HEC MONTRÉAL

Affiliée à l'Université de Montréal

Three essays on sustainable forest management

par

Pablo Andrés Domenech

Thèse présentée à la Faculté des études supérieures

en vue de l'obtention du grade de Philosophiae Doctor (Ph.D.) en administration

option méthodes quantitatives

Montréal, Québec, Canada

Février 2012

HEC MONTRÉAL

Affiliée à l'Université de Montréal

Cette thèse intitulée:

Three essays on sustainable forest management

présentée par

Pablo Andrés Domenech

a été évaluée par un jury composé des personnes suivantes:

Pierre-Olivier Pineau
président-rapporteur

Georges Zaccour
co-directeur de recherche

Guiomar Martín-Herrán
co-directrice de recherche

Hassan Benchekroun
membre du jury

Katheline Schubert
examinatrice externe

Germain Belzile
représentant du doyen

Sommaire

Cette thèse traite du problème de la déforestation excessive et des externalités négatives qui en résultent. L'objectif des trois essais présentés est d'identifier et de mettre en évidence les causes de la déforestation ainsi que de proposer un nouvel ensemble de politiques forestières compatibles avec un développement économique durable.

Le problème de la déforestation est étudié d'un point de vue microéconomique et, par la suite, ses effets à grande échelle sur l'accumulation des gaz à effet de serre sont analysés. On insiste sur la nécessité de traiter simultanément le problème de l'accumulation de gaz à de effet de serre et celui de la déforestation et on propose un ensemble de stratégies durables.

La notion du développement durable de la forêt est présente tout au long de ces trois essais mais les objets étudiés ainsi que les techniques utilisées varient. Dans le premier essai on analyse le problème de la disparition de la forêt en modélisant le système productif d'une communauté traditionnelle représentative - Les Tandroy à Madagascar- dont la subsistance dépend entièrement de l'utilisation et de l'exploitation de la forêt. Les Tandroy déboisent pour survivre et l'impact négatif de leurs activités sur la forêt augmente au fur et à mesure que leur population croît.

On montre que leur système productif n'est pas durable et on en analyse les causes. On propose un ensemble de politiques forestières qui doivent être implémentées pour assurer la viabilité de leur système.

Dans le deuxième essai on analyse le problème de la déforestation à une échelle globale et on étudie le lien qu'il existe avec un autre problème : l'accumulation de gaz à effet de serre (GES) dans l'atmosphère. On montre qu'actuellement la déforestation tout comme les émissions sont loin d'être viables, et un certain nombre de scenarios alternatifs sont considérés. En utilisant la théorie de la viabilité on obtient des conclusions sur le rapport à long terme entre la déforestation et l'accumulation de GES ainsi qu'un ensemble de politiques dites durables. L'importance qu'ont les transferts monétaires pour achever ces objectifs est également mise en évidence.

Dans le dernier essai on travaille dans un cadre de théorie des jeux avec un modèle à deux agents où on considère de façon explicite, cette foi ci, les coûts et bénéfices de différentes politiques forestières.

On obtient les politiques optimales pour différents scenarios et on montre dans quelles conditions, les résultats obtenus sont durables. La conservation de la forêt implique nécessairement l'existence de coopération.

Mots clés: Forêt, durabilité, théorie de la viabilité, théorie des jeux, jeux dynamiques, contrôle optimal, émissions, transferts, déforestation, Madagascar.

Summary

This dissertation deals with the problem of forest depletion and the negative externalities that result from it. The objective of these three essays is to indentify and account for the causes that are responsible for deforestation and propose a new set of forest-management strategies that are compatible with a sustainable economic development.

We address the problem of deforestation from a microeconomic point of view and analyze its large scale effects on the accumulation of greenhouse gases in the atmosphere. We argue on the need to address the problem of greenhouse gases accumulation and forest depletion jointly and give a number of policy recommendations as to what strategies yield sustainable outcomes.

The notion of sustainable forest management is present throughout the three essays but the objects studied as well as the techniques used vary. In the first essay we capture the problem of forest depletion by modelling the productive system of a representative traditional community -The Tandroy in Madagascar- whose subsistence entirely relies on the use and exploitation of the forest. The Tandroy deforest to subsist and, as their population expands, the negative impact that their activities have on the forest increases. We show that their productive system is not sustainable, analyze the causes and propose a set of sustainable forest-management strategies that need to be implemented in order to slow down the rapid deforestation witnessed in the region.

In the second essay we analyze the problem of deforestation at a global scale and study its link with another environmental problem: the accumulation of greenhouse gases (GHGs) in the atmosphere. We show that current net deforestation and world emissions are far from being sustainable and consider a number of alternative policy scenarios. By means of viability theory techniques we obtain insights on the long run impact of global forest depletion on GHG accumulation and retrieve a set of viable policies. We show the importance of monetary transfers to achieve these goals.

In the third essay we propose a game theoretic framework with a two agent model where we account for both the costs and the benefits of applying different forest policies. We obtain the optimal policies for different scenarios and show under which conditions the outcomes are also sustainable. Forest conservation necessarily requires cooperation.

Key words: Forest management, sustainability, viability theory, game theory, dynamic games, optimal control, emissions, transfers, deforestation, Madagascar.

Resumen

Esta tesis trata el problema de la desaparición de los bosques y de las externalidades negativas que de ella resultan. El objetivo de estos tres ensayos es identificar las causas de la deforestación y al mismo tiempo proponer un conjunto de estrategias forestales que sean compatibles con un desarrollo económico sostenible.

Abordamos el problema de la deforestación desde un punto de vista microeconómico y se analizan sus efectos a gran escala sobre la acumulación atmosférica de gases de efecto invernadero. Defendemos la necesidad de resolver el problema de la acumulación de gases de efecto invernadero y de la desaparición de los bosques de manera conjunta y realizamos una serie de recomendaciones sobre qué estrategias generan resultados sostenibles.

La noción de desarrollo sostenible de los bosques está presente a lo largo de los tres ensayos; pero, tanto los objetivos estudiados como la metodología utilizada varían. En el primer ensayo se estudia el problema de la desaparición de los bosques. Para ello se modeliza el sistema productivo de una comunidad tradicional -el pueblo Tandroy en Madagascar- cuya subsistencia depende del uso y la explotación del bosque. Los Tandroy deforestan para subsistir y a medida que su población aumenta, aumenta también el impacto negativo de sus actividades sobre el bosque. En nuestro estudio mostramos que su sistema productivo no es sostenible, analizamos las causas y proponemos un conjunto de políticas forestales sostenibles que han de ser aplicadas para reducir la deforestación en la región.

En el segundo ensayo analizamos el problema de la deforestación a escala global y estudiamos su relación con otro problema medioambiental: la acumulación atmosférica de gases de efecto invernadero. Mostramos que la tasa de deforestación y las emisiones mundiales están lejos de ser sostenibles y consideramos una serie de escenarios alternativos. Además mostramos la importancia que tienen las transferencias monetarias para lograr estos objetivos.

En el tercer ensayo proponemos un modelo de dos agentes dentro de un marco de teoría de juegos que nos permite dar cuenta de los costes y beneficios de aplicar diferentes políticas forestales. Obtenemos las políticas óptimas para diferentes escenarios (cooperativo y no-cooperativo) y mostramos en qué casos los resultados, además de óptimos, son sostenibles. Nuestros resultados indican que para lograr conservar los bosques es necesario que exista cooperación.

Palabras clave: Bosques, sostenibilidad, teoría de la viabilidad, teoría de juegos, juegos dinámicos, control óptimo, emisiones, transferencias, deforestación, Madagascar.

Table of Contents

List of Tables

List of Figures

List of Abbreviations

BAU	Business as usual.
DEF-ME	Direction des Eaux et Forêts, Ministère de l'Environnement de Madagascar.
EIA	Energy Information Administration.
FAO	Food and Agriculture Organization of the United Nations.
FO	Forest owners.
GDP	Gross domestic product.
GHG	Greenhouse gas.
IMF	International Monetary Fund.
INSTAT	Institut National de la Statistique de Madagascar.
IPCC	Intergovernmental Panel on Climate Change.
NFO	Non-forest owners.
NGO	Non-governmental organization.
NIOSH	National Institute for Occupational Safety and Health.
NOAA	National Oceanic & Atmospheric Administration.
PPM	Parts per million.
PPMV	Parts per million by volume.
RM	République de Madagascar.
TCU	Tropical cattle unit.
USAID	United States Agency for International Development.
WB	World Bank.
WCED	World Commission on Environment and Development.
WFP	World Food Programme.

To my parents, for all the sacrifices they made

so that Lucía and I had the best possible education.

Acknowledgements

I would like to thank my thesis supervisor, Georges Zaccour, for giving me the possibility to come to Canada and work in such a stimulating and friendly environment. I also want to thank him for his moral and economic support, for his great availability and sympathy, for his humanity and his unforgettably pedagogical lessons.

I am very grateful to my co-supervisor, Guiomar Martín-Herrán, for her permanent help and availability, as well as for her patience, kindness and understanding nature. She encouraged me to take this road and I will always feel in debt towards her.

I would like to thank Patrick Saint-Pierre for all that he has taught me, for making me feel part of his family and for being an example of what a mentor, a father and a human being should be.

I am very thankful to GERAD for supporting my research and to its staff for their technical help and kindness. I would also like to thank all my colleagues and student companions at GERAD and HEC for their valuable company and friendship.

Completing a Ph.D. is a lengthy process and many people have shared my journey throughout these years. I am very grateful to Hugo Pous, Alessandro Zanarini and Anthony Guillou for always being there for me. I also want to thank all the members of GCC football team for the wonderful moments we shared.

Very special thanks to my Spanish friends Miguel Martínez García, David Pérez Gustavo, David Rodríguez Solás and Manuel Navarro Tébar with whom I shared the joy of feeling at home while being abroad. And to my colleagues Kakeu Kegne Justin Johnson, Tania Briceño and David Salgado Locela who always offered me their advice and friendship.

I cannot miss thanking all the members of my family for always encouraging me to pursue my dreams and support me all along: To Lucía, my parents, Cosé, Maína, Luis, Amalia, David, Rocío and to Alessia for being beside me and supporting me all these years.

General Introduction

Forests provide us with a number of services and intangible goods, and according to FAO (2006): "Forests host biological diversity, mitigate climate change, protect land and water resources, provide recreation facilities, improve air quality and help alleviate poverty." Forests also provide shelter, nourish people and constitute the main source of income in many rural areas. However, total world forest area is decreasing at an alarming rate and every year an area of forest equivalent to the size of Costa Rica is lost (FAO, 2010).

According to FAO, rapid deforestation is related to a number of factors such as rural poverty, population increase, forest ownership, agricultural expansion, excessive -and often illegal- logging, industrial development and increasing demand of forest products. The study of the link between deforestation and all these factors has generated a vast literature that can be broadly divided into three groups: the theoretical literature, the empirical literature and a third branch that we could define as the *case-study* literature.

The theoretical literature has proposed models that explain the link between deforestation and one or several of these deforestation-related factors. Some well known examples are: Angelsen (1999) on population expansion and property rights, Larson and Bromley (1990) on property rights, Pattanayak et al. (2006) on poverty, Barbier and Burgess (1997), Ehui et al (1990) and Van Soest and Lensink (2000) on agricultural expansion, Barbier et al. (1995) on industrial development and demand of forest products.

This theoretical literature has been well supported by the empirical literature which has focused on explaining deforestation using these same factors. See e.g., Deacon (1999) on poverty, Cropper and Griffiths (1994) on poverty and population increase, Panayotou and Sungsuwan (1994) on forest ownership and demand of forest products, Reis and Guzmán (1994) on population, agricultural expansion and logging.

The papers in these two strands are complementary and identify the qualitative aspects that relate to deforestation. The general conclusion arising from them is that deforestation is a very complex problem where, typically, several of these factors are present and their individual contribution is hard to disentangle.

Because deforestation is a complex phenomenon and cannot be de-coupled from other social, cultural or political factors, the so called *case-study* literature has proved particularly useful at contextualizing the problem and at identifying other deforestation-related factors. Some notable examples are Sunderlin and Rodríguez (1996) for the context of Honduras, Reis and Guzmán (1994) for the Amazonia, Bernard (2011) for central Madagascar, Ehui and Hertel (1989) for the Ivory Coast, Cropper and Griffiths (1999) for Thailand, etc.

The first main objective of our research is to gain insight on how all these factors relate to deforestation, to model deforestation and to explain its causes. Our approach borrows aspects from these three branches of the literature. First, we have built a theoretical model to explain the evolution of forest surface area. Second, our theoretical model also addresses the link between deforestation and poverty, population expansion, property rights, agricultural expansion and demand of forest products. Third, we have estimated the relevant variables and parameters of our theoretical model and obtained both qualitative and quantitative conclusions. Fourth, we have followed a case-study approach to analyze and explain deforestation at the microeconomic level.

The second main objective of this research is to model the consequences from deforestation at the macroeconomic level. Forest destruction is not just a local problem, it has rather become an issue of great international concern for several reasons: First, world forests have great ecological value both as carbon tanks and carbon sinks. Second, forests host much of the world's biodiversity. Third, forests protect land and water resources and help prevent land erosion and desertification. Here, we concentrate mainly on the first of these three aspects even if the other two are also considered.

In this thesis we account for the ecological value that forests have in mitigating climate change and study the link between deforestation and long-term accumulation of greenhouse gases (GHGs) in the atmosphere.

Forests are a major carbon tank and store 283 gigatons of carbon in living matter, and more than 600 gigatons if we include the carbon in the soil below (FAO, 2006). This amount constitutes 35% of the approximate 800 gigatons of carbon (3000 $GtCO_2$) in the atmosphere. Forests also behave as carbon sinks. During photosynthesis trees absorb (i.e., sequestrate) carbon that they convert into organic matter (wood mainly). As trees grow in volume, they sequestrate a proportional amount of carbon from the atmosphere, and roughly half the dry weight of biomass is carbon (IPCC, 2000). Current anthropogenic emissions are approximately 9 gigatons of carbon and increasing. Of these, forests sequestrate worldwide, in average, 2.6 GtC every year (Le Quéré et al., 2009). However, carbon sequestration by the world's forests may be affected in the long run by forest depletion. A tree that is cut cannot grow and thus cannot sequester carbon. Hence, a significant reduction in the forest stock, may bring a reduction in total carbon sequestration.

The third main objective of this thesis is to analyze deforestation from a sustainable viewpoint. The notion of sustainability is present throughout this work. We have made use of the seminal definition of sustainable development given by the Brundtland Comission, according to which development is said to be sustainable if it "meet(s) the needs of the present generations without compromising the ability of future generations to meet their own needs." (WCED, 1987) We have articulated this definition in the form of (economic and environmental) constraints that measure the fulfilment or not of those needs.

The methodology used in this thesis serves well to the purpose of capturing this notion of sustainability. We call upon viability theory (Aubin, 1991) because its framework is particularly well suited to give answers to the following two simple questions:

- Is a sustainable development possible for some given economic and environmental requirements?

- Is this sustainable development possible for the current economic and environmental state of the system?

Viability theory was developed as a tool to analyze the evolution of complex dynamic and constrained systems and has been used for economic and environmental applications in the field of natural and renewable resources. Some of these applications are: Aubin et al. (2004) on renewable resources; Béné and Doyen (2008) on biodiversity, Bernard (2011) on deforestation and poverty issues; De Lara et al. (2007), Doyen et al. (2007) and Martinet and Blanchard (2009) on fisheries and marine ecosystems; Martinet and Doyen (2007) on exhaustible resources.

The first two essays of this thesis use viability theory. In the first essay we have concentrated on the causes of deforestation. We analyze the impact that rural poverty, increasing population, forest ownership and agricultural expansion create on deforestation. In particular, we have considered the traditional economic system of the Tandroy people in Madagascar for theirs is a paradigmatic case of a poor and rural society whose economy heavily relies on the forest. In Tandroy society, trees and more generally forests are seen and used as a common property resource and the Tandroy cut and sell them as an additional source of revenue.

The Tandroy's production system is characterized by the slash-and-burn farming agriculture (*hatsake*) and the burning of cacti (*ororaketa*) used as fodder to feed the livestock.

These two practices, together with timber harvest, fuelwood and charcoal production induce land degradation and forest depletion. Deforested land is used to grow maize, manioc, sweet potatoes and a variety of legumes. The combined effect of an increasing Tandroy population and its reluctance to change their traditional lifestyle and production system has led to great ecological pressure on the Androy forest.

We have built a theoretical model to account for forest evolution, the various causes of deforestation at the microeconomic level and the specificity of Tandroy's productive system. We have introduced a number of economic and environmental requirements and, with the help of viability theory, obtained valuable insights on the reasons why their economic system is not sustainable. We have analyzed what changes are necessary to achieve a sustainable development and proposed some alternative ways of achieving those requirements.

In the second essay of this thesis we move away from the causes of deforestation to analyze some of its consequences. To do so we enlarge our scope and consider greenhouse gases accumulation. Forests prevent or delay climate change by absorbing carbon. We focus on the long term impact of forest depletion on the atmospheric accumulation of GHGs. To do so we have modelled the dynamics of three key variables, namely, GHG emissions, the stock of emissions in the atmosphere and world's total forest area. In continuation with the first essay, we examine the trade-offs faced to comply with environmental and economic objectives. We try to give answers to the following research questions:

- Is the current emissions-deforestation-afforestation model sustainable?

- Which policies are part of a sustainable development?

These questions are related to the notion of sustainability and, again in this case, we make use of viability theory to give answers to them. We find that forest depletion has an important impact on GHG accumulation in the long run and that the current emissions-deforestation-afforestation model is far from being sustainable. A detailed sensitivity analysis is performed to determine which are the policy changes needed to achieve a sustainable development and it is shown that monetary transfers may ease the environmental problem in some cases.

In the two first essays we focus on the feasibility or achievability of a set of desirable economic and environmental objectives. This being said, the results obtained somehow presuppose the existence of a central planner that can drive the system in the desirable direction. In the third essay we move away from this optics and clearly differentiate the existence of two types of agents: forest owners and non forest owners. These two types of agents have somewhat conflicting objectives. On the one hand forest owners care about the revenues that they obtain from forest exploitation. On the other non-forest owners produce goods and emit GHGs as a by-product but also suffer from GHG accumulation. This specification allows capturing the negative externality that deforestation creates on non-forest owners due to the reduced carbon sequestration induced by forest depletion.

The methodology used in this third essay is different. Both forest owners and non-forest owners optimize their payoffs which are mutually influenced. To account for this we make use of dynamic game theory for it offers a natural environment to model this interaction. Dynamic game theory has been extensively used to model deforestation problems, see e.g., Van Soest and Lensink (2000), Fredj et al. (2004), Fredj et al. (2006), Martín-Herrán and Tidball (2005) and Martín-Herrán et al. (2006). In all these cases there is an environmentally aware player or *donor* who is willing to compensate the environmentally unaware player, *forest owner*, for implementing a *greener* environmental policy (lower deforestation). In all these cases the reasons why the *donor* is willing to compensate forest owners are somewhat abstract. The novelty of our third essay with respect to the existing literature is that we explicitly model the evolution of the accumulation of the GHGs in the atmosphere and relate it to the evolution of forests worldwide. In so doing we are explicitly linking these two issues and capturing the negative externality that deforestation creates on atmospheric GHG accumulation. As a consequence the philanthropy of the donor is not taken for granted and depends, rather, on the state of the system.

Part of the novelty of this essay consists precisely on building a bridge between the existing literature on deforestation and that on the control of emissions. Within this framework we answer to the following research questions:

- What are the optimal non-cooperative emissions and net deforestation policies?

- Are they sustainable?

- Can cooperation bring economic and environmental gains?

We show that the optimal non-cooperative strategies are far from being sustainable and analyze in which cases the cooperative solution is strictly welfare improving and sustainable from the environmental point of view.

To summarize, this thesis contributes to the existing literature by analyzing numerous causes and some of the consequences of deforestation and by modelling these processes both at the local (for the case of the Androy Region in Southern Madagascar) and global level. At the same time this thesis goes beyond by giving real figures to forest area, population, agricultural expansion, deforestation, demand for forest products, etc. We have also defined a number of economic and environmental objectives and, working with these real figures, investigated the conditions under which these objectives can be simultaneously attained. Our results show the importance of addressing both types of objectives jointly.

We address this complex environmental problem by dividing it into three parts. The first part is dedicated to modelling and understanding the relationship between poverty and deforestation. The second is devoted to accounting for the link between deforestation, emissions and greenhouse gases accumulation. In the third we consider the colliding objectives in terms of forest policy that different economic agents have. We also consider the negative environmental externalities that some agents create on others and explore under what conditions a decentralized but coordinated solution can yield both optimal and sustainable outcomes. These three parts correspond each to one of the three essays presented.

Bibliography

[1] Angelsen, A. (1999). Agricultural expansion and deforestation: modelling the impact of population, market forces and property rights. *Journal of Develoment Economics*, 58, 185-218.

[2] Aubin, J. P. (1991). *Viability Theory, Systems and Control: Foundations and Applications*. Boston: Birkhäuser.

[3] Aubin, J. P., Kropp, J., Scheffran J., & Saint-Pierre, P. (2004). *An Introduction to Viability Theory and Management of Renewable Resources, Process of Artificial Intelligence in Sustainable Science*. Commack: Nova.

[4] Barbier, E. B., Bockstael, N., Burgess, J. C., & Stand, I. (1995). The linkages between the timber trade and tropical deforestation-Indonesia. *World Economy, 18*(3), 411-442.

[5] Barbier, E. B., & Burgess, J. C. (1997). The economics of tropical forest land use options. *Land Economics, 73*(2),174-195

[6] Béné, C., & Doyen, L. (2008). Contribution values of biodiversity to ecosystem performances: A viability perspective. *Ecological Economics, 68*, 14-23.

[7] Bernard, C. (2011). *La théorie de la viabilité au service de la modélisation mathématique du développement durable. Application au cas de la forêt humide de Madagascar.* Université Blaise Pascal.

[8] Bruckner, T., Petschel-Held, G., Leimback, M., & Toth, F. (2003). Methodological aspects of the tolerable windows approach. *Climatic Change, 56*(1-2), 73-89.

[9] Cropper, M., & Griffiths, C. (1999). Roads, population pressures, and deforestation in Thailand, 1976-1989. *Land Economics, 75*(1), 58-73.

[10] Cropper, M., & Griffiths, C. (1994). The interaction of population growth and environmental quality, *American Economic Review, 84*(2), 250-254.

[11] De Lara, M., Doyen, L., Guilbaud, T., & Rochet, M. J. (2007). Is a management framework based on spawning-stock biomass indicators sustainable? A viability approach. *ICES Journal of Marine Science, 64*(4), 761-767.

[12] Deacon, R. T. (1999). Deforestation and ownership: evidence from historical accounts and contemporary data. *Land Economics, 75*(3), 341-359.

[13] Doyen, L., De Lara, M., Ferraris, J., & Pelletier, D. (2007). Sustainability of exploited marine ecosystems through protected areas: A viability model and a coral reef case study. *Ecological Modelling, 208*(2-4), 353-366.

[14] Ehui, S. K., & Hertel, T. W. (1989). Deforestation and agricultural productivity in the Côte d'Ivoire. *American Journal of Agricultural Economics, 71*(3), 703-711.

[15] Ehui, S. K., Hertel, T. W., & Preckel, P. V. (1990). Forest resource depletion, soil dynamics, and agricultural development in the tropics. *Journal of Economics and Environmental Management, 18*(2), 136-154.

[16] Food and Agriculture Organization of the United Nations (FAO) (2006). *Forest Resources Assessment 2005.*

[17] Food and Agriculture Organization of the United Nations (FAO) (2006). *Forest Resources Assessment 2010.*

[18] Fredj, K., Martín-Herrán, G., & Zaccour, G. (2004). Slowing deforestation rate through subsidies: A differential game. *Automatica, 40*(2), 301-309.

[19] Fredj, K., Martín-Herrán, G., & Zaccour, G. (2006). Incentive mechanisms to enforce sustainable forest exploitation. *Environmental Modeling & Assessment, 11*(2), 145-156.

[20] Intergovernmental Panel on Climate Change (IPCC) (2000). *Land use, land-use change and forestry. Special report.* Cambridge: Cambridge University Press.

[21] Larson, B. A., & Bromley, D. W. (1990). Property rights, externalities and resource degradation: locating tragedy. *Journal of Development Economics, 33*(2), 235-260.

[22] Le Quéré, C., Raupach, M. R., Canadell, J. G., Marland, G., et al. (2009). Trends in the sources and sinks of carbon dioxide. *Nature Geoscience, 2*, 831-836.

[23] Martín-Herrán, G., Cartigny, P., Motte, E., & Tidball, M. (2006). Deforestation and foreign transfers: a Stackelberg differential game approach. *Computers & Operations Research, 33*(2), 386-400.

[24] Martín-Herrán, & G., Tidball, M. (2005). Transfer mechanisms inducing a sustainable forest exploitation. In C. Deissenberg & R. Hartl (Eds.) *Optimal Control and Dynamic Games: Applications in Finance* (pp. 85-103). *Management Science and Economics.* The Netherlands: Springer.

[25] Martinet, V., & Blanchard, F. (2009). Fishery externalities and biodiversity: Trade-offs between the viability of shrimp trawling and the conservation of Frigatebirds in French Guiana. *Ecological Economics, 68*, 2960-2968.

[26] Martinet, V., & Doyen, L. (2007). Sustainability of an economy with an exhaustible resource: A viable control approach. *Resource and Energy Economics, 29*(1), 17-39.

[27] Panayotou, T., & Sungsuwan, S. (1994). An econometric Analysis of the causes of tropical deforestation: the case of Northeast Thailand. In K. Brown and D. W. Pearce (Eds.) *The Causes of Tropical Deforestation: The Economic and Statistical Analysis of Factors Giving Rise to the Loss of Tropical Forests* (pp. 192-210). London: University College London Press.

[28] Pattanayak, S., Dickinson, K., Corey, C., Murray, B., Sills, E., Kramer., R. (2006) *Deforestation, malaria and poverty. Sustainability: Science, Practice, & Policy, 2*(2), 45-56.

[29] Reis, E., & Guzmán, R., (1994). An econometric model of Amazonian deforestation. In K. Brown and D. W. Pearce (Eds.), *The Causes of Tropical Deforestation.* London: University College London Press.

[30] Sunderlin, W. D. & Rodríguez, J. A. (1996). Cattle, Broadleaf Forests and the Agricultural Modernization Law of Honduras. CIFOR Occasional Paper No. 7, Center for International Forestry Research, Jakarta, Indonesia.

[31] Van Soest, D., & Lensink, R. (2000). Foreign transfers and tropical deforestation: what terms of conditionality? *American Journal of Agricultural Economics, 82*(2), 389-399.

[32] World Commission on Environment and Development (1987). *Our Common Future*. Oxford: Oxford University Press.

Essay 1

Sustainability of the dry forest in Androy: A viability analysis

Essay information

This essay was submitted for publication to *Ecological Economics*:

Andrés-Domenech, P., Fanokoa, P. S., Saint-Pierre, P., & Zaccour, G. Sustainability of the dry forest in Androy: A viability analysis. *Ecological Economics*, under revision.

Abstract

We investigate the dynamic effect that the Tandroy's unsustainable practices have on the forest. The Tandroy people lives in Androy, a region located in the southern part of Madagascar. They are mainly an agricultural and cattle herding society whose subsistence relies on the slash-and-burn farming agriculture (*hatsake)* and the burning of cacti which are given as fodder to the livestock (*ororaketa*). These activities generate ecological pressure on the surrounding dry forest and socio-economic risks related to the lack of sustainability of these practices in the long run. In this paper we address the notion of sustainability and confront it with the Tandroy's current productive and economic system. By means of viability theory, we characterize the actions and scenarios that are compatible with a sustainable use of the forest in the region.

Key Words: Viability theory, sustainability, deforestation, Androy, Madagascar.

1 Introduction

Androy is a region located in the southern part of Madagascar and is home to one of the most unique and biologically rich dryland areas on Earth (Olson and Dinerstein, 2002; USAID, 2008). It contains a relatively thick vegetation cover dominated by several species of *Didiereaceae*, *Euphorbia* and *Adansonia za* and hosts locally endemic succulent plants that have evolved under arid and poor-soil conditions (Elmqvist et al., 2007).

Figure 1.1: The Androy region is located in the south of Madagascar

In Madagascar, 85% of the population are farmers whose main economic activities are farming and cattle raising. The Androy region is no exception and its population, the Tandroy, are characterized by their reliance on slash-and-burn farming agriculture (*hatsake*) and the burning of cactus for livestock (*ororaketa*). These two activities, along with fuelwood and charcoal production, are directly related to land degradation and forest depletion. Forest trees are seen as a common-property resource, and the Tandroy cut and sell them as an additional source of revenue. Deforested land is used to grow maize, manioc, sweet potatoes and a variety of legumes (e.g., *antake*, catjang, beans and lentils) which are essential to people's livelihoods, particularly in the south.

The Tandroy population has more than doubled in the last 30 years and the combined effect of an increasing population and of the reluctance to change their traditional lifestyle and production system has led to great ecological pressure on the dry forest. As both human and cattle populations increase, *hatsake* and *ororaketa* activities require ever-increasing land space. This has resulted in great environmental degradation. Forest cover in the region has declined: It occupied only 23% of Androy's total area in 2005 compared to more than 30%

in 1960 (Humbert and Cours-Darne, 1965; RM, 2008; USAID, 2007). Rapid forest depletion imperils Tandroy's whole production system and has negative impacts on soil productivity and the hydrological cycle, causing recurrent famines, increased aridity, drought, migration and cultural changes (Fanokoa, 2007).

Our main research question is whether there is something that the Tandroy can do to stop or slow down this relentless process, and ultimately, whether sustainability can be achieved. To formalize the notion of sustainability, we look at a number of socioeconomic indicators, such as consumption, capital accumulation and timber demand for heating, and define minimum thresholds. We build an economic model to explain the evolution of three key variables (i.e., the Tandroy population, forest cover in the Androy region and physical capital) and we define sustainability thresholds with respect to all three.

We make use of viability theory (Aubin, 1991) to assess the sustainability of the Tandroy economic system. Viability theory is particularly well-suited to model environmental problems and applications (e.g., Aubin et al., 2011; Aubin et al., 2005; Béné and Doyen, 2008; Doyen et al., 2007; Martinet and Blanchard, 2009; Martinet and Doyen, 2007) and has already been used to model deforestation problems (Andrés-Domenech et al., 2011) and to assess the sustainability in other regions of Madagascar (Bernard, 2011).

Using viability theory, we find that the Tandroy's current production system is clearly not sustainable. We show that neither the human nor the cattle population can continue to increase at the current levels. We compute the system's maximum carrying capacity for human and cattle populations in the long run and show that there is a trade-off between forest area and human and cattle carrying capacities. Our results also acknowledge the importance of reducing the current net deforestation rate in order to achieve sustainable outcomes.

Social aspects, such as open access to the resource, may make it imposible to slow down deforestation. To tackle this problem, we propose an alternative solution: increased afforestation on abandoned agricultural and farming land. We show that the current deforestation rate, though very high, may be part of a sustainable solution, provided that the afforestation rate increases greatly.

Total current population is too high and the current state of the system is not sustainable. To achieve sustainability, either the population has to decrease or the Tandroy have to greatly reduce their current consumption rate. Deforestation in Androy is mainly caused by poverty; hence, asking the Tandroy to reduce their consumption will probably exacerbate the problem of forest depletion.

We analyze the role that monetary transfers can play in easing the economic-environmental problem. We determine the amount with which the Tandroy should be compensated so that they are able to (i) guarantee their current consumption levels, and (ii) ensure that the forest may be preserved. According to our results, it will take roughly 65 billion ariary[1] per year (approximately 31M 2011$US) to preserve the dry forest in Androy.

The remainder of this paper is organized as follows: In Section 2, we model Tandroy economic system and Androy forest evolution. In Section 3, we specify the economic and environmental objectives and in Section 4, we explicitly define the states, the set of controls and their bounds. Our main findings are presented in Section 5. Section 6 summarizes and concludes.

[1]The national currency in Madagascar.

2 Model

2.1 State dynamics

Time is discrete and denoted by t. The Tandroy economic and environmental system is described by three state variables, namely, the Tandroy population N, the forest area in the Androy region F, and the physical capital K.

Population dynamics: The Tandroy population evolves according to the following difference equation:

$$N_{t+1} - N_t = N_t \cdot (\alpha_t - \beta + \gamma_t), \tag{1}$$

where α_t denotes the birth rate at time t, β the death rate and γ_t is the net migration rate at t.

We model the birth rate as a linear process formed of a given fixed component $\underline{\alpha}$ and a controllable one b_t, that is,

$$\alpha_t = \underline{\alpha} + b_t. \tag{2}$$

One can think of b_t as the increment or reduction in the fertility rate due to some governmental policy (e.g., birth-control campaigns, tax cuts to support larger families). Because the Tandroy's lifestyle remains traditional and seems not to change significantly with higher levels of wealth or income, we assume that the mortality rate β is constant over time. The sign of the migration rate γ_t depends on the difference between the number of immigrants and the number of emigrants. We measure the Androy region's attraction level at time t by the difference between the per capita income in the region and the corresponding figure in the rest of Madagascar. More specifically, we denote by R_t the total revenue in the Androy region and by M the per capita income in Madagascar. Then, the migration rate is given by

$$\gamma_t = \theta \cdot \left(\frac{R_t}{N_t} - M \right), \tag{3}$$

where θ is a positive parameter measuring the propensity to migrate. Admittedly, our specification is not the most general. Indeed, the migration rate may also depend on sociological and political factors. Our implicit assumption is that these factors are somehow embedded in parameter θ. In the same vein, we could have considered the magnitude of the propensity to migrate to vary with the sign of the difference $\frac{R_t}{N_t} - M$. Further, our assumptions that M is constant and that it corresponds to the per capita income in the whole country, as opposed to in Madagascar excluding Androy, are simplifying assumptions, which are mainly dictated by data availability. We refer the reader to the Appendix for more details on all of the model's parameters.

Forest dynamics: The forest area in the Androy region is measured in hectares. The evolution of its size depends on decisions about afforestation[2] and deforestation. Denote by a_t the afforested area at t and by d_t the size of deforested area. Forests also grow and spread naturally. According to Bennett (1983), Chen (1988), Sugita and Tsukada (1982) and Tsukada (1981, 1982) tree expansion is non-linear and the area occupied by a given taxon can be well approximated by exponential or logistic functions. We follow this line of thought and assume the expansion of forest area follows a logistic growth function with a maximum carrying capacity $F_{\max}$ to which the forest naturally converges. The value of $F_{\max}$ is given by the maximum size of land suitable for forest colonization and is set equal to the forest area in 1960 (Humbert and Cours-Darne, 1965). The evolution of the forest

[2] In this paper we use the term afforestation to denote "[...] planting of trees on land that was not previously forested" (FAO, 2010).

area is then governed by the following difference equation:

$$F_{t+1} - F_t = a_t - d_t + \eta \cdot \left(1 - \frac{F_t}{F_{\max}}\right) \cdot F_t, \tag{4}$$

where η is a positive parameter. The above specification was also adopted in Andrés-Domenech et al. (2011).

Note that there exist several reasons for deforestation; namely, *hatsake*, *ororaketa*, and demand for fuelwood and timber. Hence, the total deforested area d_t, in equation (4) is allocated to three different uses.

We denote the *hatsake* area by da_t; the deforested area for wood production by dw_t; and, the *ororaketa* area by dz_t. The three together give the identity below:

$$d_t = da_t + dw_t + dz_t. \tag{5}$$

Capital dynamics: One of the most unique characteristics of the Tandroy traditional society is the dual role played by zebus, as both productive capital and stock of value. Zebu is a type of domestic cattle of the bovid family that is widespread across Madagascar. Zebus are used in Androy to harvest, as a means of transport, as source of milk and food, as stock of wealth and as source of social status. Because they are central to Tandroy life and have all the characteristics of physical capital, we have considered the number of zebus in the region to be a good proxy for the stock of capital in Tandroy society.[3] Denote by K the number of heads of zebu, and by μ the difference between the birth and mortality rate of the zebu population. A first source of variation in the stock of zebus is captured by the term μK. A second source is trade and consumption. Denote by c_t the per capita consumption at time t in monetary units. The economic surplus (or deficit) in the Androy

[3] Other types of cattle in Androy (e.g., goats or sheep) have minor importance compared to the zebu.

region is then given by $R_t - c_t \cdot N_t$. Denote by P_z the constant price of a zebu in an adjacent region. This assumption of constant price is reasonable because the Androy region can be considered an atomistic player in the zebu trade.[4] Assuming that the surplus is entirely devoted to the acquisition of zebus, then the capital dynamics are given by

$$K_{t+1} - K_t = \mu \cdot K_t + (R_t - c_t \cdot N_t)\frac{1}{P_z}. \tag{6}$$

Note that if the Tandroy society is witnessing a deficit, that is, if $R_t - c_t \cdot N_t$ is negative, this would mean that the Tandroy either kill or sell off part of their cattle in order to feed themselves. Note also that given the nature of capital (number of zebu heads) our specification makes sense for integer values of K.

