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Foreign aid, inequality and fiscal policy in developing countries

par
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Résumé

Cette thèse est constituée de trois (3) essais qui analysent les effets macroéconomiques de l'aide étrangère, et la cyclicité de la politique fiscale dans les pays en développement.

Le premier article intitulé Foreign Aid, Human Capital, and Welfare, analyse les effets de l'aide étrangère sur la croissance économique et le bien-être des pays récipiendaires. Pour ce faire, nous développons un modèle néoclassique dans lequel les agents peuvent accumuler du capital humain à travers l'éducation. La totalité de l'aide au développement peut être non liée ou allouée dans le but d'accroitre le stock de capital ou encore pour financer les dépenses publiques en éducation. Dans ces derniers cas, on dit alors de l'aide qu'elle est liée. D'après nos résultats, utiliser toute l'aide pour financer les dépenses en éducation procure le niveau de bien-être le plus élevé, comparativement aux deux scénarios alternatifs consistant à ne pas lier l'aide ou à l'octroyer dans le but d'augmenter le stock de capital. D'après nos résultats l'allocation optimale requiert que la majeure partie de l'aide soit liée au financement des dépenses en éducation. Enfin, les résultats sur la transition dynamique indiquent que les flux d'aide au développement qui sont entièrement ou partiellement alloués à l'éducation, occasionnent des pertes de bien-être dans le court-terme.

Le deuxième article intitulé Does Foreign Aid Raise Inequality?, examine les liens qui pourraient exister entre l'aide étrangère et les inégalités dans les pays en développement. Nous commençons par fournir une évidence empirique selon laquelle les augmentations de flux d'aide seraient associées avec des hausses subséquentes d'inégalités. Dans la seconde partie de l'article, nous développons un modèle avec agents hétérogènes qui permet également d'analyser la relation entre la croissance économique et les inégalités. Les résultats indiquent que les transitions dynamiques des inégalités de revenu et de richesse, ainsi que la relation entre la croissance économique et les inégalités, dépendent de la manière dont l'aide est allouée.

Le troisième article, intitulé Can Progressive Taxation Explain Fiscal Policy in Developing Countries?, propose une nouvelle explication du constat empirique stipulant que la politique fiscale est procyclique dans les pays en développement, alors qu'elle est acyclique ou contracyclique dans les pays développés. En nous basant sur un échantillon de 36 pays développés et en voie de développement, nous confirmons dans un premier temps les faits stylisés concernant les propriétés cycliques de la politique fiscale. Contrairement à la littérature existante, nous fournissons des faits nouveaux sur les propriétés cycliques des principales catégories de dépenses publiques. Nous montrons que la principale différence entre les pays développés et en développement se situe au niveau du comportement cyclique des transferts sociaux (tels que les prestations de sécurité sociale) qui sont contracycliques dans le premier groupe de pays, alors qu'ils sont procycliques dans le second. Les autres catégories de dépenses publiques sont procycliques dans les deux groupes de pays. Dans la seconde partie de l'article, nous développons un modèle théorique comportant des mécanismes de stabilisateurs automatiques qui est en mesure d'expliquer ces observations empiriques.

Mots clés: Aide étrangère, croissance économique, capital humain, bien-être, inégalités, politique fiscale procyclique, taxation progressive, transferts sociaux.

Méthodes de recherche: Recherche théorique basée sur le développement de modèles macroéconomiques d'équilibre général et recherche quantitative basée sur des simulations numériques.

Abstract

This thesis is composed of three (3) essays which analyze the macroeconomic effects of foreign aid and the cyclical behavior of fiscal policy in developing countries.

The first paper entitled Foreign Aid, Human Capital, and Welfare, analyzes the question of whether foreign aid should be tied or not. We study this question in the context of a dynamic growth model of a developing economy in which agents can accumulate human capital through education. We compare the growth and welfare implications on the recipient economy of three polar scenarios in which foreign aid is either (i) completely untied, (ii) exclusively tied to public investment in infrastructure, or (iii) exclusively tied to public spending on education. Our results indicate that, under plausible parameter values, tying aid to education is more beneficial from a long-run welfare perspective than the two alternative scenarios. We also compute the optimal allocation of foreign aid and find that the largest fraction of aid flows ought to be tied to public spending on education. Finally, we study the transitional dynamics of the recipient economy following an aid inflow and find that aid programs that are tied (entirely or partially) to public spending on education generally entail some welfare losses in the short run.

The second paper, entitled Does Foreign Aid Raise Inequality?, studies the relationship between foreign aid and inequality. I start by providing new and robust evidence that increases in aid flows are associated with subsequent increases in income inequality in the recipient countries. I then attempt to rationalize this empirical observation by proposing an heterogeneous-agent growth model of a developing economy. The model implies that untied aid reduces income inequality whereas tied aid raises it with a delay regardless of whether aid programs are tied to physical or human capital. To the extent that aid flows to developing countries are mostly tied, the model's predictions are therefore consistent with the empirical evidence. The results indicate that foreign aid improves average welfare but increases its dispersion across private agents.

The third paper, entitled Can Progressive Taxation Explain Fiscal Policy in Developing Countries?, proposes a novel explanation for the empirical puzzling fact that fiscal policy is procyclical in developing countries, while it is acyclical or countercyclical in developed countries. Based on a sample of 36 developing and developed countries, I first revisit the stylized facts about the cyclical behavior of fiscal policy. While existing studies have focused almost exclusively on the cyclical properties of total public spending, I provide some new insights on the cyclicality of the main sub-categories of public spending. I show that only social transfers (such as benefits from social security and assistance) exhibit a different cyclical behavior across developed and developing countries, being countercyclical in the former group of countries and procyclical in the latter. The remaining sub-categories are procyclical in both groups. In the second part of the paper, I develop a theoretical model with automatic stabilizer mechanisms that successfully accounts for the empirical evidence.

Keywords: Foreign aid, growth, human capital, welfare, inequality, procyclical fiscal policy, progressive taxation, social transfers.

Research methods: Theoretical research focused on general equilibrium models and quantitative research based on numerical simulations.

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Introduction générale

Cette thèse est composée de trois essais qui examinent différents thèmes en économie du développement sur le plan théorique et empirique. De façon spécifique, nous nous intéressons aux effets de l'aide étrangère sur la croissance économique, le bien-être ou les inégalités et examinons les propriétés cycliques de la politique fiscale dans les pays en développement. Dans chacun des essais, nous développons un modèle théorique pour expliquer des faits empiriques précis que l'on observe dans les pays en développement.

Le premier article de notre thèse a été co-écrit avec Hafedh Bouakez et analyse la question de l'aide au développement qui est passée de 4.8 milliards en 1960, à 141.3 milliards de dollars en 2010. Selon la Banque Mondiale, ces transferts représentent 8.7% du Produit Intérieur Brut (PIB) des pays les moins avancés et environ 15% de la formation brute du capital dans les pays à faibles revenus. Cet article est motivé par le fait qu'il existe une grande littérature empirique traitant des effets de l'aide étrangère sur la croissance économique, alors que très peu de travaux théoriques ont analysé la question de savoir si l'aide devrait être liée ou non. Dans une série d'articles, Chatterjee et Turnovsky (2004, 2007) étudient la question en développant un modèle de croissance endogène, dans lequel l'aide au développement est non liée ou bien liée à des investissements publics en capital physique. Selon leurs résultats, lier l'aide au capital physique est la meilleure alternative d'un point de vue croissance et bien-être. Cependant, cette série de papiers ne tient pas compte du capital humain, ni du fait que l'aide étrangère pourrait être liée aux dépenses publiques en éducation. Cet oubli, est peut-être dû au fait que la proportion des flux d'aide alloués à l'éducation est historiquement faible, se situant en moyenne autour de 10%, ce qui est quand même surprenant étant donné la reconnaissance universelle du capital humain en général et de l'éducation en particulier comme étant des moteurs de croissance économique.

Nous avons donc développé un modèle néoclassique de croissance, dans lequel l'aide étrangère peut être non liée, ou allouée de manière à accroitre les dépenses publiques en éducation, ou les investissements en capital physique. Puisque le gouvernement fourni deux types de biens, il est possible de calculer l'allocation optimale d'aide étrangère, à savoir la composition de l'aide qui maximise le bien-être social. Nos résultats indiquent que lier l'aide aux dépenses publiques en éducation procure le niveau de bien-être le plus élevé, comparativement aux deux autres cas extrêmes où l'aide est complètement non liée ou exclusivement liée au capital physique. Lorsqu'on calcule l'allocation optimale d'aide, les résultats montrent que le bien-être social est maximisé lorsque la plus grande proportion, à savoir 60% des flux d'aide sont alloués aux dépenses en éducation et la fraction restante est liée au capital physique.

Le second article analyse les effets de l'aide étrangère sur les inégalités de richesse et de revenus. Cette étude est motivée par les deux faits empiriques suivants au sujet des pays en voie de développement : (i) l'aide étrangère représente une part non négligeable des sources de financement des pays en voie de développement; (ii) l'indice de Gini est presque deux fois plus élevé dans les pays à faibles ou moyens revenus que dans les pays membres de l'Organisation de Coopération et de Développement Économiques (OCDE).

Le principal objectif de cet article est de comprendre à la fois empiriquement et théoriquement, les liens qui pourraient exister entre l'aide étrangère et les niveaux d'inégalités. Pour ce faire, nous fournissons dans un premier temps, une évidence selon laquelle, les flux d'aide étrangère reçus quelques années auparavant, constituent un facteur déterminant des niveaux courants d'inégalités de revenus. Nos résultats empiriques sont robustes et ne dépendent pas de la spécification empirique ni des différentes variables de contrôle. Notre approche est différente de celle des précédents articles empiriques qui mettaient plutôt l'emphase sur la relation contemporaine entre l'aide étrangère et les inégalités, et trouvaient des effets non significatifs. En fait, nos résultats montrent que lorsqu'on inclue des variables d'aide retardées, dans la spécification empirique, elles se révèlent être des déterminants du niveau contemporain d'inégalité.

À la lumière de ces résultats, nous avons donc élaboré un modèle théorique dans le but de comprendre le mécanisme par lequel l'aide étrangère pourrait avoir un effet sur les inégalités. Nous considérons une économie en développement peuplée par des individus qui sont hétérogènes en ce sens qu'ils diffèrent dans leurs dotations initiales en capital physique, en dette étrangère et en capital humain, lequel est accumulé à travers un processus d'apprentissage par la pratique. Cette économie reçoit des flux d'aide qui peuvent être liés ou non. Par aide liée, nous faisons référence à une aide étrangère qui est octroyée dans le but de financer des projets d'investissement spécifiques. C'est ainsi que dans le modèle, l'aide étrangère peut être complétement non liée ou liée soit à des investissements publics en capital physique, ou à des dépenses publiques en capital humain. Cette distinction est très importante, parce que les effets de l'aide étrangère sur les inégalités de revenu et de richesse dépendent de façon cruciale des schémas d'allocation des flux d'aide. En effet, nos résultats théoriques montrent que l'aide étrangère réduit les inégalités lorsqu'elle est non liée, mais la même aide augmente les inégalités au cours des périodes suivantes, lorsqu'elle est complètement liée. L'intuition économique est la suivante: l'aide non liée équivaut à un simple transfert à l'économie, elle engendre donc un effet de richesse qui pousse les agents économiques à allouer plus de temps aux loisirs. Mais cet effet de richesse n'affecte pas de la même manière les pauvres et les riches. Cette différence vient du fait que comparativement aux pauvres, les riches valorisent moins une unité supplémentaire de richesse. Ils choisissent donc d'allouer plus de temps aux loisirs que les pauvres, ce qui a pour effet de réduire les inégalités. L'aide liée par contre, augmente la productivité marginale des facteurs de production, ce qui incite donc les riches à accumuler relativement plus de capital physique et humain que les pauvres, augmentant de ce fait les inégalités. Le capital étant une variable prédéterminée, les effets de l'aide liée sont plus élevés au cours des périodes suivant le choc d'aide. Le modèle théorique a également d'importantes implications concernant la relation entre croissance et inégalités. Selon nos résultats, cette relation dépend de la manière dont l'aide est allouée. En effet, lorsque l'aide est non liée, il existe une corrélation négative entre la croissance et les inégalités, mais cette relation devient positive ou est représentée par une courbe en U inversée, lorsque l'aide étrangère est complétement liée.

Le troisième article de notre thèse analyse les propriétés cycliques de la politique fiscale dans les pays en développement. Il existe une évidence empirique selon laquelle, les dépenses publiques sont procycliques dans les pays en développement alors qu'elles sont plutôt acycliques ou contracycliques dans les pays développés (Ilzetzki (2011), Alesina et co-auteurs (2008), Talvi and Vegh (2005), Kaminsky et co-auteurs (2004), Lane (2003), Gavin and Perotti (1997)).

En nous basant sur un échantillon de 36 pays développés et en voie de développement, nous confirmons dans un premier temps les faits stylisés concernant les propriétés cycliques de la politique fiscale. Contrairement à la littérature existante, les dépenses publiques totales ont été désagrégées en trois catégories, à savoir, la consommation publique, l'investissement public et les transferts sociaux. Cette distinction est importante puisqu'elle nous a permis de présenter des faits nouveaux sur les propriétés cycliques des principales catégories de dépenses publiques. Nous montrons que la principale différence entre les pays développés et en développement se situe au niveau du comportement cyclique des transferts sociaux (tels que les prestations de sécurité sociale) qui sont contracycliques dans le premier groupe de pays, alors qu'ils sont procycliques dans le second. Les autres catégories de dépenses publiques sont procycliques dans les deux groupes de pays.

Ce résultat empirique suggère que toute tentative d'expliquer la procyclicité de la politique fiscale dans les pays en développement, devrait comporter une théorie au sein de laquelle les transferts sociaux joueront un rôle central, mais aussi un rôle qui dépendra du niveau de développement de l'économie. Dans la seconde partie de l'article, nous développons un modèle théorique qui est en mesure d'expliquer ces observations empiriques. Notre explication est élaborée sur les différences de progressivité des systèmes de taxation que l'on peut observer au sein des pays développés et ceux en voie de développement. En effet, il est reconnu que les systèmes de taxation dans les pays en développement sont moins progressifs que ceux des pays développés (Schmitt (2003)). Les gouvernements des pays pauvres collectent beaucoup moins d'impôts sur les revenus et obtiennent la grande majorité de leurs revenus des impôts sur les ventes de biens et services, lesquels sont très régressifs. Rao et Weller (2008) montrent qu'au cours de la période entre 1981 et 2002, la réduction des taux marginaux d'imposition sur les plus riches a été plus importante que celle des taux moyens d'imposition, suggérant ainsi que le système de taxation était devenu moins progressif au fil du temps.

Nous modélisons une économie peuplée par un continuum de ménages identiques composés de membres de familles qui vivent éternellement. À l'instar de Diamond (1982a, b) et Mortensen et Pissarides (1994), le processus d'appariement et la recherche d'emploi sont sujets à des frictions sur les marchés de travail. Le modèle est utilisé pour analyser la politique fiscale optimale après une augmentation du prix de l'énergie, lorsque l'on considère différents degrés de progressivité du système de taxation. Nos résultats montrent que dans les pays développés et en voie de développement, les réponses à un choc négatif du prix de l'énergie impliquent des ajustements à la baisse de la consommation gouvernementale et de l'investissement public, ce qui réduit le stock de capital public. Les propriétés cycliques des transferts sociaux sont consistantes avec l'analyse empirique, puisqu'ils augmentent dans l'économie développée caractérisée par un système d'imposition progressif, mais diminuent dans le pays en développement. L'intuition économique de ces résultats repose sur les effets stabilisateurs que le système de taxation progressif exerce sur plusieurs variables telles que la consommation privée, les heures travaillées, l'investissement privé et l'accumulation du capital. Lorsque le système d'imposition est uniforme, les agents privés répondent au choc négatif en travaillant moins et en investissant moins dans le capital physique et ces ajustements à la baisse sont beaucoup plus importants que ceux qui sont enregistrés lorsque le système d'imposition est progressif. Cela suggère donc que les pertes de revenus sont beaucoup plus importantes dans le pays en développement, induisant le gouvernement à réduire toutes les composantes de ses dépenses publiques dans le but d'équilibrer son budget. Cependant, dans le cas du pays développé, le gouvernement décide de façon optimale de renforcer les effets stabilisateurs du système d'imposition progressif en réduisant la consommation et l'investissement publics, tout en octroyant plus de transferts sociaux aux agents économiques privés.

Nos résultats indiquent une corrélation positive entre le PIB et la consommation et l'investissement publics, qui augmente avec le degré de progressivité du système d'imposition, alors que la corrélation entre le PIB et les transferts sociaux diminue et devient négative lorsque le degré de progressivité dépasse le seuil de 0.1. Les transferts sociaux sont moins procycliques et deviennent contracycliques au fur et à mesure que le degré de progressivité du système d'imposition augmente. Ces résultats suggèrent que lorsqu'on considère des valeurs plausibles pour les différents paramètres, la faiblesse des mécanismes de stabilisateurs automatiques peut être une importante variable explicative de la procyclicité de la politique fiscale.

1 Essay 1 [Foreign Aid, Human Capital and Welfare]

Abstract

Should foreign aid be tied or untied? We study this question in the context of a dynamic growth model of a developing economy in which agents can accumulate human capital through education. We compare the growth and welfare implications on the recipient economy of three polar scenarios in which foreign aid is either (*i*) completely untied, (*ii*) exclusively tied to public investment in infrastructure, or (*iii*) exclusively tied to public spending on education. Our results indicate that, under plausible parameter values, tying aid to education is more beneficial from a long-run welfare perspective than the two alternative scenarios. We also compute the optimal allocation of foreign aid and find that the largest fraction of aid flows ought to be tied to public spending on education. Finally, we study the transitional dynamics of the recipient economy following an aid inflow and find that aid programs that are tied (entirely or partially) to public spending on education generally entail some welfare losses in the short run.

1.1 Introduction

Foreign aid to developing countries has reached its highest level in several decades. Official Development Assistance (ODA) increased from US$4.8 billions in 1960 to US$141.3 billions in 2010,[1] representing, on average, around 15% of the Gross Domestic Product of the Least Developed Countries (LDCs). Foreign aid can be completely untied or tied either by source, when the recipient country is obliged to spend the received transfer in the donor country, or by end-use, when aid is linked to a specific commodity or a specific investment project in the recipient country. While a large literature has attempted to evaluate the effectiveness of foreign aid mainly from an empirical standpoint,[2] little theoretical research has been done to address the question of whether aid flows should be tied or untied.

The first attempt to address this question was made by Chatterjee, Sakoulis, and Turnovsky (2003), who develop a dynamic optimizing growth model in which aid flows can take the form of pure transfers or be tied to public investment in physical capital (or infrastructure). They find that tying aid to public capital is more

[1] These numbers are extracted from OECD Stat (2011) and represent the gross disbursements of the Development Assistance Committee (DAC) countries.

[2] Example of empirical studies include those by Levy (1988), Burnside and Dollar (2000), Hansen and Tarp (2000, 2001), Collier and Dollar (2001, 2002), and Dalgaard, Hansen, and Tarp (2004).

beneficial from a growth and a welfare perspective. Chatterjee and Turnovsky (2004, 2007) extend this framework by allowing for a more general production technology and for endogenous labor supply, and show that these extensions can alter the relative merits of tied and untied aid. For example, greater substitutability between private and public capital in production and more elastic labor supply tend to mitigate the long-run welfare gain of tied aid, potentially rendering it less preferable to untied aid. Under plausible parameter values, however, tied aid remains generally more advantageous than untied aid. Agénor and Yilmaz (2008) develop an endogenous growth model in which the government provides two categories of public services, infrastructure and health, and aid can be tied to either one of these categories or completely untied. They find that foreign aid will achieve the highest welfare and the fastest growth if it is tied to public spending on health.

Perhaps surprisingly, however, none of these studies account for human capital accumulation, nor for the possibility that foreign aid be tied to public spending on education. Conversely, while there is an extensive literature on the growth implications of human capital in general and education in particular (e.g., Lucas 1988; Mankiw, Romer, and Weil 1992, Glomm and Ravikumar 1992, 1998, 2003; Blankenau and Simpson 2004), this literature generally abstracts from foreign aid. This neglect might be due to that fact that the fraction of total aid flows intended for education has historically been low, averaging less than 11% between 1971 and 2010. The latter observation is certainly perplexing given the widespread recognition of the importance of human capital as a growth engine, which is perhaps best exemplified by the international community's commitment in 2000 to provide universal primary education by 2015.[3]

In this paper, we investigate whether and to what extent tying foreign aid (or a fraction of it) to public spending on education improve social welfare in the recipient economy. For this purpose, we develop an optimizing growth model of a small open economy in which foreign aid can be completely untied or tied either to public investment in infrastructure or to public spending on education.[4] In the absence of foreign aid, public investment in infrastructure and spending on education are chosen so as to maintain a constant ratio of each of these expenditures to GDP.[5] The government runs a balanced budget in every period so when foreign aid is not tied to public investment or to education, it is redistributed as a transfer to agents. Education increases human capital, which is an input in the production function of the private sector, just as labor and public capital. The allocation of time between

[3] See the Millennium Development Goals at http://www.un.org/millenniumgoals/.

[4] Thus, as in Chatterjee et al. (2003) and Chatterjee and Turnovsky (2004, 2007), our definition of tied aid is based on the "end-use" criterion.

[5] If the government were able to allocate resources optimally, then obviously untied aid would always be optimal.

work, education and leisure is chosen optimally by agents. Human capital accumulation depends on government spending on education and on the stock of public capital. Hence, public spending on education affects welfare both directly through its effect on leisure time and indirectly through its effect on the marginal productivity of private inputs.

The model is used to analyze the welfare and growth implications of foreign aid under different allocations schemes, both in the short and the long run. In addition to analyzing the three polar scenarios in which aid is either completely untied or is tied exclusively to public infrastructure or to education, we compute the optimal allocation of aid flows, i.e., the share that should be allocated to each category of spending so as to maximize social welfare in the recipient economy. Due to the complexity of the model, the analysis is carried out via numerical simulations.

Our results indicate that under plausible parameter values, tying aid to public spending on education achieves the largest welfare gain compared with the two alternative scenarios in which aid is completely untied or tied exclusively to public investment in infrastructure. Unlike untied aid, which operates mainly through a wealth effect, both forms of tied aid affect the productive capacity of the recipient economy, albeit through different mechanisms. When aid is tied to public investment in infrastructure, it acts primarily as an externality in the production function, raising the marginal productivity of all private inputs. Compared with the initial (zero-aid) equilibrium, the recipient economy grows at a much faster rate in the long run and agents enjoy more consumption but less leisure. When aid is tied to education, on the other hand, it acts as an externality in the human capital accumulation process, improving the efficiency with which education increases human capital. In the long run, the stock of human capital rises, which in turn increases the marginal productivity of private capital and labor. This increase, however, is not as large as in the case where aid is tied to public infrastructure, which results in a relatively smaller increase in the growth rate. But given that leisure time barely falls compared with the initial equilibrium and that consumption rises most when aid is tied to public spending on education, this form of aid proves to be more beneficial from a welfare standpoint than the two alternative scenarios.

Notwithstanding the latter result, we find that the optimal allocation of foreign aid does not involve tying it exclusively to any of the spending categories. Instead, the welfare-maximizing scenario requires that 60 percent of aid flows be devoted to public spending on education, and the remaining 40 percent to public investment in infrastructure.

We also study the transitional dynamics of the recipient economy following the received aid under each of the three polar scenarios described above as well as under the optimal allocation scheme. We find that the time paths of key macroeconomic variables differ markedly across the different cases. For example, while hours worked increase non-monotonically and converge to their new-equilibrium level from above when aid is tied to public investment in infrastructure, they fall initially and converge to the new equilibrium from below under an aid program that is tied to education. Inversely, the consumption-output ratio remains permanently below its initial level in the former case, but temporarily exceeds it in the latter. Importantly, our results show that when foreign aid is untied or entirely tied to public investment in infrastructure, it improves welfare both in the short and the long run. In contrast, when a sufficiently large fraction of aid is tied to public spending on education, an intertemporal trade-off in welfare arises, as the recipient economy enjoys a welfare gain in the long run but incurs a welfare loss in the short run. This suggests that aid programs that are tied to education become beneficial only gradually.

In the last part of the paper, we conduct a sensitivity analysis by considering alternative values for the key parameters of the model, namely those characterizing the human capital accumulation process and the production technology, the weight of leisure in utility, and the average tax rate in the recipient economy. We find our main results regarding the relative merits of the different forms of aid and the optimal allocation of aid flows to be generally robust.

The rest of the paper is organized as follows. Section 2 presents the model economy. In Section 3, we derive the balanced growth path. Section 4 discusses the effects of a permanent aid shock, both in the long run and along the transitional path towards the new equilibrium. Section 5 performs a sensitivity analysis. Section 6 concludes.

1.2 The Economy

We consider a small open economy populated by an infinitely lived representative agent who produces a single traded good. The good can be used either for consumption or for investment. The agent has a unit of time that can be allocated between labor, education and leisure. Time spent on education raises the stock of human capital, which is a productive input, just as labor and physical capital. The government undertakes two types of discretionary expenditures, namely, investment in public capital and spending on education, which are financed by income taxation and foreign aid flows.

1.2.1 Private sector

The infinitely-lived agent maximizes a lifetime utility function given by

$$U_0 = \int_0^\infty e^{-\rho t} \mathcal{U}(C, L, E) dt, \tag{1}$$

where C is consumption, L and E are the fractions of time devoted to work and education, respectively, and $\rho \in (0,1)$ is the subjective discount rate. The instantaneous utility function, $\mathcal{U}(.)$ is assumed to satisfy the Inada conditions. Time spent on education increases the agent's stock of human capital, which evolves according to

$$\dot{H} = \mathcal{H}(I_e, K_g, H, E) - \delta_h H, \tag{2}$$

where K_g is the stock of public capital, I_e is public spending on education, and δ_h $(0,1)$ is the depreciation rate of human capital. The human capital production technology, $\mathcal{H}(.)$ is increasing in all its arguments, concave in time devoted to education, and exhibits constant returns to scale with respect to I_e, K_g and H.[6]

Output, Y, is produced using labor, L, human capital, H, private physical capital, K, and public capital, K_g

$$Y = \mathcal{F}(L, H, K_g, K), \tag{3}$$

where the production function, $\mathcal{F}(.)$ is increasing in all its arguments and exhibits constant returns to scale in both the private factors, L and K, and the reproducible factors, H, K_g, and K. Denoting the rate of depreciation by $\delta_k \in (0,1)$ the law of motion of private capital is

$$\dot{K} = I - \delta_k K, \tag{4}$$

Capital accumulation is subject to adjustment costs given by

$$\mathcal{C}(I, K),$$

where $\mathcal{C}(.)$ is increasing and convex in I and decreasing in K.

The representative agent pays lump-sum taxes and income taxes to the government. It has access to a world capital market and its cost of borrowing includes a premium that increases with the economy's debt-capital ratio. Therefore, the agent's budget constraint is given by

$$\dot{D} = C + i(D/K)D + I + \mathcal{C}(I, K) + T - (1-\tau)\mathcal{F}(L, H, K_g, K), \tag{5}$$

[6] In this specification, K_g measures public infrastructure (roads, school buildings, equipment, etc.), while I_e represents public spending on teachers' salaries and educational material.

where $\tau \in (0,1)$ is the income-tax rate and T is lump-sum taxes (or transfers).

When solving the optimization problem, the agent takes public policies and the borrowing rate, $i(D/K)$, as given. The agent has no influence on the interest rate because the latter depends on the country's aggregate debt-capital ratio. Denoting by κ, ν, and ζ the Lagrange multipliers associated with the constraints (2), (4), and (5), respectively, the optimality conditions with respect to the agent's choice of C, L, E, I, D, K, and H are, respectively

$$\mathcal{U}_C = \zeta, \tag{6}$$

$$\mathcal{U}_L = -\zeta(1-\tau)\mathcal{F}_L, \tag{7}$$

$$\mathcal{U}_E = m\mathcal{H}_E, \tag{8}$$

$$1 + \mathcal{C}_I = q, \tag{9}$$

$$\rho - \frac{\dot{\zeta}}{\zeta} = i, \tag{10}$$

$$\frac{1}{q}\left[\mathcal{C}_K + (1-\tau)\mathcal{F}_K\right] - \delta_k + \frac{\dot{q}}{q} = i, \tag{11}$$

$$\frac{1}{m}\left[(1-\tau)\mathcal{F}_H\right] + \left[\mathcal{H}_H - \delta_h\right] + \frac{\dot{m}}{m} = i, \tag{12}$$

The budget constraint (5) and the following transversality conditions must hold:

$$\lim_{t\to\infty} \zeta D\ e^{-\rho t} = 0; \quad \lim_{t\to\infty} \nu K\ e^{-\rho t} = 0; \quad \lim_{t\to\infty} \kappa H\ e^{-\rho t} = 0, \tag{13}$$

In the equations above, $\mathcal{U}_C$,$\mathcal{U}_L$, and $\mathcal{U}_E$ are the derivatives of $\mathcal{U}$ with respect to C, L, and E; $\mathcal{F}_L$, $\mathcal{F}_K$, and $\mathcal{F}_H$ are the derivatives of the production function with respect to L, K, and H; $\mathcal{H}_E$ and $\mathcal{H}_H$ are the derivatives of $\mathcal{H}$ with respect to E and H; $\mathcal{C}_I$ and $\mathcal{C}_K$ are the derivatives of $\mathcal{C}$ with respect to I and K; $m \equiv \frac{\kappa}{\zeta}$ is the market price of human capital and $q \equiv \frac{\nu}{\zeta}$ is the market price of private capital. Equation (6) equates the marginal utility of consumption to the shadow value of wealth, ζ. Equation (7) equates the marginal rate of substitution between consumption and labor to the (after-tax) marginal product of labor. Equation (8) equates the marginal product of education (in terms of accumulated human capital) to the marginal rate of substitution between consumption and time spent on education. Equation (9) equates the marginal cost of an additional unit of investment to the market price of private capital. Equations (10), (11), and (12) are the usual arbitrage relationships that equate the returns on consumption, investment in private capital and investment in human capital to the cost of borrowing from abroad.

1.2.2 Government

Government spending in the recipient economy consists of two categories: investment in public capital (infrastructure), I_g, and spending on education, I_e. The government receives a flow of foreign aid, $\mathcal{A}$, which can be a pure transfer (untied) or tied either to public investment in infrastructure or to public spending on education.

$$I_g = \beta_g Y + \phi_g \mathcal{A}, \tag{14}$$

and

$$I_e = \beta_e Y + \phi_e \mathcal{A}, \tag{15}$$

where β_g and β_e are the domestic co-financing parameters, which satisfy $0 \leq \beta_g, \beta_e \leq 1$, ϕ_g and ϕ_e are the fractions of aid tied to, respectively, public investment in infrastructure and public spending on education. The remaining fraction, $1 - \phi_g - \phi_e$, represents untied aid. As in Chatterjee and Turnovsky (2007), the flow of aid transfers, $\mathcal{A}$, is tied to the scale of the recipient economy:

$$\mathcal{A} = \mu Y, \tag{16}$$

where μ is a positive parameter.

Letting $\delta_g \in (0, 1)$ be the depreciation rate, the stock of public capital evolves over time according to

$$\dot{K}_g = I_g - \delta_g K_g, \tag{17}$$

We assume that the gross investment in public capital is subject to adjustment costs

$$\mathcal{G}(I_g, K_g),$$

where $\mathcal{G}(.)$ has the same properties as $\mathcal{C}(.)$.

The government collects lump-sum taxes, T, income tax revenues, τY, and receives foreign transfers, $\mathcal{A}$, to finance its expenditures and the adjustment costs related to public capital accumulation, in such a way that it runs a balanced budget in every period.[7] Thus, its budget constraint is given by

$$T + \tau Y + \mathcal{A} = I_g + \mathcal{G}_g(I_g, K_g) + I_e, \tag{18}$$

Combining equations (5) and (18), the national budget constraint (balance of payment equation) can be written as

$$\dot{D} = C + iD + I + \mathcal{C} + I_g + \mathcal{G} + I_e - \mathcal{F}(L, H, K_g, K) - \mathcal{A}, \tag{19}$$

[7] When foreign aid is partially or completely untied, agents receive lump-sum transfers from the government.

1.2.3 Functional Forms

Before describing the macroeconomic equilibrium, we need to specify the functional forms of $\mathcal{U}(.)$,.$\mathcal{H}(.)$, $\mathcal{F}(.)$, $\mathcal{C}(.)$, $\mathcal{G}(.)$, and $i(.)$.

We assume that preferences are described by the following isoelastic utility function:[8]

$$\mathcal{U}(C, L, E) = \frac{1}{\gamma}(C(1 - L - E)^{\theta})^{\gamma}, \tag{20}$$

where the parameter $\gamma < 0$ determines the intertemporal elasticity of substitution (which is given by $\frac{1}{1-\gamma}$), and $\theta > 0$ reflects the weight assigned to leisure in the utility function. The human capital formation function is assumed to be

$$\mathcal{H}(I_e, K_g, H, E) = I_e^{\omega_1} K_g^{\omega_2} H^{1-\omega_1-\omega_2} E^{\eta}, \tag{21}$$

where $\omega_1, \omega_2 \in (0, 1)$ and η is a positive parameter.

The production function is assumed to be Cobb-Douglas and is given by

$$\mathcal{F}(L, H, K_g, K) = ZH^{\alpha_1} K_g{}^{\alpha_2} L^{\alpha_1+\alpha_2} K^{1-\alpha_1-\alpha_2}, \tag{22}$$

where $\alpha_1, \alpha_2 \in (0, 1)$ and Z represents technological progress.[9]

The capital-adjustment-cost functions for private and public capital are given by, respectively

$$\mathcal{C}(I, K) = \pi_1 \frac{I^2}{2K} \qquad \text{and} \qquad \mathcal{G}(I_g, K_g) = \pi_2 \frac{I_g^2}{2K_g},$$

where π_1 and π_2 are positive parameters.

Finally, as in Chatterjee and Turnovsky (2005, 2007), the borrowing rate, i, grows with the ratio of foreign debt to capital according to

$$i(D/K) = i^* + \exp(a\frac{D}{K}) - 1,$$

where i^* is the exogenous risk-free world interest rate and a is a positive parameter.

[8] Some authors, such as Ladrón-de-Guevara et al (1999) and De Hek (2003), pointed out that, under certain conditions, assuming that the stock of human capital does not affect symmetrically the time spent on the different activities can be a source of nonconvexities in the model and may lead to multiple steady states. We checked that this problem, which has been shown in the context of a closed economy, does not occur in our open-economy model.

[9] The production function can be re-written as:

$$\mathcal{F}(L, H, K_g, K) = Z(LH)^{\alpha_1}(LK_g)^{\alpha_2} K^{1-\alpha_1-\alpha_2},$$

where the terms L and LH can be interpreted as, respectively, raw and skilled labor.

1.3 Balanced Growth Path

In a balanced growth path, C, K, K_g, H, Y and D grow at the same constant rate, which we denote by Γ, while q, m, L, and E remain constant. That is

$$\frac{\dot{C}}{C} = \frac{\dot{K}}{K} = \frac{\dot{K}_g}{K_g} = \frac{\dot{H}}{H} = \frac{\dot{Y}}{Y} = \frac{\dot{D}}{D} = \Gamma,$$

and

$$\dot{q} = \dot{m} = \dot{L} = \dot{E} = 0,$$

The balanced growth path will be defined using the functional forms described above. From (9), we can express the growth rate of private capital as a function of q:

$$\frac{\dot{K}}{K} \equiv \Gamma_k = \frac{q-1}{\pi_1} - \delta_k, \tag{23}$$

Taking logs and differentiating condition (6) with respect to time yields

$$(\gamma - 1)\frac{\dot{C}}{C} - \theta\gamma\frac{\dot{L} + \dot{E}}{1 - L - E} = \frac{\dot{\zeta}}{\zeta},$$

which, using (10), gives an expression for the growth rate of consumption

$$\frac{\dot{C}}{C} \equiv \Gamma_c = \frac{1}{1-\gamma}\left[i - \rho - \theta\gamma\frac{\dot{L} + \dot{E}}{1 - L - E}\right], \tag{24}$$

Substituting (14) and (16) into (17) we obtain the growth rate of public capital

$$\frac{\dot{K}_g}{K_g} \equiv \Gamma_g = (\beta_g + \phi_g \mu)\frac{Y}{K_g} - \delta_g, \tag{25}$$

Using (15) and (16), the growth rate of human capital is given by

$$\frac{\dot{H}}{H} \equiv \Gamma_h = \left((\beta_e + \phi_e \mu)\frac{Y}{H}\right)^{\omega_1} \left(\frac{K_g}{H}\right)^{\omega_2} E^{\eta} - \delta_h, \tag{26}$$

Finally, equation (19) yields

$$\frac{\dot{D}}{D} = \frac{C}{D} + i + \frac{I}{D}\left(1 + \frac{\pi_1}{2}\frac{I}{K}\right) + \frac{I_g}{D}\left(1 + \frac{\pi_2}{2}\frac{I_g}{K_g}\right) + \frac{I_e}{D} - \frac{Y}{D} - \frac{\mathcal{A}}{D}, \tag{27}$$

Let $k_g \equiv \frac{K_g}{K}$ denote the ratio of public to private capital stock, $h \equiv \frac{H}{K}$ denote the ratio of human to private capital stock and $d \equiv \frac{D}{K}$ denote the ratio of national debt to private capital stock. Then, we have

$$\frac{\dot{k}_g}{k_g} = \frac{\dot{K}_g}{K_g} - \frac{\dot{K}}{K},$$

$$\frac{\dot{h}}{h} = \frac{\dot{H}}{H} - \frac{\dot{K}}{K},$$
$$\frac{\dot{d}}{d} = \frac{\dot{D}}{D} - \frac{\dot{K}}{K},$$

The dynamic behavior of the economy can be described by a system of six differential equations in k_g, h, d, q, L, and E. As shown in the Appendix A we get the following system:

$$\frac{\dot{k}_g}{k_g} = (\beta_g + \phi_g \mu)\frac{y}{k_g} - \delta_g - \left(\frac{q-1}{\pi_1} - \delta_k\right), \tag{28}$$

$$\frac{\dot{h}}{h} = \left((\beta_e + \phi_e \mu)\frac{y}{h}\right)^{\omega_1} \left(\frac{k_g}{h}\right)^{\omega_2} E^{\eta} - \delta_h - \left(\frac{q-1}{\pi_1} - \delta_k\right), \tag{29}$$

$$\frac{\dot{d}}{d} = i + \frac{1}{d}\left[c + \frac{q^2-1}{2\pi_1} + (\beta_g + \phi_g \mu)y\left(1 + \frac{\pi_2}{2}(\beta_g + \phi_g \mu)\frac{y}{k_g}\right) + (\beta_e + \phi_e \mu)y - (1+\mu)y\right] - \left(\frac{q-1}{\pi_1} - \delta_k\right), \tag{30}$$

$$\dot{q} = iq - \left[\frac{(q-1)^2}{2\pi_1} + (1-\tau)(1-\alpha_1-\alpha_2)y\right] + \delta_k q, \tag{31}$$

$$\dot{L} = \left[\frac{\Omega_2}{\Psi_2 \Psi_3} - \left(\Omega_1 + \frac{\Omega_2}{\Psi_2}\right)\frac{\Psi_1}{\Psi_4}\right] L, \tag{32}$$

$$\dot{E} = \left(\frac{\Omega_1 \Psi_2}{\Psi_4} + \frac{\Omega_2}{\Psi_4}\right) E, \tag{33}$$

where the variables y, c, m, and i are determined by, respectively

$$y \equiv \frac{Y}{K} = A(Lh)^{\alpha_1}(Lkg)^{\alpha_2}, \tag{34}$$

$$c \equiv \frac{C}{K} = \left(\frac{(1-\tau)(\alpha_1+\alpha_2)}{\theta}\right)\left(\frac{1-L-E}{L}\right) y, \tag{35}$$

$$m = \frac{\theta c}{\eta(1-L-E)}\left((\beta_e + \phi_e \mu)y\right)^{-\omega_1} k_g^{-\omega_2} h^{\omega_1+\omega_2-1} E^{1-\eta}, \tag{36}$$

$$i = i^* + \exp(ad) - 1, \tag{37}$$

and the composite parameters Ψ_1, Ψ_2, Ψ_3, Ψ_4, Ω_1, and Ω_2 are given by, respectively

$$\Psi_1 = (1-\gamma(1+\theta))\frac{E}{1-L-E}, \tag{38}$$

$$\Psi_2 = \left[(1-\alpha_1-\alpha_2)(1-\gamma) + (1-\gamma(1+\theta))\frac{L}{1-L-E}\right], \tag{39}$$

$$\Psi_3 = \left[(1-\gamma(1+\theta))\frac{L}{1-L-E} - (1-\gamma)\omega_1(\alpha_1+\alpha_2)\right], \tag{40}$$

$$\Psi_4 = \left[(\eta-1)(1-\gamma) - (1-\gamma(1+\theta))\frac{E}{1-L-E}\right]\Psi_2 + \Psi_3\Psi_1, \tag{41}$$

$$\begin{aligned}\Omega_1 \quad = \quad & i - \rho - (1-\gamma)\left[i(d) - \frac{\alpha_1(1-\tau)y}{mh} - \left((1-\omega_1-\omega_2)\left((\beta_e+\phi_e\mu)\frac{y}{h}\right)^{\omega_1}\left(\frac{k_g}{h}\right)^{\omega_2} E^{\eta} - \delta_h\right)\right. \\ & \left. + \ \omega_1(1-\alpha_1-\alpha_2)\Gamma_k + (\omega_1\alpha_1 + 1 - \omega_1 - \omega_2)\Gamma_h + (\omega_1\alpha_2 + \omega_2)\Gamma_g\right],\end{aligned} \tag{42}$$

$$\Omega_2 = \Psi_3[\rho - i(d) + (1-\gamma)\left(\alpha_1\Gamma_h + \alpha_2\Gamma_g + (1-\alpha_1-\alpha_2)\Gamma_k\right)], \tag{43}$$

Equations (28)-(33) form a dynamic system with three state variables (k_g, h, and d) and three jump variables (q, L, and E). The remaining variables and the long-run growth rate are known once k_g, h, d, q, L, and E are determined. The stationary solution of this system is found by setting $\dot{k}_g = \dot{h} = \dot{d} = \dot{q} = \dot{L} = \dot{E} = 0$.[10]

1.4 Macroeconomic Effects of Foreign Aid

In this section, we study the macroeconomic effects of a permanent increase in the ratio of foreign aid to GDP, μ. We evaluate these effects at the new long-run equilibrium and along the transitional path towards that equilibrium. Given that the complexity of our model precludes analytical solutions, we resort to numerical methods to derive the equilibrium. To this end, numerical values must be assigned to the model parameters. In the following subsection, we discuss the calibration of the model and the implied initial steady state.

1.4.1 Calibration and initial steady state

We calibrate the model at an annual frequency and choose the parameter values so that the model represents a low-income developing economy. The benchmark economy is assumed to start out at an initial zero-aid steady state (that is, $\mu = 0$). Some of the model parameters are quite common in the literature and little uncertainty exists about their values. Other parameters, however, are less straightforward to calibrate either because the empirical literature does not provide tight direct estimates or because there are likely significant disparities between the different countries regarding certain aspects of the data. For those parameters, we conduct an extensive sensitivity analysis, which we discuss in Section 5.

