HEC MONTRÉAL

The Electric Bus Rostering and Charging Scheduling Problem with Uncertain Energy Consumption: a Two-Stage Stochastic Programming Approach

par

Kyra Ann D'Ignazio

Jorge E. Mendoza Gimenez
HEC Montréal
Directeur de recherche

Sciences de la gestion
(Spécialisation Global Supply Chain Management)

Mémoire présenté en vue de l'obtention
du grade de maîtrise ès sciences
(M. Sc.)

July 2024

Résumé

Les véhicules électriques sont l'une des technologies prometteuses utilisées dans le secteur des transports pour contribuer à réduire les impacts négatifs associés aux technologies traditionnelles. Afin d'aider à l'intégration des véhicules électriques dans les transports publics, cette étude se focalise sur l'analyse de la manière dont l'incertitude liée à la consommation des batteries électriques affecte les coûts d'exploitation associés aux décisions de recharge. Cette recherche formule le problème d'affectation d'itinéraires et confection des plans de recharge sous la forme d'un modèle de programmation stochastique. Les résultats établissent une corrélation entre le niveau d'incertitude de la consommation et la difficulté à résoudre le problème (c'est-à-dire le temps de résolution). Cette recherche contribue au domaine de la gestion de flottes de véhicules électriques en fournissant une méthodologie complète pour optimiser l'attribution des itinéraires de bus et les protocoles de recharge en tenant compte de la variabilité réelle de la consommation d'énergie.

Mots-clés

logistique de transport, recharge intelligente, programmation stochastique à deux étapes, véhicules électriques

Méthodes de recherche

Programmation linéaire en nombres entiers mixtes, programmation stochastique à deux étapes, problème d'alignement et de planification des bus électriques

Abstract

Electric vehicles are one of the promising technologies that are being used in the transportation sector to help reduce the negative impacts associated with traditional technologies. To foster the integration of electric vehicles into public transportation, this study focuses on the examination of how energy uncertainty in consumption impacts operating costs associated with charging decisions. This research formulates the energy uncertainty challenge as a two-stage stochastic programming model by employing mixed-integer linear programming. The model optimizes bus to route assignments and charging decisions such as when to charge, while accounting for the potential deviations in energy requirements that may result in an e-bus running out of energy which would require the recourse action to be used (i.e., deploying a diesel bus) in certain scenarios. The results establish a correlation between the level of uncertainty in consumption and the difficulty to solve the problem (i.e., CPU time). This research contributes to the realm of electric vehicle fleet management by providing a comprehensive methodology to optimize bus route assignments and charging protocols considering the real-world variability in energy consumption.

Keywords

transportation logistics, smart charging, two-stage stochastic programming, electric vehicles

Research Methods

Mixed Integer Linear Programming, Two-Stage Stochastic Programming, Electric Bus Rostering and Scheduling Problem

Contents

List of Figures

List of Acronyms

AC Alternating Current

CV Coefficient of Variation

DC Direct Current

E-BRCSPUEC Electric Bus Rostering and Charging Scheduling Problem with Uncertain Energy Consumption

e-bus electric-bus

EV Electric Vehicle

FEV Fully Electric Vehicle

LCA Life Cycle Analysis

OR Operations Research

SoC State of Charge

STM La Société de transport de Montréal

V Volt

VSP Vehicle Scheduling Problem

Acknowledgements

I would like to give thanks to my supervisor Professor Jorge E. Mendoza Gimenez for his enduring support and much-appreciated advice throughout my dissertation. Without his invaluable guidance, this dissertation would not have been possible. I want to thank my mother, my sister Tara and my boyfriend Sebastian for their support. I would also like to thank my research colleagues Mateo Espitia and Daniel García de Vicuña for their endless support.

Chapter 1

Introduction

Climate change is the defining problem of this century (United Nations, 2024). It has many profound effects on society, such as worsening air pollution, environmental degradation, energy insecurity, and other adverse effects. There are three strategies that can be deployed to mitigate climate change: switch to cleaner energy sources, reduce energy consumption, and build more energy-efficient systems (David Suzuki Foundation, 2024). One of the sectors where switching to cleaner energy sources may yield high benefits is transportation. The transportation industry is known to be very energy intensive and a heavy polluting industry. The transportation sector alone accounts for 20% of the world's primary energy consumption and whereby fossil fuels are the main fuel source that is widely available (Pustějovská et al., 2023).

Many different renewable energy sources are being studied at the moment to help decarbonize the transportation sector such as hydrogen and bio-gas (Pustějovská et al., 2023). Electricity emerges as a popular choice for decarbonizing the transportation sector (International Energy Agency, 2021). Many governments such as Canada, Norway, and China have implemented fiscal incentives such as purchase subsidies to help consumers be able to purchase electric cars (International Energy Agency, 2021). These initiatives have helped with the rise of purchases of electric cars. In 2022, electric cars made up 14% of all car sales, showcasing the growing global shift towards electric vehicles (International

Energy Agency, 2023). The rise in popularity makes electric cars a valuable alternative energy vehicle to research.

Readily available renewable technologies for electric vehicles (i.e., EVs) are split into two main categories: hybrid vehicles (i.e., diesel tank and electric battery) and fully EVs (i.e., electric battery only). The difference in carbon output between the mentioned technologies is due to the fuel sources. A hybrid vehicle has an internal combustion engine and an electric motor which uses electricity that is stored in a battery system (U.S. Department of Energy, 2023d). Hybrid vehicles belong to different families based on how they use the internal combustion engine and battery system together. They fall into two categories: mild or full hybrid (Racz et al., 2015). If the electric motor can drive the vehicle without the usage of an internal combustion engine during certain conditions, then it is considered a full hybrid vehicle (Racz et al., 2015). On the other hand, the mild hybrid vehicle always has to use the internal combustion engine to power the vehicle, while the electric motor just assists the former.

Fully EVs (i.e., FEVs) do not use internal combustion engines but rather they are completely powered by one or more electric motors. These electric motors carry out a need for a large battery pack to store energy and then power them (U.S. Department of Energy, 2023c). Based on this, FEVs are usually considered a better choice because they do not produce carbon emissions whereas a hybrid or diesel vehicle produce greenhouse gas emissions. For example, a diesel vehicle will produce 120 g of CO2 per km (Martins et al., 2013) versus a FEVs produces 0 g of CO2 per km. Moreover, FEVs require less maintenance because they do not need oil changes, transmission fluid replacements, or any other typical combustion engine maintenance (Optiom, 2024). Additionally, FEVs boost instant torque and acceleration, which translates into a smoother, more responsive and a quieter drive (Optiom, 2024).

As mentioned before, FEVs require charging the electric battery packs to power the electric motor. The most commonly used batteries in FEVs are lithium-ion batteries, nickel-metal hydride batteries, lead-acid batteries, and ultracapacitators (U.S. Department of Energy, 2023a). The batteries in FEVs store chemical energy that is then converted into electricity (Racz et al., 2015). The durability of a battery is a critical factor for assessing its life cycle. The life cycle of a battery represents the amount of charging-discharging cycles it can undergo (Racz et al., 2015). Moreover, the temperature of a battery can affect its life cycle (Racz et al., 2015). Constant exposure to extreme temperatures, due to inefficient charging practices like allowing the battery to fully deplete, can degrade the battery quality and reduce its lifespan. Demand-related decisions for FEVs involve managing and optimizing the energy demanded during the charging process of batteries. Inefficient charging and scheduling of FEVs could lead to fast and high-demand charging cycles which can put stress on the battery, potentially leading to faster degradation over time. This is why demand-related decisions are very important to study to understand how to best plan the charging schedules of FEVs batteries to avoid battery degradation.

Battery packs in FEVs are charged by plug-in chargers versus diesel vehicles which charge at a petrol station. On average, it takes about 5-10 minutes to fill up a diesel tank, whereas charging FEVs can take anywhere from 30 minutes to 10 hours, depending on the charging station type and battery size (Charged Future, 2020). This is why the charging decisions associated with FEVs are more complicated than those of diesel or hybrid vehicles.

The charging equipments for FEVs are categorized by the rate at which the batteries are charged. There are three categories of chargers for FEVs: Level 1 chargers, Level 2 chargers and direct current (DC) fast chargers (U.S. Department of Energy, 2023b). Level 1 chargers use alternating current (AC) and provides charging through a 120 volt (V) AC plug (U.S. Department of Energy, 2023b). Most homes can accommodate a Level 1 charger and can satisfy most individual driver's charging needs of a FEV. Level 2 chargers offers charging through 240 V (typical in residential homes) or 208 V (typical in public charging stations) electrical service. Level 2 chargers charge faster than Level 1 chargers. For example, for one hour of charging, a Level 1 charger can charge for 5 miles of range while a Level 2 charger can charge for 25 miles of range. DC fast charging equipment can charge approximately 100 to 200 plus miles of range per 30 minutes of charging (U.S. Department of Energy, 2023b). The type of charger used influences the charging strategy deployed to charge a FEV.

The two main charging strategies for FEVs are opportunity charging and overnight charging. Overnight charging is when FEVs charge at the depot overnight. Opportunity charging refers to "on-route charging" and "fast charging" whereby the FEV charges outside the depot (Paulraj, 2022). Level 1 and 2 chargers are often times used with overnight charging strategies due to the slow charging nature of this technology. Overnight at the depot, FEVs have time to charge using Level 1 and 2 chargers. Alternatively, DC fast charging equipment is used with opportunity charging strategies because the charging decisions can be done swiftly on-route. Although FEVs are a popular alternative fuel-powered option that have been adopted globally they come with a host of operational challenges related to their charging schedules. While charging technologies like Level 2 chargers are essential for supporting the adoption of FEVs, addressing the challenges of urban sprawl and inefficient public transportation in Canada is equally crucial in promoting sustainable commuting solutions.

Many Canadians use cars as their main mode of transportation due to the large nature of Canada's geographical size (Statistics Canada, 2023). This problem is only getting larger due to the growing trend of Canadians moving to suburbs around cities. Since 1996, 3.1 million Canadians moved to the suburbs (Gilmore, 2017). Canadians who live in the suburbs must commute to city centers to work via individual cars or public transportation systems. Many Canadians decide to commute to work in cars due to long commute times and unreliable public transportation services in less densely populated areas. To further illustrate this point, as of May 2023, eight out of ten Canadians mainly use a car, truck, or van to travel to work (Statistics Canada, 2023). In fact, only 12% of Canadians used public transportation to commute to work in 2017 (Gilmore, 2017). Efficient logistics of public transportation systems is crucial to increase the rate of Canadians using public transportation to commute to work. One solution to tackle unsustainable public transportation is efficient public transportation fueled by clean energy sources (Mead, 2024). Efficient sustainable public transportation systems are important for the Canadian society because they lead to less carbon emissions, a healthier population, fewer road accidents and less traffic congestion on roads (Gilmore, 2017). Therefore, operations research related to public transportation that uses clean energy sources such as FEVs is crucial for the Canadian society and other developed nations to transition toward a sustainable and healthy future. Moreover, the transportation industry is facing great pressure globally to switch to renewable energy sources. Legislation is making public institutions transition towards a fully electric fleet. For example, la Société de transport de Montréal (STM) aims to have only FEVs in its fleet by 2025. Moreover, the STM aims to have a minimum of 147 new FEVs in their fleet between 2025 and 2027 demonstrating the growing adoption of FEVs (Société de transport de Montréal, 2023). This trend demonstrates that FEVs are being incorporated into fleets of public transportation systems in urban cities. The operations research community has started to look at how they can solve operational issues related to the implementation of FEVs in public transportation systems.

Operations research related to the charging of FEVs aims to deal with the following issues:

- Limited charging infrastructure
- Short driving ranges for battery systems
- Long recharging time of battery systems
- Reduce operational charging costs
- Energy consumption uncertainty

The operations research (i.e., OR) community has started to research how to help guide charging-related decisions for FEVs in a public transportation system. OR can help optimize FEVs charging schedules to ensure smooth operations and avoid FEVs running out of charge. This translates into a reliable public transportation system that avoids delays and breakdowns, which could help increase the usage rate of public transportation in city centers.

Additionally, there are often times more FEVs than chargers. Therefore, efficient planning and scheduling of charging FEVs is crucial due to the limited number of chargers available at the depot.

Another motivation to do research on charging schedules is that even if one had enough chargers to charge the entire fleet of FEVs, it would be expensive cost-wise. Electricity costs are often based on the maximum energy used. These costs are often called *demand charges*. Demand charges represent the peak demand energy required from the entire facility from the grid. Therefore, if a fleet of FEVs were charging at the same time at the depot, this would increase the total amount of energy needed causing a large peak in demand from the grid which would translate into a very costly electricity bill due to the inefficient charging strategy used.

Lastly, electric batteries in FEVs can only cover a certain amount of distance on a single charge. For example, an 18-meter electric bus with a battery capacity of 350 kWh on a full charge can only cover a range in between 190 and 210 kilometres (Sustainable Bus, 2022). Optimal charging and bus-to-route assignment decisions are critical to having efficient operations of FEVs and avoid running out of charge while completing a route. Many factors can alter the energy used when performing a route such as traffic and weather (Gallet et al., 2018). This is why studying uncertainty in the energy consumption of FEVs performing routes is crucial to successfully implementing FEVs in public transportation systems.

One of the things that the OR community has focused on is solving the electric bus rostering and scheduling problem. The electric bus rostering and scheduling problem involves efficiently planning the routes and timetables for electric buses in a transportation system. It aims to maximize the utilization of electric buses, considering factors like charging times, energy constraints, and service reliability. This problem requires balancing the operational needs of the bus fleet with the limitations imposed by electric vehicle technology (i.e., charging technology and battery capacity limits), while ensuring a reliable public transportation service. The goal is to create schedules that minimize energy consumption and operational costs while meeting passenger demand and maintaining a consistent and punctual bus service (Perumal et al., 2022). The majority of the approaches in the literature have focused on versions of this problem where energy consumption is assumed to be perfectly known. This research relaxes that assumption and we explicitly model the natural uncertainty of energy consumption.

