Distributional Robustness and Inequity Mitigation in Disaster Preparedness of Humanitarian Operations *

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Abstract

Problem definition: In this paper, we study a predisaster relief network design problem with uncertain demands. The aim is to determine the prepositioning and reallocation of relief supplies. Motivated by the call of the International Federation of Red Cross and Red Crescent Societies (IFRC) to leave no one behind, we consider three important practical aspects of humanitarian operations: shortages, equity, and uncertainty. **Methodology/results:** We first employ a form of robust satisficing measure, which we call the *Shortage Severity Measure*, to evaluate the severity of the shortage caused by uncertain demand in a context with limited distribution information. Because shortages often raise concerns about equity, we then formulate a mixed-integer lexicographic optimization problem with non-convex objectives and design a new branch-and-bound algorithm to identify the exact solution. We also propose two approaches for identifying optimal postdisaster adaptable resource reallocation: an exact approach and a conservative approximation that is more computationally efficient. Our case study considers the 2010 Yushu earthquake, which occurred in northwestern China, and demonstrates the value of our methodology in mitigating geographical inequities and reducing shortages. **Managerial implications:** In our case study, we show that (i) incorporating equity in both predisaster deployment and postdisaster reallocation can produce substantially more equitable shortage prevention strategies while sacrificing only a reasonable amount of total shortage; (ii) increasing donations/budgets may not necessarily alleviate the shortage suffered by the most vulnerable individuals if equity is not fully considered; (iii) exploiting disaster magnitude information when quantifying uncertainty can help alleviate geographical inequities caused by uncertain relief demands.

1 Introduction

The number of disasters reported worldwide and their impact on the population has increased in recent decades. Extreme events such as tornadoes, earthquakes, or hurricanes can strike a community without warning and cause massive damage and many casualties. For example, the Emergency Events Database has recorded 7,348 natural disasters over the last twenty years

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(2000-2019), affecting over 4 billion people (many on more than one occasion), and causing economic losses of \$2.97 trillion around the world (EM-DAT 2020). The massive-scale social and economic damages caused by disasters have brought increasing attention to the need for effective disaster relief management.

Prepositioning of emergency supplies can be an effective mechanism for improving response to natural disasters. In the fall of 2019, hurricane Dorian was estimated to have caused up to \$3 billion in losses in the Caribbean (CNBC 2019), and highlighted the inadequacy of existing prepositioning strategies. This paper focuses on the prepositioning strategy in disaster relief systems and considers the Predisaster Relief Network Design Problem (PRNDP) to prepare for sudden disasters. This problem determines the locations and capacities of the response facilities and the inventory levels at each facility, as well as reallocations of relief supplies to distribution locations in order to improve the effectiveness of the postdisaster relief operations.

A good location and emergency inventory prepositioning strategy is critical for disaster relief operations, since lack of relief supplies may cause suffering and loss of life among victims. However, disaster preparedness is subject to considerable uncertainty because it is not known where events will occur (or if they will occur at all). As a result, disaster-stricken areas often face shortages of emergency supplies. For example, the Fritz Institute reported that there was a massive shortage of supplies and medical personnel during the 2004 tsunami in Southeast Asia (Fritz Institute 2005). Food and water shortages also appeared in the Philippines after being hit by typhoon Haiyan in 2013 (Uichanco 2022). Furthermore, in the winter of 2016/2017, an extreme dzud (a kind of dreaded severe weather) exposed more than 255,000 herders in Mongolia to water and food shortages and killed millions of animals (BBC 2016). To alleviate the suffering caused by shortages, since 2011, appeals for funding by humanitarian organizations (HOs) have steadily increased, but more than 55% of the requirements have still not been met (Besiou and Van Wassenhove 2020).

Shortages of relief supplies often raise concerns about the equity of disaster relief systems. Specifically, on 1 January 2016, the world officially began implementation of the 2030 Agenda for Sustainable Development and pledged to leave no one behind. In addition, many large international HOs, such as the IFRC, began to call on leaving no one behind in humanitarian response (IFRC 2018). However, there is little research to-date on the equity regarding predisaster deployment decisions. We attempt to bridge this gap in our research.

1.1 Research Questions

This study mainly attempts to answer the following research questions:

- *How to measure the severity of possible shortages in the presence of demand uncertainty?* Predisaster deployment decisions are often constrained by demand uncertainty and limited budgets, which may lead to shortages of supplies in certain affected areas after a disaster. Moreover, shortages would differently affect people in affected areas. In New York City, for example, three weeks after Hurricane Sandy, people in high-poverty census tracts remained significantly more worried about food and medicine than those in wealthy neighborhoods possessing the coping capacity for disaster recovery (Subaiya et al. 2014). Therefore, humanitarian practitioners need to measure the severity of the shortage according to local conditions under demand uncertainty to make effective predisaster deployment decisions. To answer this question, we employ a form of robust satisficing measure to mitigate the severity of shortages under uncertainty.

- *How to allocate limited resources equitably among beneficiaries to reduce the impact of shortages?* Equity is an essential requirement in humanitarian operations and has received widespread attention. Starr and Van Wassenhove, writing in the Special Issue for the Board of the POMS College on Humanitarian Operations and Crisis Management (Starr and Van Wassenhove 2014), state "Humanitarians need to bring relief items to all beneficiaries in an equitable fashion, even if this is far from being efficient". They continue "There is an obvious need to consider equity in addition to classical efficiency or cost minimization objectives." Therefore, HOs need to give a more formal treatment of equity in predisaster deployment and humanitarian response (Besiou and Van Wassenhove 2020). In this study, we characterize the concepts of an equitable solution and formulate a lexicographic optimization problem to mitigate inequity caused by shortages.

- *How to address the inherent difficulty of quantifying postdisaster demand to improve on predisaster deployment and postdisaster response decisions?* The initial prepositioning deployment decisions are difficult to make in the presence of demand uncertainty. For the HOs to deploy high inventory levels in each possible disaster area will be too costly for their limited budgets and donations, especially if postdisaster demands are relatively small (Stauffer and Kumar 2021). If prepositioning of supplies is not enough, then during a major disaster, many regions may suffer from shortages and even secondary casualties. Thus, HOs want to find a trade-off between small initial deployment levels that are within limited resources and large initial deployment levels that avoid serious shortages. Furthermore, a reasonable description of uncertain demands is a key issue in finding such a balance (Uichanco 2022). In response to this question, we develop a distributionally robust optimization model that adapts the ambiguity set to the magnitude of the disaster.

1.2 Our Contributions

1. *Model*: Our model considers three important practical aspects of humanitarian operations (i.e., shortages, equity, and uncertainty). First, to the best of our understanding, this is the first paper to use the *Shortage Severity Measure* (SSM) to control the uncertain relief shortages. Specifically, as an example of *satisficing measure* (Brown and Sim 2009), the SSM is an axiomatically motivated way of measuring the severity of random shortages when compared to targeted maximum shortage thresholds. Second, we propose a model that ensures that the allocations of the limited resources are equitably distributed among disaster-prone regions. This is done by formulating a mixed-integer lexicographic optimization problem with non-convex objectives. Third, to account for the inherent difficulty of quantifying postdisaster demand distributions, we employ a two-stage robust stochastic optimization model that relies on event-wise moment information. Overall, we consider our model to take an important step toward bridging the gap between theory and practice in humanitarian operations.
2. *Solution approach*: First, we discuss two approaches for identifying optimal adaptable resource reallocation: an exact approach and a conservative approximation that is based on affine decision rules and allows us to solve instances of realistic size. Empirically, the latter also appears surprisingly accurate with a maximum measured optimality gap of 8%. Second, we handle the *lexicographic minimization* aspect of the model using a new branch-and-bound algorithm. This algorithm corrects for a deficiency found in the procedure proposed by Qi (2017). In fact, our algorithm appears to be the first iterative scheme with the guarantee of finding exact solutions to non-convex, mixed-integer, lexicographic optimization problems.
3. *Managerial insights*: Our case study involving a real earthquake case, provides three interesting insights. First, compared to approaches that minimize the total shortage without consideration of equity, decisions that incorporate equity in pre- and post-disaster management can, in some cases, achieve higher levels of equity at the cost of only slightly increasing the total shortage. Second, if equity is ignored, the shortages experienced by some beneficiaries may not be alleviated with an increase in donations. Third, disaster magnitude information, if properly segmented, can alleviate inequities caused by uncertain relief demands.

The paper is organized as follows. In Section 2, we provide a brief literature review. In Section 3, we discuss three important practical aspects of humanitarian operations and present the model. In Section 4, we describe the solution procedure. In Section 5, we perform several numerical studies. We finally conclude in Section 6. All proofs can be found in appendix.

2 Literature Review

In this section, we present a review of the related literature, i.e., uncertainty, shortage, and equity. After that, we discuss the distinction between our paper and the existing literature.

2.1 Predisaster Relief Network Design under Uncertainty

Predisaster relief network design involves decisions regarding facility locations, inventory prepositioning and resource reallocation under uncertainty. The PRNDP as a whole was first studied by Balcik and Beamon (2008), where the authors propose a scenario-based model to maximize the benefits provided to affected people. Rawls and Turnquist (2010) formally introduce the PRNDP that simultaneously determines the decisions of facility location, inventory prepositioning and resource distribution under demand uncertainty. They formulate a risk-neutral two-stage stochastic programming model and propose a Lagrangian L-shaped method. Following their path, stochastic programming is widely used to address uncertainty in PRNDP. In addition, some literature begins to focus on the measurement of risk by applying concepts such as the conditional value-at-risk (CVaR) (Noyan 2012) and probabilistic constraints (Rawls and Turnquist 2011, Hong et al. 2015). Elçi and Noyan (2018) further develop a chance-constrained two-stage stochastic programming model that combines quantitative risk (CVaR) and qualitative risk (probabilistic constraints).

All of the above literature is scenario-based stochastic programming, which has to deal with difficulties in selecting scenarios. This challenge motivates research on the robust optimization method as an alternative. Ni et al. (2018) propose a min-max robust model that integrates the three-part decision making of predisaster relief network design. To address uncertainties in supply, demand and road link capacity, they construct budgeted uncertainty sets and develop computationally tractable reformulations based on the budgeted uncertainty sets. Similarly, Velasquez et al. (2019) apply the budgeted uncertainty set to model demand uncertainty and propose a robust model for prepositioning relief items. Paul and Wang (2019) further develop a two-stage robust optimization model and consider two types of robustness, one of which is the budgeted uncertainty set.

To hedge uncertainty in humanitarian operations, stochastic relief network design often assume uncertainty parameters following fully known distributions and use a finite number of scenarios to model uncertainties. This mainly faces two challenges in practical implementation: (i) computational difficulties for instances with abundant scenarios; (ii) sampling difficulties for uncertainty parameters of high dimension. On the other hand, although robust optimization with budgeted uncertainty sets does not require any knowledge of distributions except for their support, it tends to produce overly conservative solutions due to hedging against the rare worst case.

2.2 Shortage

To reduce shortages in disaster areas, the existing optimization models are mainly described from two perspectives: the objective function and the conditions on the constraints. As for the former, shortage is added to the objective function as a penalty cost. Most optimization models for predisaster relief network design use cost minimization as their objective to reduce shortages. Specifically, in the widely used two-stage stochastic and/or robust programming models, shortage cost is usually considered in the second stage, whereas facility cost and ordering cost are incurred in the first stage. For postdisaster resource reallocation, Altay (2013) considers both the quantity and capability of multiple resources, minimizing the total capability shortages in the objective function. From the perspective of constraint conditions, in order to control shortages, probabilistic constraints are typically employed to ensure that all relief demands can be satisfied with a prescribed high probability (see, for instance, Ozbay and Ozguven 2007, Hong et al. 2015). In addition, Rawls and Turnquist (2011) introduce *service quality* constraints that ensure that the probability of meeting all demand is not lower than a given target level.

Because the penalty cost of shortage is difficult to determine in practice, some HOs will not use cost to guide their decision. Gralla et al. (2014) study five attributes (e.g., quantity, type, location, speed, and cost) based on the preferences of experts toward humanitarian logistics, and the results show that quantity delivered is the most valued objective while cost is the least important. In addition, although we can ensure that the satisfaction of demands is maintained at a certain level by probabilistic constraints, we may ignore the unmet demands and the seriousness of their impact.

2.3 Equity

In the fields of operations research and management science (OR/MS), there are many different ways of incorporating equity into decision making. For a comprehensive overview of equity in OR/MS problems, we refer readers to the recent reviews provided by Karsu and Morton (2015). One of the most common ways of modeling equity is the lexicographic optimization approach, which is applied in many fields, such as resource allocation (Luss 1999), network flows (Nace and Orlin 2007) and appointment scheduling (Qi 2017). Moreover, the solution derived by the lexicographic approach is sometimes considered to be the *most equitable* solution (Karsu and Morton 2015).

In humanitarian operations, equity concerns play a role in the reallocation of relief items to beneficiaries. To provide relief resources to demand locations in a fair manner, the existing literature mainly studies equity from the perspectives of response time and supply quantity. Huang et al. (2012) employ a convex disutility function to study the arrival time equity in humanitarian logistics problems. Holguín-Veras et al. (2013) introduce a new concept called *deprivation cost*, which depends on the deprivation time. They quantify equity concerns for postdisaster humanitarian logistics by using social costs, which is the sum of logistic and deprivation costs. Following their direction, some studies have employed *deprivation costs* in their model to ensure equitable relief delivery operations in some sense (Ni et al. 2018, Yu et al. 2018). Gutjahr and Fischer (2018) show that minimizing the total deprivation cost given a budget may yield inequitable solutions and they propose to extend the deprivation cost objective with the Gini inequity index. In terms of equitable allocation quantities, Noyan et al. (2016) compute the *maximum proportion of unsatisfied demand* among demand locations and apply a proportional allocation policy to ensure equitable allocation of resources in the last mile. Velasquez et al. (2019) introduce equity constraints to ensure that relief items are distributed proportionately to the demand. Huang and Rafiei (2019) investigate equitable resource allocation by balancing the delivery quantities and times to different locations. Arnette and Zobel (2019) apply a measure of relative risk and develop a risk-based objective function to ensure equitable allocations of assets in advance of a natural disaster. Recently, Uichanco (2022) propose a stochastic programming model for typhoon preparedness with two objectives, one of which is a fair strategy by minimizing the expected largest proportion of unmet demand.

2.4 Distinction of Our Work from Past Literature

Motivated by special features that HOs face in practice, our paper considers three important aspects of humanitarian operations: shortages, equity, and uncertainty. Previous studies on predisaster relief network design mainly use a cost criteria (e.g., setup cost, purchase cost, transportation cost, shortage cost, etc.), while some HOs will not use cost to guide their decision (Uichanco 2022). Besiou and Van Wassenhove (2020) also argue that it is not easy to evaluate the performance of humanitarian operations through cost. Furthermore, as we mentioned before, HOs usually put the cost in the least important position and the amount of supply delivered in the first place (Gralla et al. 2014). Therefore, we use the amount of supply shortage to measure the severity of shortages and formulate a mixed-integer lexicographic optimization problem with non-convex objectives.

Although there has been significant progress in addressing lexicographic optimization models (see, for instance, Marchi and Oviedo 1992, Nace and Orlin 2007), these approaches cannot be applied to non-convex models with discrete and continuous decisions, which have received less attention in the literature. Nace and Orlin (2007) provide a polynomial approach for linear lexicographic optimization problems and prove its optimality. Ogryczak et al. (2005) propose a reformulation based on conditional means for lexicographic optimization problems with non-convex feasible sets. Since our model has a non-convex objective function, this method will lead to a mixed-integer non-convex programming reformulation, which we expect to quickly become intractable for large-scale problems. Recently, Letsios et al. (2021) propose a branch-and-bound algorithm for a scheduling problem to obtain exact lexicographic scheduling. Because the scheduling problem belongs to a typical combinatorial optimization problem, they enumerate all possible job-to-machine assignment, which cannot be applied to problems that include both discrete and continuous decisions. In this stream of literature, the most relevant work to ours is Qi (2017), who propose a lexicographic minimization procedure for mixed-integer lexicographic optimization model with non-convex objectives. However, we find that such a procedure cannot guarantee optimality for our problems. Therefore, our paper corrects for a deficiency found in Qi (2017). Specifically, we propose a new branch-and-bound algorithm for solving non-convex, mixed-integer, lexicographic optimization problems and prove its optimality.

3 Problem Formulation

In this section, we begin by introducing some notational conventions used in the remaining sections. We then present a multi-objective two-stage stochastic optimization model for the PRNDP. After that, we introduce a measure to evaluate shortages in the presence of demand uncertainty. Finally, we formulate a lexicographic minimization problem to address equity concerns.

3.1 Notation

We use boldfaced characters to represent vectors (e.g., $\boldsymbol{x} \in \mathbb{R}^n$). We use $|\mathcal{L}|$ to denote the cardinality of a set $\mathcal{L}$ and $(x)^+$ to denote $\max(x, 0)$. We use $\mathscr{P}_0(\mathbb{R}^n)$ to represent the set of all probability distributions on $\mathbb{R}^n$. A random variable, $\tilde{\mathbf{d}}$, is denoted with a tilde sign, and we use $\tilde{\mathbf{d}} \sim \mathbb{P}$, $\mathbb{P} \in \mathscr{P}_0(\mathbb{R}^n)$ to define $\tilde{\mathbf{d}}$ as a n-dimensional random variable with distribution $\mathbb{P}$. We assume that $\mathbb{P}$ lies in a distributional ambiguity set $\mathcal{F} \subset \mathscr{P}_0(\mathbb{R}^n)$ and denote $\mathbb{E}_{\mathbb{P}}[\cdot]$ as the expectation over the probability distribution $\mathbb{P}$.

3.2 A Two-stage Model

Given a set of potential demand locations $\mathcal{L}$, we assume that a facility can be opened at any such demand locations. Each location $i \in \mathcal{L}$ represents a geographical area (e.g., state, county, district, etc.) with a random relief demand $\tilde{d}_i : \Omega \to \mathbb{R}_+$ on a probability space $(\Omega, \Sigma, \bar{\mathbb{P}})$. Let $\mathcal{K}$ denote the discrete set of possible facility sizes, indexed by κ, and let $M_\kappa > 0$ denote the capacity of facility of category κ. Associated with each candidate location i and each size category κ is a fixed location cost $c_{i\kappa} > 0$. We consider a single type of inventory unit that consists of a bundle of critical relief supplies, including prepackaged food, medical kits, blankets, and water. The total amount of these emergency supplies, denoted by $R > 0$, is determined by predisaster donations. Let $B > 0$ denote the total budget for opening the facilities. In the predisaster operations, HOs need to decide where to set up the facilities for prepositioning emergency supplies and how much inventory to preposition in each facility that has been opened. After a disaster, the reallocation operation of emergency supplies should be able to adjust adaptively. To formulate the model, let $x_{i\kappa} \in \{0, 1\}$ denote whether or not a facility of size category $\kappa \in \mathcal{K}$ is opened at location $i \in \mathcal{L}$ and let $r_i \geq 0$ denote the number of supplies prepositioned at location $i \in \mathcal{L}$. In addition, let $\tilde{y}_{ij} : \Omega \to \mathbb{R}_+$ be an adaptive strategy indicating the amount of supplies reallocated to location $j \in \mathcal{L}$ from location $i \in \mathcal{L}$ under each possible outcome $\omega \in \Omega$ and similarly $\tilde{u}_i : \Omega \to \mathbb{R}_+$ be a random variable denoting the planned amount of unsatisfied demand at location $i \in \mathcal{L}$. The parameters and decision variables for the model are summarized in Table 1.

Table 1: Model Parameters and Decision Variables

Sets	
$\mathcal{L}$	Set of demand locations
$\mathcal{K}$	Set of facility size categories
Parameters	
$c_{i\kappa}$	Fixed cost of opening a facility of size category κ at location i
M_κ	Capacity of a facility of size category κ
R	A total amount of emergency supplies
B	Budget limit for opening the facilities
τ_i	The tolerance threshold of supply shortage for demand location i
$\tilde{d}_i$	Random relief demand at location i
Decision variables	
$x_{i\kappa}$	1, if a facility of size category κ is opened at location i; 0 otherwise
r_i	The amount of supplies prepositioned at facility location i
$\tilde{y}_{ij}$	The amount of supplies allocated to location j from location i
$\tilde{u}_i$	The amount of unmet demand (supply shortage) at location i

To determine predisaster deployment decisions, we formulate the following constraints:

$$\sum_{\kappa\in\mathcal{K}} x_{i\kappa} \leq 1, \qquad i\in\mathcal{L}, \tag{1a}$$

$$r_i \leq \sum_{\kappa\in\mathcal{K}} M_\kappa x_{i\kappa}, \qquad i\in\mathcal{L}, \tag{1b}$$

$$\sum_{i\in\mathcal{L}} r_i \leq R, \tag{1c}$$

$$\sum_{i\in\mathcal{L}}\sum_{\kappa\in\mathcal{K}} c_{i\kappa}x_{i\kappa} \leq B. \tag{1d}$$

According to constraint (1a), not more than one facility can be opened at any demand location. Constraint (1b) states that the quantity of prepositioned relief items cannot exceed facility capacity. Constraint (1c) specifies that the total amount of the emergency supplies is R. Constraint (1d) ensures that the construction of response facilities are within the given budget. Note that one also can decide whether to consider other costs in the budget constraint according for different settings. Furthermore, the shortage can be represented by a convex piecewise linear function as defined next.

