HEC Montréal

Exchange Rates, Value-at-Risk Models, Quantile Regressions and Deviations from Fundamentals

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Science de la gestion

(Économie financière appliquée)

Mémoire présenté en vue de l'obtention du grade de

Maîtrise ès sciences en gestion

(M. Sc.)

Mai 2014

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Abstract

As the exchange rate market trades trillions of dollars on a daily basis, it is a prime candidate for proper risk management. Common practice for financial institutions is to calculate Value-at-Risks (VaRs). We propose using alternative techniques for exchange rate VaRs, which have previously been used for equity risk. We compare GARCH models, models derived from quantile regressions and conditional autoregressive value at risks (CAViaRs). We also include deviations from macroeconomic "fundamentals" as additional regressors to lagged returns, in order to better estimate the VaRs. Our results suggest that CAViaR techniques with proper regressors are appropriate for exchange rate risk as well as using macroeconomic variables as independent variables. Furthermore, our results also seem to support the continued use of semi-parametric approaches to VaR modeling.

Keywords:

Risk management, Value-at-Risk, Quantile regression, Nonlinear regression, Exchange rates, Macroeconomic fundamentals, Backtesting

Sommaire

Le marché des taux de change transige des milliards de dollars en transactions par jour, ce qui justifie une bonne utilisation de pratiques de gestion de risque. Les institutions financières évaluent habituellement le risque en estimant des valeurs-à-risque (VaR). Nous proposons des techniques alternatives de VaR pour le risque de taux de change, qui sont généralement utilisées en modélisant des risques de marchés. En plus des modèles traditionnels GARCH, nous utilisons des modèles à base de régressions quantiles, ainsi que modèles conditionnels autorégressifs de valeur-à-risque (CAViaR). En plus de ces techniques, nous ajoutons des déviations de fondamentaux macroéconomiques aux retards des rendements des taux de change afin de mieux estimer les VaRs. Nos résultats empiriques démontrent que ces techniques alternatives sont en effet appropriées pour le risque de taux de change, ainsi que semblent justifier l'utilisation de variables macroéconomiques en tant que variables indépendantes. Par ailleurs, nos résultats semblent aussi supporter l'utilisation continue de modèles semi-paramétriques dans la modélisation de VaRs.

Mots clés:

Gestion de risque, Valeur-à-Risque, Régression quantile, Régression non linéaire, Taux de change, Fondamentaux macroéconomiques, Backtests.

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Acknowledgements

First and foremost, I would like to thank my director, Professor Simon van Norden. It has been more than a pleasure to work under his supervision and to share ideas with him. Without his guidance, his knowledge and support, this would not have been possible. It has been an incredible experience and working for M. van Norden has allowed me to enrich my academic growth.

I would also like to thank my friends and family. They have supported me for years and continually challenged me on a daily basis. I would like to thank my parents who have always been there for me and allowed me to follow my own path to happiness.

I would like to thank my colleagues who have also been a part of this experience and who helped me get through it all. I have not forgotten the discussions and debates, many of which have been useful to the completion of this work. I would like to single out Sophie, whose help was especially significant, particularly in the early stages.

I would like to thank Emmanuelle who has been incredible through this process, her help, her lending ear and her suggestions have been integral to this completion. Her help was invaluable and will not be forgotten.

Finally, I would like to thank my economics and mathematics teachers from High School. Despite my difficulties, there were encouragements that did not fall on deaf ears and life lessons that have not been forgotten.

Last but not least, I would like to thank the members of the committee for accepting to evaluate this work.

1 Introduction

The importance of effective market risk management practices continues to grow. One of the most popular methods for evaluating risk is with Value-at-Risk (VaR) models. We consider different VaR models for foreign exchange risk. According to the Bank of International Settlements (BIS), trading in the foreign exchange markets averaged more than $3 trillion per day since 2007 and has steadily increased year over year to $5.3 trillion per day in April 2013. Such a high trading volume combined with exchange rate volatility can lead to devastating potential losses. Furthermore, the continued use of carry trade and investments into emerging markets, made profitable in part by the “forward premium puzzle” (Brunnermeier et al., 2008), has continued to increase exchange rate exposure. This thesis will evaluate different VaR models and attempt to find the best suited for foreign exchange risk.

Key innovations in this thesis is the use of quantile regression VaR methods for exchange rate risk. More precisely, we estimate CAViaR models on foreign exchange data with macroeconomic variables as explanatory variables. Quantile regressions have been getting more attention over the last decades since Koenker & Basset (1978) seminal paper and are now widely used in many fields. The concept of quantile regressions is to estimate parameters using the least absolute deviations (LAD) methodology, based on a particular quantile and putting weights on the quantiles. Assuming a given quantile θ, we use a weighted sum of absolute deviations, where a quantile θ is chosen and used for positive deviations while $(1-\theta)$ is then used for negative deviations. There are now many different algorithms to solve the linear programming problem associated to quantile regressions which we will mention in greater detail in sections 2 & 4. A distinctive feature of such models is that they allow for different behaviors for each quantile of interest.

This thesis is therefore based on comparing different VaR models estimated by different techniques. We first include more classical models, such as standard GARCH models. Then, we construct different VaR models based on quantile regressions. Finally, we build on the Conditional Autoregressive Value at Risk (CAViaR) models by Engle & Manganelli (2004), more complex models based on quantile regressions, models that have not been widely studied for foreign exchange rate risk. Also, in the comparison of models, we estimate each one with different regressors, including macroeconomic variables: deviations from fundamental PPP levels, interest rate differentials and exchange market pressures (EMP). Engle & Manganelli use a lag of returns in their market risk analysis and we also include the lag change variable in our different models.

Part of the innovation of this work on exchange rate risk is the addition of macroeconomic factors that influence exchange rates. The purchasing power parity (PPP) has been studied and analysed for many years, a concept that stipulates that the exchange rate between two currencies is directly linked to the prices of a basket of goods. If the PPP holds true, then it should be equivalent to purchase the same number of goods in a foreign country, starting with domestic currency. The general consensus is that it should hold true in the very long run, while having big volatility swings in the short run (Rogoff, 1996). There has also been a lot written on the effects of the deviations from the PPP on exchange rates, which will be further detailed in section 2.

While the CAViaR model is now more widely used and reproduced, it has been done seemingly exclusively for equity risk. There seems to be little to no literature on the use of the CAViaR models in foreign exchange risk. The same can be said about simple quantile regressions in estimating VaRs. While Nikolaou (2008) used quantile regressions with exchange rates, she looked at the individual quantiles and their likelihood of mean reversion. This is an important paper in giving us fuel to pursue quantile regressions as a method for estimating VaR models, as well as wanting to use deviations from fundamentals as a way of potentially observing mean reversion in the tails.

As such, we use the daily changes in different exchange rates over the US dollar from 1995 to 2012 inclusively. In order to ensure that the proper data was available for all of our models, we selected 6 countries: Canada, Japan, Norway, South Africa, Switzerland and United Kingdom. Using the returns facilitates our assumption that the data is stationary and most suitable to manipulate our different models. We use the common PPP formulas in order to find the real exchange rates, formulas that are available in section 3.

After estimating the different models, we put them through extensive backtesting methodology. We use a combination of different tests that are both regularly used and more suited for the use of quantile regressions. We decided to use the traditional ones established by Kupiec (1995), Christoffersen (1998) as well as the more recent tests by Engle & Manganelli (2004) and Gaglianone et al. (2011). The former (Unconditional, Independent and Conditional coverage tests) are the more common backtests used and the latter (Dynamic Quantile and VaR by Quantile Regression tests) are adapted for quantile regressions, but not exclusive to them. We will document them further in section 4.

The results seem to confirm the usefulness of quantile regressions in VaR modeling both with and without the inclusion of deviations. As well, the results indicate that the use of deviations from fundamentals in VaR modeling may be more effective when modeling by quantile regressions rather than with traditional GARCH models, where we see little additional impacts by the deviations. In addition, the deviations from fundamentals as regressors in quantile regressions seem to perform fairly well in estimating VaRs, especially at the 1% level, despite certain caveats that will be explored.

Furthermore, when observing more complex quantile regression models we see that the CAViaR model seems to perform the best and outperforms the IGARCH, a traditional method of estimating VaRs. Additionally, at the 5% and 1% quantiles, the model that performs the best according to our criteria, is the model that includes the lagged returns and the deviations from the fundamental PPP. Moreover, while this does not seem to be the case at the 20% and 10% quantiles, it may in fact indicate a stronger importance of the effects of fundamentals on the tails of the returns.

All in all, we demonstrate that it may be useful to use semi-parametric techniques to estimate VaRs rather than parametric methods. Furthermore, we demonstrate the usefulness of additional regressors that may prove to be especially relevant in estimating particular quantiles of exchange rate risk.

The rest of the thesis is organized as follows. In the next section, we will go over a literature review of general VaR models, present the CAViaR models and their extensions, as well as previous research on deviations from fundamentals and different types of market pressure. The third section will demonstrate in further detail our data. The fourth section will show the models in more depth as well as the methodology used to estimate the different models. Finally, section 5 presents empirical results to the study, followed by a brief conclusion.

2 Literature Review

In this following section, we will introduce previous work that has been the basis of this thesis, as well as information pertinent to our models and methodology, in addition to our assumptions. We begin with an overview of VaRs in general, followed by a presentation of the CAViaR models and a few extensions. Then, we continue with a review of empirical work done on exchange rates. Within that subsection, we first mention deviations from the PPP, continue with a review of exchange market pressure (EMP) and finish with a description of potential conflicts between different types of investors.

2.1 VaR

As mentioned, VaR models are now the most common measure for risk management, by financial institutions and their regulators. VaR is a measure of how much a certain portfolio can lose in a given time period at a given confidence level. We obtain VaR_t such that:

$$\Pr\,[r_t \;<\; VaR_t \mid \Omega_t] = \theta \tag{1}$$

where r_t are returns and Ω_t denotes the information set available at time *t*. Also, with the equation and our notation, it is clear the VaR[1] is essentially calculating the θ th quantile of r_t based on Ω_t.

Models to calculate VaRs may also be separated into two different classes. The first class is the parametric approach, such as RiskMetrics (1996) for example, which must model the conditional volatility. The use of a GARCH or especially an IGARCH model is often used. However, in order to properly forecast using this method, there is still the need to assume a certain distribution of the shocks, which is difficult to do accurately. The second class is a non-parametric approach, which does not necessarily assume any particular distribution. However, it usually makes the assumption that the returns are iid, something

[1] We will use a negative VaR as opposed to a positive one.

that is also uncommon. This class of technique requires many observations to be able to measure accurately the probability of extreme but rare events. As both classes have their pros and cons, the methods which will be generally used throughout this paper is a compromise between these two types of models: a semi-parametric approach based on the work by Engle & Manganelli (2004) called CAViaR and quantile regressions[2].

2.2 Conditional Autoregressive Value at Risk by Regression Quantiles (CAViaR)

Engle & Manganelli (2004) build on quantile regressions in a way that allows them to directly model the evolution of the quantiles $Q_t(\theta)$, without having to first estimate the distribution. They thus proposed a conditional autoregressive quantile specification for value at risk, CAViaR. The general specification of the CAViaR model is the following:

$$Q_t^{CAV}(\theta) = \beta_0 + \sum_{i=1}^{k} \beta_i Q_{t-i}(\theta) + \sum_{j=1}^{r} \beta_j h(x_{t-j}) \quad (2)$$

where h is a function of the regressor and X_{t-j} is a vector of independent variables, the notation being adapted for this paper. The autoregressive terms of the quantile allows the VaR to change smoothly over time. In their adaptation of the CAViaR model, Engle & Manganelli (2004) provide four different versions of the model but we will first focus on two: the adaptive and the asymmetric slope. We expect that negative returns should have a greater impact on the VaR estimates than positive returns, a result that would seem expected by prior research. Engle & Manganelli (2004) demonstrate this effect on their equity data with News Impact Curves (Engle & Ng, 1993). For a given estimated VaR, the News Impact Curves (NIC) show how the VaR changes as the regressors vary. Engle & Manganelli (2004) find that there is a strong asymmetry in the NIC of the asymmetric slope VaRs, which supports our hypothesis that negative returns might have a greater impact on the VaRs than positive returns.

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[2] There is also the semi-parametric approach by extreme value theory (EVT), for which VaR models try to find the specific behavior of large returns and looks to provide a parametric overview of the distribution of the extremes. There has been quite a bit of work on EVT (see among others, Danielsson & de Vries, 2000; McNeil & Frey, 2000; de Jesus, Ortiz, & Cabello, 2013) but it will not be the focus here.
Engle & Manganelli (2004) also show that their method is preferable to the EVT methodology due to unresolved assumptions that are made.
See Engle & Manganelli (2004) for a more technical justification of quantile regressions vs EVT.

The adaptive model is on that is capable of adjusting based on whether the VaR threshold was hit or not. It is defined as:

$$Q_t^{CAV}(\theta) = Q_{t-1}^{CAV}(\theta) + \beta\,\{[1 + exp(\kappa[r_{t-1} - Q_{t-1}^{CAV}(\theta)])]^{-1} - \theta\} \qquad (3)$$

where κ is a positive finite number, which allows the model to be a logistically-smoothed version of a step function. As a functions, whenever the series exceeds the VaR, its level should be increased. However, when the series does not exceed the VaR, its level should be decreased slightly.

The asymmetric slope model simply allows for asymmetric impacts of the returns, where the positive and negative returns can have different effects:

$$Q_t^{CAV}(\theta) = \beta_0 + \beta_1 Q_{t-1}(\theta) + \beta_2 \max(r_{t-1}, 0) + \beta_3\, min(r_{t-1}, 0) \qquad (4)$$

The parameters of either model are then estimated as the solution to:

$$\min_{\beta} \frac{1}{T}\sum_{i=1}^{T}[\theta - I(r_t < Q_t(\theta))]\,[r_t - Q_t(\theta)] \qquad (5)$$

where I is the indicator function. This can be minimized using many different methods such as the interior point algorithm proposed by Koenker & Bassett (1978) or simplex algorithms.

However, while it is our assumption that the two previous models would be the best of the four CAViaR models, the other two are nonetheless estimated and are: the symmetric slope model and the GARCH type model. The difference with these last two CAViaR models is that they treat positive and negative returns with simple equal weights as opposed to the first two. We continue with the section where we will also further justify our preference for the first two models. The equations for the other two CAViaR models are shown in section 4, Eq. (9) and Eq. (10).

In using the CAViaR models, we ensure that each quantile $Q_t^{CAV}(\theta)$ is dependent on its lagged quantile. This allows to mitigate shocks and not to over adjust the VaRs in response to potential shocks observed by regressors.

Engle & Manganelli (2004) then demonstrate how CAViaR estimators are also consistent and asymptotically normal, using earlier proofs where White (1994) established that nonlinear regression quantiles are consistent in both iid and stationary dependent cases[3].

2.3 Extensions of CAViaR

One of the main extensions to the use of CAViaR models is for volatility forecasting. Taylor (2005) describes the models from which he builds his research and further justifies the use of CAViaR over using GARCH models, based on Poon & Granger (2003) who show that the latter fail to properly account for tail thickness, even when using a Student t distribution with maximum likelihood. Taylor (2005) mentions that the simplest way to obtain the volatility forecasts is to proceed with two steps. The first, is to estimate the quantile parameters of the quantile models (for example by using the CAViaR models) and the second is to plug the results into the corresponding expressions for the volatility forecasting.

In his work, Taylor (2005) focuses on individual stocks and stock indices and like Engle & Manganelli (2004), he uses lagged returns. In the end, according to his work, the best results were achieved by the asymmetric slope CAViaR model, which demonstrates the different impacts between the positive and the negative returns on the VaRs. Engle & Manganelli (2004), however, reported their best results from the adaptive model. Both of these models demonstrate the notion that returns do not follow symmetric properties when estimating the VaRs. Other authors, such as Nikolaou (2008), Huang et al. (2011),

[3] For further explanation and proofs, see the Appendix in Engle & Manganelli (2004).
Taylor (2005) also shows how the CAViaR model has the same robust properties as simple quantile regressions.

Altavilla & de Grauwe (2013) and de Jesus, Ortiz, & Cabello (2013), have also noted the different impacts of positive and negative returns with exchange rates. This reinforces our original assumption of focusing primarily on the adaptive and asymmetric slope models.

Huang et al. (2011) built off the work of Taylor (2005) but looked to apply the CAViaR model towards exchange rate volatility[4]. To differentiate exchange rates from other financial assets, they point out that exchange rate volatility seems to be more volatile for emerging markets than for developed countries[5] and that there can be bodies at work that look to stabilize price movements[6]. This also reinforces the idea of using different types of countries and their currencies[7]. The authors found that overall, on average, the regressions by quantile with adaptive function and their weighted composition perform the best, as the model had the highest explanatory power. However, while their results seem encouraging, the authors mention that no model was overwhelmingly more appropriate than the others in estimating the volatility.

Jeon & Taylor (2013) also provided an extension to the CAViaR model, also using it with implied volatility, in order to predict volatility and VaR estimation. They extended the model in order to add a predictive variable, which is also what we are looking to do. Their results are encouraging as an additional regressor may help in the predictability of the models and can further support this thesis as we also look to add additional regressors to the CAViaR models. An interesting result of their work on the S&P 500, is that the explanatory power of the implied quantiles increased as they got further into the left tail but decreased as they got further into the right tail. Again, this is further evidence

[4] They tried to model a uniformly spaced series of quantiles, which they believe allows them to follow the whole distributional pattern rather than only reflect the tail behaviors. This could be interesting for further research but was not used for this paper.
[5] This result was also reported by Rufino & de Guia (2011), who used different models.
[6] Nikolaou (2008) presents and shows other articles that speculate on Central bank intervention.
Vigfusson (1997), Altavilla & De Grauwe (2010) among others, on the other hand, offer the alternative of a sort of game between different types of exchange rate traders (trading): fundamentalits and chartists (technical analysis).
[7] While we use a mix of currencies, comparing different blocks, or types of currencies may be a notable extension to this work.

supporting our assumption that there may be a greater impact by negative returns as opposed to positive returns in the estimation of VaRs.

2.4 Fundamentals and exchange rates

In this section, there are two subsections, each generally developed historically and demonstrating a certain evolution in the literature, pertinent for this thesis. The first subsection will reflect the nature of the deviation from the fundamental levels of the PPP and its effects on exchange rates. We begin by mentioning different views on the importance of different impacts to the exchange rates by deviations from the PPP. We then move on to articles that identify different portions of the models used in this thesis, to establish the utility of using the deviations as a regressor on changes in exchange rates. At the end of the subsection, we also focus on two articles that explain why we choose to observe different quantiles and their expected effects on the deviations from fundamental levels.

The second subsection will touch on different types of market pressures. We begin by presenting exchange market pressure, a technique attempting to model pressures in the exchange rates. Then, we mention the market pressures that ensue from two different classes of investors: fundamentalists and chartists. Establishing that there are different types of investors, we are trying to further justify the importance of observing results at different quantiles. This will allow us to expect different results by quantile and be able to make links to other areas of research that could extend this work, such as bubbles and crises.

2.4.1 Deviation from fundamentals - PPP

There is a lot of information linking the use of economic fundamentals to exchange rates, some of it conflicting, as it often is. Engel and West (2005) reported that it was difficult to find evidence that economic fundamentals, such as real money balances, outputs, interest rates, etc. can Granger-cause exchange rates. However, they acknowledge that other papers, such as MacDonald & Taylor (1994), Mark (1995), Mark & Sul (2001) and Groen (2000) seem to find some evidence in the predictability of the exchange rate by the use of fundamentals, most notably through long horizons and panel data. There is nonetheless a seemingly general consensus that there is little to no linear relationship between exchange rates and fundamentals.

Taylor, Peel, & Sarno (2001) wrote a widely-cited article demonstrating the nonlinear relationship between exchange rates and their deviation from the PPP[8]. They look at the real exchange rate and the level of mean reversion when it gets far from its fundamental levels. Their results demonstrated that the level and speed of mean reversion is faster when it is estimated in a nonlinear fashion than when it is estimated linearly. Their results also first concluded that in their limited testing, there seems to be a stronger sense of mean reversion when the deviations are further from the fundamental levels.

