HEC MONTRÉAL
École affiliée à l'Université de Montréal

Three Essays on Economics of Pollution Control

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Thèse présentée en vue de l'obtention du grade de Ph.D en administration
(Option, Méthodes quantitatives de gestion)

Décembre 2013

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École affiliée à l'Université de Montréal

Cette thèse intitulée:

Three Essays on Economics of Pollution Control

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Résumé

Cette thèse est composée de trois articles qui appliquent la théorie des jeux dynamiques pour traiter certains problèmes de contrôle de la pollution. Dans le premier article, nous considérons un jeu différentiel à deux joueurs représentant les groupes de pays développés et les pays en voie de développement (PVD). On élargit le concept d'un coût de dommage environnemental à l'idée plus vaste de préoccupations environnementales qui prennent de l'ampleur dans les PVD à travers le temps. Nous caractérisons et comparons les solutions coopératives et non coopératives. Nous obtenons, contrairement à ce que pourrions penser, qu'il n'est pas nécessairement de mise de faire pression sur les PVD pour qu'ils réduisent leurs émissions à court terme. Aussi, nous obtenons que la coopération en matière de contrôle des émissions puisse ne pas produire suffisamment de bénéfices à court terme pour qu'elle soit attrayante par rapport à un mode de jeu non coopératif.

Dans le deuxième article, nous introduisons une incertitude écologique et de l'apprentissage dans un jeu dynamique de contrôle de la pollution à l'échelle internationale à n joueurs. Nous caractérisons et comparons analytiquement les stratégies non-coopératives en boucle fermée sous trois différentes hypothèses informationnelles, à savoir : information complète (scenario de référence), apprentissage par anticipation et apprentissage par adaptation. Nous trouvons que l'incertitude liée à l'apprentissage par anticipation réduit les émissions totales tandis qu'une incertitude structurelle peut soit augmenter ou réduire ou ne pas avoir d'impact sur les émissions individuelles dépendamment de la forme fonctionnelle de la distribution et des croyances. En outre, nous trouvons que si les croyances d'un joueur tendent à devenir plus optimistes, ou s'il pense que la situation est moins risquée, il augmentera ses émissions tandis que d'autres réagiront à ce changement et réduiront leur émissions. Cette réduction n'est pas suffisante pour compenser l'augmentation induite par cet optimisme, et de ce fait les émissions totales augmentent toujours.

L'objectif du troisième article est de comparer les deux politiques fréquemment utilisées pour induire les firmes à réduire leurs émissions de polluants, à savoir, les taxes et les quotas. Nous effectuons cette comparaison dans le contexte où le régulateur fait face à une incertitude de marché et une incertitude écologique. Nous supposons que la firme et le régulateur ont des informations asymétriques et que le régulateur apprend à travers le temps. Nous obtenons que les niveaux espérés d'émissions sont les mêmes sous les deux politiques, mais elles diffèrent en termes de bien-être social, avec un avantage pour les taxes. L'article discute de l'impact de l'incertitude et de l'apprentissage, ainsi que des croyances du régulateur sur les résultats.

Mots clés: Contrôle de pollution; Jeux différentiels; Jeux dynamiques; Préoccupations environnementales; Accords environnementaux internationaux; Incertitude; Apprentissage; Politiques publiques.

Abstract

This thesis is composed of three articles which apply game theoretic methodology to address some pollution control problems. In the first article, we consider a two-player differential game of international emissions to represent the interactions between developed and developing countries. We adopt a broader-than-usual definition of environmental cost for developing countries to account for their evolving involvement in tackling with environmental externalities. Cooperative and non-cooperative solutions are characterized and contrasted. The main contribution of this paper is about providing new insights in the environmental behavior of developing countries, and in clarifying the difficulties and challenges that may arise in achieving cooperation. We find that cooperation may not create enough dividend in the short term to be implementable. We also show that asking developing countries to take environmental cost into account sooner is not necessarily the best course of action. Finally, we characterized the conditions under which the cooperation is easier to be implemented.

In the second article we introduce ecological uncertainty and learning in an N-player dynamic game of international pollution. We analytically find and compare the feedback non-cooperative strategies of players under three different information assumptions: informed, as our benchmark; anticipated learner; and adaptive learner. We find out that while uncertainty due to anticipation of learning ends up to a decrease in total emissions, depending on functional form of distributions and beliefs, the effect of structural uncertainty could be either an increase, decrease, or even no change in the emissions of individual players and also total emissions. Moreover, we find out that if a player's beliefs change toward more optimistic views or he feels that the situation is less risky, he will increase his emissions while others will react to this change and decrease their emissions but the latter effect never overcomes the former and, as a result, total emissions will increase.

The third article compares price-based policies, taxes, and quantity-based policies, quotas, for controlling emissions in a dynamic setup when the regulator faces two sources of uncertainty: (i) market related uncertainty; and (ii) ecological uncertainty and he is an anticipated learner. Our results suggest that taxes and quotas lead to the same expected emissions level. But comparison of the total benefits related to these policies suggests that taxes dominate quotas which means taxes will provide more social welfare. Even though taxes have some benefits over quotas, neither learning nor ecological uncertainties affect our choice of policy, i.e. the only effective source is uncertainty in instantaneous net benefits of emissions. Besides, the more volatile this uncertainty is the more benefits of taxes over quotas.

Keywords: Pollution Control; Differential and Dynamic Games; International Environmental Agreements; Uncertainty; Learning; Policy Choice

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To: Mehrab, Mom and Dad

Mahtab, Abolfazl and their families

Acknowledgements

I would like to express my very great appreciation to my supervisor, Professor Georges Zaccour, as my support base in writing of this thesis, for his patient guidance, enthusiastic encouragement and useful critiques of this research work. In addition, I would like to especially thank Professor Marc Santugini. His willingness to motivate me and to give his time so generously contributed tremendously to my work. My grateful thanks are also extended to the rest of my committee members: Professor Michele Breton and Professor Hassan Benchekroun for their valuable and constructive suggestions. I also would like to thank my exterminators, Professor Pierre-Olivier Pineau and Professor Santiago J. Rubio.

I also wish to thank Mehrab and my family as my inspirational figures whose help, support, and encouragement have been unquantifiable.

Chapter 1

Introduction

The most up-to-date information we have about climate change is the IPCC's *Fifth Assessment Report*. According to this report, "[w]arming of the climate system is unequivocal" and

> "[h]uman influence has been detected in warming of the atmosphere and the ocean, in changes in the global water cycle, in reductions in snow and ice, in global mean sea level rise, and in changes in some climate extremes. ... It is extremely likely that human influence has been the dominant cause of the observed warming since the mid-20th century." IPCC (2013), SPM-12.

The largest contribution to the total radiative forcing is a result of the increase in the atmospheric concentration of CO2 since 1750. That humans have contributed to the observed changes and the extreme weather and climate events is very likely. Furthermore, while the likelihood of further changes in early 21st century is likely, the likelihood of further changes in the late 21st century is *virtually certain*. Such changes in the climate system impose huge costs on agriculture, forestry, ecosystems, major water resources, human health, industry, and society as a whole. Some examples of the potential types of damage include reduced agricultural yields in warmer regions due to heat stress; increased danger of wildfire; water shortages; water quality problems; increased risk of heat-related mortality, especially for the elderly, chronically sick and very young; increased risk of deaths, injuries and infectious, respiratory and skin diseases; reduction in quality of life for people in warm areas without appropriate housing; reduced hydropower generation potentials, etc. IPCC (2013).[1]

In short, we are certain that climate change is occurring and that it has negative impacts; and, more importantly, we know its main driver which is the increasing concentrations of greenhouse gases in the atmosphere. As a result, the direct solution would be controlling the concentration of GHGs. However, while this much is clear, it is a huge challenge to implement any policy to control

[1]However, the severity of the results depend on the vulnerabilities of different areas, which may be characterized by magnitude, timing, persistence/reversibility, potential for adaptation, and so on. For example, in very cold areas, climate change may increase agricultural yields and reduce human mortality rates resulting from cold exposure IPCC (2013).

emissions of these gases. Indeed, for a number of reasons, there is a considerable debate over how controls should be designed and how much control is enough. Some of reasons for this include the complexities involved in forcing all polluters, especially in an international context, to emit less; uncertainties in different aspects of this problem; and the nature of the environmental goods that encourage the classic problem of free-riding. These complexities will be dealt with in greater detail in this chapter and in the upcoming chapters. Above all, so far, most of the approaches to mitigate climate change have failed. The Kyoto Protocol is one important example given that some countries including the United States declined to ratify the agreement, many others, which had ratified the treaty at the beginning, did not reduce their emissions and others, for example Canada, finally left the agreement. At best, the Kyoto Protocol, just reduced GHG emissions very slightly. Such a failure is a sign that even if, in recent decades, a huge body of literature has addressed the pollution-control problem and improved our understanding in this regard significantly, still there are some characteristics of this problem that need a deeper analysis and more precise clarifications. In other words, this line of research is a relevant and extremely timely one. This thesis is composed of three essays addressing three crucial issues related to the pollution-control problem. The first and second articles deal with this problem at an international level. The third article addresses the pollution control problem from a national or local perspective.

The international dimension of the pollution-control problem has become one of the main concerns of the last decade and has attracted the attention of many researchers. Indeed, since the concentrations of the long-lived GHGs are the main drivers of climate change, no matter where and when (almost) they were emitted, they contribute to the total pollution concentrations and thus cause the changes in the climate system. In other words, to confront climate change, we face an international problem requiring the collaboration of all countries. However, there is no supranational authority to force countries to cooperate, and some countries are not interested in controlling pollution. Most of the developing countries (DCs) are in this group. They have important reasons for not being interested in controlling emissions, such as: (i) industrialized countries (ICs) are mainly responsible for the current state of the environment, and it would be unfair to constrain developing countries' emissions when it is their turn to industrialize; and (ii) compared to other economic issues, such as extreme poverty, offering essential services (education, health care, etc.) and building infrastructure, the environment is seen as a luxury service or a problem with a low level of urgency that developing countries cannot really afford in the short term. The historical responsibility of ICs is recognized in international political circles, and was actually accepted in the Kyoto Protocol, where DCs were not asked to reduce their emissions. Further, the argument that reaching a certain level of development has priority over environmental and possibly other social and political concerns appeals to a large community of scholars and decision makers because it corresponds to the successful role-model path that was followed by the developed countries themselves. Following on this idea, in the first essay, we model the evolvement in environmental considerations of developing countries by adopting a broader-than-usual definition of environmental cost. So, we provide new insights into the environmental behaviour of developing countries, and clarify the difficulties and challenges that may arise in achieving cooperation between ICs and DCs. We suppose that developing countries: (i) need a period of time $[0, T]$ to accomplish a desired level of development, during which they disregard environmental damage, either partially or completely, when making their production decisions; and (ii) fully internalize their environmental externalities after T.

This paper belongs to the significant differential-games literature in environmental economics and international environmental agreements (IEA): see, e.g., the recent surveys by Jørgensen et al. (2010); Long (2010). The dynamic IEA literature can be divided into two streams. In the first stream, the papers characterize and contrast cooperative and non-cooperative solutions and demonstrate the benefits of cooperation. Early contributions in this stream are Van der Ploeg and de Zeeuw (1992); Long (1992). The (static) noncooperative-game approach to IEA started two decades ago, and a recurrent result in this literature is that full cooperation is hard to achieve, albeit urgently needed to curb emissions and avoid some possible catastrophic events. Recently, we have seen contributions that include pollution accumulation and membership dynamics in their models; see, e.g., Rubio and Casino (2005); Rubio and Ulph (2007); de Zeeuw (2008); Bahn et al. (2009); Breton et al. (2010).

A main contribution of this paper lies in its different treatment of the two players in terms of their environmental concerns, and consequently, in modeling their damage cost functions. Indeed, we suppose that DCs adopt a gradual-involvement approach in dealing with the environmental externality. In some sense, we are using a broader-than-usual notion of environmental-damage costs by considering the idea of "environmental concerns" (EC). Roughly speaking, by EC we mean any kind of feeling, economic problem, health issue, etc., that could result in an attempt (a costly one) and react to emissions or their accumulation. Precisely modeling an EC function is a task with multiple levels of challenges, including the specification of such a function, the measurement of the variables that are supposedly involved and the choice of a functional form. As a first exploratory step in this direction, we suppose that EC can be represented by a (damage-cost) function whose arguments are the pollution stock (as in any standard model) and the accumulated revenues.

Another contribution with respect to the dynamic-games literature on pollution control lies in our analysis of emissions strategies and payoffs during the first period of the game. In summary, we find that throughout the game, ICs emit less under cooperation than under non-cooperation. The same is observed for DCs but only in the long term, that is, after reaching the threshold level of development. In the shorter term, the result depends on the degree of environmental concern in the DCs. For a large region of the parameter space, cooperation would lead to a longer period of low or no involvement by the DCs in dealing with environmental damage than would noncooperation. Also, in the short term, cooperation may not create enough dividends to be an attractive mode of play. Moreover, we find that it may not be the best option for ICs to press DCs to engage in abatement efforts in the short term.

Moreover, any effort that takes place in order to control climate change will include decisions for the future; and the main costs and benefits will also occur in the future. Thus, there is almost always uncertainty over the future costs and benefits of adopting any particular policy. Taking into consideration the different uncertainties in our knowledge about the future makes the pollution-control problem far more difficult. Indeed, even though it is widely accepted that GHG concentrations are responsible for global climate change, any policy and decision involves a great deal of uncertainty from many different sources, including a lack of information about or randomness in, for example: the exact amount of accumulated pollution; nature's ability to decay pollution; the exact effect of greenhouse gases on climate change; the precise change in terms of degrees of temperature we may face in the future; the exact impact of these changes on the economy; and, even, how our production and abatement technologies and processes will develop and improve in terms of their efficiency (for a survey, see Pindyck (2007)).

On the other hand, in the presence of uncertainty there is a chance and a possibility of learning about the unknowns. Indeed, decision makers always seek new information to take into consideration in their beliefs about the unknown characteristics of the problem they are tackling. In the second essay, we focus on an international pollution problem, in which some neighboring countries emit a pollutant, e.g. CO2, and the accumulation of this pollutant damages the environment in these countries, while the exact evolution of the pollution is not known. We call such a situation ecological uncertainty since it happens due to inadequate information about one of the model's ecological variables. We contribute to the literature by embedding the environmental uncertainty and learning in a dynamic international pollution-control problem. We also decompose the effects of different sources of uncertainty on the players' decisions. The main contribution of this paper is answering the following research question: does ecological uncertainty alleviate the emissions problem or aggravate it?

This second paper belongs to three different streams of the literature: (i) the literature that tries to shed light on different aspects of uncertainty in environmental problems (see, e.g., Pindyck (2000, 2007); Yeung and Petrosyan (2008); de Zeeuw and Zemel (2012)); (ii) dynamic and differential games in international environmental management and economics problems (see, e.g., Dockner and Long (1993); Long (1992); Rubio and Casino (2005), and for a survey see: Jørgensen et al. (2010)); and, (iii) a body of the economics literature that addresses uncertainty and learning (see, e.g., Cogley and Sargent (2008); Koulovatianos et al. (2009); Agbo (2011)).

Based on our assumption about players' information on unknown parameters of the model, we can have two general groups. The first group includes those who know all the parameter values. Following Levhari and Mirman (1980) we call these players fully informed players. This assumption is used in the standard literature related to uncertainty (see e.g. Bramoullé and Treich (2009); de Zeeuw and Zemel (2012); Arrow and Fisher (1974); Pindyck (2012)). The alternative assumption is to assume that players do not know the exact values of some of the model parameters, but that whenever they receive new information about them, they learn and update their beliefs, i.e., the players are learners. We analytically find and compare the feedback non-cooperative strategies of players under these information assumptions.

We find out that while uncertainty due to anticipation of learning leads to a decrease in total emissions, depending on the functional form of the distributions and beliefs, the effect of structural uncertainty could be either an increase, a decrease, or even no change in the emissions of individual players and also the total emissions. While the first observation is totally in line with the available literature, the latter results are rather new and more controversial. We also present the conditions under which a player may emit more when he faces uncertainty due to anticipation than when he doesn't face this source of uncertainty. Besides, if a player's beliefs change for more optimistic views or if he feels the situation is less risky, he will increase his emissions, while others will react to this change and decrease their emissions; however, the latter effect never overcomes the former and, as a result, total emissions will increase.

In the last essay, we consider the choice of national or local emissions-control policy when the regulator faces environmental and economic uncertainty. To curb pollution emissions, regulators could adopt either quantity-based instruments (e.g., quotas) or price-based ones (e.g., taxes). When there is one externality (here, pollution) and the context is deterministic, quotas and taxes are equivalent in terms of controlling that externality (emissions, in an environmental-economics context); see, e.g., Weitzman (1974)). The main objective of this paper is to verify whether this result carries over to the case where there are environmental and economic uncertainties and the regulator is not fully informed. This question is natural as it is widely accepted in the scientific community that pollutant-accumulation processes, as well as their impacts on climate and the economy are affected by random disturbances. The idea that uncertainty or inadequate information could advantage one policy instrument over another was emphasized long ago by Weitzman (1974).

There exists a large body of literature about uncertainty and policy choices. For example, using a static model of public commodities (goods) with externalities, Weitzman (1974) shows that if the cost of production or social benefits are affected by an unobserved and unknown disturbance term or random variable, then "neither instrument yields an optimum *ex post*". However, depending on the curvature of the cost and benefit functions around the optimal output level, one of the two instruments (quotas or taxes) may provide a higher expected social payoff. Fishelson (1976) also shows that the difference between taxes and quotas in terms of expected social gains depends upon the relative slopes of the marginal benefit and cost functions.

As environmental damage is mainly caused by the accumulation of pollution over time, and since the sources, nature and magnitude of the uncertainties are not the same in the short and long terms, scholars have also compared taxes to quotas in a dynamic setting. For example, Karp and Zhang (2012) find, under the assumptions of an endogenous abatement cost and asymmetric information about this cost, that taxes have some advantages (again in terms of expected payoff for society) over quotas.

In this paper, we adopt a dynamic-game model involving a regulator and a representative firm, and compare quotas and taxes in terms of emissions and expected social payoffs. We retain two sources of uncertainty, namely, consumption uncertainty and ecological (or environmental) uncertainty. Following the literature (see, e.g., Karp and Zhang (2006); Hoel and Karp (2002, 2001)), we assume that there is asymmetry of information between the regulator and the firm, that is, that the firm knows the realization of the random consumption at the time of making its decision, whereas the regulator does not have access to this information at the time of announcing the policy. Finally, we consider and compare the results in two scenarios, namely, where the regulator is fully informed and where the regulator is not but learns over time.

A paper that is closely related to ours is Karp and Zhang (2006), where they also consider uncertainty and learning in a dynamic policy choice model. In their contribution, the anticipated learning (i) increases the optimal level of emissions, and (ii) favors the use of taxes. We differ from Karp and Zhang (2006) in two important aspects. First, in their model, learning starts in the current period and continues for n predetermined consecutive periods. Here, we do not introduce a presumed end-time for learning, which is less restrictive than imposing one. Further, allowing learning to continue indefinitely allows for more flexibility in tracing the effects of either a change in beliefs or the occurrence of a shock at any time during the planning period. Second, they use a log-normal distribution and find their analytical results implicitly. Here, we do not select a particular distribution and work with a general probability density function, where the information can be summarized by a finite-dimensional vector of sufficient statistics. This method was introduced by Koulovatianos et al. (2009) for an optimal-growth model; and Agbo (2011) used the same methodology in a dynamic game of resource consumption. Using this methodology allows us to recognize and decompose different sources of uncertainty that affect the emissions rules under the learning and no-learning scenarios. Another contribution of our paper to the literature is the introduction and analysis of the effects of ecological uncertainties in an infinite-horizon model.

Our main results in the third article can be summarized as follows: Ecological uncertainties and learning do not affect the choice of policy, and the only source that matters is uncertainty in the instantaneous net benefits of emissions (consumption shock). But, ecological uncertainty leads to a difference between the emissions rule under the informed and anticipated-learning assumptions. However, the direction of this difference depends on the regulator's belief bias with regard to ecological shock. In the presence of uncertainty and in a dynamic setting, taxes yield a higher social welfare than do quotas. A change in the regulator's beliefs toward more optimistic views or a less risky situation will increase the emissions.

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Chapter 2

A Differential Game of International Pollution Control with Evolving Environmental Costs[1]

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Abstract

We consider a two-player differential game of international emissions to represent the interactions between two groups of countries, namely, developed and developing countries. We adopt a broader-than-usual definition of environmental cost for developing countries to account for their evolving involvement in tackling environmental externalities. Cooperative and non-cooperative solutions are characterized and contrasted. We obtain that it may not be the best course of action to push

[1]This paper is accepted for publication by Environment and Development Economics journal. We wish to thank the two anonymous reviewers and the editor for their very helpful comments. Research supported by NSERC, Canada.

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developing countries to reduce their emissions in the short term, and that cooperation may not create enough dividend, also in the short term, to be implementable.

Key Words: International Pollution; Environmental Concerns, International Environmental Agreements; Differential Games.

2.1 Introduction

It has often been advocated that developing countries (DCs) be given more time before being asked to make serious efforts to reduce their greenhouse gas (GHG) emissions. The two underlying reasons for granting them such a delay are: (i) it is industrialized countries (ICs) that are mainly responsible for the current state of the environment, and it would be unfair to constrain DCs' emissions when it is their turn to industrialize; and (ii) compared to other pressing economic issues, such as eradicating extreme poverty, offering essential services to their citizens (education, health care, etc.) and building infrastructure, the environment is seen as a luxury service that developing countries cannot really afford in the short term. The historical responsibility of ICs is recognized in international political circles, and was actually accepted in the Kyoto Protocol, where DCs were not asked to reduce their emissions. Further, the argument that reaching a certain level of development has priority over environmental and possibly other social and political concerns appeals to a large community of scholars and decision makers because it corresponds to the successful role-model path followed by the developed countries themselves. The rationale behind this type of thinking is that "As incomes rise, the demand for improvements in environmental quality will increase, as will the resources available for investment" World Bank (1992) p. 39. In fact, this idea of a relationship between welfare (prosperity or development) and environmental concerns has been around for some time, and has been supported by different arguments ranging from the purely materialistic to the post-materialistic and the psychological ones. The interested reader may refer to the survey in Martínez-Alier (1995) for a complete discussion of the links between economic growth and environmental goods.

Suppose that DCs: (i) need a period of time $[0, T]$ to accomplish a desired level of development, during which they disregard environmental damage, either partially or completely, when making their production decisions; and (ii) fully internalize their environmental externalities after T. Then, the objective of this paper is to address the following research questions:

1. How do cooperative and non-cooperative emissions strategies compare during this first period $[0, T]$ and in the long run?
2. Can cooperation between DCs and ICs reduce T?
3. Under which conditions cooperation is collectively better than noncooperation during the time interval $[0, T]$?

We believe that these questions are relevant to the negotiations of all environmental agreements involving both types of countries. To answer these questions, we adopt a two-player differential-game model, where player 1 represents developed countries and fully accounts for the environmental damage cost, and player 2 represents developing countries and does not consider the environment a top priority during an initial period of time, whose terminal date is endogenous. The choice of a dynamic-game framework to deal with the above questions is quite natural. Indeed, the transboundary nature of pollution emissions calls for a game-theoretic approach to capture the strategic interactions between players. Further, as economic development and environmental concerns are typically long-term problems that cannot be solved overnight, the model must reflect this dynamic feature.

This paper belongs to the significant differential-games literature in environmental economics, and more specifically, to its part dealing with international environmental agreements (IEA); see, e.g., the recent surveys by Jørgensen et al. (2010); Long (2010). The dynamic IEA literature can be divided into two streams. In the first stream, the papers characterize and contrast the cooperative and non-cooperative solutions and demonstrate the benefits of cooperation. A pending issue when the game is cooperative is how to allocate the total payoff among them. Typically, this is done by adopting a cooperative-game solution, e.g., core and Shapley value. The early contributions in this stream are Van der Ploeg and de Zeeuw (1992); Long (1992). An impressive list of papers followed, which we will not review but instead refer the reader to the above-cited surveys. The second stream sees the signing of an international environmental agreement as a non-cooperative game that ends when a stable coalition is reached, that is, no outsider would like to join the agreement, and no insider would like to leave it. The (static) noncooperative-game approach to IEA started two decades ago, and a recurrent result in this literature is that full cooperation is hard to achieve, albeit it is urgently needed to curb emissions and avoid some possible catastrophic events. Recently, we have seen some contributions that include pollution accumulation and membership dynamics in their models; see, e.g., Rubio and Casino (2005); Rubio and Ulph (2007); de Zeeuw (2008); Bahn et al. (2009); Breton et al. (2010).

A main contribution of this paper lies in its different treatment of the two players in terms of their environmental concerns, and consequently, in modeling their damage cost functions. Indeed, we depart from the often-retained assumption that DCs either fully account for their environmental costs from the outset of the game or never account for them, by supposing that they adopt a gradual-involvement approach in dealing with the environmental externality. By doing this, we hope to capture in a more realistic way the structural asymmetry between the two groups of countries, and not only the asymmetry in parameter values. In some sense, we are extending here the notion of environmental-damage costs to the broader idea of "environmental concerns" (EC). Roughly speaking, by EC we mean any kind of feeling, economic problem, health issue, etc., which could result in an attempt (a costly one) and react to emissions or their accumulation. Of course, precisely modeling an EC function is a highly challenging task at all levels, including the specification of such a function, the measurement of the variables that are supposedly involved and the choice of a functional form. As a first exploratory step in this direction, we suppose that EC can be represented by a (damage-cost) function whose arguments are the pollution stock (as usual) and accumulated revenues. By letting this function positively depend on revenues till a certain threshold is reached, we are in fact operationalizing in the most parsimonious possible way the World-Bank idea quoted above. Another contribution with respect to the dynamic-games literature on pollution control lies in our analysis of emissions strategies and payoffs during the first period of the game. Short-term considerations are generally overlooked in this literature, which has focused only on long-term solutions, that is, on cooperative and non-cooperative steady states in infinite-horizon games.

Our main results can be summarized as follows:

1. Throughout the game, ICs emit less under cooperation than non-cooperation. The same is observed for DCs but only in the long term, that is, after reaching the threshold level of development. In the shorter term, the result depends on the degree of environmental concern in DCs.

