HEC MONTRÉAL

Firm leverage and the pricing of equity options

by
Deandra D’Arcangelo

Master of science (MSc) – Financial Engineering

Thesis presented to obtain a Master of Science degree (M. Sc.)

February 2025

ABSTRACT

The compound option pricing model derived by Geske (1979) is studied to identify whether incorporating leverage can improve the performance in call option pricing. The sample data consists of 188 firms listed on the NASDAQ-100 from 2001 to 2023. The implied volatility is calculated for both the compound option (CO) model and the Black Scholes (BS) model. The pricing improvements are analyzed with regards to moneyness, time to maturity and the debt-equity ratio. Overall, the compound option model performed better than the Black Scholes model.

The compound option model contains two extra variables compared to the Black Scholes model. The two extra variables capture the leverage effect and the time to maturity of debt. The relationship between the implied volatility and the leverage ratio is observed, specifically for firms with higher leverage.

The overall results indicate that the compound option model is better at pricing options than the Black Scholes model. The Black Scholes model has difficulty pricing the implied volatility for options with greater leverage, as evidenced by the greater percent error of implied volatility. The results observed show that the compound option model works better than Black Scholes because it can capture changes in equity volatility. The changes in equity volatility are captured because volatility varies with changes in leverage.

This research is an extension of the Geske, Subrahmanyam and Zhou (2016) paper, which evaluated the pricing performance of the compound option model.

Keywords
Compound option pricing, Black Scholes, Leverage

TABLE OF COTENTS

ACKNOWLEDGEMENTS

I would like to thank the supervisor of my thesis, Professor Piotr Orlowski, for his expertise on option pricing. As well as, for his guidance and support throughout my thesis. Thank you for steering me in the right direction through endless iterations of code, as well through the writing process of the thesis. The journey was long but a great learning experience.

I would like to thank my family for their constant encouragement. Rosanna and Ella Rose, thank you for your moral support, it truly helped me bring this thesis to completion. Mom, thank you for your unwavering belief in me, and for the strength you gave me. Lastly to Dad, I started this academic journey with you by my side, unfortunately you didn’t get to read the final product, but this is for you.

DECLARATION

I declare that I used ChatGPT (https://chat.openai.com/) to occasionally verify grammatical correctness and synonyms. I used the following prompts: “Is this sentence grammatically correct: ...”. I declare that the content of writing is my own creation. I also made use of ChatGPT on occasion in my code to understand the possible reasons for the error. For example, I used the following prompts “ValueError: could not convert string to float: '2019-06-19'.

1. INTRODUCTION

There are numerous theories and models that exist to price equity options. This inevitably leads to the question of whether there is a specific factor in a model that contributes to the accurate valuation of the option. Interest in the study of option pricing goes as far back as 1900, when Bachelier first introduced the concept of a stochastic process in option pricing (Wu et al., 2023). Samuelson (1973, 1965) then argued that the stock price depends on the real probability measure and that it can be simulated by a geometric Brownian motion (Wu et al., 2023). It was only later that Black and Scholes (1973), by employing the replication argument, derived a partial differential equation for derivative pricing. The equation yielded the eminent Black-Scholes (BS) option pricing formula. In the BS model, the stock price follows a geometric Brownian motion (Black & Scholes, 1973) and this led to the risk-neutral framework and the risk-neutral probability. In the risk-neutral framework, the investor's concerns are embedded for risk in the probability distribution (Black & Scholes, 1973). Therefore, option pricing involves taking the expected value of the future payoff discounted at the risk-free rate under a risk neutral probability measure (Black & Scholes, 1973). This approach is central to the Black Scholes pricing model (Black & Scholes, 1973).

Although the Black-Scholes model is widely used, it fails empirically. This is because to derive the BS formula, the model assumes several market conditions for both the stock and the option (Black & Scholes, 1973). Two examples of these assumptions are that i) a stock has a constant volatility, ii) the stock prices follow a lognormal distribution (Black & Scholes, 1973). Due to the fact that these conditions do not occur in the market, discrepancies arise between the actual option prices and those predicted from the BS model. As a result, this incited research in option pricing to further evolve and include models with stochastic volatility, stochastic interest rates, jump risk (Bates, 2000) and compound options.

The compound option (CO) model considers a stock to be an option on a levered firm, and therefore, an option on a stock is an option on an option (i.e. a compound option) (Geske et al., 2016). The value of a call option, as a compound option is derived as a function of the value of the firm, and the stock is viewed as an option on the value of the firm (Geske, 1979). Robert Geske derived the option pricing formula for compound options based on Merton's application of the Black-Scholes model to price a call option on stock (Geske, 1979). What distinguishes the compound option model from the BS model is that the variance of the rate of return of the stock is not constant. The CO model manages to capture changes in the equity volatility because the model incorporates leverage (Geske, 1979). Research by Choi and Richardson (2016) determined that financial leverage does have an impact on equity volatility. Their study was able to control time-varying asset volatility through GARCH-type effects and to isolate the partial correlation between leverage and equity volatility (Choi & Richardson, 2016). Furthermore, the advantage of the compound option model (CO) is that unlike stochastic volatility models, it is less computationally demanding to implement.

The following research examines the compound option pricing model and whether incorporating leverage can improve the performance in call option pricing. The analysis involves calculating the percent error of implied volatility for both the Black Scholes no arbitrage model, and the CO model, and comparing their pricing performance. The research also analyses the option pricing performance of the firms that have a higher leverage ratio. The compound option pricing model (CO) is derived from the Merton (1973) model (Geske & al., 2016). Similarly, the model treats a stock as an option on a levered corporation, therefore following a nested sequence of options on options (Geske & al., 2016).

This research is inspired by the following papers: “Capital Structure Effects on the Prices of Equity Call Options” (Geske & al., 2016) and “The Effects of Leverage on the Pricing S&P500 Index Call Options” (Geske & Zhou, 2007). These studies measured asset returns, and estimated the volatility of a firm’s assets, to investigate the role leverage has on equity volatility (Geske & al., 2016). Similarly, I analyse and discuss the option pricing behaviour of the compound option model (CO) and its pricing performance, especially for the firms with greater leverage. Throughout this research paper, I analyse how the CO pricing model performs on the 188 firms in the data sample, which are all companies listed on the NASDAQ-100 between 2001 to 2023. The paper is organized as follows: in section IV, I present the results for the implied volatility error for both the BS model and CO model. In section V, I discuss the significance of the results obtained along with the limitations of the compound option model. Lastly, in section VI, I conclude the paper with the findings of the research regarding the pricing performance of the compound option model.

2. LITERATURE REVIEW

The following research analyses the compound option pricing model and whether incorporating leverage can improve the performance in call option pricing. To appreciate the pricing improvements of an option model, it is important to understand what an option is. Black and Scholes (1973) define an option as a security giving the right, but not the obligation to buy or sell a stock, subject to predefined conditions and maturity. The term “strike price” defines the amount paid for the asset when it is exercised (Black & Scholes, 1973). An “American option” is a contract that allows the holder to exercise the option at any time, up to the maturity date. On the contrary, a “European option” can solely be exercised at maturity (Black & Scholes, 1973).

The following sections highlight the research in the field of option pricing that provided developments in the world of financial economics and offers context to the pricing dynamics of both the Black Scholes model and compound option model.

2.1. Discrete time varying model

The Binomial Tree is the most famous discrete time-varying model for option pricing (Boudreault & Renaud, 2019). The model assumes a frictionless market and is made up of only two assets, a risk-free asset, which evolves according to the risk-free interest rate, and a risky asset (Boudreault & Renaud, 2019). The Binomial Tree model is mainly used because of its versatility and ability to price American options (Chiarella et al.,2015). Research by Chiarella et al, (2015) states that the discrete-time approach is less accurate than continuous models, especially for options that are sensitive to small changes in asset price. Rather, the discrete-time models are better for qualitative and statistical analysis (Chiarella et al., 2015). However, continuous time models are more convenient because they are stochastic and provide explicit solutions and formulas to price options (Yan, 2018). Interestingly, the Black Scholes partial differential equation, can be discretized to ultimately yield the binomial model (Chiarella et al., 2015). This implies that for the Binomial tree method, if the number of time steps increases, it will converge to a lognormal distribution (Boudreault & Renaud, 2019). However, in that case, the Binomial Tree model becomes less efficient at pricing long time to maturity options, therefore continuous time models are preferred (Chiarella et al.,2015).

2.2. Black Scholes model

In 1973, Fischer Black and Merton Scholes derived the renowned Black-Scholes formula, and their research did empirical tests on the valuation formula on call option data. The BS formula is structured around several ideal conditions. These ideal conditions include the assumption that the stock price follows a continuous path throughout time, and that the instantaneous volatility of the stock rate of return is not stochastic (Black & Scholes, 1973).

Therefore, under the ideal assumptions made in the BS formula, an option's value is dependent only on the price of the stock, time to maturity, strike of the option and on specific variables that are taken as constants (Black & Scholes, 1973). Their results show, that the actual price for which an option is bought or sold can deviate in certain systematic ways to the predicted price obtained by the BS formula (Black & Scholes, 1973). Black and Scholes (1973) observed that option buyers pay more that the price predicted by the formula. The Black-Scholes model is widely used and studied because of its straightforwardness and computational ease, but there are other models available that are better at capturing option pricing dynamics.

2.3. Compound Option model

Robert Geske's (1979) paper titled "The Valuation of Compound Options" developed the framework for deriving the compound option model as an extension of the Merton (1973) model. The difference between the compound option model and the Black Scholes model is that the CO formula accounts for the firm's debt position (Geske, 1979). The advantage of compound options is that the variance of equity is not assumed to be constant, but instead depends on the firm's leverage and the total value of the firm (Geske, 1979). This is a contradiction to the Black Scholes model, which assumes that the stock has a constant volatility (Geske, 1979). This implies that the CO model can correct biases set by the Black Scholes formulation (Geske, 1979). In addition, Geske (1979) proposes that a firm's debt position alters the total risk or volatility of equity because the market reevaluates the cash flow of the firm. This indicates there is a relationship between leverage and the equity of a firm.

