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La contagion financière : une étude empirique de la corrélation dans un modèle hybride de risque de crédit avec changement de régime de Markov

par

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La contagion financière : une étude empirique de la corrélation

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Résumé

Ce mémoire présente une extension du modèle hybride avec changement de régime de Bégin et al. (2014) afin d'étudier la contagion financière à l'intérieur d'un portefeuille composé de 24 grandes banques américaines. Plus précisément, le projet consiste à implémenter la version bivariée du modèle hybride avec changement de régime permettant de capter les corrélations endogènes entre chaque paire de firmes composant le portefeuille à l'étude. En utilisant des séries chronologiques de primes de CDS de Janvier 2005 à Décembre 2012, le modèle déduit les ratios d'endettement et les régimes basé sur le filtre de Kalman inodore et l'estimation du maximum de vraisemblance. Ensuite, les corrélations endogènes entre les firmes sont estimées à partir des co-mouvements des ratios d'endettement. Les résultats de l'analyse empirique stipulent que les entreprises sont plus corrélées pendant le régime de forte volatilité suggérant l'existence de contagion au sein du secteur financier américain au cours de la dernière crise de 2007-2009. De plus, le modèle est capable de dériver les séries chronologiques des probabilités de défaut qui indiquent la tendance du risque au niveau individuel. Bref, ces résultats présentent des implications majeures pour la gestion du risque de crédit puisque la contagion financière peut avoir des conséquences importantes dans les portefeuilles sensibles au crédit.

Mots clés : changement de régime, risque de crédit, probabilité de défaut, estimation de corrélations, contagion financière, filtre de Kalman inodore.

Abstract

This thesis presents an empirical analysis of contagion within the financial sector in the United States during the last global crisis of 2007-2009. To this end, we propose a multivariate extension of the Markov-switching hybrid credit risk model of Bégin et al. (2014), which allows to study the interdependence among a group of major U.S. financial institutions. Using time series of credit default swap (CDS) premiums from January 2005 to December 2012, the model infers the market log-leverage ratios and regimes, and allows to endogenously capture pairwise correlations from log-leverages' co-movements based on the unscented Kalman filter (UKF) and quasi-maximum likelihood estimation. The results of the empirical analysis show that firms are more correlated during high volatility regime suggesting the existence of contagion within the U.S. financial sector during the last crisis. In addition, the model is able to derive the firms' probabilities of default as well as the recovery risk that indicate the risk trend at the individual level. These results present major implications for risk management practices since financial contagion can lead to important consequences in credit-sensitive portfolios.

Keywords : Regime switching ; Portfolio credit risk ; Probability of default ; Correlation ; Contagion ; Credit ratings ; Unscented Kalman filter (UKF).

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Introduction

Le risque de crédit est défini comme étant le risque de perte financière inhérent au défaut d'un emprunteur par rapport au remboursement de ses dettes. Depuis la fin des années 1990, le marché du risque de crédit est en pleine expansion. Dans ce marché, des produits dérivés financiers sont échangés dans le but d'obtenir une protection contre le risque de défaut d'une ou de plusieurs contreparties, généralement des entreprises. Parmi ces produits dérivés, le Credit Default Swap (CDS) est le plus répandu. Ce dernier est un contrat d'échange par lequel le vendeur de protection s'engage à dédommager l'acheteur en cas de défaut d'une contrepartie et ce, contre le paiement d'une prime. En raison de la popularité toujours grandissante des CDS, les recherches se sont multipliées quant à la modélisation et à l'évaluation du risque de crédit. Il n'en demeure pas moins que la crise financière de 2007-2009 a témoigné d'un énorme manque d'efficacité des modèles déjà existants. En effet, le défaut d'importants vendeurs de protection a affecté de nombreux participants de marché et a entrainé des effets de domino en raison de l'interdépendance entre les différentes entreprises. Par conséquent, la modélisation du risque de crédit doit impérativement considérer l'interdépendance qui existe entre les grandes institutions financières afin d'assurer la stabilité du secteur bancaire.

Dans le cadre du présent projet de recherche, l'implémentation d'une extension bivariée du modèle hybride avec changement de régime de Markov proposé par Bégin et al. (2014) permet d'estimer le risque de contagion de 24 grandes firmes américaines durant la période 2005-2012. Pour ce faire, deux paramètres clé sont considérés afin de déterminer le profile de risque du portefeuille. Tout d'abord, le premier paramètre indique la tendance du risque au niveau individuel et il réfère à la probabilité de défaut de l'entreprise ainsi qu'à son inter-relation avec le taux de recouvrement. Ensuite, le deuxième paramètre consiste en l'interdépendance entre les firmes et il est mesuré à partir de la structure de corrélation des log-ratios d'endettements.

Le modèle hybride avec changement de régime de Markov est basé sur l'approche de Boudreault et al. (2013) dans laquelle le taux de recouvrement, advenant un défaut, est endogène. Ce dernier permet de capter l'effet de surprise dans le moment de défaut ainsi que le processus d'intensité qui dépend de la valeur des actifs et des passifs de l'entreprise. À partir de ce modèle, Bégin et al. (2014) introduisent une dynamique de changement de régime de Markov au log-ratio d'endettement permettant de distinguer les périodes stables des périodes à haute volatilité.

La contribution principale de ce mémoire consiste en une nouvelle approche d'analyse de l'interdépendance dans le secteur financier aux États-Unis pendant la dernière crise de 2007-2009. Basé sur une version bivariée à changement de régime, le modèle infère les log-ratios d'endettement et les régimes à partir de primes de CDS observées sur le marché. Les paires de corrélations sont ensuite estimées à partir des co-mouvements dans les séries chronologiques des log-ratios d'endettement en utilisant des techniques de filtrage ainsi que la méthode du maximum de vraisemblance. Finalement, l'application empirique à un groupe d'importantes institutions financières américaines montre une augmentation de la corrélation pendant le régime à haute volatilité, ce qui suggère l'existence de la contagion durant la dernière crise financière.

Correlation Analysis of Financial Contagion in a Markov-Switching Hybrid Credit Risk Model

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Abstract

This paper presents an empirical analysis of contagion within the financial sector in the United States during the last global crisis of 2007-2009. To this end, we propose a multivariate extension of the Markov-switching hybrid credit risk model of Bégin et al. (2014), which allows to study the interdependence among a group of major U.S. financial institutions. Using time series of credit default swap (CDS) premiums from January 2005 to December 2012, the model infers the market log-leverage ratios and regimes, and allows to endogenously capture pairwise correlations from log-leverages' co-movements based on the unscented Kalman filter (UKF) and quasi-maximum likelihood estimation. The results of the empirical analysis show that firms are more correlated during high volatility regime suggesting the existence of contagion within the U.S. financial sector during the last crisis. In addition, the model is able to derive the firms' probabilities of default as well as the recovery risk that indicate the risk trend at the individual level. These results present major implications for risk management practices since financial contagion can lead to important consequences in credit-sensitive portfolios.

Keywords: Regime switching; Portfolio credit risk; Probability of default; Correlation; Contagion; Credit ratings; Unscented Kalman filter (UKF).

1 Introduction

The recent financial crisis of 2007-2009 has highlighted serious negative consequences for the global economy due to interconnectedness of large financial institutions. Indeed, the crisis clearly

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demonstrated the lack of adequate indicators for monitoring and measuring the risk of contagion within the financial industry. Since Basel II failed to provide a regulatory framework at the macro-prudential level, Basel III introduced enhanced requirements to mitigate the risk of contagion from the banking sector throughout the whole financial system.

Given the importance of the topic, there is an extensive literature focusing on contagion analysis during periods of financial distress. On one hand, numerous studies are devoted to find evidence to the existence of contagion. On the other hand, other researches try to uncover the drivers of transmission of the crisis across different firms or countries. For complete surveys on contagion analysis methodologies, we refer to Pericoli and Sbracia (2003), Dungey et al. (2005) and Forbes (2012). In addition, an extensive review of the literature is presented in the next section. However, the starting point of all models measuring and monitoring financial contagion is to set a credit risk framework allowing to capture the main determinants of default. To this end, various credit risk models have been proposed in the literature that have been historically divided into two categories. First, structural models link the credit events to the firm's economic fundamentals assuming default occurs when the firm's value falls through some boundary. Second, reduced-form models consider the surprise element of the default trigger, which is exogenously given through a default rate or intensity. Although they provide a better fit to market data than structural approach does, reduced-form methodology lacks the economic and financial intuition of the structural framework.

To overcome the limitations of both traditional approaches while retaining the main strengths of each, hybrid credit risk models have recently emerged in the literature. Duffie and Lando (2001) have introduced one major class of hybrid models, known as incomplete information models. Their approach mainly addresses the structural framework's issue about the unavailability of observations on the firm's value process. Then, the authors propose a first-passage time model where market participants have noisy estimates of the firm value and the default intensity depends on observable variables related to the balance-sheet fundamentals of the firm. In a similar way, Çetin et al. (2004) study the case where investors receive a reduced version of the information available to firm's managers rather than noisy information. Other significant contributions to this class of models include papers of Giesecke and Goldberg (2003) and Giesecke (2006), where the default threshold, considered random or constant respectively, is unobserved. However, the main idea of incomplete information models is that the firm's assets and/or the default barrier are not observable data to investors.

Other innovations have been devoted to hybrid credit risk models. Among others, let us mention the papers of Madan and Unal (2000) and Boudreault et al. (2013). In the former paper, authors propose a model based on the reduced-form methodology with the additional feature that directly links the hazard rate of default event to the current firm's value. This implies that the likelihood of default increases as the firm's value approaches some threshold. In the latter paper, authors describe a hybrid credit risk model in which both the recovery rate and the default intensity, which is defined from a simple transformation of the firm's leverage, are driven by a Markov-process. However, inspired by the structural framework, both papers link the default intensity to the firm's fundamentals.

Although numerous single-firm approaches exist for measuring the credit risk, it is now well known that the stability of the banking system cannot be guaranteed by simply considering the individual bank level. Indeed, distress of a single bank may cause a cascade of defaults throughout the financial system that results in greater damage to the banking system than the initial bank distress. Therefore, measuring dependence between firms is an essential feature of a credit risk model. As a result, several studies have investigated the interrelations between different entities in a portfolio approach. For example, Boudreault et al. (2014) consider two layers of dependence within a bond portfolio. First, authors analyze the presence of correlations among firm leverage ratios as well as the interrelation between default probabilities and recovery rates. Second, a Clayton copula is used to further create dependence between credit events. The main finding is that correlations between leverage ratios lead to default clusters along with low recovery rates responsible for extreme loss events, which are mainly observed in the extreme tail of the distribution. However, the model assumes that correlation among firms is constant over time. Another contribution to this strand of literature includes the work of Duffie and Gârleanu (2001). Within a portfolio of financial contracts, the authors consider a reduced-form model and address the dependence among defaults through correlated intensities driven by common factors. However, this approach assumes a conditional independence structure, i.e. defaults are independent conditionally on the evolution of the common variables and therefore, contagion effects cannot be captured. In addition to academic researches, other contributions include professional models such as Moody's-KMV and CreditMetrics.

In this paper, we propose a multivariate extension of the Markov-switching hybrid model of Bégin et al. (2014) for measuring the credit risk among 24 major U.S. financial institutions during the sample period 2005-2012. Three key parameters are considered to determine the risk profile of the portfolio. The first ones indicate the risk trend at the individual level and refer to the probability of default and the recovery risk, as well as the link between both of them. The third parameter is related to the interdependence among institutions and is measured through regime-dependent correlations between firm's log-leverages.

The Markov-switching hybrid credit risk model is based on the shell approach of Boudreault et al. (2013). Essentially, the shell allows for an endogenous recovery rate to capture the surprise element in the default trigger, as well as an intensity process that depends on the firm's assets and liabilities. However, Bégin et al. (2014) introduce Markov-switching dynamics to the firm's log-leverage allowing the model to capture stable and turbulent time periods, depending on low and high volatility respectively [1]. Moreover, the model is forward-looking since it uses current market information that reflects future prospects of the firms.

The main contributions of this paper are as follows. First, a novel approach is proposed to analyze the interdependence within the financial sector in the United States during the last global crisis. Based on a bivariate regime-switching framework, the model infers the market log-leverage ratios and regimes from CDS time series, and allows to endogenously capture pairwise correlations from log-leverages' co-movements using filtering techniques and maximum likelihood estimation.

1. According to Ericsson et al. (2009), the main determinants of default are firm leverage, volatility and the riskless interest rate.

Second, the empirical application on a set of top U.S. financial institutions shows that firms are more correlated during high volatility regime suggesting the existence of contagion during the last financial crisis.

The remainder of this paper is organized as follows. Section 2 provides a review of the literature devoted to contagion analysis methodologies. Section 3 describes the bivariate framework of the Markov-switching hybrid credit risk model. Section 4 first introduces the data and estimation procedure. Then, empirical results based on a portfolio constituted of 24 major U.S. financial institutions are presented. Finally, the last section concludes.

2 Review of the Literature

There is an extensive literature focusing on contagion analysis during periods of financial distress. While views on the precise definition of contagion differ in the empirical literature, there is a consensus that the phenomenon is generally associated with the increased co-movements in the returns of risky assets. According to Forbes and Rigobon (2002), contagion occurs when the correlation between two economic or financial entities increases significantly after a financial shock in comparison to the stable period preceding that shock. However, the authors mention that tests for contagion may be biased since crises tend to generate high volatility and therefore, stronger co-movements do not necessarily reflect transmission shocks. Without correcting for the effects of heteroskedasticity, the increase in correlation may simply represent an increase in market volatility. More precisely, Boyer et al. (1999) specify that bias occurs when a data sample is split according to the observed values alone, without respect to the stationarity properties of the sample. In order to avoid such problems in terms of correlation tests for contagion analysis, the authors mention, among other methods, the use of Markov regime switching models with different parameters for both stable and turbulent time periods.

Following the definition of contagion mentioned above, numerous papers have contributed to contagion phenomena analysis during periods of financial distress. For example, Chiang et al. (2007) apply a multivariate GARCH model to measure time-varying conditional correlations during the Asian crisis of the late 1990s. The shift in the level as well as in the variance of correlation coefficients during the crisis indicates the existence of contagion in the Asian market. Another contribution is the paper of Corsetti et al. (2005) that uses a standard factor model to study the international transmission of shocks from the Hong Kong stock market crisis in 1997. The authors find empirical evidence of contagion during the crisis by analyzing correlations of market returns in Hong Kong with the market returns for other emerging economies and G7 countries. There are also several studies on contagion covering times of financial turmoil that rely on the use of copulas to add dependence in credit events. A major contribution is the work of Rodriguez (2007) that uses a copula approach with regime-switching parameters based on returns of stock indices in Asia and Latin America. In terms of Forbes and Rigobon's (2002) definition, the author provides evidence of contagion during the Asian and Mexican crises explained by significant changes in the dependence structure between stock market returns. More recently, Brechmann et al. (2013) use vine copulas to explore contagion and interdependencies of CDS spreads in the global financial market. By studying an international portfolio of 20 insurers and 18 banks, authors find that the default of a bank seems to constitute a more important systemic risk than the default of an insurer.

As contagion phenomena are usually associated to systemic risk, many empirical studies devoted to that field are relevant to our paper. According to Bandt and Hartmann (2000), « the notion of contagion is at the heart of the systemic risk concept and can be defined as a particularly strong propagation of failures from one institution, market or system to another ». In what follows, the empirical systemic risk literature focusing on financial contagion is divided into two parts: models based on the network approach and models studying co-movements in the returns of risky assets.

First, models based on network analysis have recently become very popular in the empirical systemic risk literature. By considering the structure of financial linkages between banks, network models allow for easy tracking of the domino effects triggered by individual bank failures and provide quantitative methodology for capital losses estimation. To date, several interesting results have emerged from this research area. First, Chan-Lau (2010) finds that highly interconnected institutions are a major source of contagion during the last financial crisis. In the same vein, Markose et al. (2012) claim that applying stress in the core of the network, which presents the highest density of interconnections, has extremely bad consequences on the financial system while peripheral stress has little effect. Second, Nier et al. (2008) argue that the banking system is more resilient against contagion defaults when institutions are better capitalized. Furthermore, authors show evidence, along with Gai and Kapadia (2010), that greater connectivity within networks may reduce the probability of contagion, but it can also increase the shocks' spread when failures occur. Network models also highlight that bank size is part of the main sources of contagion, but needs to be considered along with other determinants such as interconnectedness and concentration of exposures across counterparties [see Cont et al. (2010) and Moussa (2011)] or firm-specific probability of default and exposures to common risk factors [see Tarashev et al. (2010)]. This result is in accordance with a recent study of Fund et al. (2009) arguing that systemically important financial institutions are not limited to those that are the largest, but also financial institutions that present higher interconnectedness. Finally, let us mention the recent contribution of Billio et al. (2012). By using principal components analysis and Granger-causality networks, authors quantify the interdependence during the recent crisis between four groups of financial institutions: hedge funds, brokers, banks and insurers. As mentioned by authors, such indirect measures are capable to capture periods of market dislocation and distress. Their empirical results suggest that the banking and insurance sectors are more important sources of connectedness than other financial institutions. Although network approach presents interesting results, it suffers from the major drawback that detailed information about interbank exposures is required to construct the financial linkages between banks. As a result, such models have to deal with extremely complex networks of relationships. Moreover, the behaviour of financial institutions is generally considered static over time.

Second, there are several studies on systemic risk during periods of financial distress that rely, directly or indirectly, on the degree of co-movements in the returns of risky assets. For example, Huang et al. (2009) and Huang et al. (2012) construct a systemic risk indicator based on the probability of default of individual banks and the asset return correlations among banks, which are estimated from CDS spreads and equity price co-movements, respectively. Both studies cover the last financial crisis and are conducted on 12 large U.S. banks and 22 large Asia-Pacific banks, respectively. The first paper shows evidence of a substantial increase in systemic risk corresponding to the inception of the subprime crisis. The second paper validates that the size effect is a major factor in assessing the systemic importance of individual banks.

Finally, important contributions in systemic risk literature rely on other mechanisms than contagion to study the role played by interconnectedness across the U.S. financial sector during the last global crisis. For example, Adrian and Brunnermeier (2009) introduce the concept of CoVaR that measures the value at risk (VaR) of the financial system conditional on the distress of a specific firm. By using correlation, this measure captures spillovers of risk across all financial institutions. However, authors do not take into consideration the different behaviour of CoVaR during stable and crisis periods over which correlations are measured. Another contribution in that field is the systemic expected shortfall (SES) proposed by Acharya et al. (2010) that measures the expected loss to each institution conditional on the undercapitalization of the entire financial system. Based on an empirical study conducted among 102 U.S. financial firms, authors demonstrate the ability of SES to predict systemic risk during the last financial crisis.

In light of the extensive review of methodologies exploring financial contagion and systemic risk, this paper contributes to the existing literature in twofold. First, the empirical study on contagion focuses exclusively on the financial sector in the United States and covers three time periods: a pre-crisis period, a crisis period and a post-crisis period. Based on CDS spreads, the paper analyzes the systematic risk rather than systemic risk as generally investigated in the current literature. More specifically, the portfolio's systematic risk refers to an increase in risk that occurs through an uniform reaction at the firm level following a common shock. For the purpose of contagion analysis, a second contribution of this paper consists in the multivariate extension of the Markov-switching hybrid credit risk framework providing a complete and flexible tool in terms of risk management. Indeed, while being able to model at a firm level an endogenous stochastic recovery rate negatively correlated with the probability of default (both depending on the leverage ratio), the framework is also capable to capture the contagion effect in a portfolio approach.

3 Bivariate Markov-Switching Model

The bivariate Markov-switching model combines the regime-switching univariate framework of Bégin et al. (2014), as well as the portfolio hybrid default risk approach of Boudreault et al. (2014). Therefore, the first part of this section is devoted to the bivariate extension of the univariate Markov-switching framework, while the second part describes the portfolio hybrid credit risk model.

3.1 Bivariate Markov-Switching Framework

As a starting point to the model, assume that the total value of the assets of a specific firm is described by the discrete-time stochastic process $\{A_t : t \in \{1, 2, ..., T\}\}$, and its obligations toward creditors are represented by the liabilities discrete-time stochastic process $\{L_t : t \in \{1, 2, ..., T\}\}$. Moreover, the behaviour of both assets and liabilities is assumed to be driven by a hidden Markov process $\{s_t : t \in \{1, 2, ..., T\}\}$. Suppose now that we have K firms across the portfolio and assume that s_t are K independent first-order Markov chains. If $p_{11}^{(i)}$ denotes $\mathbb{P}\Big(s_t^{(i)} = 1 \mid s_{t-1}^{(i)} = 1\Big)$ and $p_{22}^{(i)}$ denotes $\mathbb{P}\Big(s_t^{(i)} = 2 \mid s_{t-1}^{(i)} = 2\Big)$ for a specific firm i with $i \in \{1, ..., K\}$, the regime state $s_t^{(i)}$ has the following transition matrix

$$\mathbf{P}^{(i)} = \begin{bmatrix} p_{11}^{(i)} & 1 - p_{11}^{(i)} \\ 1 - p_{22}^{(i)} & p_{22}^{(i)} \end{bmatrix}. \tag{1}$$

Then, define $\mathcal{G}_t = \sigma(A_u, L_u, s_u : u \in \{1, 2, ..., t\})$ as the σ–field generated by A, L, and s with the usual regularity conditions. Thus, the leverage ratio $X_t \equiv L_t/A_t$ results in a $\mathcal{G}_t$–adapted discrete-time stochastic process $\{X_t : t \in \{1, 2, ..., T\}\}$. Finally, the model also assumes that the log-leverage time series of a specific firm i, denoted by $x_t^{(i)} \equiv \ln(X_t^{(i)})$, follows a first-order two-state Markov-switching process such as

$$x_t^{(i)} = x_{t-1}^{(i)} + \left(\mu^{(i)} - \frac{1}{2}\Big(\sigma_{s_t^{(i)}}^{(i)}\Big)^2\right)\Delta t + \sigma_{s_t^{(i)}}^{(i)}\sqrt{\Delta t}\epsilon_t^{(i)}, \; i \in \{1, ..., K\}, \tag{2}$$

where $s_t^{(i)} \in \{1, 2\}$ is the regime state at time t of firm i, Δt represents the elapsed time between two consecutive observations, and $\Big(\epsilon_t^{(i)}\Big)_{t=1}^{\infty}$ is a standardized Gaussian noise series. The drift $\mu^{(i)}$ as well as regime diffusions $\sigma_1^{(i)}$ and $\sigma_2^{(i)}$ are firm-specific parameters to be estimated. Recall that, depending on the modelling objective, the log-leverage dynamics evolves either under risk-neutral pricing measure $\mathbb{Q}$, or under physical measure $\mathbb{P}$ for risk management purposes.