2. For a large region of the parameter space, cooperation would lead to a longer period of low or total lack of involvement by the DCs in dealing with environmental-damage than would noncooperation. Also, in the short term, cooperation may not create enough dividend to be an attractive mode of play.

3. It may not be the best option for ICs to press DCs to engage in abatement efforts in the short term.

The rest of the paper is organized as follows: In Section 2, we introduce the model, and in Section 3, we characterize emissions strategies under cooperative and non-cooperative modes of play, and perform a sensitivity analysis. In Section 4, we compare emissions and outcomes under the two regimes and discuss the feasibility of cooperation in the short term. Section 5 briefly concludes.

2.2 Model

We consider a two-player differential game, where player 1 represents the group of developed countries characterized by high environmental concern levels, and player 2 represents less developed countries for whom environmental issues are not yet at the top of their economic agenda.[3] Time t is continuous, with $t \in [0, \infty)$. Denote by $e_i(t)$ the emissions of player i at time t. Emissions are a by-product of industrial production activities $y_i(t)$, and are given by $e_i(t) = h_i(y_i(t))$. We suppose that $h_i(\cdot)$ is a smooth function that can be inverted, and write $y_i(t) = h_i^{-1}(e_i(t)) \triangleq f_i(e_i(t))$, that is, we can express production (or revenue) as a function of emission levels. We make the standard assumption that $f_i(e_i)$ is concave and increasing.

Denote by $S(t)$ the stock of pollution, whose evolution is described by the following differential equation:

$$\dot{S}(t) = \mu(e_1(t) + e_2(t)) - \delta S(t), \qquad S(0) = S_0, \tag{2.1}$$

[3] An alternative approach is to assume that these two players are neighboring countries.

where μ is a positive scaling parameter, δ is the positive natural absorption rate of pollution, and S_0 is the initial given pollution stock.

2.2.1 Environmental Cost

In the transboundary pollution literature, (see, e.g, Jørgensen et al. (2010)), the usual assumption is that, from the outset, all players account for the full environmental damage-cost when making their production decisions. The damage cost of player i, denoted $D_i(S)$, is assumed to be an increasing convex function of the stock of pollution. We retain this assumption for player 1, the representative of developed countries who can afford to fully internalize the environmental cost. For player 2, we assume that this internalization of the environmental cost is gradual, and will become complete only when a threshold level of economic development has been achieved. To reflect this, the damage cost is modeled as a function of the pollution stock and of the level of development during a first period of time, and a function of only the pollution stock after that period. Adequately measuring a country's development level is a difficult task and requires the use of a long list of variables, e.g., available infrastructure, consumption per capita, life expectancy, etc. To keep the model as parsimonious as possible without however, losing much qualitative insight into the problem we are dealing with, we avoid taking this route and simply assume that the level of development can be well approximated by cumulative revenues. Note that we could add environmental awareness (or information acquisition) as a determinant of environmental concerns, but refrain from doing so, assuming that cumulative revenues and awareness are highly correlated variables, and that therefore, there is no need to retain both of them. In fact, our simple reasoning is consistent with who found, using data from 149 countries for the period 1960-90, that "income has the most consistently significant effect on all indicators of environmental quality".

We assume that both players discount future gain using the same constant rate of time preference $\rho > 0$. Denote by $Y_2(t)$ the cumulative discounted revenues of player 2 by time t, which is given by

$$Y_2(t) = \int_0^t e^{-\rho z} f_i(e_i(z))dz,$$

and by $\bar{Y}_2$ the threshold value of cumulative revenues before this player starts to fully account for the environmental damage. The damage-cost function of player 2 is then specified as

$$D_2(S(t), Y_2(t)) = \begin{cases} d_2(S(t), Y_2(t)), & \forall Y_2(t) < \bar{Y}_2, \\ D_2(S(t)) & \forall Y_2(t) \geq \bar{Y}_2, \end{cases}$$

with

$$\frac{\partial D_2(S, Y_2)}{\partial S} > 0, \quad \frac{\partial^2 D_2(S, Y_2)}{\partial S^2} \geq 0, \quad \frac{\partial D_2(S, Y_2)}{\partial Y_2} \geq 0.$$

Finally, we note that there is a common element between our approach and the well-known environmental Kuznets curve (EKC) approach, namely, the existence of a link between income and environmental quality.

2.2.2 Payoffs and Functional Forms

We will henceforth skip the time argument when no ambiguity may arise. We suppose that each player's objective is to maximize his discounted stream of welfare, given by the difference between production revenues and the damage cost. The optimization problems of the two players are then as follows:

$$\begin{aligned}
\max_{e_1} W_1 &= \int_0^\infty e^{-\rho t}\left(f_1\left(e_1\right) - D_1\left(S\right)\right)dt,\\
\max_{e_2} W_2 &= \int_0^T e^{-\rho t}\left(f_2\left(e_2\right) - d_2\left(S, Y_2\right)\right)dt + \int_T^\infty e^{-\rho t}\left(f_2\left(e_2\right) - D_2\left(S\right)\right)dt,\\
\text{subject to} &: \quad \dot{S} = \mu(e_1 + e_2) - \delta S, \qquad S\left(0\right) = S_0,
\end{aligned}$$

where T is the date satisfying the following equality:

$$\int_0^T e^{-\rho t} f_i(e_i) dt = \bar{Y}_2. \tag{2.2}$$

Remark 2.1 *Since there is a one-to-one correspondence between T and $\bar{Y}_2$, we can express, with a slight abuse of notation, the damage cost of player 2 as a function of time instead of writing it as a function of cumulative revenues, that is,*

$$D_2\left(S\left(t\right), t\right) = \begin{cases} d_2\left(S\left(t\right), t\right), & \forall t < T, \\ D_2\left(S\left(t\right)\right), & \forall t \geq T. \end{cases}$$

To illustrate the type of insight that can be derived using our model, from now on, we adopt the following functional forms:

$$f_i(e_i) = \alpha_i e_i - \frac{1}{2} e_i^2, \tag{2.3}$$

$$D_1(S) = \beta_1 S, \tag{2.4}$$

$$D_2\left(S\left(t\right), Y_2\left(t\right)\right) = \begin{cases} \frac{t}{T}\gamma\beta_2 S, & \forall Y_2\left(t\right) < \bar{Y}_2, \\ \beta_2 S & \forall Y_2\left(t\right) \geq \bar{Y}_2. \end{cases} \tag{2.5}$$

In the revenue function in (2.3), the parameter $\alpha_i, i = 1, 2$, is strictly positive, and the implicit assumption is that $f_i(e_i)$ is strictly positive for all possible actual pollution values. This quadratic form is common in the literature; see, e.g., Smala Fanokoa et al. (2011); Breton et al. (2005); Rubio and Casino (2005); Breton et al. (2010); Dockner and Long (1993). The damage-cost functions of both players are assumed linear in the stock, with parameters β_1 and β_2 being strictly positive. Admittedly, the linearity assumption of the damage cost is a considerable simplification. However, we expect that the impact of this choice on the results to be quantitative and not qualitative. We note that our linearity assumption is not uncommon in the modeling literature (see, e.g., Breton et al. (2010)), and is also supported in some empirical contributions (see, e.g., Labriet and Loulou

(2003)). This being said, it would be of interest to extend the analysis to a non-linear damage cost in future works. Further, we suppose that $\gamma \in \{0, 1\}$. A value $\gamma = 0$ means that player 2 completely ignores the environmental damage before reaching T. This is meant to represent an extreme case where this player is experiencing harsh development problems, and is not willing to account for environmental externalities when making his production decisions, till he reaches the sought-for threshold of cumulative wealth $\bar{Y}_2$. This is somehow in the spirit of the Kyoto Protocol, where developing countries were not requested to reduce their emissions. The case $\gamma = 1$ corresponds to a gradual internalization of the damage cost. We now contrast the results of the two cases.

2.2.3 Solution Concepts

We will characterize and compare the cooperative and non-cooperative solutions to the two-player differential game introduced above. We suppose that the information structure is feedback, that is, the pollution strategy of each player is a function of the state variable (pollution stock). The cooperative game corresponds to the scenario where the two players sign an international environmental agreement (IEA) aiming at controlling pollution. In such a context, the optimal pollution levels are obtained by jointly maximizing the welfare of the two players. The results will be superscripted by C (for cooperation). In the absence of an agreement, the game is played non-cooperatively, and the two players seek a Nash equilibrium. The results will be superscripted by N (for Nash equilibrium).

Because of the assumption made about the damage-cost function of player 2, the determination of the two solutions is much more complicated than in the case where the damage cost does not vary over time. Indeed, both cooperative and non-cooperative solutions will depend on the date at which player 2 becomes fully vulnerable to the environmental damage. To handle this, we proceed backward, that is, we assume that we know $T^j, j = C, N$, and solve the infinite-horizon problem defined on $[T^j, \infty)$. Next, we deal with the first-period problem defined on $\left[0, T^j\right]$, using the second-period value functions as salvage values in the overall optimization problem defined on $[0, \infty)$. At this stage, we have the information to compute T^j and $S\left(T^j\right)$, which at the beginning, were assumed given, by inserting the optimal strategies into (2.2). Note that the date T^j will not only depend on the mode of play, but also on the value of the parameter γ. To reflect this, we will write T^j when a result holds for both values of γ, and subscript T^j with γ when the result holds for a specific value of γ, that is, T^j_γ, with T^j_0 and T^j_1 corresponding to $\gamma = 0, 1$, respectively.

2.3 Results

2.3.1 Non-Cooperative and Cooperative Emissions

To save on notation, define by

$$\begin{aligned} F\left(t;T^{N}\right) &= \frac{1+t\left(\rho+\delta\right)-e^{\left(\rho+\delta\right)\left(t-T^{N}\right)}}{\left(\rho+\delta\right)T^{N}}, \qquad G\left(t;T^{N}\right)=1-e^{\left(\rho+\delta\right)\left(t-T^{N}\right)}, \\ F\left(t;T^{C}\right) &= \frac{1+t\left(\rho+\delta\right)-e^{\left(\rho+\delta\right)\left(t-T^{C}\right)}}{\left(\rho+\delta\right)T^{C}}, \qquad G\left(t;T^{C}\right)=1-e^{\left(\rho+\delta\right)\left(t-T^{C}\right)}. \end{aligned}$$

We first characterize the feedback-Nash equilibrium, and next determine the cooperative solution.

Proposition 2.1 *Assuming an interior solution, the feedback-Nash equilibrium emissions are given by*

$$\begin{aligned} e_1^N\left(t\right) &= \alpha_1-\frac{\mu\beta_1}{\rho+\delta}, \qquad \forall t\in[0,\infty), \\ e_2^N\left(t\right) &= \begin{cases} \alpha_2-\frac{\mu\beta_2}{(\rho+\delta)}\left(\gamma F\left(t;T^N\right)+e^{\left(\rho+\delta\right)\left(t-T^N\right)}\left(1-\gamma\right)\right), & \textit{for } t\in\left[0,T^N\right], \\ \alpha_2-\frac{\mu\beta_2}{\rho+\delta}, & \textit{for } t\in[T^N,\infty), \end{cases} \end{aligned}$$

where T^N is the solution to the equation

$$\int_0^{T^N}\left(\alpha_2 e_2^N-\frac{1}{2}(e_2^N)^2\right)e^{-\rho t}dt=\bar{Y}_2. \tag{2.6}$$

Proof. See Appendix 2.7.1. ■

The results in the above proposition call for four observations. First, the equilibrium strategies are independent of the state variable (pollution stock). This is a by-product of the linear-state structure of the adopted differential game. Second, the strategy of player 1 is the same before and after T^N, which is expected since this player is fully vulnerable to the environmental damage from the beginning of the game. Third, the given expression for e_2^N is valid for any $\gamma\geq 0$. Specializing the results in each value of γ, we get the following emissions trajectory for player 2 for $t\leq T_\gamma^N$:

$$e_2^N\left(t;\gamma\right)=\begin{cases} \alpha_2-\frac{\mu}{(\rho+\delta)^2}\frac{\beta_2}{T_1^N}\left(1+t\left(\rho+\delta\right)-e^{\left(\rho+\delta\right)\left(t-T_1^N\right)}\right), & \text{for } \gamma=1, \\ \alpha_2-\frac{\mu\beta_2}{(\rho+\delta)}\left(e^{\left(\rho+\delta\right)\left(t-T_0^N\right)}\right), & \text{for } \gamma=0. \end{cases}$$

The sign of the difference

$$e_2^N\left(t;0\right)-e_2^N\left(t;1\right)=\frac{\beta_2\mu}{T_1^N\left(\rho+\delta\right)^2}\left(1-e^{\left(t-T_1^N\right)\left(\rho+\delta\right)}+\left(\rho+\delta\right)\left(t-T_1^N e^{\left(t-T_0^N\right)\left(\rho+\delta\right)}\right)\right), \tag{2.7}$$

cannot be unambiguously determined, as it depends on the parameter values, and on the dates T_0^N and T_1^N. We will come back to this later. Fourth, the emissions level of player 2 are monotonically decreasing over time during the time interval $\left[0, T^N\right]$, and reach their lowest value at T^N, that is, when this player becomes fully vulnerable to the environmental damage. Indeed, computing the derivative of e_2^N with respect to time, we get

$$\dot{e}_2^N(t) = -\frac{\mu}{(\rho+\delta)}\frac{\beta_2}{T^N}\left(\gamma\left(1-e^{(\rho+\delta)\left(t-T^N\right)}\right)+e^{(\rho+\delta)\left(t-T^N\right)}\left(T^N(\rho+\delta)(1-\gamma)\right)\right)<0.$$

Note that $t = T_\gamma^N$, $\gamma \in \{0,1\}$, the emissions are given by

$$e_2^N\left(T_\gamma^N;\gamma\right) = \alpha_2 - \frac{\mu\beta_2}{(\rho+\delta)},$$

that is, the same value as the one computed for the long run, i.e., after T^N.

To solve for the dynamics in (2.1), that is, to determine the equilibrium pollution-stock trajectory and the steady-state value, we design the following algorithm and use it in the next section:

Step 1: Compute T_γ^N for $\gamma \in \{0,1\}$ using (2.2), that is,

$$\int_0^T e^{-\rho t} f_2(e_2^N(t;\gamma))dt = \begin{cases} \int_0^T e^{-\rho t}\left(\alpha_2 e_2^N(t;1) - \frac{1}{2}\left(e_2^N(t;1)\right)^2\right)dt = \bar{Y}_2, & \gamma = 1, \\ \int_0^{T_0} e^{-\rho t}\left(\alpha_2 e_2^N(t;0) - \frac{1}{2}\left(e_2^N(t;0)\right)^2\right)dt = \bar{Y}_2, & \gamma = 0. \end{cases}$$

Step2: For $t \in \left[0, T_\gamma^N\right]$, $\gamma \in \{0,1\}$, solve the differential equation

$$\dot{S} = \begin{cases} \mu(e_1^N(t) + e_2^N(t;1)) - \delta S, & S(0) = S_0, \quad \gamma = 1, \\ \mu(e_1^N(t) + e_2^N(t;0)) - \delta S, & S(0) = S_0, \quad \gamma = 0. \end{cases}$$

Let $S_{T_\gamma^N}$ be the value of the pollution stock at $T_\gamma^N, \gamma \in \{0,1\}$.

Step 3: For $t \in [T_\gamma^N, \infty), \gamma \in \{0,1\}$, solve the differential equation

$$\dot{S} = \begin{cases} \mu(\alpha_1 + \alpha_2 - \frac{\mu}{\rho+\delta}(\beta_1+\beta_2)) - \delta S, & S\left(T_1^N\right) = S_{T_1^N}, \quad \gamma = 1, \\ \mu(\alpha_1 + \alpha_2 - \frac{\mu}{\rho+\delta}(\beta_1+\beta_2)) - \delta S, & S\left(T_0^N\right) = S_{T_0^N}, \quad \gamma = 0. \end{cases}$$

The above equation has as its solution

$$S^N(t) = \begin{cases} \frac{1}{\delta}\mu(\alpha_1+\alpha_2-\frac{\mu}{\rho+\delta}(\beta_1+\beta_2))\left(1-e^{\delta\left(T_1^N-t\right)}\right) + e^{\delta\left(T_1^N-t\right)}S_{T_1^N}, & \gamma = 1, \\ \frac{1}{\delta}\mu(\alpha_1+\alpha_2-\frac{\mu}{\rho+\delta}(\beta_1+\beta_2))\left(1-e^{\delta\left(T_0^N-t\right)}\right) + e^{\delta\left(T_0^N-t\right)}S_{T_0^N} & \gamma = 0. \end{cases}$$

The steady-state value is independent of γ, and is given by

$$S_{ss}^N = \lim_{t\to\infty} S^N(t) = \frac{\mu}{\delta}(\alpha_1+\alpha_2-\frac{\mu}{\rho+\delta}(\beta_1+\beta_2)).$$

The following proposition characterizes cooperative emissions strategies.

Proposition 2.2 *Assuming an interior solution, the optimal emissions of player $i, i = 1, 2$, are given by*

$$e_i^C(t) = \begin{cases} \alpha_i - \frac{\mu\beta_1}{\rho+\delta} - \frac{\mu\beta_2}{(\rho+\delta)}\left(\gamma F\left(t;T^C\right) + e^{(\rho+\delta)\left(t-T^C\right)}(1-\gamma)\right), & \text{for } t \in \left[0, T^C\right], \\ \alpha_i - \mu\frac{\beta_1+\beta_2}{\rho+\delta}, & \text{for } t \in [T^C, \infty), \end{cases}$$

where T^C is the solution to equation

$$\int_0^{T^C} \left(\alpha_2 e_2^C - \frac{1}{2}(e_2^C)^2\right) e^{-\rho t} dt = \bar{Y}_2.$$

Proof. See Appendix 2.7.2. ■

The two main differences with the non-cooperative case are: (i) player i now accounts for player j's marginal damage cost β_j, which is a standard result in cooperative environmental games; and (ii) player 1's emissions are now a function of time during the interval $\left[0, T^C\right]$, that is, till the date at which player 2 reaches his full environmental vulnerability. Further, we note that both players decrease their emissions by the same rate over time during the interval $\left[0, T^C\right]$. Indeed, computing the derivative of $e_i^C(t)$ with respect to time yields

$$\dot{e}_i^C(t) = -\frac{\mu\beta_2}{T^C(\rho+\delta)}\Bigg(\gamma\left(1 - e^{(\rho+\delta)\left(t-T^C\right)}\right) + e^{(\rho+\delta)\left(t-T^C\right)}\left(T^C(\delta+\rho)(1-\gamma)\right)\Bigg) < 0, \quad i = 1, 2.$$

As in the non-cooperative case, T^C depends on γ, and will be denoted by T_γ^C. To compute this date and the optimal pollution-stock trajectory, we follow the same algorithm as in the non-cooperative scenario. It can easily be shown that the steady state here is given by

$$S_{ss}^C = \frac{\mu}{\delta}(\alpha_1 + \alpha_2 - \frac{2\mu}{\rho+\delta}(\beta_1+\beta_2)).$$

Remark 2.2 *We assume that the parameter values are such that all equilibrium and optimal emissions and steady-state pollution values are strictly positive. In particular, this requires imposing the following condition:*

$$\alpha_1 + \alpha_2 > \frac{2\mu}{\rho+\delta}(\beta_1+\beta_2). \tag{2.8}$$

2.3.2 Comparative Statics

Table 2.1 summarizes the effect of the model's parameters on the players' strategies for the whole time horizon, and on the steady-state pollution stock under cooperative and non-cooperative modes of play. In the non-cooperative scenario, a higher marginal cost β_i leads to lower emissions e_i^N, $i = 1, 2$, and has no effect on e_j^N, $j \neq i$. Under cooperation, an increase in either marginal damage cost has a negative effect on both players' emissions. This is because each member of the coalition

Table 2.1: Parameter effects on emissions and steady-state stock

	$e_2^N(t;\gamma)$, $t \in \left[0, T^C\right]$	$e_1^N(t;\gamma)$, $t \in \left[0, T^C\right]$	e_i^{NT} $t \in \left(T^C, \infty\right)$		$e_i^C(t)$					S_{ss}^N	S_{ss}^C
			$i=1$	$i=2$	$t \in \left(T^C, \infty\right)$	$t \in \left[0, T^C\right]$ $i=1$		$t \in \left[0, T^C\right]$ $i=2$			
						$\gamma=0$	$\gamma=1$	$\gamma=0$	$\gamma=1$		
β_1	0	$-$	$-$	0	$-$	$-$	$-$	$-$	$-$	$-$	$-$
β_2	$-$	0	0	$-$	$-$	$-$	$-$	$-$	$-$	$-$	$-$
α_1	$+$	$+$	$+$	0	$+$	$+$	$+$	0	0	$+$	$+$
α_2	0	0	0	$+$	$+$	0	0	$+$	$+$	$+$	$+$
δ	$+$	$+$	$+$	$+$	$+$	$+$	$+$	$+$	$+$	*	*
ρ	$+$	$+$	$+$	$+$	$+$	$+$	$+$	$+$	$+$	$+$	$+$
μ	$-$	$-$	$-$	$-$	$-$	$-$	$-$	$-$	$-$	$-$	$-$

accounts for the damage to all players when determining his pollution strategy. The impact of varying α_i is, as expected, positive on the emissions of player i. A higher discount rate, i.e., attaching less value to future damages, encourages players to increase their today's revenues by emitting more, and consequently, we obtain a higher steady-state pollution. Further, the impact of varying the natural absorption on the steady-state of the pollution stock is a priori ambiguous. Indeed, increasing δ is synonymous with an improvement in abatement technology, and hence, with a higher reduction in the existing stock. On the other hand, a higher δ is an incentive to increase today's emissions. However, under condition (2.8), we obtain that the net impact of increasing δ on the non-cooperative steady state is negative. The same holds true for cooperative steady state for sufficiently low δ.[4]

2.4 Comparison

In this section, we first compare cooperative and non-cooperative emissions, and then, the corresponding outcomes.

[4]Most of the results in Table 1 are straightforward. To save space, we do not report the derivatives. Only the results regarding the variation of the steady state with respect to δ merits some detail. The derivatives are given by

$$\begin{aligned}
\frac{\partial S_{ss}^C}{\partial \delta} &= -\frac{\mu\left((\alpha_1+\alpha_2)(\delta+\rho)^2 - 2\mu(\beta_1+\beta_2)(2\delta+\rho)\right)}{\delta^2(\delta+\rho)^2}, \\
\frac{\partial S_{ss}^N}{\partial \delta} &= -\frac{\mu\left((\alpha_1+\alpha_2)(\delta+\rho)^2 - \mu(\beta_1+\beta_2)(2\delta+\rho)\right)}{\delta^2(\delta+\rho)^2}.
\end{aligned}$$

Clearly, we have

$$\frac{\partial S_{ss}^N}{\partial \delta} < 0 \Leftrightarrow \alpha_1+\alpha_2 > \mu(\beta_1+\beta_2)\frac{(2\delta+\rho)}{(\delta+\rho)^2},$$

which always holds true under condition (2.8). For $\frac{\partial S_{ss}^C}{\partial \delta}$ to be negative, we need

$$\alpha_1+\alpha_2 > 2\mu(\beta_1+\beta_2)\frac{(2\delta+\rho)}{(\delta+\rho)^2},$$

which is satisfied if δ is sufficiently small.

2.4.1 Comparison of Emissions

The next two propositions compare the non-cooperative and cooperative emissions of player 1 and 2, respectively.

Proposition 2.3 *Player* 1 *emits more in the non-cooperative game than under cooperation at all instants of time, that is,*

$$e_1^N(t) > e_1^C(t), \qquad \forall t \in [0, \infty).$$

Proof. We need to show the result for two time intervals, namely, $\left[0, T^C\right]$ and $[T^C, \infty)$. For $t \in \left[0, T^C\right]$, the difference in emissions is given by

$$\begin{aligned} e_1^N(t) - e_1^C(t) &= \frac{\mu\beta_2}{T^C(\rho+\delta)^2}\Big(\gamma\left(1 + t(\rho+\delta) - e^{(\rho+\delta)\left(t-T^C\right)}\right) \\ &\quad + e^{(\rho+\delta)\left(t-T^C\right)}\left(T^C(\delta+\rho)(1-\gamma)\right)\Big), \end{aligned}$$

which is clearly positive.

For $t \in [T^C, \infty)$, we have

$$e_1^N(t) - e_1^C(t) = \frac{\mu\beta_2}{\rho+\delta} > 0.$$

■

Proposition 2.4 *Player* 2*'s cooperative and non-cooperative emissions compare as follows:*

1. *For all* $t \geq \max\left\{T^N, T^C\right\}$*, we have*

$$e_2^N(t) - e_2^C(t) = \frac{\mu\beta_1}{\rho+\delta} > 0. \tag{2.9}$$

2. *For all* $t \in \left[\min\left\{T^N, T^C\right\}, \max\left\{T^N, T^C\right\}\right]$,
   (a) *If* $T^N < T^C$*, then* $\forall t \in \left[T^N, T^C\right]$*, we have*

$$e_2^N(t) - e_2^C(t) = \begin{cases} \frac{\mu}{\rho+\delta}\left[\beta_1 - \beta_2\left(1 - F\left(t; T^C\right)\right)\right] > 0 \Leftrightarrow \frac{\beta_1}{\beta_2} > 1 - F\left(t; T^C\right), & \gamma = 1, \\ \frac{\mu}{\rho+\delta}\left[\beta_1 - \beta_2 G\left(t; T^C\right)\right] > 0 \Leftrightarrow \frac{\beta_1}{\beta_2} > G\left(t; T^C\right), & \gamma = 0. \end{cases}$$

   (b) *If* $T^N > T^C$*, then for all* $t \in \left[T^C, T^N\right]$*, we have*

$$e_2^N(t) - e_2^C(t) = \begin{cases} \frac{\mu}{\rho+\delta}\left[\beta_1 + \beta_2\left(1 - F\left(t; T^N\right)\right)\right] > 0, & \gamma = 1, \\ \frac{\mu}{\rho+\delta}\left[\beta_1 + \beta_2 G\left(t; T^N\right)\right] > 0, & \gamma = 0. \end{cases}$$

3. For all $t \in \left[0, \min\left\{T^N, T^C\right\}\right]$, *we have*

$$e_2^N(t) - e_2^C(t) = \begin{cases} \begin{array}{c} \frac{\mu}{\rho+\delta}\left[\beta_1 + \beta_2\left(F\left(t;T^C\right) - F\left(t;T^N\right)\right)\right] > 0 \Leftrightarrow \\ \frac{\beta_1}{\beta_2} > F\left(t;T^N\right) - F\left(t;T^C\right) \end{array} & \gamma = 1, \\ \begin{array}{c} \frac{\mu}{\rho+\delta}\left[\beta_1 + \beta_2\left(e^{(\rho+\delta)\left(t-T^C\right)} - e^{(\rho+\delta)\left(t-T^N\right)}\right)\right] > 0 \Leftrightarrow \\ \frac{\beta_1}{\beta_2} > e^{(\rho+\delta)\left(t-T^N\right)} - e^{(\rho+\delta)\left(t-T^C\right)} \end{array} & \gamma = 0. \end{cases}$$

Proof. Straightforward algebraic manipulations lead to the results. ■

Based on the results in the above two propositions, the answer to our first research question is as follows:

1. In the long run, cooperation leads to lower emissions. This result is fully in line with the related literature. Indeed, when each player accounts for the damage cost of all players, he emits less. Consequently, the steady-state value of the pollution stock in the non-cooperative game is higher than in its cooperative counterpart, with the difference being given by

$$S_{ss}^N - S_{ss}^C = \frac{\mu^2\left(\beta_1 + \beta_2\right)}{\delta\left(\rho + \delta\right)} > 0.$$

2. In the short run, there is no definite conclusion regarding player 2's emissions. The relationship between cooperative and non-cooperative emissions depends, in a complex way, on the parameter values. The message here is that cooperation may not necessarily lead to a reduction in emissions by all parties. However, under some conditions stated in the corollary below, we obtain that player 2's non-cooperative emissions are greater than his cooperative emissions for all instants of time, that is, including in the short run.