Kilian & Taylor (2003) find some evidence of a nonlinear relationship between exchange rates and economic fundamentals[9]. Their evidence was strongest in the longer run (2 to 3 years) and less clear over shorter horizons. They also found that there seems in fact to be a nonlinear mean reversion tendency in real exchange rates. They note that the mean reversion is particularly strong when exchange rates are far from their fundamental values and weak to non-existent when closer to their fundament levels, in a case where the real exchange rate is a function estimated by nonlinear least squares. We can thus hope to see the deviations have an effect on the estimation of VaRs, at least in the outer quantiles where the deviations from fundamental levels are bigger and there may be greater pressure of mean reversion.

Nikolaou (2008) used quantile regressions in order to demonstrate the behaviour of real exchange rates in more depth. Like other authors, she looked at the possibility of mean

[8] They look at Smooth Transition Autoregressive (STAR) models and their variations, LSTAR, ESTAR.
[9] Their work also focused on forecasting the real exchange rate with nonlinear models.

reversion but noted the possibility of asymmetries in the mean reversion process, which could differ by quantiles. She mentions that this could be due to interventions by central banks[10]. Her results are similar to those of Kilian & Taylor (2003) in that she also found evidence of mean reversion as a whole. However, the use of quantile regressions allowed her to decompose the mean reversion process by quantile, thus where the distribution of the mean reversion process is nonlinear.

Her work is based on two methods, one that observes the size of the shock effect (semi-parametric) and the other for the size of both the shocks and the deviation effects (non-parametric)[11]. The first is based on a standard autoregressive, AR(1)[12] model that she then turns into a quantile autoregressive, QAR(1) model, which she writes in the form of Koenker & Xiao (2004)[13]. Her second model is a method of quantile smoothing splines with total variation roughness penalty (Koenker et al., 1994)[14].again leads us to believe

Nikolaou (2008) found that in her results there is some evidence of mean reversion, but its presence was not detected in every quantile. In fact, the results demonstrate that mean reversion is mainly present in the outside quantiles. In the middle quantiles, the estimators would statistically and consistently be results close to unity – indicating the possible presence of unit roots, and thus not rejecting the idea that a random walk model could be the most efficient model. This also seems to confirm that the presence of a mean reversion process would usually take place when there are big shocks or when the deviations get further from their fundamental values. Furthermore, Nikolaou (2008) was also able to demonstrate a possible asymmetric relationship in the mean reversion process. This asymmetric relationship is established by the fact that even with the presence of

[10] See further down and the work by Altavilla & De Grauwe (2010).
[11] The introduction of a non-parametric model is to offer greater flexibility within each quantile and allow for nonlinearity.
[12] $r_t = \alpha r_{t-1} + \varepsilon_t$, where $r_t = q_t - \mu$ with q_t the log of the real exchange rate and μ its unconditional mean. She also follows the typical definition for the real exchange rate where $q_t = s_t - p_t + p_t^*$, where s_t is the log of the nominal exchange rate and p_t and p_t^*, respectively the log of the domestic and foreign prices.
[13] $Q_t(\theta) = \varepsilon(\theta) + \beta Q_{t-1}(\theta)$ in our notation where $\varepsilon(\theta)$ captures the magnitude of the shocks.
[14] This problem is a trade-off between "fidelity" and "roughness" for a certain level of smoothness.

symmetric (positive or negative) shocks, their impacts appear asymmetric. This was shown by the positive shocks (depreciation) that seem to demonstrate stronger mean reversion properties than by the negative shocks (appreciation)[15]. All in all, the author demonstrates the advantages of using quantile regressions, further establishes the potential importance of deviations from the PPP at different quantiles and the greater impacts of negative returns over the impact of positive returns.

Finally, Altavilla & de Grauwe (2010) again looked for nonlinearity in the relationship between the exchange rate and fundamentals. This nonlinear relationship is a case where the expected value of the exchange rates could be explained in part by macroeconomic variables. Continuing on the extensions proposed by Taylor, Peel, & Sarno (2001) and by Sarno et al. (2007)[16], Altavilla & de Grauwe (2010) also look to include theoretical framework. The basis of their work is to include different agents that forecast future exchange rates differently: with fundamentals, and with charts (technical analysis). They then chose to focus on three fundamental values: the relative GDP, the relative inflation rate and the interest rate differential. They first built a standard linear vector error correcting model (VECM) of the form:

$$\Delta x_t = c + \sum_{i=1}^{k-1} \Gamma_i \Delta x_{t-i} + \Pi x_{t-k} + \Upsilon \varepsilon_t$$

$$x_t = [y_t \; \pi_t \; i_t \; e_t]'$$

where Γ, Π and Υ are estimators, y_t is the GDP differential, π_t is the inflation rate differential, i_t is the short term interest rate differential and e_t is the euro-dollar exchange rate. Also, $\Pi = \alpha\beta'$ where α and β are m by n matrices, where m is the number of variables and n is the number of cointegrating relationships, containing the adjustment coefficient and the cointegrating vector.

[15] This could be due to central bank intervention.
This could also be due to heterogeneous agent models (HAMs), a review in Hommes (2006).

[16] They used a Markov-switching vector error correcting model (MS-VECM). There was an earlier paper by Vigfusson (1997) that also used a Markov regime-switching approach between chartists and fundamentalists.

In their nonlinear model to analyze exchange rate dynamics, they found three different regimes and demonstrate the model as follows:

$$\Delta y_t = c(s_t) + \sum_{i=1}^{k-1} \Gamma_i(s_t)\Delta y_{t-i} + \alpha(s_t)\beta' y_{t-k} + \Upsilon(s_t)\varepsilon_t$$

where s_t is a latent state variable that allows to capture the different regimes in all the terms: the intercept, the autoregressive coefficients, the speed of the adjustment component in the cointegration matrix and the variance-covariance matrix. In their analysis, the regime-dependent cointegrating vector provides information about the long-term relationship between the exchange rate and economic fundamentals, and its evolution across different periods of time.

The results by Altavill & de Grauwe (2010) demonstrate that for each of the three different regimes, there is a specific conclusion and impact. In the first regime, a shock to the equilibrium is adjusted by the GDP differential and the interest rate differential. The first regime also seems to coincide with a period where there has been an appreciation of the exchange rate. In the second regime, only the exchange rate has an impact on the equilibrium and thus not any economic fundamentals. In the third regime, a shock is adjusted by the inflation rate differential and interest rate differential. The last regime also seems to coincide with a period where there has been an exchange rate depreciation. The authors also find that with this framework, the exchange rates seem to also be affected by economic fundamentals in the short and medium run. However, this leads the authors to conclude that when the exchange rates are close to their fundamental values, exchange rate movements are not based on economic fundamentals, rather it is when they are based on chartist speculation, expectations and self-fulfilling beliefs. Once again, this further emphasises the observations of different quantiles and the subsequent impacts of the deviations from fundamentals.

Similarly to Nikolaou (2008), Altavill & de Grauwe (2010) find that a random walk model is more efficient when the exchange rate is close to its equilibrium value but that the nonlinear models are more accurate when the exchange rates are far from their fundamental value. Two of their three regimes could be directly linked to fundamental levels, while the other would be little or unaffected by the fundamentals. These results further support the idea of the presence of a nonlinear relationship between exchange rates and fundamentals. It would seem that when the deviations are far from their fundamental value, they have a greater impact on exchange rates, something that we hope to capture in modelling VaRs with quantile regressions.

These are a few of the reasons why we are using economic fundamentals in trying to estimate the CAViaR model in connexion with exchange rates of various countries. The use of this technique will allow us to obtain a nonlinear look at the different quantiles based on the exchange rates and economic fundamentals, while not having to contend with estimating distributions. Also, being able to use a time varying model could not only help with the link between exchange rates and economic fundamentals, but also be important for the calculation of VaRs.

2.4.2 Market Pressures

In this subsection, we look at two different types of market pressures. We begin with a review of a few different ways of analysing exchange market pressures with regards to exchange rates. Then, we focus on a type of pressure that confronts different types of investors. We turn our attention to fundamentalists and chartists, which includes technical analysis.

2.4.2.1 Exchange Market Pressure (EMP)

Exchange market pressure was pioneered by Girton & Roper (1977), which tackles the issue of shocks to foreign exchange markets. In theory, the EMP should usually be close to, or at zero. As the exchange rates suffer a greater deviation from their market levels, there may be an adjustment made by reserves and absorbed by the EMP. We speculate that the adjustment can come from countries themselves, or from the market pressures between investors, which will be developed in the next subsection. The first model of EMP was summarized by Hall et al. (2013) as:

$$EMP_t = -(-\Delta s_t + \Delta Res_t)$$

where Δs_t is the log change in exchange rate and $\Delta Res_t = \frac{S_t res_t - S_{t-1} res_{t-1}}{M_{t-1}}$ is the change in reserves (minus gold) in regard to the money supply M_t. EMP can be referred to as a magnitude of money market disequilibrium (Weymark, 1998) and Balakrishnan et al. (2011) establish that the IMF monitors EMP as one of its five components to measure financial stress. We follow the work by Aizenman & Pasricha (2012)[17] and define EMP for this exercise as:

$$EMP_t = \left(\frac{s_t - s_{t-1}}{s_t} - \frac{res_t - res_{t-1}}{res_t}\right) * 100 \qquad (6)$$

This method was also used by Aizenman & Hutchison (2012) and Feldkircher et al. (2013), the former who established that it had a high significant correlation rate (0.63) with the Weighted EMP used by the IMF's World Economic Outlook (2009) [18].

As EMP is a measure of financial stress, we hope to establish a link between EMP and the changes in exchange rates, in the estimation process of VaRs. We may expect to see a greater impact on the outer quantiles of the VaRs, as EMP is more likely to capture the greater swings of the changes in exchange rates.

2.4.2.2 Fundamentalists and Chartists

The issue of the two classes, fundamentalists and chartists, has been studied for some time. Frankel & Froot (1986) were among the first to emphasize the differences between the two. They also started to investigate the possibility of speculative bubbles in exchange rates and the ability of switching investment strategies (Frankel & Froot, 1988, 1990). Kirman (1991, 1993) then continued the work of Frankel & Froot (1986) with the addition of a micro-foundation of asset demand. He concluded that in periods where the market is

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[17] We used the nominal exchange rates as opposed to nominal exchange rates against Special Drawing Rights (SDR). Also, the reserves can be scaled for total money supply minus gold.

[18] Some EMP models choose to include interest rates which we did not. Among other reasons, Tanner (2001) establishes that interest rates can be considered more of a response variable than an indicator.

driven by fundamentalists, the values are more stable and will be pushed towards fundamental levels. However, when the market is driven by chartists, the exchange rate is driven by a stable, random-walk process[19]. Based on subsection 2.4.1, we hypothesize that the market is more likely to be driven by fundamentalists in the tails of deviations from fundamental levels. We will continue to develop this thought with the following articles.

Continuing on the switches between chartists and fundamentalists, Vigfusson (1997) used a Markov regime-switching approach to try and further model what Frankel & Froot (1988) began. Concentrating on the Canada-US exchange rate, Vigfusson (1997) found some evidence confirming the separation of the fundamentalists and the chartists. He contends that a moving average (MA) model outperforms the autoregressive (AR) model for chartists but finds no evident model preference for the fundamentalists. He also mentions how it could be pertinent to use ARCH effects in order to further improve his model specification.

Brock & Hommes (1998) developed an adaptive belief system (ABS) to continue to model the different strategies[20]. They took three agents: fundamentalists, optimists and pessimists, who can invest in risky or risk-free assets[21]. The authors demonstrated that even when there are no information costs to fundamentalists, chartists tend to cause different 'bifurcations' where different steady states arise deviating from fundamentals. Furthermore, the changes between states becomes irregular and seems to be caused by chartists.

[19] Kirman (1993) mentions a parallel between ants and the markets, how investors are not always rational, even when there are no noise traders.

[20] An ABS is a standard discounted value asset pricing model derived from mean-variance maximization, with the possibility of heterogeneous beliefs and the ability of being formulated in terms of deviations from fundamentals, see Hommes (2006).

[21] Here, both optimists and pessimists are types of chartists.

Hommes (2006) also points out the importance of realising and explaining that the fundamentalists and chartists also switch between their different strategies. He expresses that the switches seem to be led by expected or realized excess profits. Goodhart (1988) mentioned that this asymmetry may push certain traders towards chartist strategies. Also, Boswijk et al. (2007), found further evidence insisting that the performance of the chartists' strategy incites more investors to choose that method over fundamental analysis, even forcing some to switch strategies. These factors could seemingly further explain the 'forward premium puzzle', one that permits positive expected gains from carry trade, where certain currencies do not seem to revert back toward their uncovered interest parity (UIP) fundamentals (Brunnermeier et al. (2008). The author also observed that this could be consistent when the deviations are small from the fundamental levels. This further supports our idea of also including possible models with interest rate differentials. We would thus expect that the UIP would behave in a similar manner than the PPP in regards to affecting changes in exchange rates.

Boswijk et al. (2007) expanded on the work by Brock & Hommes (1998) with a model based on the deviations from market fundamentals[22]. Also investigating the differences between a linear and a nonlinear world, they found that in the nonlinear world, the mean reversion process begins later than in a linear world in response to good news. The investors seem to have a greater tendency to overreact and continue to overreact in the periods following the news. Furthermore, they also have the ability to neglect or forget the news and focus more and more on chartist strategies while neglecting fundamentals. While the fundamentalists continue to expect the prices to fall (increase), the chartists can continue to drive the market.

We can continue to link the previous articles to little bubbles, as mentioned by Boswijk et al. (2007). When the exchange rates are close to the fundamental levels, their changes are seemingly unpredictable and can be affected in a self-fulfilling manner, which can

[22] Working with stock prices, they used the simple Gordon model and a dynamic version of the model.

lead them to continue to deviate from fundamental levels. However, once the exchange rates gravitate far from their fundamentals and the mini-bubble bursts, the exchange rates will return (faster) to their fundamental levels, as if a shock had sent them back to their fundamental levels. Concentrating on the bubble effect, if all the investors decided that the exchange rates were too far from their fundamental levels, the investors could immediately burst the bubble and return the exchange rates closer to their fundamental value. However, investors prefer to continue to take advantage of potential gains until the last possible moment. This is a known issue with bubbles that is due to available information and common knowledge (Abreu and Brunnermeir, 2003). However, Boswijk et al. (2007) also contend that the further the absolute deviation from fundamental levels, the higher the probability that the bubble collapses[23]. Finally, the authors mention that while the bubbles are usually triggered by the allure of short term profits, there is the odd possibility of creating a much longer lasting bubble. Boswijk et al. (2007) mention the Dot-com bubble as an example of a long lasting bubble and how its probability of bursting kept growing.

[23]For further analysis, see van Norden & Schaller (1999), who also mention the greater presence of speculative behavior when observed in a nonlinear world and the greater chances of collapse of a bubble when the deviation from the fundamental levels is high in the previous period.

3 Data

The time series that are used to estimate the different models are available from Bloomberg and The Organization for Economic Co-operation and Development (OECD). We found pertinent data for the following 7 countries: United States of America (USA), Canada (CAD), Japan (JAP), Norway (NOK), South Africa (SA), Switzerland (SWZ) and United Kingdom (UK). The observed sample window is from 1995 to 2012, inclusively. This allows us to have a fairly large sample that can include different economic periods, including the 2007-2008 crisis that saw a flight towards safe haven currencies such as the American dollar (Brunnermeir et al., 2008). All the VaRs that are estimated from the changes in exchange rates are reported on a daily basis and were calculated using all, less 1000 observations that were reserved for out-of-sample testing. This leaves us with an out-of-sample window from March 2009 to December 2012. In using 1000 observations, we are doubling the out-of-sample size used by Engle & Manganelli (2004) in their estimation of CAViaR models on stock market data. In the figures that are shown, the vertical black line separates the in-sample and the out-of-sample testing.

3.1 Regressors

First, we obtained from Bloomberg spot exchange rates (S = country/USA), where USA is the domestic country and as such, an increase in the exchange rate S, is a depreciation of the other country's currency and an appreciation of the USA dollar (USD). We then used the log of the exchange rates ($s = \ln S$) and took the first difference to find the changes in exchange rate ($r_t = \Delta s$).

Second, the price levels for each country are necessary to construct the real exchange rate and find the deviations from the PPP. We obtained the consumer price index (P = CPI) for each of the countries from the OECD statistics library. The CPI for a given month was used for daily results for the particular month. Once the daily results were available, we again used the log of the prices ($p = \ln P$). From these first two series, we can construct the log of the daily real exchange rate $q_t = s_t - p_t + p_t^*$ where the * denotes the foreign currency. Then, from the real exchange rate, we obtained the deviations from the PPP as $d_t = q_t - \mu$ where μ is the mean of the real exchange rate.

We introduce two other regressors that required different series. First, we introduce exchange rate market pressure (EMP) as a regressor. We use the model from equation (6) that requires a country's reserves and their exchange rate. This information was also gathered from the OECD database on a monthly basis and interpolated to a daily basis[24].

Lastly, we want to observe certain effects from interest rates. As we presented a few articles that note a certain importance in changes in the UIP and changes interest rate differential. Thus, we want to observe certain deviations from interest rates and the UIP. We obtained daily interest rates for the USA from the Wharton Research Data Service (WRDS). However, it was necessary to estimate the interest rates for the remaining countries using the covered interest rate parity (CIP)[25].

Being unable to obtain daily forward rates for the next day, we use 30 day forward rates on exchange rates since they are available daily. Since short-term maturity rates are highly correlated, results are not expected to be sensitive to this choice. This allows us to obtain the interest rate differential between the USA and a given country:

$i_{t-1} - i_{t-1}^*$[26].

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[24] Once again, this poses little problem as a whole. However, in the case of fixed currencies this would not be the best solution as the reserves could evaporate in a matter of days rather than over months. Nonetheless, as the currencies that are observed are classified as de facto independent floating currencies by the IMF, changing the monthly rates into daily rates should not cause any problems. There are also no FX crisis in the sample.

[25] $F_{t+1} = S_t \frac{1+i_t}{1+i_t^*}$ where i_t is the USA's interest rate, i_t^* is the interest rate for the other country and F_t is the forward rate of the exchange rate at the next period.

[26] We also tried the opposite relation in difference and the results were largely unaffected. The main difference was in the asymmetric model where the positive and negative Betas were inverted.

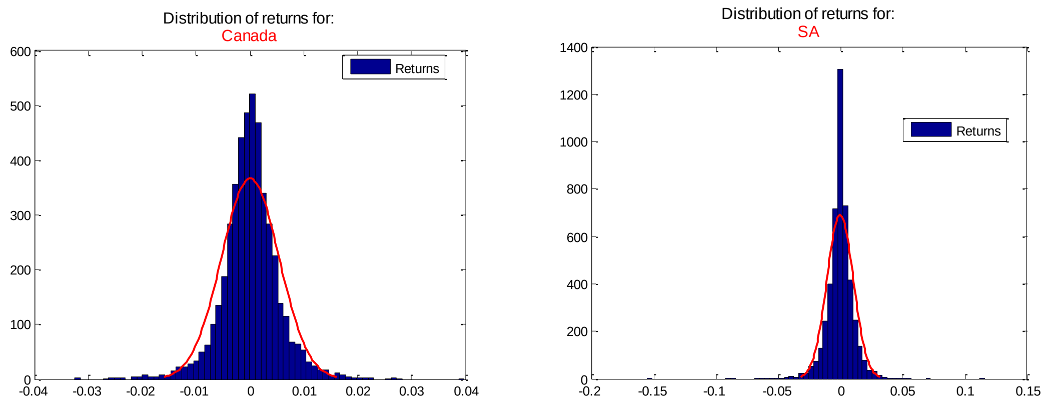
We present in Table 1 a few noteworthy statistics over the full sample. The remaining descriptive statistics for the four different series are available in Appendix A1. The statistics are available for each country.

