that any interest rate change would produce value from nothing and would violate the basic no arbitrage proposition from financial theory. In reality we observe non-parallel shifts and a more complex term structure. Furthermore, Mackauley duration (a common measure defined as the weighted average term to maturity) assumes constant cash flows and fails to be usable for cases where cashflows may be interest rate dependent or where liabilities have some built in optionalities. As for an obvious yet important similarity between all dynamic models, the performance will be greatly affected by the liquidity of the assets.

To reiterate, the duration of a financial instrument is its sensitivity to fluctuating interest rates and this information can be used to build a portfolio tailored for specific purposes. This relationship is not strictly positive. Certain instruments or positions, such as long put options, can have a negative duration (Lukner, et al., 2003). Although this is not explored explicitly in M&S, there is potentially a portfolio construction that can exploit this. In the above portfolio construction the information at hand only allows us to say for sure that duration of the assets is greater than the duration of the liabilities, not if their durations are positive or negative. M&S go on to say that there are advantages to the use of model conditioned strategies such as their "potential accuracy and the opportunity to incorporate derivative instruments in the same term structure environment". However the disadvantage of such strategies is the non-stationarity of factors, which requires monitoring to detect. Following this logic it will be interesting to see if there is a way to better incorporate derivatives within standard immunization and/or stabilize model conditioned strategies to reduce stationarity issues.

Based on some of the above sources, we conclude that the more common techniques, the most feasible practical applications and those with highest potential should all be further discussed. As for the challenges and trade-off involving stability and computation time, it will be useful to still be observing static and dynamic models in order to try to accommodate different sizes and experience of firms, the importance being that these models should be flexible enough to allow the user to input increasingly complex assets. We see with the above construction of models and their evaluations, there are some arguments for both static and dynamic techniques.

Finally, considering different types of models, we also observe what might be the characteristics of an ideal technique. According to Kusy & Ziemba (1986), the ideal strategy incorporates:

- multiperiodicity (with changing yield spreads, transaction costs associated with selling assets prior to maturity and synchronization of cash flows across time by matching maturity of assets with expected cash outflows);
- simultaneous considerations of assets and liabilities to satisfy basic accounting principles and match the liquidity of assets and liabilities;
- transaction costs that incorporate brokerage fees and other expenses incurred in buying and selling securities;
- uncertainty of cash flows that incorporates the uncertainty inherent in the depositors' withdrawal claims and deposits (to ensure that the structure of the asset portfolio gives the bank the capacity to meet these claims);
- uncertain interest rates into the decision-making process to avoid lending and borrowing decisions that may ultimately be detrimental to the financial well-being of the bank;
- legal and policy constraints appropriate to the bank's operating environment.

K&Z were able to achieve a working model with the above characteristics in 1986, and with the progress of computational power such models now can incorporate even more data and still be tractable.

Briefly, we understand that the complexity of our ALM model will be dictated by our needs. A smaller firm with little investment experience will require much less than a governmental pension fund or a large bank. We see from the above sources that, when possible and necessary, firms should lean towards a dynamic model, but for the purpose of this paper in asset selection to supplement ALM, we will focus on cash flow matching before applying these concepts to more dynamic models or more complex immunization.

Clearly, as investors, our expectations of extreme negative events have changed over the past few decades (Abourashchi, N., Clacher, I., Hillier, D., Freeman, M., Kemp, M., & Zhang, Q., 2013, Sanders, 2005), a few decades marked by an increased amount of *black swan* events. This will likely continue to impact how we perceive and model liabilities. The liabilities we are focused on in this problem are the cash flows resulting from the mortality and retirement rates of the subscribers of a pension fund. Sweeting (2007), discusses certain risks in modeling longevity. He says, that we are in a way at the mercy of this uncertainty, and our models, whether correct or not, do not guarantee accuracy in forecasting events; he talks of "the risk of getting the average wrong; and the risk of getting the average right, but being unlucky". The actuarial sciences and practices of forecasting liabilities are extremely complex. The size of a pension fund will generally dictate its needs and restrictions, it will also contribute to determining the averages mentioned above. A small entity will experience difficulty in building a statistically significant model of withdrawals due to lack of data. Actuarial consultancies have access to data for a large number of schemes and should be able to provide a workable model.

Sweeting, featured in the British Actuarial Journal, has been a strong reference for methodology of modelling assets, liabilities and risks faced in our problem. His paper has a wider scope, and gives glimpses of the differences between 3 major financial industries that deal with risk management, namely banking, insurance and defined benefit pension plans. He broadly explains the workings of the, now out dated, Basel II accord.

We take interest in describing mortality risk, for which the International Actuarial Association (IAA) defines four types. This is mostly an exercise in understanding the risks rather than planning for them directly, as our liabilities are simulated for a particular amount but without confidence intervals regarding the following risks.

- Level, uncertainty around the current average rate of mortality.
- Trend, uncertainty around the future average rate of mortality.
- Volatility, the risk that average rates will differ from the central expectation.
- Catastrophe, the risk of mortality being significantly different from the average because of concentration risk.

A highly debated and researched topic in the field of epidemiology and actuarial sciences is the relationship between economic cycles on mortality rates. Although this is not the focus of this paper and we are by no means specialists of these fields, investigating such questions can be very impactful towards the assets we choose and the compositions of our portfolios. Certain sources agree that economic expansions lead to higher mortality rates (Granados, 2005, Rolden, Bodegom, Hout & Westendorp, 2013). During economic expansions we work more, drive more, sleep less, etc. These factors and many more contribute to increased stress and less healthy habits. Most existing studies have agreed on these elements, but how exactly they relate to retirees and people of old age is still puzzling to many and yet we do observe this similar increase in mortality among older people in economic upturns. Rolden, Bodegom, Hout & Westendorp (2013) speculate that higher levels of air pollution and informal care and social support during these good economic times could be contributors to this correlation but the evidence is insufficient to confirm this belief. They also question whether this is in fact the correct conclusion to draw or perhaps that economic uptrends actually reduce mortality rate but with a lagged effect and this is particularly difficult to determine given available datasets. Another important topic for forecasting liabilities is what factors impact an employee decision to retire. According to Pang, Warshawsky & Weitzer (2010), employees with defined-contribution plans tend to retire later and their timing is sensitive to business cycles.

“Derivatives allow firms and individuals to hedge risks and take risks efficiently” according to Stulz (2004). Derivative assets have a tough reputation to break out of and, as the author goes on to say, we should not fear them but rather attempt to understand them and treat them with respect. It takes an experienced and knowledgeable individual to deal with derivatives as they can be risky if misused. We must also understand when is the appropriate time to use them.

Chen (2011) has pioneered an interesting study on the risk taking behaviors of hedge funds who invest in derivative products. He begins by acknowledging the dangers of derivatives use, by citing a few highly visible examples in the media such as Amaranth in 2006, Société Générale in 2008 and Long-Term Capital Management, but also that they can be used to hedge portfolio risk and exploit superior information.

Using a regression model and controlling for various fund characteristics, he finds that the use of derivatives in hedge funds is associated with a 27% lower market beta (from 0.20 to -0.053) and these funds experience downside and event risks 80% less than funds who do not use derivatives. He then shows that derivatives using hedge funds will engage in risk shifting less than non-users. Briefly, risk shifting is the practice of manipulating fund risks in the managers self-interest of better personal compensation. This is demonstrated by the dummy variable associated with derivatives use being linked to a 50% lower level of shifting a funds' total volatility risk. The final part of his paper addresses the failure risk of derivatives use, as introduced through the earlier high-visibility examples. He proceeds to say that in fact the use of derivatives does not help prevent fund failure when performance is particularly low, but it does mitigate the unfavorable influence of severe market conditions on fund operation. This paper also highlights the perceptions of investors on derivatives use. The explanation is not completely defined, hesitating between the consideration that investors are indifferent as long as they receive similar net-of-fee performance, that they are simply unaware of the difference in risk or that they are unfazed by the potential risk. Noteworthy to consider that the clientele for hedge funds is drastically different than that of pension funds, and that the perception of derivative products may be a deterrent for some. This perception is what we hope to challenge through our analysis.

Chen concludes that derivatives use in hedge funds does not suggest higher fund risk. Contrary to popular beliefs, or as portrayed in the media, hedge funds that engage in derivatives use display lower risk according to return volatility, market risk, downside risk and extreme event risk. Other papers he cites deal with similar topics but choose to focus on mutual funds. Deli & Varma (2002) and Koski & Pontiff (1999) conclude that there is no difference in risk and returns between mutual funds that use derivative and those that do not, rather that they mostly contribute to reducing transaction costs. Constraints of derivatives use are rather related to fund monitoring systems according to Almazan et al. (2004).

A study concerning Danish pension insitutions (this includes pension funds and insurance firms) by Ladekarl, Ladekarl, Andersen & Vittas (2007) has demonstrated that derivatives use had increased after 2001 becoming the preferred hedging instrument. It was a moment in time where, similarly to the present amidst the COVID-19 outbreak, interest rates were drastically falling, due in part to the bursting of the tech bubble. In addition, there was a modification of accounting regulations requiring mark-to-market valuation on the balance sheet. These pension institutions were forced to find a new solution and out of necessity managed to emerge with better matched asset liability positions and lower exposure interest rate risk. They now mainly engage in swaptions to hedge interest rate risk and futures contracts to hedge equity risk. Hentov, Petrov & Odedra (2018) observe a similar use of derivative in public pension funds globally. Their analysis omits derivative products despite their increased use, due to the accounting complexities and the fact that mostly, these are used for hedging rather than investing activities.

Considering cost reductions, we also observe exotic options, which are financial instruments with highly specific payoffs that are generally path dependent and less demanded. Dupont (2001), whose attention is specifically on barrier options, discusses their advantages. They are cheaper than their vanilla equivalents; therefore they make risk exposure adjustments less costly. “They allow traders who place directional bets enhance their leverage and investors who accept to keep some residual risk on their books reduce their hedging costs. More generally, barrier options allow market participants to tailor their trading strategies to their specific market views.” The primary concerns with exotic derivatives, as mentioned in Sweeting (2007) when discussing more broadly about liquidity risk, is that when it comes to calculating assets for regulatory purposes, illiquid assets may be ineligible or only partially eligible to count towards the regulatory capital of a bank or insurance company. He goes on to say that most firms can afford some level of illiquidity in their assets, if for example these assets have a good match with the liability cash flows. For the purposes of this thesis, we will mostly focus on vanilla options as they will more effectively replicate the simplicity of a benchmark zero coupon bond portfolio. However, barrier options and other exotic options could be used in a similar application and potentially to a further degree due to their reduced costs.

According to Bakshi, Cao & Chen (2000) on the pricing of long term options and hedging LEAPS, Black-Scholes is always the least effective. Stochastic volatility and stochastic volatility jump models have similar hedging errors and both perform better than stochastic interest rate model. In fact, their study suggests that once a model has accounted for stochastic volatility allowing for stochastic interest rates does not improve the performance, even for long-term options. Only for hedging very specific types of options does incorporating stochastic interest rates make any noticeable difference.

We see through this brief analysis of existing literature that options and other derivative products can have many uses, from signalling to speculating but more importantly a strong hedging tool that is underused in the pension industry. Sometimes, as in the case of the Danish pension institution case, it takes an extreme situation or a change in regulations for investors to realize there is a tool that could have been beneficial to use.

In essence, this thesis will attempt to summarize existing research on cashflow matching, immunization and portfolio allocation methods for basic defined benefit pension schemes in order to explore the possibility of including options in the asset mix and see if there are scenarios in which these products may be more optimal than more traditional fixed income products. We hope to contribute to this literature by demonstrating the hedging and profit generating abilities of options in a liability driven portfolio.

Data and Methodology

The following chapter outlines the processes used for collecting and manipulating the data used. Certain theoretical concepts and option strategies are briefly described.

Liabilities

In order to simulate liabilities we wanted to start with real data. We require a member base, retirement rates, mortality rates and salaries along with any additional info on pension payouts. All of our raw data for liability simulation was taken from the "ACTUARIAL REPORT on the Pension Plans for the CANADIAN FORCES Regular Force and Reserve Force". In this report we can find everything we need to simulate the future liabilities faced by the entity. We start by taking note of the active members. We focus on the regular force, for which the active members are split into 4 categories: Male Officers, Female Officers, Male Other Ranks, Female Other Ranks. For each category we also have current ages and years of service. Below we have one of the tables as an example to show the structure of the data, the remaining tables of active member data for the other categories are shown in Appendix C. All data is extracted from the above report using a spreadsheet generator, cleaned up and confirmed in Excel, then imported into Matlab for the upcoming manipulations.

Age	0-4	5-9	10-14	15-19	20-24	25-29	30-34	35+	All Years of Service
15-19	70								70
	$18,933								$18,933
20-24	236	110							346
	$30,580	$58,096							$39,328
25-29	152	250	92						494
	$59,986	$74,911	$81,514						$71,549
30-34	80	214	239	60					593
	$61,483	$87,643	$89,088	$96,830					$85,626
35-39	36	88	173	176	46				519
	$67,267	$87,392	$100,078	$102,562	$112,466				$97,591
40-44	16	57	102	113	107	11			406
	$66,056	$92,202	$95,114	$100,922	$107,713	$116,710			$99,082
45-49	12	28	52	67	100	84	6		349
	$75,100	$83,837	$102,693	$98,757	$115,016	$111,264	$109,894		$105,193
50-54	4	12	22	20	27	61	34	3	183
	$69,483	$103,734	$88,893	$93,673	$105,995	$114,735	$114,707	$104,196	$106,148
55-59	2	3	7	14	8	8	9	2	53
	$82,440	$187,915	$92,308	$97,916	$112,927	$99,265	$123,466	$114,456	$109,118
60+				1					1
				$17,914					$17,914
All Ages	608	762	687	451	288	164	49	5	3,014
	$45,068	$80,021	$92,792	$100,097	$110,992	$112,335	$115,726	$108,300	$84,230

Table 1: Active member quantities and average annual earnings; Female Officers 2013

Next, we take into account the retirement rates for every possible category and age combination as based on the assumptions and calculations of the actuarial report. These details are shown in Table 2. The active members retire in t = 0 according to their respective rates.

Qualifying Service	Male Officer	Female Officer	Male Other Rank	Female Other Rank
20-24	0.061	0.058	0.063	0.096
25-29	0.071	0.071	0.071	0.099
30-34	0.098	0.107	0.082	0.134
35+	0.396	0.396	0.328	0.406

Table 2: Modified table of assumed retirement rates per category and service bracket

We then have the now retired members age and die according to the mortality rates defined in the report and shown in Table 3. Every year, we observe the amount of people in each category, at each age for each bracket of years of service and store this value. Every year, the members age and remain in the same bracket, age and move to the next bracket, or die and are removed from the count. As the retirees age, they maintain the same payout that was entitled to them, but move on to the next category for mortality meaning they become increasingly likely to die. The information for females was not split by employment so we continue with the assumption that they have the same mortality rates for both officers and other ranks.