Definition 1. ***(Supply Shortage)*** *For any fixed decision* $(\boldsymbol{x},\boldsymbol{r},\boldsymbol{y})$ *and realization* $\mathbf{d}$*, the relief supply shortage for location* $i\in\mathcal{L}$ *is defined by the function*

$$f(\boldsymbol{x},\boldsymbol{r},\boldsymbol{y},\mathbf{d}) := \left(d_i + \sum_{j\in\mathcal{L}\setminus i} y_{ij} - (r_i + \sum_{j\in\mathcal{L}\setminus i} y_{ji})\right)^+ .$$

This gives rise to the following multi-objective optimization problem under uncertainty:

$$(\mathrm{P_{MOU}}) \qquad \underset{\boldsymbol{x},\boldsymbol{r},\tilde{\boldsymbol{y}},\tilde{\boldsymbol{u}}}{\text{minimize}} \quad \{\tilde{u}_i\}_{i\in\mathcal{L}} \tag{2a}$$

$$\text{s.t.} \quad \tilde{u}_i = \left(\tilde{d}_i + \sum_{j\in\mathcal{L}\setminus i}\tilde{y}_{ij} - (r_i + \sum_{j\in\mathcal{L}\setminus i}\tilde{y}_{ji})\right)^+, \qquad \text{a.s., } i\in\mathcal{L}, \tag{2b}$$

$$\sum_{j\in\mathcal{L}\setminus i}\tilde{y}_{ij} \leq r_i, \qquad \text{a.s., } i\in\mathcal{L}, \tag{2c}$$

$$\tilde{y}_{ij} \geq 0, \qquad \text{a.s., } i,j\in\mathcal{L}, \tag{2d}$$

$$(1a) - (1d),$$

where "a.s." stands for almost surely. Constraint (2b) ensures the exact calculation of the relief supply shortages. Constraint (2c) restricts that the amount of supply delivered from each facility does not exceed the level of prepositioned supplies. Note that the objective of problem $\mathrm{P_{MOU}}$ is both multiple, as it attempts to minimize the shortage at each location $i \in \mathcal{L}$, and uncertain, as the shortages depend on random demand. Hence, in the next two sections, we will discuss how we propose to control the risk of excessive shortage and trading off between the different locations in an equitable way. In doing so, it will be useful to summarize problem $\mathrm{P_{MOU}}$ using:

$$\underset{\tilde{\boldsymbol{u}} \in \mathscr{U}}{\text{minimize}} \quad \tilde{\boldsymbol{u}}\,,$$

where $\mathscr{U}$ represents the set of random shortage vectors that can be produced in problem $\mathrm{P_{MOU}}$, i.e.

$$\mathscr{U} := \{\tilde{\boldsymbol{u}} \mid \exists \boldsymbol{x}, \boldsymbol{r}, \tilde{\boldsymbol{y}}, (1\text{a}) - (1\text{d}), (2\text{b}) - (2\text{d})\}.$$

3.3 Shortage Severity Measure

For any feasible decisions $(\boldsymbol{x}, \boldsymbol{r})$, we can first consider in isolation how to treat the uncertainty about supply shortages $\tilde{u}_i$ at each location i. This will be done by assuming throughout this section that $\mathcal{L} = 1$, so that $\boldsymbol{u}$ will be referred as the random shortage $\tilde{u}$. The classical stochastic relief network design approaches assume a known probability setting and employ a risk measure such as the expected supply shortage (see, for instance, Rawls and Turnquist 2010). On the other hand, classical robust optimization tends to produce overly conservative solutions because of ignoring any knowledge regarding the distribution except for its support. Therefore, we apply an alternative modeling paradigm known as distributionally robust optimization (DRO) and assume that $\mathbb{P}$ is only known to belong to a convex ambiguity set $\mathcal{F}$ that is characterized by partial distribution information estimated from historical data (see Bertsimas et al. 2019 and references therein). As a result, we can seek the worst-case distribution to protect the risk measure by hedging against all probability distributions in $\mathcal{F}$. In our model, we will consider a generic risk constraint in the form

$$\sup_{\mathbb{P} \in \mathcal{F}} \rho\left(\tilde{u}\right) \leq \tau,$$

where $\rho(\tilde{u})$ denotes a risk measure of supply shortages, τ represents a bounding threshold for the risk of supply shortages.

Next, we discuss the specific form of risk measure. When the coherent risk measure CVaR is specified as the risk measure, we can set $\rho\left(\tilde{u}\right) := \mathrm{CVaR}_{1-\alpha}\left(\tilde{u}\right)$. We refer interested readers to Rockafellar and Uryasev (2000) and references therein for more details and examples of modeling and optimization problems using CVaR. In this setting, the CVaR is the expected shortage given that it falls beyond its $1-\alpha$ quantile. Hence, intuitively, this setting ensures that the CVaR of the supply shortage with $1-\alpha$ confidence remains under τ for all distributions in the set $\mathcal{F}$. In addition, CVaR has been adopted as a preferred measure in disaster management. For example, Elçi and Noyan (2018) suggest that CVaR is a reliable measure for the supply shortages. Alem et al. (2016) compare different risk measures, showing that the CVaR concept leads to higher demand satisfaction.

With the worst-case CVaR as a risk measure, one can impose

$$\sup_{\mathbb{P} \in \mathcal{F}} \mathrm{CVaR}_{1-\alpha}\left(\tilde{u}\right) \leq \tau \tag{3}$$

to keep the worst-case CVaR of the supply shortages below a threshold τ. Note that the worst-case CVaR is still a coherent risk measure and exhibits some valuable properties (Zhu and Fukushima 2009). We propose an equivalent representation of the worst-case CVaR of shortages in Lemma 1.

Lemma 1. *The bounded worst-case CVaR constraint* (3) *is equivalent to:*

$$\inf_{\eta \geq 0} \left(\eta + \frac{1}{\alpha} \sup_{\mathbb{P} \in \mathcal{F}} \mathbb{E}_{\mathbb{P}} \left[\left(\tilde{u} - \eta\right)^{+} \right] \right) \leq \tau. \tag{4}$$

With the worst-case CVaR of shortages in hand, we next introduce a measure to evaluate the uncertain relief supply shortage in Definition 2.

Definition 2. ***(Shortage Severity Measure)*** *Assume an uncertain supply shortage denoted by the random variable $\tilde{u} \in \mathscr{U}$ and a tolerance threshold $\tau \geq 0$. If we only know that the true distribution $\mathbb{P}$ lies in a distributional ambiguity set $\mathcal{F}$, we define the SSM $\rho_\tau : \mathscr{U} \to [0,1]$ as follows:*

$$\rho_\tau(\tilde{u}) := \begin{cases} \inf\limits_{\alpha \in (0,1]: \mathcal{V}_{1-\alpha}(\tilde{u}) \leq \tau} \alpha & \text{if } \mathcal{V}_0(\tilde{u}) \leq \tau, \\ 1 & \text{if } \mathcal{V}_0(\tilde{u}) > \tau, \end{cases}$$

where $\mathcal{V}_{1-\alpha}(\tilde{u})$ is the worst-case $CVaR_{1-\alpha}$ of the $\tilde{u}$ defined as

$$\mathcal{V}_{1-\alpha}(\tilde{u}) := \inf_{\eta \geq 0} \left\{ \eta + \frac{1}{\alpha} \sup_{\mathbb{P} \in \mathcal{F}} \mathbb{E}_\mathbb{P}\left[(\tilde{u} - \eta)^+\right] \right\}, \ \alpha \in (0,1],$$

so that $\mathcal{V}_0(\tilde{u}) := \sup_{\mathbb{P} \in \mathcal{F}} \mathbb{E}_\mathbb{P}[\tilde{u}]$.

Intuitively, for any given random shortage, SSM quantifies the risk of excessive shortage using a number between 0 and 1. If there is no possibility of shortage beyond τ, then the value of the SSM is 0. When the supply shortage is on average close or beyond τ, the SSM value will be close to 1. It is desirable to have an uncertain supply shortage with the smallest SSM value, because it implies that even for those unlikely massive demands, their worst-case CVaR can still be no more than the tolerance threshold.

The further interpretation is that the SSM fixes the shortage threshold τ and then identifies the smallest level of risk tolerance α such that the worst-case $\text{CVaR}_{1-\alpha}$ of shortages evaluated at that level is acceptable. Given that $\text{CVaR}_{1-\alpha}(X)$ is known to always provide an upper bound for $\text{VaR}_{1-\alpha}(X)$, SSM can also be interpreted as a tractable method for minimizing the probability of violating the shortage threshold (see Brown and Sim 2009 for more details).

We note that the SSM falls within the framework of *satisficing measures* proposed by Brown and Sim (2009) and is analogous to the *Delay Unpleasantness Measure* in Qi (2017) and the *buffered probability of exceedance* in Mafusalov and Uryasev (2018). To the best of our knowledge, this is the first time that a satisficing measure has been used in disaster management. Specifically, SSM can be regarded as $\rho_\tau(\tilde{u}) := 1 - S(\tau - \tilde{u})$, where S is a satisficing measure. We further present several important properties in Proposition 2.

Proposition 2. *Given $\tilde{u}, \tilde{u}_1, \tilde{u}_2 \in \mathscr{U}$, the SSM satisfies the following properties:*

i) Monotonicity: if $\mathbb{P}(\tilde{u}_1 \leq \tilde{u}_2) = 1$ for all $\mathbb{P} \in \mathcal{F}$, then $\rho_\tau(\tilde{u}_1) \leq \rho_\tau(\tilde{u}_2)$.

ii) Satisfaction: $\rho_\tau(\tilde{u}) = 0$ if and only if $\mathbb{P}(\tilde{u} \leq \tau) = 1$ for all $\mathbb{P} \in \mathcal{F}$.

iii) Dissatisfaction: if $\sup\limits_{\mathbb{P} \in \mathcal{F}} \mathbb{E}_\mathbb{P}[\tilde{u}] > \tau$, then $\rho_\tau(\tilde{u}) = 1$.

iv) Quasi-convexity: for any $\hat{\alpha} \in (0,1]$, the set $\mathscr{U}(\hat{\alpha}) := \{\tilde{u} | \rho_\tau(\tilde{u}) \leq \hat{\alpha}\}$ is closed and convex.

The *monotonicity* property implies that the smaller the shortage, the lower the risk. This is consistent with disaster management practices because the decision maker prefers smaller shortages. The *satisfaction* property ensures that, if an uncertain shortage is fully tolerable in the affected area, then the value of the measure is zero. This property also indicates that shortages below the tolerable threshold are the most preferred. The *dissatisfaction* property shows that, if an uncertain shortage exceeds the tolerable threshold of the affected area in worst-case expectation, then the severity of the shortage reaches the limit, which should be avoided in disaster management. The *quasi-convexity* is an appealing property in optimization that, under certain conditions such as the convexity of $\mathcal{U}$, allows us to efficiently obtain a global SSM minimizer.

3.4 Equity Modeling

Equity is an essential issue in humanitarian operations and providing relief supplies to every affected individual in an equitable manner has become a social consensus. While there are many concepts with regard to equity, typically they can be divided into *horizontal* equity and *vertical* equity (Karsu and Morton 2015). In the context of disaster management, *horizontal* equity refers to every individual or group being given the exact same resources to meet their needs (an example of which can be seen in Figure 1(a)), while *vertical* equity allocates the resources based on the different needs of the recipients (see Figure 1(b)). Since HOs need to consider relief demands of all victims from a global perspective and provide treatment accordingly, we mainly focus on *vertical* equity. Moreover, supply shortages often raise concerns about equity, so we characterize the concept of an equitable solution (see, for instance, Luss 1999) from the perspective of shortages in Definition 3.

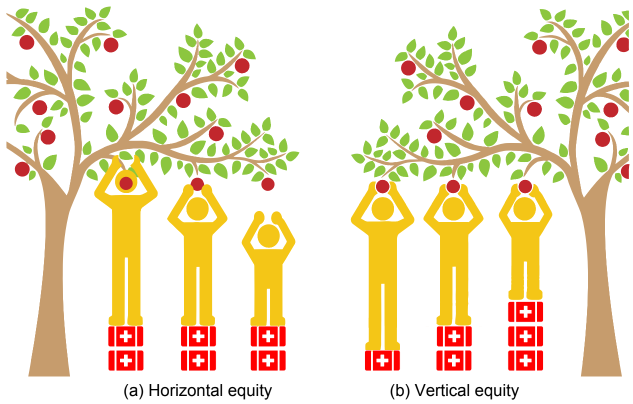


Figure 1: Horizontal Equity and Vertical Equity (Adapted from © 2014, Saskatoon Health Region.)

Definition 3. ***(Equitable Solution)*** *A solution is called equitable if no affected area can reduce its SSM value without raising an already equal or higher SSM value of another area.*

To obtain such an equitable solution, we can formulate a lexicographic minimization problem according to the Rawlsian principle of justice (Rawls 1971), as follows:

$$(\mathrm{P_{LM}}) \quad \tilde{\boldsymbol{u}}^* \in \arg \operatorname*{leximin}_{\tilde{\boldsymbol{u}} \in \mathscr{U}} \boldsymbol{\rho_\tau}(\tilde{\boldsymbol{u}}), \tag{5}$$

where $\boldsymbol{\rho_\tau}(\tilde{\boldsymbol{u}}) := \left[\rho_{\tau_1}(\tilde{u}_1), \rho_{\tau_2}(\tilde{u}_2), ..., \rho_{\tau_{|\mathcal{L}|}}(\tilde{u}_{|\mathcal{L}|})\right]^\top$, and where $\tilde{u}_i$ and τ_i respectively denote uncertain shortage and tolerable shortage thresholds for location $i \in \mathcal{L}$. Let $\tilde{\boldsymbol{u}}^*$ denote the optimal lexicographic solution that provides the lexicographically minimal vector $\boldsymbol{\alpha}^* := \boldsymbol{\rho_\tau}(\tilde{\boldsymbol{u}}^*)$. To keep this paper self-contained, we briefly provide the definition of lexicographic order in Definition 4.

Definition 4. ***(Lexicographic Order)*** *Given $\boldsymbol{\delta} \in \mathbb{R}^{|\mathcal{L}|}$, let $\vec{\boldsymbol{\delta}} \in \mathbb{R}^{|\mathcal{L}|}$ denote the vector $\boldsymbol{\delta}$ with its indices reordered so that the components are in non-increasing order. The vector $\boldsymbol{\delta} \in \Delta$ is lexicographically less than $\boldsymbol{\delta}' \in \Delta$, denoted by $\boldsymbol{\delta} \preceq \boldsymbol{\delta}'$ if either $\vec{\boldsymbol{\delta}} = \vec{\boldsymbol{\delta}}'$ or there exists a $k \in \{1, \ldots, |\mathcal{L}|\}$, such that $\vec{\boldsymbol{\delta}}_i = \vec{\boldsymbol{\delta}}'_i$ for all $i < k$ and $\vec{\boldsymbol{\delta}}_k < \vec{\boldsymbol{\delta}}'_k$. Furthermore, a vector $\boldsymbol{\delta} \in \Delta$ is said to be lexicographically minimal in some $\Delta \subseteq \mathbb{R}^{|\mathcal{L}|}$ if for every vector $\boldsymbol{\delta}' \in \Delta$, $\boldsymbol{\delta} \preceq \boldsymbol{\delta}'$.*

Remark 1. *In problem P_{LM}, we note that our definition of lexicographic ordering does not impose an a-priori ordering on which elements of $\boldsymbol{\rho_\tau}(\tilde{\boldsymbol{u}})$ should be minimized first. It rather should be understood as minimizing in order from 1 to $|\mathcal{L}|$ the terms of the sorted (in decreasing order) $\vec{\boldsymbol{\rho}}_{\boldsymbol{\tau}}(\tilde{\boldsymbol{u}})$.*

With this objective in hand, we can present our proposed predisaster relief network design problem with equity (PRNDP-E):

$$\text{(PRNDP-E)} \qquad \underset{\boldsymbol{x},\boldsymbol{r},\tilde{\boldsymbol{y}},\tilde{\boldsymbol{u}}}{\text{leximinimize}} \quad \boldsymbol{\rho_{\tau}}(\tilde{\boldsymbol{u}}) \tag{6a}$$

$$\text{s.t.} \quad (1a)-(1d), (2b)-(2d). \tag{6b}$$

where "leximinimize" refers to our search for the minimal feasible solution in terms of lexicographic ordering. Mathematically, $\boldsymbol{x}$ is discrete while $\boldsymbol{r}$ is continuous, both $\tilde{\boldsymbol{y}}$ and $\tilde{\boldsymbol{u}}$ are adaptable, and each $\rho_{\tau}(\cdot)$ is quasi-convex, we say that our problem belongs to the class of distributionally robust mixed-integer non-convex two-stage lexicographic optimization problems. Furthermore, as we show in Section 2.4 and Appendix A, we find that the *lexicographic minimization procedure* proposed by Qi (2017) for such problems cannot guarantee optimality. This motivates us to develop an efficient computational method with the guarantee of finding exact solutions to non-convex, mixed-integer, lexicographic optimization problems.

Remark 2. *PRNDP-E offers a way of handling shortage risk with equity in mind. Alternative formulations can also be proposed that employ other risk metric than worst-case SSM, e.g. the worst-case expectation or CVaR. We will explore these alternative formulations later in the paper.*

4 Solution Approach

In this section, we first propose a new branch-and-bound algorithm to address the *lexicographic minimization* aspect of the PRNDP-E. We then handle demand–distribution ambiguity by the robust stochastic optimization approach. Finally, we discuss two approaches for identifying optimal adaptable resource reallocation.

4.1 The Branch-and-Bound Algorithm

We first focus on proposing a branch-and-bound algorithm (see Algorithm 1) for solving non-convex mixed-integer lexicographic optimization models of the form presented in problem P_{LM}. For convenience, we refer to the lexicographic minimal vector as $\boldsymbol{\alpha}^* := \boldsymbol{\rho_{\tau}}(\tilde{\boldsymbol{u}}^*) \in \mathbb{R}^{|\mathcal{L}|}$, to the lexicographic order as $\preceq$, and to $\bar{\boldsymbol{\alpha}}$ and $\underline{\boldsymbol{\alpha}}$ as respective upper and lower bounds for $\boldsymbol{\alpha}^*$ if $\underline{\boldsymbol{\alpha}} \preceq \boldsymbol{\alpha}^* \preceq \bar{\boldsymbol{\alpha}}$. The algorithm starts by minimizing a worst-case SSM over all locations $i \in \mathcal{L}$. Then the procedure branches according to locations imposing that the SSM for this location does not exceed the minimax value that was identified at its parent node. Hence, each node n in the enumeration tree $\mathscr{N}$ corresponds to a minimax problem (7) in which the SSM for some locations in $\bar{\mathcal{L}}^n$ cannot exceed $\underline{\alpha}_i^n$, while the worst SSM is minimized for the other locations (i.e., $i \in \mathcal{L}/\bar{\mathcal{L}}^n$). A lower bound $\underline{\boldsymbol{\alpha}}^n$ for the children of each node is also maintained and compared to the best solution $\bar{\boldsymbol{\alpha}}$ found so far in order to trim the node if no improvement can be achieved.

Remark 3. *We note that Algorithm 1 might in the worst case expand a number of nodes that is factorial with respect to the number of locations. In order to keep the list of expandable nodes as small as possible, in our implementation we make two changes to Algorithm 1. First, in the step of branching (i.e., Step 16), one can alternatively obtain a lower bound using the following model for all $i \in \mathcal{L}/\bar{\mathcal{L}}^{n'}$:*

$$\begin{aligned}
\underline{\alpha}_i^{n'} := \min_{\tilde{\boldsymbol{u}} \in \mathscr{U}} & \rho_{\tau_i}(\tilde{u}_i) \\
\textit{s.t. } & \rho_{\tau_j}(\tilde{u}_j) \leq \underline{\alpha}_j^n, && j \in \bar{\mathcal{L}}^n, \\
& \rho_{\tau_j}(\tilde{u}_j) \leq v_n^*, && j \in \mathcal{L}/\bar{\mathcal{L}}^n.
\end{aligned}$$

Second, in Step 6, we select the node with lowest lower bound first. With these changes, we observed empirically in our case study that there is never a need to expand a number of nodes that is comparable to $|\mathcal{L}|!$.

Algorithm 1 Branch and Bound Algorithm for Lexicographic Optimization Problem $\mathrm{P_{LM}}$

Input: A set $\mathcal{U}$ and vectored risk measure $\boldsymbol{\rho} : \mathcal{U} \to \mathbb{R}^{|\mathcal{L}|}$
Output: The lexicographic minimal vector $\boldsymbol{\alpha}^*$ and a lexicographic minimal solution $\boldsymbol{u}^*$.
Set $\bar{\alpha}_i := \infty$ for all i, and some arbitrary $\bar{\boldsymbol{u}} \in \mathcal{U}$.
Initialize enumeration tree $\mathscr{N} := \{n_0\}$ with $\bar{\mathcal{L}}^{n_0} := \emptyset$, $\underline{\boldsymbol{\alpha}}_i^{n_0} := -\infty$ for all i.
while $\mathscr{N} \neq \emptyset$ **do**
    Select a node n in the enumeration tree $\mathscr{N}$ and remove n from $\mathscr{N}$ (*Node selection*)
    if $\underline{\boldsymbol{\alpha}}^n \prec \bar{\boldsymbol{\alpha}}$ **then** (*Node expansion*)
        Solve the following minimax problem associated with node n,

$$\min_{\tilde{\boldsymbol{u}} \in \mathscr{U}} \max_{i \in \mathcal{L}/\bar{\mathcal{L}}^n} \rho_{\tau_i}(\tilde{u}_i) \tag{7a}$$

$$\text{s.t. } \rho_{\tau_i}(\tilde{u}_i) \leq \underline{\alpha}_i^n, \ i \in \bar{\mathcal{L}}^n, \tag{7b}$$

        Let v_n^* and $\tilde{\boldsymbol{u}}_n^*$ be optimal value and the minimizer of problem (7)
        Construct $\bar{\boldsymbol{\alpha}}^n$ as follows:

$$\bar{\boldsymbol{\alpha}}_i^n := \begin{cases} \underline{\alpha}_i^n & \text{if } i \in \bar{\mathcal{L}}^n \\ v_n^* & \text{otherwise} \end{cases}$$

        if $\bar{\boldsymbol{\alpha}}^n \preceq \bar{\boldsymbol{\alpha}}$ **then** (*Update best solution*)
            Let $\bar{\boldsymbol{\alpha}} := \bar{\boldsymbol{\alpha}}^n$ and $\bar{\boldsymbol{u}} := \tilde{\boldsymbol{u}}_n^*$.
        end if
        if $|\mathcal{L}/\bar{\mathcal{L}}^n| > 1$ **then** (*Branching*)
            for $j \in \mathcal{L}/\bar{\mathcal{L}}^n$ **do**
                Create new node n' with $\bar{\mathcal{L}}^{n'} := \bar{\mathcal{L}}^n \cup j$ and

$$\underline{\alpha}_i^{n'} := \begin{cases} \underline{\alpha}_i^n & \text{if } i \in \bar{\mathcal{L}}^n \\ v_n^* & \text{if } i = j \\ -\infty & \text{otherwise} \end{cases}$$

                Append the new node n' to $\mathscr{N}$.
            end for
        end if
    end if
end while
return $\boldsymbol{\alpha}^* := \bar{\boldsymbol{\alpha}}$ and $\boldsymbol{u}^* := \bar{\boldsymbol{u}}$.

We now prove the optimality of the output vector $\boldsymbol{\alpha}^*$ in Theorem 1.