2.2 Revenues

As we have seen, Tandroy revenues have an impact on the evolution of both the population (through the migration rate γ_t) and the capital stock. The total revenues can be decomposed into four main categories or sources: First are the revenues from agricultural activities (e.g., growth of corn, manioc, chickpeas, potatoes). These revenues are related to *hatsake* and are denoted by RA_t. Second, the revenues from selling wood and charcoal are denoted by RW_t. Third, revenues from cattle breeding and *ororaketa*-related activities (e.g., milk, meat) are denoted by RZ_t. Fourth, we have revenues from monetary transfers T_t. These transfers are almost negligible nowadays, but could play a crucial role in the future to ensure sustainability, as we will see in Section 5. Total revenues R_t can then be written as

$$R_t = RA_t + RW_t + RZ_t + T_t. \tag{7}$$

[4]The Tandroy population is roughly 550 000 people, whereas the Malgache population is over 20 million. Extending this specification to the national level, however, would necessarily require general-equilibrium considerations on the price and availability of zebus in the market.

Revenues from agriculture: For simplicity, we suppose that agricultural revenues RA_t come from the growth of a single (or composite) good whose price is given by P_A. The total yield depends on the area available for agriculture, that is, $F_{\max} - F_t - \delta K_t$, and on the productivity of this land, denoted X_t. Therefore, we have

$$RA_t = P_A \cdot X_t \cdot [F_{\max} - F_t - \delta K_t] \,. \tag{8}$$

Note that, in our specification, we have made the assumption that the total available area for either forest, agriculture or pasture is given by $F_{\max}$. The extension of agricultural land is then defined as the land that is not occupied by neither forest, F_t, nor zebu pastures, δK_t. Parameter δ captures the area of pasture land needed per zebu.

The productivity of the agricultural land, X_t, depends on the fertility of the soil, which is affected by *hatsake* activities. Indeed, the Tandroy slash and burn the forest and grow mostly corn, manioc and potatoes on the fertile ashes that remain. The productivity of agriculture land is approximated as follows:

$$X_t = \overline{X} + \lambda_t \cdot da_t - \varphi \cdot \frac{F_{\max} - F_t}{F_{\max}}, \tag{9}$$

where $\overline{X}$ is the average productivity of this land. The term $\lambda_t \cdot da_t$ captures the productivity gained by burning an area da_t, and the term $-\varphi \cdot \frac{F_{\max} - F_t}{F_{\max}}$ represents the reduction in productivity due to forest depletion, with φ being a positive parameter. The time-varying coefficient λ_t is given by

$$\lambda_t = \frac{\psi \cdot \overline{X}}{F_{\max} - F_t - \delta K_t}, \tag{10}$$

where ψ is a positive parameter. In words, λ_t measures the normalized increase in productivity due to burning land.

Revenues from wood: In Androy, people log trees and gather fallen branches from the forest and use them primarily for heating and cooking. Denote by q_t the quantity of wood retrieved at time t; by c_w the annual wood consumption per capita; and by P_w the price of timber. The revenues from selling wood are then given by

$$RW_t = p_w \cdot (q_t - c_w \cdot N_t) . \tag{11}$$

Note that whenever $q_t - c_w \cdot N_t < 0$ there is not enough wood to cover the Tandroy society's minimum needs for heating and cooking. Such a deficit is not realistic for as long as the forest exists, but it could happen in the future if the forest is depleted and/or the population continues to increase. If that were the case, then the Tandroy would need to import wood and, rather than a source of revenue, the term RW would become a cost.

Tandroy society obtains wood through logging, which is done by men, and the gathering of fallen branches by women. Denote by n the amount of wood per hectare of forest, measured in cubic metres. Then, the product $n \cdot dw_t$ gives the total timber yield from deforestation. Let τ be the forest patches where the Tandroy do not enter for religious or superstition reasons. These sacred parts are known as taboo forest (Ferguson, 2007). The total quantity of wood harvested at time t can then be approximated by

$$q_t = n \cdot dw_t + n \cdot \varepsilon \cdot (F_t - \tau) , \tag{12}$$

where ε is a *small* positive parameter capturing the fraction of trees and branches that fall every period.

Revenues from zebus: Denote by ω the estimated yearly revenues obtained from milk and meat. The revenues from zebus are simply given by

$$RZ_t = \omega \cdot K_t . \tag{13}$$

To summarize, we have a model with three state variables, namely population N_t, forest area F_t, and capital stock K_t, and seven control variables: afforestation a_t, a control on the birth rate b_t, per capita consumption c_t, the deforestation rate for agriculture da_t, the deforestation rate for wood dw_t, the deforestation rate for cattle breeding dz_t, and total transfers T_t. Note that the deforestation rate d_t is controlled through its components, not directly. Also, it is interesting to point out that these control variables do not belong to a single economic actor but rather to a number of them. We are concerned with the possibility or impossibility to commonly drive the system to a desirable (i.e. sustainable) set of states.

3 Sustainability objectives

Tandroy society heavily relies on the dry forest. However, its increasing population, traditional lifestyle and extensive production system (characterized by *hatsake* and *ororaketa* activities) has great destructive impact. Consequently, forest area continues to decrease alarmingly. It appears natural that their economic system will be sustainable only if it encompasses a sustainable use of the forest. This being said, it is quite arbitrary to define a threshold (in terms of forest area) beyond which Tandroy development stops being sustainable. Due to the difficulty of defining a minimum sustainable level of forest area, we instead enumerate a number of desirable properties that any so-called sustainable development must meet.

According to the definition given by the Brundtland Commission (WCED, 1987), a development is said to be sustainable if it "meet(s) the needs of the present generations without compromising the ability of future generations to meet their own needs." We shall put this definition in practice through a number of desirable economic requirements that, in our view, reflect the needs (the basic needs at least) of the Tandroy. We analyze the extent to which these requirements can be satisfied in time. In particular, following Ekins et al. (2003), we are interested in the underlying relation between compliance with these economic requirements and the state of Androy forest. As it turns out, we do not need to define explicit threshold levels for the stock of forest. Rather, the viability techniques used will tell us what initial levels of our system's three state variables (and not just the forest) are compatible with the perennial satisfaction of such needs.

Finding a set of initial states that allow the Tandroy to forever satisfy such needs is equivalent to saying that a sustainable development (both economic and environmental) is achievable for those initial states of the system. More precisely, what viability theory does is to give a clear-cut yes or no answer to the following two questions:

1. Can we find a subset of initial states such that the economic needs of the Tandroy can be satisfied forever?

2. Is the current situation (i.e., current population, forest area and capital stock) part of this set of initial states?

If the answer to the two questions is positive, then both economic and environmental sustainability are achievable. Else one has to identify the necessary policy changes that could lead to sustainable development.

We express the needs of Tandroy society in terms of consumption, production of timber and capital. In line with the definition of sustainability given above, we require the future levels of these variables to be at least as high as their current ones. More specifically, we adopt the following constraints:

$$c_t \geq \underline{c}, \tag{14}$$

$$q_t \geq c_w \cdot N_t, \tag{15}$$

$$K_t \geq K_{2005}, \tag{16}$$

$$\frac{K_t}{N_t} \geq \frac{K_{2005}}{N_{2005}}. \tag{17}$$

Constraint (14) states that the per capita level of consumption c_t at any period t must be at least equal to the current per capita consumption $\underline{c}$.[5] In (15), we impose that the production of wood covers at least the basic needs of the Tandroy population at any time period. Given the great importance of zebus in Tandroy society in terms of source of food, revenues, wealth and social status, the constraints in (16) and (17) require maintaining both the absolute and relative levels of capital, at least, at their 2005-levels. The year 2005 is chosen as a benchmark because it is the most recent year for which we have reliable data on many of our model's key variables.

4 State and control variables

4.1 State variables

Recalling that our model has three state variables, namely, population (N), forest area in Androy (F), and capital (K), our state space is defined as follows:

$$\mathbb{k} := \{x = (N, F, K) \,/ N \in [N_{\min}, N_{\max}], F \in [F_{\min}, F_{\max}], K \in [K_{\min}, K_{\max}]\},$$

where x denotes a point in this set $\mathbb{k}$.

Forest area in Androy is bounded. Current forest area (453 561 hectares in 2005) is 37% less than it used to be before massive deforestation began in the 1950s-1960s (Fanokoa, 2007). According to Humbert and Cours-Darne (1965), forest area in 1960 was equal to

[5] Current per capita consumption is measured as the per capita consumption in the region in 2005.

623 000 hectares. We use their estimation as a proxy for the upper bound of forest area. This upper bound, $F_{\max}$, is set as equal to the forest's carrying capacity.

To determine $F_{\min}$, recall that part of the forest is taboo and that the Tandroy do not enter it. Taboo forest covers an area approximately equal to 60 000 hectares (Ferguson, 2007). Therefore, we shall use $F \in [F_{\min}, F_{\max}] = [60\,000,\ 623\,000]$.

Once we have the bounds on F, we can endogenously determine the bounds for the other two state variables as a function of F itself. Recall from equation (8) that parameter δ denotes the area of pasture required by a head of cattle. Suppose that the whole available area in the model was turned into pasture land. Then, maximum cattle population would be given by the expression $\frac{1}{\delta} \cdot (F_{\max} - F_{\min}) = 1\,555\,248$ zebus. We thus have $K \in [K_{\min}, K_{\max}] = [0,\ 1\,555\,248]$.

We can determine the upper bound on population in a similar manner. Minimum per capita consumption is given by inequality (14) and revenues are given by (7). This means that $N_{\max} = \frac{1}{\underline{c}} \max[R]$. From (8), if all available land was dedicated to agriculture, then revenues would equal $P_A \cdot X_t \cdot (F_{\max} - F_{\min})$. Conversely, if all available land was dedicated to cattle herding, then, according to expression (13), cattle-herding revenues would be $\omega \cdot K_{\max}$. Finally, if all available land was used to obtain and sell wood, then timber revenues would equal $P_w \cdot n \cdot \varepsilon \cdot (F_{\max} - \tau)$. Given the fact that monetary transfers T are set equal to zero, the upper bound on population can be computed as follows:

$$N_{\max} = \frac{1}{\underline{c}} \max\left[P_A \cdot \left(\overline{X} - \varphi\right) \cdot (F_{\max} - \tau)\,, \omega \cdot K_{\max}, P_w \cdot n \cdot \varepsilon \cdot (F_{\max} - \tau)\right]. \tag{18}$$

If we substitute all variables and parameters in (18) by their values, we obtain that agriculture is the most efficient way, per unit of land, of creating revenue and that: $N_{\max} =$

$\frac{1}{\underline{c}} \cdot P_A \cdot \left(\overline{X} - \varphi\right) \cdot (F_{\max} - \tau) = 1\,836\,128$ people. We thus have that $N \in [N_{\min}, N_{\max}] = [0, 1\,836\,128]$.

The bounds on the three state variables are summarized in Table 1.I below.

Table 1.I: State variables

State	Lower Bound	Current	Upper Bound
N	0	548 418 ps	1 836 128 ps
F	60 000 ha	453 561 ha	623 000 ha
K	0	290 000 zb	1 555 249 zb

4.2 Control variables

As stated above, our model includes seven control variables. Table 1.II provides their current values as well as the adopted upper and lower bounds.

Table 1.II: Control variables

Control	Lower Bound	Current	Upper Bound
a_t	0	69 ha yr^{-1}	100 ha yr^{-1}
b_t	0 %	0 %	1 %
c_t	298 600 ar yr^{-1}	298 600 ar yr^{-1}	597 200 ar yr^{-1}
d_t	0	5 708 ha yr^{-1}	6 000 ha yr^{-1}
da_t	0	$---$	6 000 ha yr^{-1}
dw_t	0	$---$	6 000 ha yr^{-1}
dz_t	0	$---$	6 000 ha yr^{-1}
T_t	0	0	0 ar yr^{-1}

Control a_t denotes the afforestation rate. We have chosen an upper bound on the afforestation rate that is quite low or pessimistic. We shall assess later how the solutions change when we allow for a larger afforestation rate, i.e., higher $a_{\max}$. Control b_t denotes the variation in the fertility rate that can be induced by local authorities. To start with, we have kept a lower bound on b_t that is equal to zero. This means that the birth rate cannot be reduced. Again, we shall analyze later the impact of letting b_t take negative values (i.e., $b_{\min} < 0$). With respect to the per capita consumption variable, c_t, we have set the lower bound exactly equal to current per capita consumption in order to be consistent with the definition of sustainable development put forward above. The deforestation rate d_t has been included in the table even if it is not a true control variable. We have no current observation for da, dw and dz. However we do have an estimate of the current value of d_t. Recall from (5) that $d = da + dw + dz$; and, hence, that the upper bound is shared for all four variables even if da, dw and dz cannot simultaneously have the maximum value of 6000 hectares per year. Although it has not been made explicit in Table 1.II, deforestation is limited by the availability of forest land, i.e., $d \in [0, \min(F - \tau,\ 6000)]$. Finally, we set the monetary transfers T_t equal to zero and, in the sequel, check the impact of having positive values for T_t. All monetary values are stated in ariary, hereafter denoted "ar". Note that for all of our control and state variables, the current observation lies somewhere between these two bounds.

To wrap up, our three-dimensional dynamic system, to which we shall refer to as $\mathcal{S}$, consists of

$$(\mathcal{S}) \quad \begin{cases} N_{t+1} - N_t & = & N_t \cdot (\alpha_t - \beta + \gamma_t) \\ F_{t+1} - F_t & = & b(t) + \eta \cdot \left(1 - \frac{F(t)}{F_{\max}}\right) \cdot F(t) - d(t), \\ K_{t+1} - K_t & = & \mu \cdot K_t + (R_t - c_t \cdot N_t) \frac{1}{P_z}, \\ \text{with } u & := & (a, b, c, da, dw, dz, T) \in U, \end{cases} \tag{19}$$

where U is the feasible set of controls

$$\begin{aligned} U \quad = \quad & [a_{\min}, a_{\max}] \times [b_{\min}, b_{\max}] \times [c_{\min}, c_{\max}] \times [da_{\min}, da_{\max}] \times \\ & [dw_{\min}, dw_{\max}] \times [dz_{\min}, dz_{\max}] \times [T_{\min}, T_{\max}] \,. \end{aligned}$$

For future reference, we generically write the three first lines of $\mathcal{S}$ as $x_{t+1} = g(x_t, u_t)$ with $x_t = (N_t, F_t, K_t)$, $t \in \mathbb{N}$.

5 Results

5.1 Feasibility set

In the previous section, we introduced the set $\Bbbk$, which is our computational area. Denote now by $\Bbbk_C$ the feasibility set, that is, the subset of $\Bbbk$ for which our constraints (14)-(17) can be met.

The first constraint (14) refers to the minimum per capita consumption and was used to obtain the boundary $N_{\max}$.

The second constraint (15) refers to the minimum wood consumption level necessary to satisfy the Tandroy demand for heating and cooking. If we plug (12) in (15), we obtain the following expression: $n \cdot dw_t + n \cdot \varepsilon \cdot (F_t - \tau) \geq c_w \cdot N_t$. From this last expression, we can obtain the minimum forest area required to meet our demand as a function of the population N and dw:

$$\underline{F} \geq \frac{c_w \cdot N - n \cdot dw + n \cdot \varepsilon \cdot \tau}{n \cdot \varepsilon}. \tag{20}$$

Bearing in mind that $dw \in [0, \min(F - \tau, 6000)]$, substitute N and dw by their maximum in expression (20) to obtain the minimum forest area required to guarantee the heating and cooking needs of the Tandroy, that is,

$$\underline{F} \geq \frac{c_w \cdot N_{\max} + n \cdot \tau \cdot (\varepsilon + 1)}{n \cdot (\varepsilon + 1)}. \tag{21}$$

We can compute the value of $\underline{F}$ by inserting the values of our parameters into expression (21). $\underline{F}$ equals 62 319 ha. Now define $\Bbbk_q$ as the subset of $\Bbbk$ for which condition (21) can be satisfied in the absence of timber commerce: $\Bbbk_q := \{(N, F, K) \,/\, F \geq \underline{F}\}$. Figure 1.2 is a plot of set $\Bbbk_q$. The set comprises all the states above the horizontal plane at the bottom of the figure. As it appears, this constraint is only binding when forest area is very low.

The third constraint (16) refers to the minimum level of capital K. We require the absolute level of wealth to be at least as high as it was in 2005. Denote by $\Bbbk_K$ the subset of $\Bbbk$ for which condition (16) can be satisfied: $\Bbbk_K := \{(N, F, K) \ / \ K \geq K_{2005}\}$. Figure 1.3 is a plot of set $\Bbbk_K$. This set includes all the states to the right of K_{2005}.

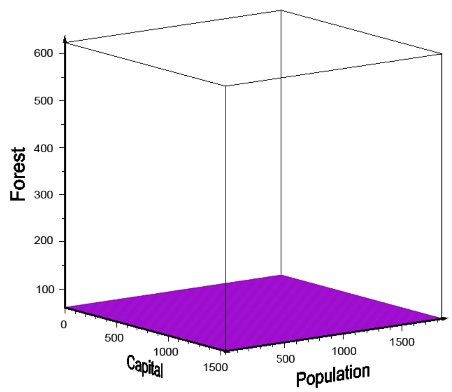

Figure 1.2: Set $\Bbbk_q$ is above the horizontal plane

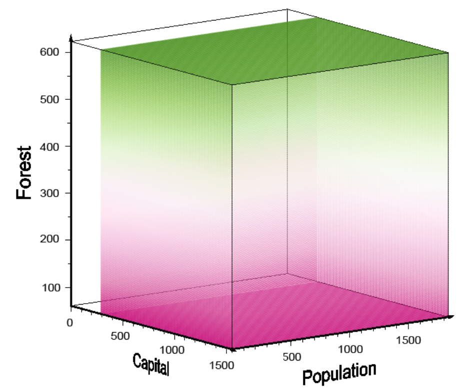

Figure 1.3: Constraint on capital: Set $\Bbbk_K$

Our fourth constraint refers to the minimum per capita level of capital. We require that wealth in per capita terms always be at least as high as in year 2005. Denote now by $\Bbbk_{K_{pc}}$ the subset of $\Bbbk$ for which condition (17) can be satisfied: $\Bbbk_{K_{pc}} := \{(N, F, K) \ / \ \frac{K}{N} \geq \frac{K_{2005}}{N_{2005}}\}$. Figure 1.4 is a plot of set $\Bbbk_{K_{pc}}$.

By merging all our constraints together, we obtain our feasibility set $\Bbbk_C$. We can now formally define $\Bbbk_C$ in the following manner:

$$\Bbbk_C := \{x = (N, F, K) \ / \ \Bbbk_q \cap \Bbbk_K \cap \Bbbk_{K_{pc}}\}. \tag{22}$$

Figure 1.5 is a plot of set $\Bbbk_C$.

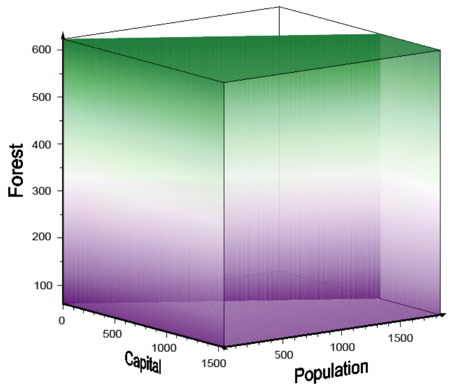

Figure 1.4: Constraint on capital per capita: Set $\mathbb{k}_{K_{pc}}$

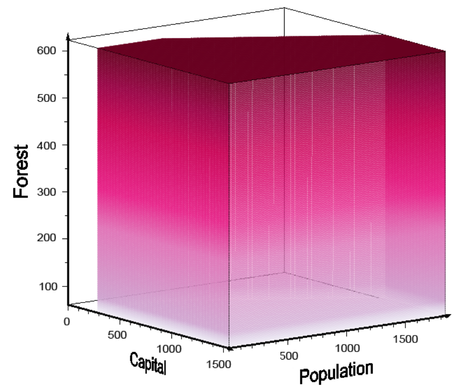

Figure 1.5: Feasibility set: $\mathbb{k}_C$

5.2 Benchmark case: $Viab_{\mathcal{S}}(\mathbb{k}_C)$

Set $\mathbb{k}_C$ does not involve the system dynamics; it is a static set. Our three state variables N, F, K are not constant and evolve in time. To determine whether or not Tandroy's productive system is sustainable we refer to viability theory. Viability theory provides us with the concept of the viability kernel. Denote by $Viab_{\mathcal{S}}(\mathbb{k}_C)$ the viability kernel of set $\mathbb{k}_C$ for system S. The viability kernel $Viab_{\mathcal{S}}(\mathbb{k}_C)$ is the largest subset of $\mathbb{k}_C$ for which one can find at least one control rule such that its resulting trajectory remains in $\mathbb{k}_C$ forever (such a trajectory is then called viable).

For our application, set $Viab_{\mathcal{S}}(\mathbb{k}_C)$ can be understood as the set of initial states for which sustainable development is possible, in the sense that an evolution that always remains in $\mathbb{k}_C$ is an evolution for which our constraints can always be satisfied. If the set $Viab_{\mathcal{S}}(\mathbb{k}_C)$ is empty, then this would mean that there is no initial state in $\mathbb{k}_C$ from which a viable evolution starts. In such case Tandroy productive system is said unsustainable.

More formally, denote by $x^0 = (N, F, K)$ any point in $\Bbbk_C$. Denote now by $S_{\mathcal{S}}(x^0)$ the set of all evolutions $x(\cdot)$ for the dynamic system (29) which emanate from x^0, that is,

$$S_{\mathcal{S}}(x^0) := \left\{ (x_n)_{n\in\mathbb{N}} / \exists\, (u_n)_{n\in\mathbb{N}} \in U^{\mathbb{N}} \text{ such that } x_{n+1} = g\,(x_n, u_n)\,,\ x_0 = x^0 \right\}.$$

Then, the viability kernel is given by

$$Viab_{\mathcal{S}}(\Bbbk_C) := \{x \in \Bbbk_C \ / \ \exists x(\cdot) \in S_{\mathcal{S}}(x), \forall n \in \mathbb{N},\ x_n \in \Bbbk_C\}\,.$$

Using the viability algorithm proposed in Saint-Pierre (1994),[6] we obtain that $Viab_{\mathcal{S}}(\Bbbk_C)$ is an empty set. This means that there are no states in $\Bbbk_C$ for which all our constraints can be satisfied forever.

For this particular case, it is easy to identify the reason why sustainability fails, i.e., $Viab_{\mathcal{S}}(\Bbbk_C)$ is empty. Substituting for α_t and γ_t, from (2) and (3), in the population dynamics (1) we obtain

$$\frac{N_{t+1} - N_t}{N_t} = \underline{\alpha} + a_t - \beta + \theta \cdot \left(\frac{R_t}{N_t} - M \right).$$

Given that R_t is positive, and that $a_t \in [a_{\min}, a_{\max}]$, then we have the following lower bound on the population's variation rate:

$$\frac{N_{t+1} - N_t}{N_t} \geq \underline{\alpha} + a_{\min} - \beta - \theta M. \tag{23}$$

Substituting for the parameters values in (23), we get $\frac{N_{t+1}-N_t}{N_t} \geq 0.01$, i.e., total population will increase by at least 1% every year. As the set $\Bbbk_C$ is bounded and the population can only increase, then, clearly, all evolutions will leave $\Bbbk_C$ in finite time. Consequently, the set $Viab_{\mathcal{S}}(\Bbbk_C)$ is empty and this means that Tandroy's productive system is not sustainable.

[6] The viability algorithm is a set-valued algorithm to numerically approximate the viability kernel. VIMADES Inc. developed this algorithm and allows us to use a version that is well adapted to environmental problems. All the viability kernels appearing in this paper were computed with this type of algorithm.

Faced with this negative result, the relevant question becomes the following: Can the Tandroy do something to turn their system into a sustainable one? Our previous results show that the current birth rate is too high, so we begin by exploring what changes in the birth rate are required in order to have a non-empty viability kernel.

5.3 Reduction in the birth rate

Recall that the net birth rate in (2) is given by two terms: a fixed one $\underline{\alpha} = 0.026$ and a variable one a_t. A necessary, though not sufficient, condition for the viability kernel to be non-empty is that $\frac{N_{t+1}-N_t}{N_t} \leq 0$ is feasible. This is possible if $a_{\min} \leq -0.010$. Denote by $\mathcal{S}_1$ the dynamic system that results when we let $a_{\min} = -0.015$. The viability kernel of $\Bbbk_C$ for the dynamic system $\mathcal{S}_1$ can be denoted by $Viab_{\mathcal{S}_1}(\Bbbk_C)$. In Table 1.III, we specify the bounds for all of our control and state variables for system $\mathcal{S}_1$. We also include the specification used for all the different scenarios computed throughout this paper:

Table 1.III: Bounds on control variables for all scenarios studied

System	a_t	b_t	c_t	d_t	T_t
$\mathcal{S}$	$[0, 100]$	$[\mathbf{0}, 0.01]$	$[298\,600, 597\,200]$	$[0, 6000]$	$[0, 0]$
$\mathcal{S}_1$	$[0, 100]$	$[-0.015, 0.01]$	$[298\,600, 597\,200]$	$[0, 6000]$	$[0, 0]$
$\mathcal{S}_2$	$[0, 100]$	$[-0.015, 0.01]$	$[298\,600, 597\,200]$	$[\mathbf{5708}, \mathbf{5708}]$	$[0, 0]$
$\mathcal{S}_3$	$[0, \mathbf{3000}]$	$[-0.015, 0.01]$	$[298\,600, 597\,200]$	$[\mathbf{5708}, \mathbf{5708}]$	$[0, 0]$
$\mathcal{S}_4$	$[0, 100]$	$[-0.015, 0.01]$	$[298\,600, 597\,200]$	$[0, 6000]$	$[0, 0]$
$\mathcal{S}_{5.5}$	$[0, 100]$	$[-0.015, 0.01]$	$[\mathbf{180\,000}, 597\,200]$	$[0, 6000]$	$[0, 0]$
$\mathcal{S}_6$	$[0, 100]$	$[-0.015, 0.01]$	$[298\,600, 597\,200]$	$[0, 6000]$	$[0, \mathbf{6.5 \cdot 10^{10}}]$

The units for all variables in Table 1.III have been omitted for simplicity. The values in bold indicate a change with respect to $\mathcal{S}_1$. Monetary transfers are constant and equal to zero for most of the scenarios considered. Scenarios $\mathcal{S}_4$-$\mathcal{S}_6$ were computed requiring an extra constraint on the total forest area ($F \geq F_{2005}$).

Figure 1.6 below is a plot of $Viab_{\mathcal{S}_1}(\Bbbk_C)$.

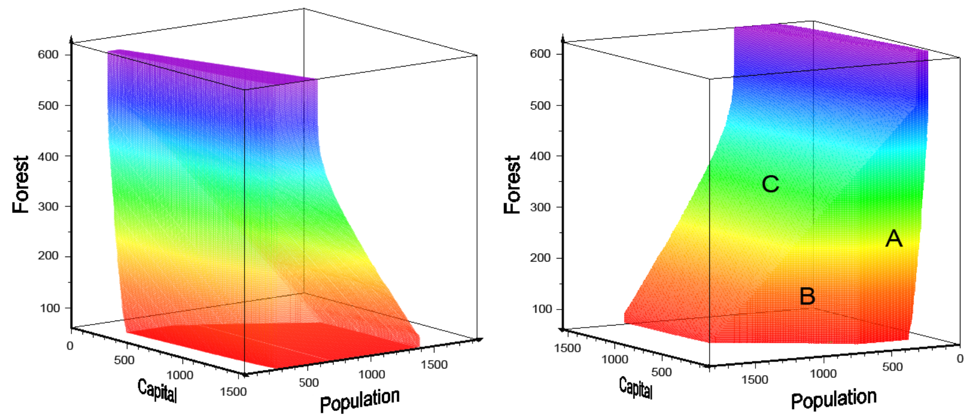

Figure 1.6: Two views of set $Viab_{\mathcal{S}_1}(\mathbb{k}_C)$

Figure 1.6 shows two views of $Viab_{\mathcal{S}_1}(\mathbb{k}_C)$ from two different perspectives.[7] Set $Viab_{\mathcal{S}_1}(\mathbb{k}_C)$ is made of all the points contained within the coloured volume. There are at least three distinct regions in the cube that do not belong to $Viab_{\mathcal{S}_1}(\mathbb{k}_C)$. We have labelled these regions with letters A, B and C in the figure. Region A is determined by the constraint on minimum capital (16). Region B is related to the existence of constraint (17) on the minimum *per capita* capital. Finally, region C is also determined by constraint (17), but here, the relation is more subtle. Recall that if the population increases, capital must also increase in order to satisfy (17). Recall also that every head of cattle requires δ hectares of pasture land. The inclined surface that we observe relates these three facts. Take any point in the boundary of $Viab_{\mathcal{S}_1}(\mathbb{k}_C)$ in region C. For a variation in population equal to ΔN, capital must increase at least $\Delta N \frac{K_{2005}}{N_{2005}}$ units, i.e., $\Delta K \geq \Delta N \frac{K_{2005}}{N_{2005}}$. Now, if capital increases ΔK units, then pasture land must increase (i.e., forest area must decrease) at

[7] Most of the figures representing the viability kernels plotted in this paper consist of two views of the same set from two different perspectives. The sets are complex and this double representation allows the reader to better seize the geometry of the set. It is also worth noting that the axes representing capital and population are inverted in each of these views.

least δ hectares for every additional head of cattle: $\Delta F \leq -\delta \cdot \Delta K$. We thus have that $\Delta F \leq -\delta \cdot \Delta N \frac{K_{2005}}{N_{2005}}$. This explains the shape of the frontier of $Viab_{\mathcal{S}_1}(\mathbb{k}_C)$ in region C.

5.4 Forest ownership and afforestation

Until now, we have supposed that the deforestation rate in Androy can be controlled. One of the reasons why $Viab_{\mathcal{S}_1}(\mathbb{k}_C)$ was not empty is that low deforestation rates are feasible. Figure 1.7 is a plot of the highest possible deforestation rate, d, along the boundary of the previously computed $Viab_{\mathcal{S}_1}(\mathbb{k}_C)$.

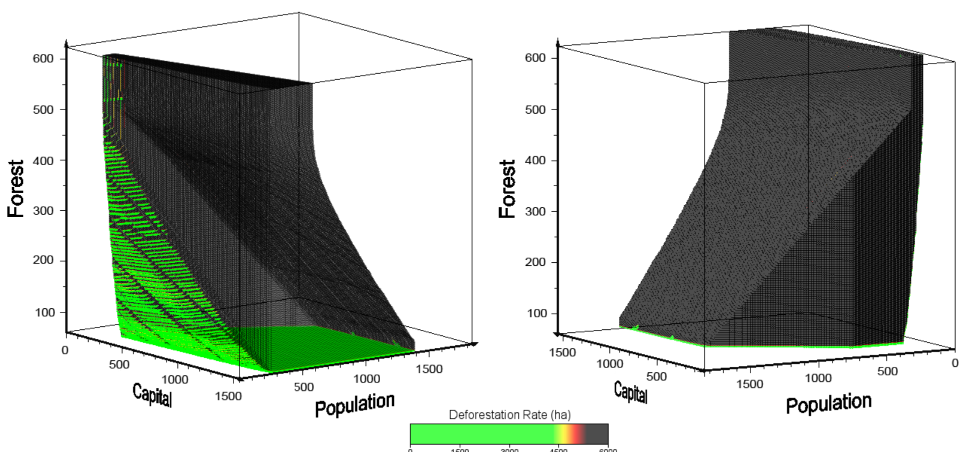

Figure 1.7: Highest viable deforestation rate along the boundary of $Viab_{\mathcal{S}_1}(\mathbb{k}_C)$

Figure 1.7 shows that the Tandroy can afford to have a high deforestation rate for as long as they are not close to the lower boundary of $Viab_{\mathcal{S}_1}(\mathbb{k}_C)$.

Forest in Androy is, in practice, an open-access resource. Whoever clears and subsequently occupies a piece of land is identified as its *de facto* owner. This type of behaviour poses a great problem and makes the task of reducing deforestation quite hard -if at all possible- to achieve.

Now suppose that it was not possible to reduce the deforestation rate. That is, suppose d is constant and equal to its current rate (i.e., $d = d_{current} = 5708$ ha yr^{-1}). This raises an interesting question: Can Tandroy's economic system be sustainable if deforestation cannot be reduced? Put differently: Would the viability kernel ($Viab_{\mathcal{S}_2}(\Bbbk_C)$) be non-empty if d were fixed and equal to the current deforestation rate?

Our results show that $Viab_{\mathcal{S}_2}(\Bbbk_C)$ is empty. In fact, it is easy to prove analytically that the current deforestation rate is not sustainable. Consider equation (4) and substitute d_t by its current value to obtain

$$F_{t+1} - F_t = a_t + \eta \cdot \left(1 - \frac{F_t}{F_{\max}}\right) \cdot F_t - d_{current}. \tag{24}$$

The left-hand side of (24) is maximized when $F = \frac{F_{\max}}{2}$ and $a = a_{\max}$ and can be expressed as follows:

$$F_{t+1} - F_t \leq a_{\max} + \frac{\eta \cdot F_{\max}}{4} - d_{current}. \tag{25}$$

Substituting for the parameters' values, we obtain that $F_{t+1} - F_t \leq -2390$ ha yr^{-1}. Hence, if the deforestation rate is set constant and equal to its current level, the forest area will decrease by at least 2390 ha every year. The current afforestation rate in Androy ($a_{current}$) is equal to 69 ha yr^{-1}(RM, 2003). Up to now, the upper bound used for the afforestation rate, $a_{\max}$, was equal to 100 ha yr^{-1}. Now suppose that the yearly afforestation rate could be as high as 3000 ha and denote by $\mathcal{S}_3$ the dynamic system that results when we allow for $a \in [0, 3000]$. Figure 1.8 below is a plot of $Viab_{\mathcal{S}_3}(\Bbbk_C)$.

There is not a great difference between Figure 1.6 and Figure 1.8. In fact the two sets almost coincide. This means that if we are able to fully control deforestation and afforestation is low ($Viab_{\mathcal{S}_1}(\Bbbk_C)$), we obtain a similar result as when we are not able to reduce current deforestation levels but the afforestation rate is high ($Viab_{\mathcal{S}_3}(\Bbbk_C)$). This

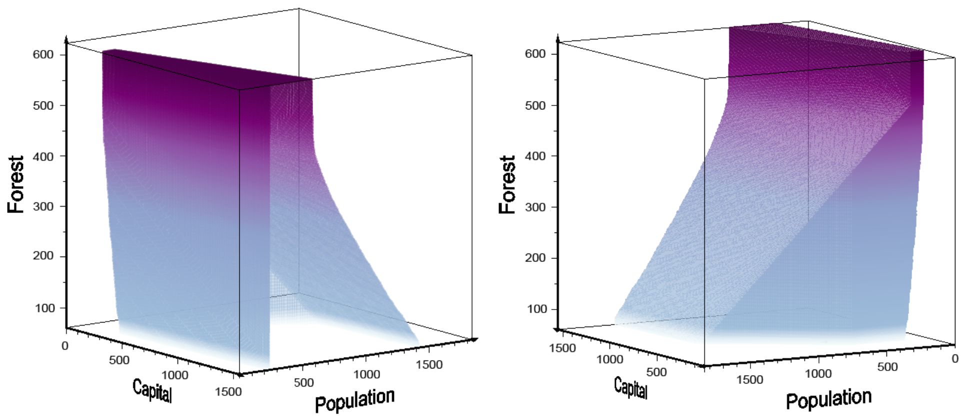

Figure 1.8: $Viab_{\mathcal{S}_3}(\Bbbk_C)$: Impact of increasing the upper bound on reforestation

is a positive message for local forestry authorities: Implementing a high afforestation rate yields similar results as controlling the deforestation rate, and it is probably less difficult and costly to monitor.

5.5 Keeping the forest intact

Figure 1.8 provides an optimistic message: Sustainability is achievable even if the current deforestation rate remains high and unchanged. This same figure, however, also conveys a negative message: Sustainability can be achieved even when the forest area is quite low. As we saw in Figure 1.8, forest area can converge to $F_{\min}$ and still be part of a sustainable solution. In other words, it is sustainable to virtually deplete the forest.