Age Last Birthday	Male Officer	Female Officer	Male Other Rank	Female Other Rank
30	0.0005	0.0004	0.0007	0.0004
40	0.0006	0.0005	0.0009	0.0005
50	0.0011	0.0013	0.0024	0.0013
60	0.0041	0.0037	0.0079	0.0037
70	0.0118	0.0116	0.0201	0.0116
80	0.0442	0.035	0.0621	0.035
90	0.1434	0.1092	0.1605	0.1092
100	0.3117	0.2863	0.3252	0.2863
110	0.4997	0.4921	0.4997	0.4921

Table 3: Assumed mortality rates per category and age

For each year, we take the remaining amount of people from a specific bucket and pay them the appropriate amount of their pension, resulting in the total cash liability for every year in the period of interest.

Although there are many types of payouts indicated in the actuarial report based on countless conditions and types of termination, we simplify by considering each pensioner will receive a deterministic percentage of pensionable earnings. The most common payouts are an option between a deferred annuity and the transfer value if the member is under age 50 or an immediate annuity. We decide to consider all retirements resulting in immediate annuities with a payout of 2% of average pensionable earnings annually. We also choose to simplify and ignore any disability, surviving spouse or other payments that are not a result of retirement.

This entire method is based on the premise of observing a company at any single point in time and forecasting what will be the decision over the next year when the following batch of employees decide to retire. Of course the overall company has more employees retiring every year, and the overall liability cashflows would have to be added up. In this scenario we observe only the one group of retirees and track their evolution over the max horizon of 3 years and our method can be expanded to observe longer horizons with no modification. In essence, we could include the more realistic scenario of new hires and new retirees using the other assumptions and data provided in the same actuarial report used thus far with a relatively simple modification. Starting with a full active employee member base we could run the existing method each year and sum the amount of retirees to consider. In year 1, we would only have Batch 1 of retirees as we have already done. In year 2, we would have a new Batch 2 of retirees according to the existing active member base in addition to Batch 1, which may have been reduced by some deaths. In year 3 the total group continues to grow as described, assuming that every year more people retire than die from the previous batches of retirees. Some additional code must be included to add new hires to the active employee member base every year. In short, this is not a complex addition when considering the same assumed hiring, retiring and death rates year over year. However we do not consider these assumptions to add enough value to be pursued for the length of our period of interest. This would more likely be considered if we were looking at 10 year horizon or more, as the liability cash flows would vary more and we could update the various rate assumptions throughout by consulting the CAF Armed Forces actuarial reports, which are published every 3 years. By implementing the described methodology including new hires and new retirees would only impact the asset side of the problem by causing additional cash flow matching portfolios to be bought every time a new batch of employees retires. Our focus lies in the analysis of the asset side of this problem therefore the liabilities are simplified to facilitate this portion of the research. The simplification also helps provide a more clear snapshot of the assets used and the resulting payoffs by ignoring new purchases.

Assets

Zero coupon bonds are assumed to exist for any face value and any maturity. Their prices are simulated using a standard discounting formula:

$$ZC = \frac{F_t}{(1+r)^t} \tag{1}$$

The face value represents the value of the liability cash flow at time t. The time t will depend on when the cash flow is expected. The rates are determined for each of the three scenarios to represent a low, mid and high rate environment and are 0.5%, 1.5% and 2.5% respectively. The total cost of acquiring these bonds in $t = 0$ is the total cost of our benchmark portfolio. The benchmark portfolio is the simplest cashflow matching strategy and results in no solvency risk. We know at maturity this portfolio will give us the exact liquidity to cover the simulated liability cash flow, assuming a 100% forecasting accuracy.

For the risky assets, we screen for any Canadian indexes within Bloomberg with price data that goes back pre-1999. The goal is to incorporate the largest crashes (2000, 2008) and have a reasonable universe of assets that can be traded easily in the open market and derive their value from a variety of sources or asset classes. Going as far back as January 1995, we have daily prices for 201 assets to draw information from. Additionally, the indexes selected inherently have less idiosyncratic risk than individual stocks as they are essentially portfolios of assets with a certain theme. The list of indexes selected is featured in the results section of this paper and has assets with various themes ranging from exposure to specific commodities, markets, volatility and more.

In order to simulate our asset prices we start by finding the distribution of the daily returns over our historical period and its probability density function. From here we are able to draw at random from this distribution for every day in our simulation period. As we can see these charts are quite noisy due to the amount of data, their purpose is primarily to demonstrate that there are some more or less correlated assets at various price levels rather than to determine the price of a specific asset at a specific time. Our entry and exit dates are non-flexible and do not involve any particular timing strategies, as such we can be vulnerable to daily volatility of prices but we are not concerned for this analysis.

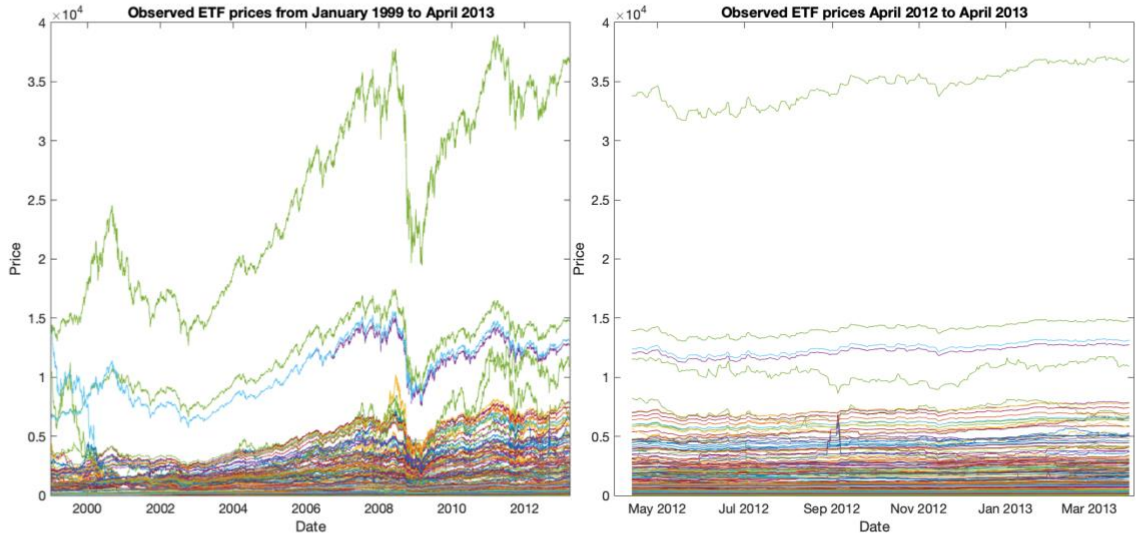

Figure 1: Historical Risky Asset Price Data – Custom filter on Canadian ETFs with specific characteristics and available data for time period, closing day prices, obtained from Bloomberg

Figure 1 shows the historical prices for the 201 risky assets we selected for this analysis. The first graph shows the entire period of available prices in an attempt to include larger financial events, of which we note the 2008 crisis has definitely had a significant impact on many of the observed assets. The second graph shows just the final year of prices leading up to our observation period. The observed prices in Figure 1 also show that there have been major price movements over the 14 year period, but the 1 year period leading up to our initial observation period was mostly flat, perhaps with a slight upward trend. From these prices we can gather the daily returns for our entire period and observe the distribution of these daily returns for each asset. By including more data we try to account for the larger less frequent movements.

Index	Mean	Standard Deviation	Skewness	Kurtosis
SPTSX	0.00025	0.0118	-0.4595	11.4899
SPTSX3	0.00025	0.0118	-0.4595	11.4899
SPTSX4	0.00025	0.0118	-0.4595	11.4899
SPTSX2	0.00025	0.0118	-0.4595	11.4899
x0000AR	0.00034	0.0118	-0.4554	11.4784
MXCA	0.00028	0.0126	-0.3651	11.0202
GDDLCA	0.00037	0.0126	-0.3618	11.0141
NDDLCA	0.00035	0.0126	-0.3627	11.0161
MSDLCA	0.00028	0.0126	-0.3651	11.0201
SPTSX60	0.00026	0.0126	-0.4111	11.3693
FRCANTR	0.00046	0.0110	0.0220	13.4885
FRCAN	0.00036	0.0114	0.1444	21.3449
STENRS	0.00057	0.0163	-0.2363	10.5348
STBANKX	0.00038	0.0136	0.3803	13.5847
STFINL	0.00030	0.0125	0.2359	16.1126
STINFT	-0.00001	0.0269	-0.4751	11.6984
STGOLD	0.00043	0.0253	0.7117	12.7375
STUTIL	0.00021	0.0090	-0.2490	10.7346
STCONS	0.00034	0.0077	-0.1367	7.0920
STOILP	0.00057	0.0185	-0.1696	10.7483
STMATR	0.00043	0.0179	0.0325	15.2084
STCOND	0.00013	0.0100	-0.1361	7.2120
STINDU	0.00030	0.0130	-0.1472	7.2926
STHLTH	0.00021	0.0166	-0.2542	10.8712
STTELS	0.00035	0.0133	-0.2238	18.1348
STLIFE	0.00017	0.0177	0.1946	14.2491
STRLST	0.00030	0.0121	-0.3510	10.1239
STFDSR	0.00030	0.0084	-0.0751	6.1503
STDBNK	0.00038	0.0136	0.3810	13.5861
STFRST	0.00037	0.0205	-3.4200	72.5427
STIOIL	0.00064	0.0185	-0.2379	9.8417
STENRE	0.00051	0.0178	-0.1216	7.1932
STINSU	0.00002	0.0162	-0.3748	19.1747
STTELSX	0.00035	0.0133	-0.2239	18.1387
STREAL	0.00030	0.0121	-0.3510	10.1239
STTRAN	0.00058	0.0145	-0.0034	7.0004
STPAFO	0.00005	0.0205	-3.4706	71.5811
STRAIL	0.00071	0.0158	0.0531	6.7196
STMETL	0.00046	0.0201	0.3649	15.0377
STDIVM	0.00083	0.0231	-0.0516	8.2917
STCBNK	0.00038	0.0136	0.3804	13.5823
STMATRX	0.00043	0.0179	0.0308	15.2248
STOILG	0.00058	0.0164	-0.2161	10.4846
STTECH	-0.00006	0.0318	-0.4131	10.2122
STOILE	0.00051	0.0168	0.0763	7.4818
STRETL	0.00016	0.0124	-0.0223	7.4244
STCHEM	0.00070	0.0204	-0.4229	16.0589
STITEL	0.00038	0.0138	-0.7897	31.3798
STSFTW	0.00029	0.0212	1.6463	82.7441
STHOTRX	0.00035	0.0142	0.3565	8.3339
STAUCO	0.00018	0.0189	0.4395	9.7249
STFDPR	0.00021	0.0130	0.1619	7.6916
STCODU	0.00053	0.0193	-1.7193	34.4708
STMDREX	0.00033	0.0152	-0.2278	11.0116
STCSTEX	0.00067	0.0189	0.0415	11.7076
STDIVF	0.00030	0.0119	-0.3219	10.4423
STMEDAX	0.00007	0.0112	0.0112	8.1092
STMOVI	0.00031	0.0179	-0.1281	17.1131
STAERO	0.00013	0.0256	-0.0797	7.7103
STAUTC	0.00019	0.0193	0.5964	12.3004
STHCES	0.00038	0.0175	-0.2131	11.1975
STHOTR	0.00034	0.0139	0.2645	7.6716
STMEDA	0.00007	0.0112	0.0109	8.1154
STBEVG	-0.00022	0.0193	-4.1930	108.6502
STENRSX	0.00057	0.0163	-0.2361	10.5346
STMRET	0.00033	0.0144	0.0179	7.5223
STSPRE	-0.00025	0.0192	0.3461	14.4493

Index	Mean	Standard Deviation	Skewness	Kurtosis
STAEROX	0.00013	0.0256	-0.0798	7.7093
STAIRLX	0.00036	0.0208	1.2784	18.5545
STCOMS	-0.00016	0.0139	0.0016	9.6554
STCSTE	0.00067	0.0189	0.0415	11.7076
STELUT	0.00028	0.0107	0.3147	8.8072
STMUTI	0.00032	0.0119	-0.1571	9.6049
STAIRL	0.00038	0.0218	1.1932	18.6832
STAUTP	0.00018	0.0189	0.4401	9.7148
STBRDC	0.00045	0.0146	1.0756	21.6484
STCMDT	0.00041	0.0211	1.8853	46.5740
STCOMSX	-0.00028	0.0143	-0.1452	9.2454
STCPGS	0.00010	0.0164	-0.1652	7.5364
STCSTF	0.00065	0.0185	0.3287	6.7739
STDIVT	0.00035	0.0138	-0.7978	31.3769
STDVFS	0.00037	0.0127	-0.4773	8.9385
STELUTX	0.00028	0.0107	0.3148	8.8076
STFDBT	0.00044	0.0113	0.1259	8.8819
STFDRE	0.00032	0.0096	-0.0929	8.2195
STFDSRX	0.00030	0.0085	-0.0763	5.9956
STFERT	0.00079	0.0224	-0.3162	13.6950
STGENM	0.00038	0.0173	0.1209	8.1272
STHCFA	-0.00019	0.0282	-10.8044	403.5657
STHCPS	-0.00002	0.0163	-0.1018	8.3388
STINSUX	0.00002	0.0162	-0.3747	19.1712
STMACH	0.00026	0.0173	0.1859	6.7175
STMLIN	0.00022	0.0223	0.1410	17.0945
STMUTIX	0.00032	0.0119	-0.1582	9.6141
STROAD	0.00064	0.0156	0.0311	6.7542
STSTEL	0.00075	0.0201	0.1885	11.1318
STTRUC	-0.00030	0.0255	-0.3290	22.3917
STUTILX	0.00021	0.0090	-0.2492	10.7386
STSOFD	-0.00006	0.0252	-1.5092	40.7012
STWIREX	0.00023	0.0216	-0.0805	11.5235
STWIRE	0.00023	0.0215	-0.0826	11.5256
STSOFT	0.00043	0.0220	0.8412	14.5630
SPTSXC	0.00028	0.0113	-0.4215	12.0329
TXEQ	0.00026	0.0118	-0.4436	11.5040
STPHRM	0.00027	0.0198	0.0242	12.2402
STELEIX	0.00022	0.0303	0.7974	14.0696
STAPLS	0.00044	0.0226	0.7757	13.6894
STENVR	0.00024	0.0512	37.6534	1964.9251
DWCA	0.00026	0.0116	-0.4463	11.4618
DWCAT	0.00034	0.0116	-0.4401	11.5069
TRIBCN	0.00019	0.0167	1.4430	56.3795
M4CARWGT	0.00038	0.0090	-0.3309	11.8208
M5CARWGT	0.00041	0.0090	-0.3298	11.8130
M6CARWGT	0.00031	0.0090	-0.3342	11.8289
MXCA00LV	0.00026	0.0116	-0.0345	13.5923
M5CALC	0.00033	0.0128	-0.3376	11.5480
M4CAIM	0.00034	0.0117	-0.4162	11.7932
M4CAVW	0.00039	0.0114	-0.2954	12.4309
M5CAVW	0.00042	0.0114	-0.2940	12.4238
M6CAVW	0.00032	0.0114	-0.2993	12.4499
M4CAIVW	0.00041	0.0108	-0.3535	13.1786
M5CAIVW	0.00044	0.0109	-0.3512	13.1706
M6CAIVW	0.00035	0.0108	-0.3601	13.1996
MXCA0EN	0.00057	0.0168	-0.2220	10.1624
MXCA0UT	0.00026	0.0114	0.0724	9.5777
MXCA0FN	0.00033	0.0128	0.2629	16.0370
MXCA0CD	0.00016	0.0123	0.0367	7.5526
MXCA0TC	0.00051	0.0148	0.5071	14.0486
MXCA0HC	0.00015	0.0204	-0.0011	12.8770
MXCA0IN	0.00033	0.0151	-0.1506	7.6119
MXCA0IT	-0.00004	0.0301	-0.3691	10.9922
MXCA0MT	0.00045	0.0183	0.1257	15.0936
MXCA0CS	0.00031	0.0097	-0.0119	6.7086
MXCA000G	0.00039	0.0264	15.3513	771.7437