Theorem 3. *The vector $\boldsymbol{\alpha}^*$ returned by Algorithm 1 is lexicographically minimal, i.e. $\boldsymbol{\alpha}^* \preceq \boldsymbol{\rho_\tau}(\tilde{\boldsymbol{u}})$ for all $\tilde{\boldsymbol{u}} \in \mathscr{U}$.*

Since we need to solve a series of similar minimax problems (7) repeatedly, we then mainly focus on solving these problems. By Definitions 1 and 2, we can reorganize problem (7) as the following distributionally robust optimization (DRO) problem:

$$
\begin{aligned}
(\mathrm{P_{DRO}})\ v_n^* := \inf_{\boldsymbol{x},\boldsymbol{r},\boldsymbol{\eta},\alpha,\tilde{\boldsymbol{y}},\tilde{\boldsymbol{u}}} \ & \alpha && && \text{(8a)}\\
\text{s.t.} \quad & \eta_i + \frac{1}{\alpha} \sup_{\mathbb{P}\in\mathcal{F}} \mathbb{E}_{\mathbb{P}}\left[(\tilde{u}_i - \eta_i)^+\right] \le \tau_i, && i \in \mathcal{L} \setminus \bar{\mathcal{L}}^n, && \text{(8b)}\\
& \eta_i + \frac{1}{\underline{\alpha}_i^n} \sup_{\mathbb{P}\in\mathcal{F}} \mathbb{E}_{\mathbb{P}}\left[(\tilde{u}_i - \eta_i)^+\right] \le \tau_i, && i \in \bar{\mathcal{L}}^n, && \text{(8c)}\\
& \tilde{u}_i = \left(d_i + \sum_{j\in\mathcal{L}\setminus i} \tilde{y}_{ij} - (r_i + \sum_{j\in\mathcal{L}\setminus i} \tilde{y}_{ji})\right)^+, && \text{a.s. under all } \mathbb{P}\in\mathcal{F},\ i \in \mathcal{L}, && \text{(8d)}\\
& \sum_{j\in\mathcal{L}\setminus i} \tilde{y}_{ij} \le r_i, && \text{a.s. under all } \mathbb{P}\in\mathcal{F},\ i \in \mathcal{L}, && \text{(8e)}\\
& \tilde{y}_{ij} \ge 0, && \text{a.s. under all } \mathbb{P}\in\mathcal{F},\ i,j \in \mathcal{L}, && \text{(8f)}\\
& \text{(1a)} - \text{(1d)}, \alpha \in (0,1], \boldsymbol{\eta} \ge \mathbf{0}. && && \text{(8g)}
\end{aligned}
$$

We note that $\eta_i + \frac{1}{\alpha} \sup_{\mathbb{P}\in\mathcal{F}} \mathbb{E}_{\mathbb{P}}[\tilde{u}_i - \eta_i]^+$ is non-increasing in α which implies that the set of feasible α's has the form $[\alpha^*, 1]$. Hence, one can solve problem $\mathrm{P_{DRO}}$ by performing a bisection search for α^*, which at each step tests for the feasibility $\mathrm{P_{DRO}}$ when α is fixed to some value. The latter reduces to verifying the feasibility of a convex optimization problem. However, we still face two challenges in solving problem $\mathrm{P_{DRO}}$: (i) how to construct an ambiguity set $\mathcal{F}$; and (ii) how to develop a tractable formulation so that a feasible solution can be quickly found.

4.2 The Robust Stochastic Optimization Approach

To handle problem $\mathrm{P_{DRO}}$, we deploy the framework of robust stochastic optimization (RSO), where the uncertainty associated with problems is described by an event-wise ambiguity set.

4.2.1 Event-wise Ambiguity Set.

The uncertain demands are strongly correlated with covariates (e.g., the Seismic magnitude scales), so we can raise the level of forecasting for uncertain demands by using such data. It follows that we need to consider uncertain covariates and uncertain demands to construct an event-wise ambiguity set (see Chen et al. 2020). For example, in the context of earthquake preparedness, the *Richter scale* ($\mathrm{M_L}$) can be used as a covariate to construct an event-wise ambiguity set: when an earthquake reaches magnitude $\mathrm{M_L}$7.0, it will cause more damage than an earthquake of magnitude $\mathrm{M_L}$5.0 and the demand for relief supplies will also be higher. For simplicity, we further divide the space of covariates into $|\mathcal{S}|$ non-overlapping regions to form $|\mathcal{S}|$ scenarios. This gives rise to the outcome space $\Omega := \{(\tilde{\mathbf{d}}, \tilde{s}) \in \mathbb{R}^{|\mathcal{L}|} \times |\mathcal{S}|\}$. Let s represent a scenario taking values in $\mathcal{S}$ and p_s denote the probability of scenario s happening. Furthermore, we have $\mathbb{P}(\tilde{s} = s) = p_s$ and $\sum_{s\in\mathcal{S}} p_s = 1$, where $\tilde{s}$ represents a set of random scenarios whose realization probabilities may be uncertain. The joint distribution of $(\tilde{\mathbf{d}}, \tilde{s})$ is denoted by $\mathbb{P} \in \mathcal{F}$.

Now, we specify the event-wise ambiguity set $\mathcal{F}$ as

$$\mathcal{F} := \left\{ \mathbb{P} \in \mathscr{P}_0(\mathbb{R}^{|\mathcal{L}|} \times |\mathcal{S}|) : \begin{array}{ll} (\tilde{\mathbf{d}}, \tilde{s}) \sim \mathbb{P} & \\ \mathbb{P}(\tilde{\mathbf{d}} \in \mathcal{D}^s | \tilde{s} = s) = 1, & s \in \mathcal{S} \\ \mathbb{E}_{\mathbb{P}}[\tilde{\mathbf{d}} | \tilde{s} = s] = \boldsymbol{\mu}^s, & s \in \mathcal{S} \\ \mathbb{E}_{\mathbb{P}}[|\tilde{\mathbf{d}} - \boldsymbol{\mu}^s| | \tilde{s} = s] \leq \boldsymbol{\nu}^s, & s \in \mathcal{S} \\ \mathbb{P}(\tilde{s} = s) = p_s, & s \in \mathcal{S} \end{array} \right\},$$

where $\mathcal{D}^s$ is the support set defined as

$$\mathcal{D}^s := \left\{ \tilde{\mathbf{d}} \in \mathbb{R}^{|\mathcal{L}|} : \underline{\mathbf{d}}^s \leq \tilde{\mathbf{d}} \leq \overline{\mathbf{d}}^s \right\}.$$

In $\mathcal{F}$, it is natural to incorporate the mean, the mean absolute deviation and the support set of the random variable $\tilde{\mathbf{d}}$. Specifically, for different scenarios, the bounded support set defined in the first set of equality constraints may differ. Conditioning on the scenario realization, the mean of $\tilde{\mathbf{d}}$ is specified in the second set of equality constraints, while the third set of inequality constraints provides upper bounds on the mean absolute deviation of $\tilde{\mathbf{d}}$. The last set of equality constraints specifies the probability of each scenario. $\mathcal{F}$ requires simple descriptive statistics from data and allows us to model a rich variety of structural information about the uncertain demand.

4.2.2 Model Reformulation.

Note that the event-wise ambiguity set $\mathcal{F}$ involves nonlinear moment constraints, so we introduce an auxiliary probability space $\Omega' := \{(\tilde{\mathbf{d}}, \tilde{\mathbf{z}}, \tilde{s}) \in \mathbb{R}^{|\mathcal{L}|} \times \mathbb{R}^{|\mathcal{L}|} \times |\mathcal{S}|\}$ to reformulate the ambiguity set $\mathcal{F}$ as the projection of a lifted ambiguity set $\mathcal{G}$:

$$\mathcal{G} := \left\{ \mathbb{Q} \in \mathscr{P}_0(\mathbb{R}^{|\mathcal{L}|} \times \mathbb{R}^{|\mathcal{L}|} \times |\mathcal{S}|) : \begin{array}{ll} (\tilde{\mathbf{d}}, \tilde{\mathbf{z}}, \tilde{s}) \sim \mathbb{Q} & \\ \mathbb{Q}((\tilde{\mathbf{d}}, \tilde{\mathbf{z}}) \in \bar{\mathcal{D}}^s | \tilde{s} = s) = 1, & s \in \mathcal{S} \\ \mathbb{E}_{\mathbb{Q}}[\tilde{\mathbf{d}} | \tilde{s} = s] = \boldsymbol{\mu}^s, & s \in \mathcal{S} \\ \mathbb{E}_{\mathbb{Q}}[\tilde{\mathbf{z}} | \tilde{s} = s] \leq \boldsymbol{\nu}^s, & s \in \mathcal{S} \\ \mathbb{Q}(\tilde{s} = s) = p_s, & s \in \mathcal{S} \end{array} \right\},$$

where $\bar{\mathcal{D}}_s$ is the lifted support set defined as

$$\bar{\mathcal{D}}^s := \left\{ (\tilde{\mathbf{d}}, \tilde{\mathbf{z}}) \in (\mathbb{R}^{|\mathcal{L}|} \times \mathbb{R}^{|\mathcal{L}|}) : \begin{array}{l} \underline{\mathbf{d}}^s \leq \tilde{\mathbf{d}} \leq \overline{\mathbf{d}}^s \\ |\tilde{\mathbf{d}} - \boldsymbol{\mu}^s| \leq \tilde{\mathbf{z}} \end{array} \right\}.$$

Compared with the original ambiguity set $\mathcal{F}$, the lifted ambiguity set $\mathcal{G}$ is a set of distributions of random triplet ($\tilde{\mathbf{d}}$,$\tilde{\mathbf{z}}$, $\tilde{s}$). Furthermore, following recent results in DRO (see Bertsimas et al. 2019), we can define the ambiguity set $\mathcal{F}$ as the set of marginal distributions over $(\tilde{\mathbf{d}}, \tilde{s})$ for all $\mathbb{Q} \in \mathcal{G}$. That is, $\mathcal{F} = \prod_{\tilde{\mathbf{d}}, \tilde{s}} \mathcal{G}$. With the lifted ambiguity set $\mathcal{G}$ in hand, we transform problem $\mathrm{P_{DRO}}$ in Lemma 4.

Lemma 4. *The distributionally robust optimization problem P_{DRO} with $\mathcal{F}$, i.e., $\mathcal{F} = \prod_{\tilde{\mathbf{d}}, \tilde{s}} \mathcal{G}$,* *is equivalent to the following adjustable robust optimization (ARO) problem:*

$$(P_{ARO}) \quad v_n^* :=$$

$$\inf_{\substack{\boldsymbol{x},\boldsymbol{r},\boldsymbol{\eta},\alpha,\{\boldsymbol{y}^s(\cdot)\}_{s\in\mathcal{S}}\\ \boldsymbol{\pi}^1,\boldsymbol{\pi}^2,\boldsymbol{\pi}^3\geq \mathbf{0}}} \alpha \tag{9a}$$

$$\text{s.t.} \quad \eta_i + \frac{1}{\alpha}\sum_{s\in\mathcal{S}}(\pi_{si}^1 + \big(\boldsymbol{\pi}_{si}^2)'\boldsymbol{\mu}^s + (\boldsymbol{\pi}_{si}^3)'\boldsymbol{\nu}^s\big) \leq \tau_i, \qquad i\in\mathcal{L}\setminus\bar{\mathcal{L}}^n, \tag{9b}$$

$$\eta_i + \frac{1}{\underline{\alpha}_i^n}\sum_{s\in\mathcal{S}}\big(\pi_{si}^1 + (\boldsymbol{\pi}_{si}^2)'\boldsymbol{\mu}^s + (\boldsymbol{\pi}_{si}^3)'\boldsymbol{\nu}^s\big) \leq \tau_i, \qquad i\in\bar{\mathcal{L}}^n, \tag{9c}$$

$$\pi_{si}^1 + (\boldsymbol{\pi}_{si}^2)'\mathbf{d} + (\boldsymbol{\pi}_{si}^3)'\mathbf{z} \geq 0, \qquad (\mathbf{d},\mathbf{z})\in\bar{\mathcal{D}}^s, s\in\mathcal{S}, i\in\mathcal{L}, \tag{9d}$$

$$\pi_{si}^1 + (\boldsymbol{\pi}_{si}^2)'\mathbf{d} + (\boldsymbol{\pi}_{si}^3)'\mathbf{z} \geq -p_s\eta_i, \qquad (\mathbf{d},\mathbf{z})\in\bar{\mathcal{D}}^s, s\in\mathcal{S}, i\in\mathcal{L}, \tag{9e}$$

$$\pi_{si}^1 + (\boldsymbol{\pi}_{si}^2)'\mathbf{d} + (\boldsymbol{\pi}_{si}^3)'\mathbf{z} \geq p_s(d_i + \sum_{j\in\mathcal{L}\setminus i} y_{ij}^s(\mathbf{d},\mathbf{z}) - (r_i + \sum_{j\in\mathcal{L}\setminus i} y_{ji}^s(\mathbf{d},\mathbf{z})) - \eta_i), \qquad (\mathbf{d},\mathbf{z})\in\bar{\mathcal{D}}^s, s\in\mathcal{S}, i\in\mathcal{L}, \tag{9f}$$

$$\sum_{j\in\mathcal{L}\setminus i} y_{ij}^s(\mathbf{d},\mathbf{z}) \leq r_i, \qquad (\mathbf{d},\mathbf{z})\in\bar{\mathcal{D}}^s, s\in\mathcal{S}, i\in\mathcal{L}, \tag{9g}$$

$$y_{ij}^s(\mathbf{d},\mathbf{z}) \geq 0, \qquad (\mathbf{d},\mathbf{z})\in\bar{\mathcal{D}}^s, s\in\mathcal{S}, i,j\in\mathcal{L} \tag{9h}$$

$$(1a)-(1d), \alpha\in(0,1], \boldsymbol{\eta}\geq\mathbf{0}. \tag{9i}$$

where each $y_{ij}^s : \mathbb{R}^{|\mathcal{L}|}\times\mathbb{R}^{|\mathcal{L}|}\to\mathbb{R}$, and where π_{si}^1, $\boldsymbol{\pi}_{si}^2$ and $\boldsymbol{\pi}_{si}^3$ are the dual variables associated with first, second, and third constraints that define $\mathcal{G}$.

Note that problem P_{ARO} is a semi-infinite programming problem with an infinite number of constraints and adaptive decision variables. Specifically, the adaptive decisions y_{ij}^s can be seen as general functions of random vectors $(\tilde{\mathbf{d}}, \tilde{\mathbf{z}})$. Therefore, problem P_{ARO} is not yet directly solvable.

4.3 Identifying Optimal Adaptable Resource Reallocation

4.3.1 Exact Solution.

Based on a vertex enumeration (VE) method, we first present an exact linear programming reformulation for problem P_{ARO} in Proposition 5.

Proposition 5. *The adjustable robust optimization problem P_{ARO} is equivalent to the following* *mixed-integer linear program:*

$$(P_{VE})\ v_n^* := \inf_{\substack{\boldsymbol{x},\boldsymbol{r},\boldsymbol{\eta},\alpha,\{\boldsymbol{y}^s(\cdot)\}_{s\in\mathcal{S}}\\ \boldsymbol{\pi}^1,\boldsymbol{\pi}^2,\boldsymbol{\pi}^3\geq \mathbf{0}}} \alpha \tag{10a}$$

$$\text{s.t.}\quad \eta_i + \frac{1}{\alpha}\sum_{s\in\mathcal{S}}\left(\pi^1_{si} + (\boldsymbol{\pi}^2_{si})'\boldsymbol{\mu}^s + (\boldsymbol{\pi}^3_{si})'\boldsymbol{\nu}^s\right) \leq \tau_i, \qquad i\in\mathcal{L}\setminus\bar{\mathcal{L}}^n, \tag{10b}$$

$$\eta_i + \frac{1}{\underline{\alpha}_i^n}\sum_{s\in\mathcal{S}}\left(\pi^1_{si} + (\boldsymbol{\pi}^2_{si})'\boldsymbol{\mu}^s + (\boldsymbol{\pi}^3_{si})'\boldsymbol{\nu}^s\right) \leq \tau_i, \qquad i\in\bar{\mathcal{L}}^n, \tag{10c}$$

$$\pi^1_{si} + (\boldsymbol{\pi}^2_{si})'\mathbf{d}(\omega) + (\boldsymbol{\pi}^3_{si})'\mathbf{z}(\omega) \geq 0, \qquad \omega\in\Omega^s, s\in\mathcal{S}, i\in\mathcal{L}, \tag{10d}$$

$$\pi^1_{si} + (\boldsymbol{\pi}^2_{si})'\mathbf{d}(\omega) + (\boldsymbol{\pi}^3_{si})'\mathbf{z}(\omega) \geq -p_s\eta_i, \qquad \omega\in\Omega^s, s\in\mathcal{S}, i\in\mathcal{L}, \tag{10e}$$

$$\pi^1_{si} + (\boldsymbol{\pi}^2_{si})'\mathbf{d}(\omega) + (\boldsymbol{\pi}^3_{si})'\mathbf{z}(\omega) \geq p_s(d_i(\omega) + \sum_{j\in\mathcal{L}\setminus i} y^s_{ij}(\omega) - (r_i + \sum_{j\in\mathcal{L}\setminus i} y^s_{ji}(\omega)) - \eta_i), \qquad \omega\in\Omega^s, s\in\mathcal{S}, i\in\mathcal{L}, \tag{10f}$$

$$\sum_{j\in\mathcal{L}\setminus i} y^s_{ij}(\omega) \leq r_i, \qquad \omega\in\Omega^s, s\in\mathcal{S}, i\in\mathcal{L}, \tag{10g}$$

$$y^s_{ij}(\omega) \geq 0, \qquad \omega\in\Omega^s, s\in\mathcal{S}, i,j\in\mathcal{L}, \tag{10h}$$

$$(1a) - (1d), \alpha\in(0,1], \boldsymbol{\eta}\geq \mathbf{0}. \tag{10i}$$

where each $y^s_{ij} : \Omega_s \to \mathbb{R}$, and where, for all $s\in\mathcal{S}$, the set $\{\mathbf{d}(\omega)\}\}_{\omega\in\Omega^s}$ contains all vertices of the bounded polyhedron $\bar{\mathcal{D}}^s_b$ defined as:

$$\bar{\mathcal{D}}^s_b := \left\{(\tilde{\mathbf{d}},\tilde{\mathbf{z}})\in\mathbb{R}^{|\mathcal{L}|}\times\mathbb{R}^{|\mathcal{L}|} : \underline{\mathbf{d}}^s \leq \tilde{\mathbf{d}} \leq \overline{\mathbf{d}}^s,\ |\tilde{d}_i - \mu^s_i| \leq \tilde{z}_i \leq \max\{\overline{d}^s_i - \mu^s_i, \mu^s_i - \underline{d}^s_i\}\ \forall i\in\mathcal{L}\right\}.$$

Note that the number of vertices indexed by Ω^s for all $s\in\mathcal{S}$ grows exponentially with the number of locations. In practice, we can employ a column-and-constraint generation (C&CG) method to speed up the resolution. We however consider further investigation of acceleration schemes to fall beyond the scope of this paper, and instead derive in the next subsection a conservative approximation that takes the form of a more reasonably sized mixed-integer linear programs (MILP).

4.3.2 Affinely Adjustable Robust Counterpart.

We apply the idea of an affine decision rule to address adaptive decisions and solve an approximate problem. More specifically, for each scenario $s\in\mathcal{S}$, we approximate the adaptive decision by an affine function $\boldsymbol{y}^s(\cdot)\in\mathscr{A}$, where

$$\mathscr{A} := \left\{\boldsymbol{y} : \mathbb{R}^{|\mathcal{L}|}\times\mathbb{R}^{|\mathcal{L}|}\to\mathbb{R}^{|\mathcal{L}|\times|\mathcal{L}|} :\ \begin{array}{l}\exists \boldsymbol{y}^0, \boldsymbol{y}^1_l, \boldsymbol{y}^2_l \in\mathbb{R}^{|\mathcal{L}|\times|\mathcal{L}|}, \forall l\in\mathcal{L}\\ \boldsymbol{y}(\mathbf{d},\mathbf{z}) = \boldsymbol{y}^0 + \sum_{l\in\mathcal{L}}\boldsymbol{y}^1_l d_l + \sum_{l\in\mathcal{L}}\boldsymbol{y}^2_l z_l\end{array}\right\}.$$

We then have the following affinely adjustable robust counterpart (AARC) of problem $\mathrm{P}_{\mathrm{ARO}}$:

$$
\begin{aligned}
(\mathrm{P}_{\mathrm{AARC}})\ v_n^{\mathrm{AARC}} := \\
\inf_{\substack{\boldsymbol{x},\boldsymbol{r},\boldsymbol{\eta},\alpha,\{\boldsymbol{y}^s(\cdot)\}_{s\in\mathcal{S}}\\ \boldsymbol{\pi}^1,\boldsymbol{\pi}^2,\boldsymbol{\pi}^3\geq\mathbf{0}}} \quad & \alpha && && (11a)\\
\text{s.t.}\quad & \eta_i + \frac{1}{\alpha}\sum_{s\in\mathcal{S}}\left(\pi_{si}^1 + (\boldsymbol{\pi}_{si}^2)'\boldsymbol{\mu}^s + (\boldsymbol{\pi}_{si}^3)'\boldsymbol{\nu}^s\right) \leq \tau_i, && i\in\mathcal{L}\setminus\bar{\mathcal{L}}^n, && (11b)\\
& \eta_i + \frac{1}{\underline{\alpha}_i^n}\sum_{s\in\mathcal{S}}\left(\pi_{si}^1 + (\boldsymbol{\pi}_{si}^2)'\boldsymbol{\mu}^s + (\boldsymbol{\pi}_{si}^3)'\boldsymbol{\nu}^s\right) \leq \tau_i, && i\in\bar{\mathcal{L}}^n, && (11c)\\
& \pi_{si}^1 + (\boldsymbol{\pi}_{si}^2)'\mathbf{d} + (\boldsymbol{\pi}_{si}^3)'\mathbf{z} \geq 0, && (\mathbf{d},\mathbf{z})\in\bar{\mathcal{D}}^s, s\in\mathcal{S}, i\in\mathcal{L}, && (11d)\\
& \pi_{si}^1 + (\boldsymbol{\pi}_{si}^2)'\mathbf{d} + (\boldsymbol{\pi}_{si}^3)'\mathbf{z} \geq -p_s\eta_i, && (\mathbf{d},\mathbf{z})\in\bar{\mathcal{D}}^s, s\in\mathcal{S}, i\in\mathcal{L}, && (11e)\\
& \pi_{si}^1 + (\boldsymbol{\pi}_{si}^2)'\mathbf{d} + (\boldsymbol{\pi}_{si}^3)'\mathbf{z} \geq p_s\big(d_i + \sum_{j\in\mathcal{L}\setminus i} y_{ij}^s(\mathbf{d},\mathbf{z}) \\
& \quad - (r_i + \sum_{j\in\mathcal{L}\setminus i} y_{ji}^s(\mathbf{d},\mathbf{z})) - \eta_i\big), && (\mathbf{d},\mathbf{z})\in\bar{\mathcal{D}}^s, s\in\mathcal{S}, i\in\mathcal{L}, && (11f)\\
& \sum_{j\in\mathcal{L}\setminus i} y_{ij}^s(\mathbf{d},\mathbf{z}) \leq r_i, && (\mathbf{d},\mathbf{z})\in\bar{\mathcal{D}}^s, s\in\mathcal{S}, i\in\mathcal{L}, && (11g)\\
& y_{ij}^s(\mathbf{d},\mathbf{z}) \geq 0, && (\mathbf{d},\mathbf{z})\in\bar{\mathcal{D}}^s, s\in\mathcal{S}, i,j\in\mathcal{L}, && (11h)\\
& \boldsymbol{y}^s(\cdot)\in\mathscr{A}, && s\in\mathcal{S} && (11i)\\
& (1a)-(1d), \alpha\in(0,1], \boldsymbol{\eta}\geq\mathbf{0}. && && (11j)
\end{aligned}
$$

Since problem $\mathrm{P}_{\mathrm{AARC}}$ has an infinite number of constraints, it cannot be solved directly. One can alternatively view some constraints in problem $\mathrm{P}_{\mathrm{AARC}}$ as robust counterparts of the linear optimization problem under the lifted support set $\bar{\mathcal{D}}_s$ for all $s\in\mathcal{S}$. Hence, we can transform it into a linear optimization problem via standard techniques from the robust literature. For presentation brevity, the derivation of the resulting MILP is relegated to Appendix B.6.

