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Navigating Through Momentum Crashes: An Empirical Study of Momentum Returns

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In Partial fulfilment of the requirements for
the Degree of Master of Science (M.Sc.)

December 2022

1 Abstract

Momentum investment strategies have proven themselves profitable in terms of high returns and a higher Sharpe ratio than the market portfolio, across most markets and financial asset classes. They are, however, known for having left-skewed returns with a high excess kurtosis. This is reflected in the infrequent but severe crashes that happen right after extreme market downturns. In a chaotic market state, volatility soars, which causes momentum portfolios to act like short call options on the market. As a consequence of this feature, the momentum portfolio is vulnerable to big losses that could wipe out most of the gains when the market turns around. The fact that these momentum crashes can be partly foreseen motivates the development of a dynamic strategy that reduces the intensity of these adverse events. A dynamic strategy consists of finding time-varying weights of the WML portfolio in a way that adjusts the investor's exposure to the forecasted market condition. This paper introduces a dynamic momentum model that relies on the realized volatility of the S&P500, which in fine reduces the intensity of the momentum crashes and delivers higher cumulative P&L and risk-adjusted return measures than the benchmark's dynamic momentum model.

keywords: Momentum; crashes; realized volatility

2 Résumé

Les stratégies d'investissement momentum se sont avérées rentables en termes de rendement élevé et de ratio de Sharpe supérieur à celui du portefeuille de marché, sur la plupart des marchés et des classes d'actifs financiers. Cependant, elles ont également la réputation d'avoir des rendements asymétriques à gauche, associés à un excès d'aplatissement élevé. Cela se traduit par des krachs peu fréquents mais sévères qui se produisent juste après des baisses extrêmes du marché. En fait, dans un état de marché chaotique, la volatilité monte en flèche, ce qui fait que les portefeuilles momentum agissent comme une option d'achat courte sur le marché. Cette caractéristique spécifique expose le portefeuille momentum à des pertes extrêmes qui pourraient anéantir la plupart des gains accumulés lorsque le marché se redresse. Le fait que ces krachs momentum puissent être partiellement prévus motive le développement d'une stratégie dynamique qui réduit l'intensité de ces événements négatifs. Une stratégie dynamique consiste à trouver des pondérations du portefeuille WML variables dans le temps de manière à ajuster l'exposition de l'investisseur en fonction de l'état prévu du marché. Cet article présente un modèle momentum dynamique qui s'appuie sur la volatilité réalisée du S&P500 et qui réduit in fine l'intensité des crashs momentum ainsi fournissant des mesures de P&L cumulé et de rendement ajusté au risque plus élevées que le modèle benchmark.

mots-clés: Momentum; crashs; volatilité réalisée

3 Acknowledgement

My deepest and sincerest gratitude goes to Geneviève Gauthier for being a mentor, a great professor, and an amazing friend. I learned from her that mathematics is a language that speaks through equations and that math is better understood if done with colored pens. I thank her very much for all the hours and patience that she has dedicated to the achievement of this research.

My sincere thanks and appreciation go to all my professors, HEC Montreal staff, and everyone who contributes directly or indirectly to making the MSc Financial Engineering a great program and HEC Montreal a great institution.

I would like to give credit to Fin-ML create scholarship and machine learning training programs that were absolutely appreciated.

I would like to thank my family and friends for their support and encouragement throughout my journey at HEC Montreal.

4 Introduction

Momentum investing has a higher Sharpe ratio than most other investment strategies, making it an appealing strategy for investors. However, negative skewness and high kurtosis are inherent features of these strategies, as they tend to severely crash in turbulent financial crises. The momentum strategy entails going long the winning portfolio and short the losing portfolio of the investment sample. The winner and loser portfolios are designated by a ranking process based on cumulative return over the formation period, which usually goes from 3 to 12 months before the investment date. The investment sample is subsequently divided and ranked from least-performing to most-performing portfolios. When it comes to stock selection and ranking, there are two types of momentum to consider: cross-sectional momentum and time-series momentum. The cross-sectional momentum strategy assigns stocks to the winner and loser portfolios based on the relative performance of the stocks in the investment sample during the formation period. This sums up to ranking the stocks compared to each other and assigning them to different momentum portfolios from bottom (least performing) to top (most performing). The time-series momentum strategy, on the other hand, ranks and assigns stocks based on their absolute performance. This means that stocks are considered winners if their return is higher than a certain level that the investor sets, and losers otherwise. The resulting portfolios are either cap-weighted or equal-weighted, meaning that stocks are included in each portfolio in amounts that correspond to their total market capitalization versus investing the same amount of money in each stock in each portfolio, respectively. In this research paper, the momentum strategy we opt for is cap-weighted and cross-sectional.

Momentum strategies have proven profitable in a wide range of asset classes and financial markets around the world. However, although they are able to generate competitive returns, they are vulnerable to extreme losses when financial market crashes happen. Daniel and Moskowitz (2016), in their paper "momentum crashes," confirm the limitations of these strategies. The two researchers start by implementing a cap-weighted cross-sectional strategy by buying the top decile, or the winner portfolio, and shorting the bottom decile, or the loser portfolio. Their momentum strategy applied to the US stock market from January 1927 until March 2013 yields an astonishing 7.6% excess market return and a WML portfolio Sharpe ratio of 0.71 that almost doubles the market's of 0.4. The winner and loser portfolios score, respectively, a yearly average excess return of 15.3% and -2.5%. However, the paper points out that although the average returns are large and statistically significant, the strategy suffers from momentum crashes that took place from June 1932 until December 1939 and from March 2009 until March 2013. During these periods, the loser portfolio outperformed the winner portfolio significantly. In fact, during the first crash, the loser portfolio made more than twice the profits of the winner portfolio. During the second momentum crash, the losers outperformed the winners by 50%. Researchers and investors both wonder if it would be worth it to try to predict these crashes, given how much they affect the overall profitability of the strategy.

Motivated to find a solution to the downside of momentum investment strategies, Daniel and Moskowitz (2016) developed a dynamic momentum strategy based on maximizing the unconditional Sharpe ratio of the WML portfolio. Building on the intertemporal version of Markowitz (1952) portfolio optimization, they came up with a method to assign optimal time-varying weights to the winner and loser portfolios in order to avoid extreme negative events.

The optimal dynamic time-varying weight is obtained by calculating the conditional expected return divided by the conditional expected variance of the WML portfolio over the coming month multiplied by a time-invariant scalar that controls the unconditional risk and return of the dynamic portfolio. The conditional expected return of the WML portfolio is estimated by a regression that takes into account a bear market indicator and the variation in the market return over the past 126 days. The conditional variance is approximated using the trailing variance of the WML portfolio over the past 126 days. The intuition behind this dynamic weighting formula is that the conditional expected return is estimated taking into account the potential presence of a market crash, as indicated by both historic negative market cumulative returns and high market volatility. When the conditional variance of the WML portfolio is expected to be high, the weights get scaled down to account for the expected risk, and vice versa. This strategy provided an out-of-sample Sharpe ratio of 1.194 which greatly exceeds the Sharpe ratio of the plain momentum strategy.

Barroso and Santa-Clara (2015) found that the realized variance of daily returns is a better risk measure and indicator of volatility than the simple variance of returns in the momentum strategy portfolio. Convinced that it is possible to improve the risk-return profile of the dynamic momentum strategy developed by Daniel and Moskowitz (2016), we combine the previous two findings to build a dynamic model based on the realized volatility of the S&P500 instead of relying on the standard deviation of the market returns to forecast the conditional expected return of the momentum strategy. The reason we elected to exploit the S&P500 data to calculate the realized volatilities subsequent to this study is twofold. Firstly, the CRPS market portfolio and the S&P500 index are strongly correlated, which makes them equally useful as a volatility gauge signal in our model. Second, no data provider provides high-frequency data for the CRSP portfolio, whereas intraday data for the S&P500 is available with a frequency as low as one minute. The newly developed strategy that we refer to hereafter as the S&P500 RVol-based dynamic strategy succeeds in outperforming Daniel and Moskowitz (2016) strategy. In fact, the out-of-sample cumulative return of our strategy is superior to Daniel and Moskowitz (2016) dynamic strategy. Moreover, the new strategy delivers higher risk-adjusted returns and higher profits. In fact, the out-of-sample Sharpe ratio and the Sortino ratio for the S&P500 RVol-based dynamic strategy are 0.2473 and 1.0610, respectively, versus Daniel and Moskowitz (2016) dynamic strategy, which amount to 0.1469 and 0.2742, respectively.

The blueprint of this paper is the following: Section 5 is a detailed review of the most relevant research papers on momentum investment strategies as it discusses their uses, effectiveness, limits and how they could be improved. The literature review aims to situate the reader in the problematic context and prepare the ground for the subsequent study. Section 6 is a direct application of the vanilla momentum strategy. Section 7 is an exact replication of Daniel and Moskowitz's (2016) dynamic momentum strategy, except that we have used a shorter time horizon that goes from January 1996 until 2021. We refer to this strategy as the vanilla dynamic momentum strategy. A test of the robustness of this strategy is also applied in the same section by using an expanding window regression and a rolling window regression in order to estimate the conditional expected return of the WML portfolio. Section 8 presents the RVol-based dynamic momentum strategy model and compares its performance to the vanilla dynamic momentum strategy model. Section 9 sums up the research presented in this paper, with an emphasis on the difficulties faced during the implementation of this research and the possible future venues to further push the reflection on this investment strategy.

5 Literature Review

Since they were first applied in the US stock market by Jagadeesh and Titman in 1993, the high returns rationale for momentum investing strategies, which are portrayed as one of the most effective and widely used style investing strategies within the factor investing family, has been a source of contention. In fact, in their paper, "Returns to Buying Winners and Selling Losers: Implications for Stock Market Efficiency," Jegadeesh and Titman come to the conclusion that momentum investing strategies are responsible for generating abnormal returns in different markets. By buying past winners and shorting past losers over the past six months and holding them for the next six months, the two researchers proved that this strategy generates a Sharpe ratio that exceeds the Sharpe ratio of the market itself. From 1927 to 2011, the monthly momentum excess return in the US market amounted to 1.75%. The fact that momentum strategies can produce an abnormal return means that some level of predictability is possible, which goes against the Efficient Market Hypothesis, which says that market prices always reflect all available information. Lots of finance researchers and practitioners have tried to explain this phenomenon, but no consensus has been reached so far. However, two main branches of finance, namely behavioral finance and traditional finance, provided solid arguments in the quest to unveil the logic behind such abnormally high returns.

On the one hand, abnormal returns from momentum investing are possible due to a variety of biases that affect the investor's behavior, according to behaviorists. "A positive feedback trading strategies" bias was advanced by Delong, Shleifer, Summers, and Waldman (1990) in their article, "Positive feedback investment strategies and destabilizing rational speculation." This bias, which could be described as a "herding effect," has an impact on the market and causes prices to deviate from their fundamental value in the short run, feeding the momentum effect. "Conservative Bias" was the subject of Barberis, Shleifer, and Vishny (1998) article, "A model of investor sentiment". It is based on the idea that investors tend to take a long time to adjust their investment plans based on new market information. This delay of action contributes to the continuation of momentum in the short term. However, once the new information is fully incorporated into the market, there is no space left for stock return predictability. "overconfidence" and "self-attribution bias" are two other biases that were discussed by Daniel, Hirshleifer and Subrahmanyam (1998). They argue that overconfident investors overestimate their ability to analyze and evaluate new information. The self-attribution bias is related to overestimating private information and underestimating the possibility of forecasting errors. These two psychological patterns of overconfidence contribute to investors' overreactions, which add momentum to market returns. The list of behavioral biases is long and backed by psychological explanations that could be modeled, which constitute a solid case for explaining the momentum strategies' abnormal returns.

On the other hand, traditional finance has made several attempts to explain the abnormally high returns of momentum investing strategies. Grundy and Martin (2001) tackle the subject from a risk-return perspective, trying to decompose and subsequently explain the high returns of such strategies. When the Fama-French three-factor model is applied to the various momentum portfolios, the two researchers discover that the market betas for winners and losers portfolios are the same. However, the losers' portfolios are more sensitive to the SMB and HML factors. This sensitivity relationship between past returns on one end and the firm size and book-to-market ratio on the other indicates that the losers' portfolio is riskier compared to the winners' portfolio. Therefore, shorting the loser portfolio and going long the winner portfolio results in mitigating risk while maximizing the short-term returns. From a different perspective that is specific to industry returns, Moskowitz and Grinblatt (1999) examine the contribution of industry momentum to the overall momentum strategy portfolio. They build their momentum portfolios on a value-weighted industry basis and rank them according to past industry returns. By replacing the firms in the momentum portfolios with other firms that are not relevant to the same industry but score the same return over the formation period, they note that the random industry portfolios total near zero returns. This finding drives them to conclude that industry momentum is the motor behind the high returns of momentum strategies and not firm-specific momentum. Avramov, Cheng and Hameed (2016) on their paper "Time-Varying Liquidity and Momentum Profits" attribute the abnormally high return of the momentum strategy to the high market liquidity of the US stock market. The paper demonstrates a strong relationship between market liquidity and high momentum profit, as well as vice versa. Whatever the true reason for momentum strategies' profitability, they are prone to recurring crushes that undermine their effectiveness. For instance, the subprime mortgage crisis of 2008–2009 left momentum strategies in the red with a net return of the WML portfolio of -73.42% in three months time. Such a loss is not only huge and substantial but also impossible to recover from in the short run.

Many studies have looked at the problems with momentum strategies by trying to figure out why they keep falling apart. Kothari and Shanken (1992) defended the idea that momentum portfolios, by nature, have betas of return that are not constant since they are ranked based on past returns. This specific feature makes such portfolios exposed throughout time to systematic factors, which, in turn, makes the momentum strategy vulnerable in periods of market recession and financial market crashes. This is further illustrated by the fact that when a severe bear market occurs during the momentum strategy formation period, the winner portfolio is most likely to be composed of low beta stocks, while the loser portfolio is composed of high beta stocks. When followed by a market recovery, the momentum portfolio, which has a large negative unconditional beta most of the time, fails to catch up with the uptrend and crashes. Grundy and Martin (2001) tried to remediate the vulnerability of the momentum strategy by hedging the time-varying market and size risk exposure; however, their hedging portfolio was constructed by accounting for forward-looking betas, which makes it strongly biased since it relies on future information of the market. As pointed out by Daniel and Moskowitz (2016) Grundy and Martin's hedge strategy does not yield the same performance when based on ex ante betas. Jointly, Daniel and Moskowitz (2016) in their paper "momentum crashes", and following the Henriksson and Merton (1981) specification, calculated in bear market periods the up and down betas for momentum portfolios. They found that the up-market beta for momentum portfolios is more than double its down-market beta, with values of -1.51 and -0.7, respectively. Outside of bear-market periods, there is no real difference in betas that is statistically significant. This specific result motivated the researchers to conclude that this pattern of the loser portfolio suggests that in bear markets, momentum strategies behave like written call options on the market.

Developing models or forecasting methods to deal with these infrequent but costly crashes in the strategy imposed itself as a necessity in order to improve its overall profitability. Barroso and Santa-Clara (2015) in their paper “Momentum has its moment” were not only successful in reducing the intensity of the downside risk of the strategy but they were also able to improve its Sharpe ratio. They implement a risk-managed momentum strategy based on scaling the long-short portfolio by its realized volatility in the previous six months in order to keep the volatility of the strategy at a constant, targeted level of 12%. This scaled momentum strategy yields a Sharpe ratio of 0.97, which is almost twice the regular momentum strategy's 0.53 applied in the period of 1927–2001. Moreover, The excess kurtosis is significantly reduced from 18.24 to 2.68, and the left skew is less pronounced and gets from -2.47 to -0.42. Daniel and Moskowitz (2016) developed a dynamic momentum strategy that adjust the time-varying weights of the winner and loser portfolios in order to maximize the unconditional Sharpe ratio of the WML portfolio. The researchers estimate the conditional moments of the WML portfolio to generate the dynamic weights by relying on the predictability of both momentum premium and momentum volatility. Once the strategy is properly implemented, it outperforms the classical momentum strategy by yielding a positive alpha and a Sharpe ratio that is twice as high.

6 Vanilla Momentum Strategy Implementation

The simplest version of momentum investing consists of assigning stocks from the investment sample to different portfolios based on their past performance. As it has been pointed out in the previous sections, the past cumulative return over a certain period ranging from three months to a year is the most privileged ranking criteria. Dividing the investment sample into same-sized portfolios based on past performance is the next step in this strategy's implementation. Once the portfolios are formed the momentum strategy is implemented by going long the past winner portfolio and going short the past loser one. This section addresses in detail the steps taken from the selection of stocks and the construction of the different portfolios to the implementation of the vanilla momentum strategy according to the method followed by Daniel and Moskowitz (2016).

6.1 The Investment Sample Data

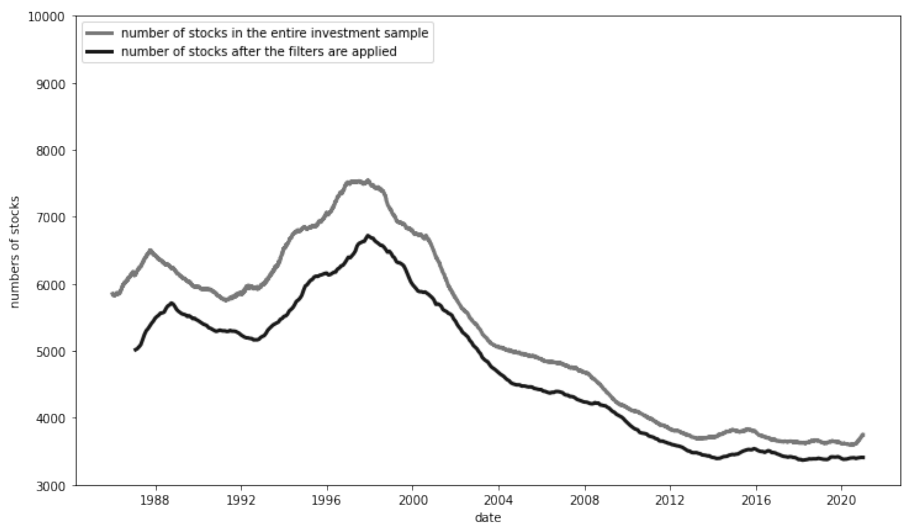
The investment sample data is based on US common equities spanning from the first of January 1986 to the end of December 2020. The source of this database is the CRPS Wharton Research Data Services, which provide day-end and month-end prices. Certain filters are applied to the database to ensure that the necessary data for the momentum strategy implementation is available.

- The first filter is global and concerns the investment sample as a whole; Stock data is restricted to only ordinary common shares enlisted on the NYSE, AMEX and NASDAQ, such shares from the CRSP database correspond to the share-code 10 and 11 and the exchange code 1, 2 and 3.

Once the stock data is selected, an adjustment is required to reflect cash dividends, stock dividends, stock splits, mergers, total liquidations, numbers of shares outstanding, and delisted stocks by following the CRSP guidelines to make the data comparable and consistent throughout time.

- The second filter is specific to each stock in the investment sample; the following conditions are mandatory in order for a stock to be kept in the database:
  - The price of the stock at month $t-13$ is available.
  - The return of the stock at month $t-2$ is available.
  - The market capitalization of the stock at month $t-1$ is available.
  - There is a minimum of 8 months returns over the past 11 months skipping the last month.[1]

Figure 1: The Investment Sample Stocks Evolution Before and After The Filters are Applied



This figure tracks the number of ordinary common shares enlisted on the NYSE, AMEX and NASDAQ, which form our entire investment data sample, before and after filters are applied. The gray curve highlights the number of stocks in the initial database throughout time, while the black curve indicates the portion of stocks that do meet the filter conditions and that are included in the implementation of the momentum strategies.

Figure 1 is a clear manifestation of the application of the discussed filters, this figure plots the evolution of the number of stocks in the investment sample throughout the investment horizon before and after the filters are applied. The gray curve in Figure 1 appears to be parallel to the black curve in a way that the filtered stock data sample drops all the stocks that do not satisfy the requirements of the studied filters. It only keeps the stocks of interest that constitute the filtered stock investment sample, which is the base of all subsequent applications of the momentum strategies.

[1] The one month gap between the investment and the formation period is adopted to avoid the short-term reversals as demonstrated by both Jegadeesh (1990) and Lehmann (1990).

6.2 Decile Portfolios Construction: A Methodology Based on Cumulative Returns

The remaining stocks are ranked from the best-performing to the worst-performing based on the monthly cumulative return of each stock over the past 11 months between months $t-12$ and $t-2$. The monthly cumulative return is calculated as:

$$R^{cum}_{i,t-12:t-2} = \frac{S_{i,t-2} - S_{i,t-12}}{S_{i,t-12}}. \tag{1}$$

where $S_{i,t-2}$ and $S_{i,t-12}$ are end of month and end of day prices of stock i at months $t-2$ and $t-12$ respectively.