Index	Mean	Standard Deviation	Skewness	Kurtosis
MXCA000V	0.00026	0.0111	-0.0971	12.8082
STCOMP	0.00003	0.0211	0.2023	18.2014
STMETG	0.00025	0.0217	-1.0734	23.0028
STCONP	0.00018	0.0214	-1.1228	23.4681
FIDCNCLT	0.00044	0.0089	-0.2811	13.7536
FIDCCQLT	0.00046	0.0108	-0.5027	12.5301
FIDCNULT	0.00019	0.0107	0.1746	9.1752
FIDCNCQP	0.00036	0.0108	-0.5107	12.5453
FIDCNCDT	0.00047	0.0099	-0.4302	11.4257
FIDCNCDP	0.00032	0.0099	-0.4289	11.4207
FIDCNCLP	0.00032	0.0089	-0.2839	13.7626
FIDCNHDN	0.00025	0.0129	0.1524	10.5819
FIDCNHDP	0.00015	0.0129	0.1514	10.5731
FIDCNHDT	0.00029	0.0129	0.1525	10.5790
FIDCNILN	0.00018	0.0079	-0.0757	8.9004
FIDCNILP	0.00007	0.0079	-0.0743	8.8700
FIDCNILT	0.00020	0.0079	-0.0765	8.9022
FIDCNINN	0.00024	0.0103	0.0199	9.5706
FIDCNINP	0.00010	0.0103	0.0213	9.5411
FIDCNINT	0.00026	0.0103	0.0200	9.5698
FIDCNRRN	0.00014	0.0125	0.1139	8.6782
FIDCNRRP	0.00006	0.0125	0.1118	8.6833
FIDCNRRT	0.00018	0.0125	0.1144	8.6753
FIDCNULN	0.00016	0.0107	0.1737	9.1804
FIDCNULP	0.00008	0.0107	0.1724	9.1964
STPHARX	0.00067	0.0237	-0.1306	12.4020
STPHRX	0.00066	0.0231	-0.1258	13.4547
RUCASCC	0.00029	0.0106	-0.7031	10.4298
RUDEVLNC	0.00008	0.0100	-0.0332	7.2862
RUDXNLNC	0.00009	0.0100	-0.2749	6.1368
MGLDCA	0.00023	0.0175	-0.9557	22.2205
MVLDCA	0.00026	0.0111	-0.0970	12.8098
FTL3CA	0.00035	0.0123	-0.3658	11.6490
STITSV	0.00043	0.0248	0.1001	21.5968
STITCS	0.00040	0.0249	0.1043	21.3023
SPTXLVPR	0.00029	0.0068	-0.6306	16.4699
SPTXHBPR	0.00002	0.0208	-0.1748	7.9421
M4CAIRW	0.00039	0.0081	-0.6282	13.6337
M5CAIRW	0.00041	0.0081	-0.6236	13.6343
M6CAIRW	0.00032	0.0081	-0.6406	13.6189
STPROP	0.00043	0.0230	-2.2066	90.7443
SPTSEN	0.00055	0.0176	-0.2793	11.0868
SPRTRE	0.00027	0.0092	-0.4918	15.9715
SPTSUT	0.00021	0.0089	-0.2154	10.3724
BBCREIT	0.00027	0.0085	-0.6930	19.2301
SPTSMT	0.00043	0.0176	0.0268	15.8339
TTUTAR	0.00038	0.0089	-0.2169	10.3433
SPTSTS	0.00021	0.0113	-0.4423	10.3620
SPTSRE	0.00029	0.0117	-0.3421	10.8161
SPRTCM	0.00031	0.0106	-1.0222	21.6299
SPTSCD	0.00015	0.0099	-0.1486	6.9498
SPTSIN	0.00029	0.0130	-0.1396	7.3441
SPTSHC	0.00016	0.0149	-0.1834	7.6142
SPTSCS	0.00035	0.0078	-0.2136	6.5401
TTCDAR	0.00021	0.0125	-5.1000	143.1655
TXSY	0.00035	0.0099	-0.5264	10.9107
M9CAQU	0.00037	0.0118	-0.5215	11.6448
M9CXBRC	0.00042	0.0105	-0.5474	9.0449
STPACK	0.00039	0.0152	0.4691	10.9494
M6CASC	0.00034	0.0114	-0.5198	9.6755
DJCASDT	0.00041	0.0092	0.0432	14.2649
M4CADY	0.00047	0.0113	0.2332	14.8755
M5CADY	0.00050	0.0113	0.2341	14.8641
M4CAEW	0.00035	0.0108	-0.3947	10.4591
M5CAEW	0.00037	0.0108	-0.3924	10.4585
M6CADY	0.00036	0.0113	0.2306	14.8975
M6CAEW	0.00029	0.0108	-0.4012	10.4608

Table 4: Descriptive Statistics of Daily Returns for All Observed Assets – We note non-normal characteristics of daily returns which is consistent with existing literature. Mostly moderate-to-high positive and negative skewness values indicate that we have assets with either high gain outlier days and high loss outlier days. We observe leptokurtic distributions on all assets with some very high values.

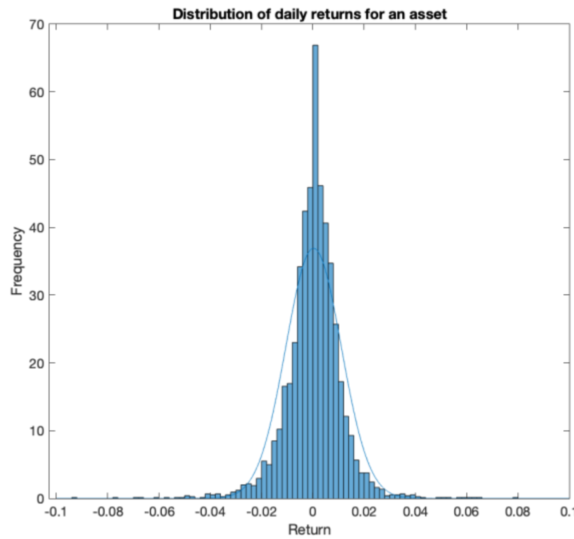

Figure 2: Distribution of Daily Returns for an Example Risky Asset – Historical data obtained from Bloomberg, demonstrates high peak at near zero along with less frequent but significant positive and negative outliers

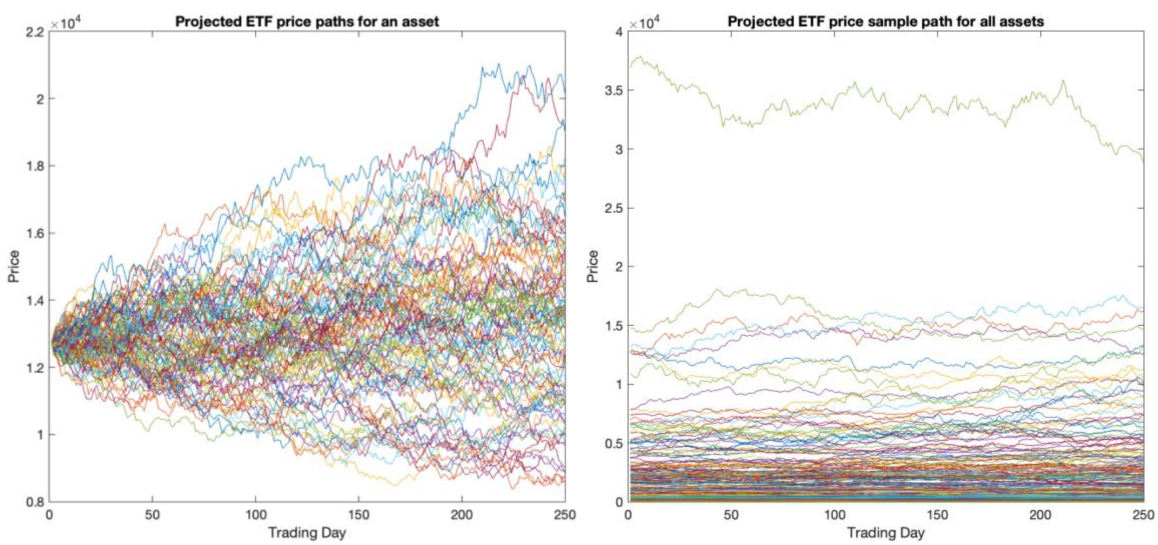

Figure 3: Simulated Asset Price Data – Markov chain Monte Carlo, drawing from the distribution of daily historical returns we show 100 paths for one example asset and 1 example path across all assets. Simulated in Matlab

Once we have the distribution of daily returns for each asset, we calculate their probability density functions, the result can be seen in Figure 2 for an example asset. We then draw from this continuous set of daily returns to generate simulated daily returns over our 1 and 3 year observation periods. These paths are stored, but our main interest is the simulated price at maturity, which is obtained in classic Monte Carlo fashion, by taking the mean of the end prices of the generated paths.

Using the above methodology, we simulate the price paths for each of the risky assets over the required maturity. The first graph in Figure 3 shows the 100 simulated daily price paths of 1 asset over the 1 year time period. This pattern clearly demonstrates that our simulation method has an assumption of heteroscedasticity, as seen by the paths radiating both up and down in a cone shape from the starting point, which is important due to our selected option pricing model and its assumptions. The second chart in Figure 3 shows one iteration of the 100 simulations but this time for all of our assets. In the multi-asset graph we do notice that our simulated paths are a little noisier than the observed prices in the multi-asset graph of Figure 1, however the volatility does not appear to be tremendously dissimilar. We also see that the multiple assets paths seem less aligned in Figure 3 than in Figure 1. This is normal as the simulation was done assuming independence between underlying assets. Although Figure 1 shows visually that there may be some correlation over longer periods (see the overall upward trend from 2004-2008 followed by the all too familiar shock of the crisis and also peaks in 2011), our analysis does not mix these underlying assets. Each cash flow matching portfolio is an already semi-diversified index with an option strategy. The simulated prices of these underlying assets can therefore be independent from each other as they will not be combined to adjust the characteristics of the cash flow matching portfolios. The desired effect is for each simulation to be driven by the return history of the asset itself while including an assumption of stochastic volatility.

This assumption of stochastic volatility is extremely significant as we must maintain consistency across the models used. In the following pages, we will discuss why we selected the Heston-Nandi GARCH approximation as our option pricing model. This model has an assumption of heteroscedasticity that we replicated, albeit independently, in our underlying asset simulation. This means that the underlying assets were simulated using a completely independent process from the option pricing model used but with the same underlying assumptions. As described above, the assets were simulated using Markov Chain Monte Carlo based on the historical return data. For these reasons, we assume the simulated prices of underlying assets and the calculated option prices are sufficiently consistent.

We now have established a liability cash flow that we are trying to secure at a specific time in the future, we have a benchmark portfolio, we know the observed risky asset prices at initiation and simulated risky asset price at maturity. We need to determine the details of our options to complete our test portfolio.

We know we want our test portfolio to have the same payoffs as the benchmark portfolio until the final year. In the years until the final year, the growth portfolio also uses zero coupon bonds to match the liability cash flows. In this final year, the expected liability is covered by the growth strategy. Therefore, in the 1 year horizon we are directly comparing the test against the benchmark, whereas in the 3 year horizon, there will be 2 years where both portfolios are identical and in the final year we will observe the difference between the option strategy and the benchmark zero coupon bonds.

Option Pricing & Strategies

We want to take advantage of the leverage of options to hedge the risk of our growth assets, resulting in overall savings for our asset-liability strategy. To do so we considered a variety of option strategies.

Married puts were studied, they are essentially providing the same kind of coverage as our zero coupon bonds. With these we are able to address the risk that our growth asset will not generate the expected returns needed to cover the liability payment in the final year and we secure this minimum cashflow. Of course this comes at a cost; the premium we pay for this safety should not exceed the cost of the benchmark portfolio. Even if it is the exact same cost, we gain from the fact that we are not limiting the potential gain of the growth asset. Covered calls reduce the initial cost of purchase of our risky asset, but they offer no downside protection and limit our upside potential. The collar strategy, which is essentially a combination of the two, may be the solution. With the collar, we protect our downside risk with the purchase of puts and we offset some of the cost of this safety through the sale of calls.

As discussed in previous papers (Kusy & Ziemba, 1986), in order to implement a specific ALM model, we need to know which assets we can invest in and the point estimates of the returns on these assets. We maintain as discussed in the above literature review that BS is considered to be an over simplified and unrealistic model and some form of heteroscedasticity should be implemented. Due to the asset data used, and the unavailability of long term option prices we decide to use the H&N model which avoids the complexity of calibrating Heston parameters using observed option prices. This model provides a closed form solution for option prices using underlying historical price data.

Heston-Nandi GARCH

We generate option prices using the Heston-Nandi GARCH model to account for more realistic assumptions than the BS model, and eliminating the challenge of not having existing point estimates for the long term option prices required for the Heston stochastic volatility model. In addition, at long maturities the Heston-Nandi model should tend towards the Heston SV model (Heston & Nandi, 2000). The model is estimated using the characteristic function:

$$C = \frac{1}{2}S_t + \frac{e^{-r(T-t)}}{\pi}\int_0^\infty Re\left[\frac{K^{-i\varphi}\, f^*(i\varphi+1)}{i\varphi}\right]d\varphi - Ke^{-r(T-t)}\left(\frac{1}{2}+\frac{1}{\pi}\int_0^\infty Re\left[\frac{K^{-i\varphi}\, f^*(i\varphi)}{i\varphi}\right]d\varphi\right) \tag{2.1}$$

$$f(\varphi) = E_t\left[S_t^{\varphi}\right] = S_t^{\varphi}\exp\left(A_t + B_t h_{t+1}\right) \tag{2.2}$$

$$A_t = A_{t+1} + \varphi r + B_{t+1}w - \frac{1}{2}\log(1-2aB_{t+1}) \tag{2.3}$$

$$B_t = \varphi(\lambda + c) - \frac{1}{2}c^2 + bB_{t+1} + \frac{\frac{1}{2}(\varphi - c)^2}{1 - 2aB_{t+1}} \tag{2.4}$$

The inputs for the model consist of the historical price data on the selected growth assets. We of course have the starting stock price and the unconditional return volatility, which we use to compute the autoregressive future conditional volatility as a method of distancing ourselves from the BS assumption of constant volatility. We consider that this model is a good choice for our needs as it provides a closed form solution for option prices, does not require any complex calibration or observed market option prices and assumes heteroscedastic volatility, which is a big leap towards realism from the classic BS model. As mentioned in the literature review, the use of stochastic volatility is a huge improvement in the pricing models for long term options, and including stochastic interest rates add little to the efficiency of pricing once stochastic volatility is already accounted for. There may be more optimal pricing models, but for our application, the concision and assumptions of this model make it a desirable choice.