Therefore, we ask ourselves if it would be feasible to maintain the forest untouched provided it were possible to monitor and control the deforestation rate. Once more, we can translate this question into the language of viability theory. Denote by $\Bbbk_{C_F}$ the following set: $\Bbbk_{C_F} := \{\Bbbk_C \cap \{(N, F, K) \ / \ F \geq F_{2005}\}\}$. Now denote by $Viab_{\mathcal{S}_4}(\Bbbk_{C_F})$ the viability kernel for set $\Bbbk_{C_F}$ where $\mathcal{S}_4 \equiv \mathcal{S}_1$. We want to determine whether or not $Viab_{\mathcal{S}_4}(\Bbbk_{C_F})$ is empty. Note that, by construction, $Viab_{\mathcal{S}_4}(\Bbbk_{C_F}) \subseteq Viab_{\mathcal{S}_4}(\Bbbk_C) \cap \Bbbk_{C_F}$. Figure 1.9 is a plot of $Viab_{\mathcal{S}_4}(\Bbbk_{C_F})$.

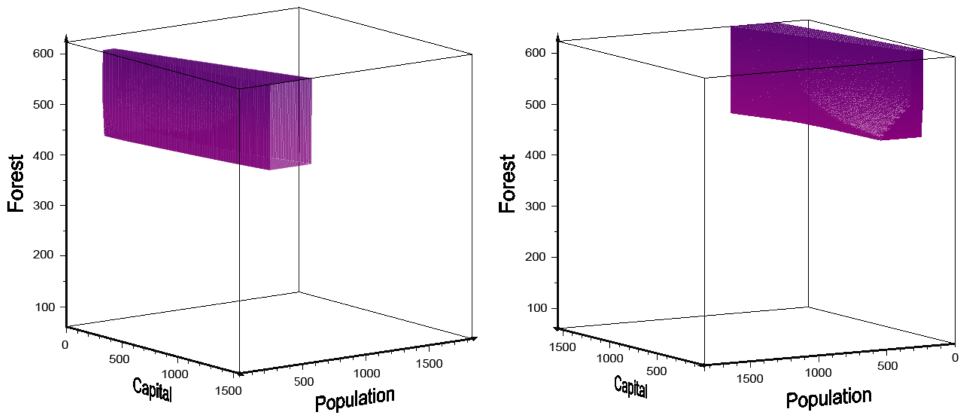

Figure 1.9: Two views of $Viab_{\mathcal{S}_4}(\Bbbk_{C_F})$. Maintaining current forest area

One can observe that $Viab_{\mathcal{S}_4}(\Bbbk_{C_F})$ is not empty. In fact it is equal to the intersection between $Viab_{\mathcal{S}_1}(\Bbbk_C)$ and $\Bbbk_{C_F}$. The main message we get is that if one can control the deforestation rate then it is possible to satisfy all constraints without harming the forest.

5.6 Current state of the Tandroy economy

Up to now we have computed a number of viability kernels. Some of them were empty, while others were not. However, we have not yet addressed our second research question, namely, is the Tandroy economy's current state sustainable? Or put differently, does $x_{2005} = (N_{2005}, F_{2005}, K_{2005})$ belong to any of the kernels that we have computed so far?

The answer to this question is negative. Figure 1.10 and Figure 1.11 are a plot of $Viab_{\mathcal{S}_1}(\Bbbk_C)$ and $Viab_{\mathcal{S}_4}(\Bbbk_{C_F})$ respectively. The initial state x_{2005} does not belong to either.

Figure 1.10 is a plot of the previously computed $Viab_{\mathcal{S}_1}(\Bbbk_C)$ from a different perspective. One can observe that $x_{2005} \notin Viab_{\mathcal{S}_1}(\Bbbk_C)$. Figure 1.11 shows the same set as in Figure 1.9.

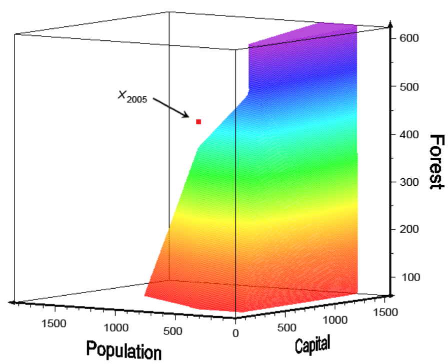

Figure 1.10: State x_{2005} does not belong to $Viab_{\mathcal{S}_1}(\mathbb{k}_C)$

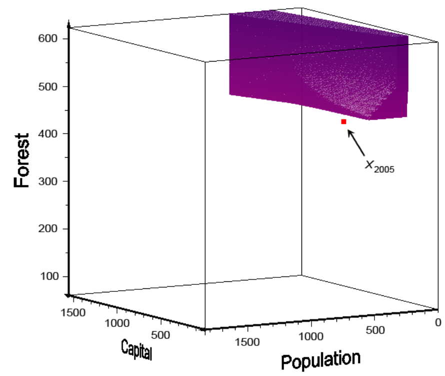

Figure 1.11: State x_{2005} does not belong to $Viab_{\mathcal{S}_4}(\mathbb{k}_{C_F})$

We have now added x_{2005} and again show that $x_{2005} \notin Viab_{\mathcal{S}_4}(\mathbb{k}_{C_F})$. What these two figures tell us is that, even if Tandroy's economic system is sustainable for some initial states, this is not the case for the current state.

The main reason why x_{2005} is not sustainable in any of the scenarios studied is that overall Tandroy consumption, $\underline{c} \cdot N_{2005}$, is not sustainable in time. Recall that parameter $c_{\min} = \underline{c}$ is equal to 298 600 ar yr^{-1}. Figure 1.12 is a plot of the highest feasible (i.e., affordable) consumption along the boundary of our previously computed $Viab_{\mathcal{S}_4}(\mathbb{k}_{C_F})$. We show two views of this set from different perspectives.

Figure 1.12 shows the obvious fact that as population increases the highest affordable consumption must decrease. It also suggests that if the lower bound in consumption, $c_{\min}$, decreases (for a constant level of capital and forest area) then the boundary of the viability kernel would shift and include states with greater population.

To clarify this last point, we have performed a sensitivity analysis and computed the viability kernel for several scenarios. Each scenario has a different lower bound on the per

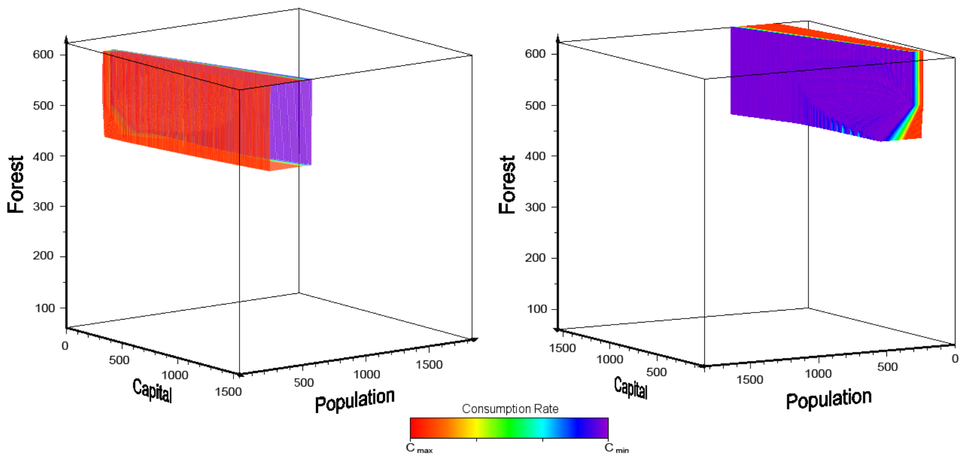

Figure 1.12: Highest affordable consumption along the boundary of $Viab_{\mathcal{S}_4}(\Bbbk_{C_F})$

capita consumption variable, as specified in Table Table 1.IV. The idea is to obtain the highest lower bound on the consumption variable for which the currrent state of the system is part of the viability kernel.

Table 1.IV: Different lower bounds in consumption per capita

Scenario	C1	C2	C3	C4	C5
c_{min}	298 600	270 000	240 000	210 000	180 000

Denote by $Viab_{\mathcal{S}_{5.1}}(\Bbbk_{C_F})$ the viability kernel of scenario C1. The minimum consumption rate in scenario C1 is exactly the same one that we used to compute $Viab_{\mathcal{S}_4}(\Bbbk_{C_F})$. In fact, $Viab_{\mathcal{S}_{5.1}}(\Bbbk_{C_F})$ is exactly the same set as the previously computed $Viab_{\mathcal{S}_4}(\Bbbk_{C_F})$. Now, denote by $Viab_{\mathcal{S}_{5.2}}(\Bbbk_{C_F})$-$Viab_{\mathcal{S}_{5.5}}(\Bbbk_{C_F})$ the viability kernels corresponding to scenarios C2 to C5. The viability kernel for C5 is the only one of the of the five proposed scenarios that includes x_{2005} inside its boundaries. In this scenario, $c_{\text{min}} = 180\,000$ ar yr^{-1}.

Figure 1.13 is a plot of the kernel for scenario C5. The size of the viability kernel increases when c_{min} decreases. Since x_{2005} lies at the boundary of $Viab_{\mathcal{S}_{5.5}}(\Bbbk_{C_F})$, we can write that, *ceteris paribus*, $c_{\text{min}} = 180\,000$ is the largest minimum consumption rate for

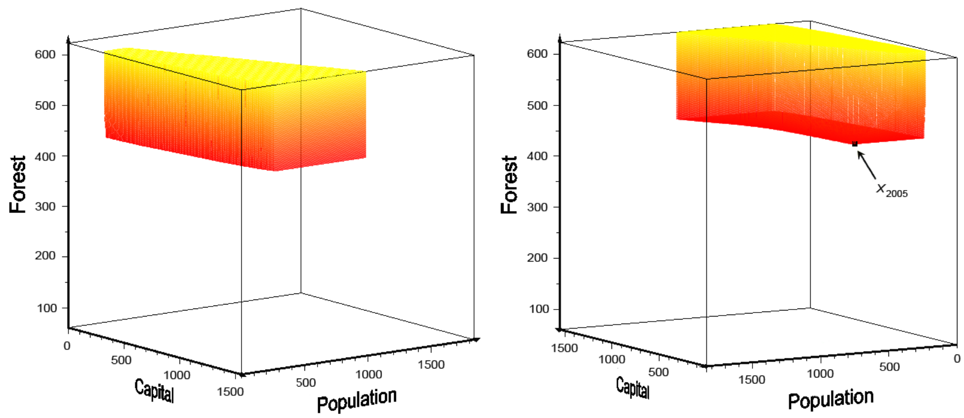

Figure 1.13: State x_{2005} belongs to $Viab_{\mathcal{S}_{5.5}}(\mathbb{k}_{C_F})$

which the current state of the system is viable. In other words, the current per capita consumption rate in Androy must be reduced by 40% in order for x_{2005} to be viable.

5.7 Monetary transfers

The viability kernel computed in Figure 1.13 somehow presupposes that reducing consumption is feasible. The current consumption rate is already very low and the Tandroy perceive the forest as something to be exploited if needed. Therefore, it is not clear that the Tandroy would be willing to sacrifice 40% of their consumption in order to be viable and preserve the forest. It is important to mention that reducing consumption is not the only way to ensure that x_{2005} is included in the viability kernel. Another possibility would be using monetary transfers.

So far we have considered monetary transfers as being equal to zero. It is possible, however, that national authorities or international organizations or countries would be willing to compensate the Tandroy for their keeping the forest intact. The question then becomes how much money would have to be transferred in order that there are no decreases in either the forest area or the current consumption rate.

To answer this question, we need to review our previous findings: $Viab_{\mathcal{S}_4}(\mathbb{k}_{C_F})$ is not empty; therefore it is possible to preserve the current forest area and current consumption levels, although not at the current population levels. Analogously $Viab_{\mathcal{S}_{5.5}}(\mathbb{k}_{C_F})$ contains x_{2005}, which means that it is possible to preserve the current forest area and the current population, but not the current consumption rate. The only difference between these two sets is the lower bound in consumption, $c_{\min}$, used to obtain them.

Denote by $\widetilde{T}$ the minimum value of transfers that guarantees that current consumption is sustainable for the current Tandroy population. One can compute $\widetilde{T}$ analytically as the product between current population and the difference in the $c_{\min}$ used for S_4 and $S_{5.5}$:

$$\widetilde{T} = N_{2005}\,(\underline{c} - c_{C5}) = 6.5 \cdot 10^{10}\, ar \cdot yr^{-1},$$

where $\underline{c}$ and c_{C5} are the minimum per capita consumption rates allowed in scenarios S_4 and $S_{5.5}$, respectively. Now denote by S_6 the dynamic system that results when we use $T \in [0, \widetilde{T}]$. Figure 1.14 below is a plot of $Viab_{\mathcal{S}_6}(\mathbb{k}_{C_F})$.

State x_{2005} is included in the boundary of set $Viab_{\mathcal{S}_6}(\mathbb{k}_{C_F})$ by construction. Hence, for transfers as high as $6.5 \cdot 10^{10}$ ar $\cdot$ yr^{-1} (31 M2011\$US yr^{-1}) it is possible for the Tandroy to maintain their current per capita consumption, and for their valuable forest to be preserved. It is also worth noting that $Viab_{\mathcal{S}_{5.5}}(\mathbb{k}_{C_F})$ and $Viab_{\mathcal{S}_6}(\mathbb{k}_{C_F})$ do not strictly coincide. We have that for $N > N_{2005}$ / $N \cdot (\underline{c} - c_{C5}) > \widetilde{T}$. Hence $Viab_{\mathcal{S}_{5.5}}(\mathbb{k}_{C_F})$ includes some states with high population levels that do not belong to $Viab_{\mathcal{S}_6}(\mathbb{k}_{C_F})$. Conversely, for $N < N_{2005}$ / $N \cdot (\underline{c} - c_{C5}) < \widetilde{T}$. Hence $Viab_{\mathcal{S}_6}(\mathbb{k}_{C_F})$ includes some states with low population levels that do not belong to $Viab_{\mathcal{S}_{5.5}}(\mathbb{k}_{C_F})$.

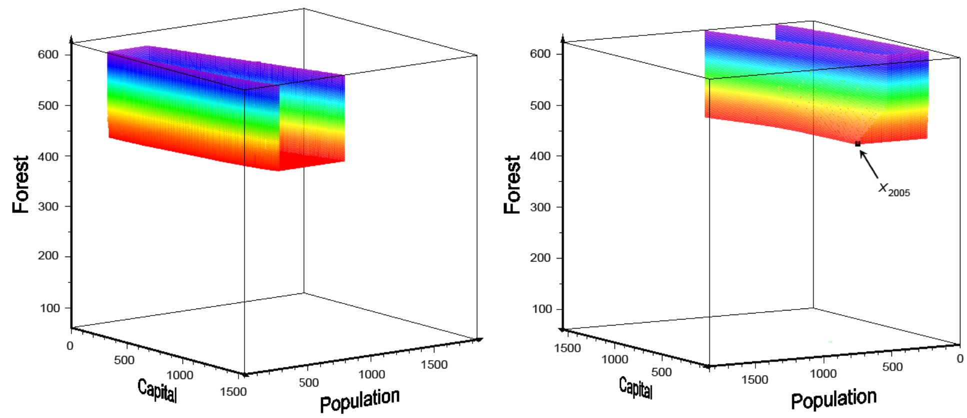

Figure 1.14: Monetary transfers can make x_{2005} viable: $Viab_{\mathcal{S}_6}(\mathbb{k}_{C_F})$

6 Conclusions

The dry forest in Southern Madagascar has great ecological importance and much of its biodiversity is unique. In this paper we modelled the economic and productive system of the Tandroy people and showed that it is far from being sustainable. We proposed a number of policy changes that are necessary to recover sustainability. The Tandroy need to reduce their birth rate, else population will continue to grow and put pressure on the forest resource. Reducing natality is necessary, but not sufficient.

The current deforestation rate is also far from being sustainable. Forests in Androy are an open-access resource and it is not clear that deforestation can be reduced. If reducing deforestation is beyond reach, or if it is too costly to monitor and control deforestation, then it may be desirable to boost afforestation by means of public programs to offset part of the deforestation damage and help stabilize the area of forest.

We have found a number of requirements (e.g., low birth rates, lower deforestation rates than the current levels, low consumption rate) that make the Tandroy production system sustainable for a small set of states. However, the current population in Androy is high and is not part of the set of initial states for which sustainability can be guaranteed. One way to ensure a sustainable development for the current level of population (more than 550 000 people) is to reduce per capita consumption in the region by roughly 40% with respect to 2005 levels.

Reducing consumption can have devastating consequences, though. Deforestation is mainly caused by the mix of poverty and ill-defined property rights over the forest. It is thus very unlikely that the forest will be spared if the Tandroy's consumption rate is reduced. On the contrary, the deforestation problem may worsen if consumption decreases or if population increases persist over time.

Malgache national authorities, developed countries and NGOs are showing increasing interest in preserving the dry forest in Androy. We have computed the amount of money that would be required from them on a yearly basis to preserve the forest as it currently is. According to our estimates, as much as 65 billion ariary (roughly US$ 31M), are required to maintain the current level of consumption for the region's population.

Appendix: Variables description

State Variables

N: Tandroy population

Tandroy population in the Androy region. The data on the Tandroy population that we used was forecasted by RM (2003). According to their forecasts Tandroy population in 2005 totalled 548 418 individuals. State variable N can take values within two bounds $N \in [N_{\min}, N_{\max}]$. $N_{\min} = 0$ is exogenously determined and $N_{\max}$ is determined endogenously as the largest population that can consume their subsistence level $\underline{c}$:

$$N_{\max} = \frac{1}{\underline{c}} \max\left[RA, RW, RZ\right] = \frac{1}{\underline{c}} P_A \overline{X} \left(F_{\max} - \tau\right) = 1\,836\,128,$$

where the terms inside the brackets represent the maximum agricultural yield, the maximum yield from cattle farming, and the maximum timber yield, respectively, when the whole available area is allocated to either of the three activities.

F: Forest area in Androy

The forest stock in our model is measured as the total area of dry forest in the Androy region given in hectares. RM (2008) and USAID (2007) estimate that the forest in 2005 covered 453 561 ha. Forest area can take values in the range $F \in [F_{\min}, F_{\max}]$ where $F_{\min}$ is set equal to the size of the taboo forest (see estimate of parameter τ below) and parameter $F_{\max}$ denotes the maximum extent of the Androy forest measured in hectares. $F_{\max}$ was set equal to the original pristine size of the forest. The oldest and most accurate estimate we have access to is the one by Humbert and Cours-Darne (1965) who estimated that the forest in Androy covered 623 000 ha in 1960.

K: Capital stock in Tandroy society

The stock of capital is proxied by the zebu population. The current zebu population in Androy for 2005 is 290 000 heads (Fanokoa, 2007). The capital stock is bounded in our model and belongs to the set $K \in [K_{\min}, K_{\max}]$. $K_{\min}$ is set equal to zero and represents a physical constraint (we do not allow for banking or negative capital assets) and $K_{\max}$ is the maximum achievable cattle stock that is determined endogenously according to the region's maximum carrying capacity. A zebu needs, on average, an area of δ ha for pasture.[8] If we suppose

[8] The value of δ is estimated below.

that all available land, $F_{\max} - \tau$, is used for pastures, we then obtain $K_{\max} = 1\,555\,994$ zebus.

Control Variables

a: **Afforestation rate**

Total afforested area measured in hectares. Total afforestation in the region is very small. RM (2003) estimates the afforestation for 2005 to be 69 *ha*. The afforestation rate in our model belongs to the interval $a \in [a_{\min}, a_{\max}] = [0\ , 100]$.

b: **Control on the birth rate**

Ability that the governmental institutions have to influence the base birth rate with respect to its natural value $\underline{\alpha}$. A positive value indicates a policy that aims at increasing natality (e.g., tax cuts for larger families) and *vice versa* (e.g., advertising and educational campaigns for the use of contraceptive measures). For our benchmark case, we consider $b \in [b_{\min}, b_{\max}] = [0\ ,\ 0.01]$.

c: **Consumption per capita**

Yearly per capita consumption in the region. We have allowed consumption to fluctuate around the following interval: $c \in [c_{\min}, c_{\max}] = [\underline{c}\ ,\ 2\underline{c}]$, where $\underline{c}$ denotes current consumption (estimated below). The upper bound chosen for maximum consumption, $c_{\max}$, is completely arbitrary but has little or no impact on the solution. We simply chose a value that doubles the current per capita consumption. This upper bound is just required for mathematical tractability.

d: **Deforestation rate**

Total human deforestation measured in hectares. We do not have direct measures or estimations of current deforestation in the region. Nevertheless, we have access to observations of the yearly variation in total forest area in Androy (USAID, 2007). According to USAID, the total forest area decreased at a rate of 3098 ha yr^{-1} during the period 2000-2005, meaning that average forest loss in equation (4) equals 3098 ha yr^{-1}. We know all the other terms in equation (4). We can thus compute current deforestation as a residual. Substituting for parameters and current values of the afforestation rate, we obtain an estimate of current deforestation: $d_{2005} = 5708.4\ ha$. In our paper d is modelled as a bounded pseudo-control variable that the Tandroy can adjust, i.e., $d \in [0\ ,\ d_{\max}]$ where we rounded up and set $d_{\max}$ equal to 6000 ha so that the value of the upper bound is consistent with current deforestation figures, that is, we set the current deforestation as a worst-case scenario.

da: **Area deforested and burnt for agricultural purposes**

Total area burnt and subsequently used to grow agricultural goods. Typically, the productivity of this type of land is much higher.

dw: **Area deforested to obtain wood**

Forest area deforested to obtain wood primarily used for heating and cooking. The wood obtained can also be sold to obtain economic revenues.

dz: **Area deforested for pasture use**

The Tandroy burn the cacti and use them to feed cattle. This practice is known as *ororaketa*.

T: **Monetary transfers**

Total monetary transfers expressed in ariary. These are monetary transfers coming from the national authorities or foreign aid agencies. They can be seen as transfers or subsidies to the Tandroy people with the objective of compensating them for reducing deforestation or applying a given forest or environmental policy.

Parameters

The parameters are written in order of appearance in the text.

$\underline{\alpha}$: **Average birth rate**

The birth rate in Androy is measured in child births per person and year. We compute $\underline{\alpha}$ using the estimates from RM (2003) for the four main districts in the region in 2005. The numbers inside parentheses correspond to the birth rates while the other figures correspond to the population in each of these four districts.

$$\underline{\alpha} = \frac{146078\cdot(0.020) + 51592\cdot(0.023) + 50652\cdot(0.028) + 99194\cdot(0.032)}{146078 + 51592 + 50652 + 99194} = 0.026.$$

β: **Average mortality rate**

Average mortality rate was computed using the same source (RM, 2003) and in the same manner as $\underline{\alpha}$.

$$\beta = \frac{146078\cdot(0.006) + 51\,592\cdot(0.003) + 50652\cdot(0.009) + 99194\cdot(0.006)}{146078 + 51592 + 50652 + 99194} = 0.006.$$

θ: **Population willingness to migrate in terms of revenue**

Current migration in the region is $\gamma = -0.005$ (Fanokoa, 2007). We have calibrated θ using equation (3) to fit current migration. This gives us an estimate of $\theta = 5.956 \cdot 10^{-8}$.

M: **Per capita income for Madagascar**

Average per capita income for Madagascar measured in ariary per year for 2005. IMF (2007) estimates per capita income at US$ 309. The average exchange rate between the ariary and the US dollar for the period from 2003 to 2006 is 1238.33 ar per dollar. This gives a value of $M = 382\,644$ ariary.

η: **Natural growth rate of the forest**

Elmqvist et al. (2007) report that the Androy forest cover grew 4% in abandoned rural areas from 1993 to 2000. The authors show that these findings are also consistent with data for the period between 1984 and 1993. This change in forest area can be expressed on an annual basis to obtain a mean annual increase of 0.5619%. Considering that total area of Androy forest in 2005 was 453 561 ha, we estimate the natural regeneration of forest area to be 2548.6 ha yr^{-1}. If forest grows following expression (4), substituting, we obtain $\eta = 0.021$.

μ: **Natural net growth rate of livestock population per annum**

This parameter denotes the yearly net growth rate of zebu population when taking into account natality and mortality rates for natural causes. Note that the Tandroy often kill zebu in times of famine in order to eat. This has to be taken into account when computing μ or else the parameter could be underestimated. We used data from RM (2007) to estimate μ at 0.029.

P_z: **Price per zebu**

The average price of a zebu has been proxied by the price of the meat in an adult animal or rather what is called a tropical cattle unit (TCU). Barral (1974) and Kirsch-Jung and

Soeftestad (2006) introduce this synthetic unit to compare across zebus of different ages and types. An adult zebu weights approximately 250 kg on the hoof, while the average one is slightly smaller (approximately 25 % smaller). The price of meat being $P_{meat} = 2900$ ar/kg we estimate the price per zebu at 543 750 ariary.

δ: **Area needed for pasture per head of cattle**

This parameter is estimated by computing the ratio of current pasture land, here denoted SZ_{2005}, with respect to the current stock of cattle, i.e., $\delta = \frac{SZ_{2005}}{K_{2005}}$. To obtain SZ_{2005}, we subtract the surface area dedicated to agriculture, SA_{2005}, from the already deforested area ($F_{\max} - F_{2005}$). The area dedicated to agriculture SA_{2005} equaled 65 000 ha in 2005 (INSTAT, 2006) and $F_{\max} - F_{2005} = 170\,000$ ha, hence $SZ_{2005} = 105\,000$ ha. Parameter $\delta = 0.362$ $ha/zebu$.

$\overline{X}$: **Average land productivity**

Average land productivity is measured in kilograms of agricultural good per hectare. We use a single agricultural good in our model even if the Tandroy grow a variety of goods. To compensate for this shortcoming we computed $\overline{X}$ taking into account the three most important agricultural products in the region (i.e. corn, manioc and sweet potato). RM (2006a and 2006b) provide us with estimations of the average productivity per hectare for each of these goods, namely $\overline{X}_{corn} = 1000$ kg/ha, $\overline{X}_{manioc} = 6300$ kg/ha and $\overline{X}_{potato} = 4500$ kg/ha. Note that in one year one may grow corn and sweet potato on the same land since both growing calendars are compatible; however, manioc cannot be mixed with either of them because its growing calendar overlaps that of the other two crops. We compute the average yield in tons of crop per hectare by aggregating together corn and sweet potato on the one hand and manioc on the other: $\overline{X} = \frac{(\overline{X}_{corn}+\overline{X}_{potato})+\overline{X}_{manioc}}{2} = 5900$ kg/ha. We thus have three crops but two possibilities, for this reason we have a two in the denominator. We can do this type of aggregation as long as we are careful in computing the average price of the representative agricultural product P_A.

P_A: **Price of the representative agricultural good**

The price of the representative agricultural good in ariary per kilogram of crop. We aggregate the information on the three most important crops, to compute both the price and the average land productivity of our representative good. RM (2006a and 2006b) and WFP (2006) provide us with the price, in ariary, per kilogram of crop of corn, manioc, and sweet potato: $P_{corn} = 191\ ar/kg$, $P_{manioc} = 178.8\ ar/kg$ and $P_{potato} = 178.8\ ar/kg$. To determine the average price, P_A, we aggregate the economic value of these three products as follows:

$$P_A = \frac{P_{corn} \cdot \overline{X}_{corn} + P_{potato} \cdot \overline{X}_{potato} + P_{manioc} \cdot \overline{X}_{manioc}}{2\overline{X}} = 179.8 ar/kg.$$

φ: **Reduced agricultural productivity due to forest depletion**

Eswaran et al. (2001) estimate the loss in productivity as a consequence of land degradation, erosion and desertification. They report that the productivity loss due to such processes in Africa ranges between 2% and 40% and they provide an average estimate for the whole continent: 8.2%. We use their estimates to compute φ. Average productivity in our model is $\overline{X}$. Hence $\varphi = 0.082 \cdot \overline{X} = 483.8 kg/ha$.

$\underline{c}$: **Per capita consumption per annum**

WB (2007) estimates that per capita consumption in Androy in 2005 is 298 600 ariary.

ψ: **Increased agricultural productivity per ha deforested and burned**

This parameter is computed as a ratio with respect to average land productivity. Blanc-Pamard and Rebara (2001) estimate this increase between 50% and 100% for the first two years after the land is burnt. We have set $\psi = 0.75$.

P_w: **Average price of wood**

Price of wood per cubic meter. This parameter is determined by the ratio between the total economic value of wood (industrial round wood and fuelwood) in Madagascar and its total volume. FAO (2006) estimates the economic value of this volume at US$ 75.34 million and the total volume of wood in year 2005 at 7.03 million m^3. We compute the price as $P_w = \frac{75.3}{7.03} = 10.7\$/\mathrm{m}^3$ and translate it into the local currency. This gives $P_w = 13250$ ar/m^3.

c_w: **Per capita consumption of wood**

This parameter is measured as per capita consumption of wood per annum. In Madagascar, one person consumes 0.037 $\mathrm{m}^3/\mathrm{yr}^{-1}$ of fuelwood (FAO, 1983).

n: **Per-hectare timber yield**

The timber yield is measured in cubic metres of wood per hectare of deforested forest. Timber yield is estimated by DEF-ME (1996) at 29 m^3/ha for Eastern and Southern Africa, particularly for the *Euphorbiaceae, Acanthaceae*, etc.

ε: **Free collection of wood ratio**

This is the ratio of free collection of wood per hectare with respect to the average timber yield of deforested land. The ratio has been estimated in the literature to range between $[0.01, 0.025]$. We follow Andrés-Domenech et al. (2011) and adopt $\varepsilon = 0.01$.

τ: Total area of taboo forest

Parameter τ denotes the part of the forest that is taboo for the Tandroy for various reasons, namely, religion and superstition. The Tandroy do not enter and do not collect wood in these parts of the forest. According to Ferguson (2007), the total area of taboo or sacred forest in Androy is 60 000 *ha*.

ω: Yearly revenue per cattle unit

This parameter measures annual economic revenues in ariary coming from cattle. Zebus give milk and provide the Tandroy with meat, two revenue sources that are far from being negligible. Parameter ω can be computed as follows:

$$\omega = p_{milk} \cdot q_{milk} \cdot \frac{K_L}{K} + \frac{p_{meat} \cdot q_{meat}}{\text{life expectancy}},$$

where p_{milk} is the price of a litre of milk; q_{milk} is the number of litres of milk produced yearly by a *cow* in lactation. The ratio $\frac{K_L}{K}$ denotes the proportion of zebus in lactation. With respect to meat revenues, p_{meat} stands for the price per kilogram of meat, q_{meat} equals the amount of meat per average zebu (its weight is 75% of that of a TCU). We divide by the life expectancy to obtain an average quantity of meat on an annual basis. The values estimated for these parameters by RM (2006c) are $p_{milk} = 400$ ar/litre, $q_{milk} = 300$ litres/year, $\frac{K_L}{K} = 0.1875$, $p_{meat} = 2900$ ar/kg, $q_{meat} = 0.75 \cdot 250\ kg$, and their life expectancy is 8 years. Merging these figures together we obtain an estimation of $\omega = 90469$ ar zb^{-1} yr^{-1}.

Bibliography

[1] Andrés-Domenech, P., Saint-Pierre P., & Zaccour, G. (2011). Forest conservation and CO_2 emissions: A viable approach. *Environmental Modeling & Assessment, 16*(6), 519-539.

[2] Aubin, J. P. (1991). *Viability theory. Systems & control: Foundations & applications.* Boston: Birkhäuser.

[3] Aubin, J. P., Bayen, A., & Saint-Pierre, P. (2011). *Viability Theory. New Directions.* Dordrecht: Springer.

[4] Aubin, J. P., Bernardo, T., & Saint-Pierre, P. (2005). *The Coupling of Climate and Economic Dynamics, Essays of Integrated Assessment*. Dordrecht: Kluwer.

[5] Barral, M. (1974). *Mobilité et cloisonnement chez les éleveurs du nord de la Haute-Volta: les zones dites "d'endodromie pastorale"*. Calz. Sci. Hmz. *ORSTOM, 11*(2), 127-135.

[6] Béné, C., Doyen L. (2008). Contribution values of biodiversity to ecosystem performances: A viability perspective. *Ecological Economics, 68*, 14-23.

[7] Bennett, K. D. (1983). Postglacial population expansion of forest trees in Norfolk. U.K. *Nature, 303*, 164-167.

[8] Bernard, C. (2011). *La théorie de la viabilité au service de la modélisation mathématique du développement durable. Application au cas de la forêt humide de Madagascar.* Université Blaise Pascal.

[9] Blanc-Pamard, C., & Rebara, F. (2001). *Paysans de la commune d'Analamisampy. Agriculture Pionnière et construction du territoire en pays Masikoro (Sud-ouest de Madagascar)*. GEREM/IRD/CNRE,CNRS/EHESS.

[10] Chen, Y. (1988). Early Holocene population expansion of some rainforest tree at Lake Barrine basin, Queensland. *Australian Journal of Ecology, 13*(2), 225-233.

[11] Direction des Eaux et Forêts, Ministère de l'Environnement (DEF-ME) (1996). *Plan d'actions environnementales, inventaire ecologique forestier national, programme environnemental - Phase 1. Problématique, objectifs, méthodes, résultats, analyses et recommandations*. Madagascar.

[12] Doyen, L., De Lara, M., Ferraris, J., & Pelletier, D. (2007). Sustainability of exploited marine ecosystems through protected areas: A viability model and a coral reef case study. *Ecological Modelling, 208*(2-4), 353-366.

[13] Ekins, P., Simon, S., Deutsch, L., Folke, C., & Groot, R. D. (2003). A framework for the practical application of the concepts of critical natural capital and strong sustainability. *Ecological Economics, 44*(2), 165-185.

[14] Elmqvist, T., Pyykönen, M., Tengö, M., Rakotondrasoa, F., Rabakonandrianina E., & Radimilah, C. (2007). Patterns of loss and regeneration of tropical dry forest in Madagascar: The social institutional context. *PLoS ONE 2*(5).

[15] Eswaran, H., Lal R., & Reich, P. F. (2001). Land degradation: an overview. In E. M. Bridges, I. D. Hannam, L. R. Oldeman, F. W. T. Penning de Vries, S. J. Scherr & S. Sombatpanit (Eds.), *Response to land degradation* (pp. 20–35). Enfield, NH, USA: Science Publishers Inc.

[16] Fanokoa, P. S. (2007). *Dynamique des pratiques paysannes face à la déforestation dans l'extrême sud de Madagascar.* Université de Versailles Saint-Quentin-en-Yvelines, France.

[17] Ferguson, H. B. (2007). *Community forest management in Southern Madagascar: Local Institutions and the policy process.* University of East Anglia.

[18] Food and Agriculture Organization of the United Nations (FAO) (1983). *Fuelwood supplies in the developing countries.*

[19] Food and Agriculture Organization of the United Nations (FAO) (2006). *Forest resources assessment 2005.*

[20] Food and Agriculture Organization of the United Nations (FAO) (2010). *Forest resources assessment 2010.*

[21] Humbert, H., & Cours-Darne, G. (1965). Carte internationale du tapis végétal et des conditions écologiques. 3 coupures à 1/1.000.000 de Madagascar. *Travaux de la section scientifique et technique de l'Institut Français de Pondichery, hors série No 6, 3 maps.*

[22] International Monetary Fund (IMF) (2007). Republic of Madagascar: First review under the three-year arrangement under the poverty reduction and growth facility and request for waiver and modification of performance criteria - Staff report; Staff statement; Press release on the executive board discussion; and statement by the executive director for the Republic of Madagascar, country *report No. 07/7.*

[23] Institut National de la Statistique (INSTAT) (2006). *Enquête productivité agricole, campagne 2004-2005. Résultats et analyse.* Anatananarivo: Madagascar.

[24] Kirsch-Jung, K. P., & Soeftestad, L. T. (2006). Regulating the commons in Mauritania: Local agreements as a tool for sustainable natural resource management. *Tenth biannual conference of the International Association for the Study of Common Property.*

[25] Martinet, V., & Blanchard, F. (2009). Fishery externalities and biodiversity: Trade-offs between the viability of shrimp trawling and the conservation of Frigatebirds in French Guiana. *Ecological Economics, 68*, 2960-2968.

[26] Martinet, V., & Doyen, L. (2007). Sustainability of an economy with an exhaustible resource: A viable control approach. *Resource and Energy Economics, 29*(1), 17-39.

[27] Olson, D. M., & Dinerstein, E. (2002). The Global 200: Priority ecoregions for global conservation. *Annals of the Missouri Botanical Garden, 89*, 199-224.

[28] République de Madagascar (RM) (2003). *Monographies régionales des 18 D.I.R.R.*

[29] République de Madagascar (RM) (2006a). *Système d'information sur les marchés ruraux.* MAEP/SIMR/Bulletin n8.