TABLE 1: SELECTED CURRENCY DESCRIPTIVE STATISTICS

Data Statistics						
	Canada	Japan	Norway	South Africa	Switzerland	United Kingdom
Statistics						
Mean returns	7.40E-05	3.18E-05	4.26E-05	-1.85E-04	7.77E-05	8.27E-06
Mean deviations from PPP	1.18E-05	-6.02E-05	-6.65E-06	-7.04E-05	-2.20E-05	1.47E-07
Mean int. rate diff.	4.05E-05	0.0028	-0.0007	-0.0064	0.0019	-0.0008
Mean EMP	-0.0239	-0.0453	-0.0092	-0.0746	-0.0407	-0.0129
Kurtosis returns	6.9106	8.7618	5.8180	20.6512	10.9931	5.4205
Kurtosis deviations from PPP	1.7490	4.0082	2.6935	3.9909	2.5207	2.4367
Kurtosis int. rate diff.	6.8610	5.6211	3.8796	104.8185	19.0758	4.6241
Kurtosis EMP	156.3495	31.0328	80.7173	257.6080	324.1652	125.8357
K-S / J-B tests	Rejects null	Rejects null	Rejects null	Rejects null	Rejects null	Rejects null

Note: The ** denotes a rejection at 5%.
The K-S test is the Kolmogorov-Smirnov test of normality. The J-B test is the Jarque-Bera test of normality.

FIGURE 1: DISTRIBUTIONS



We can see from Table 1 that with both the Kolmogorov-Smirnov and the Jarque-Bera tests, for all the series we reject the null that the series are normally distributed. This further fuels our hypothesis that it is best not to require assumptions on distributions of returns and their errors. We can also see from Table 1 & Fig.1 that the returns (and all the other series) have a high kurtosis and do not quite fit the normal distribution. We also notice that all the series have a mean very close to, or at zero. From Fig.1 we also see that the SA returns can vary more than the CAD returns, but continue to be mainly around zero. While the other currencies are not shown, they all follow similar distributions that gravitate around zero but have longer tails than normal distributions.

FIGURE 2: QQ PLOTS

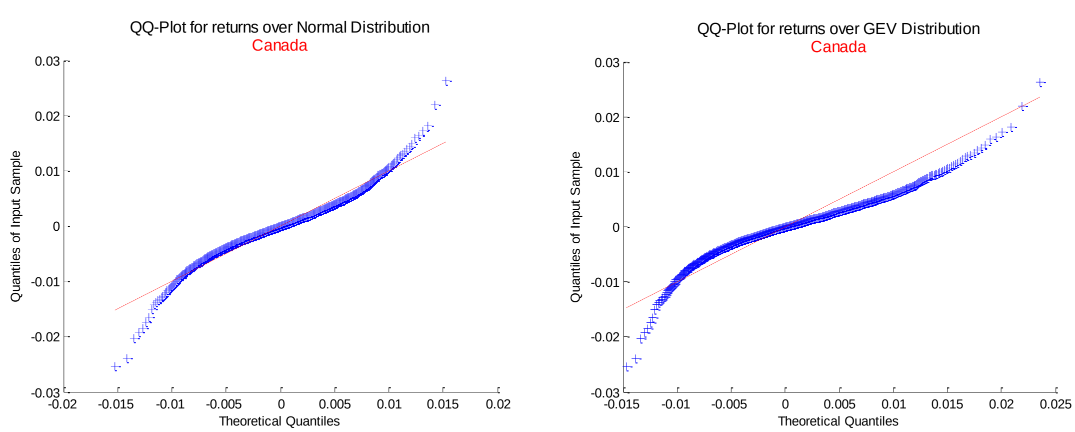


Fig.2 shows two QQ-plots for CAD/USD[27] and we can notice that they are very similar to the exchange rates of the other 5 countries. We can see that over the normal distribution, the returns are quite close in the middle quantiles but very far away in the outer quantiles. In contrast, with the Generalized Extreme Value (GEV)[28] distribution, we see that it manages to capture some of the positive outer quantiles but does poorly with the negative outer quantiles.

[27] These graphs are very similar to those of the other five countries.
[28] A generalization between the three extreme value distribution families: Gumbel, Weibull, Fréchette, it can emulate them depending on its shape parameter ξ. Here the GEV parameters were estimated by MLE with *gevfit* from MATLAB. We obtain a shape parameter of -0.14, a scale parameter of 0.006 and a location parameter of -0.002

4 Models and Methodology

4.1 Models

The final model that has not yet been presented is the GARCH type VaR model. Let us assume that the returns follow a simple AR(1) model:

$$r_t = \alpha_0 + \alpha_1 r_{t-1} + u_t \text{ where } u_t \sim iid$$

A traditional GARCH model will account for time-varying volatility clustering in the error term as follows:

$$u_t = \sigma_t \epsilon_t \text{ where } \epsilon_t \sim iid\ N(0,1)$$

$$\sigma_t^2 = \gamma + \sum_{i=1}^{q} \alpha_i u_{t-i}^2 + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2$$

where u_t is the error term of the AR(1) model. Once we estimate the volatility, in order to construct the actual VaR, we have that:

$$VaR_t = r_t - \Phi^{-1}(\theta) * \sqrt{\sigma_t^2} \qquad (7)$$

where $\Phi^{-1}(\theta)$ is the inverse of a normal cumulative density function (cdf) for a given θ.

As the most common used variable to model returns is a lag of returns, it is the basis of our models and we build on the presented AR(1) model. We then add additional regressors:

$$r_t = \alpha_0 + \alpha_1 r_{t-1} + \sum_{i=2}^{z} \alpha_i X_i + u_t$$

where X_i can be a vector of the different regressors that can include the deviation from its fundamental PPP level, the interest rate differential and the EMP, and z is the total number of regressors. In our models, we use a traditional GARCH(1,1) that follows:

$$\sigma_t^2 = \beta_0 + \beta_1 u_{t-1}^2 + \beta_2 \sigma_{t-1}^2$$

In order for the GARCH model to be weakly stationary, the sum of $\beta_1$and β_2 must be smaller than 1. We also use the IGARCH method which resembles RiskMetrics (1996) and is almost identical to the GARCH with the exception that $\beta_1$and β_2 sum to 1. The IGARCH(1,1) will be the basis of comparison for our results.

Next, we have the standard quantile regressions (QR), which do not require any assumptions about the actual distributions, as opposed to the parametric GARCH methods. Following our notations, the general model of a quantile regression in order to model a VaR is:

$$Q_t^{QR}(\theta) = \alpha_0(\theta) + \sum_{i=1}^{h} \alpha_i(\theta) X_i + u_t \qquad (8)$$

where X_i can be a vector of the regressors, including the lagged returns and α_i each of the coefficients of the quantile regression. This is the general model that we will follow for quantile regressions, from which we can see that:

$$VaR_t = Q_t^{QR}(\theta)$$

We estimated 14 different VaRs by quantile regressions with the different combinations of the regressors presented and results are available upon request as more interesting results are presented in the Appendix.

Finally, there are the traditional CAViaR models. Despite our original focus on two models, all four models were estimated at each quantile and for the different independent variables. The tables of other CAViaR estimations with certain additional regressors are available in Appendix A2. We defined the general CAViaR model in Eq.(2) and the VaRs are obtained in similarly to those of estimated by quantile regressions:

$$VaR_t = Q_t^{CAV}(\theta)$$

As we have presented the asymmetric slope model in Eq. (4), we now show the remaining two models, the symmetric slope (Eq. 9) and indirect GARCH models (Eq. 10):

$$Q_t^{CAV}(\theta) = \beta_0 + \beta_1 Q_{t-1}(\theta) + \beta_2 |r_{t-1}| + \sum_{i=3}^{w} \beta_i \, |X_i| \qquad (9)$$

$$Q_t^{CAV}(\theta) = (\beta_0 + \beta_1 Q_{t-1}^2(\theta) + \beta_2 (r_{t-1}^2) - \sum_{i=3}^{w} \beta_i \, (X_i^2))^{1/2} \qquad (10)$$

In these models, there is always the lagged changes of the exchange rates as omitting this variable caused abnormal VaR results[29]. Finally, while the adaptive model was presented in Eq.(3), adding extra variables may not be as obvious as for the other models:

$$Q_t(\theta) = Q_{t-1}(\beta) + \beta_0 \{[1 + exp(\kappa[r_{t-1} - Q_{t-1}(\theta)])]^{-1} - \theta\} + \sum_{i=2}^{w} \beta_1 \, \{[1 + exp(\kappa[X_{t-1} - Q_{t-1}(\theta)])]^{-1} - \theta\} \qquad (11)$$

4.2 Estimation Methodology

In order to estimate the models, MATLAB was used exclusively. So as to estimate the GARCH models, we used the function *estimate* after previously specifying the choice of model. However, as there is no integrated function to model an IGARCH, we used Kevin Sheppard's econometrics toolbox. Then, in order to forecast the volatility and calculate the VaRs, we used Eq.(7).

In regards to the quantile regression, there is no formal code provided by MathWorks. We therefore wrote a function that would minimize Eq.(5). We decided to utilize the Nelder-Mead Simplex algorithm provided by MATLAB's *fminsearch* as well as a quasi-newton optimizing algorithm *fminunc*[30].

Next, we were able to use the code from Engle & Manganelli (2004) for the CAViaR. However, their code was designed for two quantiles (1% and 5%) and for the single use of the lagged returns. We thus built on their code in order to accept the additional regressors, the additional quantiles and to ensure that it was robust with exchange rate

[29] In Appendix A2 there are the results for individual independent variable, but not any combination of variables that do not include the lagged returns.

[30] both of which are the algorithms used by Engle & Manganelli (2004)

returns[31]. We continued their method of using C to run the loops to compute the recursive quantile functions of the CAViaR models. Thanks to their code, we also followed their general method of optimizing the models that we discuss below.

After extensive verifications, when estimating quantile regressions, with our data as well as with Engle & Manganelli's data, we confirm that by simple use of the simplex algorithm or even both the simplex and the quasi-newton algorithm, in certain cases may be insufficient to achieve results as precise as ones obtained by different minimization methods. Furthermore, as Engle & Manganelli alternated in their estimation of the CAViaR model between the two algorithms: *fminsearch* and *fminunc*, we have ultimately verified that MATLAB will often find a false convergence, obtain results that are inferior in precision to other methods. In the end, the use of these algorithms must be with caution. The combination of the two algorithms in MATLAB can eventually yield the most precise results possible, however it can be difficult knowing when, especially if enough precautions are not taken. We discovered that it is usually in extreme quantiles that there can be the greatest problem, which can be partially solved by continuing to alternate between optimization methods, despite the software thinking it has already found results that converge[32]. In order to properly verify our suspect quantile regressions results, we compared the results with a linear programming method we wrote using *linprog* from MATLAB, as well as using other statistical software such as Stata and R, for which Roger Koenker provided his quantile regression code.

We first encountered this issue using the code provided by Engle & Manganelli (2004) without any modifications, but using our returns. While MATLAB was providing results that indicated convergence, in reality, it did not always find appropriate estimators. We found these anomalies with robustness checks by adding second and third best estimators

[31] Their direct code did not always converge for our data, even for just lagged returns.
[32] It does not seem possible to use the same algorithm alone as it will not continue to search for a better result.

found by the model, and discovered significant discrepancies[33]. In the end, in order to ensure that the program had finally properly converged towards the right estimators, it was necessary to begin with a greater number of initial conditions. For the symmetric, asymmetric, GARCH and adaptive CAViaR models, Engle & Manganelli (2004) used m initial conditions = [10, 15, 10, 5] for the four models respectively. We increased the number of initial conditions for the four models to m = [100, 25, 100, 25][34]. We also ensured that the algorithms were used at least twice as many time in order to feel satisfied with the results. To confirm that the proper estimators were found, we used our robustness verification and ensured that the three best estimators were (almost) identical.

4.3 Backtesting

Backtests, also known as diagnostic tests, are used to establish the validity of VaR models. Their goal is to test the null hypothesis, that the models are properly specified based on a given θ and thus to see if the models properly approximate the conditional quantile. The first mainstream backtests were the coverage tests by Kupiec (1995) and Christoffersen (1998). Their three tests (unconditional, independent and conditional coverage) were mainly based on binary variables, whether or not there is a "hit" and the loss exceeds the VaR or not. We can define the hit sequence H_t as follows:

$$H_t = \begin{cases} 1, if\ r_t < VaR_t \\ 0, if\ r_t > VaR_t \end{cases}$$

Of course, the probability of violating the VaR should always be:

$$Pr(H_t = 1 | \, \Omega_t) = \theta$$

[33] We confirm using our verification that Engle & Manganelli's coefficients are robust with their data, only their code did not work directly with our data.
[34] The number of initial conditions was selected arbitrarily in a manner that satisfied our verification of the estimators.

Kupiec (1995) proposed a nonparametric test based on the hit sequence and the total number of observations T. The test reflects whether $\hat{p} \equiv \sum_{t=1}^{T} H_t \,/T$ is equal to θ. As such the test verifies:

$$H_0: p = E[H_t] = \theta$$

which can be verified through a likelihood ratio (LR) test. This test is known as the unconditional coverage (uc) and is known for its lower power because it does not capture time series dependence in the violations, especially in smaller samples.

Christoffersen (1998) went beyond the unconditional coverage test and extended the previous LR statistic to see if the hit sequence is independent (ind) over time. The author argues that if there is no independence, it should be possible to construct a better model. The proposed statistic thus first includes an additional variable, T_{ij} as the number of days where state *j* occurred in one day after being in state *i* the previous day. Another variable needed for the test statistic is π_i which is the probability of observing state i, conditional on state *i* the previous day. The author then assumes that the hit sequence follows a first order Markov sequence with transition matrix:

$$\Pi = \begin{array}{cl} \text{Previous day} & \\ \begin{bmatrix} 1-\pi_0 & 1-\pi_1 \\ \pi_0 & \pi_1 \end{bmatrix} & \begin{array}{c} \text{Current day (violation)} \\ \text{No violation} \end{array} \end{array}$$

where under the null hypothesis of independence, we have $\Pi = \pi_0 = \pi_1 = (T_{01} + T_{11})/T$. From this, the LR statistic can be defined:

$$\text{LR}_{\text{ind}} = 2ln\left(\frac{(1-\pi_0)^{T_{00}}\pi_0{}^{T_{01}}(1-\pi_1)^{T_{10}}\pi_1{}^{T_{11}}}{(1-\pi)^{(T_{00}+T_{10}}\pi^{(T_{01}+T_{11})}}\right) \sim \chi^2((s-1)^2) = \chi^2(1)$$

Christoffersen (1998) also worked on a conditional coverage (cc) test that could be written as the sum of the unconditional coverage test and independence test. Berkowitz, Christoffersen & Pelletier (2009) extended and unified the existing three tests by specifying that the de-meaned violations $Hit_t = H_t - \theta$ form a martingale difference sequence (mds). By definition, it implies that Hit_t is uncorrelated at all leads and lags. This allowed them to test the violations by calculating statistics based on sample autocorrelations as opposed to only the autocorrelation of order 1 by Christoffersen (1998). This is the same approach that Engle & Manganelli (2004) used in the creation of their backtest.

Engle & Manganelli (2004) developed the Dynamic Quantile (DQ) test to have better power against certain forms of misspecifications that can arise from quantile regression models and in a way that could be incorporated and extended to suit additional models beyond quantile regressions. Their test uses the mds established earlier with respect to Ω_t and using the previous notation we have:

$$Hit_t = H_t - \theta$$

where the Hit_t function takes the value $(1-\theta)$ when r_t is less than the quantile and $-\theta$ otherwise. As Berkowitz, Christoffersen, & Pelletier (2009) confirm, since Hit_t must be uncorrelated to both lags and leads, past information can be included. Their in-sample (IS) statistic is:

$$DQ_{IS} = \frac{Hit_t' X_t [X_t' X_t]^{-1} X_t' Hit_t}{\theta(1-\theta)} \sim \chi_Q^2 \tag{12}$$

In this model, the vector of instruments X_t can include lags of Hit_t, the VaR and its lags. Berkowitz, Christoffersen, & Pelletier (2009) found that based on their Monte Carlo simulations, the DQ test was the best test for 1% VaR models and also performed quite well at the 5% level.

Lastly, Gaglianone et al. (2011) establish that the DQ test is a Lagrange Multiplier (LM) test and propose an alternative Wald-type test statistic. Their test is named VaR by quantile regressions (VQR) and is based on a quantile regression:

$$Qr_t(\theta|\Omega_{t-1}) = \alpha_0(\theta) + \alpha_1(\theta)V_t$$

where V_t is the VaR they are trying to test and should be equal to the conditional quantile if the model is properly specified. From this, their null hypothesis becomes:

$$H_0: \begin{cases} \alpha_0(\theta) = 0 \\ \alpha_1(\theta) = 1 \end{cases}$$

In order to more easily test the null, it is redefined as: H_0: $\beta(\theta) = 0$, where $\beta(\theta) = [\alpha_0(\theta), (\alpha_1(\theta) - 1)]'$. Their test statistic is then:

$$VQR(\xi) = T\left[\hat{\beta}(\theta)'(\theta(1-\theta)H(\theta)^{-1}JH(\theta)^{-1})^{-1}\hat{\beta}(\theta)\right] \quad (13)$$

where $\mathrm{J} = \operatorname*{plim}_{T\to\infty} \sum_{t=1}^{T} x_t x'_t$ and $H(\theta) = \operatorname*{plim}_{T\to\infty} \frac{1}{T} \sum_{t=1}^{T} x_t x'_t \left[f_{r_t}(\theta|x_t)\right]$. Specifying that here $x'_t = [1, V_t]$ and $f_{r_t}(\theta|x_t)$ is the conditional density of r_t at the quantile θ. Using techniques from Koenker & Machado (1999), J and $H(\theta)$ can be computed as consistent estimators[35].

Gaglianone et al. (2011) contend that the DQ test and VQR methods are asymptotically equivalent under the null hypothesis and local alternatives, but that the use of the Wald specification for the VQR seems to have more power in finite samples. We chose to use both tests on our models.

Most of the backtesting was done without any issues as the models shown in the previous section are quite straightforward. However, the VQR test requires a particular assumption that can require specific attention. The issue is finding the true distribution for r_t which is unknown. As Gaglianone et al. (2011) mention, there are many ways of estimating the distribution that are shown in Koenker & Machado (1999)[36]. They also mention the potential of using average values but we decided to use a known distribution. There are also many suitable distribution candidates that can easily be used thanks to statistical

[35] For more assumptions and proofs, see Gaglianone et al. (2011).

[36] Their most prevalent estimation technique is using a jump function as the bandwidth. Using this function allows to model plus or minus the bandwidth for a given quantile: $\hat{f}_{r_t}(\theta|x_t) = \frac{2h_T}{X_t(\boldsymbol{\beta}(\theta+h_T)-\boldsymbol{\beta}(\theta-h_T))}$. However, in the outer quantiles, $\theta - h_T$ tends to zero with their technique: $h_T = T^{-1/3} z_\theta^{2/3} \left[\frac{1.5\phi^2\left(\Phi^{-1}(\theta)\right)}{\left(2\left(\Phi^{-1}(\theta)\right)^2+1\right)}\right]^{1/3}$ where z is the absolute value of the inverse cdf of a normal distribution for a given θ.

software. We have already shown that the returns do not follow a normal distribution as they all seem to have higher kurtosis. Based on our previous results and observations, we choose to mainly continue using the generalized extreme value (GEV) distribution with the same parameters estimated earlier, such a shape parameter $\xi = -0.014$. With such a shape parameter, it allows to account for a tail that may go a little more towards the negative returns. Having a negative shape parameter allows for a finite upper bound and resembles a Weibull family distribution. Furthermore, the fact that it is bigger than -0.5 allows for potential GEV estimators to have normal asymptotic properties[37].

[37] However, we take note that the previous fit was not perfect for the returns and further work can be done to better the GEV distribution parameters chosen here; this was not the main aspect of this work. However, other distributions were also used and we obtained similar results.

5 Empirical Results

Many different VaRs were estimated in trying to establish the best possible VaR model, we estimated many different VaRs. We first show GARCH results, then we present the best quantile regression models, and finally the best CAViaR models, at each of the four quantiles.

In order to choose the best models, we first base our criteria on VaR hit percentages from out-of-sample data. Since we are dealing with six different countries, we chose the model that seemed to properly represent appropriate VaRs for all the different countries at once, as opposed to choosing a model that worked best for each country. We used our judgement in order to choose the model that we felt best fit each quantile. Our criteria in selecting each model was subjectively which has the lowest discrepancy between the maximum and minimum in hit percentages, while maintaining an average close to the desired quantile for the different countries. Finally, we compare the results across models, taking the IGARCH(1,1) as the main point of comparison.