We appreciate that there may be issues with oscillation for deep OTM options with this type of approach, using a relatively naïve method to calculate the integral of our Heston-Nandi GARCH in two shapes and we are aware of possible solutions however due to time constraints they have not been addressed in this version. Using a Fourier transform would be one method to pursue improving our option value calculation. We also appreciate the low volatility (in certain cases) and significant positive/negative moneyness of our options. The volatility stems from the assets selected, for which we wanted to have consistent return data going back at least to 1999 and assets that were less exposed to idiosyncratic risk. For this reason, we found a group of Canadian indexes, which our *client*, the Canadian Armed Forces, would have potentially been investing in. Indexes are essentially a grouping of other assets with a specific theme and inherently have less idiosyncratic risk than individual equities. As for the moneyness of our options, there is a next step in this questioning which would involve optimizing the strike prices used and could theoretically result in a completely different option combination. Due to the long term nature of this problem, and the generally short term nature of options, we believe there are still substantial advancements in long term option pricing that could further contribute to solving this type of problem. Understanding the comparison between products with similar payoffs can perhaps aid in reconciling true option values. Despite the potential difficulties in the accuracy of the pricing of these strategies, we are looking to explore the possible application of these traditionally retail oriented structured products in an institutional environment.

In all cases, we calculate the value of the call using the H&N model described above and, following their observations, find the value of the put through the simple yet reliable put call relationship:

$$C + Ke^{-rT} = S_0 + P \tag{3}$$

This relationship allows us to calculate the value of the call (C) using the H&N model and a known strike price (K), interest rate (r), maturity (T) and current underlying price (S_0). Then to compute our put value we simply isolate and solve for our last unknown, P.

Collar Strategy

We decided to pursue the collar strategy primarily due to its underlying structure. A standard collar is usually set up to get exposure to a risky asset, but while limiting potential losses and offsetting the cost by limiting potential gains. Figure 4 demonstrates all the components of the option strategy and their individual payoffs as well as the total combined payoff. As we can see the strike price of the put is lower than the strike price of the call. In the classic case, the purchase price of the underlying is between the two strikes.

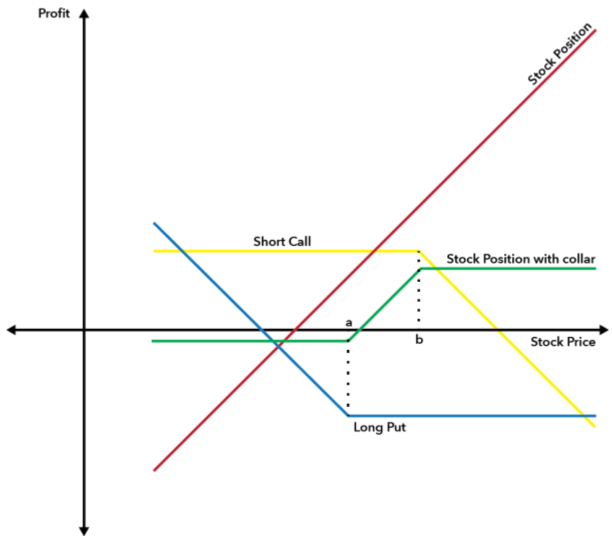

Figure 4: Components of standard collar and associated payoffs – Demonstration of payoffs of long put with strike price a and short call with strike price b, when combined along with a long stock position we obtain a collar which provides a payoff range depending on underlying price at maturity. Our collars are set in a way that the bottom range of the payoff will match the liability cashflow expected at maturity due to our purchase of ITM puts.

The protection of the put gets more expensive as we approach a positive moneyness (ITM) and go deeper ITM. It gets more costly as we set the strike price higher above the current underlying price. This cost is offset by the selling of calls, which provide less premium as we place the strike higher above the current underlying price. If the risky asset goes above our call strike, the call holder will exercise his right to buy at maturity meaning we are forced to sell. As we go deeper OTM on the calls, we experience a tradeoff between cash generated from the premium and the potential upside of the risky asset. We will potentially sell at a higher profit but receive less guaranteed premium from the initial sale of the calls.

Our collar consists of buying a growth asset at price X, purchasing a put and selling a call with the following strike prices where a & b are greater than 1 and b is always 0.01 higher than a.

$$Put\ Strike = aXT \qquad Call\ Strike = bXT$$

The pairs are set as follows: 1.05/1.06, 1.1/1.11 and 1.15/1.16. This means that in the 1 year horizon strike pair 1, we buy a put at a strike 5% above the growth asset price X and we sell a call at a strike price 6% above the growth asset price. For the 3 year horizon strike pair 1, we buy a put at strike 15% above todays growth asset price X and sell a call at strike 16% above the growth asset price. These levels were determined heuristically by setting levels around an expected average 1 year market return of 10%. The setting of *a* & *b* and the spread between them is in itself an interesting and broad question which, given more time would have to be investigated further to optimize this strategy. Such a strategy should allow us to guarantee a minimum payoff (the value of the liability cash flow) and add a limited upside relative to the benchmark. The cash flows at maturity are illustrated in Figure 5, where the zero coupon at maturity will cover the exact amount of the liability cash flow, and the collar will at the very least do the same if the price of the growth asset remains or goes under price *a* and more if it ends above. The following graph illustrates the payoffs at maturity, but does not include any comparison of the cost at initiation.

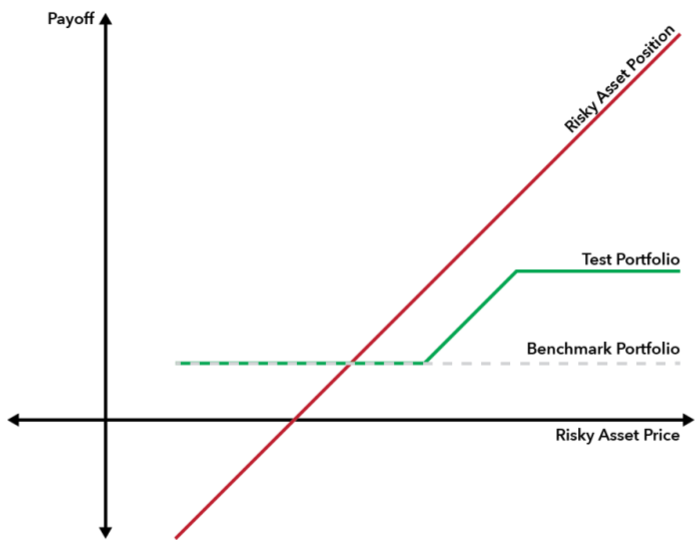

Figure 5: Expected Payoff of Portfolios at Maturity depending on Underlying Risky Asset Price – The test portfolio represents the payoff of our risky asset, long ITM put and short OTM call strategy; This secures the required payoff to cover the expected liability cashflow at maturity (through the put) and more upside potential relative to the benchmark (limited by the call) if the underlying risky asset price increases

Visually we see the test portfolio has 3 different types of payoff, identified by the 3 segments of the green line. Below we describe the possible outcomes of the test portfolio mathematically. The value of the cash generated by these outcomes at maturity is determined by the formulas below.

1. $S_T > b$: The best case is the result of our short calls being assigned. This way we collect the premium and we sell our shares at the highest possible price within our option strikes. This is associated with the leftmost flat line in Figure 5.

$$Best\ case\ cash\ flow = \frac{F_T b}{a} \tag{4.1}$$

2. $S_T < a$: The worst case is the result of exercising our puts, this is the lowest price within our option strikes, but it still allows us to cover our expected liability. This is associated with the rightmost flat line in Figure 5.

$$Worst\ case\ cash\ flow = F_T \tag{4.2}$$

3. $a < S_T < b$: The middle case is when the price of the underlying settles between our two strike prices at maturity. This is associated with the diagonal line in Figure 5.

$$Middle\ case\ cash\ flow = \frac{F_T S_T}{S_0 a} \tag{4.3}$$

The next comparison we will be making is the difference between the setup costs of both the benchmark portfolio and the collar strategy test portfolio. The savings at $t = 0$ and initial cost for each strategy is represented by the following formulas. In order to determine the bottom line difference against the benchmark we simply add the setup savings to the terminal cashflow.

$$Benchmark\ Portfolio - Test\ Portfolio = Setup\ Savings \tag{5.1}$$

$$Benchmark\ Portfolio = \sum_{t=1}^{T} \frac{F_t}{(1+r)^t} \tag{5.2}$$

$$Test\ Portfolio = \frac{F_T}{S_0 * a}(S_0 + C - P) + \sum_{t=1}^{T-1} \frac{F_t}{(1+r)^t} \tag{5.3}$$

As mentioned, the focus of this thesis lies primarily on the asset side of this problem. Our liabilities have simplified assumptions and are used mainly to simulate some future cash flows. We do this to test the efficacy of our investment decision assuming a perfect forecast of the liabilities the business would incur. Furthermore we believe that the underlying strategy of moving away from the use of standard fixed income assets, or rather adding tools that do not rely solely on fixed income, can be beneficial in exploiting different parts of the business cycle. There is evidence of negative correlation between equity assets and bonds (Abourashchi, Clacher, Hillier, Freeman, Kemp & Zhang 2013), which suggests that there is a moment in time where certain risky assets can be acquired inexpensively and another where fixed income assets are relatively cheaper. If we maintain a balanced portfolio we should experience little change on the asset side given this relationship, which is largely driven by the economic cycle and interest rates. But as in Abourashchi et al. 2013, this ignores what is happening to the present value of liabilities. As rates are falling, bond prices are increasing, offsetting any potential losses on equities sure, but the present value of liabilities is also climbing and if the sensitivities differ they can be too poorly immunized against these rate changes. This may offer an opportunity to implement this collar strategy or something similar in order to exploit such a relationship.

According to economic cycle theory, recessionary and depression periods should be followed by periods of recovery and expansion. A number of key economic variables are expected to fluctuate including national product, employment rate and inflation rate (Gabisch & Lorenz 1987), signalling an improvement in the economy. At the time of writing this paper, many are beginning to question this, but if one believes that the stock market is indicative of the economy, we should see risky assets go up. In our case this would lead to more frequently hitting the upper bound of our option strategy.

In summary, the above section describes our liability simulation model, which uses observed pension data from the Canadian Armed Forces. This simulation allows us to forecast an expected cash outflow at a known time T. We then use our Heston-Nandi GARCH option pricing model and observed historical price of various indexes to enter into a collar strategy, our test portfolio, at time $t = 0$ and whose payoff at time T is at minimum the same as our benchmark zero coupon bond portfolio. Our heteroscedastic simulation of growth assets is used to identify the probability of achieving the upper bound of our collar strategy. These cashflow matching strategies will then be compared to identify cost savings at initiation, upside potential and more. We hope that this will allow us to identify possible economic scenarios where the use of derivative products may be a useful tool to add to the repertoire of pension funds or other entities engaging in ALM.

Results

In this section we show the results of the comparison between the performance of various asset based growth portfolios with option hedges to that of the benchmark portfolio. The comments will be split for the 1 and 3 year horizons unless we draw the same conclusions, in which case that will be stated.

To reiterate some of the most important details of our comparison, we describe some key elements. In the first 2 years, the benchmark and test portfolio have the exact same composition, and therefore same cost. They are composed of zero coupon bonds which will mature to those face values at the desired time and are bought today at a discount at the price calculated using the formula in the previous chapter. In the final year the portfolios differ, therefore we understand that any differences will come from the performance of the test portfolio in its final year. We compare the difference between the benchmark with zero coupon bonds and the test portfolio, which holds in a growth portfolio composed of a risky asset and collar. The strike prices vary as described previously. The strike pairs are 1.05/1.06, 1.1/1.11 and 1.15/1.16 for the 1 year horizon and 1.15/1.16, 1.3/1.31 and 1.45/1.46 for the 3 year horizon after accounting for the time difference, these are referred to as pairs 1 through 3. We observe the performance of the strategy in 3 different interest rate environments, namely low, medium and high (respectively 0.5%, 1.5% and 2.5%).

The earlier description of the methods use should suffice to recreate this analysis, but below we include an example of the detailed results tables used for each interest rate, strike pair and investment horizon. There are many results and for this reason the larger table is split into tables 5, 6 and 7. Each table shows the name of the growth asset to purchase, the quantity (number of shares) of risky asset to purchase, the expected return of the risky asset until maturity based on our simulation and the calculated call and put premia using the Heston-Nandi GARCH pricing model. The setup savings compare the initiation cost of the test portfolio and that of the benchmark portfolio. The bottom lines show the combination of the setup savings and the cashflow at maturity relative to the benchmark and the liability. Consequently, if we had the same cost and cashflow as the benchmark, the value would be zero. All the detailed results are not included as these are simulated values and the summary tables and distributions should suffice to provide a full analysis, furthermore, the detailed results can be recreated by following the previously described methodology.