When it comes to a portfolio approach, one must consider the interrelation among firms that can lead to clusters of defaults and may significantly impact the future value distribution of the portfolio. To this end, the model captures the firms' interconnections through the correlation between noise terms of log-leverage described in Eq. (2), i.e.

$$\rho_{\mathbf{s}_t}^{(i,j)} = \mathrm{Corr}^{\mathbb{P}}(\epsilon_t^{(i)}, \epsilon_t^{(j)}), \tag{3}$$

with $\mathbf{s}_t \in \Big\{s_t^{(i)}, s_t^{(j)}\Big\}$, or $\mathbf{s}_t \in \{(1, 1), (1, 2), (2, 1), (2, 2)\}$. Thus, four correlation values have to be estimated for each pair of firms depending on their specific regimes, i.e. $\rho_{\mathbf{s}_t}^{(i,j)} = (\rho_{1,1}^{(i,j)}, \rho_{1,2}^{(i,j)}, \rho_{2,1}^{(i,j)}, \rho_{2,2}^{(i,j)})$.

3.2 Hybrid Default Risk Framework

The bivariate Markov-switching model is based on a hybrid default risk framework, which combines features of both structural and reduced-form approaches. On the one hand, the model links the credit events to the firm's capital structure, i.e. the firm's assets and liabilities. On the other hand, the model considers the surprise element of the default trigger, which is exogenously given through a default intensity process. In addition, the firm's leverage and regimes are latent variables that are implied by the term structure of CDS premiums. Finally, the model also features an endogenous stochastic recovery rate that depends on the firm's probability of default.

The model first relies on the assumption that default is driven by an intensity process H_t that depends on the leverage ratio X_t such as

$$H_t^{(i)} = \beta^{(i)} + \left(\frac{X_t^{(i)}}{\theta^{(i)}}\right)^{\alpha^{(i)}}, \tag{4}$$

where $\alpha^{(i)} > 0$, $\beta^{(i)} > 0$ and $\theta^{(i)} > 0$ are firm specific constants to be estimated. The intensity process increases with the leverage ratio, making the default more likely to occur. Furthermore, $H_t^{(i)}$ allows defining the default time as a reduced-form default trigger, that is, the first jump of a Cox process

$$\tau^{(i)} \equiv \inf \left\{ t \in \{1, 2, ..., T\} : \sum_{u=0}^{t-1} H_u^{(i)} \Delta t > E_1^{(i)} \right\}, \tag{5}$$

where $E_1^{(i)}$ is an exponential random variable with mean 1 that is independent of σ–field $\mathcal{G}_t$. Hence, default is highly correlated with the leverage ratio of the firm. Indeed, since $\alpha^{(i)}$, $\beta^{(i)}$ and $\theta^{(i)}$ are positive constants, the likelihood of default tends to increase with the leverage ratio.

The model also defines an endogenous recovery rate that depends on the capital structure of the firm at the moment of default. Considering the liquidation and legal fees as a fraction $\kappa^{(i)}$ of the market assets' value at default, one can approximate this value by $\min\Big((1-\kappa^{(i)})A_\tau^{(i)}; L_\tau^{(i)}\Big)$. Given the leverage's dynamics, the random behaviour of the recovery rate at the moment of default is

$$R_\tau^{(i)} = \min\left((1-\kappa^{(i)})\frac{1}{X_\tau^{(i)}}; 1\right). \tag{6}$$

The endogenous recovery rate distribution is consistent with the empirical literature since it is a decreasing function of the leverage ratio meaning that default probability is negatively correlated with the recovery rate at the moment of default [2]. Finally, the stochastic behaviour of the recovery rate as well as regime switching dynamics imply that corporate coupon bond prices and CDS premiums cannot be calculated in closed form. Therefore, a numerical method that is based on a trinomial lattice approach is used. Details about pricing bonds and CDSs are provided in Appendix A.

4 Estimation

4.1 Data and assumptions

Since the late 1990s, the credit risk market has substantially grown and the CDS has become the new instrument for banks to manage and measure their risk. Considering that the CDS spread is directly linked to the credit quality of the bond issuer, it is expected to reflect a pure measure of default risk. In the recent literature, many authors challenge this argument [see Friewald et al. (2012) and Bielecki et al. (2011), among others]. However, empirical researches suggest that default

2. See Altman et al. (2004) and Altman (2006).

risk is one of the most important risks involved in the CDS spread and therefore, it provides a good proxy in studying the credit risk of a firm. Indeed, Ericsson et al. (2009), Tang and Yan (2007) and Longstaff et al. (2005), among others, show that an important portion of CDS spreads can be directly attributed to credit risk. In this paper, the CDS is the underlying instrument of the Markov-switching hybrid credit risk model.

The CDS prices are provided by MarkIt for maturities of 1, 2, 3, 5, 7, and 10 years. The weekly term structure of CDS data starts on January 5, 2005 and ends on December 26, 2012 for a total of 417 observations. Prices correspond to Wednesday data since it is the least likely day to be a holiday. The CDS's tier is chosen as senior and refers to the level of debt in the capital structure of the reference entities. Furthermore, the restructuring clause is XR meaning that all restructuring events are excluded as trigger events. Note that this type of clause is generally suggested for U.S. reference entities.

Table 1 presents the descriptive statistics of CDS premiums for each entity across the portfolio. Among the firms, AIG and Washington Mutual have the widest average spreads reaching maximum values of 6052.36 for 1-year tenor and 9463.68 for 2-years tenor respectively, while minimum values are below 10. To the extent that the spread is a compensation for credit risk, this indicates that both AIG and Washington Mutual were considered the riskiest firms by the market during the sample period. This is consistent with the near collapse of AIG and the failure of Washington Mutual, which was the largest commercial bank failure in American history. Conversely, Fannie Mae and Freddie Mac, for which CDS data were considered up to September 2008, have the narrowest average spreads. Although the holders of CDS triggered the default clauses for both entities, the debt was guaranteed by the U.S. government which mitigated the risk associated to these firms in the CDS market. Finally, descriptive statistics provided in Table 1 also show very high standard deviations compared to mean values. This reflects the deterioration of the entire financial sector during the crisis.

In addition to CDS data, the model requires other inputs such as the risk-free interest rate and the initial log-leverages of the firms. First, the risk-free interest rate is assumed to be constant over time at 1.75%. This value represents the average rate of the daily 1-month and 3-month Treasury constant maturity series obtained from the Federal Reserve Bank of St. Louis. Second, the log-leverages as of January 2005 are approximated from the logarithm of the total liabilities divided by the total assets of each firm in the sample. More specifically, the firms' financial information is extracted from the Wharton Research Data Services (WRDS) Compustat database as of the fourth quarter of 2004's accounting data.

4.2 Empirical Results

4.2.1 Markov-Switching Parameters

Since log-leverage ratio time series and Markov regimes are unobservable variables, filtering techniques need to be applied in order to infer these quantities from observable CDS data. Our approach consists in a two-step estimation procedure: i) parameters for each specific firm are esti-

Table 1: Descriptive statistics of CDS premiums by firm.

Firm		CDS premium (bps) 1y	2y	3y	5y	7y	10y	Firm		CDS premium (bps) 1y	2y	3y	5y	7y	10y
1.AIG	mean	322.29	321.47	325.74	326.16	311.76	300.01	13.Goldman Sachs	mean	106.30	112.31	117.69	127.24	128.19	130.13
	std	709.45	609.56	564.35	488.65	435.97	390.17		std	117.20	109.50	105.20	100.40	95.80	90.31
	min	2.12	3.64	4.67	7.92	11.43	15.37		min	3.95	7.15	10.72	17.40	21.62	29.44
	max	6052.36	4773.37	4036.51	3336.17	2957.63	2637.87		max	620.89	574.31	591.13	579.29	577.84	561.30
2.Allstate Corp	mean	53.33	58.54	63.92	76.54	81.60	86.14	14.Hartford Finl Serv Gp	mean	149.99	158.12	168.65	186.81	188.52	191.31
	std	83.25	77.01	71.63	65.56	63.08	60.28		std	253.72	225.33	209.59	195.15	181.85	171.31
	min	2.40	3.73	4.38	8.96	11.91	16.16		min	2.05	3.66	5.16	9.13	13.22	16.98
	max	467.95	444.41	423.89	397.02	369.22	375.00		max	1583.29	1317.44	1209.27	1097.68	974.20	906.28
3.American Express	mean	89.41	92.37	96.05	103.72	103.49	104.83	15.JPMorgan Chase & Co	mean	40.35	48.25	56.84	72.59	76.44	81.44
	std	151.48	139.86	130.25	113.11	100.71	91.69		std	41.82	40.73	42.15	45.91	44.87	43.68
	min	1.79	3.27	4.18	7.84	9.92	12.73		min	2.65	3.84	6.51	10.87	14.71	20.29
	max	780.54	724.32	703.09	629.16	573.71	544.35		max	242.68	219.41	206.71	208.19	204.93	206.34
4.Bank of America	mean	102.32	111.21	119.25	134.75	136.27	137.96	16.Lehman Brothers	mean	92.42	84.52	80.00	77.12	76.81	79.65
	std	119.48	110.71	106.10	103.35	100.69	97.10		std	174.22	141.89	120.45	92.23	81.20	71.46
	min	1.89	3.14	4.41	7.61	10.66	14.77		min	3.73	6.73	9.79	17.20	21.66	29.73
	max	540.71	513.59	500.61	481.58	470.93	455.50		max	1113.71	861.76	719.60	551.29	501.11	466.71
5. BB&T Corp	mean	41.93	43.53	50.32	77.28	78.41	85.15	17.Merrill Lynch & Co	mean	85.60	81.96	80.67	80.45	80.18	82.70
	std	45.72	38.32	36.80	59.31	54.89	56.13		std	131.14	118.24	107.53	92.47	83.69	75.37
	min	2.98	4.67	7.11	11.94	12.97	20.27		min	2.37	5.70	9.06	14.62	18.11	24.59
	max	199.10	163.13	142.18	254.00	255.00	301.92		max	670.88	613.55	573.05	522.19	493.52	470.89
6.Bear Stearns	mean	60.17	59.47	58.98	60.73	63.00	67.67	18.Morgan Stanley	mean	162.40	162.05	162.15	167.51	165.17	164.84
	std	123.11	108.41	94.20	74.89	68.29	60.65		std	215.77	182.92	166.29	149.26	139.11	129.11
	min	3.84	8.20	10.99	17.68	22.03	29.83		min	3.94	6.92	9.94	16.55	20.81	29.93
	max	970.44	857.88	709.18	558.29	528.51	495.77		max	2005.80	1488.52	1346.15	1103.65	990.91	892.21
7.Citigroup	mean	108.96	116.23	123.82	137.33	138.36	140.14	19.Schwab Charles Corp	mean	31.49	37.84	44.43	52.91	57.95	64.13
	std	151.74	136.17	127.22	121.12	113.41	106.77		std	24.93	27.63	31.65	30.72	30.67	30.07
	min	1.32	2.53	3.45	6.54	9.50	13.61		min	2.26	5.54	8.82	14.70	18.98	25.28
	max	889.04	767.96	694.63	638.32	577.47	545.77		max	121.00	120.20	140.75	135.08	139.84	145.23
8.Chubb Corp	mean	23.69	30.28	36.71	50.65	56.86	62.62	20.SunTrust Banks	mean	86.20	97.15	109.10	125.96	128.93	134.45
	std	19.69	21.67	23.59	28.99	30.17	29.93		std	84.79	81.37	91.64	95.07	92.10	92.37
	min	2.12	3.50	4.98	8.96	12.28	16.96		min	3.85	4.23	5.58	9.07	13.30	17.02
	max	107.79	107.77	113.21	143.52	146.26	138.17		max	364.53	304.40	336.45	360.45	358.45	361.45
9.Countrywide Finl Corp	mean	251.04	209.40	185.87	163.51	156.22	151.07	21.U S Bancorp	mean	40.09	52.63	56.88	72.03	79.29	88.22
	std	524.17	380.17	321.25	226.69	204.97	179.47		std	37.65	43.59	46.55	52.23	53.89	61.37
	min	5.26	8.95	12.51	20.92	25.97	33.47		min	1.44	1.89	4.46	7.10	10.98	15.26
	max	4777.24	3209.60	2839.90	1609.43	1656.96	1468.73		max	187.30	203.00	230.14	249.00	275.13	345.07
10.Capital One Finl Corp	mean	126.01	127.96	128.79	136.35	133.26	132.40	22.Wachovia Corp	mean	48.90	53.71	58.13	64.24	65.93	69.18
	std	169.31	151.19	132.66	108.45	91.33	77.32		std	102.46	103.39	101.60	98.73	94.11	89.02
	min	4.40	7.46	11.75	21.04	26.28	34.67		min	2.41	4.15	5.31	9.08	12.30	16.74
	max	718.12	643.70	589.06	518.66	472.91	434.57		max	778.30	752.51	736.09	719.23	706.56	676.50
11.Freddie Mac	mean	8.71	11.80	14.17	19.36	21.83	25.33	23.Washington Mutual	mean	307.12	274.15	246.99	202.26	185.16	177.74
	std	10.91	14.14	15.46	18.13	17.12	16.43		std	882.10	848.38	740.15	496.87	413.61	370.18
	min	1.27	2.06	2.45	4.04	5.85	8.06		min	4.44	7.32	11.32	19.54	23.62	30.03
	max	47.76	65.41	67.53	82.66	82.90	83.69		max	7434.28	9463.68	8489.77	5207.79	4199.61	3446.94
12.Fannie Mae	mean	9.35	12.05	15.01	20.78	23.57	26.92	24.Wells Fargo & Co	mean	48.11	53.89	60.43	74.63	77.51	81.24
	std	11.99	13.39	15.23	17.48	16.54	15.94		std	59.21	54.68	53.27	54.66	51.71	49.57
	min	1.47	2.53	3.57	5.66	7.99	10.22		min	1.66	2.46	3.52	5.90	8.57	11.60
	max	57.58	62.42	68.30	80.51	80.96	82.09		max	416.28	350.98	326.41	305.84	272.13	246.27

This table shows descriptive statistics of C-storeDS premiums for each firm across the portfolio for maturities of 1, 2, 3, 5, 7, and 10 years. The weekly term structure of CDS data is provided by MarkIt on May 20, 2013 and covers the time period from January 5, 2005 through December 26, 2012 for a total of 417 observations. Note that the number of observations is smaller for firms that defaulted, or firms that have been acquired during the sample period. The reported summary statistics are the average CDS spread (mean), the standard deviation (std), and minimum/maximum values (min/max).

mated in an univariate manner, and ii) correlation coefficients for each possible pair of firms across the portfolio are obtained by a bivariate approach. In both univariate and bivariate steps, model parameters are recovered by applying filtering techniques and quasi-likelihood maximization.

First, the model parameters are obtained on a firm-by-firm basis by applying M parallel unscented Kalman filters (UKF) described in Appendix B. For a detailed version of the regime-switching hybrid default risk model, we refer to Bégin et al. (2014). The set of Markov-switching parameters to be estimated for each firm in the first stage is

$$\phi_1 = (\mu^{\mathbb{P}}, \mu^{\mathbb{Q}}, \sigma_1, \sigma_2, p_{11}^{\mathbb{P}}, p_{22}^{\mathbb{P}}, p_{11}^{\mathbb{Q}}, p_{22}^{\mathbb{Q}}, \alpha, \beta, \theta, \kappa, \delta^{(1)}, \delta^{(2)}, \delta^{(3)}, \delta^{(5)}, \delta^{(7)}, \delta^{(10)}),$$

where $\delta^{(1)}$, $\delta^{(2)}$, $\delta^{(3)}$, $\delta^{(5)}$, $\delta^{(7)}$, and $\delta^{(10)}$ are standard errors of the noise terms for tenors of 1, 2, 3, 5, 7 and 10 years respectively. To obtain $\hat{\phi}_1$, the quasi-maximum likelihood estimation (QMLE) approach is used. Table 2 presents the model parameters for each financial institution under study. First, empirical results show strong persistence for both low and high volatility regimes. Indeed, transition probabilities $p_{11}^{\mathbb{P}}$ and $p_{22}^{\mathbb{P}}$ are greater than 92% for all firms, with the majority exceeding 98%. In particular, Bear Stearns, Merrill Lynch and Washington Mutual transition probabilities $p_{22}^{\mathbb{P}}$ reach 100%, suggesting permanent regime changes during the crisis. This is because CDS data are truncated at the effective acquisition date, which corresponds to the high volatility regime. Recall that these three firms were acquired during the late 2008 by JPMorgan Chase and Bank of America. Second, the positive drift of the leverage ratio indicates the tendency of the firm to contract debt or could be associated to a deterioration of the firms' assets. In both cases, it means that the riskiness increases over time. In the sample under study, both Merrill Lynch and Countrywide present the highest drifts of 12.9% and 9.5% respectively. Recall that Bank of America acquired both failing companies in 2008 preventing their potential bankruptcy.

Univariate step procedure also allows to estimate the firm specific constants α, β and θ that define the intensity process H_t of Eq. (4). Parameters α and θ gauge the sensitivity of the firm's survival toward its leverage ratio. The convexity of the default intensity is guided by α, while the critical leverage threshold is defined by θ. The parameter β captures a part of the default drivers, and ensures that H_t is a positive function when $\beta > 0$. With all other variables being the same, the larger the β, the greater the intensity and default probability. As shown in Table 2, all firms have positive values for each constant. The estimated α has minimum and maximum values of 4.6 and 16.6 respectively, implying that the intensity process is strongly convex with the leverage ratio. The critical leverage value θ lies between 1.1 and 1.9, which is realistic since a part of the default risk is captured by the parameter β and the standardized leverage ratio X/θ affects the default intensity in a non-linear fashion. Finally, the estimated β has a minimum value of around 0.06% and is generally smaller than 1%. However, four firms present larger values, i.e. Fannie Mae, Freddie Mac, Goldman Sachs, and Schwab Charles, which reflect the financial difficulties encountered by those firms during and/or after the global crisis. Fannie Mae and Freddie Mac have been placed in government conservatorship in September 2008 in order to ensure their financial soundness. Moreover, the federal takeover of both companies resulted in a bankruptcy status in contracts traded in the CDS market. Financial trouble also appeared in 2010 for Goldman Sachs after fraud charges related to the U.S. subprime mortgage crisis. The Securities and Exchange Commission (SEC),

Table 2: First step parameter estimates.