Table 2: IGARCH Estimates and Relevant Statistics at 20%

IGARCH (1,1)						
	Canada	Japan	Norway	South Africa	Switzerland	United Kingdom
20% VaR						
Hit (%)	18.00%	15.90%	16.80%	18.60%	15.20%	16.70%
LR uc (p val)	0.1126	0.0007***	0.0102**	0.2708	0.0001***	0.0080***
LR ind (p val)	0.1589	0.4491	0.0907*	0.9386	0.7810	0.8064
LR cc (p val)	0.1053	0.0023***	0.0088***	0.5437	0.0005***	0.0288**
DQ (p val)	0.2907	0.0223**	0.0138**	0.8910	0.0029***	0.0716*
VQR (p val)	0.9039	0.6439	0.7784	0.7020	0.2184	0.8372

Note: The results shown were performed on out-of-sample data.
The *, **, *** denote significant coefficients or rejection of models (LR, DQ, VQR) at 10%, 5% and 1% respectively.

TABLE 3: IGARCH ESTIMATES AND RELEVANT STATISTICS AT 10%

IGARCH (1,1)						
	Canada	Japan	Norway	South Africa	Switzerland	United Kingdom
10% VaR						
Hit (%)	8.90%	6.90%	8.40%	8.70%	7.10%	8.80%
LR uc (p val)	0.2424	0.0006***	0.0855*	0.1651	0.0014***	0.2011
LR ind (p val)	0.2259	0.6994	0.2510	0.8166	0.1866	0.7640
LR cc (p val)	0.2425	0.0026***	0.1180	0.3715	0.0025***	0.4222
DQ (p val)	0.3065	0.0319**	0.2663	0.8255	0.0248**	0.7199
VQR (p val)	0.7474	0.4013	0.3183	0.4548	0.1472	0.7416

Note: The results shown were performed on out-of-sample data.
The *, **, *** denote significant coefficients or rejection of models (LR, DQ, VQR) at 10%, 5% and 1% respectively.

TABLE 4: IGARCH ESTIMATES AND RELEVANT STATISTICS AT 5%

IGARCH (1,1)						
	Canada	Japan	Norway	South Africa	Switzerland	United Kingdom
5% VaR						
Hit (%)	4.80%	3.50%	4.60%	3.90%	2.90%	4.40%
LR uc (p val)	0.7758	0.0221**	0.5614	0.0989*	0.0010***	0.3784
LR ind (p val)	0.8287	0.8272	0.5478	0.0750*	0.2649	0.4404
LR cc (p val)	0.9380	0.0712*	0.7052	0.0525*	0.0024***	0.5038
DQ (p val)	0.0561*	0.2942	0.9264	0.2632	0.0485**	0.6141
VQR (p val)	0.6366	0.0136**	0.1853	0.0089***	0.0048***	0.8106

Note: The results shown were performed on out-of-sample data.
The *, **, *** denote significant coefficients or rejection of models (LR, DQ, VQR) at 10%, 5% and 1% respectively.

TABLE 5: IGARCH ESTIMATE AND RELEVANT STATISTICS AT 1%

IGARCH (1,1)	Canada	Japan	Norway	South Africa	Switzerland	United Kingdom
1% VaR						
Hit (%)	0.60%	0.80%	0.90%	0.10%	0.70%	0.70%
LR uc (p val)	0.1705	0.5121	0.7488	0.0002***	0.3150	0.3150
LR ind (p val)	0.7877	0.7193	0.6858	0.9643	0.7533	0.7533
LR cc (p val)	0.3771	0.7562	0.8754	0.0012***	0.5745	0.5745
DQ (p val)	0.8913	0.9878	0.9953	0.1486	0.9599	0.9599
VQR (p val)	0.0000***	0.0000***	0.0000***	0.0000***	0.0000***	0.0000***

Note: The results shown were performed on out-of-sample data.
The *, **, *** denote significant coefficients or rejection of models (DQ, VQR) at 10%, 5% and 1% respectively.

Tables 2-5 present standard IGARCH results. We observe that while they have results that are fairly accurate in hit percentages for certain countries, overall there are notable exceptions and little consistency. This is especially evident across the different quantiles. For example, the IGARCH VaR models for Japan at 20%, 10% and 5% are rejected by most tests, however at 1%, the VaR model is not rejected. This also holds true for SWZ while the opposite is true for SA. Only CAD is not rejected at any quantile.

In the case of the IGARCH, the LR (uc and cc) tests are usually consistent in their rejection of the models (most VaRs are not dependant). Also, the DQ test usually follows the same rejection pattern as the LR tests. Nonetheless, the DQ test does not reject any IGARCH model at the 1% quantile, despite SA displaying a rather poor 0.1% hit percentage. In addition, the VQR seemingly has difficulties with power as it possibly incorrectly rejects no models at 20% and 10% and rejects all the models at 1% where for example the model for SA appears to be acceptable.

Tables 6-9 show the best quantile regression models, mainly according to hit percentage. For the quantile regressions and the CAViaR estimates, we present the Betas associated with the VaRs. In the case of the quantile regressions, Beta1 is the constant and the following Betas are the estimators for each regressor, listed in the order presented at the top of each table.

TABLE 6: QR ESTIMATES AND RELEVANT STATISTICS AT 20%

Quantile Regression with interest rate differential and EMP

	Canada	Japan	Norway	South Africa	Switzerland	United Kingdom
20% VaR						
Beta1 (Const)	-0.0032*** (0.0001)	-0.0049*** (0.0004)	-0.0049*** (0.0002)	-0.0055*** (0.0004)	-0.0056*** (0.0003)	-0.0037*** (0.0002)
Beta2 (Int diff)	0.4870*** (0.0950)	-0.0516 (0.1083)	0.1530** (0.0697)	0.0283 (0.0552)	0.2274** (0.0994)	0.0677 (0.1300)
Beta3 (EMP)	0.0000 (0.0001)	-0.0004*** (0.0002)	-0.0002** (0.0001)	-0.0001 (0.0001)	-0.0005*** (0.0001)	-0.0001 (0.0001)
Hit (%)	25.50%	16.60%	22.60%	26.00%	17.40%	23.30%
LR uc (p val)	0.0000***	0.0048***	0.0413**	0.0000***	0.0379**	0.0100***
LR ind (p val)	0.7511	0.0565*	0.2190	0.9565	0.0383**	0.8118
LR cc (p val)	0.0001***	0.0031***	0.0586*	0.0000***	0.0136**	0.0353**
DQ (p val)	0.0007***	0.0298**	0.1389	0.0001***	0.0823*	0.1652
VQR (p val)	0.3146	0.5219	0.7142	0.0107**	0.4952	0.6040

Note: The results shown were performed on out-of-sample data.
The *, **, *** denote significant coefficients or rejection of models (LR, DQ, VQR) at 10%, 5% and 1% respectively.

TABLE 7: QR ESTIMATES AND RELEVANT STATISTICS AT 10%

Quantile Regression with interest rate differential and EMP						
	Canada	Japan	Norway	South Africa	Switzerland	United Kingdom
10% VaR						
Beta1	-0.0052***	-0.0079***	-0.0083***	-0.0096***	-0.0092***	-0.0061***
(Const)	(0.0002)	(0.0004)	(0.0003)	(0.0007)	(0.0003)	(0.0002)
Beta2	0.7132***	-0.0634	0.3482***	0.1876**	0.4224***	0.1432
(Int diff)	(0.1359)	(0.1137)	(0.1183)	(0.0939)	(0.1242)	(0.1726)
Beta3	-0.0001	-0.0004**	-0.0002**	0.0001	-0.0005***	-0.0002
(EMP)	(0.0001)	(0.0002)	(0.0001)	(0.0002)	(0.0002)	(0.0002)
Hit (%)	15.50%	7.90%	12.90%	12.50%	8.40%	12.30%
LR uc (p val)	0.0000***	0.0167**	0.0032***	0.0105**	0.0855*	0.0183**
LR ind (p val)	0.6175	0.2416	0.7083	0.9175	0.0036***	0.7350
LR cc (p val)	0.0000***	0.0288**	0.0120**	0.0377**	0.0033***	0.0583*
DQ (p val)	0.0000***	0.0761*	0.0181**	0.0928*	0.0107**	0.1825
VQR (p val)	0.0001***	0.3861	0.0396**	0.0089***	0.0293**	0.3514

Note: The results shown were performed on out-of-sample data.
The *, **, *** denote significant coefficients or rejection of models (LR, DQ, VQR) at 10%, 5% and 1% respectively.

TABLE 8: QR ESTIMATES AND RELEVANT STATISTICS AT 5%

Quantile Regression with lagged returns, deviations and interest rate differential						
	Canada	Japan	Norway	South Africa	Switzerland	United Kingdom
5% VaR						
Beta1	-0.0084***	-0.0115***	-0.0113***	-0.0149***	-0.0125***	-0.0083***
(Const)	(0.0006)	(0.0012)	(0.0006)	(0.0023)	(0.0009)	(0.0008)
Beta2	0.0815**	-0.1213***	0.0333	0.1255***	-0.0502	0.0247
(Lag ret)	(0.0901)	(0.0711)	(0.0661)	(0.1077)	(0.0537)	(0.1094)
Beta3	-0.0147***	-0.0109***	-0.0053**	0.0235***	-0.0045**	0.0054**
(PPP)	(0.0042)	(0.0039)	(0.0047)	(0.0064)	(0.0037)	(0.0069)
Beta4	1.4294***	0.2359*	0.5778***	0.1910	0.6193***	0.6335***
(int diff)	(0.4250)	(0.3276)	(0.2811)	(0.3043)	(0.3357)	(0.5962)

Hit (%)	6.00%	2.90%	6.80%	7.50%	3.20%	6.40%
LR uc (p val)	0.1567	0.0005***	0.0128**	0.0007***	0.0054***	0.0502*
LR ind (p val)	0.7284	0.2484	0.5104	0.4367	0.0970	0.2239
LR cc (p val)	0.3454	0.0013***	0.0363**	0.0023***	0.0053***	0.0701*
DQ (p val)	0.0000***	0.0022***	0.0506*	0.0000***	0.0066***	0.1668
VQR (p val)	0.0000***	0.0000***	0.0000***	0.0013***	0.0001***	0.2663

Note: The results shown were performed on out-of-sample data.
The *, **, *** denote significant coefficients or rejection of models (LR, DQ, VQR) at 10%, 5% and 1% respectively.

TABLE 9: QR ESTIMATES AND RELEVANT STATISTICS AT 1%

Quantile Regression with deviations and interest rate differential

	Canada	Japan	Norway	South Africa	Switzerland	United Kingdom
1% VaR						
Beta1 (Const)	-0.0059*** (0.0002)	-0.0086*** (0.0006)	-0.0083*** (0.0003)	-0.0092*** (0.0010)	-0.0093*** (0.0005)	-0.0061*** (0.0003)
Beta2 (PPP)	-0.0093*** (0.0017)	-0.0067*** (0.0019)	-0.0034*** (0.0023)	0.0129*** (0.0024)	-0.0032** (0.0020)	0.0010*** (0.0030)
Beta3 (Int diff)	0.9827** (0.2161)	0.1094 (0.1493)	0.3819*** (0.1365)	0.2027*** (0.1377)	0.4271*** (0.1602)	0.1748* (0.2411)
Hit (%)	1.00%	0.90%	1.70%	1.00%	0.60%	0.50%
LR uc (p val)	0.9975	0.7488	0.0428**	0.9975	0.1705	0.0791*
LR ind (p val)	0.6529	0.6858	0.4430	0.6529	0.0246**	0.8225
LR cc (p val)	0.9038	0.8754	0.0957*	0.9038	0.0313**	0.2087
DQ (p val)	0.0778*	0.9953	0.0000***	0.1190	0.0000***	0.7698
VQR (p val)	0.0000***	0.0000***	0.0000***	0.0000***	0.0000***	0.0000***

Note: The results shown were performed on out-of-sample data.
The *, **, *** denote significant coefficients or rejection of models (LR, DQ, VQR) at 10%, 5% and 1% respectively.

We can see that for the quantile regressions, there is greater volatility in the hit percentages than for the IGARCH. It is apparent that while certain VaRs estimated by quantile regressions have proper hit percentages for certain countries at the 20%, 10% and 5% quantiles, they do not seem to outperform the IGARCH VaRs. However, at the 1% quantile, the quantile regression VaRs perform noticeably better than at the other quantiles. The models are not rejected for the majority of the countries, although the model for NOK seems to greatly overestimate the VaRs. On average, at the 1% quantile, the quantile regression estimated VaRs using deviations from the PPP and interest rate differentials perform better than with other regressors. At the 1% quantile, the VaRs estimated by quantile regressions outperforms the IGARCH for certain countries but underperforms for others according to our presented criteria.

As the VaRs presented are the best quantile regression models, it is clear that in its simple form, there are generally better options to model VaRs, especially in the middle quantiles. Nonetheless, it is noteworthy that using the additional regressors constantly outperforms the simple use of lagged returns. It is also of interest that the deviations from the PPP are important regressors at the 5% and 1% quantiles and that the interest rate differential is part of the best models at each quantile.

We can see that all the constants (Beta1) for the VaRs estimated by quantile regression are statistically significant at 1%. We also notice that for most models, the estimators for the regressors are also generally statistically significant. This is especially true for the estimators of the deviations from the PPP, who are all statistically significant at 5%. Furthermore, it is also evident that for the quantile regression VaRs, as the accuracy of the whole model increases, so does the occurrence of statistically significant estimators. We established that the 1% quantile is the most accurate of the VaR estimation by quantile regression and it is also noticeable that all but a single estimator is statistically significant at 10% significance.

Tables 10-13 demonstrate the best CAViaR models for each quantile. We can observe a few important distinctions between the four tables, such that it is not always the same model type, nor is it always the same regressors that seem to be the most accurate in the selection of the best VaRs, mainly according to hit percentage. All the results in the tables that are discussed below can be compared with certain results of the other CAViaR models, also shown as tables included in Appendix A2. While not all regressors used in estimating the different CAViaR models are shown, most models estimated with only lagged returns are presented in Appendix A2, as well as other next best CAViaR models:

TABLE 10: CAVIAR ESTIMATES AND RELEVANT STATISTICS AT 20%

Symmetric CAViaR Model with lagged returns and interest rate differential						
	Canada	Japan	Norway	South Africa	Switzerland	United Kingdom
20% VaR						
Beta1 (Const)	-0.0000 (0.0000)	-0.0001* (0.0001)	0.0000 (0.0000)	-0.0002** (0.0001)	0.0000 (0.0000)	0.0000 (0.0000)
Beta2 (VaRt-1)	0.9303*** (0.0296)	0.9279*** (0.0240)	0.9640*** (0.0159)	0.8582*** (0.0504)	0.9699*** (0.0090)	0.9455*** (0.0185)
Beta3 (Lag ret)	-0.0610*** (0.0242)	-0.0509*** (0.0156)	-0.0379*** (0.0150)	-0.1238*** (0.0519)	-0.0309*** (0.0087)	-0.0556** (0.0245)
Beta4 (Int diff)	-0.0018 (0.0125)	-0.0088 (0.0080)	0.0035 (3.94E-03)	0.0160** (8.31E-03)	-0.0060* (4.20E-03)	-0.0009 (8.56E-03)
Hit (%)	20.30%	19.60%	18.30%	20.10%	19.00%	19.80%
LR uc (p val)	0.8006	0.7033	0.1791	0.9244	0.4354	0.8867
LR ind (p val)	0.8065	0.7997	0.2345	0.9307	0.8595	0.1416
LR cc (p val)	0.9400	0.9006	0.2001	0.9918	0.7262	0.3360
DQ (p val)	0.7454	0.8811	0.2420	0.2553	0.6507	0.1573
VQR (p val)	0.9390	0.9932	0.7469	0.4143	0.7368	0.8271

Note: The results shown were performed on out-of-sample data. For the estimators, see Eq. (9)
The *, **, *** denote significant coefficients or rejection of models (LR, DQ, VQR) at 10%, 5% and 1% respectively.

TABLE 11: CAVIAR ESTIMATES AND RELEVANT STATISTICS AT 10%

Symmetric CAViaR Model with lagged returns and interest rate differential						
	Canada	Japan	Norway	South Africa	Switzerland	United Kingdom
10% VaR						
Beta1 (Const)	-0.0000 (0.0000)	-0.0001* (0.0001)	-0.0001 (0.0001)	-0.0003** (0.0001)	-0.0000 (0.0000)	0.0001 (0.0000)
Beta2 (VaRt-1)	0.9315*** (0.0239)	0.9539*** (0.0142)	0.9588*** (0.0246)	0.8699*** (0.0349)	0.9691*** (0.0095)	0.9638*** (0.0092)
Beta3 (Lag ret)	-0.0996** (0.0487)	-0.0559*** (0.0204)	-0.0591*** (0.0245)	-0.1802*** (0.0643)	-0.0468*** (0.0128)	-0.0745*** (0.0215)
Beta4 (Int diff)	0.0117 (0.0129)	-0.0027 (0.0064)	0.0074 (0.0101)	0.0180 (0.0142)	-0.0006 (0.0068)	-0.0043 (0.0090)
Hit (%)	10.10%	9.20%	9.20%	10.40%	8.90%	8.90%
LR uc (p val)	0.9078	0.3991	0.3991	0.6673	0.2424	0.2424
LR ind (p val)	0.0482**	0.5678	0.2036	0.3192	0.1346	0.2259
LR cc (p val)	0.1411	0.5953	0.3123	0.5552	0.1648	0.2425
DQ (p val)	0.2613	0.6157	0.4348	0.1193	0.3009	0.0521*
VQR (p val)	0.9687	0.7121	0.7137	0.0177**	0.5930	0.0159**

Note: The results shown were performed on out-of-sample data. For the estimators, see Eq. (9)
The *, **, *** denote significant coefficients or rejection of models (LR, DQ, VQR) at 10%, 5% and 1% respectively.

TABLE 12: CAVIAR ESTIMATES AND RELEVANT STATISTICS AT 5%

Symmetric CAViaR Model with lagged returns and deviation from fundamental PPP						
	Canada	Japan	Norway	South Africa	Switzerland	United Kingdom
5% VaR						
Beta1 (Const)	-0.0000 (0.0000)	-0.0001 (0.0001)	-0.0002 (0.0002)	-0.0004*** (0.0002)	-0.0000 (0.0001)	3.01E-05* (2.21E-05)
Beta2 (VaRt-1)	0.9232*** (0.0244)	0.9547*** (0.0154)	0.9583*** (0.0208)	0.8665*** (0.0261)	0.9755*** (0.0081)	0.9570*** (0.0102)
Beta3 (Lag ret)	-0.1579*** (0.0478)	-0.0778*** (0.0202)	-0.0613** (0.0264)	-0.2636*** (0.0639)	-0.0473*** (0.0153)	-0.1036*** (0.0242)
Beta4 (PPP)	-0.0003** (0.0002)	-0.0001 (0.0002)	0.0001 (0.0002)	0.0005* (0.0003)	0.0001 (0.0001)	0.0001 (0.0002)
Hit (%)	4.40%	5.30%	5.80%	4.50%	4.10%	4.50%

LR uc (p val)	0.3784	0.6610	0.2540	0.4652	0.1805	0.4652
LR ind (p val)	0.4404	0.9066	0.3743	0.4059	0.3359	0.4059
LR cc (p val)	0.5038	0.9021	0.3516	0.5422	0.2568	0.5422
DQ (p val)	0.2115	0.3122	0.5566	0.0507*	0.4407	0.3061
VQR (p val)	0.5851	0.1233	0.2638	0.0000***	0.4269	0.0526*

Note: The results shown were performed on out-of-sample data. For the estimators, see Eq. (9)
The *, **, *** denote significant coefficients or rejection of models (LR, DQ, VQR) at 10%, 5% and 1% respectively.