Our benchmark calibration is summarized in Table I. Starting with the preference parameters, we set the rate of time preference, ρ, to 0.04, and the parameters γ and θ to -1.5 and 1.8, respectively. The first two parameters are quite standard, but we check the sensitivity of our results to alternative values of θ. There is very little information in the empirical literature regarding the parameters of the human capital accumulation process, so we draw on earlier studies in our benchmark calibration and consider alternative values in the sensitivity analysis. More specifically, we set the elasticity of human capital formation with respect to public spending on education, ω_1, to 0.15, that with respect to public capital, ω_2, to 0.05, and that with respect to education, η to 1.[11] The steady-state level of technology, Z, is a scaling factor and is set to 0.6. The elasticity of output with respect to

[10] Appendix B provides the complete set of equations for the steady-state equilibrium.

[11] Glomm and Ravikumar (1998) set ω_1 to 0.15 and η to 0.8. Heckman et al. (1998) estimate η to be 0.9, and Lau (2000) and Fougère et al. (2006) set it to 0.8.

skilled labor, α_1, is chosen to be 0.4, as is commonly assumed in the literature. The elasticity of output with respect to public capital, α_2, is less common but the empirical literature provides estimates for this parameter that range from 0.11 to 0.26.[12] We set $\alpha_2 = 0.2$ and consider lower and larger values in our sensitivity analysis. We assume that human capital depreciates at a slower pace than physical capital, and accordingly, set δ_h to 0.01, and both δ_k and δ_g to 0.05.[13] The domestic government co-financing parameters, β_g and β_e, are both set to 0.05, consistently with empirical evidence.[14] The income tax rate, τ, is set to 0.15, but this value is likely to differ substantially across countries. We therefore allow for alternative values in our sensitivity analysis. The exogenous risk-free interest rate, i^*, is set to equal 5% while the borrowing-premium parameter, a, is chosen so as to ensure a plausible debt-output ratio. Finally, we set the adjustment cost parameters, π_1 and π_2, to 10.

The second column of Table II presents the resulting initial equilibrium of the economy. The steady state ratio of public to private capital is 0.42, the fraction of time devoted to labor is 0.27 while that devoted to learning is 0.06. Thus, leisure represents roughly 2/3 of the time available to agents. The consumption-output ratio is 0.71, the capital-output ratio is 1.87 and the ratio of human to private capital is 8.35. The debt-output ratio is 0.29 and yields an interest rate of 7.34%, implying that the equilibrium premium on borrowing is equal to 2.34%. The long-run growth for the benchmark economy is 1.34%.

1.4.2 Long-run steady state effects

The benchmark economy is now allowed to receive a permanent flow of foreign aid that is equivalent to 5% of its steady-state GDP (that is, μ rises from 0 to 0.05). In what follows, we compare the growth and welfare consequences of this inflow under three polar scenarios about its allocation. In the first scenario, the received aid is left completely untied (that is, $\phi_g = \phi_e = 0$) so that the government continues to allocate fractions β_g and β_e of GDP to investment in infrastructure and to spending on education, respectively. In the second scenario, aid is completely tied to public investment in physical capital (that is, $\phi_g = 1$, $\phi_e = 0$), while in the third scenario, aid is entirely devoted to public spending on education (that is, $\phi_g = 0$, $\phi_e = 1$). We also compute the optimal

[12] For example, Dessus and Herrera's (2000) estimates of the income share of public capital range from 0.11 to 0.13 for a sample of 28 developing countries over the period 1981-1991. Aschauer (2000) reports an estimate of 0.24 using data from 46 low- and middle-income countries from 1970 to 1990. Arslanalp et al. (2010) estimate the output elasticity of public capital equal to be 0.26 for a sample of developing countries.

[13] Unfortunately, we did not find empirical estimates of the deprecition rate of human capital for developing countries. However, our calibrated value is consistent with the estimates reported by Johnson and Hebein (1974), Arrazola et al. (2005), and Weber (2012) for, respectively, the U.S., Spain, and Switzerland.

[14] Arestoff and Hurlin (2006) estimate the average ratio of public expenditure on capital to GDP to be 4.42% for a group of 19 developing over the period 1974-1997. According to World Bank (2008) CD-ROM, the ratio of public spending on education to GDP was 4.25% in 2005 for Sub-Saharian Africa while a 2004 Report of the United Nations Secretariat for states that this ratio was 4.6% for Africa and 4.1% for developing countries over the period 1990-2002.

allocation of foreign aid, i.e., the fractions $\hat{\phi}_g$ and $\hat{\phi}_e$ that achieve the largest long-run welfare gain. In all of these calculations, the welfare gain is measured as the equivalent percentage variation in the private stock of capital, that is, the percentage change in the initial stock of capital that would leave agents as well off as in the new equilibrium. As we show in Appendix C, this quantity is given by

$$\varphi = \left(\frac{c_f l_f^{\theta}}{c_b l_b^{\theta}}\right)\left(\frac{\rho - \gamma\Gamma_b}{\rho - \gamma\Gamma_f}\right)^{\frac{1}{\gamma}} - 1,$$

where variables with subscripts b and f pertain to the benchmark (initial) and after-shock equilibrium, respectively. For given benchmark values, c_b, l_b^{θ} and Γ_b, φ is increasing in the ratio of consumption to the private stock of capital, in leisure, and in the long-run growth rate.

Untied aid The effects of a permanent untied aid shock are presented in the third column of Table II. When aid is untied, it amounts to a pure transfer payment that the government of the recipient economy redirects to agents. This transfer gives rise a positive wealth effect that induces agents to increase their consumption and to enjoy more leisure as they devote less time to work and to education. Moreover, agents reduce their foreign borrowing, which leads to a fall in the aggregate debt-output ratio, and thus the nominal interest rate, in the new steady state. As a result of the fall in non-leisure activities, the marginal productivity of private capital and the ratios k_g and h all decline while the ratio of private capital to output rises in the new equilibrium. In turn, the fall in the reproducible factors, k_g and h, causes the economy to grow at a slower rate of 1.16%. Nonetheless, the increase in the consumption-capital ratio and in leisure time generates a welfare gain of 4.1% relative to the initial equilibrium.

Aid tied to public investment in infrastructure The fourth column of Table II presents the long-run effects of an increase in aid tied to public investment in infrastructure. As expected, the ratio of public to private capital increases substantially, more than doubling in the new equilibrium. The received aid increases the marginal productivity of all private inputs and, to a lesser extent, the return to education, i.e., the efficiency with which education is converted into human capital. In the long run, these effects lead agents to substitute away from leisure and to devote more time to work and to education. Because foreign aid affects the productive capacity of the recipient economy, it has a larger impact on output than on consumption, implying a fall in the consumption-output ratio. Nonetheless, the ratio of consumption to the private stock of capital, c, increases to 0.4. On the other hand, the ratio of foreign debt to output rises, as the economy increases its external borrowing to finance its investment needs. The growth rate increases to 1.72% in the new equilibrium and the welfare gain is 2.5 times larger than in the untied-aid case. Therefore, the recipient economy is better off in the long run when all aid is tied to public investment in capital than when it is left completely untied. This result corroborates the conclusion reached by Chatterjee and Turnovsky (2007) under their baseline calibration.

Aid tied to public spending on education We now turn to the analysis of the long-run effects of a permanent increase in aid tied to public spending on education, which are shown in the fifth column of Table II. In this case, foreign aid acts primarily as an externality to the human capital accumulation process, which increases the efficiency with which education increases human capital. As the stock of human capital rises, it raises the marginal productivity of private capital and labor and causes the capital-output ratio to decline. While the increase in public spending on education induces agents to devote more time to it, the effect on labor supply is ambiguous. On the one hand, agents would like to work more because the marginal productivity of labor has increased, but on the other hand, agents' desire to devote more time to education and the positive income effect stemming from the increase in the marginal product of labor tend to reduce labor supply. It turns out that, given our calibration, these three motives offset each other exactly, leaving hours worked unchanged in the long run. As is the case when aid is tied to public investment in infrastructure, the shock has a larger effect on output than on consumption, leading to a fall in the consumption-output ratio, but the ratio of consumption to the private stock of capital rises to 0.41. Tying aid to education allows the recipient economy to grow at a faster rate (relative to the initial equilibrium) of 1.66% and to achieve the largest welfare gain compared to the two preceding scenarios.

It is worth emphasizing that the theoretical predictions discussed above are consistent with available empirical evidence on the effects of foreign aid to education. For example, Michaelowa (2004), Michaelowa and Weber (2007), and Dreher, Nunnenkamp, and Thiele (2008) find a positive and significant relationship between aid to education and primary school enrollment rates. Moreover, Asiedu and Nandwa (2007) document that aid targeted at education fosters economic growth in low- and middle-income countries.

Optimal allocation of aid So far, we have considered three "polar" cases in which aid is either untied, tied to public investment in infrastructure or tied to spending on education. But although the latter scenario was found to yield the largest welfare gain among the three alternative scenarios, this need not imply that it necessarily dominates any convex combination of the three polar allocation schemes. Put differently, it may well be the case that a larger welfare gain can be achieved by leaving a fraction of the aid flows untied and splitting the remaining fraction between investment in public capital and spending on education. To verify whether this is indeed the case, we compute the long-run welfare gain for all admissible values of ϕ_g and ϕ_e. The results are depicted in Figure 1. The figure shows that it is never optimal, from a long-run perspective, to leave a fraction of foreign aid completely untied and that, consequently, all aid should be tied. Furthermore, the figure shows that dividing foreign aid between investment in infrastructure and public spending on education can yield a larger welfare gain than tying aid to either one of these spending categories. In particular, welfare is maximized when $\hat{\phi}_e = 0.6$ and $\hat{\phi}_g = 0.4$. The last column of Table II shows that under this scenario, the new steady-state values of most of the key variables lie in between the values implied by the two polar cases in which aid is either tied to public capital or to education. One notable exception is the marginal productivity of private capital, which increases most when aid is optimally allocated. As a result, the recipient economy achieves the highest growth rate in addition to enjoying the largest welfare gain in the new equilibrium.

1.4.3 Transitional dynamics

The analysis above has focused on the long-run implications of foreign aid under different assumptions about its allocation. While foreign aid is found to improve long-run welfare under all scenarios, this may not be the case in the short run, i.e., along the transitional path towards the new equilibrium. More importantly, a form of aid that is more beneficial than another form from a long-run perspective may prove to entail a larger welfare cost in the short run. To investigate this issue, this section studies the dynamic adjustment of the economy during its transition from the initial to the new equilibrium under the different scenarios about aid allocation described above. Figures 2–5 show these transitional paths while Table III reports the instantaneous response of key variables following the aid shock.[15]

Untied aid The dynamic adjustment of the economy following an untied aid shock is depicted in Figure 2. The welfare effect generated by the transfer leads consumption to temporarily grow at a faster rate. The transfer also increases leisure, as agents devote less time to work and to education. This in turn reduces the marginal productivity of private, public, and human capital, causing their growth rates to fall to 1.25%, 1.26%, and 0.89%, respectively, in the period following the shock. As a result, the growth rate of output initially declines to 0.69%, thus implying that the consumption-output ratio rises.

[15] To solve for the dynamic response of the economy, we use the reverse-shooting algorithm developed by Atolia and Buffie (2009).

Figure 2 also shows that although labor supply and time devoted to education both fall initially, their transitional paths differ sharply. Indeed, while labor continues to fall until it reaches its new steady-state level, time devoted to education rises in the subsequent periods, converging to its new steady-state level from below. It is worth noting that under this scenario, foreign aid improves social welfare both in the short run (along the transitional path) and in the long run, so there is no intertemporal trade-off along this dimension.

Aid tied to public investment in infrastructure The transitional dynamics in the case where aid is tied to public investment in infrastructure are displayed in Figure 3. The received aid immediately raises the growth rate of public capital to 7.82%. In the subsequent period, the ratio of public to private capital starts to increase gradually towards its new steady-state level, thereby raising future marginal products of capital and hence the shadow price of private capital. As a result, the growth rate of private capital initially increases to 1.4%. Labor responds in a hump-shaped manner, continuing to rise for a few periods after the initial jump, before converging to its new steady-state level from above. An opposite path is displayed by time devoted to education, which falls initially, continues to decline for a few periods and converges to its new long-run level from below. Given the large increase in the growth rate of public capital and the resulting increase in the growth rate of productive inputs, the growth rate of output increases substantially after the shock (3.38%) and exceeds that of consumption, causing the ratio of consumption to output to fall. These dynamic paths diverge markedly from those implied by the untied aid scenario. However, as in the untied-aid case, welfare increases both in the short and the long run.

Aid tied to public spending on education Figure 4 shows the transitional paths following a foreign aid shock tied to public spending on education. The transfer raises the growth rate of human capital immediately after the shock and in the subsequent periods. As a result, the ratio of human to physical capital, h, increases monotonically over time until it reaches its new steady-state level. Because the transfer raises the return to education, agents work less and devote more time to education. The fall in labor reduces the marginal productivity of private capital, whose growth rate initially declines to 0.91%. Nonetheless, owing to the large increase in the growth rate of human capital, the growth rate of output increases to 2.25%. Although the shock initially lowers the growth rate of consumption, it has a larger effect on the *level* of consumption than on that of output. As a result, the consumption-output ratio jumps immediately to 0.74 before declining gradually towards its new steady-state level.

Two important observations about the transitional dynamics of the economy in the case of a transfer tied to spending on education are worth emphasizing. First, the short-run effects of this transfer are opposite to those generated by a foreign transfer that is tied to public investment. Second, although aid tied to education unambiguously raises long-run welfare in the recipient economy, the transition towards the new equilibrium involves a sizable welfare loss, which amounts to a 9% fall in the initial stock of capital in the period following the shock (see the last row of Table III). Our simulations indicate that, in the case of a transfer tied to public spending on education, the recipient economy will suffer welfare losses during eight consecutive years after receiving the aid, before starting to achieve welfare gains.

Optimal allocation of aid As stated above, the largest welfare gain in the long run is achieved when 60% of aid is tied to education and the balance is devoted to public investment in physical capital. The transitional paths under this scenario, shown in Figure 5, are generally similar to those obtained in the case when aid is completely tied to public spending on education. The only notable exceptions concern the response of public capital, whose growth rate rises substantially in the former case whereas it decreases in the latter, and the response of labor, which exceeds its initial level in the former case but remains permanently below it in the latter. Importantly, we find that although this scenario about aid allocation achieves the highest long-run welfare, its instantaneous effect is welfare deteriorating, as is the case when aid is tied exclusively to public spending on education. When aid is optimally allocated, however, the recipient economy incurs welfare losses only during the first five years after the shock.

1.5 Sensitivity Analysis

This section performs a sensitivity analysis in order to determine whether the relative (long-run) merits of the different scenarios about aid allocation, as well as the optimal allocation scheme, will change when we consider alternative values of the model parameters. We focus on the parameters of the human capital accumulation process, the output elasticity of public capital, the weight of leisure in utility, and the income-tax rate. In all cases, and to facilitate comparison, the results obtained under the benchmark calibration are reproduced.

1.5.1 Parameters of the human capital accumulation process $(\omega_1, \omega_2, \eta)$

We start by studying the sensitivity of our results to alternative values of the parameter ω_1, which measures the elasticity of human capital formation with respect to public spending on education. Table IV reports the results for the values of 0.1 and 0.2, which are respectively lower and higher than the value used in the benchmark calibration (0.15). Panel A of the table shows that the welfare gain associated with foreign aid is increasing in ω_1 regardless of how aid flows are allocated. However, the increase is larger when aid is tied to education than when it is untied or tied to public spending on education. This result is intuitive as higher values of ω_1 mean that higher levels of human capital (and thus output) are reached for a given amount of time spent acquiring education. As a corollary, the welfare-maximizing fraction of aid that ought to be tied to public spending on education, $\widehat{\phi}_e$, is also increasing in ω_1. Importantly, however, $\widehat{\phi}_e$ is still roughly equal to one half even when ω_1 is as low as 0.1, which indicates that the conclusion that a substantial fraction of aid should be assigned to education is robust to the choice of ω_1.

Second, we vary the parameter ω_2 within the range $0 - 0.15$. When ω_2 is equal to 0, the stock of public capital does not enter as an input in the production of human capital. The results, shown in Table V, indicate that the welfare benefit from aid is increasing in ω_2 when aid is left untied or is tied to public investment in infrastructure, while it decreases with ω_2 when aid is tied to public spending on education. When aid is tied to public investment in infrastructure, higher values of ω_2 imply a larger decline in leisure, but the positive effect on consumption is strongly amplified. As ω_2 increases, tying aid to public infrastructure becomes relatively more beneficial both from a growth and a welfare standpoint and, as a result, the optimal allocation of aid assigns a smaller fraction to public spending on education, which does not exceed 7% when $\omega_2 = 0.15$.

Finally, we consider alternative values of the parameter η. Table VI reports the results for the cases $\eta = 0.7$ and $\eta = 0.85$. Panel A of the table indicates that the direction in which the parameter η affects the welfare gain associated with foreign aid also depends on how the latter is allocated. When aid is tied to public investment in infrastructure, the associated welfare gain increases monotonically with η, but the opposite is true when all aid is untied. In contrast, when aid is tied to public spending on education, there is no monotonic relationship between the resulting welfare gain and the value of η. In all cases, however, the magnitude of the welfare gain changes very little with η regardless of the category of spending to which aid flows are tied. As a result, the optimal allocation of aid is also fairly insensitive to the value of $\eta :$ the welfare-maximizing fraction devoted to public spending on education decreases from 68 to 60 percent as η increases from 0.7 to 1.

1.5.2 Output elasticity of public capital (α_2)

In our benchmark calibration, we set the elasticity of output with respect to public capital, α_2 to 0.2. In this experiment, we consider two alternative values: 0.1 and 0.3, which correspond to the lower and upper bounds of the range of available estimates, respectively. The results, shown in Table VII, indicate that when aid is tied to public investment in infrastructure, the associated welfare gain increases with α_2 due to a smaller decline in leisure and to a larger increase in consumption and in the growth rate of the economy. This larger gain reflects the larger impact of foreign aid on the marginal productivity of private inputs. On the other hand, the benefit associated with untied aid falls with α_2, while that associated with an aid program that is tied to public spending on education displays little sensitivity with respect to α_2. As a consequence, the optimal fraction of aid flows that should be tied to public investment in infrastructure is increasing in α_2. Nonetheless, even for an output elasticity of public spending as high as 0.3, it is still optimal, from a welfare perspective, to allocate 36 percent of aid flows to public spending on education.

1.5.3 Weight of leisure in utility (θ)

Next, we vary the value of the parameter θ, setting it first to 0 and then to 3, and present the results in Table VIII. When θ is equal to 0, agents do not value leisure and non-leisure activities are supplied inelastically. As θ increases, the importance of leisure in utility increases. Table VIII shows that the long-run welfare gain from foreign aid decreases with θ if the transfer is untied or tied to public spending on education. In contrast, the effect is non-monotonic when aid is tied to public investment in infrastructure, with the welfare gain rising sharply as θ increases from 0 to 1.8, but falling slightly as θ increases from 1.8 to 3. This in turn implies that the optimal fraction of aid that should be tied to public spending on education is a non-linear function of θ, declining rapidly in the neighborhood of 0 and becoming relatively flat for larger values of θ. For example, when θ increases from 1.8 to 3, this fraction drops only by 2 percentage points (from 60 to 58 percent).

1.5.4 Income tax rate (τ)

Finally, we discuss the sensitivity of our results to alternative values of the average income-tax rate, τ. We allow this rate to be as low as 0.1 and as high as 0.3, and report the results in Table IX. Higher tax rates unambiguously reduce the welfare gain associated with untied aid because the wealth effect of the transfer becomes smaller as τ increases, as one can deduce from the smaller increase in consumption and leisure, which in turn implies a smaller decline in the long-run growth rate.

When aid is tied, on the other hand, its effects on consumption and leisure move in opposite directions as τ increases. With higher tax rates, the increase in consumption becomes smaller but leisure declines less, implying a smaller increase in the long-run growth rate. The net effect of a higher tax rate on the long run welfare gain therefore depends on the extent to which the reduced benefits of foreign aid on consumption and the growth rate are offset by the lesser adverse effect on leisure. This net effect turns out to be positive when aid is tied to public investment in infrastructure, but negative when aid is tied to public spending on education. In both cases, however, the welfare gain displays little sensitivity with respect to the tax rate. As a result, the relative merits of the different forms of aid and the optimal allocation scheme change very little with τ. As τ increases from 0.1 to 0.3, the welfare maximizing fraction of aid that should be allocated to education drops by only 5 percentage points (from 61% to 55%).

1.6 Conclusion

This paper has studied the optimal allocation of foreign aid in a dynamic growth model with human capital accumulation. Under a plausible parametrization of the model, we find that from a long-run welfare perspective, tying a large fraction of aid to public spending on education achieves a better economic outcome for the recipient economy than a scenario in which aid is mostly untied or mostly tied to public investment in infrastructure. This paper's results, therefore, have crucial policy implications regarding the conditionality of official development assistance to low-income countries.

These results, however, have been derived in the context of a representative-agent setting, which implies that all agents in the recipient economy are identically impacted by the received aid. Since this is unlikely to be the case in reality, it would be interesting to reexamine the question of the optimal allocation of aid in a model in which the government weighs differently the different classes of agents (for example, privileging the poor). An equally important question is the effect of foreign aid on the wealth and income distribution in the recipient economy. Addressing these questions would require extending the theoretical model developed in this paper to allow for heterogeneity across agents. We leave this task for future research.

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1.8 Appendix

1.8.1 Appendix A: Derivation of the Balanced Growth Path

Differentiating the optimality condition (8) yields

$$\frac{\dot{E}}{E}=\frac{1}{\eta-1}\left[\frac{\dot{C}}{C}+\frac{\dot{L}+\dot{E}}{1-L-E}\right]+\frac{1}{1-\eta}\left[\frac{\dot{m}}{m}+\omega_1\frac{\dot{I}_e}{I_e}+\omega_2\frac{\dot{K}_g}{K_g}+(1-\omega_1-\omega_2)\frac{\dot{H}}{H}\right], \tag{A.1}$$

From (15) we have $\frac{\dot{I}_e}{I_e}=\frac{\dot{Y}}{Y}$. Differentiating (22) gives

$$\frac{\dot{Y}}{Y}=\alpha_1\frac{\dot{H}}{H}+\alpha_2\frac{\dot{K}_g}{K_g}+(\alpha_1+\alpha_2)\frac{\dot{L}}{L}+(1-\alpha_1-\alpha_2)\frac{\dot{K}}{K}, \tag{A.2}$$

Using equation (24) we obtain after some algebra

$$\begin{aligned}\frac{\dot{E}}{E}\left[(\eta-1)(1-\gamma)-(1-\gamma(1+\theta))\frac{E}{1-L-E}\right] \quad &= \quad i-\rho-(1-\gamma)[\frac{\dot{m}}{m}+\omega_1(1-\alpha_1-\alpha_2)\frac{\dot{K}}{K}\\ &+(\omega_1\alpha_1+1-\omega_1-\omega_2)\frac{\dot{H}}{H}+(\omega_1\alpha_2+\omega_2)\frac{\dot{K}_g}{K_g}]\\ &+\left[(1-\gamma(1+\theta))\frac{L}{1-L-E}-(1-\gamma)\omega_1(\alpha_1+\alpha_2)\right]\frac{\dot{L}}{L},\end{aligned} \tag{A.3}$$

Substituting (6) into (7) yields

$$\theta C(1-L-E)^{-1}=(1-\tau)(\alpha_1+\alpha_2)\frac{Y}{L}, \tag{A.4}$$

Taking logs and differentiating equation (A.4) with respect to time yields

$$\frac{\dot{C}}{C}-\frac{\dot{Y}}{Y}=-\left(\frac{\dot{L}+\dot{E}}{1-L-E}\right)-\frac{\dot{L}}{L}, \tag{A.5}$$

Combining equations (24), (A.2), and (A.5), we obtain

$$\begin{aligned}\left[(1-\gamma(1+\theta))+(1-\alpha_1-\alpha_2)(1-\gamma)\frac{1-L-E}{L}\right]\dot{L} \quad &= \quad (1-L-E)[\rho-i(d)\\ &+(1-\gamma)(\alpha_1\frac{\dot{H}}{H}+\alpha_2\frac{\dot{K}_g}{K_g}+(1-\alpha_1-\alpha_2)\frac{\dot{K}}{K})]\\ &-(1-\gamma(1+\theta))\dot{E},\end{aligned} \tag{A.6}$$

Equations (A.3) and (A.6) form a system of two equations in $\dot{L}$ and $\dot{E}$ and solving this system yields equations. (32) and (33).

The Balanced Growth Path can be expressed as a system of six equations in k_g, h, d, q, L and E:

$$\frac{\dot{k}_g}{k_g}=(\beta_g+\phi_g\mu)\frac{y}{k_g}-\delta_g-\left(\frac{q-1}{\pi_1}-\delta_k\right), \tag{A.7}$$

$$\frac{\dot{h}}{h} = \left((\beta_e + \phi_e\mu)\frac{y}{h}\right)^{\omega_1}\left(\frac{k_g}{h}\right)^{\omega_2} E^{\eta} - \delta_h - \left(\frac{q-1}{\pi_1} - \delta_k\right), \tag{A.8}$$

$$\frac{\dot{d}}{d} = i + \frac{1}{d}\left[c + \frac{q^2-1}{2\pi_1} + (\beta_g + \phi_g\mu)y\left(1 + \frac{\pi_2}{2}(\beta_g + \phi_g\mu)\frac{y}{k_g}\right) + (\beta_e + \phi_e\mu)y - (1+\mu)y\right] - \left(\frac{q-1}{\pi_1} - \delta_k\right), \tag{A.9}$$

$$\dot{q} = iq - \left[\frac{(q-1)^2}{2\pi_1} + (1-\tau)(1-\alpha_1-\alpha_2)y\right] + \delta_k q, \tag{A.10}$$

$$\dot{L} = \left[\frac{\Omega_2}{\Psi_2\Psi_3} - \left(\Omega_1 + \frac{\Omega_2}{\Psi_2}\right)\frac{\Psi_1}{\Psi_4}\right]L, \tag{A.11}$$

$$\dot{E} = \left(\frac{\Omega_1\Psi_2}{\Psi_4} + \frac{\Omega_2}{\Psi_4}\right)E, \tag{A.12}$$

where the variables y, c, m, and i are determined by, respectively

$$y \equiv \frac{Y}{K} = A(Lh)^{\alpha_1}(Lkg)^{\alpha_2}, \tag{A.13}$$

$$c \equiv \frac{C}{K} = \left(\frac{(1-\tau)(\alpha_1+\alpha_2)}{\theta}\right)\left(\frac{1-L-E}{L}\right)y, \tag{A. 14}$$

$$i = i^* + \exp(ad) - 1, \tag{A.15}$$

and the composite parameters Ψ_1, Ψ_2, Ψ_3, Ψ_4, Ω_1, and Ω_2 are given by, respectively

$$\Psi_1 = (1-\gamma(1+\theta))\frac{E}{1-L-E}, \tag{A.16}$$

$$\Psi_2 = \left[(1-\alpha_1-\alpha_2)(1-\gamma) + (1-\gamma(1+\theta))\frac{L}{1-L-E}\right], \tag{A.17}$$

$$\Psi_3 = \left[(1-\gamma(1+\theta))\frac{L}{1-L-E} - (1-\gamma)\omega_1(\alpha_1+\alpha_2)\right], \tag{A.18}$$

$$\Psi_4 = \left[(\eta-1)(1-\gamma) - (1-\gamma(1+\theta))\frac{E}{1-L-E}\right]\Psi_2 + \Psi_3\Psi_1, \tag{A.19}$$

$$\begin{aligned}\Omega_1 \quad = \quad & i - \rho - (1-\gamma)\left[i(d) - \frac{\alpha_1(1-\tau)y}{mh} - \left((1-\omega_1-\omega_2)\left((\beta_e+\phi_e\mu)\frac{y}{h}\right)^{\omega_1}\left(\frac{k_g}{h}\right)^{\omega_2}E^{\eta} - \delta_h\right)\right.\\ & \left. + \ \omega_1(1-\alpha_1-\alpha_2)\Gamma_k + (\omega_1\alpha_1 + 1 - \omega_1 - \omega_2)\Gamma_h + (\omega_1\alpha_2 + \omega_2)\Gamma_g\right],\end{aligned} \tag{A.20}$$

$$\Omega_2 = \Psi_3[\rho - i + (1-\gamma)(\alpha_1\Gamma_h + \alpha_2\Gamma_g + (1-\alpha_1-\alpha_2)\Gamma_k)], \tag{A.21}$$

$$m = \frac{\theta c}{\eta(1-L-E)}((\beta_e+\phi_e\mu)y)^{-\omega_1} k_g^{-\omega_2} h^{\omega_1-\omega_2-1} E^{1-\eta}, \tag{A.22}$$

and the growth rates of the different types of capital are

$$\frac{\dot{K}}{K} \equiv \Gamma_k = \frac{q-1}{\pi_1} - \delta_k, \tag{A.23}$$

$$\frac{\dot{K}_g}{K_g} \equiv \Gamma_g = (\beta_g + \phi_g\mu)\frac{y}{k_g} - \delta_g, \tag{A. 24}$$

$$\frac{\dot{H}}{H} \equiv \Gamma_h = \left((\beta_e+\phi_e\mu)\frac{y}{h}\right)^{\omega_1}\left(\frac{k_g}{h}\right)^{\omega_2} E^{\eta} - \delta_h, \tag{A.25}$$

1.8.2 Appendix B: Steady-state equilibrium

The steady-state equilibrium is determined when $\dot{k}_g = \dot{h} = \dot{d} = \dot{q} = \dot{L} = \dot{E} = 0$, which implies that

$$\frac{\dot{C}}{C} = \frac{\dot{K}}{K} = \frac{\dot{K}_g}{K_g} = \frac{\dot{H}}{H} = \frac{\dot{Y}}{Y} = \frac{\dot{D}}{D} \equiv \Gamma,$$

Imposing $\dot{k}_g = \dot{h} = \dot{d} = \dot{q} = \dot{L} = \dot{E} = 0$ in equations (A.7)–(A.12) and using (A.13), (A.14), (A.22)–(A.25), and (12) we obtain the following system of equations representing the steady-state:

$$(\beta_g + \phi_g \mu)\frac{y}{k_g} - \delta_g - \left(\frac{q-1}{\pi_1} - \delta_k\right) = 0, \tag{B.1}$$

$$\left((\beta_e + \phi_e \mu)\frac{y}{h}\right)^{\omega_1} \left(\frac{k_g}{h}\right)^{\omega_2} E^{\eta} - \delta_h - \left(\frac{q-1}{\pi_1} - \delta_k\right) = 0, \tag{B.2}$$

$$i + \frac{1}{d}\left[c + \frac{q^2-1}{2\pi_1} + (\beta_g + \phi_g \mu)y\left(1 + \frac{\pi_2}{2}(\beta_g + \phi_g \mu)\frac{y}{k_g}\right) + (\beta_e + \phi_e \mu)y - (1+\mu)y\right] - \left(\frac{q-1}{\pi_1} - \delta_k\right) = 0, \tag{B.3}$$

$$iq - \left[\frac{(q-1)^2}{2\pi_1} + (1-\tau)(1-\alpha_1-\alpha_2)y\right] + \delta_k q = 0, \tag{B.4}$$

$$\frac{[i(d) - \rho]}{1-\gamma} - \left(\frac{q-1}{\pi_1} - \delta_k\right) = 0, \tag{B.5}$$

$$c = \frac{(1-\tau)(\alpha_1+\alpha_2)}{\theta}\left(\frac{1-L-E}{L}\right)y, \tag{B.6}$$

$$y = A(Lh)^{\alpha_1}(Lk_g)^{\alpha_2}, \tag{B.7}$$

$$m = \frac{\theta c}{\eta(1-L-E)}\left((\beta_e + \phi_e \mu)y\right)^{-\omega_1} k_g^{-\omega_2} h^{\omega_1+\omega_2-1} E^{1-\eta}, \tag{B.8}$$

$$im - \left[(1-\tau)\alpha_1 \frac{y}{h}\right] - m\left[(1-\omega_1-\omega_2)\left((\beta_e + \phi_e \mu)\frac{y}{h}\right)^{\omega_1}\left(\frac{k_g}{h}\right)^{\omega_2} E^{\eta} - \delta_h\right] = 0, \tag{B.9}$$

Solving this system provides the steady-state values of $\overline{k}_g$, $\overline{h}$, $\overline{d}$, $\overline{q}$, $\overline{L}$, $\overline{E}$, $\overline{m}$, $\overline{c}$, $\overline{y}$ and the long-run equilibrium growth rate, $\overline{\Gamma}$.

1.8.3 Appendix C: Derivations of Welfare Changes

Welfare changes are measured by the equivalent variations in the private stock of capital following an aid shock, i.e., the percentage changes in the initial stock of capital necessary to make agents as well off in the benchmark equilibrium as in the after-shock equilibrium. Given the form of the utility function, and assuming that the economy is initially on a balanced growth path with a constant growth rate equal to Γ_b, the benchmark intertemporal welfare is given by

$$W_b = \int_0^\infty \frac{1}{\gamma}(c_b l_b^{\theta} K_b(t))^{\gamma} e^{-\rho t} dt, \tag{C.1}$$

$$= \int_0^\infty \frac{1}{\gamma}(c_b l_b^{\theta} K_0)^{\gamma} e^{(\gamma\Gamma_b - \rho)t} dt, \tag{C.2}$$

where variables with subscript b pertain to the benchmark equilibrium, and K_0 is the stock of capital at $t = 0$. Since c_b and l_b are constant along the initial balanced growth path, the expression above reduces to

$$W_b = W_b(c_b, l_b, K_0) = \frac{1}{\gamma} \frac{\left(c_b l_b^\theta K_0\right)^\gamma}{\rho - \gamma \Gamma_b}, \tag{C.3}$$

The intertemporal welfare in the after-shock equilibrium is given by

$$W_f = \int_0^\infty \frac{1}{\gamma} (c_b l_b^\theta K_f(t))^\gamma e^{-\rho t} dt, \tag{C.4}$$

$$= \frac{1}{\gamma} \frac{\left(c_f l_f^\theta K_0\right)^\gamma}{\rho - \gamma \Gamma_f}, \tag{C.5}$$

where variables with subscript f pertain to the new equilibrium. Evaluating the percentage change in the initial stock of capital, K_0, that would make the representative agent as well off in the benchmark equilibrium as in the new steady state amounts to finding the quantity φ such that $W_b(c_b, l_b, (1+\varphi)K_0) = W_f$. This quantity is given by

$$\varphi = \left(\frac{c_f l_f^\theta}{c_b l_b^\theta}\right) \left(\frac{\rho - \gamma \Gamma_b}{\rho - \gamma \Gamma_f}\right)^{\frac{1}{\gamma}} - 1, \tag{C.6}$$

Table I – Benchmark Calibration

Description	Parameter	Value
Preferences		
Discount factor	ρ	0.04
Preference Parameter	γ	-1.5
Preference Parameter	θ	1.8
Human capital accumulation		
Elasticity w.r.t. public spending on education	ω_1	0.15
Elasticity w.r.t. public capital	ω_2	0.05
Elasticity w.r.t. education	η	1
Production		
Output elasticity of effective labor	α_1	0.4
Output elasticity of raw labor and public capital	α_2	0.2
Steady-State level of technology	Z	0.6
Depreciation rates		
Private capital	δ_k	0.05
Public capital	δ_g	0.05
Human capital	δ_h	0.01
Foreign Aid		
Aid-output ratio	μ	0
Fraction of aid tied to investments in infrastructure	ϕ_g	0
Fraction of aid tied to public spending on education	ϕ_e	0
Policy parameters		
Income tax rate	τ	0.15
Co-financing parameter (investment in infrastructure)	β_g	0.05
Co-financing parameter (spending on education)	β_e	0.05
Others		
Adjustment-cost parameter (private capital)	π_1	10
Adjustment-costs parameter (public capital)	π_2	10
Risk-free world interest rate	i^*	0.05
Borrowing-premium parameter	a	0.15

Table II – Long-Run Effects of an Increase in Foreign Aid

	Initial Equilibrium ($\mu = 0$)	New Equilibrium ($\mu = 0.05$)			
		Untied $\phi_g = \phi_e = 0$	Tied to Infrastructure $\phi_g = 1, \phi_e = 0$	Tied to Education $\phi_g = 0, \phi_e = 1$	Optimal Allocation $\widehat{\phi}_g = 0.4, \widehat{\phi}_e = 0.6$
L	0.2668	0.2548	0.2712	0.2668	0.2686
E	0.0642	0.0595	0.0691	0.0674	0.0685
H/K	8.3474	8.0564	7.2009	10.0463	8.8034
K_g/K	0.4213	0.4139	0.8747	0.4346	0.6131
C/Y	0.7105	0.7624	0.6893	0.7069	0.6992
K/Y	1.8734	1.9600	1.7005	1.7285	1.6944
D/Y	0.2887	0.2469	0.3689	0.3562	0.3716
$i(\%)$	7.3388	6.9074	8.3076	8.1397	8.3447
$\Gamma(\%)$	1.3355	1.1629	1.7231	1.6559	1.7379
$\Delta W(\%)$	$--$	4.0869	10.8523	12.5214	14.0616

Table III – Short-Run Effects of an Increase in Foreign Aid

	Initial Equilibrium ($\mu = 0$)	New Equilibrium ($\mu = 0.05$)			
		Untied $\phi_g = \phi_e = 0$	Tied to Infrastructure $\phi_g = 1, \phi_e = 0$	Tied to Education $\phi_g = 0, \phi_e = 1$	Optimal Allocation $\widehat{\phi}_g = 0.4, \widehat{\phi}_e = 0.6$
$\Gamma_k(0)\%$	1.3355	1.2472	1.4012	0.9074	1.0285
$\Gamma_h(0)\%$	1.3355	0.8868	0.8139	2.5405	1.9189
$\Gamma_g(0)\%$	1.3355	1.2563	7.8181	1.1296	3.3564
$\Gamma_y(0)\%$	1.3355	0.6883	3.3818	2.2543	2.6067
$\Gamma_c(0)\%$	1.3355	1.3599	1.5913	1.2507	1.3303
$L(0)$	0.2668	0.2612	0.2720	0.2525	0.2581
$E(0)$	0.0642	0.0519	0.0498	0.0881	0.0750
$h(0)$	8.3474	8.3474	8.3474	8.3474	8.7836
$k_g(0)$	0.4213	0.4213	0.4213	0.4213	0.4337
$\frac{c(0)}{y(0)}$	0.7105	0.7452	0.7065	0.7399	0.7320
$\Delta W(0)\%$	$--$	6.9542	4.2211	-8.9865	-4.6918

Table IV – Sensitivity Analysis: Alternative Values of ω_1

	Untied $\phi_g = \phi_e = 0$	Tied to Infrastructure $\phi_g = 1, \phi_e = 0$	Tied to Education $\phi_g = 0, \phi_e = 1$	Optimal Allocation	$(\widehat{\phi}_g, \widehat{\phi}_e)$
			A. $\Delta W(\%)$		
$\omega_1 = 0.1$	4.05	9.58	9.12	11.22	$(0.53, 0.47)$
$\omega_1 = 0.15$	4.09	10.85	12.52	14.06	$(0.40, 0.60)$
$\omega_1 = 0.2$	4.21	11.79	15.33	16.50	$(0.33, 0.67)$
			B. $\Delta l(\%)$		
$\omega_1 = 0.1$	2.62	-1.57	-0.36	-1.06	
$\omega_1 = 0.15$	2.49	-1.38	-0.48	-0.92	
$\omega_1 = 0.2$	2.41	-1.29	-0.60	-0.91	
			C. $\Delta c(\%)$		
$\omega_1 = 0.1$	2.36	6.34	5.85	7.29	
$\omega_1 = 0.15$	2.55	6.88	7.83	8.80	
$\omega_1 = 0.2$	2.74	7.25	9.37	10.07	
			D. $\Delta\Gamma$		
$\omega_1 = 0.1$	-0.0022	0.0047	0.0029	0.0044	
$\omega_1 = 0.15$	-0.0017	0.0039	0.0032	0.0040	
$\omega_1 = 0.2$	-0.0014	0.0033	0.0032	0.0038	

Table V – Sensitivity Analysis: Alternative Values of ω_2

	Untied $\phi_g = \phi_e = 0$	Tied to Infrastructure $\phi_g = 1, \phi_e = 0$	Tied to Education $\phi_g = 0, \phi_e = 1$	Optimal Allocation	$(\widehat{\phi}_g, \widehat{\phi}_e)$
			A. $\Delta W(\%)$		
$\omega_2 = 0$	3.88	5.17	14.13	14.13	$(0, 1)$
$\omega_2 = 0.05$	4.09	10.85	12.52	14.06	$(0.40, 0.60)$
$\omega_2 = 0.15$	4.39	19.99	10.25	20.05	$(0.93, 0.07)$
			B. $\Delta l(\%)$		
$\omega_2 = 0$	2.57	-1.27	-0.55	-0.70	
$\omega_2 = 0.05$	2.49	-1.38	-0.48	-1.59	
$\omega_2 = 0.15$	2.39	-1.58	-0.40	-2.61	
			C. $\Delta c(\%)$		
$\omega_2 = 0$	2.32	3.43	8.92	8.92	
$\omega_2 = 0.05$	2.55	6.88	7.83	8.80	
$\omega_2 = 0.15$	2.88	12.15	6.31	12.18	
			D. $\Delta\Gamma$		
$\omega_2 = 0$	-0.0020	0.0028	0.0041	0.0041	
$\omega_2 = 0.05$	-0.0017	0.0039	0.0032	0.0040	
$\omega_2 = 0.15$	-0.0013	0.0050	0.0022	0.0050	

Table VI – Sensitivity Analysis: Alternative Values of η

	Untied $\phi_g = \phi_e = 0$	Tied to Infrastructure $\phi_g = 1, \phi_e = 0$	Tied to Education $\phi_g = 0, \phi_e = 1$	Optimal Allocation	$(\widehat{\phi}_g, \widehat{\phi}_e)$
			A. $\Delta W(\%)$		
$\eta = 0.7$	5.89	9.51	12.63	13.65	$(0.32, 0.68)$
$\eta = 0.85$	4.99	10.16	12.47	13.76	$(0.37, 0.63)$
$\eta = 1$	4.09	10.85	12.52	14.06	$(0.40, 0.60)$
			B. $\Delta l(\%)$		
$\eta = 0.7$	2.48	-1.62	-0.41	-0.86	
$\eta = 0.85$	2.47	-1.46	-0.43	-0.88	
$\eta = 1$	2.49	-1.38	-0.48	-0.92	
			C. $\Delta c(\%)$		
$\eta = 0.7$	3.59	6.29	8.07	8.77	
$\eta = 0.85$	3.09	6.57	7.89	8.73	
$\eta = 1$	2.55	6.88	7.83	8.80	
			D. $\Delta \Gamma$		
$\eta = 0.7$	-0.0020	0.0055	0.0045	0.0056	
$\eta = 0.85$	-0.0018	0.0046	0.0038	0.0047	
$\eta = 1$	-0.0017	0.0039	0.0032	0.0040	