Moreover, AARC can also lead to improvements in computational performance. Empirically, according to a preliminary study in Appendix D.3, AARC appears surprisingly accurate with a maximum measured optimality gap of 8%. Regarding computation time, AARC returns a solution in 3 seconds on average for experimental instances consisting of 13 locations, while VE takes more than 2 hours for $|\mathcal{L}|\geq 9$. Therefore, in the following real-world case study, we apply AARC to solve problem $\mathrm{P}_{\mathrm{ARO}}$ to achieve the right trade off between quality of solution and speed of execution.

5 Computational Results

In this section, we conduct a series of numerical studies based on a real earthquake case that occurred in Yushu County, Qinghai Province, China in 2010. This case consists of 13 locations and 15 links. The deterministic parameters (e.g., cost, capacity) and demand-related parameters are from Ni et al. (2018). Moreover, as discussed in Section 4.2.1, we utilize the *Richter scale* to construct the event-wise ambiguity set. For presentation brevity, most details of the numerical studies are pushed to Appendix D.1. In what follows, we compare the performance of different predisaster relief network design approaches in terms of equity, total shortage, and deployment plan. We also investigate the impact of the size of total donation (and budget) and of the choice of uncertainty model on performance. As we have mentioned in Remark 2, the effects of three choices of risk measures are compared in Appendix D.4. Appendix D.5 finally offers an additional sensitivity analysis regarding the threshold vector $\boldsymbol{\tau}$. All experiments are carried out on a PC with a 3.6-GHz processor and 16 GB RAM. The models are coded in JAVA and solved using IBM ILOG CPLEX Optimization Studio 12.7.1. Finally, all adjustable robust optimization problems are solved using their respective affinely adjustable robust counterpart.

5.1 Comparison of Equity Performance

We denote the model that considers equity in both predisaster deployment and postdisaster reallocation by PRNDP-E and use PRNDP-NE to refer to the benchmark model without equity in both stages (shown in Appendix C). We also investigate the setting where the decision maker considers equity only when reallocating supplies after the disaster (with predisaster deployment fixed to solutoin of PRNDP-NE), denoted by PRNDP-NE-E. We test three models—PRNDP-E, PRNDP-NE-E, and PRNDP-NE—and compare their out-of-sample performances under the five commonly used equity indices (shown in Appendix D.2). Specifically, we first solve the models to optimality and obtain the optimal decisions $(\boldsymbol{x}^*, \boldsymbol{r}^*)$. After that, given each of the solutions $(\boldsymbol{x}^*, \boldsymbol{r}^*)$ and an observed sample $(s, \mathbf{d})$ pair, for PRNDP-E and PRNDP-NE-E we solve a deterministic lexicographic optimization problem to minimize the shortage of each location, while for PRNDP-NE we minimize the total shortage. We then examine (i) the Maximum Shortage Gap (MSG); (ii) the Relative Mean Deviation (RMD); (iii) the Variance (VAR); (iv) the Sum of Pairwise Absolute Differences (SPAD); and (v) the Gini coefficient (Gini) of 1,000 test samples.

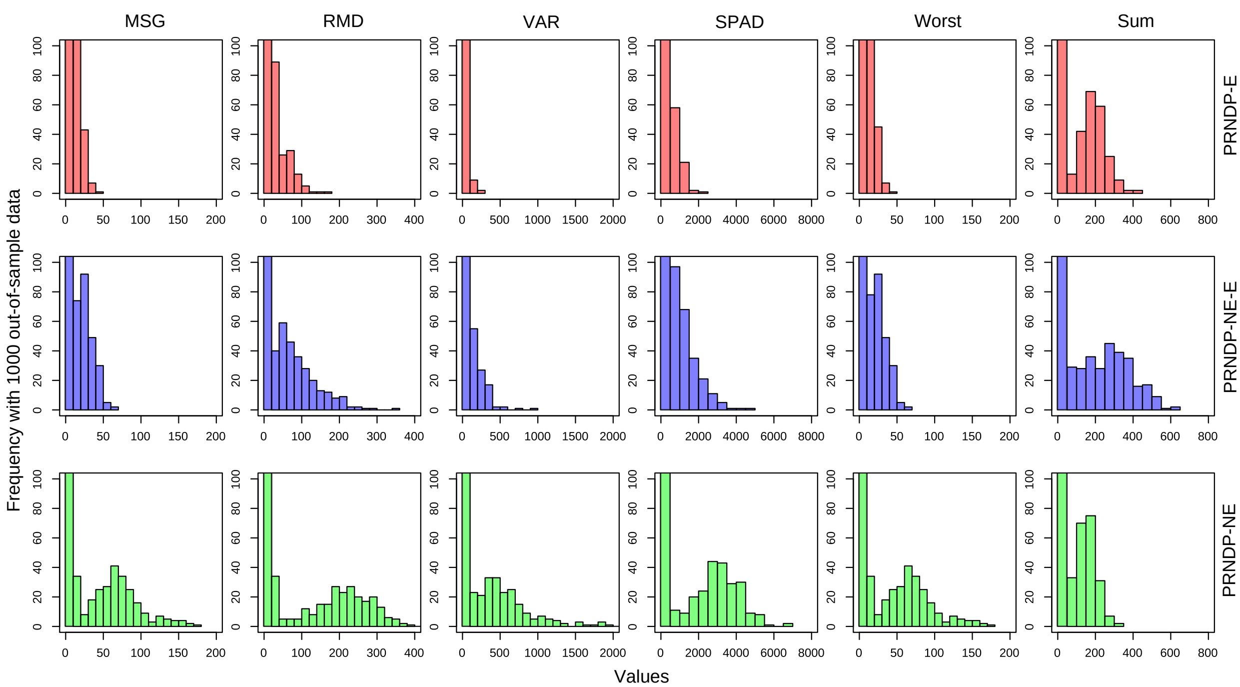


Figure 2: Distribution of Equity-Related Indices of Shortage Observed Out-of-Sample

Figure 2 presents the distributions of equity indices for 1,000 test samples under the PRNDP-E, the PRNDP-NE-E, and the PRNDP-NE solutions. It also presents the distribution of worst shortage and sum of shortages. First, note that, for all tested equity indices, the PRNDP-E and the PRNDP-NE-E perform better than the PRNDP-NE because a smaller index value is more desirable. It indicates the importance of incorporating equity in the allocation of relief resources. Second, we can observe that the performance of PRNDP-E is better than PRNDP-NE-E. The result shows that incorporating equity already in the stage of predisaster deployment can further improve the equity of resource allocation. This confirms that HOs can draw real value from jointly considering deployment and reallocation strategies when searching for the most equitable allocation of relief supplies. More precisely, when it comes to the mean, the 95%VaR, the 99%VaR, and the standard deviations (STD), the PRNDP-E approach also outperformed the other two approaches (see Table 2 for detailed statistics). Moreover, all considered models can be solved within reasonable amount of time. The average computational times to solve the PRNDP-E, the PRNDP-NE, and PRNDP-NE-E were respectively 586, 3, and 3 seconds.

We further conduct experiments on the Gini coefficient. Since the Gini coefficient is often represented graphically through the Lorenz curve, Figure 3 shows the Lorenz curve of shortage distributions by plotting the location percentile by shortages on the x-axis and cumulative shortages on the y-axis. We can observe that the curve of PRNDP-E is closer to the line of perfect equality with a Gini coefficient equal to 0.08, which is smaller than the results of the other two approaches.

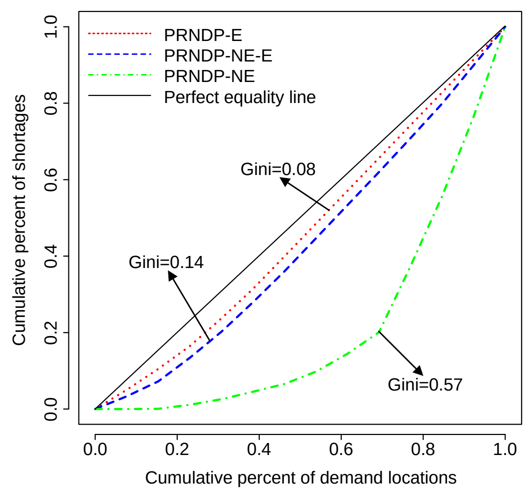


Figure 3: Gini Coefficient (Measured Out-of-Sample) under PRNDP-E, PRNDP-NE-E, and PRNDP-NE

To capture more of the impact of equity, we conduct more numerical experiments. Specifically, we first add a perturbation Δ to the mean of the out-of-sample demand distribution, that is $\boldsymbol{\mu}^s_{out} := (1+\Delta)\boldsymbol{\mu}^s$ for all $s \in \mathcal{S}$. Then we generate 1,000 test samples from the out-of-sample distribution according to the value of Δ (see Appendix D.1 for details). In Table 2, we summarize the out-of-sample performance of equity indices under the PRNDP-E, the PRNDP-NE-E, and the PRNDP-NE. We observe that on the average and in the extremes (VaR@95% and VaR@99%) the PRNDP-E continues to outperform the other two approaches with regard to all equity indices. This observation suggests that PRNDP-E does help address equity concerns in humanitarian operations.

Table 2: Out-of-Sample Statistics of Equity Index under PRNDP-E, PRNDP-NE-E and PRNDP-NE

		PRNDP-E				PRNDP-NE-E				PRNDP-NE			
Δ	Statistic	MSG	RMD	VAR	SPAD	MSG	RMD	VAR	SPAD	MSG	RMD	VAR	SPAD
-0.10	Mean	0.44	1.42	0.44	18.51	2.64	6.23	5.22	96.74	3.57	7.97	6.98	107.58
	Var@95%	3.02	9.15	1.15	118.92	18.04	49.43	43.44	633.80	25.26	54.24	49.04	740.64
	Var@99%	10.66	36.08	14.79	469.03	28.74	53.43	107.43	1,034.60	37.69	63.08	118.37	1,251.48
	STD	1.80	6.18	32.81	80.38	6.50	18.85	280.38	178.14	8.76	19.96	306.21	271.43
-0.05	Mean	1.69	4.89	2.73	63.71	4.84	19.07	19.16	250.35	8.50	23.22	35.43	322.19
	Var@95%	11.99	36.85	16.37	479.00	28.61	121.38	133.17	1,596.64	49.46	137.75	208.64	1,941.40
	Var@99%	19.56	73.78	53.46	959.19	41.29	181.62	277.25	2,371.20	74.52	214.92	537.53	3,042.16
	STD	4.49	14.48	144.08	188.36	10.05	41.90	727.23	551.33	17.83	50.66	1,261.12	709.21
0.00	Mean	3.23	8.53	6.51	111.60	7.25	25.94	31.01	341.42	17.59	51.02	130.17	727.48
	Var@95%	20.00	58.38	39.26	758.99	37.05	138.79	196.94	1,804.22	90.37	277.77	732.13	3,976.16
	Var@99%	28.03	94.89	102.31	1,233.51	47.49	207.78	359.79	2,701.20	142.62	339.63	1,508.01	5,086.36
	STD	7.17	21.09	270.41	276.09	13.13	50.49	1,049.24	667.98	33.93	97.39	3,992.42	1,400.84
0.05	Mean	4.45	11.27	11.02	148.03	9.44	31.59	43.20	415.80	25.64	76.53	267.16	1,104.63
	Var@95%	28.61	75.43	76.52	985.44	45.27	154.51	266.72	2,047.20	136.54	392.51	1,607.64	5,718.00
	Var@99%	36.73	115.09	151.12	1,500.59	58.47	231.00	534.87	3,090.96	163.06	468.51	2,148.72	6,811.64
	STD	9.71	26.51	419.41	347.61	15.94	55.96	1,338.42	737.92	46.92	141.67	7,390.43	2,058.87
0.10	Mean	5.70	14.14	15.50	185.58	11.57	35.18	50.97	464.15	33.54	104.70	446.90	1,526.94
	Var@95%	34.65	79.45	98.29	1,081.31	49.02	157.20	268.66	2,055.68	163.81	520.96	2,616.08	7,610.88
	Var@99%	43.10	135.41	206.89	1,760.28	62.97	213.50	517.99	2,799.72	182.17	587.16	3,096.05	8,519.88
	STD	11.65	30.45	575.97	399.74	18.13	56.36	1,443.08	743.67	58.29	187.71	11,796.93	2,761.86

5.2 Comparison of Total Shortage Performance

To compare the performance in terms of total shortages under the PRNDP-E, the PRNDP-NE-E, and the PRNDP-NE, we return to Figure 2, where the last two columns of subfigures present the distributions of worst shortage among all the locations and the distributions of total shortage, both measured out-of-sample. Specifically, we find that, whereas the distribution of total shortage for PRNDP-E is similar to PRNDP-NE (only 22% more total shortage on average based on Table 3, see when $\Delta = 0$), its distribution of worst shortages is significantly improved (nearly 5 times less for the average). This observation confirms that PRNDP-E supports the most vulnerable groups without necessarily incurring a large loss in terms of total shortage. In contrast, while the PRNDP-NE-E does reduce the worst shortage (nearly 3 times less for the average), this comes at a heavy price in terms of total shortage (doubling it on average).

Additionally, Table 3 provides a summary of out-of-sample shortage performance data with different perturbation Δ of the mean of the out-of-sample distribution. We note that the previous observation remain stable: PRNDP-E significantly reduces the worst shortage while only slightly increasing the total shortage, unlike PRNDP-NE-E. The reason for this seems to be that the preparation of PRNDP-NE-E only considers total shortages, thus changing the objective to equity after the disaster occurs leads to excessive unforeseen costs.

Table 3: Out-of-Sample Statistics of Shortage under PRNDP-E, PRNDP-NE-E, and PRNDP-NE

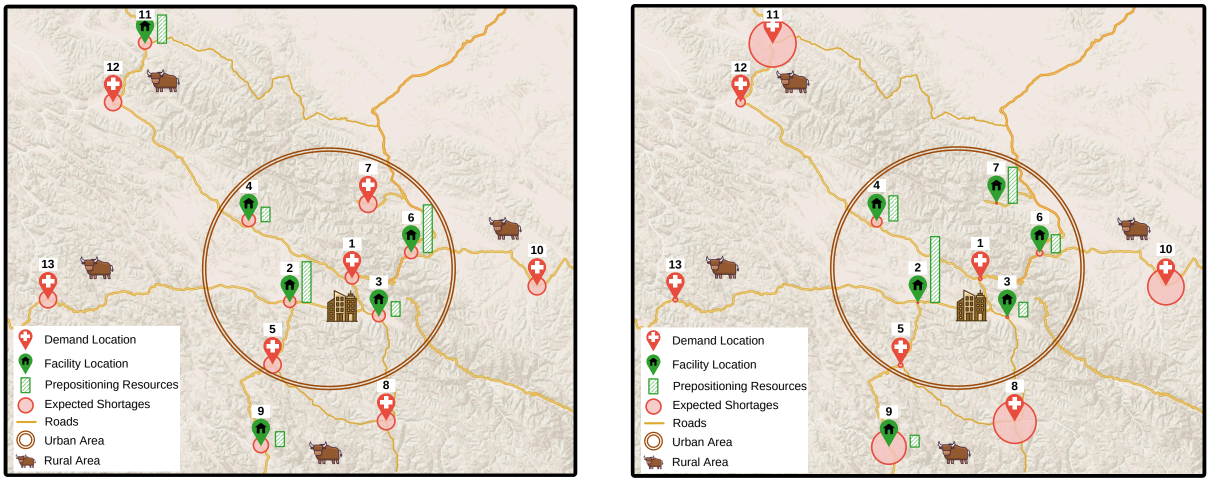
		PRNDP-E		PRNDP-NE-E		PRNDP-NE	
Δ	Statistic	Worst	Sum	Worst	Sum	Worst	Sum
-0.10	Mean	0.46	5.20	2.64	27.10	3.57	4.70
	Var@95%	3.41	38.06	18.04	190.60	25.26	31.25
	Var@99%	10.66	117.26	28.74	290.07	37.69	57.12
	STD	1.81	20.00	6.50	65.88	8.76	12.00
-0.05	Mean	1.84	21.03	4.85	50.97	8.50	14.94
	Var@95%	12.36	143.24	28.61	296.95	49.46	92.25
	Var@99%	19.56	221.19	41.29	421.58	74.52	148.82
	STD	4.58	51.60	10.04	104.30	17.83	33.55
0.00	Mean	3.68	42.83	7.31	72.78	17.59	35.03
	Var@95%	20.04	234.48	37.05	392.10	90.37	192.89
	Var@99%	28.03	311.33	47.49	500.50	142.62	238.45
	STD	7.45	85.74	13.14	140.31	33.93	68.08
0.05	Mean	5.28	62.17	9.50	104.03	25.64	53.89
	Var@95%	28.65	327.09	45.27	493.90	136.54	279.80
	Var@99%	36.73	411.40	58.47	604.14	163.06	327.09
	STD	10.29	120.37	15.95	174.63	46.92	101.43
0.10	Mean	7.22	85.67	11.64	129.96	33.54	76.03
	Var@95%	35.26	426.86	49.02	549.36	163.81	385.02
	Var@99%	43.10	487.46	62.97	697.44	182.17	438.39
	STD	12.95	153.73	18.15	204.44	58.29	139.53

5.3 Differences in Deployment Plans

We further investigate deployment patterns of PRNDP-E and PRNDP-NE (the same for PRNDP-NE-E). As shown in Figure 4 (see Appendix D.7 for additional details), we depict the inventory and expected shortages at each location, where the height of the rectangle next to each open facility captures its proportion of total emergency supplies, and the radius of the circle under each location captures its proportion of total shortage. It suggests that, the PRNDP-E prefers a more decentralized deployment of emergency supplies by establishing more facilities in rural areas (e.g., location 11), while the PRNDP-NE centralizes everything to minimize the total shortage. Moreover, for the PRNDP-E, the amount of resources prepositioned in open facilities is between 155 to 498, while the PRNDP-NE varies from 127 to 685. As a result, each region can have access to a portion of the supplies under the PRNDP-E so that there will not be severe shortages in each region. For PRNDP-NE, however, it sacrifices some high shortages in some rural regions (e.g., location 8-11) to be able to match the shortage in a large number of places.

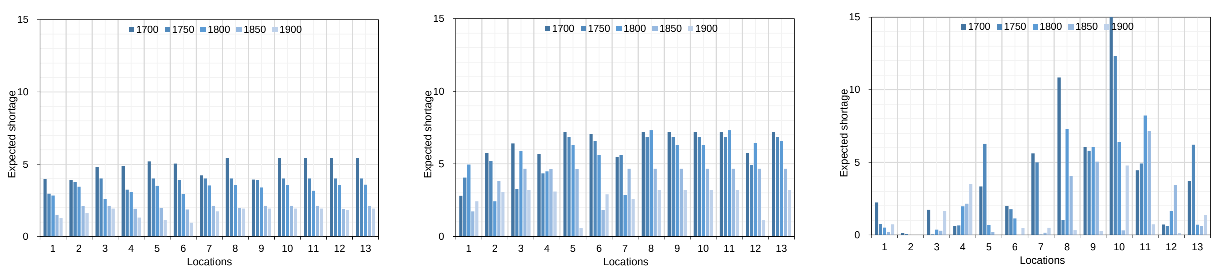
5.4 Impact of Donations and Budget Constraints

Donations and budgets are major concerns of the HOs in predisaster deployment; they also directly affect the shortage situation after the disaster occurs. If the HOs do not receive adequate donations and funds for the prepositioning of emergency supplies, the vulnerable population will inevitably face a relief shortage after a disaster. Our numerical study shows that donations and budgets have similar effects, so we focus in this section on the impact of varying the amount of total donations on shortages, whereas we refer interested readers to Appendix D.6 for a study on the impact of budget. Specifically, we set the total amount of supplies to $R = \{1700, 1750, 1800, 1850, 1900\}$ and evaluate the impact of different donations.



(a) PRNDP-E (b) PRNDP-NE

Figure 4: Deployment Plan and Expected Shortage (Measured Out-of-Sample) at Each Location



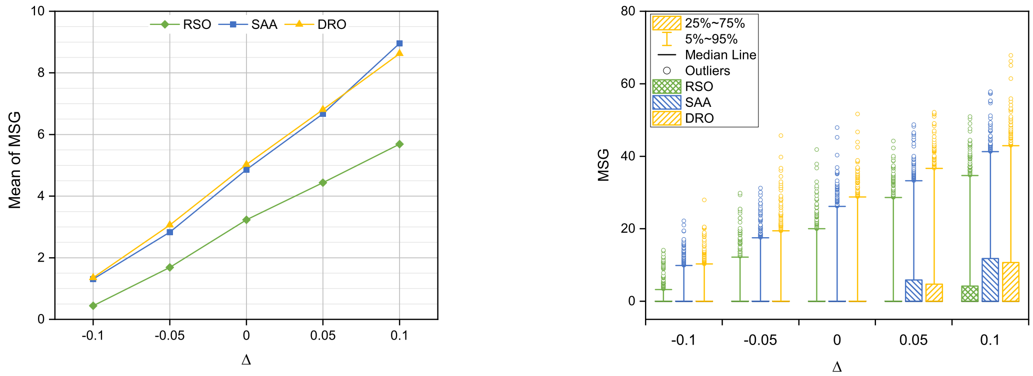
(a) PRNDP-E (b) PRNDP-NE-E (c) PRNDP-NE

Figure 5: Expected Shortage (Measured Out-of-Sample) at Each Location under Different Donation Levels

Figure 5 reports the out-of-sample expected shortage under different donation levels. First, in Figure 5(a), the expected shortage at each location shows a decreasing trend as donations increase. However, if equity is not fully considered, then we find that increasing donation does not necessarily reduce shortages and even increase shortages in some locations. It indicates that HOs cannot simply increase the amounts of supplies to reduce the shortages at each location, but must instead do so through equitable and reasonable resource allocation. As stated by Besiou and Van Wassenhove (2020): "Reduced funding calls for careful prioritization and cost-effectiveness, but it should be noted that this may conflict with equity and other ethical considerations." Our work can give an equitable solution when the donation is reduced. Second, we can find that the expected shortage reduction is not symmetric across all locations and not necessarily proportional to the percentage increase in donations, which is consistent with Observation 6 in Stauffer and Kumar (2021). Third, the shortage gaps between demand locations under PRNDP-E is smaller than under PRNDP-NE-E and PRNDP-NE, which demonstrates the equity efficiency of PRNDP-E from another perspective.

5.5 Impact of Uncertainty Model

The RSO framework unifies both scenario-tree-based stochastic optimization (SAA) and distributionally robust optimization (DRO). To further illustrate the performance of RSO model, we compare the out-of-sample performance of the three models in dealing with equity concerns.