TABLE 13: CAVIAR ESTIMATES AND RELEVANT STATISTICS AT 1%

Asymmetric CAViaR Model with lagged returns and deviation from fundamental PPP

	Canada	Japan	Norway	South Africa	Switzerland	United Kingdom
1% VaR						
Beta1 **(Const)**	-0.0001 (0.0002)	-0.0009*** (0.0003)	-0.0010* (0.0008)	-0.0002 (0.0006)	-0.0000 (0.0001)	-0.0002 (0.0005)
Beta2 (VaRt-1)	0.9393*** (0.0327)	0.8966*** (0.0358)	0.8846*** (0.0834)	0.9074*** (0.0354)	0.9809*** (0.0195)	0.9233*** (0.0644)
Beta3 (Lag ret)+	-0.1640*** (0.0689)	-0.3101*** (0.0759)	-0.0468 (0.0886)	-0.1706* (0.1156)	-0.0590** (0.0333)	-0.2077** (0.1152)
Beta4 (Lag ret)-	-0.1703*** (0.0442)	-0.0209 (0.0903)	-0.2933 (0.2742)	-0.4448*** (0.0696)	-0.0403 (0.0591)	-0.2336* (0.1564)
Beta5 (PPP)+	-2.74E-05 (0.0007)	-0.0014 (0.0013)	-0.0020 (0.0017)	-0.0007 (0.0028)	-0.0003 (0.0007)	0.0002 (0.0009)
Beta6 (PPP)-	0.0005* (0.0004)	0.0022** (0.0011)	0.0003 (0.0008)	-0.0009 (0.0008)	-0.0003 (0.0004)	0.0004 (0.0009)
Hit (%)	0.80%	0.80%	1.20%	0.90%	0.80%	0.90%
LR uc (p val)	0.5121	0.5121	0.5356	0.7488	0.5121	0.7488
LR ind (p val)	0.7193	0.7193	0.5891	0.6858	0.7193	0.6858
LR cc (p val)	0.7562	0.7562	0.7133	0.8754	0.7562	0.8754
DQ (p val)	0.9838	0.9959	0.9779	0.1063	0.9878	0.9937
VQR (p val)	0.0662*	0.0000***	0.0000***	0.0000***	0.0293**	0.2192

Note: The results shown were performed on out-of-sample data. For the estimators, see Eq. (4)
The *, **, *** denote significant coefficients or rejection of models (LR, DQ, VQR) at 10%, 5% and 1% respectively.

In tables 10-12 (20%, 10% and 5% quantiles), we can see that the model that seems to best estimate VaRs is the symmetric CAViaR model[38]. For the symmetric model, the Beta1 is the constant, Beta2 is the autoregressive property, Beta3 is the estimator for lagged returns and Beta4 is the estimator for the additional regressor, in this case either the interest rate differential or the deviations from the PPP. Nonetheless, we can also see that at the 1% quantile, the asymmetric slope model seems to perform best in estimating the VaRs. For the asymmetric slope model, Beta2 and Beta 3 are positive and negative lagged returns, and Beta4 and Beta5 are the positive and negative additional regressor, in this case the deviations from the PPP.

In tables 10 and 11 (20% and 10%), the model that seems to best satisfy our criteria for the VaRs is the symmetric model with lagged returns and with the interest differential as regressors. However, in tables 12 and 13 (5% and 1%), the most appropriate models seem to be the symmetric and asymmetric models respectively, both with the lagged returns and the deviation from the PPP as regressors. While the differences are not immense with other CAViaR models and other independent variables, these are the regressors that provide the models that best satisfy the hit percentage. Although the estimators for the macroeconomic variables (deviations from the PPP in tables 12 and 13) are but sporadically statistically significant, as opposed to those of the lagged returns, it does not diminish the importance added by these variables to the models. The addition of the extra variables beyond only using the lagged returns not only modifies the actual estimators but also improves the average accuracy of out-of-sample testing and minimizes the variance of hit percentage across the countries. As we can see in Appendix A2, other models can perform well for individual countries, however the above tables demonstrate the best overall performance for the different quantiles across the countries[39].

[38] This was a result that was not anticipated when we began this exercise. We had originally believed that the symmetric model would not be as interesting as the asymmetric and adaptive models, as it treats the positive and negative changes in the exchange rate with the same weight. We had anticipated that there could have been a greater importance on negative returns than positive returns.

[39] There was a debate about the best 1% CAViaR model. The issue in question is the debate between over or under estimating the hit percentages. While Table 13 mostly underestimates the percentages, the GARCH

While the CAViaR estimators are not as frequently statistically significant as those estimated by simple quantile regressions, it does not hinder the overall effectiveness of the CAViaR models. A possible hypothesis, is that the autoregressive property of the CAViaR manages to capture most of the importance of the changes in exchange rates, a portion that is missed by simple quantile regressions. Nonetheless, as it was mentioned, the importance of the other regressors must not be overlooked in the CAViaR models, as we establish that they improve the overall performance of hit percentages[40].

The last main result to be taken away from tables 11-13 is that they all seem to perform well in estimating the VaRs out-of-sample at the different quantiles, based on hit percentage. The results however, are a little murkier when it comes to the backtest scores. When using the traditional backtests with Likelihood Ratios (LR) that do not account for the intricacies of quantile regressions, the tests seem to almost never reject any of the models presented by the different CAViaRs at each quantile[41]. The tests reject one model, for independence at 5% significance. However, there are a slightly more rejections when using the other two backtests (DQ and VQR). Similarly to the LR tests, the DQ test only rejects two models at 10% significance, but overall seems to not reject the other models with high test coefficients. Conversely, while the VQR rejects no model at the 20% quantile, it was more frequent with the rejection of models at the other quantiles. We hypothesise that it is due to the assumption of a particular distribution in the test, which has the greatest impact at the 1% quantile, where it rejects five out of the six models.

Taking a closer look at the estimated coefficients, the most noticeable result is that Beta2, the autoregressive property is always statistically significant at 1% in every model, at

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CAViaR with lagged returns, deviations and interest rate differential from Table 33 often slightly overestimates as can be seen in Appendix A2. This was a case of judgement where it is difficult to assess the different relative costs. Because the deviations from the PPP are also present, it still demonstrates the importance of the estimator in the tails, as it was necessary for the two best models, both models are nonetheless close to the 1% hit percentage.

[40] Subjectively, we believe that the interest rate differential and the deviations from PPP may be the two most important regressors. While the deviations from PPP seems most useful in the tails, the interest rate differentials may offer explanatory power at all quantiles.

[41] Especially the unconditional coverage (uc) and the conditional coverage (cc) tests from section 4.3

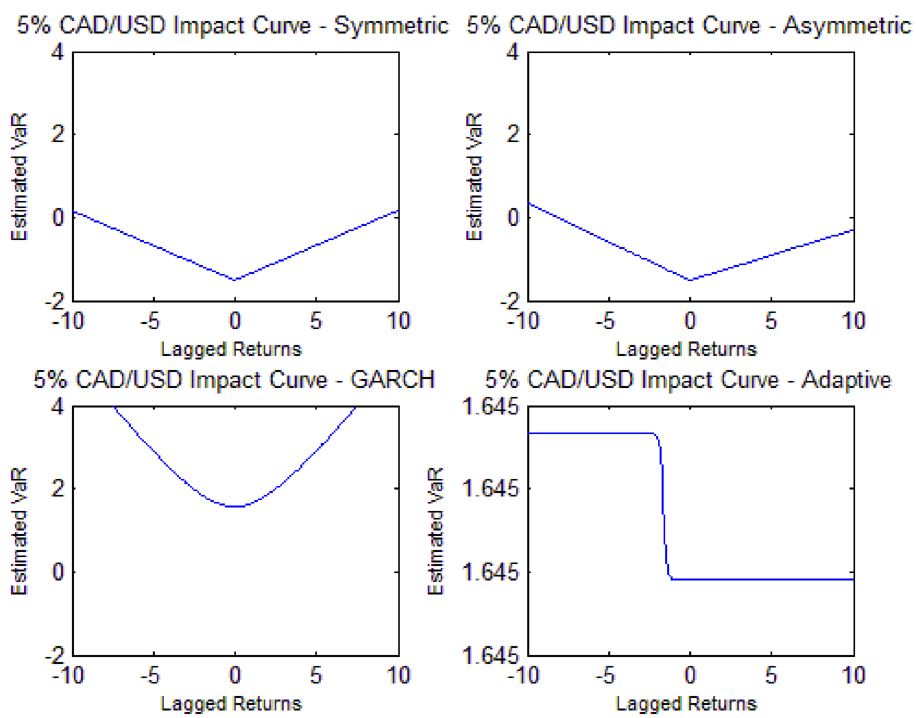
every quantile. Furthermore, most of those estimates are close to unity and can thus be of some interest. While in their original paper, Engle & Manganelli (2004) also had autoregressive Betas close to unity, they were not quite as close as ours. This particular estimator is of great interest as it is the one that represents the lagged VaR. This obviously raises questions about the possibility that the true path of the models follow is a random walk and can thus be difficult to predict. Although, as the IGARCH equation resembles that of a random walk, it could warrant further attention. It could be interesting to use IGARCH equations with a CAViaR model, which could allow another way of accounting for persistence in volatility shocks. Nonetheless, the existing CAViaR models all generally performed better, according to our standards than did the IGARCH(1,1), especially in the tails.

We also observe that the Beta3 estimator, the lagged change in exchange rate, is also almost always statistically significant. There is one model where this estimator is not statistically significant at 10% significance, but most are usually statistically significant at 1% significance. We also notice that the estimators are always negative. This means that positive returns will decrease the VaR. Furthermore, the coefficients grow in absolute value as we move into the tails. Thus, as we go further into the tails, we expect a bigger negative impact of a positive return.

In regards to the 1% asymmetric CAViaR model, it is also interesting to notice the different impacts by the positive and negative returns. First, we notice that both the estimators for positive and negative lagged returns are always negative, as the lagged return (Beta3) estimators in the symmetric models. Nevertheless, we notice that the estimators have different impacts for different countries. For CAD, NOK, SA and the UK, the impacts are stronger for negative returns than they are for positive returns. However, for JPY and SWZ we notice that the positive returns have a stronger impact than the negative returns. This is an interesting result because both the JPY and the SWZ are safe haven currencies that have historically had lower interest rates. This may demonstrate the distinctions between countries with historically lower interest rates and those with historically higher rates. This could potential be a way of distinguishing countries that are be used for carry trade and further reason to group certain countries together.

As previously mentioned, while the additional regressors do not have significant impacts individually, they improve the hit percentage for each country as opposed to the equivalent model with just lagged returns as a regressor. Moreover, as it has been mentioned, the VQR test continues to seemingly over reject models at the 1%[42], even when the hit percentage is equal to the desired quantile. It is noteworthy that the test performed similarly with many different types of distributions. Beyond the GEV, the test was also performed with normal, Student t and Weibull distributions[43]. Something that we can take away from these results, is that this may demonstrate certain limits of using known distributions and further strengthens the use of semi-parametric methods in estimating VaRs, rather than relying on purely parametric models.

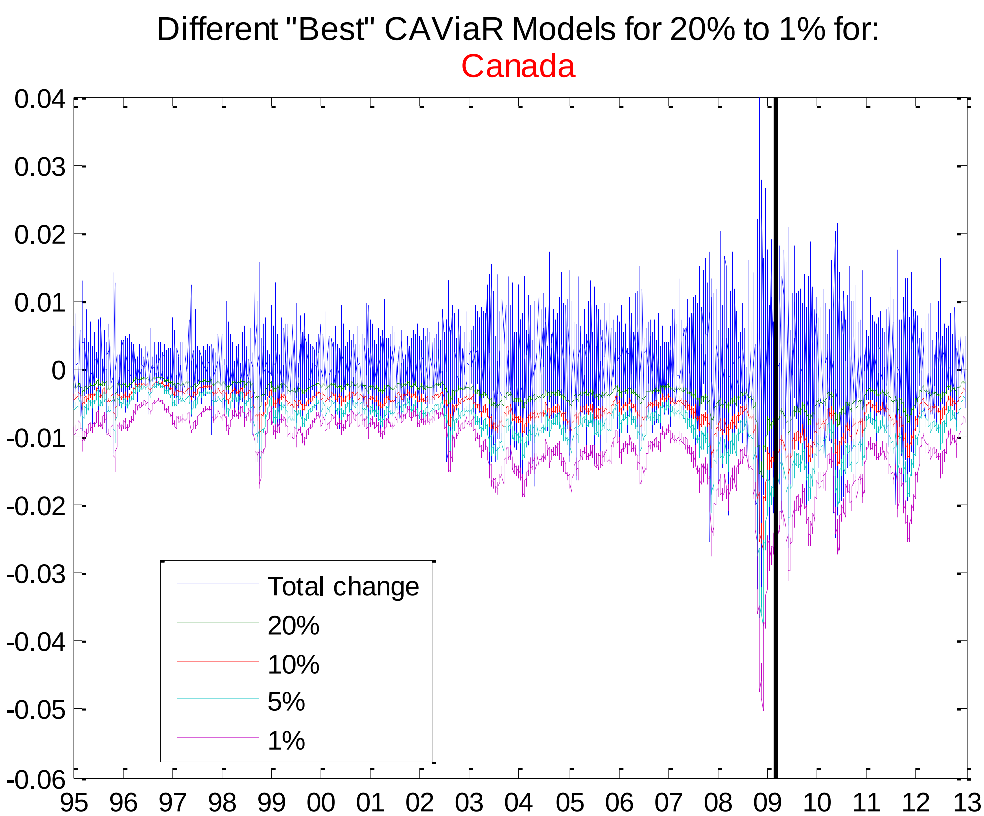
Figure 3: News Impact Curves for Canada at 5%



[42] More work should be conducted as the hit percentages are not the only criteria of evaluation.
[43] As we have used the GEV distribution throughout, it is the only one presented in the tables.

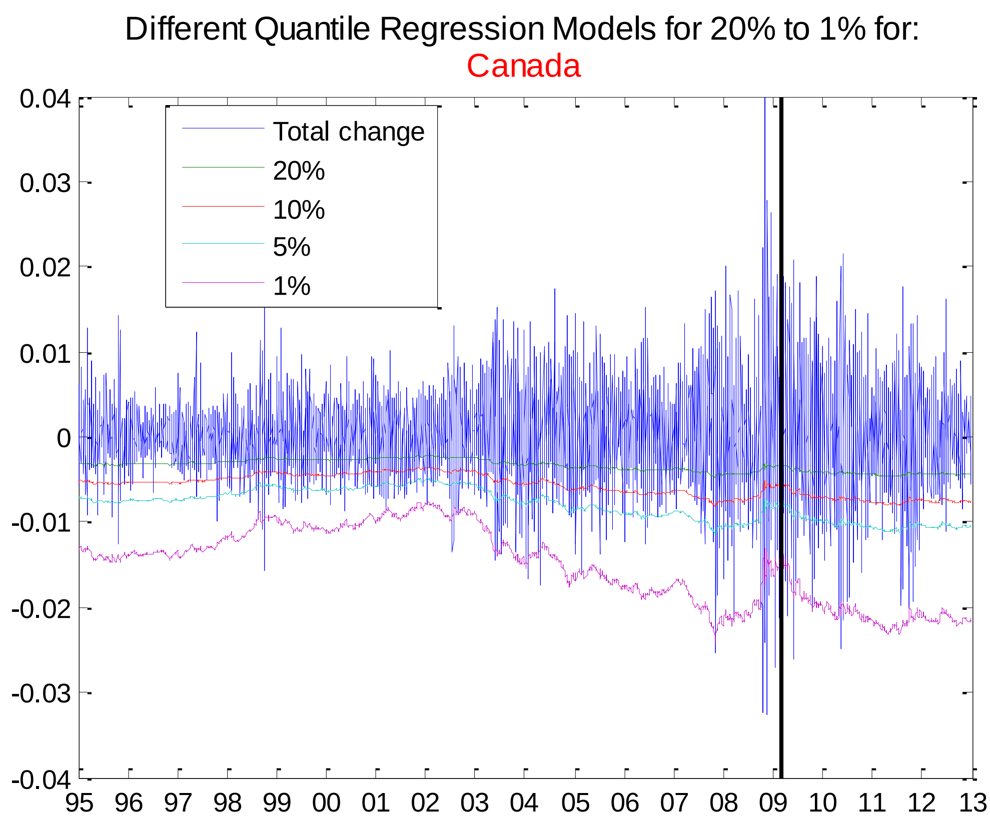
In order to further compare our results with the ones from Engle & Manganelli (2004), we also plot News Impact Curves (NIC) in the same manner as they used them on stock market data. Fig.3 shows CAViaR NIC for Canada with lagged returns and see how the estimated VaRs change as the regressors vary. While the results from the NIC found in Engle & Manganelli (2004) led us to focus on the asymmetric and adaptive models, we can see in these NIC that there is little to no difference in the impact between the positive and the negative returns. The NIC results are similar for different quantiles and with the use of different regressors. These NIC also seem to explain the results from tables 10-12 showing the symmetric model as the "best" CAViaR model at three different quantile. Based on the NIC, the impacts of positive and negative returns are very similar and almost not discernable. It is thus understandable that the results obtained by the symmetric and asymmetric models are not too different and seems to dispel our original assumption that negative returns would always have a stronger impact than positive returns.

FIGURE 4: BEST CAVIAR MODELS FOR CAD



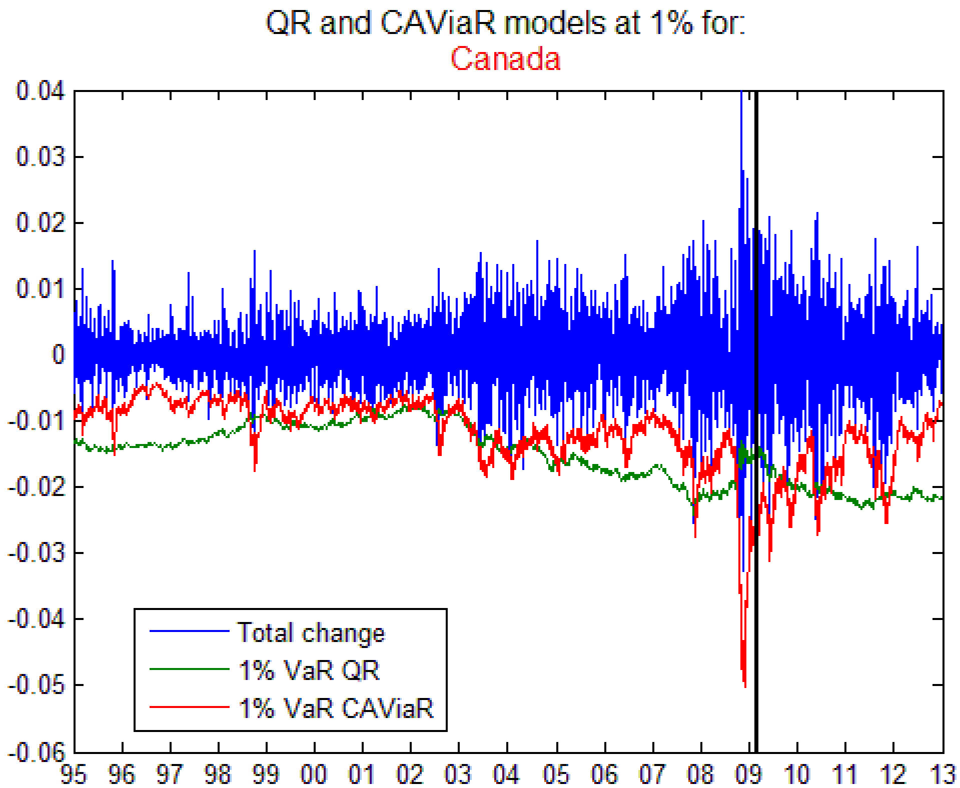
We can see in Fig. 4 how well-behaved the CAViaRs are per quantile. First, the estimated CAViaRs do not cross over by quantile. Second, we notice that the estimated CAViaRs become more volatile as they get closer to the tails. This is something that can also be observed for the different countries. We also notice that there is no visible discernable difference between the in-sample CAViaR estimation and the out-of-sample estimation.

FIGURE 5: BEST QR VARS FOR CAD



Also, as we can see in Fig. 5, the QR VaR models also become more volatile as they go further into the outer quantiles, thus the 1% percent quantile is the most volatile of the QR VaR models that we have estimated. This is something that is more apparent for the QR VaR models than for the CAViaRs because the former are generally much less volatile. Nonetheless, the QR VaRs display some of the same general well-behaved properties shown by the CAViaRs.