[30] République de Madagascar (RM) (2006b). *Enquête productive agricole. Campagne 2004-2005. Résultats et analyse.* MAEP/MEFB/USAID/INSTAT.

[31] République de Madagascar (RM) (2006c). *Le rapport économique et financier 2005-2006.*

[32] République de Madagascar (RM) (2007). *Cheptel animal. Recensement de l'agriculture. Campagne agricole 2004-2005. MAEP/DMEE/SSA Tome IV.*

[33] République de Madagascar (RM) (2008). *Couverture forestière Ministère de l'environnement des forêts et du tourisme.*

[34] Saint-Pierre, P. (1994). Approximation of the viability kernel. *Applied Mathematics and Optimization, 29*(2), 187-209.

[35] Sugita, S., & Tsukada, M. (1982). Late quaternary dynamics of pollen influx at Mineral Lake, Washington. *Journal of Plant Research, 95*(4), 401-418.

[36] Tsukada, M. (1981). Cryptomeria japonica D. Don, 1. Pollen dispersal and logistic forest expansion. *Japanese Journal of Ecology, 31*, 371-383.

[37] Tsukada, M. (1982). Late-Quaternary development of the Fagus forest in the Japanese archipelago. *Japanese Journal of Ecology, 32*, 113-118.

[38] United States Agency for International Development (USAID) (2007). *Forest Cover and Change: Madagascar, 1990-2000-2005.*

[39] United States Agency for International Development (USAID) (2008). *Mise à jour 2008 de l'évaluation des menaces et opportunités pour l'environnement à Madagascar.*

[40] World Bank (WB) (2007). *Les implications structurelles de la libéralisation sur l'agriculture et le développement rural. Première phase : Synthèse nationale madagascar.* EPP- PNDR/APB Consulting/Coopération Française.

[41] World Commission on Environment and Development (WCED) (1987). *Our common future.* Oxford: Oxford University Press.

[42] World Food Programme (WFP) (2006). *Madagascar: Profil des marchés pour les évaluations d'urgence de la sécurité alimentaire.* Catholic University of Leuven.

Essay 2

Forest conservation and CO_2 emissions: A viable approach

Essay information

This essay was published in *Environmental Modeling & Assessment*:

Andrés-Domenech, P., Saint-Pierre, P., & Zaccour, G. (2011). Forest conservation and CO_2 emissions: A viable approach. *Environmental Modeling & Assessment, 16*(6), 519-539.

Abstract

We adopt viability theory to assess the sustainability of the world's forests while taking into account some of the competing economic, social and environmental uses of these forests; namely, timber production, poverty alleviation through agriculture, and air quality, as well as the negative externalities that these uses create. We provide insights on the different trade-offs faced to achieve sustainability and draw some policy implications as to what is the path leading to sustainability in the long run.

Key Words: Viability theory, sustainability, forest management, emissions, deforestation, renewable resources.

1 Introduction

For the Food and Agriculture Organization "[Forests] are to provide renewable raw materials and energy, maintain biological diversity, mitigate climate change, protect land and water resources, provide recreation facilities, improve air quality and help alleviate poverty" (see, Global Forest Resources Assessment 2005 (FAO, 2006)). The report also states that "Competing interests in the benefits of forest resources and forest land are omnipresent, and the need for a sound basis for analysis and conflict resolution has never been greater." The aim of this paper is to assess sustainability of the world's forests by taking into account some of these competing economic, social and environmental uses; namely, timber production, poverty alleviation through agriculture, and air quality, as well as the negative externalities that these uses create.

Both the provision of timber and agricultural use of deforested land bring economic revenues in the short run. However, an excessive exploitation of forests (and subsequent reduction of the forest mass) may make it harder, if possible at all, to fulfil either economic or environmental goals in the long run. As world forests grow smaller not only timber production falls, but also the capacity of world forests to absorb (i.e., sequester) carbon (FAO, 2001, 2006). World's forests *clean the air* by sequestering carbon from the atmosphere and releasing oxygen as they grow. This carbon is fixed in the form of organic matter. World forests thus behave both as a carbon tank, storing carbon in form of biomass, and as a carbon sink, sequestering carbon from the atmosphere as they grow. A decrease in forest biomass implies a reduction in the total amount of carbon stored, however, it is often neglected that the size of forest biomass may also affect the ability of forests to work as a carbon sink.

Forest expansion depends on total forest land. Some forestry studies have shown that the speed at which forests expand is far from being constant or even linear (see e.g., Bennett (1983), Chen (1988), Sugita and Tsukada (1982), Tsukada (1981, 1982)). This non-linearity implies that forest depletion has an impact on the role of forests to behave as a carbon sink. A tree that is cut cannot grow and thus cannot sequester carbon. Deforestation not only reduces total forest biomass, but also total carbon sequestration.

Based on this simple idea we have built a model that accounts for the exacerbating effect that deforestation has on the accumulation of greenhouse gases, namely CO_2, in the atmosphere. Increases in the atmospheric concentration (i.e., stock) of greenhouse gases due to emissions cause problems like global warming (see IPCC[9] (2001, 2007), The Royal Society (2005)) and a number of heart and breathing diseases (NIOSH[10], 2007).

We have modelled deforestation as a means to get revenues from timber production as well as from agricultural use of deforested land. To model these two sources of revenues we took stock on the forestry model presented in Van Soest and Lensink (2000) and added some new elements, i.e., afforestation effort and natural growth of the forest to account for its renewable nature.

Together with the forestry model, we have considered CO_2 emissions and the stock of CO_2 in the atmosphere. The main objective of this paper is to link deforestation to the issue of accumulation of greenhouse gases in the atmosphere.

We have introduced a number of economic and environmental requirements and characterized the set of controls (deforestation and afforestation rates, emissions and eventual monetary transfers to forest owners) and initial states of a dynamic system that are compatible with the compliance of such requirements.

[9]IPCC: Intergovernmental Panel on Climate Change.

[10]NIOSH: National Institute for Occupational Safety and Health.

In particular, we question ourselves about the sustainability of the current emissions-forestry system. To tackle this question we call upon viability theory, see e.g., Aubin (1991), which offers a natural framework to assess sustainability by retrieving the set of initial states of a system for which it is possible to find a control path that satisfies a number of requirements. This approach proves to be extremely useful when working with complex dynamic systems where analytical solutions are beyond reach. Recent applications of viability theory to resources and environmental economics include, among others, De Lara and Doyen (2008), De Lara et al. (2007), Doyen et al. (2007), Martinet and Doyen (2007), Aubin et al. (2005), Aubin et al. (2004).

Our results show that sustainability requires larger afforestation levels, decreases in emissions, and convergence, in the long term, to emissions 18% lower than its 1990 levels.

The remainder of this paper is organized as follows. In Section 2 we present the model. Section 3 serves to introduce the notion of sustainability and relate it to viability theory. In Section 4 we obtain a first set of results that is complemented in Section 5 with a series of scenarios where the values of main model's parameters are modified. Section 6 concludes.

2 Model

In this section, we introduce the dynamical system, the control variables and the constraints. The forest model that we use was originally proposed in Ehui et al. (1990) and later modified and specified in Van Soest and Lensink (2000). We enrich the version in Van Soest and Lensink (2000) by capturing new elements of the forest dynamics such as afforestation and the natural growth of the forest. We have included the dynamics of both CO_2 emissions and the stock of CO_2 in the atmosphere. As a result, we have a dynamical system involving three state variables. The four control variables retained in our model are the rate of deforestation, the rate of afforestation, the speed of adjustment of CO_2 emissions and, eventually, monetary transfers to land owners.

2.1 Dynamical system

Although it is trivial to state that a forest is a renewable resource that grows and expands overtime, it is a highly challenging issue to measure how fast it expands. The reason lies in the fact that "There is little information, on the population dynamics of trees [...] and any significant changes in populations usually occur over at least several decades, making field observation and experimentation very difficult" (Chen, 1988). Because of this difficulty to model the expansion -surface wise- of different taxa, a number of studies have used pollen data to track their historical evolution (see, e.g., Bennett (1983), Chen (1988), Sugita and Tsukada (1982), Tsukada (1981, 1982)). The conclusion to which most of these studies arrive is that the expansion of the analyzed taxa depends positively on their size, and can be well modelled through an exponential or logistic law.

Although in this paper we are interested in modelling the growth rate of the forest itself, and not just a few species, we shall take stock on the above cited studies in modelling the expansion of the global forest. Indeed, we assume this expansion to be non-linear and limited by the availability of conquerable land.[11] As a forest expands, total available conquerable land diminishes until a point where land scarcity matters and slows down forest expansion, exactly as it happens with individual tree taxa. We approximate this process by a logistic function exhibiting a carrying capacity. Denote by F the world's total forest area, and by ρ and d the instantaneous reforested and deforested area, respectively. The evolution of

[11] Conquerable land can be broadly defined as the land that is suitable for forest colonization. In our case conquerable land is proxied by world total forest area prior to industrialization.

forest area is described by the following differential equation

$$\dot{F}(t) = \rho(t) + \eta \cdot \left(1 - \frac{F(t)}{F_{\max}}\right) \cdot F(t) - d(t), \tag{26}$$

where η is a positive parameter[12], and $F_{\max}$ is the maximum carrying capacity of the forest. Previous studies have estimated the total size of world's forests before pre-industrial levels at roughly 42% of total land (that is 5 500 million hectares). We have set $F_{\max}$ at this value. The above dynamics is a generalization of the forest dynamics used in Van Soest and Lensink (2000) and Fredj et al. (2006), where $\rho = \eta = 0$ in the first and $\rho = 0$ in the second.

Let $E(t)$ represent the flow of anthropogenic CO_2 emissions. We suppose that emissions take time to adjust. Indeed, it seems unrealistic to assume that emissions can be changed (i.e., reduced) drastically overnight, for many reasons, among them inertia in consumption habits and technology. Denote by v the rate of variation of emissions, i.e., $\frac{\dot{E}(t)}{E(t)} = v(t)$. Our assumption that emissions take time to adjust implies that v is bounded. Note that saying that emissions are a flow control variable whose rate of change is bounded, is entirely equivalent to the expression below, where, for simplicity, we have modelled emissions as a state variable and v as a bounded control variable:

$$\dot{E}(t) = v(t) \cdot E(t), \tag{27}$$

where $v \in [v_{\min}, v_{\max}]$,with $v_{\min} < 0$ and $v_{\max} > 0$. The lower and upper bounds of v, discussed in the appendix, can be seen as technological caps. We have kept $v_{\min}$ constant, for it is the simplest specification that allows us to capture the increasing difficulty to abate emissions in absolute terms as emissions decrease. One could think of more sophisticated specifications where $v_{\min}$ and $v_{\max}$ are parameters that depend on technology or emissions

[12] All parameters values and references used to obtain them are provided in the appendix.

themselves. However, in this exploratory study, we keep $v_{\min}$ and $v_{\max}$ constant for simplicity.

Denote by S the cumulated stock of CO_2 in the atmosphere. This stock increases with the release of emissions and decreases with carbon sequestration by world's forests and oceans.[13] Trees sequester carbon as they grow and IPCC (2007) and FAO (2006) estimate that roughly 50% of the dry weight of forest biomass is carbon. The total amount of carbon sequestered is proportional to the variation in volume of biomass in world forests. However, measuring this variation is far from being an easy task. Further, other factors, e.g., the value of forest captures, need to be considered to avoid underestimating the carbon sequestration effect by world forests. To overcome these difficulties, we make the simplifying assumption that there exists a representative and homogeneous forest with a constant growth rate -in terms of volume- per unit of area. Considering the growth rate of biomass as given enables us to express total carbon sequestration in terms of forest area alone. This implies, *ceteris paribus*, that carbon sequestration increases with forest area. It is tempting to assume, for the sake of generality, that the marginal rate of sequestration depends on the availability of CO_2, and, therefore, have the rate of sequestration given by $\varphi(S) \cdot F$. However, due to lack of conclusive evidence showing larger absorption rates in environments with greater CO_2 concentration,[14] we suppose that total carbon sequestration by the forests is given by $\varphi \cdot F$, where φ is a positive parameter that denotes the amount of carbon sequestered per unit of area and time.

The second carbon sink considered in this paper are oceans. Denote by $W(t)$ the carbon sequestered by oceans per unit of time. To keep the numerical computations manageable, we assume W to be constant, i.e., $W(t) = W$. This assumption, however, is in line with

[13] For simplicity land use change emissions have been neglected in this paper.

[14] See, e.g., Norby et al. (2002).

empirical evidence showing that carbon uptake during the last few years has remained relatively stable despite small year to year variations due to el Niño effects (Le Quéré et al. (2009)). The evolution of the stock of pollution is, then, given by the following differential equation:

$$\dot{S}(t) = E(t) - \varphi \cdot F(t) - W. \tag{28}$$

To wrap up, our three-dimensional dynamical system, to which we shall refer to as $\mathcal{D}$, consists of:

$$(\mathcal{D}) \quad \begin{cases} \dot{F}(t) & = & \rho(t) + \eta \cdot \left(1 - \frac{F(t)}{F_{\max}}\right) \cdot F(t) - d(t), \\ \dot{E}(t) & = & v(t) \cdot E(t), \\ \dot{S}(t) & = & E(t) - \varphi \cdot F(t) - W, \\ \text{with } u & := & (\rho, d, v) \in U, \end{cases} \tag{29}$$

where U is the feasible set of controls, that is,

$$U = [\rho_{\min}, \rho_{\max}] \times [d_{\min}, d_{\max}] \times [v_{\min}, v_{\max}].$$

For future reference, we generically write the first three lines of $\mathcal{D}$ as

$$\dot{x}(t) = f\left(x(t), u(t)\right),$$

with $x(t) = (F(t), E(t), S(t))$.

2.2 Control variables and revenue

The forest in our model is owned and managed by some agents (private or public) who get revenues from three different sources: forest exploitation (i.e., selling of timber), agricultural activities and monetary transfers. Denote by R the revenues generated from these three sources. Timber exploitation yields a revenue given by $P \cdot q$, where P is the price and q the quantity of timber put on the market. We assume that the prevailing price of timber is given by the following linear inverse demand function

$$P = \bar{P} - \theta \cdot q, \tag{30}$$

where $\bar{P}$ is the choke price (maximum price consumers are willing to pay) and $\theta > 0$ is the slope of timber demand.

As in Van Soest and Lensink (2000), we assume that timber can be obtained from two sources: clear felling and selective logging. Clear felling a given forest area d provides a quantity of timber $n \cdot d$, where n stands for the per hectare timber yield. Timber can also be retrieved by selectively logging a few trunks per hectare, which has little impact on the stock of forest. The per hectare yield of selectively-logged forest land equals $n \cdot \gamma \cdot F$, where parameter γ stands for the efficiency of selective logging with respect to clear felling. We have included an additional parameter δ to account for the fact that only a fraction of the world's forests is currently being used for productive purposes (FAO, 2007).[15] The quantity of timber available in the market is thus given by

$$q = n \cdot d + n \cdot \gamma \cdot \delta \cdot F. \tag{31}$$

Revenues from agriculture depend on the quantity of good produced and its price. We suppose that the price of the agricultural good is a given constant P_A. The harvest depends on the size of the land available for agriculture, that is $F_{\max} - F$, and on its productivity, denoted by Z. The productivity of agricultural land is determined by three components, a fixed element, and two variable ones:

$$Z = \bar{Z} + \alpha \cdot d - \beta \, \frac{F_{\max} - F}{F_{\max}}. \tag{32}$$

[15] Van Soest and Lensink (2000) set $\delta = 1$.

The term $\alpha \cdot d$ stands for the enhanced soil productivity of newly deforested land, and $-\beta \frac{F_{\max}-F}{F_{\max}}$ captures the positive externality generated by forests that serve as shelter and source of nutrients and rain to nearby agricultural land. Note that this positive externality becomes a negative one as the forest is depleted. We use α to remain consistent with Van Soest and Lensink's notation; however, here α is a variable, rather than a fixed parameter, and is given by

$$\alpha = \frac{\psi \cdot \bar{Z}}{F_{\max} - F}. \tag{33}$$

The rationale for having this variable term, instead of a fixed one as in Van Soest and Lensink (2000), is to normalize the impact of d among all agricultural land $F_{\max} - F$, and recognize that newly deforested land is more productive by a factor ψ. In fact, omitting this normalization term in the denominator would imply overestimating the impact of deforestation on productivity.

The third source of revenues for forest owners comes from eventual monetary transfers T. The determination of the latter has been the subject of a number of papers (see, e.g., Barbier and Rauscher (1994), Stähler (1996), Van Soest and Lensink (2000), Fredj et al. (2004, 2006), Martín-Herrán et al. (2006)). Basically, the idea in this literature is to let the monetary transfers by industrialized countries ($North$) be contingent on deforestation rate or size of the tropical forest in the developing countries ($South$). We do not address here the issue of designing a payment scheme having some desirable properties (e.g., time consistency). We focus rather on the determination, if necessary, of the monetary transfer that can be part of a sustainable environmental and economic regime.

Taking into account the three above-mentioned sources of revenues together we obtain the following total net revenue function:

$$R = P \cdot q + P_A \cdot Z \cdot [F_{\max} - F] + T - \kappa_1 \rho - \kappa_2 d, \tag{34}$$

where P_A is the price of the agricultural good,[16] transfers T unless otherwise specified are taken constant and equal to zero, and κ_1 and κ_2 denote the per-hectare afforestation and deforestation costs respectively.

3 Defining sustainability

In 1987 the World Commission on Environment and Development of the United Nations released the famous report entitled "Our Common Future" where sustainable development was formally defined for the first time as: "Development that meets the needs of the present without compromising the ability of future generations to meet their own needs." It is now admitted that the term sustainable development embodies three components, namely environmental, economical and social. This paper deals with the first two aspects in the context of exploitation of the world forests.

Before we formally define what we intend by environmental sustainability, we introduce the state-space used, that is made of three state variables: forest area, CO_2 emissions and atmospheric concentration of emissions. The state space to which we shall refer to is a subset of $\mathbb{R}^3_+$:

$$K := \{x = (F, E, S) \;/\; F \in [F_{\min}, F_{\max}]\,,\; E \in [E_{\min}, E_{\max}]\,,\; S \in [S_{\min}, S_{\max}]\}\,.$$

The set is a cube with upper and lower bounds for each state variable. The upper and lower bounds that we use for the forest area F represent physical rather than normative

[16] Note that P_A, unlike the price of timber P, is constant. In fact, our model accounts for all wood production and it makes sense to have a variable P. On the contrary, the agricultural production in our model represents only a fraction of world agricultural production. For this reason, we make the simplifying assumption that P_A is constant.

constraints. The upper bound on F denotes world forest area at pre-industrial levels and the lower bound $F_{\min}$ is set to zero.[17]

The bounds on emissions E can be seen as reasonable economic constraints. It is not very realistic to think that emissions can be totally eradicated. For this reason we have chosen a lower bound for emissions, $E_{\min}$, that is different from zero. This lower bound on emissions can be identified with a minimal size of the economy. In particular we have chosen $E_{\min}$ equal to the level of emissions in 1990 ($E_{\min} = E_{1990}$), since this figure has found large consensus in the economic literature as a benchmark level following the Kyoto Protocol. The upper bound on emissions, $E_{\max}$, that we have retained is two times $E_{\min}$. This choice, though arbitrary, is set sufficiently large so that its impact on the solutions retrieved throughout is negligible. In fact the only reason to have an upper bound on emissions is mathematical tractability (i.e., need to work with bounded states).

Finally, the upper and lower bounds on S do have a clear and important environmental interpretation. The lower bound reflects pre-industrial atmospheric carbon levels, i.e., 284 ppm; while the upper bound, $S_{\max}$, accounts for carbon concentration levels beyond which serious climate issues may arise. We have used the upper bound of 650 ppm CO_2 as a benchmark level.[18] The benchmark 650 ppm CO_2 concentration is equivalent to roughly 700-750 ppm CO_2-equivalent and, according to the IPCC, such concentration will lead, by the end of the 21st century, to a very likely increase in temperature above 1.8^{o}C (most probably 4.3^{o}C) and a sea level rise between 0.26 to 0.59 meters mostly due to the thermal expansion of the water. As any choice is somehow arbitrary, we shall also run the model with a different upper bound (550 ppm), and assess the impact on the results.

[17]Note that $F_{\min} = 0$ has to be seen as a physical constraint and we, by no means, think that a world without forests is sustainable.

[18]The selection of the lower and upper bounds for each state variable is further discussed in the Appendix.

Our approach of restricting the control and state variables values to lie in pre-defined intervals, is similar to the so-called Tolerable Windows Approach (see, e.g., Petschel-Held et al. (1999), Bruckner et al. (2003) and Aubin et al. (2005a)). Actually, the bounds on the controls simply reflect the idea that decisions cannot be changed drastically overnight, while the bounds on the states are related to physical, technological or environmental constraints. Table 2.I summarizes the benchmark bounds that will be used for the state and control variables of our model in the scenarios showed in sections 3 and 4. In section 5, we have performed some sensitivity analysis with respect to these key variables and thresholds for the sake of completeness.

Table 2.I: State and control variables' bounds

Variable	Lower Bound	Upper Bound
F	0 ha	5.5 Gha
E	21.4 $GtCO_2$	42.8 $GtCO_2$
S	284 ppm	650 ppm
ρ	0 ha	3 Mha
d	0 ha	15 Mha
v	-0.015	0.03

The benchmark upper bound values used for ρ and d correspond to the current or business-as-usual (BAU) ones. With respect to v we have allowed for a lower bound of up to -1.5% that is consistent with what is achievable, but is far from the BAU scenario of 1%-3% yearly emissions increase.

3.1 Environmental sustainability

Once our working environment, K, has been defined, our goal is to find a control policy such that the system remains within K, and more particularly within the bounds $[S_{\min}, S_{\max}]$. Denote by $x_0 = (F_0, E_0, S_0)$ any point in K. Any such point x_0 is said to be viable if and only if we can find at least one evolution whose trajectory remains inside K forever.

Denote by $S_{\mathcal{D}}(x_0)$ the set of all evolutions $x(\cdot)$ for the dynamical system (29) which emanate from x_0, that is,

$$S_{\mathcal{D}}(x_0) := \{x(\cdot)/\exists u(\cdot) \ \in U \text{ such that } \dot{x}(t) = f\left(x(t), u(t)\right), \ x(0) = x_0\}\,. \tag{35}$$

To generalize this notion from one single point (as in (35)) to a whole set of states we call upon viability theory and its definition of the viability kernel. The viability kernel is the set of points x_0 for which there exists at least one evolution $x\left(\cdot\right) \in S_{\mathcal{D}}(x_0)$, which remains in K:

$$Viab_{\mathcal{D}}(K) := \{x_0 \in K \ / \ \exists x(\cdot) \in S_{\mathcal{D}}(x_0), \forall t \geq 0, \ x(t) \in K\}\,.$$

The concept of $Viab_{\mathcal{D}}(K)$ is particularly useful since it coincides with the set of points denoted as environmentally sustainable. By construction, for all points (states) in $Viab_{\mathcal{D}}(K)$ it is possible to find a control rule such that our environmental criterion (i.e., $S \in [S_{\min}, S_{\max}]$) can be met at all times. Although we do not impose *a priori* a minimum level of F,[19] this does not mean that having a small-sized forest is desirable (or sustainable). In this paper, forests are seen as an instrument to control pollution and we do not actively require that forest area remains above a specified level. This being said, if the forest area becomes too small, then the stock of greenhouse gases accumulates quickly (see equation (28)) and sustainability may not be achieved. In other words, we let the model determine endogenously which is the minimum size of the forest that is compatible with a sustainable development.

3.2 Economic sustainability

To account for economic sustainability we have retained two indicators: the level of revenues R and the quantity of wood produced q. Economic revenues measure the income of forest owners and we use them as a proxy to their welfare, while the quantity of wood q accounts

[19] Recall that $F_{\min} = 0$.

for the needs in terms of furniture and heating. We operationalize economic sustainability through the following two constraints:

$$q\,(x,u) \geq \underline{q}, \tag{36}$$

$$R\,(x,u) \geq \underline{R}, \tag{37}$$

that is, we require that forest revenues and supply of timber always be greater or equal than some pre-specified thresholds. We have equated these lower bounds $\underline{q}$ and $\underline{R}$ to current timber production and economic revenues. Current levels are used here as a proxy to account for the needs of the present. At the same time, by requiring q and R to be always greater or equal than these current threshold levels, we are operationalizing the definition of sustainable (economic) development put forward before. This being said, the choice of any pair $\left(\underline{q}, \underline{R}\right)$ may be subject to a long debate. In any event, one can modify these values at will and verify if under the new ones, sustainability is still feasible.

3.3 Global sustainability

After having introduced the two notions of sustainability, one can further define *global* sustainability as the simultaneous fulfilment of both economic and environmental objectives. Again, this notion of global sustainability can be easily dealt with by the existing tools in viability theory.

The functions q and R, defined respectively by (31) and (34), depend on the state x and the control u. They must satisfy the constraints (36) and (37).

For any $x = (F, E, S)$, define the set-valued map

$$x \to U_C\,(x) := \left\{u \in U \text{ such that } q\,(x,u) \geq \underline{q} \text{ and } R\,(x,u) \geq \underline{R}\right\}. \tag{38}$$

Note that if there is no pair (x, u) verifying the economical constraints then $U_C(x)$ is empty. Introduce now the corresponding constrained dynamical system:

$$(\mathcal{D}_C) \quad \left\{ \begin{array}{lcl} \dot{F}(t) & = & \rho(t) + \eta \cdot \left(1 - \frac{F(t)}{F_{\max}}\right) \cdot F(t) - d(t), \\ \dot{E}(t) & = & v(t) \cdot E(t), \\ \dot{S}(t) & = & E(t) - \varphi \cdot F(t) - W, \\ \text{with } u & := & (\rho, d, v) \in U_C(F, E, S). \end{array} \right. \tag{39}$$

We now define the viability kernel of K for the above dynamical system:

$$Viab_{\mathcal{D}_C}(K) := \{x_0 \in K,\ \exists x(\cdot) \in S_{\mathcal{D}_C}(x_0), \forall t \geq 0, x(t) \in K\}.$$

By construction, all the elements x_0 belonging to $Viab_{\mathcal{D}_C}(K)$ are environmentally and economically sustainable. By comparing (29) and (39) we observe that for all $x \in K$, $U_C(x) \subset U$, therefore we have

$$Viab_{\mathcal{D}_C}(K) \subseteq Viab_{\mathcal{D}}(K). \tag{40}$$

4 Results

Step 1: Computation of $Viab_{\mathcal{D}}(K)$

Consider the system $\mathcal{D}$, that is, the vector of controls, u plus the three state equations governing the evolution of forest area, emissions and stock of emissions, (26)-(28); and consider $Viab_{\mathcal{D}}(K)$ the subset of K for which at least one evolution remains in K forever. The set $Viab_{\mathcal{D}}(K)$ is made of the states for which there exists a control rule such that our environmental requirements can be sustained.

The first insight that we get is that $Viab_{\mathcal{D}}(K)$ is empty. Recall that our system D is made of three dynamic equations, where the third one is $\dot{S}(t) = E - \varphi F - W$ and with $F \in [0, F_{\max}], E \in [E_{\min}, E_{\max}]$ and W constant. We observe that, for $E = E_{\min}$ and $F = F_{\max}$, $\dot{S}$ is positive for our parameters' values. A fortiori, $\dot{S}$ will be positive for any other values of E and F. Now, if $\dot{S} > 0$ is always positive for any point $x = (F, E, S) \in K$ and K is a bounded set, this means that any solution starting from any point in K, leaves K in finite time. Put differently, a necessary condition to recover viability is that there exists a state $x \in K$ for which the following condition applies:

$$E - \varphi F - W \leq 0. \tag{41}$$

The highest emissions level for which inequality (41) can be satisfied is obtained by substituting F by its maximum value. We denote this emissions level by $E^{\#}$:

$$E^{\#} = \varphi F_{\max} + W.$$

We define the new set $K^{\#}$ accordingly as:

$$K^{\#} := \left\{ x = (F, E, S) \ / \ F \in [F_{\min}, F_{\max}] \, , \ E \in \left[E^{\#}, E_{\max} \right] , \ S \in [S_{\min}, S_{\max}] \right\} ,$$

where $K^{\#}$ is the minimum set for which one can find a non-empty viability kernel. By this we mean that emissions $E^{\#}$ is the highest level of emissions for which environmental sustainability is achievable.

From now onwards we work with $K^{\#}$ rather than with K. Figure 2.1 below is a plot of $Viab_{\mathcal{D}}(K^{\#})$, where $Viab_{\mathcal{D}}(K^{\#})$ is the region above the coloured surface.

From Figure 2.1 we draw a number of conclusions: (i) The larger the initial stock of CO_2 in the atmosphere, the larger the minimum initial forest area needed to guarantee sustainability. (ii) States with too low forest levels and high stock of GHGs (bottom-right) or states with high emissions and high stock levels of CO_2 (top-right corner) are not sustainable, regardless of the values of the other state variables: If the forest area is too small and GHG concentrations are high, then one cannot reforest fast enough to counter balance the rapid increase of greenhouse gases due to high emissions levels. Similarly, if both

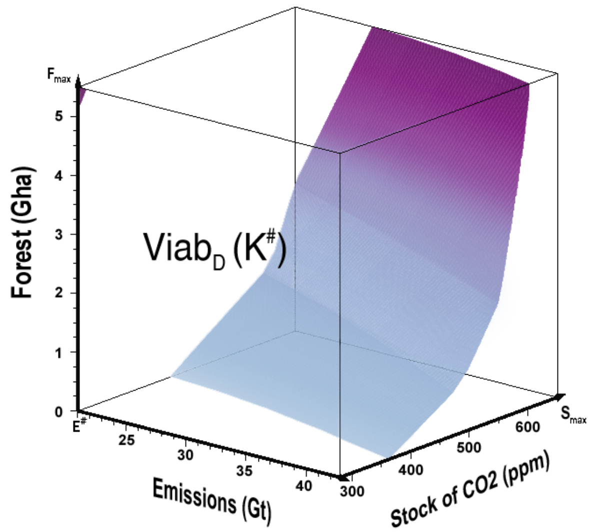

Figure 2.1: Viability kernel of set $K^{\#}$: $Viab_{\mathcal{D}}(K^{\#})$

emissions and the stock of CO_2 are too high, then, whatever the forest area, one cannot reduce emissions quickly enough to ensure sustainability. In either case, sustainability cannot be achieved because of a too large accumulation of CO_2 in the atmosphere.

Note that in Figure 2.1 we have the paradoxical result that some low-forest states *are* sustainable. This result, however, has to be interpreted with great care. In fact, forest area needs to converge to $F_{\max}$ in the long run, else we have a perennial accumulation of greenhouse gases in the atmosphere. Hence, forests play a key role in maintaining GHG concentrations within the pre-specified limits, even if forest area is small in the short run. What this figure suggests is that such low-initial forest-area scenarios may still be viable in the long run provided that we reforest fast enough and forest surface increases sufficiently fast.

Step 2: Introduction of the economic constraints

Let us define

$$K_C^{\#} := \left\{ x \in K^{\#} \text{ such that } U_C(x) \neq \varnothing \right\}. \tag{42}$$

In this step we retrieve $K_C^{\#}$, which is the subset of $K^{\#}$ for which one can find a set of controls for which it is possible to satisfy our economic constraints. The only state variable involved in constraints (36) and (37) is forest area F. The aim here is to determine the lowest and highest value of F (further denoted $\underline{F}$ and $\overline{F}$ respectively) for which it is possible to satisfy these two constraints at any time. Unlike with $Viab_{\mathcal{D}}(K^{\#})$ one can now retrieve analytically the half spaces where each of these two constraints are satisfied. From (31) and (36), the supply of timber reads

$$n \cdot d + n \cdot \gamma \cdot \delta \cdot F \geq \underline{q}. \tag{43}$$

Note that the greater the deforestation rate d, the lower the stock of forest required to achieve the minimum wood supply $\underline{q}$. The above condition gives

$$F \geq \underline{F} := \frac{\underline{q} - n \cdot d_{\max}}{n \cdot \gamma \cdot \delta}. \tag{44}$$

Inequality (44) provides the minimum value of F for which there exists a control ($d_{\max}$ in this case) that satisfies $q \geq \underline{q}$. We show the half-space $K_{\underline{q}}^{\#} := \{(F, E, S) \ / \ F \geq \underline{F}\}$ in Figure 2.2 below.

The value of $\underline{F}$ (3.3 billion hectares) in Figure 2.2 is obtained by substituting all the values of the parameters in (44). This represents approximately 85% of the world's current forest area (3.95 billion hectares in 2005 (FAO, 2006)).

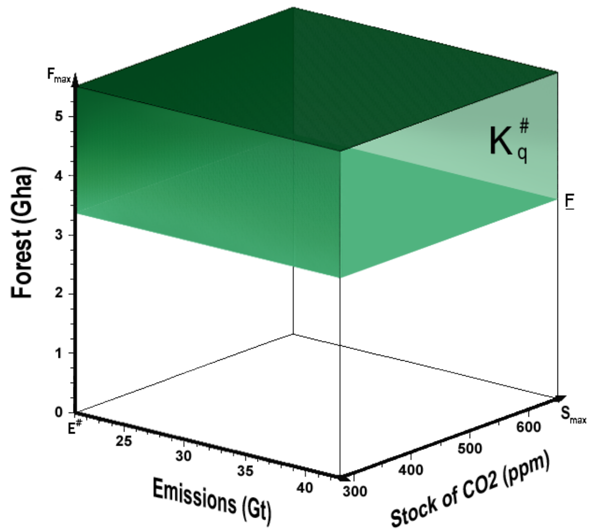

Figure 2.2: Addressing our timber constraint: Set $K_q^{\#}$

To retrieve $\overline{F}$ we proceed analogously. Substitute expressions (30)-(33) in (34) and then consider constraint (37). We obtain the following expression:

$$R = \left[-\frac{P_A\beta}{F_{\max}} - \theta n^2\gamma^2\delta^2\right]F^2 + \left[-2\theta n^2\gamma\delta d + \bar{P}n\gamma\delta - P_A\bar{Z} + 2P_A\beta\right]F$$
$$-\theta n^2 d^2 + [\bar{P}n + P_A\psi\bar{Z}]d + P_A[\bar{Z}F_{\max} + \beta F_{\max}] - \kappa_1\rho - \kappa_2 d + T \geq \underline{R}. \tag{45}$$

Solving expression (45) for F and substituting the values of our parameters gives us the maximum forest size, $\overline{F}$, that can secure $\underline{R}$.[20] Once we have $\overline{F}$, we can define the set $K_R^{\#}$ $:= \{(F, E, S) \ / \ F \leq \overline{F}\}$ drawn in Figure 2.3. The values of F that satisfy $R \geq \underline{R}$ are those inside the shaded area.

By looking at Figure 2.3 it may seem counter-intuitive that our revenue constraint cannot be met for high levels of forest area. The rationale behind this result is that an expansion of forest area necessarily implies a reduction in the surface area available for agriculture. Reducing the agricultural surface area, however, has a pronounced negative

[20] Note that, for the time being, T is kept constant and equal to zero.

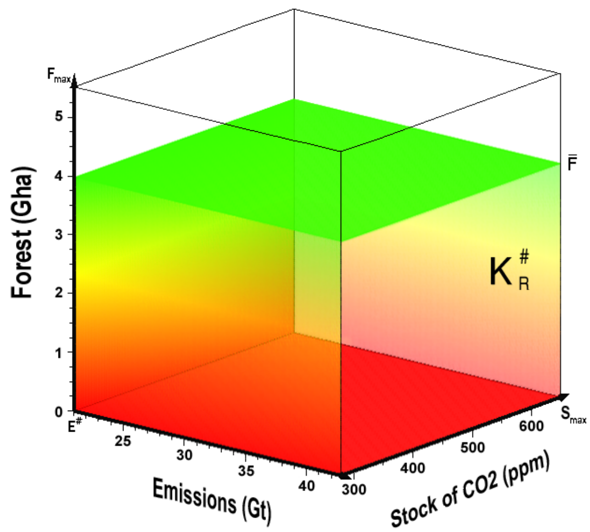

Figure 2.3: Addressing our revenue constraint: Set $K_R^{\#}$

impact on economic revenues since agriculture, rather than timber, is the main source of revenue for land owners. This result is in line with other empirical evidence, see, e.g., Barbier and Rauscher (1994), Barbier and Burgess (2001) or FAO (2006).

The value of $\overline{F}$ obtained equals 3.96 billion hectares which is only slightly greater than current forest area. In other words, forest area cannot increase beyond $\overline{F}$ due to the pressure set by agricultural revenues. As we will later see, the only possible way to increase forest area is by compensating forest owners with monetary transfers.

Figure 2.2 and Figure 2.3 represent the subsets $K_q^{\#}$ and $K_R^{\#}$ where inequalities (36) and (37) are respectively satisfied.