Geske, Subrahmanyam and Zhou's research (2016) explored the impact that a firm's leverage has on pricing options and suggested that it is a vital factor in evaluating equity call options. Their research found that the CO model outperformed both in the money and out of the money options in comparison to the BS model (Geske, et al., 2016). The implication of their research is that incorporating the leverage in financial models improves option pricing. Furthermore, Geske (1979) observed that changes in a firm’s equity value inherently influences leverage, and that the variance of the stock returns increases in a stable manner as the firm's leverage increases.

There exists on average a negative relationship between the volatility of the rate of return on equity and the value of equity. Researchers like Christie (1982) have studied this relationship in order to provide an explanation for this behaviour. Christie (1982) studied the impact several variables have on the variance of equity returns and proposed that equity variances have a strong correlation with debt. His research implies that “volatility is an increasing function of financial leverage” and this can cause the volatility, with regards to the value of equity, to be negative under a range of conditions (Christie, 1982).

Consequently, if financial leverage has an impact on option pricing, it is important to understand what financial leverage is. Financial leverage is an investment strategy that involves using borrowed capital to expand a firm’s asset base and generate returns on risk capital (Adrian & Song Shin, 2010). The capital structure of a firm is the nature in which the firm funds its operations using both debt and equity. In addition, Geske et al. (2016) evaluated whether equity options traded for individual firms are impacted by the firm’s capital structure. They found that “a firm’s debt influences the values of securities held by the firm’s equity holders […] therefore debt must influence options on equity” (Geske, et al., 2016).

Moreover, the CO model for a call option implies that if the firm has debt obligations that are included in the pricing model, and the firm's volatility is deterministic, the volatility of the stock will be greater than the volatility of the firm (Geske et al., 2016). This is because the stock volatility doesn't only depend on the price of the stock but also on a stochastic pattern (Geske et al., 2016). This pattern is evident by the following behaviour; if the price of a stock falls, this will inevitably cause the debt-equity ratio to increase (Geske et al., 2016). This then increases the riskiness of the firm's stock, which will be observable by the variance in the returns on the stock (Geske et al., 2016).

The compound option pricing model is the focus of my research because of its ability to capture the changes in equity volatility through changes in leverage. Although, there are other option pricing models that capture equity volatility, however their variability is not attributed because the debt value is included as a parameter within the model.

2.4. Stochastic volatility models

As previously stated, the Black Scholes model make several ideal assumptions in their formula (Fouque et al., 2011). By relaxing these ideal assumptions and permitting volatility to vary randomly, the observed differences between the option prices obtained from the BS model and those observed in the market can be better explained using stochastic volatility models (Fouque et al., 2011). Stochastic volatility models account for the implied volatility skew, which allows volatility to fluctuate as opposed to remaining constant (Fouque et al., 2011). Stochastic volatility models can also explain the volatility smile and term structure effects to describe complex financial markets (Ben-Zhang et al., 2020).

2.4.1. Single Factor Stochastic Volatility Models

Single factor stochastic volatility models follow a one-dimensional Itô process, governed by a stochastic differential equation and driven by a second Brownian motion (Fouque et al., 2011). A one factor model is used for its mean-reversion properties (Fouque et al., 2011).

The Heston model is an example of a stochastic volatility model. The Heston model is a bivariate system of stochastic differential equations (Rouah, 2015). The model follows a Black Scholes stochastic process for the underlying stock price, but with a stochastic variance that follows a Cox, Ingersoll and Ross process (Rouah, 2015). The Heston model is a stochastic volatility model known to provide the correct smile or skew for implied volatility (Choi & al., 2016). Furthermore, the benefit of the Heston model is that the option pricing formula is derived from a computable and explicit integral, which is suitable for calibration purposes (Choi & al., 2016).

Anderson et al. (2002) state that a drawback of the Heston model is its inability to capture the full kurtosis (Jones, 2003). Single factor stochastic volatility models can provide the correct smiles and smirks. However, because the correlation between the variance and stock returns are constant overtime, the Heston model is unable to capture the time varying nature of the smirk (Christoffersen et al., 2009). This explains why the model does not capture the full dynamics of volatility (Christoffersen et al., 2009). This limitation is observed by the discrepancy between the predicted and market prices for certain moneyness options (Choi & al., 2016). Moreover, Mikhailov and Noegel (2003) observe that in the Heston model, the implied volatility skew differs when compared to the implied volatility observed directly from the market, especially for short term maturities (Mikhailov & Noegel, 2003). As well, research by Jones (2003) states that because of the Heston model's square root process, the model has difficulty capturing periods of high volatility.

Stochastic volatility models can explain asymmetry and high kurtosis but, as mentioned by Cont and Tankov (2004), Brownian models with stochastic volatility, cannot explain jumps in price due to the continuous nature of the paths (Hainaut & Moraux, 2019).

2.4.2. Multi Factor Stochastic Volatility Models

Multi factor stochastic volatility models, like the one proposed by Fouque et al. (2003), present volatility as a two factor mean reverting diffusion process. Christoffersen et al. (2009) developed a two-factor stochastic volatility model, referred to as a double Heston, which has two independent variance processes. They proposed to model fluctuations in the slope of the smirk using a two-factor stochastic model since the factors have distinct correlations with market returns (Christoffersen et al., 2009). The model generates a stochastic correlation between volatility and stock returns and provides more flexibility for the modeling of the time variation in the smirk (Christoffersen et al., 2009). Although, these models capture the behaviour of volatility, their mathematical complexity make them difficult to adopt.

It’s important to note, that nor the stochastic volatility models or the Black Scholes model mentioned above directly consider debt in their option pricing formula. Therefore, if debt is a relevant but omitted variable in option pricing, this can explain the pricing errors that are visible in other option pricing models (Geske et al., 2016).

The following paper specifically analyses option pricing using the compound option model. The CO model is less complex to use than stochastic volatility models, however it is still capable of capturing the changes in equity volatility through the changes in the value of debt.

3. METHODS

3.1. Overview of Compound option model

For the compound option pricing model (CO) a stock is an option on a levered company, and therefore an option on a stock is an option on an option (Geske et al., 2016). The CO model involves two correlated options, one to repay the debt and the other to exercise the stock (Geske et al., 2016).

The firms observed in this research, all have debt that they use as working capital to invest in their future growth (Chen & Murry, 2022). In the CO model, the debt obligation is represented as the strike price of the firm's option to default on its debt (Geske et al., 2016). Furthermore, all 188 companies are financed by both debt and equity, therefore the model assumes that the total volatility for the firm is smaller and less volatile than the stock (Geske et al., 2016). The reason being is because debt offers a lower return on an investment and is less risky compared to a stock (Maverick & Catalano, 2021).

The compound option model is derived from a partial equilibrium, self-financing and arbitrage free portfolio (Geske et al., 2016). This portfolio consists of the option, the firm and a risk-free bond (Geske et al., 2016). The formula for the value of the call option (C) in the compound option model is derived as a function of the firm value (V), and the firm's stock is viewed as an option on the value of the firm (Geske, 1979). In the CO model, for each firm, their debt is considered a zero-coupon bond with time to expiration equal to the imputed duration of the firm's debt (Geske et al., 2016).

The CO formula for a call option in continuous time assumes an environment where there is constant demand and trading available for the options, and that all markets are competitive (Geske, 1979). Furthermore, it assumes that the risk-free rate of interest is observable and constant over time (Geske, 1979). The model also assumes that trading is continuous and that a firm's change in value follows a random walk in continuous time with a variance rate that is proportional to the square of the value of the firm (Geske, 1979). The CO model captures that the volatility of the stock is time-varying without being a stochastic volatility model.

Equation 1 below, is the formula stated by Geske (1979) to price a stock option using the compound option model:

$$C = VN_2\left(h_1 + \sigma_{v_{Ti}}, h_{2+} + \sigma_{v_{Td}} ;\ \rho\right) - Me^{-r_{T_d}(T_d - t)} N_2(h_1, h_2;\ \rho) - Ke^{-r_{T_i}(T_i - t)} N_1(h_1), \quad \textbf{(1)}$$

Which can also be expressed as

$$C = (S + D)N_2\left(h_1 + \sigma_{v_{Ti}}, h_{2+} + \sigma_{v_{Td}} ;\ \rho\right) - Me^{-r_{T_d}(T_d - t)} N_2(h_1, h_2;\ \rho) - Ke^{-r_{T_i}(T_i - t)} N_1(h_1)$$

where,

$$h_1 = \frac{\ln(V/_{V^*}) + (\, r_{T_i} - 0.5\sigma^2{}_{v_{T_i}}(T_i - t)}{\sigma_{v_{T_i}}\sqrt{(T_i - t)}}$$

$$h_2 = \frac{\ln(V/_{M}) + (\, r_{T_d} - 0.5\sigma^2{}_{v_{Td}}(T_d - t)}{\sigma_{v_{T_d}}\sqrt{(T_d - t)}}$$

$$\rho = \sqrt{\frac{(T_i - t)}{(T_d - t)}}$$

The input variables of the Geske (1979) compound option model are the following:

C= stock option call value
S= stock market value
V= implied market value of a firm (S+D)
V*= critical total market value of a firm
M= face value of market debt
K= strike price of the option
r_{Ti}= risk-free rates of interest to dates T_i
r_{Td}= risk-free rates of interest to dates T_d
$\sigma_{v_{Ti}}$= instantaneous firm volatility at T_i
$\sigma_{v_{Td}}$= instantaneous firm volatility at T_d
t= current time
T_i= specific expiration date of option
T_d= date of maturity of debt
N_1= bivariate cumulating normal distribution
N_2= bivariate cumulating normal distribution
ρ= correlation coefficient between the asset value at T_i and T_d

For the compound option model, the boundary condition to exercise the call option depends on the value of the firm (V) and the critical total market value of a firm (V*) (Geske et al., 2016). The call option on equity expires at T_i, however the debt option expires at T_d (Geske et al., 2016). This suggests that all events that occur during the time between the expiration date of the call option but before the debt default option expires, can affect the value of the equity option (Geske et al., 2016).