The research examines charging-related decisions and the bus-to-route assigment decisions for a fleet of FEVs. The resulting problem is known as the electric bus rostering and charging scheduling problem with uncertain energy consumption (E-BRCSPUEC). We formulate the E-BRCSPUEC as a two-stage stochastic program and we solve it using Gurobi, a commercial solver. We explore how energy uncertainty in the energy consumption of electric batteries in e-buses affects the operating costs associated with charging decisions and the difficulty to solve the model. To measure the latter, we use the optimality gap given by Gurobi which is the difference between the best integer solution and the best lower bound.

The contributions of this research are as follows:

- Provide an overview of the technology, process, methods and costs associated with the E-BRCSPUEC
- Formulate the E-BRCSPUEC as a two-stage stochastic programming model
- Implement a MILP model
- Solve the model under various parameters
- Analyze the output of the model, and make managerial recommendations for charging decisions

This research used instances provided by GIRO, a Montreal-based company specializing in software for public transit and postal service planning. We enriched those instances by adding energy consumption scenarios. We could not solve any of the instances to optimality but these results provided us with some interesting insights. Our experiments allowed us to establish the correlation between the number of scenarios and the level of uncertainty in consumption and the difficulty to solve the problem (i.e., CPU time). Furthermore, the results demonstrated that when the coefficient of variation increases in the scenarios created, the optimality gap decreases demonstrating better results found. Our results also shed some light into possible decomposition approaches to solve the problem efficiently.

This document is organized as follows: Chapter 1 is the introduction followed by Chapter 2 which contains the literature review. Chapter 3 introduces the problem description. Chapter 4 presents the mathematical model which formulates the E-BRCSPUEC problem as a mixed integer linear programming model. Chapter 5 describes the computational experiments. Chapter 6 presents the results obtained from the experiments and analyzes the output. Chapter 7 is the conclusion, the limitations of the proposed model, and further areas of research.

Chapter 2

Literature Review

The literature review will start with an overview of the different renewable energy sources available to fuel the transportation sector. Afterwards, a review of the three levels of decision making in supply chain management will be introduced. The rest of the literature review is divided into two sections. The first section pertains to OR related to FEVs and research that is relevant to the E-BRCSPUEC. In the first section, we will cover the vehicle scheduling problem and the charging scheduling problem. The second section is related to OR in general where we cover stochastic programming. Under the stochastic programming section we cover two stage stochastic programming and chance constrained programming.

2.1 Renewable Energy Sources in Transportation

The transportation industry in the future will face an energy crisis due to the limited nature of fossil fuels and their negative impact on society (WWF, 2023). Therefore, current research efforts are focusing into looking for a viable clean energy source that is cost effective to fuel the transportation sector. In 2023, researchers Pustějovská et al. (2023) conducted a review of alternative sources of energy applicable in the transportation sector using the life cycle analysis (LCA) framework. LCA is a tool that allows for the evaluation of the environmental impacts associated with a product, process, or service throughout its entire life cycle, from raw material extraction to disposal.

The main alternative sources of energy in transportation are: (1) bio-diesel from various oils, (2) bio-gas and bio-methane, (3) hydrogen, (4) alcohol blends, and (5) electricity. To begin, bio-diesel is made from various oils. The European Union is focusing on bio-diesel made from oil waste and fats such as palm oil (Pustějovská et al., 2023). Bio-diesel is made from certain vegetable oils such as rapeseed oil share similar characteristics with diesel. Research has shown that the use of bio-diesel has demonstrated a reduction in emissions of carbon dioxide and smoke but an increase in nitrogen oxide emissions (Singh Verma et al., 2022).

Furthermore, bio-gas can be upgraded into bio-methane. When bio-gas is purified it becomes bio-methane, which has similar characteristics to natural gas. Therefore, bio-methane can be used with existing natural gas infrastructure. A drawback of bio-methane is that it is very costly to produce, which may hinder its wider adoption as an alternative fuel source globally (Prussi et al., 2019).

Additionally, hydrogen is another promising alternative energy source due to its favorable characteristics. It releases a large amount of energy when reacting with oxygen and the only byproduct is steam (Pustějovská et al., 2023). However, the majority of hydrogen production is fueled by fossil fuels (Pustějovská et al., 2023). Moreover, hydrogen poses challenges related to the storage, transportation and use.

Alcohol blends are another prospective category of renewable fuels for internal combustion engines. The two main alcohol blends are methanol and ethanol. Methanol is a low-carbon intensity electro-fuel and ethanol is a low-carbon intensity bio-fuel (Pustějovská et al., 2023). One main benefit of alcohol blends, such as wet ethanol 80, is that they can be produced at a cost comparable to other renewable fuels. Furthermore, production of alcohol blends, such as wet ethanol 80, can be carried with a lower carbon intensity than fossil fuels and they produce slightly less nitrogen oxide emissions (Pustějovská et al., 2023).

Lastly, electricity is the alternative energy source that has gained the most popularity in the transportation sector. In 2022, the US market saw an increase of 55% in electric car sales (International Energy Agency, 2023). The main benefit of using electricity as an alternative energy source is that it does not emit direct air pollutants during operations if the vehicle is fully electric (Pustějovská et al., 2023). A cause for concern when evaluating electricity as an alternative energy source is what fuel source is used to produce the electricity. If coal or other fossil fuels are being used to produce electricity, then the emissions released during the production phase of the electricity outweigh the potential benefits in reducing emissions in operation on the road.

In Quebec, hydro-electricity is used to produce electrical energy making the whole production cycle of electricity renewable. Hydro-electricity is generated by harnessing the kinetic energy of flowing or falling water to turn turbines, converting mechanical energy into electrical energy. In 2009, the electricity sector accounted for only 0.6% of green house gas emissions in Québec (Hydro-Québec, 2024). To ensure that electricity is a clean and renewable energy source, its whole production process has to omit the use of fossil fuels.

Considering the review of the different alternative energy sources, electricity is a promising alternative that can help transition society to a carbon neutral transportation sector (Environment and Climate Change Canada, 2022).

2.2 Supply Chain Management Decision Levels

Supply chain management research operates at three levels of decision making: strategic, tactical, and operational. The strategic level deals with long-term problems that usually involve large investment decisions. One example of strategic-level research in supply chain management is buying and placing electric chargers for FEVs. This field of research deals with facility location problems. Facility location problems are mathematical optimization problems that aim to determine the optimal locations for a set of facilities to serve either a group of customers or demand points (Drezner & Hamacher, 2004). The main objective of facility location problems is to minimize either some cost or distance while meeting certain constraints. Facility location problems aim to help guide decision-makers to know where to place electric chargers based on electric demand needs. Furthermore, strategic level research in supply chain management also tackles financial decisions associated with investments related to charging stations, such as whether to buy a fast-charging or slow-charging electric charger. Tzamakos et al. (2023) created a model that locates fast wireless chargers in an electric bus network. The researchers modelled the problem of where to locate the fast chargers as an integer linear programming model that minimizes the investment cost of purchasing and locating an electric charger.

To continue, tactical-level research in the field of supply chain management deals with medium-term problems such as buying a fully electric vehicle or a hybrid vehicle for a bus fleet. This type of problem is called a fleet management problem. For example, Pelletier et al. (2019) looked at which type of vehicles (i.e., electric vs. hybrid) a bus fleet should invest in. They modelled their fleet transition problem as a mixed integer linear model and their research concluded that in all scenarios realized, buying fully electric buses was cost-effective (Pelletier et al., 2019).

Finally, the operational level of supply chain management research deals with day-to-day operational decisions such as the deployment of buses to fulfill routes which is a vehicle scheduling problem. The vehicle scheduling problem is a logistics and optimization challenge that involves determining the most efficient way to assign buses to routes (Cokyasar et al., 2023). The E-BRCSPUEC belongs to the operational level of supply chain management research. Therefore, the literature review will mostly be focused on operational level research related to transportation research. Specifically, problems related to FEVs.

2.3 Vehicle Scheduling Problem

The Vehicle Scheduling Problem (i.e., VSP) is a combinatorial optimization problem. The VSP involves the allocation of a predetermined set of scheduled trips to a fleet of vehicles, ensuring that each trip is assigned to a specific vehicle while minimizing a cost function (Cokyasar et al., 2023). The E-BRCSPUEC model decides how to assign a set of predetermined scheduled trips to a set a fleet of FEVs. These bus-to-route assignment decisions fall under the category of the VSP.

The vehicle scheduling problem aims to:

- Assign a set of vehicles to set of scheduled trips
- Minimize a cost function or maximize vehicle utilization
- Serve all routes while reducing delays in scheduled trips

VSPs encompass various objective functions, such as the minimization of fixed costs or the maximization of vehicle utilization, and are mathematically modeled using integer programming techniques. The generic VSP can be extended to consider: time windows, vehicle capacities, and service time constraints (Cokyasar et al., 2023).

The Electric Vehicle Scheduling Problem (E-VSP) is an extension of the VSP. The E-VSP deals with a set of time tabled trips which is assigned to a set of EVs with limited driving ranges based at different depots (Wen et al., 2016). The E-VSP can be conceptualized as a Multi-Depot VSP with imposed distance constraints and charging opportunities. Within the framework of the E-VSP, every vehicle trip commences at a specified locations and concludes at a predetermined time (Wen et al., 2016). Notably, each vehicle is afforded the option to undergo either full or partial recharging at any designated recharging station. It is assumed that the recharging time is a linear function of the amount of energy to be replenished (Wen et al., 2016). The E-VSP has to assign FEVs to time-tabled trips while making charging decisions at the same time. The charging element of the E-VSP adds complexity to the VSP. Charging a fleet of FEVs is a very complex issue due to the constraints that reality inflicts on this problem such as the limited number of chargers available at the depot.

2.4 Charging Scheduling Problem

There are three different charging strategies: the dumb charging, profit maximization, and power factor control strategy. In the dumb charging strategy, a term coined by Sachan et al. (2020), the vehicle owner just plugs-in and charges the FEVs as soon as a connecting port is made available. In the profit maximization strategy, the FEVs are directed to charge when the electricity tariff prices are low and discharge back to the grid when the prices rise in order to make a profit (Sachan et al., 2020). In the power factor control strategy, if the voltage at any point is too low relative to the required electricity, the power factor control system will decrease the charging. To fix this, some extra energy (i.e., reactive power) is sent back to the main power supply grid. This is done to make sure that the voltage at that specific point stays above a certain level (Sachan et al., 2020). The term "smart charging" denotes a technologically advanced charging system tailored for FEVs, with the principal objective of optimizing the charging process (Sachan et al., 2020). The overarching aim of smart charging strategies is to augment the efficiency, reliability, and sustainability of the infrastructure employed for charging FEVs. Among the strategies discussed above, only two belong to the smart charging family: the profit maximization scenario and the power factor control scenario.

The following section covers research related to smart charging strategies that deal with charging schedules of FEVs. The study developed by Liu et al. (2022) focuses on developing an optimization method for charging plans for electric buses considering time-of-day (TOD) electricity tariffs. This study proposes two optimization models, one for deterministic trip travel times and another for stochastic trip travel times. The objective is to minimize the electricity costs of daily operations by optimizing the charging time. Researchers Houbbadi et al. (2019) introduce an optimal charging strategy for a fleet of electric buses based on charging schedule planning and modulation of charging power. The study emphasizes the need for controlled charging to avoid grid instability and high operating costs.

Bie et al. (2021) propose a vehicle scheduling method for electric bus routes considering stochastic volatilities in trip travel time and energy consumption. The study by Bie et al. (2021) develops a model for estimating trip energy consumption based on field-collected data and the probability distribution function of the trip energy consumption to determine the stochastic volatility (Bie et al., 2021). The paper by Bie et al. (2021) proposes a novel approach to charging buses during idle times, a strategy aimed at optimizing electric bus operations. In their study, they delve into the intricate dynamics of stochastic volatilities, particularly their effects on factors such as departure times. By employing an optimization model, Bie et al. (2021) not only seek to minimize bus procurement costs but also aim to reduce the expectation of delayed trip departure times and the overall energy consumption. Their findings highlight the significance of optimizing scheduling plans to identify feasible idle times, effectively mitigating the accumulation of stochastic volatility within the system.

Sassi and Oulamara (2017) address the Electric Vehicle Scheduling and Optimal Charging Problem, focusing on optimizing the assignment of EVs and Combustion Engine Vehicles to tours while minimizing EV charging cost. The problem considers operational constraints related to chargers, the electricity grid, and EV driving range. The authors Sassi and Oulamara (2017) prove that the Electric Vehicle Scheduling and Charging Problem (EVSCP) is NP-hard and present a mixed-integer linear programming formulation solved with CPLEX for small and medium instances, along with two proposed heuristics for large instances: a Sequential Heuristic and a Global Heuristic . The efficiency of the proposed approaches is evaluated through computational results on a diverse set of real and randomly generated test instances, including larger instances with up to 200 EVs and 320 tours.

2.5 Stochastic Programming

2.5.1 Two Stage Stochastic Programming

Two-stage stochastic programming is a decision-making framework designed to tackle uncertainty in complex optimization problems (Higle, 2005). In this approach, decisions are made in two sequential stages. In the first stage, decisions are made based on available information before uncertainty is realized. These decisions involve resource allocation, investment choices, or similar strategic determinations. After these initial decisions are made, the second stage comes into play, usually including a recourse action, which includes a recourse cost. This stage accounts for the uncertainty that unfolds, typically represented by scenarios that follow a given probability distribution (Küçükyavuz & Sen, 2017). Adjustments are made in response to the realized uncertainty, aiming to minimize costs, maximize benefits, or achieve other optimization goals. Two-stage stochastic programming thus enables decision-makers to create robust plans that consider both the inherent uncertainty of the future and the ability to adapt strategies in response to actual outcomes. It finds application in diverse fields, such as assignment, network problems, energy, transportation, and supply chain management (Sen & Higle, 1999). Two-stage stochastic programming provides a valuable framework for making effective decisions in complex and uncertain environments.

Two-stage stochastic programming is especially useful for problems where data elements are difficult to predict such as energy consumption for electric buses serving a particular route (Higle, 2005). Two-stage stochastic programming tackles the issue of predicting these data element values by the usage of scenarios.