Firm	$\hat{x}_{0\|0}$	$\mu^{\mathbb{P}}(\%)$	$\mu^{\mathbb{Q}}(\%)$	σ_1	σ_2	$p_{11}^{\mathbb{Q}}(\%)$	$p_{11}^{\mathbb{P}}(\%)$	$p_{22}^{\mathbb{Q}}(\%)$	$p_{22}^{\mathbb{P}}(\%)$
1. AIG	0.89	0.00	0.17	0.0794	0.3373	99.71	99.06	96.80	96.47
2. Allstate Corp	0.85	0.53	-0.39	0.0738	0.3381	99.35	99.56	93.61	99.40
3. American Express	0.92	0.20	-0.90	0.0736	0.3050	99.30	99.62	95.70	99.38
4. Bank of America	0.91	2.77	-0.18	0.0613	0.2712	99.48	98.45	94.75	99.00
5. BB&T Corp	0.89	2.74	-1.20	0.0891	0.3502	98.96	98.75	98.53	96.42
6. Bear Stearns	0.96	2.06	-0.27	0.0918	0.4962	99.83	99.23	94.31	100.00
7. Citigroup	0.93	3.30	-0.15	0.0819	0.3395	99.34	98.74	94.38	98.99
8. Chubb Corp	0.77	-1.66	-0.55	0.0914	0.3911	99.37	99.76	93.55	99.55
9. Countrywide Finl Corp	0.92	9.46	-0.74	0.1278	0.4524	99.89	98.48	95.22	98.25
10. Capital One Finl Corp	0.84	0.01	-0.86	0.1131	0.3341	99.79	99.26	94.40	99.25
11. Freddie Mac	0.96	-7.65	-0.64	0.0483	0.4376	98.71	98.05	97.73	99.97
12. Fannie Mae	0.96	-4.85	-2.12	0.0552	0.3720	98.08	96.23	98.00	99.61
13. Goldman Sachs	0.95	0.38	-1.57	0.0973	0.3105	95.87	94.56	95.19	96.27
14. Hartford Finl Serv. Gp.	0.95	-0.06	-0.11	0.0654	0.2894	99.51	99.19	96.65	98.13
15. JPMorgan Chase & Co	0.91	-5.70	-0.68	0.0898	0.3636	99.48	99.20	94.62	98.99
16. Lehman Brothers	0.96	2.15	-0.42	0.0759	0.3538	99.82	99.83	93.72	99.98
17. Merrill Lynch & Co	0.95	12.94	-0.73	0.0966	0.3879	99.78	99.03	93.56	100.00
18. Morgan Stanley	0.96	-7.11	0.09	0.0792	0.4776	99.38	98.64	95.88	97.67
19. Schwab Charles Corp	0.91	0.00	0.01	0.0919	0.3011	95.49	95.84	96.45	94.42
20. SunTrust Banks	0.90	0.10	-1.03	0.0425	0.3213	95.99	98.67	89.69	92.66
21. U S Bancorp	0.90	-0.07	-0.53	0.0513	0.3567	98.60	98.46	94.60	92.82
22. Wachovia Corp	0.90	-0.79	-0.58	0.0689	0.3541	99.16	97.29	95.84	99.54
23. Washington Mutual	0.93	5.96	-1.03	0.1082	0.3872	99.91	99.31	95.07	100.00
24. Wells Fargo & Co	0.91	-2.18	-1.12	0.0891	0.3078	99.56	98.47	91.88	98.90

Firm	α	β (%)	θ	κ	$\delta^{(1)}$	$\delta^{(2)}$	$\delta^{(3)}$	$\delta^{(5)}$	$\delta^{(7)}$	$\delta^{(10)}$
1.	14.8211	0.2063	1.5066	0.4444	0.3023	0.1564	0.0933	0.0401	0.0185	0.0536
2.	13.9489	0.2557	1.4033	0.4463	0.2449	0.1656	0.1133	0.0401	0.0009	0.0383
3.	11.8259	0.1463	1.4938	0.4987	0.2681	0.1593	0.1060	0.0316	0.0319	0.0579
4.	15.5117	0.0971	1.4653	0.4942	0.2525	0.1632	0.1026	0.0544	0.0025	0.0422
5.	8.1043	0.5949	1.7016	0.6152	0.6259	0.3766	0.2299	0.1006	0.1390	0.1621
6.	10.1853	0.0754	1.2463	0.5580	0.1683	0.1137	0.0649	0.0454	0.0301	0.0493
7.	11.6777	0.2227	1.4868	0.4776	0.2201	0.1344	0.0770	0.0332	0.0257	0.0462
8.	11.7652	0.2195	1.3253	0.5405	0.2285	0.1432	0.0960	0.0420	0.0145	0.0314
9.	12.9686	0.7768	1.3704	0.3409	0.2029	0.1426	0.0877	0.0381	0.0451	0.0730
10.	11.2272	0.2849	1.5056	0.3226	0.1996	0.1213	0.0836	0.0011	0.0440	0.0706
11.	10.7463	1.6248	1.6364	0.7000	0.4027	0.1700	0.0927	0.0678	0.0236	0.0598
12.	14.4007	2.1155	1.8595	0.5500	0.4084	0.1553	0.0950	0.0611	0.0289	0.0562
13.	4.6301	3.4856	1.4367	0.6028	0.3320	0.1162	0.0366	0.0919	0.1195	0.1531
14.	9.7247	0.0828	1.6456	0.5555	0.2758	0.1505	0.0884	0.0363	0.0101	0.0348
15.	12.1609	0.1174	1.5010	0.5422	0.2677	0.1873	0.1165	0.0495	0.0170	0.0507
16.	14.0191	0.1426	1.2084	0.3944	0.1818	0.0988	0.0564	0.0311	0.0405	0.0604
17.	12.3407	0.2098	1.4649	0.3584	0.1375	0.0687	0.0354	0.0317	0.0532	0.0787
18.	9.9342	0.4239	1.1129	0.6639	0.3079	0.1673	0.0936	0.0273	0.0441	0.0775
19.	6.4679	4.1490	1.5000	0.6837	0.3313	0.1997	0.1121	0.0717	0.0514	0.0804
20.	6.3683	0.2716	1.4660	0.6285	0.2688	0.2255	0.1083	0.0656	0.1090	0.0851
21.	5.6271	0.2178	1.3201	0.6638	0.2427	0.2150	0.1290	0.0955	0.0900	0.1221
22.	7.2337	0.1156	1.2386	0.6840	0.1844	0.1098	0.0558	0.0393	0.0341	0.0635
23.	11.9365	0.1485	1.3690	0.4092	0.1493	0.1130	0.0555	0.0373	0.0497	0.0727
24.	16.6132	0.0577	1.7067	0.5284	0.2517	0.1612	0.1060	0.0304	0.0375	0.0620

The table shows firms' parameters estimates obtained from CDS data from January 2005 to December 2012 by applying filtering techniques and quasi-likelihood maximization. More specifically, the following parameters are reported: the initial debt $\hat{x}_{0|0}$ calculated from Compustat accounting information, the drifts μ under both measures $\mathbb{P}$ and $\mathbb{Q}$, the diffusions σ for each regime, p_{11} and p_{22} which are the probabilities to stay in low and high volatility regime respectively, the constants α, β and θ that define the intensity process, the liquidation and legal fees parameter κ, and finally the standard errors of the trading noise for tenors of 1, 2, 3, 5, 7 and 10 years.

as well as the UK's Financial Services Authority, fined Goldman Sachs with the biggest fine ever imposed. Finally, Schwab Charles also encountered financial difficulties in early 2008 when one of its fund lost hundreds of millions of dollars due to the default of many mortgage bonds. Because the fund was marketed as being safe, when in fact the major part of the fund's assets were invested in mortgage-backed securities, Schwab Charles settled SEC allegations that it misled investors about the fund's risks.

Table 2 also shows the parameter κ, which is related to liquidation and legal fees. The estimated value across firms is around 30%-70% and represents a fraction of the market assets' value at default. Therefore, the value of κ tends to increase with riskiness.

Finally, standard errors of the trading noise are relatively low for tenors longer than two years with an average value lying between 4.4% and 9.3%. However, short tenors have higher variations that may be related to lower trading frequency of 1- and 2-years CDS contracts. One can also mention the very high volatility period of time during which the analysis is performed, implying higher standard errors than stable period would generate. Furthermore, since the model estimates credit derivative prices for seven maturities simultaneously, it is possible that the fitting step has an impact on the quality of the fit of each derivative.

The estimation of univariate parameters presented above is based on the Markov-switching hybrid credit risk model described in Bégin et al. (2014). In what follows, results obtained from the bivariate extension of the univariate Markov-switching framework are discussed. With the objective to test the bivariate approach used in this study, a validation procedure (not reported here) has been performed. This procedure consists in the simulation of correlated log-leverage ratios to obtain CDS time series, which are then used as inputs to the model in order to validate that results are unbiased. As a starting point to the bivariate step, suppose that we have K firms across the portfolio and correlations are recovered from log-leverage ratios of all possible pair of firms (i, j), with $1 \leq i, j \leq K$. Thus, the number of estimated values is $K(K-1)/2$ for each regime state leading to $2K(K-1)$ total values. The set of parameters for the bivariate estimation stage is

$$\phi_2 = (\rho_{1,1}^{(i,j)}, \rho_{1,2}^{(i,j)}, \rho_{2,1}^{(i,j)}, \rho_{2,2}^{(i,j)}).$$

Since the leverage ratio time series are inferred from the set of CDS prices by the UKF method, recovering correlation from smoothed leverage data would result in under-estimated coefficients. Therefore, dependence among firms must be captured endogenously or prior to the filtering process. Details about UKF equations are provided in Appendix B, while details on endogenous correlation coefficients are presented in Appendix C.

Table 3 presents correlation estimates between log-leverage ratios for each possible pair of firms across the portfolio depending on the regime state. Figure 2 of Appendix D also depicts the filtered log-leverages and regimes for each firm, which allows to observe the co-moving trend between time series.

First, panels (a) and (b) of Table 3 report the correlation structure amongst financial institutions for $s_t^{i,j} = \{(1,1), (2,2)\}$ respectively. The results highlight positive pairwise correlations when both firms are in the same regime, with some exceptions for Schwab Charles (SCH) and Washington Mu-

Table 3: Second step parameter estimates.

	AIG	ALL	AXP	BACORP	BBT	BSC	C	CB	CCR	COF	FHLMC	FNMA	GS	HIG	JPM	LEH	MER	MWD	SCH	STI	USB	WB	WM	WFC
AIG	1.00																							
ALL	0.60	1.00																						
AXP	0.57	0.65	1.00																					
BACORP	0.64	0.45	0.71	1.00																				
BBT	0.12	0.37	0.21	0.09	1.00																			
BSC	0.45	0.45	0.60	0.69	0.43	1.00																		
C	0.60	0.62	0.76	0.83	0.68	0.85	1.00																	
CB	0.52	0.84	0.67	0.46	0.58	0.67	0.70	1.00																
CCR	0.40	0.48	0.76	0.59	0.28	0.80	0.84	0.69	1.00															
COF	0.47	0.56	0.76	0.61	0.29	0.71	0.66	0.68	0.76	1.00														
FHLMC	0.23	0.25	0.38	0.26	0.06	0.32	0.29	0.32	0.28	0.29	1.00													
FNMA	0.28	0.14	0.31	0.54	0.05	0.29	0.25	0.28	0.23	0.34	0.63	1.00												
GS	0.74	0.55	0.75	0.88	0.51	0.94	0.86	0.56	0.78	0.51	0.30	0.41	1.00											
HIG	0.66	0.64	0.67	0.62	0.14	0.60	0.71	0.60	0.63	0.47	0.30	0.31	0.60	1.00										
JPM	0.49	0.66	0.68	0.70	0.64	0.85	0.80	0.72	0.71	0.69	0.34	0.36	0.69	0.63	1.00									
LEH	0.51	0.56	0.65	0.65	0.62	0.97	0.88	0.75	0.79	0.71	0.33	0.24	0.89	0.66	0.90	1.00								
MER	0.52	0.59	0.64	0.63	0.63	0.90	0.87	0.76	0.80	0.73	0.29	0.23	0.90	0.69	0.84	0.93	1.00							
MWD	0.53	0.39	0.41	0.70	0.25	0.75	0.60	0.40	0.54	0.41	0.33	0.23	0.79	0.47	0.69	0.76	0.69	1.00						
SCH	0.19	0.17	0.27	0.15	0.09	0.69	0.34	0.18	0.85	0.71	-0.25	-0.03	0.43	0.15	0.34	0.67	0.81	0.04	1.00					
STI	0.28	0.30	0.24	0.30	0.17	0.58	0.60	0.33	0.73	0.45	0.11	0.35	0.45	0.30	0.41	0.65	0.57	0.36	-0.10	1.00				
USB	0.48	0.50	0.70	0.52	0.41	0.70	0.67	0.63	0.73	0.60	0.05	0.27	0.42	0.47	0.53	0.71	0.72	0.39	0.06	0.34	1.00			
WB	0.41	0.41	0.56	0.82	0.53	0.77	0.84	0.47	0.68	0.56	0.22	0.24	0.90	0.54	0.79	0.71	0.69	0.55	0.81	0.51	0.51	1.00		
WM	0.48	0.60	0.74	0.64	0.50	0.77	0.87	0.79	0.92	0.78	0.28	0.34	0.99	0.70	0.79	0.83	0.86	0.49	0.23	0.72	0.69	0.68	1.00	
WFC	0.50	0.62	0.70	0.80	0.81	0.79	0.89	0.75	0.84	0.71	0.22	0.19	0.67	0.58	0.87	0.85	0.74	0.40	0.55	0.53	0.54	0.82	0.88	1.00

(a) Correlation estimates when all firms are in regime 1 (ρ_{11})

	AIG	ALL	AXP	BACORP	BBT	BSC	C	CB	CCR	COF	FHLMC	FNMA	GS	HIG	JPM	LEH	MER	MWD	SCH	STI	USB	WB	WM	WFC
AIG	1.00																							
ALL	0.56	1.00																						
AXP	0.67	0.88	1.00																					
BACORP	0.58	0.81	0.86	1.00																				
BBT	0.56	0.66	0.62	0.63	1.00																			
BSC	0.72	0.74	0.79	0.84	0.58	1.00																		
C	0.58	0.78	0.82	0.92	0.70	0.82	1.00																	
CB	0.61	0.95	0.88	0.80	0.70	0.79	0.79	1.00																
CCR	0.35	0.17	0.26	0.49	0.24	0.63	0.57	0.26	1.00															
COF	0.57	0.89	0.93	0.89	0.66	0.82	0.86	0.90	0.52	1.00														
FHLMC	0.56	0.53	0.41	0.56	0.28	0.61	0.64	0.52	0.28	0.62	1.00													
FNMA	0.62	0.55	0.43	0.59	0.38	0.67	0.65	0.53	0.27	0.62	1.00	1.00												
GS	0.86	0.47	0.66	0.82	0.50	0.89	0.84	0.49	0.37	0.51	0.84	0.79	1.00											
HIG	0.69	0.83	0.77	0.68	0.56	0.64	0.62	0.78	0.22	0.70	0.48	0.46	0.56	1.00										
JPM	0.65	0.85	0.86	0.96	0.71	0.88	0.93	0.85	0.56	0.92	0.59	0.60	0.58	0.70	1.00									
LEH	0.83	0.84	0.83	0.89	0.75	0.88	0.91	0.85	0.56	0.86	0.49	0.50	0.92	0.73	0.93	1.00								
MER	0.61	0.70	0.77	0.87	0.71	0.77	0.81	0.72	0.51	0.89	0.49	0.49	0.54	0.58	0.90	0.94	1.00							
MWD	0.74	0.76	0.84	0.87	0.70	0.88	0.90	0.78	0.45	0.95	0.60	0.62	0.81	0.66	0.92	0.93	0.80	1.00						
SCH	0.89	0.53	0.63	0.46	0.19	0.15	0.67	0.42	0.13	0.44	0.62	0.71	0.59	0.13	0.76	0.18	0.08	0.45	1.00					
STI	0.65	0.57	0.77	0.72	0.53	0.81	0.77	0.78	0.28	0.72	0.31	0.33	0.64	0.57	0.73	0.79	0.80	0.80	0.19	1.00				
USB	0.48	0.58	0.77	0.75	0.88	0.62	0.69	0.67	0.64	0.74	0.26	0.30	0.50	0.58	0.75	0.61	0.54	0.79	0.59	0.70	1.00			
WB	0.69	0.27	0.47	0.81	0.75	0.82	0.72	0.39	0.49	0.55	0.50	0.49	0.80	0.32	0.82	0.87	0.71	0.73	0.41	0.48	0.64	1.00		
WM	0.34	0.70	0.65	0.64	0.62	0.81	0.65	0.73	0.62	0.84	0.54	0.50	0.26	0.52	0.72	0.87	0.79	0.49	-0.12	-0.07	0.47	0.66	1.00	
WFC	0.58	0.86	0.88	0.96	0.73	0.84	0.93	0.86	0.57	0.93	0.63	0.65	0.76	0.67	0.98	0.92	0.91	0.91	0.69	0.54	0.73	0.77	0.76	1.00

(b) Correlation estimates when all firms are in regime 2 (ρ_{22})

Table 3: Second step parameter estimates (cont'd).

	AIG	ALL	AXP	BACORP	BBT	BSC	C	CB	CCR	COF	FHLMC	FNMA	GS	HIG	JPM	LEH	MER	MWD	SCH	STI	USB	WB	WM	WFC
AIG	1.00	0.17	0.28	0.20	0.15		0.21	0.07	-0.11	0.16	0.73	0.71	0.33	0.12	0.11		-0.06	0.22	0.32	0.11	0.13	-0.37		0.11
ALL	-0.81	1.00		0.45	0.06		0.57	-0.95			0.52	0.52	0.70	-0.69	0.19			0.51	-0.38	0.86	0.11	0.11		0.19
AXP	-0.93	0.14	1.00	0.67	0.19		0.20	0.09		-0.81	0.53	0.48	0.29	-0.89	0.28			0.25	0.33	0.36	0.12	0.11		0.07
BACORP		-0.24		1.00	0.20		-0.02	-0.11			0.49	0.49	0.25						-0.32		-0.15			0.17
BBT	0.49	0.09	0.18	0.12	1.00		0.23	0.07	0.15	0.16	0.37	0.30	0.27	0.36	0.12	0.26	0.22	0.27	0.79	0.39	0.08	0.34	0.45	0.15
BSC	0.33	0.21	0.25	0.27	0.56	1.00	0.22	0.24			0.59	0.56	0.57	0.26	0.28				0.56	0.62	0.58	0.29		0.25
C				0.35	0.35		1.00				0.76	0.74	0.00					-0.37	0.80		0.29			-0.01
CB	-0.78			0.68	0.06		0.77	1.00			0.75	0.74	0.11	-0.90	0.26			0.64	0.79	0.99	0.10	0.14		0.36
CCR	0.28	-0.03	-0.59	0.30	0.31		0.08	-0.04	1.00	0.95	0.50	0.48	0.84	0.24	0.34			-0.86	0.47	0.28	0.36	0.25	0.93	0.21
COF	0.28	0.06	0.25	0.63	0.33	0.32	0.33	0.07	0.68	1.00	0.49	0.53	0.82	0.13	0.28	0.98	-0.24	0.39	0.45	0.59	0.45	0.26		0.19
FHLMC											1.00	0.06	0.50						0.07					
FNMA												1.00	0.97											
GS		0.54	0.21	0.79	-0.07		0.74	0.40		0.39			1.00		0.71			0.66	0.83	-0.82	0.56			0.60
HIG		0.22	0.37	0.25	0.02		0.27	0.19		0.27	0.68	0.70	0.12	1.00	0.19			0.25	0.53	0.09	0.12			0.14
JPM		-0.39		0.36	0.04		0.69	-0.81			0.65	0.67	0.19		1.00			-0.77	-0.14		-0.45			-0.45
LEH	0.22	0.10	0.07	0.38	0.56	-0.55	0.30	0.08	0.50		0.65	0.65	0.90	0.16	0.28	1.00			0.58	0.42	0.43	0.41		0.39
MER	0.30	0.12	0.16	0.61	0.65	0.25	0.22	0.13	0.57		0.64	0.62	0.75	0.20	0.50		1.00		0.58	0.60	0.58	0.44		0.12
MWD	0.03	-0.01		-0.14	0.02		0.08	-0.01			0.31	0.32	-0.15	-0.39	0.02			1.00	0.00	0.72	0.01	-0.22		0.00
SCH	0.58	-0.80	-0.34	0.03	0.80	0.64	-0.65	0.84	-0.07	0.57	-0.21	-0.22	0.42	0.80	0.65	0.64	0.72	0.90	1.00	0.98	-0.66	-0.01	0.52	-0.76
STI	0.77	0.86	0.58	0.62	0.14	0.02	0.77	0.88	0.78	0.82	0.69	0.70	0.45	0.61	0.78	0.84	0.72	0.34	0.50	1.00	0.80	0.81	0.98	0.22
USB	0.53	0.19	0.36	0.24	0.40	-0.08	0.33	0.21	-0.08	0.40	0.25	0.22	0.54	0.46	0.28	0.32	0.14	0.48	0.44	0.31	1.00	0.33	0.45	0.17
WB							0.22				0.65	0.58	0.56					0.75	-0.51			1.00		0.36
WM	0.21	0.06	0.17	0.31	0.25	0.25	0.22	0.04	0.55		0.50	0.50	0.77	0.09	0.33	0.80	-0.16	0.23	0.72	0.48	0.48	0.27	1.00	0.29
WFC				0.58	0.20		0.00				0.64	0.72	0.59					0.53	0.89		0.38			1.00

(c) Correlation estimates when pairwise firms are in different regimes (ρ_{12}: lower left matrix, ρ_{21}: upper right matrix)

This table shows correlation estimates among the 24 firms for each regime state $s_t^{i,j} = \{(1,1),(1,2),(2,1),(2,2)\}$. Based on the UKF and quasi-maximum likelihood estimation, the results are obtained endogenously from log-leverages' co-movements inferred from the term structure of CDS prices from January 2005 to December 2012. The first two matrices present 24x23/2 = 276 correlation estimates for $s_t^{i,j} = \{(1,1),(2,2)\}$ respectively, while the last matrix combining correlation estimates for $s_t^{i,j} = \{(1,2),(2,1)\}$ shows missing values due to unavailable observations. After removing all missing values, 375 correlation estimates form the matrix (without diagonal values).

Table 4: Descriptive statistics of correlation estimates.