To price a call option on a firm's stock, the CO model needs to solve four unknowns (V, V*, $\sigma_{v_{Ti}}$ and $\sigma_{v_{Td}}$) that are not readily observable in the market and need to be implied using an optimizer (Geske et al., 2016). The market value of the firm is referred to as V and it is the sum of the equity price (S) and the implied debt value (D) (Geske et al., 2016). The equity price (S) is observable in the market, unlike the implied debt (D) which needs to be solved (Geske et al., 2016). Furthermore, in the compound option model, the volatility of the stock is random and inversely related to the equity and leverage of the firm (Geske et al., 2016).

The compound option model proposes an approach that allows for both the implied firm volatility and implied market debt to be observed from option and equity prices (Geske et al., 2016). The implementation of the model and the optimization of the four unknowns are detailed in the subsequent section.

3.2. Implementation overview

The following research evaluates the improvements in option pricing between the Black Scholes model and the compound option model through the percent error of implied volatility. The percent error of implied volatility is the difference between the actual implied volatility from Optionmetrics and the one obtained by both the BS and CO models. I implement a similar approach as Geske et al. (2016) did in their research titled the "Capital structure effects on the prices of equity call options", to analyse the option pricing improvements between the Black Scholes model and the compound option model.

3.2.1. KMV Merton Model

To determine whether the CO model can accurately price options when leverage is incorporated in the option pricing model, certain variables need to be inferred, since they are not observable from the dataset. More specifically, there are four unknown variables that need to be solved using optimization to solve the compound option model. These four unknowns are the implied debt value (D = V - S), the critical total market value of the debt (D*= V* - S*), instantaneous firm return volatility at T_i ($\sigma_{v_{Ti}}$) and the instantaneous firm volatility at T_d ($\sigma_{v_{Ti}}$) .

The dataset is composed of daily security prices and debt. The CO model assumes that at any time, the market value of a firm's debt (D) is less than the risk-free present value of the firm's debt D < Me^{-rT_d}. Therefore, the initial assumption for the implied market value of debt is D= Me^{-rT_d}, which makes the initial guess for the current firm value V=S + Me^{-rT_d}. I then use the KMV Merton model on each day and for each firm to determine the instantaneous firm return volatility at expiration T_d ($\sigma_{v_{Td}}$).The results from the KMV model are used as initial values for two out of the four unknown variables in the CO model.

According to the Black Scholes Merton model in **Equation 2,** there is a relationship between the volatility of a firm's equity (σ_E), $and\ \sigma_{v_{Td}}$ which is represented as follows:

$$E = V \times N(d_1) - e^{-r_{T_d}} \times M \times N(d_2).$$

$$d_1 = \frac{\ln(V/_M) + \left(r_{T_d} + 0.5\sigma_{v_{T_d}}{}^2\right)T_d}{\sigma_{v_{Td}}\sqrt{T_d}}$$

$$d_2 = d_1 - \sigma_{v_{Td}}\sqrt{T_d}$$

Where the relationship between σ_E and $\sigma_{v_{Td}}$ is

$$\sigma_E = \frac{V}{E} \times N(d_1) \times \sigma_{v_{Td}} \quad \textbf{(2)}$$

The data that is used as input variables in the KMV equation to solve for $\sigma_{v_{Td}}$, are the value of the firm's equity (E), the face value of debt (M) , the volatility of a firm's equity (σ_E), time to maturity of debt (T_d) and the risk-free interest at T_d (r_{T_d}) .

3.2.2. Implementation of the CO Model

The condition for which to exercise the option ($V \geq V^*$) is dependent on the value of the firm (V) as well as the critical total market value of a firm (V*). For the critical total market value of a firm, V* is represented as the sum of the S^* (also known as the strike price of the option) and the D^* (the critical market value of debt).

The maturity of the debt (T_d) is calculated using the Macaulay duration formula. For each option observed, the specific expiration date of the option (T_i) is less than the date of the maturity of debt (T_d). Therefore, the debt expires after the option's expiration date. This allows to observe the effect of debt on the calculation of the implied volatility of the call option value. There is also a correlation on whether the option is exercised and if the firm defaults within the time to maturity of the option. Therefore, this correlation is represented by ρ in the compound option model, to highlight that there exists two associated exercise opportunities.

Since there are four unknowns to solve, (D, D*, $\sigma_{v_{Ti}}$, $\sigma_{v_{Td}}$), to be able to calculate the compound option model for a call option, four equations are used to infer them. Three of the equations are from the Geske's (1979) compound option model (**Equation 1**). The equation uses options for a specific firm that are listed on the same day, with the same time to maturity, however, have different strike prices (K1, K2 and K3). I also apply Merton's (1974) equation for stock (S) as an option on the assets of the firm V (**Equation 3** see below) .

$$S = V \times N(d_1) - e^{-r_{T_d}} \times M \times N(d_2) \quad \textbf{(3)}$$

$$d_1 = \frac{\ln(V/_M) + \left(r_{T_d} + 0.5\sigma_{v_{T_d}}{}^2\right)T_d}{\sigma_{v_{T_d}}\sqrt{T_d}}$$

$$d_2 = d_1 - \sigma_{v_{T_d}}\sqrt{T_d}$$

To sort the dataset and run the optimization to solve for the four unknowns, I calculate for each option the difference between the strike price and price of the underlying stock, from data obtained from OptionMetrics and CRSP respectively. This allows to identify the options which are close to the at the money options (ATM). Then for each firm, on each day, for options with the same time to maturity (TTM), I group them together in my dataset. I then sort them to ensure that they are in order of the smallest "closest to atm", and I then select for each firm for each day and TTM, the three smallest “closest to atm ” options with different strike prices. Using the four equations (3x Equation 1, 1x Equation 3) with the four market prices (C1,C2,C3,S), I then run the model to solve for the four unknowns (D, D*, $\sigma_{v_{Ti}}$, $\sigma_{v_{Td}}$). This is done using a minimization optimizer with the SLSQP method, to minimize the difference in option prices.

Once I obtain for each firm the values of D, D*, $\sigma_{v_{Ti}}$, $\sigma_{v_{Td}}$, I then assign those values to all the options with the same date and TTM. Then for each option, the compound option price from **Equation 1** is calculated using the values (D, D*, $\sigma_{v_{Ti}}$, $\sigma_{v_{Td}}$) obtained from the optimization. Subsequently, the implied volatility for each option using the CO price is calculated. The calculated implied volatility from the CO model is then compared for each option with the implied volatility from OptionMetrics. The percent error is then calculated and used to compare the performance of the CO model to the BS model.

To run the optimization in Python, there are certain constraints that are applied to the model. One being that the implied market value of debt is less than the present face value of debt. Another, is that the instantaneous firm volatility at T_i is less than then the Black Scholes volatility, since there is the component of debt. It is also assumed that the instantaneous firm volatility at T_d is less than the instantaneous firm volatility at T_i. Furthermore, for each option (and their respective strike prices K1, K2, K3) the constraint V > V^* is applied.

3.2.3. Black Scholes Model

The Black-Scholes model is essentially a special case of the CO model that assumes that the firm's debt value is negligible (M=0, V=S), and that the volatility of the firm is equal to the volatility of the stock ($\sigma_v = \sigma_s$) (Geske et al., 2016).

By determining the percent error of the implied volatility for the Black Scholes model, and then comparing it to the compound option model, I was able to evaluate what the effect of adding leverage to option modeling can have on the ability to price options.

The Black Scholes formula is the following (**Equation 4**):

$$C = S \times N(d_1) - Ke^{-r_{T_i}}N(d_2) \quad \textbf{(4)}$$

$$d_1 = \frac{\ln(S/K) + \left(r_{T_i} + 0.5\sigma_{s_{T_i}}{}^2\right)T_i}{\sigma_{s_{T_i}}\sqrt{T_i}}$$

$$d_2 = d_1 - \sigma_{s_{T_i}}\sqrt{T_i}$$

4. EMPIRICAL RESULTS

4.1. Data Collection

The data collected consists of 188 firms listed on the NASDAQ-100 index from January 1st, 2001 to August 31st, 2023 and it is comprised of a total of 2,827,415 call options. The price of each security as well as the debt value from each firm's balance sheet is gathered. Furthermore, call option prices for each firm are also collected, along with the continuously compounded zero-coupon interest rates that are obtained from the zero-coupon yield curve.

4.1.1. CRSP

Data is collected for each firm from the Center for Research in Security Prices (CRSP) database. Data consists of the i) *Price or Bid/Ask Average (*which is the closing price for a trading day), ii) the *Volume (* which is the total number of shares of a stock sold on specific day, expressed in the units of one share). Furthermore, to be able to calculate the market capitalization of each firm, the iii) *Shares Outstanding* (the number of shares that are publicly held) are also collected.

4.1.2. Compustat

Annual balance sheet information for each firm is collected from Compustat-Capital IQ . This data is used to calculate the face value of debt for each firm. Compustat is also used to obtain data to calculate the Macaulay duration of debt, which I use as the maturity of debt (T_d).

The complete list of variables that are used to calculate the face value of debt are the following: *i)* the total current liabilities (LCT), *ii)* the debts that mature in year one through five. Furthermore, I gather the *iii)* total long-term debt (DLTT), which is the reported debt with a maturity longer than five years. I gather data for the *iv)* accrued expense and deferred income (AEDI), *v)* deferred charges (DC), *vi)* notes payable (NP), *vii)* the debt of the consolidated subsidiary (DCS), *viii)* the finance subsidiary (DFS), *ix)* notes debt (DN), *x)* other liabilities (LO), *xi)* debentures (DD), *xii)* contingent liabilities (CLG), *xiii)* mortgage debt and other secured debt (DM), *xiv)* long-term debt tied to the prime rate (DLTP), *xv)* total assets (AT). I also collect the data for *xvi)* the capitalized lease obligation (DCLO), which is due to expire in the seventh year, and *xvii)* the federal, foreign, and state deferred tax.

Similar, to Geske et al. (2016), I refrain from using all data which has convertible debt (DCVT) with more than 3% of the total assets (AT). In addition, data which has finance subsidiary (DFS) greater than 5% of total assets is also excluded from the analysis (Geske et al, 2016). Furthermore, if the total long-term debt (DLTT) and the debt maturities from year one to five are not available, the data is omitted (Geske et al, 2016).