FIGURE 6: QR & CAVIAR MODELS FOR CAD AT 1%



We can confirm from Fig. 6 that between the QR VaR and the CAViaR, the CAViaR model is the more volatile of the two. As a result of being more volatile but correctly sized, the CAViaR is almost always closer to zero than the QR VaRs. This is a property that demonstrates the usefulness of the additional conditioning information in the CAViaR model. It would also seem to generally be a more affordable way to cover positions than the other models, notably than the QR VaRs. We can see that the two models follow different paths, despite both of them being efficient in hit percentages. Moreover, the QR VaR model seems to drift off in the out-of-sample period, something that does not happen with the CAViaR model. This may be in part due to the autoregressive property of the CAViaR model. While the drift may diminish the accuracy of the QR VaR model as the out-of-sample increases, this does not seem to always hurt the overall validity of using quantile regressions in estimating VaRs, especially in the outer quantiles. While the CAViaR is overall a better choice than the QR VaR as a coverage technique, it is not always the case for each country. As the QR VaR generally is less accurate in terms of hit percentage and has a higher cost to cover positions, it can still outperform the CAViaR out of sample in terms of hit percentages, as it does here for the CAD (1% to 0.8% hit percentage). This result may be due to the lower volatility of the QR VaR and the fact that the VaR remains far from zero.

FIGURE 7: DIFFERENCE BETWEEN GARCH MODELS AT 1%

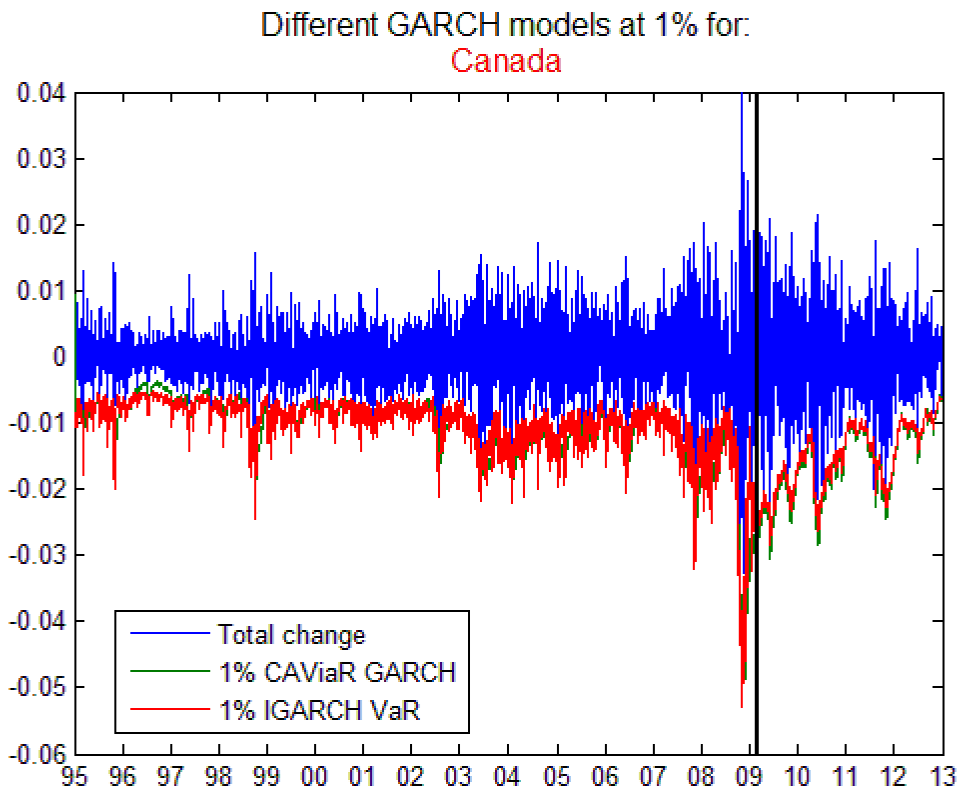


Figure 7 shows the visual differences (similarities) between the IGARCH(1,1) and the CAViaR indirect GARCH model. It is very difficult to discern any differences graphically, especially in the in-sample portion of the data. However, in the out-of-sample portion we notice a few differences in the spikes. It is noteworthy that the CAViaR indirect GARCH performs better or as well as the IGARCH(1,1) in hit percentage in all but one of the countries. The reason why the CAViaR GARCH model was not selected as the best CAViaR 1% model is that it has a slightly bigger range than the asymmetric slope model over the six countries.

6 Conclusion

This thesis demonstrates previously untested VaR techniques such as the CAViaR models used on exchange rate data, combined with macroeconomic variables as regressors. The goal was not only to utilize these techniques, but also to ensure the proper regressors are used when estimating the models. In attempting to model exchange rate risk, we used four independent variables: lagged returns, deviations from fundamental PPP levels, interest rate differential, and exchange market pressure.

As a method of comparing our results, we used six different currencies over the USD. We also compared our results against a baseline method for exchange rate risk, an IGARCH(1,1). The results demonstrate that while quantile regressions can be a powerful tool in the estimation of VaRs, they may lack the autoregessive property observed by the CAViaR. This may be a reason that the CAViaR models performed on average, better than the QR VaRs and the IGARCH VaRs.

However, when it comes down to the best CAViaR model, it is less conclusive. Based on our criteria, we found that for three of the four quantiles, the symmetric model was the best of the four CAViaR models, mainly based on hit percentage. As it was mentioned, especially with the indirect GARCH and the asymmetric model, the differences were sometimes fairly small and it came down to judgement, based on the mix of the six different countries. Furthermore, the best regressors used was also difficult to select. Nonetheless, it is our conclusion that at the 20% and 10% quantiles, the regressors that performed the best are the lagged returns with the interest rate differential. However, at the 5% and the 1% quantiles, the regressors that performed the best are the lagged returns with the deviations from fundamental PPP levels[44].

[44] This is also true if the GARCH model is selected over the asymmetric model at the 1% quantile.

These results seem to confirm our hypothesis that the deviations from the PPP could have a predictive power on the estimation of VaRs, especially in the tails of the changes of exchange rates. The regressors that were chosen as part of the best QR VaR models also seem to demonstrate this conclusion. Similarly, the CAViaR models, at 5% and 1% demonstrate the usefulness of using the deviation from PPP levels as independent variables. However, at the 20% and 10% quantiles, the deviations from the PPP are not part of the most useful regressors.

Another conclusion taken from the results is that it may be challenging to use parametric models with known distributions. They must be carefully used, as we have shown that not only normal distributions do not seem to properly fit the returns, neither do distributions for extreme values such as the GEV.

Finally, it would be interesting to continue the search for appropriate VaR estimation techniques for exchange rate risk. While we have demonstrated that CAViaR estimates the best type of model in modeling VaRs between those tested, no specific CAViaR model and no combination of regressors have clearly stood out ahead of the others for all the countries together. Certain models capture almost perfectly the VaRs for particular countries, but have difficulties with others. It would be interesting to further investigate the relationship between different types of countries and different currency types.

Also, while our work on existing distributions demonstrate certain caveats, it would be interesting to explore more work that is done in EVT in connexion with our current models. A problem that we mentioned and was not resolved, is the difference between observing the unconditional and conditional distributions. It seems as though these returns have very high kurtosis that suffer certain extreme events, something that is never easy to model.

Appendix A: Tables

A1: Statistics

Data Statistics						
	Canada	Japan	Norway	South Africa	Switzerland	United Kingdom
Statistics						
Median returns	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001
Median deviations from PPP	-0.0282	-0.0060	0.0252	0.0153	-0.0119	0.0016
Median int. rate diff.	0.0000	0.0031	-0.0009	-0.0059	0.0018	-0.0005
Median EMP	0.0000	-0.0125	0.0000	0.0000	0.0000	0.0000
Std. Dev. returns	0.0054	0.0070	0.0075	0.0106	0.0071	0.0055
Std. Dev. deviations from PPP	0.1407	0.1321	0.1292	0.1860	0.1408	0.0801
Std. Dev. int. rate diff.	0.0010	0.0020	0.0019	0.0032	0.0015	0.0009
Std. Dev. EMP	1.0296	0.8685	1.4950	2.0541	1.5206	0.9790
Skewness returns	-0.1293	0.3741	-0.0723	-0.8716	-0.2611	-0.2301
Skewness deviations from PPP	0.1448	0.7345	-0.5992	-0.7828	-0.1094	-0.1391
Skewness int. rate diff.	0.4679	0.3524	-0.1717	3.6496	-0.2413	-0.2053
Skewness EMP	0.5317	-1.9956	2.6978	3.5037	0.0030	1.5430

A2: CAViaR

Lagged Returns

TABLE 14: APPENDIX - SYMMETRIC MODEL WITH LAGGED RETURNS AT 20%

Symmetric CAViaR Model with lagged returns						
	Canada	Japan	Norway	South Africa	Switzerland	United Kingdom
20% VaR						
Beta1 (Const)	-3.15E-05*** (0.0000)	-0.0001*** (0.0000)	0.0000 (0.0000)	-0.0001** (0.0000)	-4.32E-05* (0.0000)	0.0000 (0.0000)
Beta2 (VaRt-1)	0.9320*** (0.0284)	0.9172*** (0.0157)	0.9555*** (0.0141)	0.8652*** (0.0329)	0.9654*** (0.0120)	0.9448*** (0.0143)
Beta3 (Lag ret)	-0.0566** (0.0263)	-0.0547*** (0.0088)	-0.0416*** (0.0147)	-0.1127*** (0.0362)	-0.0266*** (0.0092)	-0.0492*** (0.0137)
Hit (%)	21.60%	18.20%	18.00%	22.00%	17.00%	20.60%
LR uc (p val)	0.5707	0.1322	0.2708	0.3789	0.0672	0.8006
LR ind (p val)	0.5751	0.6724	0.3267	0.6454	0.8898	0.1492
LR cc (p val)	0.7277	0.2945	0.3371	0.6108	0.1854	0.3424
DQ (p val)	0.8215	0.2608	0.4352	0.4512	0.1110	0.6550
VQR (p val)	0.9634	0.8999	0.9298	0.4272	0.8027	0.9064

Note: The results shown were performed on out-of-sample data.
The *, **, *** denote significant coefficients or rejection of models (LR, DQ, VQR) at 10%, 5% and 1% respectively.

TABLE 15: APPENDIX - ASYMMETRIC MODEL WITH LAGGED RETURNS AT 20%

Asymmetric CAViaR Model with lagged returns						
	Canada	Japan	Norway	South Africa	Switzerland	United Kingdom
20% VaR						
Beta1 (Const)	-3.16E-05*** (0.0000)	-0.0005*** (0.0000)	0.0000 (0.0000)	-0.0001** (0.0000)	0.0000 (0.0000)	0.0000 (0.0000)
Beta2 (VaRt-1)	0.9281*** (0.0235)	0.8216*** (0.0442)	0.9570*** (0.0169)	0.8730*** (0.0233)	0.9659*** (0.0179)	0.9409*** (0.0143)
Beta3 (Lag ret)+	-0.0553*** (0.0169)	-0.1217*** (0.0241)	-0.0373** (0.0184)	-0.0751*** (0.0245)	-0.0403*** (0.0139)	-0.0553*** (0.0184)

Beta4 (Lag ret)-	-0.0663** (0.0307)	-0.0499** (0.0247)	-0.0436** (0.0212)	-0.1433*** (0.0346)	-0.0107 (0.0171)	-0.0488*** (0.0126)
Hit (%)	21.20%	16.80%	18.40%	21.40%	17.00%	20.40%
LR uc (p val)	0.6818	0.0672*	0.3470	0.3789	0.0462**	0.9874
LR ind (p val)	0.5491	0.7379	0.2576	0.9343	0.6104	0.1048
LR cc (p val)	0.7683	0.1770	0.3385	0.6768	0.1203	0.2682
DQ (p val)	0.9224	0.1644	0.4733	0.8519	0.1207	0.7085
VQR (p val)	0.9472	0.8619	0.9363	0.4058	0.8676	0.9037

Note: The results shown were performed on out-of-sample data.
The *, **, *** denote significant coefficients or rejection of models (LR, DQ, VQR) at 10%, 5% and 1% respectively.

TABLE 16: APPENDIX - GARCH MODEL WITH LAGGED RETURNS AT 20%

GARCH CAViaR Model with lagged returns

	Canada	Japan	Norway	South Africa	Switzerland	United Kingdom
20% VaR						
Beta1 (Const)	0.0000** (0.0000)	9.31E-07*** (0.0000)	0.0000 (0.0000)	1.57E-07** (0.0000)	0.0000 (0.0000)	0.0000 (0.0000)
Beta2 (VaRt-1)	0.9309*** (0.0065)	0.9036*** (0.0087)	0.9617*** (0.0054)	0.8556*** (0.0119)	0.9640*** (0.0076)	0.9450*** (0.0069)
Beta3 (Lag ret)	0.0358 (0.0341)	0.0334 (0.0300)	0.0229 (0.0342)	0.0776* (0.0601)	0.0176*** (0.0067)	0.0314* (0.0209)
Hit (%)	21.60	17.40	18.20	21.40	16.80	21.00
LR uc (p val)	0.4693	0.0954	0.4354	0.4227	0.0803*	0.6252
LR ind (p val)	0.7406	0.3545	0.1343	0.2698	0.9511	0.2836
LR cc (p val)	0.7286	0.1621	0.2404	0.3944	0.2162	0.4994
DQ (p val)	0.8215	0.1438	0.3423	0.4022	0.0999*	0.7352
VQR (p val)	0.9560	0.9141	0.9426	0.4092	0.5795	0.8726

Note: The results shown were performed on out-of-sample data.
The *, **, *** denote significant coefficients or rejection of models (LR, DQ, VQR) at 10%, 5% and 1% respectively.

TABLE 17: APPENDIX - SYMMETRIC MODEL WITH LAGGED RETURNS AT 10%

Symmetric CAViaR Model with lagged returns						
	Canada	Japan	Norway	South Africa	Switzerland	United Kingdom
10% VaR						
Beta1 (Const)	-3.53E-05** (0.0000)	-0.0001** (0.0001)	-0.0001* (0.0000)	-0.0002** (0.0001)	0.0000 (0.0001)	0.0000 (0.0000)
Beta2 (VaRt-1)	0.9322*** (0.0080)	0.9511*** (0.0117)	0.9512*** (0.0099)	0.8759*** (0.0172)	0.9646*** (0.0134)	0.9524*** (0.0132)
Beta3 (Lag ret)	-0.0968*** (0.0095)	-0.0559*** (0.0120)	-0.0659*** (0.0108)	-0.1694*** (0.0199)	-0.0487*** (0.0138)	-0.0779*** (0.0148)
Hit (%)	11.4000	9.2000	9.2000	11.6000	8.8000	9.4000
LR uc (p val)	0.5237	0.2424	0.6791	0.3985	0.2891	0.4621
LR ind (p val)	0.0220**	0.7127	0.3296	0.5750	0.1553	0.2966
LR cc (p val)	0.0593*	0.4718	0.5708	0.5983	0.2077	0.4426
DQ (p val)	0.4442	0.2874	0.0752*	0.7511	0.2083	0.9558
VQR (p val)	0.9643	0.6673	0.7764	0.0138**	0.8698	0.1570

Note: The results shown were performed on out-of-sample data.
The *, **, *** denote significant coefficients or rejection of models (LR, DQ, VQR) at 10%, 5% and 1% respectively.

TABLE 18: APPENDIX - ASYMMETRIC MODEL WITH LAGGED RETURNS AT 10%

Asymmetric CAViaR Model with lagged returns						
	Canada	Japan	Norway	South Africa	Switzerland	United Kingdom
10% VaR						
Beta1 (Const)	0.0000 (0.0000)	-0.0001** (0.0001)	-0.0001** (0.0000)	-0.0002*** (0.0001)	0.0000 (0.0000)	0.0000 (0.0000)
Beta2 (VaRt-1)	0.9256*** (0.0154)	0.9450*** (0.0154)	0.9577*** (0.0105)	0.8910*** (0.0177)	0.9697*** (0.0108)	0.9527*** (0.0101)
Beta3 (Lag ret)+	-0.0903*** (0.0207)	-0.0747*** (0.0137)	-0.0475*** (0.0194)	-0.1011*** (0.0381)	-0.0552*** (0.0154)	-0.0617*** (0.0155)
Beta4 (Lag ret)-	-0.1277*** (0.0323)	-0.0433*** (0.0146)	-0.0671*** (0.0123)	-0.2000*** (0.0276)	-0.0265* (0.0177)	-0.0909*** (0.0198)
Hit (%)	11.00%	8.20%	9.20%	11.20%	8.00%	9.20%

LR uc (p val)	0.6673	0.0400**	0.7587	0.5237	0.1341	0.2891
LR ind (p val)	0.0787*	0.8556	0.2163	0.2585	0.1717	0.3975
LR cc (p val)	0.1944	0.1193	0.4441	0.4310	0.1280	0.3986
DQ (p val)	0.7728	0.0433**	0.0855*	0.3467	0.0966	0.9569
VQR (p val)	0.9777	0.6900	0.7123	0.1148	0.7964	0.2300

Note: The results shown were performed on out-of-sample data.
The *, **, *** denote significant coefficients or rejection of models (LR, DQ, VQR) at 10%, 5% and 1% respectively.

TABLE 19: APPENDIX - GARCH MODEL WITH LAGGED RETURNS AT 10%

GARCH CAViaR Model with lagged returns

	Canada	Japan	Norway	South Africa	Switzerland	United Kingdom
10% VaR						
Beta1 (Const)	0.0000 (0.0000)	8.46E-07*** (0.0000)	0.0000 (0.0000)	6.90E-07** (0.0000)	0.0000 (0.0000)	0.0000 (0.0000)
Beta2 (VaRt-1)	0.9239*** (0.0069)	0.9506*** (0.0034)	0.9471*** (0.0065)	0.8843*** (0.0111)	0.9616*** (0.0041)	0.9555*** (0.0060)
Beta3 (Lag ret)	0.1152 (0.1745)	0.0501 (0.0475)	0.0725** (0.0433)	0.1546** (0.0776)	0.0495** (0.0298)	0.0753 (0.0811)
Hit (%)	11.00%	8.40%	9.20%	13.20%	8.20%	10.00%
LR uc (p val)	0.5237	0.0855	0.8407	0.0498**	0.3413	0.7587
LR ind (p val)	0.0592	0.7052	0.4076	0.7202	0.1783	0.8793
LR cc (p val)	0.1377	0.2123	0.6956	0.1370	0.2570	0.9430
DQ (p val)	0.8215	0.2608	0.4352	0.4512	0.1110	0.6550
VQR (p val)	0.9693	0.7504	0.6167	0.0482**	0.4168	0.1798

Note: The results shown were performed on out-of-sample data.
The *, **, *** denote significant coefficients or rejection of models (LR, DQ, VQR) at 10%, 5% and 1% respectively.

TABLE 20: APPENDIX - SYMMETRIC MODEL WITH LAGGED RETURNS AT 5%

Symmetric CAViaR Model with lagged returns

	Canada	Japan	Norway	South Africa	Switzerland	United Kingdom
5% VaR						
Beta1 (Const)	0.0000 (0.0000)	-0.0001** (0.0001)	-0.0002*** (0.0001)	-0.0003** (0.0002)	0.0000 (0.0000)	0.0000 (0.0000)
Beta2 (VaRt-1)	0.9170*** (0.0207)	0.9534*** (0.0128)	0.9459*** (0.0205)	0.8776*** (0.0279)	0.9738*** (0.0067)	0.9533*** (0.0071)
Beta3 (Lag ret)	-0.1675*** (0.0388)	-0.0702*** (0.0243)	-0.0822** (0.0381)	-0.2319*** (0.0526)	-0.0493*** (0.0089)	-0.1025*** (0.0129)
Hit (%)	5.40%	5.20%	6.40%	5.60%	2.60%	5.80%
LR uc (p val)	0.5615	0.8900	0.2540	0.9942	0.0492**	0.7758
LR ind (p val)	0.9601	0.6958	0.3743	0.7298	0.2064	0.3132
LR cc (p val)	0.8438	0.9176	0.3516	0.9421	0.0651	0.5774
DQ (p val)	0.2020	0.0884*	0.1576	0.2083	0.3034	0.8132
VQR (p val)	0.7354	0.2705	0.3345	0.0000***	0.3649	0.1942

Note: The results shown were performed on out-of-sample data.
The *, **, *** denote significant coefficients or rejection of models (LR, DQ, VQR) at 10%, 5% and 1% respectively.