	ρ_{11}				ρ_{12}				ρ_{21}				ρ_{22}				$\rho_{22}-\rho_{11}$			
Firm	mean	std	min	max	mean	std	min	max	mean	std	min	max	mean	std	min	max	mean	std	min	max
AIG	0.46	0.15	0.12	0.74	0.18	0.24	-0.37	0.73	0.11	0.55	-0.93	0.77	0.63	0.13	0.34	0.89	0.16	0.19	-0.14	0.70
ALL	0.50	0.17	0.14	0.84	0.12	0.55	-0.95	0.86	0.08	0.35	-0.80	0.86	0.67	0.20	0.17	0.95	0.18	0.18	-0.32	0.41
AXP	0.58	0.18	0.21	0.76	0.08	0.47	-0.93	0.67	0.15	0.30	-0.59	0.58	0.72	0.18	0.26	0.93	0.13	0.21	-0.50	0.53
BACORP	0.58	0.21	0.09	0.88	0.08	0.29	-0.32	0.49	0.38	0.24	-0.14	0.79	0.76	0.15	0.46	0.96	0.18	0.16	-0.10	0.54
BBT	0.37	0.23	0.05	0.81	0.27	0.17	0.07	0.79	0.26	0.24	-0.07	0.80	0.59	0.17	0.19	0.88	0.22	0.18	-0.08	0.54
BSC	0.68	0.19	0.29	0.97	0.39	0.16	0.21	0.62	0.12	0.37	-0.55	0.64	0.74	0.16	0.15	0.89	0.06	0.20	-0.55	0.38
C	0.70	0.19	0.25	0.89	0.32	0.40	-0.37	0.80	0.28	0.32	-0.65	0.77	0.76	0.12	0.57	0.93	0.07	0.17	-0.27	0.41
CB	0.58	0.18	0.18	0.84	0.31	0.56	-0.90	0.99	0.08	0.44	-0.95	0.88	0.70	0.18	0.26	0.95	0.12	0.18	-0.43	0.45
CCR	0.66	0.20	0.23	0.92	0.27	0.44	-0.86	0.95	0.33	0.36	-0.11	0.78	0.41	0.16	0.13	0.64	-0.25	0.18	-0.71	0.03
COF	0.58	0.15	0.29	0.78	0.38	0.27	-0.24	0.98	0.32	0.51	-0.81	0.95	0.75	0.16	0.44	0.95	0.17	0.18	-0.27	0.53
FHLMC	0.25	0.16	-0.25	0.63	0.21	0.25	0.06	0.50	0.53	0.22	-0.21	0.76	0.54	0.17	0.26	1.00	0.28	0.17	0.01	0.87
FNMA	0.28	0.14	-0.03	0.63	0.97	-	0.97	0.97	0.50	0.24	-0.22	0.74	0.55	0.16	0.27	1.00	0.27	0.17	-0.02	0.74
GS	0.67	0.20	0.30	0.99	0.43	0.45	-0.82	0.83	0.47	0.30	-0.15	0.97	0.65	0.18	0.26	0.92	-0.02	0.25	-0.72	0.54
HIG	0.53	0.17	0.14	0.71	0.28	0.20	0.02	0.70	0.04	0.52	-0.90	0.80	0.58	0.17	0.13	0.83	0.06	0.18	-0.41	0.42
JPM	0.66	0.17	0.34	0.90	-0.03	0.55	-0.81	0.69	0.33	0.21	0.02	0.78	0.79	0.13	0.56	0.98	0.13	0.14	-0.14	0.42
LEH	0.71	0.18	0.24	0.97	0.34	0.31	-0.55	0.90	0.64	0.29	0.26	0.98	0.78	0.19	0.18	0.94	0.07	0.18	-0.49	0.32
MER	0.70	0.18	0.23	0.93	0.42	0.22	0.12	0.75	0.19	0.39	-0.24	0.72	0.69	0.20	0.08	0.94	0.00	0.23	-0.74	0.26
MWD	0.49	0.19	0.04	0.79	0.04	0.25	-0.39	0.72	0.26	0.49	-0.86	0.90	0.75	0.15	0.45	0.95	0.27	0.17	-0.10	0.53
SCH	0.32	0.31	-0.25	0.85	0.23	0.59	-0.80	0.98	0.38	0.42	-0.51	0.89	0.42	0.26	-0.12	0.89	0.10	0.46	-0.74	0.87
STI	0.40	0.20	-0.10	0.73	0.64	0.25	0.02	0.98	0.44	0.43	-0.82	0.99	0.58	0.23	-0.07	0.81	0.19	0.30	-0.79	0.53
USB	0.51	0.19	0.05	0.73	0.30	0.17	-0.08	0.54	0.22	0.36	-0.66	0.80	0.62	0.15	0.26	0.88	0.12	0.19	-0.22	0.52
WB	0.61	0.19	0.22	0.90	0.37	0.43	-0.51	0.75	0.21	0.28	-0.37	0.81	0.62	0.18	0.27	0.87	0.01	0.17	-0.39	0.28
WM	0.68	0.21	0.23	0.99	0.33	0.24	-0.16	0.80	0.66	0.26	0.45	0.98	0.57	0.26	-0.12	0.87	-0.11	0.25	-0.79	0.26
WFC	0.66	0.20	0.19	0.89	0.50	0.27	0.00	0.89	0.13	0.28	-0.76	0.60	0.79	0.13	0.54	0.98	0.12	0.18	-0.27	0.51

This table reports the mean, standard deviation, minimum and maximum of all pairwise correlations depending on the firms' regimes, i.e. both firms are in the stable period (ρ_{11}), firms are in different regimes (ρ_{12} and ρ_{21}), or both firms are in the highly volatile period (ρ_{22}). Moreover, this table shows descriptive statistics of the increase in correlation between regime state $s_t^{i,j} = (1,1)$ and $s_t^{i,j} = (2,2)$. Note that when FNMA is in the first regime and all other firms are in the second one, data is available only for GS firm leading to one pairwise correlation of 0.97.

tual (WM). Results also display a higher degree of interdependence when regime switches from stable to volatile for both entities. In particular, Fannie Mae (FNMA) and Freddie Mac (FHLMC) are significantly more correlated with the others during the crisis, reflecting the linkages between subprime mortgages and the financial sector downturn. For example, the increase in correlation is as large as 74% and 87% between FNMA-SCH and FHLMC-SCH respectively. Strong interdependence changes also arise, among others, between AIG-SCH, AXP-STI, BACORP-BBT, MWD-COF, USB-SCH and WFC-MWD, for which coefficients increase by more than 50%. For further analysis, Table 4 provides descriptive statistics of pairwise correlation estimates for each state, as well as descriptive statistics of the average increase in correlation between $s_t^{i,j} = (1,1)$ and $s_t^{i,j} = (2,2)$. As pointed out earlier, results suggest a strong potential channel for transmitting shocks from FNMA and FHLMC to other firms. Table 4 strengthens this finding by showing that mean correlations of both government-sponsored entities during the crisis are approximately double what they are in the stable regime. Morgan Stanley (MWD) also exhibits a significant increase in correlation during the turmoil displaying a rise of 27%, followed by BBT (22%), STI (19%), BACORP and ALL (18%), COF (17%), AIG (16%), AXP and JPM (13%), CB, USB and WFC (12%), as well as SCH (10%). Although the degree of interdependence between institutions is generally higher during the highly volatile period, three firms within the entire portfolio provide different results. As reported in Table 4, Countrywide Financial (CCR) presents an estimated mean correlation with other firms of 66% prior to the crisis, while it decreases to 41% afterwards. This is explained by CCR's leverage tendency to decrease starting from the beginning of 2008, while other institutions seem to react in the opposite direction, as shown in Figure 2 of Appendix D. The decrease in leverage is due to the announcement of Bank of America (BACORP) to purchase CCR in January 2008. Then, Table 4 reveals that WM is also less correlated with other entities in the sample during the highly volatile period. The mean pairwise correlation reaches 68% when the regime state is $s_t^{i,j} = (1,1)$ in comparison with a coefficient of 57% for $s_t^{i,j} = (2,2)$. In this case, WM's leverage tends to rise during the crisis much faster than other firms' leverages, as illustrated in Figure 2. Finally, descriptive statistics also suggest that Goldman Sachs is in average 2% less correlated with the portfolio during the volatile period, but this difference is not as significant as for CCR and WM.

Second, Table 3 (c) combines correlation estimates for $s_t^{i,j} = \{(1,2),(2,1)\}$ in the lower left matrix and upper right matrix respectively. In some specific cases, CDS price dynamics as well as univariate parameter estimates are such that the joint probability of pairwise firms to be in different regimes is too low. Consequently, the estimation procedure is unable to adequately recover the coefficients leading to missing values in the correlation matrix. As an example, one can see in Figure 2 that during the period of time FNMA and FHLMC are in the first regime, few institutions belong to the second one. Conversely, when both government-sponsored entities are in the highly volatile period, the model is able to estimate correlations for almost all pairwise combinations. Interestingly, coefficients are quite similar between FNMA/FHLMC and the rest of the portfolio reflecting the similar role both firms had within the U.S. financial sector. Table 3 (c) also reports larger correlation spreads than Table 3 (a) and Table 3 (b). As expected, the pattern of co-movements across log-leverage ratios of firms in different regimes is associated with wider spreads in the correlation structure. This is also shown in Table 4 by larger standard deviations for $\rho_{1,2}$ and $\rho_{2,1}$ in comparison to $\rho_{1,1}$ and $\rho_{2,2}$. In addition, the mean value is generally lower reflecting less inter-connectedness among financial institutions in different regimes.

In summary, empirical results clearly show that firms are more correlated during the high volatility regime suggesting the existence of contagion within the U.S. financial sector during the last crisis. These results present major implications for risk management practices since financial contagion can lead to important consequences in credit-sensitive portfolios.

4.2.2 Probabilities of Default

Here, the evolution of default probabilities estimated by the model is investigated. More precisely, we examine how well the model derives these probabilities by focusing our analysis on important dates such as the near default of AIG, the bankruptcy of Lehman Brothers, as well as mergers and acquisitions. For this purpose, Table 5 summarizes the major events of U.S. financial institutions under study during the global crisis of 2007-2009. This analysis also covers the default probability time series inferred from Moody's databases of historical default frequencies.

First, the hybrid default risk framework links default probabilities to firms' leverage ratios through the intensity process described in Equation (4). Since the leverage of the firm is not directly observable from market data, CDS premiums are used to infer the model's latent variables (i.e. hidden regimes and leverages). Therefore, the model estimates a forward-looking measure of the firm-specific default probability. In contrast, Moody's approach is based on historical data rather than current market prices. In this approach, default counts are aggregated over time by rating category across all industries and are provided in transition matrices, which can be compounded for multiple periods to produce n-year default probabilities. In this analysis, Moody's default probabilities for all credit ratings are obtained from monthly transition matrices using the generator estimation approach with a window length of 3 years ex ante data (see Dionne et al. (2010) for more details). Finally, Moody's transition matrices include banking, finance and insurance industries from January 2002 to December 2012.

To visualize the evolution of default probabilities for each firm across the portfolio, we depict the time series in Figure 3 of Appendix D. For sake of comparison, default probabilities generated by both the model and Moody's are illustrated for time horizons of 1, 5, and 10 years. Note that each event described in Table 5 is labeled by a letter in order to simplify the notation in Figure 3. The main observation is that model's estimates are significantly higher for the entire period of time under study, and especially prior to major events that happened during the crisis. First, this observation supports the fact that CDS-implied default probabilities have a predictive power. Second, one can observe that Moody's estimates tend to increase starting from the beginning of 2009 (when data is available). This might be due to rating changes that may capture credit quality deterioration with a certain time lag, which is incorporated in Moody's default probabilities. Although both time series seem to get closer over time, the difference is significantly high for the entire sample period but the logarithmic scale makes large differences appear proportionally much less. To have a better idea of the term structure of default probabilities retrieved by both the model and Moody's, Table 6 shows the descriptive statistics. The large differences between both approaches' estimates can easily be illustrated by taking two examples: the bankruptcy of Lehman Brothers (LEH) and the near default of AIG. For both firms, the model estimates default probabilities as large as 43% and 34% respectively for 10-year horizon. In contrast, the rating agency displays values close to zero

Table 5: Timeline of major events during the financial crisis of 2007-2009.
Source: Federal Reserve Bank of St. Louis website.

Event A	January 11, 2008	Bank of America announces that it will purchase Countrywide Financial in an all-stock transaction worth approximately $4 billion.
Event B	March 14, 2008	The Federal Reserve Board approves the financing arrangement between JPMorgan Chase and Bear Stearns.
Event C	March 24, 2008	The Federal Reserve Bank of New York announces a term financing to facilitate JPMorgan Chase's acquisition of Bear Stearns. A limited liability company (Maiden Lane) is formed to control $30 billion of Bear Stearns assets that are pledged as security for $29 billion in term financing from the New York Fed at its primary credit rate. JPMorgan Chase will assume the first $1 billion of any losses on the portfolio.
Event D	June 05, 2008	The Federal Reserve Board announces approval of the notice of Bank of America to acquire Countrywide Financial Corporation.
Event E	July 13, 2008	The Federal Reserve Board authorizes the Federal Reserve Bank of New York to lend to the Federal National Mortgage Association (Fannie Mae) and the Federal Home Loan Mortgage Corporation (Freddie Mac), should such lending prove necessary. The U.S. Treasury Department announces a temporary increase in the credit lines of Fannie Mae and Freddie Mac and a temporary authorization for the Treasury to purchase equity in either GSE if needed.
Event F	July 15, 2008	The Securities Exchange Commission (SEC) issues an emergency order temporarily prohibiting naked short selling in the securities of Fannie Mae, Freddie Mac, and primary dealers at commercial/investment banks.
Event G	September 7, 2008	The Federal Housing Finance Agency (FHFA) places Fannie Mae and Freddie Mac in government conservatorship. The U.S. Treasury Department announces 3 additional measures to complement the FHFA's decision: 1) Preferred stock purchase agreements between the Treasury/FHFA and Fannie Mae and Freddie Mac to ensure GSEs positive net worth; 2) new secured lending facility which will be available to Fannie Mae, Freddie Mac, and the Federal Home Loan Banks; and 3) temporary program to purchase GSE MBS.
Event H	September 15, 2008	Bank of America announces its intent to purchase Merrill Lynch & Co. for $50 billion. Lehman Brothers Holdings Incorporated files for Chapter 11 bankruptcy protection.
Event I	September 16, 2008	The Federal Reserve Board authorizes the Federal Reserve Bank of New York to lend up to $85 billion to the American International Group (AIG) under Section 13(3) of the Federal Reserve Act.
Event J	September 19, 2008	The Federal Reserve Board announces the creation of the Asset-Backed Commercial Paper Money Market Mutual Fund Liquidity Facility (AMLF) to extend non-recourse loans at the primary credit rate to U.S. depository institutions and bank holding companies to finance their purchase of high-quality asset-backed commercial paper from money market mutual funds. The Federal Reserve Board also announces plans to purchase federal agency discount notes (short-term debt obligations issued by Fannie Mae, Freddie Mac, and Federal Home Loan Banks) from primary dealers.
Event K	September 21, 2008	The Federal Reserve Board approves applications of investment banking companies Goldman Sachs and Morgan Stanley to become bank holding companies.
Event L	September 25, 2008	The Office of Thrift Supervision closes Washington Mutual Bank. JPMorgan Chase acquires the banking operations of Washington Mutual in a transaction facilitated by the FDIC.
Event M	September 29, 2008	The FDIC announces that Citigroup will purchase the banking operations of Wachovia Corporation. The FDIC agrees to enter into a loss-sharing arrangement with Citigroup on a $312 billion pool of loans, with Citigroup absorbing the first $42 billion of losses and the FDIC absorbing losses beyond that. In return, Citigroup would grant the FDIC $12 billion in preferred stock and warrants.
Event N	October 3, 2008	Wells Fargo announces a competing proposal to purchase Wachovia Corporation that does not require assistance from the FDIC.
Event O	October 12, 2008	The Federal Reserve Board approves an application of Wells Fargo & Co to acquire Wachovia Corporation.
Event P	October 24, 2008	PNC Financial Services Group Inc. purchases National City Corporation, creating the 5th largest U.S. bank.
Event Q	November 17, 2008	Three large U.S. life insurance companies seek TARP funding: Lincoln National, Hartford Financial Services Group, and Genworth Financial announce their intentions to purchase lenders/depositories and thus qualify as savings and loan companies to access TARP funding.
Event R	November 20, 2008	Fannie Mae and Freddie Mac announce that they will suspend mortgage foreclosures until January 2009.
Event S	November 23, 2008	The U.S. Treasury Department, Federal Reserve Board, and FDIC announce an agreement with Citigroup to provide a package of guarantees, liquidity access, and capital. Citigroup will issue preferred shares to the Treasury and FDIC in exchange for protection against losses on a $306 billion pool of commercial/residential securities held by Citigroup. The Federal Reserve will backstop residual risk in the asset pool through a non-recourse loan, and the Treasury will invest an additional $20 billion in Citigroup from the TARP.
Event T	November 26, 2008	The Federal Reserve Board announces approval for Bank of America to acquire Merrill Lynch & Co.
Event U	December 15, 2008	The Federal Reserve Board announces that it has approved the application of PNC Financial Services to acquire National City Corporation.
Event V	January 5, 2009	The Federal Reserve Bank of New York begins purchasing fixed-rate mortgage-backed securities guaranteed by Fannie Mae, Freddie Mac and Ginnie Mae under a program first announced on November 25, 2008.
Event X	January 16, 2009	The U.S. Treasury Department, Federal Reserve, and FDIC announce a package of guarantees, liquidity access, and capital for Bank of America. The U.S. Treasury and the FDIC will enter a loss-sharing arrangement with Bank of America on a $118 billion portfolio of loans, securities, and other assets in exchange for preferred shares. In addition, and if necessary, the Federal Reserve will provide a non-recourse loan to back-stop residual risk in the portfolio. Separately, the U.S. Treasury will invest $20 billion in Bank of America from the TARP in exchange for preferred stock.

Table 6: Descriptive statistics of default probability estimates.

	1-YR PD				5-YR PD				10-YR PD			
Firm	mean (%)	std (%)	min (%)	max (%)	mean (%)	std (%)	min (%)	max (%)	mean (%)	std (%)	min (%)	max (%)
AIG	5.07	8.81	0.21	49.83	22.46	19.52	2.40	83.33	33.68	21.29	7.42	88.16
ALL	2.10	2.89	0.27	16.55	16.04	8.54	4.72	43.58	28.37	9.21	13.43	53.78
AXP	2.07	3.53	0.15	19.08	13.83	10.37	2.43	47.72	24.58	11.26	8.04	57.43
BACORP	3.80	3.72	0.12	18.48	30.66	12.94	9.23	55.07	48.31	12.74	26.24	68.51
BBT	0.75	0.21	0.60	1.50	7.65	4.12	3.39	19.51	20.21	9.02	9.43	41.44
BSC	3.03	4.61	0.54	27.59	24.73	7.57	18.15	55.06	39.24	4.78	34.02	61.78
C	3.59	3.94	0.24	21.18	26.83	13.50	7.02	56.38	43.75	14.05	21.31	68.95
CB	1.00	0.96	0.23	4.94	9.38	5.42	2.72	23.07	16.04	5.71	7.17	29.91
CCR	9.15	11.10	1.62	61.66	50.36	10.46	36.05	85.87	71.39	5.99	62.16	91.97
COF	4.18	4.68	0.47	19.79	24.91	9.46	10.96	49.02	38.48	8.37	24.05	58.78
FHLMC	1.61	0.01	1.61	1.65	8.51	0.65	7.98	10.94	16.70	1.00	15.67	20.15
FNMA	2.09	0.00	2.09	2.12	10.91	0.69	10.30	13.45	21.36	1.15	20.12	25.16
GS	4.54	1.06	3.52	9.39	24.38	5.84	17.30	43.53	43.64	7.11	33.59	62.99
HIG	2.44	3.68	0.10	21.72	15.57	11.90	1.89	53.09	27.11	15.11	6.59	63.95
JPM	1.01	0.96	0.13	4.94	7.97	3.83	2.38	18.35	12.07	4.37	5.12	22.97
LEH	4.80	7.38	0.33	33.11	23.14	15.96	10.29	63.04	42.88	10.23	31.45	70.72
MER	6.87	8.33	0.89	34.82	57.10	5.93	47.47	75.09	77.29	2.65	67.40	85.50
MWD	2.53	2.89	0.44	23.68	11.04	6.63	3.44	41.42	15.92	7.38	6.92	46.30
SCH	4.08	0.01	4.07	4.11	19.04	0.25	18.79	19.82	35.39	0.84	34.40	37.66
STI	1.41	1.01	0.31	4.52	9.23	6.17	1.82	23.85	19.01	11.12	4.54	40.01
USB	0.92	0.56	0.29	2.80	6.65	3.77	1.85	17.33	14.90	7.04	4.94	32.61
WB	1.08	1.96	0.16	15.64	11.13	8.22	4.91	46.40	20.03	9.03	12.55	55.45
WM	8.56	14.22	0.69	80.04	42.44	13.51	29.01	91.48	62.86	7.37	53.77	93.84
WFC	2.14	2.04	0.09	10.90	16.40	6.96	5.09	34.02	24.97	7.64	11.75	42.36

(a) Model's estimates

	1-YR PD				5-YR PD				10-YR PD			
Firm	mean (%)	std (%)	min (%)	max (%)	mean (%)	std (%)	min (%)	max (%)	mean (%)	std (%)	min (%)	max (%)
AIG	0.02	0.04	0.00	0.09	0.62	0.85	0.00	2.13	2.81	3.72	0.00	9.71
ALL	0.20	0.25	0.00	0.52	1.53	1.83	0.00	4.49	5.00	5.83	0.04	15.45
AXP	0.20	0.25	0.00	0.52	1.53	1.83	0.00	4.49	5.00	5.83	0.04	15.45
BACORP	0.21	0.25	0.00	0.52	1.71	2.01	0.00	5.30	5.52	6.54	0.00	16.92
BBT	0.16	0.24	0.00	0.52	1.39	1.81	0.00	4.49	4.69	5.88	0.00	15.45
BSC	0.00	0.00	0.00	0.00	0.03	0.08	0.00	0.36	0.21	0.37	0.04	1.75
C	0.20	0.24	0.00	0.52	1.57	1.81	0.00	4.49	5.05	5.83	0.00	15.45
CB	0.20	0.25	0.00	0.52	1.53	1.83	0.00	4.49	5.00	5.83	0.04	15.45
CCR	0.00	0.00	0.00	0.02	0.09	0.14	0.00	0.53	0.43	0.56	0.04	2.14
COF	0.40	0.43	0.00	1.08	3.54	3.94	0.01	10.50	10.05	10.94	0.04	29.90
FHLMC	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.01	0.00	0.05
FNMA	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.01	0.00	0.05
GS	0.18	0.24	0.00	0.52	1.47	1.83	0.00	4.49	4.83	5.90	0.00	15.45
HIG	0.41	0.44	0.00	1.08	3.53	4.03	0.00	10.50	9.84	11.29	0.04	29.90
JPM	0.03	0.03	0.00	0.09	0.67	0.84	0.00	2.13	2.97	3.66	0.00	9.71
LEH	0.00	0.00	0.00	0.00	0.02	0.02	0.00	0.07	0.14	0.12	0.04	0.50
MER	0.04	0.14	0.00	0.50	0.22	0.73	0.00	2.71	0.52	1.59	0.00	6.34
MWD	0.22	0.25	0.00	0.52	1.64	1.81	0.00	4.49	5.17	5.80	0.00	15.45
SCH	0.20	0.25	0.00	0.52	1.53	1.83	0.00	4.49	5.00	5.83	0.04	15.45
STI	0.16	0.24	0.00	0.52	1.39	1.81	0.00	4.49	4.68	5.89	0.00	15.45
USB	0.03	0.04	0.00	0.09	0.69	0.86	0.00	2.13	3.09	3.75	0.00	9.71
WB	0.04	0.14	0.00	0.50	0.22	0.73	0.00	2.71	0.51	1.59	0.00	6.34
WM	0.47	3.21	0.00	21.74	1.46	9.40	0.00	63.84	2.11	11.72	0.04	79.74
WFC	0.17	0.24	0.00	0.52	1.41	1.82	0.00	4.49	4.72	5.90	0.00	15.45

(b) Moody's estimates

This table shows descriptive statistics of 1-, 5-, and 10-year default probabilities for each firm across the portfolio over the period of time 2005-2012. Model's time series are inferred from CDS premiums market data. Moody's time series are obtained from monthly transition matrices for banking, finance and insurance industries using the generator estimation approach with window length of 3 years ex ante default data.

and therefore, do not reflect the financial situation of distressed firms during the turmoil. This is explained by the rarity of the observed default events that constitutes the main difficulty in the Moody's empirical estimation.