The construction of the 10 decile portfolios consists of assigning the filtered stocks, which have been ranked from the most performing to the least performing based on cumulative returns over the past 11 months, to ten equisized portfolios ranging from the losers' stock composition to the winners' stock selection. The decile portfolio's composition changes from one month to the next but remains constant throughout the month. The rebalancing of the portfolios takes place on the first day of the month based on the decision criteria established on the last day of the previous month.[2]

6.3 The Vanilla Momentum Strategy Implementation

The vanilla momentum strategy adopted by Daniel and Moskowitz (2016) consists of going long the winner or the top decile of the ordered portfolios based on past performance and going short the loser portfolio or the bottom decile, while placing an amount equivalent to the shorted amount (which is the same amount invested in the long portfolio) in a risk-free investment or a bank account. This momentum strategy finds its foundation in the fact that momentum does not die out in the short term, as it has been shown in literature, so the strategy continues to be profitable over the foreseeable future. Implementing such a strategy is regulated and can only be achieved with the intervention of a broker that allows for short selling while requiring collateral for an amount of the short sale that is remunerated at the risk-free rate over the one month period until the next rebalancing.[3] The value of the momentum strategy is self-financed and will evolve smoothly through time.

$$V^{van-mom}_{t+1} = |V^{van-mom}_{t}|(1 + R^{D_{10}}_t - R^{D_1}_t + R^F_t). \tag{2}$$

where $V^{van-mom}_{t}$ is the value of the vanilla momentum strategy at the end of the month t, $R^{D_{10}}_t$ and $R^{D_1}_t$ are respectively the winner decile and the loser portfolio monthly value-weighted returns respectively, while R^F_t is the monthly risk free rate.

Equation (2) should be interpreted based on the following case scenarios:

[2]Given that the data sample starts in January 1986 and ends in December 2020, the criteria decision date is the end of January 1987, and the investment date is the start of February 1987. The ten deciles portfolios for the month of February are based on the ranked stocks at the end of January 1987 and so on.

[3]The risk-free rate used in this thesis is the one-month Treasury bill rate retrieved from the Ken French data library.

- $V_t^{van-mom} > 0$: implies that the a dollar value equal to $V_t^{van-mom}$ is invested by going long the winner portfolio. An equivalent dollar value of $-V_t^{van-mom}$ is retrieved by shorting the loser portfolio. While a dollar amount of $V_t^{van-mom}$ is placed in a bank account or a risk free investment.

- $V_t^{van-mom} \leq 0$: implies that a dollar value of $-V_t^{van-mom}$ is invested in the winner portfolio. The proceeds of an equivalent dollar amount of $V_t^{van-mom}$ are collected from the short sale of the loser portfolio. While a risk free loan which value in dollars is $V_t^{van-mom}$ is contracted.

More precisely if $V_t^{D_k}$ is the value of the decile portfolio number k at time t where $k \in \{1, ..., 10\}$, $n_{i,t-1}$ is the number of shares of stock i in portfolio $D_{k,t}$ at the end of month $t-1$ and $S_{i,t}$ is the price of stock i at the end of month t, the monthly value-weighted return of decile D_k is

$$\begin{aligned} R_t^{D_k} &= \frac{V_t^{D_k} - V_{t-1}^{D_k}}{V_{t-1}^{D_k}} \\ &= \frac{\sum_{i \in D_k} n_{i,t-1} S_{i,t} - \sum_{i \in D_k} n_{i,t-1} S_{i,t-1}}{\sum_{i \in D_k} n_{i,t-1} S_{i,t-1}} \\ &= \sum_{i \in D_k} \frac{n_{i,t-1} S_{i,t-1}}{\sum_{i \in D_k} n_{i,t-1} S_{i,t-1}} \frac{S_{i,t} - S_{i,t-1}}{S_{i,t-1}} \\ &= \sum_{i \in D_k} \theta_{i,t-1} R_{i,t}. \end{aligned} \tag{3}$$

where

$$\theta_{i,t-1} = \frac{n_{i,t-1} S_{i,t-1}}{\sum_{i=1}^{n} n_{i,t-1} S_{i,t-1}}. \tag{4}$$

and

$$R_{i,t} = \frac{S_{i,t-1} - S_{i,t-1}}{S_{i,t-1}}. \tag{5}$$

$\theta_{i,t-1}$ refers to the weight of the particular stock i in the decile portfolio number k at the end of month $t-1$ and $R_{i,t}$ refers to the monthly return of stock i at time t.

Hereafter the winner and loser portfolio are referred to as R_t^W and R_t^L respectively, instead of $R_t^{D_{10}}$ and $R_t^{D_1}$ respectively.

It is opportune to define the market portfolio, as it is paramount for the subsequent study. In fact, each and every firm in the CRSP with a valid data set is included in the calculation of the market return, which is calculated in the same way as in Equation (3) with the only difference that each stock i belongs to the $Market$ universe of valid data sets.

$$R_t^M = \sum_{i \in Market} \theta_{i,t-1} R_{i,t}. \tag{6}$$

Table 1 reports some relevant statistics of the studied portfolios. From this table, it can be seen that, on average, the vanilla momentum strategy portfolio outperforms the winner decile of the investment sample, but it fails to stand out when it comes to the Sharpe ratio measure. The vanilla momentum portfolio has a smaller Sharpe ratio compared to the winner decile portfolio due to its high historical volatility, measured by the standard deviation of returns that approaches 10%. This is due to the vanilla momentum strategy's poor defenses against the financial crashes in the market. This viewpoint gives rise to the idea of improving this vanilla momentum strategy and fortifying its defenses against extreme market movement in order to keep the historical accumulated returns rather than losing them due to financial turmoil.

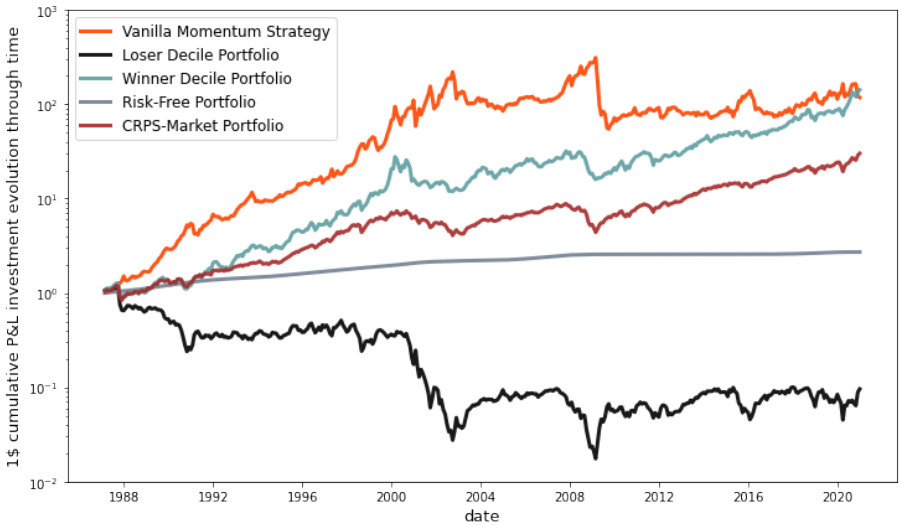
Table 1: Monthly Return Statistics Of The Different Portfolios

	Vanilla Momentum deciles portfolios											
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	VM	MK
$\overline{r - r_f}$	-0.23	-0.03	0.3	0.4	0.4	0.38	0.5	0.6	0.7	1.2	1.44	0.69
std	11	8.1	6.5	5.4	4.7	4.4	4.1	4.3	4.7	6.7	9.6	4.47
SR	-2	-0.3	4.2	7.6	8.6	8.6	12	14	14.2	17.8	14.9	15.5
Skewness	0.6	0.08	-0.08	-0.14	-0.52	-0.67	-0.88	-0.69	-0.84	-0.17	-1.09	-0.87
Kurtosis	3.07	3.36	3.68	3.12	2.01	2.77	3.97	2.27	3.28	2.74	4.61	2.7

This table exhibits the monthly return statistics of the 10 decile portfolios, the vanilla momentum portfolio, and the market portfolio. The statistics calculated in percentage terms over the investment sample horizon, that is, from the start of February 1987 until the end of December 2020, are the mean excess return, the standard deviation, the Sharpe ratio denoted by SR, the skewness and the Kurtosis of the monthly returns of the studied portfolios. The decile portfolios are referred to by numbers in parenthesis from 1 until 10, while VM refers to the vanilla momentum strategy portfolio, and finally, MK refers to the CRPS market portfolio. The decile portfolios are constructed from our data sample, which is formed of ordinary common shares listed on the NYSE, AMEX and NASDAQ.

The cumulative P&L of the vanilla momentum strategy throughout the investment horizon largely outperforms the CRSP market portfolio; however, the strategy suffers from occasional crashes that last several months. This can be seen in Figure 2 which plots the 1$ cumulative P&L of the vanilla momentum strategy, the winner decile portfolio, the CRSP market portfolio, the risk-free portfolio, and the loser decile portfolio side by side and invested from the start of February 1986 until the end of December 2020. Although the vanilla momentum strategy delivers superior results in terms of cumulative P&L throughout the investment horizon compared to all other portfolios plotted in Figure 2, it still drops a big part of the cumulative returns acquired from the start of the investment horizon when faced with financial crashes, as can be clearly seen around the subprime crises of 2008-2009. For this specific reason, the idea of conceiving a momentum dynamic strategy saw the light and was implemented by Daniel and Moskowitz (2016) in their paper momentum crashes.

Figure 2: The 1$ Cumulative P&L Performance of The Different Portfolios



This figure plots the cumulative P&L of 1$ invested at the start of February 1987 until the end of December 2020. The 1$ cumulative P&L vanilla momentum portfolio is presented alongside the top decile portfolio, the bottom decile portfolio, the market portfolio, and the risk-free portfolio. The vanilla momentum portfolio cumulative P&L is far superior than any other portfolio however it is not immune from crashes that affect the market.

7 The Vanilla Dynamic Strategy

This section explores the vanilla dynamic strategy adopted by Daniel and Moskowitz (2016) that aims to calibrate the time-varying weights assigned to the vanilla momentum strategy from one month to the next in order to make it more profitable and less vulnerable to recurrent market crashes. The idea behind this dynamic strategy finds its building blocks in the work done by Markowitz (1952) in the intertemporal version of portfolio optimization; it basically relies on the maximization of the Sharpe ratio relative to the portfolio composed of two assets the vanilla momentum portfolio and the risk-free asset. This can be equivalently presented as a maximization under constraint problem as presented in the system of Equation (7).

$$\begin{aligned} \max_{W} \quad & \frac{W^T\mu - R_f}{\sqrt{W^T \Sigma W}}. \\ \text{s.t} \quad & 1^T W = 1. \end{aligned} \tag{7}$$

where W denotes the normalized weights of the two assets in the optimization portfolio, $W^T\mu$ stands for the expected return of the optimization portfolio, while $\sqrt{W^T\Sigma W}$ is the volatility of this portfolio.

The advantage of maximizing the Sharpe ratio is that such an optimization is independent of the risk preference of the investor and that there is only one Pareto-optimal frontier portfolio that achieves the maximum Sharpe ratio.

7.1 The Optimization Problem

The objective of the optimization problem is to find the optimal weight of the vanilla momentum strategy portfolio that maximizes the Sharpe ratio of the two-asset portfolio composed of the vanilla momentum strategy portfolio and the risk-free asset. Since the rebalancing of the portfolio only takes place monthly, a discrete time modeling is suited in this case, with T representing the total number of months in the investment sample time series horizon. The optimization portfolio is the following:

$$P_t^{opt} = w_t P_t^{Van-Mom} + (1 - w_t) P_t^{Risk-free}. \tag{8}$$

where P_t^{opt} denotes the optimization portfolio value at time t, $P_t^{Van-Mom}$ denotes the vanilla momentum strategy portfolio value at time t, $P_t^{Risk-free}$ stands for the risk free investment portfolio, while w_t is the normalized weight at month t of the vanilla momentum strategy portfolio in the optimization portfolio.

Let $R_{P_t^{opt}}$ denote the monthly return of the optimization portfolio, $R_t^{Van-Mom}$ stands for the monthly return of the vanilla momentum strategy portfolio, R_t^F the monthly return of the risk free investment, and let R_t^{WML} represent the monthly return of the WML portfolio. It follows that:

$$\begin{aligned} R_{P_t^{R^{opt}}} &= w_t R_t^{Van-Mom} + (1 - w_t) R_t^F \\ &= w_t (R_t^W - R_t^L + R_t^F) + (1 - w_t) R_t^F \\ &= w_t (R_t^W - R_t^L) + R_t^F \\ &= w_t R_t^{WML} + R_t^F. \end{aligned} \tag{9}$$

where

$$R_t^{WML} = R_t^W - R_t^L.$$

So the Sharpe ratio of the optimization portfolio can be equivalently expressed as:

$$SP_t = \frac{E[R_{t+1}^{P^{opt}} | \mathcal{F}_t] - R_{t+1}^{RF}}{\sqrt{Var[(R_{t+1}^{P^{opt}} - R_{t+1}^{RF}) | \mathcal{F}_t]}}. \tag{10}$$

where $\mathcal{F}_t$ is a filtration that contains the market stock data until time t.

The conditional expected return and the conditional variance of the vanilla momentum excess return portfolio (or the winner minus loser deciles portfolio, WML) over period $t+1$ that goes from t to $t+1$ are referred to as μ_t and $\eta^2_{WML,t}$ respectively.

$$\mu_t = E_t[\tilde{R}_{t+1}^{WML} | \mathcal{F}_t]. \tag{11}$$

and

$$\eta^2_{WML,t} = E_t[(\tilde{R}_{t+1}^{WML} - \mu_t)^2 | \mathcal{F}_t]. \tag{12}$$

where $\tilde{R}_t^{WML}$ is the excess return of the WML portfolio.

The Sharpe ratio optimization problem[4] yields the optimal monthly time varying weight of the risky asset or the vanilla momentum portfolio as follows:

$$w_t^* = \frac{1}{2\lambda}\frac{\mu_t}{\eta^2_{WML,t}}. \tag{13}$$

where λ is a constant relative to the unconditional risk and return of the vanilla dynamic portfolio. Particularly, λ is determined by setting the in-sample annualized volatility of the dynamic strategy equal to the corresponding period annualized volatility of the vanilla momentum strategy, in a way that only past data is used.[5]

$$\begin{aligned}
\eta^{VanMom}_{WML,t} &= \eta^{DynMom}_{WML,t} \\
&= \sqrt{\frac{1}{t}\sum_{u=1}^{t}(R_u^{DynMom} - \bar{R}^{DynMom})^2} \\
&= \sqrt{\frac{1}{t}\sum_{u=1}^{t}(w_u^* R_u^{WML} - \frac{1}{t}\sum_{u=1}^{t} w_u^* R_u^{WML})^2} \\
&= \sqrt{\frac{1}{t}\sum_{u=1}^{t}\left(\frac{1}{2\lambda}\frac{\mu_u}{\eta^2_{WML,u}}R_u^{WML} - \frac{1}{t}\sum_{u=1}^{t}\frac{1}{2\lambda}\frac{\mu_u}{\eta^2_{WML,u}}R_u^{WML}\right)^2}.
\end{aligned}$$

which yields

$$\lambda = \frac{\sqrt{\frac{1}{2t}\sum_{u=1}^{t}(\frac{\mu_u}{\eta^2_{WML,u}}R_u^{WML} - \frac{1}{t}\sum_{u=1}^{T}\frac{\mu_u}{\eta^2_{WML,u}}R_u^{WML})^2}}{\eta^{VanMom}_{WML,u}}. \tag{14}$$

Now that we know how to calculate the time-varying weight of the vanilla dynamic momentum strategy, the next step is to estimate the conditional expected return and variance of the winner-minus-loser portfolio over the next period spanning from t to $t+1$. Once this is done, the vanilla dynamic strategy is easily implemented.

7.2 The Conditional Expected Return and the Conditional Variance Of The WML Portfolio Estimation

Based on the study of time-varying market betas of the Winner and Loser portfolio[6], Daniel and Moskowitz (2016) noticed that the market beta of the loser portfolio surges in periods of

[4]The complete proof of the Sharpe ratio optimization problem is available in Daniel and Moskowitz (2016) Momentum crashes paper in Appendix C.

[5]Opposed to Daniel and Moskowitz (2016) who used future data by setting λ in a way that the in-sample annualized volatility of the Dynamic momentum portfolio equal to 19% which is the CRSP value-weighted Index over the full investment sample.

[6]Daniel and Moskowitz (2016) use 10 daily lags of market return to estimate the market beta of each decile portfolio following a rolling window daily regression $\tilde{r}^e_{i,t} = \beta_0\tilde{r}^e_{m,t} + \beta_1\tilde{r}^e_{m,t-1} + ... + \beta_{10}\tilde{r}^e_{m,t-10} + \tilde{\epsilon_{i,t}}$ where the market beta of each decile is approximated as the sum of the coefficients. $\tilde{\beta_0} + \tilde{\beta_1} + ... + \tilde{\beta_{10}}$ (more detail can be found in the Momentum Crashes (2016) section 3).

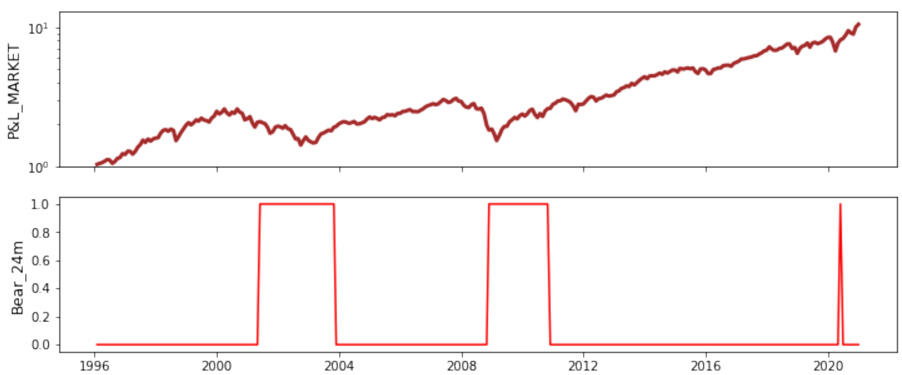
high volatility to extreme values reaching sometimes a value of 5, while the winner portfolio market beta only reaches a value of 2 after a bull market. Further exploration around this point[7] have led the two researchers to conclude that after a bear market the WML portfolio suffers a negative impact when the market rebounds. This special property relative to the WML portfolio inspired the idea that holding such a portfolio is similar to being short a call option on the market, and this is mainly driven by the loser portfolio behavior.

Capitalizing on the previous points, Daniel and Moskowitz (2016) adopted the following model to estimate the conditional expected return of the WML portfolio at month t:

$$\tilde{R}_t^{WML} = \alpha_0 + \alpha_b 1_{24B,t-1} + \alpha_{\nu_M} \sigma^2_{126M,t-1} + \alpha_{int} 1_{24B,t-1} \sigma^2_{126M,t-1} + \epsilon_t. \tag{15}$$

where $1_{24B,t-1}$ is the bear market indicator that takes 0 when the market cumulative return over the past 24 months is positive and 1 when it is negative. $\sigma^2_{126M,t-1}$ is the historical annualized market variance of daily returns calculated based on the past 126 days data (approximately 6 months data).

Figure 3: The Bear Market Indicator Dummy Variable



This figure plots the 1$ investment cumulative P&L of the market portfolio referred to as P&L-MARKET alongside the bear marker indicator dummy variable when its calculation is linked to the past 24 months market cumulative return referred to as Bear-24m.

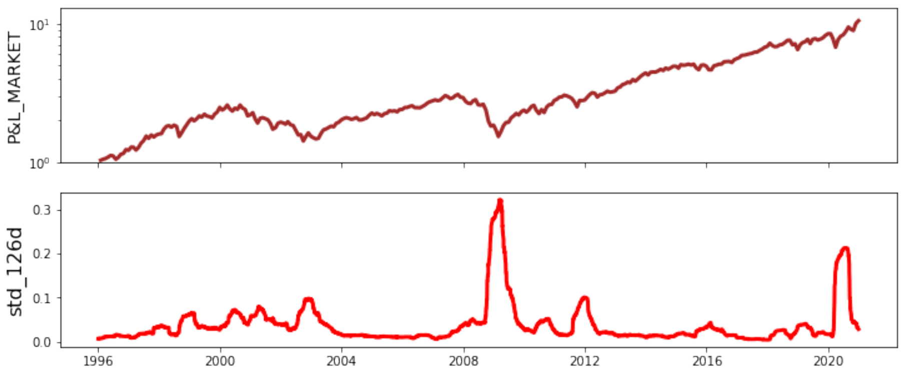
Figure 3 plots the bear market indicator $1_{24B,t-1}$ in Equation (15), it can clearly be seen from this figure that this dummy variable does a good job in covering periods of market uncertainty and financial crashes. The first time this variable takes a value of 1 corresponds to the 2001-2002 burst of the internet bubble, the second time this variable takes a value of 1 corresponds to the 2008-2009 subprime bubble burst, and the last time this variable briefly takes the value 1 corresponds to the covid-19 financial crisis of March 2020.