Table VII – Sensitivity Analysis: Alternative Values of α_2

	Untied $\phi_g = \phi_e = 0$	Tied to Infrastructure $\phi_g = 1, \phi_e = 0$	Tied to Education $\phi_g = 0, \phi_e = 1$	Optimal Allocation	$(\widehat{\phi}_g, \widehat{\phi}_e)$
			A. $\Delta W(\%)$		
$\alpha_2 = 0.1$	4.39	6.24	12.58	12.62	$(0.07, 0.93)$
$\alpha_2 = 0.2$	4.09	10.85	12.52	14.06	$(0.40, 0.60)$
$\alpha_2 = 0.3$	3.90	15.44	12.40	16.81	$(0.64, 0.36)$
			B. $\Delta l(\%)$		
$\alpha_2 = 0.1$	2.65	−1.45	−0.57	−0.65	
$\alpha_2 = 0.2$	2.49	−1.38	−0.48	−0.92	
$\alpha_2 = 0.3$	2.39	−1.37	−0.44	−1.11	
			C. $\Delta c(\%)$		
$\alpha_2 = 0.1$	2.76	4.11	8	8.03	
$\alpha_2 = 0.2$	2.55	6.88	7.83	8.80	
$\alpha_2 = 0.3$	2.41	9.52	7.63	10.33	
			D. $\Delta\Gamma$		
$\alpha_2 = 0.1$	−0.0022	0.0034	0.0038	0.0040	
$\alpha_2 = 0.2$	−0.0017	0.0039	0.0032	0.0040	
$\alpha_2 = 0.3$	−0.0014	0.0042	0.0027	0.0042	

Table VIII – Sensitivity Analysis: Alternative Values of θ

	Untied $\phi_g = \phi_e = 0$	Tied to Infrastructure $\phi_g = 1, \phi_e = 0$	Tied to Education $\phi_g = 0, \phi_e = 1$	Optimal Allocation	$(\widehat{\phi}_g, \widehat{\phi}_e)$
			A. $\Delta W(\%)$		
$\theta = 0$	7.88	7.85	14.07	14.49	$(0.19, 0.81)$
$\theta = 1.8$	4.09	10.85	12.52	14.06	$(0.40, 0.60)$
$\theta = 3$	3.89	10.19	11.43	12.96	$(0.42, 0.58)$
			B. $\Delta l(\%)$		
$\theta = 0$	0	0	0	0	
$\theta = 1.8$	2.49	-1.38	-0.48	-0.92	
$\theta = 3$	1.66	-0.84	-0.36	-0.62	
			C. $\Delta c(\%)$		
$\theta = 0$	7.88	2.87	8.80	8.65	
$\theta = 1.8$	2.55	6.88	7.83	8.80	
$\theta = 3$	2.26	6.47	7.12	8.10	
			D. $\Delta\Gamma$		
$\theta = 0$	0	0.0078	0.0078	0.0086	
$\theta = 1.8$	-0.0017	0.0039	0.0032	0.0040	
$\theta = 3$	-0.0014	0.0026	0.0022	0.0028	

Table IX – Sensitivity Analysis: Alternative Values of τ

	Untied $\phi_g = \phi_e = 0$	Tied to Infrastructure $\phi_g = 1, \phi_e = 0$	Tied to Education $\phi_g = 0, \phi_e = 1$	Optimal Allocation	$(\widehat{\phi}_g, \widehat{\phi}_e)$
			A. $\Delta W(\%)$		
$\tau = 0.1$	4.56	10.63	12.56	14	$(0.39, 0.61)$
$\tau = 0.15$	4.09	10.85	12.52	14.06	$(0.40, 0.60)$
$\tau = 0.3$	2.82	11.21	12.23	14	$(0.44, 0.56)$
			B. $\Delta l(\%)$		
$\tau = 0.1$	2.67	−1.52	−0.52	−0.99	
$\tau = 0.15$	2.49	−1.38	−0.48	−0.92	
$\tau = 0.3$	1.98	−1.03	−0.40	−0.74	
			C. $\Delta c(\%)$		
$\tau = 0.1$	2.67	6.88	7.92	8.87	
$\tau = 0.15$	2.55	6.88	7.83	8.80	
$\tau = 0.3$	2.29	6.75	7.48	8.48	
			D. $\Delta\Gamma$		
$\tau = 0.1$	−0.0018	0.0041	0.0034	0.0043	
$\tau = 0.15$	−0.0017	0.0039	0.0032	0.0040	
$\tau = 0.3$	−0.0015	0.0031	0.0026	0.0033	

Figure 1 – Long-run welfare as a function of ϕ_g and ϕ_e

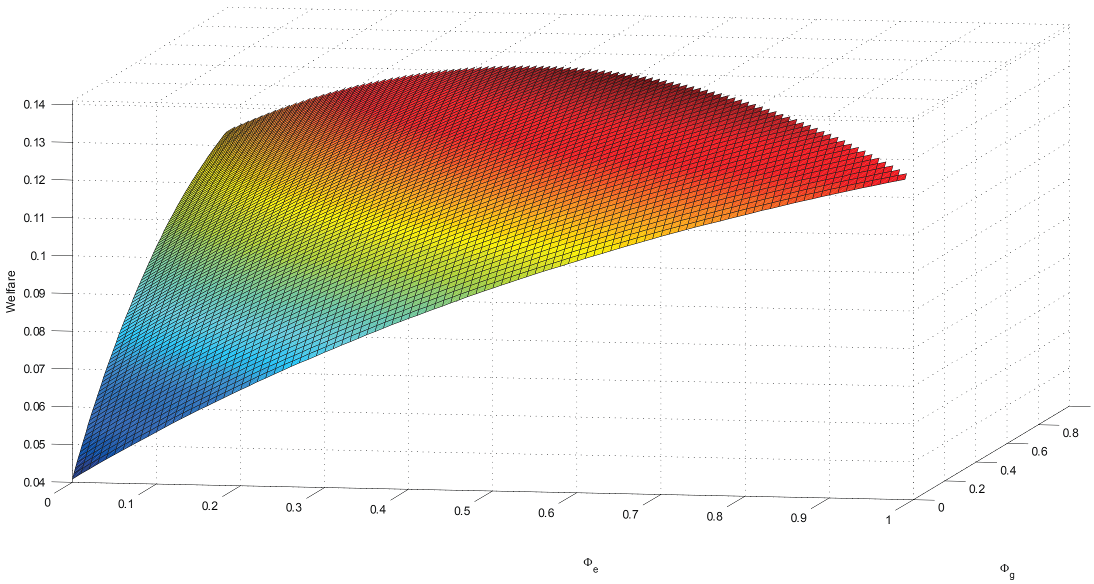


Figure 2 – Transitional dynamics: Untied aid

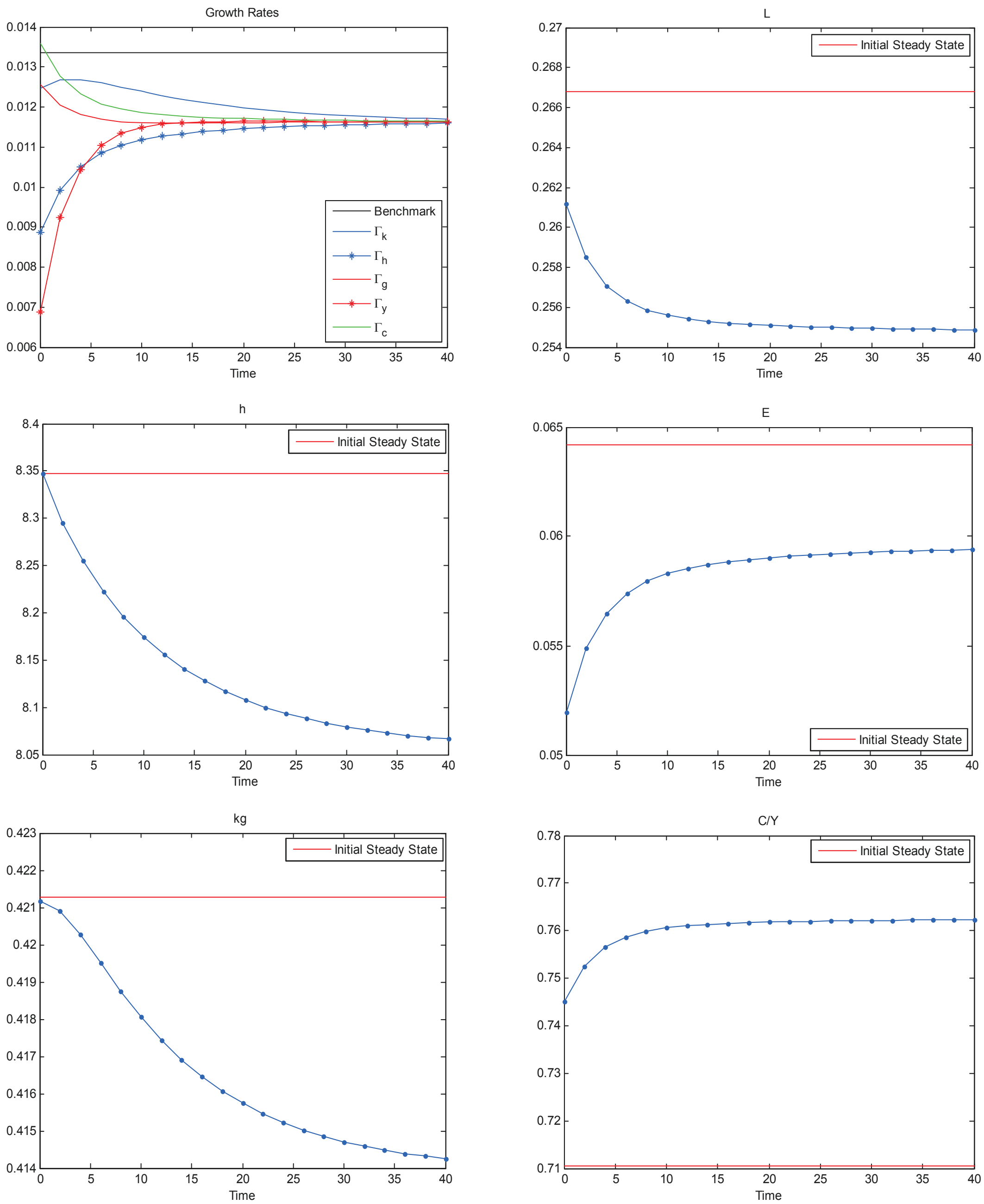


Figure 3 – Transitional dynamics: Aid tied to public investment in infrastructure

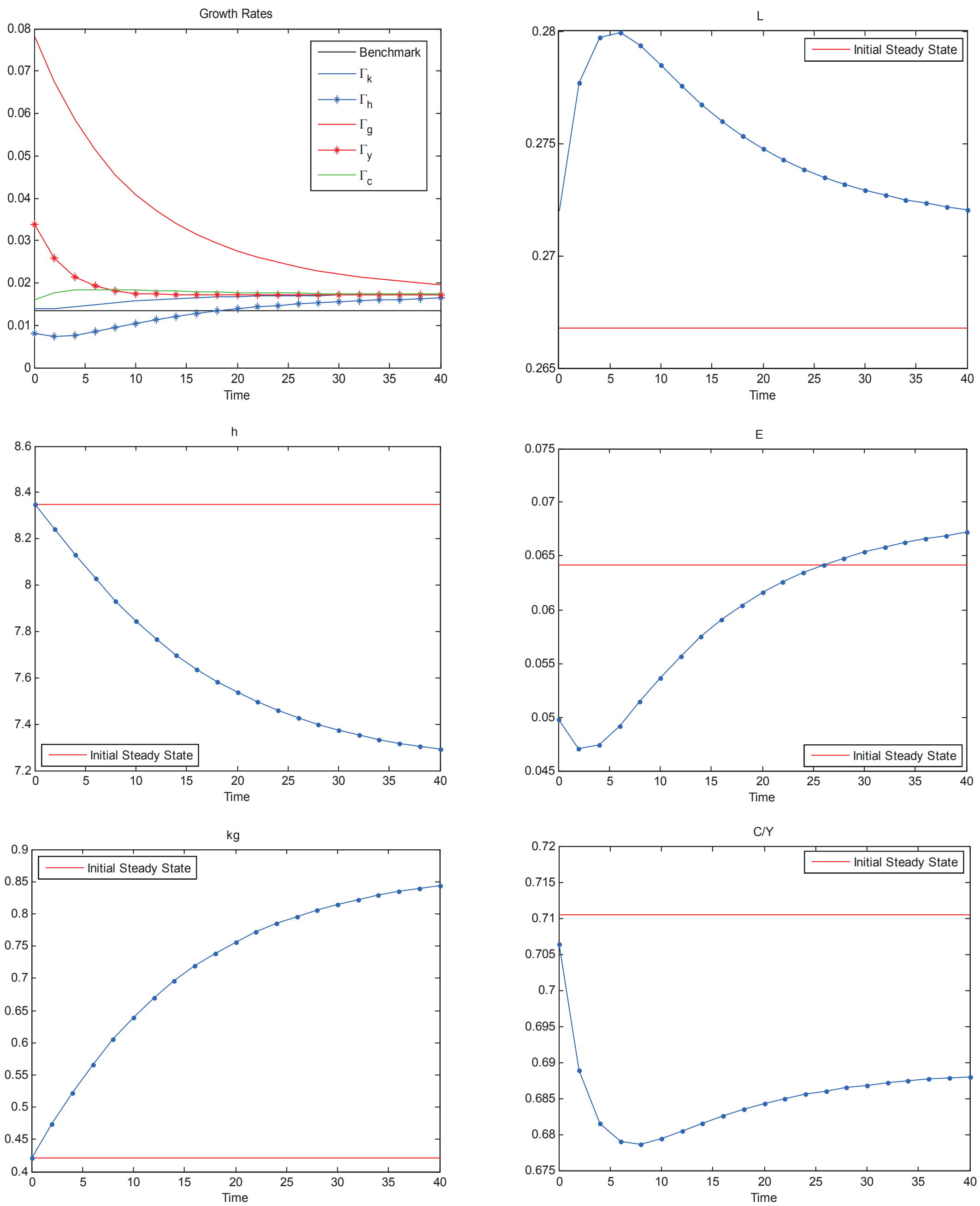


Figure 4 – Transitional dynamics: Aid tied to public spending on education

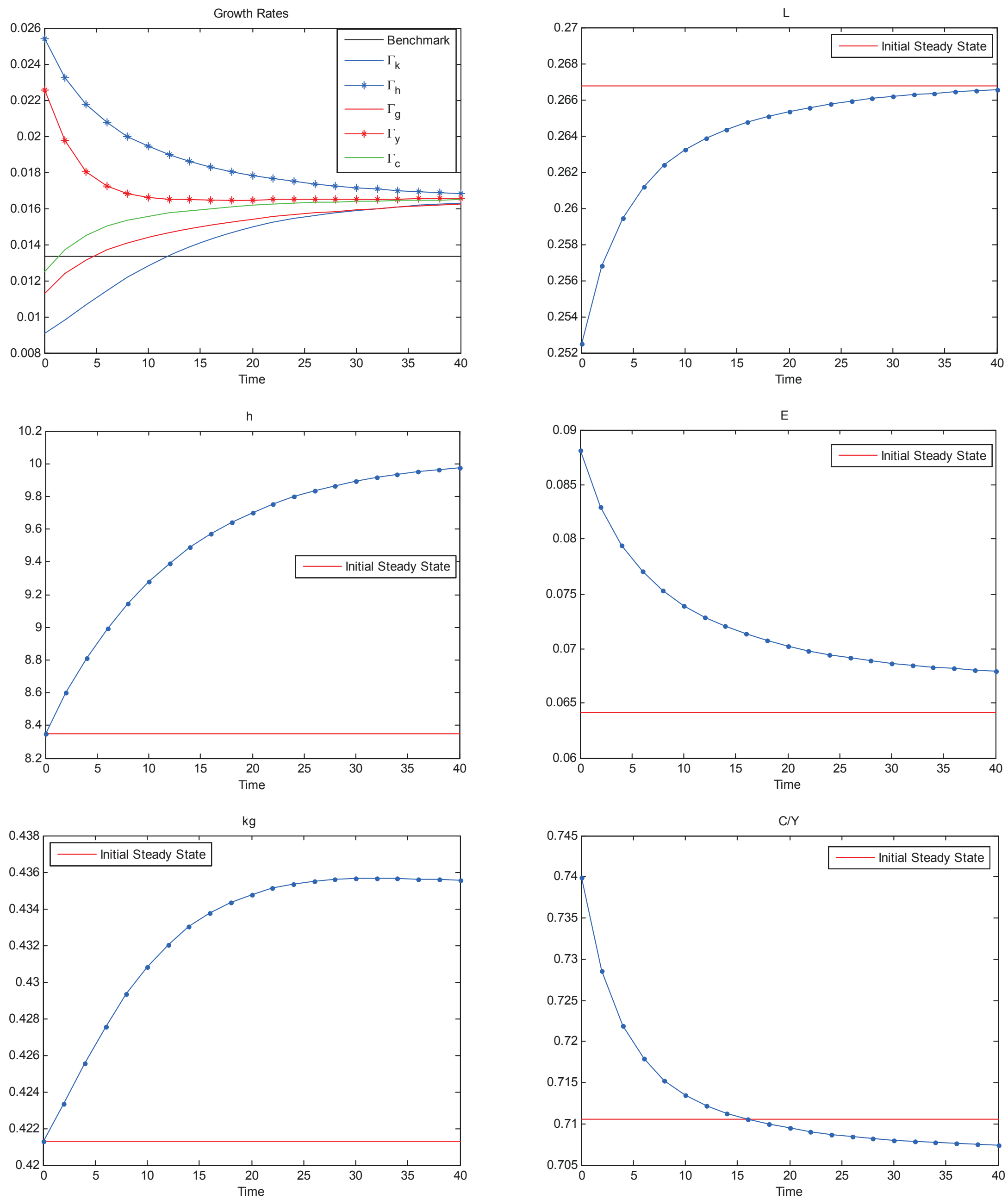
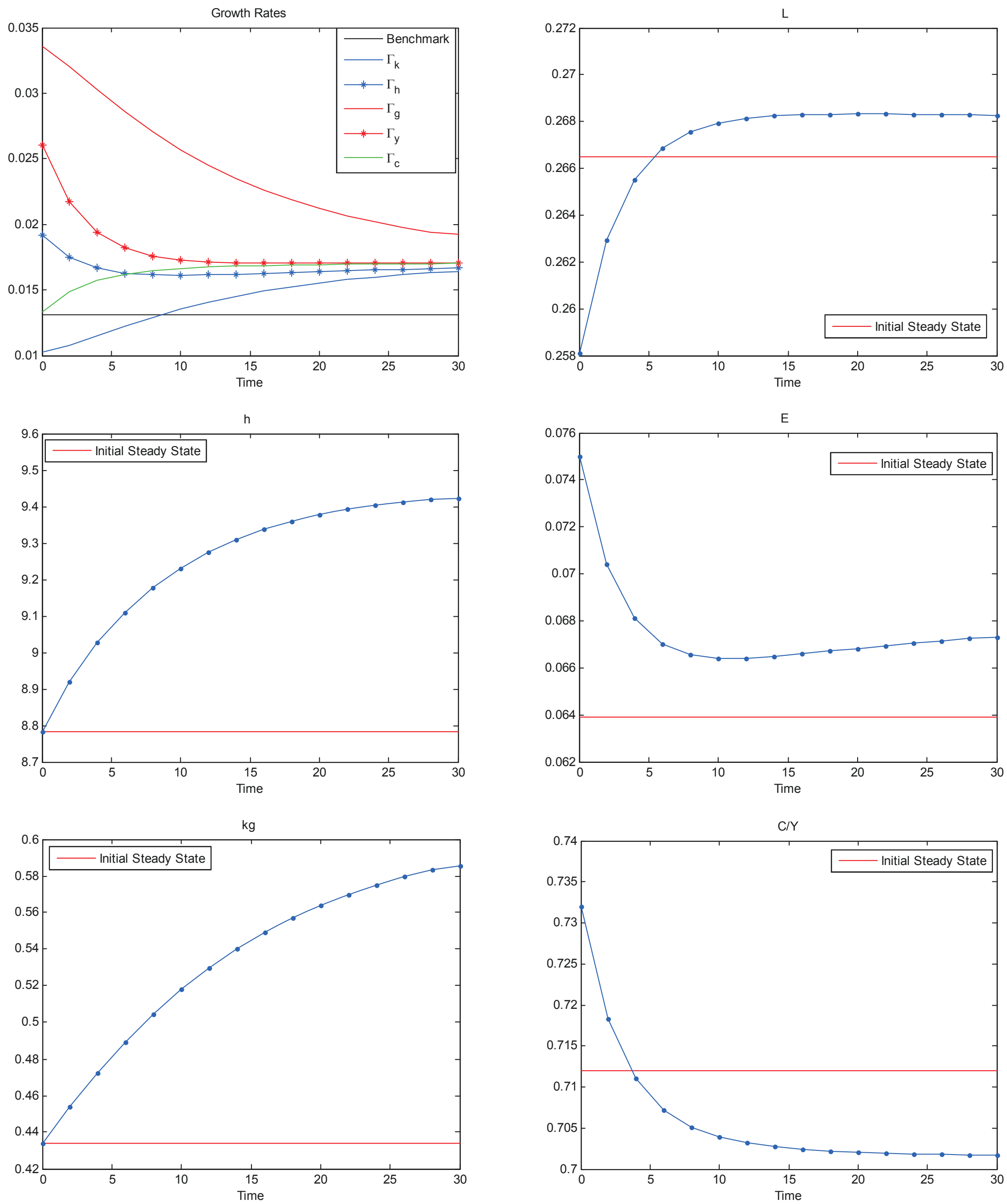


Figure 5 – Transitional dynamics: Optimal allocation of foreign aid



2 Essay 2 [Does Foreign Aid Raise Inequality?]

Abstract

This paper studies the relationship between foreign aid and inequality. I start by providing new and robust evidence that increases in aid flows are associated with subsequent increases in income inequality in the recipient countries. I then attempt to rationalize this empirical observation by proposing an heterogenous-agent growth model of a developing economy. Foreign aid received by the economy can be completely untied or tied to public investment in physical or human capital. The model implies that the dynamics of wealth and income inequality depend on the status of foreign aid. In particular, I show that untied aid reduces income inequality whereas tied aid raises it with a delay regardless of whether aid programs are tied to physical or human capital. To the extent that aid flows to developing countries are mostly tied, the model's predictions are therefore consistent with the empirical evidence. The results indicate that foreign aid improves average welfare but increases its dispersion across private agents. I also study the implications of foreign aid for the growth-inequality relationship.

> *The top 10 percent no longer takes in one-third of our income – it now takes half. Whereas in the past, the average CEO made about 20 to 30 times the income of the average worker, today's CEO now makes 273 times more. And meanwhile, a family in the top 1 percent has a net worth 288 times higher than the typical family, which is a record for this country. So the basic bargain at the heart of our economy has frayed. In fact, this trend towards growing inequality is not unique to America's market economy. Across the developed world, inequality has increased. Some of you may have seen just last week, the Pope himself spoke about this at eloquent length. "How can it be," he wrote, "that it is not a news item when an elderly homeless person dies of exposure, but it is news when the stock market loses two points?"*

President Barack Obama, 2013

2.1 Introduction

Developing countries exhibit greater wealth and income inequality than developed economies. The Gini index is generally 1.5 to twice as large in the former group of countries than in the latter (Deininger and Squire (1996), Solt (2009), Ortiz and Cummins (2011)). Clearly, this gap can be attributed to a multitude of factors pertaining both to the socioeconomic structure of relatively poor countries and to the nature of shocks impinging on them. In this regard, a distinctive feature of developing countries is that a significant fraction of their resources is in the form of foreign aid. As an illustrative statistic, foreign aid represents roughly 15.26 percent of the GDP of the least developed countries.[16] A natural question therefore is whether foreign aid flows partly explain the relatively high level of inequality in developing countries.

The present paper addresses this question in two parts. In the first part, I provide new evidence that foreign aid leads to a subsequent widening in income inequality in the recipient economy: After controlling for other factors that may also affect income inequality, such as economic growth, population growth, and the level of democracy, I find that the Gini index is positively associated with past aid flows. This evidence is robust to the lag at which foreign aid is measured, to splitting the sample by income level, and to the use of instruments to account for the potential endogeneity of the regressors.

In the second part, and in order to understand the distributional effects of foreign aid, I develop a neoclassical growth model of a developing economy with heterogeneous agents. The aid flows received by the economy can be either tied to public spending or completely untied. In the former case, aid is intended for productive investments in physical or human capital, whereas in the latter, it takes the form of a pure transfer to households. The allocation of time between work and leisure is chosen optimally by households who can accumulate human capital through a learning-by-doing process. The model is used to analyze the dynamics of income and wealth inequality following an inflow of aid, conditionning the analysis on the allocation scheme of aid flows.[17]

In this respect, the main contribution of this paper is to bring together two bodies of literature that have so far remained largely disconnected: the literature on the macroeconomic effects of foreign aid and the one on inequality. The literature on foreign aid is mainly empirical and little theoretical research has been done.[18] In a series of papers, Chatterjee and Turnovsky (2004, 2005, 2007) analyze the impact of tied versus untied aid on economic growth and welfare under various assumptions related to the labor market and production technology. Bouakez and Gouba (2016) expanded Chatterjee and Turnovsky's framework to account for human capital accumulation.

[16] World Bank online database (1960-2010).

[17] García-Peñalosa and Turnovsky (2006, 2007, 2008) develop a general equilibrium framework that allows to jointly analyze the dynamics of wealth and income inequality. However, that earlier work does not study foreign aid and allows for a single source of heterogeneity, namely, the initial endowments of physical capital. In contrast, the present paper introduces two additional sources of heterogeneity related to the initial endowments of human capital and foreign debt.

[18] Examples of empirical studies include those by Papenek (1973), Levy (1988), Hadjimichael, et. al. (1995), Durbarry et. al. (1998), Burnside and Dollar (2000), Hansen and Tarp (2000), Dalgaard and Hansen (2000), Hansen and Tarp (2000 and 2001), Lensink and White (2001), Collier and Dollar (2001, 2002) and Dalgaard, et.al. (2004)

They analyze the growth and welfare implications on the recipient economy, when foreign aid is completely untied or tied either to public investment in infrastructure, or to public spending on education. However, these authors conduct their analysis within a representative agent framework and do not study the effects of foreign aid on wealth and income inequality.

Inequality, on the other hand, has been the subject of a vast literature in development economics, and one major strand of that literature focused on the relationship between inequality and economic growth. Following the seminal work of Kuznets (1955), who specifies an inverted-U relationship between income inequality and the level of development,[19] several papers have investigated this issue, but no consensus has been reached.[20] Very few papers, however, have empirically examined the direct link between inequality and foreign aid (e.g., Herzer and Nunnenkamp (2012) and Bjørnskov (2010)), and to my knowledge, no theoretical research has been done on this subject. In addition to addressing this largely overlooked question in the literature, the present paper can bring new and useful insights on the relationship between economic growth and inequality, namely, how this relationship depends on foreign aid and its allocation scheme.

I find that the dynamics of wealth and income inequality depend on whether foreign aid is tied or not. An untied aid program has positive wealth effects that increase aggregate leisure and reduce wealth inequality. Wealthier agents enjoy more leisure while poorer ones increase their labor supply, thereby reducing income inequality. While both wealth and income inequality decrease monotonically following an untied aid program, they increase with a delay when all aid is tied. In the latter case, the aid shock initially leaves wealth inequality essentially unaffected and slightly decreases income inequality, but both variables increase monotonically in the subsequent periods. To the extent that aid flows to developing countries are mostly tied, the model's predictions are therefore consistent with the empirical evidence. Regardless of how the transfer is allocated, foreign aid improves welfare in the recipient economy. On the other hand, it increases welfare inequality (the dispersion of average welfare). When the transfer is completely tied to human capital, the recipient economy achieves the largest welfare gain and records the lowest dispersion of welfare, compared to the alternative allocation schemes.

[19] That is, at low levels of development, an increase in per capita income rises inequality, but beyond a certain threshold, further increases in income reduce inequality.

[20] Some papers find a negative relationship, between inequality and economic growth, which has been explained by (*i*) credit market imperfections, which reduce productive investments in physical or human capital (Galor and Zeira (1993); Fishman and Simhon (2002)), (*ii*) the effects of inequality on redistributive taxation (Alesina and Rodrik (1994); Persson and Tabellini (1994); Benabou (1996b)) and (*iii*) the sociopolitical consequences of inequality (Benhabib and Rustichini (1996)). Other papers find a positive inequality-growth relationship, which has been attributed to (*i*) technological changes, which tend to concentrate high-skilled workers in advanced sectors (Galor and Tsiddon (1997a, 1997b)), (*ii*) a positive correlation between inequality and changes in taxation rate (Saint-Paul and Verdier (1993); Li and Zou, (1998)) and (*iii*) the process of democratization when political participation is determined by education (Bourguignon and Verdier (2000)).

Moreover, the time profile of wealth and income inequality depends on the category of public spending to which aid is tied. Following an aid program that is tied to productive investment in human capital, wealth and income inequality increase gradually whereas they have hump-shaped transitional paths when aid is tied to public capital. On the one hand, tied aid increases labor supply, which raises the accumulation of human and physical capital (public and private) and improves the productive capacity of the recipient economy. On the other hand, it increases external borrowing which reduces aggregate wealth in the long run. Depending on the category of spending to which aid is tied, one of these two effects will dominate. When foreign aid is tied to human capital, foreign debt rises more than physical capital, which raises monotonically wealth inequality. Agents with above-average wealth increase their labor supply whereas poorer agents enjoy more leisure, leading to greater income inequality. Following an aid program that is tied to public capital, foreign debt rises for a few periods, and wealth and income inequality increase. But as the economy accumulates physical and public capital and relies less on external borrowing, the situation reverses and both wealth and income inequality start to decline.

The theoretical framework delivers some important implications regarding the growth-inequality link. More specifically, this relationship is not invariant to the allocation of foreign aid: the model implies a negative relationship between wealth and income inequality and economic growth when all aid is untied. Following an aid shock tied to human capital, a positive relationship is found whereas an inverted-U curve is obtained after an aid shock tied to public capital. During the transition towards the new equilibrium, an untied aid program increases per capita output, but public and private capital accumulation does not rise sufficiently to translate into a higher stock of capital in the long-run. Poorer agents increase their labor supply and invest in private capital while wealthier agents do the opposite and choose to enjoy more leisure. As a result, inequality decreases as the growth rate of output increases. An aid shock tied to productive investments triggers capital accumulation, which increases the growth rate of per capita output. Following a foreign aid program that is tied to human capital, the larger stock of capital gradually improves the marginal productivity of average labor supply, which increases in the long-run. As the equilibrium wage rate decreases due to higher labor supply, inequality rises since wealthier people accumulate capital, while poorer ones desinvest. After an aid shock that is tied to public capital, the economy grows at a higher rate and the inverted-U curve reflects the hump-shaped transitional paths of wealth and income inequality.

The rest of the paper is organized as follows: Section 2 reports the preliminary empirical analysis. Section

3 presents the economy and derives the aggregate dynamics. Section 4 examines the distributional dynamics of wealth and income. Calibration and simulations related to the aggregate dynamics and the distributional effects of a permanent aid shock are discussed in Section 5. Section 6 discusses the growth-inequality relationship while Section 7 concludes.

2.2 Empirical Analysis

This section analyzes the relationship between foreign aid and income inequality from an empirical standpoint. For this purpose, I estimate the following panel regression:

$$Inequality_{it} = \beta_1 Aid_{it-\tau} + \beta_2 X_{it} + \alpha_i + \epsilon_{it}, \tag{44}$$

where subscripts i and t index countries and years, respectively, τ is the lag at which Aid is measured; $Inequality$ is measured by the Gini coefficient, Aid is measured by the real Official Developement Assistance (ODA) as a percentage of GDP; X_{it} is a matrix of control variables; α_i are country dummy variables and ϵ_{it} is the error term. As controls, I consider the growth rate of per capita real Gross Domestic Product (GDP), a measure for democracy represented by the Polity IV score, the growth rate of total population, the value added of agricultural sector as a percentage of GDP, the percentage of total population that is younger than 15 years.

The inclusion of democracy is motivated by the fact that some empirical papers such as those by Reuveny and Li (2003) and Bjørnskov (2010) find a significant relationship between inequality and democracy. I control for population growth because developing countries, which exhibit higher levels of inequality than developed countries, also record higher growth rates of their total population. Furthermore, it is often argued that employment in agriculture can affect income inequality. Due to the limited availability of data, however, the value added in agricultural sector as a percentage of GDP is used as a proxy. Finally, I control for youth population represented by population younger than 15 years since a link between income inequality and youth population has also been established in the literature. Summary statistics for dependent and control variables are reported in Table X.

I first run a fixed-effect estimation of Eq.(44), for $\tau = 0, 1, 2, 3$. As a robustness check, and in order to account for the potential endogeneity of the regressors, I also estimate this equation using Arellano and Bond (1991)'s generalized method of moments (GMM) differenced estimator, which consists in running the following regression:

$$\Delta Inequality_{it} = \beta_1 \Delta Aid_{it-\tau} + \beta_2 \Delta X_{it} + \Delta\epsilon_{it}, \tag{45}$$

where Δ is the first-difference operator.

Data

Empirical studies on income distribution are often limited by the quality and the availability of data. Several papers rely on Gini data coming from the the Luxembourg Income Study (LIS) database or the UNU-WIDER World Income Inequality Database (WIID). However, the use of these data sets involves some tradeoffs. In fact, the LIS has generated the most comparable inequality observations, but it has a limited coverage across countries and over time. The WIID data provides a larger set of income-inequality statistics, but with a significant loss of comparability. In this paper, data on Gini coefficient are taken from the Standardized World Income Inequality Database (SWIID), developed by Solt (2009). The SWIID combines information from LIS and WIID data to generate an improved data set, which maximizes the coverage and the comparability of income inequality observations. The GINI index used in this paper is calculated from the net income. ODA as a share of GDP is collected from the DAC database (OECD, 2012).

The series of control variables are taken from the World Development Indicator online database, except for the democracy variable (Polity IV) which comes from the Income and Democracy Data.[21] Income growth is measured by the growth rate of real per capita GDP.

Empirical Results

Estimation is performed using an unbalanced panel dataset of 34 developing countries over the period 1965-2009 (44 years). The sample is restricted to countries for which continuous annual data on foreign aid and income inequality are available for a sufficiently long period of time. Table XI reports the results from the fixed effects and first differenced GMM estimations for the complete sample. Models 1 through 4 correspond to values of τ ranging from 0 to 3, respectively.

Starting with the fixed effects estimation, the results indicate that the estimated coefficient on current aid is small and statistically insignificant. In contrast, when a lagged measure of aid is instead included in the regression, its effect is found to be positive and significant at the 1 percent level. More specifically, an increase in current aid by 1 percent of GDP raises income inequality in subsequent years by around 0.3 percentage points. Inequality is also found to be negatively associated with the level of democracy, the share of agriculture in GDP, and youth population.

[21] Available on the internet at: economics.mit.edu/files/5000

These results remain largely unchanged when using the first differenced GMM estimation. In particular, they indicate a positive and delayed effect of foreign aid on inequality. The latter still negatively depends on democracy and the share of agriculture in GDP, but the effect of youth population is no longer significant. It is worth noting that for all of the estimated versions (Models 1 through 4), the Sargan test does not reject the null hypothesis of instrument validity and the Arellano-Bond test for autocorrelation rejects the null hypothesis of first order serial correlation for the AR(1) and AR(2) types.

The robustness of these results is checked by disaggregating the complete sample according to countries' income level. Following the classification of the World Bank, countries are characterized as Low Income or Middle Income Countries.[22] Table XII presents the fixed-effect estimation results for each of these two groups. Results based on Arellano and Bond's methodology are reported in Table XIII. Both tables show that foreign aid raises income inequality with a delay in both sub-samples, although the effect is generally larger in low- than in middle-income countries. Hence, the positive relationship between past aid and income inequality appears to be a robust empirical fact. In the following section, I develop a theoretical model that attemps to rationalize it.

2.3 Theoretical Framework

I consider a small open economy populated by heterogeneous households who differ in their initial endowments of physical capital, human capital and foreign debt. Human capital enters as an input in the production function and is accumulated through a learning-by-doing mechansim. The economy receives foreign aid, which can be either untied or tied to public investment in physical or human capital.

2.3.1 Description of the economy

Technology

It is assumed that aggregate output, Y, is produced according to a Cobb-Douglas technology

$$Y = F(l, K_G, H, K) = A((1-l)K_G)^{\omega_1}((1-l)H)^{\omega_2}K^{1-\omega_1-\omega_2},\ A > 0 \tag{46}$$

where $\omega_1, \omega_2 \in (0,1)$, K and K_G denote the per capita stock of private capital and the per capita stock of public capital respectively, $(1-l)H$ is effective labor and $(1-l)$ denotes the per capita raw labor supply. Owing to constant returns to scale in both the private factors (K and $(1-l)$) and the reproducible factors (K_G, H and K),

[22] Middle Income Countries include lower and upper middle income countries

the production function can generate permanent endogenous growth. The wage rate, w, and the rental price of capital, r, are given by

$$w(l, K_G, H, K) = (\omega_1 + \omega_2)\frac{Y}{1-l},$$

$$r(l, K_G, H, K) = (1 - \omega_1 - \omega_2)\frac{Y}{K}.$$

Households

The economy is populated by N heterogeneous households indexed by i. There are three main sources of heterogeneity: the households' initial endowments of private capital, K_{i0}, human capital, H_{i0}, and net foreign debt, D_{i0}. I define the units of private capital, human capital and foreign debt held at time t by household i by $K_i(t)$, $H_i(t)$ and $D_i(t)$ respectively. Summing over all households yields the total stock of physical capital, human capital and the national foreign debt

$$\begin{aligned} K_T(t) &= \int_0^N K_i(t)di, \\ H_T(t) &= \int_0^N H_i(t)di, \\ D_T(t) &= \int_0^N D_i(t)di. \end{aligned}$$

The relative shares of capital and foreign debt held by household i are defined as

$$k_i(t) = \frac{K_i(t)}{K(t)} \text{ and } d_i(t) = \frac{D_i(t)}{D(t)},$$

where $K(t)$, and $D(t)$ are average quantities. The relative capital and foreign debt have a mean equal to 1 and their initial standard deviation, σ_{k0} and σ_{d0} are given.[23]

Each household i has a unit of time and devotes the proportion l_i to leisure, while the balance $(1 - l_i)$ is allocated to labor. Household i maximizes an isoelastic utility function

$$U(C_i, l_i) = \int_0^\infty \frac{1}{\gamma}(C_i l_i^\theta)^\gamma e^{-\rho t} dt, \tag{47}$$

where C_i is consumption, $\rho \in (0, 1)$ is the discount rate, $\frac{1}{1-\gamma}$ determines the intertemporal elasticity of substitution, and $\theta > 0$ reflects the weight assigned to leisure in the utility function.

[23] The mean of the relative shares of capital and foreign debt are
$\frac{1}{N}\int_0^N k_i(t)di = \frac{1}{NK(t)}\int_0^N K_i(t)di = \frac{K_T(t)}{NK(t)} = 1,$
$\frac{1}{N}\int_0^N d_i(t)di = \frac{1}{ND(t)}\int_0^N D_i(t)di = \frac{D_T(t)}{ND(t)} = 1.$

In each period, household i receives an after-tax income which finances consumption, interest payments on debt, capital accumulation and lump-sum taxes. It is assumed that households have access to a world capital market, but being a developing economy, they encounter some restrictions when borrowing from abroad. Originally proposed by Bardhan (1967), these constraints are characterized by a premium which increases with the national debt-capital ratio. The borrowing rate, $i(D/K)$, is of the form[24]

$$i(D/K) = i^{*} + \exp(a\frac{D}{K}) - 1,$$

where i^{*} is the exogenous risk-free world interest rate and a is a positive parameter.

Household i accumulates private capital, which depreciates at the constant rate δ_K and the gross investment in capital, I_i, is subject to convex costs of adjustment. Thus, household's private capital evolves according to

$$\dot{K}_i = I_i - \delta_K K_i, \qquad \delta_K \in (0, 1). \tag{48}$$

The budget constraint faced by households is then given by

$$\dot{D}_i = C_i + i(D/K)D_i + I_i + \Pi_K + T_i - (1-\tau)Y_i \tag{49}$$

where τ, T_i and $\Pi_K = \phi_1 \frac{I_i^2}{2K_i}$ are, respectively, the income tax rate, some exogenous lump-sum taxes and convex costs of adjustment.

Households can also accumulate human capital through a learning-by-doing effect stemming from the production process. I follow Krugman (1987) and Lucas (1988) by assuming that this effect is completely external to each household. As a result, the household's human capital accumulation depends on the average labor supply, which no agent can affect. Hence, the household's stock of human capital evolves according to

$$\dot{H}_i = BG_H^{\eta}((1-l)H_i)^{1-\eta} - \delta_H H_i, \quad B > 0, \quad \eta,\ \delta_H \in (0, 1), \tag{50}$$

where δ_H denotes the depreciation rate of human capital and G_H is the current flow of public spending on human capital, which can be seen as a measure of quality.[25] The human capital technology is increasing and concave in the aggregate labor supply and exhibits constant returns to scale with respect to G_H and H_i.

Public Sector

[24] This specification has been used in Chatterjee and Turnovsky (2005, 2007).

[25] Heyneman (1984) and Card and Krueger (1992) provide empirical evidence of positive relationship between the quality of school and the rate of return to education.

The government collects lump-sum taxes and income tax revenues and receives foreign transfers in the form of aid. It spends on education services, invests in public capital and, as stated above, these expenses are all productive. In each period, the public sector runs a balanced budget constraint given by

$$T + \tau Y + Aid = G_H + G + \Pi_G \tag{51}$$

where $\Pi_G = \phi_2 \frac{G^2}{2K_G}$ are convex costs of adjustment related to public capital accumulation.

It is assumed that a constant share of output is spent on the provision of education services and public capital. Moreover, foreign aid may be untied (pure transfers) or tied either to public investment in capital or to public spending on human capital. That is,

$$G_H = \lambda_H Y + \alpha_H Aid, \tag{52a}$$

$$G = \lambda_G Y + \alpha_G Aid, \tag{52b}$$

where $(\lambda_H, \lambda_G) \in (0,1)$, α_H represents the fraction of aid flows tied to human capital, α_G is the share of foreign aid invested in public capital and the balance $(1 - \alpha_H - \alpha_G)$ is the share of untied aid.

Aid transfers from abroad are assumed to be equal to a constant fraction of output:

$$Aid = \mu Y, \qquad \mu \in (0,1). \tag{53}$$

Gross investments increase the stock of public capital, which depreciates at a constant rate, δ_G and evolves over time according to

$$\dot{K}_G = G - \delta_G K_G, \qquad \delta_G \in (0,1). \tag{54}$$

Substituting (52b) and (53) into (54) yields the growth rate of public capital

$$\frac{\dot{K}_G}{K_G} \equiv \Psi_G = (\lambda_G + \alpha_G \mu)\frac{Y}{K_G} - \delta_G. \tag{55}$$

Finally, defining the wealth of household i and the aggregate wealth in the economy by V_i and V, respectively, I assume that[26]

$$\frac{T_i(t)}{T(t)} = \frac{V_i(t)}{V(t)},$$

which ensures that

$$\int_0^N T_i di = \frac{T}{V}\int_0^N V_i di = T.$$

[26] This assumption rules out any direct distributional effects coming from lump-sum taxes, which are supposed to be arbitrary and exogenous.