In what follows, we examine how well the model derives the probabilities of default (PD) by focusing our analysis on important events during the crisis. Let first take the case of AIG that almost defaulted on September 16, 2008. One month prior to that date, the model derives 1-year PD of approximately 7% (28% and 41% for 5- and 10-year PD respectively) [3]. Then, the estimate reaches 11% (35% and 48% for 5- and 10-year PD respectively) on September 10, followed by a spike of 50% (73% and 79% for 5- and 10-year PD respectively) one week later. The same behaviour is observed for LEH prior to its collapse on September 15, 2008. Indeed, high levels are reached four months prior to the bankruptcy event (average values of 21%, 54% and 63% for 1-, 5-, and 10-years PD respectively), followed by a jump in the probability of default of approximately 10% for all time horizons on September 10, 2008. When it comes to acquired firms such as Bear Stearns (BSC), Countrywide (CCR), Merrill Lynch (MER), Wachovia (WB), and Washington Mutual (WM), the same characteristic jump pattern is displayed close to major events preceded by relatively large PD. Moreover, one can observe higher estimates when examining PD measures of firms that have been acquired during the crisis and distressed firms in comparison to the others in the sample. Indeed, the maximum values displayed in Table 6 a) correspond to the ones of purchased institutions BSC, CCR, MER, and WM as well as to AIG (62%, 92%, 86%, 94%, and 88% respectively for 10-year PD). Although default is more likely to occur over a long time horizon, some acquired firms also outline large 1-year PD. For example, CCR and WM display maximum values of 62% and 80%, while other acquired firms such as BSC and MER present values exceeding 25%.

In summary, we conclude that major events of financial difficulties are well reflected in the model's empirical results by an upward trend of the default probabilities. These results are consistent with the idea of financial contagion between distressed institutions since the unpredicted default of some firm affects the remaining firms, whose default probabilities are more likely to increase. Furthermore, the term structure estimated by the model appears to contain additional information in comparison to the one generated by Moody's. Therefore, we conclude that Moody's PD estimation is not well adapted in the context of our analysis considering the crisis period.

5 Conclusion

The empirical study presented in this paper finds supportive evidence of contagion within the financial sector in U.S. during the last global crisis of 2007-2009. The proposed framework is a bivariate extension of the Markov-switching hybrid credit risk model of Bégin et al. (2014) that is able to endogenously capture pairwise correlations from log-leverages' co-movements by UKF processing and quasi-maximum likelihood estimation. The methodology is based on correlation analysis which allows to study the interdependence among 24 major U.S. financial institutions. The results of the empirical analysis show an increase in correlation during the high volatility regime

3. Note that weekly time series of default probabilities are not reported here. However, their descriptive statistics are provided in Table 6.

in comparison to the stable regime for 21 firms within the portfolio. In particular, Fannie Mae and Freddie Mac are significantly more correlated with the others during the crisis, reflecting the linkages between subprime mortgages and the financial sector downturn. These empirical results suggest the existence of contagion among the U.S. financial institutions under study during the last crisis. In addition, the model derives the firms' probabilities of default that indicate the risk trend at the individual level. The results show larger default probabilities for distressed firms reflecting their financial difficulties in the turmoil period. To conclude, the empirical analysis studied in this paper presents major implications for risk management practices since financial contagion can lead to important consequences in credit-sensitive portfolios.

Appendix A Credit derivative pricing

This appendix presents the numerical method used to price credit derivatives. Since the framework assumes stochastic endogenous recovery rates and regime switching dynamics, CDS premiums cannot be calculated in closed form. Therefore, a numerical method that is based on a trinomial lattice approach is used.

A.1 Credit default swaps

Since the estimation procedure is based on time series of CDS premiums, a pricing equation is required for such credit sensitive derivatives. Basically, a CDS contract is an agreement between two counterparties whereby the protection seller insures the buyer against a possible credit event of an underlying bond issuer. In exchange for the insurance, the buyer pays a premium (the CDS spread) to the seller until the contract expires or the bond issuer's default occurs. In the latter case, the seller compensates the buyer and takes possession of the defaulted loan. The CDS spread is usually fixed such that the expected present value of losses equals the expected present value of premiums. According to Duffie and Singleton (1999), the expected present value of losses, given a CDS contract that matures at T and considering a constant risk-free rate r, is

$$\begin{aligned}
&\mathbb{E}^{\mathbb{Q}}\Big[\exp\Big(-r(\tau-t)\Big)(1-R_\tau)\mathbb{I}_{\{t<\tau\leq T\}} \mid \mathcal{F}_t\Big] \\
&\quad=\mathbb{E}^{\mathbb{Q}}\Bigg[\sum_{t\leq t_k<T}(1-R_k)\exp\Big(-r(t_k-t)\Big)\exp\Bigg(-\sum_{t\leq t_u<t_k}H_u\Delta t\Bigg)\Big(1-\exp(-H_k\Delta t)\Big) \mid \mathcal{G}_t\Bigg]\mathbb{I}_{\{\tau>t\}}.
\end{aligned} \tag{7}$$

Assume now that a premium of 1 is paid. The expected present value of premiums is

$$\begin{aligned}
&\mathbb{E}^{\mathbb{Q}}\Big[\sum_{t_i}\exp\Big(-r(t_i-t)\Big)\mathbb{I}_{\{t\leq t_i<\tau\}} \mid \mathcal{F}_t\Big] \\
&\quad=\mathbb{E}^{\mathbb{Q}}\Bigg[\sum_{t_i}\exp\Big(-r(t_i-t)\Big)\exp\Bigg(-\sum_{t\leq t_u<t_i}H_u\Delta t\Bigg) \mid \mathcal{G}_t\Bigg]\mathbb{I}_{\{\tau>t\}},
\end{aligned} \tag{8}$$

where t_i are premium payment dates and the time step is 1 week (i.e. $\Delta t = 1/52$). Then, it follows that the periodic premium is the ratio of (7) over (8). Numerical methods are required to price CDS contracts since R_k and H_k are dependent random variables and therefore, no closed-form solution is available.

A.2 Trinomial lattice approach

This appendix is highly inspired from Bégin et al. (2014) and describes the numerical method used to price credit default swaps and corporate bonds. The method is based on the trinomial lattice approach proposed by Yuen and Yang (2010) for Markov-switching dynamics. The branching structure when two regimes are considered is shown in Figure 1. The idea of the multi-state trinomial tree model is to change the risk neutral probability measure if the regime state changes rather than increasing the branches of the tree.

The actual value of the log-leverage at a typical node in the tree is x. Assuming constant risk-free interest rate and volatility, the log-leverage is allowed either to remain unchanged to $x_m = x$, increase to $x_u = x\,\mathrm{e}^{\sigma\sqrt{\Delta t}}$, or decrease to $x_d = x\,\mathrm{e}^{-\sigma\sqrt{\Delta t}}$. Furthermore, the volatility for a two-regime trinomial tree is

$$\max_{1\leq i\leq 2}\Big(\sigma_i + (\sqrt{1.5}-1)\bar{\sigma}\Big), \tag{9}$$

where $\bar{\sigma}$ is the arithmetic mean of σ. Then, the risk neutral probabilities corresponding to each branch for the regime i are

$$\begin{aligned}
\pi_m^i &= 1 - \frac{1}{\lambda_i^2},\\
\pi_u^i &= \frac{\mathrm{e}^{\mu_i^{\mathbb{Q}}\Delta t} - \mathrm{e}^{-\sigma\sqrt{\Delta t}} - \Big(1-\frac{1}{\lambda_i^2}\Big)\Big(1-\mathrm{e}^{-\sigma\sqrt{\Delta t}}\Big)}{\mathrm{e}^{\sigma\sqrt{\Delta t}} - \mathrm{e}^{-\sigma\sqrt{\Delta t}}},\\
\pi_d^i &= \frac{\mathrm{e}^{\sigma\sqrt{\Delta t}} - \mathrm{e}^{\mu_i^{\mathbb{Q}}\Delta t} - \Big(1-\frac{1}{\lambda_i^2}\Big)\Big(\mathrm{e}^{\sigma\sqrt{\Delta t}} - 1\Big)}{\mathrm{e}^{\sigma\sqrt{\Delta t}} - \mathrm{e}^{-\sigma\sqrt{\Delta t}}},
\end{aligned} \tag{10}$$

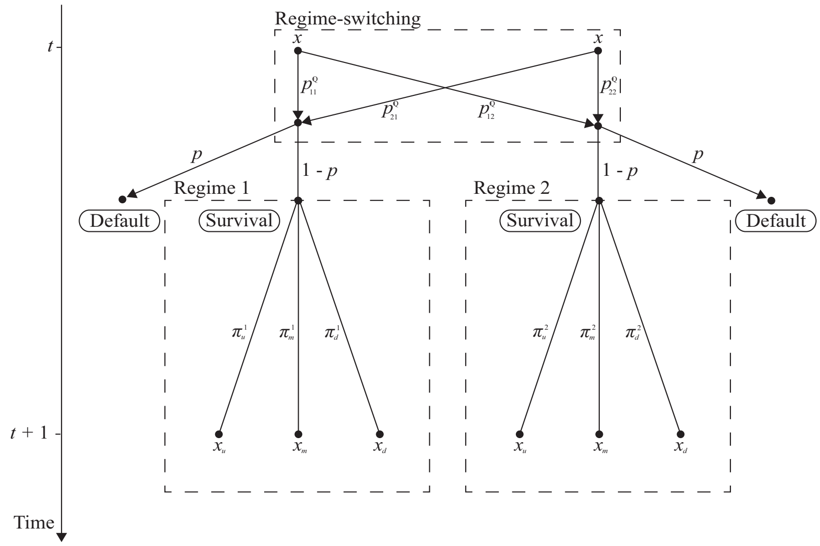
where $\lambda_i = \frac{\sigma}{\sigma_i}$.

Finally, the key contribution of Bégin et al. (2014) is the addition of a default branch in the trinomial tree so that survival and default payoffs are taken into consideration when pricing derivatives. The idea behind this additional branch is similar to the one applied to the standard trinomial tree in Schönbucher (2002). At each node, the default probability is given by

$$p = 1 - \exp\Big(-H_t\Delta t\Big), \tag{11}$$

where H_t is the intensity process from Equation (4).

Figure 1: Two-state trinomial lattice branching structure (source: Bégin et al. (2014)).
This figure shows the branching structure at a typical node in the tree when the number of regimes is set to two. Note that both trees represent the same lattice: only the weights change across different regimes.



Appendix B Estimation based on filtering techniques

This appendix outlines the unscented Kalman filter (UKF) equations that have been applied to estimate parameter sets and model's latent variables (i.e. leverage ratios and hidden regimes) on a firm-by-firm basis.

B.1 State-space representation

Filtering techniques require that the model is represented in state space form. First, the state equation that describes the evolution of leverage ratios depending on the hidden regime is given in Equation (2). Second, the measurement equation linking the state of the model to CDS observations is a non-linear function of the log-leverage x_k that depends on the regime s_k, and is given by

$$\begin{aligned} y_k^{(m)} &= f^{(m)}(x_k, s_k) + v_k^{(m)} \\ &= \ln\Big(g^{(m)}(\exp(x_k), s_k)\Big) + v_k^{(m)}, \end{aligned} \tag{12}$$

where $m \in \{1, 2, 3, 5, 7, 10\}$ refers to CDS tenors used in the estimation, and $v_k^{(m)}$ is a Gaussian random variable with standard deviation $\delta^{(m)}$ that is assumed to be independent across maturities. Moreover, $g^{(m)}$ is the theoretical price of an m-year CDS, in which dynamics of leverage ratios evolve under the risk neutral measure $\mathbb{Q}$.

B.2 Unscented Kalman filter

The standard Kalman filter is generally limited to linear transformations. Given the non-linearity of the measurement equation, one needs to apply other statistical and recursive algorithms to obtain the best estimate of state variables (i.e. leverage ratios and regimes). According to Christoffersen et al. (2013), the UKF produces better state estimation and parameter identification than both extended Kalman filter (EKF) and particle filter in many applications in finance. In this paper, $M = N^d$ parallel UKFs are applied to filter both unobserved variables, where N is the number of possible regime states in the model and d is determined by computation and storage capabilities. This method is based on an extension of the Tugnait (1982)'s detection-estimation algorithm, which is described in more details in Bégin et al. (2014). The bivariate framework implies $N = 4$ regime states, while $d \geq 4$ produces robust results, and therefore $d = 4$ is selected for providing the best processing efficiency.

The unscented transform (UT) refers to a method that allows to propagate mean and covariance information through non-linear transformations. The main idea consists in producing a set of weighted points, namely sigma points, around the current state estimate based on its covariance. Then, the sigma points are propagated through the non-linear system to get more accuracy on the mean and covariance estimates. Next, the UKF equations are described based on Boudreault (2009), but adapted for the bivariate framework. As a starting point, let define the augmented state vector on which the UT is applied

$$(\mathbf{x}_t^a)^\top \equiv \Big[x_t, \epsilon_t^*, \mathbf{v}_t^\top\Big], \tag{13}$$

where $x_t = (x_t^{(i)}, x_t^{(j)})$, $\epsilon_t^* \equiv \sigma\sqrt{\Delta t}\epsilon_t$ with $\sigma = (\sigma_{s_t}^{(i)}, \sigma_{s_t}^{(j)})$ $\epsilon_t = (\epsilon_t^{(i)}, \epsilon_t^{(j)})$, $\mathbf{v}_t = (\mathbf{v}_t^{(i)}, \mathbf{v}_t^{(j)})$ is the noise vector, and i and j refer to the pairwise firms. Note that for simplification purposes, the following notation is considered next

$$Y_t = \{\mathbf{y}_1^{(i)}, ..., \mathbf{y}_{t-1}^{(i)}, \mathbf{y}_t^{(i)}, \mathbf{y}_1^{(j)}, ..., \mathbf{y}_{t-1}^{(j)}, \mathbf{y}_t^{(j)}\}.$$

Define now the first two moments of the augmented state vector as

$$\begin{aligned}(\hat{\mathbf{x}}_{t|u}^a) &\equiv \mathbb{E}\Big[\mathbf{x}_t^a \mid Y_u\Big],\\ (\mathbf{P}_{t|u}^a) &\equiv \mathbb{C}\mathrm{ov}\Big[\mathbf{x}_t^a \mid Y_u\Big],\end{aligned} \tag{14}$$

for $u = t-1$ or t. Then, define the set of sigma points of the augmented state vector $\mathbf{x}_{t-1}^a$ as

$$S_{t-1|t-1} \equiv \hat{\mathbf{x}}_{t-1|t-1}^a \oplus \Big[-\sqrt{2(D+2)+\lambda} \times \mathrm{chol}(\mathbf{P}_{t-1|t-1}^a), \mathbf{0}, \sqrt{2(D+2)+\lambda} \times \mathrm{chol}(\mathbf{P}_{t-1|t-1}^a)\Big], \tag{15}$$

where D refers to the number of CDS maturities available for each firm, λ is an UKF-specific parameter[4], and chol is the Cholesky decomposition of a positive-definite symmetric square matrix. The operator $\oplus$ means that the column vector $\hat{\mathbf{x}}_{t-1|t-1}^a$ is added to each column of the matrix on its right-hand side, leading to a sigma points matrix of dimension $2(D+2) \times (4(D+2)+1)$. Therefore, the first two rows of $S_{t-1|t-1}$ correspond to the propagation of x_t, the next two rows are the propagation of ϵ_t^*, and the last $2D$ rows are the propagation of the noise vector $\mathbf{v}_t$.

4. The parameter λ depends on UKF-scaling parameters and is given by $\lambda = \alpha_{UKF}^2(D+2+\kappa_{UKF}) - (D+2)$. Note that scaling parameters have been assumed to be $\kappa_{UKF} = 2$, $\alpha_{UKF} = 0.1$, and $\beta_{UKF} = 0$.

In what follows, the prediction and update steps of the UKF are presented. The first step allows to predict *a priori* state estimate $x_t \mid Y_{t-1}$ and its covariance using a discrete sample of points provided by $\mathcal{X}_{t|t-1}$. Then, the second step approximates the measurement residual and its covariance, which allow to update *a posteriori* state estimate $\mathbf{y}_t \mid Y_{t-1}$ and its covariance using $\mathcal{Y}_{t|t-1}$. In the literature, $\mathcal{Y}_{t|t-1}$ refers to weighted sigma points.

As a starting point to the prediction stage, let define the c-th element of the column vector $\mathcal{X}_{t|t-1}$ as

$$\mathcal{X}^{(c)}_{t|t-1} = S^{(1:2,c)}_{t-1|t-1} + \Big(\mu - \frac{1}{2}\sigma^2\Big)\Delta t + S^{(3:4,c)}_{t-1|t-1}, \tag{16}$$

where $c = (1, 2, ..., 4(D+2)+1)$, and upper index $(l1 : l2, c)$ refers to rows from $l1$ to $l2$ and column c of elements in matrix $S_{t-1|t-1}$. Applying the UT allows to obtain the predicted value of x_t given the weighted average of its sigma points $\mathbf{y}_1, ...\mathbf{y}_{t-1}$, i.e.

$$\hat{x}_{t|t-1} = \mathbb{E}\Big[\mathbf{x}_t \mid Y_{t-1}\Big] \cong \sum_{c=1}^{4(D+2)+1} W^{(n)}_c \mathcal{X}^{(c)}_{t|t-1}, \tag{17}$$

where the weights $W^{(n)}_c$ are defined as

$$W^{(n)}_c = \begin{cases} \frac{\lambda}{2(D+2)+\lambda} & \text{if } c = 2(D+2)+1 \\ \\ \frac{1}{2(2(D+2)+\lambda)} & \text{otherwise} \end{cases}.$$

Equation (17) is similar to a discretization of the random variable $x_t \mid \mathbf{y}_1, ..., \mathbf{y}_{t-1}$ with realizations $\mathcal{X}^{(c)}_{t|t-1}$ and associated probabilities $W^{(n)}_c$. Then, the variance of the predicted state estimate is

$$P_{t|t-1} = \mathbb{C}\text{ov}\Big[\mathbf{x}_t \mid Y_{t-1}\Big] \cong \sum_{c=1}^{4(D+2)+1} W^{(z)}_c \Big(\mathcal{X}^{(c)}_{t|t-1}\hat{x}_{t|t-1}\Big)^2, \tag{18}$$

where the weights $W^{(z)}_c$ are defined as

$$W^{(z)}_c = \begin{cases} \frac{\lambda}{2(D+2)+\lambda} + (1 - \alpha^2_{UKF} + \beta_{UKF}) & \text{if } c = 2(D+2)+1 \\ \\ W^{(n)}_c & \text{otherwise} \end{cases},$$

where $\lambda = \alpha^2_{UKF}\Big(2(D+2)+\kappa_{UKF}\Big) - 2(D+2)$. Once again, Equation (18) is similar to the variance of a discrete random variable $x_t \mid Y_{t-1}$ with probabilities $W^{(z)}_c$.