Figure 4 graphs the $\sigma^2_{126M,t-1}$ which is the second independent variable in the regression in Equation (15). This figure highlights the role of this variable by depicting the historical volatility

[7] Daniel and Moskowitz (2016) study the following regression to assess the reaction of the WML portfolio after the market trends up after a bear period, $\tilde{R}_t^{WML} = (\alpha_0 + \alpha_b 1_{B,t-1}) + (\beta_0 + 1_{B,t-1}(\beta_B + \tilde{1}_{B,t}\beta_{B,U}))\tilde{R}_{m,t} + \tilde{\epsilon_t}$, proving that the WML portfolio performs badly when the market recovers after a bear market founding that $\tilde{\beta}_0 + \tilde{\beta}_B + \tilde{\beta}_{B,U} = -1.796$ more detail is available at Daniel and Moskowitz (2016) paper on momentum crashes.

of the market portfolio. It is deduced that in periods of crashes and market uncertainty, which we have enumerated in the previous paragraph, this variable gets inflated and reflects the increased volatility and instability in the market.

Figure 4: The Market Return Standard Deviation



This figure presents graphically the 1$ investment cumulative P&L of the market portfolio referred to as P&L-MARKET alongside the market's return standard deviation based on the past 126 days data referred to as std-126d.

Table 2: Regression Summary Of the Conditional Expected Return of WML Model

Coefficient	Variable	Estimated Coefficient (1)	(2)	(3)	(4)	(5)
$\hat{\alpha}_0$	1	0.0142	0.0227	0.0227	0.017	0.012
		(2.060)	(2.879)	(2.842)	(2.692)	(1.248)
$\hat{\alpha}_b$	$1_{24B,t-1}$	-0.0136		0.0013		0.035
		(-0.849)		(0.074)		(1.564)
$\hat{\alpha}_{\nu_M}$	$\sigma^2_{126M,t-1}$		-0.2890	-0.2933		0.093
			(-2.247)	(0.141)		(0.431)
$\hat{\alpha}_{int}$	$1_{24B,t-1}\sigma^2_{126M,t-1}$				-0.37	-0.671
					(-2.874)	(-2.367)
R^2		0.002	0.017	0.017	0.027	0.035

This table reports the OLS regression parameter estimation of the WML conditional expected return model, its t-statistics results in parenthesise and the R-square statistic using the whole data as in-sample data, that is the data from the start of January 1996 until the end of December 2020. The last five columns numbered from (1) until (5) present the 5 different combination of the variables of Equation (15).

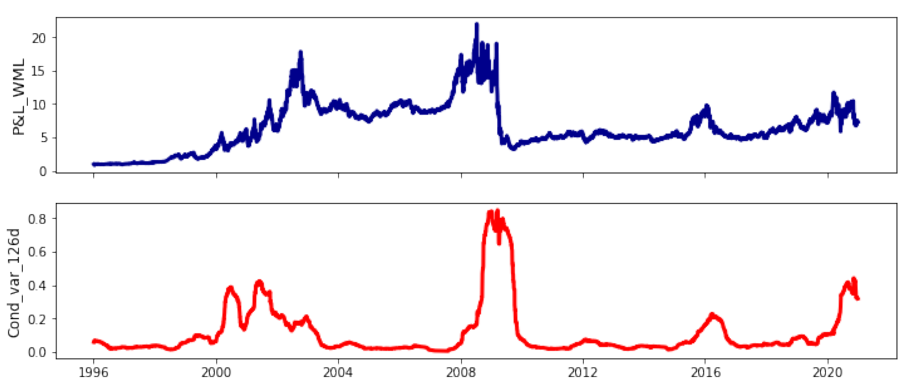
Table 2 presents the OLS regression summary of the WML conditional expected return model in Equation (15) using the whole data as in-sample data, that is the data from the start of January 1996 until the end of December 2020. In fact, Table 2 reports the regression summary of all the possible combinations of the two independent variable $1_{24B,t-1}$ and $\sigma^2_{126M,t-1}$, the last column of this table corresponds to the model in Equation (15) and it is the model with the highest coefficient of determination R^2 which means that this model is the one that best approximates the conditional expectation of the WML portfolio.

An important property of the WML portfolio's conditional expected return can be observed from the last column of Table 2 where the interaction coefficient of the model in Equation (15) is negative. This means that when the dummy variable which is the bear market indicator $1_{24B,t-1}$ comes to life in periods of financial recession, the interaction term is then activated as well and pushes the conditional expected return of the WML portfolio down which in turn scales down the weight of the WML portfolio in favor of the risk free investment. This crucial property is what makes the vanilla dynamic strategy resilient to financial crashes, as it allows the investor to navigate through rough market movement.

Having detailed the procedure relevant to the estimation of the conditional expected return of the WML portfolio, the spotlight is now directed towards the missing variable essential to the calculation of the time-varying weights of the vanilla dynamic strategy, which is the conditional variance of the WML portfolio. The conditional variance of the WML portfolio is approximated using the annualized historical variance of the daily WML portfolio returns over the past 126 days of data and is given by the following equation:

$$\tilde{\eta}^2_{WML,t} = 252\eta^2_{Van-Mom,t}. \tag{16}$$

Figure 5: The Conditional Variance of the WML Portfolio



This figure plots the cumulative P&L of the WML portfolio and the conditional variance of the WML portfolio when its calculation is based on the past 126 days of data. P&L-WML refers to the cumulative return of the WML portfolio and the cond-var-126d refers to the conditional variance based on the past 126 days data.

Figure 5 exhibits graphically the conditional variance of the WML portfolio when its calculation is based on the past return data of the last 126 days. It is observed that this variable takes on extremely high levels when the WML portfolio suffers from financial crashes. This is a desired property specific to the vanilla dynamic strategy, since the conditional variance of the WML portfolio is the denominator in Equation (13), which means that when its value increases in periods of uncertainty, the time-varying weights go down and shield the past accumulated returns of the vanilla dynamic strategy from extreme loss. In other words, thanks to this property, the vanilla dynamic strategy investor's exposure to the risky WML portfolio is scaled down when there is high market uncertainty in favor of the risk-free investment and vice versa.

7.3 Implementation of the Vanilla Dynamic Momentum Strategy

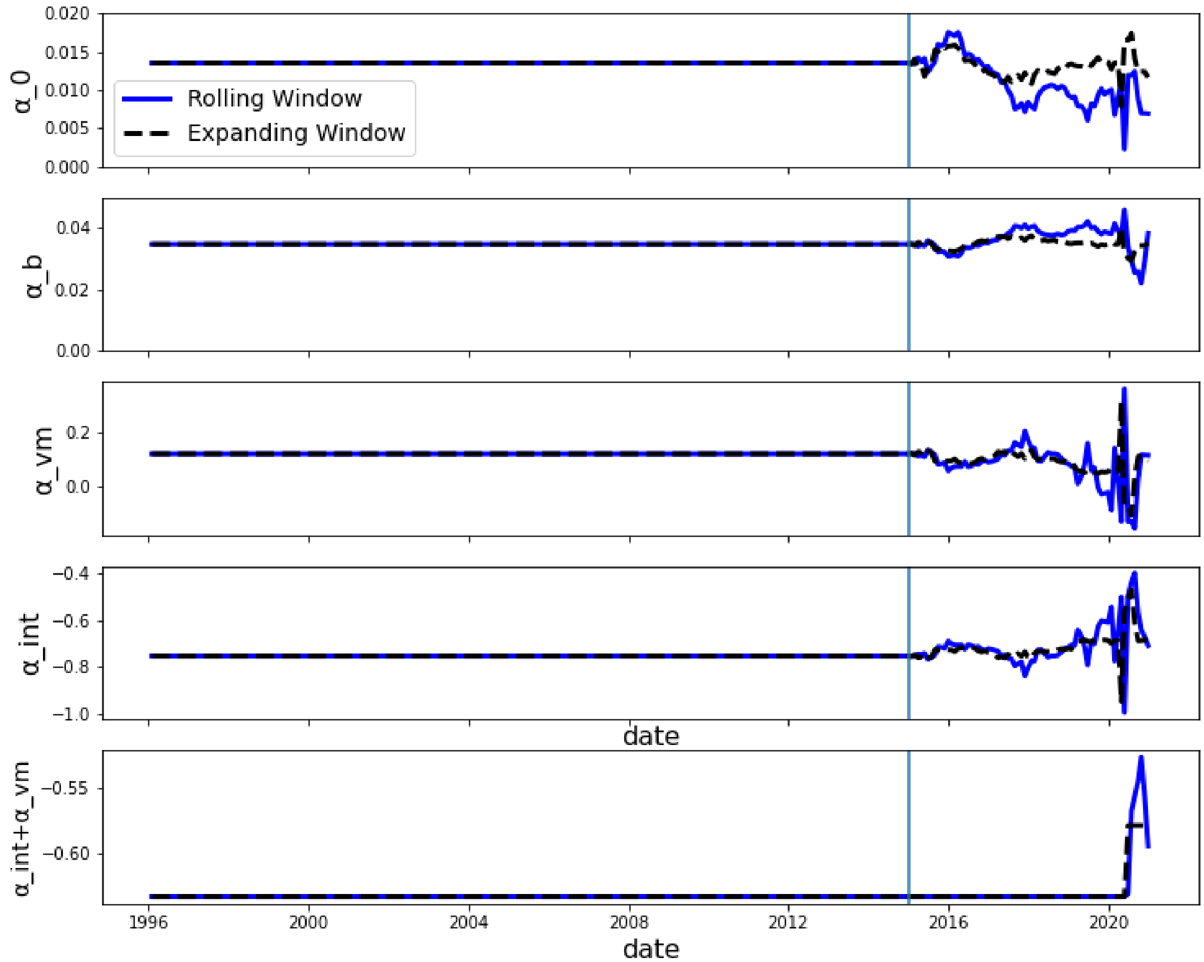
In order to implement the vanilla dynamic momentum strategy in a more realistic way, only past data should be used. As a result, the parameters of Table 2 cannot be used to estimate the conditional expected return of the WML portfolio. In this section, we estimate the parameters of Equation (15) using only past data and according to different estimation windows. In fact, having featured the advantages of the vanilla dynamic momentum strategy and the technicalities behind the calculation of its time varying weights that change from one month to the next and that adapt the exposure of the investor to the WML risky portfolio dynamically, this part focuses on the implementation of the vanilla dynamic momentum strategy based on the estimation of the WML portfolio conditional expected return model in Equation (15) in three different ways. Firstly, the conditional expected return model is estimated using a simple OLS using the whole sample data from the start of January 1996 until the end of December 2020. Secondly, the model is estimated using an expanding window regression starting from January 1996 until December 2014 as the initial in-sample base while adding one month of data on each run. Finally, the model is estimated using a rolling window regression starting from January 1996 until the end of December 2014 as initial in-sample data, which slides by one month on each run.

Graphically presenting the time-varying weights and the cumulative P&L of the vanilla dynamic momentum strategy following the three types of regressions mentioned in the previous paragraph is essential to the assessment of the profitability and the behavior of the output components of the strategy. However, before presenting such results, it is opportune to picture the rolling window and expanding window OLS regression coefficients.

From Figure 6, it is observed that the coefficients of the rolling and expanding window regressions of the WML portfolio conditional expected return in the model of Equation (15) are dynamic outside the first 19 year period. In fact, since both regressions take the window period of 19 years from the start of January 1996 to the end of December 2014 as in-sample, the estimated coefficient only starts to diverge after the expiration of the in-sample period, which is at the start of January 2015. These coefficients are relatively stable throughout the investment horizon for both regressions; however, they tend to fluctuate heavily during the covid-19 crisis of March 2020. This is explained by the increased volatility of the data throughout the Covid-19 crash.

Moreover, from Figure 6, it is observable that α_{ν_M} and α_{int} which are the coefficients of the variables $\sigma^2_{126M,t-1}$ and $1_{24B,t-1}\sigma^2_{126M,t-1}$ respectively add up to a negative value when the dummy variable $1_{24B,t-1}$ is activated. This phenomenon influences the conditional expected return of the WML portfolio to take negative values in periods of financial recession as indicated by the dummy variable $1_{24B,t-1}$ taking a value of 1. This particular fact reverses the initial

Figure 6: Coefficients of the Rolling and Expanding Window OLS Regressions

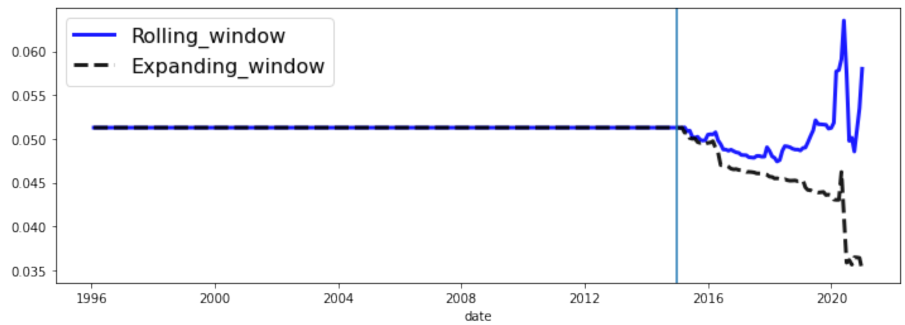


This figure plots the evolution of the coefficients in Equation (15) when estimated using a rolling window regression with a one-month sliding window starting from January 1996 until December 2014 as the initial in-sample base (figure in blue) and when the coefficients are estimated using a one-month expanding window regression with a base window starting from January 1996 until December 2014. In fact, the learning set goes from January 1996 until December 2014, spanning 228 months. The vertical lines separate the learning set from the testing set.

trivial logic of the strategy, which consists of going long the winning portfolio and going short the losing one. Thus, we become long the loser portfolio and short the winner portfolio by a certain time-varying weight particular to the month or the succession of months where the dummy variable is activated. The same phenomenon is reproduced in the R-squared statistic evolution in Figure 7, since the R-squared statistic diverges between the two regressions at the start of the Covid-19 pandemic. The rolling window regression scores a higher R-squared statistic since it relies only on more relevant recent data and gets rid of the older data, compared to the expanding window that does not drop old data.

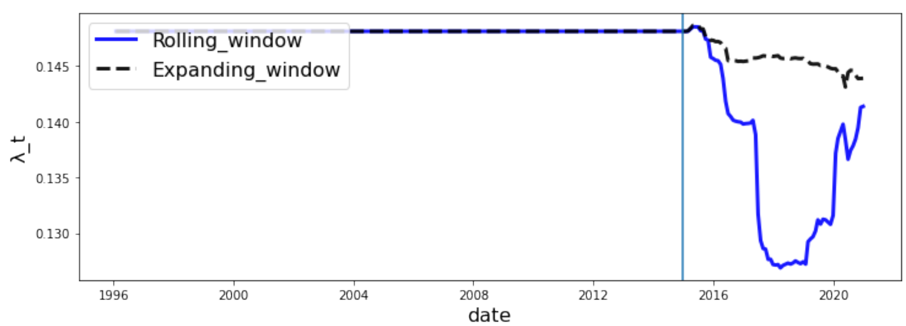
Figure 8 plots the evolution of the lambda in Equation (14), this variable stops being constant

Figure 7: The R-Squared Statistic of Rolling Window and the Expanding Window Regressions



This figure plots the evolution of the R-squared statistic of the WML portfolio's conditional expected return when it is estimated using a one-month rolling window regression and a one-month expanding window regression with a learning set that goes from January 1996 until December 2014, spanning 228 months. The vertical lines separate the learning set from the testing set.

Figure 8: The lambda of Rolling Window and the Expanding Window Regressions

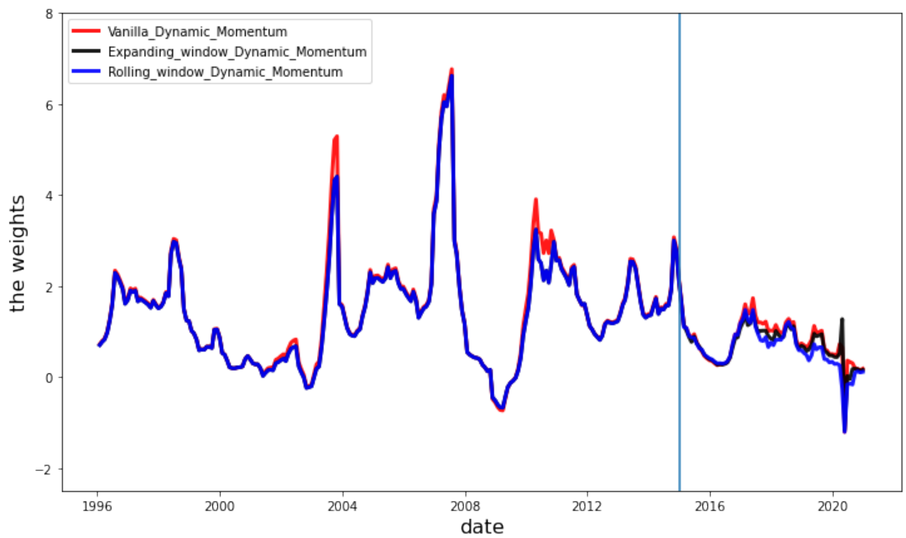


This figure plots the evolution of the lambda in Equation (14) when it is estimated using a one-month rolling window regression and a one-month expanding window regression with a learning set that goes from January 1996 until December 2014, spanning 228 months. The vertical lines separate the learning set from the testing set.

outside the learning set period since its calculation depends on the window that we choose as in-sample. Indeed, for the rolling and expanding window regressions, we set the in-sample annualized volatility of the dynamic strategy, which corresponds to 19 years from January 1996 until December 2014, equal to the corresponding period annualized volatility of the vanilla momentum strategy. Past this 19 year period, we keep rolling this window by one month for the rolling regression and expanding it by one month for the expanding regression window.

Figure 9 plots the time-varying weights of Equation (13), while Figure 10 plots the 1$ cumulative P&L of the vanilla dynamic momentum strategy given that the WML portfolio conditional expected return is estimated using the three types of estimation approaches studied. The red curve in both figures presents the vanilla dynamic momentum strategy that relies on the whole sample data and only serves as a benchmark because it includes future information, so the estimation of the weights of the vanilla dynamic momentum strategy is over-fit. The two other

Figure 9: The Time Varying Weights of the Vanilla Dynamic Momentum Strategy



This figure plots the time varying weights of the vanilla dynamic momentum strategy when the WML portfolio conditional expected return is estimated using the three types of regressions: the simple OLS using the whole data as in-sample, using a one-month rolling window regression, and using a one-month expanding window regression with a learning set that goes from January 1996 until December 2014, spanning 228 months. The vertical lines separate the learning set from the testing set.

vanilla momentum strategies use only past data to estimate the conditional expected return of the WML portfolio, so comparing their performances is an educational way to determine which one outperforms the other. From Figure 10 it can be deduced that both the black and blue curves almost finish the race at the same place, although a slight superiority is noted in favor of the rolling window regression.

Figure 10: The Cumulative P&L of the Vanilla Dynamic Momentum Strategy

This figure plots the 1$ cumulative P&L of the vanilla dynamic momentum strategy when the WML portfolio conditional expected return is estimated using the three types of estimation approaches: the simple OLS using the whole data as in-sample, using a one-month rolling window regression, and using a one-month expanding window regression with a learning set that goes from January 1996 until December 2014, spanning 228 months. The vertical lines separate the learning set from the testing set.

8 Realized Volatility Dynamic Momentum Strategy

The aim of this section is to improve the estimation of the variables that drive the time varying weights in Equation (13), the conditional return of the WML portfolio μ_t and the conditional variance of the WML portfolio $\eta^2_{WML,t}$. In appendix A, a rigorous sensitivity analysis of variable $\tilde{R}^{WML}_t$ and $\eta^2_{WML,t}$ have been conducted and it has been concluded that the benchmark model in Equation (15) is the most appropriate in the estimation of the conditional expected return of the WML portfolio. However, it has been concluded that $\eta^2_{WML,t}$ is optimal and impacts positively the return of the dynamic strategy through its impact on the time varying weights when its calculation is based on the past 22 days data instead of the benchmark's 126 days data.

Subsequently in this section, the attention is directed towards the improvement of the estimation of the conditional expected return of the WML portfolio μ_t. This is done by taking Daniel and Moskowitz (2016) model's given by Equation (15) as a starting point. The idea is to replace the independent variables of Equation (15), $1_{xB,t-1}$ and $\sigma^2_{yM,t-1}$, by other variables that do relatively the same job by relying on more advanced technologies that in fine perform a better and more accurate estimation of the conditional expectation of the WML portfolio. The statistic of interest on which the spotlight is directed in order to improve the benchmark model is the realized volatility measure and its application. Despite the fact that realized volatility is still a historical measure of volatility, it is more advantageous as it incorporates intraday data at a frequency that can go as low as one minute, which makes it a more reflective measure of volatility than the return's standard deviation.

8.1 A Review on the Realized volatility Measure

Given the increased availability of financial assets' high-frequency price data and the rapid development of machines' computational power, market participants resort more and more to exploiting these intraday data to produce an ex-post volatility measure called realized volatility.