The set $K_C^{\#}$ -defined in (42)- is contained in the intersection of these two sets, i.e., $K_C^{\#} \subseteq \left\{K_q^{\#} \cap K_R^{\#}\right\}$. We have computed $K_C^{\#}$ and, in this case, it coincides with the intersection of the two (i.e., $K_C^{\#} = \left\{K_q^{\#} \cap K_R^{\#}\right\}$) since there is no conflict between the simultaneous fulfilment of the two constraints. Figure 2.4 is a plot of subset $K_C^{\#}$.

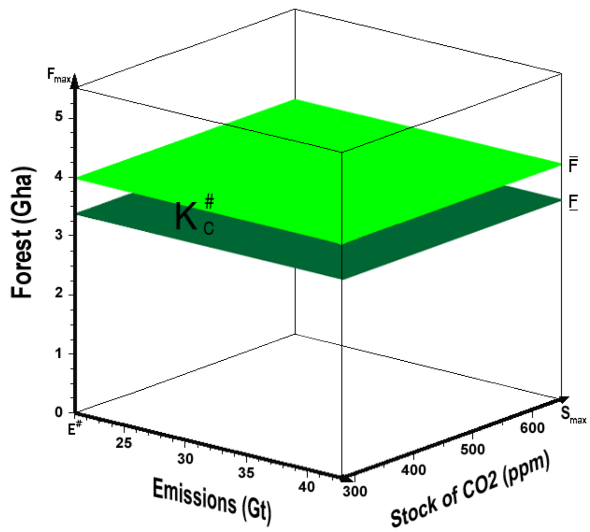

Figure 2.4: Addressing both economic constraints jointly: Set $K^{\#}_{C}$

To summarize, if the forest stock grows beyond the top plane in Figure 2.4 then revenues are too low and inequality (37) is violated. Conversely, if the forest size is below the bottom plane, wood demand cannot be met and inequation (36) is not satisfied.

Step 3: Combining environmental and economic constraints

In this third step, we compute the Viability Kernel of $K^{\#}_{C}$ for the dynamical system D_C, denoted $Viab_{\mathcal{D}_C}(K^{\#}_{C})$. Note that by construction

$$Viab_{\mathcal{D}_C}(K^{\#}_{C}) = Viab_{\mathcal{D}_C}(K^{\#}) = Viab_{\mathcal{D}}(K^{\#}_{C}).$$

This set includes all the points that are sustainable both in economic and environmental terms.

Before computing $Viab_{\mathcal{D}_C}(K^{\#}_{C})$, recall that in Step 1 we showed that 1990 emissions levels are not compatible with a long run stabilization of GHG concentrations. We then defined the set $K^{\#}$ that is given by the highest emissions level that is compatible with the non-emptiness of the viability kernel, and subsequently computed $Viab_{\mathcal{D}}(K^{\#})$. Recall also that, when defining both $E^{\#}$ and $K^{\#}$, sustainability relied on the possibility to have forests grow and reach $F_{\max}$.

Then, in Step 2, we reduced the area of interest to region $K_C^{\#}$. That is, the states of the system for which it is possible to find controls that allow the satisfaction of the economic constraints. We showed that the forest area that is compatible with the satisfaction of such economic constraints ($\underline{F} \leq F \leq \overline{F}$) is much lower than $F_{\max}$ (i.e., $\overline{F} < F_{\max}$).

Recall that for $Viab_{\mathcal{D}_C}(K_C^{\#})$ not to be empty we need, by construction, the following things : (i) That $F_{\max}$ is reachable and reached or else GHGs will not cease to accumulate. (ii) That $F < \overline{F}$ or else the revenue constraint cannot be addressed. The two requirements cannot be met simultaneously. Therefore, it is trivial to conclude that $Viab_{\mathcal{D}_C}(K_C^{\#})$ must be empty. In other words, environmental and economic sustainability enter in contradiction. The main reason being that emissions $E^{\#}$, are not low enough to guarantee a non-empty viability kernel for $\overline{F}$.

From the computation of $K_C^{\#}$ we have that $\overline{F}$ is the largest economically viable forest area. Let us denote by $E^{\flat}$ the maximum emissions level for which both the revenue constraint can be satisfied and $\dot{S}(t) \leq 0$ is feasible. Then $E^{\flat}$ is given by the following expression:

$$E^{\flat} = \varphi \overline{F} + W.$$

A necessary but not sufficient condition to have a non-empty $Viab_{\mathcal{D}_C}(K_C^{\#})$ is to have a new lower bound on emissions not higher than $E^{\flat}$.

This being said, $E^{\flat}$ emissions are not enough to guarantee the stability of the atmospheric concentration of GHGs. It is also required that forest area $\overline{F}$ is sustainable over time. Let us recall that $\underline{F}$ and $\overline{F}$ represent threshold levels, but nothing prevents that the forest gets depleted while we are in the region $K_C^{\#}$.

In what comes next we show that there exist other additional reasons that cause $Viab_{\mathcal{D}_C}(K_C^{\#})$ to be empty. These are indeed related to the fact that not only $\overline{F}$ is not sustainable but also that F decreases throughout $K_C^{\#}$ due to a too high deforestation rate and/or a too low afforestation rate.

Recall from inequality (44) that, for the threshold level $\underline{F}$, quantity $\underline{q}$ can be supplied only if deforestation is set at its maximum level, $d_{\max}$. As we move towards the interior of $K_C^{\#}$ (i.e., as F increases) the minimum deforestation required to satisfy constraint (36) will decrease. Now, let us define d_q as the minimum deforestation rate required to guarantee a supply of timber $\underline{q}$ throughout $K_C^{\#}$:

$$d_q := \inf \ \left\{ d \ / \ \forall x \in K_C^{\#}, \quad q \geq \underline{q} \right\}. \tag{46}$$

where q is given by equation (31).

If we now link (46) with our forest dynamics (eq. (26)) we obtain that if d_q is too large it will force F to decrease:

$$\forall F \in K_C^{\#}, \quad d_q > \rho_{\max} + \eta \left(1 - \frac{F}{F_{\max}} \right) F \ \Rightarrow \dot{F} < 0. \tag{47}$$

Equation (47) gives us an explanation of why $Viab_{\mathcal{D}_C}(K_C^{\#})$ is necessarily empty if the timber constraint is too stringent: If the minimum deforestation required to ensure $\underline{q}$ (d_q) is always larger than maximum afforestation, $\rho_{\max}$, plus the natural growth of the forest, $\eta \left(1 - \frac{F}{F_{\max}} \right) F$, then forest area is doomed to decrease. In such case, compliance with $\underline{q}$ ultimately leads to forest depletion thus creating a vicious cycle such that complying with

$\underline{q}$ becomes always harder. As a result, all the possible evolutions starting from any point inside $K_C^{\#}$ will necessarily leave $K_C^{\#}$ in finite time.

This is exactly what we observe. We are not sustainable since all the possible control policies have to face the dilemma of depletion of the forest *vs* non compliance of economic objectives: While the system remains within region $K_C^{\#}$ the forests keep being depleted but the economic constraints can still be satisfied. But as soon as the stock of forest (i.e., forest area) goes below $\underline{F}$ the economic crisis begins[21] and if we continue trying to address our economic constraints during the crisis we will further deplete the forest until it completely disappears. That is, until leaving K, and not just $K_C^{\#}$.

According to FAO (2006) the average world net forest loss equals 7.3 million hectares every year, our model closely replicates this stylized fact. Figure 2.5 shows the evolution of forest area supposing that the business-as-usual (BAU) scenario prevails. If the tendency is not reverted and one extrapolates the current data on deforestation and afforestation, it will take roughly 80 years to trespass the threshold level $\underline{F}$.

The qualitative message that we get is that if we want to satisfy the economic objectives we may do so in the short run, but only at the price of creating an ever growing environmental breach that will ultimately lead to forest depletion and impossibility to satisfy those same objectives in the long run. Furthermore, from (46) we know that deforestation cannot be reduced or else the economic constraints are not met. If deforestation cannot be reduced, then total forest area is bound to decrease unless afforestation can compensate for it.

Afforestation affects the forest dynamics (eq (26)) just as deforestation does. Since afforestation and deforestation are linked, one can compute the minimum necessary afforestation in terms of d_q. We can rewrite condition (47) to obtain a necessary, though not

[21]The term *economic crisis* is used here to denote the non fulfilment of one or more of the economic constraints.

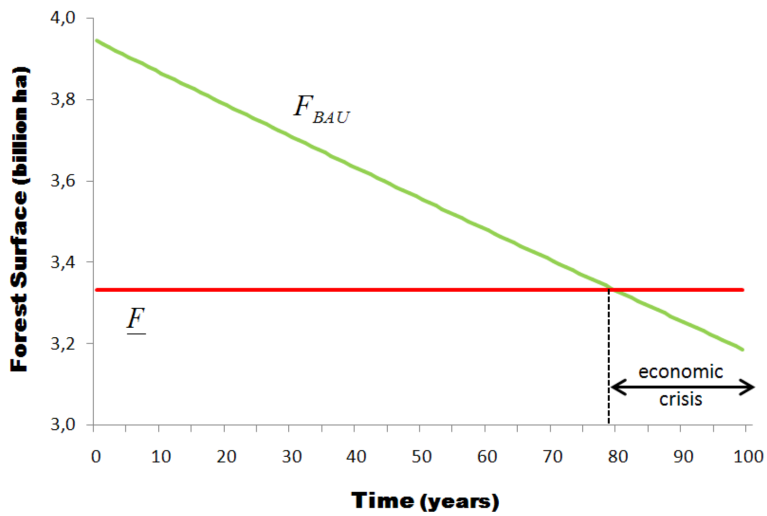

Figure 2.5: Forest surface evolution when applying the BAU policy

sufficient, condition for $Viab_{\mathcal{D}_C}(K_C^{\#})$ not to be empty:

$$\exists F \in K_C^{\#} / \quad \rho_{\max} > d_q - \eta \left(1 - \frac{F}{F_{\max}}\right) F \tag{48}$$

We need afforestation ρ to be at least greater than the right-hand side of expression (48), to avoid having an empty $Viab_{\mathcal{D}_C}(K_C^{\#})$. Since $\eta \left(1 - \frac{F}{F_{\max}}\right) F$ is decreasing in F in the interval $[\frac{\overline{F}}{2}, \overline{F}]$ and we have that $\frac{\overline{F}}{2} < \underline{F}$ we thus have that the natural growth rate of the forest is decreasing in F throughout $[\underline{F}, \overline{F}]$, i.e., throughout $K_C^{\#}$. Making use of this monotonicity property on the natural growth rate of the forests we can rewrite our last necessary condition for $Viab_{\mathcal{D}_C}(K_C^{\#})$ not to be empty in simpler terms:

$$\rho_{\max} > d_q - \eta \left(1 - \frac{\overline{F}}{F_{\max}}\right) \overline{F} \tag{49}$$

Substituting d_q by its value in (49) gives $\rho_{\max} > 9.4$ million hectares per year. Other way put, if $\rho_{\max} < 9.4$ million hectares then $Viab_{\mathcal{D}_C}(K_C^{\#})$ will surely be empty since $\dot{F}$ will be negative. According to FAO (2006) current world afforestation is 2.8 million hectares per year. The upper bound for afforestation used so far in this paper is 3 million hectares.

To summarize, there are several reasons why the business-as-usual (BAU) system is not sustainable. Among them, the following three: First, deforestation is too large and is leading to forest depletion. Second, afforestation is too low to counterbalance the negative effect of deforestation. Third, emissions create excessive accumulation of CO_2 in the atmosphere and emissions $E^{\#}$ are not low enough to ensure sustainability since compliance with the economic constraints prevents forest area from expanding beyond $\overline{F}$.

Facing this gloomy perspective, a natural question to ask is whether there is something that can be done about it. The answer is positive. Indeed, if we can fulfil all the following requirements sustainability can be recovered:

1. Deforestation is too high and causes forest depletion. However, deforestation cannot decrease or else our constraint on timber supply cannot be met. This means that afforestation must increase to compensate for the high deforestation rates.

2. Yearly afforestation has to increase beyond 9.4 million hectares. Otherwise F decreases ($\dot{F} < 0$) and minimum timber supply (31) will not be achievable in the long run due to forest depletion.

3. Emissions as low as $E^{\flat}$ need to be achievable. That is, $E_{\min} \leq E^{\flat}$.

4. Emissions have to be at least non-increasing. This means that the lower bound on the speed of variation of emissions, $v_{\min}$, has to be smaller than zero. Emissions may be constant provided that they do not exceed $\varphi \cdot \overline{F} + W$. Since current world emissions surpass by more than 50% the boundary $\varphi \cdot \overline{F} + W$, emissions must decrease.

Figure 2.6 below is a plot of a non-empty viability kernel computed with the following parameter specification:

$$v_{\min} = -1.5\%, \ \ \delta = 0.30, \ \rho_{\max} = 11 \cdot 10^6 \text{ ha.}$$

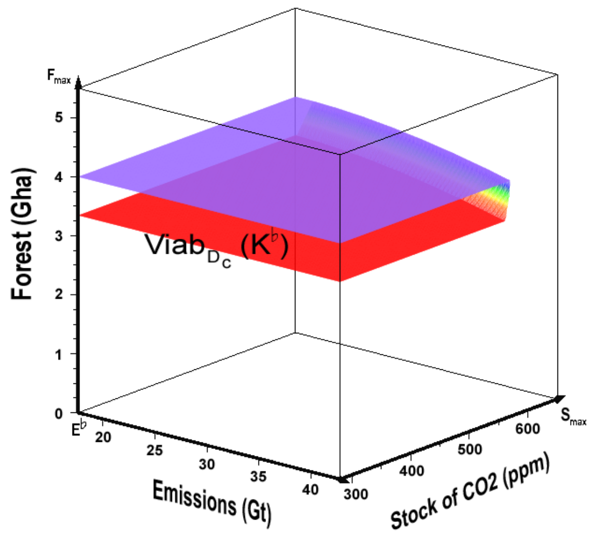

Figure 2.6: A viable solution: Set $Viab_{\mathcal{D}_C}(K_C^\flat)$

The lower bound on emissions has also been changed and we will apply $E_{\min} = E^\flat = 17.5$ Gt CO_2. The new set with which we shall work is denoted by $K^\flat$:

$$K^\flat := \left\{ x = (F, E, S) \ / \ F \in [F_{\min}, F_{\max}] \,, \ E \in \left[E^\flat, E_{\max} \right] \,, \ S \in [S_{\min}, S_{\max}] \right\}.$$

Analogously the constrained set $K_C^\flat := \left\{ x \in K^\flat \text{ such that } U_C\left(x\right) \neq \varnothing \right\}$.

Figure 2.6 is a plot of $Viab_{\mathcal{D}_C}(K_C^\flat)$. This set contains all the initial states of the system for which one can find both an economic and environmentally sustainable development path. Needless to say, sustainability is achievable as long as the correct controls are applied. It is worth noting, also, that the current state of the system: ($F = 3.95$ Gha, $E = 28$ Gtons CO_2, $S = 379$ ppm) lies within this viable set. This is good news. However, implementing these sustainable policies is obviously a challenging task. Indeed, to achieve sustainability, the current rate of afforestation has to more than triple; emissions, rather than increasing, need to decrease. It is also necessary that emissions are reduced in the long run to levels sensibly lower (18% lower) to what they were in 1990.

Figure 2.7 shows three viable trajectories among many possible ones. Each of them departing from $(F_{2005}, E_{2005}, S_{2005})$ but with different properties

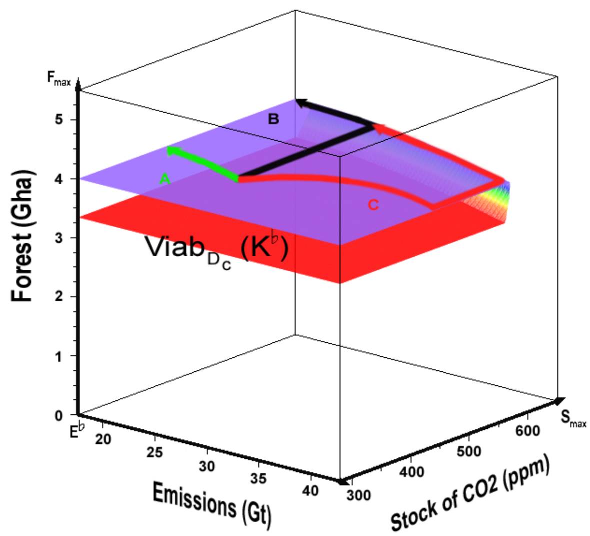

Figure 2.7: Three examples of viable trajectories

Trajectory A is obtained by applying the following vector of constant controls $u = (v = v_{\min}, \rho = 10 \cdot 10^6$ ha, $d = 13 \cdot 10^6$ ha). Trajectory A quickly converges to a stable equilibrium point characterized by a low concentration of CO_2 in the atmosphere. On the other hand, trajectory B has been obtained by applying the vector of controls $u' = (\ v = 0, \rho = 10 \cdot 10^6$ ha, $d = 13 \cdot 10^6$ ha). Emissions are constant, the stock of emissions increases and the system moves towards the frontier of the set. Then, just before leaving $Viab_{\mathcal{D}_C}(K_C^\flat)$ there is a policy switch to $u'' = u = (v = v_{\min}, \rho = 10 \cdot 10^6$ ha, $d = 13 \cdot 10^6$ ha). Finally, trajectory C can be obtained by applying $u''' = (v = v_{\max}, \rho = 10 \cdot 10^6$ ha, $d = 13 \cdot 10^6$ ha). Emissions are now increasing at its BAU rate until reaching $E_{\max}$, then emissions stay constant until the threshold beyond which continuing with high emissions takes the evolution outside the Viability Kernel. For trajectories B and C the system finally converges to a new equilibrium characterized by a much higher concentration of CO_2 in the atmosphere. Trajectories B and C follow the so-called heavy evolution that prescribes that the control only changes when strictly necessary, that is, when the evolution hits the boundary of the viability kernel. For the sake of clarity, in Figure 2.8, we have plotted in 2D the trajectories of emissions and the stock of emissions for each of the three evolutions.

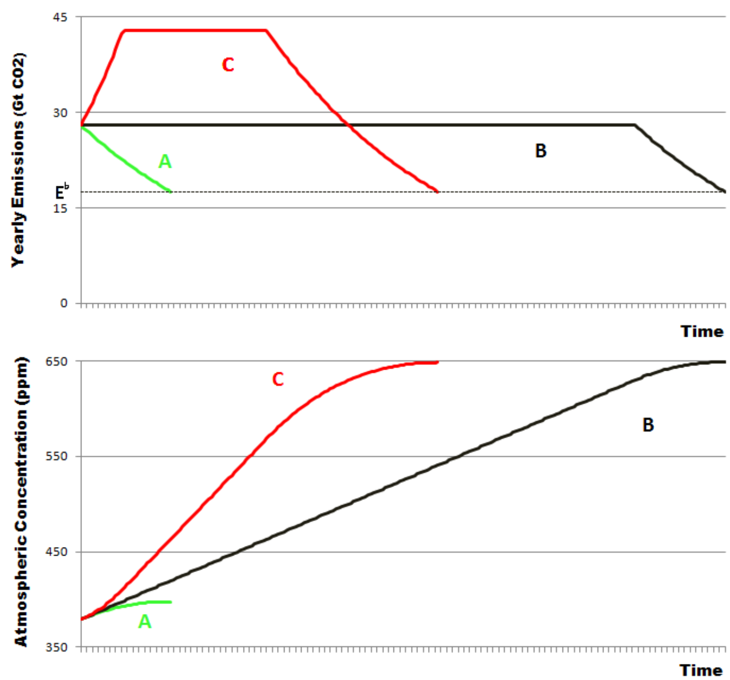

Figure 2.8: Time trajectories of the state variables

5 Sensitivity analysis and scenarios

In this section, we assess the impact of changes in some of the most important variables and their bounds on the solution. The aim of this exercise is to understand the separate contribution of each of them on $Viab_{\mathcal{D}_C}(K_C^\flat)$. To simplify the presentation, we analyze in some cases the impact on $K_C^\flat$ or on $Viab_{\mathcal{D}}(K^\flat)$ separately rather than on $Viab_{\mathcal{D}_C}(K_C^\flat)$ for exposition (pedagogical) reasons. As a benchmark we have taken the following values:

$$E_{\min} = E^\flat = 17.5 \text{ GtCO}_2,\ v_{\min} = -1.5\%,\ S_{\max} = 650 \text{ ppm},$$
$$\delta = 0.30,\ T = 0 \text{ \$},\ \rho_{\max} = 11 \text{ Mha}.$$

These are the same values that we used to obtain Figure 2.6 in the previous section. In each of the following scenarios, one of the above parameters is changed every time.

5.1 Carbon free economy

So far we have worked with the hypothesis that emissions cannot be reduced *sine die*. We have operationalized this idea by adding a constraint on the side of minimum yearly emissions. We have seen that setting the lower bound on 1990 emission levels ($E_{\min} = E_{1990}$) yields non-sustainable outcomes. Considering the slightly more optimistic case of $E_{\min} = E^\# < E_{1990}$ also fails since emissions $E^\#$ require that forest area increases and reaches pre-industrial levels ($F_{\max}$). As we saw, increasing forest size beyond $\overline{F}$ (with $\overline{F} < F_{\max}$) enters in contradiction with the economic constraints of the model. It thus takes the more optimistic $E_{\min} = E^\flat < E^\#$ (i.e., 4 GtCO_2 less than 1990 emissions levels) to be compatible with our long term goal of staying within the threshold of 650 ppm of CO_2.

In this scenario we explore yet another possibility, albeit extremely optimistic: What if in the long run anthropogenic emissions could be reduced to zero? That is, what would be the outcome if we allowed for $E_{\min} = 0$? It is straightforward to conclude that if emissions are equal to zero, then the role of forests as a carbon sink is no longer essential. This, however, does not mean that forests are now totally unnecessary. In fact, emissions reduction is slow and it will take time for current emissions to converge to zero. Following the emissions dynamics equation -eq (27)- and keeping in mind that the speed of adjustment of emissions, v, is bounded, there are still many states of the system for which the situation is irreversible. Introduce the new set $K^{\diamondsuit}$:

$$K^{\diamondsuit} := \{x = (F, E, S) \ / \ F \in [F_{\min}, F_{\max}] \, , \ E \in [0, E_{\max}] \, , \ S \in [S_{\min}, S_{\max}]\}$$

Figure 2.9 shows $Viab_{\mathcal{D}}(K^{\diamondsuit})$, that is made of the points between the two coloured surfaces depicted. By looking at Figure 2.9 we see that when we allow emissions to converge to zero, we have a new set of states (to the left of the curved surface on the left side of the figure) for which we are not sustainable. For these states with low emissions levels, carbon sequestration by forests may actually reduce carbon concentration below pre-industrial levels. This curved surface shows the threshold beyond which one cannot avoid the stock of carbon from decreasing below 284 ppm.

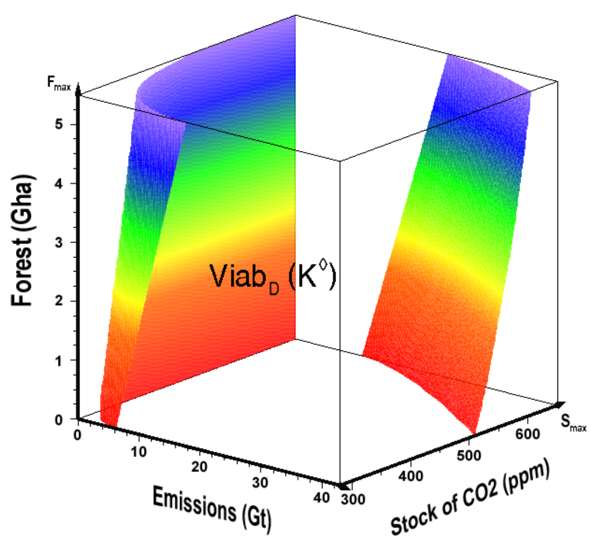

Figure 2.9: Carbon free economy: Set $Viab_{\mathcal{D}}(K^{\diamondsuit})$

Set $K^{\diamondsuit}$ is quite different from $K^{\flat}$ since $K^{\diamondsuit}$ includes states with null emissions levels, while $K^{\flat}$ only allowed for emissions as low as $E^{\flat}$. Since emissions is one of the three *state* variables, the size of both sets differs and this has an impact on the size of $Viab_{\mathcal{D}}(K^{\diamondsuit})$ and $Viab_{\mathcal{D}}(K^{\flat})$ as well. Figure 2.10 is a joint plot of the two previously computed sets, $Viab_{\mathcal{D}}(K^{\diamondsuit})$ and $Viab_{\mathcal{D}}(K^{\flat})$, using the same scale for both. Figure 2.10 gives us a fairer understanding of the differences between them.

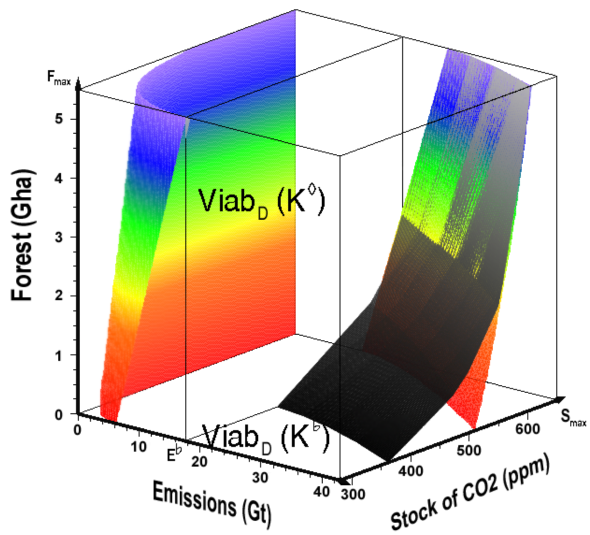

Figure 2.10: Comparison of two scenarios: $Viab_{\mathcal{D}}(K^{\diamondsuit})$ vs $Viab_{\mathcal{D}}(K^{\flat})$

The set $Viab_{\mathcal{D}}(K^{\flat})$ is made of the states above the inner dark-coloured surface. While $Viab_{\mathcal{D}}(K^{\diamondsuit})$ is made of the set within the two multicoloured surfaces that span the big cube. The vertical cut that we observe in the middle of the figure corresponds to $E^{\flat}$ and this is the reason why $Viab_{\mathcal{D}}(K^{\flat})$ does not span to the left of this vertical plane. It is clear to see that $Viab_{\mathcal{D}}(K^{\diamondsuit}) \supset Viab_{\mathcal{D}}(K^{\flat})$. This being said, if we analyze the differences on the half-cube where the two sets overlap, the most interesting findings are: First, by allowing emissions to converge to zero in the long run, one can see that sustainability is achievable for a wider range of initial states with low forest levels. And, second, applying $E_{\min} = E^{\flat}$ or $E_{\min} = 0$ as our lower bound on emissions results in no difference for states with high initial GHG concentrations and high forest levels. The main reason to explain why the boundary is common to both sets on the top right corner of the figure is simply that in this part of the cube it is short-run excessive accumulation of greenhouse gases, rather than long term convergence to null emissions, that poses a viability problem.

To sum up, the qualitative message that we get is that reducing emissions beyond $E^\flat$ does not change the situation much, except in those cases in which initial forest area is low. However, in the more realistic case where forest's area is not low and both emissions and the stock of GHGs in the atmosphere are high, then what matters is not long term convergence to null emissions, but rather being able to reduce emissions fast in the short run. This ability to reduce emissions fast depends on $v_{\min}$ rather than on $E_{\min}$ and explains why both sets overlap in the top-right part of the cube.

5.2 Rate of reduction of emissions

We now study how the previously computed $Viab_{\mathcal{D}}(K^\flat)$ varies with changes in the lower bound of CO_2 emissions adjustment rate (changes in $v_{\min}$). Recall that so far all figures have been computed with $v_t \in [v_{\min}, v_{\max}] = [-0.015, 0.03]$. The lower bound $v_{\min} = -0.015$ implies that it is possible to reduce CO_2 emissions from one year to the next at a rate of 1.5%. This lower bound is probably too optimistic. In fact, according to real data, world emissions increased roughly 1% yr^{-1} during the 1990s and 3.4% yr^{-1} between 2000 and 2008. During this time only a few economies were able to reduce their emissions. Among them, Germany is probably the most outstanding example and managed to reduce its emissions by 1.8% yr^{-1} during the 1990s. Eastern European countries and former Soviet Republics were also able to reduce their emissions, but this was mainly due to the collapse of their economic system. In fact, the number of countries that achieved a sustained economic growth while reducing their emissions is rather small.

We have considered, here, five different scenarios (see Table 2.II) ranging from the realistic business-as-usual (BAU) scenario of +3%, to the optimistic emissions-reduction scenario of −1.5% and we have computed the Viability Kernel for each to test its sensitivity to changes in $v_{\min}$. Intuitively, the value of $v_{\min}$ is related to technology. We do not claim here that such technology is already available, but look at what would be its impact if it were.

Table 2.II: Different lower bounds on the rate of adjustment of emissions

Scenarios	E1	E2	E3	E4	E5
$v_{\min}$	+3%	0%	−0.5%	−1%	−1.5%

Figure 2.11 shows the plot of the lower boundary of the viability kernel for each of these scenarios. These scenarios can be compared to E5 that shows the benchmark $Viab_{\mathcal{D}}(K^{\flat})$ computed in the previous section.

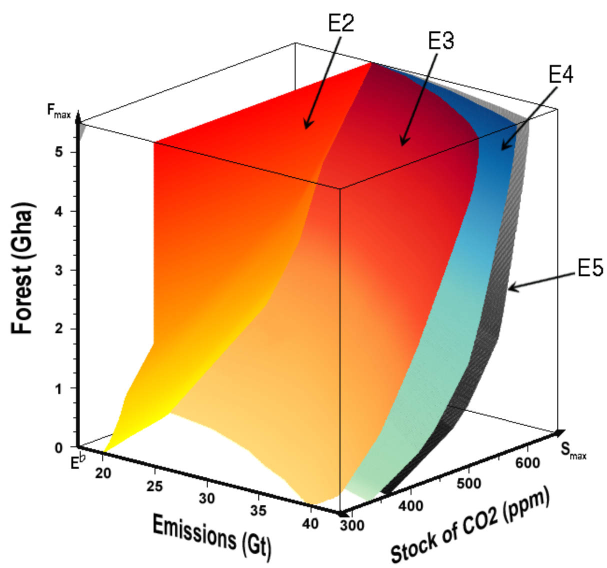

Figure 2.11: Impact on $Viab_{\mathcal{D}}(K)$ of changes in $v_{\min}$

From Figure 2.11 we draw the following conclusions: First, if emissions are, at best, strictly increasing (scenario E1) then the corresponding viability kernel is empty (unobservable in the figure) meaning that the business-as-usual scenario that describes the world's current economic system is not sustainable. Second, if emissions are not decreasing and at best constant (scenario E2) then the viability kernel is non-empty but current emissions must not exceed $\varphi \cdot F_{\max} + W$.[22] Third, if emissions can be reduced (i.e., scenarios E3, E4 and E5) then the viability kernel increases considerably with respect to E2. However, the difference among E3, E4 and E5 is not very substantial. The qualitative message that we get is that it takes some emissions reduction to have a viable environmental system.

In fact, if emissions continue to increase at the BAU rate of 3% yr^{-1}(scenario E1)the atmospheric concentration will exceed the upper bound of 650 ppm in 45 to 50 years depending on what is the forest policy implemented.

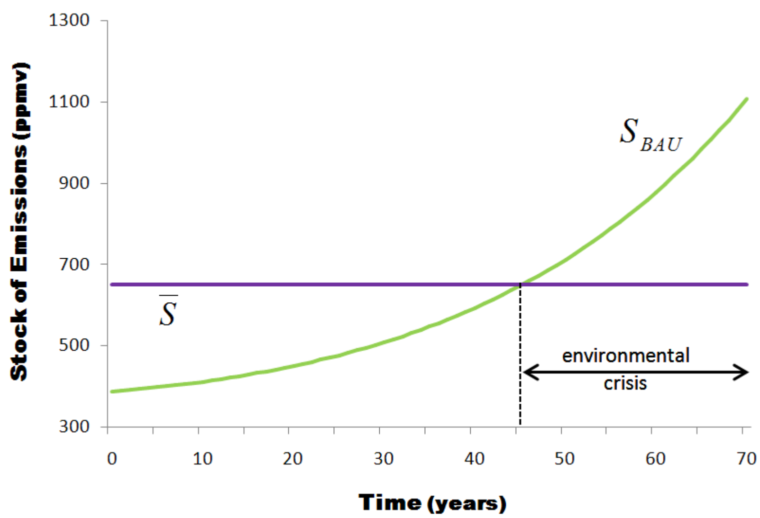

Figure 2.12: Evolution of GHGs concentration with BAU emissions

Figure 2.12 illustrates this last statement. The horizontal line denoted by $\overline{S}$ is the 650 ppm line. BAU emissions are depicted by the curve S_{BAU}. Note that if emissions continue to increase at a 3% rate, it will take roughly 45 years to go beyond the 650 ppm threshold.

[22]Note that $\varphi \cdot F_{\max} + W = 17.5$ $\mathrm{GtCO_2}$. Current world emissions are far from these figures. This rules out scenario E2 as an alternative.

5.3 More stringent environmental targets

Up to now we have worked with an upper bound on the atmospheric concentration of greenhouse gases of 650 ppm CO_2 which according to IPCC (2007) will very likely result in a temperature increase above 1.8^{o}C with respect to pre-industrial levels. This 650 ppm threshold, however, may also be too optimistic. In fact, there exists empirical evidence suggesting that beyond 560 ppm CO_2 net dissolution of reefs worldwide is possible due to the acidification of the oceans (Silverman et al., 2009).

The modelling framework used here does not consider the possibility of changes in the feedbacks governing carbon sequestration by the oceans. What we have done, instead, is perform a sensitivity analysis with respect to $S_{\max}$ and recalculated $Viab_{\mathcal{D}}(K^{\flat})$ for the case in which the upper bound on emissions is $S'_{\max} = 550$ ppm rather than the 650 ppm threshold used so far. We have computed the viability kernel for this new scenario, further denoted as $Viab_{\mathcal{D}}(K^{550\flat})$. To make the new results comparable with the previous ones, we graph together $Viab_{\mathcal{D}}(K^{\flat})$ and $Viab_{\mathcal{D}}(K^{550\flat})$ in the same figure (Figure 2.13).

The results show that $Viab_{\mathcal{D}}(K^{550\flat})$ has essentially the same shape as $Viab_{\mathcal{D}}(K^{\flat})$ apart from having its boundary shifted 100 ppm to the left. In addition to this, there are some small changes between the two sets in the bottom part of the cube. With the 650 ppm setting, most of the initial states with low forest levels were viable, whereas with 550 ppm one, they are not viable any longer. The reason why reducing $S_{\max}$ has an impact on the bottom of the cube -state values close to $F_{\min}$- is that more stringent environmental targets reduce the transition time: If we start off from a state with high emissions and a low forest area, we will now have less time to steer the economy towards sustainable levels. And considering that $v_{\min}$ is constant, a reduction in $S_{\max}$ translates into an increase of the minimum forest area necessary to stay viable.

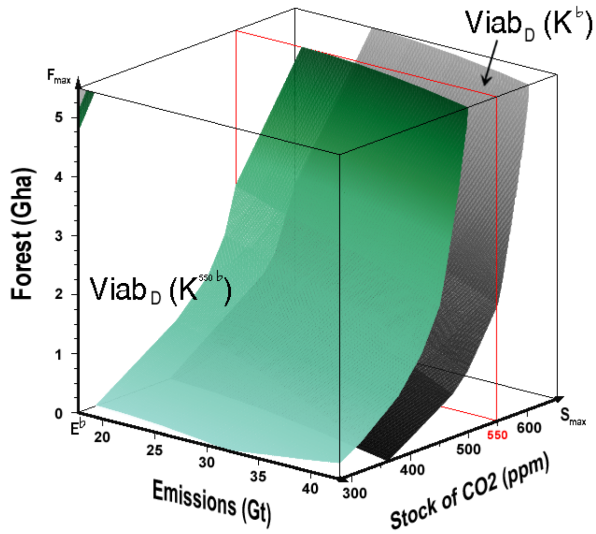

Figure 2.13: Comparing the 550 ppm & 650 ppm environmental targets

5.4 Selective logging

Selective logging is a source of timber. The assumption up to now has been that the share of world forests selectively-logged is constant and equal to its current level, i.e., $\delta = 0.30$. Nothing prevents, though, that the share of world forests used for productive purposes increases in the future. An increase in δ leads to a reduction in the minimum size of forest, $\underline{F}$, needed to supply the minimum required quantity of timber $\underline{q}$. Recall from (44) that $\underline{F} = \frac{\underline{q} - n \cdot d_{\max}}{n \cdot \gamma \cdot \delta}$. We thus have that

$$\frac{\partial \underline{F}}{\partial \delta} = -\frac{1}{\delta^2} \cdot \frac{\underline{q} - n \cdot d_{\max}}{n \cdot \gamma}.$$

The sign of the derivative is always negative since we have that $\underline{q} >> n \cdot d_{\max}$. Hence, increasing δ shifts the bottom plane ($\underline{F}$) of $K_C^\flat$ downwards.