Corollary 1 *Suppose* $\gamma = 0$. *If* $\beta_1 > \beta_2$, *then* $e_2^N(t) - e_2^C(t) > 0, \forall t \in [0, \infty)$.

Proof. For $t \geq \max\left\{T^N, T^C\right\}$, we have

$$e_2^N(t) - e_2^C(t) = \frac{\mu\beta_1}{\rho + \delta} > 0.$$

For all $t \in \left[\min\left\{T^N, T^C\right\}, \max\left\{T^N, T^C\right\}\right]$,

- If $T^N < T^C$, then $\forall t \in \left[T^N, T^C\right]$, we have

$$e_2^N(t) - e_2^C(t) = \frac{\mu}{\rho + \delta}\left[\beta_1 - \beta_2 G\left(t;T^C\right)\right],$$

  which is positive as $0 < G\left(t;T^C\right) < 1$.

- If $T^N > T^C$, then for all $t \in \left[T^C, T^N\right]$, we have

$$e_2^N(t) - e_2^C(t) = \frac{\mu}{\rho + \delta}\left[\beta_1 + \beta_2 G\left(t;T^N\right)\right] > 0.$$

For all $t \in \left[0, \min\left\{T^N, T^C\right\}\right]$, we have

$$e_2^N(t) - e_2^C(t) = \frac{\mu}{\rho+\delta}\left[\beta_1 + \beta_2\left(e^{(\rho+\delta)\left(t-T^C\right)} - e^{(\rho+\delta)\left(t-T^N\right)}\right)\right],$$

which is positive because $0 < e^{(\rho+\delta)\left(t-T^C\right)} < 1$ and $0 \leq e^{(\rho+\delta)\left(t-T^N\right)} \leq 1$. ■

Note that the condition $\beta_1 > \beta_2$ is sufficient and not necessary. Whether it is fulfilled or not is an empirical matter beyond the scope of this paper. At first glance, we may think that this condition does not hold true because DCs are probably less prepared than richer ICs to cope with the environmental damage, including those induced by rising ocean level, floods, and other extreme events.

2.4.2 Comparison of Threshold Dates and Outcomes

Our second and third research questions refer to comparative outcomes, in terms of the value of γ and the mode of play (cooperative versus noncooperative), as well as to threshold dates T^N and T^C. Unfortunately, almost all these comparisons cannot be carried out analytically and we need to resort to numerical simulations.

Our model has nine parameters, namely, α_i, β_i, μ, ρ, $\delta, \bar{Y}_2$ and $S_0, i = 1, 2$. To start with, we rely on the literature to select values for some of the parameters. Based on Nordhaus (1993), we retain $\mu = 0.64$ and $\delta = 0.0083$ as reference values, and let these parameters vary in the following intervals: $\mu \in [0.5, 1]$ and $\delta \in [0.001, 0.01]$. Further, we assume that $\rho \in [0.02, 0.2]$, which is a sufficiently large interval. The calibration of the remaining parameters, that is, $\alpha_i, \beta_i, \bar{Y}_2$ and $S_0, i = 1, 2$, must be made subject to the satisfaction of non-negativity for revenues, emissions and pollution-stock values at each instant of time. The following choice for $\alpha_i, \beta_i, i = 1, 2$, leads to a large feasible region: $\alpha_i \in [0.5, 1]$ and $\beta_i \in [0.001, 0.01]$ (note that for some combinations of parameter values, we may still get a negative value for the emissions). The values of $\bar{Y}_2$ and S_0 are selected in each simulation conditionally to the choice of the other parameters and satisfaction of the non-negativity constraints. To summarize, we retain upper bounds for α_i and μ that are twice their lower-bound values, and upper bounds for δ, ρ and β_i that are tenfold their respective lower bounds. With such a large parameter space, we believe that our numerical results are sufficiently robust to allow for some general qualitative conclusions.

Our second research question was "Can cooperation between DCs and ICs shorten the first period of low or full absence of involvement by the DCs in dealing with environmental externalities?" The answer is clear-cut in the case of $\gamma = 0$. Indeed, we prove in the proposition below that cooperation delays the point in time at which developing countries would start to fully internalize the environmental externality.

Proposition 2.5 *If $\gamma = 0$, then $T^N < T^C$.*

Proof. See Appendix 2.7.3. ■

Unfortunately, we were not able to analytically prove a similar result for $\gamma = 1$. The expressions involved are highly non-linear in T^N and T^C. We ran a large number of simulations in which we visited the whole numerical grid (in a discrete sense), and obtained either an infeasible result (i.e., negative emissions) or that $T^N < T^C$. Based on this, we also conclude that when DCs partially account for the negative environmental externality, they will reach the threshold point in terms of cumulative revenues $\bar{Y}_2$ earlier when the game is noncooperative than when it is cooperative. As this result is numerical, we state it as a claim rather than as a proposition.

Claim 1 *If $\gamma = 1$, then $T^N < T^C$.*

The intuition is as developing countries are more motivated by revenue in the short run, they will produce and consequently emit more (at lower cost) under non-cooperative setup, and consequently achieve the threshold sooner.

Moving to the comparison of outcomes, we start by looking at the impact of γ on non-cooperative payoffs (we account for cooperation later on). Recall that during the first period, DCs face a damage cost given by $\frac{t}{T}\gamma\beta_2 S$, with $\gamma \in \{0, 1\}$. We recall that the damage cost is interpreted in a broad sense and includes abatement efforts. If the total payoff when $\gamma = 1$ is larger than the total payoff when $\gamma = 0$, then an argument can be made to press, DCs to start dealing with environmental externalities from the outset.

Let Δ_γ be the difference between total non-cooperative payoffs when $\gamma = 0$ and $\gamma = 1$, i.e.,

$$\Delta_\gamma = \sum_{i=1}^{2} \left(W_i^N (\gamma = 0) - W_i^N (\gamma = 1) \right).$$

Our numerical results are summarized in Figures 2.1 to 2.3. These figures show that Δ_γ is positive for the following combinations of parameter values: (i) the discount rate and the marginal damage cost of player 1 are large, while the marginal damage cost of player 2 is small (see Figure 2.1, left panel, and Figure 2.2); (ii) the discount rate and α_i, $i = 1, 2$, are large enough (Figure 2.1, right panel); (iii) the scaling parameter μ is large enough and the absorption rate δ is small enough (Figure 2.2, right panel); and finally (iv) for a large enough initial pollution stock (Figure 2.3, right panel). A first conclusion here is that we can get $\Delta_\gamma > 0$ for very different reasons, e.g., high asymmetry in marginal damage cost, high revenues from production (emissions), and fast accumulation of pollution (high initial stock, or large μ and small δ).

In an attempt to find a commonality in the above combinations, we again look at the emissions for the two different values of γ, that is, $e_2^N(t; 0)$ and $e_2^N(t; 1)$. (The difference is given in (2.7), with T_0^N and T_1^N yet to be computed.) Although it was not possible to sign the difference $e_2^N(t; 0) - e_2^N(t; 1)$, our numerical simulations show that we have $e_2^N(t; 0) > e_2^N(t; 1)$ in the beginning, with the inequality reversed when time approaches $T_0^N < T_1^N$ (see an illustration in Figure 2.4). When $\gamma = 0$, DCs reach, in a shorter time than when $\gamma = 1$, the point when they start to fully internalize the environmental externality, but they emit more during that period. Depending on which effect dominates the other, we end up in the positive or negative region of Δ_γ. Back to the different

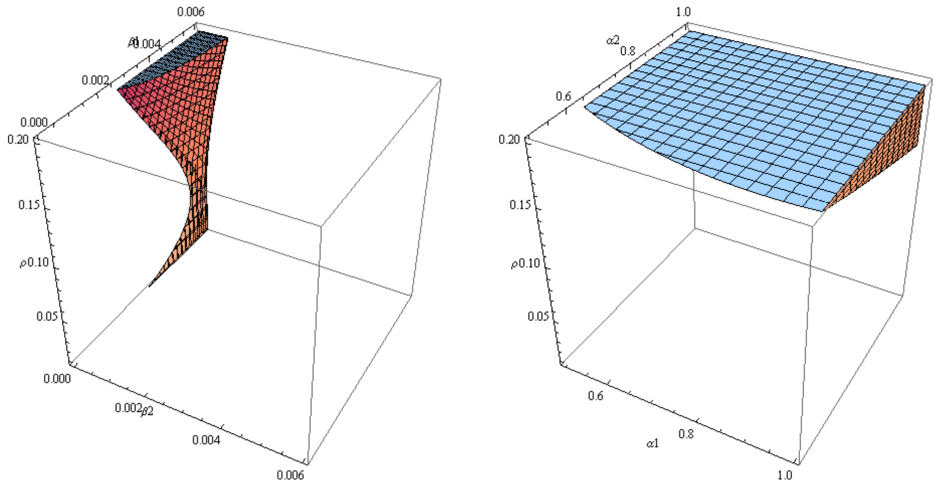

Figure 2.1: Parameter effects on the region with $\Delta_\gamma > 0$
Left panel effects of β_1, β_2 and ρ, right panel effects of α_1, α_2 and ρ, when $\delta = 0.003$, $\mu = 0.8, S_0 = 5$, and $2 < \hat{Y} < 8$ (depending on different parameter values on the figure).

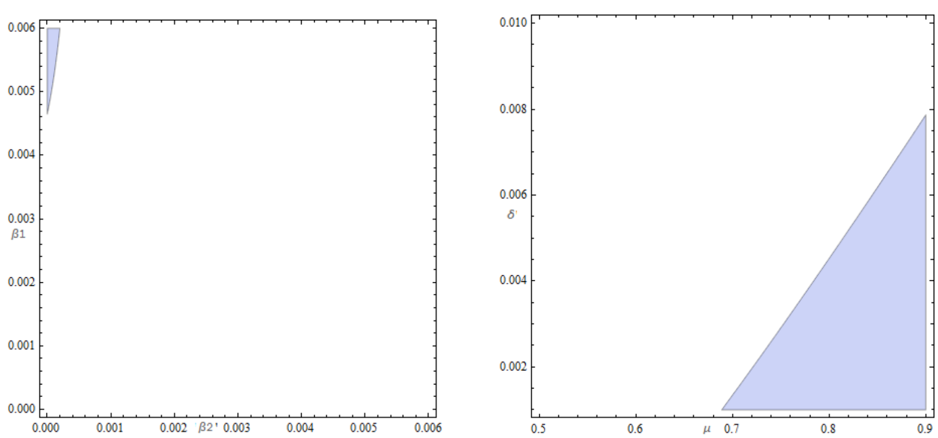

Figure 2.2: Other parameter effects on the region with $\Delta_\gamma > 0$
Left panel effects of β_1, β_2, right panel effects of μ and δ, when $\rho = .04, \delta = 0.003$, $\mu = 0.8, S_0 = 5, and 2 < \hat{Y} < 8$ (depending on different parameter values on the figure).

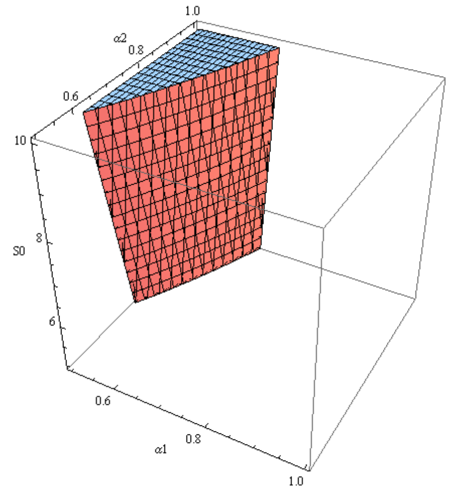

Figure 2.3: Effect of initial state, S_0, $\alpha_1$and α_2
The region with $\Delta_\gamma > 0$, when $\rho = .04, \delta = 0.003$, $\mu = 0.8, S_0 = 5{,}2 < \hat{Y} < 8$.

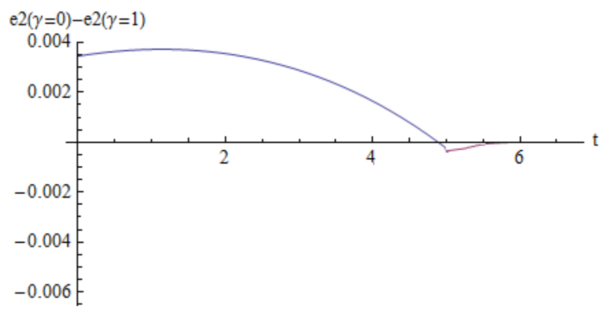

Figure 2.4: $e^N(t;0) - e^N(t,1)$
when $\beta_1 = \beta_2 = 0.003$, $\mu = 0.8$, $\rho = 0.1$, and $T_0^N = 5$.

cases identified above, it seems that when the revenue parameter α_2 is high, which may be seen as a cartoon representation of the situation in China or India, where high growth rates have been recorded for some time, then it is harder to promote much involvement in tackling environmental problems. The same can be said for low values of β_2. If the environmental externality is very small compared to the much-desired revenues, then again the case for dealing with pollution is harder to make.

To address our last research question, i.e., "Under which conditions is cooperation collectively better than noncooperation in the short run?" we compare the welfares that would be accumulated under the two regimes. As the dates T^C and T^N will not coincide in general, a first issue to deal with is the interval of time under which the comparison must be carried out. As we found (analytically) that $T^N < T^C$ for $\gamma = 0$, and that the same result also holds true (numerically) for $\gamma = 1$, it makes sense to retain the interval $\left[0, T^N\right]$ as a basis for comparing the total outcomes. After that date, by virtue of joint optimization, the sum of cooperative outcomes is higher than the sum of noncooperative outcomes on $[T^N, \infty)$. Of course, by the same principle, cooperation would also dominate noncooperation on $[0, \infty)$. This is to reiterate that the issue here is really the likely mode of play if the players only focus on short term gains. If cooperation can yield a higher dividend in this short-term, then ICs would possibly be able to offer side payments to DCs to reduce their emissions without too much delaying their economic development.

The difference in payoffs is given by

$$\begin{aligned}\triangle_W &= \left(\int_0^{T^N} e^{-\rho t}\left(f_1\left(e_1^C\right)+f_2\left(e_2^C\right)-D_1\left(S\right)-d_2\left(S^C,Y_2\right)\right)dt\right)- \\ &\quad \left(\int_0^{T^N} e^{-\rho t}\left(f_1\left(e_1^N\right)+f_2\left(e_2^N\right)-D_1\left(S\right)-d_2\left(S,Y_2\right)\right)dt\right).\end{aligned}$$

Substituting for the functional forms in the above equality, we get

$$\begin{aligned}\triangle_W &= \int_0^{T^N} e^{-\rho t}\left[\left(e_1^C-e_1^N\right)\left(\alpha_1-\frac{1}{2}\left[e_1^C+e_1^N\right]\right)+\left(e_2^C-e_2^N\right)\left(\alpha_2-\frac{1}{2}\left[e_2^C+e_2^N\right]\right)\right. \\ &\quad \left.+\left(\beta_1+\frac{t}{T^N}\gamma\beta_2\right)\left(S^N-S^C\right)\right]dt.\end{aligned}$$

The sign of $\triangle_W$ is ambiguous. Indeed, whereas the first term is always negative, the sign of the second term is ambiguous and the third term is positive. The negativity of the first term stems from our result that $\left(e_i^C-e_i^N\right)$ is negative, and the actual values of e_1^C and e_1^N, which are both less than α_1. Actually, the determination of the sign of $\triangle_W$ is very hard because the involved terms are highly non-linear in T^C and T^N, and these two dates are only given implicitly.

Figures 2.5 and 2.6 illustrate examples of regions in the parameter space where $\triangle_W$ is positive. These results, along with others not shown here,[5] seem to show that $\triangle_W$ would be positive if: (i) the time horizon of the first period T^N and the marginal damage cost β_1 of player 1 are large enough, while β_2 is small enough (Figure 2.5); (ii) the rate of discount, ρ is large enough (Figure 2.6). Note that increasing the absorption parameter δ very slightly enlarges the region where $\triangle_W$ is positive. Interestingly, these results are in line with those obtained for the characterization of the region where $\Delta_\gamma > 0$. However, importantly, varying the revenue parameters $\alpha_i, i=1,2$, and μ, does not seem to change the region where $\triangle_W$ is positive.

2.5 Conclusion

For many developing countries, urgent economic problems have relegated environmental issues to the background. However, neglecting the environment may not be a sustainable policy due to the revelation of environmental scarcity, citizens awareness about environmental problems, and improvements in economic conditions and welfare, which may lead to the introduction of environmental goods in the consumption basket of citizens. In other words, as time passes, new circumstances will quite naturally push policy makers of developing countries to consider the environment in their

[5]Results for any parameter values can be provided by the authors upon request.

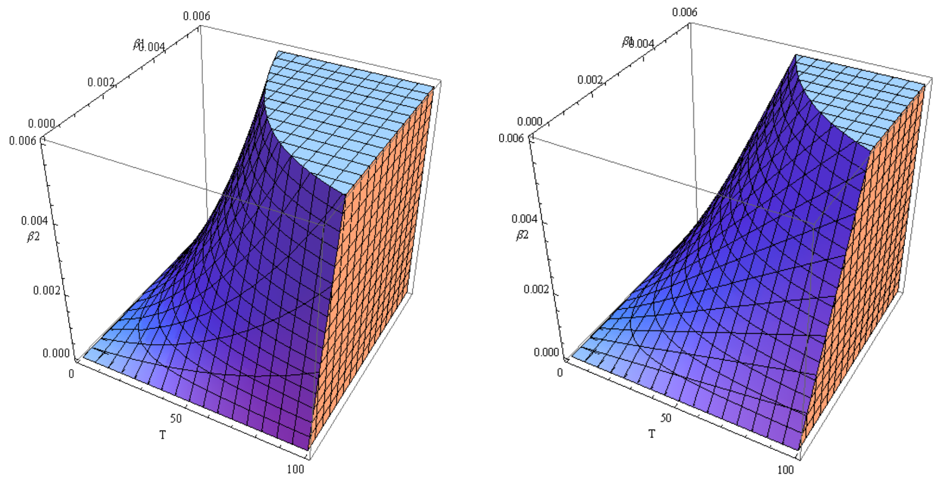

Figure 2.5: Effect of β_1, β_2 and T on the region where $\triangle_W > 0$
when $\delta = 0.002, \rho = 0.03, S_0 = 20, \mu = 0.8$. Left panel $\gamma = 0$, Right panel $\gamma = 1$.

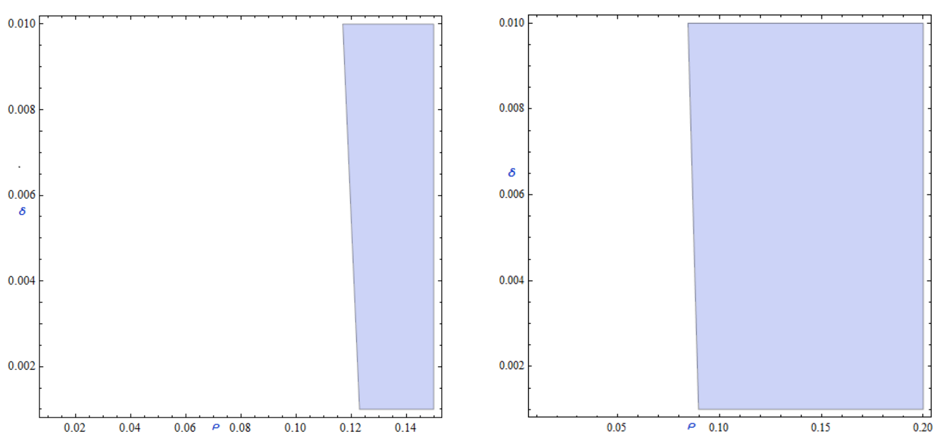

Figure 2.6: Effect of δ, ρ on the region where $\triangle_W > 0$
when, $\beta_1 = \beta_2 = 0.003, S_0 = 20, \mu = 0.8, T = 30$. Left panel $\gamma = 0$, Right panel $\gamma = 1$.

policies. To reflect this evolution in environmental concerns in an international pollution-control setup, we proposed an environmental-damage cost that is not only a function of the accumulated pollution but also a function of accumulated income and time (as proxies for level of awareness, infrastructures provision and welfare level).

The main contribution of this paper is about providing new insights in the environmental behavior of developing countries, and in clarifying the difficulties and challenges that may arise in achieving cooperation (or an international environmental agreement). We also showed that asking developing countries to take environmental cost into account sooner is not necessarily the best course of action. Finally, we characterized the conditions under which the point for cooperation is easier to be made.

As stated before, we see our analysis as an exploratory attempt to account for the complex idea of evolving environmental concerns in the most parsimonious setting. Many extensions can be envisioned in future work, including: (i) a more sophisticated modeling of the damage costs; (ii) a modeling approach where the mode of play can endogenously change from noncooperation to cooperation; and finally (iii) the inclusion of more players to account for different levels of development within the group of developing countries.

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2.7 Appendix

2.7.1 Proof of Proposition 2.1

To determine the feedback-Nash equilibrium, we proceed backward and solve the after-T^N problem, assuming that T^N is known. Denote by $v_i\left(S;T^N\right)$ the value function of player i for the game starting at T^N. This is an infinite-horizon game, and the Hamilton-Jacobi-Bellman (HJB) equation of player $i, i=1,2$, is given by

$$\rho v_i\left(S;T^N\right)=\max_{e_i}\{\alpha_i e_i-\frac{1}{2}e_i^2-\beta_i S+v_i'\left(S;T^N\right)\left(\mu(e_1+e_2)-\delta S\right)\}.$$

Given the linear-state structure of the game, we make the informed guess that the value functions have the form $v_i\left(S;T^N\right)=A_iS+B_i$, $i=1,2$. The HJB equations become

$$\rho(A_iS+B_i)=\max_{e_i}\{\alpha_i e_i-\frac{1}{2}e_i^2-\beta_i S+A_i(\mu(e_1+e_2)-\delta S)\},\quad i=1,2. \tag{2.10}$$

Assuming an interior solution, differentiating the right-hand side and equating to zero yields

$$e_i=\alpha_i+\mu A_i.$$

Substituting for e_i by its value in (2.10) leads to the following system with two equations and two unknowns:

$$\begin{aligned}\rho A_i &= -\beta_i-A_i\delta,\\ \rho B_i &= \frac{1}{2}\alpha_i^2+\frac{1}{2}\mu^2\left(-\frac{\beta_i}{\rho+\delta}\right)^2+\left(-\frac{\beta_i}{\rho+\delta}\right)\mu\left(\alpha_i+\alpha_j+\mu\left(-\frac{\beta_j}{\rho+\delta}\right)\right),\end{aligned}$$

which is equivalent to

$$
\begin{aligned}
A_i &= -\frac{\beta_i}{\rho+\delta}, \\
B_i &= \frac{1}{2\rho}\left[\alpha_i^2 + \mu^2 A_i^2 + 2A_i\mu\left(\alpha_i+\alpha_j+\mu A_j\right)\right].
\end{aligned}
$$

Consequently, the emissions strategies after T^N are given by

$$
e_i^{NT^N} = \alpha_i - \mu\frac{\beta_i}{\rho+\delta}, \quad i = 1, 2.
$$

The total value collected from T^N onward is given by

$$
v_i\left(S\left(T^N\right); T^N\right) = A_i S\left(T^N\right) + B_i,
$$

where $S\left(T^N\right)$ is the value of the pollution stock resulting from the implementation of equilibrium emissions strategies during the time interval $\left[0, T^N\right]$. These values are treated as salvage values in the overall optimization problems of the two players.