TABLE 21: APPENDIX - ASYMMETRIC MODEL WITH LAGGED RETURNS AT 5%

Asymmetric CAViaR Model with lagged returns

	Canada	Japan	Norway	South Africa	Switzerland	United Kingdom
5% VaR						
Beta1 (Const)	-0.0001* (0.0000)	-0.0001*** (0.0000)	-0.0002** (0.0001)	-0.0003** (0.0002)	0.0000 (0.0000)	0.0000 (0.0001)
Beta2 (VaRt-1)	0.9211*** (0.0138)	0.9576*** (0.0067)	0.9436*** (0.0214)	0.8860*** (0.0320)	0.9742*** (0.0064)	0.9577*** (0.0165)
Beta3 (Lag ret)$^+$	-0.1213*** (0.0241)	-0.0913*** (0.0161)	-0.0726** (0.0340)	-0.1266*** (0.0533)	-0.0518*** (0.0099)	-0.0792*** (0.0258)
Beta4 (Lag ret)$^-$	-0.1872*** (0.0152)	-0.0407*** (0.0162)	-0.1027*** (0.0332)	-0.2970*** (0.0510)	-0.0437*** (0.0144)	-0.1130*** (0.0314)
Hit (%)	4.40%	5.80%	6.40%	5.20%	2.80%	5.40%

LR uc (p val)	0.9942	0.9942	0.2010	0.9942	0.0492**	0.6655
LR ind (p val)	0.7475	0.7298	0.4144	0.7475	0.2064	0.0312**
LR cc (p val)	0.9495	0.9421	0.3164	0.9495	0.0651	0.0894
DQ (p val)	0.2019	0.0001***	0.1674	0.2326	0.3868	0.6007
VQR (p val)	0.9752	0.2244	0.2653	0.0000***	0.3121	0.1106

Note: The results shown were performed on out-of-sample data.
The *, **, *** denote significant coefficients or rejection of models (LR, DQ, VQR) at 10%, 5% and 1% respectively.

TABLE 22: APPENDIX - GARCH MODEL WITH LAGGED RETURNS AT 5%

GARCH CAViaR Model with lagged returns

	Canada	Japan	Norway	South Africa	Switzerland	United Kingdom
5% VaR						
Beta1 (Const)	3.56E-07* (0.0000)	1.23E-06*** (0.0000)	2.36E-06** (0.0000)	1.85E-06* (0.0000)	0.0000 (0.0000)	0.0000 (0.0000)
Beta2 (VaRt-1)	0.9139*** (0.0058)	0.9560*** (0.0049)	0.9340*** (0.0103)	0.8748*** (0.0112)	0.9736*** (0.0039)	0.9572*** (0.0067)
Beta3 (Lag ret)	0.2402*** (0.0834)	0.0768* (0.0557)	0.1278 (0.1361)	0.3763 (0.3294)	0.0641 (0.0558)	0.1358 (0.2289)
Hit (%)	5.40	4.60	6.00	5.60	2.60	6.00
LR uc (p val)	0.6610	0.6655	0.2010	0.5615	0.0706*	0.9942
LR ind (p val)	0.9066	0.8797	0.4144	0.5488	0.2353	0.2603
LR cc (p val)	0.9021	0.9004	0.3164	0.7059	0.0964	0.5307
DQ (p val)	0.0248**	0.2409	0.2758	0.2186	0.2974	0.7257
VQR (p val)	0.8684	0.6128	0.0923*	0.0000***	0.0076***	0.1218

Note: The results shown were performed on out-of-sample data.
The *, **, *** denote significant coefficients or rejection of models (LR, DQ, VQR) at 10%, 5% and 1% respectively.

TABLE 23: APPENDIX - SYMMETRIC MODEL WITH LAGGED RETURNS AT 1%

Symmetric CAViaR Model with lagged returns

	Canada	Japan	Norway	South Africa	Switzerland	United Kingdom
1% VaR						
Beta1 (Const)	0.0000 (0.0000)	0.0000 (0.0000)	-0.0004 (0.0005)	-0.0004** (0.0002)	0.000 (0.0001)	-0.0001*** (0.0001)
Beta2 (VaRt-1)	0.9410*** (0.0097)	0.9865*** (0.0021)	0.9157*** (0.0426)	0.8690*** (0.0328)	0.9834*** (0.0047)	0.9250*** (0.0162)
Beta3 (Lag ret)	-0.1732*** (0.0206)	-0.0478*** (0.0068)	-0.2055*** (0.0552)	-0.4295*** (0.1119)	-0.0510*** (0.0094)	-0.2147*** (0.0471)
Hit (%)	0.60%	2.40%	1.20%	1.40%	0.40%	0.60%
LR uc (p val)	0.7488	0.0428	0.9975	0.5121	0.1705	0.7488
LR ind (p val)	0.6858	0.2897	0.6529	0.7193	0.7877	0.6858
LR cc (p val)	0.8754	0.0733	0.9038	0.7562	0.3771	0.8754
DQ (p val)	0.9884	0.0000***	0.8646	0.0311**	0.8343	0.9884
VQR (p val)	0.8891	0.0000***	0.0000***	0.0000***	0.0000***	0.0050***

Note: The results shown were performed on out-of-sample data.
The *, **, *** denote significant coefficients or rejection of models (LR, DQ, VQR) at 10%, 5% and 1% respectively.

TABLE 24: APPENDIX - ASYMMETRIC MODEL WITH LAGGED RETURNS AT 1%

Asymmetric CAViaR Model with lagged returns

	Canada	Japan	Norway	South Africa	Switzerland	United Kingdom
1% VaR						
Beta1 (Const)	0.0000 (0.0001)	0.0000 (0.0000)	-0.0007*** (0.0002)	-0.0004** (0.0002)	0.0000 (0.0001)	-0.0002** (0.0001)
Beta2 (VaRt-1)	0.9415*** (0.0134)	0.9759*** (0.0051)	0.8972*** (0.0157)	0.9123*** (0.0287)	0.9848*** (0.0046)	0.9260*** (0.0130)
Beta3 (Lag ret)+	-0.1693*** (0.0396)	-0.1207*** (0.0208)	-0.1028*** (0.0330)	-0.1391* (0.1000)	-0.0492*** (0.0089)	-0.1832*** (0.0518)
Beta4 (Lag ret)-	-0.1727*** (0.0210)	-0.0142 (0.0144)	-0.2960*** (0.0363)	-0.3944*** (0.0901)	-0.0386*** (0.0151)	-0.2324*** (0.0373)
Hit (%)	0.60%	3.00%	2.00%	1.60%	0.40%	0.60%
LR uc (p val)	0.7488	0.0051***	0.1381	0.7488	0.1705	0.7488

LR ind (p val)	0.6858	0.4135	0.4989	0.6858	0.7877	0.6858
LR cc (p val)	0.8754	0.0141	0.2649	0.8754	0.3771	0.8754
DQ (p val)	0.9884	0.0000***	0.2696	0.0409**	0.8118	0.9910
VQR (p val)	0.8677	0.0000***	0.0000***	0.0000***	0.0000***	0.0111**

Note: The results shown were performed on out-of-sample data.
The *, **, *** denote significant coefficients or rejection of models (LR, DQ, VQR) at 10%, 5% and 1% respectively.

TABLE 25: APPENDIX - GARCH MODEL WITH LAGGED RETURNS AT 1%

GARCH CAViaR Model with lagged returns

	Canada	Japan	Norway	South Africa	Switzerland	United Kingdom
1% VaR						
Beta1 (Const)	0.0000 (0.0000)	0.0000 (0.0000)	1.74E-05** (0.0000)	7.63E-06** (0.0000)	0.0000 (0.0000)	1.95E-06** (0.0000)
Beta2 (VaRt-1)	0.9384*** (0.0021)	0.9754*** (0.0045)	0.8700*** (0.0239)	0.8378*** (0.0216)	0.9794*** (0.0059)	0.9132*** (0.0051)
Beta3 (Lag ret)	0.3713*** (0.0938)	0.1250 (0.1661)	0.4765*** (0.1310)	1.2676** (0.5749)	0.1143 (0.3015)	0.5018 (0.5258)
Hit (%)	0.60%	1.60%	2.00%	1.80%	0.40%	0.60%
LR uc (p val)	0.9975	0.3604	0.1381	0.5356	0.3150	0.7488
LR ind (p val)	0.6529	0.1570	0.4989	0.5891	0.7533	0.6858
LR cc (p val)	0.9038	0.2418	0.2649	0.7133	0.5745	0.8754
DQ (p val)	0.9879	0.0037***	0.1064	0.0301**	0.8288	0.9840
VQR (p val)	0.2550	0.0002***	0.0000***	0.0000***	0.0000***	0.0725*

Note: The results shown were performed on out-of-sample data.
The *, **, *** denote significant coefficients or rejection of models (LR, DQ, VQR) at 10%, 5% and 1% respectively.

Lagged Returns & Deviations

TABLE 26: APPENDIX - ASYMMETRIC MODEL WITH LAGGED RETURNS AND DEVIATIONS AT 5%

Asymmetric CAViaR Model with lagged returns and deviations from fundamental PPP

	Canada	Japan	Norway	South Africa	Switzerland	United Kingdom
5% VaR						
Beta1 (Const)	0.0000 (0.0001)	-0.0001* (0.0001)	-0.0001 (0.0002)	-0.0005* (0.0004)	0.0000 (0.0000)	-0.0000 (0.0000)
Beta2 (VaRt-1)	0.9411*** (0.0300)	0.9550*** (0.0129)	0.9574*** (0.0245)	0.8751*** (0.0354)	0.9769*** (0.0103)	0.9566*** (0.0146)
Beta3 (Lag ret)+	-0.0743** (0.0486)	-0.0981*** (0.0180)	-0.0492** (0.0268)	-0.1390** (0.0611)	-0.0487*** (0.0157)	-0.0925*** (0.0315)
Beta4 (Lag ret)-	-0.1543** (0.0886)	-0.0350** (0.0203)	-0.0753*** (0.0311)	-0.3233** (0.1723)	-0.0400** (0.0233)	-0.1183** (0.0562)
Beta5 (PPP)+	-0.0007 (0.0006)	-0.0003 (0.0006)	-0.0005 (0.0005)	0.0011 (0.0010)	-0.0001 (0.0001)	0.0000 (0.0002)
Beta6 (PPP)-	-0.0002 (0.0002)	0.0003 (0.0003)	0.0000 (0.0002)	0.0007** (0.0004)	0.0001 (0.0001)	0.0001 (0.0002)
Hit (%)	3.30%	4.40%	5.10%	4.90%	3.20%	4.80%
LR uc (p val)	0.0089***	0.3784	0.8792	0.8900	0.0054***	0.7758
LR ind (p val)	0.9280	0.4404	0.1594	0.7787	0.0970*	0.0277**
LR cc (p val)	0.0325**	0.5038	0.3673	0.9521	0.0053***	0.0851*
DQ (p val)	0.0253**	0.1373	0.3062	0.1159	0.0983*	0.1715
VQR (p val)	0.1531	0.2641	0.4123	0.0000***	0.0215**	0.0458*

Note: The results shown were performed on out-of-sample data.
The *, **, *** denote significant coefficients or rejection of models (LR, DQ, VQR) at 10%, 5% and 1% respectively.

TABLE 27: APPENDIX - GARCH MODEL WITH LAGGED RETURNS AND DEVIATIONS AT 1%

GARCH CAViaR Model with lagged returns and deviations from fundamental PPP

	Canada	Japan	Norway	South Africa	Switzerland	United Kingdom
1% VaR						
Beta1 (Const)	0.0000 (0.0000)	0.0000 (0.0000)	0.0000 (0.0000)	0.0000 (0.0000)	0.0000 (0.0000)	0.0000 (0.0000)
Beta2 (VaRt-1)	0.9389*** (0.0096)	0.9446*** (0.0096)	0.9397*** (0.0156)	0.8439*** (0.0319)	0.9665*** (0.0078)	0.9113*** (0.0198)

Beta3 (Lag ret)	0.3653 (0.2996)	0.2585** (0.1174)	0.3424*** (0.1184)	1.2852 (1.7207)	0.1414 (0.5141)	0.5244** (0.3117)
Beta4 (PPP)	0.0000 (0.0003)	0.0000 (0.0008)	0.0000 (0.0005)	0.0000 (0.0010)	0.0001 (0.0010)	0.0001 (0.0008)
Hit (%)	1.10%	1.40%	1.10%	1.00%	0.80%	1.00%
LR uc (p val)	0.7520	0.2292	0.7520	0.9975	0.5121	0.9975
LR ind (p val)	0.6207	0.5281	0.6207	0.6529	0.7193	0.6529
LR cc (p val)	0.8416	0.3978	0.8416	0.9038	0.7562	0.9038
DQ (p val)	0.9899	0.5620	0.2073	0.0940*	0.9801	0.9916
VQR (p val)	0.2182	0.0000***	0.0000***	0.0000***	0.0000***	0.0257**

Note: The results shown were performed on out-of-sample data.
The *, **, *** denote significant coefficients or rejection of models (LR, DQ, VQR) at 10%, 5% and 1% respectively.

Lagged Returns & Interest Rate Differential

TABLE 28: APPENDIX - SYMMETRIC MODEL WITH LAGGED RETURNS AND INTEREST RATE DIFFERENTIAL AT 5%

Symmetric CAViaR Model with lagged returns and interest rate differential

	Canada	Japan	Norway	South Africa	Switzerland	United Kingdom
5% VaR						
Beta1 (Const)	-0.0001 (0.0000)	-0.0001* (0.0001)	-0.0002 (0.0002)	-0.0004*** (0.0002)	0.0000 0.0001)	0.0000 (0.0000)
Beta2 (VaRt-1)	0.9198*** 0.0282	0.9559*** 0.0143	0.9518*** 0.0207	0.8740*** 0.0199	0.9763*** 0.0104	0.9550*** 0.0122
Beta3 (Lag ret)	-0.1623*** 0.0515	-0.0724*** 0.0171	-0.0735*** 0.0259	-0.2554*** 0.0463	-0.0466*** 0.0164	-0.1082*** 0.0283
Beta4 (Int diff)	0.0171 0.0243	0.0047 0.0085	0.0054 0.0078	0.0250* 0.0190	0.0013 0.0054	0.0059 0.0079
Hit (%)	5.20%	4.60%	5.80%	4.50%	3.50%	4.50%
LR uc (p val)	0.7675	0.5614	0.2540	0.4652	0.0221**	0.4652
LR ind (p val)	0.6362	0.5478	0.3743	0.4059	0.1559	0.4059
LR cc (p val)	0.8559	0.7052	0.3516	0.5422	0.0266**	0.5422
DQ (p val)	0.3134	0.4389	0.7459	0.0585*	0.2074	0.2983
VQR (p val)	0.7227	0.1266	0.3062	0.0000***	0.1917	0.0363**

Note: The results shown were performed on out-of-sample data.
The *, **, *** denote significant coefficients or rejection of models (LR, DQ, VQR) at 10%, 5% and 1% respectively.

TABLE 29: APPENDIX - SYMMETRIC MODEL WITH LAGGED RETURNS AND INTEREST RATE DIFFERENTIAL AT 1%

Symmetric CAViaR Model with lagged returns and interest rate differential

	Canada	Japan	Norway	South Africa	Switzerland	United Kingdom
1% VaR						
Beta1 (Const)	-0.0000 (0.0001)	-0.0000 (0.0000)	-0.0003 (0.0005)	-0.0004 (0.0008)	-0.0001 0.0001)	-0.0002 (0.0004)
Beta2 (VaRt-1)	0.9460*** (0.0144)	0.9853*** (0.0030)	0.9303*** (0.0491)	0.8469*** (0.0571)	0.9846*** (0.0071)	0.9208*** (0.0390)
Beta3 (Lag ret)	-0.1646*** (0.0336)	-0.0535*** (0.0092)	-0.1875*** (0.0762)	-0.5040*** (0.1804)	-0.0385*** (0.0130)	-0.2279*** (0.0899)
Beta4 (Int diff)	0.0099 (0.0474)	0.0075** (0.0042)	0.0058 (0.0316)	-0.0378 (0.0649)	0.0111 (0.0090)	0.0510 (0.0477)
Hit (%)	0.70%	1.20%	0.90%	0.90%	0.40%	0.80%
LR uc (p val)	0.3150	0.5356	0.7488	0.7488	0.0303**	0.5121
LR ind (p val)	0.7533	0.1302	0.6858	0.6858	0.8577	0.7193
LR cc (p val)	0.5745	0.2627	0.8754	0.8754	0.0941*	0.7562
DQ (p val)	0.9843	0.0665*	0.9985	0.0943*	0.3840	0.9882
VQR (p val)	0.0416*	0.0000***	0.0000***	0.0000***	0.0000***	0.0000***

Note: The results shown were performed on out-of-sample data.
The *, **, *** denote significant coefficients or rejection of models (LR, DQ, VQR) at 10%, 5% and 1% respectively.

Lagged Returns & EMP

TABLE 30: APPENDIX - SYMMETRIC MODEL WITH LAGGED RETURNS AND EMP AT 5%

Symmetric CAViaR Model with lagged returns and EMP

	Canada	Japan	Norway	South Africa	Switzerland	United Kingdom
5% VaR						
Beta1 (Const)	0.0000 (0.0001)	-0.0001 (0.0000)	-0.0001 (0.0001)	-0.0003** (0.0001)	0.0000 (0.0001)	0.0000 (0.0000)
Beta2 (VaRt-1)	0.9228*** (0.0416)	0.9619*** (0.0112)	0.9528*** (0.0189)	0.8756*** (0.0334)	0.9757*** (0.0101)	0.9557*** (0.0103)
Beta3 (Lag ret)	-0.1514** (0.0801)	-0.0327** (0.0182)	-0.0636*** (0.0226)	-0.2557*** (0.0887)	-0.0414 (0.0337)	-0.0874*** (0.0200)

Beta4 (EMP)	-0.0001 (0.0002)	-0.0003** (0.0001)	-0.0001 (0.0001)	0.0000 (0.0002)	-0.0001 (0.0003)	-0.0002*** (0.0000)
Hit (%)	5.20%	5.70%	5.80%	4.60%	3.30%	4.90%
LR uc (p val)	0.7675	0.3164	0.2540	0.5614	0.0089***	0.8900
LR ind (p val)	0.6362	0.8807	0.3743	0.3732	0.1145	0.2859
LR cc (p val)	0.8559	0.5986	0.3516	0.5682	0.0094***	0.5604
DQ (p val)	0.0713*	0.0112**	0.7446	0.0423**	0.1345	0.1323
VQR (p val)	0.5873	0.2782	0.3350	0.0000***	0.1003	0.0127**

Note: The results shown were performed on out-of-sample data.
The *, **, *** denote significant coefficients or rejection of models (LR, DQ, VQR) at 10%, 5% and 1% respectively.

TABLE 31: APPENDIX - ADAPTIVE MODEL WITH LAGGED RETURNS AND EMP AT 1%

Adaptive CAViaR Model with lagged returns and EMP

	Canada	Japan	Norway	South Africa	Switzerland	United Kingdom
1% VaR						
Beta1 (Lag ret)	-0.0046*** (0.0000)	-0.0034*** (0.0000)	-0.0106*** (0.0000)	-0.0228*** (0.0000)	-0.0039*** (0.0000)	-0.0070*** (0.0000)
Beta2 (EMP)	0.0005*** (0.0000)	0.0002*** (0.0000)	0.0005*** (0.0000)	0.0006*** (0.0000)	0.0002*** (0.0000)	0.0006*** (0.0000)
Hit (%)	1.80%	0.90%	1.10%	1.20%	0.80%	0.90%
LR uc (p val)	0.0221**	0.7488	0.7520	0.5356	0.5121	0.7488
LR ind (p val)	0.3288	0.6858	0.6207	0.5891	0.0009	0.6858
LR cc (p val)	0.0452**	0.8754	0.8416	0.7133	0.0034	0.8754
DQ (p val)	0.0164**	0.9987	0.9558	0.2934	0.0000***	0.9977
VQR (p val)	0.0000***	0.0000***	0.0003***	0.0000***	0.0000***	0.5666

Note: The results shown were performed on out-of-sample data.
The *, **, *** denote significant coefficients or rejection of models (LR, DQ, VQR) at 10%, 5% and 1% respectively.