4.1.3. OptionMetrics

Data on call options traded on each firm's underlying assets is gathered directly from OptionMetrics. The information I incorporate in my dataset is the following, i) the date the option is traded on, ii) the expiration date of the option and the iii) option's strike price (K). I also obtain the iv) closing bid price which is referred to as the best bid.

Furthermore, I gather *v)* the best offer, meaning the best closing ask price of the option, v*i)* the total volume of the option that is traded, v*ii)* the open interest per day for each option, *viii)* the delta of the option and *ix)* the implied volatility of the option, which is calculated using the Black Scholes equation.

I delete from the sample data all the options that are missing fields such as the expiration date or the open interest. I also include only options for which the bid price is positive and smaller than the offer price. In addition, if the volume of the option is zero, the options are also disregarded in the analysis. As well, like Geske et al. (2016), all options that violate the arbitrage condition $C \leq S - Ke^{-rT}$ are omitted.

4.1.4. Yield curve

The continuously compounded zero coupon interest rates and the days to maturity are obtained from the OptionMetrics databased from January 1st 2001 to August 31st 2023. Any missing interest rate are interpolated from the zero-coupon yield curve.

4.2. Analysis of Data

The results obtained from the 188 firms are analysed and compared according to the implied volatility obtained from the Black Scholes and Compound Option model. The interpolated implied volatility from the models is compared to the implied volatility from Optionmetrics (Market). The difference between the observed implied volatility and the market implied volatility is defined as the error and calculated as follows:

$$Implied\ vol.Error\ BS = \frac{|(Market - BS)|}{Market}$$

$$Implied\ vol.Error\ CO = \frac{|(Market - CO)|}{Market}$$

To compare the pricing performance, the results obtained by each model are analysed in terms of specific factors. The factors used to analyse the improvement of the compound option model to the Black Scholes model are moneyness, time to maturity and the leverage ratio.

Moneyness is the proportion of the strike price (K) to the stock market value of the underlying stock (S) and is classified in terms of ITM [0.89-0.95], ATM [0.96-1.05] and OTM [1.06-1.55]. Furthermore, the time to maturity (TTM) is the duration of time for which the option can be exercised. Lastly, since the objective of the paper is to observe whether incorporating the firm's debt can improve the performance in call option pricing it is vital to observe the leverage ratio.

$$Leverage\ ratio = \frac{Face\ Value\ of\ Debt}{Market\ Value\ of\ Equity}$$

The leverage ratio allows a better understanding of the capital structure of each firm being evaluated and how incorporating leverage in option pricing impacts the performance of the model.

4.3. Descriptive Statistics

Table I
Sample Composition

The sample contains 2,827,415 observations on options for 188 firms between 2001 and 2023. The table represent summary information on relative option strikes (K/S), Compound Option implied volatilities (%), Black-Scholes implied volatilities (in %), as well as the number of available observations according to option types (Panel A), maturities (Panel B) and moneyness (Panel C).

			Rel. Strike			Impl. Vol. CO			Impl. Vol. BS			
Panel A: All options												
	Mon	τ	Avg.	Q05	Q95	Avg.	Q05	Q95	Avg	Q05	Q95	N
Calls	All	All	1.06	0.91	1.34	36	20	58	35	20	57	2,827,415
Panel B: By Maturity												
	Mon	τ	Avg.	Q05	Q95	Avg.	Q05	Q95	Avg	Q05	Q95	N
Calls	All	[0.25-0.50)	1.06	0.91	1.33	36	19	59	35	19	58	1,735,314
	All	[0.50-0.75)	1.06	0.91	1.35	35	20	57	35	20	56	765,828
	All	[0.75-1.00]	1.08	0.91	1.39	35	21	54	34	21	54	326,273
Panel C: By Moneyness												
	Mon	τ	Avg.	Q05	Q95	Avg.	Q05	Q95	Avg	Q05	Q95	N
Calls	ITM	All	0.92	0.9	0.94	33	18	55	34	19	57	523,531
	ATM	All	0.99	0.95	1.04	33	18	55	34	19	55	1,079,351
	OTM	All	1.18	1.05	1.42	39	23	61	36	21	59	1,224,533

Table II
Leverage Composition

The sample contains 2,827,415 observations on options for 188 firms between 2001 and 2023. The table represent summary information such as the average and percentiles (5th, 25th, 75th, 95th) for the leverage ratio (%) of the sample data.

Panel A : All options								
			Leverage Ratio					
	Mon.	τ	Avg.	Q05	Q25	Q75	Q95	N
Calls	All	All	41.20	3.01	11.06	45.94	137.81	2,827,415

Graph I
Leverage Composition

The graph represents a histogram of the leverage ratio from the dataset consisting of 188 firms and displays the frequency distribution of the leverage ratio. The histogram also includes the percentiles (5th, 25th, 75th, 95th).

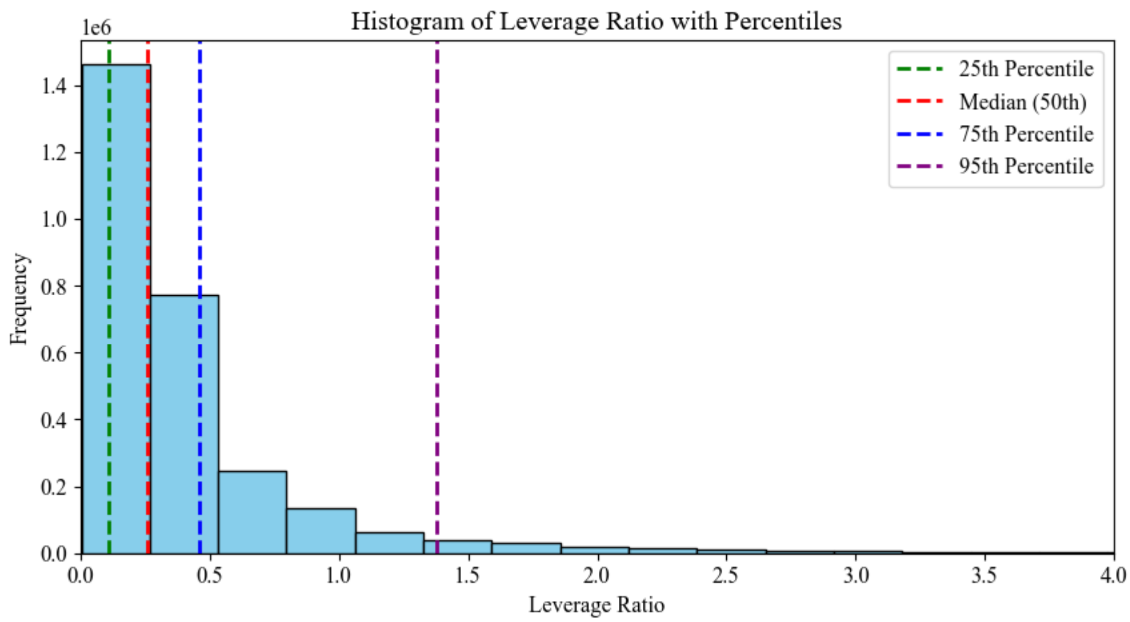


Table III
Top Ten Industries in Sample Data

The sample data contains 188 firms on the NASDAQ between 2001 and 2023. The firms within the sample vary in industry. The following table identifies the top 10 industries in decreasing order within the sample data.

SIC	Industry
7372	Prepackaged Software
3674	Semiconductor, Related Device
2834	Pharmaceutical Preparations
7389	Business Services, N.E.C
2836	Biological Products, Except Diagnostics
7370	Computer Programming, Data Process
4841	Cable and Other Pay TV Services
7371	Computer Programming Service
3572	Computer Storage Devices
3576	Computer Communications Equipment

Table IV
Summary Statistics

The sample contains 2,827,415 observations on options for 188 firms between 2001 and 2023. The table represents the average implied volatility calculated using the compound option model and their standard deviations (SD) reported in percent and annualized. Sk denotes the coefficient of skewness. The time to maturity (TTM) is observed for options between [0.25-1.00]. In addition, the moneyness (K/S) for options between [0.89-1.50].

Panel A: By Maturity					
		Call options			
Mon.	τ	Avg.	SD	sk	N
All	[0.25-0.50)	36	6.85	1.256	1,735,314
All	[0.50-0.75)	35	6.91	1.201	765,828
All	[0.75-1.00]	35	6.89	1.416	326,273

Panel B: By Moneyness					
		Call options			
Mon.	τ	Avg.	SD	sk	N
ITM	All	33	6.94	1.296	523,531
ATM	All	33	6.70	1.208	1,079,351
OTM	All	39	6.98	1.277	1,224,533