2.3.2 Household optimization

The household's optimization problem is to choose her levels of consumption, leisure, investment as well as the rates of debt, private and human capital accumulation, to maximize the intertemporal utility (47) subject to the budget constraint (49) and the accumulation equations, (48) and (50). In performing this optimization, the household takes the public policies and the borrowing rate $i(D/K)$, as given. The optimality conditions with respect to the three first decisions are

$$C_i^{\gamma-1} {l_i}^{\theta\gamma} = \xi_i, \tag{56a}$$

$$\theta C_i^{\gamma} {l_i}^{\theta\gamma-1} = \xi_i(1-\tau)w(l, K_G, H, K), \tag{56b}$$

$$\frac{I_i}{K_i} = \frac{q_i - 1}{\phi_1}, \tag{56c}$$

where q_i is the household i's shadow value of capital devided by the marginal utility of wealth, ξ_i.

From Eqs. (56a) and (56b), I obtain

$$\theta \frac{C_i}{l_i} = (1-\tau)w(l, K_G, H, K), \tag{57}$$

which states that the marginal rate of substitution between consumption and leisure must be equated to the after-tax equilibrium wage.

Using Eq. (56c), the growth rate of the household i's private capital can be expressed as follows

$$\frac{\dot{K}_i}{K_i} \equiv \frac{q_i - 1}{\phi_1} - \delta_K. \tag{58}$$

The corresponding first-order conditions with respect to D_i, K_i and H_i lead to the following usual arbitrage relationships

$$\rho - \frac{\dot{\xi}_i}{\xi_i} = i(D/K), \tag{59a}$$

$$\frac{\dot{q}_i}{q_i} + \frac{1}{q_i}\left[\frac{(q_i-1)^2}{2\phi_1} + (1-\tau)r(l, K_G, H, K)\right] - \delta_K = i(D/K), \tag{59b}$$

$$\frac{\dot{m}_i}{m_i} + \left[(1-\eta)BG_H^{\eta}\left((1-l)H_i\right)^{-\eta}(1-l) - \delta_H\right] + \frac{1}{m_i}\left[(1-\tau)\frac{\partial Y_i}{\partial H_i}\right] = i(D/K), \tag{59c}$$

where m_i is the household i's shadow value of human capital devided by ξ_i. Eqs. (59a), (59b) and (59c) state that the returns on consumption and investment in both private capital and human capital must be equated to the cost of borrowing from abroad.

Recalling (59a), the time derivative of condition (56a) yields

$$(\gamma-1)\frac{\dot{C}_i}{C_i}+\theta\gamma\frac{\dot{l}_i}{l_i}=\frac{\dot{\xi}_i}{\xi_i}=\rho-i(D/K), \tag{60a}$$

which gives an expression for the growth rate of individual i's consumption

$$\frac{\dot{C}_i}{C_i}\equiv\frac{1}{1-\gamma}\left[i(D/K)-\rho+\theta\gamma\frac{\dot{l}_i}{l_i}\right]. \tag{60b}$$

The growth rate of the stock of human capital held by household i is given by

$$\frac{\dot{H}_i}{H_i}=BG_H^{\eta}\left((1-l)H_i\right)^{-\eta}(1-l)-\delta_H. \tag{61}$$

Finally, the household's budget constraint, (49), which can be rewritten as

$$\dot{D}_i=i(D/K)D_i+I_i+\Pi_K+T_i-(1-\tau)r(l,K_G,H,K)K_i+(1-\tau)w(l,K_G,H,K)\left[\frac{l_i}{\theta}-(1-l_i)\right], \tag{62}$$

and the following transversality conditions must hold:

$$\lim_{t\to\infty}\xi_i D_i\ e^{-\rho t}=0;\quad \lim_{t\to\infty}\xi_i q_i K_i\ e^{-\rho t}=0;\quad \lim_{t\to\infty}\xi_i m_i H_i\ e^{-\rho t}=0. \tag{63}$$

2.3.3 Macroeconomic equilibrium and aggregate dynamics

Equilibrium

I need to determine the macroeconomic equilibrium and the dynamics of the aggregate economy before I obtain the distributions of wealth and income. The equilibrium in the domestic capital, labor and the world capital markets is given by

$$K(t)=\frac{1}{N}\int_0^N K_i(t)di,$$

$$L(t)\equiv 1-l(t)=\frac{1}{N}\int_0^N (1-l_i(t))di,$$

$$D(t)=\frac{1}{N}\int_0^N D_i(t)di.$$

Appendix A shows that

$$\frac{\dot{C}_i}{C_i}=\frac{\dot{C}}{C};\quad \frac{\dot{l}_i}{l_i}=\frac{\dot{l}}{l};\ \text{and } q_i=q_j=q \text{ for all } i \text{ and } j, \tag{64}$$

implying the same growth rate for consumption and leisure at both individual and aggregate levels. In addition, all individual capital stocks and the aggregate private capital grow at the same rate, i.e.,

$$\Psi_K \equiv \frac{\dot{K}}{K} = \frac{\dot{K}_i}{K_i} = \frac{q-1}{\phi_1} - \delta_K.$$

The aggregate optimality conditions corresponding to (57), (59b) and (59c) are

$$\theta\frac{C}{l} = (1-\tau)w(l, K_G, H, K), \tag{65a}$$

$$\frac{\dot{q}}{q} + \frac{1}{q}\left[\frac{(q-1)^2}{2\pi_1} + (1-\tau)r(l, K_G, H, K)\right] - \delta_K = i(D/K), \tag{65b}$$

$$\frac{\dot{m}}{m} + \left[(1-\eta)BG_H^{\eta}\left((1-l)H\right)^{-\eta}(1-l) - \delta_H\right] + \frac{1}{m}\left[(1-\tau)\frac{\partial Y}{\partial H}\right] = i(D/K). \tag{65c}$$

Aggregating over all households, (60b), (50) and (62) yield

$$\frac{\dot{C}}{C} \equiv \frac{1}{1-\gamma}\left[i(D/K) - \rho + \theta\gamma\frac{\dot{l}}{l}\right], \tag{65d}$$

$$\frac{\dot{H}}{H} = BG_H^{\eta}\left((1-l)H\right)^{-\eta}(1-l) - \delta_H, \tag{65e}$$

$$\dot{D} = i(D/K)D + I + \Pi_K + T - (1-\tau)r(l, K_G, H, K)K + (1-\tau)w(l, K_G, H, K)\left[\frac{l}{\theta} - (1-l)\right], \tag{65f}$$

where Eqs. (65d) and (65e) are the growth rates of aggregate consumption and human capital, while (65f) is the accumulation equation for national debt.

The national budget constraint can be written as

$$\dot{D} = C + I + G_H + G + \Pi_K + \Pi_G + i(D/K)D - Y - Aid, \tag{66}$$

which states that total expenses on consumption, private and public capital, education and interest payments are financed through national debt accumulation, output produced and foreign transfers received.

Aggregate dynamics

The dynamic behavior of the aggregate economy can be characterized by a system of six nonlinear differential equations in $h = H/K$, $k_g = K_G/K$, $d = D/K$, q, m and l. Consequently, the balanced-growth path is a set of functions $\{h, k_g, d, q, m, l\}_{t=0}^{\infty}$ such that the stocks of human capital and public capital and the national debt, all grow at the same constant rate, Ψ, while q, m, and l remain constant. The equilibrium growth rate of the economy is given by

$$\Psi = \frac{q-1}{\phi_1} - \delta_K.$$

As shown in Appendix A the following system is obtained

$$\frac{\dot{h}}{h} = \frac{\dot{H}}{H} - \frac{\dot{K}}{K} = B\left((\lambda_H + \alpha_H\mu)\frac{y}{h}\right)^{\eta}(1-l)^{1-\eta} - \delta_H - \left(\frac{q-1}{\phi_1} - \delta_K\right), \tag{67a}$$

$$\frac{\dot{k}_g}{k_g} = \frac{\dot{K}_G}{K_G} - \frac{\dot{K}}{K} = (\lambda_G + \alpha_G\mu)\frac{y}{k_g} - \delta_G - \left(\frac{q-1}{\phi_1} - \delta_K\right), \tag{67b}$$

$$\frac{\dot{d}}{d} = \frac{\dot{D}}{D} - \frac{\dot{K}}{K} = i(d) + \frac{1}{d}\left[c + \frac{q^2-1}{2\phi_1} + (\lambda_G + \alpha_G\mu)y\left(1 + \frac{\phi_2}{2}(\lambda_G + \alpha_G\mu)\frac{y}{k_g}\right) + (\lambda_H + \alpha_H\mu)y - (1+\mu)y\right] - \left(\frac{q-1}{\phi_1} - \delta_K\right), \tag{67c}$$

$$\frac{\dot{q}}{q} = i(d) + \delta_K - \frac{1}{q}\left[\frac{(q-1)^2}{2\phi_1} + (1-\tau)\frac{\partial Y}{\partial K}\right], \tag{67d}$$

$$\frac{\dot{m}}{m} = i(D/K) - \left[(1-\eta)B((\lambda_H + \alpha_H\mu)\frac{y}{h})^{\eta}(1-l)^{1-\eta} - \delta_H\right] - \frac{1}{m}\left[(1-\tau)\frac{\partial Y}{\partial H}\right], \tag{67e}$$

$$\frac{\dot{l}}{l} = \frac{\rho - i(d) + (1-\gamma)\left[\omega_1\frac{\dot{K}_G}{K_G} + \omega_2\frac{\dot{H}}{H} + (1-\omega_1-\omega_2)\frac{\dot{K}}{K}\right]}{\gamma(1+\theta) - 1 - (1-\gamma)(1-\omega_1-\omega_2)\frac{l}{1-l}}, \tag{67f}$$

where

$$y \equiv \frac{Y}{K} = A((1-l)k_g)^{\omega_1}((1-l)h)^{\omega_2}, \tag{68a}$$

$$c \equiv \frac{C}{K} = \frac{(1-\tau)(\omega_1+\omega_2)}{\theta}\frac{l}{1-l}y, \tag{68b}$$

$$i(d) = i^* + \exp(ad) - 1, \tag{68c}$$

and the growth rates of the different types of capital are

$$\frac{\dot{K}}{K} \equiv \Psi_K = \frac{q-1}{\phi_1} - \delta_K, \tag{69a}$$

$$\frac{\dot{H}}{H} \equiv \Psi_H = B\left((\lambda_H + \alpha_H\mu)\frac{y}{h}\right)^{\eta}(1-l)^{1-\eta} - \delta_H, \tag{69b}$$

$$\frac{\dot{K}_g}{K_g} \equiv \Psi_G = (\lambda_G + \alpha_G\mu)\frac{y}{k_g} - \delta_G. \tag{69c}$$

Eqs. (67a)-(67c) express the equilibrium growth rates of human capital, public capital and national debt, all normalized by the stock of private capital. Eqs. (67d) and (67e) specify the dynamics of the shadow prices of private and human capital, while Eq. (67f) describes the evolution of leisure. Together, these equations yield a dynamic system of three state variables (h, k_g and d) and three jump variables (q, m and l). As shown in Turnovsky and García-Peñalosa (2008), because the utility function is homogeneous, the dynamics of aggregate quantities do not depend on distributional features. The steady-state values of h, k_g, d, q, m and l are determined when $\dot{h} = \dot{k}_g = \dot{d} = \dot{q} = \dot{m} = \dot{l} = 0$.[27]

[27] The Appendix B provides the complete set of equations for the steady-state equilibrium.

2.4 Distributional Dynamics

2.4.1 Relative wealth distribution

As established in Appendix A, q is constant and identical across households. Define the wealth of household i at time t by

$$V_i(t) = qK_i(t) - D_i(t), \tag{70}$$

the corresponding aggregate wealth is

$$V(t) = qK(t) - D(t). \tag{71}$$

Denoting household i's relative wealth by $v_i(t) = \frac{V_i(t)}{V(t)}$, I obtain the following dynamic equation (details of derivations are in Appendix C)

$$\dot{v}_i(t) = \frac{(1-\tau)w(l, K_G, H, K)}{V}\left[1 - l_i\frac{1+\theta}{\theta} - \left(1 - l\frac{1+\theta}{\theta}\right)v_i\right], \tag{72}$$

where $w(.)$ and V are derived from the aggregate equilibrium and the initial relative wealth v_{i0} is given. Considering that $\frac{\dot{l}_i}{l_i} = \frac{\dot{l}}{l}$, individual and aggregate leisure are proportional:

$$l_i = \beta_i l,$$

where β_i is constant for each individual i and its average value is equal to 1.

Eq.(72) can be rewritten as follows

$$\dot{v}_i(t) = \frac{(1-\tau)w(l, K_G, H, K)}{V}\left[1 - \beta_i l\ \frac{1+\theta}{\theta} - \left(1 - l\frac{1+\theta}{\theta}\right)v_i\right] \tag{73}$$

As shown in Appendix C, the transversality condition implies

$$l > \frac{\theta}{1+\theta}. \tag{74}$$

Setting $\dot{v}_i = 0$, and recalling (74) provides the following positive relationship between relative wealth and relative leisure

$$(\bar{l}_i - \bar{l}) = \left(\bar{l} - \frac{\theta}{1+\theta}\right)(\bar{v}_i - 1). \tag{75}$$

Eq.(75) explains the mechanism by which the household's relative wealth affects the distribution of income, that is, wealthier households devote more time to leisure and work less because of their lower marginal utility of wealth.

Linearizing Eq.(73) around the steady-state yields

$$\dot{v}_i(t) = \frac{(1-\tau)w(\bar{l}, \bar{K}_G, \bar{H}, \bar{K})}{\bar{V}} \left[\left(\frac{1+\theta}{\theta}\right)(\bar{v}_i - \beta_i)\left(l(t) - \bar{l}\right) + \left(\bar{l}(\frac{1+\theta}{\theta}) - 1\right)(v_i(t) - \bar{v}_i) \right], \tag{76}$$

which describes the evolution of relative wealth. In Appendix D, I show that the stable path for v_i is

$$v_i(t) - 1 = n(t)(\bar{v}_i - 1), \tag{77a}$$

where

$$n(t) = 1 + \frac{\frac{(1-\tau)\bar{w}}{\bar{V}}\left(1 - \frac{l(t)}{\bar{l}}\right)}{\frac{(1-\tau)\bar{w}}{\bar{V}}\left(\bar{l}(\frac{1+\theta}{\theta}) - 1\right) - \epsilon}. \tag{77b}$$

Setting $t = 0$, I obtain

$$\begin{aligned} v_{i0} - 1 &= n(0)(\bar{v}_i - 1) \\ &= \left[1 + \frac{\frac{(1-\tau)\bar{w}}{\bar{V}}\left(1 - \frac{l(0)}{\bar{l}}\right)}{\frac{(1-\tau)\bar{w}}{\bar{V}}\left(\bar{l}(\frac{1+\theta}{\theta}) - 1\right) - \epsilon}\right](\bar{v}_i - 1), \end{aligned} \tag{78}$$

with v_{i0} given by the initial distribution of relative wealth. Once the time path of the aggregate economy and the distribution of initial wealth endowments are known, Eq.(78) determines the steady state distribution of wealth $(\bar{v}_i - 1)$. Having derived $(\bar{v}_i - 1)$, Eq.(75) determines household i's relative leisure time β_i, while Eqs. (77a) and (77b) provide the time path of the relative wealth, which can be expressed as follows

$$\begin{aligned} v_i(t) - \bar{v}_i &= \left(\frac{n(t) - 1}{n(0) - 1}\right)(v_{i0} - \bar{v}_i) \\ &= \left(\frac{\bar{l} - l(t)}{\bar{l} - l(0)}\right)(v_{i0} - \bar{v}_i) = (v_{i0} - \bar{v}_i)e^{\epsilon t}. \end{aligned} \tag{79}$$

The evolution of wealth and income inequality is measured and analyzed using the standard deviation. Since Eqs.(77a), (78) and (79) are linear, they can be expressed in terms of the standard deviation of the distribution of wealth. This yields

$$\sigma_v(t) = n(t)\bar{\sigma}_v, \tag{80a}$$

$$\sigma_{v0} = n(0)\bar{\sigma}_v, \tag{80b}$$

$$\sigma_v(t) - \bar{\sigma}_v = (\sigma_{v0} - \bar{\sigma}_v)e^{\epsilon t}. \tag{80c}$$

The relative wealth converges to a long run distribution with the same ranking of households as that of the initial wealth distribution. From Eqs.(80a) and (80b) one can notice that $\sigma_v(t) > \sigma_{v0}$ if and only if $n(t) > n(0)$ or equivalently, if $l(0) > l(t)$. Following an aid shock, the evolution of wealth inequality depends on how this shock affects labor supply and aggregate wealth. An aid shock that increases aggregate wealth will be associated with a reduction in wealth inequality because wealthier people who choose to enjoy more leisure choose also to accumulate capital at lower rates, while poorer people do the opposite.

2.4.2 Income distribution

Household i's total income from production at time t is defined by $Y_i(t) = r(t)K_i(t) + w(t)(1 - l_i(t))$. Thus, aggregate income is $Y(t) = r(t)K(t)+w(t)(1-l(t))$, and the relative income is denoted by $y_i(t) = \frac{Y_i(t)}{Y(t)}$. Recalling that $l_i(t) = \beta_i l(t)$, the relative income can be specified as a weighted average of the income coming from relative private capital and the relative labor income:

$$y_i - 1 = \frac{r(t)K(t)}{Y(t)}(k_i(t) - 1) + \frac{w(t)(1 - l(t))}{Y(t)} \frac{l(t)}{1 - l(t)}(1 - \beta_i), \tag{81}$$

where $\frac{r(t)K(t)}{Y(t)} = 1 - \omega_1 - \omega_2$ and $\frac{w(t)L(t)}{Y(t)} = \omega_1 + \omega_2$. Using (75), Eq.(81) can be rewritten as follows

$$y_i - 1 = (1 - \omega_1 - \omega_2)(k_i(t) - 1) - (\omega_1 + \omega_2)\frac{l(t)}{1 - l(t)}\left(1 - \frac{1}{\bar{\bar{l}}}\frac{\theta}{1+\theta}\right)(\bar{v}_i - 1), \tag{82a}$$

which together with (77a) yields

$$y_i - 1 = (1 - \omega_1 - \omega_2)(k_i(t) - 1) - (\frac{\omega_1 + \omega_2}{n(t)})\frac{l(t)}{1 - l(t)}\left(1 - \frac{1}{\bar{\bar{l}}}\frac{\theta}{1+\theta}\right)(v_i(t) - 1). \tag{82b}$$

The individual i's relative wealth can be expressed as

$$\begin{aligned} v_i - 1 &= \frac{qK}{V}(k_i - 1) - \frac{D}{V}(d_i - 1) \\ &= A_k(k_i - 1) + A_d(d_i - 1), \end{aligned} \tag{83}$$

where $A_k = \frac{qK}{V}$ and $A_d = 1 - A_k = -\frac{D}{V}$. Given the linearity of Eq.(83), it can be related to the distributions of private capital and foreign debt, σ_k, σ_d and their covariance σ_{kd}. Specifically,

$$\sigma_v = \left[A_k^2\sigma_k^2 + 2A_kA_d\sigma_{kd} + A_d^2\sigma_d^2\right]^{1/2}. \tag{84}$$

Considering the case where each household's relative debt matches her relative private capital ($d_i = k_i$) so that the heterogeneity is uniformly distributed across agents (i.e., $\sigma_k^2 = \sigma_d^2 = \sigma_{kd}$), (84) simplifies to

$$\sigma_v = \sigma_k = \sigma_d, \tag{85}$$

and Eq.(82a) implies

$$\sigma_y = \left[(1 - \omega_1 - \omega_2) - (\frac{\omega_1 + \omega_2}{n(t)})\frac{l(t)}{1 - l(t)}\left(1 - \frac{1}{\bar{\bar{l}}}\frac{\theta}{1+\theta}\right)\right]\sigma_v, \tag{86}$$

or, equivalently,

$$\sigma_y = s(t)\sigma_v. \tag{87a}$$

Since $l > \frac{\theta}{1+\theta}$, $s(t)$ is less than 1, implying that wealth is more unequally distributed than is income.

The long-run distribution of income can be expressed as

$$\bar{\sigma}_y = \bar{s}\bar{\sigma}_v, \tag{88}$$

where

$$\bar{s} = \lim_{t\to\infty} s(t) = 1 - \frac{\omega_1 + \omega_2}{(1+\theta)(1-\bar{l})}.$$

Comparing the long-run distribution of income to the inital one (σ_{y0}), I obtain

$$\frac{\bar{\sigma}_y}{\sigma_{y0}} = \frac{\bar{s}}{s_0}(\frac{\bar{\sigma}_v}{\sigma_{v0}}) = \left(\frac{(1+\theta)(1-\bar{l}) - (\omega_1+\omega_2)}{(1+\theta)(1-l_0) - (\omega_1+\omega_2)}\right)\frac{1-l_0}{1-\bar{l}}(\frac{\bar{\sigma}_v}{\sigma_{v0}}), \tag{89}$$

where σ_{v0} denotes the initial distribution of wealth. The relative income converges to a long-run distribution with the same ranking of households as that of the initial distributions of income and wealth.

The effect of a foreign aid shock on the long-run income distribution depends on two main factors: the steady-state change in the distribution of wealth, reflected by $\frac{\bar{\sigma}_v}{\sigma_{v0}}$, and time devoted to labor supply, measured by $\frac{\bar{s}}{s_0}$.

2.4.3 Welfare distribution

It is important to determine the impact of foreign aid on economic welfare, especially when the distributions of wealth and income are unequal. Since private agents are heterogeneous, I can consider the average or aggregate welfare and its dispersion across agents (welfare inequality).

Welfare measure

The welfare gain is measured as the equivalent percentage variation in the private stock of capital, that is, the percentage change in the initial stock of capital that would leave agents as well off as in the new equilibrium. As shown in Appendix E, this quantity is given by

$$\varphi = \left(\frac{c_f l_f^\theta}{c_b l_b^\theta}\right)\left(\frac{\rho - \gamma\Psi_b}{\rho - \gamma\Psi_f}\right)^{\frac{1}{\gamma}} - 1,$$

where variables with subscripts b and f pertain to the benchmark (initial) and after-shock equilibrium, respectively. For given benchmark values, c_b, l_b^θ and Ψ_b, φ is increasing in the ratio of consumption to the private stock of capital, in leisure, and in the long-run growth rate.

Distribution of welfare

The instantaneous level of welfare for individual i at time t is derived from the utility function Eq.(47) and given by

$$W_i = \frac{1}{\gamma}\left[C_i l_i^\theta\right]^\gamma,$$

which combined with Eq.(56a) yields

$$W_i = \frac{1}{\gamma}\left[\frac{1}{\theta}(1-\tau)w(l, K_G, H, K)l_i^{1+\theta}\right]^\gamma. \tag{90a}$$

The average level of instantaneous welfare is

$$W = \frac{1}{\gamma}\left[\frac{1}{\theta}(1-\tau)w(l, K_G, H, K)l^{1+\theta}\right]^\gamma. \tag{90b}$$

Using Eq.(75) the relative welfare can be expressed as follows

$$\mathbf{w}_i \equiv \frac{W_i}{W} = \left(\frac{l_i}{l}\right)^{\gamma(1+\theta)} = \left[1+\left(1-\frac{1}{\bar{l}}\frac{\theta}{1+\theta}\right)(\overline{v}_i - 1)\right]^{\gamma(1+\theta)}. \tag{91}$$

The relative welfare of individual i expressed in terms of units of wealth can be obtained by applying the following monotonic transformation $(w_i)^{\frac{1}{\gamma(1+\theta)}} \equiv w(v_i)$. Finally, welfare inequality or the dispersion of welfare across agents is given by its standard deviation, σ_w

$$\sigma_w = \left(1-\frac{1}{\bar{l}}\frac{\theta}{1+\theta}\right)\overline{\sigma}_v. \tag{92}$$

2.5 An Increase in Foreign Aid: Numerical Analysis

The theoretical framework described above is solved numerically and this section examines the aggregate dynamics as well as the distributional effects of a permanent increase in the ratio of foreign aid to GDP, μ. I begin by defining the parameter values for the benchmark economy that does not receive any aid flows. Then, I analyze the aggregate dynamics in the presence of untied versus tied aid transfers. Finally, I examine the effects of foreign aid on wealth and income inequality.

2.5.1 The Benchmark economy

The parameter values shown in Table XIV represent as much as possible the initial equilibrium (without aid, i.e., $\mu = 0$) of a small open low-income economy. Starting with the production and human capital parameters, I set the output elasticities ω_1 and ω_2 to 0.2 and 0.4 respectively. The human capital parameter, η, is set to 0.2, which implies that the learning elasticity with respect to human capital is equal to 0.8, as in Glomm and Ravikumar (1998). The scaling parameters A and B are chosen to generate plausible growth rates for developing countries. The rate of time preference, ρ, is assumed to be equal to 4 percent while the intertemporal elasticity of substitution, $\frac{1}{1-\gamma}$ is set to 0.4, consistent with the empirical evidence for low-income countries.[28] The leisure parameter, θ, is set to 1.7 and ensures a plausible time allocation in the steady state. The depreciation rate of physical capital (private and public) is assumed to be equal to 5 percent, while that of human capital is set to 1 percent.[29] The income tax, τ, is set to 0.15 and the remainder public policy parameters, λ_G and λ_H, are equal to 0.04, consistent with the empirical evidence.[30] The adjustment cost parameters, ϕ_1 and ϕ_2, are set to 10, while the exogenous risk-free interest rate is taken to be 4%. Finally, the borrowing premium parameter, a, is chosen so as to ensure a plausible debt-capital ratio.

The second row of Table XV shows the initial equilibrium of the benchmark economy. The steady-state ratio of debt to private capital is 0.18 leading to an interest rate of 6.77% and a premium on borrowing of 2.77%. The ratio of human to private capital is 11.21 and that of public to private capital is 0.33. The aggregate wealth to private capital ratio is 1.43, the ratio of consumption to private capital is 0.37 and 71% of time available to households is devoted to leisure, consistently with emprical evidence. The long-run growth rate for the benchmark economy is 1.11%.

2.5.2 Aggregate Dynamics

I now assume that the benchmark economy described above receives a permanent inflow of foreign aid that is equivalent to 5% of its GDP. Thus, the ratio of foreign aid to GDP, μ increases from 0 to 0.05. I compare the results for three polar cases where all aid is: (i) untied, (ii) tied to public spending on human capital (iii) tied to

[28] Atkeson and Ogaki (1996) find an estimate of 0.27 for the intertemporal elasticity of substitution using Indian panel data. Ogaki, et al. (1996) provide estimates of intertemporal elasticity of substitution between 0.233 and 0.441 for a group of 31 low-income countries.

[29] Johnson and Hebein (1974) find depreciation rates between 1 percent and 3.5 percent for USA and Weber (2008) gives some evidence for Switzerland with depreciation rates of 1 percent for males and 1.8 percent for females.

[30] Arestoff and Hurlin (2006) estimate the ratios of public expenditure on capital to GDP for a group of 19 developing countries. They find that during the period 1974-1997 the average ratio was 4.42%. According to World Bank (2008) CD-ROM, the ratio of public spending on education to GDP was 4.25% in 2005 for Sub-Saharian Africa.

public investment in capital.

Untied Aid The long-run effects of a permanent untied aid shock are shown in the third row of Table XV, while Figure 6a depicts the transitional dynamics. Starting with the steady-state effects, the ratio of debt to private capital decreases implying that households devote aid transfers to debt reduction. As a result, the interest rate falls while aggregate wealth normalized by private capital increases. Because households increase their consumption of normal goods both leisure and the ratio of consumption to private capital rise. The ratio of public to private capital drops very slightly, reflecting a higher private capital accumulation. The ratio of human to private capital increases slightly, implying that households accumulate human capital more than private capital, but at a slower rate since they decrease their labor time. Notwithstanding the increase in human capital, the economy grows at the slower rate of 1.01% following an untied aid shock. The increase in leisure and in the consumption-capital ratio allows the economy to experiment a global welfare improvement of almost 6% relative to the initial equilibrium. However, this welfare gain is unequally distributed among private agents. In fact, following an untied aid program, the dispersion of welfare increases to 13.84%, which represents the highest level of welfare inequality compared with the two alternative allocation schemes discussed below.

Figure 6a depicts the transitional dynamics of some key aggregate variables following an untied aid shock. The transfer leads the market price of private capital, q to decrease immediately, before it converges monotonically to its permanently lower new long-run value. The ratio of national debt to private capital starts to decrease on impact before overshooting its long-run equilibrium value. The transitional path of aggregate wealth to private capital ratio depends on the dynamic adjustments of q and d. A permanent increase in untied aid causes aggregate wealth over capital to decline instantaneously reflecting the initial fall of q while d remains unchanged. But over time, the ratio of aggregate wealth to capital increases monotonically to its new steady-state value, which is higher than its initial long-run value. Following an untied aid shock, the ratio of human to private capital increases monotonically to its new steady-state value, while the ratio of public to private capital decreases continuously after a very short increase.

Aid Tied to Human Capital The fourth row of Table XV presents the long-run effects of an increase in aid tied to public spending on human capital. Since all aid is used to improve the educational system, the ratio of human to private capital increases while time devoted to leisure remains almost unchanged. The larger stock of human capital increases the marginal productivity of public capital more than that of private capital and leads to an increase in the ratio of public to private capital. The transfer has a positive wealth effect, which increases the ratio of consumption to private capital. However, the aggregate wealth to private capital ratio decreases due to a larger effect of foreign aid on capital accumulation combined with the increase in the ratio of national debt to private capital. Since the economy accumulates debt, its premium on borrowing and interest rate increase. When foreign aid is tied to human capital, it enhances the different types of capital accumulation, which allows the economy to grow at the higher rate of 1.43%. In addition to growing at the faster rate, the economy achieves the largest welfare gain evaluated to 14.26%, following an aid program that is completely tied to human capital. Relative to the initial equilibrium, welfare inequality increases. But, compared to the other allocation schemes, this welfare improvement is the least unequally distributed among private agents, since it has the lowest dispersion (1.71%).

The dynamic adjustment following an aid shock tied to human capital are depicted in Figure 6b. The new long-run value of q is higher than its original pre-shock value and between these two steady-state levels, the market price of private capital exhibits a monotonic increase after an initial drop. The transitional paths of d, $\frac{V}{K}$ and l are opposite to those observed when all aid is untied, despite some similarities in the impact responses. Indeed, the transfer leads d to increase continuously until it reaches its new long-run value, while the ratio of aggregate wealth to private capital decreases on impact and remains declining before overshooting its new steady-state level. Although the long-run level of leisure is almost the same with its initial value, the transitional path indicates that leisure jumps on impact before declining monotonically. The ratio of human to private capital as well as that of public to private capital start to increase on impact and remain increasing towards their new steady-state levels.

Aid Tied to Public Capital The last row of Table XV shows the long-run effects of a permanent increase in aid tied to public capital. Being tied to productive investment in capital, the transfer increases the ratio of public to private capital. The marginal productivities of private inputs like capital and labor increase, leading to a slight fall in leisure time. The ratio of human to private capital and that of aggregate wealth to private capital decrease due to a larger effect of aid on private capital accumulation. The introduction of foreign aid devoted to public capital accumulation increases the ratio of national debt to private capital, which decreases the ratio of aggregate wealth to private capital and rises the interest rate. The ratio of consumption to private capital increases. Finally, the growth rate achieved by the economy after an aid shock tied to public capital (1.3%) is higher than that following an untied aid shock (1.01%) but lower than the growth rate after an aid shock tied to public spending on human capital (1.43%). Owing to a lesser impact of the transfer on the consumption-capital ratio, the economy reaches the lowest welfare gain compared with the two preceeding scenarios. Following an aid program that is completely tied to public capital, welfare inequality increases but remains much lower than that recorded when foreign is entirely untied.

Figure 6c displays the transitional dynamics following an aid shock tied to public capital. The transitional paths of q, d, $\frac{V}{K}$ and l are different from those described above. Regardless of whether aid is tied or untied, it causes q to decline on impact. After the initial fall, q increases monotonically without overshooting in the first two cases (untied aid and tied aid to human capital), whereas it has a hump-shaped trajectory following an aid shock tied to public capital. In fact, q increases and overshoots its new steady-state level, after which, it reverses and decreases towards its long-run value. The transitional path of d depends on whether foreign aid is tied or not. Indeed, it decreases when all aid is untied, increases after an aid shock tied to human capital and exhibits a hump-shaped path when all aid is tied to public capital. Owing to the transitional paths of q and d, the ratio of aggregate wealth to private capital displays a U-shaped path, declining after the initial fall on impact, before increasing continuously until it reaches its new long-run value. Irrespective of the nature of foreign aid, leisure increases on impact. After the initial jump, it remains increasing following an untied aid shock and decreases when all aid is tied. Moreover, contrasting with the tied aid to human capital case, l overshoots its new long-run value when all aid is tied to public capital. Finally, after a short-run accumulation, the ratio of human to private capital decreases over time while public capital is accumulated gradually towards the new steady-state.

2.5.3 Distributional Effects of Foreign Aid

This section analyzes the effects of foreign aid on wealth and income inequality. Figure 7 depicts the evolution of the distributions of income and wealth following an aid shock. Again, I distinguish between untied aid, aid tied to human capital and aid tied to public capital.

Untied Aid The first two graphs of Figure 7 present the transitional dynamics of wealth and income inequality following an untied aid shock. Both wealth and income inequality fall persistently over time until they reach their new steady-state values, but the shock has a stronger effect on the latter than on the former.

In order to understand the intuition underlying the dynamic responses of wealth and income inequality, it is useful to recall (75), (78) and (89), which can be rewritten as

$$(l_i - \bar{l}) = \frac{1}{n(0)} \left(\bar{l} - \frac{\theta}{1+\theta} \right) (v_{i0} - 1),$$

$$\bar{v}_i - 1 = \frac{1}{n(0)} (v_{i0} - 1),$$

$$\frac{\bar{\sigma}_y}{\sigma_{y0}} = \left(\frac{(1+\theta)(1-\bar{l}) - (\omega_1 + \omega_2)}{(1+\theta)(1-l_0) - (\omega_1 + \omega_2)} \right) \frac{1-l_0}{1-\bar{l}} (\frac{\bar{\sigma}_v}{\sigma_{v0}}),$$

where

$$n(0) \equiv \left[1 + \frac{\frac{(1-\tau)\bar{w}}{\bar{V}} \left(1 - \frac{l(0)}{\bar{l}} \right)}{\frac{(1-\tau)\bar{w}}{\bar{V}} \left(\bar{l}(\frac{1+\theta}{\theta}) - 1 \right) - \epsilon} \right].$$

The effect of foreign aid on wealth inequality depends on the impact of $l(t) - \bar{l}$ on $n(0)$. An untied aid shock has a positive wealth effect, which increases instantaneously aggregate leisure in the short run. Since the maximum effect is not reached on impact, $l(0) - \bar{l} < 0$ and $1 - \frac{l(0)}{\bar{l}} > 0$, implying that $n(0) > 1$. The increase in $n(0)$, along with the fact that during the transition $l(t) - \bar{l}$ is always negative, lead to a permanent fall in wealth inequality.

The overall effect of foreign aid on income inequality depends on its effect on wealth inequality and its impact on labor supply. When foreign aid is untied, it increases leisure immediately and during the transition towards the new steady state. Aggregate wealth increases since agents reduce their external borrowing. As leisure and aggregate wealth increase, wealthier people devote less time to labor and accumulate less capital than poorer people, which decreases wealth inequality. Anticipating this long run decline in wealth inequality, wealthier people initially enjoy more leisure, while poorer people increase their labor supply. In doing so, the relative income of wealthier people decreases whereas that of poorer people increases, leading to an initial decline in income inequality. During the subsequent periods, as leisure increases, people with above-average wealth still enjoy more leisure and income inequality continues to decline.

Aid Tied to Human Capital The middle graphs of Figure 7 depict the transitional paths of wealth and income inequality when aid is tied to human capital. Contrasting with the untied aid case, wealth inequality increases monotonically until its new long-run value. Income inequality, on the other hand, falls initially, before starting to increase in the subsequent periods.

When aid is tied to human capital, aggregate leisure increases in the short-run before declining gradually until it reaches its new long-run value, which is lower than the pre-shock level. Consequently, $l(0) - \bar{l} > 0$ and $1 - \frac{l(0)}{\bar{l}} < 0$, implying that $n(0) < 1$. The decline in $n(0)$ combined with $l(t) - \bar{l}$ being always positive during the transition, generates an overall increase in wealth inequality over time. The intuition is the following: contrary to the untied aid case, an aid program that is tied to human capital decreases leisure and aggregate wealth in the long-run (as depicted in Figure 6b). The equilibrium wage rate decreases due to higher labor supply and induces wealthier people to accumulate capital while poorer people desinvest in order to smooth their consumption. The total effect is a gradual increase in wealth inequality. Turning to income inequality, owing to the initial jump in aggregate leisure, wealthier people work less whereas poorer people increase their work time, as a result, income inequality decreases on impact. But during the subsequent periods, aggregate leisure decreases and wealthier people choose to increase their stock of capital. In doing so, the relative income coming from capital of wealthier people increases while that of poorer people decreases, and this generates a gradual increase in income inequality.

Aid Tied to Public Capital The last two graphs of Figure 7 display the dynamic responses of wealth and income inequality to an aid program that is tied to public capital. Contrary to the previous cases, the transitional paths of wealth and income inequality are non-monotonic. Indeed, following an aid shock that is tied to public capital, wealth inequality increases in a hump-shaped manner, reaching its peak after around 12 periods after the shock, before converging to its new steady-state value from above. A similar pattern is displayed by income inequality, which, however, slightly decreases on impact, before starting to increase.

Following an aid program that is tied to public capital, leisure initially rises before it eventually falls below its new long-run value. As a result, there are two opposite effects on $n(0)$: one effect is driven by positive values of $l(t) - \bar{l}$ while the other is driven by negative values of $l(t) - \bar{l}$. The net impact of $l(t) - \bar{l}$ on $n(0)$ will determine the evolution of wealth inequality. During the first stages of transition, the former effect ($l(t) - \bar{l} > 0$) dominates, $n(0) < 1$, leading to a gradual increase in wealth inequality. But, during the subsequent stages, the latter effect ($l(t) - \bar{l} < 0$) dominates so that after the early increase, wealth inequality declines, yielding a hump-shaped trajectory.

The evolution of income inequality reflects the hump-shaped time path of wealth inequality. Similar to the previous cases, income inequality falls on impact. But, as wealth inequality increases, the relative income of wealthier people increases whereas that of poorer people decreases, the overall effect is an increase in income inequality. When wealth inequality reverses and starts to decline, people with above-average wealth begin to enjoy more leisure while poorer people increase their labor supply, and income inequality declines after an early increase.

Robustness Analysis The distributional effects of foreign aid described above, depend on whether the transfer is tied or not. This section performs a robustness analysis in order to determine how sensitive this main result is to the parameters values chosen for the benchmark economy. I focus on the output elasticities (ω_1 and ω_2), the leisure and human capital parameters (θ and η).

Output elasticities (ω_1 and ω_2)

Figures 9 and 10 present the dynamic effects of foreign aid on wealth and income inequality, as the output elasticitiy, ω_1, changes between 0.1 and 0.3, while ω_2,varies between 0.3 and 0.5. The benchmark values of these parameters are 0.2 and 0.4 respectively. The aid shock can be untied or tied either to human capital or to public capital. The results of these experiments indicate that the tendency of untied and tied foreign aid to have opposite effects on inequality is robust to variations in ω_1 and ω_2. In fact, regardless of the value of output elasticities, both wealth and income inequality reduce following an untied aid shock, but inequality increases when the transfer is completely tied. Moreover, the impact of foreign aid increases with ω_1 and ω_2. As shown in Figure 9, following an untied aid shock, the largest decrease in inequality is recorded for ω_1 equal to 0.3. The same value of ω_1 is also associated with the largest increase in inequality when the transfer is completely tied.

In order to understand these results, one must recall that households' raw labor supply interacts with public capital and human capital to yield labor measured in efficiency units ($(1-l)K_G$ and $(1-l)H$). The parameters ω_1 and ω_2 represent the elasticity of output with respect to effective labor supply. As these parameters increase, the efficiency with which raw labor is transformed in effective labor supply increases, which leads to a higher improvement of the marginal productivity of private inputs following a tied aid shock. As a result, wealthier people accumulate relatively more physical and human capital than poorer people, which increases inequality. But, when the transfer is completely untied, its wealth effects on private agents increase with ω_1 and ω_2. Since wealthier people have a lower marginal utility of wealth, they enjoy more leisure than poorer people, and this reduces income and wealth inequality.

Leisure parameter (θ)

The benchmark value of the leisure parameter, θ, is set to 1.7. In the experiment, I consider two alternative values: 1.4 and 2. Figure 11 depicts the robustness of distributional dynamics to alternative values of θ. Following an untied aid program, wealth and income inequality decrease in the long-run. Moreover, the initial and long-run reduction in income inequality increases with θ. When the transfer is completely tied both wealth and income inequality increase. The hump-shaped trajectory obtained following an aid program tied to public capital, is robust to alternative values of θ.

As the importance of leisure in the utility increases, the positive wealth effects of an untied aid program on time devoted to leisure decrease, which leads poorer people to increase their labor supply. In doing so, their relative income increases and translates into the largest reduction in income inequality when θ reaches its highest value. But, when the transfer is completely tied, its adverse effects on time devoted to leisure decrease as θ increases. Since leisure weighs more in utility, private agents are less willing to reduce it following an aid program that is completely tied. As a result, the positive effects of tied aid on the marginal productivity of private inputs decline slightly. However, wealthier people still accumulate relatively more physical capital than poorer people and inequality increases.

Human capital parameter (η)

According to the literature, the learning elasticity with respect to human capital $(1-\eta)$ varies between 0.7 and 1.[31] Hence, the human capital parameter, η changes within $0.1-0.3$, with 0.2 being the benchmark value. Figure 12 displays the results of these experiments. Regardless of the value of η, an untied aid shock decreases both wealth and income inequality, whereas they increase following a tied aid shock. After an untied aid shock, the long-run decrease in wealth inequality is higher when η is lower. Income inequality decreases on impact and remains decreasing during the subsequent transition. Following an aid shock tied to human capital, wealth inequality increases gradually over time. Moreover, the transitional paths of wealth inequality, as well as the long-run levels are very close when η is higher than 0.1. After an initial fall, which increases with η, income inequality increases continuously until it reaches its new steady-state value. Irrespective of the value of η, following an aid shock tied to public capital, wealth and income inequality have hump-shaped trajectories. The long-run levels of wealth inequality increase with η, while income inequality, after an initial drop, converges to the same long-run level, regardless of the value of η.

[31] The value used by Lau (2000) and Fougère et al. (2006) is 0.7, Glomm and Ravikumar (1998) set this parameter to 0.8 and the estimate of Heckman et al. (1998b) is equal to 0.9.

2.6 The Growth-Inequality Relationship

As mentioned in the introduction, the link between inequality and economic growth has been widely examined. However, empirical findings are inconclusive. Indeed, both negative and positive linkages have been found in early growth regressions, and several theories have been elaborated to explain these results. The present model generates a mechanism that differs from existing theories on growth-inequality relationship, in the sense that it emphasizes the implications of foreign aid for labor supply and productive investments in capital. Figure 8 displays the correlation between wealth and income inequality and economic growth measured by the growth rate of per capita output, following a permanent increase in tied versus untied foreign aid.