To derive the origin of the realized volatility measure and for simplicity's sake let's suppose that asset prices (stocks in our case) are driven by a continuous time diffusion process without a jump and that markets are frictionless. The stock price is then driven by the following equation:

$$dS(t) = \mu(t)dt + \sigma(t)dB(t). \tag{17}$$

where $S(t)$ is the logarithmic price process, $\mu(t)$ is a predictable process with a finite variation, $\sigma(t)$ is also a predictible process that is strictly positive and square integrable such that $E(\int_{t-k}^{t} \sigma^2(\tau)d\tau) < \infty$, $B(t)$ is a standard Brownian motion process, while the continuously compounded return over the time interval from $t-k$ to t, $0 < k < t$ is given by the following equation:

$$\begin{aligned} r(t,k) &= S(t) - S(t-k) \\ &= \int_{t-k}^{t} \mu(s)ds + \int_{t-k}^{t} \sigma(s)dB(s). \end{aligned} \tag{18}$$

from Equation (18) stems the quadratic variance given by:

$$QV(t) = \int_{t-k}^{t} \sigma(s)^2 ds. \tag{19}$$

The quadratic variance for one day is calculated using Equation (19) where $k = \frac{1}{252}$.

The realized variance of the logarithmic price process is given by:

$$RV_t^{(d)} = \sum_{j=0}^{M-1} r_{t-j\Delta}^2. \tag{20}$$

where

$$\Delta = \frac{\frac{1}{252}}{M} \text{ and } r_{t-j\Delta} = \log\left(\frac{P(t-j\Delta)}{P(t-(j+1)\Delta)}\right)$$

such that P is the instantaneous price of the stock during the day at regular intervals of time Δ. M is the intraday frequency at which we calculate the RV.

The semi-martingale theory makes sure that the realized variance converges in probability to the return quadratic variance when the daily time interval division M increases, such that:

$$RV_t \xrightarrow{M \to \infty} QV(t) \tag{21}$$

It follows that the aggregated weekly realized variance are calculated as such:

$$RV_t^{(w)} = \frac{1}{5}(RV_{t-1d}^{(d)} + RV_{t-2d}^{(d)} + ... + RV_{t-5d}^{(d)}). \tag{22}$$

the aggregated monthly realized variance is given by the following equation:

$$RV_t^{(m)} = \frac{1}{22}(RV_{t-1d}^{(d)} + RV_{t-2d}^{(d)} + ... + RV_{t-22d}^{(d)}). \tag{23}$$

and the aggregated quarterly realized variance is given by:

$$RV_t^{(3m)} = \frac{1}{66}(RV_{t-1d}^{(d)} + RV_{t-2d}^{(d)} + ... + RV_{t-66d}^{(d)}). \tag{24}$$

while the annualized daily, the aggregated weekly, the aggregated monthly and the aggregated quarterly realized volatility's are given respectively by the following equations:

$$RVol_t^{(d)} = \sqrt{252 * RV_t^{(d)}}. \tag{25}$$

and

$$RVol_t^{(w)} = \sqrt{252 * RV_t^{(w)}}. \tag{26}$$

while

$$RVol_t^{(m)} = \sqrt{252 * RV_t^{(m)}}. \tag{27}$$

and finally

$$RVol_t^{(3m)} = \sqrt{252 * RV_t^{(3m)}}. \tag{28}$$

By abuse of language, the aggregated monthly and the aggregated quarterly realized volatility are referred to in the rest of this thesis as simply monthly and quarterly (or three month) realized volatility, respectively.

8.2 Realized Volatility of the S&P500

Given that the daily realized volatility is the building bloc for all higher frequency realized volatilities such as the weekly, monthly or even quarterly realized volatility as we have highlighted previously, we elect to proceed into the calculation and the plotting of the graph of the S&P500 realized volatility given intraday data of this index from January 1996 until December 2020.

To calculate the proofed realized variance and annualized volatility of the S&P500, we utilize the intraday data of the S&P500 using a frequency of 5-minute intervals. Since there are 6.5 trading hours per day, this translates into 390 minutes of trading per day. These two measures are calculated following the approach discussed by Zhang, Mykland and Ait-Sahalia in their 2005 paper, "A Tale of Two Time Scales: Determining Integrated Volatility With Noisy High-Frequency Data" respectively, as the following:

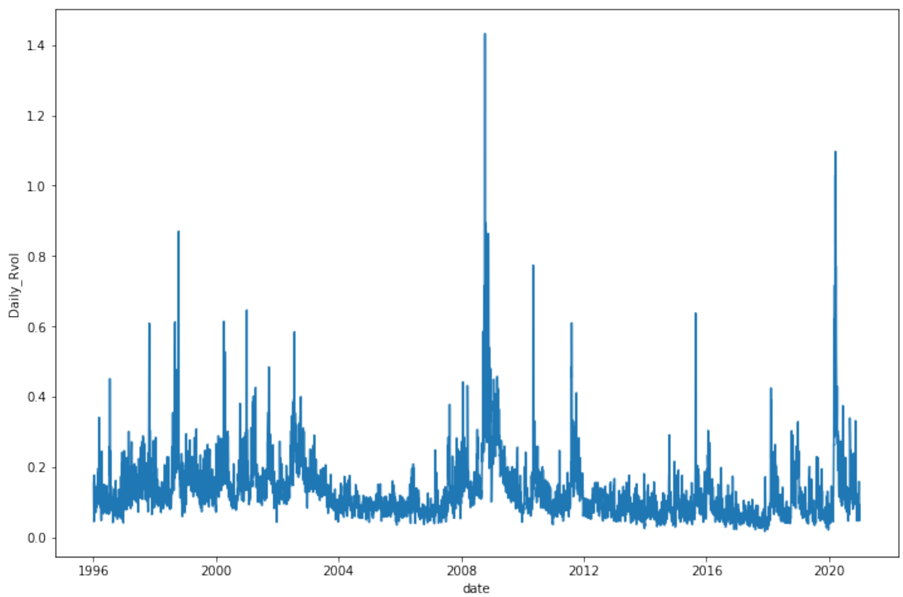
$$RV_{(S\&P500,t)}^{(d)} = \frac{1}{5}(\sum_{\tau=0}^{77} r^2_{5\tau,5(\tau+1)} + \sum_{\tau=0}^{76} r^2_{1+5\tau,1+5(\tau+1)} + \sum_{\tau=0}^{76} r^2_{2+5\tau,2+5(\tau+1)} + \sum_{\tau=0}^{76} r^2_{3+5\tau,3+5(\tau+1)} + \sum_{\tau=0}^{76} r^2_{4+5\tau,4+5(\tau+1)}). \tag{29}$$

$$RVol^{(d)}_{(S\&P500,t)} = \sqrt{252 * RV^{(d)}_{(S\&P500,t)}}. \tag{30}$$

where τ can be interpreted as the unit of time; one minute in this case.

The application of Equation (30) to the intraday data of the S&P500 yields the annualized realized volatility graph reported in Figure 11. From this figure we get to see how illustrative is the realized volatility as it soared to unprecedented values during the 2008 financial crisis and significantly increased during the covid-19 market crash of March 2020.

Figure 11: Daily Realized Volatility of the S&P500



This figure plots the daily realized volatility of the S&P500 based on 5min interval data.

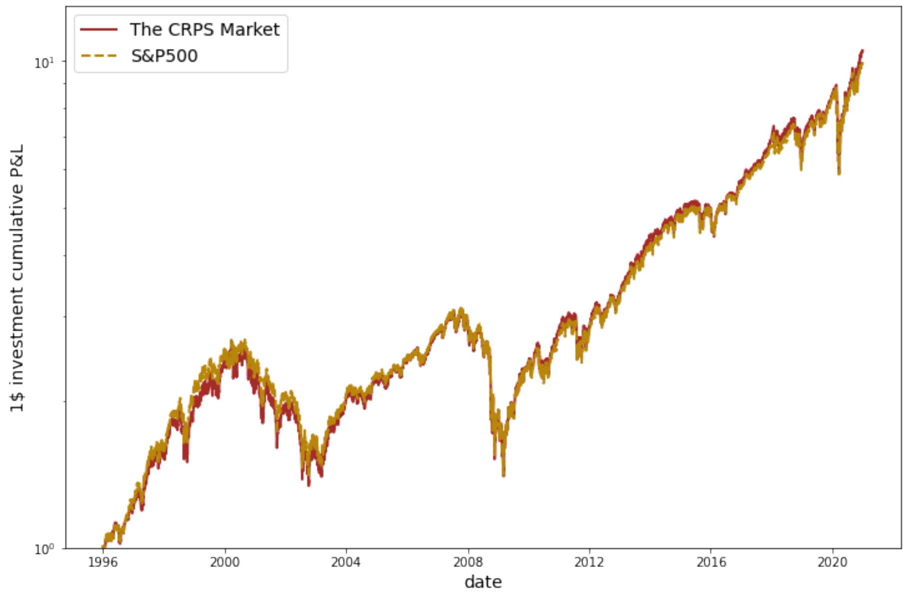
8.3 The new model based on the Realized variance of the S&P500

The benchmark model of Equation (15) relies on the CRSP market portfolio for the calculation of the independent variables $1_{xB,t-1}$ and $\sigma^2_{yM,t-1}$. Since our objective is to utilize some functions of the realized volatility measure to replace the benchmark's model independent variables, we find it impossible to calculate the realized volatility of the market portfolio since there is no intraday data available for this portfolio. As a way to circumvent this issue, we resort to the S&P500 index as a substitute for the CRPS market portfolio since they are closely correlated and exhibit the same behavior as depicted in Figure (12).

We elect to replace the bear market indicator independent variable in Equation (15), $1_{xB,t-1}$, which is a dummy variable that takes 1 when the cumulative market return over the past 24 months is negative and 0 otherwise, by a dummy variable that takes 1 when the one month lagged realized volatility of the S&P500 is above the 11.5% level[8] threshold and 0 otherwise.

[8]This threshold level has been chosen after a thorough sensitivity analysis of different thresholds as indicated

Figure 12: The 1$ Investment Cumulative P&L of the Market and S&P500



This figure plots on the same graph the 1$ investment cumulative P&L of the CRPS Market portfolio and the S&P500 portfolio.

We refer to this new independent variable by $1_{11.5\vartheta_{t-1}}$. While the other independent variable, $\sigma^2_{yM,t-1}$, which is a historical volatility measure that uses the market's returns standard deviation over the past 126 days is replaced by $RVol^{(3m)}_{t-1}$ which is the realized volatility of the S&P500 over the past 3 months. The new model responsible for the estimation of the conditional expectation of the WML portfolio μ_t becomes:

$$\tilde{R}^{WML}_t = \alpha_0 + \alpha_\delta 1_{11.5\vartheta_{t-1}} + \alpha_\nu RVol^{(3m)}_{t-1} + \alpha_{int} 1_{11.5\vartheta_{t-1}} RVol_{t-1}{}^{(3m)} + \epsilon_t.^9 \tag{31}$$

Breaking down the model at hand in Equation (31) and comparing its independent variables to the benchmark model in Equation (15), in terms of visualizing their respective independent variables in order to grasp each component's behavior and how they impact the estimation of the conditional expected return of the WML portfolio, is of great importance. Figure 3 and Figure 13 graphically present respectively the dummy variables; $1_{xB,t-1}$ and $1_{11.5\vartheta_{t-1}}$ plotted against the variable on which each is contingent upon.

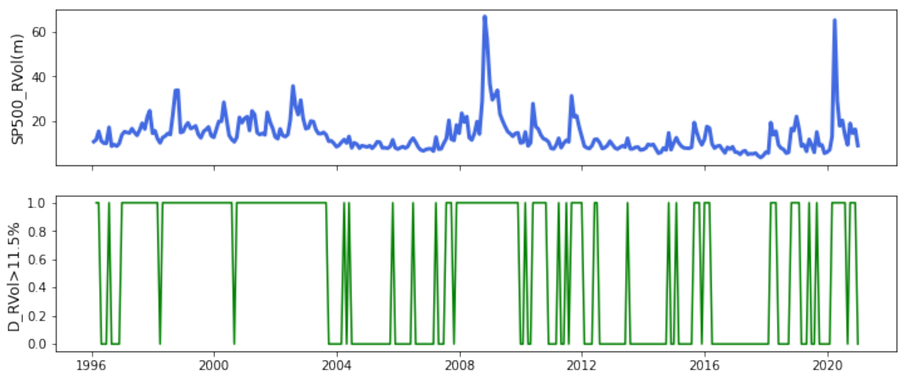
From Figure 3, it is observable that there are only three periods of time where the dummy variable has a value of 1, and they all coincide with big crashes in the financial market. As a matter of fact, the first time this variable took a value of 1 corresponds to the 2001–2002 burst of the internet bubble and the subsequent stock market downturn. The second time this dummy variable climes to 1 corresponds to the burst of the subprime mortgage bubble in the

in Appendix B.

[9] Appendix 2 presents a detailed sensitivity analysis exploring all potential models and this analysis have led to the selection of this particular model as a winner in term of good in-sample fit translated by a satisfying R^2 statistic compared to all other models including the benchmark plus a superior out-of-sample cumulative P&Ls.

United States and the following worldwide financial crisis of 2008–2009. Finally, the last time this dummy variable jumps to 1 corresponds to the covid-19 caused financial crash of March 2020. Therefore, this dummy variable, which is the bear market indicator, clearly fulfills its mission in the sense that it successfully indicates the presence of a recession and a bearish market sentiment.

Figure 13: The Dummy Variable of the S&P500 Rvol Based Model



This figure plots the one month lagged time series of the realized volatility of the S&P500 in percentage value referred to as SP500-RVol(m), alongside the dummy variables which calculation is linked to the S&P500 realized volatility when it is above 11.5% referred to as D-RVol>11.5%.

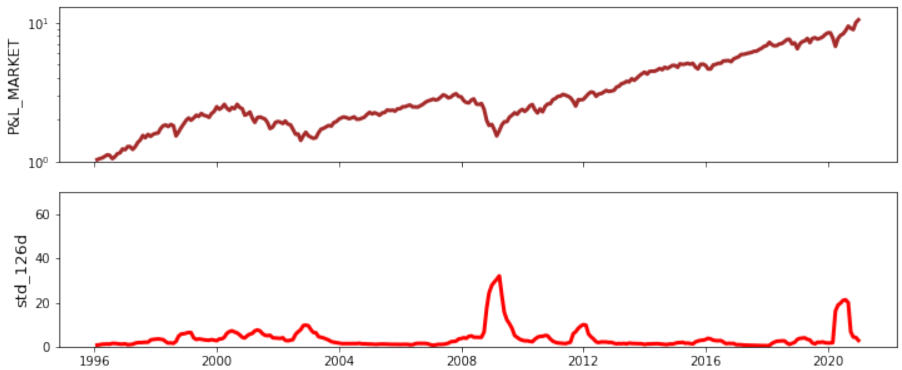
From Figure 13, it can be observed that the dummy variable in green is highly sensitive to the realized volatility movement of the S&P500 in blue. In periods of increased volatility, the dummy variable takes a value of 1, and in periods of low volatility, it takes a value of 0. It can be observed that the financial crashes of 2001–2002, 2008–2009, and 2020 are well covered by this dummy variable because it takes 1 in each of these periods. Moreover, this dummy variable reacts also to small volatility movement as long as there are above 11.5% which makes this dummy variable a good indicator to potential recessions and financial crashes.

Figures 14 and 15 plot the volatility measures of each of the benchmarks and the S&P500 RVol based model using the same y-axis scale for comparison purposes. These measures clearly inversely follow the trends of the cumulative P&Ls of the market and the S&P500 respectively, in the sense that the volatility measures become inflated when there is a financial crash and deflate when there is steady growth and financial prosperity. It is also remarkable that during the financial crashes we tackled previously, these measures increased to high levels but with a more pronounced magnitude of the volatility measure relative to the S&P500 RVol based model.

Understanding the functioning of the regression models responsible for the conditional expectation of the WML portfolio is essential to understanding the way the time variant weights of the dynamic strategy are calculated. Thus the regression output of both the benchmark model in Equation (15) and the S&P500 RVol based model in Equation (31) are presented in Table 3.

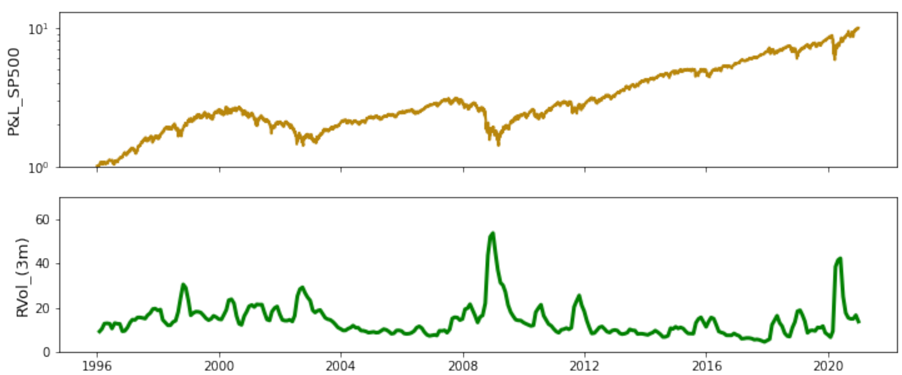
From Table 3, it is concluded that both models follow relatively the same patterns, which

Figure 14: The Market Return Standard Deviation



This figure presents graphically the 1$ investment cumulative P&L of the market portfolio referred to as P&L-MARKET alongside the market's return standard deviation in percentage value based on the past 126 days data referred to as std-126d.

Figure 15: The Realized Volatility Variables of the S&P500 based Models



This figure plots the 1$ investment cumulative P&L of the S&P500 referred to as P&L-SP500 alongside, the S&P500 realized volatility in percentage value when it is based on the past 3 months referred to as RVol-(3m).

makes the dynamic momentum strategy desirable and yields higher cumulative returns than the vanilla momentum one. This is illustrated by the fact that in periods of high volatility, that is either detected by the bear market indicator in the benchmark model or the RVol dummy variable in the S&P500 RVol-based model, and that is manifested by the coming to life of these dummy variables by taking a value of 1 and subsequently activating the interaction term in both models. Thus, the conditional expected return of the WML portfolio takes low values in such environments, which translates into reduced weight in the WML portfolio in that particular month, resulting in a low exposure to the risky asset and a high exposure to the risk-free investment.

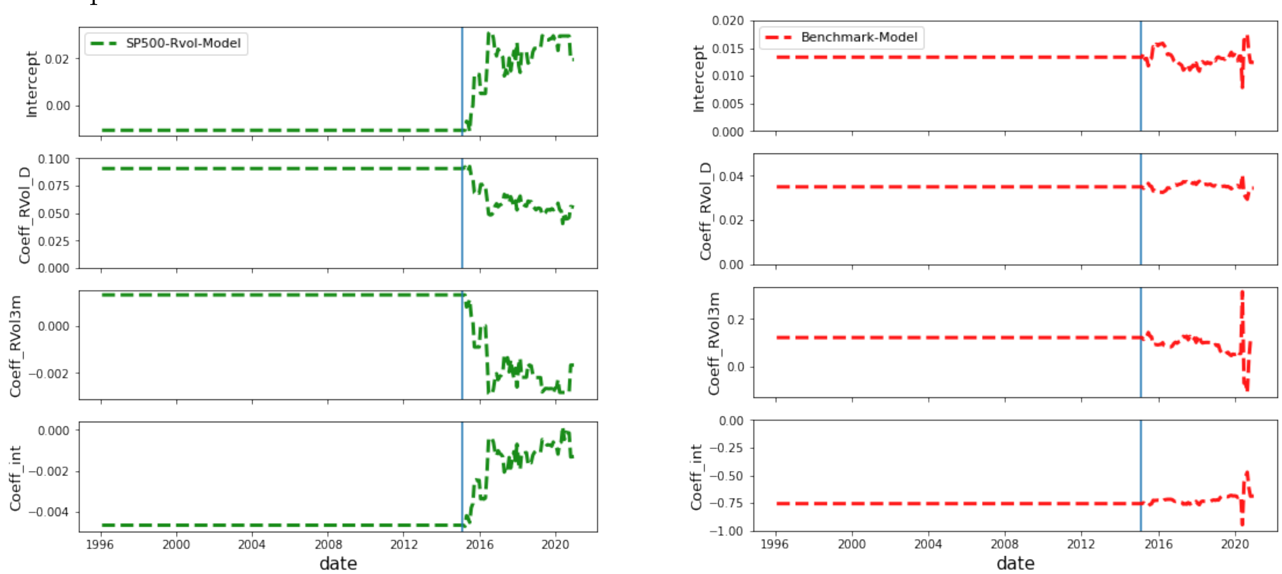
Figure 16 plot side by side the evolution of the expanding window regression coefficients of the two models in Equation (15) and (31). From this figure, it can easily be seen that the benchmark model stays relatively stable out-of-sample until the March 2020 covid-19 crisis

Table 3: The Regressions Summary of the Benchmark Model and the S&P500 RVol-Model

Regressions	Estimated params				
	$\hat{\alpha}_0$	$\hat{\alpha}_b/\alpha_\delta$	$\alpha_{\nu_M}/\alpha_\nu$	$\hat{\alpha}_{int}$	R^2
Benchmark-Model	0.0134*	0.0350*	0.1221*	-0.7554*	0.051
	(1.050)	(1.427)	(0.342)	(-1.862)	
S&P500-RVol-Model	-0.0103*	0.0906*	0.0013*	-0.0047*	0.034
	(-0.164)	(1.334)	(0.212)	(-0.717)	

This table reports the estimated parameters of the benchmark and the S&P500 RVol-Model regressions, their t-statistics results in parenthesise, while the starred parameters are significantly different than 0 at 95% confidence level, as well as the corresponding R-squared of the two following regression equations of the benchmark and the S&P500 RVol-Model repectively using the data from 01-01-1996 until 31-12-2014 as in-sample data $\tilde{R}_t^{WML} = \alpha_0 + \alpha_b 1_{24B,t-1} + \alpha_{\nu_M}\sigma^2_{126M,t-1} + \alpha_{int}1_{24B,t-1}\sigma^2_{126M,t-1} + \epsilon_t$. and $\tilde{R}_t^{WML} = \alpha_0 + \alpha_\delta 1_{11.5\vartheta_{t-1}} + \alpha_\nu RVol_{t-1}^{(3m)} + \alpha_{int} 1_{11.5\vartheta_{t-1}} {RVol_{t-1}}^{(3m)} + \epsilon_t$.