We propose four possible values for δ in Table 2.III. The current value of δ is referred to as scenario SL1, which is the same lower bound used to compute the timber constraint in Figure 2.4. We have recalculated this set analytically for each of the other three values in Table 2.III and depicted them all together in Figure 2.14 below.

Table 2.III: Different values of parameter δ

Scenarios	SL1	SL2	SL3	SL4
δ	0.30	0.32	0.34	0.36

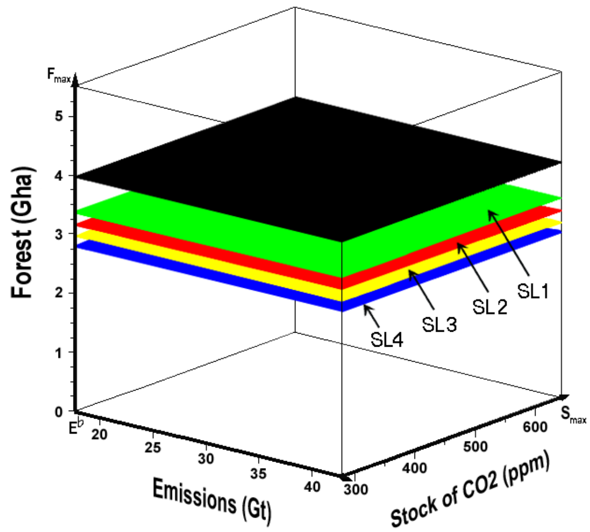

Figure 2.14: Impact of increasing selective logging on set $K_C^\flat$

By looking at Figure 2.14 we learn that increases in δ correspond to large shifts of $\underline{F}$. Greater selective logging does not impact the dynamics (i.e., does not impact $Viab_{\mathcal{D}}(K^\flat)$). Nevertheless, higher levels of δ make the forest problem less stringent: From equation (43) we have that greater values of δ imply more wood coming from selective logging, hence, less deforestation is required to achieve $\underline{q}$. Increases in δ and ρ are thus somewhat interchangeable and contribute in the same direction. In other words, increasing marginally the share of forests selectively logged can help considerably in achieving a sustainable development of the forest since it compensates deforestation.

5.5 Transfers

So far in this paper we have neglected monetary transfers and set them equal to zero. Recall from equation (34) that transfers T are a part of the revenue function and can be seen as an economic incentive given to forest owners so that they deforest less and/or reforest more; or simply as a lump sum donation to compensate them for reducing their agricultural land.

Let us consider monetary transfers as a decision variable of non-forest owners who may decide to transfer money to forest owners for as long as they implement a given desirable forest policy. In this paper we work with a very simple formulation: Non-forest owners compensate forest owners with the difference between *status quo* revenues and current revenues *before* transfers.

$$T = \underline{R} - R_{BT},$$

where revenues before transfers are equal to:

$$R_{BT} = P \cdot q + P_A \cdot Z \cdot [F_{\max} - F] - \kappa_1 \rho - \kappa_2 d.$$

Using this simple transfer mechanism -and bearing in mind that revenues are a function of forest area- we can easily retrieve the required level of transfers for whatever forest area $F > \overline{F}$ such that condition (37) is not violated. In other words, we have an economic trade-off between sustainable forest area and yearly monetary transfers. Figure 2.15 shows this trade off.

Recall that for values below $\overline{F}$ (Recall that $\overline{F} = 3.95Gha$) no compensation is required since $\overline{F}$ is, by construction, the largest forest area for which the revenue constraint can be addressed. But, as forest area increases beyond $\overline{F}$, larger economic transfers are needed in order to compensate for the reduction in agricultural land. This relation is quasi linear in F.

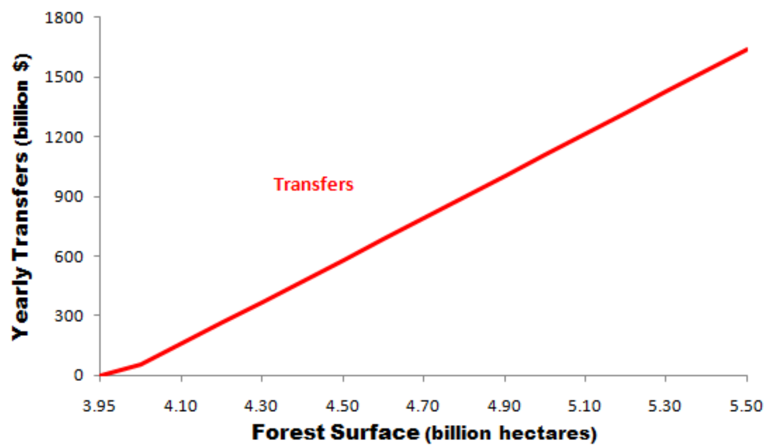

Figure 2.15: Minimum transfers to maintain forest area

Note also that, *ceteris paribus*, the greater the forest area, the greater the amount of carbon sequestered per unit of time. This implies that if the size of the forest increases so does the maximum sustainable emissions rate.

In the previous section -in Step 2- we showed that emissions $E^{\#}$ were not viable due to the impossibility to increase forest area towards $F_{\max}$ in the long run. In particular, we retrieved $E^{\flat}$ which is the maximum emissions level that is compatible with the maximum achievable forest area, $\overline{F}$.[23] If we now link these two concepts together (transfers and maximum sustainable emissions) and make use of equation (28) we can obtain for each emissions level above $E^{\flat}$ the corresponding price in terms of monetary transfers required to ensure sustainability.

Figure 2.16 illustrates the trade-off between emissions and monetary transfers. If transfers are null then $E^{\flat}$ is the maximum sustainable level of emissions, just as seen before. On the contrary, increasing economic transfers to forest owners so that they increase forest area will in turn increase the maximum sustainable level of emissions. Monetary transfers,

[23] As we saw, increasing forest area beyond $\overline{F}$ entered in contradiction with the satisfaction of our revenue constraint (37).

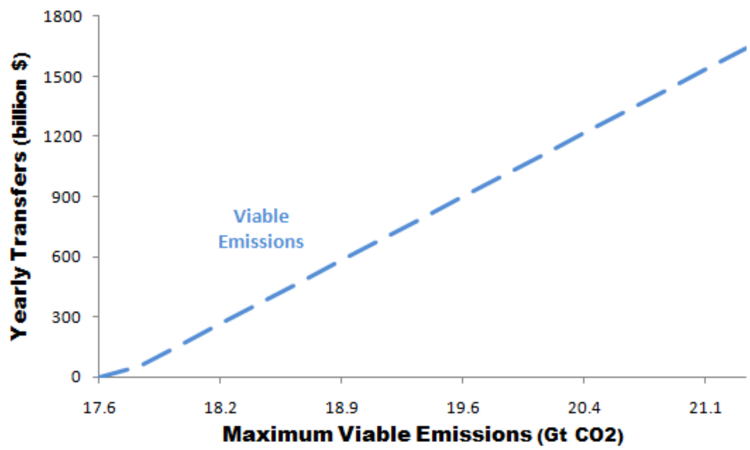

Figure 2.16: Maximum emissions as a function of transfers

however, cannot buy infinite emissions. The maximum sustainable level of emissions that one can buy with monetary transfers is $E^{\#}$. Going beyond $E^{\#}$ is not possible due to the hypothesis made that the maximum land conquerable by forests is bounded by $F_{\max}$. The line in Figure 2.16 shows all possible combinations of different maximum sustainable emissions levels and its corresponding monetary transfers. For the purpose of illustration, a yearly emissions rate of, say, 20 Gtons of CO_2 can be viable in the long run provided that world forests have roughly 5 billion hectares in size. This implies that world's forests be a billion hectares larger than nowadays. In order to achieve and sustain such large increase in forest area, yearly transfers of as much as 1 trillion dollars are necessary to compensate forest owners for their lost agricultural revenues. If these transfers are judged too onerous then the policy maker may decide among any of the other combinations presented in Figure 2.16.

It is not the purpose of this paper to determine which is best, but rather to provide a clearer view of what is feasible and what is not. This essay shows that there is no easy way out of the problem. The world has to either seriously reduce emissions and bring them at least 18% below 1990 emissions levels or else great compensation to forest owners is required. Moreover, even if compensation to forest owners is to be envisaged, some emissions reduction cannot be avoided.

This being said, the situation is not as hopeless as it appears, for the following reasons. First, transfers have been treated here globally. Actually, this is a proxy to both national effort, e.g., the Brazilian rainforest fund approved in August 2008, and international effort to maintain the world's forests, e.g., the Norwegian contribution to the Brazilian fund in September 2008. In the last years we have observed a notable increase in the contribution to these funds and, even if they are still very small, this aid is expected to continue to increase in the next years to come. Second, forest owners and forestry countries should boost other forest usages and sources of revenue like the sale of non-wood forest products, e.g., medicinal herbs and rubber. Indeed, according to the Food and Agriculture Organization (FAO, 2006), these revenues represent less than 10% of total forest revenues (i.e., \$4.7 billion in 2005) but remain a promising avenue that may help counterbalance the loss of economic revenues associated to increasing forest land.

6 Conclusions

We have focused on two interlinked environmental issues, the conservation of the world's forests and the control of the stock of CO_2 in the atmosphere within a given economic setting, and addressed the notion of sustainability from both an economic and environmental point of view.

The results obtained can be summarized as follows: The current business-as-usual emissions/deforestation system is far from being sustainable. Reducing emissions is capital; if emissions are not reduced, sustainability is not achievable. Reducing emissions, however, is not enough. As the world forests are depleted their ability to sequester carbon is decreased.

Sustainability necessarily requires high and non-decreasing levels of forest area. Deforestation cannot be reduced, or else wood supply is too low and the economic objectives cannot be met. To achieve non-decreasing levels of the forest stock, it takes greater afforestation and/or an increasing share of world forests selectively-logged. Note, however, that the assumption made here that selective logging does not affect forest biomass or forest size significantly should be taken into account to interpret these last results. Finally, monetary transfers can ease the problem of emissions reduction. Transfers both encourage forest-stock growth and are needed as an instrument to implement any desired afforestation policy. In this sense, forest transfers can be seen as a useful way to pay for part of the emissions reduction needed. This being said, the carbon-sequestration potential of forests is limited and, even under the most optimistic afforestation scenarios, emissions need to converge to levels below 1990 emissions in order to stabilize CO_2 atmospheric concentrations.

The objective of this paper was to explore what is part of a sustainable world and what is destructive. Since we have neglected some aspects, the conclusions are meant to be indications of course of action rather than definitive ones. Indeed, the model needs to be expanded to include other state variables, e.g., population dynamics, temperature dynamics. Further, we did not account here for technological progress which may help in reducing CO_2 by unit of output or lead to a more productive agriculture. These aspects need surely to be integrated in future investigations.

Appendix: Variables description

We provide in this appendix details regarding the construction and measurement of the model's variables and the estimation of parameters' values.

State variables

F: Forest surface area

Forest area in the world measured in hectares. The current stock of forest is estimated by FAO (2006) at 3952 million hectares in 2005. Parameter $F_{\max}$ has been estimated for 1750 AD at 42% of the globe's surface area[24] (i.e., 13067 million hectares excluding Antarctica and Greenland). This gives us a value for $F_{\max}$ of approximately 5500 million hectares. Consequently, we require that $F_t \in [F_{\min}, F_{\max}] = [0,\ 5.5\text{ Gha}]$.

E: Yearly emissions of CO_2

These are world yearly emissions of CO_2 measured in metric tons. In 2005, total CO_2 emissions amounted to 28.2 gigatons.[25] Minimum emissions are set to equal 1990 emission levels, i.e., 21.4 gigatons. The value of maximum emissions used plays a minor role. It has been set to equal the double of 1990 emissions. The reason to choose a maximum is simply the need to have bounded states. Hence, the constraint reads $E_t \in [E_{\min}, E_{\max}] = [21.4\text{ GtCO}_2,\ 42.8\text{ GtCO}_2]$.

S: Cumulated stock of CO_2 in the atmosphere

The cumulated stock of carbon in the atmosphere is measured in gigatons. The atmospheric stock of carbon has been estimated to be approximately equal to 800 GtC in 2005. The

[24] Source: http://www.geo.vu.nl/~renh/deforest.htm

[25] Source: EIA (2008)

current concentration of carbon measured in parts per million was approximately 379 ppm in 2005.[26] The upper bound value is set at 650 ppm i.e., 1372 GtC. The lower bound has a negligible impact on the solution. We have set it at pre-industrial levels, that is 284 ppm (i.e., roughly 591 GtC) in year 1832. In short, we want $S_t \in [S_{\min}, S_{\max}] =$ [591 GtC, 1372 GtC] $\equiv [2167 \text{ GtCO}_2, 5031\text{GtCO}_2]$.

Control variables

ρ : **Yearly afforestation** $[\rho_{\min}, \rho_{\max}] = [0,\ 3\,\text{Mha yr}^{-1}]$

For the period 1990-2005, FAO (2006) estimates world yearly afforestation at 2.8 million hectares. We let afforestation vary between no afforestation and three million hectares per year.

d : **Yearly deforestation** $[d_{\min}, d_{\max}] = [0,\ 15\,\text{Mha yr}^{-1}]$

For the same period, FAO (2006) estimates the average global deforestation rate at 13 million hectares per year. We let deforestation vary between zero and fifteen million hectares per year.

v : **Rate of adjustment of CO_2 emissions** $[v_{\min}, v_{\max}] = [-0.015,\ 0.03]$

During the decade going from 1990 to 2000 world CO_2 emissions increased roughly 1% per year.[27] Only a few countries like Germany, United Kingdom, Denmark, Finland, some Eastern European countries and former Soviet Republics were able to reduce their emissions. In particular, Germany achieved a 1.8% yearly cumulative decrease. On the other hand China's emissions increased at a rate of 3% per annum. All the other big economies lie somewhere in between. Between 2000 and 2008, however, world emissions have increased

[26] Source: NOAA (2007)
[27] Source: Bernstein et al (2006) and EIA (2008)

at a faster rate, 3.4% yr^{-1}, (Le Quéré et al., 2009) with a probable decrease during the next two years (2009-2010) due to the world crisis. Following these observations we have set both the lower and upper bounds on v. As a benchmark scenario we have chosen $v \in [v_{\min}, v_{\max}] = [-0.015, 0.03]$.

T : **Monetary transfers**

Monetary transfers to forest owners expressed in 2005 US$. Unless otherwise specified, transfers are set constant and equal to zero, $T = 0$.

Other parameters

η: **Natural growth rate of the forest**

FAO (2006) estimates net yearly afforestation (5.7 million hectares). A fraction is due to human afforestation and the rest belongs to the natural renewal of the forest. Human afforestation ρ is estimated at 2.8 million hectares, meaning that the term $\eta \cdot \left(1 - \frac{F}{F_{\max}}\right) \cdot F$ has been, on average, equal to 2.9 million hectares during this period. If we substitute F by its value for the last years and $F_{\max}$ by the value pointed above we obtain $\eta = 2.61 \cdot 10^{-3}$.

φ: **Carbon absorption rate**

This parameter is measured in metric tons of CO_2 equivalent absorbed per hectare of forest per year. According to Le Quéré et al. (2009), during the decade from 1990-2000 forests absorbed 2.6 PgC yr^{-1}(i.e., 2.6 GtC) which amounts to 9.53 $GtCO_2$. Considering that world total forest area equals 3952 million hectares, we can estimate average yearly carbon sequestration at 2.412 tonnes of CO_2 $ha^{-1}yr^{-1}$.

W: **Carbon absorption rate by oceans**

Accounts for the amount of carbon absorbed by oceans every year. According to Le Quéré et al. (2009), the oceans were able to sink on average 2.2 PgC yr^{-1} for the period 1990-2000, this is equivalent to 8.07 GtCO_2 yr^{-1}.

$\underline{R}$: **Minimum revenue of land owners**

Measured in US\$. We denote $\underline{R}$ as the current or *status quo* revenue. We compute $\underline{R}$ by substituting in equation (34) the current values of all the variables and parameters in our model.

$\underline{R}$: **Minimum timber supply**

Minimum timber supply is measured in m^3 of timber per year. Current timber supply is estimated by FAO (2006) equal to 1623 million m^3 yr^{-1} of industrial roundwood and 1777 million m^3 yr^{-1} of fuelwood. We estimate minimum timber supply to be equal to current timber supply, i.e., 3400 million m^3 yr^{-1}.

n: **Timber yield**

The timber yield is measured in m^3 per hectare and year. We use the average world figures given by FAO (2006), i.e., $n = 110$ per hectare of growing stock.

β: **Lower productivity due to forest depletion**

Lower agricultural yield measured in tons per hectare. Eswaran *et al.* (2001) estimate the loss in productivity as a consequence of land degradation, erosion and desertification. They report that the productivity loss due to such processes in Africa ranges between 2% and

40% and provide an average estimate for the whole continent to be equal to 8.2% in average. If average productivity is $\overline{Z}$, then β is equal to 0.061.

γ: **Selective logging yield, fraction of average yield**

The selective logging yield is measured as a fraction of average yield. When forests are managed for wood production they produce as much as 1-3 m^3 per hectare (in other words $n \cdot \gamma$ =1-3 m^3). We have set the value of $\gamma = 0.015$.

δ: **Fraction of forests selectively logged**

Share of the world's forests selectively-logged. Following FAO (2006), parameter δ has been calibrated at 30% (i.e., $\delta = 0.30$) to fit the world yearly production of wood $\underline{q}$.

θ: **Slope of wood demand**

According to FAO (2006), the commercial value of all wood (i.e., roundwood and fuelwood) in 2005 was $64 billion per year of which only 7 billion correspond to fuelwood. World production equals 3400 million m^3. The average price for both types of wood is 18.8 dollars per m^3. FAO (1997) gives the elasticity of demand for several countries and several types of wood.[28] A representative value of both the mean and median price elasticity of wood is -0.50. We have approximated an iso-elastic curve by a linear one in an interval of 2000 million m^3 centred at 3400 million m^3 such that the average elasticity inside the interval equals -0.50. The slope of our demand can be then computed accordingly to obtain $\theta = -2.7 \cdot 10^{-9}$.

[28] Most elasticity values are comprehended between -0.25 and -0.75

$\overline{P}$: **Choke price of wood**

With the average price of wood and the slope of demand computed above, we can retrieve the choke price of our inverse demand function and obtain $\overline{P} = 27.98$ (US\$ per m^3).

ψ: **Extra productivity of deforested land**

Parameter ψ denotes the productivity gain of land after deforestation. It is measured as a fraction of average productivity. A reasonable range for ψ suggested in the literature is $0.2 - 0.5$. We adopt $\psi = 0.3$.

κ_1: **Per hectare afforestation cost**

Parameter κ_1 denotes the afforestation cost per hectare of forest. According to the World Bank the cost for seedling is roughly 40 $ per thousand seedlings and the number of seedlings per hectare is equal to approximately 2000. This amounts to approximately 80 $ per hectare of forest just for seedling. Afforestation costs, however, also include other costs such as labour costs that change with countries. The World Agroforestry Centre gives estimates for the Philippines around 1000 $\$/ha$, other non-governmental organizations provide estimates that range between 180 $\$/ha$ for Senegal and 400-500 $\$/ha$ for other countries in Africa such as Sudan, Madagascar and Ethiopia.[29] We have chosen the round value of 500 $\$/ha$ that is representative of the average cost taking these different observations.

κ_2: **Per hectare deforestation cost**

Parameter κ_2 denotes the deforestation cost per hectare of forest. The Bureau of Business and Economic Research of Montana University estimates the costs of ground-based logging to be equal to 22.70 $ for every green ton of harvest for the year 2006.[30] Considering that a

[29]See e.g.: www.villageprojectsint.org and www.edenprojects.org
[30]www.bber.umt.edu/pubs/forest/prices/loggingCostPoster.pdf

green ton is equivalent to 2000 pounds of undried biomass material (i.e., 907 kg) and that the density of wood is typically 500 kg/m^3, then for a representative douglas fir plantation (530 kg/m^3) we obtain that the deforestation cost is equivalent to 13.26 $\$/m^3$. If the yield per hectare is equal to 110 m^3/ha, then we obtain an estimate of the deforestation cost per hectare of 1459 $\$/ha$.

P_A: **Average price of representative agricultural product**

Measured in US$ per metric ton. To determine the average price of the representative agricultural good we took four representative commodities (i.e., cocoa, coffee, cotton and sugar) from FAO (2004). Coffee and sugar in Brazil and Latin America and cotton and cacao in Africa are four crops that are related with deforestation processes. The net economic yield per hectare of crop ranges from 1660 $\$/ha$ for coffee to 771 $\$/ha$ for cocoa. The average yield equals 1141 $\$/ha$. We have chosen the price of cotton to be the representative price of the agricultural good since its prices and economic yield (1467 $ per metric ton and 1088 $\$/ha$) are the closest to the mean.

$\overline{Z}$: **Average land productivity**

Measured in tons per hectare. The average yield has been computed taking into account the yield of those same four crops in metric tons per hectare and annum as mentioned before. The estimated annual yield per hectare is equal to 0.742 metric tons per hectare.

Bibliography

[1] Aubin, J. P. (1991). *Viability Theory, Systems and Control: Foundations and Applications.* Boston: Birkhäuser.

[2] Aubin, J. P., Kropp, J., Scheffran J., & Saint-Pierre, P. (2004). *An Introduction to Viability Theory and Management of Renewable Resources, Process of Artificial Intelligence in Sustainable Science*. Commack: Nova.

[3] Aubin, J. P., Bernardo, T., & Saint-Pierre, P. (2005). *The Coupling of Climate and Economic Dynamics, Essays of Integrated Assessment*. Dordrecht: Kluwer.

[4] Barbier, E. B., & Rauscher, M. (1994). Trade, tropical deforestation and policy intervention. *Environmental and Resource Economics, 4*(1), 75-90.

[5] Barbier, E. B., & Burgess, J. C. (2001). The economics of tropical deforestation. *Journal of Economic Surveys, 15*(3), 413-433.

[6] Bennett, K. D. (1983). Postglacial population expansion of forest trees in Norfolk. U.K. *Nature, 303*, 164-167.

[7] Bernstein, P. M., Montgomery, W. D., & Tuladhar, S. D. (2006). Potential for reducing carbon emissions from non-Annex B countries through changes in technology. *Energy Economics, 28*(5-6), 742-762.

[8] Bruckner, T., Petschel-Held, G., Leimback, M., & Toth, F. (2003). Methodological aspects of the tolerable windows approach. *Climatic Change, 56*(1-2), 73-89.

[9] Chen, Y. (1988). Early holocene population expansion of some rainforest tree at Lake Barrine basin, Queensland. *Australian Journal of Ecology, 13*(2), 225-233.

[10] De Lara, M., Doyen, L., Guilbaud, T., & Rochet, M. J. (2007). Is a management framework based on spawning-stock biomass indicators sustainable? A viability approach. *ICES Journal of Marine Science, 64*(4), 761-767.

[11] De Lara, M., & Doyen, L. (2008). *Sustainable management of natural resources. Mathematical models and methods.* New York: Springer.

[12] Doyen, L., De Lara, M., Ferraris, J., & Pelletier, D. (2007). Sustainability of exploited marine ecosystems through protected areas: A viability model and a coral reef case study. *Ecological Modelling, 208*(2-4), 353-366.

[13] Ehui, S. K., Hertel, T. W., & Preckel, P. V. (1990). Forest resource depletion, soil dynamics, and agricultural development in the tropics. *Journal of Economics and Environmental Management, 18*(2), 136-154.

[14] Energy Information Administration (EIA) (2008). *International Energy Outlook 2008.*

[15] Eswaran, H., Lal R., & Reich, P. F. (2001). Land degradation: an overview. In E. M. Bridges, I. D. Hannam, L. R. Oldeman, F. W. T. Penning de Vries, S. J. Scherr & S.Sombatpanit (Eds.), *Response to land degradation* (pp. 20–35). Enfield, NH, USA: Science Publishers Inc.

[16] Food and Agriculture Organization of the United Nations (FAO) (1997). *Asia-Pacific forestry sector outlook study: Forest industry structure and the evolution of trade flows in the Asia-Pacific region - Scenarios to 2010.*

[17] Food and Agriculture Organization of the United Nations (FAO) (2001). *Forest Resources Assessment 2000.*

[18] Food and Agriculture Organization of the United Nations (FAO) (2004). *The state of food and agriculture 2003-2004.*

[19] Food and Agriculture Organization of the United Nations (FAO) (2006). *Forest Resources Assessment 2005.*

[20] Fredj, K., Martín-Herrán, G., & Zaccour, G. (2004). Slowing deforestation rate through subsidies: A differential game. *Automatica, 40*(2), 301-309.

[21] Fredj, K., Martín-Herrán, G., & Zaccour, G. (2006). Incentive mechanisms to enforce sustainable forest exploitation. *Environmental Modeling & Assessment, 11*(2),145-156.

[22] Intergovernmental Panel on Climate Change (IPCC) (2001). *Climate Change 2001: The scientific basis, third assessment report.* Cambridge: Cambridge University Press.

[23] Intergovernmental Panel on Climate Change (IPCC) (2006). *Climate Change 2007: The physical basis, fourth assessment report.* Cambridge: Cambridge University Press.

[24] Le Quéré, C., Raupach, M. R., Canadell, J. G., Marland, G., et al. (2009). Trends in the sources and sinks of carbon dioxide. *Nature Geoscience, 2*, 831-836.

[25] Martín-Herrán, G., Cartigny, P., Motte, E., & Tidball, M. (2006). Deforestation and foreign transfers: a Stackelberg differential game approach. *Computers & Operations Research, 33*(2), 386-400.

[26] Martinet, V., & Doyen, L. (2007). Sustainability of an economy with an exhaustible resource: A viable control approach. *Resource and Energy Economics, 29*(1), 17-39.

[27] National Institute for Occupational Safety and Health (NIOSH) (2007). *Carbon dioxide: IDLH Documentation.*

[28] National Oceanic & Atmospheric Administration (NOAA) (2007). *Trends in atmospheric carbon dioxide.*

[29] Norby, R. J., Hanson, P. J., O'Neill, E. G., Tschaplinski, T. J., Weltzin, J. F., Hansen, R. A., et al. (2002). Net primary productivity of a CO2-enriched deciduous forest and the implications for carbon storage. *Ecological Applications, 12*(5), 1261-1266.

[30] Petschel-Held, G., Schelnhuber, H., Bruckner, T., Toth, F., & Hasselmann, K. (1999). The tolerable windows approach: Theoretical and methodological foundations. *Climatic Change, 41*(3-4), 303-331.

[31] The Royal Society (2005). *A guide to facts and fiction about climate change.*

[32] Saint-Pierre, P. (1994). Approximation of the viability kernel. *Applied Mathematics and Optimization, 29*(2),187-209.

[33] Silverman, J., Lazar, B., Cao, L., Caldiera, K., & Erez, J. (2009). Coral reefs may start dissolving when atmospheric CO_2 doubles. *Geophysical Research Letters, 36,* L05606

[34] Stähler, F. (1996). On international compensations for environmental stocks. *Environmental and Resource Economics, 8*(1), 1-13.

[35] Sugita, S., & Tsukada, M. (1982). Late Quaternary Dynamics of Pollen Influx at Mineral Lake, Washington. *Journal of Plant Research, 95*(4), 401-418.

[36] Tsukada, M. (1981). Cryptomeria japonica D. Don, 1. Pollen dispersal and logistic forest expansion. *Japanese Journal of Ecology, 31*, 371-383.

[37] Tsukada, M. (1982). Late-quaternary development of the Fagus forest in the Japanese archipelago. *Japanese Journal of Ecology, 32,* 113-118.

[38] Van Soest, D., & Lensink, R. (2000). Foreign transfers and tropical deforestation: what terms of conditionality? *American Journal of Agricultural Economics, 82*(2), 389-399.

[39] World Commission on Environment and Development (1987). *Our Common Future.* Oxford: Oxford University Press.

Essay 3

Towards an optimal and more sustainable management of the forest

Essay information

Essay in preparation to be submitted:

Andrés-Domenech, P., Martín-Herrán, G., & Zaccour, G. Towards an optimal and more sustainable management of the forest.

Abstract

We model the role of the world's forests as a major carbon sink and consider the impact that forest depletion has on the accumulation of CO_2 in the atmosphere. We consider two types of agents: Forest owners who exploit the forest and draw economic revenues in the form of timber and agricultural use of deforested land; and non-forest owners who pollute and suffer the negative externality of having a decreasing forest stock. We retrieve the cooperative solution for this game and show in which cases cooperation allows to partly reduce the negative externality. We analyze when it is jointly profitable to abate emissions, when it is profitable to reduce net deforestation; and when it is optimal to do both things (abate and reduce net deforestation).

Key Words: Game theory, dynamic games, optimal control, deforestation, forest management, emissions, renewable resources.

1 Introduction

World forests cover nearly one third of planet Earth's surface. However, total world forest area is decreasing at an alarming rate. Every year an area equivalent to the size of Sierra Leone is deforested (FAO, 2006). World deforestation has become an issue of great international environmental concern for a number of reasons: First, world forests have great ecological value as carbon sinks. Second, forests host much of the world's biodiversity. Third, forests protect land and water resources and help prevent land erosion and desertification. In this paper we concentrate mainly on the role of forests as carbon sinks, although the framework used here could be extended to include the other two.

We view forests as a provider of somewhat competing economic and environmental goods. While forest logging brings economic revenues from both timber and agriculture on deforested land in the short run (FAO, 2006), excessive logging can exacerbate the problem of greenhouse gases (GHGs) accumulation in the long run. We have built a model where we account for the accumulation of GHGs in the atmosphere. We propose a GHG accumulation dynamics in terms of both anthropogenic emissions and carbon sequestration by the world's forests. The framework used allows us to (i) evaluate the impact that forest depletion has on atmospheric GHG accumulation through the so called *reduced-carbon-sequestration effect*; and (ii) compare short-term rewards from high emissions and intensive deforestation policies with its long-term costs due to excessive GHG accumulation and forest depletion.

There exist many papers that deal with the role of excessive GHG accumulation in the atmosphere within a dynamic setting (see, e.g., the early papers by Van der Ploeg and De Zeeuw (1992), and the literature review by Jørgensen et al. (2010)). In this literature, emissions are a control variable and the issue is to determine the optimal emissions rate so as to reduce the environmental damage coming from the excessive accumulation of GHGs.

Typically, these models concentrate on the difficulty to coordinate on the optimal level of emissions, while treating carbon sequestration as exogenously given.

In this paper, we extend the literature and explicitly account for endogenous carbon sequestration by modelling the role of forests as a carbon sink. We consider forests as a renewable resource whose evolution has an impact on the accumulation of GHGs in the atmosphere.

There exist a number of papers in the literature that deal with the issue of forest depletion using a dynamic-game approach (e.g., Van Soest and Lensink (2000), Fredj et al. (2004), Fredj et al. (2006), Martín-Herrán and Tidball (2005) and Martín-Herrán et al. (2006)). In these articles, the players are forest owners who exploit the forest to obtain economic revenues, and a donor community, or an environmentally-aware player, that is willing to compensate forest owners who engage in preservation efforts of the resource.

We have merged these two strands of the literature. We have built a dynamic optimization problem where two economic agents interact. On the one hand we have forest owners who exploit (and eventually deplete) the forest. Their actions have an environmental impact on the atmospheric accumulation of GHGs. On the other hand we have non-forest owners who derive utility from production (i.e., emissions) and disutility from the accumulation of GHGs in the atmosphere. In this setting, it is this disutility that they experience that, in some cases, may turn them into donors who seek for forest conservation.

In our model, forest owners have an incentive to deforest since deforestation increases their economic revenues. Conversely, non-forest owners have an interest to preserve forests for their value as a carbon sink. This modelling framework allows us to capture both the high opportunity cost to reduce deforestation and the negative economic externality that forest owners inflict on non-forest owners as a consequence of their deforestation policy.

Unlike the other papers aforementioned we do not focus solely on forest conservation but also on its impact on GHG accumulation. Non-forest owners are to decide what is their optimal level of emissions. The parameters of the model have been calibrated to fit real data. We compare jointly optimal outcomes with non-cooperative or business-as-usual policies. We show that cooperation allows to partly reduce the negative externality. We analyze when it is profitable to abate emissions, when it is profitable to reduce net deforestation; and when it is optimal to do both things (abate and reduce net deforestation).

The remainder of the paper is organized as follows: in Section 2, we present the model and the economic problem for the two types of agents. In Section 3, we characterize analytically the non-cooperative optimal policies for each player. In Section 4, we compute the cooperative optimal policies, and compare them to their non-cooperative counterparts. We also perform a sensitivity analysis. Our results are summarized in Section 5.

2 The model

We consider two types of agents: forest owners and non-forest owners. We model forest owners as environmentally unconcerned agents who only care about the forest revenues obtained with deforestation. We suppose forest owners to neglect the environmental impacts of their actions, i.e., they do not consider the consequences that their deforestation policy brings out in terms of GHG accumulation. On the other hand, non-forest owners get revenues from the production of economic goods. Their productive activity generates emissions and non-forest owners do take into account the negative effects of current emissions policies on the accumulation of GHGs in the atmosphere. This way of modelling allows us to capture the negative externality that forest owners create on non-forest owners through the so called *reduced-carbon-sequestration effect*.

We present the objectives of the two players. In what follows we use subscript FO to denote *forest owners* and subscript NF to denote *non-forest owners.*

2.1 The problem of forest owners

Forest owners maximize their discounted stream of net revenues. Forest revenues depend on their afforestation and deforestation rates $\rho(t)$ and $D(t)$, respectively, as well as on the existing forest area $F(t)$ measured in hectares. Net revenues are discounted at rate r_{FO} throughout a fixed and finite time horizon, given by time T. The rate r_{FO} can be viewed as an intertemporal rate of substitution. Net revenues include gross revenues $R(t)$, afforestation costs $\kappa_1\rho(t)$ and deforestation costs $\kappa_2 D(t)$, where κ_1 and κ_2 are respectively the per-hectare afforestation and deforestation costs. The objective of forest owners is the following:

$$\max_{\rho(t),D(t)} \int_0^T e^{-r_{FO}t}\left[R(t)-\kappa_1\rho(t)-\kappa_2 D(t)\right]\,dt, \tag{50}$$

where $\rho(t)\in[0,\rho_{\max}]$ and $D(t)\in[0,D_{\max}]$. The upper bounds for afforestation ($\rho_{\max}$) and deforestation ($D_{\max}$), reflect the idea that there is a physical limit in the short term to afforestation and that deforestation is subject to some regulation that allows for it within some limits. The value of $D_{\max}$ is set to fit the observed deforestation world figures provided by FAO (2006). The definitions of all parameters, their values and their sources are provided in Appendix A.

We assume that the evolution over time of the forest area can be well approximated by the following linear-differential equation:

$$\dot{F}(t)=\rho(t)+\eta F(t)-D(t), \quad F(0)=F_0, \tag{51}$$

where η is a positive parameter, and F_0 is the initial world's forest area in 2005 (FAO, 2006) and equals nearly four billion hectares. Equation (51) is an extension of Van Soest and Lensink (2000) and Fredj et al. (2006), where $\rho = \eta = 0$ in the first and $\rho = 0$ in the second. The specification used in (51) is linear for simplicity and approximates reasonably well forest expansion within a large interval around current world forest area $F(0)$. The linear form of the dynamics will enable us to obtain analytical solutions.

Forest owners obtain revenues from selling timber and agriculture products. Denote by $q(t)$ the quantity of timber put on the market at time t, and let the price $P(t)$ be given by the following linear-inverse demand:

$$P(t) = \overline{P} - \theta q(t), \tag{52}$$

where $\overline{P}$ is the choke price that makes demand equal to zero, and θ is the average price elasticity of demand. The values of parameters $\overline{P}$ and θ have been calibrated using data given by FAO on timber prices and quantities.