Two-stage stochastic programming has many applications in operations research, specifically in transportation research. Hu et al. (2022) developed a two-stage stochastic programming model to locate fast chargers at certain bus stops, considering uncertainties of passenger demand and energy consumption during bus operation. They decide the location of the fast chargers in the first stage before information is known about passenger demand and energy consumption. In the second stage, they introduce passenger demand and energy consumption by using scenarios. The corresponding cost of recourse in the second stage is the penalty caused by related charging activities. Hu et al. (2022) solved the model by using the L-shaped method and comparing those results to the commercial solver Gurobi, demonstrating better performance and efficiency. This research demonstrates that optimizing the location of fast charging chargers reduces passenger waiting time and the corresponding penalty cost can be achieved.

2.5.2 Chance Constrained Programming

Chance Constrained Programming optimizes decisions while ensuring that certain constraints are satisfied with a specified probability (Li et al., 2008). Chance Constrained Programming is an appropriate stochastic programming method to apply when: (1) the equivalent two-stage stochastic program is too hard to solve, (2) the cost structure is unknown or the cost cannot be approximated for the recourse action cost, or (3) the problem has no need for a recourse action.

Chance Constrained Programming has many applications in the energy field. A bi-level programming model was introduced by researcher Zhou et al. (2020) to address the planning challenges of fast-charging stations in electrified transportation networks, the model considers uncertain charging demands (i.e., demand for EV charging is unpredictable). The upper level minimizes planning costs while the lower level determines the spatial and temporal distribution of plug-in electric vehicle flows. Robust chance constraints are formulated to ensure service abilities under uncertain charging demands. They reformulated the model into a single-level mixed integer second-order cone programming model to enhance its efficiency in finding solutions (Zhou et al., 2020).

2.6 Solution Methods in Operations Research

For solving problems in operations research, there are generally three main categories of solution methods: 1. exact algorithms, 2. heuristics and meta-heuristics and 3. hybrid approaches.

Exact algorithms are computational methods that guarantee finding the optimal solution to a problem by exploring all possible solutions within a feasible search space. Exact algorithms include algorithms such as, Simplex which are used to solve continuous linear programs for small to medium sized instances. To continue, to solve Mixed Integer programs (i.e., programs with both continuous and integer variables) exact algorithms such as, branch-and-bound and branch-and-cut are used (Asghari & Mirzapour Al-e hashem, 2021).

Heuristics are simple, “rule-of-thumb” techniques or strategies used to quickly find good solutions to problems without guaranteeing optimality (Asghari & Mirzapour Al-e hashem, 2021). Meta-heuristics are high-level strategies or frameworks for solving complex optimization problems that guide the search for solutions using heuristics. The main difference between heuristics and meta-heuristics is that meta-heuristics aim to efficiently explore a larger area of the search space by usage of heuristics (Asghari & Mirzapour Al-e hashem, 2021). Meta-heuristics tend to find good solutions that are close to the optimal solution but cannot guarantee finding the "optimal" solution. Therefore, only exact algorithms can guarantee the optimal solution. In situations where one is dealing with a complex NP-Hard problem, heuristics and meta-heuristics are often times deployed because they are able to find a relatively good solution in less time. Examples of heuristic and meta-heuristic algorithms are the nearest neighbour, the savings algorithm, genetic algorithms, simulated annealing, and tabu search (Montoya-Torres et al., 2015). Hybrid approaches are a combination of exact algorithms, heuristics, and meta-heuristics (Asghari & Mirzapour Al-e hashem, 2021).

The main decision in the E-BRCSPUEC are the charging decisions because the model penalizes inefficient charging of FEVs by triggering a recourse action cost of deploying a diesel bus to finish the FEVs assigned route if the FEV does not have sufficient charge. Furthermore, the E-BRCSPUEC deals with assignment decisions which falls under the umbrella of the VSP. The E-BRCSPUEC assigns pre-determined routes to FEVs within reasonable time windows in normal operating hours for a bus. Currently in OR, an important research gap is illuminated by the lack of research related to charging decisions and energy uncertainty related to successfully fulfilling routes without delays. There is a lack of research related to the VSP and Two Stage Stochastic Programming. Two-Stage Stochastic Programming is an excellent operation research method that allows researchers to study uncertain variables such as, gross electric energy consumption of FEVs fulfilling routes. By adding uncertainty into the model surrounding the gross electric energy consumption of FEVs performing their pre-determined routes that they were assigned to fulfill, we are better able to understand how charging decisions done the night before at the depot can influence the short term and long term operational costs with FEVs (i.e., maintenance costs, variable and fixed charging costs, e-buses breaking down due to a lack of electrical energy, replacing batteries etc.).

Chapter 3

Problem Description

3.1 Problem Definition

In the E-BRCSPUEC, a transit agency must i) assign e-buses to a set of tasks to be performed the next day and ii) define a charging schedule that minimizes the cost of operating the system. The set of tasks is defined by *vehicle blocks*. A vehicle block is a sequence of trips that can or cannot be separated by an out-of-depot charging operation. Each vehicle block starts and ends at the depot. We assume that each e-bus performs at most one block per day. Furthermore, the cost of operating the system is given by the sum of the cost of the energy charged into the e-buses and the expected cost of deploying diesel vehicles to complete tasks that e-buses cannot. Indeed, since the exact energy consumption of a task is uncertain, it is possible that a FEV must return to the depot before completing its task. We assume that a diesel vehicle is deployed to take over the uncompleted task in such a case.

We chose to model the E-BRCSPUEC using two-stage stochastic programming rather than chance constraint programming to better express the decision makers' risk aversion towards energy consumption uncertainty and to tailor it to individual blocks. Indeed, revising the operations during the day to dispatch an "emergency" vehicle to complete Block A may be more difficult/expensive than doing it for Block B. The recourse cost can capture these dynamics better than probabilistic thresholds (even if they are tailored by block). By using two-stage stochastic programming, we enable decision-makers to more effectively balance the trade-offs between operational costs and uncertainty in the feasibility of the solution.

Let d and d+1 denote the current and next operating day of the system. Let $\mathscr{K} = \{1,2,...,k,...,K\}$ denote the fleet of e-buses. Every e-bus has a battery with a capacity of $\mathrm{H} \in N$ kilowatt-hours (kWh). The state of charge (SoC) of an e-bus is computed as the energy (i.e., in kWh) it stores at a given time divided by the capacity H; thus, it is always a number between 0 and 1. We assume that the initial SoC of each e-bus, denoted SOC_k^D, is known at the beginning of the planning horizon. We assume that the time t_k^d at which the e-bus k arrives at the depot at the end of day d is perfectly known at the beginning of the planning horizon. Note that by convention $\min_{k\in\mathscr{K}}\{t_k^d\} = 0$ and that it marks the start of the planning horizon.

On day d+1, the fleet of electric buses must perform a set of tasks defined by *vehicle blocks*. Let $\mathscr{B}$ be the set of all vehicle blocks to be performed on d+1. We assume the fleet is sufficiently large to perform all the blocks, namely, $|\mathscr{B}| \leq |\mathscr{K}|$. Each vehicle block $i \in \mathscr{B}$ is characterized by its start time t_i^{start} (i.e., the time at which the e-bus assigned to the block must leave the depot) and end time t_i^{end} (i.e., the time at which the e-bus assigned to the block returns to the depot after performing the block). The exact energy consumption of each block is unknown, so it is represented by a set of scenarios S_i. Without loss of generality, we assume that all scenarios in S_i have the same realization probability. Each *consumption scenario* $sc \in S_i$ is defined by i) a net energy consumption $e_i^{sc} \in [0,1]$ and ii) the minimum SoC $\underline{e}_i^{sc}$ that an e-bus must-have when leaving the depot in order to be able to fully cover block i if scenario sc realizes. Parameter $\underline{e}_i^{sc}$ can also be seen as the maximum decrease in the e-bus's SoC during the execution of block i if scenario sc realizes.

To recharge the e-buses' batteries, there is a set $\mathscr{C} = \{1,2,\ldots,c,\ldots,C\}$ homogeneous chargers located at the depot, where $C \leq K$. These chargers operate overnight, within the time windows $[0,L]$ where $L = \max_{i\in\mathscr{B}}\{t_i^s\}$ is the latest departure time from the depot in d+1. For modeling purposes the continuous charging time interval $[0,L]$ is discretized into a set of $\mathscr{P} = \{0, 1, \ldots, p, \ldots, P\}$ of *charging periods*, each lasting $\delta > 0$ time units (e.g., 30 minutes). Finally, although partial charges are allowed, each bus executes at most one consecutive recharge, that is, the bus is plugged and unplugged from a charger only once between the time it arrives at the depot in d and the time it departs from the depot in *d+1*.

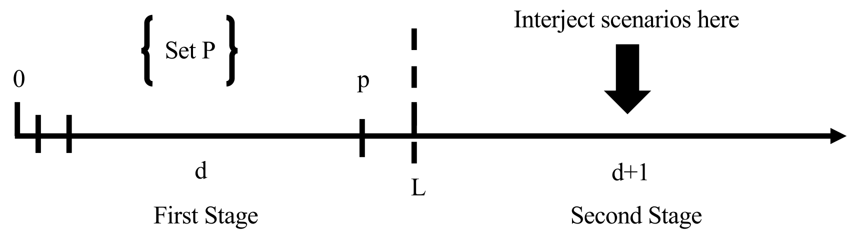

Figure 3.1: Decision timeline

As observed in Figure 3.1, all charging decisions are done overnight at the depot within $[0, L]$. Charging decisions and route assignment decisions are done at the same time. The second stage is the next operating day $(d+1)$. In the second stage, the model decides how much energy comes from the initially assigned e-bus and whether a diesel bus is needed. The model has to assess if the initially assigned e-bus does or does not have enough electric charge to finish its assigned route on each scenario, and (based on that calculation) determine if the recourse action (i.e., using a diesel bus) and it's associated costs are triggered or not.

Charging costs are split into two categories: (1) the time-of-use (TOU) electricity tariffs and (2) the facility-related demand (FRD) charges. The TOU electricity tariffs account for the time-dependent energy costs c_p ($/kW), a variable cost that depends on the time of day. The second charging cost is the FRD charges for the maximum power retrieved from the grid throughout the charging interval $[0, L]$. In reality, electricity rate plans for charging electric vehicles on company grounds usually include a maximum power restriction. Therefore, at most a power of G (units kW) can be recovered from the grid at any time during the charging interval $[0, L]$. When an e-bus is incapable of completing its assigned block (i.e., that it, leaves the depot with an SoC that is inferior to

$\underline{e}_i^{sc}$ of the consumption scenario *sc* realized during the execution of the block), the cost of the recourse action is triggered. The *recourse action* cost d_i captures the monetary costs associated with using a diesel bus to complete an unfinished route, such as the cost of diesel, coordination costs, extra driving time, and eventual inconvenience for passengers. For simplicity we assume that the depot would send a diesel bus to finish the route because all the e-buses are already dispatched because they were prioritized to fulfill these routes. Note that for simplicity, we assume that d_i is independent of the point in the route at which the reserve diesel vehicle takes over the block. Indeed, explicitly modeling that point would require either additional decision variables tracking the energy consumption within the block or a subroutine that computes the exact cost of recourse for a scenario every time an integer solution is found. Both of these alternatives will overly complicate the model and significantly increase the solution time.

The E-BRCSPUEC aims to assign buses to vehicle blocks and schedule their charging operations such that (1) each vehicle block is performed by a single e-bus, and each e-bus is assigned to at most one vehicle block, (2) the number of buses charging simultaneously at the depot overnight does not exceed the number of chargers available, (3) a single charger must consecutively charge each e-bus and a charger can serve at most one vehicle at a time, and (4) the total power retrieved by the energy grid verifies the maximum power restriction. The objective of the problem is to minimize the total costs, including the TOU electricity tariffs, the facility-related demand charges throughout the day, and the expected cost of the recourse actions.

3.2 Battery Charging Process

The battery charging behaviour is a major concern when addressing electric vehicle charging and scheduling problems. Ultimately, modelling this behaviour determines the charging time and driving range that vehicles can achieve and must be consistent with real-life practices.

Xu et al. (2019) proposed a linear piece-wise function to approximate the equivalent nonlinear resistance of a charger. They proposed a piece-wise linear function to model the charging cycle of electric vehicles at charging stations, which was used to calculate harmonic variation characteristics (Xu et al., 2019). Harmonic variation in terms of charging an electric vehicle refers to fluctuations in the electrical signals that happen during the charging process. Power control engineers are interested in understanding the harmonic variations during the charging cycle to ensure that the charging of FEVs is safe, and efficient and does not degrade the lifespan of the electric battery (Pelletier et al., 2018). The piece-wise linear function Xu et al. (2019) also calculate the maximum harmonic variations during a charging cycle, which can be seen as the peak demand charges in a charging cycle.

Bruglieri et al. (2015) used two stages to estimate the battery recharging process. In the first phase, the battery recharges almost fully and is linear on the time. The second phase of the charging process, is not linear on the time and can take several hours to achieve the full charge of the battery. For the sake of simplicity for this model, we do not consider the second phase of the recharging process like they did in their study. Therefore, we will use a linear function to model the charging process of an e-bus' battery.

The FEVs chargers usually have an adjustable power level. This means, the more power a charger applies, the faster the battery charges. On the other hand, the time-dependent energy costs and FRD charges increase as the power applied by a charger increases (Pelletier et al., 2018). In the E-BRCSPUEC, each charger can adjust the power it delivers within the range $[o_{min}, o_{max}]$, where o_{min} and o_{max} are, respectively, the minimum and maximum power (kW) that a charger can deliver to a battery, satisfying $0 < o_{min} \leq o_{max}$. Choosing the power delivered by each charger poses a trade-off between the necessary time required to charge and the total power retrieved from the grid.

The CC-CV charging scheme is a typical method for charging rechargeable batteries such as li-ion (ROHM, 2023). In the CC-CV scheme, the SoC evolution depends on the power that the charger delivers. A given power level for the charger results in a specific CC-CV battery charging scheme that a linear function can represent. An efficiency factor is applied to the linear function that is $\theta \in [0,1]$ representing the percentage of the power applied by the charger that is received by the FEV's battery.