(a) (b)

Figure 6: Out-of-Sample Performance of RSO, DRO, and SAA Models

Specifically, Figure 6(a) presents the average of out-of-sample MSG under RSO, DRO and SAA models for 1,000 test samples. We observe that the RSO model delivers a significantly better performance than the DRO and SAA models. In particular, when the out-of-sample demands have higher mean (i.e., Δ increases from -0.1 to 0.1), the gap between the RSO and the other two models becomes greater. Moreover, to compare the stability of the methods, we report the box plot of out-of-sample MSG under RSO, DRO and SAA models in Figure 6(b). We find that the values under the RSO have the smallest variation between both $(25\% - 50\%)$ and $(5\% - 95\%)$, while DRO and SAA incur a relatively large variation. Furthermore, we notice that even for outliers exceeding 95%, the performance of RSO is still better than that of DRO and SAA. The above results confirm that the event-wise ambiguity set helps to more clearly describe the uncertainty of relief demand and alleviate the inequities caused by uncertain shortages.

6 Managerial Insights and Conclusions

Our work considers three important practical aspects of humanitarian operations: shortages, equity, and uncertainty. Mathematically, we propose a new branch-and-bound algorithm for the mixed-integer lexicographic optimization problem with non-convex objectives and prove its optimality. To identify optimal adaptable resource reallocation, we propose two approaches: an exact approach and a conservative approximation that allows us to solve instances of realistic size.

Our research also proposes several managerial recommendations for the HOs:

- Decision-making that incorporates equity in predisaster deployment and postdisaster reallocation is, in some cases (see sections 5.1 and 5.2), able to significantly reduce the shortage of the most vulnerable participants while only incurring a reasonable increase in total shortage. Indeed, while the goal of HOs is usually to minimize the total shortage, the media will often focus on the most seriously affected participants implying that the HOs is inequitable in their allocation of resources. Models such as PRNDP-E can definitely help the HOs better anticipate and mitigate such issues.
- While donation and budget constraints limit the amount of relief resources available to each beneficiary, the individual shortage may not be alleviated with increased budgets and donations if equity is not fully incorporated. To respond to the needs of the most vulnerable people, HOs often request increased relief assistance. This does help reduce total shortages, but that is not necessarily the case for individuals. The results in Section 5.4 show that

the shortages experienced by some participants do not decrease proportionally (or even increase at all) with an increase in donations.

- Disaster event-wise information, if properly segmented, can help alleviate inequities caused by uncertain relief demands. While disasters are extremely unpredictable and relief demands are difficult to accurately estimate, the use of historical data and/or prior knowledge can improve decision-making effects. Numerical studies in Section 5.5 demonstrate that the event-wise ambiguity sets outperform the classical moment ambiguity sets and SAA in alleviating inequities, which may be regarded as a useful exploration of data-driven methods in disaster relief management.

In terms of future work, we identify three promising directions. First, whereas this study focuses on how to best allocate predisaster donations, it could be interesting to also model the postdisaster donations and investigate how such donation might influence optimal predisaster network design strategies. We also find relevant to further investigate whether more sophisticated acceleration schemes could be identified to accelerate the proposed branch-and-bound algorithm for lexicographic optimization. This might facilitate the deployment of lexicographic based solutions in application domains where the computational needs of our current method become prohibitive.

Finally, our current work somehow neglects (as with most of the literature on SSMs) the question of how to properly set the threshold levels for the SSM in each of the regions. In practice, each τ_i should depend on the socio-economical and infrastructural characteristics of the region that it describes. In the case of earthquake preparedness, de Goyet et al. (2006) points out that water shortage risk (see p. 1153) can be much more severe than other ones. In this case, the threshold level should be adjusted to account for the physiological limits before severe dehydration occurs, e.g. the population losing more than 10% of their body water, which is known to lead to physical and mental deterioration and even death (see Ashcroft 2002). In terms of infrastructural shortages, it is also well known that socio-economically disadvantaged regions take more time to rebuild and reinstate their services thus motivating the use of higher shortage thresholds for such regions.

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A Motivation for designing the branch-and-bound algorithm

We show the motivation to develop the branch-and-bound algorithm. Since problem $\mathrm{P_{LM}}$ can be regarded as a lexicographic minimization problem, a natural idea is whether the state-of-the-art algorithm proposed by Qi (2017) can be directly applied to problem $\mathrm{P_{LM}}$. Furthermore, we show the lexicographic minimization procedure (LMP) for problem $\mathrm{P_{LM}}$ in Algorithm S.1, which is an iterative algorithm by solving a sequence of minmax problems. In the following, we give two examples to illustrate that although the LMP can perform well on linear programming problems, it cannot guarantee optimality for mixed-integer programming problems and non-convex problems.

Algorithm S.1 Lexicographic Minimization Procedure for Problem $\mathrm{P_{LM}}$ (Qi 2017)

1: Input: A set $\mathcal{U}$ and vectored risk measure $\boldsymbol{\rho}: \mathcal{U} \to \mathbb{R}^{|\mathcal{L}|}$
2: Output: A lexicographic minimal solution $\boldsymbol{u}^*$
3: Set $h := 0$, $\tilde{\mathcal{L}}_0 := \mathcal{L}$, $\bar{\mathcal{L}}_0 := \emptyset$, $v_0^* := \infty$, and some arbitrary $\tilde{\boldsymbol{u}}_0^* \in \mathcal{U}$
4: **while** $\tilde{\mathcal{L}}_h \neq \emptyset$ **do**
5: Solve the following problem:

$$\min_{\tilde{\boldsymbol{u}} \in \mathscr{U}} \max_{i \in \tilde{\mathcal{L}}_h} \rho_{\tau_i}(\tilde{u}_i) \tag{12a}$$

$$\text{s.t. } \rho_{\tau_i}(\tilde{u}_i) \leq v_{h'}^*,\ i \in \bar{\mathcal{L}}_{h'},\ h' \in [0; h] \tag{12b}$$

6: Let v_{h+1}^* and $\tilde{\boldsymbol{u}}_{h+1}^*$ be optimal value and minimizer of problem (12)
7: Construct $\bar{\mathcal{L}}_{h+1} := \min \left\{ j \in \tilde{\mathcal{L}}_h | \rho_{\tau_j}(\tilde{u}_{h+1,j}^*) = v_{h+1}^* \right\}$
8: Set $\tilde{\mathcal{L}}_{h+1} := \tilde{\mathcal{L}}_h / \bar{\mathcal{L}}_{h+1}$, $h := h + 1$
9: **end while**
10: **return** $\boldsymbol{u}^* := \tilde{\boldsymbol{u}}_h^*$

Example 1 Consider the following feasibility problem :

$$\begin{aligned} &2x \leq \alpha_1, \\ &2 - x \leq \alpha_2, \\ &x \in \{0, 1\}, \end{aligned}$$

where $\boldsymbol{\alpha} = [\alpha_1, \alpha_2]$ is a vector. In this example, we want to find a vector $\boldsymbol{\alpha}$ that is lexicographically minimal. By the LMP, one can get $\boldsymbol{\alpha} = [2, 1]$ with $x = 1$. However, we may have $\boldsymbol{\alpha}' = [0, 2]$ with $x = 0$. Obviously, $\boldsymbol{\alpha}'$ is lexicographically less than $\boldsymbol{\alpha}$. Based on the analysis of these two solutions, we observe that, there are two sets of possible binding constraints that correspond to different solutions for the minimum scalar value $\alpha^* = 2$. Yet, the LMP is unable to decide which binding constraint to fix and instead could arbitrarily fix the first one detected. It follows that the LMP sometimes fails to find the lexicographically minimum vector.

Example 2 Consider an example of problem $\mathrm{P_{LM}}$ with $|\mathcal{L}| = 5$ and $|\mathcal{K}| = 1$. Specifically, we will let the parameters $M = 200$, $R = 300$, $B = 400$, $\boldsymbol{c} = \{200, 200, 150, 150, 300\}$, $\boldsymbol{\tau} = \{3, 33, 25, 34, 21\}$. Furthermore, as shown in Table 4, we assume that there are $|\Omega| = 5$ possible outcomes of random demands with the same probability.

Table 4: Random Demand for Each Location in Example 2

		Location				
Outcome	Prob.(%)	1	2	3	4	5
$\omega = 1$	20	40	45	42	48	32
$\omega = 2$	20	9	14	45	28	47
$\omega = 3$	20	45	30	36	92	97
$\omega = 4$	20	85	70	53	13	84
$\omega = 5$	20	54	69	12	74	11

Table 5: Detailed Optimal Solution under the LMP and the B&B

	LMP			B&B		
Location	$\boldsymbol{x}^*$	$\boldsymbol{r}^*$	$\boldsymbol{\rho}(\boldsymbol{u}^*)$	$\boldsymbol{x}^*$	$\boldsymbol{r}^*$	$\boldsymbol{\rho}(\boldsymbol{u}^*)$
1	1	150	0.00	0	0	0.00
2	1	50	0.28	0	0	0.28
3	0	0	0.00	1	150	0.00
4	0	0	0.28	1	50	0.28
5	0	0	0.28	0	0	0.00

In this example, based on the LMP, one can identify that an optimal solution takes the configuration presented in Table 5. On the other hand, we also show the details of an optimal solution obtained by our branch-and-bound (B&B) procedure in Table 5. Note that the SSM vector derived by the B&B procedure is lexicographically less than that derived by the LMP. The possible reason is that, in Step 5 of Algorithm S.1, there may be multiple optimal solutions

$\boldsymbol{u}_h^*$ at iteration h such that $\max_{i\in\tilde{\mathcal{L}}_h} \rho_{\tau_i}(u_{hi}^*) = v_h^*$ where v_h^* is the minimum scalar value, and the LMP randomly chooses one of the optimal solutions to detect an index j such that $\rho_{\tau_j}(u_{hj}^*) = v_h^*$ in Step 7. Therefore, the LMP sometimes fails to find the lexicographically minimum vector, which motivated us to develop a new algorithm. For the B&B procedure, we refer the reader to Appendix B.3 for the proof of its optimality.

B Proofs

B.1 Proof of Lemma 1.

We can first show that for any $\mathbb{P} \in \mathcal{F}$:

$$\inf_{\eta} \left(\eta + \frac{1}{\alpha}\mathbb{E}_{\mathbb{P}}\left[(\tilde{u}-\eta)^+\right]\right) \leq \tau \Leftrightarrow \inf_{0\leq\eta\leq\tau} \left(\eta + \frac{1}{\alpha}\mathbb{E}_{\mathbb{P}}\left[(\tilde{u}-\eta)^+\right]\right) \leq \tau\,.$$

This follows from the fact that any $\eta > \tau$ necessarily leads to $\eta + (1/\alpha)\mathbb{E}_{\mathbb{P}}\left[(\tilde{u}-\eta)^+\right] \geq \eta > \tau$. On the other hand, for any $\eta < 0$, we can confirm that $\bar{\eta} := 0$ is such that:

$$\eta + \frac{1}{\alpha}\mathbb{E}_{\mathbb{P}}\left[(\tilde{u}-\eta)^+\right] = \frac{(\alpha-1)\eta}{\alpha} + \frac{1}{\alpha}\mathbb{E}_{\mathbb{P}}\left[\tilde{u}\right] \geq \frac{(\alpha-1)\bar{\eta}}{\alpha} + \frac{1}{\alpha}\mathbb{E}_{\mathbb{P}}\left[\tilde{u}\right] = \bar{\eta} + \frac{1}{\alpha}\mathbb{E}_{\mathbb{P}}\left[(\tilde{u}-\bar{\eta})^+\right]\,,$$

where we used in order the fact that $\tilde{u} \geq 0$, that $(\alpha-1)/\alpha$ is negative and $\bar{\eta} > \eta$, and again $\tilde{u} \geq 0$.

We can then exploit the representation of CVaR (Rockafellar and Uryasev 2000) and follow the steps:

$$\begin{aligned}(3) &\Leftrightarrow \forall\mathbb{P}\in\mathcal{F},\ \mathrm{CVaR}_{1-\alpha}^{\mathbb{P}}(\tilde{u}) \leq \tau \Leftrightarrow \forall\mathbb{P}\in\mathcal{F},\ \inf_{\eta}\left(\eta + \frac{1}{\alpha}\mathbb{E}_{\mathbb{P}}\left[(\tilde{u}-\eta)^+\right]\right) \leq \tau \\ &\Leftrightarrow \sup_{\mathbb{P}\in\mathcal{F}} \inf_{0\leq\eta\leq\tau}\left(\eta + \frac{1}{\alpha}\mathbb{E}_{\mathbb{P}}\left[(\tilde{u}-\eta)^+\right]\right) \leq \tau \Leftrightarrow \inf_{0\leq\eta\leq\tau}\sup_{\mathbb{P}\in\mathcal{F}}\left(\eta + \frac{1}{\alpha}\mathbb{E}_{\mathbb{P}}\left[(\tilde{u}-\eta)^+\right]\right) \leq \tau \Leftrightarrow (4),\end{aligned}$$

where the fourth $\Leftrightarrow$ follows from applying Sion's minimax theorem given that η is contained in a compact set, $\mathcal{F}$ is convex and $\eta + (1/\alpha)\mathbb{E}_{\mathbb{P}}\left[(\tilde{u}-\eta)^+\right]$ is convex in η and affine in $\mathbb{P}$ (**?**). Finally, the last $\Leftrightarrow$ follows again from the fact that any $\eta > \tau$ necessarily makes $\eta + (1/\alpha)\mathbb{E}_{\mathbb{P}}\left[(\tilde{u}-\eta)^+\right] > \tau$ for any $\mathbb{P} \in \mathcal{F}$. □

B.2 Proof of Proposition 2.

The proofs of *Monotonicity*, *Satisfaction*, and *Dissatisfaction* are similar to that of Proposition 1 in Qi (2017). For the proof of *Quasi-convexity*, we refer readers to Proposition 3.4 in Mafusalov and Uryasev (2018). □

B.3 Proof of Theorem 3.

We first introduce the notation $n := (i_1, i_2, \ldots, i_k)$, with each $i_j \in \mathcal{L}$ for some $k \in \{1, \ldots, |\mathcal{L}|\}$ to describe the sequence of branching that was traversed to generate node n, which implies that $\bar{\mathcal{L}}^n := \{i_1, \ldots, i_k\}$ and that the descendants of $n = (i_1, i_2, \ldots, i_k)$ are the nodes $n' := (i_1, i_2, \ldots, i_k, j)$ with $j \in \mathcal{L}/\{i_1, \ldots, i_k\}$. Moreover, we will exploit the fact that problem (7) is equivalent to

$$\begin{aligned}&\min_{\boldsymbol{\beta}\in\mathscr{B}} \max_{i\in\mathcal{L}/\bar{\mathcal{L}}^n} \beta_i \\ &\text{s.t. } \beta_i \leq \underline{\alpha}_i^n,\ i \in \bar{\mathcal{L}}^n,\end{aligned}$$

where $\mathscr{B} := \left\{\boldsymbol{\beta} \in \mathbb{R}^{|\mathcal{L}|} : \exists \boldsymbol{u} \in \mathscr{U}, \boldsymbol{\beta} = \boldsymbol{\rho}(\boldsymbol{u})\right\}$.

Lemma 6. *Given any node $n = (i_1, i_2, \ldots, i_k)$ expanded during the resolution of Algorithm 1, we have that $\vec{\underline{\alpha}}^n_j = \underline{\alpha}^n_{i_j}$ for all $j = 1, \ldots, k$ and $\vec{\underline{\alpha}}^n_j = -\infty$ for $j > k$.*

Note that Lemma 6 and the design of Algorithm 1 allows us to use $\vec{\underline{\alpha}}^n_j = \underline{\alpha}^n_{i_j} = \vec{\bar{\alpha}}^n_j = \bar{\alpha}^n_{i_j}$ interchangeably when $n = (i_1, i_2, \ldots, i_k)$ is expanded and $j \le k$. This follows from the fact that for $j = 1, \ldots, k-1$, the optimal value $v^*_{n'}$ of problem (7) associated to $n' = (i_1, \ldots, i_{j-1})$ is necessarily greater or equal to the optimal value $v^*_{n''}$ associated to its child node $n'' = (i_1, \ldots, i_j)$:

$$\begin{aligned} \underline{\alpha}^n_{i_j} = & \min_{\boldsymbol{\beta} \in \mathscr{B}} \max_{i \in \mathcal{L}/\{i_1,\ldots,i_{j-1}\}} \beta_i & = & \min_{\boldsymbol{\beta} \in \mathscr{B}} \max_{i \in \mathcal{L}/\{i_1,\ldots,i_{j-1}\}} \beta_i \\ & \text{s.t. } \beta_i \le \underline{\alpha}^n_i,\ i \in \{i_1, \ldots, i_{j-1}\} & & \text{s.t. } \beta_i \le \underline{\alpha}^n_i,\ i \in \{i_1, \ldots, i_j\} \\ \ge & \min_{\boldsymbol{\beta} \in \mathscr{B}} \max_{i \in \mathcal{L}/\{i_1,\ldots,i_j\}} \beta_i & = & \underline{\alpha}^n_{i_{j+1}} \\ & \text{s.t. } \beta_i \le \underline{\alpha}^n_i,\ i \in \{i_1, \ldots, i_j\} & & \end{aligned}$$

where the second equality follows from the fact that we added the constraint $\beta_{i_j} \le \underline{\alpha}^n_{i_j}$ which is a relaxation of $\max_{i \in \mathcal{L}/\{i_1,\ldots,i_{j-1}\}} \beta_i \le \underline{\alpha}^n_{i_j}$, namely that the objective value is smaller or equal to its optimal value. On the other hand, by construction we have that $\underline{\alpha}^n_i = -\infty$ for all $i \notin \{i_1, \ldots, i_k\}$. □

We now prove Theorem 3 by contradiction. Suppose that there exists a vector $\bar{\boldsymbol{\beta}} \in \mathscr{B}$ such that $\bar{\boldsymbol{\beta}} \prec \boldsymbol{\alpha}^*$; that is, we have $\bar{\boldsymbol{\beta}} \preceq \boldsymbol{\alpha}^*$ but $\boldsymbol{\alpha}^* \not\preceq \bar{\boldsymbol{\beta}}$. Without loss of generality, we can assume that $\vec{\bar{\boldsymbol{\beta}}} = \bar{\boldsymbol{\beta}}$, thus $\bar{\boldsymbol{\beta}} \prec \boldsymbol{\alpha}^*$ implies that there exists a $k \in [1, |\mathcal{L}|]$ such that $\bar{\beta}_i = \vec{\alpha}^*_i$ for all $i < k$ and $\bar{\beta}_k < \vec{\alpha}^*_k$.

If $k = 1$, then $\bar{\beta}_1 < \vec{\alpha}^*_1$ should be true. By Algorithm 1, we have that $\boldsymbol{\alpha}^* \preceq \bar{\boldsymbol{\alpha}}^{n_0}$, so that $\vec{\alpha}^*_1 \le \max_i \bar{\alpha}^{n_0}_i = \min_{\boldsymbol{\beta} \in \mathscr{B}} \max_i \beta_i \le \max_i \bar{\beta}_i = \bar{\beta}_1$. This is a contradiction.

If $k > 1$, then $\bar{\beta}_i = \vec{\alpha}^*_i$ for all $i < k$ and $\bar{\beta}_k < \vec{\alpha}^*_k$ should hold. To discuss this case, we will need to make use of the following lemmas.

Lemma 7. *Given any node n expanded by Algorithm 1, it must be that $\boldsymbol{\alpha}^* \preceq \bar{\boldsymbol{\alpha}}^n$.*

This conclusion can be drawn directly from the definition of Algorithm 1. Specifically, throughout the procedure the vector $\bar{\boldsymbol{\alpha}}$ can only decrease according to the lexicographic ordering. Furthermore, when a node n is expanded, either $\bar{\boldsymbol{\alpha}}^n \succ \bar{\boldsymbol{\alpha}}$ or the latter is replaced with $\bar{\boldsymbol{\alpha}}^n$. □

Lemma 8. *If it is expanded, the node $n^* = (1, \ldots, k-1)$ is such that $\bar{\alpha}^{n^*}_i = \bar{\beta}_i$ for all $i \le k$.*

We show that $\bar{\alpha}^{n^*}_i = \bar{\beta}_i$ for all $i \le k$ by recursion using $k' \in \{0, \ldots, k\}$. Starting with $k' = 0$, we have that $\bar{\alpha}^{n^*}_1 = v^*_{n_0} = \min_{\boldsymbol{\beta} \in \mathscr{B}} \max_i \beta_i = \bar{\beta}_1$, where the first two equalities follows by construction, while the last one follows from $\min_{\boldsymbol{\beta} \in \mathscr{B}} \max_i \beta_i \le \bar{\beta}_1$ since $\bar{\boldsymbol{\beta}} \in \mathscr{B}$, and $\bar{\beta}_1 \le \min_{\boldsymbol{\beta} \in \mathscr{B}} \max_i \beta_i$ since otherwise there exists a $\bar{\boldsymbol{\beta}}' \prec \bar{\boldsymbol{\beta}}$.

We then need to show that if the lemma is true for some $k' < k$, then it is also true for $k' + 1$. In particular, letting $n = (1, \ldots, k')$ and $n' = (1, \ldots, k'+1)$, we have by construction that $\bar{\alpha}^{n'}_i = \underline{\alpha}^{n'}_i = \bar{\alpha}^n_i$ for all $i \le k'+1$ since $\bar{\mathcal{L}}^{n'} = \{1, \ldots, k'+1\}$. Yet, we have that $\bar{\alpha}^n_i = \bar{\beta}_i$ for all $i \le k'$. We are left with demonstrating that $\bar{\alpha}^n_{k'+1} = \bar{\beta}_{k'+1}$, which follows from:

$$\begin{aligned} \bar{\alpha}^n_{k'+1} = & \min_{\boldsymbol{\beta} \in \mathscr{B}} \max_{i \in \mathcal{L}/\{1,\ldots,k'\}} \beta_i & = & \min_{\boldsymbol{\beta} \in \mathscr{B}} \max_{i \in \mathcal{L}/\{1,\ldots,k'\}} \beta_i & = \bar{\beta}_{k'+1} \\ & \text{s.t. } \beta_i \le \underline{\alpha}^n_i,\ i \in \{1, \ldots, k'\} & & \text{s.t. } \beta_i \le \bar{\beta}_i,\ i \in \{1, \ldots, k'\} & \end{aligned}$$

where we exploited the fact that $\underline{\alpha}^n_i = \bar{\alpha}^n_i = \bar{\beta}_i$ for $i \le k'$, and where the last equality follows from the fact that $\bar{\boldsymbol{\beta}}$ is lexicographic minimal in $\mathscr{B}$. □

Now in the case that node $n^* = (1, \ldots, k)$ is expanded by Algorithm 1, it is clear by Lemma 7 that $\boldsymbol{\alpha}^* \preceq \bar{\boldsymbol{\alpha}}^{n^*}$, which either implies, according to Lemma 8, that $\vec{\alpha}^*_i = \bar{\alpha}^{n^*}_i = \bar{\beta}_i$ for all $i \le k$ or that there exists a $k' \le k$ such that $\vec{\alpha}^*_i = \bar{\alpha}^{n^*}_i = \bar{\beta}_i$ for all $i < k'$ while $\vec{\alpha}^*_{k'} < \bar{\alpha}^{n^*}_{k'} = \bar{\beta}_{k'}$. Both situations either lead to a contradiction with respect to the fact that $\bar{\beta}_k < \vec{\alpha}^*_k$ or the fact that $\vec{\alpha}^*_i = \bar{\beta}_i$ for $i < k$, respectively.