The quantity $q(t)$ comes from two different sources, namely, clear felling and selective logging, and is given by

$$q(t) = nD(t) + n\gamma\delta F(t), \tag{53}$$

where $nD(t)$ is the amount of wood retrieved from clear-felling an area $D(t)$ and the product $n\gamma\delta F(t)$ stands for total selective-logging yield which is lower (in per-hectare terms) than the one obtained through clear felling. Parameter n denotes the per-hectare timber yield and is typically measured in stems per hectare or cubic meters of timber per hectare. FAO (2006) provides an estimate for this parameter. We assume that clear felling an area $D(t)$ reduces total forest size by the same amount. However, unlike deforestation, selective logging is assumed here to have no impact on total forest land. "[Selective logging]*...is not necessarily* *destructive and can be done with low impact on the remaining forests, if the proper techniques are applied*".[31] Clearly, for selective logging to have a negligible environmental impact, its per hectare yield per unit of area must be much lower with respect to clear felling. This lower yield is accounted for by parameter γ ($\gamma << 1$). Finally, according to FAO (2006), roughly one third of the world's forests are used primarily for the production of wood and non-wood forest products. Parameter δ takes into account the fact that only a fraction of the world's forests are actually being exploited.[32]

Agriculture revenues are equal to prices times yields of the different crops grown. For simplicity, we suppose that forest owners grow a single agricultural good that we model as a composite good made of four representative crops that are commonly related to deforestation processes. This good is sold in international markets at a given price P_A.[33] The total yield at time t depends on the size of available (deforested) land, given by $\overline{F} - F(t)$, where $\overline{F}$ stands for the carrying capacity of the forest, and on the soil productivity $Z(t)$. As in Andrés-Domenech et al. (2011) -see also Van Soest and Lensink (2000) for a simpler version- we model $Z(t)$ as follows:

$$Z(t) = \overline{Z} + \alpha(t)D(t) - \beta\frac{\overline{F} - F(t)}{\overline{F}}. \tag{54}$$

The above expression of total productivity of land $Z(t)$ is the sum of three terms. The first one is a constant productivity term $\overline{Z}$ that measures the average yield in tons of crop per hectare of land of a representative agricultural good. The second term, $\alpha(t)D(t)$, captures the idea that newly deforested land $D(t)$ is more productive. Variable $\alpha(t)$ measures the increase in *total* average per-hectare production resulting from deforesting an area $D(t)$.

[31] Source: http://www.fao.org/forestry/news/48681/en/

[32] The equation that we have used for q is a small variation of the one presented by Van Soest and Lensink (2000). In their case γ and δ are assumed equal to one. We follow here the more comprehensive specification used by Andrés-Domenech et al. (2011).

[33] The price P_A is constant, unlike $P(t)$, due to the fact that agricultural production in deforested land represents only fraction of world total agricultural land.

The third term, $-\beta\frac{\overline{F}-F(t)}{\overline{F}}$, accounts for the positive externality that forests generate on nearby agricultural land. Forests are seen as a source of rain and a protective element to agricultural land. Parameter β measures the decrease (increase) in soil quality, and therefore in agricultural productivity, caused by forest depletion (expansion). The productivity increase of newly deforested land is given by:

$$\alpha(t) = \frac{\psi\overline{Z}}{\overline{F} - F(t)}. \tag{55}$$

Newly deforested land is more productive and parameter ψ measures the factor by which productivity is increased. However, this extra productivity needs to be normalised among all agricultural land. We divide the extra yield, $\psi\overline{Z}$, by total agricultural surface area, $\overline{F} - F(t)$, otherwise the term $\alpha(t)D(t)$ in equation (54) would overestimate the real impact that deforesting an area $D(t)$ has.[34] Note that Van Soest and Lensink (2000) assume a constant productivity increase, i.e., $\alpha(t) \equiv \alpha$.

Putting together revenues from timber sales and agricultural products, we get the following expression for gross revenue:

$$R(t) = P(t)q(t) + P_A Z(t)\left(\overline{F} - F(t)\right). \tag{56}$$

To recapitulate, forest owners maximize their net discounted economic revenues with respect to their deforestation and afforestation efforts $D(t)$ and $\rho(t)$, respectively (problem (50)), subject to the forest dynamics in (51).

[34]Agricultural revenues are obtained by multiplying productivity (54) by total agricultural land. Hence, equation (54) has to account for average *per-hectare* productivity measured in tons of crop per hectare. For this reason, the term $\alpha(t)D(t)$ cannot be understood as the extra productivity of newly deforested land, but rather as the normalised productivity increase that *newly* deforested land has on *total* agricultural land.

2.2 The problem of non-forest owners

The economic problem of non-forest owners is quite different. Non-forest owners optimize a two-part objective function. The first part consists of a short-run gain that non-forest owners derive from producing and consuming economic goods. The production of these goods generates pollution as a by product and this pollution affects their utility. For simplicity, we have supposed here that the carbon intensity of the economy is constant. Hence, *ceteris paribus*, producing more goods is equivalent to emitting more.[35] Denote by $E(t)$ the GHGs emissions by non-forest owners; and by the concave increasing function $G(E)$ the payoff generated in terms of goods production. We adopt the following functional form:

$$G(E(t)) = aE(t) - \frac{1}{2}bE^2(t), \tag{57}$$

where parameters a and b are positive and have been fixed in order to ensure that $G'(E) > 0$ for the relevant range of emissions. This specification is similar to the one proposed in e.g. Dockner and Long (1993) and Breton et al. (2005) with the only difference that we have included parameter b to calibrate $G(E(t))$ at current GDP at the world level.

The second term in the objective of non-forest owners represents an economic loss or damage related to the accumulation of emissions in the atmosphere. Denote by $S(t)$ the instantaneous stock of GHGs (e.g., stock of CO_2) in the atmosphere at a given time t. According to the IPCC (2007) increases in the atmospheric concentration of GHGs result in sea water level rising, temperatures increasing and sea water acidification. These processes are all related to economic and environmental damages. We assume that the damage cost is given by a convex increasing function $L\left(S\right)$, with $L''(S) > 0$. Although we acknowledge

[35]One could think of a more refined formulation where the carbon intensity of the economy can adjust and production increases can be compatible with constant levels of emissions or even with emission decreases.

the existence of thresholds, extreme events and jumps in the damages,[36] our formulation, which is very common in the literature (see e.g. Benchekroun and Long, 2002; Dockner and Long, 1993; Van der Ploeg and De Zeeuw, 1992; Breton et al., 2006), smooths the impacts of such phenomena rather than dealing with them explicitly. Needless to say, accounting properly for non linearities and threshold effects in the damage cost would lead to a model of much greater complexity.

This being said, for a specific function to qualify as a good candidate to model such damages we can think of yet another necessary requirement: Greenhouse gases, and most particularly CO_2, have always been present in the atmosphere and represent a basic element to the existence and development of life (e.g., plants). It is clear that more than the existence it is the *excessive* accumulation of atmospheric GHGs that poses the problem. We adopt the following specification of $L(S)$ that captures in a simple way all these elements:

$$L(S(t)) = c(S(t) - \underline{S})^2, \tag{58}$$

where $\underline{S}$ is a natural threshold, beyond which economic and environmental damages are considered to be excessive. In practical terms, choosing a reasonable value for $\underline{S}$ - given the specification above- amounts to choosing a level of atmospheric GHGs for which there is no perceived damage. We identify $\underline{S}$ with the pre-industrial level of GHGs (see, e.g., Bahn et al. (2008)).

Taking into account the gain function $G(E)$ and the damage function $L(S)$, we obtain the objective functional that non-forest owners maximize:

$$\int_0^T e^{-r_{NF}t}\,[G(E(t)) - L(S(t))]\,dt - \phi\,(S(T))\,e^{-r_{NF}T}, \tag{59}$$

[36] For instance, a small increase in the atmospheric concentration of GHGs can bring a quantitatively different damage, and large increases may trigger qualitatively different damages (e.g., massive ice cap melting, dissolution of coral reefs as a result of extreme oceanic acidification, etc).

where r_{NF} is the intertemporal rate of substitution, and $\phi(S(T))$ is a salvage value. Note that for the sake of generality, we do not require that both players discount their stream of payoffs at the same rate, i.e., r_{NF} need not be equal to r_{FO}.

Non-forest owners are modelled as forward looking agents who consider the long-term impact of their decisions. The stock of emissions accumulates slowly and then has a long-term impact on non-forest owners' payoffs. Therefore, it is sensible to have a scrap value function somehow related to the stock of emissions at the terminal date of the planning horizon. Such scrap value function can be generically written as $\phi(S(T))$. It is reasonable to think that whatever the GHG stock at the terminal date, it will strongly impact future payoffs due to the long-term persistence of greenhouse gases in the atmosphere. One could think of a more sophisticated scrap value function that also depends on the emission policy followed after the terminal date. Or even define the scrap value function as an identical problem to the one presented above in equation (59). Because we want to keep the problem as simple as possible, and because we want to be able to say something that is irrespective of what policies are chosen after the terminal date, we have chosen a formulation for $\phi(S(T))$ that depends on the terminal stock of greenhouse gases alone:

$$\phi\left(S(T)\right) = \int_{T}^{2T} e^{-r_{NF}(s-T)}\, L(S(T))\, ds. \tag{60}$$

Although the salvage function in (60) is simple, it satisfies the following intuitive requirements: (i) It reflects the idea that the terminal stock of GHGs matters and has an impact on future payoffs; (ii) it is easy to compute and does not depend on (potentially) unknown future policies; (iii) it keeps discounting in a natural way the cost of future environmental damages; and (iv) the time span considered for the scrap value function is related to the planning horizon. In fact, if the planning horizon chosen is short, then the weight given for future environmental problems will likely be short as well and vice versa.

Non-forest owners maximize their payoffs in (59) by adjusting their emissions, and their decision has an impact on the state of the system. Emissions, in our model, are supposed exclusively anthropogenic and are given entirely by non-forest owners' emissions. By this we do not mean that forest owners do not emit but rather that their contribution to global emissions is negligible. The dynamics of the emissions rate $E(t)$ is then given by:

$$\dot{E}(t) = v(t)E(t), \quad E(t) \geq 0, \; E(0) = E_0. \tag{61}$$

The dynamics of emissions in equation (61) can also be written in a more familiar way:

$$\frac{\dot{E}(t)}{E(t)} = v(t),$$

where $v(t)$ denotes the instantaneous speed of variation of emissions. For the sake of realism $v(t)$ has been modelled as a bounded control variable (i.e., $v_{\min} \leq v(t) \leq v_{\max}$), with $v_{\min} < 0$ and $v_{\max} > 0$. In the literature, it is more common to see emissions as a flow variable. As in Andrés-Domenech et al. (2011), we treat emissions as a stock and the speed of variation of emissions as a bounded control variable. This way of modelling allows us to better account for the inertia of the productive and economic system. Indeed, emissions take time to adjust and the upper and lower bounds on $v(t)$ simply reflect this idea that emissions cannot be increased or decreased at whatever rate. One can think of these bounds as being given by the existence of technical, economic and/or political constraints.

The evolution of the stock of greenhouse gases in the atmosphere depends on emissions and carbon sequestration by world forests and oceans. World forests sequester carbon as they grow and according to IPCC (2000) and FAO (2006) approximately half of the dry weight of forest biomass is carbon. To model carbon sequestration by forests one could measure the variation of total forest biomass. However, this would present two main difficulties. First, the variation in total carbon biomass is difficult to measure. And second, measuring carbon sequestration through the variation in forest biomass underestimates total carbon sequestration since timber captures are neglected. To overcome this problem, we have made the simplifying assumption that forest owners manage a representative forest whose trees grow -volume wise- at an average and constant rate. Having a representative forest whose growth rate is constant allows expressing carbon sequestration as a linear function of forest area alone (i.e., carbon sequestered per hectare of forest land and per unit of time). The advantage of having carbon sequestration in terms of forest area -rather than in terms of biomass variation- is that one can easily consider timber captures, while gaining a tractable and understandable way to measure carbon sequestration.

Further, note that by measuring carbon sequestration as a function of forests' surface area, one can account for the so called *reduced-carbon sequestration effect* that is based on the simple principle that a tree that is cut cannot grow (i.e., cannot sequester carbon). Thus, it is straightforward to see that deforestation has a negative impact on carbon sequestration due to the reduction in forest area that it induces. Expression (62) below captures the dynamics of the atmospheric concentration of carbon in terms of the forest stock, where parameter φ reflects the amount of carbon sequestered per hectare of forest and per unit of time.

We have considered a second carbon sink -the oceans- that we denote by W. We have assumed W to be constant for simplicity. Even if there exist small year-to-year variations due to el Niño effects, the carbon uptake by the oceans has remained relatively stable during the last few years (Le Quéré et al. (2009)). The evolution of the stock of pollution is, then, given by the following differential equation:

$$\dot{S}(t) = E(t) - \varphi F(t) - W, \quad S(t) \geq 0, \quad S(0) = S_0. \tag{62}$$

To wrap up, non-forest owners maximize their payoff in (59) adjusting the instantaneous variation of emissions $v(t)$, subject to equations (61) and (62) and given the fact that the solution to (51) is inherited from forest owners' problem.

3 Individual optimization

In this section we characterize the optimal strategies of the two players when they act independently. As forest owners' payoffs do not depend on emissions or GHG accumulation, their payoffs are independent of the action of non-forest owners. On the other hand, non-forest owners' payoffs are affected by forest owners' actions through the evolution of the forest stock. In this setting, where there is a one-way interaction, Nash and Stackelberg equilibria coincide. Further, open-loop and feedback information structures yield the same result. Given this, we can first solve the economic problem of forest owners, and next optimize for non-forest owners taking the evolution in the forest stock as given.

3.1 Forest owners

Forest owners maximize their revenues in (50) subject to (51)-(56). The following proposition provides the optimal solution to their control problem.

Proposition 1 *For the parameter domain defined in Appendix A the optimal control, state and co-state variables are given by:*[37]

$$\rho^*(t) = 0, \quad D^*(t) = D_{\max} \ \textit{for all}\, t \in [0, T],$$
$$F(t) = \left(F_0 - \frac{D_{\max}}{\eta}\right) e^{\eta t} + \frac{D_{\max}}{\eta}, \tag{63}$$
$$\lambda(t) = \frac{1}{\eta - r_{FO}} \left\{ (2\theta n D_{\max} - \overline{P}) n\gamma\delta + P_A(\overline{Z} - 2\beta) \right.$$
$$\left. + 2F(t) \left[\theta n^2 \gamma^2 \delta^2 + P_A \beta \frac{1}{\overline{\overline{F}}} \right] \right\} \left[1 - e^{(\eta - r_{FO})(T-t)} \right]. \tag{64}$$

Proof. See Appendix B. ■

The results show that forest-owners' optimal strategy consists in deforesting at maximal admissible level and not afforesting at all. As the problem is linear in the afforestation effort, and afforestation is a pure cost in our setting, then the optimal strategy is obviously to set $\rho(t)$ at its lowest admissible value, i.e., $\rho(t) = 0$. Further, the marginal revenue from agricultural activity is positive for all admissible values of $D(t)$, including $D_{\max}$. Therefore, there is an incentive to deforest at maximal level. These results follow from the fact that, for our parameter domain, we have $\lambda(t) \leq 0$, for all t. Indeed: (i) the term $\frac{1}{\eta - r_{FO}}$ is always negative because $\eta << r_{FO}$; (ii) $(2\theta n D_{\max} - \overline{P}) n\gamma\delta + P_A(\overline{Z} - 2\beta) > 0$, for all admissible values of $D(t)$, including $D_{\max}$; and (iii) $\left[1 - e^{(\eta - r_{FO})(T-t)}\right] \geq 0$. Note that deforestation is mainly driven by the revenues obtained from growing agricultural products on deforested land, rather than by the timber revenues that arise from deforestation itself. This is in line with other studies, e.g., Barbier and Rauscher (1994), Barbier and Burgess (2001) and FAO (2006), which suggested that deforestation for agricultural purposes is the main explanatory factor of forest depletion worldwide.

[37] The second-order sufficient optimality conditions are satisfied for this and all the problems studied in this paper.

3.2 Non-forest owners

Non-forest owners maximize their payoff given by (59) and take into account the values of the three state variables, namely forest area, F, emissions, E, and the stock of accumulated emissions in the atmosphere, S. The following proposition provides the optimal solution to the problem of non-forest owners. The optimal solution depends on the length of the planning horizon and on the intertemporal discount rate. For the values of our parameters, the solution is constant ($v^* = v_{\max}$) as long as the planning horizon (T) is less than approximately forty years, (i.e., $T \lesssim 40$).[38] The following proposition provides the optimal time paths for control and state variables in such case.

Proposition 2 *For the parameter domain defined in Appendix A and $T \lesssim 40$, the optimal control and state variables are given by:*

$$v(t) = v^* = v_{\max} \ \textit{for all} \ t \in [0, T],$$

$$E(t) = E_0 e^{v^* t}, \tag{65}$$

$$S(t) = S_0 - \frac{\varphi}{\eta} t D_{\max} - \frac{E_0}{v^*}(1 - e^{v^* t}) + \frac{\varphi}{\eta}\left(F_0 - \frac{D_{\max}}{\eta}\right)(1 - e^{\eta t}). \tag{66}$$

Proof. See Appendix C. ■

As pointed out above, the optimal control v^* depends on the planning horizon considered. For a relatively short horizon, i.e., $T \lesssim 40$, the optimal solution is constant of the type $v^* = v_{\max}$ all along. Note that the solutions showed in Proposition 2 hold for as long as there is no switching time. For longer horizons, i.e., $T \gtrsim 40$, the optimal solution is to apply the control $v = v_{\max}$ for some time and then switch to a cleaner regime. For even longer horizons, it is possible that the optimal solution consists of switching not once but

[38] The determination of the exact planning horizon beyond which Proposition 2 does not hold depends on the intertemporal discount rate. As we will see for every value of the discount rate we can obtain the maximum value of T for which Proposition 2 holds.

several times. In all cases, the different switching times and the number of switches depend on the adopted value for T. Denote by $\tilde{t}_v$ the optimal switching time. Then the optimal solution for $40 \lesssim T \leq 100$ can be summarized as follows:

$$\hat{v}(t) = \begin{cases} v_{\max}, & \text{for } t \leq \tilde{t}_v \\ v_{\min}, & \text{for } t > \tilde{t}_v. \end{cases}$$

In Appendix D we have solved the problem for the case where there is only one switching time, and characterized the first-order conditions that apply in that case. Retrieving the actual switching time, however, represents a challenge. This is mainly due to the change in the evolution of the state and co-state variables as a consequence of changes in the switching time itself. The first-order conditions before and after the switch will only be satisfied if the exact switching time is chosen. This poses a problem in determining the actual switching time since one has to try with infinitely many possibilities and the first-order conditions will only be satisfied if the exact one is chosen.

To overcome this problem we have developed an algorithm to obtain the switching time. We approximate it by the integer time at which it is best to switch. The algorithm proposed consists on evaluating the sum of payoffs for all the possible scenarios (i.e., all the possible switching times). Among them, we then select the integer time for which shifting regime (from $v_{\max}$ to $v_{\min}$) yields greater payoffs. A sketch of the algorithm can be found in Table 3.I.

Table 3.I: Sketch of algorithm used to compute the optimal switching time $\tilde{t}_v$

Fix the length of the planning horizon (T) and the discount rate (r_{NF})
for all possible integer switching times (t_v) **do**
    Payoff(t_v) = Discounted sum of payoffs before switch t_v
        + Discounted sum of payoffs after switch t_v
        + Scrap value function
end
Select the t_v whose Payoff is greater

Suppose for instance that our planning horizon and discount rate were fixed at, e.g., $T = 50$, $r_{_{NF}} = 0.02$. Figure 3.1 gives the payoffs measured in trillion dollars in the y-axis obtained for each possible switching time (x-axis). We observe, for this particular case, that switching from $v_{\max}$ to $v_{\min}$ after seventeen years ($\tilde{t_v} = 17$) is the best thing to do.

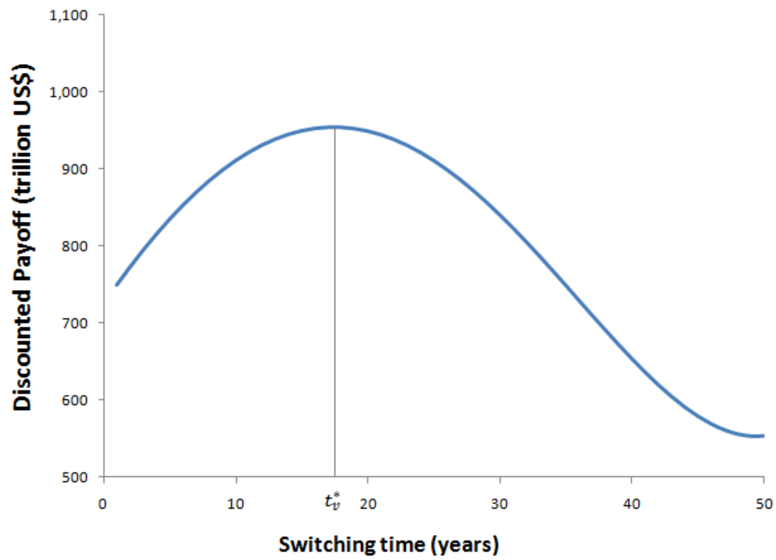

Figure 3.1: Payoffs as a function of the switching time

We can generalize the algorithm presented in Table 3.I and let the planning horizon T vary while keeping the discount rate $r_{_{NF}}$ constant. So doing we obtain the best switching time for each different planning horizon.

Figure 3.2 gives the optimal switching time for each possible planning horizon T. To better understand this figure it is important to distinguish the difference between three concepts. First, parameter T denotes the planning horizon of the problem. Second, the value T^s denotes the threshold planning horizon, that is, the minimum planning horizon beyond which there is a switch. And finally, since the switching time does not coincide with T^s; the value $\tilde{t_v}$ denotes the time in which the switch actually occurs.

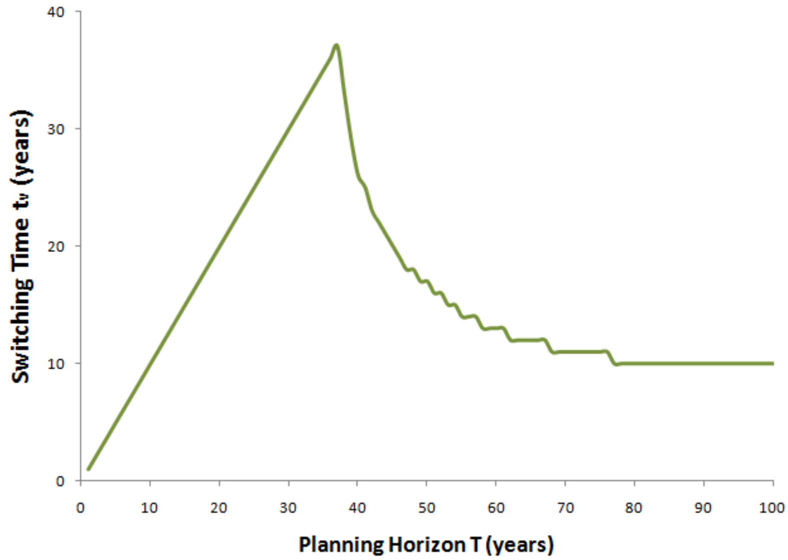

Figure 3.2: Optimal switching time for every planning horizon

Figure 3.2 illustrates the fact that it pays to emit more in the short run. It also shows that for longer planning horizons it is comparatively less attractive to apply $v_{\max}$. This result is related to the existence of the non-linear damage function $L(S)$, by which the environmental damage increases when GHGs accumulate due to excessive emissions.

In Figure 3.2, $T^s = 38$. This means that there is no switch if the planning horizon is *short* (i.e., less than 38 years). Conversely, there is a switch if $T \geq T^s$. As mentioned before, T^s and $\tilde{t_v}$ do not coincide, not even when $T = T^s$. Put differently, if the planning horizon is long enough non-forest owners recognize the need to switch to a cleaner regime, but the switch will take place some time before the terminal date. Note that the pair $\left(T = 50,\ \tilde{t_v} = 17\right)$ that we previously obtained in Figure 3.1 is now just one point of the curve displayed in Figure 3.2.

We can further generalize our algorithm and drop the assumption that the intertemporal discount rate $r_{_{NF}}$ is fixed. In the previous two figures we set $r_{_{NF}} = 0.02$ (2%). We compare our previous results with two other alternative scenarios $r_{NF} = 1\%$ and $r_{NF} = 3\%$. The results are showed in Figure 3.3.

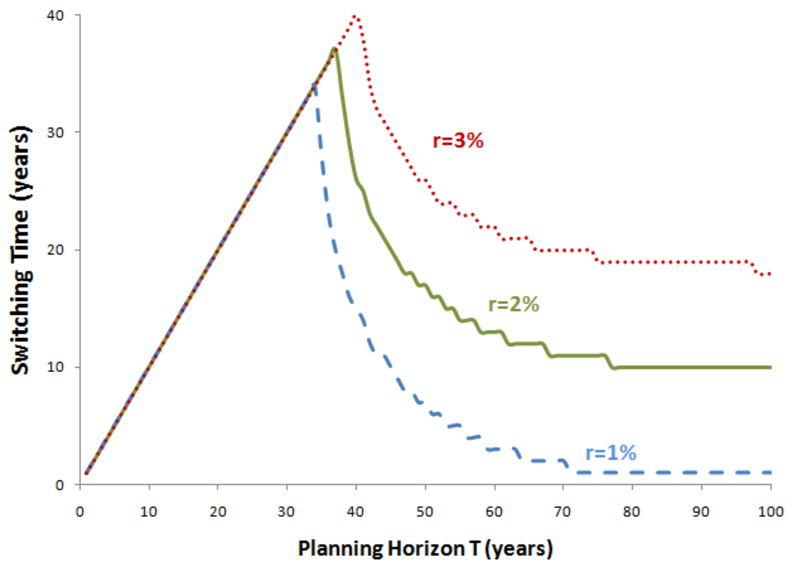

Figure 3.3: The impact of the interest rate on switching time

From Figure 3.3 we obtain a double message: First, when the discount rate is lower, non-forest owners internalize earlier the negative externality coming from the accumulation of GHGs in the atmosphere. This can be inferred from the fact that T^s is lower for lower discount rates. In particular we have that $T^s = 35$ if $r = 1\%$, $T^s = 38$ if $r = 2\%$, and $T^s = 41$ if $r = 3\%$. Second, the longer the time horizon used, the earlier the switch, i.e., the three curves are downward slopped.

To summarize, it is optimal for non-forest owners to increase emissions if $T \lesssim 40$. If $T \gtrsim 40$, it will be better to switch to a cleaner regime ($v = v_{\min}$) at some time $\tilde{t_v}$. The optimal time of the switch depends directly on the planning horizon and the discount rate. A simple folk conjecture says that the longer the planning horizon and/or the smaller the intertemporal discount rate, the sooner this switch will arrive. This is related to the existence of the damage function $L(S)$ that yields greater (cumulative) losses for lower discount rates and longer planning horizons.

We have showed how to determine the switching time. To put things into perspective we compare in Figure 3.4 the difference in payoffs between the payoff with the optimal solution with switching time (π^*) versus the payoffs $\pi(v_{\max})$ and $\pi(v_{\min})$ obtained applying the constant solutions: $v = v_{\max} \forall t \in [0, T]$ and $v = v_{\min} \forall t \in [0, T]$ respectively.

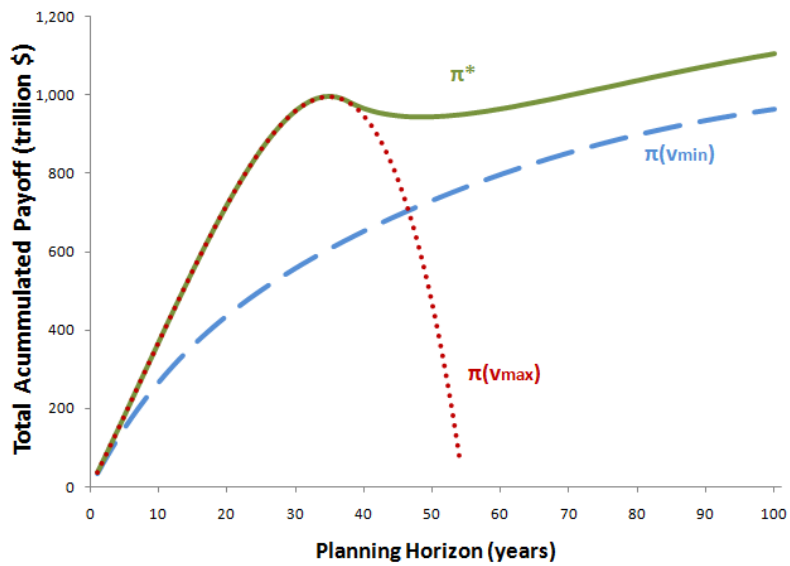

Figure 3.4: Comparing $\hat{v}$ with $v_{\min}$ and $v_{\max}$

Figure 3.4 is a plot of expression (59) along the optimal path for $E(t)$ and $S(t)$ computed for $r_{_{NF}} = 2\%$. For $T < 38$ the curve π^* and $\pi(v_{\max})$ coincide. If $T \geq 38$ the curve π^* is obtained by applying a switch.

So far we have analyzed the optimal emissions policy. Another important result that we obtain from the optimality conditions of non-forest owners regards the shadow price of the forest stock, λ_F. This shadow price is positive regardless of the time horizon and discount rate considered. The positive sign of the co-state λ_F is directly related to the ability that forests have to sequester carbon. Since the increase in forest area is directly related to the enhancement of carbon sequestration (see expression (62)); then, whatever the value of F and S,[39] increasing marginally the forest area implies marginal reductions in S, meaning smaller environmental losses (see expression (58)). This is a qualitative aspect.

At the same time, we have seen that the importance of reducing emissions is directly related to the length of the planning horizon and inversely related to the discount rate. Likewise, the marginal value that non-forest owners attach to an additional hectare of forest is greater when the planning horizon is longer and the discount rate is lower. This is a more quantitative aspect.

In short, unlike forest owners, non-forest owners are interested in increasing total forest area and this is reflected by the sign of λ_F. If we compare the different way in which forest owners and non-forest owners evaluate an additional hectare of forest, it is clear that there exists an environmental externality. As we have seen, forests have at least two uses: (i) the provision of economic revenues; and (ii) carbon sequestration. These uses are competing and somewhat excluding. Forest owners create a negative externality on non-forest owners with their net deforestation policy. Hence, the question is: Should this negative externality be corrected?

Given the existing property rights over the forest, and the fact that forest owners' payoffs are a decreasing function of total forest area, reducing net deforestation is harmful for forest owners. Therefore, the answer to this question depends on whether an additional unit of forest can generate an increase in the payoff of non-forest owners, such that it more than compensates the reduction in forest owners' revenues when they apply a more environmentally friendly deforestation/afforestation policy. If that is the case, then it will

[39] Clearly we are referring here to values of S above $\underline{S}$.

be jointly optimal to correct the externality, or at least part of it. In the next section we compute joint payoffs to give an answer to the question raised above. We also compare the cooperative scenario to the *status-quo* individual equilibrium results.

4 Cooperative solution

In the previous section we determined the non-cooperative (*status-quo*) strategies for both forest owners and non-forest owners. We saw that forest owners find it optimal to deforest as much as possible and to not afforest. On the other hand, non-forest owners suffer a negative environmental externality coming from the depletion of the forest via the *reduced-carbon-sequestration effect* that states the simple idea that a tree that is cut cannot grow and thus cannot sequester carbon. This has an impact in the concentration of GHGs in the atmosphere and leads to payoff losses to non-forest owners.

A relevant question to address is whether cooperation can improve welfare, while leading to some additional afforestation effort and/or some deforestation reduction. We assume that cooperation is achieved through joint optimization of the payoff functionals of the two players. Further, we suppose that forest and non-forest owners adopt the same discount rate r. This assumption is made to avoid giving implicitly different weights to players' streams of payoffs. Note that dealing with the general case of two different discount rates complicates the computations, but does not cause any conceptual difficulty.

The joint-optimization problem is as follows:

$$\max_{\substack{0\le\rho(t)\le\rho_{\max},\\ 0\le D(t)\le D_{\max},\\ v_{\min}\le v(t)\le v_{\max}}} \int_0^T e^{-rt}\ [R(F(t),D(t))+G(E(t))-L(S(t))]\ dt-\phi(S(T))e^{-rT}$$

$$\text{s.t.:}\qquad \dot F(t)=\rho(t)+\eta F(t)-D(t),\quad \overline{F}\ge F(t)\ge 0,\ F(0)=F_0,$$

$$\dot E(t)=v(t)E(t),\quad E(t)\ge 0,\ E(0)=E_0,$$

$$\dot S(t)=E(t)-\varphi F(t)-W,\quad S(t)\ge 0,\ S(0)=S_0,$$

where ρ, D and v are the three control variables. The joint payoff is maximized subject to the dynamics of the forest area, emissions, and stock of greenhouse gases in the atmosphere.

The Hamiltonian of the cooperative problem is:

$$\begin{aligned} H^c\left(F,E,S,\rho,D,v,\lambda_F^c,\lambda_S^c,\lambda_E^c\right) &= R(F,D)+G(E)-L(S)+\lambda_F^c[\rho+\eta F-D] \\ &\quad +\lambda_S^c[E-\varphi F-W]+\lambda_E^c vE, \end{aligned}$$

where λ_F^c, λ_E^c, λ_S^c denote the co-state variables associated with the forest stock, emissions and the stock of GHGs respectively. All the variables with a superscript c refer to cooperation as opposed to the non-cooperative outcomes retrieved before.

The Lagrangian of the cooperative problem can be written as:

$$\begin{aligned} \mathcal{L}^c\left(F,E,S,\rho,D,v,\lambda_F^c,\lambda_S^c,\lambda_E^c,w_1^c,w_2^c\right) &= H^c\left(F,E,S,\rho,D,v,\lambda_F^c,\lambda_S^c,\lambda_E^c\right) \\ &\quad + w_1^c D+w_2^c\left(D_{\max}-D\right), \end{aligned}$$

where $w_1^c(t)$ and $w_2^c(t)$ are the Lagrangian multipliers associated with the deforestation rate.

The first-order optimality conditions read:

$$\max_{\substack{0\leq\rho\leq\rho_{\max},\\ 0\leq D\leq D_{\max},\\ v_{\min}\leq v\leq v_{\max}}} \mathcal{L}^c\left(F, E, S, \rho, D, v, \lambda_F^c, \lambda_E^c, \lambda_S^c, w_1^c, w_2^c\right), \tag{67}$$

$$\dot{F} = \rho + \eta F - D, \quad \overline{F} \geq F \geq 0,\ F(0) = F_0, \tag{68}$$

$$\dot{E} = vE, \quad E \geq 0,\ \ E(0) = E_0,\ \ v \in [v_{\min}, v_{\max}],$$

$$\dot{S} = E - \varphi F - W, \quad S \geq 0,\ S(0) = S_0,$$

$$\dot{\lambda_F^c} = r\lambda_F^c - \frac{\partial \mathcal{L}^c}{\partial F}, \quad \lambda_F^c(T) = 0, \tag{69}$$

$$\dot{\lambda_S^c} = r\lambda_S^c - \frac{\partial \mathcal{L}^c}{\partial S}, \quad \lambda_S^c(T) = -\frac{d\phi(S(T))}{dS(T)}, \tag{70}$$

$$\dot{\lambda_E^c} = r\lambda_E^c - \frac{\partial \mathcal{L}^c}{\partial E}, \quad \lambda_E^c(T) = 0, \tag{71}$$

$$w_1^c D = 0,\ \ w_1^c \geq 0,\ \ D \geq 0,$$

$$w_2^c(D_{\max} - D) = 0,\ \ w_2^c \geq 0,\ \ D_{\max} \geq D.$$

The necessary condition for the maximization problem in (67) with respect to the deforestation rate reads:

$$\frac{\partial \mathcal{L}^c}{\partial D} = 0; \quad -2\theta n^2\left[\gamma\delta F + D\right] + \overline{P}n + P_A\psi\bar{Z} + w_1 - w_2 - \kappa_2 - \lambda_F^c = 0. \tag{72}$$

Because the Lagrangian function is linear in the afforestation rate, ρ, and $\frac{\partial \mathcal{L}^c}{\partial \rho} = -\kappa_1 + \lambda_F^c$, the optimal afforestation rate is a bang-bang policy as follows:

$$\rho(t) = \begin{cases} 0 & \text{if} \quad -\kappa_1 + \lambda_F^c(t) < 0, \\ \tilde{\rho} \in [0, \rho_{\max}] & \text{if} \quad -\kappa_1 + \lambda_F^c(t) = 0, \\ \rho_{\max} & \text{if} \quad -\kappa_1 + \lambda_F^c(t) > 0. \end{cases} \tag{73}$$

Just as before, λ_F^c appears in the optimality conditions for ρ and D. The only change with respect to the non-cooperative solution is that now λ_F^c captures the negative valuation of an extra hectare of forest (forest owners) as well as the positive effect that increasing forest area has on carbon sequestration (non-forest owners). Hence, now, λ_F^c can take either positive or negative values depending on which effect dominates. Furthermore, unlike in the non-cooperative case where the sign of λ_F^c was constant along the planning horizon for both players; now, nothing prevents that the sign of λ_F^c changes as time evolves. Hence, it is possible that we have a switch in either the afforestation rate, or the deforestation rate, or both throughout the planning horizon.