Growth Asset	Qty	Exp Gain (%)	Call	Put	Setup Savings	Simulated	Worst	Best
SPTXLVPRIndex	6,079.00	7.61	6.42	50.88	(4,300)	(4,300)	(4,300)	15,710
STCONSIndex	901.00	9.92	52.20	352.55	(4,600)	(4,600)	(4,600)	15,410
SPTSCSIndex	7,684.00	10.57	6.16	41.38	(4,611)	(4,611)	(4,611)	15,398
FIDCNILPIndex	11,661.00	1.94	4.19	27.40	(4,663)	(4,663)	(4,663)	15,346
FIDCNILTIndex	6,612.00	1.56	7.40	48.33	(4,664)	(4,664)	(4,664)	15,346
FIDCNILNIndex	7,070.00	6.31	6.92	45.20	(4,664)	(4,664)	(4,664)	15,346
M6CAIRWIndex	701.00	10.19	71.36	457.35	(4,700)	(4,700)	(4,700)	15,310
M4CAIRWIndex	526.00	9.96	95.20	609.65	(4,702)	(4,702)	(4,702)	15,308
M5CAIRWIndex	478.00	9.89	104.83	670.99	(4,702)	(4,702)	(4,702)	15,307
STFDSRIndex	978.00	8.46	54.30	331.17	(4,792)	(4,792)	(4,792)	15,218
BBCREITIndex	10,158.00	5.19	5.25	31.91	(4,799)	(4,799)	(4,799)	15,211
STFDSRXIndex	978.00	8.78	54.97	331.86	(4,811)	(4,811)	(4,811)	15,199
TTUTARIndex	5,156.00	11.14	10.99	63.52	(4,891)	(4,891)	(4,891)	15,119
SPTSUTIndex	9,032.00	6.60	6.29	36.28	(4,894)	(4,894)	(4,894)	15,115
FIDCNCLPIndex	4,167.00	7.60	13.66	78.67	(4,898)	(4,898)	(4,898)	15,112
FIDCNCLTIndex	2,460.00	15.16	23.14	133.24	(4,898)	(1,716)	(4,898)	15,111
M6CARWGTIndex	1,780.00	5.97	32.41	184.58	(4,918)	(4,918)	(4,918)	15,091
M4CARWGTIndex	1,683.00	5.82	34.29	195.22	(4,919)	(4,919)	(4,919)	15,091
M5CARWGTIndex	1,652.00	10.08	34.95	198.90	(4,920)	(4,920)	(4,920)	15,090
STUTILXIndex	1,045.00	6.21	55.52	314.87	(4,926)	(4,926)	(4,926)	15,084
STUTILIndex	1,045.00	7.05	55.52	314.88	(4,926)	(4,926)	(4,926)	15,084
SPRTREIndex	11,743.00	7.72	5.06	28.13	(4,963)	(4,963)	(4,963)	15,047
DJCASDTIndex	5,035.00	10.94	11.86	65.67	(4,970)	(4,970)	(4,970)	15,039
STFDREIndex	939.00	8.00	66.63	355.19	(5,039)	(5,039)	(5,039)	14,971
MXCA0CSIndex	7,577.00	8.95	8.40	44.17	(5,063)	(5,063)	(5,063)	14,946
SPTSCDIndex	18,440.00	4.87	3.51	18.21	(5,087)	(5,087)	(5,087)	14,923
FIDCNCDPIndex	4,287.00	5.47	15.15	78.38	(5,093)	(5,093)	(5,093)	14,917
FIDCNCDTIndex	2,300.00	11.46	28.26	146.11	(5,094)	(5,094)	(5,094)	14,915
TXSYIndex	6,865.00	8.13	9.52	49.01	(5,103)	(5,103)	(5,103)	14,907
RUDEVLNCIndex	1,756.00	2.12	37.40	191.79	(5,109)	(5,109)	(5,109)	14,901
RUDXNLNCIndex	1,626.00	0.45	40.56	207.25	(5,115)	(5,115)	(5,115)	14,894
STCONDIndex	1,678.00	2.15	39.47	201.04	(5,121)	(5,121)	(5,121)	14,888
FIDCNINNIndex	6,104.00	5.70	11.21	55.63	(5,169)	(5,169)	(5,169)	14,841
FIDCNINTIndex	5,752.00	8.28	11.90	59.04	(5,169)	(5,169)	(5,169)	14,841
FIDCNINPIndex	11,466.00	3.61	5.98	29.62	(5,170)	(5,170)	(5,170)	14,839
M9CXBRCIndex	487.00	10.00	142.88	699.81	(5,192)	(5,192)	(5,192)	14,817
SPRTCMIndex	10,564.00	6.59	6.65	32.32	(5,207)	(5,207)	(5,207)	14,803
RUCASCCIndex	1,052.00	5.19	66.97	324.69	(5,210)	(5,210)	(5,210)	14,799
FIDCNULPIndex	9,521.00	1.72	7.47	35.96	(5,224)	(5,224)	(5,224)	14,786
FIDCNULNIndex	6,814.00	7.09	10.44	50.24	(5,224)	(5,224)	(5,224)	14,786
FIDCNULTIndex	5,978.00	4.93	11.90	57.27	(5,224)	(5,224)	(5,224)	14,785
STELUTXIndex	845.00	8.38	84.90	406.00	(5,235)	(5,235)	(5,235)	14,775
STELUTIndex	845.00	5.33	84.90	406.00	(5,235)	(5,235)	(5,235)	14,775
FIDCCQLTIndex	2,862.00	8.05	25.19	119.96	(5,242)	(5,242)	(5,242)	14,767
FIDCNCQPIndex	4,414.00	8.86	16.34	77.78	(5,242)	(5,242)	(5,242)	14,767
M6CAEWIndex	852.00	8.52	84.78	403.18	(5,245)	(5,245)	(5,245)	14,765
M4CAEWIndex	679.00	7.91	106.44	505.97	(5,245)	(5,245)	(5,245)	14,764
M5CAEWIndex	629.00	8.12	114.83	545.75	(5,246)	(5,246)	(5,246)	14,764
M6CAIVWIndex	398.00	9.82	182.24	863.86	(5,250)	(5,250)	(5,250)	14,759
M4CAIVWIndex	288.00	11.21	252.26	1195.36	(5,251)	(5,251)	(5,251)	14,759
M5CAIVWIndex	258.00	14.80	281.14	1332.05	(5,251)	(5,251)	(5,251)	14,758
FRCANTRIndex	255.00	15.20	288.58	1354.00	(5,269)	(1,351)	(5,269)	14,741
MVLDCAIndex	1,142.00	5.42	65.40	303.00	(5,291)	(5,291)	(5,291)	14,719
MXCA000VIndex	1,142.00	10.97	65.40	303.00	(5,291)	(5,291)	(5,291)	14,719
STMEDAIndex	2,577.00	-1.14	29.26	134.52	(5,304)	(5,304)	(5,304)	14,705
STMEDAXIndex	2,577.00	1.59	29.27	134.53	(5,305)	(5,305)	(5,305)	14,705
SPTSTSIndex	16,451.00	5.88	4.60	21.09	(5,309)	(5,309)	(5,309)	14,701
STFDBTIndex	721.00	9.99	104.88	480.96	(5,309)	(5,309)	(5,309)	14,701
SPTSXCIndex	135.00	8.33	558.92	2561.58	(5,310)	(5,310)	(5,310)	14,700
M4CADYIndex	448.00	10.79	170.16	775.75	(5,319)	(5,319)	(5,319)	14,691
M6CADYIndex	661.00	10.02	115.40	526.11	(5,319)	(5,319)	(5,319)	14,691
M5CADYIndex	394.00	12.21	193.68	882.92	(5,319)	(5,319)	(5,319)	14,690
FRCANIndex	315.00	8.76	242.71	1103.82	(5,323)	(5,323)	(5,323)	14,686
M6CASCIndex	6,111.00	4.53	12.52	56.92	(5,324)	(5,324)	(5,324)	14,685
M6CAVWIndex	434.00	8.19	176.75	801.84	(5,328)	(5,328)	(5,328)	14,682
M4CAVWIndex	303.00	12.40	252.86	1146.80	(5,328)	(5,328)	(5,328)	14,682
M5CAVWIndex	269.00	12.86	284.92	1292.04	(5,328)	(5,328)	(5,328)	14,681

Table 5: Detailed Results; Low RF, Pair 1, 3Y, Table 1 of 3

Growth Asset	Qty	Exp Gain (%)	Call	Put	Setup Savings	Simulated	Worst	Best
MXCA0UTIndex	9,526.00	9.48	8.06	36.54	(5,328)	(5,328)	(5,328)	14,681
DWCATIndex	307.00	6.05	254.58	1137.13	(5,355)	(5,355)	(5,355)	14,655
DWCAIndex	481.00	6.14	162.96	727.28	(5,356)	(5,356)	(5,356)	14,653
MXCA00LVIndex	2,311.00	6.98	33.96	151.35	(5,359)	(5,359)	(5,359)	14,651
M4CAIMIndex	1,967.00	11.32	40.24	178.20	(5,370)	(5,370)	(5,370)	14,640
SPTSREIndex	8,369.00	5.08	9.46	41.89	(5,371)	(5,371)	(5,371)	14,639
SPTSXIndex	157.00	9.09	506.52	2235.50	(5,375)	(5,375)	(5,375)	14,634
SPTSX3Index	157.00	10.90	506.52	2235.50	(5,375)	(5,375)	(5,375)	14,634
SPTSX4Index	157.00	10.48	506.52	2235.50	(5,375)	(5,375)	(5,375)	14,634
SPTSX2Index	157.00	8.30	506.52	2235.50	(5,375)	(5,375)	(5,375)	14,634
x0000ARIndex	54.00	8.18	1466.16	6468.44	(5,376)	(5,376)	(5,376)	14,634
M9CAQUIndex	653.00	9.71	122.30	537.55	(5,383)	(5,383)	(5,383)	14,627
TXEQIndex	153.00	7.05	525.14	2304.02	(5,386)	(5,386)	(5,386)	14,624
STMUTIXIndex	876.00	7.21	91.67	401.50	(5,389)	(5,389)	(5,389)	14,621
STMUTIIndex	876.00	6.82	91.69	401.53	(5,389)	(5,389)	(5,389)	14,621
STDIVFIndex	904.00	6.62	89.33	389.45	(5,397)	(5,397)	(5,397)	14,613
STRLSTIndex	836.00	12.24	97.77	422.31	(5,413)	(5,413)	(5,413)	14,597
STREALIndex	836.00	9.25	97.77	422.31	(5,413)	(5,413)	(5,413)	14,597
MXCA0CDIndex	14,406.00	4.30	5.79	24.63	(5,440)	(5,440)	(5,440)	14,570
FTL3CAIndex	2,319.00	9.78	36.08	153.12	(5,443)	(5,443)	(5,443)	14,567
STRETLIndex	760.00	2.45	111.34	468.38	(5,458)	(5,458)	(5,458)	14,552
TTCDARIndex	14,016.00	8.45	6.04	25.41	(5,459)	(5,459)	(5,459)	14,551
STFINLIndex	1,105.00	6.07	76.67	322.33	(5,459)	(5,459)	(5,459)	14,551
FIDCNRRPIndex	10,327.00	0.78	8.27	34.56	(5,470)	(5,470)	(5,470)	14,540
FIDCNRRNIndex	7,201.00	7.05	11.86	49.56	(5,470)	(5,470)	(5,470)	14,540
FIDCNRRTIndex	6,254.00	5.96	13.66	57.06	(5,470)	(5,470)	(5,470)	14,540
MSDLCAIndex	1,249.00	6.95	68.49	285.89	(5,471)	(5,471)	(5,471)	14,539
MXCAIndex	1,248.00	5.75	68.50	285.93	(5,471)	(5,471)	(5,471)	14,539
NDDLCAIndex	448.00	7.42	190.78	796.35	(5,471)	(5,471)	(5,471)	14,538
GDDLCAIndex	341.00	7.05	250.98	1047.62	(5,471)	(5,471)	(5,471)	14,538
SPTSX60Index	2,736.00	9.55	31.47	130.69	(5,480)	(5,480)	(5,480)	14,530
STDVFSIndex	733.00	7.67	118.02	488.49	(5,486)	(5,486)	(5,486)	14,524
M5CALCIndex	1,985.00	11.10	43.90	180.63	(5,495)	(5,495)	(5,495)	14,514
MXCA0FNIndex	8,069.00	5.53	10.83	44.48	(5,499)	(5,499)	(5,499)	14,510
FIDCNHDTIndex	4,733.00	7.63	18.66	76.02	(5,512)	(5,512)	(5,512)	14,497
FIDCNHDNIndex	5,647.00	8.83	15.64	63.71	(5,513)	(5,513)	(5,513)	14,497
FIDCNHDPIndex	8,888.00	5.80	9.94	40.48	(5,513)	(5,513)	(5,513)	14,497
SPTSINIndex	14,382.00	9.03	6.15	25.02	(5,514)	(5,514)	(5,514)	14,496
STINDUIndex	1,181.00	8.13	74.87	304.74	(5,514)	(5,514)	(5,514)	14,496
STFDPRIndex	752.00	6.38	117.92	478.88	(5,518)	(5,518)	(5,518)	14,492
STTELSXIndex	1,696.00	11.14	53.57	213.68	(5,548)	(5,548)	(5,548)	14,462
STTELSIndex	1,696.00	9.73	53.58	213.68	(5,548)	(5,548)	(5,548)	14,462
STBANKXIndex	918.00	6.37	100.99	396.64	(5,574)	(5,574)	(5,574)	14,436
STCBNKIndex	920.00	5.06	101.00	396.30	(5,575)	(5,575)	(5,575)	14,434
STDBNKIndex	921.00	11.18	101.02	395.75	(5,578)	(5,578)	(5,578)	14,432
STDIVTIndex	1,874.00	7.38	50.33	195.23	(5,595)	(5,595)	(5,595)	14,415
STITELIndex	1,719.00	13.41	54.89	212.83	(5,595)	(5,595)	(5,595)	14,415
STHOTRIndex	1,316.00	5.36	72.10	278.52	(5,601)	(5,601)	(5,601)	14,408
STCOMSIndex	3,010.00	-5.55	31.59	121.83	(5,604)	(5,604)	(5,604)	14,405
STHOTRXIndex	1,267.00	12.61	76.56	290.94	(5,628)	(5,628)	(5,628)	14,381
STCOMSXIndex	4,852.00	-7.87	20.21	76.19	(5,642)	(5,642)	(5,642)	14,368
STMRETIndex	582.00	5.96	170.05	637.06	(5,652)	(5,652)	(5,652)	14,358
STTRANIndex	379.00	12.63	262.27	978.93	(5,658)	(5,658)	(5,658)	14,352
STBRDCIndex	936.00	12.99	107.01	397.12	(5,667)	(5,667)	(5,667)	14,343
MXCA0TCIndex	4,536.00	15.92	22.43	82.31	(5,685)	12,763	(5,685)	14,325
SPTSHCIndex	27,974.00	2.34	3.65	13.36	(5,689)	(5,689)	(5,689)	14,320
MXCA0INIndex	9,110.00	11.78	11.38	41.20	(5,707)	(5,707)	(5,707)	14,303
STPACKIndex	600.00	12.31	173.75	626.86	(5,713)	(5,713)	(5,713)	14,297
STMDREXIndex	854.00	6.97	122.06	440.07	(5,714)	(5,714)	(5,714)	14,296
STRAILIndex	297.00	19.89	365.32	1279.62	(5,738)	14,272	(5,738)	14,272
STROADIndex	302.00	15.56	352.85	1251.42	(5,761)	5,496	(5,761)	14,249
STINSUIndex	2,153.00	4.57	51.52	177.72	(5,764)	(5,764)	(5,764)	14,246
STINSUXIndex	2,153.00	-2.60	51.52	177.73	(5,764)	(5,764)	(5,764)	14,246
STHCPSIndex	2,049.00	-1.97	54.40	187.01	(5,770)	(5,770)	(5,770)	14,240
STENRSIndex	729.00	12.69	152.94	525.61	(5,770)	(5,770)	(5,770)	14,239
STENRSXIndex	729.00	15.60	152.94	525.61	(5,770)	6,286	(5,770)	14,239
STCPGSIndex	3,634.00	0.22	30.96	105.74	(5,799)	(5,799)	(5,799)	14,211