We are finally left with the possibility that n^* was not expanded by Algorithm 1. This can only occur if there is a node $\bar{n} = (1, \ldots, \bar{k})$ for some $\bar{k} \le k$, i.e. among the ancestors, that was not expanded at Step 7. Hence, $\underline{\boldsymbol{\alpha}}^{\bar{n}} \succeq \boldsymbol{\alpha}^*$ and since $\bar{n}$ is an ancestor of n^*, we also have that $\bar{\boldsymbol{\alpha}}^{n^*} \succeq \underline{\boldsymbol{\alpha}}^{\bar{n}}$. Thus $\boldsymbol{\alpha}^* \preceq \bar{\boldsymbol{\alpha}}^{n^*}$, which once again leads to a contradiction. □

B.4 Proof of Lemma 4.

By Definition 1, we first rewrite problem $\mathrm{P_{DRO}}$ with $\mathcal{F}$ (i.e., $\mathcal{F} = \prod_{\tilde{\mathbf{d}},\tilde{s}} \mathcal{G}$) as follows:

$$\inf_{\boldsymbol{x},\boldsymbol{r},\boldsymbol{\eta},\alpha,\{\boldsymbol{y}^s(\cdot)\}_{s\in\mathcal{S}}} \alpha \tag{13a}$$

$$\text{s.t.} \quad \eta_i + \frac{1}{\alpha}\sup_{\mathbb{Q}\in\mathcal{G}} \mathbb{E}_{\mathbb{Q}}\left[f(\boldsymbol{x},\boldsymbol{r},\boldsymbol{y}^{\tilde{s}}(\tilde{\mathbf{d}}),\tilde{\mathbf{d}}) - \eta_i\right]^+ \le \tau_i, \quad i \in \mathcal{L}\setminus\bar{\mathcal{L}}^n, \tag{13b}$$

$$\eta_i + \frac{1}{\underline{\alpha}_i^n}\sup_{\mathbb{Q}\in\mathcal{G}} \mathbb{E}_{\mathbb{Q}}\left[f(\boldsymbol{x},\boldsymbol{r},\boldsymbol{y}^{\tilde{s}}(\tilde{\mathbf{d}}),\tilde{\mathbf{d}}) - \eta_i\right]^+ \le \tau_i, \quad i \in \bar{\mathcal{L}}^n, \tag{13c}$$

$$\sum_{j\in\mathcal{L}\setminus i} y_{ij}^s(\mathbf{d}) \le r_i, \qquad (\mathbf{d},\mathbf{z}) \in \bar{\mathcal{D}}^s,\ s\in\mathcal{S},\ i\in\mathcal{L}, \tag{13d}$$

$$y_{ij}^s(\mathbf{d}) \ge 0, \qquad (\mathbf{d},\mathbf{z}) \in \bar{\mathcal{D}}^s,\ s\in\mathcal{S},\ i,j\in\mathcal{L}, \tag{13e}$$

$$(1a)-(1d), \alpha\in(0,1], \boldsymbol{\eta}\ge\mathbf{0}. \tag{13f}$$

where $y_{ij}^s : \mathbb{R}^{|\mathcal{L}|} \to \mathbb{R}_+$ and $\tilde{y}_{ij} := y_{ij}^{\tilde{s}}(\tilde{\mathbf{d}})$. Next, we can show that given any fixed $(\boldsymbol{x},\boldsymbol{r},\boldsymbol{\eta},\alpha)$, there exists a feasible solution for $y_{ij}^s(\mathbf{d})$ in problem (13) if and only if there is also a feasible mapping of the form $\bar{y}_{ij}^s(\mathbf{d},\mathbf{z})$. The "if" direction is straighforward given that one can simply define $\bar{y}_{ij}^s(\mathbf{d},\mathbf{z}) := y_{ij}^s(\mathbf{d})$. Regarding the "only if" part, one can use the feasible $\bar{y}_{ij}^s(\cdot)$ to define $y_{ij}^s(\mathbf{d}) := \bar{y}_{ij}^s(\mathbf{d},|\mathbf{d}-\boldsymbol{\mu}^s|)$ and show that this is a feasible assignment in problem (13) based on:

$$\begin{aligned}
&\eta_i + \frac{1}{\alpha}\sup_{\mathbb{Q}\in\mathcal{G}} \mathbb{E}_{\mathbb{Q}}\left[f(\boldsymbol{x},\boldsymbol{r},\boldsymbol{y}^{\tilde{s}}(\tilde{\mathbf{d}}),\tilde{\mathbf{d}}) - \eta_i\right]^+ = \eta_i + \frac{1}{\alpha}\sup_{\mathbb{Q}\in\mathcal{G}} \mathbb{E}_{\mathbb{Q}}\left[f(\boldsymbol{x},\boldsymbol{r},\bar{\boldsymbol{y}}^{\tilde{s}}(\tilde{\mathbf{d}},|\tilde{\mathbf{d}}-\boldsymbol{\mu}^s|),\tilde{\mathbf{d}}) - \eta_i\right]^+,\\
&\qquad \le \eta_i + \frac{1}{\alpha}\sup_{\mathbb{Q}\in\mathcal{G}} \mathbb{E}_{\mathbb{Q}}\left[f(\boldsymbol{x},\boldsymbol{r},\bar{\boldsymbol{y}}^{\tilde{s}}(\tilde{\mathbf{d}},\tilde{\mathbf{z}}),\tilde{\mathbf{d}}) - \eta_i\right]^+ \le \tau_i,\\
&\sup_{(\mathbf{d},\mathbf{z})\in\bar{\mathcal{D}}^s} \sum_{j\in\mathcal{L}\setminus i} y_{ij}^s(\mathbf{d}) = \sup_{(\mathbf{d},\mathbf{z})\in\bar{\mathcal{D}}^s} \sum_{j\in\mathcal{L}\setminus i} \bar{y}_{ij}^s(\mathbf{d},|\mathbf{d}-\boldsymbol{\mu}^s|) \le \sup_{(\mathbf{d},\mathbf{z})\in\bar{\mathcal{D}}^s} \sum_{j\in\mathcal{L}\setminus i} \bar{y}_{ij}^s(\mathbf{d},\mathbf{z}) \le r_i\\
&\inf_{(\mathbf{d},\mathbf{z})\in\bar{\mathcal{D}}^s} y_{ij}^s(\mathbf{d}) = \inf_{(\mathbf{d},\mathbf{z})\in\bar{\mathcal{D}}^s} \bar{y}_{ij}^s(\mathbf{d},|\mathbf{d}-\boldsymbol{\mu}^s|) \ge \inf_{(\mathbf{d},\mathbf{z})\in\bar{\mathcal{D}}^s} \bar{y}_{ij}^s(\mathbf{d},\mathbf{z}) \ge 0\,.
\end{aligned}$$

We are left with employing classical moment problems duality techniques to reformulate $\sup_{\mathbb{Q}\in\mathcal{G}} \mathbb{E}_{\mathbb{Q}}\left[\left(f(\boldsymbol{x},\boldsymbol{r},\boldsymbol{y}^{\tilde{s}}(\tilde{\mathbf{d}}),\tilde{\mathbf{d}}) - \eta_i\right)^+\right]$ as an infimum. Namely, one starts with the equivalent semi-infinite linear optimization problem:

$$\begin{aligned}
\max \sum_{s\in\mathcal{S}} p_s \int_{(\mathbf{d},\mathbf{z})\in\bar{\mathcal{D}}^s} &\left(\max\left\{0, -\eta_i, d_i + \sum_{j\in\mathcal{L}\setminus i} y_{ij}^s(\mathbf{d},\mathbf{z}) - (r_i + \sum_{j\in\mathcal{L}\setminus i} y_{ji}^s(\mathbf{d},\mathbf{z})) - \eta_i\right\}\right) d\mathbb{Q}_s(\mathbf{d},\mathbf{z})\\
\text{s.t.} \quad &\int_{(\mathbf{d},\mathbf{z})\in\bar{\mathcal{D}}^s} d\mathbb{Q}_s(\mathbf{d},\mathbf{z}) = 1, \qquad s\in\mathcal{S}, \qquad (\textit{dual multiplier } \pi_{si}^1 \in \mathbb{R})\\
&\int_{(\mathbf{d},\mathbf{z})\in\bar{\mathcal{D}}^s} \mathbf{d}\, d\mathbb{Q}_s(\mathbf{d},\mathbf{z}) = \boldsymbol{\mu}^s, \qquad s\in\mathcal{S}, \qquad (\textit{dual multiplier } \boldsymbol{\pi}_{si}^2 \in \mathbb{R}^{|\mathcal{L}|})\\
&\int_{(\mathbf{d},\mathbf{z})\in\bar{\mathcal{D}}^s} \mathbf{z}\, d\mathbb{Q}_s(\mathbf{d},\mathbf{z}) \le \boldsymbol{\nu}^s, \qquad s\in\mathcal{S}. \qquad (\textit{dual multiplier } \boldsymbol{\pi}_{si}^3 \in \mathbb{R}_+^{|\mathcal{L}|}),
\end{aligned}$$

where $\mathbb{Q}_s$ denote the conditional probability distribution of $(\mathbf{d},\mathbf{z})$ given that $\tilde{s} = s$.

Letting $\pi^1_{si} \in \mathbb{R}$, $\boldsymbol{\pi}^2_{si} \in \mathbb{R}^{|\mathcal{L}|}$, and $\boldsymbol{\pi}^3_{si} \in \mathbb{R}^{|\mathcal{L}|}$ be dual variables associated with constraints of the above problem, we obtain the dual formulation as

$$\begin{aligned}
\min_{\pi^1_{si}, \boldsymbol{\pi}^2_{si}, \boldsymbol{\pi}^3_{si} \geq \mathbf{0}} & \sum_{s\in\mathcal{S}} \left(\pi^1_{si} + (\boldsymbol{\pi}^2_{si})'\boldsymbol{\mu}^s + (\boldsymbol{\pi}^3_{si})'\boldsymbol{\nu}^s\right) \\
\text{s.t.} \quad & \pi^1_{si} + (\boldsymbol{\pi}^2_{si})'\mathbf{d} + (\boldsymbol{\pi}^3_{si})'\mathbf{z} \geq 0, && (\mathbf{d},\mathbf{z}) \in \bar{\mathcal{D}}^s, s \in \mathcal{S}, \\
& \pi^1_{si} + (\boldsymbol{\pi}^2_{si})'\mathbf{d} + (\boldsymbol{\pi}^3_{si})'\mathbf{z} \geq -p_s\eta_i, && (\mathbf{d},\mathbf{z}) \in \bar{\mathcal{D}}^s, s \in \mathcal{S}, \\
& \pi^1_{si} + (\boldsymbol{\pi}^2_{si})'\mathbf{d} + (\boldsymbol{\pi}^3_{si})'\mathbf{z} \geq p_s\left(d_i + \sum_{j\in\mathcal{L}\setminus i} y^s_{ij}(\mathbf{d},\mathbf{z}) - (r_i + \sum_{j\in\mathcal{L}\setminus i} y^s_{ji}(\mathbf{d},\mathbf{z})) - \eta_i\right), && (\mathbf{d},\mathbf{z}) \in \bar{\mathcal{D}}^s, s \in \mathcal{S}.
\end{aligned}$$

We thus obtain the formulation in the Lemma 4. □

B.5 Proof of Proposition 5.

By replacing $\bar{\mathcal{D}}^s$ with the bounded set $\bar{\mathcal{D}}^s_b$ in problem P_{ARO}, we obtain the following model:

$$\begin{aligned}
v^b_n := & \\
\inf_{\substack{\boldsymbol{x},\boldsymbol{r},\boldsymbol{\eta},\alpha,\{\boldsymbol{y}^s(\cdot)\}_{s\in\mathcal{S}} \\ \boldsymbol{\pi}^1,\boldsymbol{\pi}^2,\boldsymbol{\pi}^3 \geq \mathbf{0}}} \quad & \alpha && \text{(14a)} \\
\text{s.t.} \quad & \eta_i + \frac{1}{\alpha}\sum_{s\in\mathcal{S}}\left(\pi^1_{si} + (\boldsymbol{\pi}^2_{si})'\boldsymbol{\mu}^s + (\boldsymbol{\pi}^3_{si})'\boldsymbol{\nu}^s\right) \leq \tau_i, \quad i \in \mathcal{L} \setminus \bar{\mathcal{L}}^n, && \text{(14b)} \\
& \eta_i + \frac{1}{\underline{\alpha}^n_i}\sum_{s\in\mathcal{S}}\left(\pi^1_{si} + (\boldsymbol{\pi}^2_{si})'\boldsymbol{\mu}^s + (\boldsymbol{\pi}^3_{si})'\boldsymbol{\nu}^s\right) \leq \tau_i, \quad i \in \bar{\mathcal{L}}^n, && \text{(14c)} \\
& \pi^1_{si} + (\boldsymbol{\pi}^2_{si})'\mathbf{d} + (\boldsymbol{\pi}^3_{si})'\mathbf{z} \geq 0, \quad (\mathbf{d},\mathbf{z}) \in \bar{\mathcal{D}}^s_b, s \in \mathcal{S}, i \in \mathcal{L}, && \text{(14d)} \\
& \pi^1_{si} + (\boldsymbol{\pi}^2_{si})'\mathbf{d} + (\boldsymbol{\pi}^3_{si})'\mathbf{z} \geq -p_s\eta_i, \quad (\mathbf{d},\mathbf{z}) \in \bar{\mathcal{D}}^s_b, s \in \mathcal{S}, i \in \mathcal{L}, && \text{(14e)} \\
& \pi^1_{si} + (\boldsymbol{\pi}^2_{si})'\mathbf{d} + (\boldsymbol{\pi}^3_{si})'\mathbf{z} \geq p_s\big(d_i + \sum_{j\in\mathcal{L}\setminus i} y^s_{ij}(\mathbf{d},\mathbf{z}) \\
& \qquad - (r_i + \sum_{j\in\mathcal{L}\setminus i} y^s_{ji}(\mathbf{d},\mathbf{z})) - \eta_i\big) \quad (\mathbf{d},\mathbf{z}) \in \bar{\mathcal{D}}^s_b, s \in \mathcal{S}, i \in \mathcal{L}, && \text{(14f)} \\
& \sum_{j\in\mathcal{L}\setminus i} y^s_{ij}(\mathbf{d},\mathbf{z}) \leq r_i, \quad (\mathbf{d},\mathbf{z}) \in \bar{\mathcal{D}}^s_b, s \in \mathcal{S}, i \in \mathcal{L}, && \text{(14g)} \\
& y^s_{ij}(\mathbf{d},\mathbf{z}) \geq 0, \quad (\mathbf{d},\mathbf{z}) \in \bar{\mathcal{D}}^s_b, s \in \mathcal{S}, i,j \in \mathcal{L}. && \text{(14h)} \\
& \text{(1a)} - \text{(1d)}, \alpha \in (0,1], \boldsymbol{\eta} \geq \mathbf{0}. && \text{(14i)}
\end{aligned}$$

The next lemma confirms that replacing $\bar{\mathcal{D}}^s$ with $\bar{\mathcal{D}}^s_b$ does not affect the optimal solution of problem P_{ARO}.

Lemma 9. *Problem P_{ARO} is equivalent to problem* (14)*, i.e., $v^*_n = v^b_n$ and the set of optimal solutions for $\boldsymbol{x}$ and $\boldsymbol{r}$ remains unchanged.*

First we note that $v^b_n \leq v^*_n$ because problem P_{ARO} has the same objective function but more constraints than problem (14).

Second, we prove $v^b_n \geq v^*_n$. Let $\hat{\boldsymbol{x}}$, $\hat{\boldsymbol{r}}$, $\hat{\boldsymbol{\eta}}$, $\hat{\alpha}$, $\hat{\boldsymbol{\pi}}^1$, $\hat{\boldsymbol{\pi}}^2$, $\hat{\boldsymbol{\pi}}^3$, and $\hat{\boldsymbol{y}}^s(\mathbf{d},\mathbf{z})$, for all $s \in \mathcal{S}$, be a feasible solution to problem (14). It follows that the for all $s \in S, i \in \mathcal{L}, (\mathbf{d}, \mathbf{z}) \in \bar{\mathcal{D}}_b^s$, we have

$$\hat{\pi}_{si}^1 + (\hat{\boldsymbol{\pi}}_{si}^2)'\mathbf{d} + (\hat{\boldsymbol{\pi}}_{si}^3)'\mathbf{z} \geq \max\left\{0, -p_s\hat{\eta}_i, p_s\left(d_i + \sum_{j\in\mathcal{L}\setminus i}\hat{y}_{ij}^s(\mathbf{d},\mathbf{z}) - \left(\hat{r}_i + \sum_{j\in\mathcal{L}\setminus i}\hat{y}_{ji}^s(\mathbf{d},\mathbf{z})\right) - \hat{\eta}_i\right)\right\}.$$

To simplify the notation, we rewrite the above formula as $f_{si}(\hat{\boldsymbol{\pi}}, \mathbf{d}, \mathbf{z}) \geq g_{si}(\hat{\boldsymbol{r}}, \hat{\boldsymbol{\eta}}, \hat{\boldsymbol{y}}, \mathbf{d}, \mathbf{z})$. Next, we can construct a feasible solution to problem $\mathrm{P}_{\mathrm{ARO}}$ by letting $\check{\boldsymbol{x}} := \hat{\boldsymbol{x}}$, $\check{\boldsymbol{r}} := \hat{\boldsymbol{r}}$, $\check{\boldsymbol{\eta}} := \hat{\boldsymbol{\eta}}$, $\check{\alpha} := \hat{\alpha}$, $\check{\boldsymbol{\pi}}^1 := \hat{\boldsymbol{\pi}}^1$, $\check{\boldsymbol{\pi}}^2 := \hat{\boldsymbol{\pi}}^2$, $\check{\boldsymbol{\pi}}^3 := \hat{\boldsymbol{\pi}}^3$ and for all $s \in \mathcal{S}$,

$$\check{\boldsymbol{y}}^s(\mathbf{d},\mathbf{z}) := \begin{cases} \hat{\boldsymbol{y}}^s(\mathbf{d},\mathbf{z}) & \text{if } (\mathbf{d},\mathbf{z}) \in \bar{\mathcal{D}}_b^s, \\ \hat{\boldsymbol{y}}^s(\mathbf{d}, \min\{\mathbf{z}, \bar{\mathbf{z}}^s\}) & \text{if } (\mathbf{d},\mathbf{z}) \notin \bar{\mathcal{D}}_b^s, \end{cases}$$

where $\bar{z}_i^s := \max\{\overline{d}_i^s - \mu_i^s, \mu_i^s - \underline{d}_i^s\}$ for all $i \in \mathcal{L}$, and where $\min\{\mathbf{z}, \bar{\mathbf{z}}\}$ is applied termwise. We can verify that the constructed solution $(\check{\boldsymbol{x}}, \check{\boldsymbol{r}}, \check{\boldsymbol{\eta}}, \check{\boldsymbol{\pi}}, \check{\boldsymbol{y}})$ is feasible for problem $\mathrm{P}_{\mathrm{ARO}}$ by first easily confirming that it is feasible with respect to all constraints besides (9d)-(9f). Regarding, (9d)-(9f), i.e. $f_{si}(\check{\boldsymbol{\pi}}, \mathbf{d}, \mathbf{z}) \geq g_{si}(\check{\boldsymbol{r}}, \check{\boldsymbol{\eta}}, \check{\boldsymbol{y}}, \mathbf{d}, \mathbf{z})$ for all i, s and $(\mathbf{d},\mathbf{z}) \in \bar{\mathcal{D}}^s$, we discussing the following two cases:

i) If $(\mathbf{d},\mathbf{z}) \in \bar{\mathcal{D}}_b^s$, then $f_{si}(\check{\boldsymbol{\pi}}, \mathbf{d}, \mathbf{z}) = f_{si}(\hat{\boldsymbol{\pi}}, \mathbf{d}, \mathbf{z}) \geq g_{si}(\hat{\boldsymbol{r}}, \hat{\boldsymbol{\eta}}, \hat{\boldsymbol{y}}, \mathbf{d}, \mathbf{z}) = g_{si}(\check{\boldsymbol{r}}, \check{\boldsymbol{\eta}}, \check{\boldsymbol{y}}, \mathbf{d}, \mathbf{z})$, which follows from the definition of the constructed solution.

ii) If $(\mathbf{d},\mathbf{z}) \notin \bar{\mathcal{D}}_b^s$, then we have that $f(\check{\boldsymbol{\pi}}, \mathbf{d}, \mathbf{z}) = f(\hat{\boldsymbol{\pi}}, \mathbf{d}, \mathbf{z}) \geq f(\hat{\boldsymbol{\pi}}, \mathbf{d}, \min\{\mathbf{z}, \bar{\mathbf{z}}\}) \geq g(\hat{\boldsymbol{r}}, \hat{\boldsymbol{\eta}}, \hat{\boldsymbol{y}}, \mathbf{d}, \min\{\mathbf{z}, \bar{\mathbf{z}}\}) = g(\check{\boldsymbol{r}}, \check{\boldsymbol{\eta}}, \check{\boldsymbol{y}}, \mathbf{d}, \mathbf{z})$, where we used in order that $\check{\boldsymbol{\pi}} = \hat{\boldsymbol{\pi}}$, then that $\hat{\boldsymbol{\pi}}^3 \geq 0$, the fact that $(\hat{\boldsymbol{\pi}}, \hat{\boldsymbol{r}}, \hat{\boldsymbol{\eta}}, \hat{\boldsymbol{y}})$ is feasible in (14) and that $(\mathbf{d}, \min\{\mathbf{z}, \bar{\mathbf{z}}\}) \in \bar{\mathcal{D}}_b^s$.