We now turn to finding the feedback-Nash equilibrium strategies for the before-T^N game. Denote by $v_i(t,S)$ the value function of player i, with the boundary conditions

$$
v_i\left(T^N, S\right) = A_i S\left(T^N\right) + B_i, \quad i = 1, 2. \tag{2.11}
$$

The HJB equations are given by

$$
\begin{aligned}
\rho v_1(t,S) - \frac{\partial v_1}{\partial t} &= \max_{e_1}\left\{\alpha_1 e_1 - \frac{1}{2}e_1^2 - \beta_1 S + \frac{\partial v_1}{\partial S}\left(\mu(e_1+e_2) - \delta S\right)\right\}, \\
\rho v_2(t,S) - \frac{\partial v_2}{\partial t} &= \max_{e_2}\left\{\alpha_2 e_2 - \frac{1}{2}e_2^2 - \frac{t}{T^N}\gamma\beta_2 S + \frac{\partial v_2}{\partial S}\left(\mu(e_1+e_2) - \delta S\right)\right\}.
\end{aligned}
$$

Assuming an interior solution, differentiating the right-hand sides of the above equations and equating to zero yields

$$
e_i = \alpha_i + \mu\frac{\partial v_i}{\partial S}, \quad i = 1, 2.
$$

Again, we make the informed guess that the value functions are linear, with the coefficients being time functions of time, that is,

$$
v_i(t,S) = x_i(t)S + y_i(t).
$$

Consequently, the emissions are given by

$$
e_i(t) = \alpha_i + \mu x_i(t).
$$

Substituting for $v_i(t,S)$ and e_i in the HJB equations leads to

$$\begin{aligned}
\rho\left(x_1(t)S+y_1(t)\right)-x_1'(t)S-y_1'(t) &= \frac{1}{2}\alpha_1^2-\frac{1}{2}\mu^2x_1^2(t)-\beta_1S \\
&\quad +x_1(t)\left[\mu(\alpha_1+\mu x_1(t)+\alpha_2+\mu x_2(t))-\delta S\right], \\
\rho\left(x_2(t)S+y_2(t)\right)-x_2'(t)S-y_2'(t) &= \frac{1}{2}\alpha_2^2-\frac{1}{2}\mu^2x_2^2(t)-\frac{\gamma t\beta_2}{T^N}S \\
&\quad +x_2(t)\left(\mu(\alpha_1+\mu x_1(t)+\alpha_2+\mu x_2(t)))-\delta S\right).
\end{aligned}$$

To obtain the coefficients of the value functions, we need to solve for the above differential equations. By equating coefficients in orders of S in both sides we have in total 8 unknowns ($x_i, y_i, i=1,2$, and constants corresponding to the differential equations) and 8 equations (4 equations related to HJB and 4 terminal conditions). By identification (equating coefficients in orders of S in each equation), we get the following equations for x_1 and x_2:

$$\begin{aligned}
x_1'(t) &= \beta_1+(\delta+\rho)x_1(t), \\
x_2'(t) &= \frac{\gamma t\beta_2}{T^N}+(\delta+\rho)x_2(t).
\end{aligned} \tag{2.12}$$

Taking into account the boundary conditions in (2.11), the solution to the first differential equation is given by

$$x_1(t)=-\frac{\beta_1}{\rho+\delta}.$$

The second equation is a heterogeneous linear first-order ODE of the general form

$$y'+p(x)y=Q(x).$$

The above differential equation has as its solution

$$ye^{\int p(x)dx}=\int Q(x)e^{\int p(x)dx}dx+C_1,$$

which in our case is given by

$$x_2(t)e^{-(\delta+\rho)t}=\int e^{-(\delta+\rho)t}\frac{\gamma t\beta}{T^N}dt+C_1.$$

Integrating by parts the first term on the right-hand side, we get

$$x_2(t)=-\frac{\gamma\frac{\beta_2}{T^N}\left(1+t(\rho+\delta)\right)}{(\rho+\delta)^2}+e^{(\rho+\delta)t}C_2.$$

The boundary condition $v_2\left(T^N,S\right)=v_2\left(S;T^N\right)$ is equivalent to

$$A_2S+B_2=x_2\left(T^N\right)S+y_2\left(T^N\right).$$

Equating coefficients in orders of S will give us the following terminal conditions:

$$\begin{aligned} x_2\left(T^N\right) &= A_2 = -\frac{\beta_2}{\delta+\rho}, \\ B_2 &= y_2\left(T^N\right), \end{aligned}$$

i.e.,

$$-\frac{\beta_2}{\rho+\delta} = -\frac{\gamma\frac{\beta_2}{T^N}\left(1+T^N\left(\rho+\delta\right)\right)}{\left(\rho+\delta\right)^2} + e^{(\rho+\delta)T^N}C_2, \tag{2.13}$$

$$\Rightarrow C_2 = \frac{\frac{\beta_2}{T^N}}{e^{T(\delta+\rho)}}\left(\frac{T\left(\delta+\rho\right)\left(\gamma-1\right)+\gamma}{\left(\delta+\rho\right)^2}\right),$$

$$\Rightarrow x_2\left(t\right) = -\frac{\gamma\frac{\beta_2}{T^N}\left(1+t\left(\rho+\delta\right)\right)}{\left(\rho+\delta\right)^2} + e^{(\rho+\delta)(t-T)}\frac{\beta_2}{T^N}\left(\frac{T\left(\delta+\rho\right)\left(\gamma-1\right)+\gamma}{\left(\delta+\rho\right)^2}\right). \tag{2.14}$$

By reordering

$$\begin{aligned} x_2\left(t\right) = \frac{\beta_2}{T^N\left(\rho+\delta\right)^2}\Bigg[&-\gamma\left(1+t\left(\rho+\delta\right)-e^{(\rho+\delta)\left(t-T^N\right)}\right) \\ &+ e^{(\rho+\delta)\left(t-T^N\right)}\left(T^N\left(\rho+\delta\right)\left(\gamma-1\right)\right)\Bigg]. \end{aligned}$$

Consequently, we have the following emissions:

$$\begin{aligned} e_1^N\left(t\right) &= \alpha_1 - \mu\frac{\beta_1}{\rho+\delta}, \\ e_2^N\left(t\right) &= \alpha_2 + \frac{\mu}{\left(\rho+\delta\right)^2}\frac{\beta_2}{T^N}\Bigg(-\gamma\left(1+t\left(\rho+\delta\right)-e^{(\rho+\delta)\left(t-T^N\right)}\right) \\ & *7cm + e^{(\rho+\delta)\left(t-T^N\right)}T^N\left(\rho+\delta\right)\left(\gamma-1\right)\Bigg). \end{aligned}$$

As the expressions for $y_i\left(t;\gamma=1\right)$ are extremely long and do not add any useful information, we omit them.

2.7.2 Proof of Proposition 2.2

We follow the same steps as for the proof of Proposition 1, namely, we first assume that we know the date T^C at which player 2 starts being fully environmentally concerned and solve the after-T^C optimization problem. Second, we solve the overall optimization problem, using the result of the first step as a salvage value.

The after-T^C optimization problem is the following:

$$\begin{aligned}
&\max_{e_1,e_2}\int_{T^C}^{\infty}(\alpha_1 e_1-\frac{1}{2}e_1^2-\beta_1 S)e^{-\rho t}dt+\int_{T^C}^{\infty}(\alpha_2 e_2-\frac{1}{2}e_2^2-\beta_2 S)e^{-\rho t}dt,\\
\text{subject to}\quad :\quad &\dot{S}(t)=\mu(e_1(t)+e_2(t))-\delta S(t), \qquad S\left(T^C\right)=S^{T^C}.
\end{aligned}$$

Denoting by $v\left(S,T^C\right)$ the value function, the HJB equation is given by

$$\begin{aligned}
\rho v\left(S;T^C\right)=\max_{e_1,e_2}\Bigg\{&\alpha_1 e_1-\frac{1}{2}e_1^2-\beta_1 S+\alpha_2 e_2-\frac{1}{2}e_2^2-\beta_2 S\\
&+v'\left(S;T^C\right)\left(\mu\left(e_1+e_2\right)-\delta S\right)\Bigg\}. \qquad (2.15)
\end{aligned}$$

Assuming an interior solution, maximizing the right-hand side yields

$$e_i=\alpha_i+\mu v'\left(S;T^C\right).$$

We make the informed guess that the value function is linear and given by

$$v\left(S;T^C\right)=KS+L,$$

and hence obtain $e_i=\alpha_i+\mu K$. Substituting for e_i and $v\left(S;T^C\right)$ in the HJB equation, arranging terms and using the identification method leads to the following values for the coefficients of $v\left(S;T^C\right)$:

$$\begin{aligned}
K&=-\frac{\beta_1+\beta_2}{\rho+\delta},\\
L&=\frac{1}{2\rho}\left(\alpha_1^2+2\mu^2K^2+\alpha_2^2+2K\mu\left(\alpha_1+\alpha_2\right)\right).
\end{aligned}$$

For the before-T^C instant of time, the optimization problem is as follows:

$$\max_{e_1,e_2}\left\{\int_0^{T^C}\left(\alpha_1 e_1-\frac{1}{2}e_1^2-\beta_1 S+\alpha_2 e_2-\frac{1}{2}e_2^2-\gamma\frac{t}{T^C}\beta_2 S\right)e^{-\rho t}dt+e^{-\rho T^C}\left(KS+L\right)\right\},$$

subject to the dynamics of the pollution stock. The HJB equation is given by

$$\begin{aligned}
\rho v\left(S;t\right)-\frac{\partial v\left(S;t\right)}{\partial t}=\max_{e_1,e_2}\Bigg\{&\alpha_1 e_1-\frac{1}{2}e_1^2-\beta_1 S+\alpha_2 e_2-\frac{1}{2}e_2^2-\gamma\frac{t}{T}\beta_2 S\\
&+\frac{\partial v\left(S;t\right)}{\partial S}\left(\mu\left(e_1+e_2\right)-\delta S\right)\Bigg\}. \qquad (2.16)
\end{aligned}$$

Again, we make the informed guess that the value function is linear in the state variables and given by

$$v\left(S;t\right)=w\left(t\right)S+z\left(t\right).$$

Assuming an interior solution, the first-order optimality conditions give

$$e_i = \alpha_i + \mu w(t), \quad i = 1, 2. \tag{2.17}$$

Substituting for e_i and for $v(S;t)$ in (2.16), yields the following differential equation:

$$\rho w(t) - w'(t) = -\beta_1 - \gamma \frac{t}{T^C}\beta_2 - \delta w(t),$$

with the terminal condition of $w(T^C) = K = -\frac{\beta_1+\beta_2}{\rho+\delta}$. The above differential equation has the same form as differential equation 2.12 and so has as its solution

$$w(t) = -\frac{\beta_1}{\rho+\delta} - \left(\frac{\beta_2}{T^C(\rho+\delta)^2}\right)\left(\gamma(1+t(\rho+\delta)) - e^{(\rho+\delta)(t-T^C)}\left(T^C(\delta+\rho)(\gamma-1)+\gamma\right)\right).$$

As in the non-cooperative case, the expression of $z(t)$ is extremely long and there is no need to print it. Inserting $w(t)$ in (2.17) yields the emissions.

2.7.3 Proof of Proposition 2.5

If $\min\{T^N, T^C\} = T^C$ then $e^{(\rho+\delta)(t-T^N)} - e^{(\rho+\delta)(t-T^C)} < 0$ so we always have $\frac{\beta_1}{\beta_2} > e^{(\rho+\delta)(t-T^N)} - e^{(\rho+\delta)(t-T^C)}$ i.e. $e_2^N(t)$ is always larger than $e_2^C(t)$.

Recall that T^N and T^C satisfy

$$\begin{aligned}
\int_0^{T^C}\left(\alpha_2 e_2^C - \frac{1}{2}(e_2^C)^2\right)e^{-\rho t}dt &= \bar{Y}_2,\\
\int_0^{T^N}\left(\alpha_2 e_2^N - \frac{1}{2}(e_2^N)^2\right)e^{-\rho t}dt &= \bar{Y}_2,
\end{aligned}$$

that is

$$\int_0^{T^N}\left(\alpha_2 e_2^N - \frac{1}{2}(e_2^N)^2\right)e^{-\rho t}dt - \int_0^{T^C}\left(\alpha_2 e_2^C - \frac{1}{2}(e_2^C)^2\right)e^{-\rho t}dt = 0.$$

The above condition can then be rewritten as

$$\int_0^{T^C}\left(\alpha_2 e_2^N - \frac{1}{2}(e_2^N)^2\right)e^{-\rho t}dt + \int_{T^C}^{T^N}\left(\alpha_2 e_2^N - \frac{1}{2}(e_2^N)^2\right)e^{-\rho t}dt \\ - \int_0^{T^C}\left(\alpha_2 e_2^C - \frac{1}{2}(e_2^C)^2\right)e^{-\rho t}dt = 0,$$

which is equivalent to

$$\int_0^{T^C}\left[\left(e_2^N - e_2^C\right)\left(\alpha_2 + \frac{1}{2}\left(e_2^N + e_2^C\right)\right)\right]e^{-\rho t}dt + \int_{T^C}^{T^N}\left(\alpha_2 e_2^N - \frac{1}{2}(e_2^N)^2\right)e^{-\rho t}dt = 0.$$

The second integral is strictly positive by our assumption that $f_i(e_i)$ is strictly positive for all feasible e_i. Clearly, the above equality cannot hold true if $e_2^N(t) - e_2^C(t) > 0, \forall\, t \in \left[0, \min\left\{T^N, T^C\right\}\right]$. Therefore, we have a contradiction and necessarily, $T^N < T^C$.

Chapter 3

International Emissions, Uncertainty and Learning

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Abstract

We introduce learning in an N-player dynamic game of international pollution, with ecological uncertainty. We find and compare the feedback non-cooperative strategies of players under three different information assumptions, namely, informed player, as our benchmark; anticipated learner; and adaptive learner. We find out that while uncertainty due to anticipation of learning ends up to a decrease in total emissions, depending on functional form of distributions and beliefs, the effect of structural uncertainty could be either an increase, decrease, or even no change in the emissions of individual players and also total emissions. Moreover, we find out that if a player's beliefs change toward more optimistic views or he feels that the situation is less risky, he will increase his emissions while others will react to this change and decrease their emissions but the latter effect never overcomes the former and, as a result, total emissions will increase.

Keywords: International Pollution, N-Player Dynamic Games, Uncertainty, Learning

3.1 Introduction

It is widely accepted that greenhouse gas concentrations are responsible for global climate change. But, when it comes to tackling with this problem there is a great deal of uncertainty from many different sources. The typical examples of uncertainty sources are our lack of information about or the randomness in: the exact accumulation of pollutions; the abilities of nature to decay the pollutions; the exact effect of greenhouse gases on climate change; the exact change in degree of temperature which we may face in the future; the exact effects of these changes in economy; and even the development and efficiency improvements in our either production or abatement technologies and processes (for a survey see Pindyck (2007)). In other words, uncertainty is the indivisible part of environmental studies. Because of that in the environmental literature we could find a great number of studies which have addressed some sort of uncertainty: e.g. Pindyck (2000, 2007); Yeung and Petrosyan (2008); de Zeeuw and Zemel (2012).

But planners are not indifferent against uncertainty and lack of information. They always seek for new information and consider that in their beliefs about unknown characteristics of the problem they are tackling with, in other words, in real life in face of uncertainty, planners are in a learning process. This is not also hidden from the sight of economics researchers: see e.g. Cogley and Sargent (2008); Guidolin and Timmermann (2007); Koulovatianos et al. (2009); Agbo (2011) But in environment economics literature there are a few researches which partly addressed learning for instance Ulph and Maddison (1997) provide a two-period, two-player game of emissions, while damages in period one could have S possible states with given probabilities.

In this paper, we are focusing on an international pollution problem, which some neighboring countries emit a pollutant, e.g. CO2, and the accumulation of this pollutant damages the environment of these countries, while exact evolution of pollutions is not known. We call this situation ecological uncertainty since it happens due to lack of full information about one of ecological parameters of the model. Knowing that these countries behave strategically and dynamic nature of the problem, we adapt an N-player dynamic game setup. In this respect the third group of literature that this study belongs to is dynamic and differential games in international environmental management and economics: e.g. Dockner and Long (1993); Long (1992) for a survey see: Jørgensen et al. (2010). Taking into account the importance of uncertainty and learning in environmental studies, we are seeking for the answer to the following main research questions:

1. Whether uncertainty "alleviate" emissions problem? In order to answer to this question we will refer to two sub-questions as following:

    (a) In the presence of uncertainty, how emission strategies of players compare under different assumptions about their information and learning? In other words, how different sources of uncertainty affect the emissions strategies?

    (b) How changes in beliefs, e.g. more pessimistic or optimistic beliefs and feeling more risk, affect emissions strategies?

Based on our assumption about players' information about unknown parameters of the model, we can have two general groups. The first group are those who know all parameter values. Following Levhari and Mirman (1980) we call these players full information or informed players.[1] This assumption is followed in the standard literature related to uncertainty, see e.g. Bramoullé and Treich (2009); de Zeeuw and Zemel (2012); Arrow and Fisher (1974); Pindyck (2012). Another possibility is that the players not only are dealing with an uncertain variable in, but also do not know the exact values of some of the parameters of the model. In such cases the players make their own beliefs about unknown variables according to the available information. Besides, whenever they receive new information they learn about these parameters and update these beliefs.

A closely related paper to our is Breton and Sbragia (2011), they introduce uncertainty and learning in a model of international environmental agreements (IEAs). The authors model uncertainty and learning for the case that the stochastic variable is distributed normally. They find the emission rule's and size of stable IEAs numerically. They present their result under myopic assumption which is equivalent with our assumption for an informed player.

We analytically find and compare the feedback non-cooperative strategies of players under these information assumptions. One important property of this research is that following the approach presented by Koulovatianos et al. (2009)[2] we present all of our analytical results for any general probability distribution function in which the information can be summarized by a finite-dimensional vector of sufficient statistics.

Our main results can be summarized as following:

1. Uncertainty due to anticipation of learning ends up to a decrease in total emissions, i.e., it alleviates the pollutions problem

2. Depending on functional form of distributions and beliefs, the effect of structural uncertainty could be either an increase, decrease, or even no change in the emissions of individual players and also total emissions. We even present the conditions that under that a player emits more when he faces with uncertainty due to anticipation than if he doesn't. While the first observation is totally in line with the available literature the latter results are rather new and more controversial.

3. Under learning assumption, if one player changes his beliefs in a way that he feels more optimistic or less risky, while others beliefs have not changed, this player will increase his emissions and others react to this change by decreasing their emissions but, the increase in the emissions of the mentioned player will overcompensate others and in total emissions will increase. So pessimistic views or a sense of riskiness also may alleviate the pollution problem.

In the following, we first present the model setup, we proceed with introducing ecological uncertainty and then provide the non-cooperative feed-back strategies of players for three different cases:

[1] Cogley and Sargent (2008) use the term of "no learning" or "infinite-history rational expectations" for agents whom do not update their beliefs.

[2] Agbo (2011) used this approach in a game theoretic setup for the first time.

informed player; learner player; and adaptive player. After that we present comparative statics and compare these different cases. Then we will see how changes in players beliefs (getting more optimistic\pessimistic) affect their strategies. To complete our analysis, we provide the details related to effects of changes in feels about the riskiness of the situation. Finally, we wrap up with the summary and discussion.

3.2 The Model

We consider N countries indexed by $i = 1, ..., N$. Each country generates revenue by production. Production activities have a byproduct which is the emissions of a pollutant, e.g., CO2. These emissions accumulate over time and damage the environment. Thus, the model consist of three components: revenue function of each player (hereafter we use the term "player" instead of country); damage-cost functions; and dynamics of the system.

To simplify, we assume that one unit of production yields one unit of pollution. By this assumption we may define the revenue as only a function of emissions. Following the related literature, see for instance Dockner and Long (1993); Long (1992), we adapt a quadratic revenue function. Hence, player i's revenue at time t, denoted by $y_{i,t}$, is

$$y_{i,t} = \alpha_i e_{i,t} - (e_{i,t})^2 - \gamma e_{i,t} \sum_{j \neq i}^{N} e_{j,t},$$

where α and γ are constants and $i = 1, 2, .., N$, $t = 1, ..., \infty$.[3]

In addition to generating revenue, accumulation of emitted pollutants imposes environmental costs to all players. Let S_t be the stock of pollution at time t. We specify player $i's$ damage cost, denoted by $D_{i,t}$, as a linear function of the stock of pollution as following

$$D_{i,t} (S_t) = \beta_i S_t,$$

where β_i is the marginal cost of the pollution stock. The linearity assumption here is an approximation which we adopt for parsimony and simplicity of the model.[4]

The last component of the model that we need to specify is the dynamics of the pollution.

[3] In the literature it is common to ignore the mutual effect of the production of neighbors on their incomes, i.e., it is more common to put $\gamma = 0$. In such a case the only connection among these countries is their environment which is affected by the accumulation of all countries emissions. In this paper we are interested in analyzing the effects of heterogeneity among players, and also their mutual effects. In order to capture this we assume that profit of countries are interrelated and is not only a function of their own production, but also a function of all other countries' production, i.e., $\gamma \neq 0$.

[4] It is possible to be somewhat less restrictive and consider a non-linear damage cost, but only at a great expense of clarity of the results. Nevertheless, the assumption of linear environmental cost is not uncommon in the literature (See: Hoel and Schneider (1997); Breton and Sbragia (2011), for instance).

Pollution accumulates over time according to the following endogenous law of motion

$$S_{t+1} = \eta \left(\sum e_{i,t} + dS_t \right), \tag{3.1}$$

where $0 < d < 1$, such that $1 - d$ is the natural degradation rate of pollution and η is a shock variable. In the literature, η is not considered, which would be equivalent to setting its value to a constant in equation (3.1), see, e.g., Nordhaus (1991); Dockner and Long (1993). In our model, we introduce variable η in order to embed ecological uncertainty. Ecological uncertainty is one of the main sources of uncertainty in emissions control problems which arises due to lack of information about nature's abilities to absorb and decay emissions and their accumulation. Indeed, this is natural for players to be unsure about the precise motion of pollution since from one hand side it depends on highly stochastic factors like the weather condition (e.g., speed and direction of wind, temperature, humidity, etc.), and on the other side our knowledge about capacity of natural sinks of GHGs is not complete. We can even think of this uncertainty due to the possible changes in the future mitigation technologies. The variable η in equation (3.1) represents all of these random effects in our model. We will talk about this later with more details.

Now we have all elements to present player i's problem. Each player sets his own optimal emissions rule, e_i, in a way that maximizes his own expected entire life welfare, i.e., player i's problem, $i = 1, ..., N$, is given by

$$\begin{gathered} \max_{e_i} E\left[\sum_{t=0}^{\infty} \delta^t \left(\alpha_i e_{i,t} - \tfrac{1}{2}(e_{i,t})^2 - \beta_i S\right)\right], \\ s.t.(3.1), \end{gathered}$$

where δ is a common discount factor and E presents expectations operator with respect to all stochastic variables of the model.

Before presenting the solution and equilibrium concepts we need to know how much players know about the variable η, i.e., we need to clarify the information structure in the model. In our model we assume that η is a realization of a random variable, namely, $\tilde{\eta}$. We assume that the conditional probability distribution of this variable, $\tilde{\eta}$, is given by $\phi(\eta|\theta^*)$, where θ^*, $\theta^* \in \Theta \subset \mathbb{R}^k$, is the vector of sufficient parameters of probability distribution function ϕ. Let the support of this p.d.f be given by $\mathcal{H}$, such that $\eta \in \mathcal{H} \subseteq [0, 1]$. In the standard uncertainty literature without learning it is assumed that all functional forms and parameters are known by players thus, no learning happens. This is the simplest assumption we can have and following the literature we call these players informed players and we use it as our benchmark.

On the contrary, players may not know the exact value of the parameter θ^* and only have a belief about the possible values of that, let say θ. In this situation they make their own beliefs about unknown parameters according to the available information. In this situation whenever they receive new information they update their beliefs.

In the latter case, based on whether players are able to anticipate their learning in the future or not we can recognize two different cases. In case of (anticipated) learning, players update their beliefs by receiving new information and they anticipate their learning. The alternative case happens when players learn about unknown parameter whenever they receive new information but they do not anticipate their learning in the future, i.e., they assume that this is the last time they learn. We may say that such a player is bounded rational. This case is known as adaptive learning in the literature (see, e.g. Koulovatianos et al. (2009); Agbo (2011)).[5] An adaptive learner assumption may not seem appealing since this player is myopic in the sense that can not anticipate his learning in the future. But in the literature there are many reasons for using and introducing adaptive learning. For instance, adaptive learning could be used as an approximation for the more sophisticated case of learning, see e.g., Cogley and Sargent (2008). The other usefulness of this is that adaptive assumption is an intermediate case that gives us the opportunity to decompose the effects of structural uncertainty and uncertainty due to anticipation of learning. This latter characteristic of the adaptive learning assumption motivated us to consider it in this research. In the following we present the Cournot-Nash equilibrium for our dynamic game under our three different possible information assumption.

3.3 Cournot-Nash Equilibrium

We consider behavior of players under dynamic Cournot-Nash equilibrium concept. That is each player chooses his emissions ($e_{i,t}$) in a way that maximizes his own expected discounted whole-life welfare given all other players' behavior and reaction, and subject to the ecological restrictions.

As mentioned in this paper we are interested to study the effects of uncertainty and learning on emissions of players. In order to be able to decompose the effects of different sources of uncertainty, we present our results under three different assumptions about player's information which is introduced in the previous section. In the benchmark case we characterize the behavior of players and have the results for the informed players. In other words, in the benchmark we assume that players know the distribution of $\tilde{\eta}$ and the actual value of θ, that is θ^*, thus no learning happens. Then we proceed with introducing learning. Afterward, we present the intermediate case, i.e., adaptive learner and then we will have the more sophisticated case which is the (anticipated) learner case.

3.3.1 Full information

An informed decision maker knows θ^* and he never changes his beliefs during the planning horizon. In other words despite stochasticity of η, there is no structural uncertainty in this situation. For tractability reasons we assume that $\gamma = 1$. Thus, player i's problem would be as following

$$\max_{e_{i,t}} \sum_{t=0}^{\infty} \delta^t \int_{\mathcal{H}} \left(\alpha_i e_{i,t} - (e_{i,t})^2 - e_i \sum_{j\neq i}^{N} e_j - \beta_i S \right) \phi(\eta|\theta^*) d\eta, \tag{3.2}$$

[5] Anticipated utility is another term which is sometimes used for the adaptive learner; see e.g. Cogley and Sargent (2008). This term is adapted from Kreps (1998). In some other literature it is called a myopic player.

subject to (3.1). Thus, assuming a feed-back setup, each country should choose his own emissions in a way that his value function, $v_i^I(S;\theta^*)$, satisfies the following Bellman equation

$$v_i^I(S;\theta^*) = \max_{e_i} \left\{ \begin{array}{c} \alpha_i e_i - (e_i)^2 - e_i \sum_{j\neq i}^N e_j - \beta_i S + \\ \delta \int_{\mathcal{H}} \left[v_i^I(\eta(\sum_i e_i + d)S)); \theta^* \right] \phi(\eta|\theta^*) d\eta \end{array} \right\},$$

where superscript I refers to informed player and $v_i^I(S;\theta^*)$ is the value function of player i, $i \in \{1,...,N\}$. Proposition 3.1 presents the feed-back strategy of informed player i, denoted by $e_i^I(S;\theta^*)$.