In a similar manner to the prediction stage, the update step consists in first defining the l-th row and c-th column element of the matrix $\mathcal{Y}_{t|t-1}$ by

$$\mathcal{Y}^{(l,c)}_{t|t-1} = f^{(l)}\Big(\mathcal{X}^{(c)}_{t|t-1}\Big) + S^{(l+4,c)}_{t-1|t-1}, \tag{19}$$

where $l = 1, 2, ..., 2D$ and $c = (1, 2, ..., 4(D+2)+1)$. The l-th row of matrix $\mathcal{Y}_{t|t-1}$ corresponds to the discretized distribution of $y^{(l)}_t \mid Y_{t-1}$ with realizations given by each column c. Then, the predicted value of $\mathbf{y}_t$ given Y_{t-1} is the expectation of some discrete multivariate random variable such as

$$\hat{\mathbf{y}}_{t|t-1} = \mathbb{E}\Big[\mathbf{y}_t \mid Y_{t-1}\Big] \cong \sum_{c=1}^{4(D+2)+1} W_c^{(n)} \mathcal{Y}_{t|t-1}^{(c)}. \tag{20}$$

The next two equations describe the covariance between elements of $\mathcal{Y}_{t|t-1}$, as well as the covariance between $\mathcal{X}_{t|t-1}$ and $\mathcal{Y}_{t|t-1}$ respectively

$$\mathbf{P}_{\mathbf{yy}} = \sum_{c=1}^{4(D+2)+1} W_c^{(z)} \Big(\mathcal{Y}_{t|t-1}^{(c)} - \hat{\mathbf{y}}_{t|t-1}\Big)\Big(\mathcal{Y}_{t|t-1}^{(c)} - \hat{\mathbf{y}}_{t|t-1}\Big)^{\top}, \tag{21}$$

$$\mathbf{P}_{\mathbf{xy}} = \sum_{c=1}^{4(D+2)+1} W_c^{(z)} \Big(\mathcal{X}_{t|t-1}^{(c)} - \hat{x}_{t|t-1}\Big)\Big(\mathcal{Y}_{t|t-1}^{(c)} - \hat{\mathbf{y}}_{t|t-1}\Big)^{\top}. \tag{22}$$

The Kalman gain that minimizes the *a posteriori* error covariance is

$$\mathbf{K}_t = \mathbf{P}_{\mathbf{xy}} \mathbf{P}_{\mathbf{yy}}^{-1}. \tag{23}$$

Then, the predicted value of x_t given Y_t is updated to

$$\begin{aligned} \hat{x}_{t|t} &= \mathbb{E}\Big[x_t \mid Y_t\Big] \\ &= \hat{x}_{t|t-1} + \mathbf{K}_t\Big(\mathbf{y}_t - \hat{\mathbf{y}}_{t|t-1}\Big), \end{aligned} \tag{24}$$

with variance

$$\begin{aligned} P_{t|t} &= \mathbb{C}\mathrm{ov}\Big[x_t \mid Y_t\Big] \\ &= P_{t|t-1} - \mathbf{K}_t \mathbf{P}_{\mathbf{yy}} \mathbf{K}_t^{\top}. \end{aligned} \tag{25}$$

Finally, the recursion process starts at $t = 1$ with the following augmented state vector and variance

$$\begin{aligned} \hat{\mathbf{x}}_{0|0}^{a} &= \Big[\hat{x}_{0|0} \quad \mathbf{0} \quad \mathbf{0}\Big]_{1\times 2(D+2)}, \\ \mathbf{P}_{0|0}^{a} &= \begin{bmatrix} \mathbf{P}_{0|0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{\Sigma}_{\mathbf{s}_0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{R} \end{bmatrix}_{2(D+2)\times 2(D+2)}, \end{aligned} \tag{26}$$

where $\mathbf{P}_{0|0}$ is the initial covariance matrix of the predicted state variable, $\mathbf{\Sigma}_{\mathbf{s}_0}$ is the covariance matrix of leverage noise terms associated to the initial regime state $\mathbf{s}_0 = (1, 1)$, and $\mathbf{R}$ is the trading noise variance matrix. For more details on the estimation procedure, please refer to Appendix C.

Appendix C Endogenous correlation coefficients

To obtain endogenous correlation estimates, correlation coefficients are introduced in the covariance matrix of the augmented state vector on which the unscented transformation (UT) is performed. Details about UKF equations and the UT are provided in Appendix B. As a starting point, one can write the second order moment of the augmented state vector as

$$\mathbf{P}^a_{t|t} = \begin{bmatrix} \mathbf{P}_{t|t} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{\Sigma}_{\mathbf{s}_t} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{R} \end{bmatrix}_{2(D+2)\times 2(D+2)}, \tag{27}$$

where $[\mathbf{P}_{t|t}]_{2\times 2}$ is the covariance matrix of predicted state variables $(\hat{x}^{(i)}_{t|t}, \hat{x}^{(j)}_{t|t})$ updated at each time step, $[\mathbf{\Sigma}_{\mathbf{s}_t}]_{2\times 2}$ is the covariance matrix of leverage noise terms associated with regimes $\mathbf{s}_t = (s^{(i)}_t, s^{(j)}_t)$, and $[\mathbf{R}]_{2D\times 2D}$ is the trading noise variance matrix. Furthermore, dimension D refers to the number of CDS maturities available for each firm. More precisely, covariance and variance matrices can be expressed as

$$\mathbf{\Sigma}_{\mathbf{s}_t} = \begin{bmatrix} (\sigma^{(i)}_{\mathbf{s}_t})^2 & \sigma^{(i)}_{\mathbf{s}_t}\sigma^{(j)}_{\mathbf{s}_t}\rho^{(i,j)}_{\mathbf{s}_t} & (\sigma^{(j)}_{\mathbf{s}_t})^2 \end{bmatrix} \times \Delta t,$$

$$\mathbf{R} = \operatorname{diag}(\delta^2).$$

Note that $\operatorname{diag}(\delta^2)$ is the operator that creates a square matrix with diagonal elements corresponding to δ^2, and $\delta = \begin{bmatrix} \delta^{(i,1)} & \delta^{(i,2)} & \dots & \delta^{(i,10)} & \delta^{(j,1)} & \delta^{(j,2)} & \dots & \delta^{(j,10)} \end{bmatrix}$ is the vector of the noise terms' standard deviation. Maximizing the log-likelihood function, one obtains the correlation coefficients estimates.

According to Hamilton (1994) and considering the M parallel UKF in the bivariate framework, the log-likelihood function based on observations $\mathbf{y}_t = (\mathbf{y}^{(i)}_t, \mathbf{y}^{(j)}_t)$ up to time step T for all possible paths M is computed by

$$\ell(\phi_2; \mathbf{y}_T, \hat{\phi}_1) = \sum_{t=1}^{T}\sum_{l=1}^{M} \ln f(\mathbf{y}_t \mid Y_{t-1}; \phi_2), \tag{28}$$

where the conditional likelihood $f(\mathbf{y}_t \mid Y_{t-1}; \phi_2)$ given $Y_{t-1} = \{\mathbf{y}^{(i)}_1, ..., \mathbf{y}^{(i)}_{t-1}, \mathbf{y}^{(j)}_1, ..., \mathbf{y}^{(j)}_{t-1}\}$ is the probability density function of a $2D$-variate normal distribution valued at $(\mathbf{y}^{(i)}_t, \mathbf{y}^{(j)}_t)$ on path l with mean and covariance obtained from the filtering procedure. More specifically, the mean $(\hat{\mathbf{y}}^{(i)}_{t|t-1}, \hat{\mathbf{y}}^{(j)}_{t|t-1})$ is a $(1\times 2D)$ vector obtained from $\mathbb{E}[(\mathbf{y}^{(i)}_t, \mathbf{y}^{(j)}_t) \mid Y_{t-1}]$, and the covariance matrix of dimension $(2D\times 2D)$ is $\mathbf{P}_{\mathbf{yy}} = \operatorname{Cov}[(\mathbf{y}^{(i)}_t, \mathbf{y}^{(j)}_t), (\mathbf{y}^{(i)}_t, \mathbf{y}^{(j)}_t) \mid Y_{t-1}]$. Appendix B describes these equations in more details. By using Bayes rule, one can express the conditional likelihood function as

$$\begin{aligned}
f(\mathbf{y}_t \mid Y_{t-1};\phi_2) &= \frac{f(\mathbf{y}_t, Y_{t-1};\phi_2)}{f(Y_{t-1};\phi_2)} \\
&= \frac{\sum_{\mathbf{s}_t} f(\mathbf{y}_t, \mathbf{s}_t, Y_{t-1};\phi_2)}{f(Y_{t-1};\phi_2)} \\
&= \sum_{\mathbf{s}_t} f(\mathbf{y}_t \mid \mathbf{s}_t, Y_{t-1};\phi_2) \times f(\mathbf{s}_t \mid Y_{t-1};\phi_2) \\
&= \begin{bmatrix} f(\mathbf{y}_t \mid (1,1), Y_{t-1};\phi_2) \\ f(\mathbf{y}_t \mid (1,2), Y_{t-1};\phi_2) \\ f(\mathbf{y}_t \mid (2,1), Y_{t-1};\phi_2) \\ f(\mathbf{y}_t \mid (2,2), Y_{t-1};\phi_2) \end{bmatrix}^{\top} \times \begin{bmatrix} f((1,1) \mid Y_{t-1};\phi_2) \\ f((1,2) \mid Y_{t-1};\phi_2) \\ f((2,1) \mid Y_{t-1};\phi_2) \\ f((2,2) \mid Y_{t-1};\phi_2) \end{bmatrix}.
\end{aligned} \tag{29}$$

The conditional likelihood of $\mathbf{y}_t = (\mathbf{y}_t^{(i)}, \mathbf{y}_t^{(j)})$ is computed analytically using the $2D$-variate Normal density function. From the Markov property, the likelihood function given $\mathbf{y}_{t-1} = (\mathbf{y}_{t-1}^{(i)}, \mathbf{y}_{t-1}^{(j)})$ and the actual regimes $\mathbf{s}_t = (s_t^{(i)}, s_t^{(j)})$ of firms i and j can be expressed as

$$f(\mathbf{y}_t \mid \mathbf{s}_t, \mathbf{y}_{t-1};\phi_2) = \frac{1}{(2\pi)^D \left|\mathbf{P}_{\mathbf{yy}}\right|^{1/2}} \exp\left(-\frac{1}{2} e_{\mathbf{s}_t}^{\top} \mathbf{P}_{\mathbf{yy}}^{-1} e_{\mathbf{s}_t} \right), \tag{30}$$

where $e_{\mathbf{s}_t}$ is the error between observations and their forecasted values. Second, the conditional likelihood of $\mathbf{s}_t = (s_t^{(i)}, s_t^{(j)})$ given $\mathbf{y}_{t-1} = (\mathbf{y}_{t-1}^{(i)}, \mathbf{y}_{t-1}^{(j)})$ is obtained recursively. Let denote $\eta_t^{\top} = \sum_{\mathbf{s}_t} f(\mathbf{y}_t \mid \mathbf{s}_t, \mathbf{y}_{t-1};\phi_2)$ and $\xi_{t|t-1} = f(\mathbf{s}_t \mid \mathbf{y}_{t-1};\phi_2)$, one can use the following recursion equations

$$\xi_{t+1|t} = {\mathbf{P}^{(i,j)}}^{\top} \xi_{t|t}, \tag{31a}$$

$$\xi_{t|t} = \frac{\eta_t(\times)\xi_{t|t-1}}{\eta_t^{\top}\xi_{t|t-1}}, \tag{31b}$$

where $(\times)$ refers to the element-by-element multiplication and $\mathbf{P}^{(i,j)}$ is the following transition matrix

$$\mathbf{P}^{(i,j)} = \begin{bmatrix} p_{11}^{(i)}p_{11}^{(j)} & p_{11}^{(i)}p_{12}^{(j)} & p_{12}^{(i)}p_{11}^{(j)} & p_{12}^{(i)}p_{12}^{(j)} \\ p_{11}^{(i)}p_{21}^{(j)} & p_{11}^{(i)}p_{22}^{(j)} & p_{12}^{(i)}p_{21}^{(j)} & p_{12}^{(i)}p_{22}^{(j)} \\ p_{21}^{(i)}p_{11}^{(j)} & p_{21}^{(i)}p_{12}^{(j)} & p_{22}^{(i)}p_{11}^{(j)} & p_{22}^{(i)}p_{12}^{(j)} \\ p_{21}^{(i)}p_{21}^{(j)} & p_{21}^{(i)}p_{22}^{(j)} & p_{22}^{(i)}p_{21}^{(j)} & p_{22}^{(i)}p_{22}^{(j)} \end{bmatrix}. \tag{32}$$

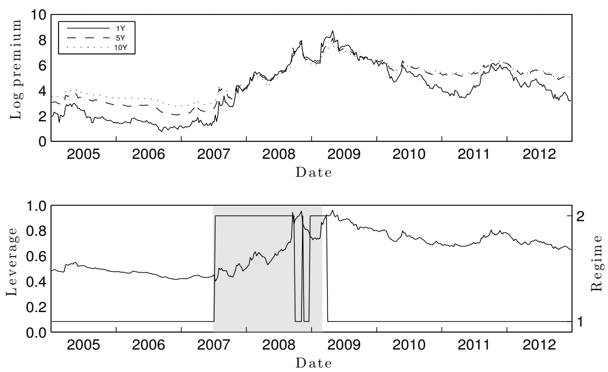
Finally, in order to obtain an estimate of ϕ_2, one can write

$$\begin{aligned}
\hat{\phi}_2 &= \operatorname{argmax} \left\{ \ell(\phi_2; \mathbf{y}_T, \hat{\phi}_1) \right\} \\
&= \operatorname{argmax} \left\{ \sum_{t=1}^{T} \sum_{l=1}^{M} \sum_{\mathbf{s}_t} \ln(\xi_{t|t-1}) - D\ln(2\pi) - \frac{1}{2}\ln\left|\mathbf{P}_{\mathbf{yy}}\right| - \frac{1}{2} e_{\mathbf{s}_t}^{\top} \mathbf{P}_{\mathbf{yy}}^{-1} e_{\mathbf{s}_t} \right\}.
\end{aligned} \tag{33}$$

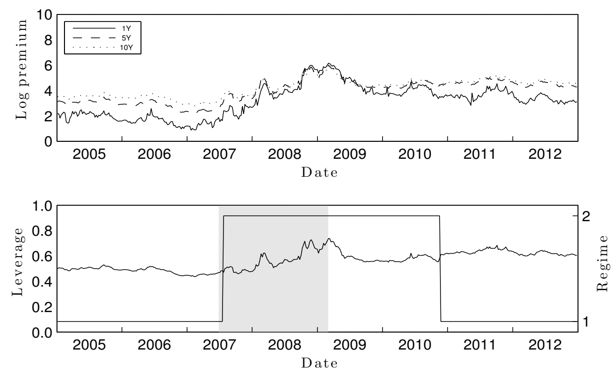
Appendix D Empirical results time series

D.1 Log premium, filtered leverage and filtered regime

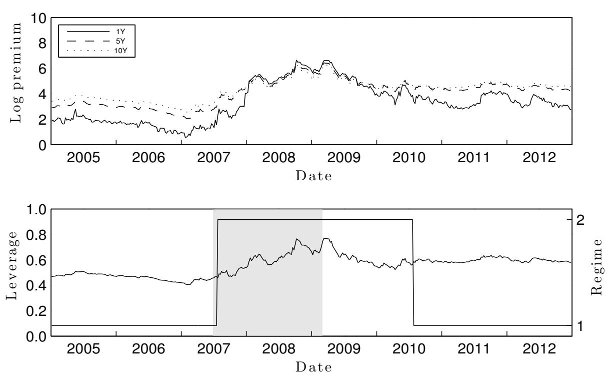
Figure 2: Log premium, filtered leverage and filtered regime.
This figure shows time series of CDS log-premium, filtered leverage and regime for each firm across the portfolio over the period of time starting on January 5, 2005 and ending on December 26, 2012. Shaded areas illustrate the crisis period that extends from June 30, 2007 to February 28, 2009. Note that filtered leverage refers to its standardized value defined as the ratio between $X_t^{(i)}$ and $\theta^{(i)}$.



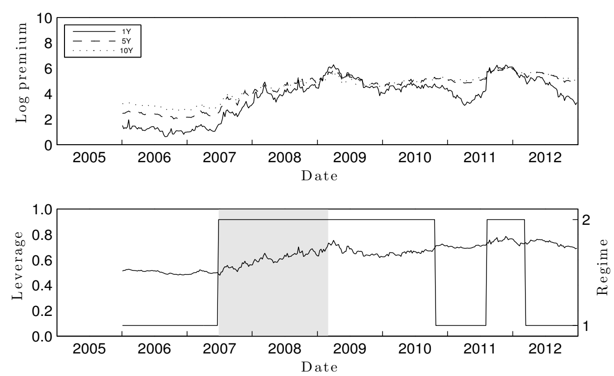
(a) AIG: American International Group



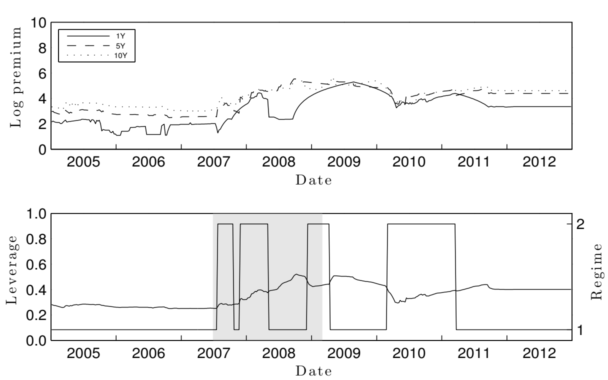
(b) ALL: Allstate Corp



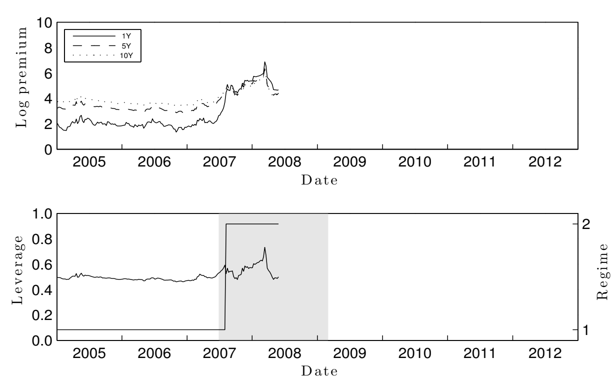
(c) AXP: American Express



(d) BACORP: Bank of America

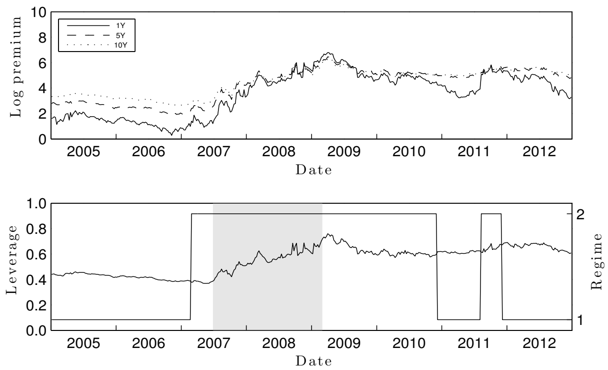


(e) BBT: BB&T Corp

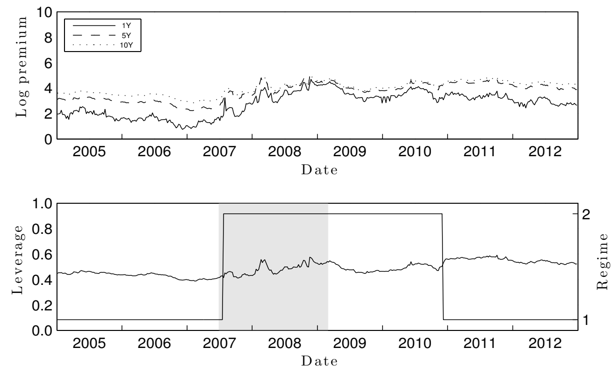


(f) BSC: Bear Stearns

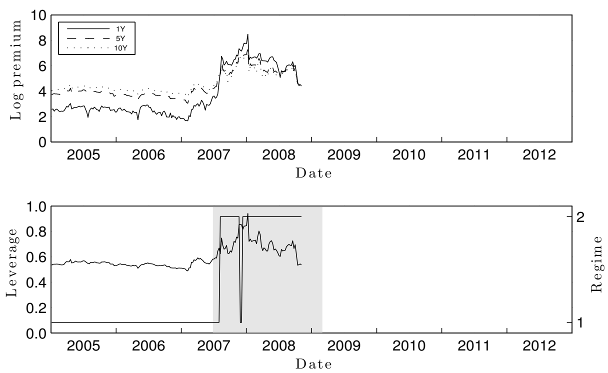
Figure 2: Log premium, filtered leverage and filtered regime (cont'd).