Figure 16: The Benchmark's and the S&P500 based model's regression parameters in and out of sample



The panel to the left plots the evolution of the coefficients of the regression in Equation (31), while the right panel presents the evolution of the coefficients of the benchmark regression in Equation (15). The learning set goes from January 1996 until December 2014, while the out-of-sample period extends from the latter date until December 2020. The horizontal blue line separates the in-sample from the out-of-sample. The regression coefficients; α_0, α_δ, α_ν and α_{int} in Equation (31) are referred to in this figure by Intercept, Coeff-RVol-D, Coeff-RVol3m and Coeff-Int respectively. while the coefficients α_0, α_b, α_{ν_M} and α_{int} in Equation (15) are referred to as Intercept, Coeff-Bear-M, Coeff-std and Coeff-int respectively.

where these coefficient became extremely volatile. Whereas, the S&P500 RVol based model coefficients are less stable than the benchmark's during the whole expanding window, this is due to the hyper sensitive dummy variable in the Equation (31) compared to the benchmark's dummy variable.

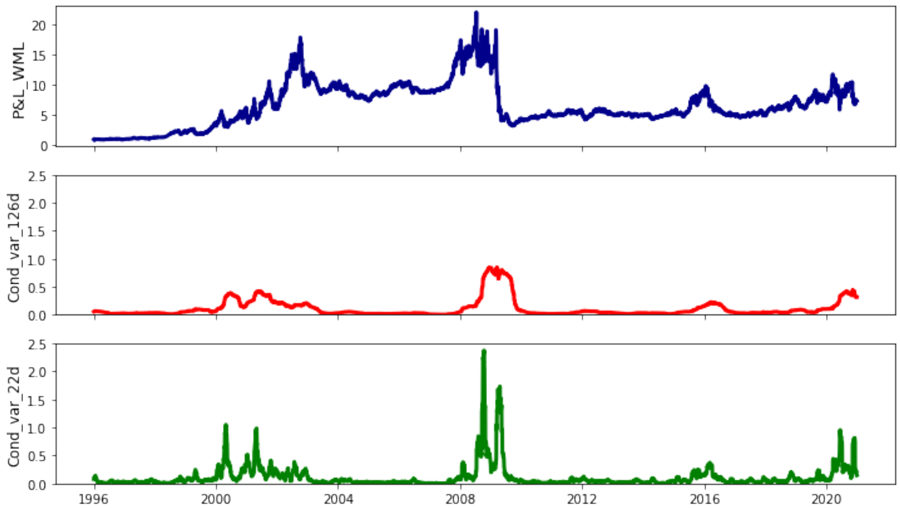
8.4 The S&P500 RVol based model Vs the Benchmark's Time Varying Weights, the In and Out of Sample P&L and the Out-Of-Sample P&L

Having identified the conditional expectation of the WML portfolio models and their parameter estimation, attention is turned towards the missing variable in the calculation of the time-varying weights in Equation (13) which is the conditional variance of the WML portfolio. It has been demonstrated in Appendix A that the dynamic strategy is more profitable in terms of cumulative returns when the conditional variance of the WML portfolio is based on the past 22 days of data rather than the past 6 months of data or the past 126 days of data.
Figure 17 provides the answer to the logic behind the advantage of the short time window in the calculation of the conditional variance and how it impacts the time varying weights of the dynamic strategy. As a matter of fact, in times of high volatility or financial recessions as shown in the P&L of the WML portfolio by the blue curve, the green curve which represents the conditional variance of the WML portfolio based on the past 22 days data increases significantly compared to the curve in red which represents the conditional variance of the WML based on the past 126 days data. This phenomenon translates into the fact that in periods of high volatility and market crashes the denominator in Equation (13) increases and drives the exposure in the risky portfolio down and the exposure to the risk free investment up which in fine avoid losing a big part of the returns of the dynamic momentum strategy accumulated in the past.

The time-varying weights of the benchmark dynamic strategy and the S&P500 RVol-based dynamic strategy are presented in Figure 17. It is noticeable that the green curve is more volatile than the red curve, which is explained by the fact that the S&P500 RVol-based dynamic strategy is more reactive to the volatility movement of the market than the benchmark dynamic strategy, as has been pointed out previously. This specific fact makes the S&P500 RVol-based dynamic strategy superior to the benchmark's because it has the ability to navigate the crashes of the financial market, which allows it to be more profitable.

Figure 18 presents the cumulative P&L of the benchmark strategy alongside the cumulative P&L of the S&P500 RVol-based dynamic strategy. The red chart of the benchmark strategy seems to deliver superior cumulative P&L, however, this figure uses both in and out sample data to plot the cumulative returns of both strategies, which makes it inconclusive. This brings us to Figure 18 which plots only the out-of-sample cumulative P&Ls of the benchmark strategy and the S&P500 RVol-based dynamic strategy. From this figure, it is visible that the S&P500 RVol based dynamic strategy delivers superior results in terms of cumulative P&L to the benchmark's strategy. This superiority in profitability is further confirmed by risk-adjusted return metrics, in particular the Sharpe ratio and the Sortino ratio. In fact, the out-of-sample Sharpe ratio and the Sortino ratio for the SP500 RVol-based dynamic strategy are 0.2473 and 1.0610, respectively, compared to the benchmark's, which amount to 0.1469 and 0.2742, respectively. A higher Sharpe ratio highlights the superiority of the overall risk-adjusted performance, while a higher Sortino ratio emphasizes the superiority of generating more return

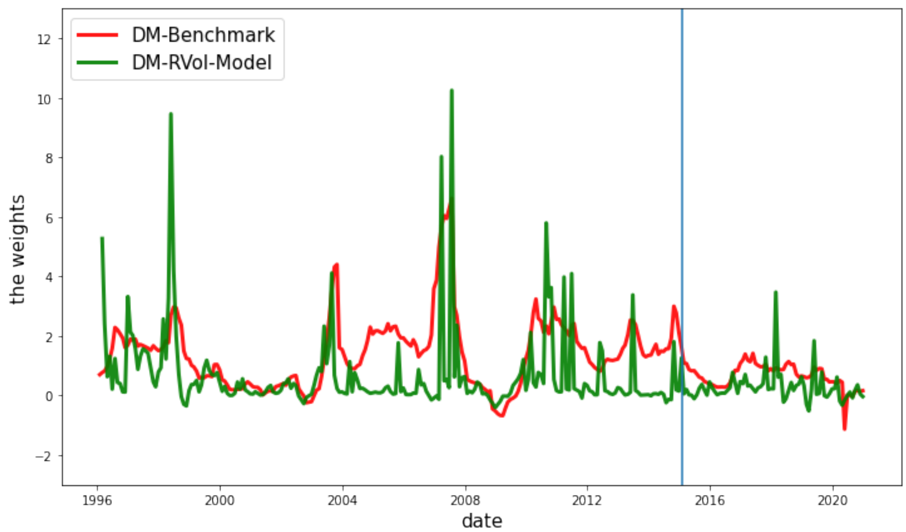
Figure 17: The Conditional Variance of the WML Portfolio



This figure plots the cumulative P&L of the WML portfolio and the conditional variance of the WML portfolio when its calculation is based on the past 126 days of data and when it is based on the past 22 days of data. P&L-WML refers to the cumulative return of the WML portfolio, cond-var-126d refers to the conditional variance based on the past 126 days data and cond-var-22d refers to the conditional variance based on the past 22 days data.

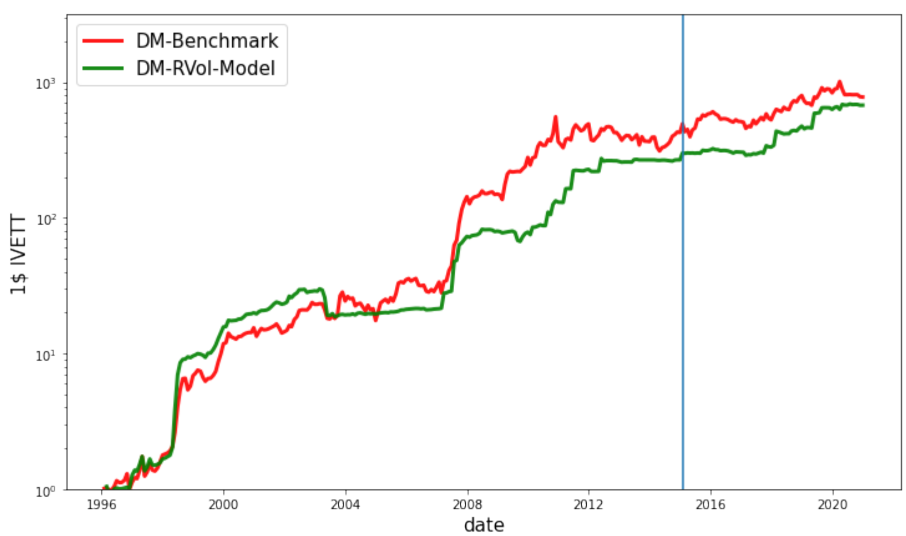
per unit of undesirable volatility or downside risk.

Figure 18: Benchmark Vs the S&P500 Rvol model Dynamic strategies time varying weights



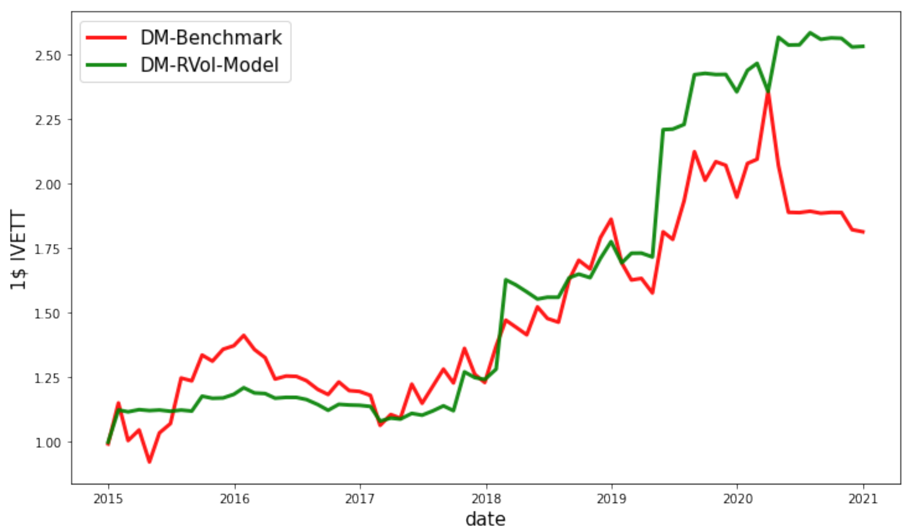
This figure maps the dynamic weights in and out-of-sample of the benchmark model when the conditional variance of the WML portfolio is based on the past 126 days and the time-varying weights of the S&P500 RVol-based model when the conditional variance of the WML portfolio is based on the past 22 days. The learning set goes form January 1996 until December 2014, while the out-of-sample period extends from the latter date until December 2020.

Figure 19: Benchmark Vs the S&P500 Rvol model Dynamic strategies cumulative P&Ls in and out of sample



This figure presents the cumulative in and out of sample P&Ls of the benchmark model and the S&P500 Rvol model when the conditional variance of the WML portfolio of the benchmark and the RVol model are based respectively on the past 126 days and 22 days data. The learning set goes from January 1996 until December 2014, while the out-of-sample period extends from the latter date until December 2020. 1$ IVETT refers to the Investment Evolution Throughout Time of one dollar.

Figure 20: Benchmark Vs the S&P500 Rvol model Dynamic strategies cumulative P&Ls out-of-sample



This figure presents the cumulative out-of-sample P&Ls of the benchmark model and the S&P500 Rvol model when the conditional variance of the WML portfolio of the benchmark and the RVol model are based respectively on the past 126 days and 22 days of data. The out-of-sample period extends from January 2015 until December 2020. 1$ IVETT refers to the Investment Evolution Throughout Time of one dollar.

9 Conclusion

Although momentum investment strategies are profitable in terms of a higher cumulative return and Sharpe ratio than the market portfolio, they embody an optionality feature that makes them crash in turbulent market states. In fact, this optionality feature makes the momentum portfolio equivalent to holding a short call option on the market portfolio. This is due to the return asymmetry of the WML portfolio compared to the market portfolio in panic market states. In this light, this thesis paper comes to reinforce the findings of the literature regarding the forecasting of potential extreme market events that render momentum investment strategies less efficient.

Building on the dynamic momentum strategy developed by Daniel and Moskowitz (2016), whose aim is to dynamically adjust the weight of the WML portfolio in order to make it less susceptible to financial crashes, we introduce new variables that replace the dynamic momentum model and improve its profitability. The building blocks of Daniel and Moskowitz (2016) model consist of calculating the time varying weights of the WML by multiplying a time-invariant scalar that controls the unconditional risk by the ratio of the conditional expected return to the conditional expected variance of the WML portfolio over the next month. The value added by this thesis is a better estimate of the conditional expected return and the conditional expected variance of the WML portfolio over the next month. These are the two most important statistical measures. This is done by relying on the realized volatility measure of the S&P500 using intraday data in the estimation of the conditional expected return of the WML portfolio. In fact, the conditional expected return of the WML portfolio is estimated using a regression that takes into account two important variables and how they interact with each other. The first variable is a dummy variable that aims to predict the arrival of a bear market; the calculation was based on the cumulative return of the market portfolio during the past 24 months, while the second variable is the market's return standard deviation over the past 126 days. In our model, the first variable is replaced by a dummy variable that takes 1 when the realized volatility of the S&P500 is above 11.5% and 0 otherwise, while the second variable is replaced by the realized volatility of the SP500 over the past 3 months. Moreover, through a sensitivity analysis, we prove that approximating the conditional expected variance of the WML portfolio is more adapted to our model when its calculation is based on the variance of the daily returns of the WML over the past 22 days instead of the past 126 days.

Our developed model succeeds in achieving a higher cumulative return and almost doubles the Sharpe ratio of the benchmark dynamic model of Daniel and Moskowitz (2016). This is essentially due to the realized volatility measure that helps better approximate the conditional expected return over the next month in the calculation of the time varying weights of the WML portfolio. The superiority of our model is mostly explained by its rebalancing power, which consists of aggressively changing the composition of our portfolio from one month to the next. Keeping this in mind, this result opens up a door for a further study on the effect of transaction costs on the profitability of our strategy.

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Appendix

A Sensitivity Analysis of the Dynamically Weighted Benchmark Model

The dynamic weighted strategy relies on the calculation of an optimal weight to invest in the WML portfolio, the time varying weights are calculated following Equation (13). Breaking down the variables μ_t and $\eta^2_{WML,t}$ by applying a sensitivity analysis to their components that drive the weights of the WML portfolio is crucial to understanding the behavior of the dynamic weighted strategy and the parameters that are responsible for its superior cumulative P&L.

A.1 Sensitivity Analysis of the Independent Variables of the Regression Model

Since the focal point of this section is the sensitivity analysis of μ_t, the $\eta^2_{WML,t}$ variable in Equation (13) is kept constant throughout this part with its calculation based on the past 126 days same as the benchmark model so that the effect of the μ_t on the time varying weights would be isolated and comparable to the benchmark model. The conditional expectation of the WML portfolio μ_t is approximated using the following equation:

$$\tilde{R}^{WML}_{(x,y),t} = \alpha_0 + \alpha_b 1_{xB,t-1} + \alpha_{\nu_M}\sigma^2_{yM,t-1} + \alpha_{int} 1_{xB,t-1}\sigma^2_{yM,t-1} + \epsilon_t. \tag{32}$$

where $1_{xB,t-1}$ is the bear market indicator that takes 0 when the market's cumulative return over the past x months is positive and 1 when it is negative. $\sigma^2_{yM,t-1}$ is the historical annualized market variance of daily returns calculated based on the past y days data.

The default case, which corresponds to the benchmark of this study elaborated by Daniel and Moskowitz (2016), for the calculation of the time varying weights is such that $x = 24$ months and $y = 126$ days. The sensitivity analysis of μ_t takes place by examining the behaviour of the dynamic strategy when x and y take the following values:

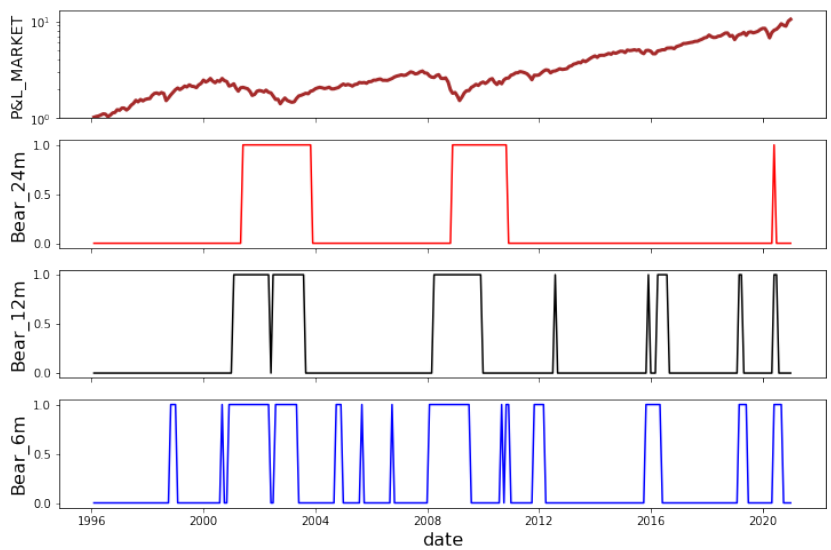
$$1_{xB,t-1} \begin{cases} 1_{24B,t-1} & x = 24 \text{ months} \\ 1_{12B,t-1} & x = 12 \text{ months} \\ 1_{6B,t-1} & x = 6 \text{ months} \end{cases}$$

$$\sigma^2_{yM,t-1} \begin{cases} \sigma^2_{252M,t-1} & y = 252 \text{ days} \\ \sigma^2_{126M,t-1} & y = 126 \text{ days} \\ \sigma^2_{63M,t-1} & y = 63 \text{ days} \end{cases}$$

Visualizing each set of variables against the cumulative P&L of the market portfolio is of great importance in the sense that we can observe firsthand the behavior of each variable with respect to the market movement and especially their coverage of the financial recessions and crashes that should be mitigated to achieve a higher cumulative P&L than the vanilla momentum strategy.

Figure 21 represents the bear market indicator variables, that clearly shows that these dummy variables take a value of 1 in periods of financial distress and recession that include the financial crashes we are eager to avoid and a value of 0 in financial prosperity and growth as it is shown by the cumulative P&L of the market portfolio. It is observable that the more this dummy variable depends on shorter periods of time, the more reactive it becomes. In fact, the 12-month and 6-month based bear market indicators, in black and blue, respectively, oscillate more frequently between 0 and 1 than the 24-month based benchmark bear market indicator, in red.