Chapter 4

Mathematical Formulation

We define the following decision variables. Let $v_{k,i} \in \{0,1\}$ indicate whether the bus $k \in \mathscr{K}$ performs vehicle block $i \in \mathscr{B}$. Let x_i be the SoC of the e-bus assigned to block $i \in \mathscr{B}$ when it leaves the depot to cover the block. Let $soc_{k,p} \in [0,1]$ be the SoC of an e-bus $k \in \mathscr{K}$ at the end of period $p \in P$. Let $o_{k,p}$ represent the power injected into an e-bus k during period p. Let $w_{k,p} \in \{0,1\}$ be equal one if bus k is charging during period p; 0 otherwise. Let $z_{k,p} \in \{0,1\}$ indicate whether e-bus $k \in \mathscr{K}$ begins a charge at the beginning of a charging period $p \in \mathscr{P}$. Let variable $r_{i,sc} \in \{0,1\}$ be 1 if the SoC of the bus assigned to block i when it leaves the depot on $d+1$ is lower than $\underline{e}_i^{sc}$ and 0 otherwise. These variables are indexed by sc in S_i. Let $e_{i,sc}^{diesel}$ represent the SoC needed to finish a route i in scenario sc supplied by a diesel bus. Finally, let $y \geq 0$ be the maximum charging power retrieved from the grid throughout the $[0,L]$ interval. The following mixed integer linear programming formulation represents the E-BRCSPUEC:

$$\min \quad \sum_{k\in\mathscr{K}} \sum_{p\in\mathscr{P}} \delta \cdot c_p \cdot o_{k,p} + F \cdot y + \sum_{i\in\mathscr{B}} \sum_{sc\in S_i} \frac{1}{|S_i|} \cdot r_{i,sc} \cdot d_i \tag{1}$$

$$\text{s.t.,} \quad \sum_{k\in\mathscr{K}} v_{k,i} = 1 \qquad \forall i \in \mathscr{B} \tag{2}$$

$$\sum_{i\in\mathscr{B}} v_{k,i} = 1 \qquad \forall k \in \mathscr{K} \tag{3}$$

$$soc_{k,p_0} = SOC_k^D \qquad \forall k \in \mathscr{K} \tag{4}$$

$$soc_{k,p+1} = soc_{k,p} + \frac{o_{k,p} \cdot \theta \cdot \delta}{H} \qquad \forall k \in \mathscr{K}, p \in \mathscr{P} \setminus \{P\} \tag{5}$$

$$soc_{k,P} = soc_{k,P-1} + \frac{o_{k,P-1} \cdot \theta \cdot \delta}{H} \qquad \forall k \in \mathscr{K} \tag{6}$$

$$x_i \geq soc_{k,P-1} - M(1 - v_{k,i}) \qquad \forall k \in \mathscr{K}, i \in \mathscr{B} \tag{7}$$

$$x_i \leq soc_{k,P-1} + M(1 - v_{k,i}) \qquad \forall k \in \mathscr{K}, i \in \mathscr{B} \tag{8}$$

$$\sum_{k \in \mathscr{K}} w_{k,p} \leq C \qquad \forall p \in \mathscr{P} \tag{9}$$

$$\frac{o_{k,p} \cdot \theta \cdot \delta}{H} \leq w_{k,p} \qquad \forall k \in \mathscr{K}, p \in \mathscr{P} \tag{10}$$

$$\sum_{p \in \mathscr{P} | (p+1)\delta > t_i^{\text{start}}} w_{k,p} \leq \left\lceil \frac{L - t_i^{\text{start}}}{\delta} \right\rceil (1 - v_{k,i}) \qquad \forall k \in \mathscr{K}, i \in \mathscr{B} \tag{11}$$

$$\sum_{p \in \mathscr{P}} z_{k,p} \leq 1 \qquad \forall k \in \mathscr{K} \tag{12}$$

$$(1 - z_{k,p}) \cdot p \geq \sum_{p' \in \mathscr{P} | p' < p} w_{k,p'} \qquad \forall k \in \mathscr{K}, p \in \mathscr{P} \tag{13}$$

$$z_{k,p} \geq (w_{k,p} - w_{k,p-1}) \qquad \forall k \in \mathscr{K}, p \in \mathscr{P} \setminus \{0\}, \tag{14}$$

$$z_{k,0} \geq w_{k,0} \qquad \forall k \in \mathscr{K}, \tag{15}$$

$$\sum_{k \in \mathscr{K}} o_{k,p} \leq y \qquad \forall p \in \mathscr{P} \tag{16}$$

$$0 \leq y \leq G \tag{17}$$

$$o_{\min} \cdot w_{k,p} \leq o_{k,p} \leq o_{\max} \cdot w_{k,p} \qquad \forall k \in \mathscr{K}, p \in \mathscr{P} \tag{18}$$

$$x_i + e_{i,sc}^{diesel} \geq \underline{e}_i^{sc} \qquad \forall i \in \mathscr{B}, sc \in S_i \tag{19}$$

$$e_{i,sc}^{diesel} \leq M r_{i,sc} \qquad \forall i \in \mathscr{B}, sc \in S_i \tag{20}$$

$$(1 - r_{i,sc}) \cdot \underline{e}_i^{sc} \leq x_i \qquad \forall i \in \mathscr{B}, sc \in S_i \tag{21}$$

$$x_i \geq 0 \qquad \forall i \in \mathscr{B} \tag{22}$$

$$soc_{k,p} \geq 0 \qquad \forall k \in \mathscr{K}, p \in \mathscr{P} \tag{23}$$

$$e_{i,sc}^{diesel} \geq 0 \qquad \forall i \in \mathscr{B}, sc \in S_i \tag{24}$$

$$0 \leq o_{k,p} \leq o_{max} \qquad \forall k \in \mathscr{K}, p \in \mathscr{P} \tag{25}$$

$$r_{i,sc} \in \{0,1\} \qquad \forall i \in \mathscr{B}, sc \in S_i \quad (26)$$

$$v_{k,i} \in \{0,1\} \qquad \forall k \in \mathscr{K}, b \in \mathscr{B} \quad (27)$$

$$w_{k,p} \in \{0,1\} \qquad \forall k \in \mathscr{K}\ p \in \mathscr{P} \quad (28)$$

$$z_{k,p} \in \{0,1\} \qquad \forall k \in \mathscr{K}, p \in \mathscr{P} \quad (29)$$

The objective function (1) minimizes the total costs, including the TOU electricity tariffs overnight, the FRD charges, and the expected cost of the recourse actions.

The first subset of constraints defines the vehicle/block pairings; constraints (2) ensure that each vehicle block is carried out by one e-bus. Constraints (3) force each e-bus to be assigned to one, and only one, block. Because the number of blocks may be smaller than the number of e-buses (i.e., $|\mathscr{B}| \leq |\mathscr{K}|$), we add set add $|\mathscr{K}| - |\mathscr{B}|$ *dummy* blocks to $\mathscr{B}$. The *dummy* blocks have zero electric consumption, zero charging cost and zero recourse action cost associated with them.

Constraints (4) set the SoC of each e-bus at the beginning of the planning horizon. It is assumed that the SoC of each e-bus is known at the start of the planning horizon.

Constraints (5) to (6) pertain to updating the SoC of the e-bus related to the charges delivered by the charger. Constraints (5) update the SoC of each e-bus in each period according to the power applied by the charger and constraints (6) defines the SoC for the last period P.

Constraints (7) and (8) capture the SoC of the vehicle assigned to block $i \in \mathscr{B}$ at the start of the corresponding block.

Constraints (9) ensure that the number of e-buses that charge simultaneously at the depot does not exceed the number of available chargers.

Constraints (10) link the $o_{k,p}$ variable with the $w_{k,p}$ indicator variables. The constraints ensures that if the the $o_{k,p}$ variable's value is zero, the corresponding indicator variable $w_{k,p}$ will equal zero to indicate no charging is occurring in that period p. If the value of $o_{k,p}$ is greater then zero, than the $w_{k,p}$ value must equal to one demonstrating that charging is occurring in that period p. Constraints (11) ensures that no charging occurs when e-bus k is performing block i.

Constraints (12), (13), (14), and (15) guarantee that the charging operations are not preemptive once started. Constraints (12) guarantee that the recharging of an e-bus may start in a single period overnight. Constraints (13) ensure that no charging is performed before the start of the charging operation for a given vehicle. Constraints (14) define the first period of charge and ensure that the charging periods for each e-bus are consecutive. Constraints (15) are similar to constraints (14) but consider the initial period of charge of each day.

Constraints (16) establish the maximum charging power used throughout the one-day planning horizon. Constraint (17) guarantee the maximum limit on grid power is respected. Constraints (18) bound the power injected by a charger to a bus.

Constraints (19) ensure the total energy needed to finish a route is supplied either by electric energy delivered by the e-battery on an e-bus or by a diesel-fueled bus.

Constraints (20) is a logical constraint imposed on the binary variable $r_{i,sc}$ to ensure a feasible solution. Constraints (21) ensure the electric energy for the e-bus is less than or equal to the SoC needed for the routes performed for today's operating day (i.e., d+1). Finally, (22) to (29) define the domains of the variables that are not already adequately bounded by the other constraints.

Directly solving formulation (1)-(29) with a MIP solver may not be possible for large-scale instances. As an alternative, researchers have studied decomposition methods that partition the complete problem into simpler ones. Among these methods, we highlight the Benders decomposition proposed by Benders (1962). This method is an iterative procedure that decomposes the main problem into two problems: an integer master problem and a linear sub-problem. The master problem includes a subset of constraints from the original model, usually related to first-stage decisions (Taşkin, 2011). On the other hand, the subproblem will be in charge of the second-stage decisions, which are usually scenario-dependent. Furthermore, the sub-problem will be used to create additional constraints (i.e., for the master problem) known as Benders cuts. These inequalities help ameliorate the current solution obtained from the master problem. Indeed, when these cuts are added to the master problem, the feasible region changes substantially. Although this method is not implemented in this work, it would undoubtedly be an interesting opportunity for future research around E-BRCSPUEC.

Chapter 5

Computational Experiments

In this section, we present our computational study. Section 5.1 discusses the data set we were provided with from Barahona Rojas (2022) and GIRO. In section 5.2, we discuss the experimental design to assess the performance of the proposed model. In section 5.3, we describe the generated scenarios for the E-BRCSPUEC model based on data from Barahona Rojas (2022) and GIRO.

Experiments were carried out using Gurobi's free academic version (v.10.0.0) and its Python (v.3.9.14) interface. Experiments were performed on a computer with Intel (R) Core (TM) i7–7800X CPU @ 3.50 GHz with six cores and 124GB of RAM.

5.1 Dataset

We used the instances designed by Barahona Rojas (2022) based on the data provided by GIRO. Barahona conducted a computational study on 20 instances, 10 with $|\mathscr{B}| = 20$ blocks, and 10 with $|\mathscr{B}| = 30$ blocks. In each instance, the number of chargers is half the number of vehicle blocks per day. We consider only the first day of data from the data set, even though the data set contains data for multiple days. For each vehicle block, the start time (t^{start}), end time (t^{end}), net decrease of SoC (soc_i) and maximum decrease of SoC (SOC_i^{dec}) are given. In the E-BRCSPUEC model, we omit information regarding the end time (t^{end}) because our planning horizon is only interested in charging decisions that have an impact on the next operating day.

We do not allow charges to occur when the vehicle k has left the depot and has started to perform its assigned vehicle block i. Therefore, vehicle k must finish charging before t^{start} to ensure that it can perform its vehicle block i with no delays. We have formulated the E-BRCSPUEC model this way to prevent conflicts between the charging and scheduling plans.

We changed the initial SoC of electric buses from 1 (i.e., starting each day with a fully charged electric battery) to 0.5 (i.e., half of a charge). Otherwise, recourse actions are never required because vehicle k would never run out of charge.

Furthermore, from the original data set, all data files for 5 days were removed from the experiment data set to be used for the computational experiments. Each data file provided by GIRO is labelled based on the number of blocks, days, chargers and instances. The number of blocks ranged from 20 to 90 blocks per day and incrementally change by 10 blocks. The number of chargers ranges from 10 to 45 chargers and incrementally changes by 5 chargers. The instances for each data file ranged from 0 to 9 so for each data file there were different values for the same file for the same number of blocks, days, chargers etc. We consider ten instances for each combination of blocks, days, and the number of chargers.

5.2 Experimental Design

To assess the performance of the model under varying levels of energy consumption uncertainty, we generated scenarios controlled by the coefficient of variation. The coefficient of variation (i.e., CV) measures the relative variability of a dataset by expressing the standard deviation as a percentage of the mean (Hayes, 2024).

For each data file, we created scenarios for three levels of number of scenarios: 10, 20, 30 and for three levels of CV of energy consumption (i.e., $\underline{e}_i^{sc}$) : 0.05, 0.10, 0.15. In other words, in our scenarios the standard deviation of the energy consumption of the blocks is 5% , 10%, or 15%. According to Gallet et al. (2018) and Liu et al. (2019), the variability of the energy consumption on a FEV's route ranges from 10% to 30% on average. Since our e-buses operate on fixed and repetitive routes, we can expect the variability of the energy consumption to be on the lower side of the interval; we chose the values of the coefficient of variation for our experiments accordingly.

We looked at the optimality gap provided by Gurobi to see how well the proposed model works in terms of computational efficiency. Gurobi calculates the optimality gap using the following formula:

(Upper Bound - Lower Bound)/ Upper Bound

To continue, regarding the battery charging process, each e-bus has a battery with an energy capacity of 363 kWh. Furthermore, the SoC of an e-bus must be between SOC^{min} = 0.1, and SOC^{max} = 1. Finally, as in Montoya et al. (2017) the proposed original piecewise linear charging function by Barahona Rojas (2022) has four break points where each break-point was associated respectively with a reduction factor of $\theta_1 = 1$, $\theta_2 = 0.49$ $\theta_3 = 0.15$. In our instances we took the reduction factor to be $\theta = 0.55$, which is the average slope in their piecewise linear function.