Table 6: Detailed Results; Low RF, Pair 1, 3Y, Table 2 of 3

Growth Asset	Qty	Exp Gain (%)	Call	Put	Setup Savings	Simulated	Worst	Best
STOILGIndex	700.00	23.17	160.92	549.32	(5,800)	14,210	(5,800)	14,210
STHLTHIndex	1,799.00	6.69	63.26	214.29	(5,812)	(5,812)	(5,812)	14,197
TRIBCNIndex	9,101.00	5.01	12.59	42.45	(5,819)	(5,819)	(5,819)	14,191
STOILEIndex	958.00	12.13	119.83	403.60	(5,821)	(5,821)	(5,821)	14,189
MXCA0ENIndex	4,097.00	12.11	28.10	94.44	(5,825)	(5,825)	(5,825)	14,185
STGENMIndex	387.00	5.99	304.92	1007.26	(5,850)	(5,850)	(5,850)	14,160
STMACHIndex	1,234.00	4.62	95.82	316.09	(5,852)	(5,852)	(5,852)	14,158
STHCESIndex	493.00	5.36	242.89	794.76	(5,864)	(5,864)	(5,864)	14,146
MGLDCAIndex	2,209.00	6.05	54.26	177.29	(5,866)	(5,866)	(5,866)	14,144
SPTSMTIndex	6,896.00	10.20	17.46	56.89	(5,871)	(5,871)	(5,871)	14,139
SPTSENIndex	7,903.00	15.12	15.25	49.64	(5,871)	(3,432)	(5,871)	14,138
STLIFEIndex	2,354.00	4.33	51.41	166.91	(5,876)	(5,876)	(5,876)	14,134
STENREIndex	1,335.00	14.49	90.77	294.34	(5,877)	(5,877)	(5,877)	14,132
STMATRXIndex	751.00	17.08	162.33	524.34	(5,883)	14,126	(5,883)	14,126
STMATRIndex	751.00	11.43	162.38	524.39	(5,884)	(5,884)	(5,884)	14,126
STMOVIIndex	1,485.00	9.33	82.30	265.41	(5,886)	(5,886)	(5,886)	14,124
MXCA0MTIndex	7,031.00	15.88	17.77	56.44	(5,908)	11,781	(5,908)	14,102
STCSTFIndex	386.00	20.94	326.33	1031.20	(5,915)	14,094	(5,915)	14,094
STIOILIndex	643.00	23.94	195.74	618.46	(5,916)	14,094	(5,916)	14,094
STOILPIndex	802.00	16.83	157.16	496.35	(5,916)	14,093	(5,916)	14,093
STCSTEXIndex	395.00	13.25	324.81	1013.13	(5,934)	(5,934)	(5,934)	14,076
STCSTEIndex	395.00	15.53	324.81	1013.13	(5,934)	4,748	(5,934)	14,076
STAUCOIndex	1,223.00	10.56	105.22	327.48	(5,937)	(5,937)	(5,937)	14,072
STAUTPIndex	1,223.00	3.76	105.26	327.52	(5,938)	(5,938)	(5,938)	14,072
STSPREIndex	2,666.00	-6.14	48.84	150.83	(5,948)	(5,948)	(5,948)	14,061
STBEVGIndex	17,845.00	-4.15	7.33	22.57	(5,952)	(5,952)	(5,952)	14,057
STAUTCIndex	1,223.00	7.51	107.21	329.49	(5,955)	(5,955)	(5,955)	14,054
STCODUIndex	720.00	17.55	182.22	559.66	(5,956)	14,054	(5,956)	14,054
STPHRMIndex	2,291.00	2.03	58.62	177.32	(5,977)	(5,977)	(5,977)	14,032
STMETLIndex	713.00	9.09	190.53	571.99	(5,988)	(5,988)	(5,988)	14,022
STSTELIndex	183.00	23.04	743.79	2229.63	(5,990)	14,020	(5,990)	14,020
STCHEMIndex	416.00	24.39	331.31	985.47	(6,000)	14,009	(6,000)	14,009
MXCA0HCIndex	25,300.00	6.35	5.45	16.20	(6,001)	(6,001)	(6,001)	14,008
STFRSTIndex	1,466.00	16.31	94.31	279.86	(6,004)	14,006	(6,004)	14,006
STPAFOIndex	5,129.00	7.03	26.97	80.00	(6,004)	(6,004)	(6,004)	14,005
STAIRLXIndex	3,482.00	15.33	40.18	118.30	(6,014)	572	(6,014)	13,995
SPTXHBPRIndex	47,104.00	0.37	2.97	8.74	(6,014)	(6,014)	(6,014)	13,995
STCMDTIndex	1,239.00	12.69	114.34	333.82	(6,026)	(6,026)	(6,026)	13,984
STCOMPIndex	3,880.00	-0.85	36.65	106.75	(6,029)	(6,029)	(6,029)	13,981
STSFTWIndex	1,202.00	12.57	118.77	345.01	(6,032)	(6,032)	(6,032)	13,977
STCONPIndex	696.00	3.29	206.68	597.55	(6,039)	(6,039)	(6,039)	13,971
STWIREIndex	2,312.00	3.78	62.53	180.18	(6,043)	(6,043)	(6,043)	13,967
STWIREXIndex	2,312.00	5.19	62.53	180.19	(6,043)	(6,043)	(6,043)	13,967
STMETGIndex	538.00	1.97	270.07	775.38	(6,048)	(6,048)	(6,048)	13,962
STAIRLIndex	3,482.00	8.80	41.95	120.08	(6,051)	(6,051)	(6,051)	13,958
STSOFTIndex	865.00	14.01	170.01	484.61	(6,057)	(6,057)	(6,057)	13,953
STMLINIndex	1,033.00	8.22	144.00	407.29	(6,067)	(6,067)	(6,067)	13,943
STFERTIndex	345.00	21.39	433.42	1222.68	(6,070)	13,940	(6,070)	13,940
STAPLSIndex	852.00	11.91	177.04	496.28	(6,078)	(6,078)	(6,078)	13,932
STPROPIndex	750.00	12.27	204.24	566.95	(6,090)	(6,090)	(6,090)	13,919
STPHRXIndex	1,199.00	18.65	128.00	354.94	(6,092)	13,918	(6,092)	13,918
STDIVMIndex	299.00	13.53	512.85	1422.11	(6,092)	(6,092)	(6,092)	13,918
STPHARXIndex	1,199.00	25.04	130.85	357.80	(6,109)	13,901	(6,109)	13,901
STITSVIndex	489.00	8.72	333.80	890.35	(6,138)	(6,138)	(6,138)	13,872
STITCSIndex	544.00	13.96	301.09	801.33	(6,140)	(6,140)	(6,140)	13,869
STSOFDIndex	17,845.00	-6.39	9.26	24.51	(6,147)	(6,147)	(6,147)	13,863
STGOLDIndex	936.00	18.87	177.32	468.07	(6,150)	13,860	(6,150)	13,860
STTRUCIndex	1,487.00	-12.23	112.17	295.16	(6,153)	(6,153)	(6,153)	13,857
STAEROIndex	8,760.00	11.14	19.10	50.17	(6,155)	(6,155)	(6,155)	13,854
STAEROXIndex	8,760.00	-0.43	19.10	50.17	(6,155)	(6,155)	(6,155)	13,854
MXCA000GIndex	2,209.00	10.48	77.71	200.88	(6,172)	(6,172)	(6,172)	13,838
STINFTIndex	16,145.00	4.29	10.80	27.66	(6,183)	(6,183)	(6,183)	13,827
STHCFAIndex	17,985.00	-1.08	10.08	25.21	(6,206)	(6,206)	(6,206)	13,804
MXCA0ITIndex	125,217.00	1.07	1.53	3.70	(6,235)	(6,235)	(6,235)	13,774
STELEIXIndex	8,381.00	2.74	22.95	55.42	(6,239)	(6,239)	(6,239)	13,771
STTECHIndex	37,868.00	-10.34	5.28	12.47	(6,257)	(6,257)	(6,257)	13,752
STENVRIndex	3,484.00	12.07	81.55	159.70	(6,324)	(6,324)	(6,324)	13,686
AVERAGE	4,467.66	8.26	116.30	448.16	(5,538)	(3,715)	(5,538)	14,472

Table 7: Detailed Results; Low RF, Pair 1, 3Y, Table 3 of 3

Tables 5, 6 and 7 show us the most important details for all assets under one of our test scenarios. In other words, these tables show the comparison of our test portfolio versus our benchmark for just one combination of interest rate and strike pair. We have these same tables created for each of the 18 scenarios. Our most interesting observations come from comparing across the 3 interest rates, 3 strike pairs and 2 time horizons that we can observe the sensitivity of the strategies' performance to these factors. What is driving the data that makes up these performances are the different assets, more specifically, the return and volatility history of these assets. We see that in this particular scenario the average quantity of shares purchased is 4,468 while the max is 125,217 shares and the minimum is 54 shares. We do not observe any direct relationship between the quantity of shares purchased and the performance of the strategy as the strategy was designed to achieve a certain minimum payoff and the quantity is related to this minimum payoff and the starting share price. The expected gain for column is based on our initial risky asset simulations using Markov chain Monte Carlo methods and is used to determine the payoff of our options. By determining the final price we can see which of the 3 payoff scenarios we are in at maturity and how our shares will be sold for these covered options.. Either the price finishes under the strike of the put and we will exercise it (right to sell at the strike), above the strike of our call and we will be assigned (forced to sell at the strike) or somewhere in between causing both options to expire and we sell our shares in the market. The call and put premiums are shown here as it was insightful to see the ratio and how it varies. Briefly, it shows how much of our protective put is funded directly by the sale of calls, more on this in the analysis and Table 10. The setup savings show us the cost of initiating the test portfolio versus the cost of initiating the benchmark portfolio. A negative number indicates that it is more expensive to initiate the test portfolio than the benchmark. The remaining 3 columns show us the same information, the final payoff of the strategy, in different payoff scenarios. Worst case shows us the final payoff given a risky asset market price below the put option strike price, meaning we exercise our put. This is the lowest amount we can get as a final payoff and is therefore named Worst case. Best case shows us the final payoff given a risky asset market price above our call strike price, meaning we are assigned on our calls, this is the highest possible payoff for the strategy. The simulated column shows us where our risky asset is expected to finish, which may result in the worst case, the best case or something in between. According to our simulation this is what we would expect to see as an actual payoff. We can observe the in the majority of assets, we are hitting the worst case. In this particular scenario we hit the lower bound for 88.56% of assets, the upper bound for 6.97% of assets and we end up between both for 4.48% of assets.

Our detailed results are presented in a more concise format within Tables 8 and 9 of the analysis. They present the average value of all the assets at each column and combine the 9 observed interest rate and strike pair combinations and will be discussed further starting on page 45. Additionally Figures 6 and 7 are used to complete these summary statistics by showing the distribution of the individual assets. The main variable we are ranking these assets on is their performance in the worst case, the case of put execution. Our analysis in the next chapter is based mostly on these averages rather than the performance of individual assets within each of our observed scenarios, but their distribution is considered to seek additional insight.

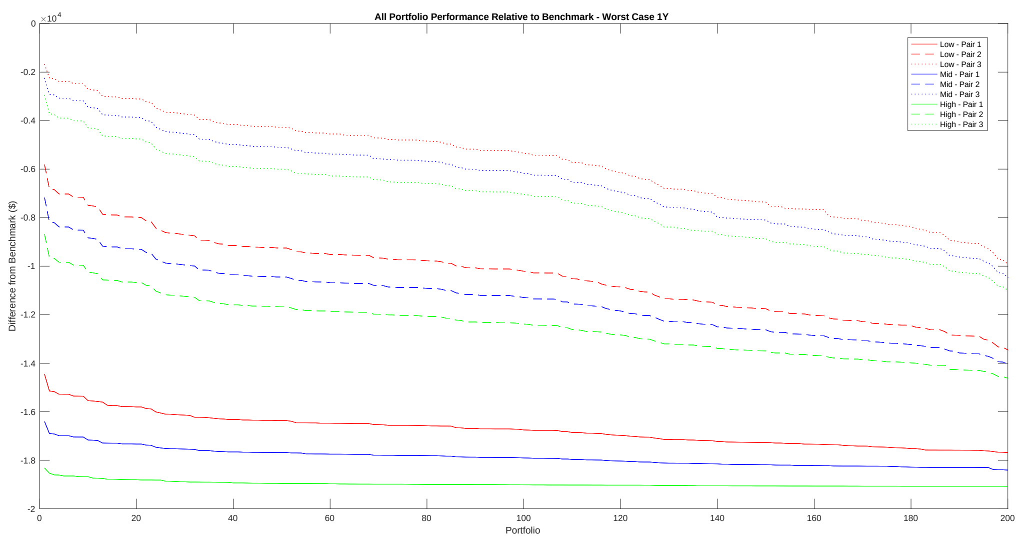

Figure 6: All Portfolio Performance Relative to Benchmark – Worst Case 1Y – Portfolios using our bullish collar strategy for different underlying assets ranked by worst case performance and organized by scenario (interest rate + option strike pair)

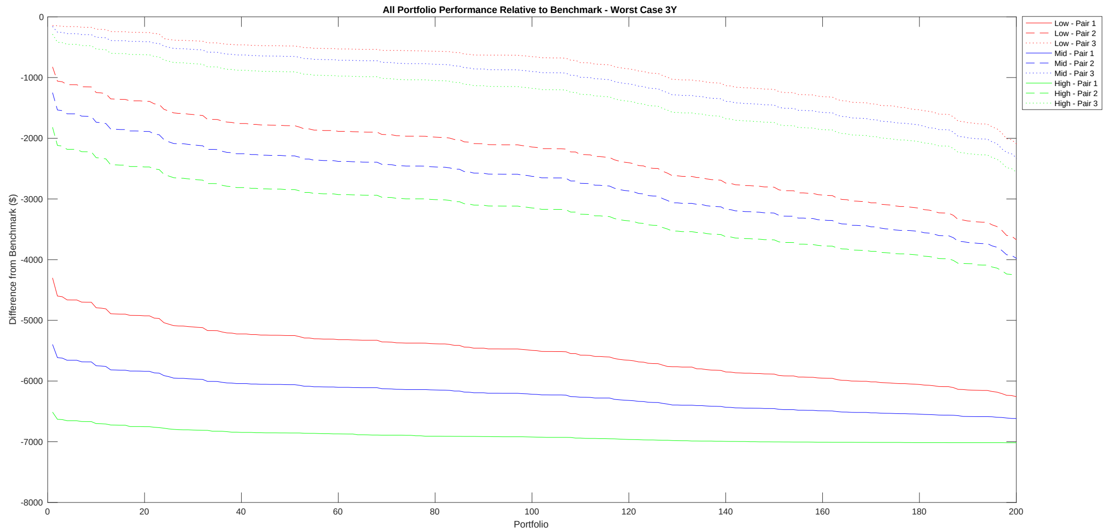

Figure 7: All Portfolio Performance Relative to Benchmark – Worst Case 3Y – Portfolios using our bullish collar strategy for different underlying assets ranked by worst case performance and organized by scenario (interest rate + option strike pair)

From the above tables and graphs we can highlight some of the most valuable details before continuing onto the analysis in the next section. Figures 6 and 7 are a simple visualization of each asset presented on a curve, ranked by Worst Case payoff and presenting the Worst Case payoff. The legend shows the 9 different scenarios, combinations of interest rate environments and strike pairs. The strike pairs, as described in the methodology, are percentages of the starting risky asset prices and are used to select our strike prices for our put purchases and call sales respectively. Therefore, in the Low – Pair 1 for the 1 year horizon, we are referring to the low interest rate of 0.5%, and the lowest strike pair, which is 105% of the starting underlying asset price for the put and 106% of the starting underlying asset price for the call. We can see at a glance from the graphs that the pattern is very similar between all of the 1Y curves and the 3Y curves. The distribution of the portfolio within each strike pair and interest rate is essentially the same, however at each of these, the results of the 3Y graph show that the portfolio offers a better worst case return relative to the benchmark. We also see that in both time horizons, strike pair 1 consistently has the worst performance and strike pair 3 has the best performance. Furthermore, portfolios evaluated in the low interest rate environment have outperformed portfolios in the high interest rate environment. We also see that there is an overall steepening effect as we move from Pair 1 to Pair 3, meaning there is an increasing difference between the top performing portfolio and the bottom performing portfolio.