Figure 21: The Bear Market Indicator Variables

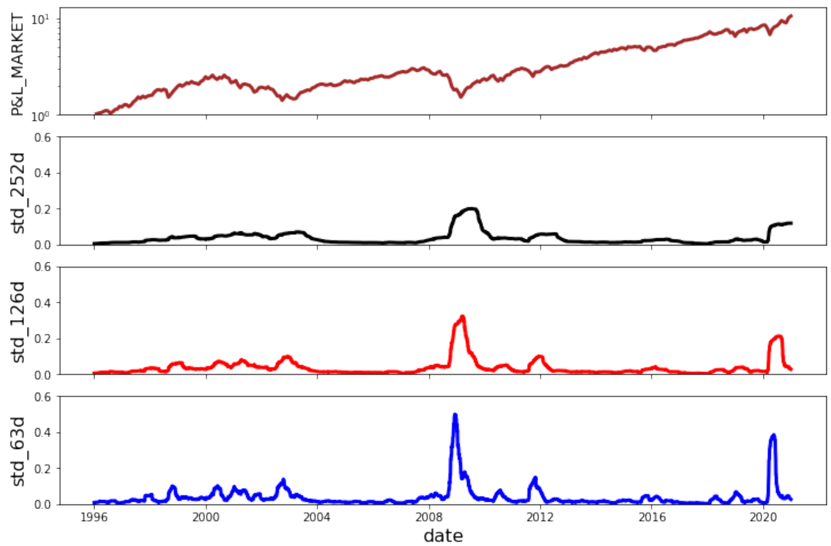


This figure depicts the behavior of the dummy variable when its calculation is based on the past 24, 12, and 6 months of cumulative market returns, respectively. It is referred to in this figure as Bear-24m, Bear-12m and Bear-6m respectively. While the 1$ investment cumulative P&L of the market portfolio is presented by the first sub-figure and referred to by P&L-MARKET.

Figure 22 shows how the volatility measure, which is the historical standard deviation of the market returns, is inversely related to market crashes in the sense that this measure increases when there is a financial crash. In fact, this is illustrated in Figure 22, where it can be seen that during the financial crash of 2008–2009 and during the covid crash of March 2020, this measure of volatility substantially increased. Moreover, the less the measure relies on historical data, the more amplified this statistic becomes. In fact, the bottom panel in Figure 22 proves that when the market return standard deviation relies on the past 63 days of data, its values become larger. From the other two panels, it can be concluded that this measure becomes less and less substantial when its calculation is based on the past 126 days of data and the past 252 days of data, respectively.

Table 4 represents the nine expanding window regressions displaying all possible combinations of the two independent variables of interest, $1_{xB,t-1}$ and $\sigma^2_{yM,t-1}$ in the estimation of the WML portfolio's conditional expected return. These regressions are run using the data from 01-01-1996 until 31-12-2014 as in-sample, while the sample from 01-01-2015 until 31-12-2020 is

Figure 22: The Market Return standard Deviation Variables



This figure presents the market return standard deviation when it is based on the past 252, 126, and 63 days, respectively, and it is referred to in this figure by std-252d, std-126d, and std-63d, respectively. While the 1$ investment cumulative P&L of the market portfolio is presented by the first sub-figure and referred to by P&L-MARKET.

considered out-of sample. According to the results of Table 4, the model with $(x = 24, y = 252)$ have the highest value of R^2, followed by the model with $(x = 12, y = 252)$, both models have an R^2 statistic that is higher than the benchmark's model. Statistically speaking they should better approximate the conditional expectation of the WML portfolio than the rest of the models since they are the ones that fit the available data best.

Besides the nine regression's parameters estimation and the reporting of their R-squared statistic, Figure 23 plots the evolution of these parameters in and out of sample data for the benchmark case as an illustration. From Figure 23, it is clear that the regression's coefficients stay relatively stable out-of-sample starting from January 2015, however the Covid-19 crisis of March 2020 made them oscillate excessively in a short period of time.

Figure 24, Figure 25, and Figure 26 present an ample picture of how each model of the regressions presented in Table 4 behaves in all its components of interest; where the sub-figures present respectively the 1$ investment cumulative P&L in and out of sample starting from January 1996, the 1$ investment cumulative P&L out-of-sample starting from January 2015 and time varying weights of the dynamic strategy in and out of sample. It is important to point out that sometimes the time-varying weights of the WML portfolio become negative, this means that the investment is inverted and that we go long the loser portfolio and short the winner portfolio in that particular month.

From Table 4, Figure 24, Figure 25, and Figure 26 and for comparison's sake, it is observable that the regression models (x = 24, 12 and 6; y = 252) are the ones that outperform the benchmark in terms of the out-of-sample cumulative P&L and are the ones with the highest coefficient of determination R-squared. However, this superiority remains slim in terms of the

Table 4: The Studied Regression Models Parameters Estimation

Regressions	Estimated params $\hat{\alpha}_0$	$\hat{\alpha}_b$	$\hat{\alpha}_{\nu_m}$	$\hat{\alpha}_{int}$	R^2
(x = 24, y = 252)	0.0095*	0.0782	0.2745*	-1.3838	0.078
	(0.642)	(2.641)	(0.580)	(-2.552)	
(x = 24, y = 126)	0.0134*	0.0350*	0.1221*	-0.7554*	0.051
	(1.050)	(1.427)	(0.342)	(-1.862)	
(x = 24, y = 63)	0.0163*	0.0006*	0.0144*	-0.2584*	0.014
	(1.467)	(0.025)	(0.057)	(-0.845)	
(x = 12, y = 252)	0.0078*	0.0814	0.3810*	-1.4936	0.077
	(0.503)	(2.581)	(0.730)	(-2.533)	
(x = 12, y = 126)	0.0155*	0.0305*	0.0807*	-0.6658*	0.045
	(1.126)	(1.148)	(0.184)	(-1.385)	
(x = 12, y = 63)	0.0196*	-0.0109*	-0.0816*	-0.0739*	0.011
	(1.575)	(-0.450)	(-0.225)	(-0.186)	
(x = 6, y = 252)	0.0261	0.0450*	-0.3807*	-0.6229*	0.059
	(2.186)	(1.725)	(-1.301)	(-1.511)	
(x = 6, y = 126)	0.0234*	0.0289*	-0.3553*	-0.2227*	0.043
	(1.572)	(1.140)	(-0.684)	(-0.403)	
(x = 6, y = 63)	0.0188*	0.0038*	-0.1615*	-0.0434*	0.009
	(1.360)	(0.162)	(-0.348)	(-0.088)	

This table reports the estimated parameters of the different regressions as well as the R-squared of each regression, following the equation $\tilde{R}^{WML}_{(x,y),t} = \alpha_0 + \alpha_b 1_{xB,t-1} + \alpha_{\nu_m}\sigma^2_{yM,t-1} + \alpha_{int} 1_{xB,t-1}\sigma^2_{yM,t-1} + \epsilon_t$ and using the data from 01-01-1996 until 31-12-2014 as in-sample data. The values in parenthesis represent the t-statistic of the corresponding coefficient, while the starred values are significantly different than 0 at the 0.05 level.

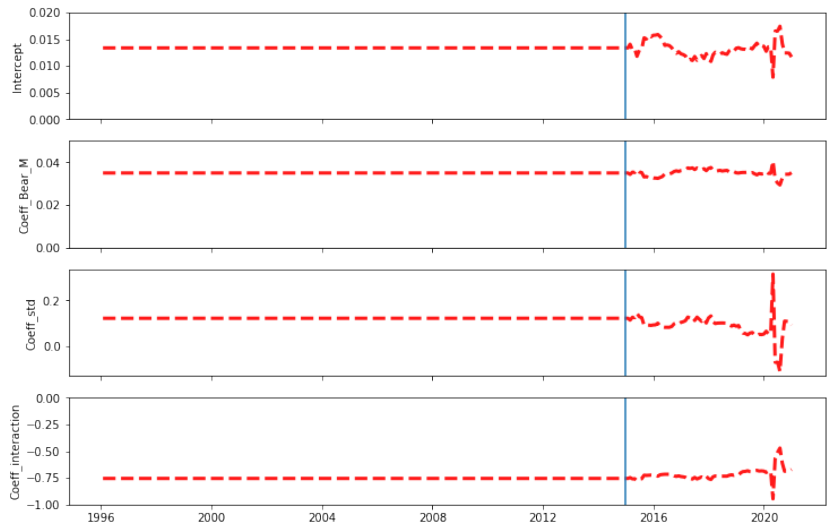
added cumulative return to the benchmark strategy. On top of that, the in- and out-of-sample data relied on in this sensitivity analysis is considerably short in comparison to the data used in the original paper by Daniel and Moskowitz (2016), which span from January 1927 until March 2013.

A.2 Sensitivity Analysis of the Conditional Variance of the WML $\eta^2_{WML,t}$

The benchmark proxies the conditional variance of the WML in the dynamic weights calculation by the variance of the daily returns of the WML over the past 126 days. In addition to this case, two additional ones will be studied, such as:

$$\eta^2_{zWML,t} \begin{cases} \eta^2_{126WML,t} & z = 126 \text{ days} \\ \eta^2_{63WML,t} & z = 63 \text{ days} \\ \eta^2_{22WML,t} & z = 22 \text{ days} \end{cases}$$

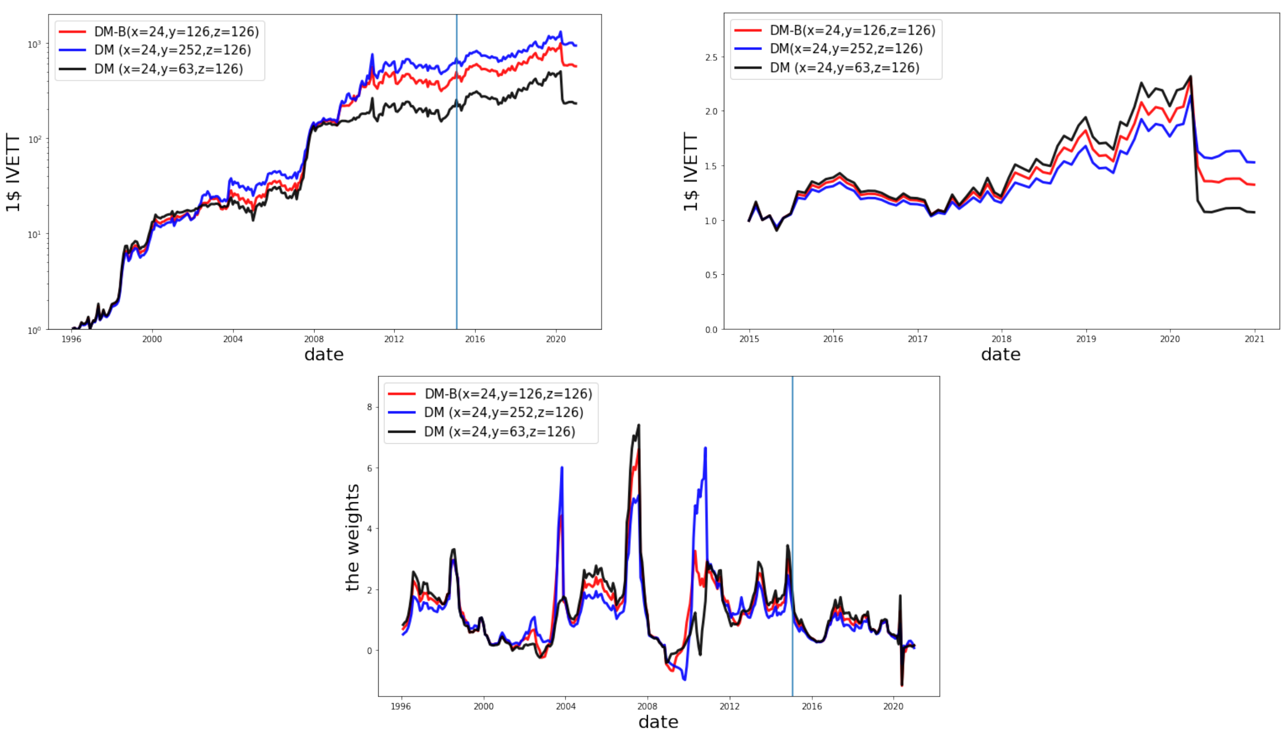
Figure 23: The Benchmark Regression's Parameters In-Sample and Out-of-Sample



This figure illustrates the expanding window regression coefficients of the benchmark model in and out of sample from January 1996 until 2020. The in-sample period extends from January 1996 until December 2015, while the rest of the data is considered out-of-sample. The vertical blue line separates the in-sample observations from the out-of-sample's. These regression coefficients; α_0, α_b, α_{ν_M} and α_{int} in Equation (32) are referred to in this figure by Intercept, Coeff-Bear-M, Coeff-std and Coeff-Int respectively.

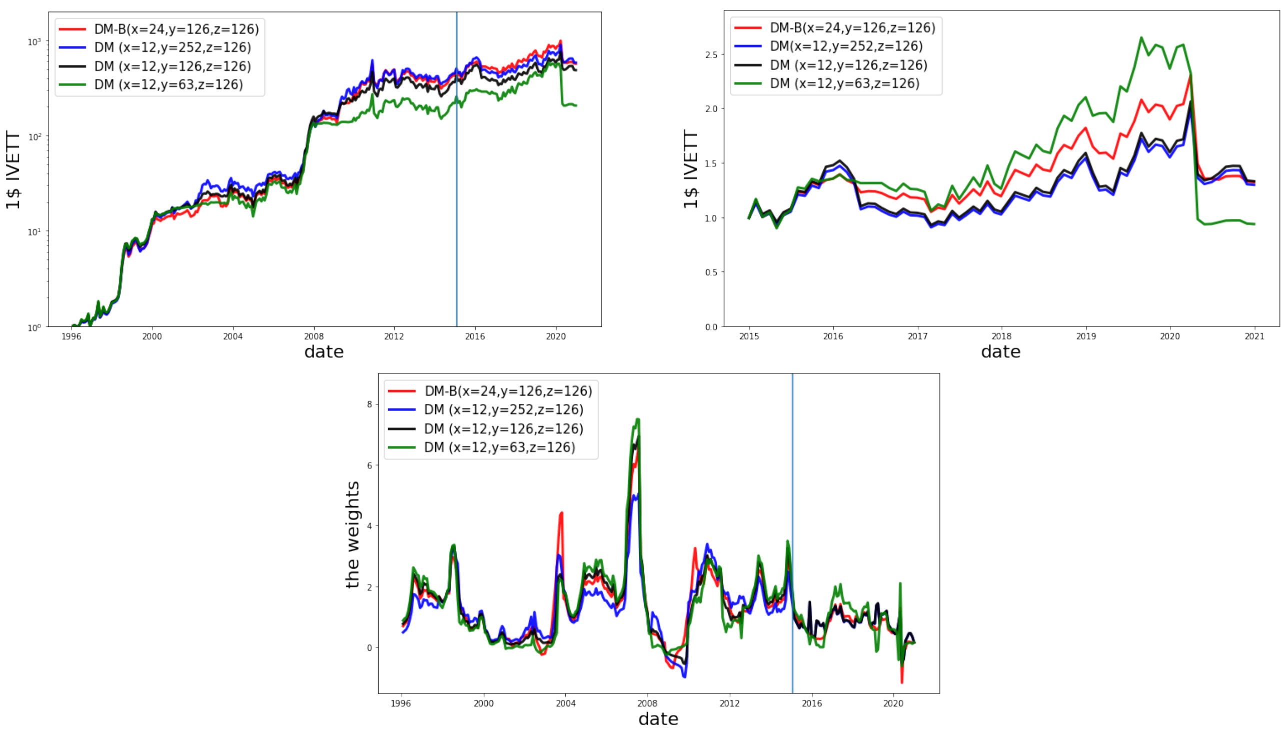
Following the same logic as the previous section, the visualization of the conditional variance is depicted in Figure 27. Since the conditional variance of WML portfolio is the denominator in the time varying weight Equation (13), so no matter what model we use in the nominator which is the conditional expectation of the WML portfolio the conclusion regarding what z value is optimal in the calculation of the time varying weights is the same for all models. Thus, Figure 28 presents the behaviour of the benchmark model's cumulative P&L in and out of sample from January 1996, cumulative P&L out-of-sample starting from 2015 and the time varying weights in and out of sample, when z takes values of 126 days (the default case) and 63 and 22 days. Basing the conditional WML variance of daily return on the past 22 days outperforms the benchmark model that is based on the past 126 days WML return variance in terms of cumulative P&L as it is clearly illustrated by the green curve in the out of sample cumulative P&L in Figure 28.

Figure 24: Cumulative P&Ls and the Time Varying Weights the First Three Regression Models in Table (4)



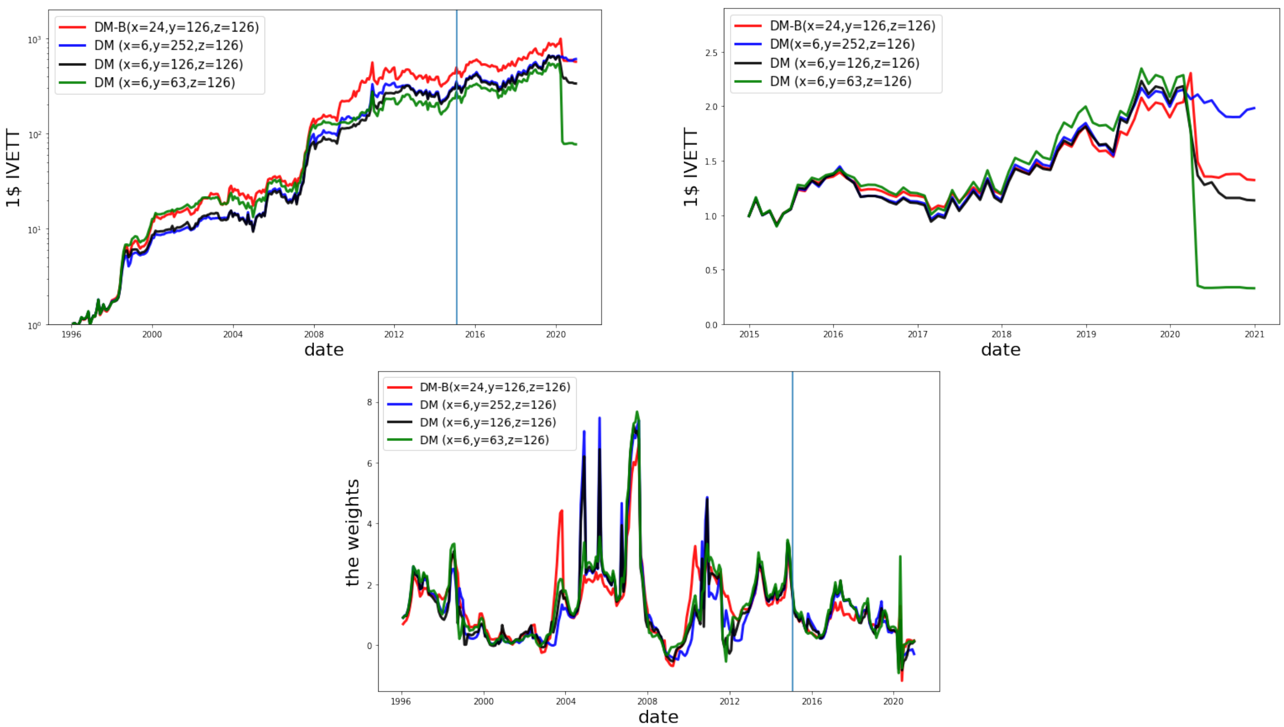
This figure depicts the behavior of a 1$ investment cumulative P&L in and out of sample, a 1$ investment cumulative P&L out-of-sample, and the dynamic weights of the dynamic strategy when the dummy variable is based on the past 24 months and the market standard deviation is based on the past 252, 126, and 63 days, all other things being equal. The red curve represents the benchmark model. The conditional expected return of the WML portfolio μ_t in Equation (13) is estimated using an expanding window regression where the in-sample period extends from January 1996 until December 2015, while the rest of the data is considered out-of-sample. 1$ IVETT refers to the Investment Evolution Throughout Time of one dollar.

Figure 25: Cumulative P&Ls and the Time Varying Weights the Benchmark and the second Three Regression Models in Table (4)



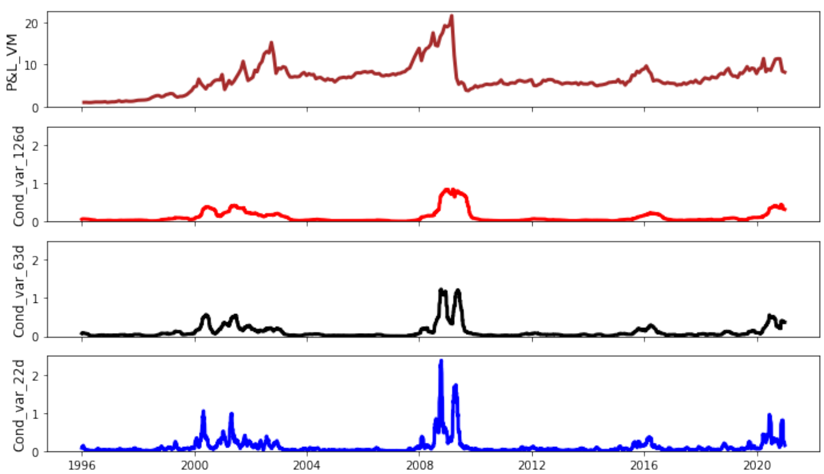
This figure depicts the behavior of a 1$ investment cumulative P&L in and out of sample, a 1$ investment cumulative P&L out-of-sample, and the dynamic weights of the dynamic strategy when the dummy variable is based on the past 12 months and the market standard deviation is based on the past 252, 126, and 63 days, all other things being equal. The red curve represents the benchmark model. The conditional expected return of the WML portfolio μ_t in Equation (13) is estimated using an expanding window regression where the in-sample period extends from January 1996 until December 2015, while the rest of the data is considered out-of-sample. 1$ IVETT refers to the Investment Evolution Throughout Time of one dollar.