4.4. Implied Volatility error of BS model and CO model by year

Table V
Mean Implied Volatility error for BS and CO by Year & TTM

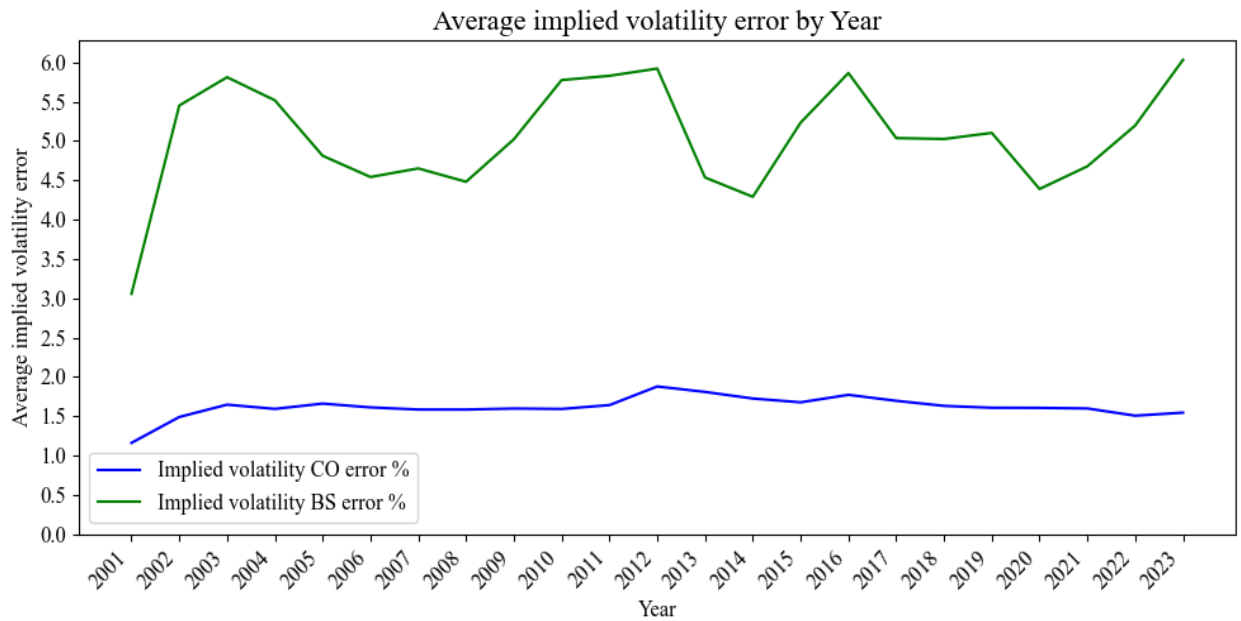
The sample contains 2,827,415 observations on options for 188 firms between 2001 and 2023. The following table reports the average percent implied volatility errors of Black and Scholes (BS) and compound option (CO) model valuations by calendar year from 2001 to 2023, as well as by time to expiration. The percent implied volatility error is defined as $\frac{|(Market-Model\ (BS\ or\ CO)|}{Market}$.

Panel A: Volatility error of BS model and CO model by year

	Black-Scholes model			Compound Option model			
Year	[0.25-0.50)	[0.50-0.75)	[0.75-1.00]	[0.25-0.50)	[0.50-0.75)	[0.75-1.00]	N
2001	3.46	2.55	1.88	1.11	1.27	1.22	14,040
2002	5.79	4.69	4.73	1.33	1.79	2.09	9,788
2003	6.11	5.23	3.85	1.54	1.87	2.21	17,689
2004	5.75	5.09	5.04	1.53	1.68	1.86	32,146
2005	5.09	4.27	4.16	1.64	1.66	1.83	39,810
2006	4.84	3.97	4.12	1.61	1.58	1.80	57,174
2007	4.84	4.32	4.30	1.56	1.58	1.87	74,431
2008	4.70	4.12	3.90	1.56	1.61	1.71	45,948
2009	5.26	4.34	4.12	1.58	1.63	1.72	57,750
2010	6.09	4.93	5.59	1.57	1.61	1.92	95,446
2011	6.18	5.16	5.24	1.60	1.66	1.90	127,199
2012	6.26	5.26	5.57	1.86	1.89	1.93	128,982
2013	4.72	4.14	4.42	1.83	1.72	1.94	134,332
2014	4.71	3.70	3.40	1.78	1.60	1.71	151,884
2015	5.50	4.82	4.79	1.68	1.62	1.82	131,744
2016	6.15	5.44	5.31	1.75	1.78	1.87	123,100
2017	5.26	4.64	4.75	1.72	1.58	1.83	187,360
2018	5.42	4.35	4.56	1.67	1.50	1.73	210,582
2019	5.40	4.61	4.61	1.61	1.50	1.83	193,987
2020	4.62	4.05	4.09	1.61	1.51	1.80	290,156
2021	5.12	4.17	3.82	1.63	1.45	1.75	324,308
2022	5.50	4.79	4.73	1.43	1.49	1.81	235,269
2023	6.36	5.82	5.31	1.51	1.54	1.68	144,290

Graph II
Average Implied Volatility Error by Year

The graph below shows the average percent error of implied volatility that is calculated using both the compound option model and the Black Scholes model



4.5. Implied Volatility error of BS model and CO model by leverage & TTM

Table VI
Mean Implied Volatility error for BS and CO by Leverage & TTM

The sample contains 2,827,415 observations on options for 188 firms between 2001 and 2023. Panel A reports the mean percent volatility errors of Black and Scholes (BS) and the compound option (CO) model valuations by leverage ratio (sample data divide in quantiles), as well as by time to expiration (TTM) between three months and one year for all moneyness. The sample data leverage ratio by TTM was also observed by moneyness ITM (Panel B), ATM (Panel C) and OTM (Panel D). The percent implied volatility error is defined as $\frac{|(Market - Model\ (BS\ or\ CO)|}{Market}$.

Panel A: All

Mon.	τ	Leverage Quantiles	Mean error: Black-Scholes model	Mean error: Compound Option model	N
All	All	0-25%	4.67	1.42	706, 874
All	All	26-50%	4.97	1.53	706, 847
All	All	51-75%	5.30	1.63	706, 873
All	All	76-100%	5.26	1.98	706, 821

Panel B: By ITM

	Mean error						
	Black-Scholes model			Compound Option model			
Leverage Quantiles	[0.25-0.50)	[0.50-0.75)	[0.75-1.00]	[0.25-0.50)	[0.50-0.75)	[0.75-1.00]	N
0-25%	3.81	3.26	3.01	1.29	1.47	2.02	121,360
26-50%	4.12	3.59	3.27	1.47	1.44	1.98	124,116
51-75%	4.80	4.12	3.52	1.66	1.54	2.18	130,506
76-100%	5.36	4.79	4.48	2.46	2.26	2.13	147,549

Panel C: By ATM

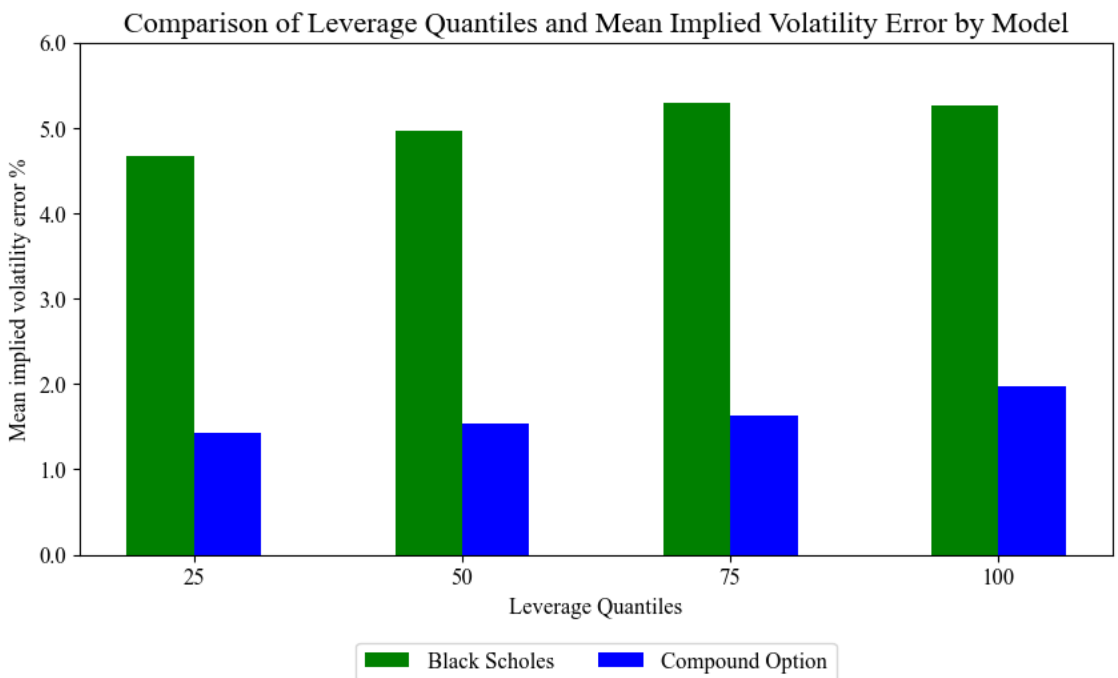
	Mean error						
	Black-Scholes model			Compound Option model			
Leverage Quantiles	[0.25-0.50)	[0.50-0.75)	[0.75-1.00]	[0.25-0.50)	[0.50-0.75)	[0.75-1.00]	N
0-25%	1.18	0.95	0.90	0.80	0.92	1.09	242,562
26-50%	1.32	1.06	0.94	1.03	1.11	1.50	262,587
51-75%	1.59	1.27	1.02	1.25	1.27	1.56	279,386
76-100%	1.85	1.49	1.29	1.79	1.72	1.77	294,816

Table VI (continued)

Panel D: By OTM

Leverage Quantiles	Mean error						N
	Black-Scholes model			Compound Option model			
	[0.25-0.50)	[0.50-0.75)	[0.75-1.00]	[0.25-0.50)	[0.50-0.75)	[0.75-1.00]	
0-25%	8.16	6.85	6.34	1.80	1.77	1.87	342,952
26-50%	9.14	7.60	7.23	1.91	1.78	1.91	320,144
51-75%	10.15	8.45	7.53	1.98	1.83	1.96	296,981
76-100%	9.82	8.52	8.03	2.07	1.88	1.92	264,456

Graph III
Comparison of Leverage Quantiles and Mean Implied Volatility Error by Model



4.6. Implied Volatility error of BS model and CO model by moneyness & TTM

Table VII
Mean Volatility error for BS and CO by Moneyness & TTM

The sample contains 2,827,415 observations on options for 188 firms between 2001 and 2023. Panel A reports the mean percent implied volatility errors of Black Scholes (BS) and the compound option (CO) model valuations by moneyness, as well as by time to expiration (TTM) between three months and one year. The percent implied volatility error is defined as $\frac{|(Market-Model\ (BS\ or\ CO)|}{Market}$.