2.6.1 Untied aid

The first two plots of Figure 8a depict a negative correlation when all aid is untied. That is, after an untied aid shock, as economy grows both wealth and income inequality decrease. In order to better understand this negative correlation, I compute the transitional paths of the growth rates of public, human and private capital, and that of per capita output. Following an untied aid shock, the long-run growth rate is lower than its benchmark value. The transitional dynamics indicate that the transfer decreases immediately the growth rates of the different types of capital, which leads to a decrease in the growth rate of per capita output on impact. During the subsequent transition, public and private capital are accumulated gradually whereas the growth rate of human capital remains declining. The overall effect causes the growth rate of output to increase monotonically until its new long-run level, which is lower than the initial one. Being untied, foreign aid is labelled as pure transfers to households and affects their decisions on time allocation and capital accumulation. So, people with below (above) average wealth, not only increase (decrease) their labor supply, but also increase (decrease) their stock of capital. As a result, both wealth and income inequality decrease as the growth rate of output increases.

2.6.2 Aid tied to human capital

According to the first two graphs of Figure 8b, a positive correlation is found when all aid is tied to human capital. So, both wealth and income inequality are growth-enhancing. The long-run growth rate following an aid shock tied to human capital is higher than it is in the benchmark economy. Being tied to human capital, the transfer increases immediately the growth rate of human capital which overshoots its new long-run value. After an initial drop in the growth rates of public and private capital, both of them increase smoothly during the subsequent transition. The larger stock of capital, increases the growth rate of per capita output on impact and over time. Following an aid shock tied to human capital, leisure increases on impact before it reverses and decreases over time (see Figure 6b), while private capital is gradually accumulated. During the transition, as leisure decreases, wealthier people accumulate capital whereas people with below average wealth decrease their stock of capital. Hence, wealth and income inequality increase with the growth rate of output, yielding a positive relationship between inequality and economic growth.

2.6.3 Aid tied to public capital

The first two plots of Figure 8c, depict an inverted-U correlation when all aid is tied to public capital. Wealth and income inequality are initially growth-enhancing before they reverse and decrease as economy grows. Following, an aid shock tied to public capital, the economy grows at a higher rate due to capital accumulation. Since all aid is devoted to productive investments in capital, the growth rate of public capital increases to over 8% on impact, overshooting by this way, its new long-run value. The growth rates of human and private capital both decrease on impact, but their transitional paths are different. Indeed, during the subsequent transition, the growth rate of human capital increases monotonically towards its new long-run value, while that of private capital exhibits a hump-shaped path, increasing at first and then declining over time. The overall effect leads the growth rate of per capita output to increase on impact, after which it decreases over time. After an aid shock tied to public capital, leisure increases on impact before it decreases and overshoots its new long-run level (see. Figure 6c). At the same time, private capital is accumulated first at an increasing rate and then at a decreasing rate over time. During the transition, as leisure decreases and remains higher than its long-run level, people with above (below) average wealth increase (decrease) their relative capital income leading to an initial positive relationship between inequality and economic growth.

But, as leisure overshoots and becomes lower than its long-run level, wealthier people enjoy more leisure and accumulate less capital than poorer people yielding therefore a negative linkage between inequality and economic growth.

2.7 Conclusion

This paper has examined the relationship between foreign aid and inequality. Panel data regressions show that foreign aid raises income inequality with a certain delay. The theoretical framework presented in this paper assumes homogeneous preferences, which allow to express the macroeconomic equilibrium using a representative-consumer setup and analyze the distributional effects of foreign aid in a tractable way. More precisely, the impact of foreign aid on wealth and income inequality depends on whether the transfer is tied or not. In particular, I show that untied aid reduces income inequality whereas tied aid raises it with a delay regardless of whether aid programs are tied to physical or human capital. To the extent that aid flows to developing countries are mostly tied, the model's predictions are consistent with the empirical evidence. Moreover, foreign aid increases average welfare gain and its dispersion across private agents.

The present framework can be extended to account for some *ex post* heterogeneity, which could be introduced by assuming that households face uninsurable idiosyncratic shocks affecting their earnings (labor income or capital income). The *ex post* heterogeneity along with restrictions on borrowing and incomplete markets would help generate a volatility of income inequality that is consistent with empirical data. Another possible extension can be related to the introduction of political considerations in foreign aid allocations. According to Boone (1996), foreign aid is distributed in a way that favors the "high-income political elite." This mechanism can explain why foreign aid may lead to higher inequality. Finally, one can assume foreign aid programs that are targeted or designed in order to help poorer people. This extension will allow to address the issues related to the implementation of pro-poor growth or development strategies that both foster economic growth and reduce inequality.

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2.9 Appendix

2.9.1 Appendix A: Derivation of macroeconomic equilibrium

The time derivative of (56a) yields

$$\frac{\dot{C}_i}{C_i} - \frac{\dot{l}_i}{l_i} = \frac{w_L L}{w}\frac{\dot{L}}{L} + \frac{w_H H}{w}\frac{\dot{H}}{H} + \frac{w_K K}{w}\frac{\dot{K}}{K} + \frac{w_{K_G} K_G}{w}\frac{\dot{K}_G}{K_G} \tag{A.1}$$

where $L = 1 - l$.

Applying equations (60a) and (A.1) to households i and j I obtain

$$(\gamma - 1)\left(\frac{\dot{C}_i}{C_i} - \frac{\dot{C}_j}{C_j}\right) + \theta\gamma\left(\frac{\dot{l}_i}{l_i} - \frac{\dot{l}_j}{l_j}\right) = 0 \tag{A.2}$$

$$\left(\frac{\dot{C}_i}{C_i} - \frac{\dot{C}_j}{C_j}\right) - \left(\frac{\dot{l}_i}{l_i} - \frac{\dot{l}_j}{l_j}\right) = 0 \tag{A.3}$$

which implies

$$\frac{\dot{C}_i}{C_i} = \frac{\dot{C}_j}{C_j};\text{ and } \frac{\dot{l}_i}{l_i} = \frac{\dot{l}_j}{l_j} \text{ for all } i \text{ and } j \tag{A.4}$$

Summing over all households and using Eq. (A4) yields

$$\frac{\dot{C}_i}{C_i} = \frac{\dot{C}}{C} \text{ and } \frac{\dot{l}_i}{l_i} = \frac{\dot{l}}{l} \text{ for all } i \tag{A.5}$$

The individual i's growth rate of private capital is denoted by

$$\Psi_i(t) \equiv \frac{\dot{K}_i}{K_i} = \frac{q_i - 1}{\phi_1} - \delta_K \tag{A.6a}$$

This allows to express the evolution of each household's capital stock as

$$K_i(t) = K_{i0}e^{\int_0^t \Psi_i(s)ds} \tag{A.6b}$$

Using (59a) and (A.6b), I can rewrite the transversality condition for the private capital stock as follows

$$\lim_{t\to\infty} q_i(t)\xi_{i0}e^{(\rho - i(D/K))t}K_{i0}e^{\int_0^t \Psi_i(s)ds}e^{-\rho t} = \lim_{t\to\infty} q_i(t)\xi_{i0}K_{i0}e^{\int_0^t \Psi_i(s)ds - i(D/K)t} = 0 \tag{A.6c}$$

which implies that $\int_0^t \Psi_i(s)ds < i(D/K)t$.

The marginal product of private capital can be derived from (59b) and substracting the corresponding capital returns for households i and j yields the following equation

$$(\dot{q}_i - \dot{q}_j) + \frac{(q_i - 1)^2 - (q_j - 1)^2}{2\phi_1} = (i(D/K) + \delta_K)(q_i - q_j) \tag{A.7a}$$

Defining $X \equiv q_i - q_j$, (A.7a) can be rewritten as

$$\dot{X} + \frac{X(q_i + q_j - 2)}{2\phi_1} = (i(D/K) + \delta_K) X$$

and using (A.6a) I obtain the following solution for $X(t)$

$$X(t) = X_0 e^{i(D/K)t - \frac{1}{2}\int_0^t (\Psi_i(s) + \Psi_j(s))ds} \tag{A.7b}$$

Considering that $\int_0^t \Psi_i(s)ds < i(D/K)t$ for all i, the unique stable solution to Eq. (A.7b) is $X(t) \equiv 0$, from which I infer that $q_i = q_j = q$ for all households i and j. Hence the aggregate private capital and individual capital stocks grow at the same rate, specifically

$$\Psi_K \equiv \frac{\dot{K}}{K} = \frac{\dot{K}_i}{K_i} = \frac{q - 1}{\phi_1} - \delta_K$$

Taking logs and differentiating Eqs. (65a) and (46) with respect to time yields

$$\frac{\dot{C}}{C} - \frac{\dot{Y}}{Y} = \frac{\dot{l}}{l} + \frac{\dot{l}}{1 - l} \tag{A.8a}$$

$$\frac{\dot{Y}}{Y} = \omega_1 \left(\frac{\dot{K}_G}{K_G} - \frac{\dot{l}}{1 - l} \right) + \omega_2 \left(\frac{\dot{H}}{H} - \frac{\dot{l}}{1 - l} \right) + (1 - \omega_1 - \omega_2)\frac{\dot{K}}{K} \tag{A.8b}$$

Combining the last two equations with (65d) gives the dynamic equation for aggregate leisure, (67f) in the text.

The aggregate dynamics can be described by a system of six equations in h, k_g, d, q, m and l

$$\frac{\dot{h}}{h} = \frac{\dot{H}}{H} - \frac{\dot{K}}{K} = B\left((\lambda_H + \alpha_H \mu)\frac{y}{h} \right)^{\eta} (1 - l)^{1-\eta} - \delta_H - \left(\frac{q - 1}{\phi_1} - \delta_K \right) \tag{A.9a}$$

$$\frac{\dot{k}_g}{k_g} = \frac{\dot{K}_G}{K_G} - \frac{\dot{K}}{K} = (\lambda_G + \alpha_G\mu)\frac{y}{k_g} - \delta_G - \left(\frac{q-1}{\phi_1} - \delta_K\right) \tag{A.9b}$$

$$\frac{\dot{d}}{d} = \frac{\dot{D}}{D} - \frac{\dot{K}}{K} = i(d) + \frac{1}{d}\left[c + \frac{q^2-1}{2\phi_1} + (\lambda_G + \alpha_G\mu)y\left(1 + \frac{\phi_2}{2}(\lambda_G + \alpha_G\mu)\frac{y}{k_g}\right) + (\lambda_H + \alpha_H\mu)y - (1+\mu)y\right] - \left(\frac{q-1}{\phi_1} - \delta_K\right) \tag{A.9c}$$

$$\frac{\dot{q}}{q} = i(d) + \delta_K - \frac{1}{q}\left[\frac{(q-1)^2}{2\phi_1} + (1-\tau)\frac{\partial Y}{\partial K}\right] \tag{A.9d}$$

$$\frac{\dot{m}}{m} = i(D/K) - \left[(1-\eta)B((\lambda_H + \alpha_H\mu)\frac{y}{h})^{\eta}(1-l)^{1-\eta} - \delta_H\right] - \frac{1}{m}\left[(1-\tau)\frac{\partial Y}{\partial H}\right] \tag{A.9e}$$

$$\frac{\dot{l}}{l} = \frac{\rho - i(d) + (1-\gamma)\left[\omega_1\frac{\dot{K}_G}{K_G} + \omega_2\frac{\dot{H}}{H} + (1-\omega_1-\omega_2)\frac{\dot{K}}{K}\right]}{\gamma(1+\theta) - 1 - (1-\gamma)(1-\omega_1-\omega_2)\frac{l}{1-l}} \tag{A.9f}$$

where

$$y \equiv \frac{Y}{K} = A((1-l)k_g)^{\omega_1}((1-l)h)^{\omega_2} \tag{A.10a}$$

$$c \equiv \frac{C}{K} = \frac{(1-\tau)(\omega_1+\omega_2)}{\theta}\frac{l}{1-l}y \tag{A.10b}$$

$$i(d) = i^* + \exp(ad) - 1 \tag{A.10c}$$

and the growth rates of the different types of capital are:

$$\frac{\dot{K}}{K} \equiv \Psi_K = \frac{q-1}{\phi_1} - \delta_K \tag{A.11a}$$

$$\frac{\dot{H}}{H} \equiv \Psi_H = B\left((\lambda_H + \alpha_H\mu)\frac{y}{h}\right)^{\eta}(1-l)^{1-\eta} - \delta_H \tag{A.11b}$$

$$\frac{\dot{K}_g}{K_g} \equiv \Psi_G = (\lambda_G + \alpha_G\mu)\frac{y}{k_g} - \delta_G \tag{A.11c}$$

2.9.2 Appendix B: Steady-state equilibrium

Applying $\dot{h} = \dot{k}_g = \dot{d} = \dot{q} = \dot{m} = \dot{l} = 0$ to Eqs. (A.9a)-(A.9f) and using (A.10a), (A.10b), (A.11a), (A.11b), (A.11c) and Eq. (65c) I get the following system of equations that represent the steady-state:

$$B\left((\lambda_H + \alpha_H\mu)\frac{y}{h}\right)^{\eta}(1-l)^{1-\eta} - \delta_H - \left(\frac{q-1}{\phi_1} - \delta_K\right) = 0 \tag{B.1}$$

$$(\lambda_G + \alpha_G\mu)\frac{y}{k_g} - \delta_G - \left(\frac{q-1}{\phi_1} - \delta_K\right) = 0 \tag{B.2}$$

$$i(d) + \frac{1}{d}\left[c + \frac{q^2-1}{2\phi_1} + (\lambda_G + \alpha_G\mu)y\left(1 + \frac{\phi_2}{2}(\lambda_G + \alpha_G\mu)\frac{y}{k_g}\right) + (\lambda_H + \alpha_H\mu)y - (1+\mu)y\right] - \left(\frac{q-1}{\phi_1} - \delta_K\right) = 0 \tag{B.3}$$

$$i(d) + \delta_K - \frac{1}{q}\left[\frac{(q-1)^2}{2\phi_1} + (1-\tau)\frac{\partial Y}{\partial K}\right] = 0 \tag{B.4}$$

$$i(d) - \left[(1-\eta)B((\lambda_H + \alpha_H\mu)\frac{y}{h})^{\eta}(1-l)^{1-\eta} - \delta_H\right] - \frac{1}{m}\left[(1-\tau)\frac{\partial Y}{\partial H}\right] = 0 \tag{B.5}$$

$$\frac{[i(d) - \rho]}{1-\gamma} - \left(\frac{q-1}{\phi_1} - \delta_K\right) = 0 \tag{B.6}$$

$$y = A((1-l)k_g)^{\omega_1}((1-l)h)^{\omega_2} \tag{B.7}$$

$$c = \frac{(1-\tau)(\omega_1 + \omega_2)}{\theta}\frac{l}{1-l}y \tag{B.8}$$

Solving this system provides the steady-state values of $\bar{h}$, $\bar{k}_g$, $\bar{d}$, $\bar{q}$, $\bar{m}$, $\bar{l}$, $\bar{c}$, $\bar{y}$ and the long-run equilibrium growth rate, $\bar{\Psi}$.

2.9.3 Appendix C: Wealth distribution

The time derivative of Eq.(70) yields

$$\dot{V}_i = q\dot{K}_i - \dot{D}_i \tag{C.1}$$

Using (56c) and the fact that q is identical and constant across households, the individual i's budget constraint (62) can be rewritten as

$$\dot{D}_i = i(D/K)D_i + \left(\frac{q^2-1}{2\phi_1} - (1-\tau)r(l, K_G, H, K)\right) K_i + (1-\tau)w(l, K_G, H, K)\left(\frac{l_i}{\theta} - (1-l_i)\right) + T_i \quad \text{(C.2)}$$

Having derived the steady-state value of $(1-\tau)r(L, H, K, K_G)$ from (65b), it can be substituted into (C2). Then, the last equation and Eq.(48) can be replaced into (C.1), namely

$$\begin{aligned}\dot{V}_i \quad = \quad & \left(\frac{q(q-1)}{\phi_1} - q\delta_K - \frac{q^2-1}{2\phi_1} + i(D/K)q + q\delta_K - \frac{(q-1)^2}{2\phi_1}\right) K_i \\ & -(1-\tau)w(l, K_G, H, K)\left(\frac{l_i}{\theta} - (1-l_i)\right) - T_i - i(D/K)D_i\end{aligned}$$

which implies that the growth rate of household i's wealth is

$$\frac{\dot{V}_i}{V_i} = i(D/K) - \frac{(1-\tau)w(l, K_G, H, K)}{V_i}\left(\frac{l_i}{\theta} - (1-l_i)\right) - \frac{T_i}{V_i} \quad \text{(C.3)}$$

Summing (C.3) over all households yields the growth rate of aggregate wealth,

$$\frac{\dot{V}}{V} = i(D/K) - \frac{(1-\tau)w(l, K_G, H, K)}{V}\left(\frac{l}{\theta} - (1-l)\right) - \frac{T}{V} \quad \text{(C.4)}$$

Defining the individual i's relative wealth by $v_i \equiv \frac{V_i}{V}$, and since taxes are such that $\frac{T_i}{V_i} = \frac{T}{V}$, using Eqs.(C.3) and (C.4), I can express the dynamics of v_i as follows

$$\dot{v}_i \equiv \frac{\dot{V}_i}{V_i} - \frac{\dot{V}}{V} = \frac{(1-\tau)w(l, K_G, H, K)}{V}\left[1 - l_i\frac{1+\theta}{\theta} - \left(1 - l\frac{1+\theta}{\theta}\right)v_i\right] \quad \text{(C.5)}$$

that is the equation (72) in the text.

For l_i and l both constants, (C.5) is a simple linear differential equation which depends upon the characteristics of the coefficient in front of $v_i(t)$. These properties can be obtained from the transversality conditions. Deducting the transversality condition for the foreign debt from that related to the stock of capital yields

$$\lim_{t\to\infty} \xi_i(qK_i - D_i)e^{-\rho t} = \lim_{t\to\infty} \xi_i V_i e^{-\rho t} = 0$$

The aggregate condition is obtained by summing over all households

$$\lim_{t\to\infty} \xi V e^{-\rho t} = \lim_{t\to\infty} \xi_0 V e^{-i(D/K)t} = 0$$

implying that $\frac{\dot{V}}{V} < i(D/K)$. Since $\frac{T}{V} > 0$, it follows from (C.4) that

$$l > \frac{\theta}{1+\theta} \tag{C.6}$$

2.9.4 Appendix D: Dynamics of the relative wealth

Linearizing (73) yields the following equation

$$\dot{v}_i(t) = \frac{(1-\tau)w(\bar{l}, \bar{K}_G, \bar{H}, \bar{K})}{\bar{V}} \left[\left(\frac{1+\theta}{\theta}\right)(\bar{v}_i - \beta_i)\left(l(t) - \bar{l}\right) + \left(\bar{l}(\frac{1+\theta}{\theta}) - 1\right)(v_i(t) - \bar{v}_i) \right] \tag{D.1}$$

The stable solution to (D.1) can be expressed as

$$v_i(t) = \bar{v}_i + \frac{\frac{(1-\tau)\bar{w}}{\bar{V}}\left[(\frac{1+\theta}{\theta})(\bar{v}_i - \beta_i)\right]}{\epsilon - \frac{(1-\tau)\bar{w}}{\bar{V}}\left(\bar{l}(\frac{1+\theta}{\theta}) - 1\right)}(l(0) - \bar{l})e^{\epsilon t} \tag{D.2}$$

where $\epsilon < 0$ is the stable eigenvalue. Setting $t = 0$ in equation (D.2) and recalling that v_{i0} is given, yields

$$v_{i0} = \bar{v}_i + \frac{\frac{(1-\tau)\bar{w}}{\bar{V}}\left[(\frac{1+\theta}{\theta})(\bar{v}_i - \beta_i)\right]}{\epsilon - \frac{(1-\tau)\bar{w}}{\bar{V}}\left(\bar{l}(\frac{1+\theta}{\theta}) - 1\right)}(l(0) - \bar{l}) \tag{D.3}$$

From Eq.(75) I derive the following expression for $\bar{v}_i - \beta_i$

$$\bar{v}_i - \beta_i = -\frac{(1-\bar{v}_i)}{\bar{l}(\frac{1+\theta}{\theta})}$$

which can be replaced in (D.3) to give

$$v_{i0} = \bar{v}_i + \frac{\frac{(1-\tau)\bar{w}}{\bar{V}}(1-\bar{v}_i)}{\frac{(1-\tau)\bar{w}}{\bar{V}}\left(\bar{l}(\frac{1+\theta}{\theta}) - 1\right) - \epsilon}\left(\frac{l(0) - \bar{l}}{\bar{l}}\right) \tag{D.4}$$

Having determined the time path of the aggregate economy and since v_{i0} is given, (D.4) specifies the long-run distribution of wealth. The evolution of the relative wealth is given by

$$v_i(t) = \bar{v}_i + \frac{\frac{(1-\tau)\bar{w}}{\bar{V}}(1-\bar{v}_i)}{\frac{(1-\tau)\bar{w}}{\bar{V}}\left(\bar{l}(\frac{1+\theta}{\theta}) - 1\right) - \epsilon}\left(\frac{l(0) - \bar{l}}{\bar{l}}\right)e^{\epsilon t} \tag{D.5}$$

or equivalently

$$v_i(t) = \bar{v}_i + \frac{\frac{(1-\tau)\bar{w}}{\bar{V}}}{\frac{(1-\tau)\bar{w}}{\bar{V}}\left(\bar{l}(\frac{1+\theta}{\theta}) - 1\right) - \epsilon}(1 - \frac{l(t)}{\bar{l}})(\bar{v}_i - 1) \tag{D.6}$$

which can be re-arranged to obtain Eq.(77a) in the text.

2.9.5 Appendix E: Derivations of Welfare Changes

Welfare changes are measured by the equivalent variations in the private stock of capital following an aid shock, i.e., the percentage changes in the initial stock of capital necessary to make agents as well off in the benchmark equilibrium as in the after-shock equilibrium. Given the form of the utility function, and assuming that the economy is initially on a balanced growth path with a constant growth rate equal to Ψ_b, the benchmark intertemporal welfare is given by

$$W_b = \int_0^\infty \frac{1}{\gamma}(c_b l_b^\theta K_b(t))^\gamma e^{-\rho t} dt, \tag{E.1}$$

$$= \int_0^\infty \frac{1}{\gamma}(c_b l_b^\theta K_0)^\gamma e^{(\gamma\Psi_b - \rho)t} dt, \tag{E.2}$$

where variables with subscript b pertain to the benchmark equilibrium, and K_0 is the stock of capital at $t = 0$. Since c_b and l_b are constant along the initial balanced growth path, the expression above reduces to

$$W_b = W_b(c_b, l_b, K_0) = \frac{1}{\gamma}\frac{\left(c_b l_b^\theta K_0\right)^\gamma}{\rho - \gamma\Psi_b}. \tag{E.3}$$

The intertemporal welfare in the after-shock equilibrium is given by

$$W_f = \int_0^\infty \frac{1}{\gamma}(c_b l_b^\theta K_f(t))^\gamma e^{-\rho t} dt, \tag{E.4}$$

$$= \frac{1}{\gamma}\frac{\left(c_f l_f^\theta K_0\right)^\gamma}{\rho - \gamma\Psi_f}, \tag{E.5}$$

where variables with subscript f pertain to the new equilibrium. Evaluating the percentage change in the initial stock of capital, K_0, that would make the representative agent as well off in the benchmark equilibrium as in the new steady state amounts to finding the quantity φ such that $W_b(c_b, l_b, (1+\varphi)K_0) = W_f$. This quantity is given by

$$\varphi = \left(\frac{c_f l_f^\theta}{c_b l_b^\theta}\right)\left(\frac{\rho - \gamma\Psi_b}{\rho - \gamma\Psi_f}\right)^{\frac{1}{\gamma}} - 1. \tag{E.6}$$

Table X – Summary Statistics

Variable	Definition	Obs.	Mean	Std. Dev.	Min.	Max.
Inequality	Gini coefficient	1147	44.786	7.353	25.239	72.180
Income Growth	Growth rate of real GDP per capita	1113	1.634	5.173	-33.588	42.116
Democracy	Polity IV normalized 0-1	954	0.565	0.357	0.05	1
Population Growth	Growth rate of total population	1113	2.089	1.882	-2.532	9.531
Agricultural Share	Value added in agricultural sector as a percentage of GDP	989	20.183	13.496	4	69
Youth Population	Population aged between 0-14 as a percentage of total population	1147	38.717	6.417	23	50
Aid	Real ODA as a percentage of GDP	1147	0.826	2.308	0.0001	25.782

Table XI – Aid and Inequality: Panel Estimation Results

	Fixed Effects (FE)				Diff-1 GMM			
	Model 1	Model 2	Model 3	Model 4	Model 1	Model 2	Model 3	Model 4
Constant	47.314***	51.264***	51.354***	51.413***				
	(26.48)	(24.69)	(25.39)	(25.71)				
Income Growth	0.019	0.004	0.012	0.017	0.355	-0.050	-0.031	-0.075
	(0.73)	(0.16)	(0.41)	(0.61)	(0.76)	(-0.46)	(-0.26)	(-0.65)
Democracy	-1.293**	-1.052	-1.082*	-1.026	-9.194	-8.614	-9.819***	-8.766***
	(-2.06)	(-1.64)	(-1.73)	(-1.65)	(-1.29)	(-3.33)	(-3.20)	(-3.01)
Population Growth	-0.006	-0.036	-0.008	-0.043	-0.697	-0.221	-0.414	-0.215
	(-0.09)	(-0.43)	(-0.11)	(-0.54)	(-0.45)	(-0.53)	(-0.80)	(-0.39)
Agricultural share	0.033	-0.071**	-0.101***	-0.114***	-0.500	-0.346***	-0.386***	-0.321**
	(0.92)	(-2.33)	(-3.37)	(-3.77)	(-1.20)	(-2.81)	(-2.76)	(-2.60)
Youth Population	-0.076	-0.178***	-0.170***	-0.168***	0.053	-0.020	0.013	-0.047
	(-1.32)	(-3.21)	(-3.14)	(-3.13)	(0.15)	(-0.12)	(0.07)	(-0.28)
Aid	0.066				0.393			
	(0.69)				(1.23)			
Aid(t-1)		0.320***				0.362**		
		(3.82)				(2.30)		
Aid (t-2)			0.301***				0.337**	
			(3.88)				(2.27)	
Aid(t-3)				0.288***				0.323***
				(3.96)				(2.63)
Sargan Test					0.987	0.883	0.969	0.899
AR(1)					0.630	0.675	0.734	0.882
AR(2)					0.446	0.055	0.112	0.106
R^2	0.77	0.69	0.71	0.72				
Observations	793	793	779	764	753	753	739	725

Notes: Dependent variable is Inequality measured by the gini coefficient. t-stats are in brackets. Diff-1 GMM is the one step difference GMM estimation. The reported values for the Sargan test are the p-values for the null hypothesis of instrument validity. The reported values for AR(1) and AR(2) are the p-values for the Arellano-Bond test of autocorrelation in the first differences equations. The absence of first (second) order autocorrelation of residuals is tested under the null hypothesis. *** significant at 1% level, ** significant at 5% level and * significant at 10% level.

Table XII – Aid and Inequality: Fixed Effects Estimations by Country Wealth

	Low Income Countries				Middle Income Countries			
	Model 1	Model 2	Model 3	Model 4	Model 1	Model 2	Model 3	Model 4
Constant	50.841***	50.598***	51.740***	51.516***	36.852***	44.127***	44.535***	45.033***
	(3.80)	(3.80)	(4.17)	(4.29)	(18.30)	(22.27)	(22.43)	(22.53)
Income Growth	-0.008	-0.019	-0.007	-0.003	0.032	0.033	0.034	0.037
	(-0.14)	(-0.33)	(-0.12)	(-0.06)	(1.16)	(1.02)	(1.08)	(1.18)
Democracy	-5.432***	-5.365***	-5.635***	-5.470***	-0.246	0.097	0.033	-0.028
	(-2.83)	(-2.82)	(-3.16)	(-3.15)	(-0.39)	(0.15)	(0.05)	(-0.04)
Population Growth	-0.185	-0.184	-0.096	-0.076	0.041	0.047	0.051	0.008
	(-0.89)	(-0.89)	(-0.50)	(-0.40)	(0.59)	(0.58)	(0.65)	(0.11)
Agricultural share	-0.061	-0.055	-0.114	-0.122	0.070*	0.028	0.0003	-0.004
	(-0.73)	(-0.69)	(-1.55)	(-1.65)	(1.74)	(0.63	(0.01)	(-0.08)
Youth Population	-0.338	-0.337	-0.325	-0.316	-0.146**	-0.087	-0.090	-0.105*
	(-1.08)	(-1.08)	(-1.11)	(-1.12)	(-2.56)	(-1.42)	(-1.45)	(-1.66)
Aid	0.937				0.071			
	(1.35)				(0.89)			
Aid (t-1)		0.958				0.209***		
		(1.39)				(2.82)		
Aid (t-2)			1.338**				0.216***	
			(2.08)				(3.03)	
Aid (t-3)				1.464**				0.237***
				(2.20)				(3.45)
R^2	0.75	0.75	0.79	0.81	0.80	0.73	0.74	0.74
Observations	159	159	154	148	634	634	625	616

Notes: Dependent variable is Inequality measured by the gini coefficient. t-stats are in brackets. *** significant at 1% level, ** significant at 5% level and * significant at 10% level.

Table XIII – Aid and Inequality: Diff-1 GMM Estimations by Country Wealth

	Low Income Countries				Middle Income Countries			
	Model 1	Model 2	Model 3	Model 4	Model 1	Model 2	Model 3	Model 4
Income Growth	0.107	0.040	-0.026	0.135	0.085	0.053	0.085	-0.0001
	(0.95)	(0.59)	(-0.24)	(1.49)	(0.97)	(0.54)	(0.99)	(-0.00)
Democracy	-13.961***	-9.263***	-7.998**	-11.083***	-8.996***	-7.838**	-12.307***	-8.880***
	(-2.67)	(-3.40)	(-2.44)	(-3.33)	(-3.80)	(-2.49)	(-4.34)	(-3.66)
Population Growth	0.073	-0.108	0.018	0.072	0.021	0.179	-0.035	0.141
	(0.24)	(-0.63)	(0.09)	(0.32)	(0.08)	(0.67)	(-0.13)	(0.57)
Agricultural share	-1.285***	-0.893***	0.195	-1.159***	0.205*	0.242*	0.300*	0.230*
	(-3.42)	(-5.16)	(0.68)	(-4.61)	(1.80)	(1.82)	(1.74)	(1.95)
Youth Population	0.620	1.085***	-1.081**	1.211**	-0.524***	-0.586***	-0.714***	-0.567***
	(0.97)	(2.88)	(-2.03)	(2.54)	(-2.97)	(-2.66)	(-3.16)	(-3.07)
Aid	0.890				0.149			
	(0.69)				(1.29)			
Aid (t-1)		1.489				0.244**		
		(1.64)				(2.06)		
Aid (t-2)			3.496*				0.211**	
			(1.68)				(2.31)	
Aid (t-3)				0.886				0.201**
				(0.96)				(2.04)
Sargan Test	1	0.784	0.751	0.999	0.468	0.742	0.946	0.581
AR(1)	0.097	0.202	0.102	0.102	0.006	0.241	0.185	0.019
AR(2)	0.651	0.099	0.360	0.236	0.193	0.233	0.565	0.136
Observations	150	150	145	139	603	603	594	586

Notes: Dependent variable is Inequality measured by the gini coefficient. t-stats are in brackets. Diff-1 GMM is the one step difference GMM estimation. The reported values for the Sargan test are the p-values for the null hypothesis of instrument validity. The reported values for AR(1) and AR(2) are the p-values for the Arellano-Bond test of autocorrelation in the first differences equations. The absence of first (second) order autocorrelation of residuals is tested under the null hypothesis. *** significant at 1% level, ** significant at 5% level and * significant at 10% level.

Table XIV – Benchmark Economy Parameters Values

	Benchmark Values
Production and Human Capital	$\omega_1 = 0.2$, $\omega_2 = 0.4$, $\eta = 0.2$, $A = 0.5$, $B = 0.2$
Preferences	$\rho = 0.04$, $\gamma = -1.5$, $\theta = 1.7$
Depreciation Rates	$\delta_K = 0.05$, $\delta_G = 0.05$, $\delta_H = 0.01$
Public Policy	$\tau = 0.15$, $\lambda_G = 0.04$, $\lambda_H = 0.04$
Foreign Aid	$\mu = 0$, $\alpha_G = 0$, $\alpha_H = 0$
Other	$\phi_1 = 10$, $\phi_2 = 10$, $i^* = 0.04$, $a = 0.15$

Table XV – Long-run effects of 5% Permanent Increase in Foreign Aid

	$\frac{H}{K}$	$\frac{K_G}{K}$	$\frac{D}{K}$	$\frac{V}{K}$	$\frac{C}{K}$	l	$i(\%)$	$\Psi(\%)$	$\Delta W(\%)$	$d\sigma_w(\%)$
Benchmark Equilibrium ($\mu = 0$, $\alpha_G = 0$, $\alpha_H = 0$)	11.2108	0.3292	0.182	1.4287	0.3662	0.7084	6.7676	1.1068	–	–
Untied Aid ($\mu = 0.05$, $\alpha_G = 0$, $\alpha_H = 0$)	11.3552	0.3259	0.1665	1.4347	0.3818	0.7221	6.5289	1.0114	5.8909	13.8411
Tied to Human Capital ($\mu = 0.05$, $\alpha_G = 0$, $\alpha_H = 1$)	13.5701	0.3403	0.2345	1.4087	0.3969	0.7074	7.5801	1.432	14.2588	1.708
Tied to Public Capital ($\mu = 0.05$, $\alpha_G = 1$, $\alpha_H = 0$)	8.1825	0.7554	0.2129	1.4169	0.3759	0.7033	7.245	1.2981	4.7882	2.5909

Note: The dispersion of welfare (welfare inequality) is reported as percentage variation relative to its pre-shock and benchmark level: $d\sigma_w = \left(\frac{\overline{\sigma}_w - \overline{\sigma}_{w0}}{\overline{\sigma}_{w0}}\right) \times 100$

Figure 6a – Aggregate Dynamics following an Untied Aid Shock

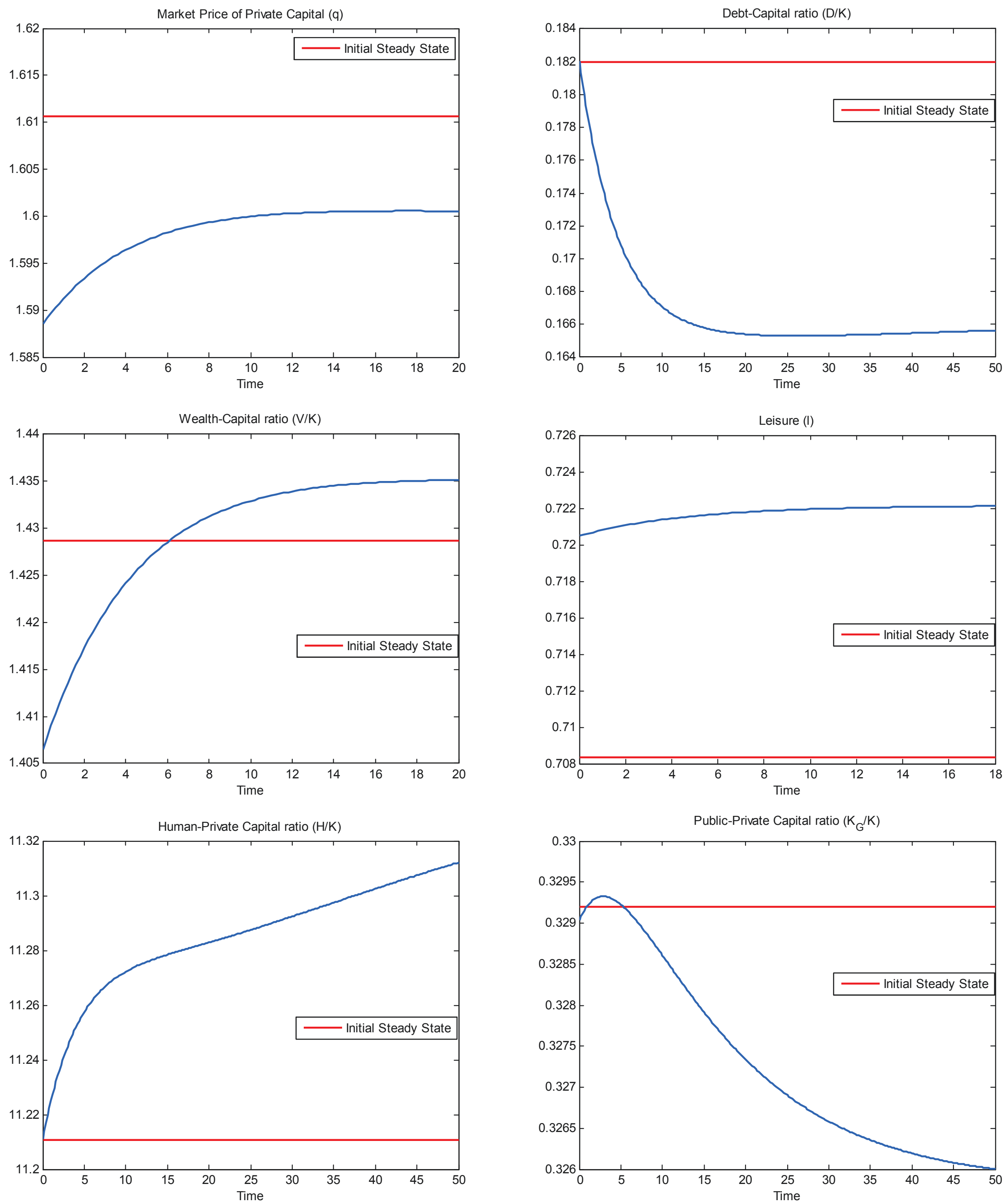


Figure 6b – Aggregate Dynamics following an Aid Shock Tied to Human Capital

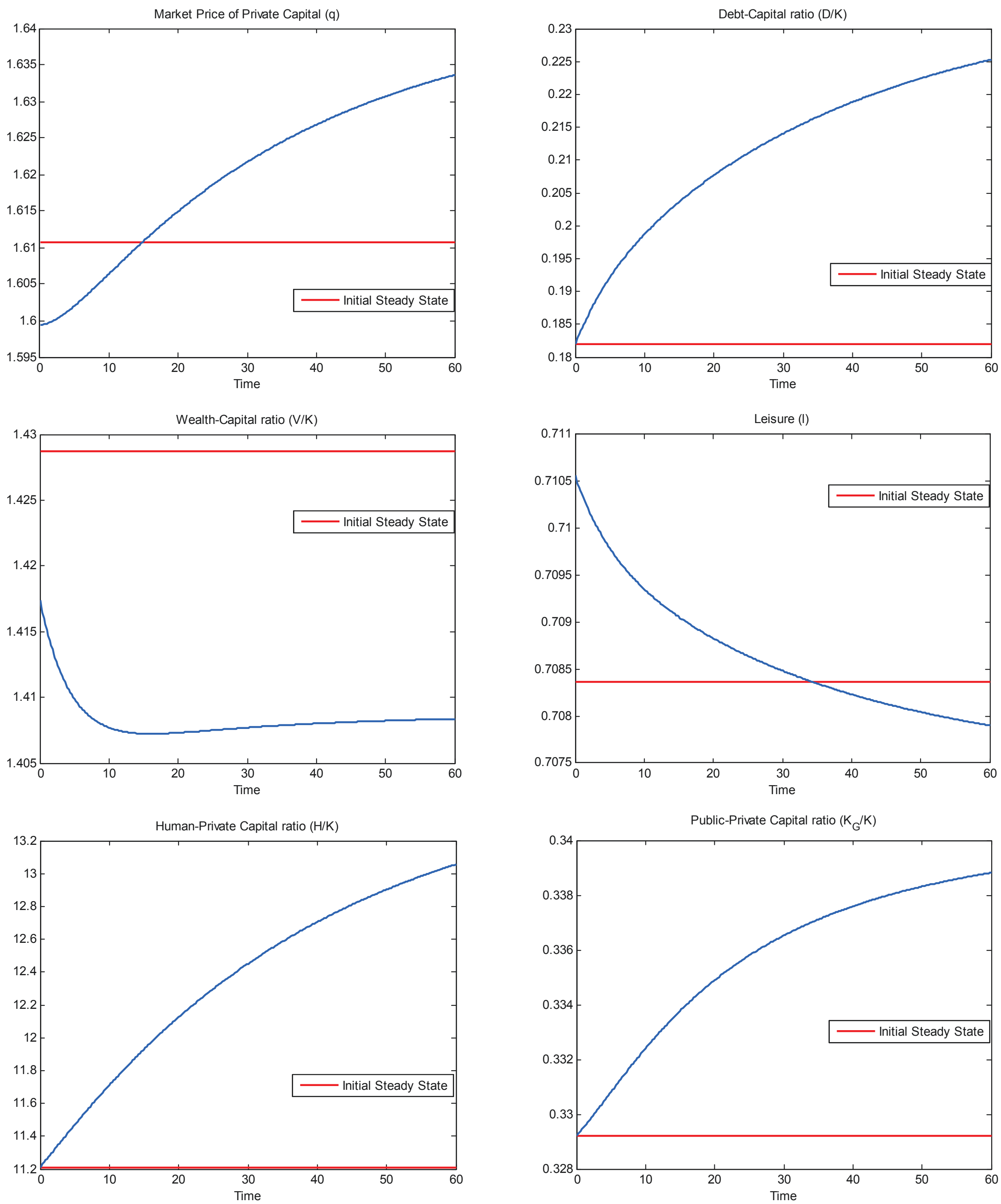


Figure 6c – Aggregate Dynamics following an Aid Shock Tied to Public Capital

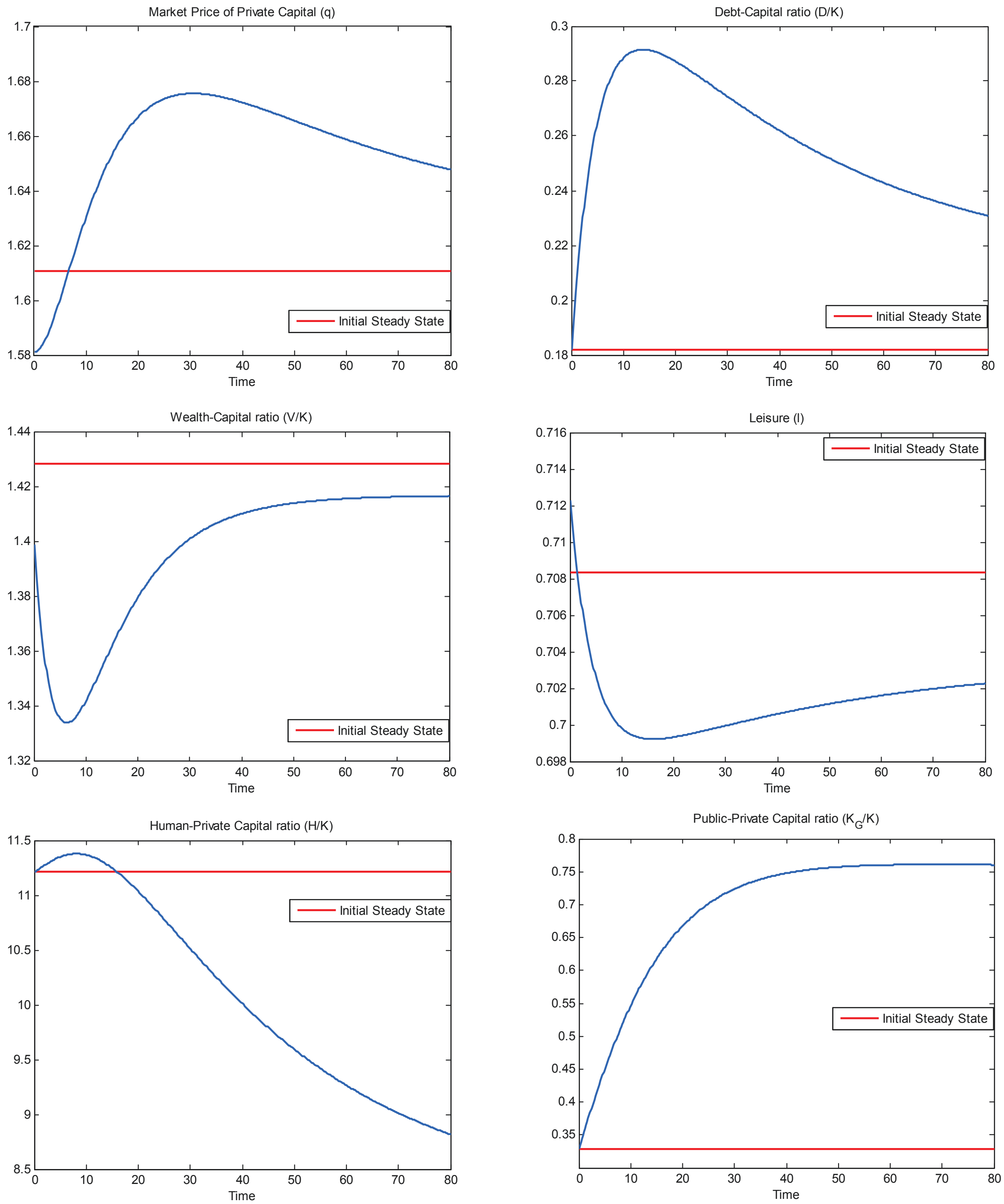
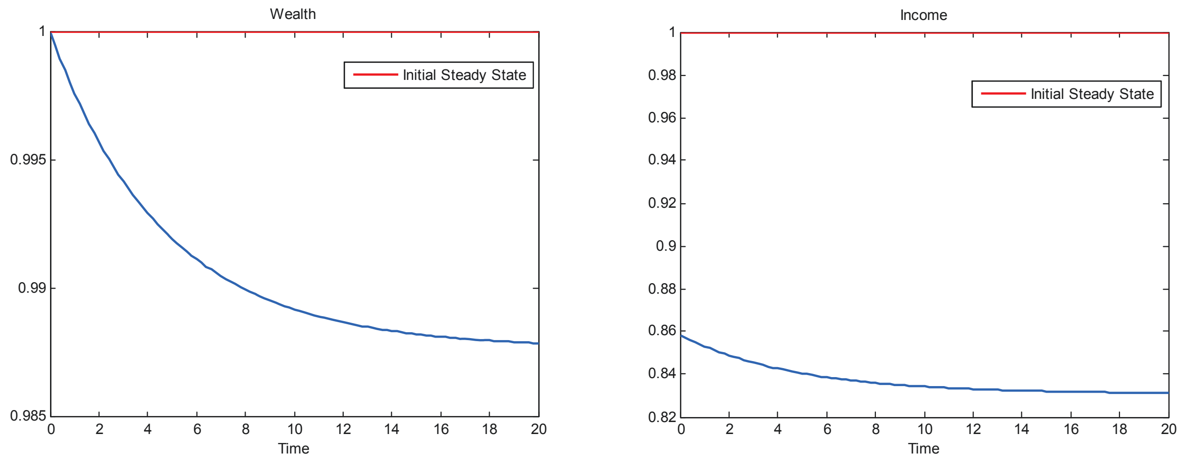
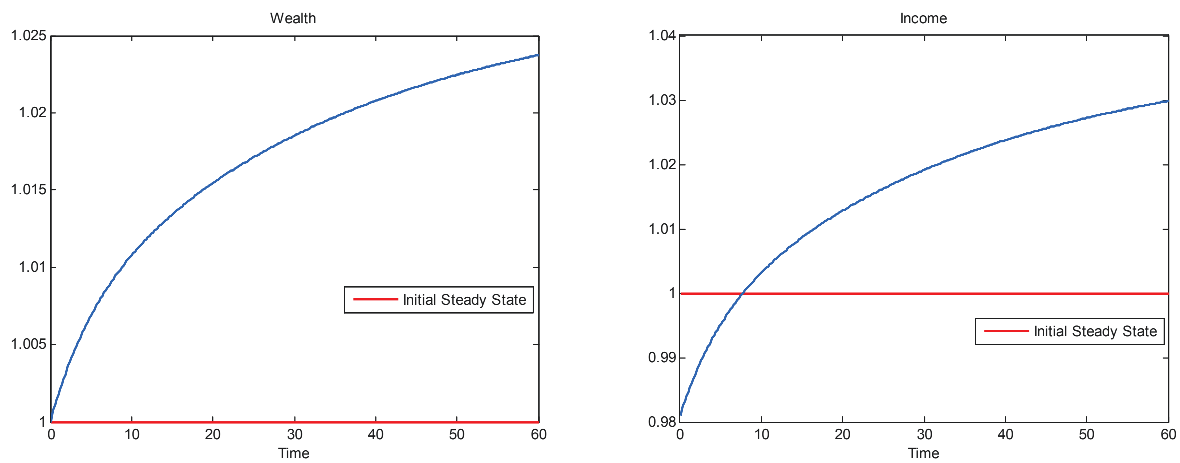