More on the distribution of the portfolio performances, we see that some of the curves in Figures 6 and 7 have tails towards either end. This shows us that in certain tests we have more pronounced gains or losses in the top or bottom performing portfolios. The below charts in Figures 8 and 9 show these same portfolio performances but organized as histograms and then fitting a smoothed probability distribution. This is done in an effort to better visualize the tail events in each scenario and demonstrates when we have excessive winners, excessive losers or both, relative to the mean. Again, as for most previous tables and graphs, these are organized by time horizon, interest rate and strike pair. As we can observe the curves do not vary much in shape as we move from the low interest rate to the high interest rate. However we do see a quite pronounced change between the strike pairs, and this change is consistent across all interest rates observed. We see at the 1 year horizon that the pair 1 distribution is rather noticeably right skewed, the majority of the observations are lower performances, but we do have a small amount of higher performers in the batch. Pairs 2 and 3 have a mostly symmetrical distribution. They are not quite normal due to their 2 peaks, but do not demonstrate any significant skew in either direction. At the 3 year horizon we see the Pairs 1 and 2 have basically the same shapes as the shorter test, but this time Pair 3 has changed. In this observation period, Pair 3 shows a much more pronounced left skew. Probabilistically, this shows us that given the same expected performance, if we are just considering distributions, most investors would likely prefer the left skewing Pair 3. The left skewed shape has more frequent “good” payoffs relative to less frequent “bad” tail events, where the payoffs are less appealing but can be more pronounced. In our scenario this rather translates to the cost of the strategy relative to the benchmark, as what these figures are showing is the distribution of Worst Case portfolio performances relative to the benchmark. We know that our payoff at maturity will cover our required cashflow, so these distributions more so indicate the cost of initiating our strategy relative to the benchmark.

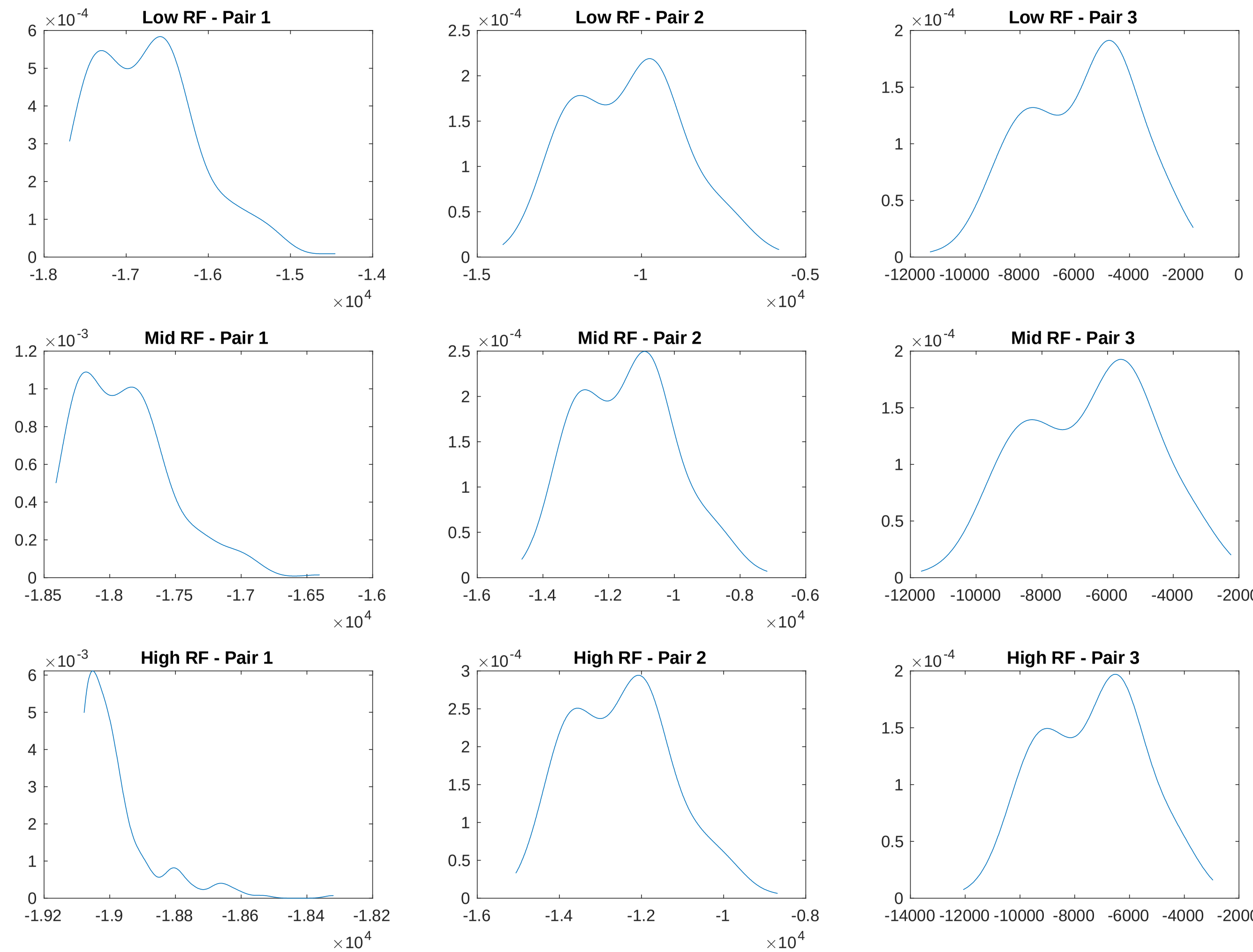

Figure 8: Distribution of Test Portfolio Performance Relative to Benchmark – 1Y – Portfolio Performances relative to benchmark organized in a histogram and smoothed using a kernel method in order to create a simple curve and compare across different scenarios. This allows us to observe if we can expect more positive or negative outliers, comment on skew and more.

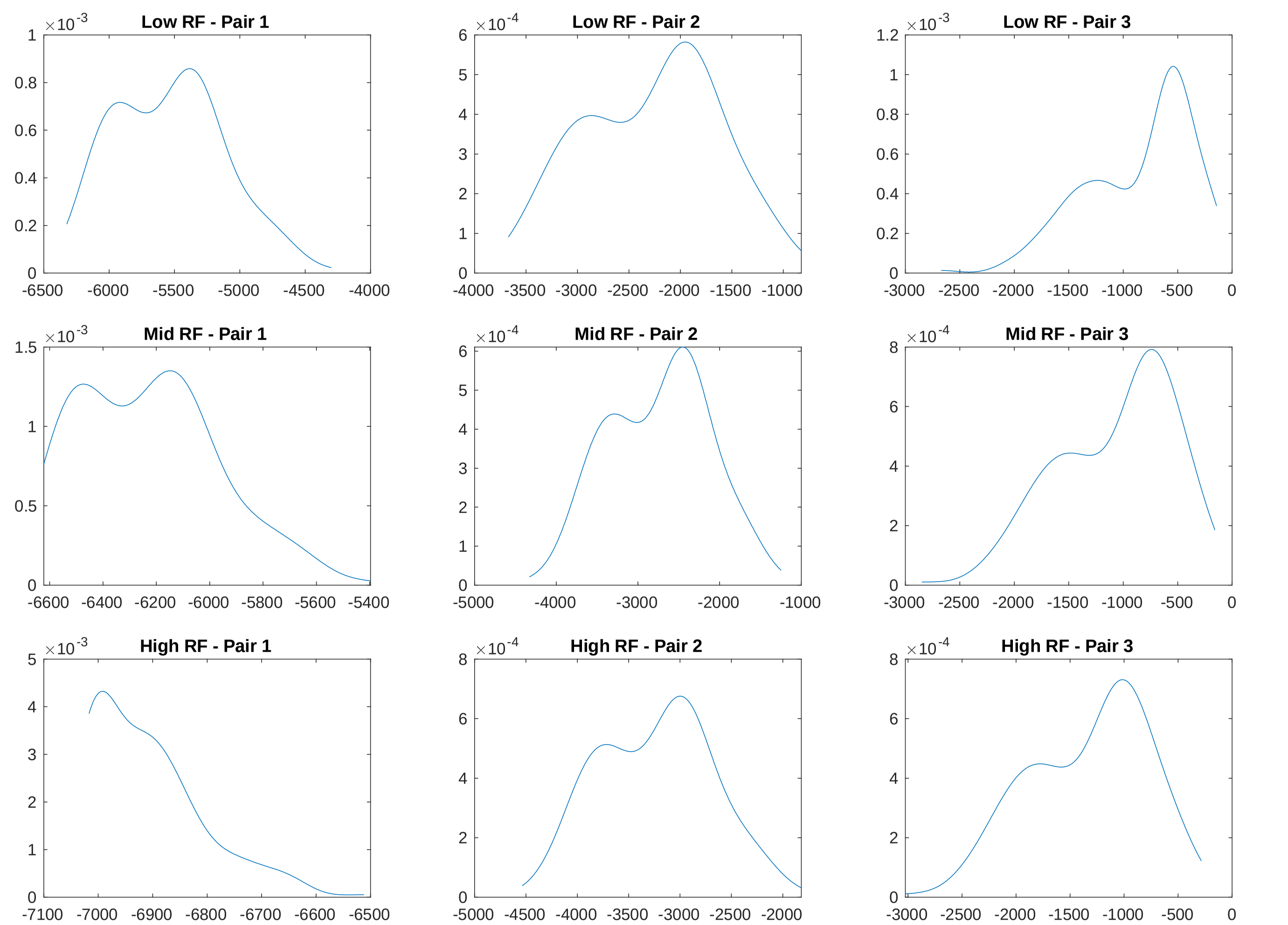

Figure 9: Distribution of Test Portfolio Performance Relative to Benchmark – 3Y – Portfolio Performances relative to benchmark organized in a histogram and smoothed using a kernel method in order to create a simple curve and compare across different scenarios. This allows us to observe if we can expect more positive or negative outliers, comment on skew and more.

Analysis and Discussion

Before getting into the quantitative analysis presented, we observe some of the details mentioned earlier regarding ideal models for, and issues faced in, a pension fund or classic asset liability management problem. We are looking at a simplified model, which is simple cash flow matching, for this analysis. The larger goal is to see if options can be used to supplement or improve a more complex ALM strategy.

We know that options will function in a multiperiod model. In our analysis we use plain vanilla European options, but these can be sold at any time prior to maturity and if we want to add the possibility of executing them we would have the availability of American options. Additionally, for the types of options we are using, there should be clear fees, that is until we start exploring the ultralong durations. Standard options can vary in maturity and typically do not extend past 1 year. However there is increasing demand for products called Long Term Equity Anticipation Securities (LEAPS) which are available as calls and puts and have maturities over 2 years. Although we believe there is still a ways to go when it comes to the pricing of longer term options, there does seem to be an appetite for these products in the retail market. This brings us to the fact that there may be liquidity issues when it comes to these products as they require a counterparty willing to take an opposite view to yours. As these become increasingly specific or long term, this can contribute to the lack of viable counterparties willing to partake at reasonable prices. Another issue that may arise is that certain entities have restrictions on the types of products they can use. We believe that given the proper strategies and supporting evidence of the advantages of derivative products, this will potentially change, but as with all change in large organizations, this could be a lengthy process.

To reiterate the purpose of this thesis, we are exploring the use of option strategies in asset liability management. Specifically, we are comparing the performance of a bullish collar strategy to that of a zero coupon bond strategy in a cash flow matching scenario. The primary goal is therefore to obtain the desired cashflow at the desired time. In our case, we are holding the options to maturity and depending on the price of the underlying asset at this time we will have one of three cashflows. In the worst case, when our put is exercised, we do obtain the desired cashflow, therefore our primary goal is achieved. However, despite the cost difference not being extremely large, we see that there is no situation that we observed where our option portfolio costs less than our benchmark portfolio. Of all the observed scenarios, our option portfolio cost at t=0 is closest to our benchmark portfolio cost at the 3 year horizon, low interest rate, high strike pair, where it only costs 850$ (or 0.0184% of our expected liability cashflow) more than our benchmark portfolio. Our worst option portfolio cost is the 1 year, high interest rate, low strike pair, where it costs 18,890$ or 0.4089% of our expected liability cashflow. These amounts can be seen in Tables 8 and 9 which are tables summarizing the detailed results shown in Tables 5, 6 and 7 as well as the remaining detailed results of all the other scenarios for which the tables were not included in this thesis.

Interest rate	Strike pair	Call	Put	Setup Savings	Simulated	Worst	Best
LOW	1	80.52	199.35	(16,754.92)	14,376.47	(16,754.92)	27,240.83
	2	44.06	281.27	(10,408.81)	1,476.02	(10,408.81)	31,587.15
	3	22.11	378.42	(5,787.50)	(2,358.65)	(5,787.50)	34,382.53
MID	1	90.36	184.65	(17,887.23)	13,244.17	(17,887.23)	26,108.55
	2	50.73	262.19	(11,440.28)	444.55	(11,440.28)	30,555.66
	3	26.10	355.32	(6,552.18)	(3,123.33)	(6,552.18)	33,617.87
HIGH	1	100.87	171.09	(18,980.29)	12,151.11	(18,980.29)	25,015.43
	2	58.09	244.30	(12,492.91)	(608.09)	(12,492.91)	29,503.02
	3	30.65	333.33	(7,368.07)	(3,939.23)	(7,368.07)	32,801.99

Table 8 : Summary details of 1Y performance

Interest rate	Strike pair	Call	Put	Setup Savings	Simulated	Worst	Best
LOW	1	116.30	448.16	(5,537.97)	(3,715.11)	(5,537.97)	14,471.70
	2	46.78	736.40	(2,288.02)	(2,288.02)	(2,288.02)	15,412.75
	3	17.36	1,066.55	(850.97)	(850.97)	(850.97)	15,018.75
MID	1	140.75	392.36	(6,231.46)	(4,408.60)	(6,231.46)	13,778.20
	2	59.78	658.38	(2,742.55)	(2,742.55)	(2,742.55)	14,958.32
	3	23.19	970.46	(1,065.54)	(1,065.54)	(1,065.54)	14,804.23
HIGH	1	168.16	342.66	(6,911.57)	(5,088.70)	(6,911.57)	13,098.08
	2	75.38	586.54	(3,235.87)	(3,235.87)	(3,235.87)	14,464.96
	3	30.63	880.01	(1,326.70)	(1,326.70)	(1,326.70)	14,543.03

Table 9 : Summary details of 3Y performance

As in the previous results tables but this time showing average values, Tables 8 and 9 show the scenario (rate and strike pair), the option premiums for each component of our strategy, specifically the long put and the short call, the savings at initiation as well as the worst case, best case and simulated payoffs at maturity. For each of the 201 assets observed, we create a test portfolio buying the asset, buying a put and selling a call to secure a payoff range at maturity.