Along the lines of Pelletier et al. (2018), we consider a discretization with periods of $\delta = 30$ minutes, a time-dependent cost ($/kW) according to Southern California Edison (2024) summer rates, and a monthly (i.e., 22 days) FRD cost of 13.2 $/kW. A charger's minimum and maximum is $o_{min} = 48$ kW and $o_{max} = 80$ kW, respectively. The number of electric buses is set to K = 1.1 $|\mathscr{B}|$. Finally, we assumed that G = $o_{max} \cdot C \cdot 0.9$ and that the time-dependent energy cost during the day is set to the highest c_p value in that period p for Set $\mathscr{P}$. The c_p rates vary from 0.05, 0.12 to 0.29 depending on what period the FEV is charging in. To estimate the cost of recourse, we determine what is the point at which fully charging an electric bus becomes more expensive than deploying a diesel bus. The recourse action cost d_i is calculated by charging all the e-buses in the fleet at the highest c_p rate (0.29) into the charging formula and charging them to the max power o_{max} that is allowed by the charger while respecting the e-bus battery limit and grid capacity constraints. We are essentially charging all the e-buses to 100% charge at the highest available charging rate. The charging formula is as follows:

$$\sum_{k\in\mathcal{K}}\sum_{p\in\mathcal{P}}\delta\cdot c_p\cdot o_{k,p}+F\cdot y$$

This is how we calculated the recourse action cost to be the worst case scenario realized financially. The recourse action was calculated this way because in reality most bus fleets would already have diesel buses in their fleet ready to be deployed. Therefore, the cost to deploy a diesel bus should only be triggered when it is more expensive to charge the e-buses to the max power at the highest charging rate.

5.3 Scenario Generation Method

The generation of instances involves a progressive connection across the different levels of number of scenarios. An instance is the result of creating a certain number of scenarios by using certain CV. The scenarios were created by altering the gross electric consumption by using a normal distribution curve individually. The gross electric consumption is used as the mean value while the selected CV sets the variance for the normal distribution function.

The gross electric consumption variable is altered because the model is interested in different energy usage conditions expressed by the different scenarios, and how that will interplay with the recourse action (i.e., deploying a diesel bus if the e-bus runs out of charge). The gross electric consumption variable represents the amount of energy needed to perform block i. In order to illustrate the scenario generation method, an example is given as follows:

Taking three sets of different number of scenarios (i.e., 10, 20, and 30 scenarios) and the given CV of 15% as an example, the process unfolds as follows:

- Level 1 (10 scenarios, 15% CV):
  - The first 10 scenarios are generated for a specific data file with a CV of 15%

- Level 2 (20 scenarios, 15% CV):

  - The initial 10 scenarios from Level 1 are included in the set of 20 scenarios with a 15% CV for the same data file
  - An additional 10 scenarios are generated, maintaining a 15% CV

- Level 3 (30 scenarios, 15% CV):

  - The last 20 scenarios from Level 2 are included in the set of 30 scenarios with a 15% CV
  - An additional 10 scenarios are generated, still maintaining a 15% CV

In summary, this process ensures a progression in the number of scenarios while maintaining a consistent coefficient of variation for each level. This hierarchical approach facilitates the creation of instances from various data files, systematically building upon the scenarios generated at the preceding levels.

Chapter 6

Results

In this section, we describe the experiments performed and assess the results produced from running the instances adopted from Barahona Rojas (2022) on the model. In Sections 6.1 and 6.2 we describe the experiments, present the computational results and assess the performance of the proposed model. Furthermore, this section will discuss how well the model performed in terms of optimization performance metrics such as the achieved optimality gap.

6.1 Experiment 1: the impact of the number of blocks

The aim of the first experiment is to assess the impact the number of blocks has on the difficulty of solving the model. We did this by testing the three levels of number of scenarios (i.e., 10, 20 and 30) and three levels of CV (i.e., 0.05, 0.1 and 0.15) with eight data files ranging from 20 to 90 blocks. The terminal condition for each run of the experiment is to achieve the optimal solution (i.e., achieve an optimality gap of zero) or reach a time limit set as four hours (i.e., 14,400 seconds). The total number of runs was 72 and the whole experiment was performed continuously meaning each run was run one after another without breaks.

Figures 6.1, 6.2, 6.3 demonstrate the output of Experiment 1. Figures 6.1, 6.2, 6.3 shows the mean values of the achieved optimality gap across the three sets of scenarios tested (i.e., 10, 20 and 30 scenarios which is represented by n=3 in the graph) and the corresponding confidence intervals. Figures 6.1, 6.2, 6.3 allow us to visualize how the model is performing in terms of achieved optimality gap compared to the number of blocks considered for each run.

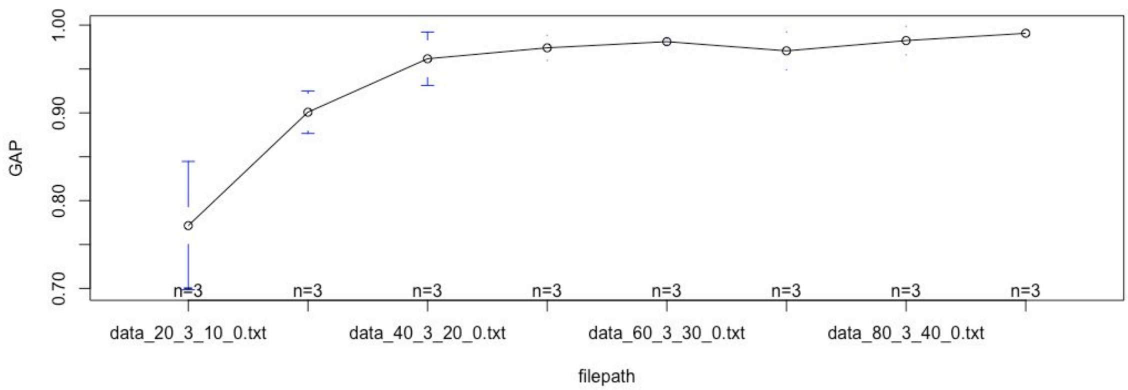

Figure 6.1: C.V of 5 %

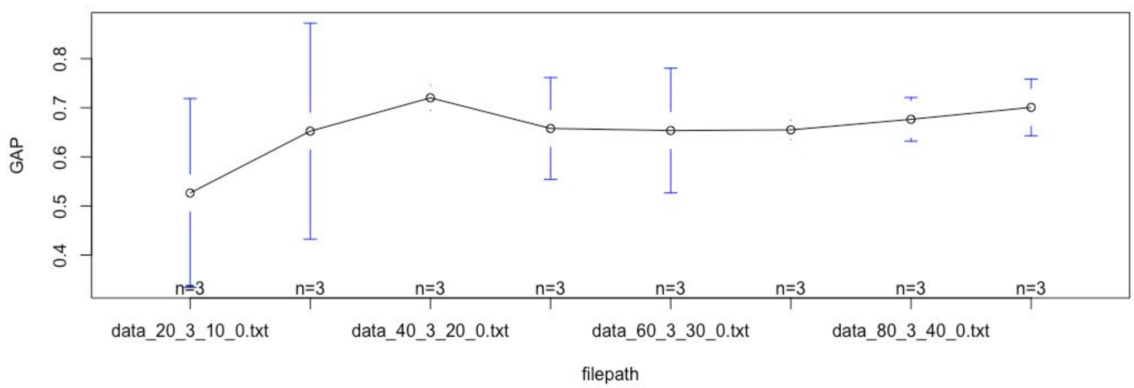

Figure 6.2: C.V of 10 %

Figure 6.1 shows a clear upwards trend for 20 to 40 at a CV of 5%. After 40 blocks, Figure 6.1 shows that the increase in optimality gap achieved is minimal and the Figure 6.1 seems to plateau after 40 blocks. Figure 6.2 shows a clear upwards trend for the model solving for blocks 20 to 40 but again the trend starts to go downwards and plateau around the 40 block mark for a CV of 10%. Figure 6.3 shows an upwards trend for the optimality gap achieved for the blocks 20 to 40 for a CV of 15%. Across all three levels of variation, the model achieves a better optimality gap for data files between 20 and 40

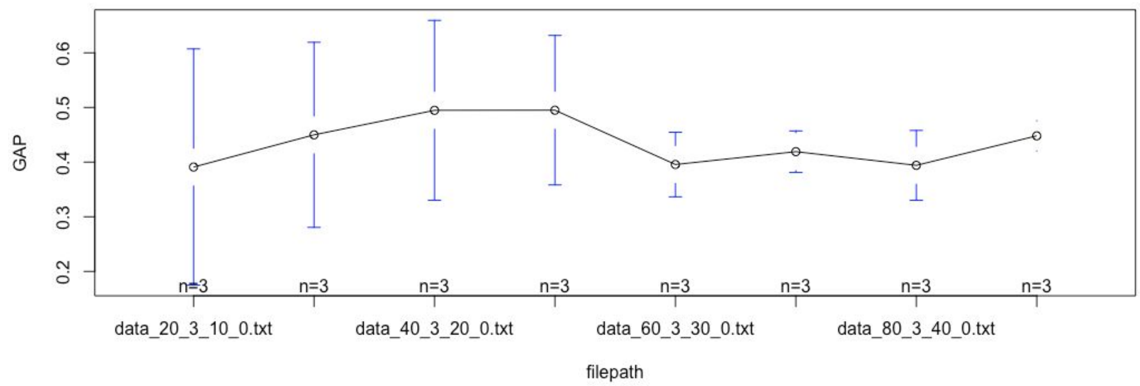

Figure 6.3: C.V of 15 %

blocks which makes sense because the size of the instances is much smaller rather than for 80 blocks. Therefore, the model performs better on smaller instances (i.e., blocks less than 40) because the achieved optimality gap is lower. It appears that this occurs because with smaller instances, they tend to have fewer variables and constraints. This reduction in complexity means the solver has to explore a smaller solution space to find the optimal solution. Therefore, due to the fewer variables and constraints, this typically leads to faster convergence to the optimal solution. Which we see through the lower optimality gap achieved with smaller instances. In Figures 6.1, 6.2, 6.3 data files pertaining to 20 blocks produce the lowest optimality gap achieved demonstrating that the model achieves more robust solutions with smaller instances which is expected.

For Experiment 2 we decided to keep only the data files that are between the range of 20 and 40 blocks to further investigate how the model works with these instances.

6.2 Experiment 2: the impact of the number of scenarios and the CV

The second experiment aims to measure the impact of the number of scenarios and the level of uncertainty. The second experiments tests how the two parameters altered impact the achieved optimality gap.

We tested the three levels of scenarios (i.e., 10, 20 and 30) and the three levels of CV (i.e., 0.05, 0.1 and 0.15). The 30 data files selected were chosen because a clear trend was observed that across all three levels of variation: 0.05, 0.10 and 0.15, it became harder for the model to obtain a better optimality gap after 40 blocks. We choose to further investigate what was occurring when the model was trying to find an optimal solution for data files ranging between 20 and 40 blocks.

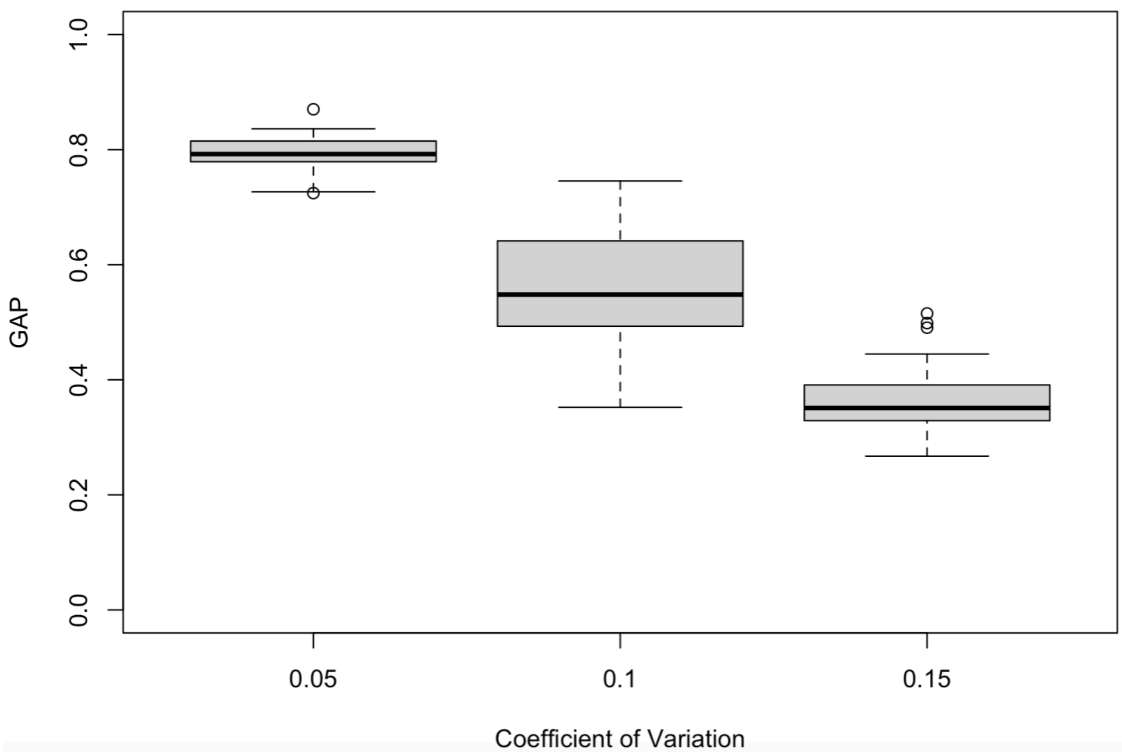

Figure 6.4: The optimality gap at various CVs for 20 blocks

The results for 20 blocks demonstrate a clear correlation between an increase in the CV and a decrease in the optimality gap achieved as demonstrated in Figure 6.4. The median value is represented by the middle line in the box plot Figures 6.4. The median value for the gap goes from around 0.8 to 0.3 as the CV increases from 5% to 15% in Figure 6.4. The outliers present in Figure 6.4 represent solutions whereby the recourse action cost was not utilized. Moreover, in Figure 6.5 it appears that an increase in the number of scenarios does not have a large impact on the achieved optimality gap when solving