Figure 26: Cumulative P&Ls and the Time Varying Weights the Benchmark and the Last Three Regression Models in Table (4)



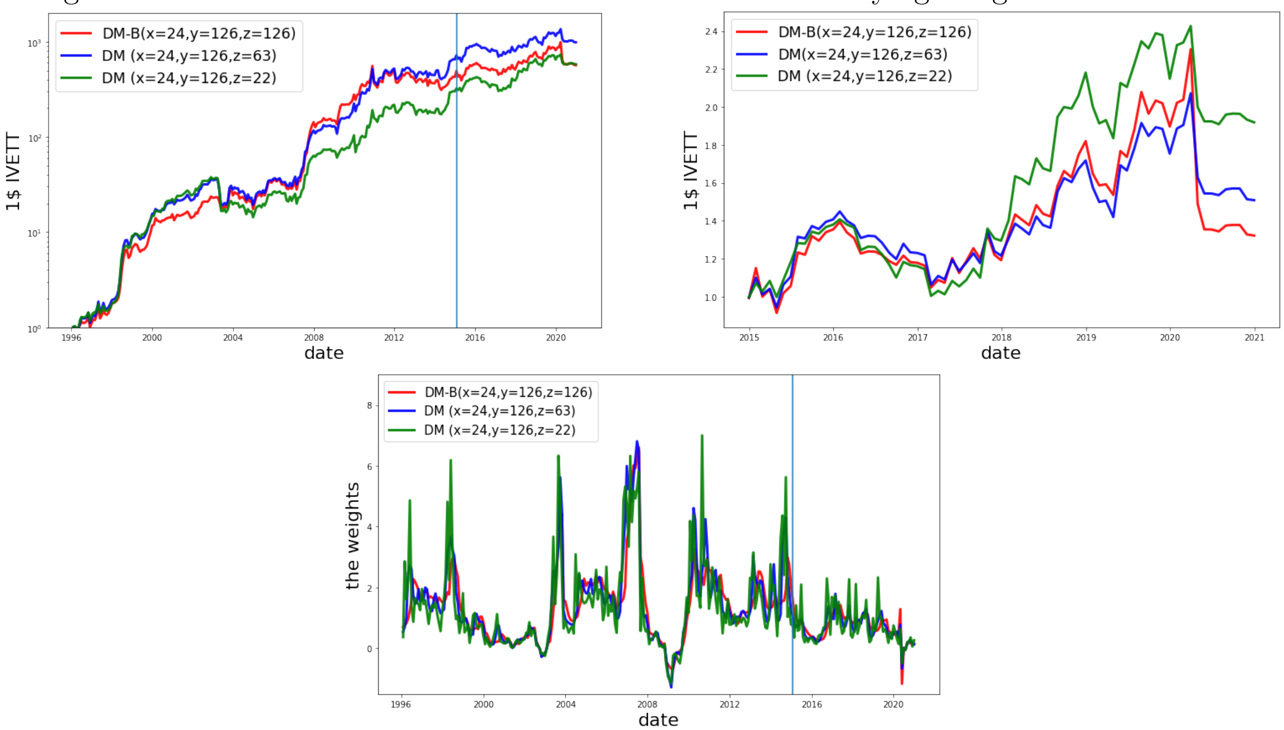
This figure depicts the behavior of a 1$ investment cumulative P&L in and out of sample, a 1$ investment cumulative P&L out-of-sample, and the dynamic weights of the dynamic strategy when the dummy variable is based on the past 6 months and the market standard deviation is based on the past 252, 126, 63 days, all other things being equal. The red curve represents the benchmark model. The conditional expected return of the WML portfolio μ_t in Equation (13) is estimated using an expanding window regression where the in-sample period extends from January 1996 until December 2015, while the rest of the data is considered out-of-sample. 1$ IVETT refers to the Investment Evolution Throughout Time of one dollar.

Figure 27: The Conditional Variance of the WML portfolio Variables



This figure depicts the behavior of the conditional variance of the WML portfolio when its calculation is based on the past 126, 63, and 22 days of return data, and it is referred to by Cond-var-126d, Cond-var-63d, and Cond-var-22d, respectively. Whereas P&L-VM refers to a 1$ cumulative P&L of an investment in the Vanilla Momentum portfolio.

Figure 28: Benchmark's Cumulative P&Ls and the Time Varying Weights when z Varies



This figure maps the behavior of the 1$ investment cumulative P&L in and out of sample, the 1$ investment cumulative P&L out-of-sample and the dynamic weight of the dynamic strategy when the conditional variance of the WML portfolio returns in based on the past 126, 63 and 22 days. The conditional expected return of the WML portfolio μ_t in Equation (13) is estimated using an expanding window regression where the in-sample period extends from January 1996 until December 2015, while the rest of the data is considered out-of-sample. 1$ IVETT refers to the Investment Evolution Throughout Time of one dollar.

B Sensitivity Analysis of the WML Portfolio's Conditional Expected Return of the model based on the S&P500

In the same spirit as the previous appendix, we continue the sensitivity analysis of the dynamically weighted models, where their time-varying weights are calculated following Equation (13). Although the conclusion of the last appendix is essentially that the conditional variance of the daily returns of the WML portfolio $\eta^2_{zWML,t}$ in Equation (13) is optimal when it is based on the past 22 days ($z = 22$), the following analysis is conducted using the past 126 days data as input for the calculation of this statistic ($z = 126$), same as the benchmark model which is the landmark of this study. Therefore, the results would be comparable to the benchmark and across the distinct models discussed subsequently. However, once the best model for the calculation of the conditional expected return of the WML portfolio in Equation (13) is selected, we utilize 22 days as a base for the calculation of the conditional variance of the daily returns of the WML portfolio as concluded in the previous appendix.

This appendix focuses on the sensitivity analysis of WML portfolio's conditional expected return of the model based on the realized volatility of the S&P500 given by the following equation:

$$\tilde{R}^{WML}_{(x,y),t} = \alpha_0 + \alpha_\delta 1_{x\vartheta_{t-1}} + \alpha_\nu RVol^{(ym)}_{t-1} + \alpha_{int} 1_{x\vartheta_{t-1}} RVol_{t-1}{}^{(ym)} + \epsilon_t. \tag{33}$$

where $1_{x\vartheta_{t-1}}$ is a dummy variable that takes 0 when the past month realized volatility of the S&P500 is above $x\%$ and 0 otherwise. Whereas $RVol_{t-1}{}^{(ym)}$ is the annualized realized volatility of the S&P500 which calculation is based on the past y months data.

A sensitivity analysis is conducted hereafter where the model given by Equation (33) is subject to different values of x and y, the aim is to determine the set of values that delivers superior cumulative P&Ls and higher regression's explanatory power R^2 than the dynamically weighted benchmark model of Daniel and Moskowitz (2016) given by Equation (15).

B.1 Sensitivity Analysis of the independent variables

In order to understand which variable positively affects the new model's conditional expected return of the WML portfolio in comparison to the benchmark model, in the sense that it increases its explanatory power and in fine delivers higher cumulative P&Ls of the dynamic strategy, we initially proceed by gradually changing the benchmark model in Equation (15) and shocking the new explanatory variables relying on the model given by the following equation:

$$\tilde{R}^{WML}_{(x,y),t} = \alpha_0 + \alpha_D 1_{x,t-1} + \alpha_V V_{y,t-1} + \alpha_{int} 1_{x,t-1} V_{y,t-1} + \epsilon_t. \tag{34}$$

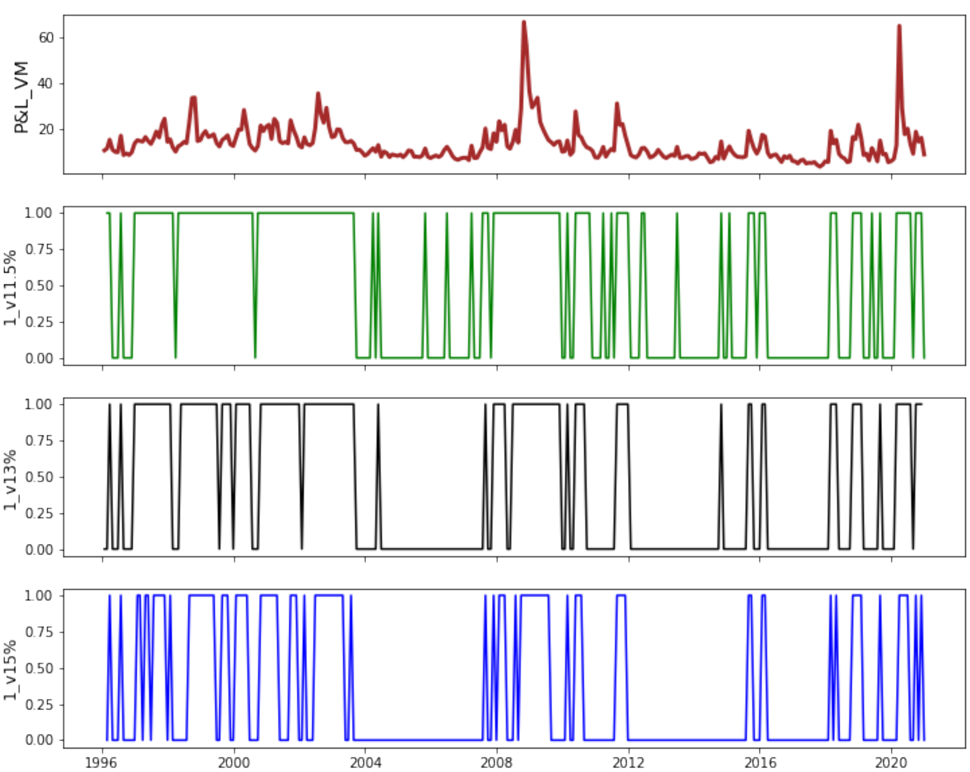
Four case scenarios of each of the independent variables, $1_{x,t-1}$ and $V_{y,t-1}$, are explored such as:

$$1_{x,t-1}\begin{cases}1_{xB,t-1} & x=24\text{ months}\\ 1_{x\vartheta_{t-1}} & x=11.5\%\\ 1_{x\vartheta_{t-1}} & x=13\%\\ 1_{x\vartheta_{t-1}} & x=15\%\end{cases}\quad \text{while } V_{y,t-1}\begin{cases}\sigma^2_{yM,t-1} & y=126\text{ days}\\ RVol_{t-1}{}^{(ym)} & y=1\text{ month}\\ RVol_{t-1}{}^{(ym)} & y=2\text{ months}\\ RVol_{t-1}{}^{(ym)} & y=3\text{ months}\end{cases}$$

The model in Equation (34) generates 16 distinct models, which constitute the object of the expected return of the WML portfolio sensitivity analysis.

The function of the regression's independent variables is highlighted when they are plotted alongside the variables they stem from. The bear market indicator variable, $1_{24B,t-1}$, should come to life and take a value of 1 in periods of market recession and uncertainty that lead to financial crashes, while the dummy variable, $1_{x\vartheta_{t-1}}$, takes values of 1 when the one month lagged realized volatility of the S&P500 exceeds a certain threshold x. While the volatility measure variables, $\sigma^2_{126M,t-1}$ and $RVol^{(ym)}_{t-1}$, should inversely follow the trend of the cumulative P&L of the market and the S&P500, respectively, to reflect the level of volatility in times of recession and financial crashes.

Figure 29: The Dummy Variables of the S&P500 Based Models

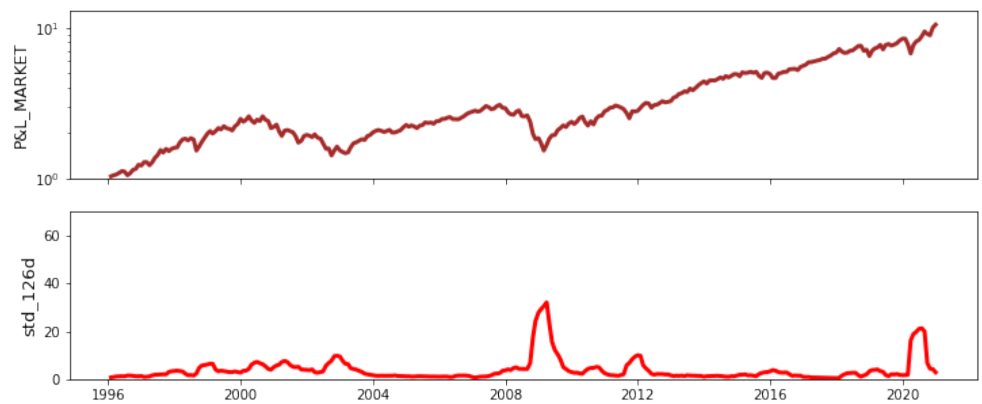


This figure plots the one month lagged time series of the realized volatility of the S&P500 in percentage value referred to as S&P500-RVol(m), alongside the dummy variables which calculation is linked to the S&P500 realized volatility when it is above 11.5%, 13% and 15% respectively and referred to as $1_{11.5\vartheta_{t-1}}$, $1_{13\vartheta_{t-1}}$ and $1_{15\vartheta_{t-1}}$ respectively.

Figure 3 demonstrates that in periods of market recessions and financial crashes the variable $1_{24B,t-1}$ takes a value of 1 while in periods of market growth it takes a value of 0. While from Figure 29, it is observable that the higher the threshold applied to the one month lagged realized volatility of the S&P500, the more the dummy variable oscillates between 0 and 1 and the more it reacts to the slightest volatility movements.

Figure 30 and Figure 31 plot the volatility measures of each of the models studied in this section, where the y-axis is the same in the two figures so that the magnitude of the two variables could be compared. These measures clearly inversely follow the trends of the cumulative P&Ls of the market portfolio and the S&P500, respectively, in the sense that the volatility measures become inflated when there is a financial crash and deflate when there is steady growth and financial prosperity. It can also be concluded from these two figures that the more the volatility measure relies on historical data, the less ample this measure is.

Figure 30: The Market Return Standard Deviation of the Benchmark Model



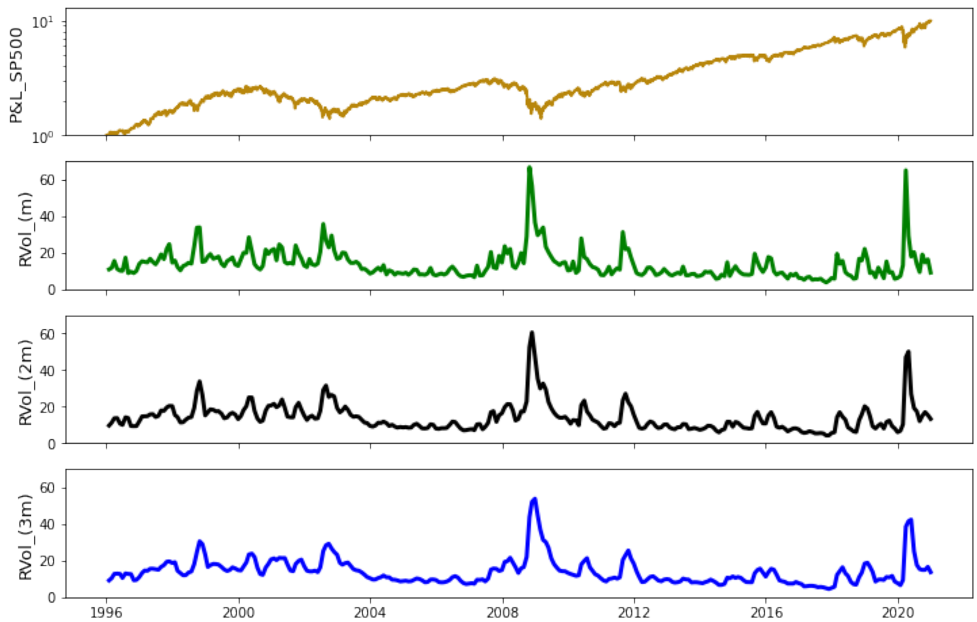
This figure presents graphically the 1$ investment cumulative P&L of the market portfolio referred to as P&L-MARKET alongside the market's return standard deviation based on the past 126 days data referred to as std-126d.

Table 5 represents the sixteen possible regressions of all possible combinations of the independent variables $1_{x,t-1}$ and $V_{y,t-1}$ in Equation (34). It exhibits the regression's parameters estimation using the data from 01-01-1996 until 31-12-2014 as in-sample while the data from 01-01-2015 until 31-12-2020 is considered out-of-sample.

According to the results of Table 5, the model with $(x = 11.5, y = 126)$ has the highest value of R^2 of 0.063, followed by the model with $(x = 13, y = 126)$ and the benchmark model with $(x = 24, y = 126)$ with a value of 0.051, while all the other models have smaller values of this statistic. Statistically speaking, the model with $(x = 11.5, y = 126)$ should better approximate the conditional expected return of WML portfolio than the rest of the models since it is the one that fits the available data the best.

Since the regressions are expanding window that take the data from 01-01-1996 until 31-12-2014 as in-sample and keep expanding the window by one month as time passes and the observation in question is revealed, Figure 32 plots the evolution of the regression's coefficients

Figure 31: The Realized Volatility Variables of the S&P500 based Models



This figure plots the 1$ investment cumulative P&L of the S&P500 referred to as P&L-SP500 alongside, the S&P500 realized volatility when it is based on the past month, the past 2 months and the past three months respectively and referred to as RVol-(m), RVol-(2m) and RVol-(3m) respectively.

for the models with the highest R^2, which are model with $(x = 11.5, y = 126)$ and the model with $(x = 13, y = 126)$, and compares it to the benchmark's coefficients of the model with $(x = 24, y = 126)$.

As can be deducted from Figure 32 the regression's parameters of all models stay relatively stable over time out-of-sample however during the covid-19 crises of March 2020 the coefficients suffer from instability which is translated by immoderate fluctuations.

The time varying weights, the 1$ cumulative P&L in and out of sample, and the 1$ cumulative P&L out of sample of the dynamic strategy based on the studied models in Table 5 are presented in Figures 33 and Figure 34. In an effort to isolate the models that deliver a superior out-of-sample cumulative P&L to the benchmark, Figure 35 plots the 1$ investment cumulative P&L in and out of sample, the 1$ cumulative P&L out-of-sample, and finally the time-varying weights of the dynamic strategies following each of the sixteen studied models. All of the model's cumulative P&Ls are plotted against the cumulative P&L of the benchmark dynamic strategy so that the winner models could be identified. From Figure 35, it can be clearly asserted that the winner of winners is the model in indigo color with $(x = 11.5, y = 3)$, this model has a R^2 statistic equal to 0.034 which does not correspond to the highest coefficient of determination. This can be explained by the fact that the R^2 represents how well the data is fitted by the model in-sample but not out-of-sample.