Panel A: By All							
	Mean error						
	Black-Scholes model			Compound Option model			
Mon.	**[0.25-0.50)**	**[0.50-0.75)**	**[0.75-1.00]**	**[0.25-0.50)**	**[0.50-0.75)**	**[0.75-1.00]**	**N**
ITM	4.58	3.97	3.58	1.76	1.69	2.08	523,531
ATM	1.51	1.20	1.04	1.25	1.26	1.49	1,079,351
OTM	9.25	7.79	7.25	1.93	1.81	1.92	1,224,533

4.7. Implied Volatility error of BS model and CO model by leverage & moneyness

Table VIII
Mean Volatility error for BS and CO by Leverage & Moneyness

The sample contains 2,827,415 observations on options for 188 firms between 2001 and 2023. The table reports the mean percent volatility errors of Black and Scholes (BS) and the compound option (CO) model valuations by leverage quantiles as well as by moneyness across all time to maturities ranging from 3 months to one year. The percent volatility error is defined as $\frac{|(Market - Model(BS\ or\ CO)|}{Market}$.

Panel A: By All

	Mean error						
	Black-Scholes model			Compound Option model			
Leverage Quantiles	ITM	ATM	OTM	ITM	ATM	OTM	N
0-25%	3.58	1.09	7.59	1.42	0.87	1.80	706,874
26-50%	3.89	1.21	8.47	1.52	1.10	1.88	706,847
51-75%	4.47	1.44	9.31	1.68	1.29	1.94	706,873
76-100%	5.14	1.71	9.28	2.38	1.77	2.01	706,821

5. DISCUSSION

5.1. Empirical Findings

The results from the sample data consisting of 2,827,415 options and representing 188 firms listed on the NASDAQ-100 from January 2001 and August 2023, are observed in the various tables and graphs in section *4-Empirical Results* of this research paper. The percent error of implied volatility of both the Black Scholes (BS) and the compound option (CO) model are analysed and compared.

5.1.1. Sample Data Analysis

Panel A: All options of Table I- Sample Composition, presents the average and the quantiles (5th and 95th) for the relative option strike prices (K/S) and the implied volatility obtained from both the CO model and BS model. The results infer that the average implied volatility is 36% for CO and 35% for BS with 5th and 95th percentiles of roughly 20% and 58% respectively. Panel B, which analyzes the results by maturity and Panel C by moneyness, also show that the average implied volatility and percentiles for the two models are similar.

Table II- Leverage Composition shows that there is a diverse range of financial leverage among the firms, as evidenced by the varying leverage ratios. The mean leverage is 41.20%, indicating that on average the firms in the sample data have a market value of equity that is greater than the face value of debt. The 75th percentile is evaluated at 45.94%, indicating that three-quarters of the firms have a leverage ratio of that level or below. In contrast, at the 95th percentile there is an increase in the leverage ratio which is 137.81%. The 95% percentile of the data point is a small portion of the sample that consists of highly levered firms in comparison to the average firms observed.

Panel A of *Table IV- Summary Statistics* represents the summary statistics for call options categorized by time to maturity across all moneyness. The results show that there are stable standard deviations (6.85, 6.91, 6.89), revealing steady market conditions and pricing behaviors for the compound option model. The skewness values (1.256, 1.201, 1.416) indicate an asymmetry, with a rightward skewness with a greater likelihood of significant positive price movements of the underlying stock as the maturity increases, particularly for options within the maturity range ([0.75-1.00]).

Panel B of *Table IV* categorizes call options based on their moneyness—In-the-Money (ITM), At-the-Money (ATM), and Out-of-the-Money (OTM) and presents data on the average implied volatility (Avg), Standard Deviation (SD), and Skewness (sk). It reveals that expected returns are relatively uniform across the different moneyness categories, which range from 33% and 39%. The OTM options are showing slightly higher average volatility. The standard deviation is also comparable across the categories, although slightly higher for OTM options at 6.98, versus 6.70 for ATM and 6.94 for ITM options, reflecting the increased risk and sensitivity of OTM options to price changes in the underlying asset. Skewness across all categories is positive, suggesting a rightward skew in the distribution. The ATM options showing the lowest skewness at 1.208, meanwhile, ITM and OTM options are indicating a higher probability of achieving significantly above-average volatility.

5.1.2. Pricing variation across time

To observe the benefit of incorporating firm leverage in option pricing, the Black Scholes and compound option model are compared to each other by observing certain behaviours and patterns across the years. *Table V* reveals the implied volatility error for both the Black Scholes model and the compound option model by year and time to maturity (TTM). The data reveals that from 2001 to 2023, the CO model predicts the implied volatility more accurately, as indicated by the consistently lower percent errors. It is observed that over the years the CO model performs better for the shorter maturity buckets (0.25-0.50), in comparison to the longer time to maturities (0.75-1.0). Proposing that the improved model performance for shorter maturing options is due to less uncertainty for changes in the debt-equity levels in the short term, therefore they are less volatile.

Furthermore, for the CO model the results show that there is no consistent pattern of increase or decrease of implied volatility error from one year to the next. Therefore, the accuracy for pricing may not vary specifically from one year to the other or in terms of time. However, it is observed that in the years for which there was financial distress (e.g. 2002, 2003, 2008, 2020), the percent errors are greater during those periods. This is consistent with *Graph II- Average Implied Volatility Year*, which shows that the average implied volatility error during those specific years is higher. That greater variability can be the cause for the increase in percent error that is observed.

5.1.3. Economic events impacting price variation

Between 2001 and 2023, there have been several economic events that have impacted the financial markets. Notably, in 2001, the dot-com bubble burst, which was a result of a decline in the linear relationship between earnings and stock returns for firms in the technology sector (Morris & Alam, 2012). During that period, there was an increase in IPOs for dotcom firms without much understanding of their debt equity structure and the viability of their businesses to generate profit (Magin & DeLong, 2006). The NASDAQ experienced substantial loss between February 2000 to October 2002, where it lost nearly three quarter of its value, resulting in one of the most significant declines in technology stocks (Magin & DeLong, 2006). This phenomenon can explain the results in *Graph II*, where we see an increase in the implied volatility error in 2002. The increase in implied volatility is more pronounced in the Black Scholes model whose recovery began only as of 2003. In addition, this noticeable implied volatility error is expected since the firms within the sample data are mainly technological companies, evident from *Table II -Top Ten Industries in Sample Data.*

Furthermore, *Graph II*, shows that in 2012 there was an increase in the implied volatility error. This could be explained by the U.S fiscal cliff, which occurred during that period and threatened tax increases within the United States along with spending cuts, which caused uncertainty in the market (Brown, 2012). The fiscal cliff refers to the end of the Bush era tax cuts, the 2% reduction in payroll taxes, and other tax breaks that were coming to expiration during that period (Brown, 2012). During this time, fear of a recession was looming which caused uncertainty in the US economy (Brown, 2012) and is evident from the increase in implied volatility error for both the BS and CO models.

The compound option model results show that it is less impacted by the events mentioned above, primarily because both incidences revolve around debt. The firms mainly impacted tend to be those that are heavily levered. The CO model incorporates leverage, therefore, the model accounts for that uncertainty. The changes in the debt-equity ratio alters the total risk and in turn it is reflected in the implied volatility results obtained from the CO model.

5.1.4. The leverage effect in option pricing

Graph I-Leverage Composition, illustrates debt-equity ratio among the firms from the data sample. Many of the firms have a leverage ratio below fifty percent. Furthermore, most of the firms are moderately levered, while a few outliers are highly levered, substantially inflating the average. The sample data supports the reality that there is significant variability in how firms manage their leverage from conservative to a highly levered approach.

The 188 firms from the sample data represent 81 different industries according to their respective standard industrial classification (SIC) code. It is apparent in *Table III-Top Ten Industries in Sample Data*, which lists the top 10 industries from the data, that there is a large number of firms in the technology sector. Moreover, the results shows that 49.5% of the firms within the sample focus mainly on industries that are at the forefront of technological advancements as well as in research and development. These results are to be expected, since the firms on the NASDAQ-100 are mainly in the technology sector.

Panel A from *Table VI* reports the mean percent volatility errors for all options under both the Black and Scholes (BS) model and the compound option (CO) model valuations. The results observe the leverage ratios which are divided into quantiles. The results show that the CO model consistently has lower mean error values in comparison to the Black-Scholes model across the leverage quantiles. The results suggest that the compound option model is better suited at handling the sample data across the different leverage buckets. In addition, as the leverage quantiles increase from 0-25% to 51-75%, both models experience an increase in the percent error, suggesting that higher debt-equity ratios cause more uncertainty for both option pricing models. For the CO model, the greater mean error is identified to be in the 75-100% percentile, suggesting the model is sensitivity to firms with very high leverage, although it is noticeable from the error values it is still significantly less than the BS model (the BS model error is on average 3.4% greater).

Panel B of *Table VI* observes the mean percent implied volatility errors of the Black Scholes model and the compound option model by quantile leverage ratio, as well as by time to expiration (TTM) for ITM options. For the CO model, the results show that for the options in time buckets (0.75-1.0), the mean percent volatility errors are slightly greater than for shorter maturities, which was not the same behavior as the results for the implied volatility errors from the Black Scholes model. Although, both models did show the pattern that as the leverage quantiles increased so did the implied volatility error. This indicates that both models are sensitive to firms with greater debt to equity ratios for ITM options. Although, the BS model implied volatility errors are nearly double those of the CO model. Therefore, the compound option model pricing performance is still better at pricing ITM options.

The results for the out of the money (OTM) options that are in Panel D of *Table VI,* indicate that the compound option model consistently outperforms the Black Scholes model in terms of mean implied volatility error. The BS model has greater error for the mid to high leverage buckets. While the CO model maintains a more uniform error distribution.

Panel C of Table VI shows the implied volatility errors of the Black Scholes (BS) model and the compound option (CO) model by leverage ratio quantiles, time to expiration (TTM) for at the money options (ATM). Both models perform best for ATM options because they are calibrated and optimized with these options. This explains the relatively low percent implied volatility error. Similarly, Panels B and D of Table VI, which are the results for the ITM and OTM options respectively, have higher degree of implied volatility error at higher leverage buckets. Overall, the results from Table VI show that based on the low implied volatility errors, that the CO model can capture leverage dynamics better than BS, in particular for the options of firms in the higher leverage quantiles.