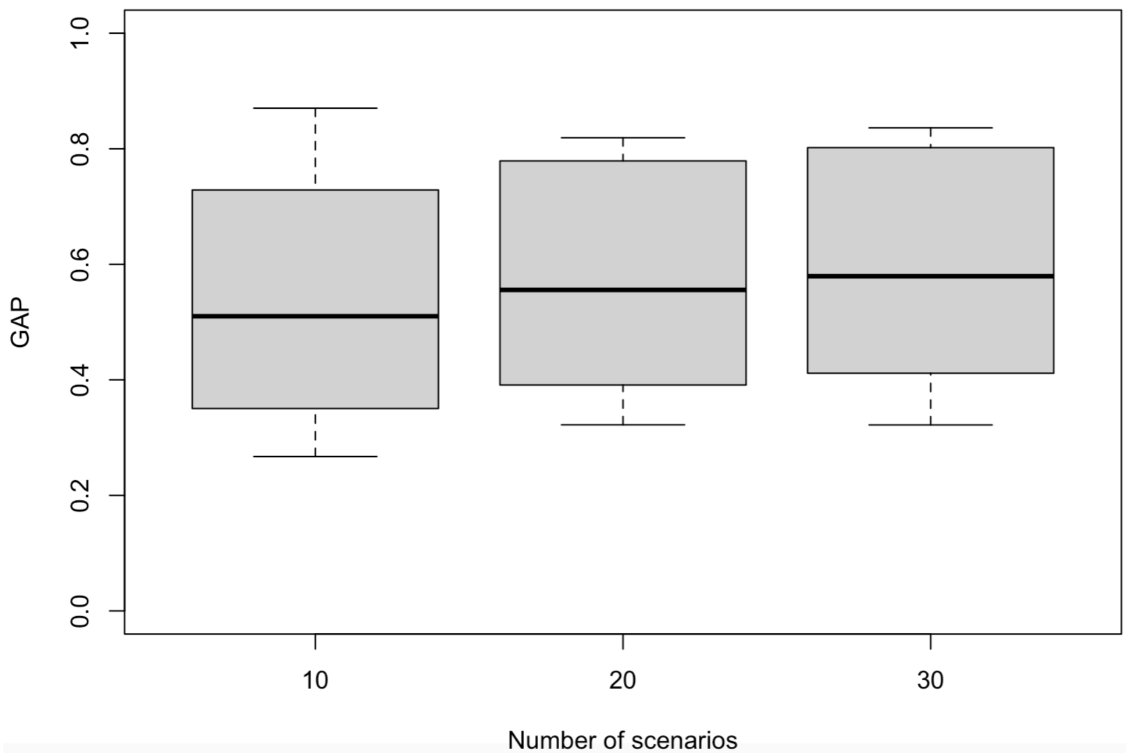

Figure 6.5: The optimality gap at various numbers of scenarios for 20 blocks

the model. The median values in Figure 6.5 marginally differ between the three different levels of number of scenarios tested in the second experiment. Similarly, as the number of scenarios increases, the achieved optimality gap increases slightly demonstrating that the solution quality deteriorates.

As shown in Figure 6.6, results for 30 blocks present a correlation between an increase in the CV and a decrease in the optimality gap achieved. Therefore, in scenarios were we have a large uncertainty related to the energy consumption the model behaves better. In Figure 6.6, the box plots are very small and demonstrate a short range in results. The outliers presented in Figure 6.6 represent solutions in which the recourse action cost was not utilized. Moreover, in Figure 6.7 it appears that an increase in the number of scenarios does not have a large impact on the optimality gap achieved when solving the model. In Figure 6.7, 50% of the results had an optimality gap greater than 0.6 at the three levels of scenarios tested. This demonstrates that as we increase the number of scenarios, the

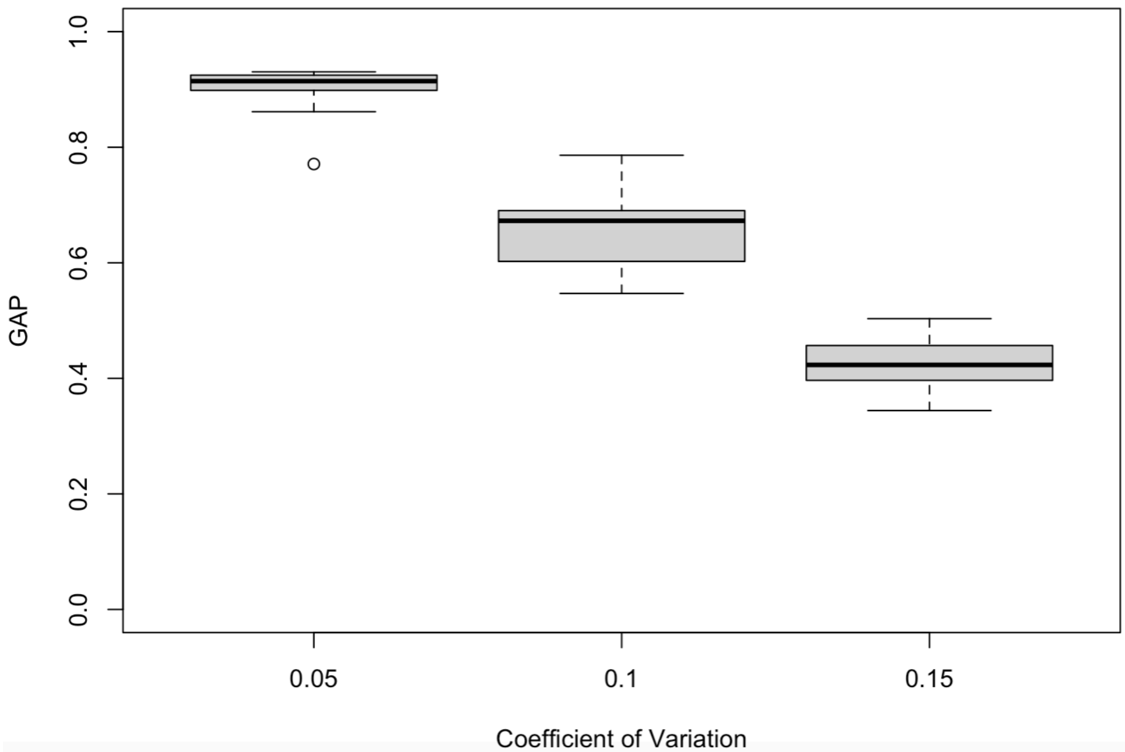

Figure 6.6: The optimality gap at various CVs for 30 blocks

results tend not to achieve a better optimality gap.

Figure 6.8 shows a clear correlation between an increase in the CV and a decrease in the optimality gap achieved for 40 blocks. Therefore, the greater the uncertainty related to energy consumption, the quality of the solution increases. The outliers present in Figure 6.8 represent solutions in which the cost of recourse action was not utilized. Most of the outliers depicted in Figure 6.8, appear at a CV of 5% and 10%. With such a small amount of uncertainty (i.e., a CV of 5%) regarding the gross electric consumption of e-buses does not substantially affect the mean consumption value, hence the recourse action cost remains unused. On average, with a CV of 5% or 10%, there are particular cases where the e-buses will have enough electrical charge to finish their assigned route. Figure 6.9 shows that the median value stabilizes around 0.6 for 10, 20 and 30 scenarios. It would seem that an increase in the number of scenarios does not make the problem harder to solve.

As demonstrated in all three figures for 20, 30 and 40 blocks, the CV has a significant

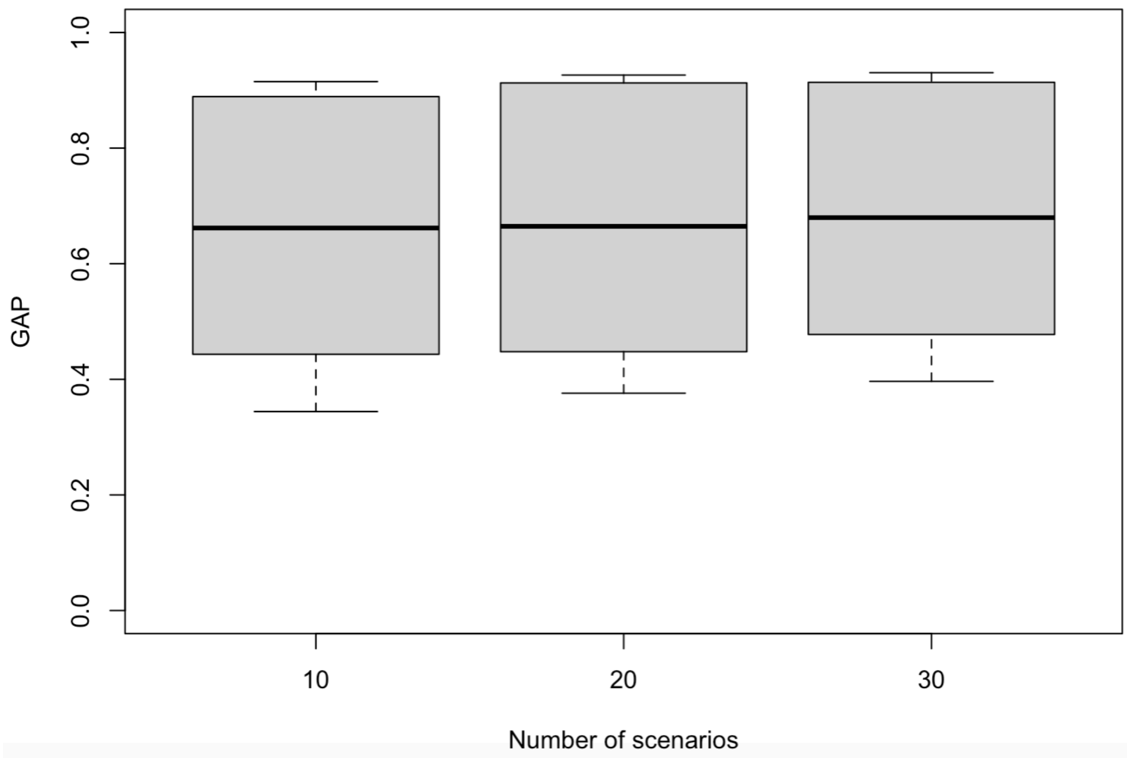

Figure 6.7: The optimality gap at various number of scenarios for 30 blocks

effect on the achieved optimality gap. Therefore, in scenarios were we have a large uncertainty related to the energy consumption the model behaves better by generating better quality solutions. The solutions with lower optimality gaps are achieved by the model because one could speculate that as the CV is larger it accounts for a broader range of scenarios. Therefore, the model can allocate resources more efficiently, reducing the optimality gap by ensuring that sufficient backup resources are available to handle uncertain situations. For example, the model may balance the cost of e-buses charging against the cost of deploying a diesel bus in uncertain conditions, leading to solutions that are closer to optimality across a range of scenarios. Furthermore, in all three figures for 20, 30 and 40 blocks we are able to see that as we increase the number of scenarios, the achieved optimality gap either does not change or increases, which produces worse solutions.

To continue, we will now look at the achieved optimality gap across varying number of instances. In the following figures we will be analyzing the results from Experiment

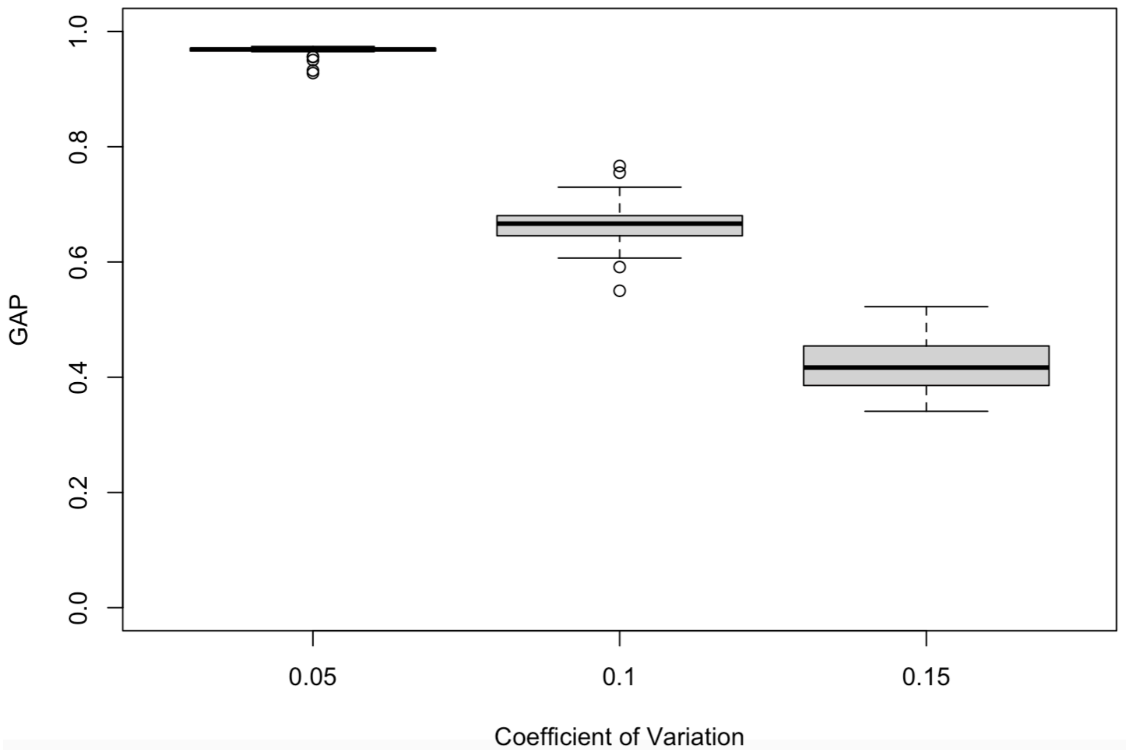

Figure 6.8: The optimality gap at various CVs for 40 blocks

2 looking at 30 scenarios across the three tested coefficients of variation [0.05, 0.10, and 0.15].