We compare this strategy against a benchmark composed exclusively of zero coupon bonds which also secures a specific payoff. Tables 8 and 9 show us the average results across all of the 201 assets for each scenario and allow us to better see the effect of the interest rate, strike price and investment horizon on our overall performance.

Considering that there is an increased cost to this strategy relative to the benchmark, what is its main advantage? Effectively, the way this strategy is designed, we secure our minimum cashflow as described in the previous paragraph, when our put is exercised, and we allow for more upside potential up to our call option strike price. The setup savings show the difference in cost between opening the zero portfolio and the test portfolio. In every single case the test portfolio is more expensive at t = 0, the moment of initial positioning. At the 1 year horizon the cost difference varies between $5,787 and $16,754 for the portfolios in the low rate environment, $6,552 and $17,887 for the mid rate and $7,368 and $18,980 for the high rate. For the 3 year horizon we observe that the cost difference varies between $850 and $5,537 for the portfolios in the low rate environment, $1,065 and $6,231 for the mid rate and $1,326 and $6,911 for the high rate. This shows us that the 1 year horizon has nearly double the variance from lowest to highest setup savings per interest rate environment. In other words, there is a wider difference in the setup cost between the strike pair nearest the money and the strike pair furthest from the money for the 1 year horizon than the 3 year horizon.

This analysis considers the interest rate at the time of purchase and European options that we execute, meaning we do not benefit from the price change in the options. However, if we do consider option sensitivities to interest rates, we can observe that puts have a negative duration, and calls have a positive duration. Meaning, as interest rates go up, the price of puts decrease and the price of calls will increase, resulting in overall improvement in our strategy. We spend less to acquire our protective puts and receive more for our written calls. We can see this is confirmed by looking at the option prices in the results tables, the average price of our puts goes down and the average price of our calls goes up as rates increase. We do observe however that this does not directly translate to our strategy performing better. In fact we see the opposite, as rates increase the overall performance of our strategy is reduced. This is because we are always comparing our strategy to a benchmark which also varies with interest rates, and benefits more than our strategy from an equal increase in interest rates. In other words, the reduction in cost to acquire our benchmark is greater than the improvement in savings of our option strategy as rates increase. This is consistent with the literature, as we expect zero coupon bonds to have a relatively high duration due to the lump sum nature of the product.

Pair 3 consistently gives the best results. This may seem counterintuitive because as we move away positively from our starting asset price the put premiums we are paying will get more expensive and the call premiums we are collecting become less valuable resulting in a net higher cost per unit. This is confirmed by the premiums shown in the results table and also summarized in the below table which shows the ratio of call premium to put premium for each scenario.

Interest rate	Strike pair	1Y C/P ratio	3Y C/P ratio
LOW	1	0.40	0.26
	2	0.16	0.06
	3	0.06	0.02
MID	1	0.49	0.36
	2	0.19	0.09
	3	0.07	0.02
HIGH	1	0.59	0.49
	2	0.24	0.13
	3	0.09	0.03

Table 10 : Call to Put Premium Ratio by Scenario and Horizon

If our ratio was 1, the value of our income on the call options would completely cover the cost of acquiring the put options. This means that the cost of our portfolio would only be the composed of the cost of acquiring the risky asset at its starting price. As we see the ratio decreases as we enter the highest strike pairs which is consistent with what we would expect and causing a net more expensive strategy per unit. However because the strike price is higher, we need less units to cover our expected liability cashflow. This balance between strike price and quantity is what contributes to the increase performance of the third strike pair relative to the first. This is also where there is the most possibility more improvement in the actual strategy for real applications and further analysis. Given our current results we observe that the ratio decreases as we move deeper into the money, meaning we are reducing our cost offsetting to a lesser degree, and it increases with interest rates, leading to the contrary and improving our cost offsetting. Simultaneously, the performance of the strategy increases as we move deeper into the money with our strike pairs and decreases with interest rates.

We also note that the ranges tighten in the extended maturities as indicated by the standard deviation of the bottom lines. In essence, the longer we extend this strategy the more the payoff becomes similar to a zero coupon bond, meaning we would know what the payoff will be with no real additional upside. We also observe the range tightens between the best and worst case scenarios as we increase the strike price of the options. This is demonstrated in Table 11 just below by taking the difference between the best case performance and the worst case performance in each scenario.

Strike pair	1Y B-W Range	3Y B-W Range
1	43,995.74	20,009.67
2	41,995.96	17,700.78
3	40,170.03	15,869.71

Table 11 : Best-Worst Case Range

To return to the distributions of our portfolio performances relative to the benchmark, we saw that the distributions of our performance will vary more by strike pair than by rate. Figures 8 and 9 allowed us to observe the skews of the portfolio performance distributions in different scenarios and are also consistent with our findings. Pair 3 in the low interest rate environment at the 3 year horizon is the best performing as, in a majority of cases we obtain higher performance worst case bottom lines. That being said, this scenario, does not offer the highest best case scenario payoff and an investor who is using that as his metric for selecting a strategy will not necessarily have the same opinion as an investor with a different utility.

Of course we must address some of the pragmatic flaws in this strategy. The quantities involved in acquiring positions in the growth assets, puts and calls can be large. This would likely require some sort of block trades at the risk of averaging up the costs. At the very least this requires an order for each of the 3 products compared to the one necessary for the zero coupon bond transaction. As mentioned earlier, the market may not be sizeable enough to accommodate these kinds of trades efficiently as we need counterparties willing to take the other side of these payoffs. This is probably the biggest current issue, as there may not be any counter party willing, or rather able, to offer these same *theoretical* prices.

Additional considerations include modifying the option strategy used. In our case we are buying deep ITM puts, this is not necessarily a common action as the prices are higher and the liquidity at these levels may be lower, therefore we may be getting less of the leverage effect of options than if we were considering cheaper (ATM or OTM) options.

For tax considerations, we look at the differences brought on by our new strategy. The benchmark portfolio is composed of only zero coupon bonds; imputed interest on zero coupon bonds known as phantom interest is subject to income tax, which accrues each year. For the collar portfolio, we have 3 possibilities.

- **Best case:** We experience a capital gain on the risky asset at maturity, a capital loss on the expired puts at maturity and a capital gain on the written exercised calls at the time of maturity added to the proceeds of the risky asset;
- **Worst case:** We experience a capital gain on the risky asset at maturity, that gain is reduced by the cost of the puts at maturity and a capital gain on the revenue of the written calls;
- **Middle case:** All of the options expire and trigger their capital gains and losses, which are still used to adjust the capital gains of the risky asset.

In essence, this shows us that there doesn't seem to be any tax advantage to either strategy. Both will trigger taxable events at maturity and the only difference is that the option strategy has a range of possible payoffs between the worst and best case.

All of the above observations and considerations show us that theoretically there is some possible use for derivatives in an ALM, specifically cash flow matching environment. What is more difficult to observe however are the management constraints as described in Dempster, Germano, Medova & Villaverde 2003. Although we tried to touch on some of these, this includes any frictional costs such as commissions, bid-ask spreads, taxes, liquidity and more.

Conclusion

In summary, the collar strategy for cash flow matching can outperform the zero coupon bond strategy in terms of pure cash output in certain market conditions and time horizons. The collar strategy for cash management works best within lower interest rate environments. This of course is demonstrated with theoretical prices for zero coupon bonds and derivatives, but we would expect real market pricing to have the same conclusion.

Our analysis shows that this strategy works best within third strike pair (1.15/1.16) (assuming a pension fund who's main responsibility is to obtain a specific cash flow at a specific date), which is the furthest from ATM. This is counter intuitive as we would expect the strategy to become more costly as we move positively further away from the money. The cost of puts becomes more expensive and the premium received for the calls is reduced. As mentioned our pricing method would benefit from more optimization. Due to oscillation issues with the current option pricing model it is possible that we are underpricing our puts or over pricing our calls resulting in a more advantageous cost for this strategy. Given more available long term pricing data we could increase the robustness and accuracy of our option pricing. It would also be interesting to go deeper into the optimization of strike prices used; How exactly could we improve our position management through widening or narrowing the difference between the strike prices for example. Growth assets, in this case equities and equity indexes, are thought to have a long term upward trend and so it would make most sense to implement this strategy at longer horizons. In fact, we find that at longer horizons the effect of the strategy changes slightly. At the 3 year horizon, the spread between the best and worst case scenarios tightens. This leads us to believe that at even longer horizons, this type of strategy gets closer and closer to replicating the payoff of a zero coupon bond. This demonstrates that there is potentially a use for this strategy as we are able to replicate (or approach replication of) the cash flow of a zero coupon bond with reduced explicit sensitivity to interest rates.

The results show that this strategy, with these specific parameters, is costlier than the benchmark. It also requires the use of more transactions than the benchmark and this, in a market that has not fully accepted these products at longer maturities, which may cause liquidity/pricing issues. The benefit of this collar strategy for cash flow matching and ALM is that it can be tailored to the needs and forecasts of different fund managers and has more possible variables than a simpler zero coupon bond. This means, if expectations are more bullish equities or overestimated liabilities, the manager can decide to reduce the strike price on the put to increase savings even more but increasing their risk of failing to meet their cash flow requirement. There is also much room for optimization of strike prices and exposure to different underlying assets or portfolios. The additional revenue generated in the best case scenario is a humble amount but may be attractive to certain investors. Additional considerations include the possibility of a regime switching approach. To exploit the negative correlation of stocks and bonds, this strategy can benefit from the anticipation of rising rates and stock prices. We will probabilistically more frequently hit the upper bound of our strategy and benefit from the advantage of the higher rate environment.

We observe that, although cost to initiate is higher, there is more upside potential when using the bullish collar for cash flow matching within a pension fund. This strategy is primarily desirable to a pension fund with a need to increase their solvency ratio or may also be suitable for a bank's LDI strategies, whose mandate includes growing the value of the fund. This is more of a hybrid strategy, combining an explicit liability hedging strategy and a profit-seeking strategy.

As suggested, in addition to optimizing the variables already observed (strike, underlying, etc.) we also have the ability to look at more complex derivatives. This could further contribute to reducing costs upfront, but would require a more sophisticated forecast of the underlying assets movements or a more concrete view of the future market conditions. The next logical step for future work, given what we have observed, is to explore more robust long term option pricing models and to compare portfolios containing derivative products against more sophisticated ALM models. Similar products used in an immunization model as opposed to a cash flow matching model would approach a more common industry practice and allow to see the benefits and drawbacks of this logic in a more dynamic setting.

Appendix A – Pension Data

The following pages contain the remaining tables of pension data for the starting assumptions of the liability cash flow calculations.

Age	0-4	5-9	10-14	15-19	20-24	25-29	30-34	35+	All Years
15-19	55								55
	$34,361								$34,361
20-24	546	115							661
	$46,538	$57,625							$48,467
25-29	546	683	81						1310
	$49,420	$58,113	$61,643						$54,708
30-34	295	614	556	21					1486
	$48,429	$58,918	$63,550	$64,127					$58,643
35-39	177	369	536	216	9				1307
	$49,471	$58,883	$65,058	$64,858	$70,040				$61,205
40-44	92	241	327	284	340	18			1302
	$49,397	$58,980	$61,605	$63,355	$69,185	$75,045			$62,803
45-49	54	141	214	149	224	221	22		1025
	$47,806	$58,828	$61,178	$63,337	$68,646	$72,749	$78,763		$64,968
50-54	14	46	91	79	72	124	84	4	514
	$46,545	$56,916	$60,129	$61,237	$66,423	$70,087	$77,441	$75,207	$65,872
55-59	7	11	19	17	28	21	17	3	123
	$50,467	$55,229	$57,806	$55,341	$62,505	$66,460	$79,444	$82,212	$62,950
60+									0
									$0
All Ages	1786	2220	1824	766	673	384	123	7	7,783
	$47,848	$58,539	$63,051	$63,400	$68,444	$71,653	$77,954	$78,209	$59,450

Table 12: Active member quantities and average annual earnings; Female Other Ranks 2013

Age	0-4	5-9	10-14	15-19	20-24	25-29	30-34	35+	All Years
15-19	319								319
	$19,394								$19,394
20-24	1039	345							1384
	$30,825	$56,949							$37,337
25-29	656	992	257						1905
	$56,975	$72,595	$82,210						$68,513
30-34	295	783	814	197					2089
	$65,766	$84,709	$91,005	$100,213					$85,950
35-39	120	416	641	750	184				2111
	$67,235	$86,612	$97,010	$103,243	$108,707				$96,502
40-44	107	161	334	441	841	171			2055
	$74,801	$86,982	$96,025	$101,628	$108,869	$112,469			$102,039
45-49	71	102	139	193	523	851	179		2058
	$79,433	$94,024	$98,001	$100,955	$111,052	$112,669	$119,408		$108,684
50-54	48	60	92	77	173	533	677	81	1741
	$83,863	$93,780	$99,417	$105,911	$112,152	$114,235	$117,806	$127,753	$113,352
55-59	14	20	39	45	65	90	168	122	563
	$110,936	$127,509	$105,169	$96,936	$105,845	$109,827	$120,563	$111,610	$112,260
60+		2	11	3	1	2	1		20
		84246	92615	$103,854	83067	76629	95156		$91,515
All Ages	2669	2881	2327	1706	1787	1647	1025	203	14,245
	$45,815	$78,431	$93,404	$102,172	$109,685	$112,956	$118,516	$118,052	$88,970

Table 13: Active member quantities and average annual earnings; Male Officers 2013

Age	0-4	5-9	10-14	15-19	20-24	25-29	30-34	35+	All Years
15-19	565								565
	$35,508								$35,508
20-24	5871	1063							6934
	$47,551	$58,184							$49,181
25-29	3844	5555	579						9978
	$49,877	$59,544	$63,161						$56,030
30-34	1358	3317	3725	238					8638
	$50,306	$60,516	$63,515	$66,737					$60,375
35-39	545	1174	2393	2167	143				6422
	$49,779	$60,695	$63,456	$67,077	$71,854				$63,199
40-44	249	486	845	1455	2599	320			5954
	$49,931	$60,626	$62,792	$67,139	$71,417	$74,128			$67,514
45-49	132	201	340	457	1233	2357	380		5100
	$49,861	$59,718	$62,199	$65,560	$69,805	$74,785	$78,901		$70,983
50-54	42	82	107	144	265	749	1281	145	2815
	$48,475	$60,238	$60,827	$63,083	$67,439	$72,746	$78,203	$82,913	$73,579
55-59	14	24	43	34	43	62	149	124	493
	$47,957	$59,197	$60,567	$63,729	$66,254	$69,471	$76,004	$80,858	$71,745
60+			1		4	1			6
			$71,082		$56,975	$78,103			$62,847
All Ages	12620	11902	8033	4495	4287	3489	1810	269	46,905
	$48,188	$59,858	$63,289	$66,772	$70,657	$74,194	$78,168	$81,966	$60,855

Table 14: Active member quantities and average annual earnings; Male Other Ranks 2013

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