5.1.5. Pricing variation by moneyness and TTM

The results from *Table VII,* display the mean implied volatility error for both the Black Scholes model and compound option model by moneyness and time to maturity. It is observed that the compound option model performs better than the Black Scholes model across all moneyness and TTM. The Black Scholes model struggles at pricing options that are out of the money, evident by the greater implied volatility error. The BS model has greater, implied volatility error, specifically for options that are both in the money and at the money. Meanwhile, the CO model, generally has the same behaviour throughout the moneyness and TTM buckets, suggesting the model is not sensitive to these factors.

Table VIII shows the results for the mean percent volatility errors for both the Black Scholes and the compound option model by leverage quantiles and by moneyness across all TTM. The Black Scholes model has a higher mean implied volatility error throughout but specifically for the heavily levered firms who have options that are OTM. The compound option model also displays greater error at the higher leverage quantiles (76-100%), however the CO model has significantly less implied volatility error than the BS model, especially for OTM options.

5.1.6. Overall Summary of Results

The overall results of my research imply that the compound option model is better at pricing options than the Black Scholes model. The CO model has lower implied volatility errors, especially when compared to the BS model in terms of moneyness, time to maturity and the leverage ratio quantiles.

The results show that the compound option model works better than the Black Scholes model because it can capture changes in the equity volatility. The changes in the equity volatility are captured because it appears to vary with leverage. This is evident in both *Tables VI and VIII*. The Black Scholes model has difficulty pricing options for the firms with greater leverage, evidenced by the greater percent error of implied volatility. Meanwhile, the CO model has more stable errors throughout the leverage quantiles. The results obtained are in accordance with Geske, Subrahmanyam and Zhou's (2016) research. Their results showed that the compound option model performed better at pricing over the Black Scholes model, especially for the firms with greater leverage ratios (151-200%) (Geske et al, 2016). Therefore, the improvement observed from the results of the compound option pricing model is the effect of incorporating leverage on asset prices as the strike price (Geske et al, 2016).

The objective of the following research was to analyse the compound option pricing model and determine whether incorporating leverage can improve the performance in call option pricing. The results do suggest that the CO model captures the variance in equity better than the BS model especially for the firms with higher leverage. Moreover, what is interesting is that the results show the compound option model's pricing performance for firms that are mainly in the technology sector. These are firms that tend to have a lot of variation of leverage over time. The compound option model can capture the changes in equity volatility successfully because financial leverage alters the volatility of equity as the market attempts to continuously reevaluate the firm's cash flow (Geske, 1979).

5.2. Limitations of Compound Option Model

Overall, it is fair to state that the CO model is a reliable model that can be used to price options as it takes into account the leverage effect. However, one of the model's weaknesses is the sensitivity of the optimizer with respect to the initial starting values for the four unknowns it is solving. If the starting values are too distant from the actual values, the solution generates a greater error. To mitigate this negative impact, the initial values are tested to ensure they reflect market conditions. Furthermore, another weakness is that the options from the dataset are American options, but the model treats them as European options.

Another drawback of the compound option model is that it requires four additional parameters (D, D*, $\sigma_{v_{Ti}}$, $\sigma_{v_{Td}}$) to price the option. These parameters need to be implied and solved for in order to price the call option using the CO model. This is more complex and longer to compute in comparison to the more simplistic Black Scholes model. In addition, the extra parameters also make it easier to fit the options prices, therefore large data is required to avoid overfitting the sample.

The CO model solves the four unknowns (D, D*, $\sigma_{v_{Ti}}$, $\sigma_{v_{Td}}$) using a python optimizer. To solve the four unknowns, three closest to the ATM options are used. In some cases, within a certain time bucket, there are only a limited number of options for a firm. Hence, the model can optimize the four unknown values to those options better and therefore the error for those options is minimal.

Furthermore, the volatility of the stock in the compound option model is time-varying, this is however not the same as for stochastic volatility models. This is because the volatility only depends on the leverage and the leverage in the end only depends on the value of the assets. This implies that volatility is a function of the assets and is driven by the same source of risk. This risk can only be hedged if the assets are sold, which means that it cannot really be hedged away. Therefore, it suggests that volatility is driven by a separate source of risk which is captured in stochastic volatility models and not in the CO model.

To fully evaluate the option pricing performance of the compound option model in relation to the leverage effect, it would be more comprehensive if a stochastic volatility model is also considered.

Previous studies have shown that the asset price volatility is stochastic, which suggests that the underlying forces for the return variation includes an unobservable shock (Veraart & Veraart, 2012).

If the behaviour of a stock price satisfies a stochastic differential equation, then the stock price can be described by a stochastic model (Fouque & al, 2000) such as:

$$dX_t = \mu X_t dt + f(Y_t) X_t dW_t$$

$$dY_t = bY_t dt + \sigma Y_t dZ_t$$

The W and Z are Brownian motions, which are the components that the model changes randomly and continuously over very small intervals of time (Veraart & Veraart, 2012).

To fully appreciate and evaluate the option pricing performance of the compound option model, it would be interesting to compare the CO model to a stochastic volatility model using the same sample set. A stochastic volatility model to consider would be the Heston model.

5.2.1. The Heston model

The Heston model is a stochastic volatility model which was presented in a research paper by Heston (1993) titled “A Closed-Form Solution for Options with Stochastic Volatility with Applications to Bond and Currency Options”. It is a commonly used stochastic volatility model, primarily because it allows for the random correlation between volatility and spot asset returns (Heston, 1993). As per Heston’s research, the correlation between the volatility and the spot asset price, is responsible for explaining return skewness and biases in the strike price from the Black Scholes (1973) model (Heston, 1993).

The Heston model assumed that the stock price (S_t) is lognormally distributed and that the volatility (V_t) follows a Cox-Ingersoll-Ross process (CIR process) (Yang, 2013). The model is as follows (Yang, 2013):

$$dS_t = \mu S_t dt + \sqrt{V_t} X_t dW_t$$

$$dV_t = x(\theta - V_t)dt + \sigma\sqrt{V_t} dZ_t$$

$$dW_t dZ_t = \rho dt$$

The following parameters are used:

$\mu = drift\ coefficient\ for\ the\ stock\ price$

$\theta = long\ term\ mean\ variance$

$x = mean\ reversion$

$\sigma = volatility\ of\ volatility$

The Heston model also considers the leverage effect, for which the stock returns and implied volatility are negatively correlated (Yang, 2013). As well, the Brownian motions W_t and Z_t are also correlated processes considered by the correlation coefficient ρ (Yang, 2013).

The advantages of the Heston model is that it can better explain the stock price when it demonstrates a non-Gaussian distribution (Yang, 2013). Furthermore, it is better at fitting the implied volatility surface of option prices, and takes into account the negative correlation between stock price and volatility (Yang, 2013). Due to the component that considers stochastic leverage, it also incorporates an additional factor of randomness in the model to better predict market volatility (Yang, 2013).

Furthermore, the Heston model also has its disadvantages. It is sensitive to the variations of its parameters, therefore the Heston model requires calibration (Yang, 2013). As well, for options with short maturity, the Heston model has difficulty capturing the skew that reflects the market (Mikhailov & Nogel, 2003).

Inconclusion, all models, whether it be the Heston model or the compound option model, these models are not able to always perfectly capture the market behaviour and the option prices obtained may deviate from to the actual option prices.

6. CONCLUSION

The following research analyses the compound option pricing model and whether incorporating leverage can improve the performance in call option pricing. This research paper collected data from 188 firms and analysed a total of 2,827,415 options between the years, 2001 and 2023. This research is inspired by the paper titled “Capital structure effects on the prices of equity call options” by Geske, Subrahmanyam and Zhou (2016), who also tested and analyzed the performance of the compound option model.

What distinguishes the compound option model from the Black Scholes model is that the CO model accounts for debt and the expiration of debt obligations (Geske et al., 2016). The Black Scholes model on the other hand, considers that the firm has no debt in its pricing model (Geske et al., 2016). For the purpose of this research, the analysis involves calculating the percent error of implied volatility for both the Black Scholes no arbitrage option pricing model and the compound option model and compares their pricing performance for firms listed on the NASDAQ-100.

The percent error of implied volatility for both models are compared according to time in years, moneyness, time to maturity of the option and to the debt-equity ratio of the firms. Overall, the results show that there is an advantage in using the compound option model as it outperforms the Black Scholes pricing model, evident by the smaller percent error of implied volatility.

Furthermore, the results show that as the leverage ratio increases, the implied volatility error also increases. The results indicate that there is a relationship where the options of firms with a greater debt-equity ratio will have a greater variation in implied volatility, evident from the increase in percent error of the implied volatility. However, the compound option model performs overall better than the Black Scholes model, even for the higher levered firms. The reason for this improvement is that the compound option model can capture changes in equity volatility, which is not the case for the Black Scholes model. Also, an interesting observation from this research paper is the pricing performance of the compound option model on options of firms that are primarily in the technology sector. These firms are known to have a lot of variation in leverage over time, however the CO model can capture this behaviour well.

The results obtained further supports that the improvement of the compound option model to price options is because the CO model includes financial leverage as the strike price in its pricing formula. The inclusion of the debt value allows to capture the time variation of the volatility of the firm (Geske et al., 2016) as seen from the performance of the model from the results.

The implication of this research is that it supports the findings from other research papers on how the compound model's pricing performance is better than the Black Scholes model. Furthermore, it shows how debt is an important variable to capture the stock's stochastic process and if omitted can generate pricing errors (Geske et al., 2016).

7. BIBLIOGRAPHY

Adrian, T., & Song Shin, H. (2010). Liquidity and leverage. *Journal of Financial Intermediation*, *19*(3), 418-437. https://doi.org/10.1016/j.jfi.2008.12.002

Andersen, T. G., Benzoni, L., & Lund, J. (2002). An Empirical Investigation of Continuous-Time Equity Return Models. *The Journal of Finance*, *57*(3), 1239-1284. https://doi.org/10.1111/1540-6261.00460

Apergis, N., Mustafa, G., & Malik, S. (2023). The role of the covid-19 pandemic in us market volatility: evidence from the vix index. *Quarterly Review of Economics and Finance, 89*, 27–35. https://doi.org/10.1016/j.qref.2023.03.004

Bachelier, L. (1900). Théorie de la spéculation. *Annales Scientifiques de l'École Normale Supérieure, 17,* 21-86.