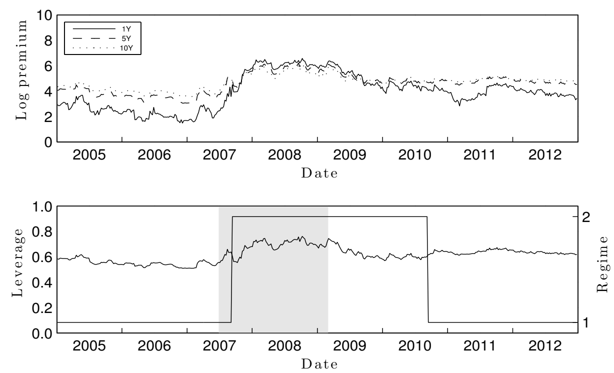
(g) C: Citigroup



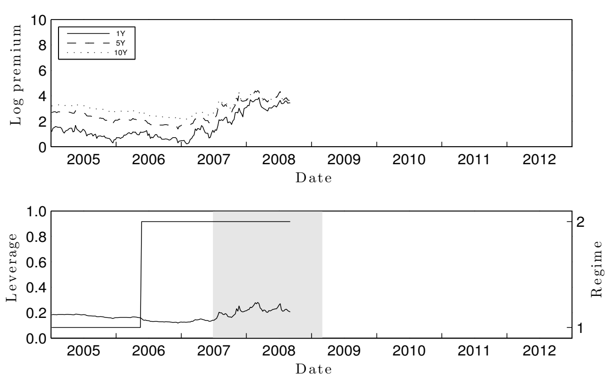
(h) CB: Chubb Corp



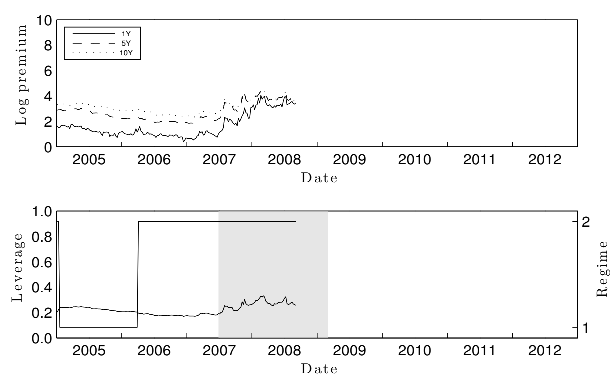
(i) CCR: Countrywide Finl Corp



(j) COF: Capital One Finl Corp

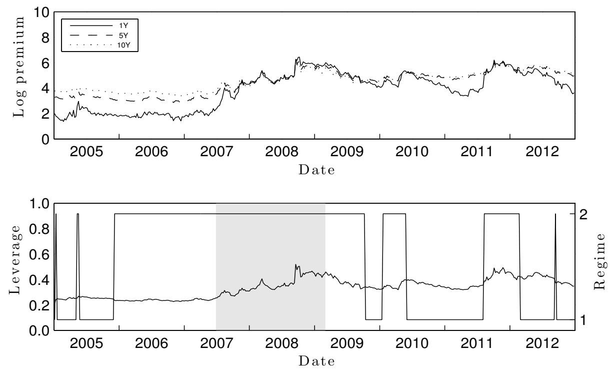


(k) FHLMC: Freddie Mac

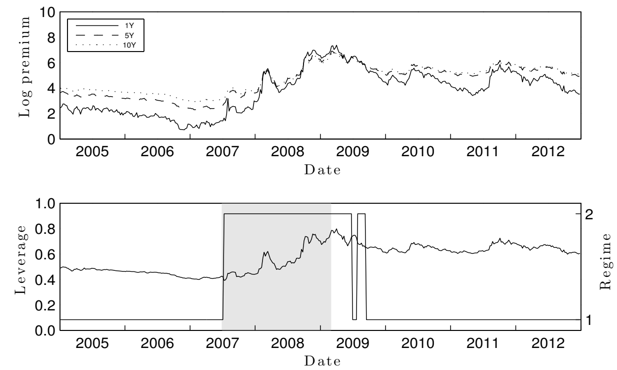


(l) FNMA: Fannie Mae

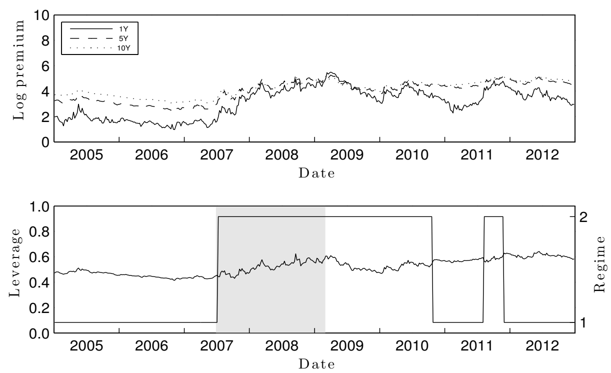
Figure 2: Log premium, filtered leverage and filtered regime (cont'd).



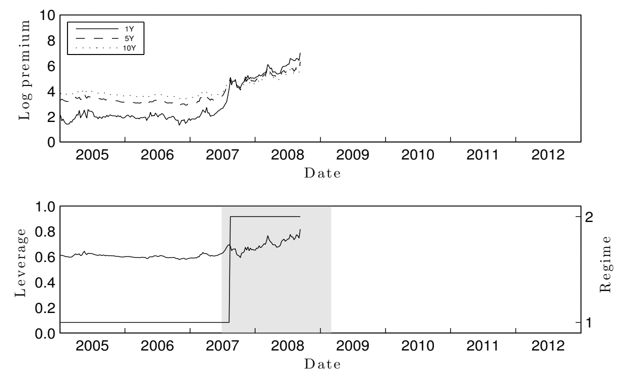
(m) GS: Goldman Sachs



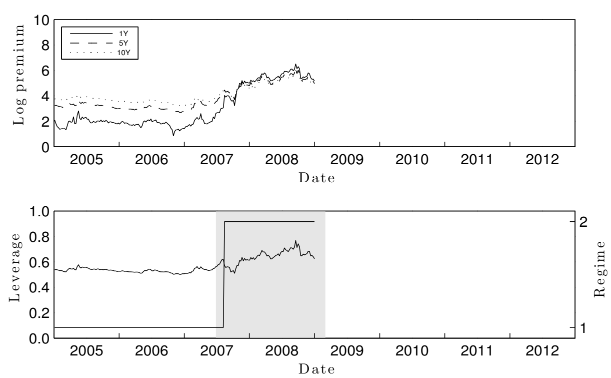
(n) HIG: Hartford Finl Serv Group



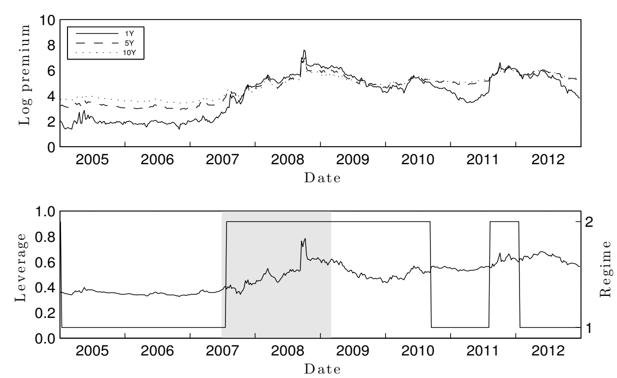
(o) JPM: JPMorgan Chase & Co



(p) LEH: Lehman Brothers

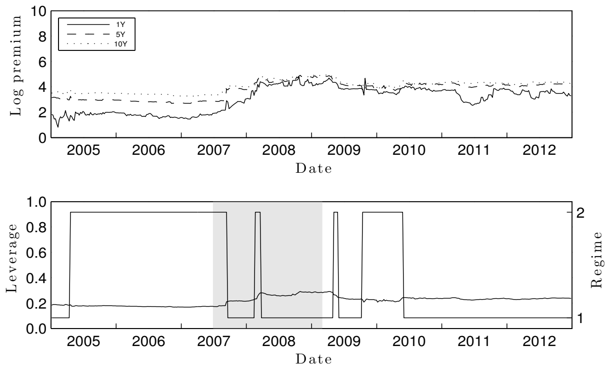


(q) MER: Merrill Lynch & Co

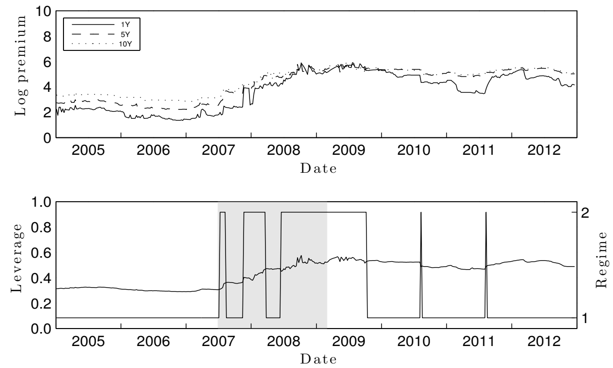


(r) MWD: Morgan Stanley

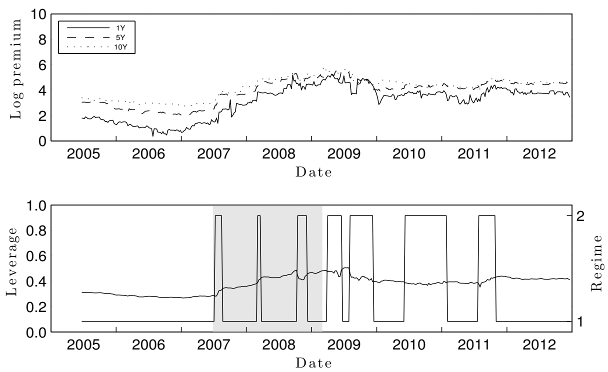
Figure 2: Log premium, filtered leverage and filtered regime (cont'd).



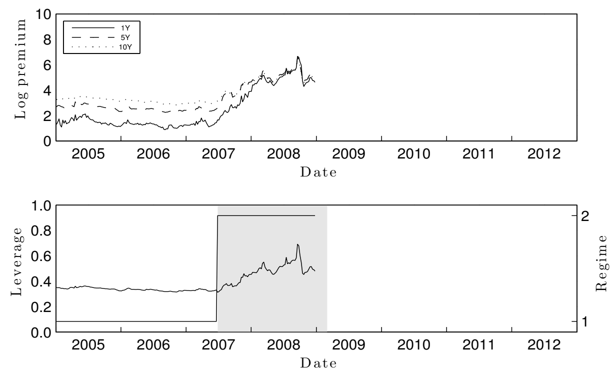
(s) SCH: Schwab Charles Corp



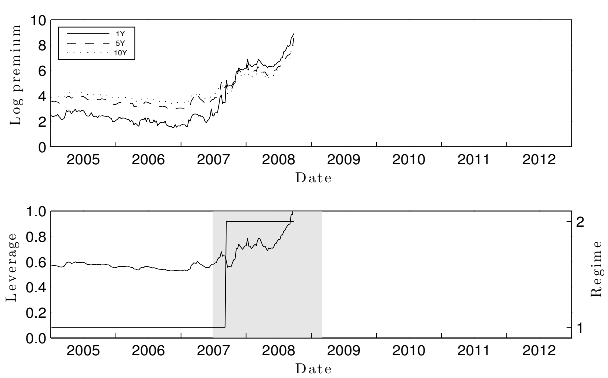
(t) STI: SunTrust Banks



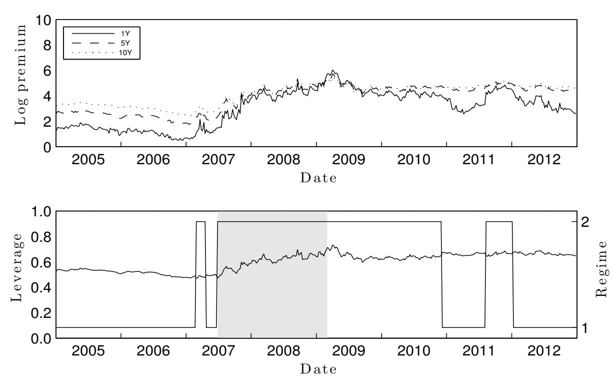
(u) USB: U.S. Bancorp



(v) WB: Wachovia Corp



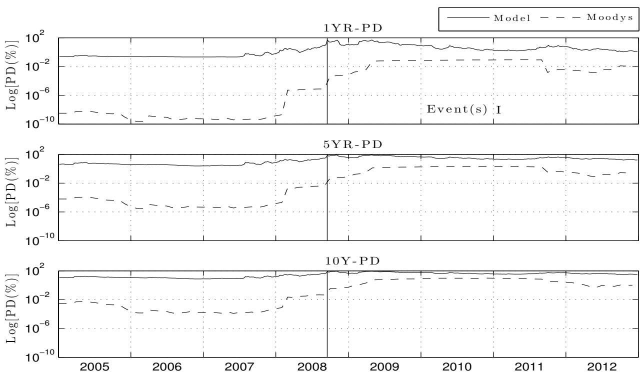
(w) WM: Washington Mutual



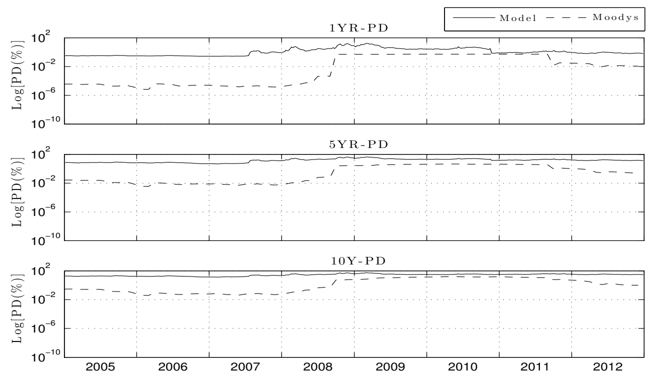
(x) WFC: Wells Fargo & Co

D.2 Probabilities of default

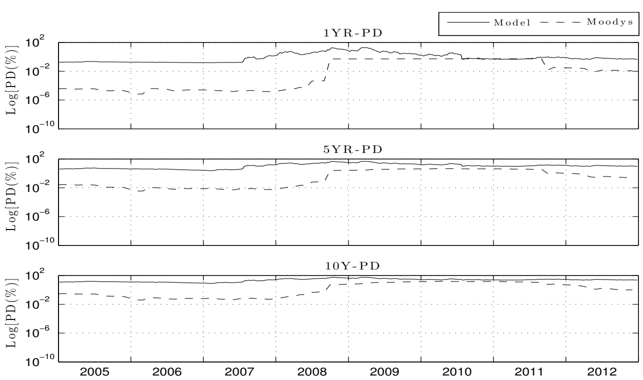
Figure 3: Model versus Moody's probabilities of default.
This figure shows time series of 1-, 5-, and 10-year default probabilities for each firm across the portfolio over the period of time starting on January 5, 2005 and ending on December 26, 2012. Model's time series are inferred from CDS premiums market data. Moody's time series are obtained from monthly transition matrices for banking, finance and insurance industries using the generator estimation approach with window length of 3 years ex ante default data.



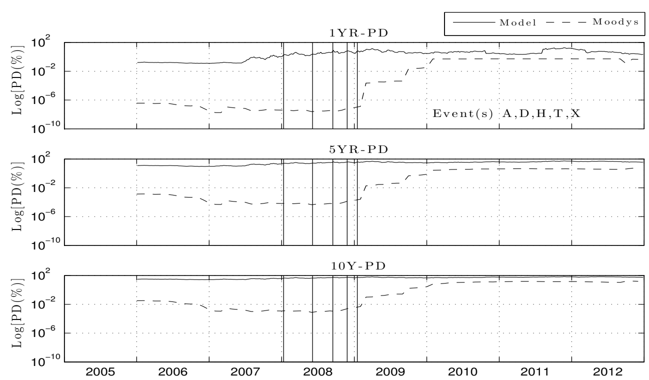
(a) AIG: American International Group



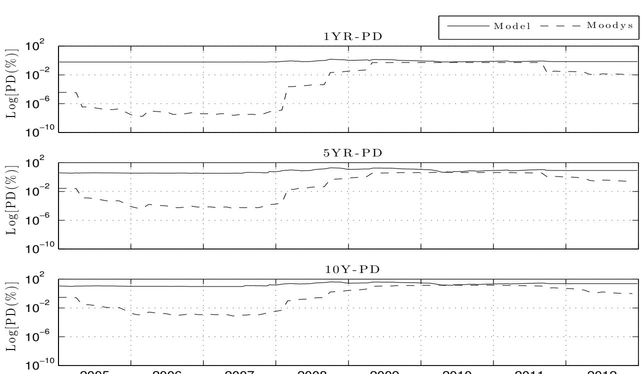
(b) ALL: Allstate Corp



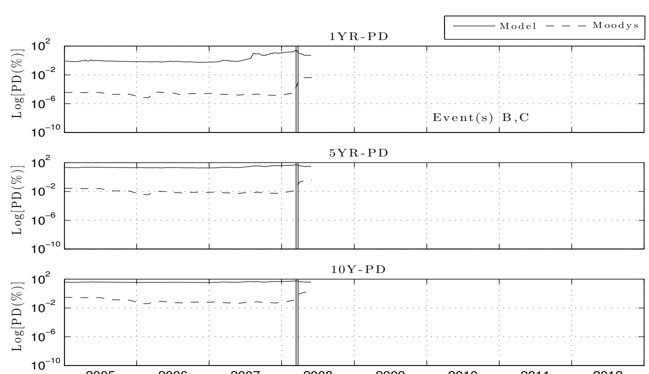
(c) AXP: American Express



(d) BACORP: Bank of America

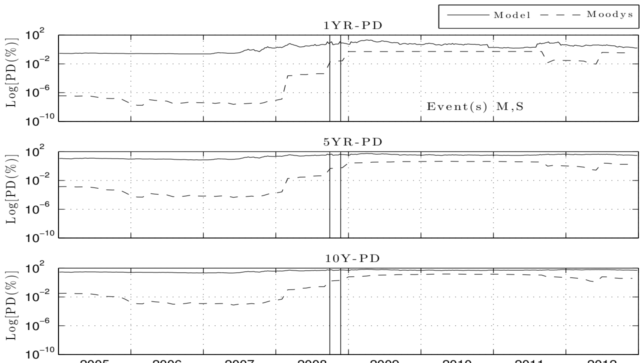


(e) BBT: BB&T Corp

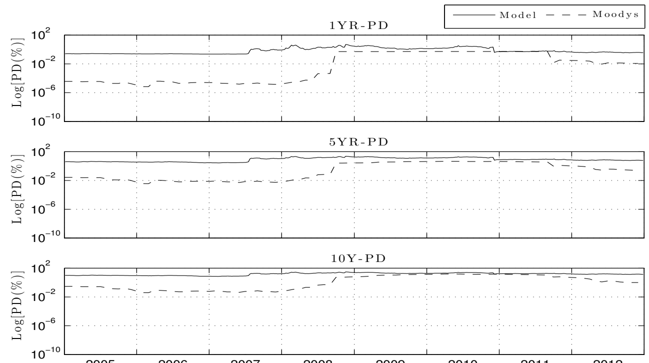


(f) BSC: Bear Stearns

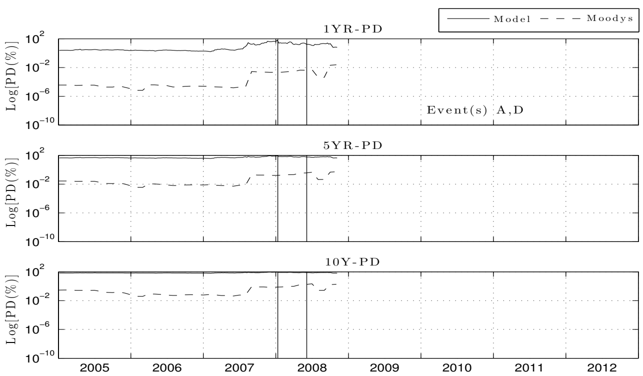
Figure 3: Model versus Moody's probabilities of default (cont'd).



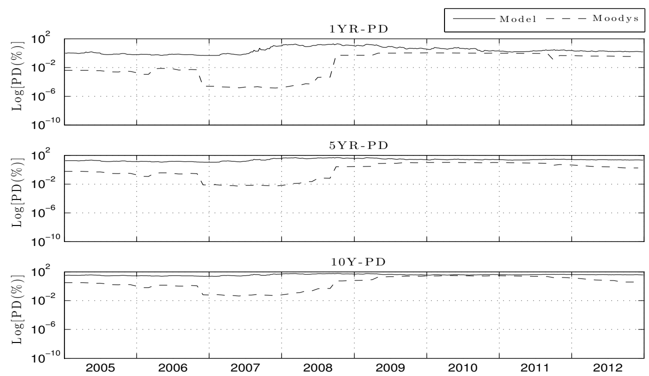
(g) C: Citigroup



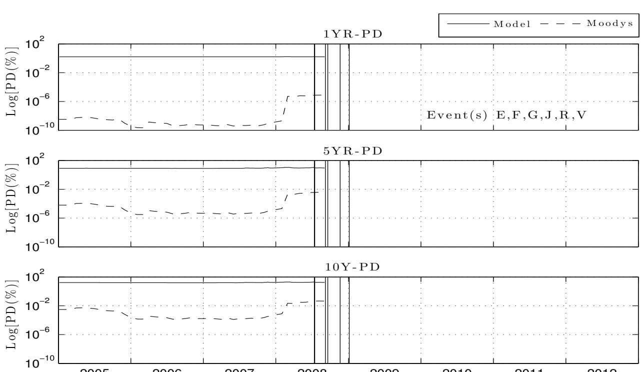
(h) CB: Chubb Corp



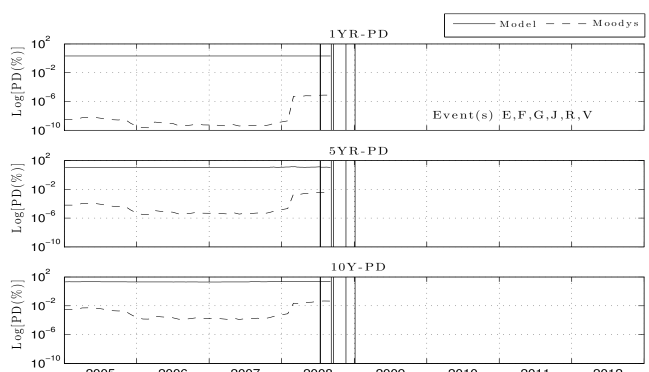
(i) CCR: Countrywide Finl Corp



(j) COF: Capital One Finl Corp

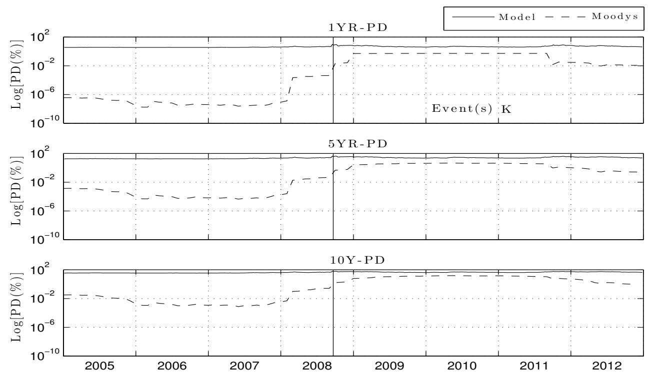


(k) FHLMC: Freddie Mac

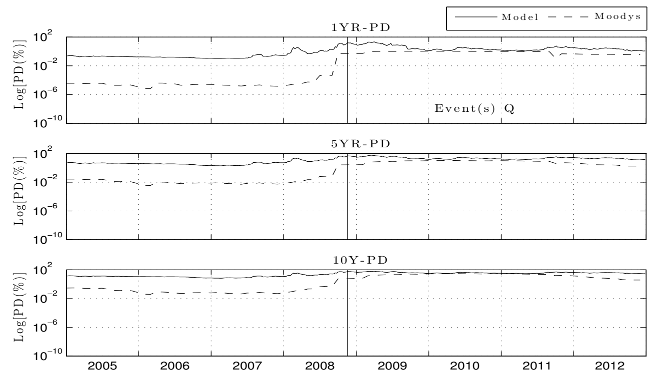


(l) FNMA: Fannie Mae

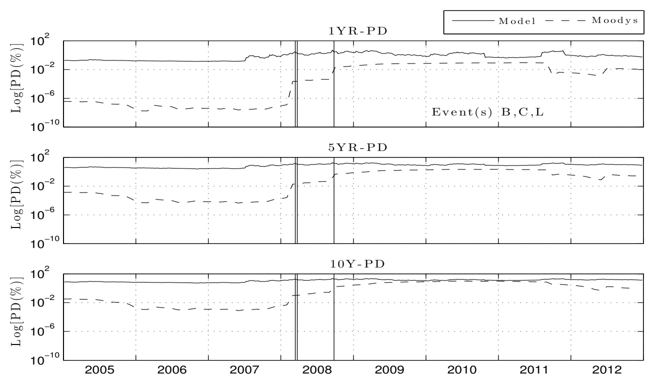
Figure 3: Model versus Moody's probabilities of default (cont'd).