Lagged Returns, Deviations & Interest Rate Differential

TABLE 32: APPENDIX - SYMMETRIC MODEL WITH LAGGED RETURNS, DEVIATIONS AND INTEREST RATE DIFFERENTIAL AT 5%

Symmetric CAViaR Model with lagged returns, deviations from PPP and interest rate differential						
	Canada	Japan	Norway	South Africa	Switzerland	United Kingdom
5% VaR						
Beta1 (Const)	0.0000 (0.0001)	-0.0001* (0.0001)	-0.0002* (0.0001)	-0.0005*** (0.0002)	0.0000 (0.0000)	0.0000 (0.0000)
Beta2 (VaRt-1)	0.9247*** (0.0241)	0.9581*** (0.0119)	0.9582*** (0.0188)	0.8738*** (0.0417)	0.9766*** (0.0060)	0.9552*** (0.0148)
Beta3 (Lag ret)	-0.1550*** (0.0428)	-0.0698*** (0.0296)	-0.0618*** (0.0238)	-0.2463*** (0.1026)	-0.0460*** (0.0078)	-0.1076*** (0.0394)
Beta4 (PPP)	-0.0003 (0.0002)	-0.0001 (0.0001)	0.0001 (0.0001)	0.0004 (0.0004)	0.0001 (0.0001)	0.0000 (0.0003)
Beta5 (Int diff)	-0.0014** (0.0238)	0.0076*** (0.0076)	0.0026*** (0.0091)	0.0269** (0.0174)	0.0014*** (0.0044)	0.0038** (0.0149)
Hit (%)	4.50%	4.50%	5.80%	4.40%	4.00%	4.40%
LR uc (p val)	0.4652	0.4652	0.2540	0.3784	0.1351	0.3784
LR ind (p val)	0.5015	0.5015	0.3743	0.4404	0.3000	0.4404
LR cc (p val)	0.6111	0.6111	0.3516	0.5038	0.1914	0.5038
DQ (p val)	0.2624	0.5792	0.5562	0.0603*	0.3944	0.3664
VQR (p val)	0.1606	0.2845	0.0000***	0.2748	0.0411**	0.0106**

Note: The results shown were performed on out-of-sample data.
The *, **, *** denote significant coefficients or rejection of models (LR, DQ, VQR) at 10%, 5% and 1% respectively.

TABLE 33: APPENDIX - GARCH MODEL WITH LAGGED RETURNS, DEVIATIONS AND INTEREST RATE DIFFERENTIAL AT 1%

GARCH CAViaR Model with lagged returns, deviations from PPP and interest rate differential						
	Canada	Japan	Norway	South Africa	Switzerland	United Kingdom
1% VaR						
Beta1 (Const)	0.0000 (0.0000)	0.0000 (0.0000)	0.0000 (0.0000)	0.0000 (0.0000)	0.0000 (0.0000)	0.0000 (0.0000)
Beta2 (VaRt-1)	0.9386*** (0.0031)	0.9484*** (0.0070)	0.9427*** (0.0220)	0.8383*** (0.0276)	0.9705*** (0.0055)	0.9046*** (0.0052)
Beta3 (Lag ret)	0.3649*** (0.1034)	0.2374* (0.1508)	0.3196*** (0.1030)	1.3013** (0.7025)	0.1265 (0.3104)	0.5594* (0.3461)
Beta4 (PPP)	0.0000 (0.0002)	0.0000 (0.0004)	0.0000 (0.0007)	0.0000 (0.0004)	0.0001 (0.0007)	0.0001 (0.0017)

Beta5 (Int diff)	-0.1925 (0.2236)	-0.0676 (1.0130)	0.0340 (0.4027)	0.0786 (0.4992)	-0.0838 (0.3553)	-0.2624 (2.9942)
Hit (%)	1.10%	1.30%	1.10%	1.10%	0.80%	0.90%
LR uc (p val)	0.7520	0.3604	0.7520	0.7520	0.5121	0.7488
LR ind (p val)	0.6207	0.5582	0.6207	0.6207	0.7193	0.6858
LR cc (p val)	0.8416	0.5545	0.8416	0.8416	0.7562	0.8754
DQ (p val)	0.9899	0.8069	0.2045	0.0989*	0.9837	0.9544
VQR (p val)	0.0000***	0.0000***	0.0000***	0.0000***	0.0226**	0.0252**

Note: The results shown were performed on out-of-sample data.
The *, **, *** denote significant coefficients or rejection of models (LR, DQ, VQR) at 10%, 5% and 1% respectively.

Lagged Returns, Deviations & EMP

Table 34: Appendix - Symmetric Model with Lagged Returns, Deviations and EMP at 5%

Symmetric CAViaR Model with lagged returns, deviations from PPP and EMP

	Canada	Japan	Norway	South Africa	Switzerland	United Kingdom
5% VaR						
Beta1 (Const)	0.0000 (0.0000)	0.0000 (0.0001)	-0.0001* (0.0001)	-0.0004 (0.0003)	0.0000 (0.0000)	-0.0000 (0.0000)
Beta2 (VaRt-1)	0.9264*** (0.0245)	0.9631*** (0.0090)	0.9545*** (0.0220)	0.8677*** (0.0332)	0.9754*** (0.0077)	0.9561*** (0.0197)
Beta3 (Lag ret)	-0.1395*** (0.0408)	-0.0282 (0.0516)	-0.0612** (0.0338)	-0.2587*** (0.0799)	-0.0428*** (0.0101)	-0.0873** (0.0468)
Beta4 (PPP)	-0.0003* (0.0002)	0.0000 (0.0002)	0.0000 (0.0001)	0.0005 (0.0007)	0.0001 (0.0001)	0.0001 (0.0002)
Beta5 (EMP)	-0.0001*** (0.0001)	-0.0003 (0.0006)	-0.0001 (0.0001)	0.0000 (0.0004)	-0.0001 (0.0001)	-0.0001*** (0.0000)
Hit (%)	4.70%	6.00%	5.80%	4.50%	3.50%	4.80%
LR uc (p val)	0.6655	0.1567	0.2540	0.4652	0.0221**	0.7758
LR ind (p val)	0.8797	0.4566	0.3743	0.4059	0.1559	0.3132
LR cc (p val)	0.9004	0.2781	0.3516	0.5422	0.0266	0.5774
DQ (p val)	0.4412	0.0045***	0.7457	0.0512*	0.2111	0.1903
VQR (p val)	0.6680	0.1217	0.3405	0.0000***	0.2898	0.0106**

Note: The results shown were performed on out-of-sample data.
The *, **, *** denote significant coefficients or rejection of models (LR, DQ, VQR) at 10%, 5% and 1% respectively.

TABLE 35: APPENDIX - ASYMMETRIC MODEL WITH LAGGED RETURNS, DEVIATIONS AND EMP AT 1%

Asymmetric CAViaR Model with lagged returns, deviations from PPP and EMP

	Canada	Japan	Norway	South Africa	Switzerland	United Kingdom
1% VaR						
Beta1 (Const)	-0.0001 (0.0001)	-0.0001 (0.0002)	-0.0011** (0.0005)	-0.0003 (0.0003)	0.0000 (0.0000)	-0.0001 (0.0001)
Beta2 (VaRt-1)	0.9489*** (0.0066)	0.9502*** (0.0103)	0.8782*** (0.0432)	0.9120*** (0.0410)	0.9822*** (0.0053)	0.9319*** (0.0143)
Beta3 (Lag ret)+	-0.1126*** (0.0222)	-0.1573 (0.1451)	0.0212 (0.0980)	-0.0540 (0.1166)	-0.0614*** (0.0127)	-0.1941** (0.1171)
Beta4 (Lag ret)-	-0.1746*** (0.0170)	0.2127* (0.1315)	-0.2925*** (0.0741)	-0.3957*** (0.1063)	-0.0191** (0.0092)	-0.1524 (0.1262)
Beta5 (PPP)+	0.0001 (0.0004)	0.0001 (0.0002)	-0.0019 (0.0020)	0.0001 (0.0009)	-0.0003 (0.0002)	0.0000 (0.0007)
Beta6 (PPP)-	0.0004* (0.0003)	0.0003 (0.0004)	0.0006 (0.0007)	-0.0003 (0.0010)	-0.0003** (0.0002)	0.0005 (0.0008)
Beta7 (EMP)+	-0.0002 (0.0005)	-0.0002 (0.0009)	-0.0007 (0.0009)	-0.0013 (0.0016)	0.0001** (0.0000)	0.0000 (0.0001)
Beta8 (EMP)-	0.0001* (0.0001)	-0.0024*** (0.0009)	-0.0001 (0.0001)	0.0001*** (0.0000)	-0.0003* (0.0002)	-0.0005 (0.0014)
Hit (%)	0.80%	2.20%	1.20%	1.00%	0.40%	0.90%
LR uc (p val)	0.5121	0.0010	0.5356	0.9975	0.0303**	0.7488
LR ind (p val)	0.7193	0.5054	0.5891	0.6529	0.8577	0.6858
LR cc (p val)	0.7562	0.0035***	0.7133	0.9038	0.0941*	0.8754
DQ (p val)	0.9871	0.0000***	0.9778	0.1421	0.1974	0.9941
VQR (p val)	0.0989*	0.0000***	0.0000***	0.0000***	0.0000***	0.1474

Note: The results shown were performed on out-of-sample data.
The *, **, *** denote significant coefficients or rejection of models (LR, DQ, VQR) at 10%, 5% and 1% respectively.

Lagged Returns, Interest Rate Differential & EMP

TABLE 36: APPENDIX - SYMMETRIC MODEL WITH LAGGED RETURNS, INTEREST RATE DIFFERENTIAL AND EMP AT 5%

Symmetric CAViaR Model with lagged returns, interest rate differential and EMP						
	Canada	Japan	Norway	South Africa	Switzerland	United Kingdom
5% VaR						
Beta1 (Const)	0.0000 (0.0001)	-0.0001* (0.0001)	-0.0001* (0.0001)	-0.0004* (0.0002)	0.0000 (0.0001)	0.0000 (0.0000)
Beta2 (VaRt-1)	0.9231*** (0.0253)	0.9616*** (0.0079)	0.9535*** (0.0235)	0.8758*** (0.0243)	0.9765*** (0.0068)	0.9528*** (0.0130)
Beta3 (Lag ret)	-0.1488*** (0.0378)	-0.0317* (0.0239)	-0.0638** (0.0375)	-0.2465*** (0.0571)	-0.0398*** (0.0067)	-0.0942*** (0.0317)
Beta4 (Int diff)	0.0095 (0.0227)	0.0033 (0.0066)	0.0071 (0.0106)	0.0253* (0.0165)	0.0019 (0.0052)	0.0053 (0.0154)
Beta5 (EMP)	-0.0001** (0.0001)	-0.0003*** (0.0001)	-0.0001 (0.0001)	-0.0001 (0.0003)	-0.0001 (0.0000)	-0.0002*** (0.0000)
Hit (%)	5.10%	4.90%	5.80%	4.50%	3.10%	4.90%
LR uc (p val)	0.8792	0.8900	0.2540	0.4652	0.0032***	0.8900
LR ind (p val)	0.6822	0.6958	0.3743	0.4059	0.0814*	0.2859
LR cc (p val)	0.9091	0.9176	0.3516	0.5422	0.0028***	0.5604
DQ (p val)	0.0629**	0.3341	0.7457	0.0596*	0.0735*	0.1265
VQR (p val)	0.5758	0.3035	0.3788	0.0000***	0.0463**	0.0090***

Note: The results shown were performed on out-of-sample data.
The *, **, *** denote significant coefficients or rejection of models (LR, DQ, VQR) at 10%, 5% and 1% respectively.

TABLE 37: APPENDIX - SYMMETRIC MODEL WITH LAGGED RETURNS, INTEREST RATE DIFFERENTIAL AND EMP AT 1%

Symmetric CAViaR Model with lagged returns, interest rate differential and EMP						
	Canada	Japan	Norway	South Africa	Switzerland	United Kingdom
1% VaR						
Beta1 (Const)	0.0000 (0.0000)	-0.0001* (0.0001)	-0.0003* (0.0004)	-0.0004* (0.0004)	0.0000 (0.0001)	-0.0002 (0.0001)
Beta2 (VaRt-1)	0.9463*** (0.0037)	0.9460*** (0.0081)	0.9228*** (0.0346)	0.8509*** (0.0736)	0.9864*** (0.0039)	0.9208*** (0.0113)
Beta3 (Lag ret)	-0.1513*** (0.0124)	0.1013*** (0.0322)	-0.1904*** (0.0684)	-0.4286** (0.2565)	-0.0533*** (0.0108)	-0.2239*** (0.0320)
Beta4 (Int diff)	0.0266* (0.0196)	0.0088 (0.0137)	0.0198 (0.0390)	-0.0021 (0.0342)	0.0049 (0.0070)	0.0494** (0.0221)
Beta5 (EMP)	-0.0002*** (0.0000)	-0.0024*** (0.0003)	-0.0002* (0.0001)	-0.0008* (0.0005)	0.0001*** (0.0000)	-0.0001** (0.0000)
Hit (%)	0.70%	1.60%	0.80%	1.00%	1.10%	0.90%
LR uc (p val)	0.3150	0.0788*	0.5121	0.9975	0.7520	0.7488
LR ind (p val)	0.7533	0.2528	0.7193	0.6529	0.0043***	0.6858
LR cc (p val)	0.5745	0.1110	0.7562	0.9038	0.0161**	0.8754
DQ (p val)	0.9842	0.0535*	0.9954	0.1170	0.0000***	0.9911
VQR (p val)	0.1470	0.0000***	0.0000***	0.0000***	0.0000***	0.0000***

Note: The results shown were performed on out-of-sample data.
The *, **, *** denote significant coefficients or rejection of models (LR, DQ, VQR) at 10%, 5% and 1% respectively.

Lagged Returns, Deviations, Interest Rate Differential & EMP

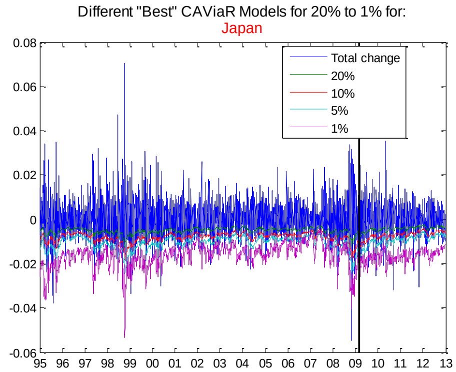
TABLE 38: APPENDIX - ASYMMETRIC MODEL WITH LAGGED RETURNS, DEVIATIONS, INTEREST RATE DIFFERENTIAL AND EMP AT 5%

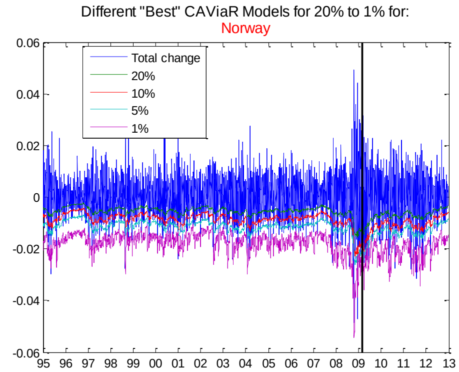
Asymmetric CAViaR Model with lagged returns, deviations from PPP, int. rate. diff and EMP

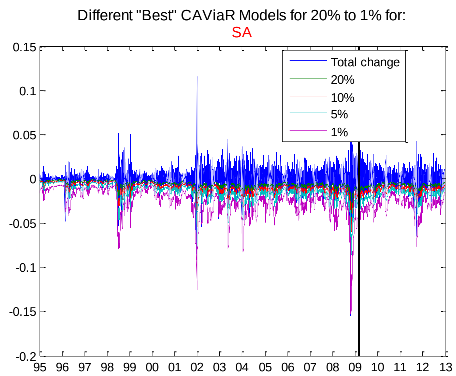
	Canada	Japan	Norway	South Africa	Switzerland	United Kingdom
1% VaR						
Beta1 (Const)	0.0000 (0.0000)	-0.0001** (0.0001)	-0.0001* (0.0001)	-0.0004 (0.0003)	0.0000 (0.0000)	4.13E-05* (0.0000)
Beta2 (VaRt-1)	0.9420*** (0.0068)	0.9531*** (0.0135)	0.9592*** (0.0065)	0.8902*** (0.0165)	0.9786*** (0.0054)	0.9624*** (0.0041)
Beta3 (Lag ret)+	-0.0521 (0.0473)	-0.1082 (0.1092)	-0.0430** (0.0219)	-0.0955* (0.0699)	-0.0454*** (0.0161)	-0.0644** (0.0299)
Beta4 (Lag ret)-	-0.1527*** (0.0233)	0.0201 (0.1073)	-0.0730*** (0.0306)	-0.3037*** (0.0229)	-0.0131 (0.0223)	-0.0616** (0.0302)
Beta5 (PPP)+	-0.0007*** (0.0003)	-0.0002 (0.0007)	-0.0005* (0.0003)	0.0007 (0.0007)	0.0000 (0.0002)	-0.0001 (0.0002)
Beta6 (PPP)-	-0.0002 (0.0002)	0.0002 (0.0003)	0.0001 (0.0001)	0.0002 (0.0007)	0.0001 (0.0001)	0.0001 (0.0002)
Beta7 (Int diff)+	0.0020 (0.0212)	0.0040 (0.0147)	0.0237 (0.0290)	0.3840*** (0.1396)	0.0006 (0.0070)	0.0389 (0.1007)
Beta8 (Int diff)-	-0.0265 (0.0503)	-0.5251 (0.6261)	0.0026 (0.0086)	0.0088 (0.0156)	-0.0086 (0.6724)	-0.0016 (0.0126)
Beta9 (EMP)+	-0.0002*** (0.0001)	0.0000 (0.0007)	0.0000 (0.0001)	-0.0003*** (0.0000)	0.0000 (0.0000)	-0.0002*** (0.0001)
Beta10 (EMP)-	0.0000 (0.0001)	-0.0005 (0.0010)	0.0000 (0.0002)	0.0000 (0.0001)	-0.0002** (0.0001)	-0.0003** (0.0002)
Hit (%)	3.10%	4.40%	4.70%	4.80%	1.30%	4.60%
LR uc (p val)	0.0021	0.0032	0.4704	0.4652	0.0492	0.0221
LR ind (p val)	0.5001	0.9683	0.5712	0.5015	0.2064	0.1109
LR cc (p val)	0.0329	0.0129	0.6564	0.6111	0.0651	0.0205
DQ (p val)	0.0088***	0.0346**	0.3914	0.0732*	0.0000***	0.1819
VQR (p val)	0.0000	0.0003	0.1386	0.0000	0.1400	0.0000

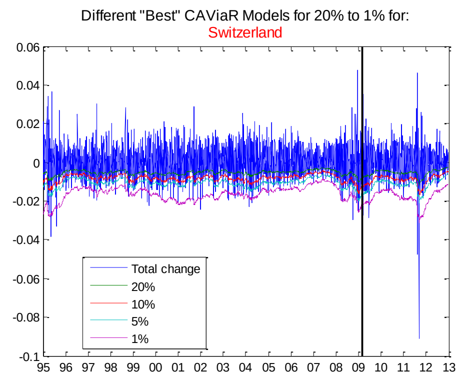
Note: The results shown were performed on out-of-sample data.
The *, **, *** denote significant coefficients or rejection of models (LR, DQ, VQR) at 10%, 5% and 1% respectively.

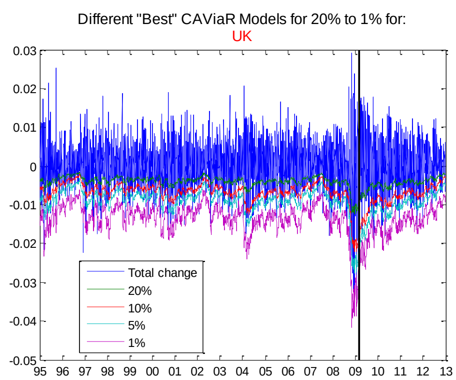
Appendix B: Figures











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