Baker, H. K., & Martin, G. S. (2011). *Capital structure and corporate financing decisions : theory, evidence, and practice*. John Wiley & Sons.

Bakshi, G., Cao, C., & Chen, Z. (1997). Empirical performance of alternative option pricing models. *The Journal of Finance, 52*(5), 2003–2049.

Bates, D. S. (2000). Post-'87 crash fears in the s&p 500 futures option market. *Journal of Econometrics, 94*(1), 181–238. https://doi.org/10.1016/S0304-4076(99)00021-4

Ben-Zhang, Y., Lu, X., Ma Guiyuan, & Song-Ping, Z. (2020). Robust Portfolio Optimization with Multi-Factor Stochastic Volatility. *Journal of Optimization Theory and Applications, 186*(1), 264–298. https://doi.org/10.1007/s10957-020-01687-w

Black, F., & Scholes, M. (1973). The pricing of options and corporate liabilities. *Journal of Political Economy, 81*(3), 637–654.

Boudreault, M., & Renaud, J.-F. (2019). *Actuarial finance : derivatives, quantitative models and risk management*. John Wiley & Sons, Inc.

Bowman, R. G. (1980). The importance of a market-value measurement of debt in assessing leverage. *Journal of Accounting Research, 18*(1), 242–242.

Brown, S. J. (2012). Addressing the fiscal cliff. American Bankers Association. *ABA Banking Journal, 104*(12), 8.

Chacko, G., Sjöman Anders, Motohashi, H., & Dessain, V. (2015). *Credit derivatives: a primer on credit risk, modeling, and instruments (Revised)*. Pearson Education, from http://proquest.safaribooksonline.com/?fpi=9780133249217.

Chen, J., & Murry, C. (2022, May 28). How debt financing works, examples, costs, pros & cons. *Investopedia*. https://www.investopedia.com/terms/d/debtfinancing.asp

Chiarella, C., He, XZ., Nikitopoulos, C.S. (2015). Derivative Security Pricing. *Dynamic Modeling and Econometrics in Economics and Finance*, 21, 371-387, Springer. https://doi-org.proxy2.hec.ca/10.1007/978-3-662-45906-5_17

Choi, J., & Richardson, M. (2016). The volatility of a firm's assets and the leverage effect. *The Journal of Financial Economics*, 254-277.

Choi, S.-Y., Kim, J.-H., & Yoon, J.-H. (2016). The Heston model with stochastic elasticity of variance. *Applied Stochastic Models in Business and Industry, 32*(6), 804–824. https://doi.org/10.1002/asmb.2203

Christie, A. A. (1982). The Stochastic Behavior of Common Stock Variances: Value, Leverage and Interest Rate Effects. *Journal of Financial Economics, 10*(4), 407.

Christoffersen, P., Heston, S., Jacobs, K. (2009). The shape and term structure of the index option smirk: Why multifactor stochastic volatility models work so well. *Management Science, 55*(12):1914–1932. DOI: 10.1287/mnsc.1090.1065.

Cont, R., & Tankov, P. (2004). *Financial modelling with jump processes*. Chapman & Hall/CRC. http://www.myilibrary.com?id=22636

Dennis, P., & Mayhew, S. (2002). Risk-neutral skewness: evidence from stock options. *Journal of Financial and Quantitative Analysis, 37*(3), 471–493.

Derman, E., Kani, I., (1994). Riding on a smile. Risk 7, 32-39.

Duffie, D., Pan, J., & Singleton, K. (2000). Transform analysis and asset pricing for affine jump-diffusions. *Econometrica*, *68*(6), 1343–1376.

Dumas,B., Fleming, J., & Whaley, R. E. (1998). Implied volatility functions: Empirical tests. *The Journal of Finance*, *53*(6), 2059–2106.

Dupire, B (1994). Pricing with a Smile, Risk, 7(1), 18-20.

Engle, R. F. (1982). Autoregressive conditional heteroscedasticity with estimates of the variance of United Kingdom inflation. *Econometrica* (Pre-1986), 50(4), 987–987.

Eriotis, N., Vasiliou, D., & Ventoura-Neokosmidi, Z. (2007). How firm characteristics affect capital structure: An empirical study. *Managerial Finance*, *33*(5), 321-329.

Fouque, J.-P., Papanicolaou, G. and Sircar, K.R. (2000) Derivatives in Financial Markets with Stochastic Volatility. *Cambridge University Press*, Cambridge.

Fouque, J.-P., Papanicolaou, G., Sircar, R., & Solna, K. (2003). Multiscale Stochastic Volatility Asymptotics. *Multiscale Modeling & Simulation*, 2(1), 21. https://doi.org/10.1137/030600291

Fouque, J.-P. (2011). *Multiscale stochastic volatility for equity, interest rate, and credit derivatives*. Cambridge University Press.

Geske, R. (1979). The Valuation of Compound Options*. *Journal of Financial Economics, 7*, 63-81.

Geske, R.,Subrahmanyam, A., & Zhou, Y.(2016). Capital Structure effects on the prices of equity call options. *Journal of Financial Economics, 121*, 231-253.

Geske, R. L., & Zhou, Y. (2007). *The Effects of Leverage on the Pricing S&P 500 Index Call Options*. https://ssrn.com/abstract=956708

Hainaut, D., & Moraux, F. (2019). A switching self-exciting jump diffusion process for stock prices. *Annals of Finance*, *15*(2), 267–306. https://doi.org/10.1007/s10436-018-0340-5

Heston, S. L. (1993). A Closed-Form Solution for Options with Stochastic Volatility with Applications to Bond and Currency Options. *The Review of Financial Studies*, *6*(2), 327–343.

Hull, J., & White, A. (1987). The pricing of options on assets with stochastic volatilities. *The Journal of Finance, 42*(2), 281–281.

Isidore, C. (2018, December 31). *2018 was the worst for stocks in 10 years*. CNN Business. https://www.cnn.com/2018/12/31/investing/dow-stock-market-today/index.html

Jones, C. S. (2003). The dynamics of stochastic volatility: Evidence from underlying and options markets. *Journal of Econometrics*, *116*(1-2), 181-224. https://doi.org/10.1016/S0304-4076(03)00107-6

Kenton, W., & Silberstein, S. (2023, July 5). Moneyness Definition and Intrinsic Value of Options. Investopedia. https://www.investopedia.com/terms/m/moneyness.asp

Korteweg, A. (2010). The Net Benefits to Leverage. *The Journal of Finance*, *65*(6), 2137-2169. https://doi.org/https://doi-org.proxy2.hec.ca/10.1111/j.1540-6261.2010.01612.x

Magin, K., DeLong, J. B., & National Bureau of Economic Research. (2006). *A Short Note on the Size of the Dot-Com Bubble*. National Bureau of Economic Research. http://papers.nber.org/papers/w12011

Mandelbrot, B. (1963). The variation of certain speculative prices. *The Journal of Business (Pre-1986), 36*(4), 394–394.

Maverick, J., & Catalano, T. J. (2021, December 13). *Debt Market vs. Equity Market: What's the Difference? Investopedia.* https://www.investopedia.com/ask/answers/071415/what-are-differences-between-debt-and-equity-markets.asp#:~:text=Investments%20in%20debt%20securities%20typically,the%20first%20to%20be%20paid.

Merton, R. C. (1973). Theory of rational option pricing. *The Bell Journal of Economics and Management Science, 4*(1), 141–183. https://doi.org/10.2307/3003143

Merton, R. C. (1974). On the pricing of corporate debt - the risk structure of interest-rates. *The Journal of Finance, 29*(2), 449–449.

Mikhailov, S., & Noegel, U. (2003). Heston’s stochastic volatility model implementation, calibration and some extensions. *Wilmott Magazine*, 401-413.

Murphy, J. E., Jr. (1968). Effect of Leverage on Profitability, Growth and Market Valuation of Common Stock. *Financial Analysts Journal*, *24*(4), 121-123. https://www.jstor.org/stable/4470380

Morris, J. J., & Alam, P. (2012). Value relevance and the dot-com bubble of the 1990s. *Quarterly Review of Economics and Finance, 52*(2), 243–243.

Rouah, F. (2015). *The Heston model and its extensions in VBA*. Wiley.

Rubinstein, M., (1994). Implied binomial trees. Journal of Finance, 49, 771-818.

Samuelson, P. (1965). Proof that properly anticipated prices fluctuate randomly. *Industrial Management Review*, 6, 41-50.

Samuelson, P. (1973). Mathematics of speculative prices. *SIAM Review*, 15, 1-42.

Soini, V., & Lorentzen, S. (2019). Option prices and implied volatility in the crude oil market. *Energy Economics, 83*, 515–539. https://doi.org/10.1016/j.eneco.2019.07.011

Thurner, S., Farmer, J. D., & Geanakoplos, J. (2012). Leverage causes fat tails and clustered volatility. *Quantitative Finance*, *12*(5), 695–707.https://doi.org/10.1080/14697688.2012.674301

Toarna, A., & Cojanu, V. (2015). The 2008 crisis: causes and future direction for the academic research. *Procedia Economics and Finance, 27*, 385–393. https://doi.org/10.1016/S2212-5671(15)01010-2

Toft, K., & Prucyk, B. (1997). Options on Leveraged Equity: Theory and Empirical Tests. *The Journal of Finance*, *52*(3), 1151-1180. https://doi.org/10.1111/j.1540-6261.1997.tb02728.x

Veraart, A. E. D., & Veraart, L. A. M. (2012). Stochastic volatility and stochastic leverage. *Annals of Finance, 8*(2-3), 205–233. https://doi.org/10.1007/s10436-010-0157-3

Xu, X., & Taylor, S. J. (1994). The term structure of volatility implied by foreign exchange options. *Journal of Financial and Quantitative Analysis, 29*(1), 57–57.

Yan, J. (2018). *Introduction to stochastic finance*. Springer.

Yang, Y. (2013). *Valuing a European Option with the Heston Model [Master's Thesis]*. Rochester Institute of Technology School of Mathematical Sciences College of Science.

Wu, H., Ni, Y., & Yang, X. (2023). Compound option pricing problem in uncertain environment. *Journal of Ambient Intelligence and Humanized Computing* , 593-605.