Because problem $\mathrm{P}_{\mathrm{ARO}}$ and problem (14) have the same objective function, we conclude that $v_n^b \geq v_n^*$. Together with $v_n^b \leq v_n^*$, the proof is complete. □

In our proof of Proposition 5, for all $s \in \mathcal{S}$, we also exploit some convexity property of the following operator:

$$G^s(\boldsymbol{r}, \alpha, \boldsymbol{\eta}, \boldsymbol{\pi}, \mathbf{d}, \mathbf{z}) := \begin{cases} 0 & \text{if } \mathcal{Y}^s(\alpha, \boldsymbol{\eta}, \boldsymbol{\pi}, \mathbf{d}, \mathbf{z}) \neq \emptyset \\ \infty & \text{otherwise} \end{cases},$$

where set $\mathcal{Y}^s(\boldsymbol{r}, \alpha, \boldsymbol{\eta}, \boldsymbol{\pi}, \mathbf{d}, \mathbf{z})$ is defined as:

$$\mathcal{Y}^s(\boldsymbol{r}, \alpha, \boldsymbol{\eta}, \boldsymbol{\pi}, \mathbf{d}, \mathbf{z}) := \left\{ \boldsymbol{y} \in \mathbb{R}^{|\mathcal{L}|\times|\mathcal{L}|} : \begin{array}{ll} (14\text{b}) - (14\text{c}) & \\ \pi_{si}^1 + (\boldsymbol{\pi}_{si}^2)'\mathbf{d} + (\boldsymbol{\pi}_{si}^3)'\mathbf{z} \geq 0, & i \in \mathcal{L}, \\ \pi_{si}^1 + (\boldsymbol{\pi}_{si}^2)'\mathbf{d} + (\boldsymbol{\pi}_{si}^3)'\mathbf{z} \geq -p_s\eta_i, & i \in \mathcal{L}, \\ \pi_{si}^1 + (\boldsymbol{\pi}_{si}^2)'\mathbf{d} + (\boldsymbol{\pi}_{si}^3)'\mathbf{z} \geq & \\ \quad p_s(d_i + \sum_{j\in\mathcal{L}\setminus i} y_{ij} - (r_i + \sum_{j\in\mathcal{L}\setminus i} y_{ji}) - \eta_i) & i \in \mathcal{L}, \\ \sum_{j\in\mathcal{L}\setminus i} y_{ij} \leq r_i, & i \in \mathcal{L}, \\ y_{ij} \geq 0, & i, j \in \mathcal{L}, \end{array} \right\},$$

and which is considered empty if any constraint in $\mathcal{Y}^s(\boldsymbol{r}, \alpha, \boldsymbol{\eta}, \boldsymbol{\pi}, \mathbf{d}, \mathbf{z})$ is violated.

Then, we can reorganize the constraints in problem (14) and equivalently rewrite it in the following general form:

$$\inf\left\{\alpha \in (0,1] \,\middle|\, \begin{array}{c} \exists \boldsymbol{x}, \boldsymbol{r}, (1\text{a}) - (1\text{d}) \\ \inf_{\boldsymbol{\eta}\geq 0, \boldsymbol{\pi}} \max_s \sup_{(\mathbf{d},\mathbf{z})\in\bar{\mathcal{D}}_b^s} G^s(\boldsymbol{r}, \alpha, \boldsymbol{\eta}, \boldsymbol{\pi}, \mathbf{d}, \mathbf{z}) \leq 0 \end{array}\right\} \tag{15}$$

Lemma 10. *$G^s(\boldsymbol{r}, \alpha, \boldsymbol{\eta}, \boldsymbol{\pi}, \mathbf{d}, \mathbf{z})$ is convex jointly in $\mathbf{d}$, $\mathbf{z}$.*

Let $(\mathbf{d}_1, \mathbf{z}_1)$ and $(\mathbf{d}_2, \mathbf{z}_2)$ be arbitrary elements in $\mathbb{R}^{|\mathcal{L}|} \times \mathbb{R}^{|\mathcal{L}|}$. For some fixed $\boldsymbol{r}$, α, $\boldsymbol{\eta}$, $\boldsymbol{\pi}$ and any $\lambda \in (0,1)$, we analyze the following two cases to evaluate $G^s(\boldsymbol{r}, \alpha, \boldsymbol{\eta}, \boldsymbol{\pi}, \mathbf{d}_\lambda, \mathbf{z}_\lambda)$, where $(\mathbf{d}_\lambda, \mathbf{z}_\lambda) = \lambda(\mathbf{d}_1, \mathbf{z}_1) + (1-\lambda)(\mathbf{d}_2, \mathbf{z}_2)$.

First, when $G^s(\boldsymbol{r},\alpha,\boldsymbol{\eta},\boldsymbol{\pi},\mathbf{d}_1,\mathbf{z}_1)=\infty$ and/or $G^s(\boldsymbol{r},\alpha,\boldsymbol{\eta},\boldsymbol{\pi},\mathbf{d}_2,\mathbf{z}_2)=\infty$, we have

$$\lambda G^s(\boldsymbol{r},\alpha,\boldsymbol{\eta},\boldsymbol{\pi},\mathbf{d}_1,\mathbf{z}_1)+(1-\lambda)G^s(\boldsymbol{r},\alpha,\boldsymbol{\eta},\boldsymbol{\pi},\mathbf{d}_2,\mathbf{z}_2)\geq\infty\geq G^s(\boldsymbol{r},\alpha,\boldsymbol{\eta},\boldsymbol{\pi},\mathbf{d}_\lambda,\mathbf{z}_\lambda),$$

since the value of $G^s(\boldsymbol{r},\alpha,\boldsymbol{\eta},\boldsymbol{\pi},\mathbf{d}_\lambda,\mathbf{z}_\lambda)$ is either 0 or infinity.

Second, when $G^s(\boldsymbol{r},\alpha,\boldsymbol{\eta},\boldsymbol{\pi},\mathbf{d}_1,\mathbf{z}_1)=0$ and $G^s(\boldsymbol{r},\alpha,\boldsymbol{\eta},\boldsymbol{\pi},\mathbf{d}_2,\mathbf{z}_2)=0$, there exist $\boldsymbol{y}_1$ and $\boldsymbol{y}_2$ that satisfy all constraints defined in $\mathcal{Y}^s(\boldsymbol{r},\alpha,\boldsymbol{\eta},\boldsymbol{\pi},\mathbf{d}_1,\mathbf{z}_2)$ and $\mathcal{Y}^s(\boldsymbol{r},\alpha,\boldsymbol{\eta},\boldsymbol{\pi},\mathbf{d}_2,\mathbf{z}_2)$ respectively. Next, we can construct $\boldsymbol{y}_\lambda$ for $G^s(\boldsymbol{r},\alpha,\boldsymbol{\eta},\boldsymbol{\pi},\mathbf{d}_\lambda,\mathbf{z}_\lambda)$ by $\boldsymbol{y}_\lambda=\lambda\boldsymbol{y}_1+(1-\lambda)\boldsymbol{y}_2$. We can easily verify that $\boldsymbol{y}_\lambda$ satisfies constraints in $\mathcal{Y}^s(\boldsymbol{r},\alpha,\boldsymbol{\eta},\boldsymbol{\pi},\mathbf{d},\mathbf{z})$. Thus, we have $\boldsymbol{y}_\lambda\in\mathcal{Y}^s(\boldsymbol{r},\alpha,\boldsymbol{\eta},\boldsymbol{\pi},\mathbf{d}_\lambda,\mathbf{z}_\lambda)$ such that $G^s(\boldsymbol{r},\alpha,\boldsymbol{\eta},\boldsymbol{\pi},\mathbf{d}_\lambda,\mathbf{z}_\lambda)=0$, which implies that

$$\lambda G^s(\boldsymbol{r},\alpha,\boldsymbol{\eta},\mathbf{d}_1,\mathbf{z}_1)+(1-\lambda)G^s(\boldsymbol{r},\alpha,\boldsymbol{\eta},\mathbf{d}_2,\mathbf{z}_2)=0\geq 0=G^s(\boldsymbol{r},\alpha,\boldsymbol{\eta},\mathbf{d}_\lambda,\mathbf{z}_\lambda).$$

Therefore, $G^s(\boldsymbol{r},\alpha,\boldsymbol{\eta},\mathbf{d},\mathbf{z})$ is convex jointly in $\mathbf{d}$ and $\mathbf{z}$. □

We proceed with the proof of Proposition 5.

Recall that $\bar{\mathcal{D}}^s_b$ is a convex hull of $\{\mathbf{d}(\omega)\}_{\omega\in\Omega^s}$. By Lemma 10, we know that $G^s(\boldsymbol{r},\alpha,\boldsymbol{\eta},\mathbf{d},\mathbf{z})$ is a jointly convex function in $\mathbf{d}$ and $\mathbf{z}$. Hence, according to the theory of concave minimization (see, for instance, **?**), problem (15) can be replaced with

$$\inf\left\{\alpha\in(0,1]\;\middle|\;\begin{array}{c}\exists\boldsymbol{x},\boldsymbol{r},(1\mathrm{a})-(1\mathrm{d})\\ \inf\limits_{\boldsymbol{\eta}\geq 0,\boldsymbol{\pi}}\max\limits_{s}\max\limits_{\omega\in\Omega^s}G^s(\boldsymbol{r},\alpha,\boldsymbol{\eta},\boldsymbol{\pi},\mathbf{d}(\omega),\mathbf{z}(\omega))\leq 0\end{array}\right\}\tag{16}$$

As a result, given a sample ω, the adaptive variables $\boldsymbol{y}(\mathbf{d},\mathbf{z})$ reduce to $\boldsymbol{y}(\omega)$. This naturally gives rise to P_{VE} as an equivalent problem to $\mathrm{P}_{\mathrm{ARO}}$. □

B.6 MILP Reformulation of problem $\mathrm{P}_{\mathbf{AARC}}$.

We present the reformulation of problem $\mathrm{P}_{\mathrm{AARC}}$ by the following proposition.

Proposition 11. *Problem P_{AARC} can be solved by the following mixed-integer linear program:*

$$
\begin{aligned}
(P_{MILP})\quad v_n^{AARC} := & \inf_{\substack{\boldsymbol{x},\boldsymbol{r},\boldsymbol{\eta},\alpha\\ \boldsymbol{\pi}^1,\boldsymbol{\pi}^2,\boldsymbol{\pi}^3\geq \mathbf{0},\boldsymbol{y}^0,\boldsymbol{y}^1,\boldsymbol{y}^2\\ (\boldsymbol{w}^1,\boldsymbol{w}^2,\boldsymbol{w}^3,\boldsymbol{w}^4,\boldsymbol{w}^5)\geq\mathbf{0}}} \alpha \\
\text{s.t.}\quad & \eta_i + \frac{1}{\alpha}\sum_{s\in\mathcal{S}}\left(\pi^1_{si} + (\boldsymbol{\pi}^2_{si})'\boldsymbol{\mu}^s + (\boldsymbol{\pi}^3_{si})'\boldsymbol{\nu}^s\right) \leq \tau_i, && i\in\mathcal{L}\setminus\bar{\mathcal{L}}^n,\\
& \eta_i + \frac{1}{\underline{\alpha}^n_i}\sum_{s\in\mathcal{S}}\left(\pi^1_{si} + (\boldsymbol{\pi}^2_{si})'\boldsymbol{\mu}^s + (\boldsymbol{\pi}^3_{si})'\boldsymbol{\nu}^s\right) \leq \tau_i, && i\in\bar{\mathcal{L}}^n,\\
& \pi^1_{si} + (\boldsymbol{w}^{11}_{si})'\underline{\mathbf{d}}^s - (\boldsymbol{w}^{12}_{si})'\overline{\mathbf{d}}^s - (\boldsymbol{w}^{13}_{si} - \boldsymbol{w}^{14}_{si})'\boldsymbol{\mu}^s \geq 0, && s\in\mathcal{S}, i\in\mathcal{L},\\
& \pi^1_{si} + (\boldsymbol{w}^{21}_{si})'\underline{\mathbf{d}}^s - (\boldsymbol{w}^{22}_{si})'\overline{\mathbf{d}}^s - (\boldsymbol{w}^{23}_{si} - \boldsymbol{w}^{24}_{si})'\boldsymbol{\mu}^s \geq -p_s\eta_i, && s\in\mathcal{S}, i\in\mathcal{L},\\
& \pi^1_{si} + (\boldsymbol{w}^{31}_{si})'\underline{\mathbf{d}}^s - (\boldsymbol{w}^{32}_{si})'\overline{\mathbf{d}}^s - (\boldsymbol{w}^{33}_{si} - \boldsymbol{w}^{34}_{si})'\boldsymbol{\mu}^s \geq p_s(\sum_{j\in\mathcal{L}\setminus i} y^{0s}_{ij} \\
&\qquad - (r_i + \sum_{j\in\mathcal{L}\setminus i} y^{0s}_{ji}) - \eta_i) && s\in\mathcal{S}, i\in\mathcal{L},\\
& \sum_{j\in\mathcal{L}\setminus i} y^{0s}_{ij} - \left((\boldsymbol{w}^{41}_{si})'\underline{\mathbf{d}}^s - (\boldsymbol{w}^{42}_{si})'\overline{\mathbf{d}}^s - (\boldsymbol{w}^{43}_{si} - \boldsymbol{w}^{44}_{si})'\boldsymbol{\mu}^s\right) \leq r_i, && s\in\mathcal{S}, i\in\mathcal{L},\\
& y^{0s}_{ij} + (\boldsymbol{w}^{51}_{sij})'\underline{\mathbf{d}}^s - (\boldsymbol{w}^{52}_{sij})'\overline{\mathbf{d}}^s - (\boldsymbol{w}^{53}_{sij} - \boldsymbol{w}^{54}_{sij})'\boldsymbol{\mu}^s \geq 0, && s\in\mathcal{S}, i,j\in\mathcal{L},\\
& \boldsymbol{w}^{11}_{si} - \boldsymbol{w}^{12}_{si} - (\boldsymbol{w}^{13}_{si} - \boldsymbol{w}^{14}_{si}) \leq \boldsymbol{\pi}^2_{si}, && s\in\mathcal{S}, i\in\mathcal{L},\\
& \boldsymbol{w}^{21}_{si} - \boldsymbol{w}^{22}_{si} - (\boldsymbol{w}^{23}_{si} - \boldsymbol{w}^{24}_{si}) \leq \boldsymbol{\pi}^2_{si}, && s\in\mathcal{S}, i\in\mathcal{L},\\
& w^{31}_{sil} - w^{32}_{sil} - (w^{33}_{sil} - w^{34}_{sil}) \leq v^2_{sil} - \sum_{j\in\mathcal{L}\setminus i} p_s(y^{1s}_{ijl} - y^{1s}_{jil}), && s\in\mathcal{S}, l,i\in\mathcal{L}, l\neq i,\\
& w^{31}_{sil} - w^{32}_{sil} - (w^{33}_{sil} - w^{34}_{sil}) \leq v^2_{sil} - \sum_{j\in\mathcal{L}\setminus i} p_s(y^{1s}_{ijl} - y^{1s}_{jil}) - p_s, && s\in\mathcal{S}, l,i\in\mathcal{L}, l = i,\\
& \boldsymbol{w}^{41}_{si} - \boldsymbol{w}^{42}_{si} - (\boldsymbol{w}^{43}_{si} - \boldsymbol{w}^{44}_{si}) \leq -\sum_{j\in\mathcal{L}\setminus i} \boldsymbol{y}^{1s}_{ij}, && s\in\mathcal{S}, i\in\mathcal{L},\\
& \boldsymbol{w}^{51}_{sij} - \boldsymbol{w}^{52}_{sij} - (\boldsymbol{w}^{53}_{sij} - \boldsymbol{w}^{54}_{sij}) \leq \boldsymbol{y}^{1s}_{ij}, && s\in\mathcal{S}, i,j\in\mathcal{L},\\
& \boldsymbol{w}^{13}_{si} + \boldsymbol{w}^{14}_{si} \leq \boldsymbol{\pi}^3_{si}, && s\in\mathcal{S}, i\in\mathcal{L},\\
& \boldsymbol{w}^{23}_{si} + \boldsymbol{w}^{24}_{si} \leq \boldsymbol{\pi}^3_{si}, && s\in\mathcal{S}, i\in\mathcal{L},\\
& \boldsymbol{w}^{33}_{si} + \boldsymbol{w}^{34}_{si} \leq \boldsymbol{\pi}^3_{si} - \sum_{j\in\mathcal{L}\setminus i} p_s(\boldsymbol{y}^{2s}_{ij} - \boldsymbol{y}^{2s}_{ji}), && s\in\mathcal{S}, i\in\mathcal{L},\\
& \boldsymbol{w}^{43}_{si} + \boldsymbol{w}^{44}_{si} \leq -\sum_{j\in\mathcal{L}\setminus i} \boldsymbol{y}^{2s}_{ij}, && s\in\mathcal{S}, i\in\mathcal{L},\\
& \boldsymbol{w}^{53}_{sij} + \boldsymbol{w}^{54}_{sij} \leq \boldsymbol{y}^{2s}_{ij}, && s\in\mathcal{S}, i,j\in\mathcal{L},\\
& (1a)-(1d), \alpha\in(0,1], \boldsymbol{\eta}\geq\mathbf{0}.
\end{aligned}
$$

For all $s\in\mathcal{S}$, we represent the adaptive decisions by an affine functions $\boldsymbol{y}^s(\cdot)\in\mathscr{A}$. For example, we can rewrite constraint (11f) as

$$
\pi^1_{si} + \min_{(\mathbf{d},\mathbf{z})\in\bar{\mathcal{D}}_s}\left\{\left(\boldsymbol{\pi}^2_{si} - \sum_{j\in\mathcal{L}\setminus i} p_s(\boldsymbol{y}^{1s}_{ij} - \boldsymbol{y}^{1s}_{ji})\right)'\mathbf{d} + \left(\boldsymbol{\pi}^3_{si} - \sum_{j\in\mathcal{L}\setminus i} p_s(\boldsymbol{y}^{2s}_{ij} - \boldsymbol{y}^{2s}_{ji})\right)'\mathbf{z} - p_s d_i\right\} \geq
$$
$$
p_s\left(\sum_{j\in\mathcal{L}\setminus i} y^{0s}_{ij} - (r_i + \sum_{j\in\mathcal{L}\setminus i} y^{0s}_{ji}) - \eta_i\right), \qquad s\in\mathcal{S}, i\in\mathcal{L}.
$$

We now focus on the above minimization subproblem. For all $s \in \mathcal{S}, i \in \mathcal{L}$, the subproblem can be rewritten as follows:

$$\begin{aligned}
\min_{\mathbf{d},\mathbf{z}} \ & \left(\boldsymbol{\pi}^2_{si} - \sum_{j\in\mathcal{L}\setminus i} p_s(\boldsymbol{y}^{1s}_{ij} - \boldsymbol{y}^{1s}_{ji})\right)' \mathbf{d} + \left(\boldsymbol{\pi}^3_{si} - \sum_{j\in\mathcal{L}\setminus i} p_s(\boldsymbol{y}^{2s}_{ij} - \boldsymbol{y}^{2s}_{ji})\right)' \mathbf{z} - p_s d_i \\
\text{s.t.} \ & \underline{\mathbf{d}}^s \le \mathbf{d} \le \overline{\mathbf{d}}^s, \qquad \text{(dual multipliers } \boldsymbol{w}^{31}_{si}, \boldsymbol{w}^{32}_{si} \in \mathbb{R}^{|\mathcal{L}|}_+) \\
& -\mathbf{z} \le \mathbf{d} - \boldsymbol{\mu}^s \le \mathbf{z}, \qquad \text{(dual multipliers } \boldsymbol{w}^{33}_{si}, \boldsymbol{w}^{34}_{si} \in \mathbb{R}^{|\mathcal{L}|}_+)
\end{aligned}$$

By the strong duality, we present the dual formulation of the above subproblem as:

$$\begin{aligned}
\max_{(\boldsymbol{w}^{31}_{si},\boldsymbol{w}^{32}_{si},\boldsymbol{w}^{33}_{si},\boldsymbol{w}^{34}_{si})\ge\mathbf{0}} \ & (\boldsymbol{w}^{31}_{si})'\underline{\mathbf{d}}^s - (\boldsymbol{w}^{32}_{si})'\overline{\mathbf{d}}^s - \left(\boldsymbol{w}^{33}_{si} - \boldsymbol{w}^{34}_{si}\right)' \boldsymbol{\mu}^s \\
\text{s.t.} \ & w^{31}_{sil} - w^{32}_{sil} - (w^{34}_{sil} - w^{33}_{sil}) \le v^2_{sil} - \sum_{j\in\mathcal{L}\setminus i} p_s(y^{1s}_{ijl} - y^{1s}_{jil}), \qquad l \in \mathcal{L}/i, \\
& w^{31}_{sii} - w^{32}_{sii} - (w^{34}_{sii} - w^{33}_{sii}) \le v^2_{sii} - \sum_{j\in\mathcal{L}\setminus i} p_s(y^{1s}_{iji} - y^{1s}_{jii}) - p_s, \\
& \boldsymbol{w}^{33}_{si} + \boldsymbol{w}^{34}_{si} \le \boldsymbol{\pi}^3_{si} - \sum_{j\in\mathcal{L}\setminus i} p_s(\boldsymbol{y}^{2s}_{ij} - \boldsymbol{y}^{2s}_{ji}).
\end{aligned}$$

For other constraint sets in problem $\mathrm{P_{AARC}}$, we can perform the similar conversion. □

C PRNDP-NE Formulation

We formulate the predisaster relief network design problem without equity (PRNDP-NE) as follows:

$$\text{(PRNDP-NE)} \quad \min_{\boldsymbol{x},\boldsymbol{r},\tilde{\boldsymbol{u}},\tilde{\boldsymbol{y}}} \sup_{\mathbb{P}\in\mathcal{F}} \mathbb{E}_{\mathbb{P}}\left[\sum_{i\in\mathcal{L}} \tilde{u}_i\right] \quad \text{s.t.} \quad \text{(1a)} - \text{(1d)}, \text{(2b)} - \text{(2d)}. \tag{17}$$

Similar to Lemma 4, the PRNDP-NE with $\mathcal{F}$, i.e., $\mathcal{F} = \prod_{\tilde{\mathbf{d}},\tilde{s}} \mathcal{G}$, is equivalent to the following optimization problem:

$$\min_{\substack{\boldsymbol{x},\boldsymbol{r},\{\boldsymbol{y}^s(\cdot)\}_{s\in\mathcal{S}} \\ \boldsymbol{\pi}^1,\boldsymbol{\pi}^2,\boldsymbol{\pi}^3\ge\mathbf{0}}} \sum_{s\in\mathcal{S}} \left(v^1_s + (\boldsymbol{\pi}^2_s)'\boldsymbol{\mu}^s + (\boldsymbol{\pi}^3_s)'\boldsymbol{\nu}^s\right) \tag{18a}$$

$$\text{s.t.} \quad \pi^1_s + (\boldsymbol{\pi}^2_s)'\mathbf{d} + (\boldsymbol{\pi}^3_s)'\mathbf{z} \ge p_s \sum_{i\in\mathcal{L}} f(\boldsymbol{x},\boldsymbol{r},\boldsymbol{y}^s(\mathbf{d},\mathbf{z}),\mathbf{d}) \quad (\mathbf{d},\mathbf{z}) \in \bar{\mathcal{D}}_s, s\in\mathcal{S}, \tag{18b}$$

$$\sum_{j\in\mathcal{L}\setminus i} y^s_{ij}(\mathbf{d},\mathbf{z}) \le r_i, \quad (\mathbf{d},\mathbf{z}) \in \bar{\mathcal{D}}_s, s\in\mathcal{S}, i\in\mathcal{L}, \tag{18c}$$

$$y^s_{ij}(\mathbf{d},\mathbf{z}) \ge 0, \quad (\mathbf{d},\mathbf{z}) \in \bar{\mathcal{D}}_s, s\in\mathcal{S}, i,j\in\mathcal{L}, \tag{18d}$$

$$\text{(1a)} - \text{(1d)},$$

where each $y^s_{ij} : \mathbb{R}^{|\mathcal{L}|} \times \mathbb{R}^{|\mathcal{L}|} \to \mathbb{R}$, and where π^1_s, $\boldsymbol{\pi}^2_s$ and $\boldsymbol{\pi}^3_s$ are the dual variables associated with first, second, and third constraints that define $\mathcal{G}$.