Figure 7 – Distributional Dynamics

Untied Aid



Aid Shock Tied to Human Capital



Aid Shock Tied to Public Capital

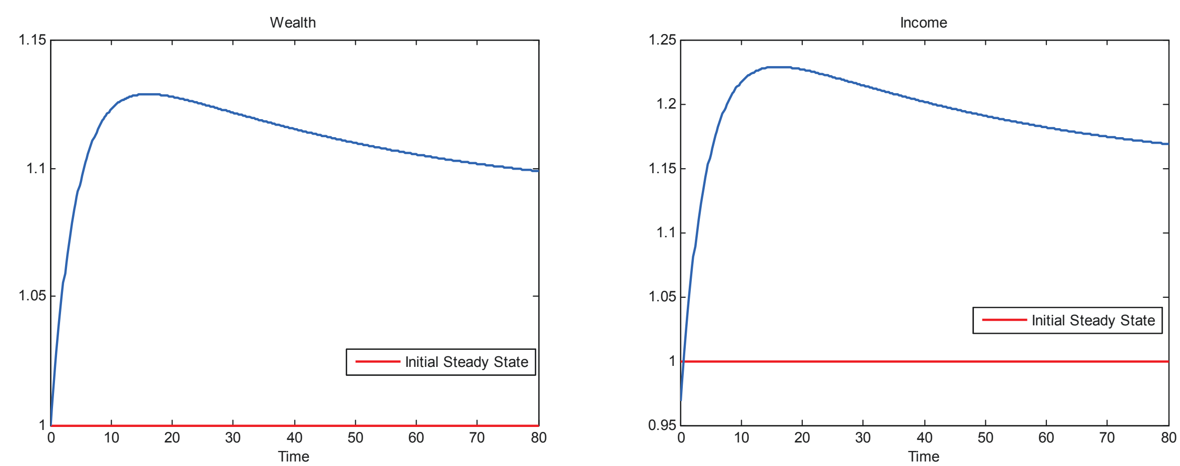


Figure 8a – Growth-Inequality Relationship following an Untied Aid Shock

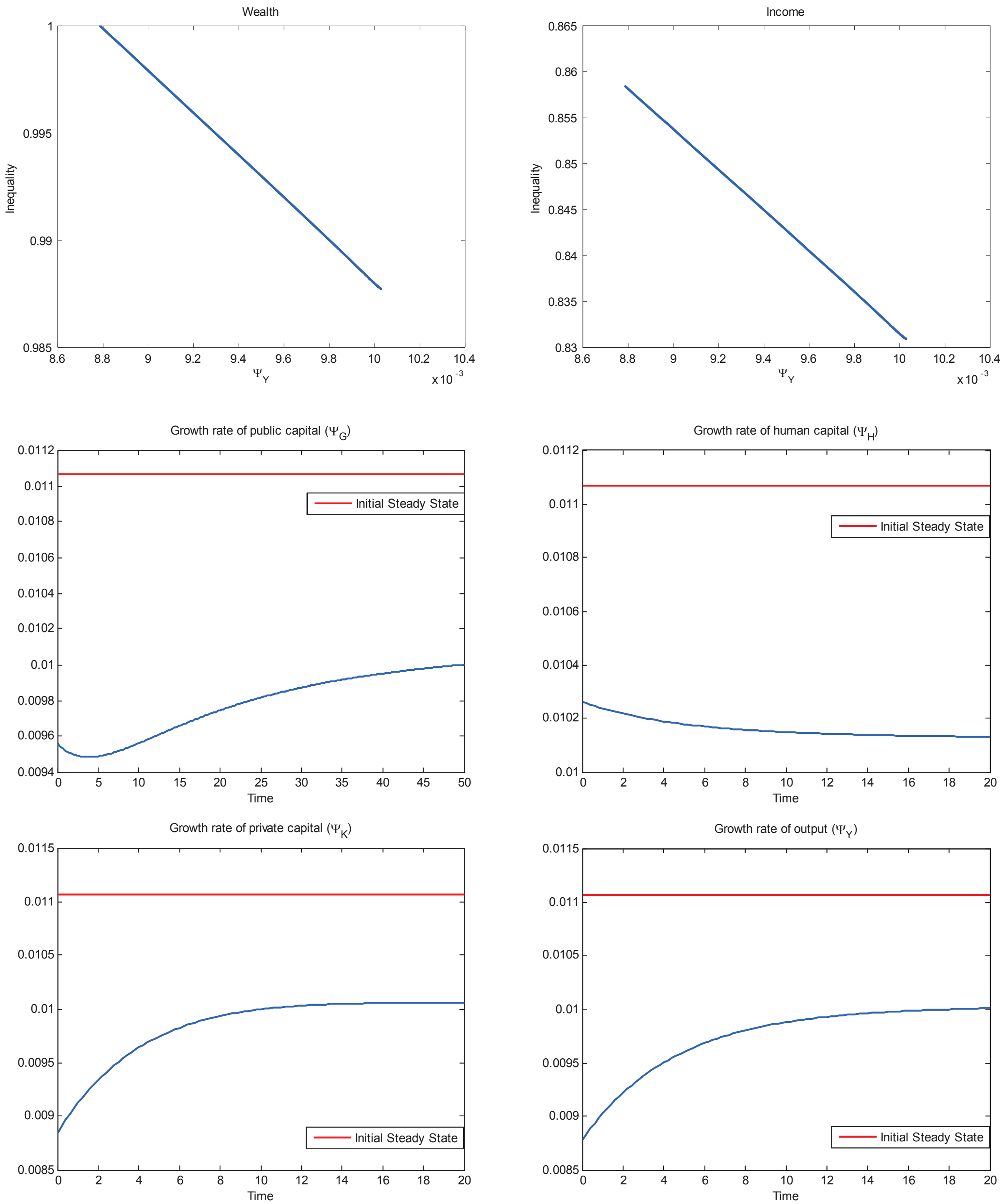


Figure 8b – Growth-Inequality Relationship following an Aid Shock Tied to Human Capital

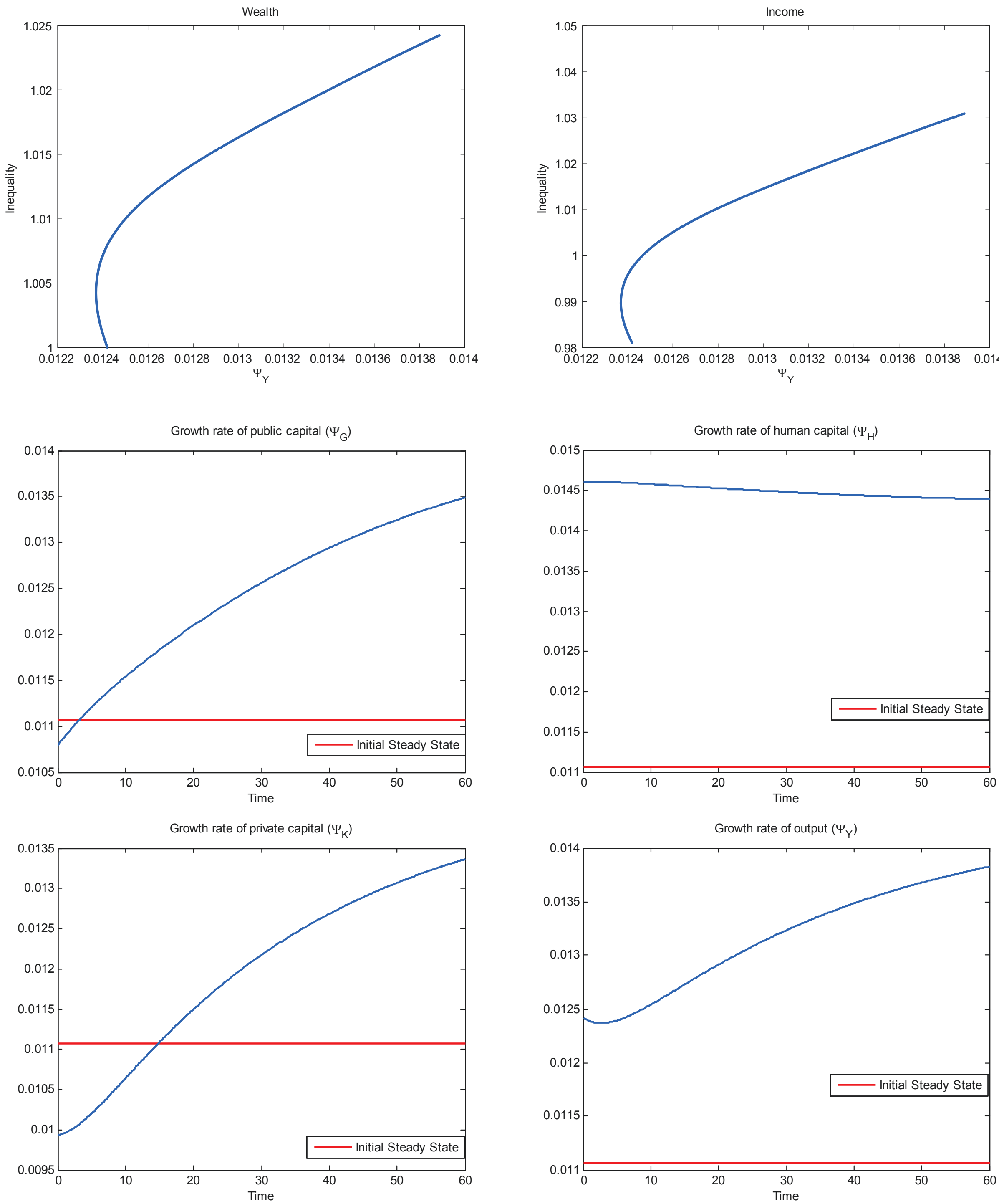


Figure 8c – Growth-Inequality Relationship following an Aid Shock Tied to Public Capital

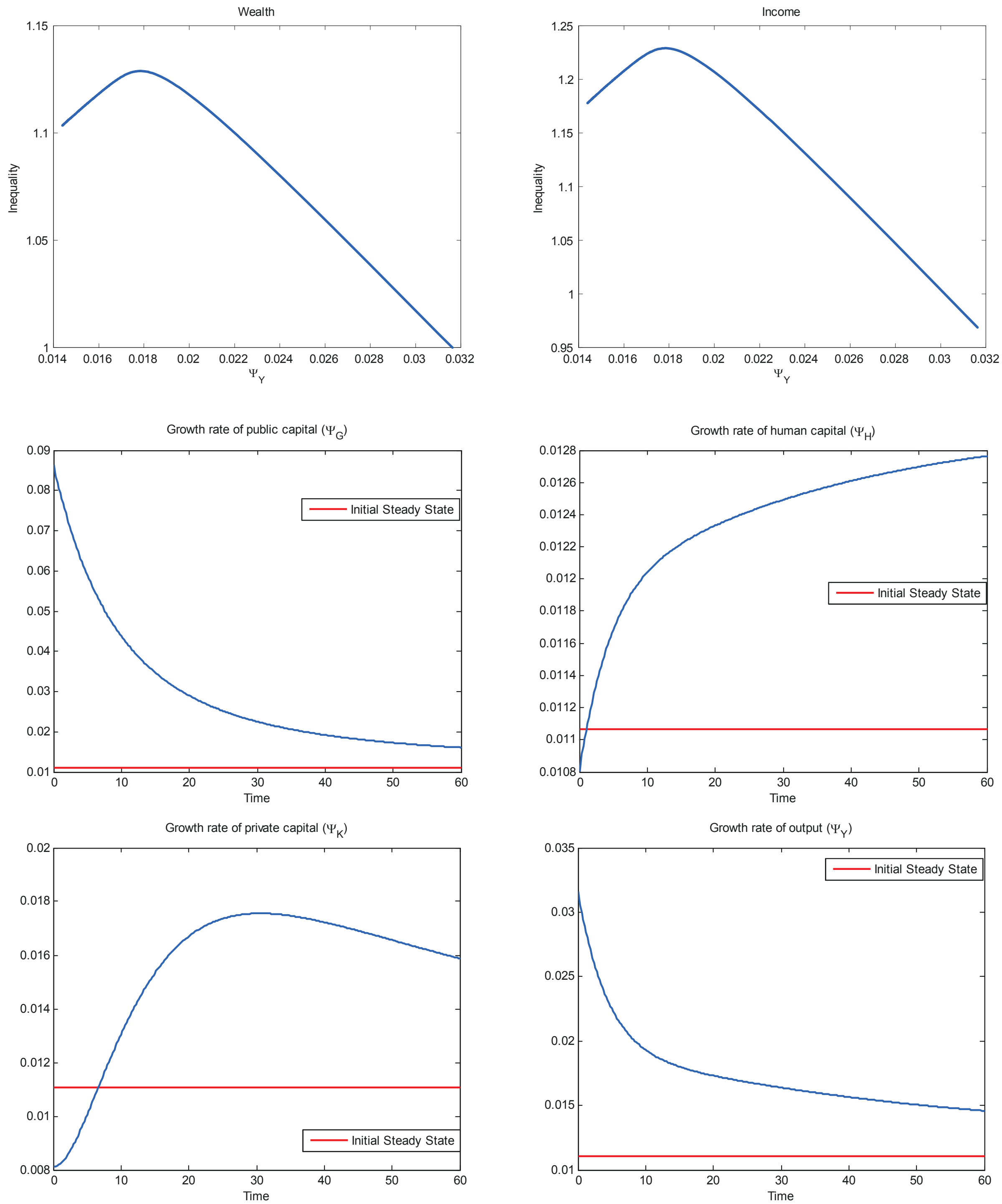
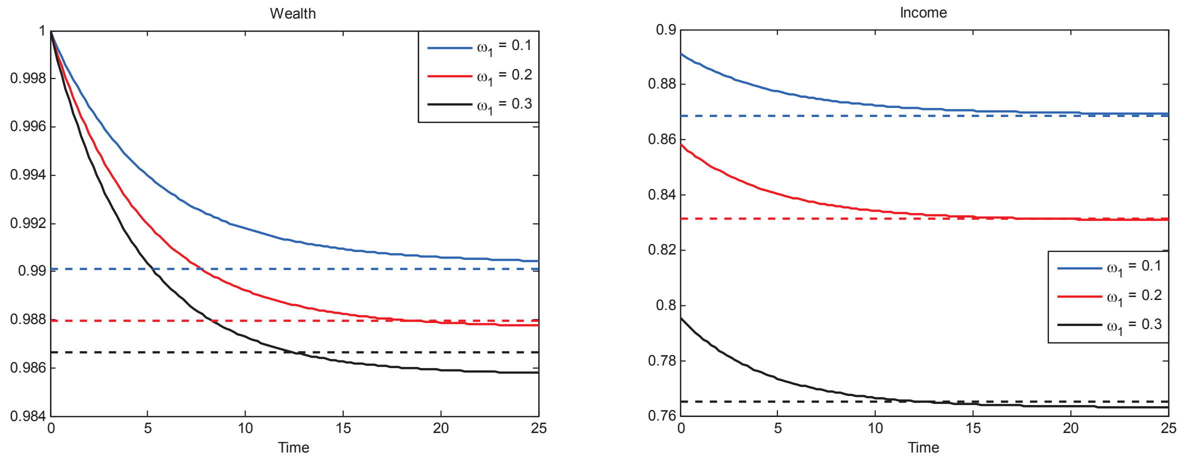
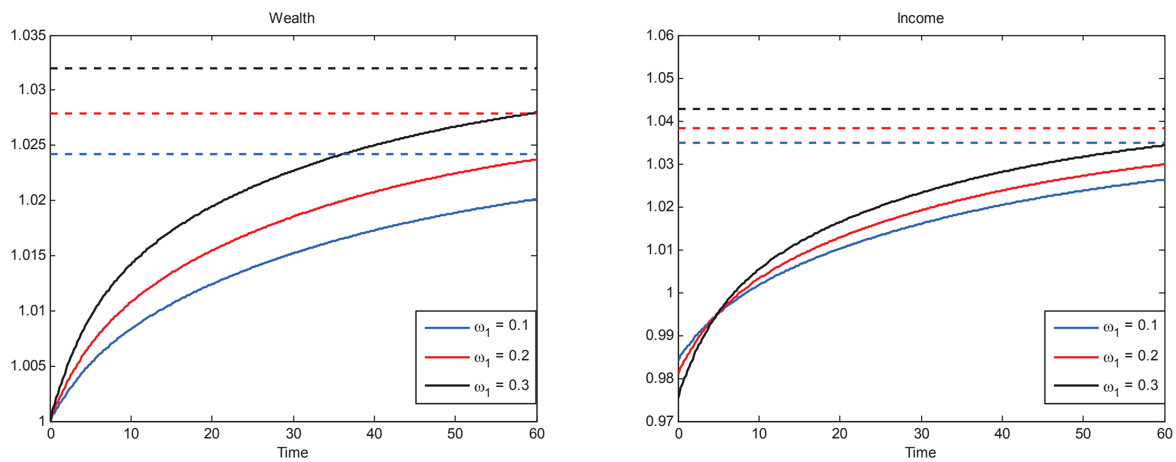


Figure 9 – Robustness Checks with respect to the output elasticity, ω_1

Untied Aid



Aid Shock Tied to Human Capital



Aid Shock Tied to Public Capital

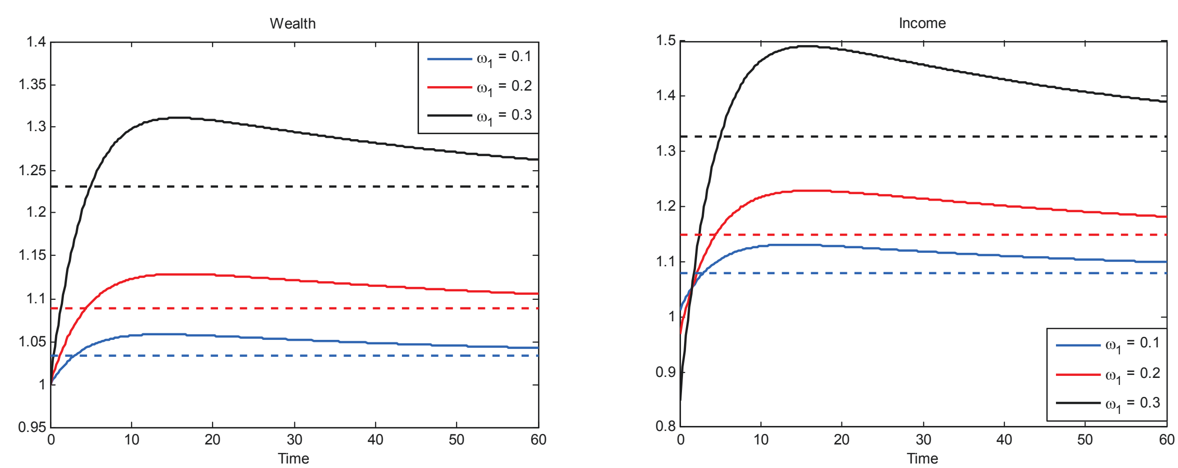
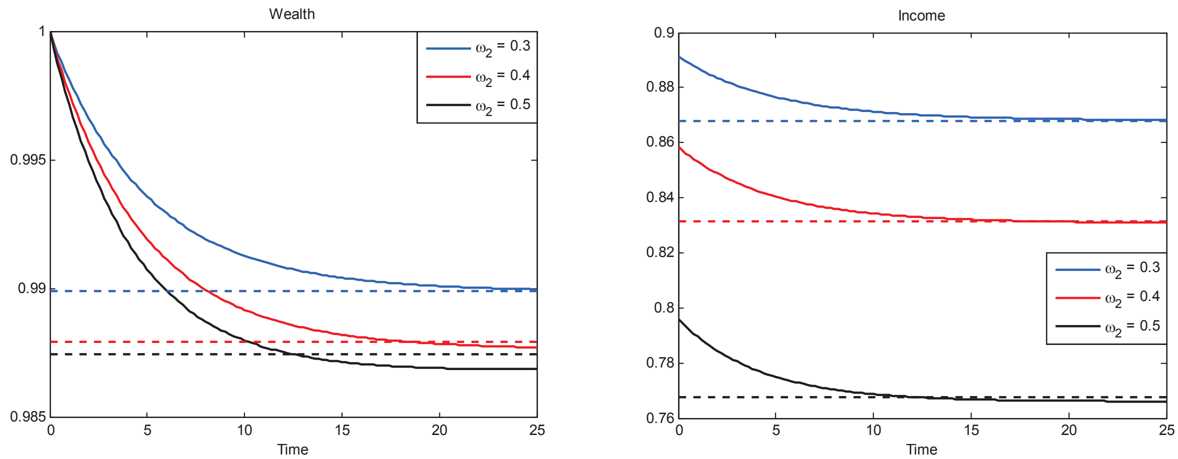
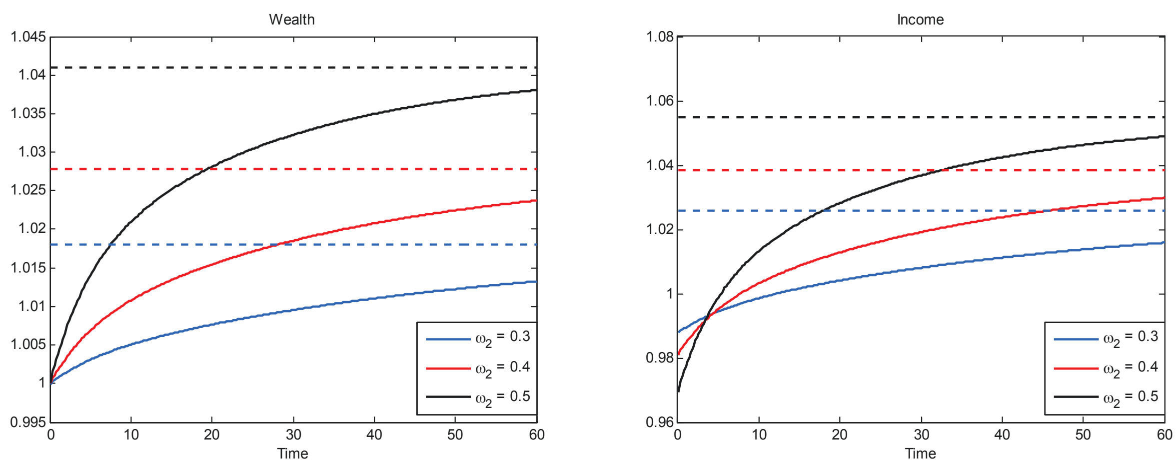


Figure 10 – Robustness Checks with respect to the output elasticity, ω_2

Untied Aid



Aid Shock Tied to Human Capital



Aid Shock Tied to Public Capital

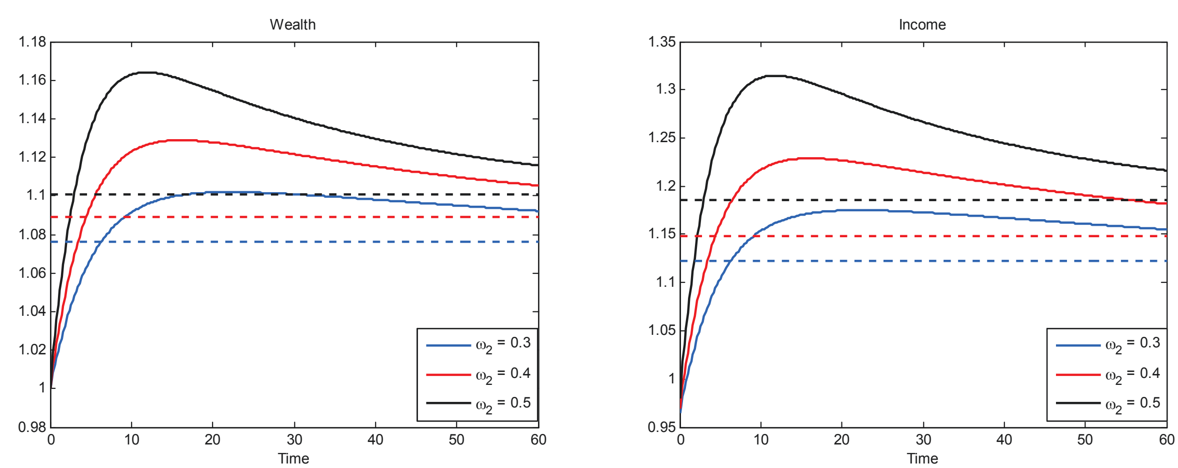
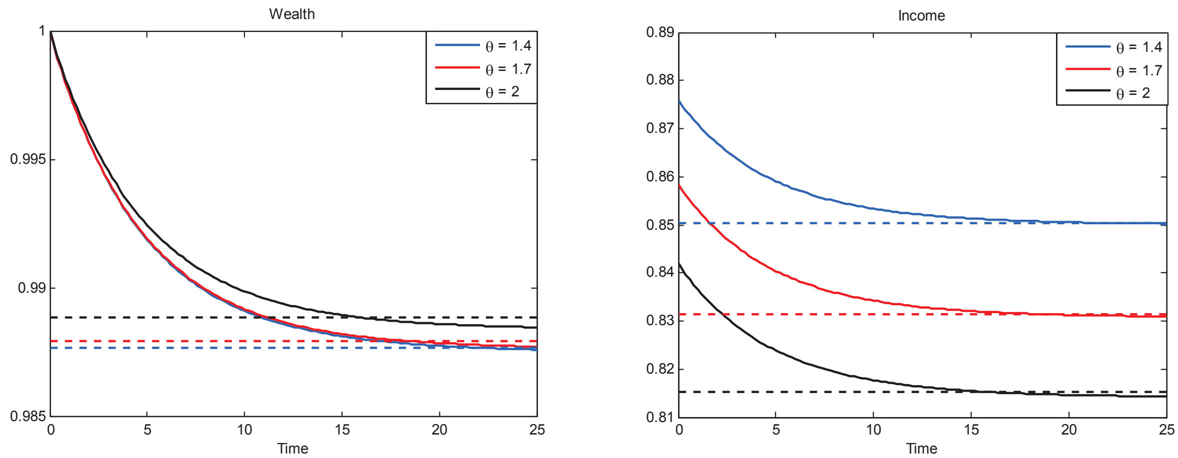
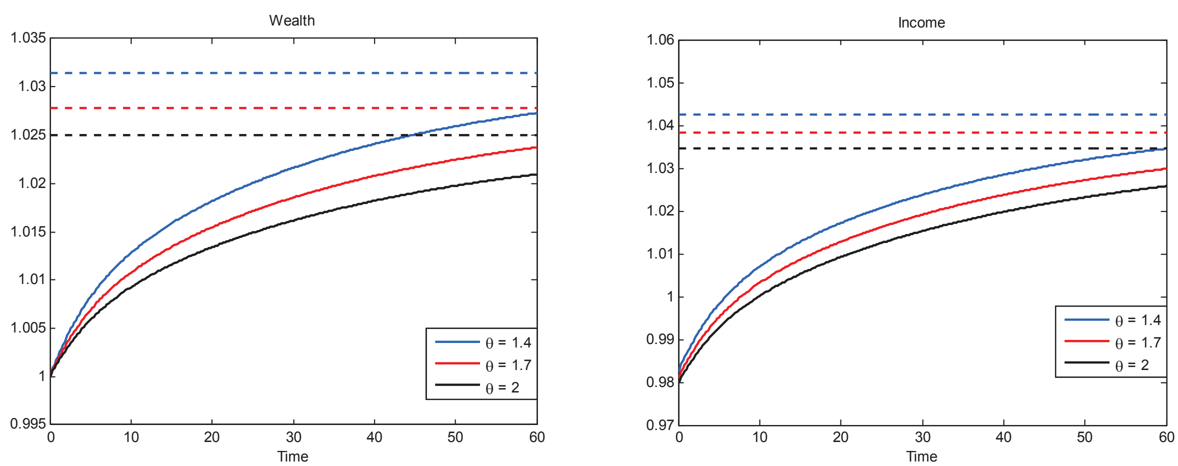


Figure 11 – Robustness Checks with respect to the leisure parameter, θ

Untied Aid



Aid Shock Tied to Human Capital



Aid Shock Tied to Public Capital

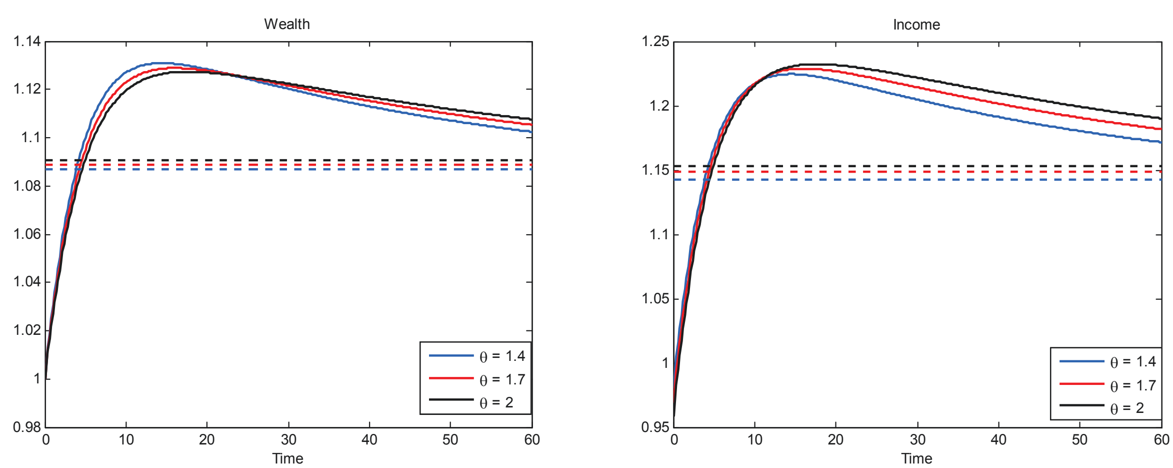
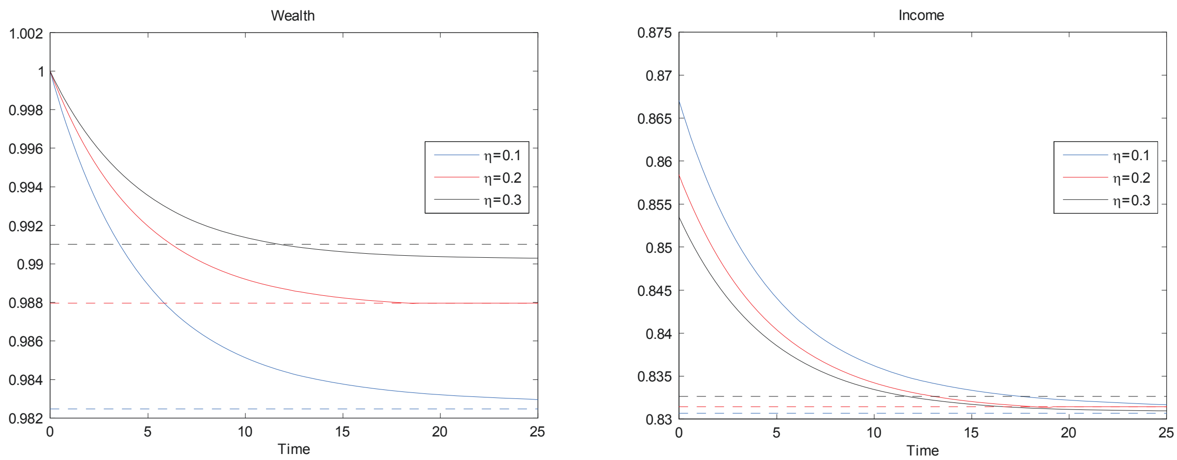


Figure 12 – Robustness Checks with respect to the human capital parameter, η

Untied Aid



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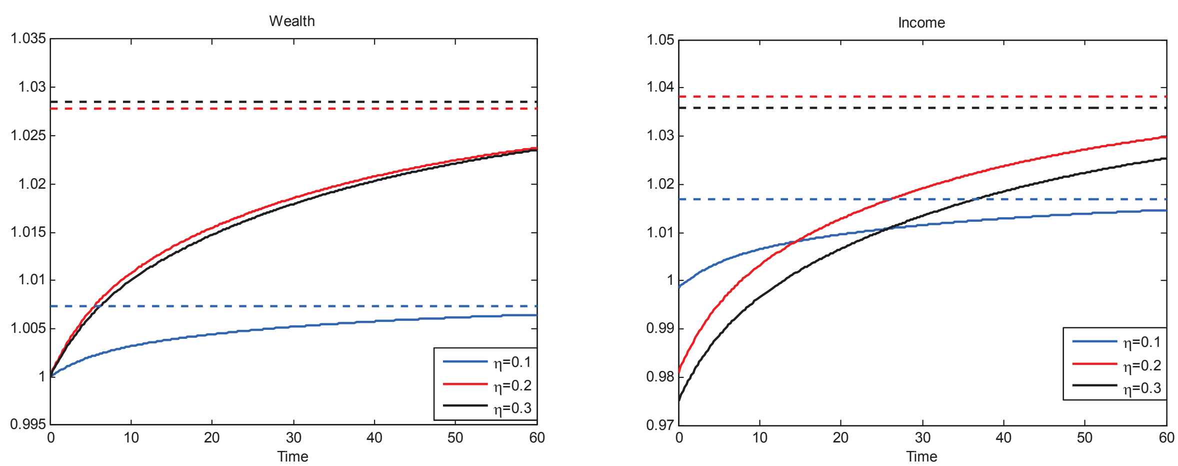

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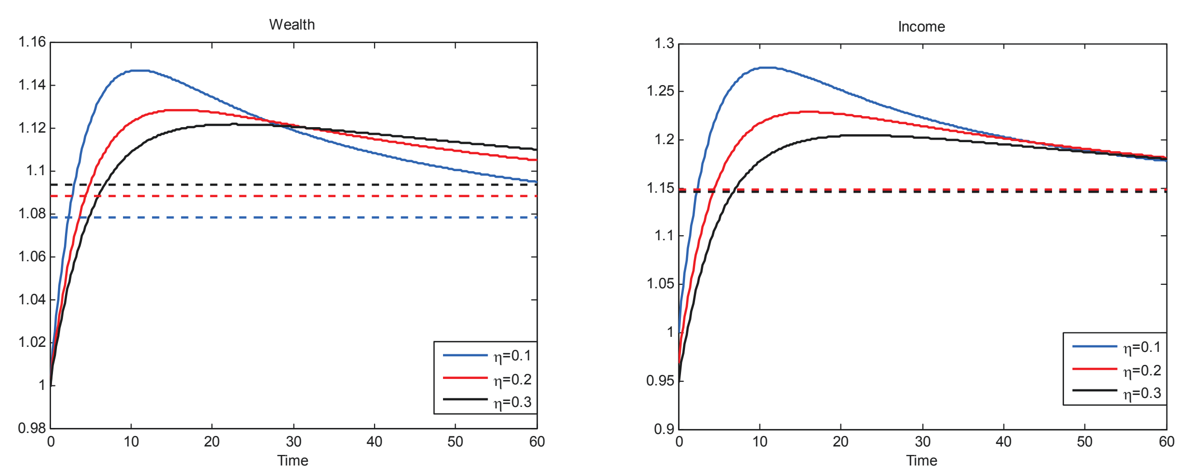


3 Essay 3 [Can Progressive Taxation Explain Fiscal Policy in Developing Countries?]

Abstract

This paper proposes a novel explanation for the empirical puzzling fact that fiscal policy is procyclical in developing countries, while it is acyclical or countercyclical in developed countries. Based on a sample of 36 developing and developed countries, I first revisit the stylized facts about the cyclical behavior of fiscal policy. While existing studies have focused almost exclusively on the cyclical properties of total public spending, I provide some new insights on the cyclicality of the main sub-categories of public spending (i.e., public consumption, public investment, social transfers, etc.). I show that only social transfers (such as benefits from social security and assistance) exhibit a different cyclical behavior across developed and developing countries, being countercyclical in the former group of countries and procyclical in the latter. The remaining sub-categories are procyclical in both groups. In the second part of the paper, I develop a theoretical model with automatic stabilizer mechanisms – through the progressivity of the tax schedule – that successfully accounts for the empirical evidence.

3.1 Introduction

Several papers provide evidence that fiscal policy is procyclical in developing countries whereas it is acyclical or countercyclical in developed countries (Ilzetzki (2011), Alesina et al.(2008), Talvi and Vegh (2005), Kaminsky et al.(2004), Lane (2003), Gavin and Perotti (1997)). In this respect, Governments in developing countries increase public spending and decrease tax rates during good times (expansions), and do the opposite during bad times (recessions).

Many researchers have attempted to explain why it could be optimal for a developing country to conduct a procyclical fiscal policy and their results can be classified in two main strands. The first one is based on a political-economy argument according to which governments in developing countries face higher political pressure from different interest groups and lobbies during good times when they record budget surpluses. The increased competition for public resources leads governments to run procyclical fiscal policy by overspending and lowering tax rates during booms (see Ilzetzki (2011), Talvi and Vegh (2005), Alesina et al. (2008), Lane and Tornell (1999)). The second explanation invokes the financial constraints faced by developing countries and their limited access to capital markets, which can force them to raise tax rates and cut spending during economic recessions (see Kuralbayeva (2013), Demirel (2010), Cuadra et al (2010), Gavin and Perotti (1997)).[32]

In this paper, I start by providing new evidence about the cyclical properties of government spending, which brings new insights about the conduct of fiscal policy in developing countries. I then develop a theoretical model that rationalizes the documented facts. My empirical analysis is based on a sample of 36 developing and developed countries over the period 1929-2011. In line with existing studies, I find that fiscal policy is generally procyclical in developing countries and countercyclical in developed countries. However, in contrast to these earlier studies, which have focused almost exclusively on the cyclical properties of total public spending, I also examine the cyclicality of the main sub-categories of public spending (i.e., public consumption, public investment, social transfers, etc.). I show that only social transfers (such as benefits from social security and assistance) exhibit a different cyclical behavior across developed and developing countries, being countrercyclical in the former group of countries and procyclical in the latter. The remaining sub-categories are procyclical in both groups.