The following figures show that, in general terms, an augmentation in the CV corresponds to a diminished optimality gap achieved. Figures 6.10, 6.11 and 6.12 show an upwards trend in the achieved optimality gap compared to the instance. As the number of blocks increases, the achieved optimality gap increases demonstrating the solution quality degrading. It could be argued that as the number of blocks increases, this would increase the solution space making it harder for the solver to explore this area and find an optimal solution. In Figure 6.10, the optimality gap ranges from 0.7 to around 0.9. As the number of blocks gets closer to 40, the achieved optimality gap veers towards 0.9 which translates to 70 percent and 90 percent optimality gap achieved. Figure 6.11 shows that most of the results achieved an optimality gap in the range of 0.5 to 0.8. When we compare Figure 6.10 to Figure 6.11, the best solution found in Figure 6.10 had an optimality gap of around

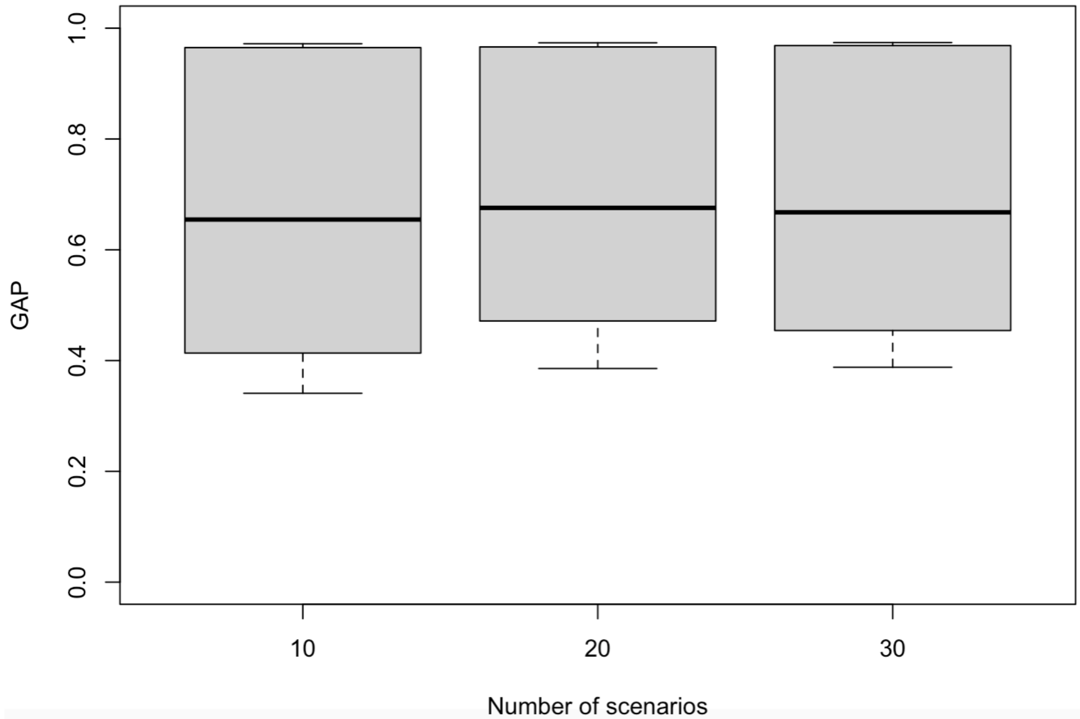

Figure 6.9: The optimality gap at various number of scenarios for 40 blocks

0.8 which is the worst solution found for Figure 6.11. These finding demonstrates that in scenarios were we have a large uncertainty related to the energy consumption the model generates better solutions. Figure 6.12 shows that at a CV of 15 % the results achieved have an optimality gap that ranges from 0.3 to 0.5. As the CV increases the achieved optimality gap decreases which translates into better solutions. Figure 6.11 results hover around 0.6 on average, in comparison Figure 6.12 show worst results that veer closer to 0.5 which is still less than the average of Figure 6.11. Figure 6.12 shows that an increase in the number the blocks does not greatly impact the achieved optimality gap if the CV is large enough. In light of the evidence, it was observed that scenarios that had larger uncertainty (i.e., a larger CV value) had a better lower bound than scenarios that had less uncertainty indicative of better solutions being found. This happens because larger uncertainty can result in a higher lower bound, which helps the model find better solutions. A better lower bound allows the model to prune the search tree more effectively, making the search for optimal solutions more efficient. Although a better lower bound doesn't directly mean we are finding better solutions, it aids the model in getting closer to the optimal solution. This is evident in the results where scenarios with higher uncertainty show a better lower bound and a resulting smaller optimality gap. The main takeaway from the results is that the larger the CV is in scenario creation the better the solutions generated will be.

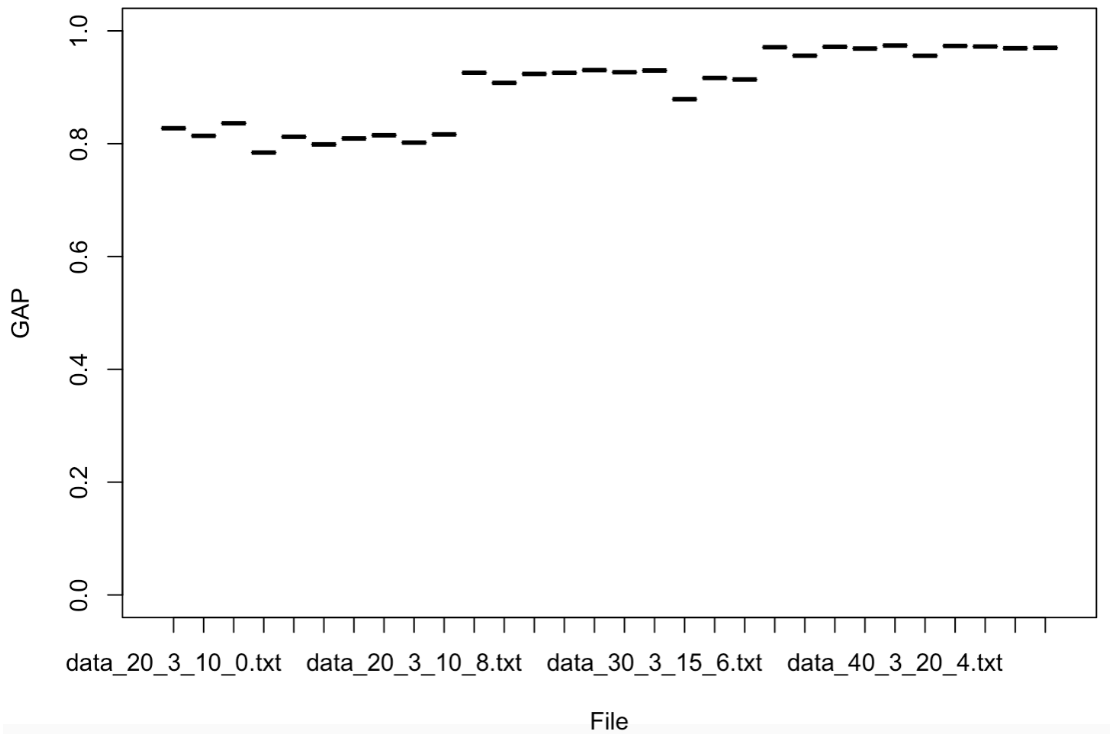

Figure 6.10: CV of 5 % - 30 scenarios

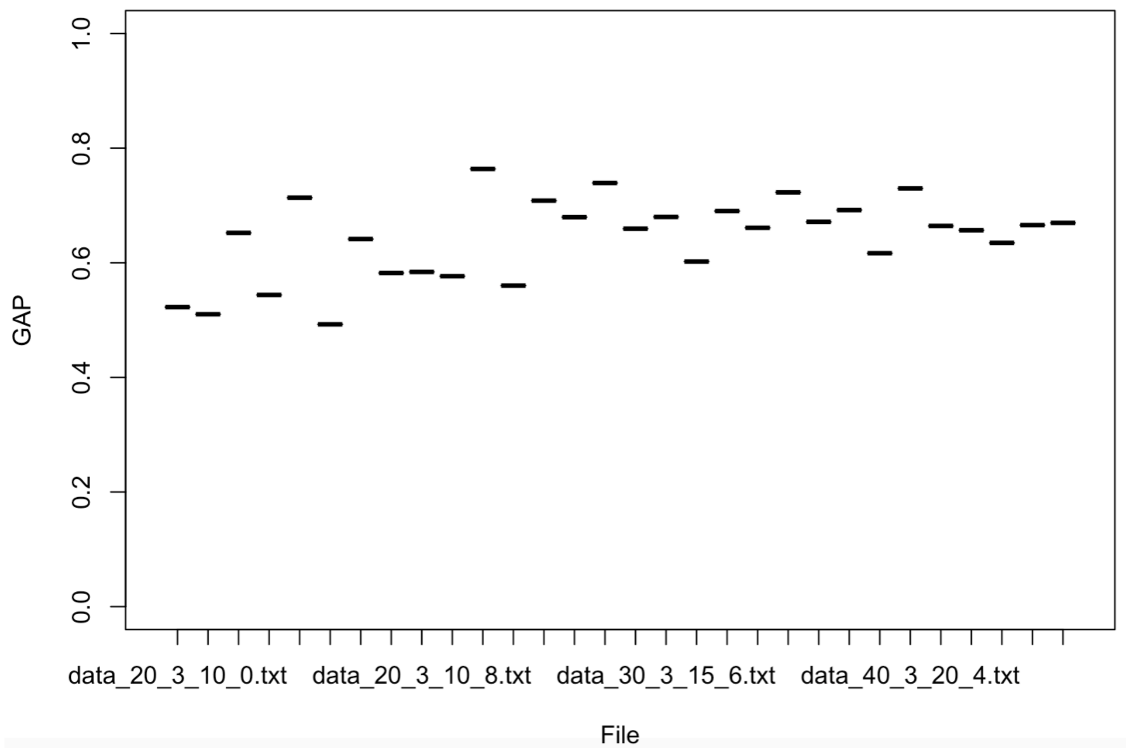

Figure 6.11: CV of 10 % - 30 scenarios

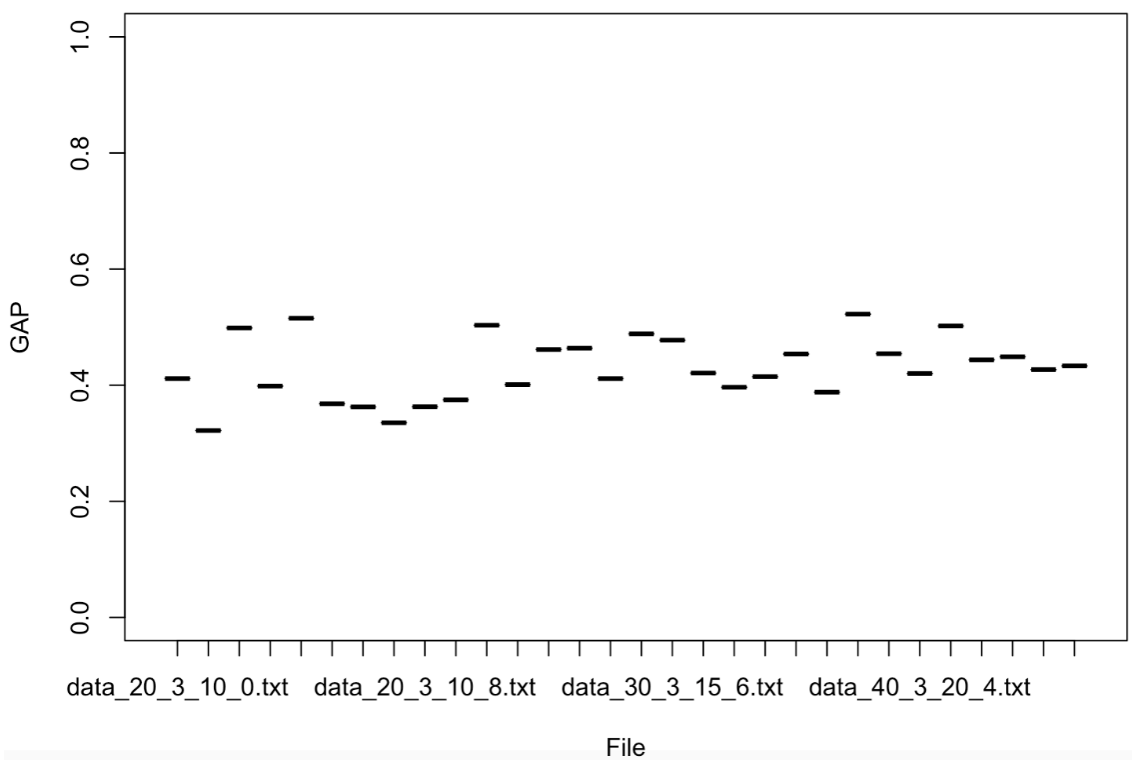

Figure 6.12: C.V of 15 % - 30 scenarios

Chapter 7

Conclusion

In conclusion, the analysis of the results reveals a consistent trend: an escalation in the coefficient of variation corresponds to a decreasing optimality gap, indicative of an increase in the quality of the solution. As the uncertainty is larger (i.e., a larger coefficient of variation) in scenario creation, this results in a lower optimality gap achieved which results in a lower objective function (i.e., lower operating costs). The findings suggest that a larger number of blocks (i.e., more than 40 blocks), on average will increase the achieved optimality gap resulting in decreased solution quality. This occurrence is speculated to be because of the resulting larger solution space created that the solver has to explore to find an optimal solution. The higher the number of blocks, the more complex the problem becomes to solve due to the larger number of variables and constraints.

The scalability of the model to achieve a reasonable optimality gap with a high number of vehicle blocks seems to be a limitation. This observation is not surprising, as exact methods tend to work for smaller instances. If one wanted to find near-optimal solutions for this model, one would need to use a metaheuristic. As previously mentioned in this document, reformulating the problem at hand as Benders decomposition is one solution that can be deployed for solving smaller instances to optimality. Benders decomposition is an exact method and works best for solving smaller instances. Solving the instances to optimality would be an interesting avenue of research that one could pursue to produce better solutions. Moreover, this model could be expanded to focus on stabilizing peak demand charges associated with charging at the depot to help reduce the total electric charging costs.