Since constraint (18b) is a sum of max function, by Definition 1, it can be written as

$$\pi^1_s + (\boldsymbol{\pi}^2_s)'\mathbf{d} + (\boldsymbol{\pi}^3_s)'\mathbf{z} \ge p_s \sum_{i\in\mathcal{A}} \left(d_i + \sum_{j\in\mathcal{L}\setminus i} y^s_{ij}(\mathbf{d},\mathbf{z}) - (r_i + \sum_{j\in\mathcal{L}\setminus i} y^s_{ji}(\mathbf{d},\mathbf{z}))\right), A \in P(\mathcal{L}), (\mathbf{d},\mathbf{z}) \in \bar{\mathcal{D}}_s, s\in\mathcal{S},$$

where $P(\mathcal{L})$ is the power set of $\mathcal{L}$. Note that we can apply the affine decision rule and obtain an affinely adjustable robust counterpart of problem (18). Finally, problem (18) can also be transformed into a MILP via standard techniques in robust optimization.

D Data, Concepts and Results of the Numerical Study

D.1 Input Parameters for the Numerical Study.

We conduct a case study using the earthquake that happened at Yushu County in Qinghai Province, PR China in 2010. This case represents an affected area by a network consisting of 13 locations and 15 links. For deterministic parameters (e.g., cost, capacity, donations, etc.), we refer to Ni et al. (2018) and show them in Table 6. As stated by **?**: "Since the physical process underlying an earthquake is very complex, we cannot express every detail of an earthquake by a single parameter. However, it would be convenient if we could find a single number that represents the overall physical size of an earthquake." Thus, we construct the event-wise ambiguity set based on the *Richter scale* ($\mathrm{M_L}$) that is a commonly used measure of the strength of earthquakes (**?**). Specifically, the *Richter scale* is divided into nine scales: felt slightly by some people less than $\mathrm{M_L}2.5$, often felt by people above $\mathrm{M_L}2.5$, can cause damage if above $\mathrm{M_L}5.0$. Therefore, we generate 5 scenarios with equal probabilities (i.e., $S = \{1, 2, 3, 4, 5\}$), in which each scenario corresponds to a scale above $\mathrm{M_L}5.0$. Intuitively, the greater the scale of the earthquake, the more demand for relief supplies there will be, and more areas will be affected, so we take this into account when generating demands-related parameters. Specifically, the underlying model is a joint distribution over $(\tilde{s}, \tilde{\mathbf{d}})$ with $\tilde{s} \sim \{1, \ldots, 5\}$ uniformly, and each $\tilde{d}_i \sim N(\bar{\mu}_i^{\tilde{s}}, \bar{\sigma}_i^{\tilde{s}}, 0, +\infty)$ for all $i \in \mathcal{L}$, where N is a truncated normal distribution, and where $\bar{\mu}_i^s = 25 + 25s$ and $\bar{\sigma}_i^s = 0.1\bar{\mu}_i^s$. We let $\tau_i = \xi_i \times \frac{\sum_{s \in S} \bar{\mu}_i^s}{|S|}$ for all $i \in \mathcal{L}$, where ξ_i is the proportion of the mean demand of location i. Unless specified otherwise, we set $\xi_i = 10\%$ for all $i \in \mathcal{L}$.

Similar to Ni et al. (2018), we first generate 50 samples from the joint distribution. We then let $\hat{p}_s$, $\hat{\boldsymbol{\mu}}^s$, $\hat{\boldsymbol{\nu}}^s$, $\hat{\boldsymbol{\sigma}}^s$,$\hat{\bar{\mathbf{d}}}^s$, $\hat{\underline{\mathbf{d}}}^s$ be, for each s, the empirical frequency of the event in the observed samples, the empirical conditional mean, conditional mean absolute deviation, conditional standard deviation, conditional minimum, and conditional maximum in the 50 observations. We also let $\hat{\boldsymbol{\mu}}$, $\hat{\boldsymbol{\nu}}$, $\hat{\bar{\mathbf{d}}}$, $\hat{\underline{\mathbf{d}}}$ be the empirical (unconditional) mean, mean absolute deviation, minimum, and maximum in the 50 observations. With these statistics in hand, we can construct the RSO model, the DRO model, and the SAA model as follows.

- For the RSO model, the ambiguity set $\mathcal{F}$ is defined using $(\hat{\boldsymbol{\mu}}^s, \hat{\boldsymbol{\nu}}^s, \hat{\bar{\mathbf{d}}}^s, \hat{\underline{\mathbf{d}}}^s)$.
- For the DRO model, the ambiguity set $\mathcal{F}$ can be regarded as a special case containing only one event, which is defined using $(\hat{\boldsymbol{\mu}}, \hat{\boldsymbol{\nu}}, \hat{\underline{\mathbf{d}}}, \hat{\bar{\mathbf{d}}})$.
- For the SAA model, we consider 100 samples drawn from $\tilde{s} \sim$ Mutlinoulli$(\hat{p}_1, \ldots, \hat{p}_5)$ and each $d_i \sim N(\hat{\mu}_i^{\tilde{s}}, \hat{\sigma}_i^{\tilde{s}}, 0, +\infty)$ for all $i \in L$, where Mutlinoulli is a multinoulli distribution.

After solving all the above models, we can obtain predisaster deployment decisions $(\boldsymbol{x}^*, \boldsymbol{r}^*)$ for each of these models. To evaluate the performance of such solutions, we generate 1,000 out-of-sample demands drawn from $\tilde{s} \sim$ Mutlinoulli$(1, \ldots, 5)$ and each $d_i \sim N(\bar{\mu}_i^{\tilde{s}}, \bar{\sigma}_i^{\tilde{s}}, 0, +\infty)$ for all $i \in L$. Moreover, to capture the possibility that the out-of-sample distribution may deviate from the underlying distribution, for each $s \in \mathcal{S}$, we add a perturbation Δ to the mean of the demand, i.e., $\bar{\boldsymbol{\mu}}_{out}^s = (1+\Delta)\bar{\boldsymbol{\mu}}^s$ and $\bar{\boldsymbol{\delta}}_{out}^s = 0.1\bar{\boldsymbol{\mu}}_{out}^s$, where Δ is enumerated from -0.1 to 0.1 with a step size of 0.05. To evaluate the impact of different donation levels, we consider R could be any value in the set {1700, 1750, . . . , 1900} and $B = \{850, 1000, \ldots, 1450\}$. For the bisection search, as it is known to terminate in $\log_2(1/\epsilon)$ iterations, we set the accuracy to $\epsilon = 10^{-6}$ in our experiments.

Table 6: Input Parameters

Number of locations	$\lvert\mathcal{L}\rvert = 13$												
Location index i	1	2	3	4	5	6	7	8	9	10	11	12	13
Fixed cost c_i	203	193	130	117	292	174	130	157	134	161	234	220	170
Facility capacity	$M = 800$												
Total supplies	$R = 1,800$												
Total Budget	$B = 1,000$												

D.2 Concepts of equity indices.

Here we list the definitions and computational methods of MSG, RMD, VAR, SPAD and Gini in detail, as follows:

- *Maximum Shortage Gap*(MSG): This is the difference between the most and least deprived parties in terms of shortage ($\max_{i\in\mathcal{L}} u_i - \min_{i\in\mathcal{L}} u_i$), which focuses on two extreme cases.
- *Relative Mean Deviation*(RMD): This is the absolute deviation from the average shortage ($\sum_{i\in\mathcal{L}} |u_i - \bar{u}|$, where $\bar{u} = \frac{\sum_{i\in\mathcal{L}} u_i}{|\mathcal{L}|}$). Compared with MSG which only considers two extremes, RMD considers other levels of shortages as well.
- *Variance*(VAR): This is the average squared deviation from the average shortage ($\frac{\sum_{i\in\mathcal{L}}(u_i-\bar{u})^2}{|\mathcal{L}|}$).
- *Sum of Pairwise Absolute Differences*(SPAD): This is the sum of absolute differences between the shortages of all pairs of demand locations ($\sum_{i,j\in\mathcal{L}} |u_i - u_j|$).
- *Gini coefficient*(Gini): This index measures the inequity among values of a frequency distribution ($\frac{\sum_{i,j\in\mathcal{L}}|u_i-u_j|}{2|\mathcal{L}|\sum_{i\in\mathcal{L}} u_i}$). The coefficient ranges from 0 to 1, with 0 representing perfect equity and 1 representing perfect inequity.

D.3 Performance of AARC

We demonstrate the performance of AARC on small instances of the Earthquake case study. For simplicity, we only consider one event in the event-wise ambiguity set. To generate parameters related to demand, we randomly sample according to the sampling range in Table 7. For other parameters, we consider the same setup as the Earthquake case study.

Table 8 reports statistics on the suboptimality gaps observed in 100 random instances with sizes $|\mathcal{L}| = \{5, 6, 7\}$. One remarks that the suboptimality gaps are quite small, namely with a 95%-VaR of less than 1% and a maximum gap of 8%. Furthermore, suboptimality gap does not seem to increase with the size of the instance.

In addition, we also compare in Table 9 the solution time of AARC, VE, and SAA (with 1,000 samples). We choose the termination criteria to be whether the optimality gap is less than 0.01%, or the CPU time exceeds 7,200 seconds. Obviously, the performance of the AARC is significantly faster to solve than VE and SAA. In particular, VE starts taking more than 2 hours to solve for $|\mathcal{L}| \geq 9$, while AARC still returns a solution in 3 seconds. One also notices that the solution time of SAA increases more rapidly than AARC when the number of locations increases.

Table 7: Parameter Setup Related to Demand at Each Location $i \in \mathcal{L}$

Parameter	Sampling range
Lower bound of demand $\underline{d}_i$	(0,50)
Upper bound of demand $\bar{d}_i$	$\underline{d}_i$+(0,100)
Mean of demand μ_i	($\underline{d}_i$,$\bar{d}_i$)
Mean absolute deviation of demand ν_i	(0,μ_i)
Threshold on demand τ_i	10% $\times$ μ_i

D.4 Comparative study of the effects of risk measures

We compare the effects of three choices of risk measures for $\boldsymbol{\rho}$ in model (5): SSM, CVaR Shortage Measure (CSM), and Expected Shortage Measure (ESM). First, we provide the definition of the two alternative measures.

Definition 5. *Given $\alpha \in (0, 1]$ and an uncertain supply shortage $\tilde{u} \in \mathscr{U}$, we define the CVaR shortage measure as the worst-case CVaR of $\tilde{u}$:*

$$\rho_{CSM}(\tilde{u}) := \mathcal{V}_{1-\alpha}(\tilde{u}) = \inf_{\eta\geq 0} \left\{ \eta + \frac{1}{\alpha} \sup_{\mathbb{P}\in\mathcal{F}} \mathbb{E}_{\mathbb{P}} \left[(\tilde{u} - \eta)^+ \right] \right\},\ \alpha \in (0, 1],$$

Table 8: Statistics of AARC Suboptimality

$\|\mathcal{L}\|$	Minimum	Median	VaR@95%	VaR@99%	Maximum
5	0.00%	0.00%	0.88%	4.14%	6.34%
6	0.00%	0.00%	0.51%	2.78%	7.98%
7	0.00%	0.00%	0.64%	3.75%	7.25%

Table 9: Average CPU Time (in sec)

$\|\mathcal{L}\|$	AARC	VE	SAA
5	< 1	8	91
6	< 1	46	182
7	< 1	514	297
8	1	5,349	497
9	1	$> 7,200$	629
10	2	$> 7,200$	801
11	2	$> 7,200$	998
12	3	$> 7,200$	1801
13	3	$> 7,200$	3154

In case that $\alpha = 1$, we retrieve the (worst-case) expected shortage measure:

$$\rho_{ESM}(\tilde{u}) := \mathcal{V}_0(\tilde{u}) = \sup_{\mathbb{P}\in\mathcal{F}} \mathbb{E}_{\mathbb{P}}\left[\tilde{u}\right].$$

Next, we randomly generate 10 instances and investigate their average performances. Specifically, for each instance, we sample from the underlying distribution, calibrate the ambiguity set, solve the lexicographic problems with different shortage measures, and evaluate the shortage performance of the worst-off participants and all participants under several considered measures. In Table 10, we summarize the frequency of shortage below the threshold, the CVaR@90% of shortage, and the expected shortage under SSM, CSM, and ESM, where we set $\alpha = 0.1$ for the CSM.

Table 10: Out-of-Sample Shortage Performance under SSM, CSM, and ESM

		Shortage performance of the worst-off participants			Average shortage performance of all participants		
Δ	Shortage measure	Frequency of shortage below the threshold (%)	CVaR@90%	Average shortage	Frequency of shortage below the threshold (%)	CVaR@90%	Average shortage
-0.10	SSM	**99.79**	3.12	0.42	**99.85**	2.75	0.33
	CSM	92.55	**1.92**	1.14	93.44	**1.56**	0.82
	ESM	93.64	9.66	**0.19**	94.47	7.64	**0.16**
-0.05	SSM	**94.86**	11.62	1.53	**95.51**	10.47	1.33
	CSM	88.50	**9.50**	2.60	89.76	**8.91**	2.03
	ESM	89.96	18.33	**1.19**	90.97	15.11	**1.09**
0.00	SSM	**86.24**	21.01	3.27	**87.07**	19.43	2.82
	CSM	80.56	**17.37**	4.51	81.28	**16.68**	3.64
	ESM	81.32	28.27	**2.69**	82.29	24.28	**2.51**
0.05	SSM	**79.21**	28.58	5.05	**80.72**	27.00	4.57
	CSM	72.17	**25.60**	6.68	73.26	**24.91**	5.53
	ESM	73.07	36.43	**4.54**	74.44	32.60	**4.26**
0.10	SSM	**75.73**	37.23	6.90	**76.90**	34.85	6.13
	CSM	68.01	**33.16**	9.17	69.11	**32.22**	7.41
	ESM	69.16	45.75	**6.17**	70.08	41.07	**5.77**

We notice several effects of different shortage measures. (1) The SSM solutions bring higher probability of being below the threshold than the two other alternatives. The reason is the SSM is related to the satisficing measure, which implicitly maximizes the success probability. (2) As the CSM focuses on reducing the CVaR of shortages, it sacrifices the performance in terms of average shortage and of probability of getting shortages below the thresholds. (3) The ESM outperforms the other two measures in terms of the average shortage. We finally note that SSM ranks second with respect to CVaR and average shortages in all experiments. Therefore, while HOs can decide of which measure to use based upon their subjective preferences, SSM does seem to come out as a legitimate choice.

D.5 Impact of Tolerance Threshold

We investigate the impact of different tolerance thresholds (τ) on the performance of PRNDP-E. We consider two settings. In the first setting, we uniformly vary the threshold values across all locations using $\tau_i := \xi_i(\sum_{s\in S}\bar{\mu}_i^s/|S|)$ where ξ_i is constant among regions and in the range $\{5\%, 10\%, 15\%, 20\%, 25\%\}$. The second setting considers, ξ_i to vary arbitrarily from location to location. Namely, inspired by the idea that structural failures are usually assumed to follow a lognormal distribution in seismic fragility analysis (see **?**), we set the each ξ_i to be randomly drawn from a lognormal distribution $LogN(10\%, \sigma^2)$ with a mean of 10% and a variance σ^2 that varies from 1% to 5%.

To capture the effects of different thresholds on equity, we slightly adjust the indices in Appendix D.2 by evaluating the excess shortages (i.e., $u_i^* = [u_i - \tau_i]^+$). As a result, we have a new version for these metrics. For instance, MSG* refers to the difference between the maximal and minimal shortages above the threshold (i.e., $\text{MSG}^* = \max_{i\in\mathcal{L}} u_i^* - \min_{i\in\mathcal{L}} u_i^*$).

Table 11, we summarize the out-of-sample performance of equity and shortage measures under the first setting. We note an improvement in the performance of equity and shortage metrics as ξ_i increases (and thereby τ_i). As discussed in Section 3.3, the SSM-based model really cares about excess shortages beyond the threshold. When the threshold increases, at some point, all locations become satisfied because they are all below the threshold, so the values of indicators that measure inequity and shortage are decreasing. It also indicates that the threshold is effectively controlling the range of shortages that the equity indices should be sensitive to.

Table 11: Out-of-Sample Performance of PRNDP-E with Identical Threshold Levels

		Equity Metrics				Shortage Metrics	
$\xi_i(\%)$	Statistic	MSG*	RMD*	VAR*	SPAD*	Worst*	Sum*
5	Mean	1.36	6.75	4.01	88.50	2.18	19.77
	Var@95%	11.09	57.72	21.58	750.31	13.96	129.04
	Var@99%	21.39	118.85	96.27	1545.04	22.44	173.45
	STD	4.32	23.01	239.63	301.10	5.10	44.38
10	Mean	1.26	6.03	2.90	80.76	1.50	9.39
	Var@95%	9.77	47.68	16.77	622.63	10.04	68.42
	Var@99%	18.01	86.30	53.20	1145.23	18.03	108.45
	STD	3.75	18.77	158.37	254.52	3.92	24.84
15	Mean	1.04	4.32	1.83	58.11	1.05	3.65
	Var@95%	8.08	34.60	9.71	462.53	8.08	28.85
	Var@99%	17.36	71.25	41.60	926.29	17.36	59.09
	STD	3.35	13.70	109.00	183.79	3.36	12.17
20	Mean	0.86	2.24	1.10	29.44	0.86	1.38
	Var@95%	7.18	17.49	4.89	227.33	7.18	9.70
	Var@99%	17.07	42.08	26.32	551.80	17.07	27.94
	STD	3.25	9.01	83.47	119.40	3.25	5.83
25	Mean	0.71	1.74	0.79	23.02	0.71	1.05
	Var@95%	4.62	15.39	2.90	200.13	4.62	9.02
	Var@99%	16.41	37.95	22.24	493.40	16.41	21.32
	STD	2.94	7.10	52.00	94.31	2.94	4.39

In Table 12, we show the results in the second setting as σ is varied. We observe that the performance of equity and shortage metrics is very close when inter-regional diversity is considered. As we have discussed in Section 3.4, the PRNDP-E approach allocates the resources based on the requirements of different participants. This means that, for different threshold levels, the PRNDP-E can provide treatment accordingly to ensure (vertical) equity between the different locations.

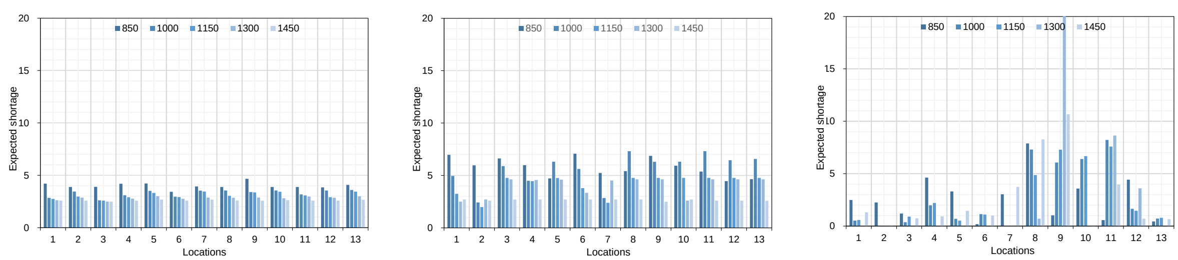
D.6 Impact of Budget

We evaluate the impact of different budget levels (B) on shortages. In this part of the computational study, we analyze the results on a base test case with $\Delta = 0$ under the following values of the varying total budget: $B \in \{850, 1000, 1150, 1300, 1450\}$. Figure 7 illustrates the impacts of budget on the out-of-sample expected shortage at each location. Similar to the results regarding the impact of donation level in Section 5.4, we notice similar advantages of the PRNDP-E

Table 12: Out-of-Sample Performance of PRNDP-E with Different Threshold Levels

		Equity Metrics				Shortage Metrics	
$\xi_i(\%)$	Statistic	MSG*	RMD*	VAR*	SPAD*	Worst*	Sum*
$logN(10,1)$	Mean	1.30	6.14	2.81	82.82	1.50	9.13
	Var@95%	10.94	58.19	23.94	799.36	11.88	72.97
	Var@99%	19.37	98.85	67.37	1339.96	19.43	114.11
	STD	4.38	21.61	188.50	295.75	4.52	26.71
$logN(10,2)$	Mean	1.24	5.73	2.59	77.07	1.44	8.57
	Var@95%	10.07	47.96	17.12	671.35	10.33	67.90
	Var@99%	18.48	90.07	65.10	1222.15	18.53	111.52
	STD	3.95	19.25	162.85	258.97	4.12	25.39
$logN(10,3)$	Mean	1.27	5.26	2.54	70.29	1.45	8.13
	Var@95%	11.59	54.95	21.98	733.48	12.67	53.25
	Var@99%	20.01	90.77	62.79	1195.92	21.74	95.57
	STD	4.58	18.54	169.37	253.25	4.62	20.90
$logN(10,4)$	Mean	1.34	5.81	2.85	77.77	1.54	8.32
	Var@95%	10.93	50.63	18.26	686.29	10.94	61.33
	Var@99%	18.83	87.51	60.09	1167.31	18.29	105.51
	STD	4.18	18.21	147.23	249.22	4.22	23.49
$logN(10,5)$	Mean	1.27	5.43	2.64	73.04	1.39	7.79
	Var@95%	10.33	43.51	15.68	581.85	9.62	57.40
	Var@99%	19.49	81.36	46.19	1101.92	19.44	114.86
	STD	3.92	152.91	129.79	226.76	3.82	22.77

compared with the other benchmarks in terms of equity and shortage.



(a) PRNDP-E (b) PRNDP-NE-E (c) PRNDP-NE

Figure 7: Expected Shortage (Measured Out-of-Sample) at Each Location for Different Budgets

D.7 Additional Details about Deployment Plans and Shortages

Table 13 lists the predisaster deployment plans under PRNDP-E, PRNDP-NE-E and PRNDP-NE. Table 14 displays the proportion of postdisaster expected shortage at each location.

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Table 13: Predisaster Deployment Decisions under PRNDP-E, PRNDP-NE-E and PRNDP-NE

Location	PRNDP-E	PRNDP-NE-E	PRNDP-NE
2	424	685	685
3	269	153	153
4	155	266	266
6	498	192	192
7	–	377	377
9	158	127	127
11	295	–	–

Note. – indicates that there are no facilities opened.

Table 14: Proportion of Expected Shortage (Measured Out-of-Sample) at Each Location

Location	PRNDP-E	PRNDP-NE-E	PRNDP-NE
1	6.63%	6.80%	1.45%
2	8.05%	3.33%	0.00%
3	6.08%	8.09%	1.05%
4	7.22%	6.16%	5.63%
5	8.21%	8.67%	1.96%
6	6.90%	7.72%	3.23%
7	8.25%	3.91%	0.06%
8	8.30%	10.05%	20.85%
9	7.93%	8.66%	17.32%
10	8.30%	8.67%	18.26%
11	7.42%	10.05%	23.47%
12	8.30%	8.86%	4.69%
13	8.39%	9.03%	2.04%

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