In the second part of the paper, I propose a novel explanation for the procyclicality of fiscal policy in developing countries, that is consistent with the empirical findings just discussed. My explanation invokes the difference in the degree of progressivity of the tax schedule across developed and developing countries. It is well known that the tax systems in developing countries are substantially less progressive than those in developed economies (Schmitt (2003)). Governments in the former group of countries indeed collect far less revenues from income taxes and rely mainly on taxation of sales of domestic goods and services, such as general-consumption and excise taxes, which are highly regressive. Rao and Weller (2008) document that over the period 1981–2002, the top marginal rates in developing countries declined more than the average tax rates, suggesting that the taxation system became less progressive over time.

I develop a model economy populated by a unit-measure continuum of identical households made of infinitely lived family members. Following Diamond (1982a,b) and Mortensen and Pissarides (1994), I introduce search-and-matching frictions in labor markets. Each family member can be either employed, in which case she receives a Nash bargained wage, or unemployed and searching for a job. There is a public sector that collects taxes in order to finance several types of discretionary expenditures, namely, payment of unemployment benefits, investment in public capital, spending on consumption good and social transfers to private agents. The model is used to

[32] This second explanation seems to be inconsistent with the fact that most developing countries do have access to international capital makets, which allowed them to accumulate large amounts of foreign debt. Data on the external debt of 12 (out of 17) developing countries included in my sample show that developing economies are still able to borrow a significant share of their GDP even during economic recessions. On average, these countries accumulated external debt equivalent to 61.2% of their GDP between 1972 and 2012. The external debt in Nicaragua soared to 206% of its GDP during this period.

analyze the optimal fiscal policy following an increase in the (exogenous) energy price under different degrees of progressivity in the tax schedule.

The dynamic responses of key endogenous variables indicate that under plausible parameter values, the adverse energy price shock generates a real contraction that increases the aggregate level of unemployment and reduces labor market tightness, which means that firms have higher probability of filling a vacancy. In both developed and developing economies, the responses to the adverse energy price shock involve some downward adjustments in government consumption and public investment, which lead to decreases in the stock of public capital. The theoretical framework delivers some important implications for the cyclical comovements of social transfers with output that are consistent with the empirical analysis. More precisely, following the adverse energy price shock, social transfers from government to private agents increase in the developed economy characterized by a progressive tax system, but fall in the developing country. The optimizing policymaker in the developing country runs a procyclical fiscal policy by cutting all components of public spending in response to the real contraction generated by the energy price shock. This adjustment to the shock contrasts with the countercyclical responses of social transfers observed in the developed economy. The intuition behind these results lies in the stabilizing effects of the progressive taxation on several variables that are directly under the control of private agents like consumption, hours worked, private investment and accumulation of capital. When the tax system is flat, private agents respond to the energy price shock by working less and decreasing their investment in capital more than they would under a progressive tax schedule. This suggests, that the tax revenue losses are far greater in the developing country, which in turn induces the government to reduce all components of public spending in order to maintain a balanced budget over time. However, in the developed economy case, the government optimally chooses to strenghten the stabilizing effects of the progressive taxation by decreasing public consumption and investment more than the fall in tax revenues while providing private agents with more social transfers.

The discussion of the correlations of output with each component of government spending shows that the model with the benchmark parametrization can replicate all procyclical fiscal policy characteristics observed in developing countries. Moreover, as the degree of tax progressivity increases, the correlations of output with government consumption and investment increase, while the output-social transfers correlation decreases and becomes negative when the degree of tax progression exceeds the threshold of 0.1. In other words, social transfers from government to private agents, become less procyclical eventually they turn countercyclical as the degree of progressivity in the tax schedule increases. These results suggest that, under plausible parameter values, the weakness of automatic stabilizers mechanisms can be an important driving force of the procyclicality of fiscal policy.

In the last part of the paper, I conduct a sensitivity analysis by considering alternative values for the key parameters of the model, namely those related to matching elasticity, the bargaining power of workers, the depreciation and utilization rates, the labor and capital tax rates and the unemployment benefits. The main finding stating that a model with different degrees of tax progressivity can explain the empirical characteristics of fiscal policy in both developing and developed ecnomies, is generally robust.

The rest of the paper is organized as follows. Section 2 presents the data and stylized facts while Section 3 describes the model economy. In Section 4, I define the ramsey problem and Section 5 discusses the baseline results related to the optimal fiscal policy following an adverse energy price shock. Section 6 performs a sensitivity analysis. Section 7 concludes.

3.2 Data and Stylized Facts

This section reviews the stylized facts about cyclical properties of fiscal policy for a sample of 36 countries, which includes 17 developing countries and 19 high-income countries. The coverage period ranging from 1929 to 2011, differs across countries.[33] For each country, I collect annual data on gross domestic product, total private consumption, public consumption, public investment, public social benefits and total public spending. Social benefits, which comprises benefits from social security and assistance serve as a proxy for public transfer payments. The sub-sample of developing countries is restricted to countries for which data on social benefits are available. Data for developing countries have been deflated using the GDP deflator, while that for developed countries has been collected directly in real terms.

My data come from various sources. For the sub-sample of developed countries, all data are from the OECD's Annual National Accounts except for the United States of America, for which they come from the Bureau of Economic Analysis. For developing countries, all data are from World Development Indicators and IMF's International Financial Statistics and Government Finance Statistics. All variables are converted to logs and Hodrick-Prescott filtered. In order to contrast the fiscal experience of developing countries to that of developed countries, I focus on standard deviation, which serves as a measure of volatility and the correlations of the

[33] Detail on coverage period for each variable and country is provided in the Appendix (see Table A.3).

macroeconomic variables cited above with output.

Table XVI presents the stylized facts about the volatility of macroeconomic and fiscal variables in developed and developing countries. Developing countries tend to have higher volatility of output than developed economies. Private consumption is more volatile than output in developing countries, while it is less volatile than output in developed countries.[34] These facts about output and private counsumption are in line with the literature (see García-Cicco et al. (2010), Aguiar and Gopinath (2007), Uribe and Yue (2006), Neumeyer and Perri (2005)). Total public spending and its components display higher volatility in developing countries when we compare to developed countries. In both groups of countries, public investment is not only more volatile than output, but also the most volatile component of total public expenditure. While in the sub-sample of developed countries, public investment is three times as volatile as output, in developing countries it is more volatile by a factor of 6. In developed countries, social benefits are slightly more volatile than output, whereas they are five times more volatile in the sub-sample of developing countries. Moreover, Latin America countries display the highest volatilities in these two categories of public spending. Finally, public consumption and total public spending are as volatile as output in developed countries, but more volatile than output in developing countries.

[34] The excess volatility of private consumption in developing countries can be explained by international borrowing limits and shocks that affect the external interest rates or the volatility of borrowing premium (Fernández-Villaverde et al (2009), Aguiar and Gopinath (2007), De Resende (2006), Neumeyer and Perri (2005))

Table XVI – Volatility of fiscal variables

Country	Standard Deviation					
	Output	Private Consumption	Public Consumption	Public Investment	Social Benefits	Total Public Spending
Developed Countries	0.03	0.02	0.03	0.09	0.04	0.03
G7	0.03	0.02	0.05	0.12	0.04	0.02
Other Developed	0.03	0.03	0.02	0.08	0.05	0.03
Developing Countries	0.04	0.06	0.10	0.23	0.18	0.07
Latin America	0.04	0.06	0.11	0.24	0.21	0.08
Other Developing	0.04	0.06	0.10	0.22	0.15	0.05

The stylized facts with respect to correlations are shown in Table XVII. In both groups of countries, public consumption and public investment are procyclical. However, there is a difference between high-income and developing countries in the comovements of social benefits and total public spending with output. Public transfer payments as well as total public expenditure, are countercyclical in developed countries, while they are procyclical in developing countries. These facts relate to the literature, since Kaminsky et al. (2004) have shown that government spending is countercyclical in high-income countries, whereas it is procyclical in Latin America and other developing countries. Furthermore, according to Ilzetzki (2011), it seems that, public transfer payments are the main driver of developed countries' countercyclical fiscal policy.

The last column of Table XVII presents the average social benefits as a percentage of GDP in each group of countries. It is worth noting that on average social benefits are more than twice higher in developed countries than they are in developing countries. It may be the case that even developing countries have countercyclical transfer payments, they are not sufficiently sizable to induce countercyclical fiscal policy (like Peru and Mauritius).[35] In contrast, a developing country such as Croatia, for which public transfer payments average 15% of its GDP, has countercyclical social benefits and public spending.

[35] Table A.2 in the Appendix

Table XVII – Comovements with output

Country	Correlations with Output					Average Social Benefits (% of Output)
	Private Consumption	Public Consumption	Public Investment	Social Benefits	Total Public Spending	
Developed Countries	0.78	0.25	0.19	-0.43	-0.22	14.98
G7	0.77	0.22	0.24	-0.49	-0.26	15.72
Other Developed	0.79	0.26	0.17	-0.40	-0.19	14.55
Developing Countries	0.62	0.39	0.34	0.22	0.31	6.23
Latin America	0.59	0.47	0.45	0.27	0.38	5.40
Other Developing	0.66	0.27	0.20	0.17	0.24	7.16

3.3 The Model Economy

I consider an economy populated by a unit-measure continuum of identical households that produces a single traded good using, capital (private and public), labor and energy. The utilization rate of private capital is chosen endogenously. It is never equal to 100% because of the costs related to depreciation and energy purchases. The single good produced can be used as a consumption good, an investment good or as a means of payment for energy purchases from abroad.

Each household is made of infinitely lived family members whose total measure is normalized to one. Each family member can be either employed, in which case she receives a Nash bargained wage, or unemployed and searching for a job. As common in the literature, I assume that unemployed individuals receive unemployment benefits from the government. There are some matching frictions in the labor market, which is also characterized by an exogenous job separation.

Finally, the government collects taxes in order to finance several types of discretionary expenditures, namely, payment of unemployment benefits, investment in public capital, spending on consumption good and social transfers to private agents.

3.3.1 Households

The infinitely lived household maximizes the following utility function

$$U_0 = E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, Cg_t, l_t, h_t),$$

with

$$U(.) = \left[u(C_t, Cg_t) - l_t v(h_t)\right], \tag{93}$$

where C_t is consumption, Cg_t is public spending on consumption, l_t is the number of employed individuals in the household, h_t is the number of hours worked by each employed individual and $\beta \in (0,1)$ is the subjective discount rate. The instantaneous utility function, $u(.)$ and disutility of work, $v(.)$ satisfy the standard conditions.

Each household accumulates private capital according to the law of motion

$$K_{t+1} = I_t + (1 - \delta(\mu_t))K_t, \quad \text{with } \delta(\mu_t) = \psi_1 \frac{\mu_t^{\gamma_1}}{\gamma_1}, \tag{94}$$

where I_t is new investment in capital, $\delta(.) \in (0,1)$, $\psi_1 > 0$ and $\gamma_1 > 1$ are parameters. As formulated in earlier work (eg. Taubman and Wilkinson (1970), Greenwood, Hercowitz, and Huffman (1988) and Finn (2000)), the depreciation rate is an increasing convex function of the utilization rate, μ.

Following Finn (2000), I assume that capital utilization requires energy, which is given by the following equation

$$e_t = a(\mu_t)K_t, \quad \text{with } a(\mu_t) = \psi_2 \frac{\mu_t^{\gamma_2}}{\gamma_2}, \tag{95}$$

where e_t is energy usage and $\psi_2 > 0$ and $\gamma_2 > 1$ are parameters. The energy usage increases with the utilization rate of capital. Housebols must supply energy along with capital services.

In each period, the households receive a total income made of their after-tax earnings, the lump-sum social transfers from government, the profits from their ownership of firms and the unemployment insurance benefits.[36] This total income finances consumption, investment and the purchases of energy from abroad. The household chooses sequences of consumption, utilization rate and capital $\{C_t, \mu_t, K_{t+1}\}$ to maximize lifetime utility function subject to the following budget constraint

$$C_t + I_t + p_t e_t = (1 - \tau_t^w) l_t w_t h_t + (1 - \tau_t^k) l_t r_t \mu_t K_t + T_t + \pi_t + b(1 - l_t), \tag{96}$$

where T_t denotes lump-sum transfers from government, π_t stands for the profit income received from firms, b is the unemployment benefit received by unemployed family members and p_t is the exogenous price of energy

[36] The social transfers refer to the payments of welfare and child benefits, the old age security pension and other income from the government. However, they do not include the benefits from employment insurance, which are represented by the variable, $b(1 - l_t)$.

expressed in terms of the final good.

The household receives labor income (y_t^n) and capital earnings (y_t^k), which are taxed at the progressive rates τ_t^w and τ_t^k , respectively. Following Guo (1999), Guo and Lansing (1998) and Mattesini and Rossi (2012), I assume that τ_t^w and τ_t^k are given by

$$\tau_t^w = 1 - \zeta_1 \left(\frac{\overline{w}}{y_t^n}\right)^{\phi}, \quad \zeta_1 \in (0,1], \quad \phi \in [0,1), \tag{97}$$

$$\tau_t^k = 1 - \zeta_2 \left(\frac{\overline{r}}{y_t^k}\right)^{\phi}, \quad \zeta_2 \in (0,1], \quad \phi \in [0,1), \tag{98}$$

where $y_t^n = l_t w_t h_t$ and $y_t^k = l_t r_t \mu_t K_t$ represent the household's taxable labor and capital income; $\overline{w}$ and $\overline{r}$ represent base levels of labor and capital earnings, which are taken as given by the household. I set these levels to the steady state levels of labor and capital income. The parameters ζ_1, ζ_2 and ϕ determine the level and the slope of the tax schedule. One can understand the progressivity of the tax schedule by distinguishing between the average tax rates, which are given by (97) and (98), and the marginal tax rates on labor and capital, which are given by

$$\tau_{m,t}^w = \frac{\partial(\tau_t^w y_t^n)}{\partial y_t^n} = 1 - \zeta_1(1-\phi)\left(\frac{\overline{w}}{y_t^n}\right)^{\phi} \tag{99}$$

$$\tau_{m,t}^k = \frac{\partial(\tau_t^k y_t^k)}{\partial y_t^k} = 1 - \zeta_2(1-\phi)\left(\frac{\overline{r}}{y_t^k}\right)^{\phi} \tag{100}$$

In order to ensure that households have an incentive to supply labor and capital to firms, I consider the case with average and marginal tax rates that are strictly less than 100%. A tax schedule is progressive when the marginal tax rate is higher than the average tax rate at any level of income. Since (99) and (100) can be rewritten as follows

$$\tau_{m,t}^w = \tau_t^w + \zeta_1 \phi \left(\frac{\overline{w}}{y_t^n}\right)^{\phi},$$

$$\tau_{m,t}^k = \tau_t^k + \zeta_2 \phi \left(\frac{\overline{r}}{y_t^k}\right)^{\phi},$$

the tax system is progressive when $\phi > 0$. When $\phi = 0$, the average and marginal tax rates on labor income are both equal to $1 - \zeta_1$, while those on capital earnings are equal to $1 - \zeta_2$, and the tax schedule is said to be "flat" or "proportional".[37] When $\phi < 0$, the tax system is said to be "regressive", which means that the tax rate is a decreasing function of the household's income.

Denoting by λ_t the Lagrange multiplier associated with the household's budget constraint, the first order conditions with respect to C_t, μ_t, and K_{t+1} are

$$u_{c,t} = \lambda_t \tag{101}$$

$$\psi_1 \mu_t^{\gamma_1 - 1} K_t + \psi_2 \mu_t^{\gamma_2 - 1} p_t K_t = l_t r_t K_t (1-\phi) \zeta_2 \left(\frac{\overline{r}}{y_t^k} \right)^{\phi} \tag{102}$$

$$\lambda_t = \beta E_t \lambda_{t+1} \left[\zeta_2 l_{t+1} r_{t+1} \mu_{t+1} (1-\phi) \left(\frac{\overline{r}}{y_{t+1}^k} \right)^{\phi} + (1 - \psi_1 \frac{\mu_{t+1}^{\gamma_1}}{\gamma_1}) - p_{t+1} (\psi_2 \frac{\mu_{t+1}^{\gamma_2}}{\gamma_2}) \right] \tag{103}$$

Equation (101) equates the marginal utility of consumption to the shadow value of wealth, λ_t. The optimal value of capital utilization rate is given by Equation (102), which equates the marginal cost of capital utilization (marginal depreciation plus energy costs) to the after tax marginal return to capital utilization. Equation (103) is a standard Euler equation that determines the intertemporal optimal condition governing capital accumulation. It equates the marginal rate of substitution in consumption to the total return to investment, which is equal to the sum of the after tax marginal return of capital and the component that did not depreciate minus capital's marginal cost of energy.

The stochastic process for the price of energy is specified as

$$\ln p_t = \rho \ln p_{t-1} + \epsilon_t \tag{104}$$

where $0 < \rho < 1$ and $\epsilon_t \sim N(0, \sigma_p^2)$.

3.3.2 Firms

Labor markets are characterized by search and matching frictions according to a standard Diamond-Mortensen-Pissarides framework. The representative firm must engage in a costly search and maintain a number v_t of job

[37] As suggested by Guo and Lansing (1998), a flat tax with the accurate personal deductions may exhibits some degrees of progressivity. In order to keep things simple, I consider a tax schedule without any deductions.

vacancies in order to hire new workers. More precisely, it faces some hiring costs equal to κv_t. The probability of finding a worker depends on a Cobb-Douglas matching technology, which converts unemployed individuals, u_t, and vacancies, v_t, into matches, $m_t = Bu_t^{\xi} v_t^{1-\xi}$, where $B > 0$ and $0 < \xi < 1$. Denoting by $\theta_t \equiv \frac{v_t}{u_t}$, the labor-market tightness, we can derive the vacancy filling rate or the total matches per vacancy, which is given by

$$q(\theta_t) = B\theta_t^{-\xi}, \tag{105}$$

and the job finding rate or the probability that unemployed workers meet vacancies, which is given by

$$\theta_t q(\theta_t) = B\theta_t^{1-\xi}. \tag{106}$$

The job finding rate is higher when the labor market is less tight, but the vacancy filling rate is a decreasing function of the labor market tightness.

The representative firm starts the period t with a stock of employment given by l_t, but firms and workers can separate exogenously with probability $\sigma \in (0, 1)$. The rate at which a vacancy is matched with unemployed agents depends on the total number of job vacancies and unemployed workers but the individual firm takes it as given. Hence, the representative firm's employment stock evolves as follows:

$$l_{t+1} = (1 - \sigma)\left[l_t + q(\theta_t)v_t\right] \tag{107}$$

The output produced by each employed worker is given by $g(K_t, Kg_t, h_t)$, which represents a production function with constant returns to scale in private inputs K_t and h_t. Kg_t denotes the stock of public capital. The total output produced by the representative firm, which depends on the production technology and the stock of employment is given by

$$Y_t = l_t g(K_t, Kg_t, h_t), \tag{108}$$

The representative firm rents capital services $\mu_t K_t$, chooses the optimal number of vacancies to post v_t, and its future employment stock l_{t+1} in order to maximize

$$\pi_t = E_0 \sum_{t=0}^{\infty} \beta^t \frac{\lambda_t}{\lambda_0} \left[l_t g(K_t, Kg_t, h_t) - l_t w_t h_t - l_t r_t \mu_t K_t - \kappa v_t\right] \tag{109}$$

subject to the law of motion of employment stock (107). I assume that firms discount future profits using the household's stochastic discount factor, $\beta^t \frac{\lambda_t}{\lambda_0}$, which comes from the household problem mentioned above.

Denoting by Ω_t the Lagrange multiplier associated with the constraint (107), the firm's first order maximization conditions are, respectively

$$r_t = g_k(K_t, Kg_t, h_t) \tag{110}$$

$$\frac{\kappa}{q(\theta_t)} = (1-\sigma)\Omega_t \tag{111}$$

$$\Omega_t = E_t\left(\beta\frac{\lambda_{t+1}}{\lambda_t}\right)\left[g(K_{t+1}, Kg_{t+1}, h_{t+1}) - w_{t+1}h_{t+1} - r_{t+1}\mu_{t+1}K_{t+1} + (1-\sigma)\Omega_{t+1}\right] \tag{112}$$

I obtain the following job-creation condition by combining the optimality conditions (111) and (112)

$$\frac{\kappa}{q(\theta_t)} = E_t\left(\beta\frac{\lambda_{t+1}}{\lambda_t}\right)(1-\sigma)\left[g(K_{t+1}, Kg_{t+1}, h_{t+1}) - w_{t+1}h_{t+1} - r_{t+1}\mu_{t+1}K_{t+1} + \frac{\kappa}{q(\theta_{t+1})}\right] \tag{113}$$

which states that, at the optimum, the vacancy creation cost incurred by the firm per current match is equated to the expected discounted value of profits from the match. Profits from a successful match take into account the future output produced and the cost of that match in terms of the wage paid as well as the rental of capital services.

3.3.3 Nash Bargaining

As is common in the literature, I assume that hours worked and the wage paid are determined through an individual Nash bargaining process between the matched worker and firm. Before any production, a worker must be matched with a firm and this successful match generates a surplus. In other words, both firms and workers are better off after successful matches than before. There is a total surplus that must be shared between the firm and the worker during the Nash bargaining process. All jobs will pay the same wage because workers and firms are identical. The total surplus of a successful match is equal to the firm's surplus plus the worker's surplus:

$$Total\ Surplus = (\mathcal{W}_t - \mathcal{U}_t) + \mathcal{J}_t \tag{114}$$

where $\mathcal{W}_t$ is the worker's value of employment, $\mathcal{U}_t$ is the worker's value of unemployment and $\mathcal{J}_t$ is the firm's value of a filled job.

The value of being employed for an individual is

$$\mathcal{W}_t = (1-\tau_t^w)w_t h_t + E_t\beta\frac{\lambda_{t+1}}{\lambda_t}\left[(1-\sigma)\mathcal{W}_{t+1} + \sigma\mathcal{U}_{t+1}\right] \tag{115}$$

The value of unemployment is given by

$$\mathcal{U}_t = \frac{v(h_t)}{u_{c,t}} + b + E_t\beta\frac{\lambda_{t+1}}{\lambda_t}\left[\theta_t q(\theta_t)(1-\sigma)\mathcal{W}_{t+1} + (1-\theta_t q(\theta_t)(1-\sigma))\mathcal{U}_{t+1}\right] \tag{116}$$

According to Equation (115), a successful match provides the worker with the after tax wage income in the current period. During the next period, with the probability $(1-\sigma)$ the worker does not separate from the job and receives the continuing value of employment, but with the remaining probability, the worker does separate and receives in this case the value of unemployment. The net present value of the worker's value of unemployment is given by Equation (116). $\frac{v(h_t)}{u_{c,t}} + b$ represents the worker's outside option, which depends on the utility value of leisure and the unemployment benefit, b. But during the next period, the unemployed worker may make the transition from unemployment to employment or remain unemployed. With the probability $\theta_t q(\theta_t)(1-\sigma)$, the unemployed agent finds a successful match and receives the future value of employment, she remains unemployed and receives the continuing value of unemployment with the remaining probability.

The value of a filled job can be expressed as follows

$$\mathcal{J}_t = g(K_t, Kg_t, h_t) - w_t h_t - r_t\mu_t K_t + E_t\beta\frac{\lambda_{t+1}}{\lambda_t}(1-\sigma)\mathcal{J}_{t+1} \tag{117}$$

Following a successful match, the firm gets the value of the output produced by the worker, net of labor and capital costs plus the future expected discounted continuation value of the match in case of non separation (or non destruction of the job), which occurs with the probability $(1-\sigma)$. The Nash bargaining process solves the following optimization problem

$$\max_{w_t,h_t} \mathcal{J}_t{}^{1-\varepsilon}(\mathcal{W}_t - \mathcal{U}_t)^{\varepsilon}$$

where $\varepsilon \in (0,1)$ is the worker's bargaining power and $(1-\varepsilon)$ is the bargaining power of the firm. The first order condition with respect to w_t is

$$(1-\varepsilon)(\mathcal{W}_t - \mathcal{U}_t)\frac{\partial \mathcal{J}_t}{\partial w_t} + \varepsilon \mathcal{J}_t \left(\frac{\partial \mathcal{W}_t}{\partial w_t} - \frac{\partial \mathcal{U}_t}{\partial w_t} \right) = 0 \tag{118}$$

where

$$\frac{\partial \mathcal{J}_t}{\partial w_t} = -h_t$$

$$\frac{\partial \mathcal{W}_t}{\partial w_t} = (1-\phi)(1-\tau_t^w)h_t$$

$$\frac{\partial \mathcal{U}_t}{\partial w_t} = 0$$

From the expression for $\frac{\partial \mathcal{W}_t}{\partial w_t}$, we see that individuals account for the degree of progressivity in the tax schedule (ϕ) during wage negotiations. More precisely, $\frac{\partial \mathcal{W}_t}{\partial w_t}$ is a decreasing function of ϕ, which implies a smaller net return to the worker from a marginal increase in the wage rate when the tax schedule becomes more progressive. In other words, if ϕ is positive, a rise in the wage rate will have a declining impact on the value of being employed because of the higher marginal tax rate.

The first order condition gives the following Nash sharing rule

$$\mathcal{W}_t - \mathcal{U}_t = \frac{\varepsilon}{1-\varepsilon}(1-\phi)(1-\tau_t^w)\mathcal{J}_t$$

The share of the matching surplus appropriated by workers increases with their bargaining power but depends negatively on the degree of tax progressivity, ϕ, and the labor tax rate, τ_t^w.

Using the expressions for $\mathcal{W}_t$ and $\mathcal{U}_t$ and the sharing rule we obtain the Nash bargained wage payment, which is given by

$$\begin{aligned} w_t h_t \quad &= \quad (1-\varepsilon)\left\{ \frac{1}{(1-\varepsilon\phi)(1-\tau_t^w)} \left[\frac{v(h_t)}{u_{c,t}} + b \right] \right\} \\ &+\varepsilon \left\{ \frac{(1-\phi)}{1-\varepsilon\phi} \left[g(K_t, Kg_t, h_t) - r_t \mu_t K_t + (1-\sigma) E_t \beta \frac{u_{c,t+1}}{u_{c,t}} \mathcal{J}_{t+1} \left(\frac{1-\tau_t^w - (1-\theta_t q(\theta_t))(1-\tau_{t+1}^w)}{1-\tau_t^w} \right) \right] \right\} \end{aligned} \tag{119}$$

The wage bill per worker is a weighted average of the worker's outside option and the firm's value of a successful match, which is given by the net contribution of the worker to the match plus the expected continuation value of the match to the firm. The weights are given by the bargaining power of the worker and the firm. During wage negotiations, the value of the worker's outside option is amplified by the factor $\frac{1}{(1-\varepsilon\phi)(1-\tau_t^w)}$, which is higher than 1 if $\phi > 0$. But, the firm's value of a successful match is scaled by the factor $\frac{(1-\phi)}{1-\varepsilon\phi}$, which is less than 1 when the tax schedule is progressive. In other words, the worker's outside option becomes more relevant while the expected continuation value of a filled job to the firm gets smaller when taxes are progressive. When the degree of progressivity increases, workers value more their outside option (or reservation wage) and depending on their bargaining power, firms must pay higher wages to induce them to work. If $\phi = 0$, the labor tax rate is constant and the tax schedule does not have dynamic effects on the Nash bargained wage payment, so assuming a progressive tax schedule or endogenous taxes drives an additional wedge in the bargaining process.

Turning to the determination of hours worked in a successful match, the first order condition with respect to h_t is

$$(1-\varepsilon)(\mathcal{W}_t - \mathcal{U}_t)\,(g_{h,t} - w_t) + \varepsilon \mathcal{J}_t \left(((1-\phi)(1-\tau_t^w)w_t) - \frac{v_{h,t}}{u_{c,t}} \right) = 0 \tag{120}$$

Hours worked are determined by

$$\frac{v_{h,t}}{u_{c,t}} = (1-\phi)(1-\tau_t^w)g_{h,t} \tag{121}$$

where $v_{h,t}$ is the marginal disutility of work. Hours worked depend on the degree of progressivity of the tax system in addition to being determined by an intratemporal optimality condition, which equates the marginal rate of substitution between consumption and leisure to the the firm's after tax marginal revenue. As stated by Eqs. (119) and (121), the determination of the wage rate and hours worked depends on the degree of tax progressivity

3.3.4 Government

Government spending in the economy consists of four categories: spending on consumption, Cg_t, investment in public capital (infrastructure), Ig_t, lump-sum social transfers to private agents, T_t,and spending on the unemployment insurance program, $b(1-l_t)$. Following Kuralbayeva (2013), I assume that only public investments involve some adjustment or additional costs, which may be explained by the presence of political distorsions or the losses of revenues due to an inefficient tax collection process

The government collects income taxes to finance its expenditures and the adjustment costs related to public capital accumulation, in such a way that it runs a balanced budget in every period. Thus its budget constraint is given by

$$Cg_t + Ig_t + \mathcal{C}(Ig_t, Kg_t) + T_t + b(1 - l_t) = \tau_t^w l_t w_t h_t + \tau_t^k l_t r_t \mu_t K_t \tag{122}$$

where $\mathcal{C}(Ig_t, K_{gt})$ denotes adjustment costs that are increasing and convex in Ig_t and decreasing in Kg_t. Letting $\delta_g \in (0, 1)$ be the depreciation rate, the stock of public capital evolves over time according to

$$Kg_{t+1} = Ig_t + (1 - \delta_g)Kg_t. \tag{123}$$

Combining equations (96), (109) and (122), the aggregate resource constraint can be written as:

$$Y_t = C_t + I_t + p_t e_t + Cg_t + Ig_t + \mathcal{C}(Ig_t, Kg_t) + \kappa u_t \theta_t \tag{124}$$

where $\kappa u_t \theta_t$ stands for total costs of posting vacancies after making the substitution $v_t = u_t \theta_t$.

3.3.5 Macroeconomic Equilibrium

The macroeconomic equilibrium is defined as follows

Definition 1 *A competitive equilibrium is a sequence of allocations $\{C_t, \mu_t, h_t, u_t, \theta_t, I_t, K_t\}_{t=0}^{\infty}$, a price system $\{w_t, r_t, p_t\}_{t=0}^{\infty}$, government policies $\{Cg_t, Ig_t, Kg_t, T_t\}_{t=0}^{\infty}$, such that*

1. *Individual variables equal average aggregate variables.*
2. *Given government policies and prices, households maximize their utility (93) subject to (96).*
3. *Given government policies and prices, firms maximize their profits (109) subject to (107).*
4. *Given allocations and prices, government policies satisfy the government budget constraint.*
5. *Wages are given by (119).*
6. *The market price of private capital is given by (110).*
7. *The price of energy services is given by (104).*
8. *Private capital and public capital accumulate according to (94) and (123), respectively.*
8. *Individual agents take the vacancy filling and job finding rates, $q(\theta_t)$ and $\theta_t q(\theta_t)$ as given.*

3.4 Ramsey Problem

The government behaves as a benevolent Ramsey policy maker. It chooses $\{Cg_t, Ig_t, Kg_t, T_t\}_{t=0}^{\infty}$in order to maximize the expected life-time utility of the household subject to the aggregate resource constraint, the implementability constraints imposed by the definition of the competitive equilibrium and taking K_{-1} and Kg_{-1}

as given. However, this dynamic optimization problem is nonstationary because of constraints (103), (113) and (119), which include expected values of future endogenous variables that affect current allocations. To overcome this problem, I need to reformulate the optimization problem in a recursive framework, following the approach described in Marcet and Marimon (2011). When we apply the law of iterated expectations, the government's objective function is equivalent to the following four equations:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[U(C_t, Cg_t, l_t, h_t) + (\Upsilon_t^1 + \Upsilon_t^2 + \Upsilon_t^3) u_{c,t} \right] \tag{125}$$

$$\Gamma_t^1 = \Upsilon_t^1 + \Gamma_{t-1}^1 \left[\zeta_2 l_t r_t \mu_t (1-\phi) \left(\frac{\overline{r}}{y_t^k} \right)^{\phi} + (1 - \psi_1 \frac{\mu_t^{\gamma_1}}{\gamma_1}) - p_t (\psi_2 \frac{\mu_t^{\gamma_2}}{\gamma_2}) \right], \quad \Gamma_{-1}^1 = 0 \tag{126}$$

$$\Gamma_t^2 \left(\frac{\kappa}{q(\theta_t)} \right) = \Upsilon_t^2 + \Gamma_{t-1}^2 (1-\sigma) \left[g(K_t, Kg_t, h_t) - w_t h_t - r_t \mu_t K_t + \frac{\kappa}{q(\theta_t)} \right], \quad \Gamma_{-1}^2 = 0 \tag{127}$$

$$\Gamma_t^3 \Lambda_t^1 = \Upsilon_t^3 - \Gamma_{t-1}^3 \Lambda_t^2, \quad \Gamma_{-1}^3 = 0 \tag{128}$$

where

$$\Lambda_t^1 = \left[w_t h_t - \left(\frac{1-\varepsilon}{(1-\varepsilon\phi)(1-\tau_t^w)} \left[\frac{v(h_t)}{u_{c,t}} + b \right] \right) - \left(\frac{\varepsilon(1-\phi)}{1-\varepsilon\phi} \left[g(K_t, Kg_t, h_t) - r_t \mu_t K_t + \frac{\kappa}{q(\theta_t)} \right] \right) \right]$$

$$\Lambda_t^2 = \left(\frac{\varepsilon(1-\sigma)(1-\theta_t q(\theta_t))}{(1-\varepsilon\phi)(1-\tau_t^w)} \right) \left[(1-\tau_t^w)(1-\phi) \left[g(K_t, Kg_t, h_t) - w_t h_t - r_t \mu_t K_t + \frac{\kappa}{q(\theta_t)} \right] \right]$$

Eqs. (126), (127) and (128) summarize the evolution of the Lagrange multipliers Γ_t^1, Γ_t^2 and Γ_t^3 associated with constraints (103), (113) and (119), respectively. These co-state variables ensure that optimal policy is consistent by summarizing the government's commitments to pre-announced policies.

The Ramsey problem expressed in a recursive framework following the approach described in Marcet and Marimon (2011) can be written as follows:

$$\max E_0 \sum_{t=0}^{\infty} \beta^t \left[U(C_t, Cg_t, l_t, h_t) + (\Upsilon_t^1 + \Upsilon_t^2 + \Upsilon_t^3) u_{c,t} \right]$$

subject to Eqs. (94), (95), (102), (104), (105), (107), (108), (110), (121), (122), (123), (124), (126), (127) and (128).

3.5 Optimal Fiscal Policy: Discussion of Baseline Results

In this section, I will state the functional forms and examine the responses of the economy to an adverse energy price shock under the Ramsey policy. I will present the impulse response functions and the correlations of output with some key endogenous variables as functions of the degree of tax progressivity, ϕ.

3.5.1 Calibration and Functional Forms

I assume that preferences are described by the following constant relative risk aversion utility function

$$U(C_t, Cg_t, l_t, h_t) = \frac{C_t^{1-\eta_1}}{1-\eta_1} + \chi_1 \frac{Cg_t^{1-\eta_2}}{1-\eta_2} - \chi_2 \frac{h_t^{1+\eta_3}}{1+\eta_3}, \tag{129}$$

The production function is assumed to be Cobb-Douglas and is given by

$$Y_t = l_t g(K_t, Kg_t, h_t) = l_t \left(A(\mu_t K_t)^{1-\alpha} h_t^{\alpha} Kg_t^{\omega} \right), \tag{130}$$

The adjustment cost function for public capital is given by

$$\mathcal{C}(Ig_t, Kg_t) = \varsigma \frac{Ig_t^2}{2Kg_t},$$

The effects of an increase in the price of energy are analyzed numerically. For this reason, I calibrate the model in order to represent a low-income developing economy with a flat tax schedule. The unit time length is equal to one quarter. The benchmark calibration is summarized in Table XVIII. I must calibrate the preference parameters $\{\beta, \eta_1, \eta_2, \eta_3, \chi_1, \chi_2\}$, the production parameters $\{\alpha, \omega, A\}$, the labor market parameters $\{B, \xi, \sigma, \varepsilon, \kappa\}$, the private capital utilization parameters $\{\psi_1, \psi_2, \gamma_1, \gamma_2\}$, the tax schedule parameters $\{\zeta_1, \zeta_2, \overline{w}, \overline{r}, \phi\}$, the depreciation rate of public capital, δ_g, the unemployment benefit rate, b, and the public investment adjustment cost parameter, ς.

I set the discount factor to 0.97, which implies a quarterly real interest rate of 3%. Following the standard range of estimates, the risk aversion coefficient, η_1, is equal to 2, the intertemporal elasticity of substitution, $1/\eta_2$, is set to 1.5, while the Frisch elasticity of labor supply $1/\eta_3$ is set to 1. The values of the preference parameters, χ_1 and χ_2 are chosen to ensure that the model's steady state ratio of government consumption to private consumption, Cg/C, and hours worked, h, are equal to 0.2 and 0.35, respectively. The steady-state level of technology, A, is a scaling factor and is set to 1.45. The elasticity of output with respect to labor, α, is chosen to be 0.6, as is commonly assumed in the literature. The elasticity of output with respect to public capital,

ω, is set to 0.1 according to the empirical literature, which provides estimates for this parameter ranging from 0.11 to 0.26.[38]. The parameters related to labor market frictions are less straightforward to calibrate because the empirical literature does not provide tight direct estimates for developing countries. For those parameters, I conduct an extensive sensitivity analysis. I assume that the usual Hosios condition for search efficiency is satisfied and set the elasticity of matches with respect to the number of unemployed individuals, ξ, and workers' Nash bargaining power, ε to 0.5. I consider lower and larger values in the sensitivity analysis. I calibrate the matching efficiency parameter, B, and the unit cost of vacancy posting, κ, to match the steady-state probability of filling a vacancy, $q(\theta) = 0.7$ and the probability of finding a job, $\theta q(\theta) = 0.6$, as reported in Shimer (2005).

[38] For example, Dessus and Herrera's (2000) estimates of the income share of public capital range from 0.11 to 0.13 for a sample of 28 developing countries over the period 1981-1991. Aschauer (2000) reports an estimate of 0.24 using data from 46 low- and middle-income countries from 1970 to 1990. Arslanalp et al. (2010) estimate the output elasticity of public capital equal to be 0.26 for a sample of developing countries.

Table XVIII – Benchmark Calibration

Description	Parameter	Value
Preferences		
Discount factor	β	0.97
Preference parameter	η_1	2
Preference parameter	η_2	1.5
Preference parameter	η_3	1
Preference parameter	χ_1	0.1
Preference parameter	χ_2	13.6
Production		
Output elasticity of labor	α	0.6
Output elasticity of public capital	ω	0.1
Steady-State level of technology	A	1.45
Labor Market		
Macthing efficiency parameter	B	0.648
Macthing elasticity	ξ	0.5
Bargaining power	ε	0.5
Job separation rate	σ	0.1
Vacancy cost parameter	κ	0.1208
Private Capital Utilization		
Depreciation parameter	ψ_1	0.0902
Utilization parameter	ψ_2	0.0247
Depreciation parameter	γ_1	1.4303
Utilization parameter	γ_2	2.2017
Tax schedule		
Tax progressivity parameter	ϕ	0
Labor tax rate parameter	$\overline{w}$	0.579
Labor tax rate parameter	ζ_1	0.85
Capital tax rate parameter	$\overline{r}$	0.4
Capital tax rate parameter	ζ_2	0.85
Others		
Depreciation rate of public capital	δ_g	0.05
Adjustment cost parameter (public capital)	ς	10
Unemployment benefit rate	b	0.1714
Standard deviation of energy price shock	σ_p	0.01
The degree of persistence of energy price shock	ρ	0.85

The resulting values are $B = 0.648$ and $\kappa = 0.1208$.The quarterly exogenous separation rate, σ, is set to 0.1, which is consistent with estimates obtained for the US economy by Hall (1995), Davis et al. (1996) and Shimer (2005). The depreciation and utilization parameters are calibrated to hit the constant private capital depreciation rate of 0.05 in the steady-state, as reported in Chatterjee and Turnovsky (2007) and Van Der Ploeg and Venables (2011) and a steady-state value of capacity utilization equal to 85 percent for μ. The resulting parameter values are $\psi_1 = 0.0902$ and $\gamma_1 = 1.4303$. The remaining parameters, ψ_2 and γ_2 are set respectively to 0.0247 and 2.2017. I consider alternative values for these parameters in the sensitivity analysis. The tax schedule in the benchmark economy is flat, which implies that the tax progressivity parameter, ϕ, is equal to 0. For the sake of simplicity, I assume that labor and capital tax rate parameters, ζ_1 and ζ_2 are both equal to 0.85, implying average tax rates of 0.15, as reported in Chatterjee and Turnovsky (2007). I allow for alternative values in the sensitivity analysis. The steady state levels of labor and capital income, $\overline{w}$ and $\overline{r}$ are equal to 0.579 and 0.4, respectively. The depreciation rate of public capital, δ_g, is set to 0.05, while the adjustment cost parameter, ς, is set to 10, which is consistent with the consensus range of 10 to 16. I set the unemployment benefits, $b = 0.1714$, so that the model's average replacement ratio, $b/(wh)$, is equal to 0.25. Finally, the exogenous process for the energy price shock, p_t, follows an AR(1) and is calibrated so that its standard deviation, σ_p, is set to 0.01 and its persistence, ρ, to 0.85.

In the benchmark equilibrium, the ratio of private consumption to output is 0.6, the capital-output ratio is equal to 3.83 and the ratio of public capital to private capital is equal to 0.168. The steady-state level of unemployment rate is equal to 0.062 while the equilibrium value of the labor market tightness is 0.857. Finally the average energy share of output in the benchmark economy is equal to 3 percent.

3.5.2 Analysis of Dynamic Responses

Figure 13 exhibits the impulse responses of key variables to a 1% increase in the price of energy under two assumptions about the tax schedule. I consider a benchmark developing economy without any progressivity in its tax system, in which case the parameter, ϕ, is equal to 0 and a typical developed economy characterized by its progressive tax schedule. According to Chen and Guo (2013), the degree of progressivity in the U.S tax system was estimated to 0.1156, on average, over the period between 1966 and 2005. In order to be consistent with this estimate, I assume that the tax slope parameter, ϕ, is equal to 0.12 in the typical developed economy. All variables are absolute deviations from their steady-state values.