Having identified the winner model in term of out-of-sample cumulative P&L that best approximates the conditional expected return of the WML portfolio, which is given by the fol-

Table 5: The Studied Regression Models Parameters Estimation

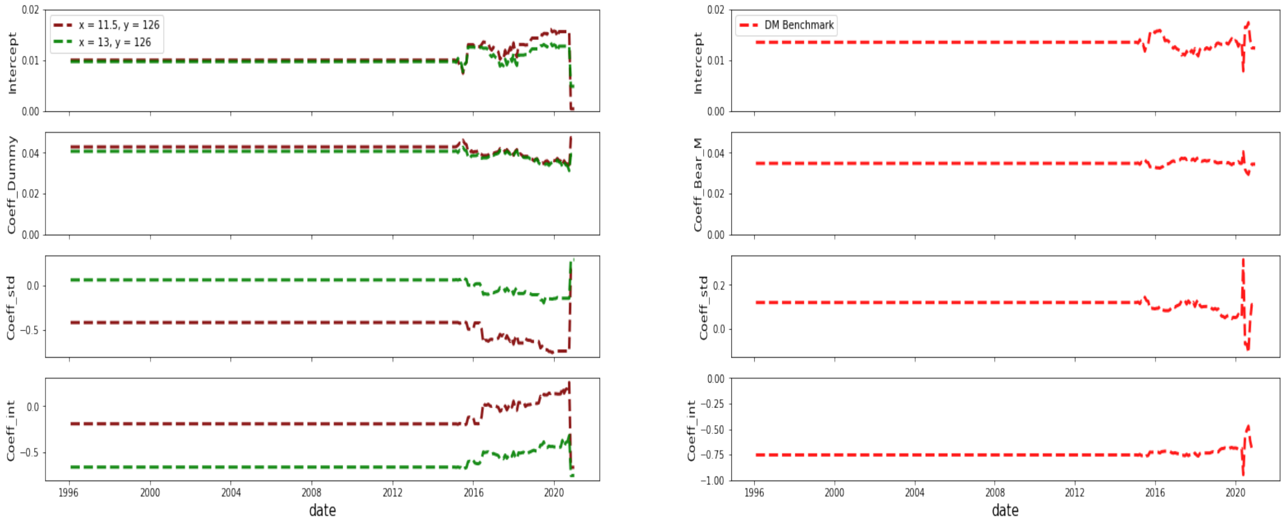
Regressions	Estimated params				
	$\hat{\alpha}_0$	$\hat{\alpha}_b$	$\hat{\alpha}_{\nu_d}$	$\hat{\alpha}_{int}$	R^2
(x = 24, y = 126)	0.0134*	0.0350*	0.1221*	-0.7554*	0.051
	(1.050)	(1.427)	(0.342)	(-1.862)	
(x = 11.5, y = 126)	0.0099*	0.0429*	-0.4151*	-0.1922*	0.063
	(0.519)	(1.852)	(-0.460)	(-0.210)	
(x = 13, y = 126)	0.0096*	0.0408*	0.0682*	-0.6656 *	0.051
	(0.555)	(1.805)	(0.095)	(-0.900)	
(x = 15, y = 126)	0.0148*	0.0139*	0.2969*	-0.7876*	0.047
	(0.996)	(0.604)	(0.586)	(-1.464)	
(x = 24, y = 1)	0.0133*	0.0314*	0.0003*	-0.0027*	0.013
	(0.735)	(0.772)	(0.208)	(-1.253)	
(x = 11.5, y = 1)	-0.0183*	0.0842*	0.0023*	-0.0049*	0.025
	(-0.232)	(1.019)	(0.269)	(-0.553)	
(x = 13, y = 1)	-0.0672*	0.1270	0.0081*	-0.0104*	0.023
	(-1.194)	(2.021)	(1.411)	(-1.766)	
(x = 15, y = 1)	-0.0814	0.0918*	0.0098	-0.0105	0.045
	(-2.053)	(1.751)	(2.673)	(-2.665)	
(x = 24, y = 2)	0.0135*	0.0516*	0.0002*	-0.0036*	0.024
	(0.665)	(1.275)	(0.172)	(-1.689)	
(x = 11.5, y = 2)	-0.0104*	0.0970*	0.0014*	-0.0050*	0.042
	(-0.155)	(1.351)	(0.199)	(-0.702)	
(x = 13, y = 2)	-0.0104*	0.0970*	0.0014*	-0.0050*	0.042
	(-0.155)	(1.351)	(0.199)	(-0.702)	
(x = 15, y = 2)	-0.0464*	0.1325	0.0057*	-0.0094*	0.037
	(-0.934)	(2.305)	(1.177)	(-1.853)	
(x = 24, y = 3)	0.0064*	0.0545	0.0008	-0.0039	0.021
	(0.290)	(1.297)	(0.506)	(-1.720)	
(x = 11.5, y = 3)	-0.0103*	0.0906*	0.0013*	-0.0047*	0.034
	(-0.164)	(1.334)	(0.212)	(-0.717)	
(x = 13, y = 3)	-0.0349*	0.1133	0.0044*	-0.0077*	0.028
	(-0.744)	(2.046)	(1.001)	(-1.655)	
(x = 15, y = 3)	0.0188*	0.0038*	-0.1615	-0.0434	0.009
	(0.0095)	(0.0782)	(0.2745)	(-1.3838)	

This table reports the estimated parameters of the different regressions, their t-statistics results in parenthesise, while the starred parameters are significantly different than 0 at 95% confidence level, as well as the corresponding R-squared of each regression following the equation $\tilde{R}^{WML}_{(x,y),t} = \alpha_0 + \alpha_D 1_{x,t-1} + \alpha_V V_{y,t-1} + \alpha_{int} 1_{x,t-1} V_{y,t-1} + \epsilon_t$ and using the data from 01-01-1996 until 31-12-2014 as in-sample data.

lowing equation:

$$\tilde{R}^{WML}_t = \alpha_0 + \alpha_\delta 1_{11.5\vartheta_{t-1}} + \alpha_\nu RVol^{(3m)}_{t-1} + \alpha_{int} 1_{11.5\vartheta_{t-1}} RVol_{t-1}{}^{(3m)} + \epsilon_t. \tag{35}$$

Figure 32: The Benchmark's and the two models' with the highest R^2 parameters in and out of sample

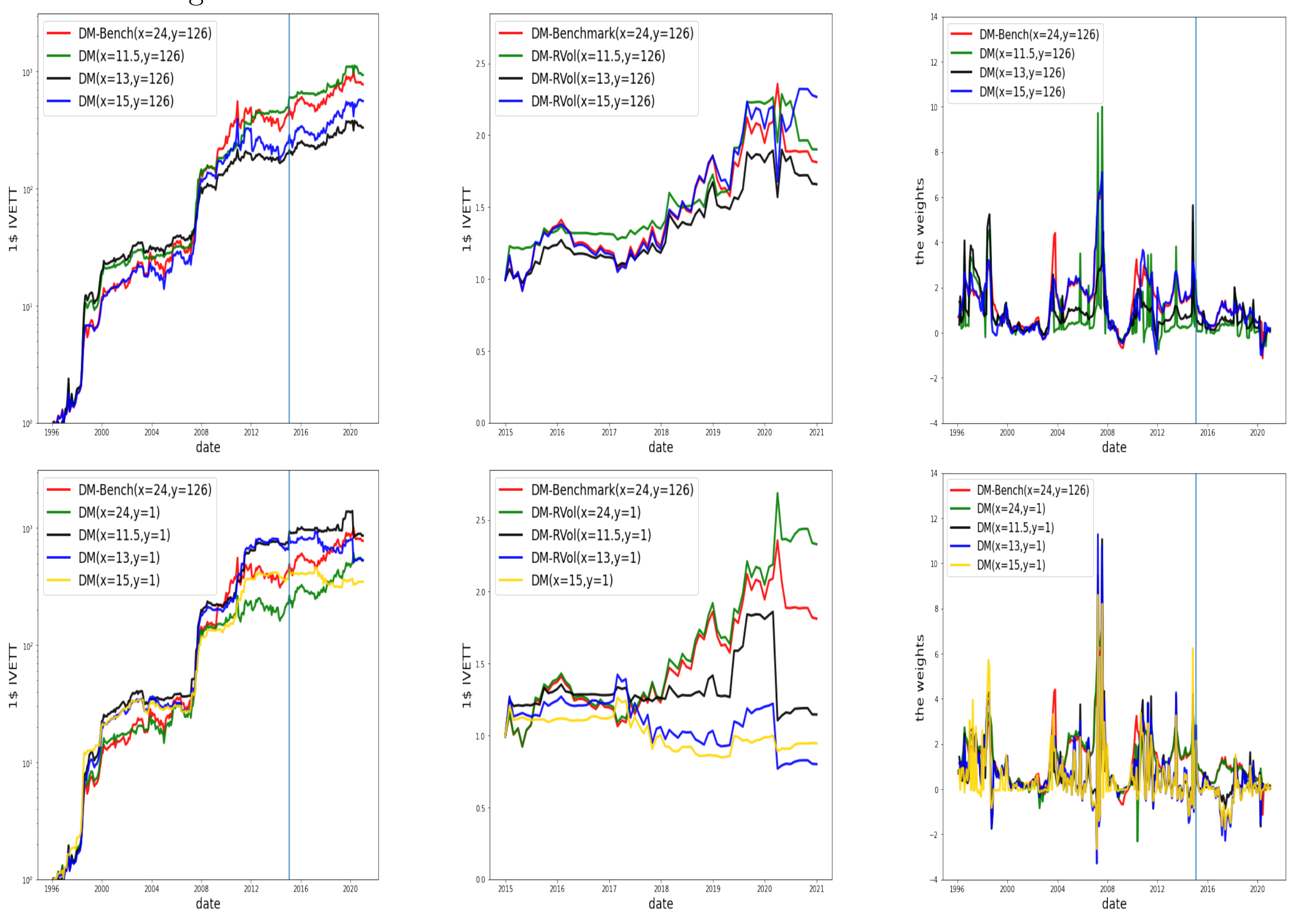


The panel to the left plots the evolution of the coefficients of the regression in Equation (34) when x takes 11.5%, 13% while y is equal to 126 days, while the right panel presents the evolution of the coefficients of the benchmark regression in Equation (15). The conditional expected return of the WML portfolio μ_t in Equation (13) is estimated using an expanding window regression where the in-sample period extends from January 1996 until December 2015, while the rest of the data is considered out-of-sample. The regression coefficients; α_0, α_b, α_{ν_M} and α_{int} in Equation (15) are referred to in this figure by Intercept, Coeff-Bear-M, Coeff-std and Coeff-Int respectively. while the coefficients α_0, α_D, α_V and α_{int} in Equation (34) are referred to as Intercept, Coeff-Dummy, Coeff-std and Coeff-int respectively.

and combining the previous finding regarding which number of days is optimal in the calculation of the WML conditional variance, such that the time varying weights are calculated following Equation (13) where $\eta^2_{zWML,t}$ is based on z = 22 days.

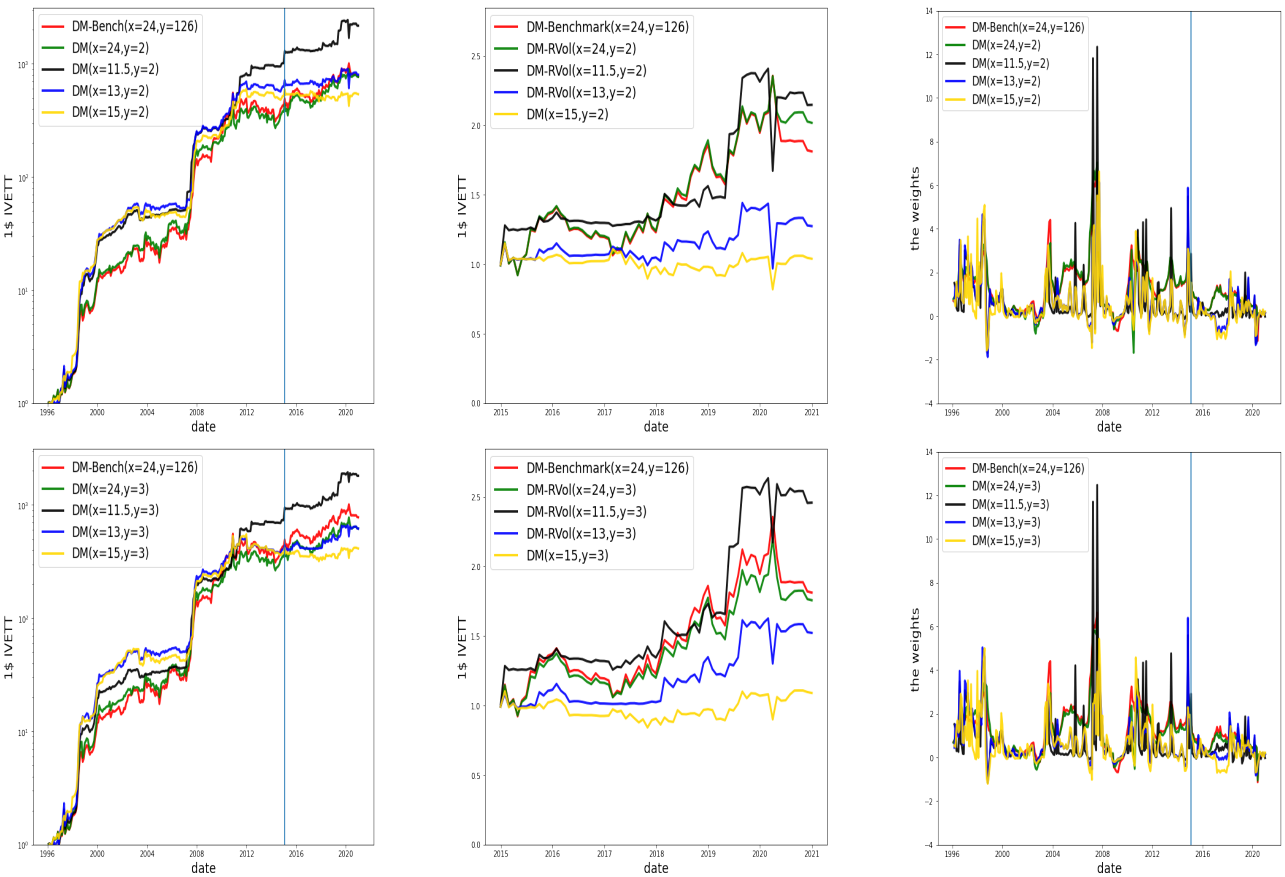
Figure 36 plots the cumulative P&L of the RVol-based model (x=11.5, y=3) with z=22 and z=126, its time-varying weights, and the out-of-sample cumulative P&L alongside the benchmark model using z = 126. It is concluded from Figure 36 that the green figure that corresponds to the model with (x=11.5,y=3,z=22) is superior in terms of out-of-sample cumulative P&L and given that it provides a good in-sample fit as well, it is the preferred model.

Figure 33: The cumulative P&Ls and the time varying weights of the dynamic strategy based on the first eight studied models in Table 5



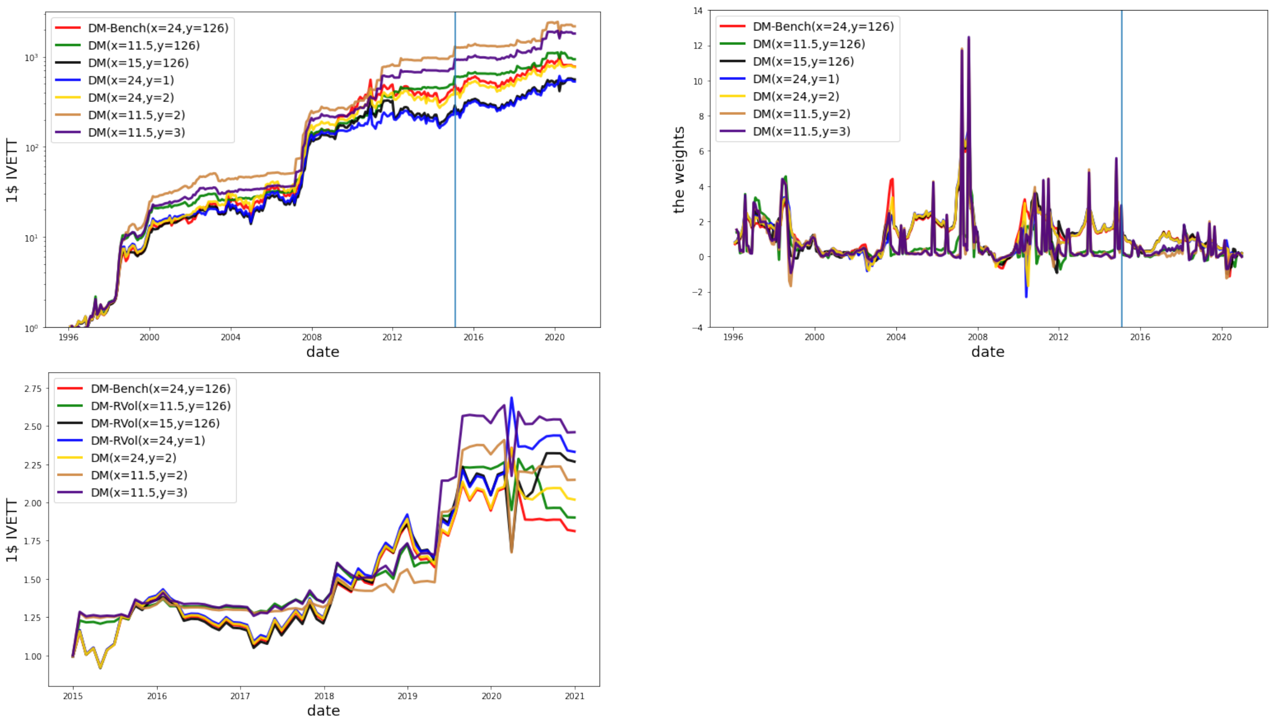
This figure plots the 1$ investment cumulative P&L in and out of sample, the 1$ investment out-of-sample cumulative P&L and the time varying weights of the dynamic strategies based on the first eight studied models presented in Table 5. The conditional expected return of the WML portfolio μ_t in Equation (13) is estimated using an expanding window regression where the in-sample period extends from January 1996 until December 2015, while the rest of the data is considered out-of-sample. 1$ IVETT refers to the Investment Evolution Throughout Time of one dollar.

Figure 34: The cumulative P&Ls and the time varying weights of the dynamic strategy based on the last eight studied models in Table 5



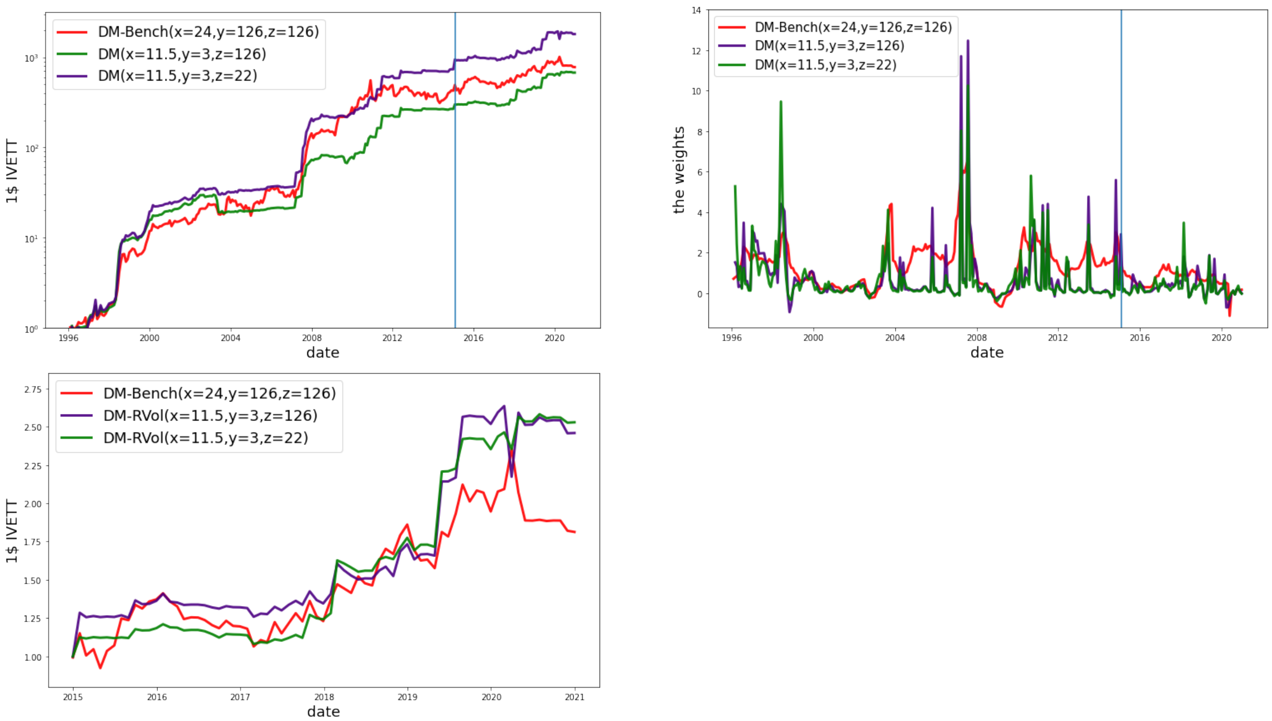
This figure plots the 1$ investment cumulative P&L in and out of sample, the 1$ investment out-of-sample cumulative P&L and the time varying weights of the dynamic strategies based on last eight studied models presented in Table 5. The conditional expected return of the WML portfolio μ_t in Equation (13) is estimated using an expanding window regression where the in-sample period extends from January 1996 until December 2015, while the rest of the data is considered out-of-sample. 1$ IVETT refers to the Investment Evolution Throughout Time of one dollar.

Figure 35: The Benchmark Vs the Winners Dynamic strategy cumulative P&Ls and their time varying weights based on the Studied models



This figure plots the 1$ investment cumulative P&L in and out of sample, the 1$ investment cumulative P&L out-of-sample, and the time varying weights of the dynamic strategies based on the winner models in terms of superior out-of-sample cumulative P&L to the benchmark model. The conditional expected return of the WML portfolio μ_t in Equation (13) is estimated using an expanding window regression where the in-sample period extends from January 1996 until December 2015, while the rest of the data is considered out-of-sample. 1$ IVETT refers to the Investment Evolution Throughout Time of one dollar.

Figure 36: The Benchmark Vs the Winner model's Dynamic strategy cumulative P&Ls and their time varying weights based on the studied models



This figure maps the cumulative P&L in and out of sample, the dynamic weight, and the out-of-sample cumulative P&L of the winner dynamic strategy when the conditional variance of the WML portfolio is based on the past 22 and 126 days. The conditional expected return of the WML portfolio μ_t in Equation (13) is estimated using an expanding window regression where the in-sample period extends from January 1996 until December 2015, while the rest of the data is considered out-of-sample. 1$ IVETT refers to the Investment Evolution Throughout Time of one dollar.