Proposition 3.1 *The feedback-Nash equilibrium emissions of informed player i, $i \in \{1,...,N\}$ is given by*

$$e_i^I(S;\theta^*) = \frac{1}{N+1}\left(N\alpha_i - \sum_{j\neq i}\alpha_j - \delta\frac{\mu(\theta^*)}{1-\delta d\mu(\theta^*)}\left(N\beta_i - \sum_{j\neq i}\beta_j \right)\right),$$

where $\mu(\theta^*) = \int_{\mathcal{H}} \eta\phi(\eta|\theta^*)d\eta$.

Proof. See Appendix 3.8.1. ■

3.3.2 Learning

The alternative assumption about players' information is that they do not know θ^*. In this case the players are facing a form of structural uncertainty that is not knowing one of the parameters of the model. However, we assume that based on available information, each player has his own prior beliefs, denoted by $\xi_i(\theta)$, about possible values of θ^*, denoted by θ. To make our results more general, we let the prior beliefs of players be heterogeneous but we assume these beliefs are common knowledge.

We assume that players only observe the actual value of η at the beginning of each period and by this new information, each one updates his own beliefs about the value of θ. We assume that these players are Bayesian learners, so they update their beliefs according to the Bayes rule, i.e.,

$$\hat{\xi}_i(\theta|\eta) = \frac{\phi(\eta|\theta)\xi_i(\theta_t)}{\int_\phi \phi(\eta|x)\xi_i(x)dx}. \tag{3.3}$$

We already have explained that in case of learning players might anticipate their learning and be learners or may not and be adaptive learner.

3.3.2.1 learner player

The difference between a learner player and an informed one is that this player's decision is affected by the beliefs. Thus, each player chooses his strategy, e_i^L, in a way that, his value function, $v_i^L(S;\xi)$, $\forall i = 1,..N$, satisfies the learner Bellman equation as following:

$$v_i^L(S;\xi) = \max_{e_i} \left\{ \begin{array}{c} \alpha_i e_i - (e_i)^2 - e_i \sum_{j\neq i}^N e_j - \beta_i S + \\ \delta \int_{\mathcal{H}} \left(\begin{array}{c} v_i^L\left(\eta\left(+e_i + d\right)S\right); \hat{\xi}_i(\theta|\eta)\right) \times \\ \left[\int_\Theta \phi(\eta|\theta)\xi_i(\theta)d\theta\right] \end{array} \right) d\eta \end{array} \right\}, \tag{3.4}$$

where superscript L refers to learner player. Proposition 3.2 provides formally the Nash non-cooperative equilibrium strategy of learner player i.

Proposition 3.2 *The feedback-Nash equilibrium emissions of learner player i, $i \in \{1, ..., N\}$ is given by:*

$$e_i^L = \frac{1}{N+1}\left(N\alpha_i - \sum_{j\neq i}\alpha_j - \delta\left(\begin{array}{c} N\beta_i \int_\Theta \frac{\mu(\theta)}{1-\delta d\mu(\theta)}\xi_i(\theta)d\theta - \\ \sum_{j\neq i}\beta_j \int_\Theta \frac{\mu(\theta)}{1-\delta d\mu(\theta)}\xi_j(\theta)d\theta \end{array}\right)\right),$$

where $\mu(\theta) = \int_{\mathcal{H}} \eta\phi(\eta|\theta)d\eta$.

Proof. See Appendix 3.8.2. ■

Notice that in this case each player faces with two more sources of uncertainties in compare to an informed player: (i) structural uncertainty or the uncertainty caused by not knowing θ^*; and (ii) uncertainty caused by anticipation of learning. As mentioned in introduction, we can use the adaptive learner concept which is an intermediate case to decompose these two sources. Indeed, since adaptive learner learns but does not anticipate his learning, he only faces with structural uncertainty and not the uncertainty related to anticipation of learning.

3.3.2.2 Adaptive learner

An adaptive learner learns about θ and updates his beliefs whenever he receives new information, but unlike a learner player, he does not anticipate his learning. In other words, he uses his today's beliefs to assess expected future payoffs. Thus, he chooses his strategy, e_i^A, in a way that his value function, v_i^A, satisfies the following Bellman equation:

$$v_i^A(S;\xi) = \max_{e_i} \left\{ \begin{array}{c} \alpha_i e_i - (e_i)^2 - e_i \sum_{j\neq i}^N e_j - \beta_i S + \\ \delta \int_{\mathcal{H}} v_i^A\left(\eta\left(\sum_i e_i + d\right)S\right); \xi_i(\theta|\eta)\right) \left[\int_\Theta \phi(\eta|\theta)\xi_i(\theta)d\theta\right] d\eta \end{array} \right\}, \tag{3.5}$$

where superscript A refers to the adaptive learner. Notice that the difference between equation (3.4) and equation (3.5) is in the beliefs they use to find the expected value of the future payoffs. As a learner uses his updated beliefs, that is $\hat{\xi}_i$, but an adaptive one uses his today beliefs or ξ_i. Proposition 3.3 presents the strategy of players when they are adaptive learners and play non-cooperatively.

Proposition 3.3 *The feedback-Nash equilibrium emissions of adaptive learner player i, $i \in \{1, ..., N\}$ is given by:*

$$e_i^A = \frac{1}{N+1}\left(\begin{array}{c} N\alpha_i - \sum_{j\neq i}\alpha_j \\ -\delta\left(N\beta_i \frac{\int_\Theta \xi_i(\theta)\mu(\theta)d\theta}{1-\delta d\int_\Theta \xi_i(\theta)\mu(\theta)d\theta} - \sum_{j\neq i}\beta_j \frac{\int_\Theta \xi_j(\theta)\mu(\theta)d\theta}{1-\delta d\int_\Theta \xi_j(\theta)\mu(\theta)d\theta}\right)\end{array}\right), \tag{3.6}$$

Proof. See Appendix 3.8.3. ■

3.4 Comparative statics

Before proceeding to the comparisons, let's have some simplifying definitions. Let's $R(x) = x(1-\delta dx)^{-1}$. So we can rewrite the strategy of player i for different cases as following:

$$e_i^L = \frac{1}{N+1}\left(\begin{array}{c} N\alpha_i - \sum_{j\neq i}\alpha_j - .. \\ ..\delta\left(N\beta_i \int_\Theta R(\mu(\theta))\xi_i(\theta)d\theta - \sum_{j\neq i}\beta_j \int_\Theta R(\mu(\theta))\xi_j(\theta)d\theta\right)\end{array}\right), \tag{3.7}$$

$$e_i^A = \frac{1}{N+1}\left(\begin{array}{c} N\alpha_i - \sum_{j\neq i}\alpha_j \\ -\delta\left(N\beta_i R\left(\int_\Theta \xi_i(\theta)\mu(\theta)\,d\theta\right) - \sum_{j\neq i}\beta_j R\left(\int_\Theta \xi_j(\theta)\mu(\theta)\,d\theta\right)\right)\end{array}\right), \tag{3.8}$$

$$e_i^I = \frac{1}{N+1}\left(N\alpha_i - \sum_{j\neq i}\alpha_j - \delta R(\mu(\theta^*))\left(N\beta_i - \sum_{j\neq i}\beta_j\right)\right). \tag{3.9}$$

Given heterogeneity in our model, we can not present a general statement to compare the strategies under learner and adaptive learner assumption for each individual. Proposition 3.4 clarifies how an increase in uncertainty, due to anticipation of learning, affects emissions under homogeneity assumption. Proposition 3.5 also provides the results under general heterogeneity assumption for total emissions.

Proposition 3.4 *If players are homogenous with regard to their beliefs and the marginal cost of pollution, i.e., $\xi_i(\theta) = \xi(\theta)$ and $\beta_i = \beta$, $\forall i = 1, ..., N$, then a learner player emits less in compare to an adaptive learner player, i.e. $\forall i \in \{1, .., N\}$*

$$e_i^L(S,\xi) < e_i^A(S,\xi).$$

Proof. If all $\beta_i = \beta$ and $\xi_i(\theta) = \xi(\theta)$, $\forall i = 1, ..., N$, then
$e_i^L = \frac{1}{N+1}\left(N\alpha_i - \sum_{j\neq i}\alpha_j - \delta\beta\left(\int_\Theta R(\mu(\theta))\xi(\theta)d\theta\right)\right)$ and
$e_i^A(S;\xi_i) = \frac{1}{N+1}\left(N\alpha_i - \sum_{j\neq i}\alpha_j - \delta\beta\left(R\left(\int_\Theta \xi(\theta)\mu(\theta)\,d\theta\right)\right)\right)$.

Since $0 < \delta, d < 1$, and $0 < \mu(\theta) < 1$, then R is an increasing convex function for acceptable values of our model parameters ($R' > 0$, $R" > 0$, $\forall x \in [0,1]$), so by Jensen's inequality we have

$$\int_\Theta R(\mu(\theta))\xi(\theta)d\theta > R\left(\int_\Theta \xi(\theta)\mu(\theta)\,d\theta\right),$$

i.e.,

$$e_i^L(S,\xi) < e_i^A(S,\xi).$$

■

Proposition 3.5 *In total, players will emit less if they are learner players in compare to being adaptive players, i.e.,*

$$\sum_{i=1}^{N} e_i^L (S,\xi) < \sum_{i=1}^{N} e_i^A (S,\xi).$$

Proof. See Appendix 3.8.4. ■

Propositions 3.4 and 3.5 are always true. In other words generated uncertainty by anticipation always decreases total emissions. We may interpret it as an increase in precautionary behavior which leads to less emissions. But what would be the behavior of individuals if they are heterogeneous? Or is it possible to have players which emit more under learner assumption than adaptive assumption? To answer this question given equilibrium emissions (equations 3.7 and 3.8) we have $e_i^L (S,\xi) > e_i^A (S,\xi)$ if

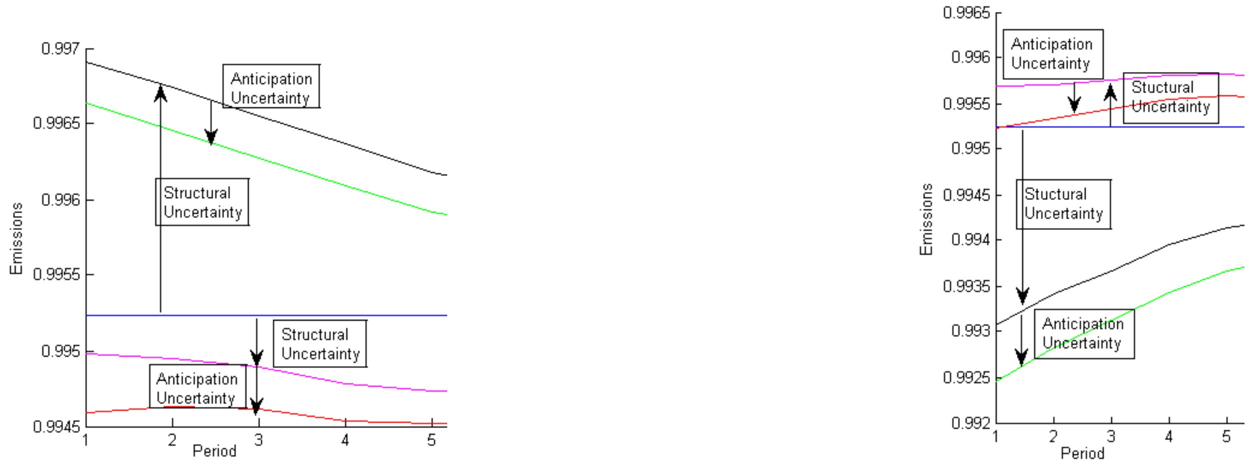
$$\begin{array}{c} N\beta_i \left(\int_\Theta R\left(\mu\left(\theta\right)\right)\xi_i(\theta)d\theta - R\left(\int_\Theta \xi_i(\theta)\mu\left(\theta\right)d\theta\right)\right) < \\ \sum_{j\neq i}\beta_j \left(\int_\Theta R\left(\mu\left(\theta\right)\right)\xi_j(\theta)d\theta - R\left(\int_\Theta \xi_j(\theta)\mu\left(\theta\right)d\theta\right)\right). \end{array} \tag{3.10}$$

Note that both sides of this equation are positive. Indeed, the left side provides the weighted self-precautionary effect of increase in uncertainty (generated by anticipation) and the right side is the same effect for all other players. In other words, if all other players decrease their emissions more in compare to player i in a way that overcompensate his precautionary behavior this player may increase his emissions.

Bramoullé and Treich (2009) in a static model showed that introducing uncertainty in the model always reduce emissions due to risk-considering. As you see propositions 3.4 and 3.5 are in line with this statement, because by definition a learner player faces more uncertainty than an adaptive player. We show that in case of heterogeneity still there could be players which increase their emissions moving from an adaptive to learner player. Besides, considering whether beliefs about mean of stochastic parameter is unbiased or biased, we can compare emissions under full information and adaptive assumption, i.e., we can analyze the effect of structural uncertainty on behavior of players. The results provided formally in proposition 3.6 also have a different message from Bramoullé and Treich (2009) general conclusion. The proposition tells us, depending on beliefs bias, dealing with structural uncertainty may increase, decrease or even not affect the emissions strategies.

Proposition 3.6

1. *If the beliefs about the mean of ecological shock η is unbiased, $\mu(\theta^*) = \int_{\mathcal{H}} \mu(\theta)\xi_i(\theta)d\theta$, then:*
   $e_i^I = e_i^A$,
2. *if $\mu(\theta^*) > \int_\Theta \mu(\theta)\xi_i(\theta)d\theta$, then:*
   $e_i^I < e_i^A$,



(a) Unknown parameter: α (b) Unknown parameter: β

Figure 3.1: Decomposing the effects of structural uncertainty and uncertainty due to anticipation for a Beta distribution.

3. if $\mu(\theta^*) < \int_\Theta \mu(\theta)\,\xi_i(\theta)\,d\theta$*, then:*

$e_i^I > e_i^A$.

Proof. Proof is straightforward by comparing equation (3.8) and (3.9). ■

Introducing the intermediate case of adaptive learner gives us the opportunity to find out that different sources of uncertainty (structural uncertainty and uncertainty due to anticipation) could even have opposite effects on emission strategies . For instance for a beta distribution, See Figure 3.1 which draws the emissions trajectory for two players with different prior beliefs, we found out that whenever structural uncertainty causes a decrease in emissions in compare to the full information case (solid straight line in the figures), uncertainty due to anticipation adds to this effect and decreases emissions more, but if structural uncertainty increases emissions uncertainty due to anticipation moderate this effect. In other words, as propositions 3.4 and 3.5 propose, at least in total, uncertainty due to anticipation may "alleviate" the emissions problem but effect of structural uncertainty depends on the model assumptions, more specifically on slope and curvature of the distribution function. Proposition 3.7 gives more precise details mathematically in special case of homogeneity and unbiasness of beliefs.

Proposition 3.7 *Assume beliefs are unbiased about the parameter, i.e.,* $\theta^* = \int_\Theta \theta\xi(\theta)\,d\theta$*, and beliefs are homogeneous, then:*

if $\mu" > 0$*, then* $\sum e_i^I(S;\theta^*) > \sum e_i^A(S,\xi)$,
if $\mu" = 0$*, then* $\sum e_i^I(S;\theta^*) = \sum e_i^A(S,\xi)$,
if $\mu" < 0$*, then* $\sum e_i^I(S;\theta^*) < \sum e_i^A(S,\xi)$.

Proof. Since $\mu(\theta^*) = \mu(E(\theta))$ if $\mu" > 0$ ($\mu" < 0$), i.e., μ is convex (concave) then by Jensen's inequality $\mu(E(\theta)) > E(\mu(\theta))$ ($\mu(E(\theta)) < E(\mu(\theta))$) which readily gives the results. ■

3.4.1 More optimistic/pessimistic beliefs and Bayesian learners

When players encounter uncertainty they form their beliefs according to the information they receive from different sources, like their observations, the results of scientific researches or activities of environmentalist groups. So by receiving new information their prior beliefs may change. Depending on the kind of changes that happens in the beliefs, their strategies will be modified and they may emit more or less. In this section we analyze the effect of such changes. Before that let's introduce useful concept of first-order strict stochastic dominance which could be used to compare better and worse situations.

Definition 3.1 *Assume two probability density functions ξ_i^1 and ξ_i^2 then ξ_i^1 first-order strict stochastically dominates ξ_i^2 , $\xi_i^1 \succ_1 \xi_i^2$, if for any increasing function $u : \mathbb{R} \to \mathbb{R}$, $\int u(x)\, \xi_i^1(x)\, dx > \int u(x)\, \xi_i^2(x)\, dx$.*

Since beliefs are with respect to the θ and not η, we need to know how uncertain variable's mean reacts to θ. Having definition (3.1) in mind, assume that beliefs of player i changes from ξ_i^1 to ξ_i^2, while $\xi_i^1 \succ_1 \xi_i^2$ and $\mu'(\theta) > 0$. Intuitively this player's beliefs have changed in a way that he expects lower values for unknown parameter, θ, and since μ is increasing he expects lower values for unknown variable η as well. Thus, under ξ_i^2 this player expects lower levels of pollution accumulation. Lower stocks is equivalent with slower deterioration of environment and in our model ends in lower environmental cost. In other words, this player becomes more optimistic about the environment by believing in ξ_i^2 instead of ξ_i^1. In case of $\mu'(\theta) < 0$, the results is opposite and a change from ξ_i^1 to ξ_i^2 would be equivalent with moving to more pessimistic beliefs. Proposition 3.8 presents the impact of a change toward more optimistic beliefs on players decisions.

Proposition 3.8 *Assume that player i becomes more optimistic while all other players priors remain unchanged, i.e., consider two N-tuple $\xi^1 = \{\xi_1^1, \xi_2^1, ..., \xi_i^1, ..., \xi_N^1\}$ and $\xi^2 = \{\xi_1^1, \xi_2^1, ..., \xi_i^2, ..., \xi_N^1\}$ when $\xi_i^1 \succ_1 \xi_i^2$ and $\mu'(\theta) > 0$*

- $e_i^x(S, \xi^2) > e_i^x(S, \xi^1)$, $x \in \{L, A\}$;
- $e_j^x(S, \xi^2) < e_j^x(S, \xi^1)$, $x \in \{L, A\}$ $\forall j \neq i$;
- $\sum_{z=1}^N e_z^x(S, \xi^2) > \sum_{z=1}^N e_z^x(S, \xi^1)$.

Proof. See appendix 3.8.5. ■

Example 3.2 *Assume η has a uniform distribution with unknown support $[0, \theta]$, and beliefs, $\xi(\theta)$, $\theta \in [0, 1]$, then $\mu(\theta) = \frac{\theta}{2}$, so $\mu'(\theta) > 0$. By proposition 3.8 if $\xi_i^1 \succ_1 \xi_i^2$, then $e_i^L(S, \xi^2) > e_i^L(S, \xi^1)$.*

Proposition 3.8 suggests that emissions are strategic substitutes, but according to part three even though others emissions are not fixed and strategic effects do come into play, they are not strong enough to overcome the direct effect.

Corollary 2 *If all countries become more optimistic about the environment, i.e.* $\xi_i^1 \succ_1 \xi_i^2$, $\forall i \in 1,...,N$, *and* $\mu'(\theta) > 0$, *then* $\forall x \in \{L, A\}$*:*

$$\sum_{z=1}^{N} e_z^x \left(S, \xi^2\right) > \sum_{z=1}^{N} e_z^x (S, \xi^1).$$

We saw that emissions are strategic substitutes, in other words while more optimistic own beliefs intend to increase, the emissions, more optimistic outsider beliefs has the opposite effect on this player. But as it is presented by corollary 2, in total, the self-effects will overcome the outsiders-effect[6].

In our model players could have their own specific beliefs. Thus, we can have same comparison between players with different beliefs as corollary 3 states it formally.

Corollary 3 *If player* i *and* j, $i \neq j$ *are similar in their parameters, but not in their beliefs such that always* $\xi_i \succ_1 \xi_j$, *then:*

- *If* $\mu'(\theta) > 0$,

  $e_i^x (S, \xi) > e_j^x (S, \xi)$, $x \in \{L, A\}$.

- *If* $\mu'(\theta) < 0$,

  $e_i^x (S, \xi) > e_j^x (S, \xi)$, $x \in \{L, A\}$.

Proof. See appendix 3.8.5. ■

If we let players to update their beliefs about the functional form of $\phi(\eta)$, we may present changes in optimism\pessimism more directly. In fact if beliefs of a player changes and he gives higher values on average to η this player becomes more pessimistic. Proposition 3.9 presents the effect of such a change on behavior of players.

Proposition 3.9 *If* $\phi_i^1(\eta) \succ_1 \phi_i^2(\eta)$, *i.e. player* i *gives higher mean to* $\mu(\theta)$ *under* $\phi_i^1(\eta)$ *than under* $\phi_i^2(\eta)$, *then:*

- $e_i^{x2} (S, \xi) > e_i^{x1} (S, \xi)$, $x \in \{L, A\}$.

Proof. Since $\mu(\theta) = \int_{\mathcal{H}} \eta \phi(\eta|\theta) d\theta$ and η is an increasing function if $\phi_i^1(\eta) \succ_1 \phi_i^2(\eta)$, so by definition $\mu_i^1(\theta) > \mu_i^2(\theta)$, given the proposition 3.2 and proposition 3.3 complete the proof. ■

In summary, our results in this section suggest that, if players (one player or more) changes their beliefs toward more optimistic views the total emissions will increase.

[6]Individuals self-effect is $N\beta_i \left(\int_\Theta R(\mu(\theta))\, \xi_i^1(\theta) d\theta - \int_\Theta R(\mu(\theta))\, \xi_i^2(\theta) d\theta\right)$, the outsiders-effect is$\sum_{j \neq i} \beta_j \left(\int_\Theta R(\mu(\theta))\, \xi_j^1(\theta) d\theta - \int_\Theta R(\mu(\theta))\, \xi_j^2(\theta) d\theta\right)$.

3.5 More risky beliefs

If a player feels more variations in the value of a variable, i.e., if he believes that η is more variable than what he used to believe, he feels himself in a more risky situation. To present optimism/ pessimism beliefs we used the term of first order stochastic dominance since it deals with "better" vs. "worse" situations. In this section we refer to second order stochastic dominance which deals with relative riskiness between two distributions.

Definition 3.3 *Given two distribution with the same mean, distribution with p.d.f.* $\phi^1(\theta)$ *second-order stochastically dominates distribution with p.d.f.* $\phi^2(\theta)$, $\phi^1(\theta) \succ_C \phi^2(\theta)$, *(or is less risky) if for every non-decreasing concave function* $u : \mathbb{R} \to \mathbb{R}$, $\int_R u(x)\,\phi^1(x)\,dx \geq \int_R u(x)\,\phi^2(x)\,dx$.

Equality of distributions means is crucial in this definition since it lets us to distinguish the effects of more optimistic/pessimistic views from the effects related to riskiness of situation. So by definitions we can say:

- If $\phi^1(\theta) \succ_C \phi^2(\theta)$ thus ϕ^1 is less variable or less risky;
- If $\phi^1(\theta) \succ_V \phi^2(\theta)$ thus ϕ^1 is more variable or more risky.

Proposition 3.10 presents the effects of a change in beliefs toward more risky situations.

Proposition 3.10 *Let* $\mu'(\theta) > 0$. *Consider two N-tuples* $\xi^1 = \left\{\xi_1^1, \xi_2^1, ..., \xi_i^1, ..., \xi_N^1\right\}$ *and* $\xi^2 = \left\{\xi_1^1, \xi_2^1, ..., \xi_i^2, ..., \xi_N^1\right\}$. *If country i feels more risk under* ξ_i^2 *rather than under* ξ_i^1, *then:*

1. $e_i^L\left(S, \xi^2\right) \leq e_i^L(S, \xi^1)$;
2. $e_j^L\left(S, \xi^2\right) \geq e_j^L(S, \xi^1)$, $\forall j \neq i$;
3. $\sum_{z=1}^N e_z^L\left(S, \xi^2\right) \leq \sum_{z=1}^N e_z^L\left(S, \xi^1\right)$;
4. $e_z^A\left(S, \xi^2\right) = e_z^A(S, \xi^1)$, $\forall z = 1, ..., N$.

Proof. See Appendix 3.8.6. ■

According to proposition 3.10, given that unknown variable's mean is increasing in θ, if beliefs of learner player changes toward a more risky situation, while all other players' beliefs remain unchanged, he will decrease his emissions, while other players which their beliefs haven't changed will behave strategically substitute, similar to proposition 3.8, but again their reactions are not strong enough to overcome the direct effect. This results would be opposite in the case of $\mu' < 0$. Proposition 3.10 suggests that only in case of facing with uncertainty related to anticipation of learning, changes in riskiness affect the decision of players. In other words, structural uncertainty by itself (i.e., in adaptive learner case) will not lead to reaction to riskiness.

If everybody feels more risky the net effect on player i's emissions is ambiguous since the increase in own riskiness tends to decrease the emissions while the increase in opponents riskiness is an incentive to increase emissions. But the net effect in total emissions would be negative as it is presented in proposition 3.11.

Proposition 3.11 *Consider two N-tuples $\xi^1 = \left\{\xi_1^1, ..., \xi_N^1\right\}$ and $\xi^2 = \left\{\xi_1^1, ..., \xi_N^1\right\}$ and $\forall i = 1, ..., N$, country i feels more risk under ξ_i^2 than under ξ_i^1, then*

$$\sum_{i=1}^{N} e_i^L\left(S, \xi^2\right) \leq \sum_{i=1}^{N} e_i^L\left(S, \xi^1\right).$$

Proof. Straightforward given the proof of Proposition 3.10. ■

3.6 Summary and conclusion

Many different sources of uncertainty encompass the environmental issues. But as any other planning case involving uncertainty, planners gather new information and learn to deal with uncertain situations. To reflect the effects of these two important aspects of international environmental management, we proposed an Nplayer dynamic game of international pollution control which players are facing uncertainty due to their lack of information about a parameter related to the environment.