The energy use has a direct effect on output, which can be seen more precisely by rewriting Eq. (95) as follows

$$\mu_t = \left(\frac{e_t}{K_t}\right)^{\frac{1}{\gamma_2}} \left(\frac{\gamma_2}{\psi_2}\right)^{\frac{1}{\gamma_2}} \tag{131}$$

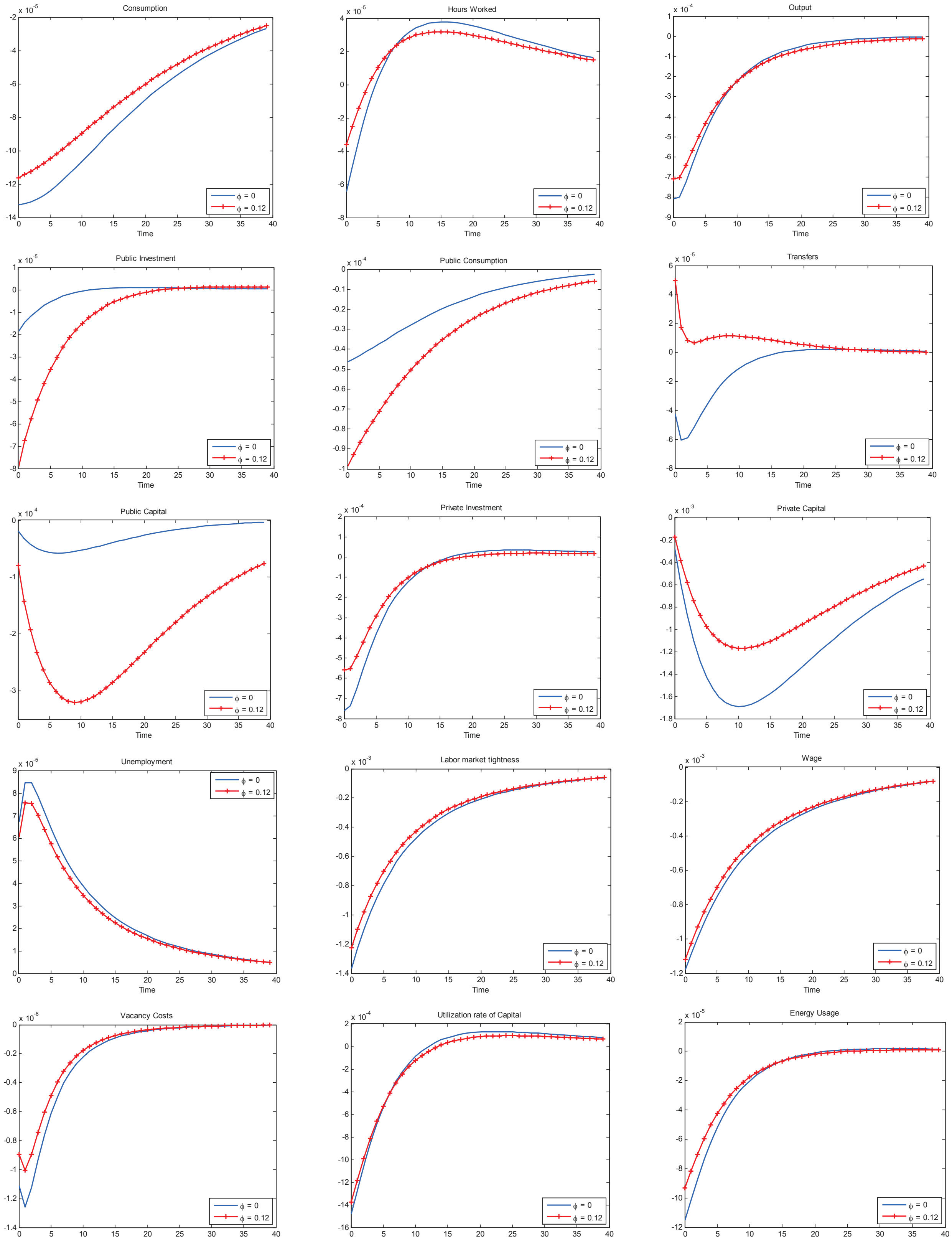
and substituting (131) into the production technology, Eq. (130) to yield

$$Y_t = (l_t A h_t^{\alpha} K g_t^{\omega}) \left[e_t^{\frac{1}{\gamma_2}} \left(\frac{\gamma_2}{\psi_2}\right)^{\frac{1}{\gamma_2}} K_t^{(1-\frac{1}{\gamma_2})} \right]^{1-\alpha} \tag{132}$$

According to Eq. (132), goods and services are produced using, labor, energy, private and public capital. The increase in the price of energy will tend to have a direct effect and indirect impact, which will work through the stock of private capital, on the level of output produced. Since energy services become more expensive, the use of this input will decrease along with the utilization rate of private capital, which requires energy, as we can see on Figure 13. Regardless of the type of economy, the output produced decreases following the energy price shock, implying a contractionary net effect on the level of production in both developed and developing economies.

The increase in the energy price affects private agents' after tax labor income, which in turn has two opposing effects on hours worked that can be labelled as the substitution and income effects. The first effect tends to decrease hours worked, while the latter effect tends to increase the number of hours devoted to work. They explain the hump-shaped response of hours worked, as displayed in Figure 13. The substitution effect is more important than the income effect during the first periods after the energy price shock, because the decline in output also means fewer taxes paid by workers on their labor income inducing them to devote less time to work.

Figure 13 – Impulse Responses to a 1% Adverse Energy Price Shock



But, as the economy recovers from the adverse energy price shock, the income effect is stronger since workers tend to pay more taxes because of their increasing labor income, which induces them to work more in order to maintain their income level. It is worth noting that regardless of the value of ϕ, the real contraction generated by the energy price shock tends to decrease the aggregate level of consumption, but the presence of tax progressivity in the developed economy, is able to stabilize private agents' consumption as this variable falls less than it does when $\phi = 0$, or in the developing economy. The degree of tax progressivity has stabilizing effects on several variables directly under the control of private agents like hours worked, private investment and accumulation of capital.

The decline in output recorded during the first periods after the adverse energy price shock is associated with higher unemployment and lower labor market tightness, which is equivalent to a higher probability of filling a vacancy for firms. But as the level of production gets higher over time, the level of unemployment decreases and labor markets become tighter. In steady-state, the bargained wage payment is

$$wh = (1-\varepsilon)\left\{\frac{1}{(1-\varepsilon\phi)(1-\tau^w)}\left[\frac{v(h)}{u_c}+b\right]\right\} + \varepsilon\left\{\frac{(1-\phi)}{1-\varepsilon\phi}\left[g(K,Kg,h)-r\mu K+\kappa\theta\right]\right\} \tag{133}$$

The worker's outside option, represented by $\frac{v(h)}{u_c}+b$ in Eq. (133), tends to decrease during economic recessions like the contraction that follows the adverse energy price shock. During these specific periods of time, it is usually harder to find a job, which explains the workers' lower reservation wage. Since hours worked per worker fall, the output per worker will also decrease so does the labor market tightness. The capital income per worker, $r\mu K$, tends to increase due to higher rental rate of capital services. After the increase in the energy price, the number of hours worked decreases along with the key expressions on the right-hand-side of Eq. (133), explaining by this way the drop in the wage rate recorded during the first periods following the shock. The lower wage rate paid by firms combined with the higher vacancy filling rate, which is represented by the decrease in the labor market tightness, will tend to reduce their vacancy costs.

Finally, the impulse responses of variables that are directly under the control of the government depend on the degree of progressivity in the tax schedule. In both the developed and developing economies, the responses to the adverse energy price shock involve some reductions in government consumption and public investment, which leads to decreases in the stock of public capital. However, the decline in these components of government spending is far greater in the developed economy, when the tax system is progressive. The difference is even more striking with respect to social transfers. Indeed, following the energy price shock, social transfers from government to private agents, increase in the developed economy characterized by a progressive tax system ($\phi = 0.12$), but they fall in the developing country. In other words, the optimizing planner runs a procyclical fiscal policy by cutting all components of public spending in response to the real contraction generated by the energy price shock, when $\phi = 0$. This adjustment to the shock contrasts with the countercyclical responses of social transfers observed in the developed economy. Regardless of the type of economy, the income taxes collected by governments tend to decrease during economic recessions but the key difference between developed and developing countries lies in their optimal adjustments to maintain a balanced budget over time. When the tax system is flat, agents are willing to devote less time to work and decrease their investment or accumulation of capital more than they would under a progressive tax schedule, which means that decreases in income taxes collected are far greater in the developing country inducing the government to reduce all of the components of public spending. In the developed economy with progressive taxation, the government optimally chooses to decrease public consumption and investment more than the fall in tax revenues, allowing the public sector to increase social transfers to agents in order to stabilize their consumption, accumulation of capital and the entire economy as well.

3.5.3 Analysis of Cyclical Comovements

The correlations of output with key policy variables, hours worked and unemployment are reported in Table XIX. To emphasize the role of tax progressivity, the correlation coefficients are simulated using different values of ϕ, varying from 0 to 0.14, where $\phi = 0$ represents the benchmark calibration. When we use the baseline parametrization, the correlations of output with each component of government spending and hours worked are positive and the correlation between output and aggregate unemployment is negative. Hence, the model with the benchmark parameter values is able to account for all procyclical fiscal policy characteristics observed in developing countries. Moreover, as the degree of tax progressivity increases, the correlations of output with government consumption and investment increase, while the output-social transfers correlation decreases and becomes negative when $\phi \geqslant 0.1$. The correlation of output with unemployment does not depend on ϕ as it remains negative. The output-hours worked correlation decreases when the tax schedule becomes more progressive, but remains largely positive. In other words, social transfers from government to private agents, become less procyclical eventually they turn countercyclical as the degree of progressivity in the tax schedule increases. These results suggest that, under plausible parameter values, the weakness of automatic stabilizer mechanisms represented by the degree of tax progressivity can be an important driving force of the procyclicality of fiscal policy.

Table XIX – Cyclical Features of Key Variables as Functions of ϕ

Degree of Tax Progressivity	Correlations with Output				
	Public Consumption	Public Investment	Social Transfers	Hours Worked	Aggregate Unemployment
$\phi = 0$	0.9676	0.9619	0.9721	0.8656	−0.9716
$\phi = 0.02$	0.9721	0.9717	0.9547	0.8493	−0.9723
$\phi = 0.04$	0.9759	0.9778	0.9175	0.8325	−0.9731
$\phi = 0.06$	0.9792	0.9822	0.8216	0.8156	−0.9738
$\phi = 0.08$	0.9821	0.9854	0.5266	0.7994	−0.9746
$\phi = 0.1$	0.9846	0.9880	−0.1506	0.7848	−0.9753
$\phi = 0.12$	0.9867	0.9901	−0.6557	0.7728	−0.9760
$\phi = 0.14$	0.9887	0.9917	−0.8350	0.7638	−0.9767

3.6 Sensitivity Analysis

According to the results obtained in the previous section, a model with different degrees of tax progressivity measured by the parameter, ϕ, can explain the empirical differences in the cyclical characteristics of fiscal policy in developing countries and developed ecnomies. Depending on the value of the parameter, ϕ, the tax schedule is progressive enough to serve as an automatic stabilizer, which in turn affects the Ramsey Planner's ability to smooth shocks and run countercyclical fiscal policies. In order to assess the robustness of these results, I consider alternative values of the model parameters and reevaluate the optimal policy problem. I focus on the matching elasticity and worker's bargaining parameters, the depreciation and utilization parameters, the labor and capital tax rate parameters and the unemployment benefit parameter. In all cases, and to facilitate comparison, the results obtained under the benchmark calibration are reproduced.

3.6.1 Matching Elasticity and Worker's Bargaining Power (ξ and ε)

In the benchmark calibration, I assume that the Hosios Condition holds and set the matching elasticity, ξ, and worker's bargaining power, ε, to 0.5. In the robustness analysis, I consider two alternative values, assuming that the Hosios Condition still holds. Table XX reports the results for the values of 0.4 and 0.6, which are respectively lower and higher than the value used in the benchmark calibration.

The correlations of output with government consumption and investment show that irrespective of the values of the matching elasticity and worker's bargaining power, these components of government spending become more procyclical as the degree of tax progression increases. However, they do not have the same patterns when we keep the degree of progressivity constant and consider different values of ξ and ε. Moreover, for a given value of tax progression, as the values of ξ and ε get higher, the cyclical comovement of government consumption with output decreases slightly while the correlation between output and government investment increases. When the tax schedule becomes more progressive, the government cuts public expenditure, namely, consumption and investment, by more than it does in the baseline specifications. On the other hand, for a given degree of progressivity, the reductions in public expenditure are predominantly cuts in government investment than in public consumption, when both the matching elasticity and worker's bargaining power get higher. According to Eq. (133), the stronger the bargaining power of workers (the higher ε), the less relevant their outside option and the higher the share of the net marginal revenue product appropriated by workers as wage payments. The cuts in public consumption are lower because they affect consumers' utility functions directly while reductions in public investment have indirect effects on consumers, which are spread over time.

The correlations of output with social transfers tend to decrease as the tax schedule becomes more progressive, regardless of the values of ξ and ε. Consistent with the results obtained in the benchmark case, social transfers become countercyclical beyond a certain threshold value of the degree of tax progressivity. In the developing country or the case without tax progressivity, all components of public spending including social transfers from government to private agents become more procyclical when the bargaining power of workers gets stronger. However, the cuts in social transfers are lower than those recorded in public investment, because they affect the private agent's budget constraint and her ability to spend on consumption or investment goods. When the tax system is progressive enough, which is the case for $\phi \geqslant 0.12$, the components of public expenditures related to consumption and investment remain largely procyclical while the countercyclicality of social transfers increases with workers' bargaining power. This result is the evidence that automatic stablizer mechanisms are working through the progressivity of the tax system.

Table XX – Correlations under Alternative Values of ξ and ε

Degree of Tax Progressivity	Correlations with Output								
	Public Consumption			Public Investment			Social Transfers		
	Matching Elasticity (ξ) and Worker's Bargaining Power (ε)								
	$\xi = 0.4$ $\varepsilon = 0.4$	$\xi = 0.5^B$ $\varepsilon = 0.5^B$	$\xi = 0.6$ $\varepsilon = 0.6$	$\xi = 0.4$ $\varepsilon = 0.4$	$\xi = 0.5^B$ $\varepsilon = 0.5^B$	$\xi = 0.6$ $\varepsilon = 0.6$	$\xi = 0.4$ $\varepsilon = 0.4$	$\xi = 0.5^B$ $\varepsilon = 0.5^B$	$\xi = 0.6$ $\varepsilon = 0.6$
$\phi = 0$	0.9813	0.9676	0.9550	0.8483	0.9619	0.9806	0.9695	0.9721	0.9842
$\phi = 0.02$	0.9833	0.9721	0.9611	0.9351	0.9717	0.9849	0.9493	0.9547	0.9741
$\phi = 0.04$	0.9850	0.9759	0.9664	0.9543	0.9778	0.9882	0.9092	0.9175	0.9514
$\phi = 0.06$	0.9864	0.9792	0.9709	0.9630	0.9822	0.9908	0.8219	0.8216	0.8837
$\phi = 0.08$	0.9876	0.9821	0.9749	0.9683	0.9854	0.9929	0.6203	0.5266	0.5778
$\phi = 0.1$	0.9886	0.9846	0.9783	0.9721	0.9880	0.9946	0.2162	−0.1506	−0.4115
$\phi = 0.12$	0.9893	0.9867	0.9814	0.9751	0.9901	0.9959	−0.2628	−0.6557	−0.8456
$\phi = 0.14$	0.9900	0.9887	0.9840	0.9777	0.9917	0.9969	−0.5725	−0.8350	−0.9369

Notes: ^{B} Benchmark value.

3.6.2 Depreciation and Utilization Parameters (ψ_1 and ψ_2)

There is very little information in the empirical literature regarding the parameters of the depreciation and utilization (ψ_1 and ψ_2). In the benchmark economy these parameters are calibrated in order to reproduce a steady state capacity utilization of 85%. In the sensitivity analysis, these parameters are calibrated to generate alternative values of the steady state capacity utilization (μ). More specifically, I set ψ_1 and ψ_2 to 0.0984 and 0.0282, respectively to hit the steady state capacity utilization of 80%, while the economy's capacity utilization is 75% in the steady state, when ψ_1 and ψ_2 are set to 0.1079 and 0.0325, respectively. Table XXI reports the results for alternative values of μ.

These experiments indicate that the main result obtained when we consider a model with different degrees of tax progressivity is robust to variations in ψ_1 and ψ_2. In fact, regardless of the value of these parameters, the correlations of output with plublic consumption and investment increase with the degree of tax progressivity, while social transfers become less procyclical as the degree of progression in the tax system increases. Moreover, irrespective of the steady state values of the capacity utilization, social transfers are countercyclical when $\phi \geqslant 0.12$.

Table XXI – Correlations under Alternative Values of μ

Degree of Tax Progressivity	Correlations with Output								
	Public Consumption			Public Investment			Social Transfers		
	Depreciation (ψ_1) and Utilization (ψ_2) Parameters for Alternative Values of μ								
	$\mu = 0.85^B$	$\mu = 0.80$	$\mu = 0.75$	$\mu = 0.85^B$	$\mu = 0.80$	$\mu = 0.75$	$\mu = 0.85^B$	$\mu = 0.80$	$\mu = 0.75$
$\phi = 0$	0.9676	0.9648	0.9591	0.9619	0.9940	0.9886	0.9721	0.9765	0.9802
$\phi = 0.02$	0.9721	0.9693	0.9640	0.9717	0.9946	0.9910	0.9547	0.9611	0.9658
$\phi = 0.04$	0.9759	0.9732	0.9683	0.9778	0.9952	0.9919	0.9175	0.9273	0.9347
$\phi = 0.06$	0.9792	0.9767	0.9719	0.9822	0.9957	0.9923	0.8216	0.8364	0.8558
$\phi = 0.08$	0.9821	0.9796	0.9625	0.9854	0.9962	0.9472	0.5266	0.5334	0.9516
$\phi = 0.1$	0.9846	0.9822	0.9778	0.9880	0.9966	0.9921	−0.1506	−0.2004	0.0682
$\phi = 0.12$	0.9867	0.9845	0.9802	0.9901	0.9969	0.9915	−0.6557	−0.6988	−0.4695
$\phi = 0.14$	0.9887	0.9865	0.9822	0.9917	0.9971	0.9906	−0.8350	−0.8597	−0.7021

Notes: ^{B} Benchmark value. The depreciation (ψ_1) and utilization (ψ_2) parameters are calibrated to generate alternative steady-state values for the capacity of utilization variable (μ): for $\mu = 0.85$, the parameters ψ_1 and ψ_2 are set respectively to 0.0902 and 0.0247; for $\mu = 0.8$, the parameters ψ_1 and ψ_2 are set respectively to 0.0984 and 0.0282; for $\mu = 0.75$, the parameters ψ_1 and ψ_2 are set respectively to 0.1079 and 0.0325.

3.6.3 Labor and Capital Tax Rate Parameters (ζ_1 and ζ_2)

Tables XXII and XXIII report the results for alternative values of the levels of labor and capital tax rates (ζ_1 and ζ_2). They measure the after tax labor and capital income received by private agents. In the benchmark case, these parameters are both equal to 0.85, but the robustness analysis is conducted by keeping one parameter constant and varying the other from 0.75 to 0.85. More precisely, I consider two lower values of 0.8 and 0.75, which raise the average tax to 0.2 and 0.25, respectively.

The correlations of output with government consumption remain higher than 0.97 regardless of the value of the labor tax level (ζ_1). When we consider the cyclical comovements between output and public investment for a given degree of tax progressivity, this component of government spending becomes slightly more procyclical as ζ_1 increases. If the level of labor tax rate remains constant, the results show that both government consumption and investment are more procyclical as the degree of tax progressivity increases, which is in line with the results obtained in the baseline calibration. The cyclical comovements of output with social transfers show an interesting pattern. Regardless of the degree of progression in the tax schedule, the correlations between these two variables increase with the level of labor tax rate. When $\phi = 0$, the correlations between output and social transfers increase from 0.88 to 0.97 as the level of labor tax increases from 0.75 to 0.85. In the developed economy case ($\phi = 0.12$), social transfers to private agents become less countercyclical as ζ_1 increases. When the value of ζ_1 increases from 0.75 to 0.85, the average labor tax rate decreases from 0.25 to 0.15, implying that government is collecting a smaller share of labor income as tax revenue, which affects its ability to increase social transfers and run countercyclical fiscal policies following adverse shocks.

Table XXII – Correlations under Alternative Values of the Labor Tax Parameter (ζ_1)

Degree of Tax Progressivity	Correlations with Output								
	Public Consumption			Public Investment			Social Transfers		
	Labor Tax Rate Parameter (ζ_1)								
	$\zeta_1 = 0.75$	$\zeta_1 = 0.80$	$\zeta_1 = 0.85^B$	$\zeta_1 = 0.75$	$\zeta_1 = 0.80$	$\zeta_1 = 0.85^B$	$\zeta_1 = 0.75$	$\zeta_1 = 0.80$	$\zeta_1 = 0.85^B$
$\phi = 0$	0.9796	0.9746	0.9676	0.9264	0.9426	0.9619	0.8831	0.9458	0.9721
$\phi = 0.02$	0.9822	0.9779	0.9721	0.9411	0.9556	0.9717	0.7686	0.9037	0.9547
$\phi = 0.04$	0.9844	0.9808	0.9759	0.9521	0.9647	0.9778	0.4871	0.8026	0.9175
$\phi = 0.06$	0.9864	0.9833	0.9792	0.9607	0.9714	0.9822	−0.0462	0.5274	0.8216
$\phi = 0.08$	0.9882	0.9856	0.9821	0.9675	0.9766	0.9854	−0.5188	−0.0596	0.5266
$\phi = 0.1$	0.9897	0.9875	0.9846	0.9730	0.9807	0.9880	−0.7460	−0.5682	−0.1506
$\phi = 0.12$	0.9910	0.9892	0.9867	0.9776	0.9841	0.9901	−0.8455	−0.7851	−0.6557
$\phi = 0.14$	0.9922	0.9906	0.9887	0.9813	0.9868	0.9917	−0.8941	−0.8728	−0.8350

Notes: ^{B} Benchmark value.

Table XXIII reports the results from experiments run using alternative values of the capital tax level (ζ_2). The results are very similar to those obtained in the sensitivity analysis regarding the parameter of the labor tax level. More precisely, public consumption and investment remain highly procyclical as the level of capital tax increases, whereas the correlations between output and social transfers increase with ζ_2.

Table XXIII – Correlations under Alternative Values of the Capital Tax Parameter (ζ_2)

Degree of Tax Progressivity	Correlations with Output								
	Public Consumption			Public Investment			Social Transfers		
	Capital (ζ_2) Tax Rate Parameter								
	$\zeta_2 = 0.75$	$\zeta_2 = 0.80$	$\zeta_2 = 0.85^B$	$\zeta_2 = 0.75$	$\zeta_2 = 0.80$	$\zeta_2 = 0.85^B$	$\zeta_2 = 0.75$	$\zeta_2 = 0.80$	$\zeta_2 = 0.85^B$
$\phi = 0$	0.9806	0.9750	0.9676	0.9968	0.9907	0.9619	0.8715	0.9510	0.9721
$\phi = 0.02$	0.9830	0.9783	0.9721	0.9969	0.9919	0.9717	0.6275	0.9018	0.9547
$\phi = 0.04$	0.9851	0.9812	0.9759	0.9970	0.9930	0.9778	−0.0752	0.7581	0.9175
$\phi = 0.06$	0.9869	0.9837	0.9792	0.9970	0.9939	0.9822	−0.6575	0.2903	0.8216
$\phi = 0.08$	0.9885	0.9858	0.9821	0.9971	0.9947	0.9854	−0.8465	−0.4393	0.5266
$\phi = 0.1$	0.9899	0.9877	0.9846	0.9970	0.9954	0.9880	−0.9123	−0.7721	−0.1506
$\phi = 0.12$	0.9910	0.9893	0.9867	0.9969	0.9960	0.9901	−0.9410	−0.8825	−0.6557
$\phi = 0.14$	0.9920	0.9907	0.9887	0.9965	0.9965	0.9917	−0.9551	−0.9274	−0.8350

Notes: ^{B} Benchmark value.

3.6.4 Unemployment Benefits (b)

The benchmark value of the unemployment benefit parameter, b, is set to 0.1714 so that the average replacement ratio is equal to 0.25. In the experiments, this parameter is calibrated to hit two alternative values for the average replacement ratio: 0.4 and 0.1. The resulting values for the unemployment benefit parameter are 0.2829 and 0.0666, respectively. Table XXIV show the cyclical comovements under alternative values of b.

Consistent with the results obtained in the benchmark case, for a given value of b, government consumption and investment are more procyclical as the degree of tax progressivity increases. However, the correlations between output and social transfers decrease when the tax schedule becomes more progressive. Furthermore, the main result stating that beyond a certain threshold in tax progression (around 0.1), social transfers are countercyclical is robust to changes in the unemployment benefit parameter.

In the developing economy case ($\phi = 0$), the correlations of output with public consumption increase slightly with the average replacement rate, while government investment and social transfers become less procyclical. In other words, cuts in government consumption are greater than those recorded in public investment and social transfers. This result can be explained by recalling that government consumption, unemployment benefits and social transfers affect consumers. The effect of public consumption occurs through the utitlity function while both unemployment benefits and social transfers affect the budget constraint of private agents. As the average replacement ratio increases, the government is substituting public consumption for more unemployment benefits allowing it to maintain a certain level of social transfers.

On the other hand, when $\phi = 0.12$, government consumption and investment remain largely procyclical while social transfers become less countercyclical as the average replacement rate increases. According to Eq. (122), social transfers from government to private agents are correlated positively to the tax revenue from labor income ($\tau_t^w l_t w_t h_t$) and negatively to the unemployment benefit payments ($b(1 - l_t)$). When the average replacement is low ($b = 0.067$), the payments of unemployment benefits are much lower than the reductions in labor tax revenue recorded after an adverse energy price shock, which allows the government to increase social transfers to private agents. However, when the average replacement rate increases, the value of workers' outside option and firms' wage bill increase, which means higher labor tax revenue collected. The fact that social transfers are less countercyclical implies that an increasing share of tax revenue is devoted to meet the payments of unemployment benefits, which increase as the average replacement rate gets higher.

Table XXIV – Correlations under Alternative Values of the Unemployment Benefits (b)

Degree of Tax Progressivity	Correlations with Output								
	Public Consumption			Public Investment			Social Transfers		
	Unemployment Benefits (b)								
	$b = 0.067$	$b = 0.171^B$	$b = 0.283$	$b = 0.067$	$b = 0.171^B$	$b = 0.283$	$b = 0.067$	$b = 0.171^B$	$b = 0.283$
$\phi = 0$	0.9604	0.9676	0.9744	0.9791	0.9619	0.8589	0.9834	0.9721	0.9469
$\phi = 0.02$	0.9661	0.9721	0.9776	0.9830	0.9717	0.9311	0.9689	0.9547	0.9231
$\phi = 0.04$	0.9709	0.9759	0.9805	0.9860	0.9778	0.9557	0.9314	0.9175	0.8783
$\phi = 0.06$	0.9750	0.9792	0.9829	0.9885	0.9822	0.9678	0.7948	0.8216	0.7861
$\phi = 0.08$	0.9786	0.9821	0.9851	0.9905	0.9854	0.9751	0.1418	0.5266	0.5836
$\phi = 0.1$	0.9817	0.9846	0.9869	0.9922	0.9880	0.9800	−0.6885	−0.1506	0.1848
$\phi = 0.12$	0.9844	0.9867	0.9885	0.9936	0.9901	0.9837	−0.8912	−0.6557	−0.2984
$\phi = 0.14$	0.9868	0.9887	0.9899	0.9947	0.9917	0.9866	−0.9451	−0.8350	−0.6148

Notes: ^{B} Benchmark value.

3.7 Conclusion

In this paper I use data from 36 developing and developed countries to corroborate the evidence stating that fiscal policy is procyclical in developing countries while it is countercyclical in developed economies. Furthermore, I break down government spending in three components, namely, public consumption, public investment and social transfers. I show that government consumption and investment are procyclical in both developed and developing countries, while social transfers behave differently over the business cycle. In fact, social transfers are procyclical in developing countries and countercyclical in developed economies. I then develop an optimal policy framework with given degrees of progressivity in the tax schedule to explain the cyclical behavior of fiscal policy in developed and developing countries. More precisely, I build a simple model in which all three components of public spending, government consumption, government investment and social transfers are treated separately and use it to analyse the goverment's optimal fiscal policy following an adverse energy price shock, when the tax schedule exhibits different levels of progressivity. The results obtained from simulations of this model are consistent with the empirical features of fiscal policy in both developed and developing countries.

The responses to the adverse energy price shock involve some cuts in government consumption and public investment, in both developed and developing economies. Moreover, the results regarding social transfers from government to private agents indicate that they increase in the developed economy whose tax schedule is progressive, but fall in the developing country, which has a flat tax system. The correlations of output with each component of government spending show that as the degree of tax progressivity increases, government consumption and investment are more procyclical, while social transfers from government to private agents, become less procyclical eventually they turn countercyclical beyond a certain threshold of progression in the tax system. These results are quite robust to alternative calibration of the model and suggest that, under plausible parameter values, the automatic stabilizer mechanisms embodied in the design of the tax schedule can explain business cycles in emerging economies.

The present analysis and framework can be extended to account for inequality, which represents another key difference between developed and developing countries. According to empirical evidence, developing countries exhibit greater wealth and income inequality than developed economies. One could develop an optimal framework with tax progressivity, different components of public expenditure including social transfers that are motivated by earnings inequality. Another possible extension can be related to the unemployment benefit programs, which can be designed in such a way that they serve as important automatic stabilizer mechanism. To this end, search and matching labor market frictions combined with unemployment risk could be embodied in a standard New Keynesian model. I leave these extensions for future research.

3.8 References

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3.9 Appendix

3.9.1 Appendix A: Cyclical aspects of fiscal policy for each country

This appendix reports for each country the aggregate statistics reported in Tables 1 and 2 in the text and the coverage period.

Table A.1

Volatility

Country	Standard Deviations					
	Output	Private Consumption	Public Consumption	Public Investment	Social Benefits	Total Spending
			Developed Countries			
G7 Countries						
Canada	0.02	0.02	0.02	0.05	0.04	0.02
France	0.01	0.01	0.02	0.05	0.01	0.02
Germany	0.02	0.02	0.02	0.07	0.02	0.03
Italy	0.02	0.02	0.02	0.09	0.02	0.02
Japan	0.02	0.02	0.01	0.06	0.02	0.03
United Kingdom	0.03	0.03	0.02	0.18	0.05	0.03
United States	0.06	0.03	0.22	0.33	0.15	0.02
Average	0.03	0.02	0.05	0.12	0.04	0.02
Other OECD Countries						
Australia	0.02	0.01	0.02	0.07	0.05	0.03
Austria	0.02	0.01	0.01	0.08	0.02	0.02
Belgium	0.02	0.02	0.01	0.09	0.03	0.03
Denmark	0.02	0.03	0.02	0.06	0.04	0.02
Iceland	0.04	0.07	0.03	0.15	0.07	0.07
Ireland	0.04	0.04	0.04	0.13	0.04	0.05
Korea, Rep.	0.03	0.03	0.02	0.10	0.12	0.05
The Netherlands	0.02	0.02	0.02	0.04	0.03	0.02
Norway	0.02	0.02	0.02	0.09	0.05	0.04
Spain	0.03	0.03	0.02	0.07	0.02	0.02
Sweden	0.02	0.02	0.02	0.05	0.03	0.02
Switzerland	0.02	0.02	0.02	0.04	0.04	0.02
Average	0.03	0.03	0.02	0.08	0.05	0.03
			Developing Countries			
Latin America						
Argentina	0.05	0.11	0.11	0.34	0.14	0.09
Bolivia	0.03	0.05	0.16	0.11	0.17	0.07
Brazil	0.03	0.06	0.07	0.12	0.13	0.03
Costa Rica	0.03	0.05	0.07	0.11	0.19	0.07
Dominican Rep.	0.04	0.04	0.21	0.35	0.86	0.11
Nicaragua	0.02	0.04	0.08	0.16	0.10	0.20
Panama	0.05	0.06	0.06	0.33	0.06	0.03
Peru	0.05	0.06	0.10	0.24	0.09	0.07
Uruguay	0.05	0.06	0.09	0.36	0.16	0.09
Average	0.04	0.06	0.11	0.24	0.21	0.08

Table A.1 (*Continued*)

Country	Standard Deviations					
	Output	Private Consumption	Public Consumption	Public Investment	Social Benefits	Total Spending
Other Developing Countries						
Bangladesh	0.02	0.07	0.17	0.07	0.08	0.02
Belarus	0.07	0.05	0.11	0.28	0.11	0.11
Bulgaria	0.05	0.07	0.14	0.23	0.14	0.10
Croatia	0.04	0.07	0.08	0.28	0.07	0.06
Mauritius	0.05	0.06	0.07	0.19	0.05	0.03
Mongolia	0.05	0.08	0.15	0.45	0.30	0.07
South Africa	0.02	0.03	0.04	0.11	0.42	0.02
Tunisia	0.03	0.03	0.04	0.16	0.05	0.02
Average	0.04	0.06	0.10	0.22	0.15	0.05

Variables are: Y, real output; C, real private consumption; GC, real public consumption; GI, real public investment; SB, real social benefits; Gtot, real total public expenditures. All variables are in logarithms and detrended using the Hodrick-Prescott filter. Data are annual from OECD, BEA (developed countries), WDI and IMF's IFS and GFS (developing countries).

Table A.2

Correlations

Country	Correlations with Output					Average Social Benefits (% of Y)
	Private Consumption	Public Consumption	Public Investment	Social Benefits	Total Spending	
Developed Countries						
G7 Countries						
Canada	0.83	0.02	−0.01	−0.52	−0.45	9.86
France	0.81	0.19	0.64	−0.43	−0.40	21.86
Germany	0.74	0.04	0.20	−0.65	−0.29	25.04
Italy	0.77	0.34	0.27	−0.15	0.02	18.34
Japan	0.86	−0.10	−0.21	−0.34	−0.36	13.83
United Kingdom	0.91	0.30	0.07	−0.76	−0.24	14.03
United States	0.46	0.74	0.69	−0.56	−0.09	7.09
Average	0.77	0.22	0.24	−0.49	−0.26	15.72
Other OECD Countries						
Australia	0.55	0.37	−0.10	−0.50	−0.32	8.10
Austria	0.73	0.25	0.01	−0.08	−0.02	21.56
Belgium	0.74	0.40	−0.17	0.04	0.11	20.69
Denmark	0.75	0.32	0.15	−0.33	−0.45	18.51
Iceland	0.91	0.50	0.76	−0.57	0.33	6.42
Ireland	0.84	0.60	0.71	−0.66	−0.60	12.28
Korea, Rep.	0.79	0.06	0.02	−0.54	0.06	2.61
The Netherlands	0.78	−0.04	0.21	0.06	−0.06	21.95
Norway	0.85	−0.23	0.13	−0.54	−0.53	15.72
Spain	0.96	0.60	0.40	−0.82	−0.32	15
Sweden	0.74	−0.06	−0.10	−0.10	0.01	20.28
Switzerland	0.83	0.38	0.02	−0.79	−0.53	11.53
Average	0.79	0.26	0.17	−0.40	−0.19	14.55

Table A.2 (*Continued*)

Country	Correlations with Output					Average Social Benefits (% of Y)
	Private Consumption	Public Consumption	Public Investment	Social Benefits	Total Spending	
	Developing Countries					
Latin America						
Argentina	0.61	0.77	0.69	0.88	0.72	7.35
Bolivia	0.48	0.40	0.25	0.09	−0.11	3.64
Brazil	0.35	0.51	0.61	0.52	0.14	7.30
Costa Rica	0.76	0.47	0.35	−0.20	0.03	5.08
Dominican Rep.	0.75	0.14	0.64	0.44	0.26	0.41
Nicaragua	0.54	0.63	0.14	−0.05	0.56	3.31
Panama	0.19	0.26	0.61	0.55	0.67	5.42
Peru	0.69	0.67	0.49	−0.59	0.31	2.62
Uruguay	0.93	0.53	0.30	0.80	0.80	13.51
Average	0.59	0.47	0.45	0.27	0.38	5.40
Other Developing Countries						
Bangladesh	0.65	0.16	0.24	0.43	0.40	1
Belarus	0.81	0.67	0.08	0.47	0.72	11.38
Bulgaria	0.66	0.59	0.02	0.55	0.65	12.45
Croatia	0.71	−0.18	0.23	−0.20	−0.41	14.55
Mauritius	0.40	0.38	0.36	−0.04	0.10	4.37
Mongolia	0.73	0.63	0.05	−0.13	0.04	4.75
South Africa	0.65	0.06	0.41	0.12	0.16	1.87
Tunisia	0.68	−0.13	0.23	0.13	0.26	6.89
Average	0.66	0.27	0.20	0.17	0.24	7.16

Variables are: Y, real output; C, real private consumption; GC, real public consumption; GI, real public investment; SB, real social benefits; Gtot, real total public expenditures. All variables are in logarithms and detrended using the Hodrick-Prescott filter. Data are annual from OECD, BEA (developed countries), and WDI and IMF's IFS and GFS (developing countries).

Table A.3

Coverage Period

Country	Time Period					
	Output	Private Consumption	Public Consumption	Public Investment	Social Benefits	Total Spending
Developed Countries						
Australia	1959-2010	1959-2010	1959-2010	1960-2009	1960-2009	1960-2009
Austria	1970-2010	1970-2010	1970-2010	1976-2010	1976-2010	1976-2010
Belgium	1970-2010	1970-2010	1970-2010	1985-2010	1985-2010	1985-2010
Canada	1970-2010	1970-2010	1970-2010	1970-2010	1970-2010	1970-2010
Denmark	1966-2010	1966-2010	1966-2010	1990-2010	1990-2010	1990-2010
France	1950-2010	1950-2010	1950-2010	1978-2010	1978-2010	1978-2010
Germany	1970-2011	1970-2011	1970-2011	1991-2010	1991-2010	1991-2010
Iceland	1970-2010	1970-2010	1970-2010	1995-2010	1995-2010	1995-2010
Ireland	1970-2010	1970-2010	1970-2010	1990-2010	1990-2010	1990-2010
Italy	1970-2010	1970-2010	1970-2010	1980-2010	1980-2010	1980-2010
Japan	1970-2009	1970-2009	1970-2009	1980-2009	1980-2009	1980-2009
Korea, Rep.	1970-2010	1970-2010	1970-2010	1970-2009	1970-2009	1970-2009
The Netherlands	1969-2010	1969-2010	1969-2010	1969-2010	1969-2010	1969-2010
Norway	1970-2010	1970-2010	1970-2010	1995-2010	1995-2010	1995-2010
Spain	1970-2010	1970-2010	1970-2010	1995-2010	1995-2010	1995-2010
Sweden	1950-2010	1950-2010	1950-2010	1993-2010	1993-2010	1993-2010
Switzerland	1970-2010	1970-2010	1970-2010	1990-2010	1990-2010	1990-2010
United Kingdom	1970-2010	1970-2010	1970-2010	1970-2010	1970-2010	1990-2010
United States	1929-2011	1929-2011	1929-2011	1929-2011	1929-2011	1960-2011
Developing Countries						
Argentina	1960-2010	1960-2010	1993-2010	1993-2006	1990-2004	1990-2001
Bangladesh	1973-2010	1973-2010	1973-2010	1981-2010	2001-2009	2001-2009
Belarus	1990-2010	1990-2010	1990-2010	1990-2008	1992-2009	1992-2009
Bolivia	1968-2010	1968-2010	1968-2010	1987-2009	1990-2007	1990-2001
Brazil	1965-2010	1965-2010	1965-2010	1983-1994	1997-2009	1997-2009
Bulgaria	1991-2010	1991-2010	1991-2010	1991-2010	1991-2009	1991-2009
Costa Rica	1960-2010	1960-2010	1960-2010	1971-1990	1990-2007	1990-2007
Croatia	1994-2010	1994-2010	1994-2010	1994-2010	1994-2009	1994-2005
Dominican Rep.	1962-2010	1962-2010	1962-2010	1980-1990	1993-2003	1990-2000
Mauritius	1960-2010	1960-2010	1960-2010	1976-2009	1990-2009	1990-2008
Mongolia	1980-2008	1980-2005	1980-2008	1995-2008	1992-2003	1992-2003
Nicaragua	1991-2009	1991-2009	1991-2009	1991-2009	1991-2001	1991-2001
Panama	1960-2009	1960-2009	1960-2009	1980-2009	1990-2001	1990-2001
Peru	1960-2010	1960-2010	1960-2010	1979-2010	1990-2009	1990-2009
South Africa	1960-2010	1960-2010	1960-2010	1980-2010	1990-2009	2000-2009
Tunisia	1961-2010	1961-2010	1961-2010	1970-2000	1990-1999	1990-2009
Uruguay	1960-2010	1960-2010	1960-2010	1970-2010	1990-2009	1990-2002

Conclusion générale

Dans cette thèse qui comporte trois essais, nous avons analysé à travers des modèles théoriques et empiriques différentes questions reliées aux effets de l'aide étrangère sur la croissance économique ou les inégalités et la procyclicité de la politique fiscale dans les pays en voie de développement.

Dans le premier essai, nous avons examiné la question de l'allocation optimale de l'aide au développement en développant un modèle de croissance endogène avec accumulation du capital humain. Lorsqu'on utilise une paramétrisation plausible, nos résultats indiquent que lier l'aide aux dépenses publiques en éducation procure le niveau de bien-être le plus élevé, comparativement aux deux autres cas extrêmes où l'aide est complètement non liée ou exclusivement liée au capital physique. Les résultats de cet article ont d'importantes implications de politiques économiques concernant la conditionnalité de l'aide au développement octroyée aux pays à faibles et moyens revenus.

Dans le second essai, nous analysons à la fois empiriquement et théoriquement, les mécanismes par lesquels l'aide étrangère pourrait affecter les niveaux d'inégalités. L'analyse empirique fondée sur des régressions à l'aide de données de panel montre que l'aide étrangère a des effets positifs mais retardés sur les inégalités de revenus. L'étude théorique est basée sur la modélisation d'une économie en développement recevant de l'aide étrangère qui peut être complétement non liée ou liée soit à des investissements publics en capital physique, ou à des dépenses publiques en capital humain. Nos résultats indiquent que l'aide au développement non liée réduit les inégalités alors que l'aide liée augmente les inégalités au cours des périodes suivantes. Dans la mesure où les flux d'aide aux pays en développement sont principalement liés à la réalisation de projets d'investissements spécifiques, les prédictions du modèle sont consistantes avec l'évidence empirique.

Dans le troisième essai, nous avons étudié les propriétés cycliques de la politique fiscale dans les pays en développement. Nous désagrégeons les dépenses publiques totales en trois catégories, à savoir, la consommation publique, l'investissement public et les transferts sociaux, ce qui nous permet de présenter des faits nouveaux sur les propriétés cycliques des principales catégories de dépenses publiques. Nous montrons que la principale différence entre les pays développés et en développement se situe au niveau du comportement cyclique des transferts sociaux qui sont contracycliques dans le premier groupe de pays, alors qu'ils sont procycliques dans le second. Les autres catégories de dépenses publiques sont procycliques dans les deux groupes de pays. Dans la seconde partie de cet article, nous développons un modèle de politique optimale avec différents degrés de progressivité du système d'imposition, pour expliquer le comportement cyclique de la politique fiscale dans les pays développés et ceux en voie de développement. Nos résultats montrent que les transferts deviennent contracycliques lorsque le degré de progressivité dépasse un certain seuil (0.1) alors que les deux autres composantes des dépenses publiques demeurent procycliques dans les deux groupes de pays. Ces résultats sont robustes et demeurent inchangés lorsque nous utilisons une calibration alternative, ce qui suggère que la faiblesse des mécanismes de stabilisateurs automatiques peut être une importante variable explicative de la procyclicité de la politique fiscale.