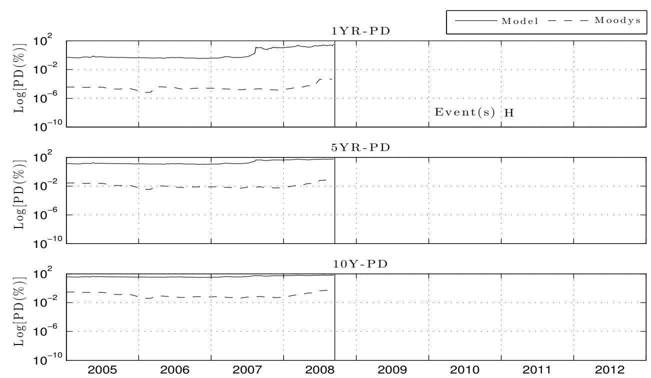
(m) GS: Goldman Sachs



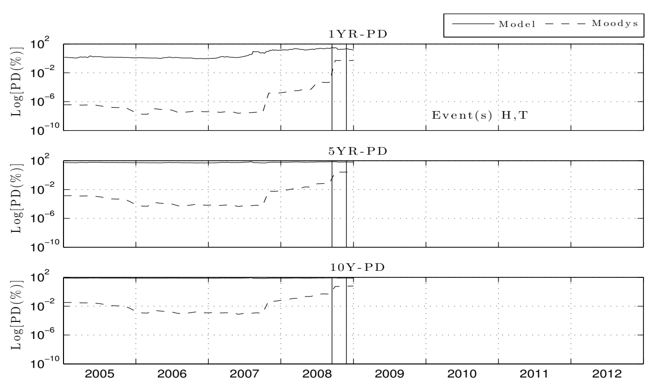
(n) HIG: Hartford Finl Serv Group



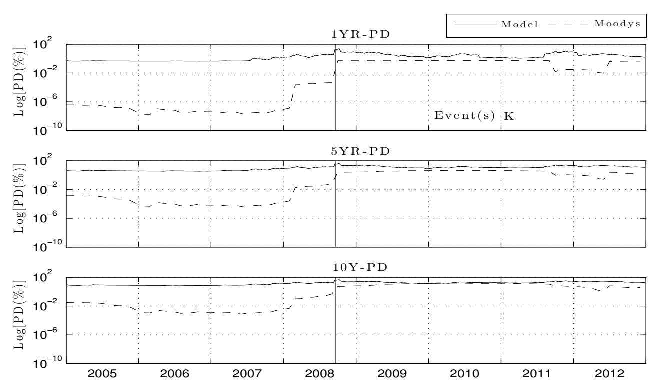
(o) JPM: JPMorgan Chase & Co



(p) LEH: Lehman Brothers

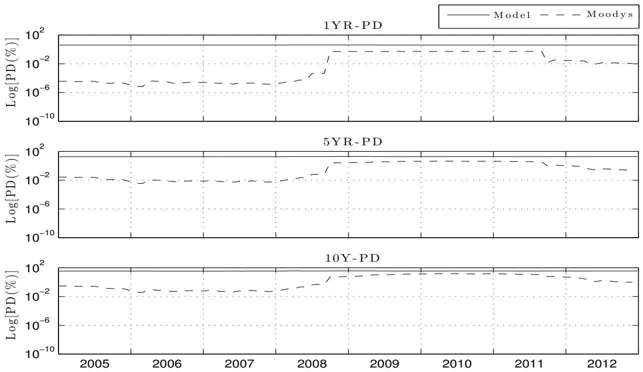


(q) MER: Merrill Lynch & Co

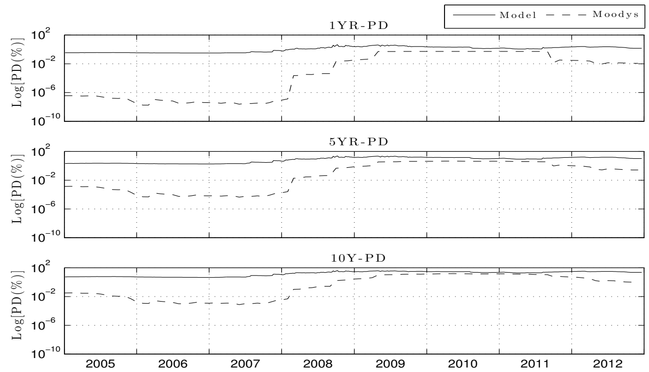


(r) MWD: Morgan Stanley

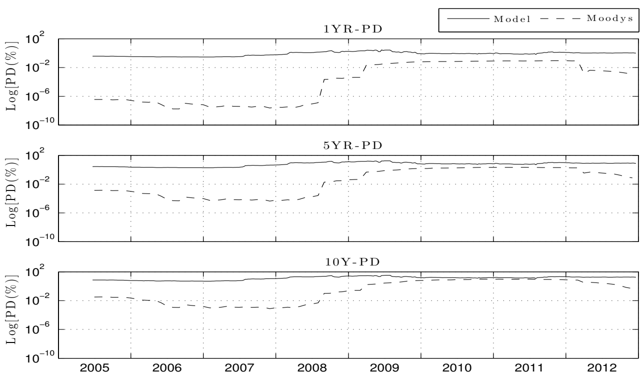
Figure 3: Model versus Moody's probabilities of default (cont'd).



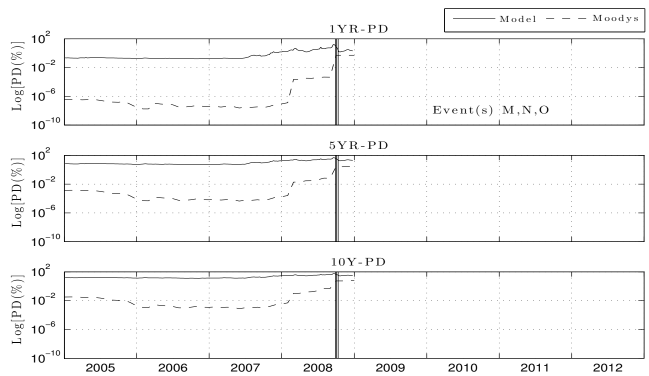
(s) SCH: Schwab Charles Corp



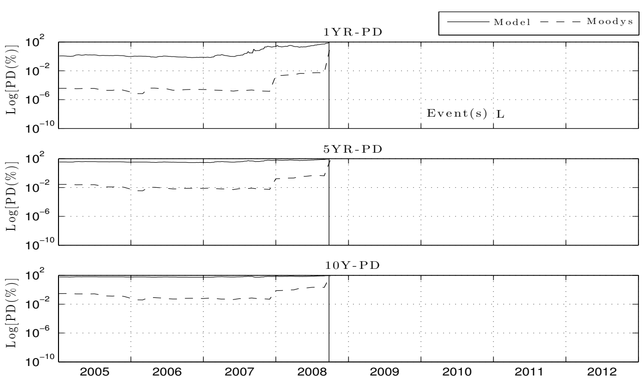
(t) STI: SunTrust Banks



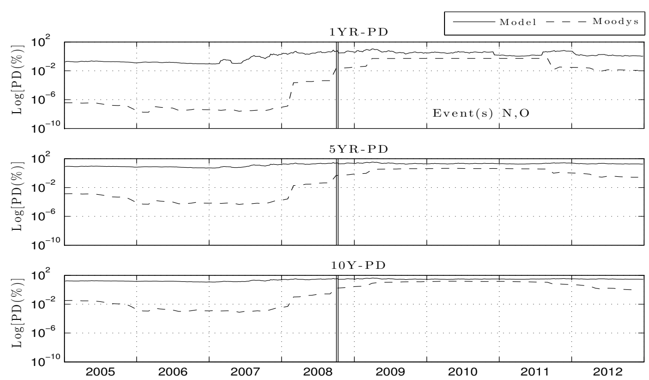
(u) USB: U.S. Bancorp



(v) WB: Wachovia Corp



(w) WM: Washington Mutual



(x) WFC: Wells Fargo & Co

Appendix E Bivariate framework validation

The aim of this appendix is to describe the validation procedure that has been implemented to evaluate the capability of the bivariate approach to estimate correlation coefficients. The first step consists in generating correlated log-leverages between two firms for specific sequences of regime states. Note that regimes are considered persistent since both $\mathbb{P}$ and $\mathbb{Q}$ transition probability matrices are concentrated over the diagonal as reported in Table 2. More precisely, 100 paths of correlated log-leverages have been simulated as per equations (2) and (3) with $K = 2$, $\Delta t = 1/52$, $\rho^{(1,2)}_{\mathbf{s}_t} = (-0.75, 0.30, -0.15, 0.60)$ and firm-specific parameters given in Table 7. Then, the time series of both log-leverages and regimes allow to price the CDS premiums for maturities of 1, 2, 3, 5, 7, and 10 years as described in Appendix A. The second step of the validation procedure consists in using these time series of CDS premiums to endogenously capture the pairwise correlations from inferred log-leverages' co-movements based on the UKF and quasi-maximum likelihood estimation. Table 8 presents the descriptive statistics of the correlation estimates obtained by the validation procedure, while Table 9 shows the detailed results for the 100 paths.

Table 7: Firms' parameters for bivariate framework validation.

Firm		$\hat{x}_{0\|0}$	$\mu^{\mathbb{P}}(\%)$	$\mu^{\mathbb{Q}}(\%)$	σ_1	σ_2	$p^{\mathbb{Q}}_{11}(\%)$	$p^{\mathbb{P}}_{11}(\%)$	$p^{\mathbb{Q}}_{22}(\%)$	$p^{\mathbb{P}}_{22}(\%)$
1. Firm1		0.77	-2.00	-0.50	0.0700	0.2000	99.40	99.00	98.00	99.40
2. Firm2		0.92	2.00	-1.00	0.0500	0.2600	99.00	99.30	92.00	95.00
Firm	α	β (%)	θ	κ	$\delta^{(1)}$	$\delta^{(2)}$	$\delta^{(3)}$	$\delta^{(5)}$	$\delta^{(7)}$	$\delta^{(10)}$
1.	13.00	2.00	1.3700	0.6000	0.2500	0.1600	0.1020	0.0540	0.0025	0.0421
2.	10.00	0.20	1.5000	0.3000	0.1800	0.1000	0.0150	0.0990	0.1007	0.1261

Table 8: Descriptive statistics of the validation procedure's correlation estimates.

	$\hat{\rho}^{(1,2)}_{(1,1)}$	$\hat{\rho}^{(1,2)}_{(1,2)}$	$\hat{\rho}^{(1,2)}_{(2,1)}$	$\hat{\rho}^{(1,2)}_{(2,2)}$
mean	-0.7898	0.3136	-0.1557	0.6122
std	0.0326	0.0807	0.1011	0.0456
min	-0.8594	0.1556	-0.4237	0.5160
max	-0.6837	0.5374	0.1049	0.7276

By means of the validation procedure, one can conclude that the bivariate framework is reliable since estimated results approach theoretical values. Indeed, $\hat{\rho}^{(1,2)}_{\mathbf{s}_t} = (-0.79, 0.31, -0.16, 0.61)$ with standard deviation of $(0.03, 0.08, 0.10, 0.05)$ respectively, while $\rho^{(1,2)}_{\mathbf{s}_t} = (-0.75, 0.30, -0.15, 0.60)$.

Table 9: Correlation estimates of the validation procedure.

Path	$\hat{\rho}^{(1,2)}_{(1,1)}$	$\hat{\rho}^{(1,2)}_{(1,2)}$	$\hat{\rho}^{(1,2)}_{(2,1)}$	$\hat{\rho}^{(1,2)}_{(2,2)}$	Path	$\hat{\rho}^{(1,2)}_{(1,1)}$	$\hat{\rho}^{(1,2)}_{(1,2)}$	$\hat{\rho}^{(1,2)}_{(2,1)}$	$\hat{\rho}^{(1,2)}_{(2,2)}$
1	-0.8133	0.1950	-0.1448	0.6620	51	-0.8176	0.3321	-0.0963	0.6386
2	-0.8322	0.2042	-0.0621	0.5777	52	-0.8291	0.4407	-0.2075	0.6270
3	-0.7747	0.2370	0.1049	0.5445	53	-0.7813	0.1810	-0.3371	0.5649
4	-0.7916	0.1556	-0.1203	0.6658	54	-0.8385	0.2725	-0.0954	0.5428
5	-0.7994	0.2095	-0.0715	0.6158	55	-0.7924	0.3151	-0.2160	0.6272
6	-0.7936	0.3482	-0.1140	0.5962	56	-0.7909	0.2987	-0.3057	0.5978
7	-0.7965	0.3251	-0.2573	0.6768	57	-0.7887	0.2642	-0.1327	0.5779
8	-0.8002	0.3580	-0.2160	0.6815	58	-0.7804	0.3781	-0.2254	0.5935
9	-0.7602	0.2808	0.0034	0.6226	59	-0.7686	0.3160	-0.1486	0.6304
10	-0.7920	0.3788	-0.0634	0.5980	60	-0.7401	0.3894	-0.1579	0.5160
11	-0.8033	0.2990	-0.0695	0.5700	61	-0.7877	0.3320	-0.1391	0.5819
12	-0.8357	0.2306	-0.1814	0.6262	62	-0.7720	0.5374	-0.2775	0.5203
13	-0.8231	0.3247	-0.2453	0.6739	63	-0.7531	0.2738	-0.1674	0.6185
14	-0.7718	0.2468	-0.1633	0.6010	64	-0.7929	0.3612	-0.1898	0.5899
15	-0.8066	0.2868	-0.3406	0.6067	65	-0.8119	0.2634	0.0051	0.5505
16	-0.8070	0.2961	-0.1456	0.5899	66	-0.7798	0.1582	-0.1276	0.5779
17	-0.8225	0.2124	-0.1160	0.6522	67	-0.7549	0.2371	0.0368	0.5853
18	-0.7650	0.3994	-0.4237	0.5523	68	-0.7712	0.3032	-0.1241	0.5918
19	-0.7207	0.2987	-0.3252	0.6268	69	-0.8004	0.3838	-0.3204	0.6018
20	-0.8435	0.2066	-0.2036	0.6859	70	-0.8086	0.3285	-0.1384	0.7046
21	-0.7914	0.3086	-0.0732	0.6478	71	-0.8546	0.3852	-0.0868	0.5969
22	-0.7557	0.2633	-0.2170	0.6066	72	-0.7403	0.2701	-0.1837	0.6078
23	-0.7469	0.4463	-0.3781	0.5342	73	-0.7552	0.4712	-0.2273	0.6394
24	-0.8387	0.3552	-0.2731	0.5863	74	-0.8383	0.2304	-0.1926	0.6083
25	-0.7209	0.2505	-0.1542	0.6302	75	-0.7555	0.3578	-0.2417	0.5738
26	-0.7815	0.4640	-0.0795	0.6389	76	-0.7983	0.3606	-0.2045	0.6610
27	-0.7279	0.3736	-0.0903	0.6371	77	-0.8290	0.3826	-0.1429	0.6042
28	-0.8312	0.3984	-0.0659	0.6125	78	-0.8019	0.4996	-0.1361	0.6237
29	-0.8103	0.2165	-0.2725	0.6574	79	-0.8093	0.3554	-0.1884	0.6425
30	-0.7828	0.3012	-0.1946	0.6874	80	-0.7737	0.3258	-0.1629	0.6678
31	-0.8594	0.3900	0.0378	0.5899	81	-0.8186	0.2927	-0.2478	0.6942
32	-0.7518	0.1779	-0.1842	0.5556	82	-0.7857	0.2830	-0.1146	0.6364
33	-0.7601	0.3452	-0.2256	0.5491	83	-0.7817	0.2940	0.0152	0.5247
34	-0.7834	0.4683	-0.1078	0.5330	84	-0.7453	0.3638	-0.1060	0.6606
35	-0.7656	0.1860	-0.0713	0.5927	85	-0.7954	0.3281	-0.2821	0.6997
36	-0.7791	0.3049	-0.3116	0.6064	86	-0.8035	0.2747	-0.1114	0.6399
37	-0.7832	0.3019	-0.2173	0.6955	87	-0.7923	0.2532	-0.2442	0.6368
38	-0.8363	0.4335	-0.2120	0.6189	88	-0.7881	0.3029	-0.2021	0.6076
39	-0.8120	0.3590	-0.1212	0.6118	89	-0.7512	0.2401	-0.1640	0.6486
40	-0.8152	0.2816	-0.0086	0.6041	90	-0.7934	0.4549	-0.2297	0.5928
41	-0.8511	0.2236	-0.0633	0.6023	91	-0.7934	0.3219	-0.1328	0.6584
42	-0.8027	0.4235	-0.2487	0.5733	92	-0.7726	0.3439	-0.2019	0.5503
43	-0.7588	0.3960	-0.1694	0.6093	93	-0.7564	0.2724	0.0684	0.5842
44	-0.7344	0.1792	-0.2542	0.5800	94	-0.8089	0.1969	0.0467	0.5620
45	-0.7765	0.2983	-0.1240	0.6413	95	-0.8448	0.3395	-0.0127	0.6532
46	-0.8332	0.2115	-0.0099	0.5820	96	-0.8173	0.4269	-0.2373	0.6209
47	-0.8167	0.3532	-0.0664	0.7276	97	-0.7999	0.3888	-0.2772	0.6312
48	-0.7629	0.2612	-0.0020	0.5216	98	-0.7700	0.3145	-0.1415	0.6237
49	-0.7800	0.2638	-0.1477	0.5804	99	-0.7615	0.2772	-0.2043	0.5600
50	-0.8001	0.4129	-0.1076	0.6640	100	-0.6837	0.2408	-0.0862	0.6716

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Conclusion

L'étude empirique présentée dans ce mémoire porte sur la présence de contagion au sein du secteur financier aux États-Unis durant la dernière crise de 2007-2009. Pour ce faire, une extension bivariée est apportée au modèle hybride de risque de crédit avec changement de régime de Bégin et al. (2014). La méthodologie est basée sur l'analyse des corrélations endogènes entre les co-mouvements des ratios d'endettement de 24 grandes institutions financières américaines permettant d'étudier leur interdépendance. Les résultats de l'analyse empirique montrent une augmentation de la corrélation dans le régime de haute volatilité en comparaison avec le régime stable pour 21 firmes du portefeuille. En particulier, les résultats stipulent que Fannie Mae et Freddie Mac sont beaucoup plus corrélés avec les autres entreprises pendant la tourmente, ce qui reflète les liens entre les prêts hypothécaires risqués et la crise du secteur financier américain durant cette période. Les résultats empiriques suggèrent donc la présence de contagion lors de la dernière crise de 2007-2009 au sein du secteur financier aux États-Unis. En plus de la structure de corrélation, le modèle permet également d'estimer les séries chronologiques des probabilités de défaut qui indiquent la tendance du risque de crédit au niveau individuel. En effet, les résultats montrent de plus grandes probabilités de défaut pour les entreprises les plus vulnérables reflétant leurs difficultés financières durant la crise. Pour conclure, les travaux de recherche menés dans le cadre de ce mémoire présentent des implications majeures dans le domaine de la gestion du risque de crédit puisque la présence de contagion financière peut avoir des conséquences importantes dans les portefeuilles sensibles au crédit.