The main contribution of this paper is providing insight about the emission strategies in presence of uncertainty and learning. While this is somehow accepted that uncertainty alleviate common problems, by our analytical results we proved that according to the model setup, introduction of uncertainty may or may not alleviate the international pollution problem. Besides, effects of different sources of uncertainty are decomposed and it is showed under which circumstances each source may amplify or weaken others. We also study the effects of changes in players beliefs, either toward more optimistic/pessimistic beliefs or more sense of riskiness. Our results suggest that more pessimistic beliefs about the pollution stock and also more sense of riskiness decrease total emissions.

One possible extension to this work is taking into account other sources of uncertainties and analyze how the results might be affected. As an example, we considered another case that the motion is deterministic, but players damage cost is affected by a random variable, i.e. marginal cost of pollution, β_i, is random.[7] Our results suggest that behavior of a learner player is equivalent to an adaptive one. This case is a simple example which shows depending on the model and the source of uncertainty, we may have cases that decision of an adaptive learner exactly matches to the Bayesian learner case.

Other possible extensions for future works could be considering a non-linear damage cost function and also introducing an active learning process.

[7]The computations are available upon request.

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3.8 Appendix

3.8.1 Proof of Proposition 3.1

The feed-back Nash strategy of player i must satisfy the following Bellman equation:

$$v_i^I\left(S;\theta^*\right)=\max_{e_i}\left\{\alpha_i e_i-(e_i)^2-e_i\sum_{j\neq i}^{N}e_j-\beta_i S+\delta\int_{\mathcal{H}}\left[v_i^I\left(\sum_i e_i+d)S\right)\right]\phi(\eta|\theta^*)d\eta\right\} \quad (3.11)$$

when $v_i^I\left(S;\theta^*\right)$ is his value function and superscript I refers to the case of informed player. Considering the linear-state structure of our model we conjecture that $v_i^I\left(S;\theta^*\right)=k_{i,1}^I S+k_{i,2}^I$. Plugging conjectured value function in (3.24) we have:

$$k_{i,1}^I S+k_{i,2}^I=\max_{e_i}\left\{\begin{array}{c}\alpha_i e_i-(e_i)^2-e_i\sum_{j\neq i}e_j-\beta_i S+\\ \delta\int_{\mathcal{H}}\left[\kappa_{i,1}^I\eta\left(\sum_i e_i+d)S\right)+\kappa_{i,2}^I\right]\phi(\eta|\theta^*)d\eta\end{array}\right\} \quad (3.12)$$

The first order condition is[8]:

$$\alpha_i - 2e_i^I - \sum_{j\neq i} e_j + \delta \int_{\mathcal{H}} \kappa_{i,1}^I \eta\phi(\eta|\theta^*)d\eta = 0$$

so the best reaction function of player i is:

$$e_i^I = \frac{1}{2}\left(\alpha_i - \sum_{j\neq i} e_j + \delta \int_{\mathcal{H}} \kappa_{i,1}^I \eta\phi(\eta|\theta^*)d\eta\right)$$

To find the coefficients of the value function, substituting e_i^I in equation (3.12) gives a system of $2N$ equations, 2 equations for each player, with $2N$ unknowns, $\kappa_{i,1}^I$ and $\kappa_{i,2}^I$ $\forall i = 1, 2$, as following, by equating coefficients in orders of S:

$$\kappa_{i,1}^I = -\beta_i + \delta(1-d)\int_{\mathcal{H}} \eta\kappa_{i,1}^I \phi(\eta|\theta^*)d\eta \tag{3.13}$$

$$\kappa_{i,2}^I = \alpha_i e_i^I - (e_i^I)^2 - e_i \sum_{j\neq i} e_j + \delta \int_{\mathcal{H}} \left[\eta\kappa_{i,1}^I \left(\sum_i e_i^I\right) + \kappa_{i,2}^I\right] \left[\phi(\eta|\theta^*)\right] d\eta \tag{3.14}$$

Since $\mu(\theta^*) = \int_{\mathcal{H}} \eta\phi(\eta|\theta^*)d\eta$, from equation (3.13)

$$k_{i,1}^I = \frac{-\beta_i}{1 - \delta d\mu(\theta^*)}$$

and from equation (3.14)

$$\kappa_{i,2}^I = \frac{1}{1-\delta}\left(\alpha_i e_i^I - (e_i^I)^2 - e_i \sum_{j\neq i} e_j + \delta\mu(\theta^*)\kappa_{i,1}^I \left(\sum_i e_i^I\right)\right)$$

We guess that strategy of player i is given by:

$$e_i^I = \frac{1}{N+1}\left(N\alpha_i - \sum_{j\neq i}\alpha_j - \delta\frac{\mu(\theta^*)}{1-\delta(1-d)\mu(\theta^*)}\left(N\beta_i - \sum_{j\neq i}\beta_j\right)\right) \tag{3.15}$$

If our guess be true it should satisfy the first order condition (equation (3.13)). Plugging the strategy of player j given by equation (3.15) in the first order condition (equation (3.13)) we have:

[8]Notice that second order condition is satisfied for the model.

$$\frac{1}{2}\begin{pmatrix} \alpha_i + \delta \int_{\mathcal{H}} \kappa^I_{i,1} \eta \phi(\eta|\theta^*) d\eta \\ -\sum_{j\neq i}\left(\frac{1}{N+1}\left(N\alpha_j - \sum_{z\neq j}\alpha_z - \delta\frac{\mu(\theta^*)}{1-\delta d\mu(\theta^*)}\left(N\beta_j - \sum_{z\neq j}\beta_z\right)\right)\right) \end{pmatrix} =$$

$$\frac{1}{2}\begin{pmatrix} \left(\alpha_i - \delta\frac{\beta_i\mu(\theta^*)}{1-\delta(1-d)\mu(\theta^*)}\right)\left(1+\frac{N-1}{N+1}\right) \\ -\frac{1}{N+1}\sum_{j\neq i}\left(N\alpha_j - \sum_{z\neq i,j}\alpha_z - \delta\frac{\mu(\theta^*)}{1-\delta d\mu(\theta^*)}\left(N\beta_j - \sum_{z\neq j,i}\beta_z\right)\right) \end{pmatrix} =$$

$$\frac{1}{2}\begin{pmatrix} 2\frac{N}{N+1}\left(\alpha_i - \delta\frac{\beta_i\mu(\theta^*)}{1-\delta d\mu(\theta^*)}\right) \\ \frac{1}{N+1}(N-N+2)\left(-\sum_{j\neq i}\alpha_j + \delta\frac{\mu(\theta^*)}{1-\delta d\mu(\theta^*)}\sum_{j\neq i}\beta_j\right) \end{pmatrix} =$$

$$\frac{1}{N+1}\left(N\alpha_i - \sum_{j\neq i}\alpha_j + \delta\frac{\mu(\theta^*)}{1-\delta d\mu(\theta^*)}\sum_{j\neq i}\beta_j - N\delta\frac{\beta_i\mu(\theta^*)}{1-\delta d\mu(\theta^*)}\right)$$

which is equal to equation (3.15).

3.8.2 Proof of Proposition 3.2:

By virtue of our linear-state model we conjecture that the value function of player i has the linear form of $v_i^L = \kappa^L_{i,1}(\xi_i)S + \kappa^L_{i,2}(\xi_i)$. Replacing the conjectured v_i^L in the Bellman equation we have:

$$v_i^L = \max_{e_i}\left\{\begin{matrix} \alpha_i e_i - (e_i)^2 - e_i\sum_{j\neq i}^N e_j - \beta_i S + \\ \delta\int_{\mathcal{H}}\begin{bmatrix} \kappa^L_{i,1}(\hat{\xi}_i)\eta\left(\sum_i e_i + d)S\right) \\ +\kappa^L_{i,2}(\hat{\xi}_i)\end{bmatrix}\left[\int_\Theta \phi(\eta|\theta)\xi^i(\theta)d\theta\right]d\eta \end{matrix}\right\} \tag{3.16}$$

By the first order conditions:

$$e_i^L = \frac{1}{2}\left(\alpha_i - \sum_{j\neq i}^N e_j^L + \delta\int_{\mathcal{H}}\left(\kappa^L_{i,1}(\hat{\xi}_i)\right)\eta\left[\int_\Theta \phi(\eta|\theta)\xi(\theta)d\theta\right]d\eta\right) \tag{3.17}$$

Plugging (3.17) into Bellman equation (3.16), gives us the following system of equations:

$$\kappa^L_{i,1}(\xi) = -\beta_i + \delta d)\int_{\mathcal{H}}\kappa^L_{i,1}(\hat{\xi}_i)\eta\left[\int_\Theta \phi(\eta|\theta)\xi^i(\theta)d\theta\right]d\eta \tag{3.18}$$

$$\kappa^L_{i,2}(\xi_i) = \begin{matrix} \alpha_i e_i - \frac{1}{2}(e_i)^2 - \frac{1}{2}e_i\sum_{j\neq i}^N e_j + \\ \delta\int_{\mathcal{H}}\left[\kappa^L_{i,1}(\hat{\xi})\eta\left(\sum_i e_i\right) + \kappa^L_{i,2}(\hat{\xi})\right]\left[\int_\Theta \phi(\eta|\theta)\xi^i(\theta)d\theta\right]d\eta \end{matrix} \tag{3.19}$$

We claim that

$$\kappa^L_{i,1}(\xi_i) = -\beta_i\int_\Theta \frac{\xi_i(\theta)}{1-\delta d\mu(\theta)}d\theta \tag{3.20}$$

and so

$$\kappa_{i,2}^L(\xi_i) = \begin{array}{c} \alpha_i e_i - \frac{1}{2}(e_i)^2 - \frac{1}{2} e_i \sum_{j \neq i}^N e_j + \\ -\beta_i \delta \left(\sum_i e_i \right) \left(\int_\Theta \frac{\mu(\theta)\xi^i(\theta)}{1-\delta d\mu(\theta)} d\theta \right) \\ \delta \int_{\mathcal{H}} \kappa_{i,2}^L(\hat{\xi}) \left[\int_\Theta \phi(\eta|\theta)\xi^i(\theta) d\theta \right] d\eta \end{array} \tag{3.21}$$

To show this let's update (3.20) for one period:

$$\kappa_{i,1}^L(\hat{\xi}) = -\beta_i \int_\Theta \frac{\hat{\xi}(\theta)}{1-\delta d\mu(\theta)} d\theta \tag{3.22}$$

Plugging (3.3):

$$\kappa_{i,1}^L(\hat{\xi}) = -\beta_i \int_\Theta \frac{1}{1-\delta d\mu(\theta)} \frac{\phi(\eta|\theta)\xi^i(\theta)}{\int_\phi \phi(\eta|x)\xi^i(x)dx} d\theta$$

Plugging back in (3.18):

$$\kappa_{i,1}^L(\xi) = \; -\beta_i \left(1 + \delta d \int_{\mathcal{H}} \int_\Theta \frac{1}{1-\delta d\mu(\theta)} \frac{\phi(\eta|\theta)\xi^i(\theta_t)}{\int_\phi \phi(\eta|x)\xi^i(x)dx} d\theta\eta \left[\int_\Theta \phi(\eta|\theta)\xi^i(\theta)d\theta \right] d\eta \right)$$

$$\kappa_{i,1}^L(\xi) = -\beta_i \left(1 + \delta d \int_\Theta \frac{\mu(\theta)}{1-\delta d\mu(\theta)} \xi^i(\theta) d\theta \right)$$

$$\kappa_{i,1}^L(\xi) = -\beta_i \left(\int_\Theta \left[\frac{1-\delta d\mu(\theta)}{1-\delta d\mu(\theta)} + \frac{\delta d\mu(\theta)}{1-\delta d\mu(\theta)} \right] \xi^i(\theta) d\theta \right)$$

Which is equal to our claim. By replacing $\kappa_{i,1}^L$ in equation (3.19) gives the equation (3.21).

Now we have all needed information for finding e_i^L. By (3.20) and first order conditions we have:

$$e_i^L = \frac{1}{2} \left(\begin{array}{c} \alpha_i - \sum_{j \neq i} e_j^L - \\ \delta\beta_i \int_{\mathcal{H}} \left(\int_\Theta \frac{1}{1-\delta d\mu(\theta)} \frac{\phi(\eta|\theta)\xi_i(\theta)}{\int_\phi \phi(\eta|x)\xi_i(x)dx} d\theta \right) \eta \left[\int_\Theta \phi(\eta|\theta)\xi_i(\theta)d\theta \right] d\eta \end{array} \right)$$

We conjecture that:

$$e_i^L = \frac{1}{N+1} \left(N\alpha_i - \sum_{j \neq i} \alpha_j - \delta \left(\begin{array}{c} N\beta_i \int_\Theta \frac{\mu(\theta)}{1-\delta d\mu(\theta)} \xi_i(\theta) d\theta - \\ \sum_{j \neq i} \beta_j \int_\Theta \frac{\mu(\theta)}{1-\delta d\mu(\theta)} \xi_j(\theta) d\theta \end{array} \right) \right)$$

To see this, plugging $\left(\kappa_{i,1}^L(\hat{\xi}_i) \right)$ in first order conditions (equation 3.17):

$$e_i^L = \frac{1}{2} \left(\begin{array}{c} \alpha_i - \sum_{j \neq i}^N e_j^L - \\ \delta \int_{\mathcal{H}} \left(\beta_i \int_\Theta \frac{1}{1-\delta d\mu(\theta)} \frac{\phi(\eta|\theta)\xi^i(\theta)}{\int_\phi \phi(\eta|x)\xi^i(x)dx} d\theta \right) \eta \left[\int_\Theta \phi(\eta|\theta)\xi(\theta)d\theta \right] d\eta \end{array} \right) \tag{3.23}$$

$$e_i^L = \frac{1}{2}\left(\alpha_i - \sum_{j\neq i}^{N} e_j^L - \delta\beta_i \int_\Theta \frac{\mu(\theta)}{1-\delta d\mu(\theta)}\xi_i(\theta)d\theta\right)$$

and also plugging the conjectured strategy of player j in equation (3.23):

$$\frac{1}{2}\left(\begin{array}{c} \alpha_i - \delta\beta_i \int_\Theta \frac{\mu(\theta)}{1-\delta d\mu(\theta)}\xi_i(\theta)d\theta \\ -\frac{1}{N+1}\sum_{j\neq i}^{N}\left(\begin{array}{c} N\alpha_j - \sum_{z\neq j}\alpha_z \\ -\delta N\beta_j \int_\Theta \frac{\mu(\theta)}{1-\delta d\mu(\theta)}\xi_j(\theta)d\theta \\ +\delta\sum_{z\neq j}\beta_z \int_\Theta \frac{\mu(\theta)}{1-\delta d\mu(\theta)}\xi_z(\theta)d\theta \end{array}\right) \end{array}\right) =$$

$$\frac{1}{2}\left(\begin{array}{c} 2\frac{N}{N+1}\left(\alpha_i - \delta\beta_i \int_\Theta \frac{\mu(\theta)}{1-\delta d\mu(\theta)}\xi_i(\theta)d\theta\right) \\ -\frac{1}{N+1}(N-N+2)\left(\sum_{j\neq i}\alpha_j - \sum_{j\neq i}\beta_j \int_\Theta \frac{\mu(\theta)}{1-\delta d\mu(\theta)}\xi_j(\theta)d\theta\right) \end{array}\right) =$$

$$\frac{1}{N+1}\left(N\alpha_i - \sum_{j\neq i}\alpha_j - \delta\left(\begin{array}{c} N\beta_i \int_\Theta \frac{\mu(\theta)}{1-\delta d\mu(\theta)}\xi_i(\theta)d\theta - \\ \sum_{j\neq i}\beta_j \int_\Theta \frac{\mu(\theta)}{1-\delta d\mu(\theta)}\xi_j(\theta)d\theta \end{array}\right)\right)$$

Which complete our proof.

3.8.3 Proof of Proposition 3.3

We follow the same procedure as we did for learner player. We conjecture that v_i^A which satisfies equation (3.5) has the linear form of $v_i^A = k_{i,1}^A(\xi_i)S + k_{i,2}^A(\xi_i)$.

Rewriting (3.5) by our conjectured value function:

$$v_i^A(S;\xi) = \max_{e_i}\left\{\begin{array}{c} \alpha_i e_i - (e_i)^2 - e_i\sum_{j\neq i}^{N} e_j - \beta_i S + \\ \delta\int_{\mathcal{H}}\left[k_{i,1}^A(\xi_i)\left(\eta\left(\sum_i e_i + dS\right)\right) + k_{i,2}^A(\xi_i)\right]\left[\int_\Theta \phi(\eta|\theta)\xi_i(\theta)d\theta\right]d\eta \end{array}\right\} \tag{3.24}$$

The first order conditions are:

$$e_i = \frac{1}{2}\left(\alpha_i - \sum_{j\neq i}^{N} e_j + \delta\int_{\mathcal{H}}\left(k_{i,1}^A(\xi_i)\right)\eta\left[\int_\Theta \phi(\eta|\theta)\xi(\theta)d\theta\right]d\eta\right)$$

Since $\int_{\mathcal{H}} \eta\phi(\eta|\theta)d\eta = \mu(\theta)$:

$$e_i = \frac{1}{2}\left(\alpha_i - \sum_{j\neq i}^{N} e_j + \delta\int_\Theta \left(k_{i,1}^A(\xi_i)\right)\mu(\theta)\,\xi(\theta)d\theta\right) \tag{3.25}$$

Plugging the first order conditions into (3.24) give us the following system of equations:

$$\kappa_{i,2}^A(\xi_i) = \begin{array}{c} \alpha_i e_i - (e_i)^2 - e_i\sum_{j\neq i}^{N} e_j + \\ \delta\int_{\mathcal{H}}\left[\kappa_{i,1}^L(\xi_i)\eta\left(\sum_i e_i\right) + \kappa_{i,2}^L(\xi_i)\right]\left[\int_\Theta \phi(\eta|\theta)\xi^i(\theta)d\theta\right]d\eta \end{array}$$

and

$$k_{i,1}^{A}(\xi) = -\beta_i + \delta d \int_{\Theta} k_{i,1}^{A}(\xi_i) \left[\int_{\mathcal{H}} \eta \phi(\eta|\theta) d\eta \right] \xi^i(\theta) d\theta$$

or

$$k_{i,1}^{A}(\xi) = -\beta_i + \delta d \int_{\Theta} k_{i,1}^{A}(\xi_i) \mu(\theta) \xi^i(\theta) d\theta \tag{3.26}$$

We now show that $k_{i,1}^{A}(\xi_i) = \frac{-\beta_i}{1-\delta d \int_{\Theta} \mu(\theta)\xi_i(\theta)d\theta}$. Plug this in (3.26):

$$k_{i,1}^{A}(\xi) = -\beta_i - \beta_i \delta d \int_{\mathcal{H}} \frac{1}{1 - \delta d \int_{\Theta} \mu(\theta) \xi(\theta) d\theta} \eta \left[\int_{\Theta} \phi(\eta|\theta) \xi^i(\theta) d\theta \right] d\eta$$

$$k_{i,1}^{A}(\xi) = -\beta_i - \beta_i \delta d \frac{\int_{\Theta} \left[\int_{\mathcal{H}} \eta \phi(\eta|\theta) d\eta \right] \xi^i(\theta) d\theta}{1 - \delta d \int_{\Theta} \mu(\theta) \xi(\theta) d\theta}$$

$$\kappa_{i,A}(\xi) = -\beta_i - \beta_i \delta \frac{\int_{\Theta} \mu(\theta) \xi^i(\theta) d\theta}{1 - \delta d \int_{\Theta} \mu(\theta) \xi(\theta) d\theta}$$

$$k_{i,1}^{A}(\xi) = -\beta_i \left[\frac{1 - \delta d \int_{\Theta} \mu(\theta) \xi(\theta) d\theta}{1 - \delta d \int_{\Theta} \mu(\theta) \xi(\theta) d\theta} + \frac{\delta d \int_{\Theta} \mu(\theta) \xi^i(\theta) d\theta}{1 - \delta d \int_{\Theta} \mu(\theta) \xi(\theta) d\theta} \right]$$

Thus here we have:

$$k_{i,1}^{A}(\xi_i) = \frac{-\beta_i}{1 - \delta d \int_{\Theta} \mu(\theta) \xi_i(\theta) d\theta}$$

To complete our proof we again claim that:

$$e_i^A = \frac{1}{N+1} \begin{pmatrix} N\alpha_i - \sum_{j \neq i} \alpha_j \\ -\delta \left(N\beta_i R \left(\int_{\Theta} \xi_i(\theta) \mu(\theta) d\theta \right) - \sum_{j \neq i} \beta_j R \left(\int_{\Theta} \xi_j(\theta) \mu(\theta) d\theta \right) \right) \end{pmatrix} \tag{3.27}$$

If our claim be true it should satisfy the first order conditions. Before that we use the following simplification notation, given $k_{i,1}^{A}(\xi_i)$ and the definition of $R(x)$:

$$\int_{\mathcal{H}} \left(k_{i,1}^{A}(\xi_i) \right) \eta \left[\int_{\Theta} \phi(\eta|\theta) \xi(\theta) d\theta \right] d\eta = -\beta_i R \left(\int_{\Theta} \xi_i(\theta) \mu(\theta) d\theta \right)$$

To proof our conjectured emission strategy we rewrite strategy of player j by equation (3.27) and plug it in the first order conditions (equation (3.8)), then we have:

$$\frac{1}{2}\left(\begin{array}{c} \alpha_i - \beta_i R\left(\int_\Theta \xi_i(\theta)\mu(\theta)\, d\theta\right) \\ -\frac{1}{N+1}\sum_{j\neq i}^N \left(\begin{array}{c} N\alpha_j - \sum_{z\neq j}\alpha_z \\ -\delta N\beta_j R\left(\int_\Theta \xi_j(\theta)\mu(\theta)\, d\theta\right) \\ +\delta\sum_{z\neq j}\beta_z R\left(\int_\Theta \xi_z(\theta)\mu(\theta)\, d\theta\right) \end{array}\right) \end{array}\right) =$$

$$\frac{1}{2}\left(\begin{array}{c} 2\frac{N}{N+1}\left(\alpha_i - \beta_i R\left(\int_\Theta \xi_i(\theta)\mu(\theta)\, d\theta\right)\right) \\ -\frac{1}{N+1}\sum_{j\neq i}^N \left(\begin{array}{c} N\alpha_j - \sum_{z\neq j,i}\alpha_z \\ -\delta N\beta_j R\left(\int_\Theta \xi_j(\theta)\mu(\theta)\, d\theta\right) \\ +\delta\sum_{z\neq j,i}\beta_z R\left(\int_\Theta \xi_z(\theta)\mu(\theta)\, d\theta\right) \end{array}\right) \end{array}\right) = \tag{3.28}$$

$$\frac{1}{2}\left(\begin{array}{c} 2\frac{N}{N+1}\left(\alpha_i - \beta_i R\left(\int_\Theta \xi_i(\theta)\mu(\theta)\, d\theta\right)\right) \\ -\frac{(N-N+2)}{N+1}\sum_{j\neq i}^N \left(\alpha_j - \beta_j R\left(\int_\Theta \xi_j(\theta)\mu(\theta)\, d\theta\right)\right) \end{array}\right) =$$

$$\frac{1}{N+1}\left(\begin{array}{c} N\alpha_i - \sum_{j\neq i}\alpha_j \\ -\delta\left(N\beta_i R\left(\int_\Theta \xi_i(\theta)\mu(\theta)\, d\theta\right) - \sum_{j\neq i}\beta_j R\left(\int_\Theta \xi_j(\theta)\mu(\theta)\, d\theta\right)\right) \end{array}\right)$$

which is the same as our claim.

3.8.4 Proof of proposition 3.5

$\sum e_i^L = \frac{1}{N+1}\left(\sum_i \alpha_i - \delta\sum_{i=1}^N \beta_i R\left(\int_\Theta \xi_i(\theta)\mu(\theta)\, d\theta\right)\right)$

$$\sum_{i=1}^N \left(e_i^L - e_i^A\right) = \frac{\delta}{N+1}\sum_{i=1}^N \left(\begin{array}{c} \left(N\beta_i R\left(\int_\Theta \xi_i(\theta)\mu(\theta)\, d\theta\right) - \sum_{j\neq i}\beta_j R\left(\int_\Theta \xi_j(\theta)\mu(\theta)\, d\theta\right)\right) \\ -\left(N\beta_i\int_\Theta R(\mu(\theta))\,\xi_i(\theta)d\theta - \sum_{j\neq i}\beta_j\int_\Theta R(\mu(\theta))\,\xi_j(\theta)d\theta\right) \end{array}\right)$$

$$\sum_{i=1}^N \left(\begin{array}{c} \left(N\beta_i R\left(\int_\Theta \xi_i(\theta)\mu(\theta)\, d\theta\right) - \sum_{j\neq i}\beta_j R\left(\int_\Theta \xi_j(\theta)\mu(\theta)\, d\theta\right)\right) \\ -\left(N\beta_i\int_\Theta R(\mu(\theta))\,\xi_i(\theta)d\theta - \sum_{j\neq i}\beta_j\int_\Theta R(\mu(\theta))\,\xi_j(\theta)d\theta\right) \end{array}\right) =$$

$$\begin{array}{c} \left(N\sum_{i=1}^N \beta_i R\left(\int_\Theta \xi_i(\theta)\mu(\theta)\, d\theta\right) - \sum_{i=1}^N\sum_{j\neq i}\beta_j R\left(\int_\Theta \xi_j(\theta)\mu(\theta)\, d\theta\right)\right) \\ -\left(N\sum_{i=1}^N \beta_i\int_\Theta R(\mu(\theta))\,\xi_i(\theta)d\theta - \sum_{i=1}^N\sum_{j\neq i}\beta_j\int_\Theta R(\mu(\theta))\,\xi_j(\theta)d\theta\right) \end{array} =$$

$$\begin{array}{c} \left(N\sum_{i=1}^N \beta_i R\left(\int_\Theta \xi_i(\theta)\mu(\theta)\, d\theta\right) - (N-1)\sum_{i=1}^N \beta_i R\left(\int_\Theta \xi_i(\theta)\mu(\theta)\, d\theta\right)\right) \\ -\left(N\sum_{i=1}^N \beta_i\int_\Theta R(\mu(\theta))\,\xi_i(\theta)d\theta - (N-1)\sum_{i=1}^N \beta_i\int_\Theta R(\mu(\theta))\,\xi_i(\theta)d\theta\right) \end{array} =$$

$$\sum_{i=1}^N \beta_i R\left(\int_\Theta \xi_i(\theta)\mu(\theta)\, d\theta\right) - \sum_{i=1}^N \beta_i\int_\Theta R(\mu(\theta))\,\xi_i(\theta)d\theta =$$

$$\sum_{i=1}^N \beta_i\left(R\left(\int_\Theta \xi_i(\theta)\mu(\theta)\, d\theta\right) - \beta_i\int_\Theta R(\mu(\theta))\,\xi_i(\theta)d\theta\right)$$

Since R is an increasing convex function, so by Jensen's inequality:

$$\int_{\Theta} R\left(\mu\left(\theta\right)\right)\xi_i(\theta)d\theta > R\left(\int_{\Theta}\xi_i(\theta)\mu\left(\theta\right)d\theta\right)$$

that is $\sum_{i=1}^{N}\left(e_i^L - e_i^A\right) < 0$, i.e.,

$$e_i^L\left(S,\xi\right) < e_i^A\left(S,\xi\right).$$

3.8.5 Proof of proposition 3.8

For learner player let $u\left(x\right) = R\left(\mu\left(x\right)\right)$. Since R is increasing so,

$$\frac{\partial u}{\partial x} = \mu' R' \Rightarrow \begin{cases} \frac{\partial u}{\partial x} > 0 & \text{if } \mu' > 0 \\ \frac{\partial u}{\partial x} < 0 & \text{if } \mu' < 0 \end{cases}$$

Recalling the total emissions under learner and adaptive learner assumptions, respectively:

$$\sum e_i^L = \frac{1}{N+1}\left(\sum_i \alpha_i - \delta\sum_{i=1}^{N}\beta_i R\left(\int_{\Theta}\xi_i(\theta)\mu\left(\theta\right)d\theta\right)\right)$$

Taking into account the definition 3.1 and equation (3.7) and (3.8) confirms the results for learner player.