To conclude, this research investigated the interplay between charging-related decisions and bus route assignment for electric vehicles, with a particular emphasis on the impact of energy uncertainty in electric battery consumption on operational costs. This research formulated the energy uncertainty problem as a two-stage stochastic programming model using mixed integer linear programming. The model optimizes bus route assignments and charging decisions, while accommodating for potential deviations in energy requirements through activating a recourse action. The findings of this research underscore the pivotal role of energy uncertainty in shaping charging strategies and influencing operational costs. This research contributes to the field of electric vehicle fleet management by providing a comprehensive methodology that optimizes bus route assignments and charging protocols while considering real-world variability in energy consumption. As mentioned previously, most research related to the electric bus rostering and scheduling problem had assumed that energy consumption was known. The E-BRCSPUEC model employs two-stage stochastic programming to address energy consumption uncertainty, enhancing robustness by testing various scenarios to inform efficient e-bus charging schedules.

Bibliography

Asghari, M., & Mirzapour Al-e hashem, S. M. J. (2021). Green vehicle routing problem: A state-of-the-art review. *International Journal of Production Economics*, *231*, 107899. https://doi.org/10.1016/J.IJPE.2020.107899

Barahona Rojas, D. S. (2022). Electric bus rostering and charging scheduling M.Sc. Thesis. *Universidad de los Andes*.

Benders, J. (1962). Partitioning procedures for solving mixed-variables programming problems. *Numerische Mathematik*, *4*, 238–252.

Bie, Y., Ji, J., Wang, X., & Qu, X. (2021). Optimization of electric bus scheduling considering stochastic volatilities in trip travel time and energy consumption. *Computer-Aided Civil and Infrastructure Engineering*, *36*(12), 1530–1548. https://doi.org/10.1111/mice.12684

Bruglieri, M., Pezzella, F., Pisacane, O., & Suraci, S. (2015). A Variable Neighborhood Search Branching for the Electric Vehicle Routing Problem with Time Windows. *Electronic Notes in Discrete Mathematics*, *47*, 221–228. https://doi.org/10.1016/J.ENDM.2014.11.029

Charged Future. (2020). Ev charging vs gas refueling comparison: evs take less of your time! [Retrieved July 13th, 2024]. https://www.chargedfuture.com/ev-charging-vs-gas-refueling/

Cokyasar, T., Verbas, O., & Auld, J. (2023). Electric Vehicle Scheduling Problem with Tour Combinations. *Procedia Computer Science*, *220*, 413–420. https://doi.org/10.1016/J.PROCS.2023.03.053

David Suzuki Foundation. (2024). Ten strategies every climate plan needs - David Suzuki Foundation [Retrieved July 14th,2024]. https://davidsuzuki.org/what-you-can-do/ten-strategies-every-climate-plan-needs

Drezner, Z., & Hamacher, H. W. (2004). Facility Location: Applications and Theory. *Springer-Verlag*.

Environment and Climate Change Canada. (2022). A clean electricity standard in support of a net-zero electricity sector: discussion paper [Retrieved July 13th,2024]. https://www.canada.ca/en/environment-climate-change/services/canadian-environmental-protection-act-registry/achieving-net-zero-emissions-electricity-generation-discussion-paper.html

Gallet, M., Massier, T., & Hamacher, T. (2018). Estimation of the energy demand of electric buses based on real-world data for large-scale public transport networks. *Applied Energy*, *230*, 344–356. https://doi.org/10.1016/j.apenergy.2018.08.086

Gilmore, J. (2017). Census in Brief: Commuters using sustainable transportation in census metropolitan areas. https://www12.statcan.gc.ca/census-recensement/2016/as-sa/98-200-x/2016029/98-200-x2016029-eng.cfm?wbdisable=true

Hayes, A. (2024). Coefficient of variation meaning and how to use it [Retrieved July 13th,2024]. https://www.investopedia.com/terms/c/coefficientofvariation.asp

Higle, J. L. (2005). Stochastic Programming: Optimization When Uncertainty Matters. *INFORMS Tutorials in Operations Research*, 30–53. https://doi.org/10.1287/EDUC.1053.0016

Houbbadi, A., Trigui, R., Pelissier, S., Iglesias, E. R., & Bouton, T. (2019). A quadratic programming based optimisation to manage electric bus fleet charging. *International Journal of Electric and Hybrid Vehicles*, *11*(4), 1. https://doi.org/10.1504/IJEHV.2019.102862

Hu, H., Du, B., Perez, P., Liu, T., & Ceder, A. A. (2022). A two-stage stochastic programming model of locating fast chargers at stops for battery electric buses. *IEEE Conference on Intelligent Transportation Systems, Proceedings, ITSC*, *2022-October*, 3679–3684. https://doi.org/10.1109/ITSC55140.2022.9922374

Hydro-Québec. (2024). Hydropower | Advantages | Hydro-Québec [Retrieved July 13th,2024]. http://www.hydroquebec.com/learning/hydroelectricite/

International Energy Agency. (2023). Executive summary – Global EV Outlook 2023 – Analysis - IEA [Retrieved July 13th,2024]. https://www.iea.org/reports/global-ev-outlook-2023/executive-summary

International Energy Agency. (2021). Policies to promote electric vehicle deployment – global ev outlook 2021 – analysis - iea. https://www.iea.org/reports/global-ev-outlook-2021/policies-to-promote-electric-vehicle-deployment

Küçükyavuz, S., & Sen, S. (2017). An Introduction to Two-Stage Stochastic Mixed-Integer Programming. *INFORMS Tutorials in Operations Research*, 1–27. https://doi.org/10.1287/EDUC.2017.0171

Li, P., Arellano-Garcia, H., & Wozny, G. (2008). Chance constrained programming approach to process optimization under uncertainty. *Computers and Chemical Engineering*, *32*, 25–45. https://doi.org/10.1016/j.compchemeng.2007.05.009

Liu, L., Kotz, A., Salapaka, A., Miller, E., & Northrop, W. F. (2019). Impact of time-varying passenger loading on conventional and electrified transit bus energy consumption. *Transportation Research Record*, *2673*, 632–640. https://doi.org/10.1177/0361198119852337

Liu, Y., Wang, L., Zeng, Z., & Bie, Y. (2022). Optimal charging plan for electric bus considering time-of-day electricity tariff. *Journal of Intelligent and Connected Vehicles*, *5*(2), 123–137. https://doi.org/10.1108/JICV-04-2022-0008

Martins, J., Brito, F., Pedrosa, D., Monteiro, V., & Afonso, J. (2013). Real-Life Comparison Between Diesel and Electric Car Energy Consumption, 209–232.

Mead, L. (2024). The Road to Sustainable Transport | International Institute for Sustainable Development [Retrieved July 13th,2024]. https://www.iisd.org/articles/deep-dive/road-sustainable-transport

Montoya, A., Guéret, C., Mendoza, J. E., & Villegas, J. G. (2017). The electric vehicle routing problem with nonlinear charging function. *Transportation Research Part B: Methodological*, *103*, 87–110. https://doi.org/10.1016/j.trb.2017.02.004

Montoya-Torres, J. R., López Franco, J., Nieto Isaza, S., Felizzola Jiménez, H., & Herazo-Padilla, N. (2015). A literature review on the vehicle routing problem with multiple depots. *Computers & Industrial Engineering*, *79*, 115–129. https://doi.org/https://doi.org/10.1016/j.cie.2014.10.029

Optiom. (2024). The advantages of electric cars over gasoline cars: unveiling the road ahead [Retrieved on July 13th,2024]. https://www.optiom.com/the-advantages-of-electric-cars-over-gasoline-cars-unveiling-the-road-ahead/

Paulraj, P. (2022). What are Depot charging & Opportunity charging for Electric Buses and Trucks - E-Mobility Simplified | Basics of Electric Vehicles and Charging. https://www.emobilitysimplified.com/2022/06/what-is-ev-depot-opportunity-charging.html

Pelletier, S., Jabali, O., & Laporte, G. (2018). Charge scheduling for electric freight vehicles. *Transportation Research Part B: Methodological*, *115*, 246–269. https://doi.org/10.1016/j.trb.2018.07.010

Pelletier, S., Jabali, O., Mendoza, J. E., & Laporte, G. (2019). The electric bus fleet transition problem. *Transportation Research Part C: Emerging Technologies*, *109*, 174–193. https://doi.org/10.1016/J.TRC.2019.10.012

Perumal, S. S., Lusby, R. M., & Larsen, J. (2022). Electric bus planning & scheduling: A review of related problems and methodologies. *European Journal of Operational Research*, *301*(2), 395–413. https://doi.org/10.1016/J.EJOR.2021.10.058

Prussi, M., Padella, M., Conton, M., Postma, E., & Lonza, L. (2019). Review of technologies for biomethane production and assessment of Eu transport share in 2030. *Journal of Cleaner Production*, *222*, 565–572. https://doi.org/10.1016/J.JCLEPRO.2019.02.271

Pustějovská, K., Janovská, K., & Jursová, S. (2023). Alternative Sources of Energy in Transport: A Review. *Processes 2023, Vol. 11, Page 1517*, *11*(5), 1517. https://doi.org/10.3390/PR11051517

Racz, A. A., Muntean, I., & Stan, S.-D. (2015). A Look into Electric/Hybrid Cars from an Ecological Perspective. *Procedia Technology*, *19*, 438–443. https://doi.org/10.1016/j.protcy.2015.02.062

ROHM. (2023). Charging Method | Charge Control ICs | Electronics Basics | ROHM [Retrieved July 13th,2024]. https://www.rohm.com/electronics-basics/battery-charge/charging-method

Sachan, S., Deb, S., & Singh, S. N. (2020). Different charging infrastructures along with smart charging strategies for electric vehicles. *Sustainable Cities and Society*, *60*, 102238. https://doi.org/10.1016/J.SCS.2020.102238

Sassi, O., & Oulamara, A. (2017). Electric vehicle scheduling and optimal charging problem: complexity, exact and heuristic approaches. *International Journal of Production Research*, *55*(2), 519–535. https://doi.org/10.1080/00207543.2016.1192695

Sen, S., & Higle, J. L. (1999). An Introductory Tutorial on Stochastic Linear Programming Models. *29*, 33–61.

Singh Verma, A., Chhabra, S., Karnwal, A., Gupta, A., & Kumar, R. (2022). A review on performance, combustion and emissions utilizing alternative fuels. *Materials Today: Proceedings*, *64*, 1459–1464. https://doi.org/10.1016/J.MATPR.2022.04.777

Société de transport de Montréal. (2023). Electric buses | Société de transport de Montréal [Retrieved July 13th,2024]. https://www.stm.info/en/about/major_projects/major-bus-projects/bus-network-electrification/electric-bus

Southern California Edison. (2024). Electric Vehicle Rates | Rates | Your Business | Home - SCE [Retrieved July 13th,2024]. https://www.sce.com/business/rates/electric-car-business-rates/business/rates/electric-car-business-rates

Statistics Canada. (2023). The daily commuting to work by car and public transit grows in 2023 [Retrieved July 13th,2024]. https://www150.statcan.gc.ca/n1/daily-quotidien/230822/dq230822b-eng.htm

Sustainable Bus. (2022). Electric bus range, focus on electricity consumption. A sum-up - [Retrieved July 13th,2024]. https://www.sustainable-bus.com/news/electric-bus-range-electricity-consumption/

Taşkin, Z. C. (2011). Benders Decomposition. *Wiley Encyclopedia of Operations Research and Management Science*. https://doi.org/10.1002/9780470400531.EORMS0104

Tzamakos, D., Iliopoulou, C., & Kepaptsoglou, K. (2023). Electric bus charging station location optimization considering queues. *International Journal of Transportation Science and Technology*, *12*(1), 291–300. https://doi.org/10.1016/J.IJTST.2022.02.007

United Nations. (2024). Climate Change | United Nations [Retrieved July 13th,2024]. https://www.un.org/en/global-issues/climate-change

U.S. Department of Energy. (2023a). Batteries for Electric Vehicles [Retrieved July 13th,2024]. https://afdc.energy.gov/vehicles/electric-batteries

U.S. Department of Energy. (2023b). Developing Infrastructure to Charge Electric Vehicles [Retrieved July 13th,2024]. https://afdc.energy.gov/fuels/electricity_infrastructure.html

U.S. Department of Energy. (2023c). How Do All-Electric Cars Work? [Retrieved July 13th,2024]. https://afdc.energy.gov/vehicles/how-do-all-electric-cars-work

U.S. Department of Energy. (2023d). How Do Hybrid Electric Cars Work? [Retrieved July 13th,2024]. https://afdc.energy.gov/vehicles/how-do-hybrid-electric-cars-work

Wen, M., Linde, E., Ropke, S., Mirchandani, P., & Larsen, A. (2016). An adaptive large neighborhood search heuristic for the Electric Vehicle Scheduling Problem. *Computers & Operations Research*, *76*, 73–83. https://doi.org/10.1016/J.COR.2016.06.013

WWF. (2023). Fossil fuels and the state of the energy crisis | WWF [Retrieved July 13th,2024]. https://wwf.panda.org/wwf_news/?7847466/fossil-fuels-energy-crisis

Xu, H., Xia, X., Liang, W., Ouyang, F., Zhu, W., & Li, H. (2019). Charging Harmonic Characteristic Analysis of Electric Vehicles Based on Simulation and Analytical Methods. *2019 3rd IEEE Conference on Energy Internet and Energy System Integration: Ubiquitous Energy Network Connecting Everything, EI2 2019*, 1166–1171. https://doi.org/10.1109/EI247390.2019.9061782

Zhou, B., Chen, G., Song, Q., & Dong, Z. Y. (2020). Robust chance-constrained programming approach for the planning of fast-charging stations in electrified transportation networks. *Applied Energy*, *262*. https://doi.org/10.1016/j.apenergy.2019.114480