For adaptive player considering $u\left(x\right) = \mu\left(x\right)$. So increasing R and total emissions as following

$$\sum e_i^A = \frac{1}{N+1}\left(\sum_i \alpha_i - \delta\sum_{i=1}^{N}\beta_i\int_{\Theta}\xi_i(\theta)R\left(\mu\left(\theta\right)\right)d\theta\right)$$

gives the results. Notice that since u for both cases is strictly increasing the relations are established in their strict forms.

3.8.6 Proof of proposition 3.10

Before proceeding to the proof of remaining parts let's have the following lemma:

Lemma 3.4 *If $E\left(X\right) = E\left(Y\right)$, when E is the expectations operator, then $X \succ_{C/V} Y$ if and only if $E\left(u\left(x\right)\right) \geq E\left(u\left(y\right)\right)$ for all concave/convex u.*

If μ be a concave function, $\mu'' \leq 0$ or this player is risk-averse, and country i's beliefs changes in a way that feels more risk, $\xi_i^1 \succ_C \xi_i^2$ i.e., $\theta^1 \succ_C \theta^2$ and so $\mu\left(\theta^1\right) \succ_C \mu\left(\theta^2\right)$. According to our definition of R is a convex function thus, $-R$ is concave. Then by lemma and having in mind that $\int_{\Theta} R\left(\mu\left(\theta\right)\right)\xi_i(\theta^2)d = E\left(R\left(\mu\right)\right)$, we have:

$$-\int_{\Theta} R\left(\mu\left(\theta\right)\right)\xi_i(\theta^1)d\theta \geq -\int_{\Theta} R\left(\mu\left(\theta\right)\right)\xi_i(\theta^2)d\theta \tag{3.29}$$

equations (3.29) and 3.7 and recalling the total emissions under learner assumption we have:

$$\sum e_i^L = \frac{1}{N+1}\left(\sum_i \alpha_i - \delta \sum_{i=1}^{N} \beta_i R\left(\int_\Theta \xi_i(\theta)\mu\left(\theta\right)d\theta\right)\right)$$

gives the results for the first three parts when the player is risk-averse. In case of risk-seeker, i.e., if $\mu'' \geq 0$, and again country i feels more risky then $\xi_i^2 \succ_V \xi_i^1$ i.e., $\theta^2 \succ_V \theta^1$ and so $\mu\left(\theta^2\right) \succ_V \mu\left(\theta^1\right)$. Since R is convex, by the lemma we have:

$$\int_\Theta R\left(\mu\left(\theta\right)\right)\xi_i(\theta^2)d\theta \geq \int_\Theta R\left(\mu\left(\theta\right)\right)\xi_i(\theta^1)d\theta \tag{3.30}$$

which which complete the proof for the learner player.

In case of adaptive learner proof is trivial since by definition we considered two mean preserve p.d.fs., i.e., $\int_\Theta \mu\left(\theta\right)\xi_i(\theta^1)d\theta = \int_\Theta \mu\left(\theta\right)\xi_i(\theta^2)d\theta$.

Chapter 4

Emissions Control Policies under Uncertainty and Learning[1]

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Abstract

We compare the use of price-based policies or taxes, and quantity-based policies or quotas, for controlling emissions in a dynamic setup when the regulator faces two sources of uncertainty: (i) market-related uncertainty; and (ii) ecological uncertainty. We assume that the regulator is an anticipated learner and the regulator and firms have asymmetric information. Our results suggest that the expected level of emissions is the same under taxes and quotas. However, the comparison of the total benefits related to these policies suggests that taxes dominate quotas, that is, they provide a higher social welfare. Even though taxes have some benefits over quotas, neither learning nor ecological uncertainty affect the choice of policy, i.e., the only factor having such an impact is

[1]Research supported by NSERC, Canada.

uncertainty in the instantaneous net emissions benefits (market-related uncertainty). Besides, the more volatile is this uncertainty, the more benefits of taxes over quotas. Ecological uncertainty leads to a difference between the emissions rule under the informed and anticipated learning assumption. However, the direction of this difference depends on the beliefs bias with regard to ecological uncertainty. We also find that a change in the regulator's beliefs toward more optimistic views will increase the emissions.

Key Words: Pollution Control, Dynamic Games, Uncertainty, Learning, Policy Choice

4.1 Introduction

To curb pollution emissions, regulators can adopt either quantity-based instruments (e.g., quotas) or price-based policies (e.g., taxes). When there is one externality (here, pollution) and the context is deterministic, quotas and taxes are equivalent in terms of controlling that externality; see, e.g., Weitzman (1974). The main objective of this paper is to verify if this result carries over to the dynamic case where there are environmental and economic uncertainties and the regulator is not fully informed. This question is natural, as it is widely accepted in the scientific community that pollutants accumulation processes, as well as their impacts on climate and on the economy are tainted by random disturbances. The idea that uncertainty or inadequate information could give one policy instrument an advantage over the other was emphasized long ago by Weitzman (1974).

There exists a large body of literature about uncertainty and policy choices, with the early papers being Weitzman (1974) and Fishelson (1976). Using a model of public commodities (goods) with externalities, Weitzman (1974) shows that if the cost of production or the social benefits are affected by an unobserved and unknown disturbance term or random variable, then "neither instrument yields an optimum *ex post*". However, depending on the curvature of the cost and benefit functions around the optimal output level, one of the two instruments (quotas or taxes), may provide a higher expected social payoff, i.e., benefits net of costs. Fishelson (1976) also shows that the difference between taxes and quotas in terms of expected social gains depends upon the relative slopes of the marginal benefit and cost functions.

In Weitzman (1974) and Fishelson (1976), the model is static. As environmental damage is mainly caused by the accumulation of pollution over time, and as the sources, nature and magnitude of the uncertainties are not the same in the short- and long-term, scholars have also compared taxes to quotas in a dynamic setting. For example, Karp and Zhang (2012) find, under the assumptions of an endogenous abatement cost and asymmetric information about this cost, that a tax has some advantages (again, in terms of expected payoff for society) over a quota.

In this paper, we adopt a dynamic game model involving a regulator and a representative firm and compare quotas and taxes in terms of emissions and expected welfare. We retain two sources of uncertainty, namely, consumption uncertainty and ecological (or environmental) uncertainty. Following the literature (see, e.g., Hoel and Karp (2001, 2002); Karp and Zhang (2006)), we assume that there is asymmetry of information between the regulator and the firm, that is, that the firm knows the realization of the random consumption at the time it makes its decision, whereas the regulator does not have access to this information when it announces its policy. Finally, we consider and compare the results in two scenarios, namely, the regulator is fully informed and where the regulator is not, but learns over time.

A closely related paper to ours is Karp and Zhang (2006), where they also consider uncertainty and learning in a dynamic policy-choice model. In their contribution, the anticipated learning (i) increases the optimal level of emissions, and (ii) favors the use of taxes. We differ from Karp and Zhang (2006) in two important aspects. First, in their model, learning starts in the current period and continues for n predetermined consecutive periods. Here, we do not introduce a presumed end-time for learning, which is less restrictive than imposing one. Further, allowing learning to continue indefinitely gives more flexibility in tracing the effects of either a change in beliefs or the occurrence of a shock at any time during the planning period. Second, they use a log-normal distribution and find their analytical results implicitly. Here, we do not select a particular distribution and work with a general probability-density function, where the information can be summarized by a finite-dimensional vector of sufficient statistics. This method was introduced by Koulovatianos et al. (2009) for an optimal-growth model, and Agbo (2011) used the same methodology in a dynamic game of resource consumption. Following this methodology allows us to recognize and decompose different sources of uncertainty which affect the emissions rules under learning and no-learning scenarios. Another contribution of our paper to the literature is the introduction and analysis of the effects of ecological uncertainties in an infinite-horizon model.

Our main results can be summarized as follows:

1. Neither learning nor ecological uncertainties affect the choice of policy, and the only source that matters is uncertainty in the instantaneous net benefits of emissions (consumption shock).

2. Ecological uncertainty leads to a difference between the emissions rule under the informed and anticipated-learning assumptions. However, the direction of this difference depends on the regulator's belief bias with regard to ecological shock.

3. Taxes and quotas lead to the same expected emissions level.

4. In the presence of uncertainty and in a dynamic setting, taxes yield a higher social welfare than do quotas.

5. The effect of a change in the regulator's beliefs toward more optimistic views is not unique and depends on the functional form of the unknown variable's mean.

The rest of the paper is organized as follows: In Section 2, we introduce the model; in Section 3, we characterize the emissions rule under the informed or no-learning case, which will act as our benchmark; and, then in Section 4, we proceed to the characterization of the emissions rule under the anticipated-learning assumption. In Section 4, we compare emission rules and policies. Section 5 provides the conclusion.

4.2 The Model

Consider a representative firm that produces a good, and as a by-product, emits a pollutant, e.g., CO_2. Time is discrete and the planning horizon is infinite. Denote by e_t the emissions at time t, $t = 0, 1, \ldots$. Assuming there is a monotone increasing relationship between the quantity produced and the pollution emitted, we can express the benefit that the firm derives from production as a function of the emissions. We suppose that the benefit function R_t is concave increasing in emissions and depends on a random parameter capturing possible shocks in consumption. We follow the literature (see, e.g., Karp and Zhang (2006); Long (1992); Dockner and Long (1993)) and retain a quadratic form for the benefit function; and in order to account for uncertainty, we let one of the parameters of this function be random,[2] that is,

$$R_t(e_t) = (\alpha + \eta_{1t})\, e_t - \frac{1}{2} e_t^2,$$

where α is a positive parameter and η_1 is a realization of the random variable $\tilde{\eta}_1$. We assume that the conditional probability distribution of $\tilde{\eta}_1$ is given by $\phi_1(\eta_1|\theta_1^*)$, with $\eta_1 \in \mathcal{H}_1 \subseteq [0, 1]$, and where θ_1^* is a finite-dimensional vector of sufficient statistics. We let $\phi_1(\eta_1|\theta_1^*)$ be a general subjective distribution for η_1. Further, we assume that the representative firm and the regulator have asymmetric information, in the sense that η_{1t} is observed and known to the firm when making its output decision, but not to the regulator at the time it announces its policy (see, e.g., Hoel and Karp (2002); Karp and Zhang (2006)).

Denote by S_t the stock of accumulated emissions at time t, and by $D_t(S_t)$ the convex-increasing damage-cost function. We assume, mainly for tractability, that this function is linear and given by

$$D_t(S_t) = \beta S_t,$$

where $\beta > 0$ is the marginal damage cost.

The evolution of the pollution stock is subject to some ecological (or environmental) randomness. To keep the model as parsimonious as possible, while still being able to capture this randomness in a reasonable way, we adopt the following multiplicative specification:

$$S_{t+1} = \eta_{2t}(e_t + dS_t), \quad S(0) = S_0, \tag{4.1}$$

where $1 - d$ is the natural pollution absorption rate, S_0 is the given initial pollution stock, and η_{2t} is a realization of the random variable $\tilde{\eta}_2$ representing environmental uncertainty in terms of, e.g., changes in wind or temperature, or in the pollutant absorption rate by forests and oceans. The

[2] This suffices to make the point we wish to establish in this paper.

conditional probability distribution of $\tilde{\eta}_2$ is given by $\phi_2(\eta_2|\theta_2^*)$, with $\eta_{2t} \in \mathcal{H}_2 \subseteq [0,1]$. As before, θ_2^* is a finite-dimensional vector of sufficient statistics for this distribution.

Moving to the players' objectives, we assume that in the absence of any environmental policy, the firm maximizes its revenues, that is, does not internalize the damage cost, whereas the regulator maximizes the total expected discounted social welfare given by

$$W = E\left(\sum_t^\infty \delta^t \left(R_t\left(e_t\right) - D_t\left(S_t\right)\right)\right), \tag{4.2}$$

where δ is the discount rate.

In order to study the effects of different sources of uncertainty and learning on policy choices, we compare quantity-based policies (quotas) and price-based policies (taxes) under two different information assumptions for the regulator, namely: (i) full-information (benchmark) case, that is, where the regulator knows all of the model's parameters and functional forms; (ii) learning case, that is, where the values of the statistics of the distributions of the two random variables (η_1 and η_2) are unknown, and the regulator bases his beliefs about them on the available information.

4.2.1 Full Information

In the full-information case, the two vectors θ_1^* and θ_2^* and the functional forms of their distributions, $\phi_1\left(\theta_1^*\right)$ and $\phi_2\left(\theta_2^*\right)$, are known. Therefore, the regulator maximizes the welfare function using certainty-equivalent values of the random variables η_1 and η_2, namely, $E\left(\eta_i\right) = \mu_i = \int_{\mathcal{H}_i} \eta_i \phi_i(\theta_i^*) d\eta_i$.

4.2.1.1 Quotas

When using quotas, the regulator solves a dynamic-programming problem that consists in selecting an e_t that maximizes (4.2), subject to the dynamics in (4.1). Denote by $v^q\left(S\right)$ the regulator's value function, and introduce the Bellman equation associated to the regulator's dynamic-optimization problem, that is,

$$v^q\left(S\right) = \max_{e^q}\left\{\ E\left(\left(\alpha + \eta_1\right)e - \tfrac{1}{2}e^2\right) - \beta S + \delta E_{\eta_1} E_{\eta_2}\left(v\left(\eta_2\left(e + dS\right)\right)\right)\ \right\}, \tag{4.3}$$

where E_{η_i} is the conditional expectation operator with respect to η_i. As the objective and the state dynamics are both linear in the state variable, we make the informed guess that the value function is linear, that is, $v^q\left(S\right) = k_1^I S + k_2^I$, where the superscript I stands for full information. Substituting for $v^q\left(S\right)$ in (4.3), we obtain

$$k_1^I S + k_2^I = \max_e\left\{\ \left(\alpha + E\left(\eta_1\right)\right)e - \tfrac{1}{2}e^2 - \beta S + \delta E_{\eta_1} E_{\eta_2}\left(k_1^I\left(\eta_2\left(e + dS\right)\right) + k_2^I\right)\ \right\}.$$

The first-order optimality condition yields

$$e^{q,I} = \alpha + E(\eta_1) + \delta E_{\eta_1} E_{\eta_2}\left(k_1^I \eta_2\right), \tag{4.4}$$

where the superscript q refers to quotas. Proposition 4.1 gives the optimal emissions when the regulator uses a quantity-based policy.

Proposition 4.1 *If the regulator is informed and adopts a quotas regime, then the optimal emissions rule is given by*

$$e^{q,I} = \alpha + E(\eta_1) - \delta\beta \frac{E(\eta_2)}{1 - \delta d E(\eta_2)}. \tag{4.5}$$

Proof. By inserting the conjectured value function into the Bellman equation (4.3), and equating the coefficients in order of S, we obtain

$$\begin{aligned} k_1^I &= -\beta + \delta E_{\eta_1} E_{\eta_2}\left(k_1^I(\eta_2 d)\right), \\ k_2^I &= (\alpha + E(\eta_1))\, e - \frac{1}{2}e^2 + \delta E_{\eta_1} E_{\eta_2}\left(k_1^I(\eta_2 e) + k_2^I\right). \end{aligned}$$

The first equation above yields

$$k_1^I = -\frac{\beta}{1 - \delta d E(\eta_2)}.$$

Substituting in (4.4) gives the optimal emissions rule in the Proposition. Finally, substituting for k_1^I in k_2^I leads to

$$k_2^I = \frac{1}{2(1-\delta)}\left(\alpha + E(\eta_1) - \frac{\delta\beta E(\eta_2)}{1 - \delta d E(\eta_2)}\right)^2. \tag{4.6}$$

■

4.2.1.2 Taxes

The regulator announces the tax rate τ, and the firm optimizes its revenues minus taxes, that is, solves the following optimization problem:

$$\max_e \left\{(\alpha + \eta_1)\, e - \tau e - \frac{1}{2}e^2\right\}.$$

Assuming an interior solution, the first-order optimality condition yields the following reaction function for the firm:

$$e^\tau = \alpha - \tau + \eta_1. \tag{4.7}$$

Substituting in the regulator's welfare function, we get

$$W = E\left(\sum_t^\infty \delta^t \left(\frac{1}{2}(\alpha - \tau + \eta_1)^2 - \beta S_t\right)\right).$$

Denote by $v^{\tau,I}(S)$ the informed regulator's value function. Recalling that η_1 is an exogenous stochastic variable, the dynamic-programing equation then reads as follows:

$$v^{\tau,I}(S) = \max_e \left\{ \tfrac{1}{2}\left(\alpha^2 - \tau^2\right) + \alpha E(\eta_1) + \tfrac{1}{2} E\left(\eta_1^2\right) - \beta S + \delta E_{\eta_1} E_{\eta_2}\left(v^\tau\left(\eta_2\left(\alpha - \tau + \eta_1 + dS\right)\right)\right) \right\}. \tag{4.8}$$

Proposition 4.2 provides the emissions trajectory for this problem under a tax policy.

Proposition 4.2 *If the regulator is informed, then the optimal tax rate is given by*

$$\tau = \delta\beta \frac{E(\eta_2)}{1 - \delta d E(\eta_2)},$$

and the emissions rule is given by

$$e^{\tau,I} = \alpha + \eta_1 - \delta\beta \frac{E(\eta_2)}{1 - \delta d E(\eta_2)}. \tag{4.9}$$

Proof. Again, we assume that the value function is linear and given by $v^{\tau,I}(S) = \lambda_1^I S + \lambda_2^I$. By substitution in (4.8) and in (4.7), we get

$$\tau = -\delta E_{\eta_1} E_{\eta_2}\left(\lambda_1^I \eta_2\right). \tag{4.10}$$

Let $z = \alpha - \tau$. Then by equating coefficients in the Bellman equation in order of S, we obtain

$$\lambda_1^I = -\beta + \delta E_{\eta_1} E_{\eta_2}\left(\eta_2 \lambda_1^I d\right), \tag{4.11}$$

$$\lambda_2^I = \frac{1}{1-\delta}\left(\alpha\left(z + E(\eta_1)\right) - \frac{1}{2}\left(z^2 - E\left(\eta_1^2\right)\right) + \delta\lambda_1^I E(\eta_2)\left(z + E(\eta_1)\right)\right). \tag{4.12}$$

From (4.11), we get

$$\lambda_1^I = -\frac{\beta}{1 - \delta d E(\eta_2)}.$$

Inserting in (4.10), we get

$$\tau = \delta E_{\eta_1} E_{\eta_2}\left(\frac{\beta\eta_2}{1 - \delta d E(\eta_2)}\right) = \frac{\delta\beta E(\eta_2)}{1 - \delta d E(\eta_2)}.$$

Substituting for τ in (4.7), we get

$$\begin{aligned} e^\tau &= \alpha + \eta_1 - \delta E_{\eta_1} E_{\eta_2}\left(\frac{\beta\eta_2}{1 - \delta d E(\eta_2)}\right), \\ &= \alpha + \eta_1 - \delta\beta \frac{E(\eta_2)}{1 - \delta d E(\eta_2)}. \end{aligned}$$

Inserting λ_1^I in (4.12) leads to

$$\lambda_2^I = \frac{1}{2(1-\delta)}\left(\left(\alpha - \frac{\delta\beta E(\eta_2)}{1-\delta d E(\eta_2)}\right)^2 + E(\eta_1)\left(\alpha - \frac{\delta\beta E(\eta_2)}{1-\delta d E(\eta_2)}\right) + E\left(\eta_1^2\right)\right). \tag{4.13}$$

■

4.2.2 Uncertainty and Learning

In this section, we consider the case where θ_1^* and θ_2^* are unknown.[3] Denote by ξ the regulator's beliefs about these unknowns, based on the available information. At each step, the regulator learns about these unknowns and uses the received information to update his beliefs according to a general learning process (Bayes' rule). The new information consists of the observed-demand and pollution-stock values from the previous period.

We introduce learning for both random variables, η_1 and η_2. We assume that η_i, $i = 1, 2$, has a distribution ϕ_i with unknown parameter θ_i (this is the beliefs about θ_i^*). The function ϕ_i could be any general probability distribution function in which the information can be summarized by a finite-dimensional vector of sufficient statistics θ_i.

4.2.2.1 Quotas

Introducing uncertainty and learning does not alter the regulator's objective, which continues to be the maximization of the total social welfare, but it does affect his information structure, and consequently, the optimal decision. Let $v^{q,L}(S)$ be the regulator's value function when he uses quotas and learns according to the process described above. The dynamic-programing equation is as follows:

$$v^{q,L}(S) = \max_e \left\{ \begin{array}{l} E_{\eta_1}\left((\alpha+\eta_1)e - \frac{1}{2}e^2;\xi_1\right) - \beta S + \\ \delta E_{\eta_1}E_{\eta_2}\left(v\left(\eta_2\left(e + dS\right)\right);\hat{\xi_1},\hat{\xi_2}\right) \end{array} \right\}, \tag{4.14}$$

where ξ_i, $i = 1, 2$, is the regulator's beliefs about parameter θ_i, and $\hat{\xi_i}$ is his updated beliefs about the same parameter. Let $\mu_2(\theta_2)$ be the conditional expected value of the random variable $\tilde{\eta}_2$, i.e., $\mu_2(\theta_2) = E[\tilde{\eta}_2|\theta] = \int_{\mathcal{H}_2} \eta_2\phi_2(\theta_2)\,\mathbf{d}\eta_2$.

Proposition 4.3 *When there are uncertainties and learning and the regulator imposes quotas, the emissions rule is given by*

$$e^{q,L} = \alpha + E(\eta_1;\xi_1) - \delta\beta\int_\Theta \frac{\mu_2(\theta_2)}{1-\delta d\mu_2(\theta_2)}\xi_2(\theta_2)\,\mathbf{d}\theta_2. \tag{4.15}$$

Proof. See Appendix 4.6.1. ■

[3] We also can assume that the distribution functions of the η_1 and η_2 are unknown.

4.2.2.2 Taxes

Introducing uncertainty and learning does not change the firm's emissions decision rule given in Section 4.2.1.2, that is, $e = \alpha - \tau + \eta_1$. However, the regulator's emissions rule will be affected. The regulator's dynamic-programming equation is as follows:

$$v^{\tau,L}(S) = \max_e \left\{ \begin{array}{c} \alpha(\alpha-\tau) + \alpha E(\eta_1;\xi_1) - \frac{1}{2}(\alpha-\tau)^2 + \frac{1}{2}E\left(\eta_1^2\right) - \beta S + \\ \delta E_{\eta_1} E_{\eta_2}\left(v^{\tau}\left(\eta_2\left(\alpha-\tau+\eta_1+dS\right)\right);\hat{\xi}_1,\hat{\xi}_2\right) \end{array} \right\},$$

where $v^{\tau,L}(S)$ is the value function.

Proposition 4.4 *When there are uncertainties and learning, the optimal tax rate is given by*

$$\tau = -\delta E_{\eta_1} E_{\eta_2}\left(\lambda_1^L \eta_2\right) = \delta\beta \int \frac{\mu_2(\theta_2)}{1-\delta d\mu_2(\theta_2)} \xi_2(\theta_2)\, \mathbf{d}\theta_2,$$

and the emissions by

$$e^{\tau,L} = \alpha + \eta_1 - \delta\beta \int \frac{\mu_2(\theta_2)}{1-\delta d\mu_2(\theta_2)} \xi_2(\theta_2)\, \mathbf{d}\theta_2. \tag{4.16}$$

Proof. See Appendix 4.6.2. ■

4.3 Comparison

In this section, we first compare the emissions rule under the learning and no-learning assumption, and next, the results obtained under the two policies (taxes and quotas).

4.3.1 Learning versus Informed

Our results suggest that it is only ecological uncertainty that leads to a difference between the emissions controls under informed and learning assumptions. On the other hand, the sign of this difference depends on the regulator's belief bias with regard to ecological shock. This is formally presented in Proposition 4.5.

Proposition 4.5

1. *If the beliefs about the mean of ecological shock η_2 is unbiased, $\mu_2(\theta_2^*) = \int_{\mathcal{H}} \mu_2(\theta_2)\xi_2(\theta_2)\,\mathbf{d}\theta_2$ and $x \in \{q,\tau\}$, then*

$$e^{x,I} > e^{x,L}.$$

2. *If $\mu_2(\theta_2^*) < \int_{\mathcal{H}} \mu_2(\theta_2)\xi_2(\theta_2)\,\mathbf{d}\theta_2$, then*

$$e^{x,I} > e^{x,L}.$$

3. *If $\mu_2(\theta_2^*) > \int_{\Theta_2} \mu_2(\theta_2)\xi_2(\theta_2)\,\mathbf{d}\theta_2$, then the relation between emissions under learning and no-learning will be ambiguous.*

Proof. By Propositions 4.3 and 4.4 we have

$$e^{x,L}-e^{x,I}=\delta\beta\left(\frac{\mu_2\left(\theta_2^*\right)}{1-\delta d\mu_2\left(\theta_2^*\right)}-\int_{\Theta_2}\frac{\mu_2\left(\theta_2\right)}{1-\delta d\mu_2\left(\theta_2\right)}\xi_2\left(\theta_2\right)\mathbf{d}\theta_2\right).$$

Let $R=\frac{z}{1-\delta dz}$, with $z=\mu_2\left(\theta_2\right)$. The above equality is then equivalent to

$$e^{x,L}-e^{x,I}=\delta\beta\left(R\left(\mu_2\left(\theta_2\right)\right)-\int_{\Theta_2}R\left(\mu_2\left(\theta_2\right)\right)\xi(\theta_2)\mathbf{d}\theta_2\right).$$

Since $0<\mu_2\left(\theta_2\right),d,\delta<1$, R is then strictly increasing and convex in z. Therefore, by Jensen's inequality, we have

$$\int_{\Theta_2}R\left(\mu_2\left(\theta_2\right)\right)\xi_2(\theta_2)\mathbf{d}\theta_2>R\left(\int_{\Theta_2}\mu_2\left(\theta_2\right)\xi_2(\theta_2)\mathbf{d}\theta_2\right). \tag{4.17}$$

Given that R is strictly increasing in the admissible domain of $E\left(\eta_2\right)$, we have the following results in the different cases:

1. As $\mu_2\left(\theta_2^*\right)=\int_{\Theta_2}\mu_2\left(\theta_2\right)\xi_2\left(\theta_2\right)\mathbf{d}\theta_2$, we have

$$R\left(\mu_2\left(\theta_2^*\right)\right)=R\left(\int_{\Theta_2}\xi_2(\theta_2)\mu_2\left(\theta_2\right)\mathbf{d}\theta_2\right),$$

and using the inequality in (4.17), we conclude that $e^{x,I}-e^{x,L}>0$ or $e^{x,I}>e^{x,L}$.

2. Applying R to $\mu_2\left(\theta_2^*\right)<\int_{\Theta_2}\mu_2\left(\theta_2\right)\xi_2\left(\theta_2\right)\mathbf{d}\theta_2$, leads to

$$R\left(\mu_2\left(\theta_2\right)\right)<R\left(\int_{\Theta_2}\xi_2(\theta_2)\mu_2\left(\theta_2\right)\mathbf{d}\theta_2\right).$$

Using the inequality in (4.17), we conclude that we have $e^{x,I}-e^{x,L}>0$ or $e^{x,I}>e^{x,L}$.

3. Applying R to $\mu_2\left(\theta_2^*\right)>\int_{\Theta_2}\mu_2\left(\theta_2\right)\xi_2\left(\theta_2\right)\mathbf{d}\theta_2$, leads to

$$R\left(\mu_2\left(\theta_2\right)\right)>R\left(\int_{\Theta_2}\xi_2(\theta_2)\mu_2\left(\theta_2\right)\mathbf{d}\theta_2\right).$$

However, in this case, the inequality in (4.17) does not enable any conclusions about the ranking of $e^{x,I}$ and $e^{x,L}$.

■

The above proposition shows that if the regulator has unbiased beliefs about η_2, then anticipated learning decreases emissions. Besides, if the regulator overestimates the accumulated pollution, he still sets the emissions rule lower than the optimal rule. Finally, we obtain an ambiguous result when the regulator underestimates the accumulated pollution. The ambiguity here arises because two opposite sources influence the difference between $e^{x,L}$ and $e^{x,I}$. The first source, which is common in all cases, is uncertainty due to the anticipation of learning. Indeed, anticipation is, in of itself, a source of uncertainty, which leads the regulator to decrease the emissions rule. Mathematically, given the functional form of R and the domain of $\mu_2(\theta_2)$, we always have

$$\int_{\Theta_2} R(\mu_2(\theta_2))\,\xi_2(\theta_2)\mathbf{d}\theta_2 > R\left(\int_{\Theta_2} \xi_2(\theta_2)\mu_2(\theta_2)\,\mathbf{d}\theta_2\right).$$

The second source is the bias in beliefs, that is, the regulator underestimating the accumulated pollution. This underestimation leads to an increase in the emissions rule (mathematically $R(\mu_2(\theta_2)) > R\left(\int_{\Theta_2} \xi(\theta_2)\mu_2(\theta_2)\,\mathbf{d}\theta_2\right)$). Consequently, the net effect will depend on the relative weights of these two sources. Karp and Zhang (2006) find that anticipated learning (unambiguously) increases the optimal level of emissions. The difference between their results and ours is due to the fact that Karp and Zhang (2006) retain a fixed and given number of periods (n) during which learning is possible. In such a situation, as n increases, learning opportunities also increase, and this leads to an increase in the emissions rule. This effect is similar to the effect of an underestimation of the pollution stock, which is only one of the sources determining the difference between the emissions rules under the informed and anticipated-learning setups in our model.

4.3.2 Taxes versus Quotas

To compare the two policies, we use two yardsticks, namely, the emissions and social welfare generated by the two policies. By virtue of our linear state model, we expect that the expected trajectories of the pollutant stock and flow will be the same under taxes and quotas, even though the evolution of the stock is stochastic under taxes and remains deterministic under quotas. Part (1) of Proposition 4.6 presents this formally.

Proposition 4.6

1. *Taxes and quotas lead to the same expected emissions level, i.e.,* $E(e^{\tau}) = E(e^{q})$.

2. *Taxes yield a higher social welfare than do quotas. Indeed,*

$$v^{\tau}(S;\xi) - v^{q}(S;\xi) = \frac{1}{1-\delta}\left(\frac{1}{2} var(\eta_1;\xi)\right) > 0. \tag{4.18}$$

Proof. Part (1) is straightforward, as $e^{\tau} = e^{q} + \eta_1 - E(\eta_1)$. For part (2), see Appendix 4.6.3. ■

The above proposition shows that both policies lead to the same expected emissions levels. The comparison of the value functions under the two policies shows, unambiguously, that taxes dominate quotas, regardless of whether or not the regulator learns. The Proposition also shows that neither the policy ranking nor the magnitude of the payoff difference depends on the information state (x_{t-1}), learning process or ecological uncertainty, which is due to the linear damage cost. However, the difference in payoffs depends on the variance of the demand shocks: the higher the volatility in economic uncertainty, the higher the benefits of taxes over quotas. This result is in line with Karp and Zhang (2006, 2012); Newell and Pizer (2003) and to some extent, with Hoel and Karp (2002). In other words, we can say that, as per the second item of Proposition 4.6 that, as regulator's beliefs become more volatile, i.e., larger $var(\eta_1)$, then by choosing taxes he gives flexibility to the firm. Since the firm can observe the shock before it occurs, it makes an optimal decision on that basis, and this, in some way, alleviates the impact of the shock.

Note that despite the similarity of the learning and no-learning results, there is a subtle but important difference. Indeed, whereas $var(\eta_1)$ is constant in the no-learning setup, it is not under anticipated learning. More specifically, under anticipated learning, $var(\eta_1)$ changes stochastically as new information is received; thus, the advantage of taxes over quotas may change, e.g., due to the expectation of a shock.

4.3.3 Learning and More Optimistic Beliefs

In this section, we study the impact of a change in the regulator's environmental beliefs on his optimal decision. In the context of our problem, if the regulator expects a lower (higher) level of pollution stock at each period, i.e., he expects lower (higher) values for η_2, then he is more optimistic (pessimistic) about the environment. To be able to present such a change mathematically, we use the "first-order strict stochastic dominance" concept. This concept formalizes the idea of "better" versus "worse" situations, and we can therefore use it to present optimistic/pessimistic beliefs.

Definition 4.1 *Consider two probability density functions ξ^1 and ξ^2; then ξ^1 strictly first-order dominates ξ^2; and we write $\xi^1 \succ_1 \xi^2$, if for any increasing function $u:\mathbb{R}\to\mathbb{R}$, we have $\int u(x)\,\xi^1(x)\,dx > \int u(x)\,\xi^2(x)\,dx$.*

We only consider optimistic/pessimistic environmental beliefs, because beliefs related to market shocks do not appear in the emissions rule.

Keeping definition (3.1) in mind, let us assume that the regulator's beliefs with respect to $\tilde{\eta}_2$ change from ξ_2^1 to ξ_2^2, where $\xi_2^1 \succ_1 \xi_2^2$, and let $\mu_2'(\theta_2) > 0$. Intuitively, this means that the regulator's beliefs have changed in a way that he now expects lower values for the random variable η_2, $(\int \mu_2(x)\,\xi^1(x)\,dx > \int \mu_2(x)\,\xi^2(x)\,dx)$, i.e., under ξ_2^2 the regulator expects a lower pollution stock. A lower stock is equivalent to a slower deterioration of the environment, which leads to a lower environmental cost. In other words, the regulator becomes more optimistic about the environment by adopting ξ_2^2 instead of ξ_2^1. Proposition 3.8 shows, the impact of a change toward more optimistic beliefs in the regulator's beliefs.

Proposition 4.7 *f the regulator becomes more optimistic about the environment, i.e., either $\mu_2'(\theta_2) > 0$ and his beliefs change from ξ_2^1 to ξ_2^2, with $\xi_2^1 \succ_1 \xi_2^2$, or $\mu_2'(\theta_2) < 0$ and his beliefs change from ξ_2^1 to ξ_2^2, with $\xi_2^2 \succ_1 \xi_2^1$, then for $x = q, \tau$, we have*

$$e^{x,L}\left(S;\xi_2^2\right) < e^{x,L}\left(S;\xi_2^1\right).$$

Proof. Let $u(x) = R(\mu(x))$. Since R is increasing, we have

$$u' = \mu' R' \Rightarrow \begin{cases} u' > 0, & \text{if } \mu' > 0 \\ u' < 0, & \text{if } \mu' < 0 \end{cases}.$$

Taking into account Definition 4.1 and equations (4.15) and (4.16) implies the results. ■

4.4 Conclusion

In a deterministic environment, choosing between a price-based policy and a quantity-based policy is more a normative question than a positive one, because these policies are equivalent in terms of the resulting social welfare. However, introducing uncertainty can break this equivalency. In this paper, we study the choice of a pollution-control instrument in uncertain and learning environments. We contribute to the literature by considering ecological uncertainty in a dynamic model; we do this by letting the pollution stock evolution depend on a random variable. One other important feature of this study that makes it different from previous ones is that we study learning for both economic and environmental uncertainties.

We also consider information asymmetries between the regulator and firms. Studying the effect of ecological uncertainty on policy choice is new in this literature, where learning about market-related parameters has also been neglected. Another feature of this study is that it considers any general probability-distribution function for the model's random variables. We only assume that the information can be summarized by a finite-dimensional vector of sufficient statistics.

In our model setup, the expected level of emissions under taxes and quotas is the same. Comparing the total benefits induced by the two policies, we obtain that taxes lead to a higher social welfare than do quotas. We also obtain that only the variance of the market-related stochastic variable is important in this comparison. Indeed, the more volatile the economic uncertainty, the larger the difference in benefits between taxes over quotas. We also find that neither information, ecological uncertainty nor even the learning process affects the choice of policy. Another finding of our study is that the effect of anticipated learning on emissions rule for taxes and quotas depends on the regulator's belief bias with respect to the mean of the ecological shock. If the regulator has unbiased beliefs in this area, then anticipated learning decreases emissions. If the regulator overestimates the amount of pollutions that will accumulate on average in the next period, he sets the emissions rule lower than is optimal. But the effect of underestimating the average of emissions accumulation is ambiguous.

As in any modeling effort, the results are a by-product of some of the assumptions made. We are aware that the simplifying assumption that the damage cost is linear plays an important role here. Since we see this study as an exploratory one, it made sense to start with the simplest possible setting that would allow analytical results to be obtained. A welcome extension would be to adopt a non-linear damage cost to verify if the results remain (qualitatively) valid. Another interesting extension is to study the case where there are N firms behaving strategically.

4.5 References

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4.6 Appendix

4.6.1 Proof of Proposition 4.3

We conjecture that the value function related to problem (4.14) has a linear form, i.e., $v^q(S) = k_1(\xi)\, S + k_2(\xi)$, where $\xi = \{\xi_1, \xi_2\}$ presents the vector of beliefs. By substitution in (4.14), we have

$$k_1 S + k_2 = \max_e \left\{ \ (\alpha + E(\eta_1;\xi_1))\, e - \tfrac{1}{2}e^2 - \beta S + \delta E_{\eta_1} E_{\eta_2} \left(k_1 (\eta_2 (e + dS)) + k_2; \hat{\xi_1}, \hat{\xi_2} \right) \ \right\}.$$

Assuming an interior solution, the first-order condition yields

$$e^{q,L} = \alpha + E(\eta_1;\xi_1) + \delta E_{\eta_1} E_{\eta_2} \left(k_1 \eta_2; \hat{\xi_1}, \hat{\xi_2} \right).$$

Inserting this in (4.14), and equating the coefficients in order of S, we obtain

$$\begin{aligned} k_1 &= -\beta + \delta E_{\eta_1} E_{\eta_2} \left(k_1 (\eta_2 d); \hat{\xi_1}, \hat{\xi_2} \right), \\ k_2 &= (\alpha + E(\eta_1;\xi_1))\, e - \tfrac{1}{2}e^2 + \delta E_{\eta_1} E_{\eta_2} \left(k_1 \eta_2 e + k_2; \hat{\xi_1}, \hat{\xi_2} \right). \end{aligned} \tag{4.19}$$

Writing the extended form for k_1, we get

$$k_1 = -\beta + \delta \int_{\mathcal{H}_2} \left(\eta_2 d k_1; \hat{\xi_2} \right) \left[\int_\Theta \phi(\eta_2|\theta_2)\xi_2(\theta_2) \mathbf{d}\theta \right] \mathbf{d}\eta_2. \tag{4.20}$$

We conjecture that the solution for k_1 is

$$k_1(\xi) = -\beta \int_\Theta \frac{\xi_2(\theta_2)}{1 - \delta d \mu_2(\theta_2)} \mathbf{d}\theta_2.$$

Updating the conjectured value with respect to ξ_2 and inserting it in equation (4.20), we have

$$k_1(\xi) = -\beta \int_\Theta \frac{1}{1 - \delta d\mu_2(\theta_2)} \frac{\phi(\eta_2|\theta_2)\xi_2(\theta_2)}{\int_{\phi_2} \phi(\eta_2|x)\xi_2(x)\mathbf{d}x} \mathbf{d}\theta_2,$$

$$\begin{aligned} k_1 &= -\beta \left(1 + \delta d \int_{\mathcal{H}_2} \eta_2 \int_\Theta \tfrac{1}{1-\delta d\mu_2(\theta_2)} \tfrac{\phi(\eta_2|\theta_2)\xi(\theta_2)}{\int_{\phi_2} \phi(\eta_2|x)\xi_2(x)\mathbf{d}x} \mathbf{d}\theta_2 \left[\int_\Theta \phi(\eta_2|\theta_2)\xi_2(\theta_2)\mathbf{d}\theta_2 \right] \mathbf{d}\eta_2 \right), \\ &= -\beta \left(1 + \delta d \int_{\mathcal{H}_2} \eta_2 \int_\Theta \tfrac{\phi(\eta_2|\theta_2)\xi(\theta_2)}{1-\delta d\mu_2(\theta_2)} d\mathbf{d}\theta_2 \mathbf{d}\eta_2 \right), \\ &= -\beta \int_\Theta \tfrac{\xi_2(\theta_2)}{1-\delta d\mu_2(\theta_2)} \mathbf{d}\theta_2, \end{aligned}$$

which is equal to the postulated value. Inserting the value of $k_1(\xi)$ in (4.19), we obtain

$$\begin{aligned} k_2 &= \tfrac{1}{1-\delta} \left((\alpha + E(\eta_1;\xi_1))\, e^{q,L} - \tfrac{1}{2} \left(e^{q,L}\right)^2 + \delta E_{\eta_2} \left(k_1 \left(\eta_2 e^{q,L}\right); \hat{\xi_2} \right) \right), \\ e^{q,L} &= \alpha + E(\eta_1;\xi_1) - \delta\beta \int_\Theta \tfrac{\mu_2(\theta_2)}{1-\delta d\mu_2(\theta_2)} \xi_2(\theta_2)\, \mathbf{d}\theta_2. \end{aligned} \tag{4.21}$$

4.6.2 Proof of Proposition 4.4

Let $z = \alpha - \tau$. Our informed guess for $v^\tau(S)$ is $v^\tau(S) = \lambda_1(\xi) S + \lambda_2(\xi)$. By plugging this guess into the Bellman equation and the first-order condition, we have

$$v^{\tau,L}(S) = \max_e \left\{ \begin{array}{c} \alpha z + \alpha E(\eta_1;\xi_1) - \frac{1}{2}z^2 + \frac{1}{2}E\left(\eta_1^2\right) - \beta S + .. \\ ..\delta E_{\eta_1} E_{\eta_2} \left(v^\tau (\eta_2 (z + \eta_1 + dS)); \hat{\xi_1}, \hat{\xi_2} \right) \end{array} \right\},$$

$$z=\alpha+\delta\int_{\mathcal{H}_1}\int_{\mathcal{H}_2}\left(\lambda_1\eta_2;\hat{\xi_1},\hat{\xi_2}\right)\left[\int_\Theta\phi(\eta|\theta)\xi(\theta)\mathbf{d}\theta\right]\mathbf{d}\eta_1\left[\int_\Theta\phi(\eta|\theta)\xi(\theta)\mathbf{d}\theta\right]\mathbf{d}\eta_2,$$

$$z=\alpha+\delta E_{\eta_1}E_{\eta_2}\left(\lambda_1\eta_2;\hat{\xi_1},\hat{\xi_2}\right),$$
$$\tau=-\delta E_{\eta_1}E_{\eta_2}\left(\lambda_1\eta_2;\hat{\xi_1},\hat{\xi_2}\right).$$

Equating the coefficients in order of S yields

$$\lambda_1\left(\xi\right)=-\beta+\delta E_{\eta_1}E_{\eta_2}\left(\eta_2\lambda_1 d;\hat{\xi_1},\hat{\xi_2}\right),$$

$$\lambda_2\left(\xi\right)=\frac{1}{1-\delta}\left(\alpha\left(z+E\left(\eta_1;\xi_1\right)\right)-\frac{1}{2}\left(z^2-E\left(\eta_1^2;\xi_1\right)\right)+\delta E_{\eta_1}E_{\eta_2}\left(\lambda_1\eta_2\left(z+\eta_1\right);\hat{\xi_1},\hat{\xi_2}\right)\right).\tag{4.22}$$

Our conjecture for λ_1, would be

$$\lambda_1\left(\xi\right)=-\beta\int_\Theta\frac{\xi_2\left(\theta_2\right)}{1-\delta d\mu_2\left(\theta_2\right)}\mathbf{d}\theta_2.$$

It suffices to follow the same steps in the proof of Proposition 4.3 to get the result.

4.6.3 Proof of Proposition 4.6

We prove part (1) of the Proposition for two cases; full-information and learning. Given this result, the proof for the other parts would be straightforward.

4.6.3.1 Informed case

We saw that, whether the regulator uses taxes or quotas, the value function is a linear function of pollution accumulation. We already know that the state variable's coefficient is equal for both policies since

$$k_1^I=\lambda_1^I=-\frac{\beta}{1-\delta dE\left(\eta_2\right)}.$$

Therefore, to find the difference between these value functions, we need to compare their intercepts, i.e., the values of k_2^I and λ_2^I. Given equations (4.6) and (4.13), we have

$$\lambda_2^I-k_2^I=\frac{1}{1-\delta}\frac{1}{2}\left(E\left(\eta_1^2;\xi_1\right)-\left(E\left(\eta_1;\xi_1\right)\right)^2\right),$$

$$\lambda_2^I-k_2^I=\frac{1}{1-\delta}\left(\frac{1}{2}\left(var\left(\eta_1;\xi_1\right)\right)\right).$$

4.6.3.2 Learning

As in regulator's informed case, the coefficient of the state variable is equal under both policies and we have

$$z^L = \alpha - \delta\beta \int_\Theta \frac{\mu_2(\theta_2)}{1-\delta d\mu_2(\theta_2)} \xi_2(\theta_2)\, \mathbf{d}\theta_2 = e^{q,L} - E(\eta_1).$$

Inserting z^L in equation 4.22 gives

$$k_2(\xi) = \frac{1}{1-\delta}\left(\alpha e^{q,L} + E(\eta_1;\xi_1) - \frac{1}{2}\left(e^{q,L}\right)^2 + \delta E_{\eta_2}\left(k_1\left(\eta_2 e^{q,L}\right);\hat{\xi}_2\right)\right)$$

$$\begin{aligned}
\lambda_2(\xi) &= \frac{1}{1-\delta}\left(\alpha z - \frac{1}{2}z^2 + \alpha E(\eta_1;\xi_1) + \frac{1}{2}E\left(\eta_1^2;\xi_1\right) + \delta E_{\eta_1}E_{\eta_2}\left(\lambda_1\eta_2(z+\eta_1);\hat{\xi_1},\hat{\xi}_2\right)\right) \\
&= \frac{1}{1-\delta}\left(\alpha e^{q,L} - \frac{1}{2}\left(e^{q,L}\right)^2 + \frac{1}{2}\left(E\left(\eta_1^2;\xi_1\right) - \left(E(\eta_1;\xi_1)\right)^2\right) + \delta e^{q,L}E_{\eta_2}\left(\lambda_1(\eta_2);\hat{\xi}_2\right)\right)
\end{aligned}$$

Consequently, we have

$$\begin{aligned}
\lambda_2(\xi) - k_2(\xi) &= \frac{1}{1-\delta}\left(\frac{1}{2}\left(E\left(\eta_1^2;\xi_1\right) - \left(E(\eta_1;\xi_1)\right)^2\right)\right), \\
&= \frac{1}{1-\delta}\left(\frac{1}{2}\left(var(\eta_1;\xi_1)\right)\right).
\end{aligned}$$

Chapter 5

General Conclusion

Climate change is one of the major international issues and tackling with this problem needs the commitment of all countries and also requires internal individual attempts. This is a paper-based thesis which is composed of three articles related to emissions control problem in international and local levels. The first paper is an exploratory attempt to account for the complex idea of evolving environmental concerns. For many developing countries, urgent economic problems have priority to environmental issues. However, as time passes, due to the revelation of environmental scarcity, citizens awareness about environmental problems, and improvements in economic conditions and welfare, policy makers of developing countries will quite naturally have to consider the environment in their policies. To reflect this evolution in an international pollution-control setup, we proposed an environmental-damage cost that is not only a function of the accumulated pollution but also a function of accumulated income and time (as proxies for level of awareness, infrastructures provision and welfare level).

Providing new insights in the environmental behavior of developing countries, and in clarifying the difficulties and challenges that may arise in achieving cooperation are the main contributions of this paper. We also show that asking developing countries to take environmental cost into account sooner is not necessarily the best course of action. Finally, we characterized the conditions under which cooperation is easier to be made.

Many extensions can be considered in future works, including: (i) a more sophisticated modeling of the damage costs; (ii) a modeling approach where the mode of play can endogenously change from noncooperation to cooperation; and finally (iii) the inclusion of more players to account for different levels of development within the group of developing countries.

The second paper embeds ecological uncertainty and learning in an international pollution control problem. We consider an N-player dynamic game of international pollution control which players are facing uncertainty due to their lack of information about a parameter related to the environment. We analyze the behavior of these players under three different informational assumptions: (i) informed; (ii) adaptive learning; and (iii) anticipated learning.

The main contribution of this paper is providing insight about the emission strategies in presence of uncertainty and learning. While this is somehow accepted that uncertainty alleviate common problems, by our analytical results we proved that according to the model setup, introduction of uncertainty may or may not alleviate the international pollution problem. Moreover, in this research effects of different sources of uncertainty are decomposed and the conditions that make any of these sources amplify or weaken each other are presented. We also study the effects of changes in players beliefs, either toward more optimistic/pessimistic beliefs or more sense of riskiness. Our results suggest that while more sense of riskiness decreases total emissions, the effects of a change toward more optimistic beliefs depends on the functional form of the distribution function of the unknown variable.

One possible extension to this work is taking into account other sources of uncertainties and analyzing how the results might be affected. As an example we considered another case that the motion is deterministic, but players damage cost is affected by a random variable, i.e. we assumed that the marginal cost of pollution is random. Our results suggest that behavior of a learner player is equivalent to an adaptive one. This case is a simple example which shows depending on the model and the source of uncertainty, we may have cases that decision of an adaptive learner exactly matches to the Bayesian learner case. Other possible extensions for future works could be considering a non-linear damage cost function and also introducing an active learning process.

Finally, in the third paper we compare taxes and quotas as two alternative policies for a regulator to control emissions. In a deterministic environment these policies are equivalent with regard to their social benefits. But introducing uncertainty could break this equivalency. In this paper we study pollution control instrument choice in a dynamic and uncertain environment when the regulator is a Bayesian learner. We contribute to the literature by considering ecological uncertainty in a dynamic model. In other words, in our model motion of the pollutions is assumed to be affected by a random variable. We also consider information asymmetries between the regulator and firms.

In our model setup, the expected level of emissions equals under taxes and quotas. But comparison of the total benefits related to these policies suggests that taxes dominate quotas which means taxes will provide more social payoffs. It also suggests that only variance of market related stochastic variable is important and affects this relative efficiency. Indeed, the more volatile economic uncertainty, the more benefits of taxes over quotas. We also find that neither information, nor ecological uncertainty and even learning process affect the choice of policy. Another finding of our study is that the effect of anticipated learning on emissions rule for taxes and quotas depends on bias of the regulator's beliefs with respect to the mean of ecological shock.

One possible extension that can be envisioned for the third paper is considering a more sophisticated damage cost. Another interesting extension would be considering firms that have partial or full market power.