The differential equation (69) for the costate variable reads:

$$\dot{\lambda}_F^c = (r-\eta)\lambda_F^c + 2\theta n^2\gamma\delta(D+\gamma\delta F) + 2P_A\beta\frac{F}{\overline{F}} + P_A(\overline{Z}-2\beta) - \overline{P}n\gamma\delta + \varphi\lambda_S^c,$$
$$\lambda_F^c(T) = 0. \tag{74}$$

In order to obtain λ_F^c we need to have an analytical expression for F that in turn depends on both ρ and D (see (68)). When we were in the non-cooperative setting it was possible to characterize analytically the solution to forest owners' problem by supposing *ex-ante* that we were in the right case of figure, and then verifying, *ex-post*, that our first order conditions were indeed satisfied (see Appendix B for more details). This type of reasoning was possible because the optimal afforestation and deforestation rates were constant. In the present case, however, we can have a policy switch on ρ and/or D at any time. The value of λ_F^c is a function of the switching time on ρ and D. Thus, the first-order conditions will be satisfied for all $t \in [0, T]$ only if the exact switching time for both variables is chosen.

With respect to the speed of adjustment of emissions, v, the Lagrangian function is linear and $\frac{\partial \mathcal{L}^c}{\partial v} = \lambda_E^c E$. Thus, the optimal speed of adjustment of emissions is a bang-bang policy as follows:

$$v(t) = \begin{cases} v_{\min} & \text{if} \quad \lambda_E^c(t)E(t) < 0, \\ \tilde{v} \in [v_{\min}, v_{\max}] & \text{if} \quad \lambda_E^c(t)E(t) = 0, \\ v_{\max} & \text{if} \quad \lambda_E^c(t)E(t) > 0. \end{cases}$$

We need to know λ_E^c to derive the optimal cooperative emissions strategy. Equations (70) and (71) can be written as:

$$\dot{\lambda}_S^c = r\lambda_S^c + 2c(S - \underline{S}), \;\; \lambda_S^c(T) = 2c\phi[\underline{S} - S(T)], \tag{75}$$

$$\dot{\lambda}_E^c = (r - v)\lambda_E^c - a + bE - \lambda_S^c, \quad \lambda_E^c(T) = 0. \tag{76}$$

From (76) we see that λ_E^c is a function of λ_S^c. And from (75) we have that S is a function of F. Therefore, to obtain λ_E^c we need to know the evolution of the forest stock and the evolution of the forest stock depends on the afforestation and deforestation policies applied. As it turns out, not only do we have a potential switch of regime for all our three controls, but the switches themselves are interdependent.

One can obtain the analytical expressions for the evolution of the state and co-state variables for all the possible cases of figure. But just as it happened with non-forest owners' problem, it is not possible to derive the exact switching times analytically.

Denote now by t_ρ^c, t_D^c and t_v^c the switching time for our three control variables ρ, D and v respectively. We have evaluated the discounted intertemporal sum of joint payoffs for all the possible combinations of integer switching times (t_ρ^c, t_D^c, t_v^c) using a similar algorithm as before. See Table 3.II for a sketch of the algorithm.

The only difference with respect to our previous algorithm is that now the computational complexity is increased as a consequence of the multiplicity of cases. Denote now by $\tilde{t}_\rho^c$, $\tilde{t}_D^c$, $\tilde{t}_v^c$ the three integer switching times that yield greater intertemporal payoffs. We have computed $\tilde{t}_\rho^c$, $\tilde{t}_D^c$, $\tilde{t}_v^c$ for $T = \{\mathbb{N} \in [1, 100]\}$ and for $r \in \{0.01, 0.02, 0.03\}$. Again, the results are linked to the length of the planning horizon and the discount rate used.

Our results, call for the following comments: The solutions obtained can be classed into four different groups that coincide with four regions of the parameter space. We denote

Table 3.II: Sketch of the algorithm used to obtain $\tilde{t}^c_\rho$, $\tilde{t}^c_D$, $\tilde{t}^c_v$

Fix the joint intertemporal discount rate r
for all integer planning horizons $T \in [1, ..., 100]$
  for all possible integer switching times t^c_ρ
    for all possible integer switching times t^c_D
      for all possible integer switching times t^c_v **do**

$$\begin{aligned} \text{JointPayoff}(t^c_\rho, t^c_D, t^c_v, T) &= \text{Discounted sum of revenues of FO} \\ &+ \text{Discounted sum of payoffs of NFO} \\ &+ \text{Scrap value function of NFO} \end{aligned}$$

      end
    end
  end
end
Select the t^c_ρ, t^c_D, t^c_v whose JointPayoff is greater for each value of T

them by Z_1 to Z_4. The boundaries of regions $Z_1 - Z_4$ are related to parameter T. We denote the limits to these regions by T_1 to T_3. Figure 3.5 is a schematic representation of the solution.

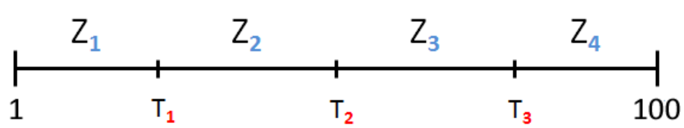

Figure 3.5: Cooperation timeline is a function of T

The results, that are summarized in Table 3.III, call for the following comments: (i) If the planning horizon is short (i.e. $T < T_1$) we are in region Z_1 and the cooperative solution coincides with the non-cooperative one (i.e. the cooperative solution brings no gain). The label not applicable ($N.A.$) is used here to denote that there is no switching time and that the solution coincides with the *status quo*. (ii) If we are in region Z_2 (i.e. $T_1 \leq T < T_2$) then it is jointly optimal to afforest at maximum rate for some time and then switch to afforestation $\rho_{\min}$ some time before the end of the planning horizon. It is not optimal to afforest all the time and we have that $\rho^* = \rho_{\max}$ if $t < \tilde{t}^c_\rho$ and $\rho^* = \rho_{\min}$ if $t \geq \tilde{t}^c_\rho$. We use the notation $\tilde{t}_\rho^c = f(T)$ to denote the fact that the switching time depends on T. Clearly for larger values of T it is optimal to switch later. The same reasoning applies for $\tilde{t}_D^c$. In this case, though, we have that $D^* = D_{\min}$ if $t < \tilde{t}_D^c$ and $D^* = D_{\max}$ if $t \geq \tilde{t}_D^c$. (iii) If we are in region Z_3 (i.e. $T_2 \leq T < T_3$) then it is optimal to apply $\rho^* = \rho_{\max}$ and $D^* = D_{\min}$ all along. We have used the notation $\tilde{t}_\rho^c = \tilde{t}_D = T$ to differentiate it from label $N.A.$ Recall that label $N.A.$ was used to denote that there is no switch and the optimal policy is identical to the *status quo* one (i.e. $\rho^* = \rho_{\min}$ and $D^* = D_{\max}$ $\forall t \in [0, T]$) whereas in region Z_3 we have that there is no switch either, but the optimal policy is to apply $\rho^* = \rho_{\max}$ and $D^* = D_{\min}$ throughout. (iv) Finally, region Z_4 is similar to region Z_3 except for the emissions policy. If $T \geq T_3$ then it is certain that we will have a jump from $v_{\max}$ to $v_{\min}$. The time of the switch is a function of T.

Table 3.III: Jointly optimal policies are a function of T

Switch	Z_1	Z_2	Z_3	Z_4
$\tilde{t}_\rho$	$N.A.$	$\tilde{t}_\rho = f(T)$	T	T
$\tilde{t}_D$	$N.A.$	$\tilde{t}_D = g(T)$	T	T
$\tilde{t}_v$	$N.A.$	$N.A.$	$N.A.$	$\tilde{t}_v = h(T)$

Cooperation is more intense and the solution is more environmentally friendly as we move from region Z_1 (no cooperation) to region Z_4 (full cooperation and emissions abatement). When the discount rate is smaller, the environmental damage is further internalized. Table 3.IV shows the values of T_1 to T_3 for our different values in the discount rates. It is not surprising that when the discount rates are smaller the threshold planning horizons between regions (T_1, T_2, T_3) are shifted downwards (See Table 3.IV).

Table 3.IV: Threshold times are a function of the discount rate

Discount	T_1	T_2	T_3
$r = 1\%$	11	19	36
$r = 2\%$	12	20	38
$r = 3\%$	12	21	41

4.1 Comparison of $\tilde{t}_v^c$ and $\tilde{t}_v$

In Table 3.III we showed that if $T \geq T_3$ it is optimal to have a switch of regime in emissions. This type of behaviour was observed in the non-cooperative case too. It is interesting to compare the differences between the two settings. In Figure 3.6 we plot the switching time for the cooperative case, $\tilde{t}_v^c$, and compare it to the non-cooperative one, $\tilde{t}_v$.

The curve in Figure 3.6 shows the one-to-one correspondence between T and $\tilde{t}_v^c$. The 45-degree diagonal indicates that no switch is applicable. The shortest planning horizon for which there is a switch - T_3- is the first element of the curve off the diagonal. $T_3 = 38$, this means that if the planning horizon (T) is shorter than 38 years it is optimal to apply $v_{\max}$ throughout and there will be no switch. Conversely, if $T \geq T_3$ we will have a switch at time $\tilde{t}_v^c$. Figure 3.6 gives in the y-axis the $\tilde{t}_v^c$ that corresponds to each T. As we saw in the non-cooperative case, the curve is downward slopped after the switch. This means that as T increases $\tilde{t}_v^c$ decreases. Otherway put, as the planning horizon increases the environmental damage coming from the accumulation of greenhouse gases is further taken into account thus making profitable that emissions reduction arrives earlier.

Comparing the cooperative and non-cooperative cases one can observe that the minimal planning horizon beyond which emissions reduction is desirable is the same for both. However we have that the cooperative curve is always above the non-cooperative one. This means that for $T \geq T_3$ it is optimal to switch later in the cooperative case than in the non-cooperative one.

To explain this difference recall that, in the cooperative setting, afforestation and reduced deforestation reduce part of the environmental damage. For this reason, it is profitable to switch to the *clean* regime later when we are in the cooperative case. In other words, cooperation allows to delay emissions abatement.

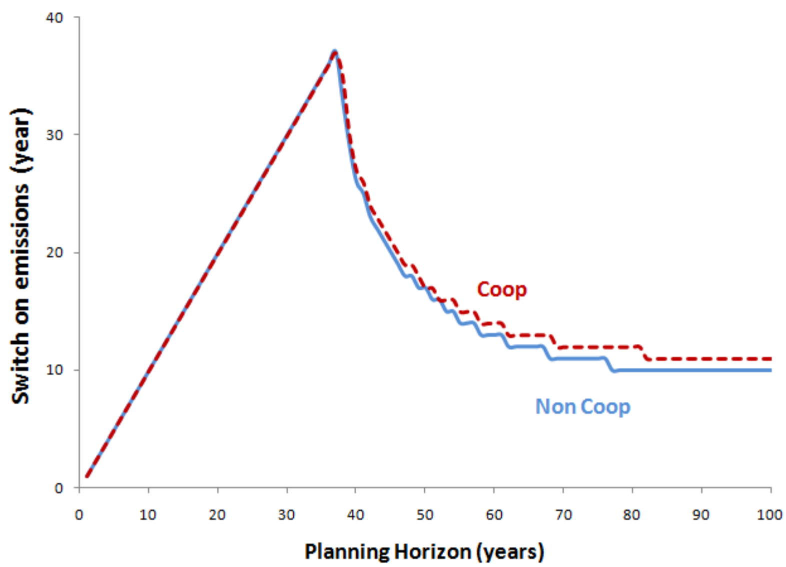

Figure 3.6: Comparison of $\tilde{t}_v^c$ and $\tilde{t}_v^{nc}$

4.2 Cooperation brings asymmetric results

We have showed that joint payoffs are greater in the cooperative setting provided that $T \geq T_1$. This is due to the damage reduction generated by increased afforestation effort and lower deforestation rates. Cooperation, however, does not bring gains to both players. Non-forest owners gain from the lower environmental damage, while forest owners lose by applying forest policies that are environmentally friendly but revenue harming.

Denote by $\pi_{NF}^c(x^c)$ the discounted sum of payoffs that non-forest owners get in the co-operative setting, where x^c denotes the state of the system along the cooperative trajectory. Analogously $\pi_{NF}^{nc}(x^{nc})$ denotes the discounted sum of payoffs that non-forest owners get in the non-cooperative setting.

The difference $\pi_{NF}^c(x^c) - \pi_{NF}^{nc}(x^{nc})$ measures the individual gain that non-forest owners obtain from cooperation. By the same token $\pi_{FO}^{nc}(x^{nc}) - \pi_{FO}^c(x^c)$ represents the loss that forest owners have in the cooperative setting *vis-à-vis* the non-cooperative one. These two quantities are a function of T and r. We compare them in Figure 3.7 for $r = 2\%$.

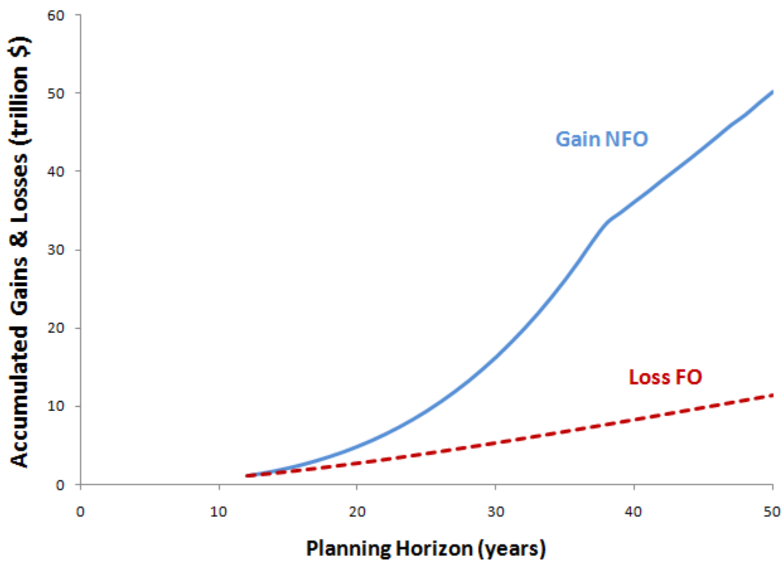

Figure 3.7: Cooperation gains and losses by NFO and FO

The cooperative gain by non-forest owners is represented by the solid line; while the loss by forest owners is represented by the dashed one. The vertical difference between these two lines measures the net cooperative gain for any given planning horizon. We observe that, for $T \geq T_1$ (with $T_1 = 11$ years) $\pi^c_{NF}(x^c) - \pi^{nc}_{NF}(x^{nc}) > \pi^{nc}_{FO}(x^{nc}) - \pi^c_{FO}(x^c)$.

It is not the issue of this paper to determine how this cooperative solution can be implemented. However, from Figure 3.7 it is clear that the implementation of the cooperative solution will require some sort of compensation from non-forest owners to forest-owners.

To sum up, the jointly optimal solution is different from the *status quo* one for T sufficiently long (i.e. $T \geq T_1$) and involves *greener* outcomes. If the planning horizon is short but not too long, i.e. $T_1 \leq T < T_3$, it will be optimal to mitigate the damage by increasing afforestation and decreasing deforestation, but not to abate emissions. If the planning horizon is sufficiently long, i.e. $T \geq T_3$, then it will be optimal both to mitigate (from the beginning) and to abate emissions (from time $\tilde{t}_v^c$ onwards). Emissions abatement has a greater cost than increasing afforestation or decreasing deforestation, this explains why it is preferable to start by applying less costly measures first and then move into more drastical changes as environmental damages increase.

4.3 Robustness analysis

Most of the parameters used in the forestry model proposed for forest owners have been obtained from FAO's Forest Resources Assessment (2006) or, when unavailable at FAO, from other sources in the literature (see Appendix A). Parameters a and b used in non-forest owners' payoff function have been calibrated to fit world GDP while making sure that emissions' gains are always increasing and concave. Finally, parameter c captures the environmental damage coming from the accumulation of greenhouse gases in the atmosphere and is key to the model. There is great uncertainty as to what is the exact impact of emissions on climate change and, therefore, on damage. For this reason attempting to estimate parameter c is a hard task. In an effort to account for part of this uncertainty on our results we have performed a sensitivity analysis with respect to it and analyzed two cases of figure. First, in Case 1 we suppose the environmental damage parameter (c) to be one third greater than the benchmark case used so far. Then, in Case 2 we suppose c to be one third below the same benchmark.

We have recomputed the cooperative outcomes obtained in the previous section for these two cases of figure. We do not observe qualitative changes. Just as before, we have four areas of interest $Z_1 - Z_4$. The behaviour in these four areas is exactly the same. The only difference that we observe is that cooperation will be more easily (i.e. earlier) achieved when the environmental damage is higher (Case 1). The results are summarized in Table 3.V below:

Table 3.V: Robustness of solution to changes in environmental damages

Discount	Benchmark			Case 1			Case 2		
	T_1	T_2	T_3	T_1	T_2	T_3	T_1	T_2	T_3
$r = 1\%$	11	19	36	8	15	31	15	25	42
$r = 2\%$	12	20	38	9	16	33	17	27	46
$r = 3\%$	12	21	41	9	17	36	18	29	49

Table 3.V shows the threshold times ($T_1 - T_3$) for Case 1 and 2. These thresholds are shifted downwards when c increases (Case 1) and upwards when c decreases (Case 2). A downward shift in T_1 indicates that the minimum planning horizon beyond which cooperation brings gains is reduced. Analogously, if T_2 and T_3 are reduced this will mean that it is optimal to enhance cooperation for shorter planning horizons than before, and vice versa for upwards shifts of the thresholds.

To sum up, our results seem quite robust to changes in parameter c and, even if the thresholds are affected, the structure of the solutions does not change and cooperation is still strictly welfare improving for all the scenarios studied regardless of the value of c used.

5 Conclusions

Forests play an important role to mitigate climate change. In this paper we have proposed a two-player model where forest owners have an incentive to deforest so as to increase their economic revenues; while non-forest owners suffer a negative externality coming from deforestation due to the so called *reduced-carbon-sequestration effect* that states that a tree that is cut cannot grow and hence cannot sequester carbon. We model the economic incentives of both types of players and explore the conditions that make environmental cooperation strictly welfare improving. We show that longer time horizons and smaller discount rates help to better account for greenhouse gas accumulation damage.

We have proposed three different mechanisms to reduce GHG accumulation: abatement of emissions, increases in afforestation, and decreases in deforestation. We show that for short planning horizons cooperation brings no gain.

For longer planning horizons it is jointly optimal to have some afforestation effort and deforestation reduction. Cooperation brings tangible economic gains that increase with the length of the planning horizon. For even longer planning horizons it is optimal to combine forestation efforts with emissions abatement. Reducing emissions is expensive but effective in offsetting the environmental damage coming from the excessive accumulation of GHGs.

Cooperation along with a sufficiently long planning horizon allows to partly internalize the positive externality that the carbon sequestration by forests creates. Cooperation brings *greener* outcomes because it helps mitigate climate change and slows down forest depletion.

Our results also convey a double positive message: First, considering the carbon sequestration potential of forests can make a significant difference to stop forest destruction. Second, international cooperation can bring sound economic and environmental gains.

The results obtained in this paper are very promising. However, there are many aspects that have not been considered and call for a critical interpretation of the results: (i) A more comprehensive dynamics of the accumulation of greenhouse gases should consider emissions related to land use change. (ii) Carbon sequestration by the oceans may be affected by the excessive acidification of the oceans. A more thorough research should integrate this aspect. (iii) We have analyzed when is it that cooperation strictly improves joint welfare. However, nothing has been said on how this cooperative solution could be implemented nor on how the surplus arising from it would be distributed. (iv) One could envisage some sort of transfer mechanism as a way to ensure cooperation. In that case it would be interesting to study the time consistency of the cooperative solution with transfers.

Appendix A: Variables & parameters description

State variables

F: **Stock of Forest**

Forest surface area of the world measured in hectares. The current stock of forest F_0 is estimated by FAO (2006) at 3952 million hectares in 2005. Parameter $\overline{F} = F_{\max}$ has been estimated for 1750 AD at 42% of the globe's surface[40] (i.e., 13067 million hectares excluding Antarctica and Greenland). This gives us a value for $F_{\max}$ of approximately 5500 million hectares. Consequently, we require that $F(t) \in [F_{\min}, F_{\max}] = \left[0 \cdot 10^9,\ 5.5 \cdot 10^9\right]$.

E: **Yearly emissions of CO_2**

These are world yearly emissions of CO_2 measured in metric tons. In 2005, total CO_2 emissions (E_0) amounted to 28.2 billion metric tons, i.e., 7.7 GtC (gigatons of carbon).[41]

S: **Cumulated quantity of CO_2 in the atmosphere**

The cumulated stock of CO_2 in the atmosphere is measured in gigatons. The current stock of CO_2 that we denote by S_0 has been estimated to be approximately at 3000 Gt of CO_2 that are equivalent to 800 GtC (383-387 ppmv) in 2007.[42]

[40]Source: http://www.geo.vu.nl/˜renh/deforest.htm
[41]Source: EIA (2008).
[42]Source: NOAA (2007).

Control variables

ρ : **Yearly afforestation** $[\rho_{\min}, \rho_{\max}] = [0,\ 3 \cdot 10^6]$

For the period 1990-2005, FAO (2006) estimates world yearly afforestation at 2.8 million hectares.

D : **Yearly deforestation** $[D_{\min}, D_{\max}] = [0,\ 13 \cdot 10^6]$

For the same period, FAO (2006) estimates the average global deforestation rate at 13 million hectares per year.

v : CO_2 **emissions adjustment rate** $[v_{\min}, v_{\max}] = [-0.015,\ 0.03]$

During the decade going from 1990 to 2000 world CO_2 emissions increased roughly 1% every year.[43] Only a few countries (e.g., Germany, United Kingdom, Denmark, Finland, some Eastern European countries and former Soviet Republics) were able to reduce their emissions. Germany was the most outstanding case and achieved a 1.8% yearly cumulative decrease. On the other hand China's emissions increased at a rate of 3% per annum. All the other big economies lie somewhere in between. Between 2000 and 2008, however, world emissions have increased at a faster rate, 3.4% yr^{-1}, (Le Quéré et al., 2009) with a probable decrease during the next two years (2009-2010) due to the world crisis. Following these observations we have set both the lower and upper bounds on v. As a benchmark scenario we have chosen $v \in [v_{\min}, v_{\max}] = [-0.015, 0.03]$.

[43] Source: Bernstein et al. (2006) and EIA (2008)

Parameters

a, b: **Emissions-output ratio parameters**

Parameters a and b are chosen to ensure that (i) $G(E)$ in 57 is increasing and concave throughout and (ii) $G(E_0)$ equals world's GDP estimate by the World Bank for the year 2008.[44] $a = 2100$, $b = 4 \cdot 10^{-9}$.

c: **Environmental damage parameter**

This parameter captures the impact of greater GHG concentration levels on the welfare of individuals. $c = 1.5 \cdot 10^{-11}$.

$\underline{S}$: **Pre-industrial CO_2 concentration level**

Parameter $\underline{S}$ has been set to match preindustrial levels, i.e., 284 ppmv in year 1832[45] that are equivalent to 587 GtC.

κ_1: **Per hectare afforestation cost**

The World Bank estimates the cost for seedling at roughly 40 US$ per thousand seedlings. The number of seedlings per hectare is equal to approximately 2000. This amounts to approximately 80 $ per hectare of forest in terms of seedling. Afforestation costs also include other costs (e.g. labour costs) that fluctuate with countries. The World Agroforestry Centre gives estimates for the Philippines around 1000 $/ha. Other NGO organisations provide estimates that range between 180 $/ha for Senegal and 400-500 $/$ha$ for other

[44] http://web.worldbank.org/
[45] Source: NOAA.

countries in Africa such as Sudan,Madagascar and Ethiopia.[46] We have chosen the round and representative value of 500 \$/ha.

κ_2: **Per hectare deforestation cost**

The Bureau of Business and Economic Research of Montana University estimates the costs of ground-based logging per green ton of harvest for the year 2006 at 22.70\$.[47] A green ton is equivalent 907 kg (2000 pounds of undried biomass material). The density of wood is typically 500 kg/m^3. For a representative douglas fir plantation (530 kg/m^3) we obtain a deforestation cost of 13.26 $\$/m^3$. If the yield per hectare, is equal to 110 m^3/ha (see the estimation of n below) then we obtain an estimate of the deforestation cost per hectare of 1459 \$/ha.

η: **Natural growth rate of the forest**

FAO (2006) estimates the average yearly natural expansion of world forests to be equal to 2.9 million hectares, i.e. $\eta\ F = 2.9 \cdot 10^6\ ha$. Parameter η is dimensionless and can be estimated accordingly: $\eta = 7.34 \cdot 10^{-4}$.

φ: **Carbon absorption rate**

This parameter is measured in metric tons of CO_2 equivalent absorbed per hectare of forest and year. According to Le Quéré et al. (2009) during the decade from 1990-2000 forests absorbed 2.6 PgC yr^{-1}(i.e., 2.6 GtC) which amounts to 9.53 $GtCO_2$. World total forest area equals 3952 million hectares. If we consider a homogeneous forest, its mean yearly carbon sequestration is 2.412 tonnes of CO_2 $ha^{-1}yr^{-1}$.

[46] See e.g.: www.villageprojectsint.org and www.edenprojects.org

[47] www.bber.umt.edu/pubs/forest/prices/loggingCostPoster.pdf

W : **Carbon absorption rate by oceans**

Le Quéré et al. (2009) estimate that oceans were able to sink, on average, 2.2 PgC yr^{-1}(8.07 GtCO$_2$ yr^{-1}) during the period 1990-2000. We have set parameter W equal to their estimate.

n : **Per hectare timber yield**

Timber yield is measured in m^3 of wood per hectare. According to FAO (2006) the mean wood content of a hectare of forest land in 2005 is equal to 110 m^3.

β : **Lower productivity due to forest depletion**

Eswaran et al. (2001) estimate the productivity loss as a consequence of land degradation, erosion, and desertification for the African continent at 8.2% of average productivity. Average land productivity is measured by $\overline{Z}$ (see the estimation below). Parameter β is thus equal to 0.061 (8.2% of $\overline{Z}$).

γ : **Selective logging yield, fraction of average yield**

The selective logging yield is measured as a fraction of average yield. When forests are managed for wood production they produce as much as 1-3 m^3 per hectare (in other words $n\gamma$ =1-3 m^3). Following Andrés-Domenech et al (2011) we set the value of $\gamma = 1.5\%$.

δ : **Fraction of forests selectively logged**

Share of the world's forests selectively-logged. Following FAO (2006), parameter δ has been calibrated at 30% to fit the current world yearly production of wood.

θ: Slope of wood demand

According to FAO (2006), the commercial value of all wood (i.e., roundwood and fuelwood) in 2005 was US$64 billion per year of which only 7 billion correspond to fuelwood. Current world production equals 3400 million m^3. The average price for both types of wood is 18.8 dollars per m^3. FAO (1997) gives the elasticity of demand for several countries and several types of wood.[48] A representative value of both the mean and median price elasticity of wood is -0.50. We have approximated an iso-elastic curve by a linear one in an interval of 2000 million m^3 centred at 3400 million m^3 such that the average elasticity inside the interval equals -0.50. The slope of our demand can be then computed and we obtain $\theta = -2.7 \cdot 10^{-9}$.

$\overline{P}$: Choke price of wood

With the average price of wood and the slope of demand computed above, we can retrieve the choke price of our inverse demand function and obtain $\overline{P} = 27.98$ (US$ per m^3).

ψ: Extra productivity of deforested land

Parameter ψ denotes the productivity gain of land after deforestation. It is measured as a fraction of average productivity. We adopt $\psi = 0.3$ following Andrés-Domenech et al. (2011).

P_A: Average price of representative agricultural product

Measured in US$ per metric ton. To determine the average price of the representative agricultural good we took four representative commodities (i.e., cocoa, coffee, cotton and sugar) from FAO (2004). These four commodities are related to deforestation processes. The net economic yield per hectare of crop ranges from 1660 $/ha for coffee to 771 $/ha

[48] Most elasticity values are comprehended between -0.25 and -0.75

for cocoa. The mean yield equals 1141 \$/ha. Cotton is the more representative of the four in terms of prices and economic yield (1467 \$ per metric ton and 1088 \$/ha). We use the price of cotton as a reference.

$\overline{Z}$: **Average land productivity**

Measured in tons per hectare. Average land productivity has been computed with the same four crops used to obtain P_A. $\overline{Z}$ is set equal to 0.742 metric tons per hectare.

Appendix B: Proof of proposition 1

The Hamiltonian of forest owners' control problem is:[49]

$$\begin{aligned} H^{FO}(F,\rho,D,\lambda) &= \left[\overline{P} - \theta\left(nD + n\gamma\delta F\right)\right] n(D+\gamma\delta F) \\ &+ P_A\left[\overline{Z} + \frac{\psi\overline{Z}}{\overline{F}-F}D - \beta\frac{\overline{F}-F}{\overline{F}}\right]\left(\overline{F}-F\right) - \kappa_1\rho - \kappa_2 D \\ &+ \lambda\left[\rho + \eta F - D\right], \end{aligned}$$

where λ denotes the co-state variable associated with the forest stock. The Lagrangian of forest owners can be written as:

$$\mathcal{L}^{FO}(F,\rho,D,\lambda,w_1,w_2) = H^{FO}(F,\rho,D,\lambda) + w_1 D + w_2\left(D_{\max} - D\right),$$

where $w_1(t)$ and $w_2(t)$ are the Lagrange multipliers associated with the non-negativity condition $D(t) \geq 0$ and $D(t) \leq D_{\max}$.[50]

[49] The time argument is eliminated when no confusion can arise.

[50] To simplify the notation, we do not include Lagrange multipliers associated with the non-negativity conditions on the other control variable, $\rho(t)$, because this variable enters in a linear way in the model and the optimal afforestation policy is bang-bang.

The first-order optimality conditions read:

$$\max_{\substack{0\leq\rho\leq\rho_{\max}\\ 0\leq D\leq D_{\max}}} \mathcal{L}^{FO}\left(F,\rho,D,\lambda,w_1,w_2\right), \tag{77}$$

$$\dot{F} = \rho + \eta F - D, \quad \overline{F} \geq F \geq 0,\ F(0) = F_0, \tag{78}$$

$$\dot{\lambda} = r_{FO}\lambda - \frac{\partial\mathcal{L}^{FO}}{\partial F}, \quad \lambda(T) = 0, \tag{79}$$

$$w_1 D = 0,\ \ w_1 \geq 0,\ \ D \geq 0,$$

$$w_2(D_{\max} - D) = 0,\ \ w_2 \geq 0,\ \ D_{\max} \geq D.$$

The necessary condition for the maximization problem in (77) with respect to the deforestation rate reads:

$$\frac{\partial\mathcal{L}^{FO}}{\partial D} = 0; \quad -2\theta n^2\left[\gamma\delta F + D\right] + \overline{P}n + P_A\psi\bar{Z} + w_1 - w_2 - \kappa_2 - \lambda = 0. \tag{80}$$

With respect to the afforestation rate, ρ, we have a Lagrangian that is linear in ρ and $\frac{\partial\mathcal{L}^{FO}}{\partial\rho} = -\kappa_1 + \lambda$. The optimal afforestation rate is a bang-bang policy as follows:

$$\rho^*(t) = \begin{cases} 0 & \text{if} \quad -\kappa_1 + \lambda(t) < 0, \\ \tilde{\rho} \in [0, \rho_{\max}] & \text{if} \quad -\kappa_1 + \lambda(t) = 0, \\ \rho_{\max} & \text{if} \quad -\kappa_1 + \lambda(t) > 0. \end{cases} \tag{81}$$

The differential equation (79) for the co-state variable reads:

$$\begin{aligned} \dot{\lambda} &= (r_{FO} - \eta)\lambda + 2\theta n^2\gamma\delta(D + \gamma\delta F) + 2P_A\beta\frac{F}{\overline{F}} + P_A(\overline{Z} - 2\beta) - \overline{P}n\gamma\delta, \\ \lambda(T) &= 0. \end{aligned} \tag{82}$$

For the values of our parameters it can be proved that a maximum deforestation rate ($D(t) = D_{\max}$ for all $t \in [0,T]$) and a minimum afforestation rate ($\rho(t) = 0$ for all $t \in [0,T]$) satisfy the optimality conditions established above. Replacing these optimal policies into the dynamics of the forest stock given in (78) we have:

$$\dot{F} = \eta F - D_{\max}, \quad F(0) = F_0.$$

The solution to this differential equation is given by (63). Plugging (63) in equation (82) leads to:

$$\begin{aligned}\dot{\lambda} &= (r_{FO} - \eta)\lambda - \overline{P}n\gamma\delta + 2\theta n^2\gamma\delta D_{\max} + P_A(\overline{Z} - 2\beta) \\ &+ 2\left[\left(F_0 - \frac{D_{\max}}{\eta}\right)e^{\eta t} + \frac{D_{\max}}{\eta}\right]\left[\theta n^2\gamma^2\delta^2 + P_A\beta\frac{1}{\overline{\overline{F}}}\right], \quad \lambda(T) = 0.\end{aligned}$$

From the integration of the above non-homogeneous linear differential equation we get the following

$$\begin{aligned}\lambda(t) &= \frac{1}{\eta - r_{FO}}\left\{-\overline{P}n\gamma\delta + 2\theta n^2\gamma\delta D_{\max} + P_A(\overline{Z} - 2\beta)\right. \\ &+ \left.2\left[\left(F_0 - \frac{D_{\max}}{\eta}\right)e^{\eta t} + \frac{D_{\max}}{\eta}\right]\left[\theta n^2\gamma^2\delta^2 + P_A\beta\frac{1}{\overline{\overline{F}}}\right]\right\} + K_\lambda e^{(r_{FO} - \eta)t},\end{aligned}$$

where K_λ denotes the constant of integration.

This constant K_λ can be retrieved using the transversality condition for the co-state variable λ, $\lambda(T) = 0$. The final expression of the co-state optimal time path reads as in (64).

For our parameter domain λ always takes negative values and increases over time to reach zero at T. Therefore, from (81), we conclude that the optimal afforestation policy is $\rho(t) = 0$ for all $t \in [0, T]$.

$$\frac{\partial \mathcal{L}^{FO}}{\partial D} = 0; \quad -2\theta n^2\left[\gamma\delta F + D\right] + \overline{P}n + P_A\psi\bar{Z} + w_1 - w_2 - \kappa_2 - \lambda = 0.$$

Finally, to show that the optimal deforestation rate D^* is indeed $D_{\max}$ for all $t \in [0, T]$ (and hence $w_1 = 0$ and $w_2 \neq 0$), we replace the optimal time paths of F and λ given by

(63) and (64), respectively, in equation (80). Given our parameters' values we observe that if $w_2 = 0$, then the LHS of equation (80) is positive -instead of null- for all feasible F and D. The only way to avoid this contradiction is by having $w_2 \neq 0$. In other words, forest owners maximize their payoffs for $D = D_{\max}$ and the forest area along the optimal path decreases with time.

Appendix C: Proof of proposition 2

The Hamiltonian of the optimal control problem of non-forest owners is:

$$H^{NF}(F, S, E, V, \lambda_F, \lambda_S, \lambda_E) = aE - \frac{1}{2}bE^2 - c\left(S - \underline{S}\right)^2 + \lambda_F\left[\rho + \eta F - D\right] + \lambda_E vE + \lambda_S\left[E - \varphi F - W\right].$$

The first-order optimality conditions read:[51]

$$\begin{aligned}
&\max_v H^{NF}, \\
\text{s.t.:}\quad &\dot{F} = \rho + \eta F - D, \quad \overline{F} \geq F \geq 0,\ F(0) = F_0, \\
&\dot{S} = E - \varphi F - W, \quad S \geq 0,\ S(0) = S_0, \\
&\dot{E} = vE, \quad E \geq 0,\ E(0) = E_0,\ v \in [v_{\min}, v_{\max}] && (83) \\
&\dot{\lambda}_F = r_{NF}\lambda_F - \frac{\partial H^{NF}}{\partial F}, \quad \lambda_F(T) = 0, && (84) \\
&\dot{\lambda}_S = r_{NF}\lambda_S - \frac{\partial H^{NF}}{\partial S}, \quad \lambda_S(T) = -\frac{d\phi\left(S(T)\right)}{dS(T)}, && (85) \\
&\dot{\lambda}_E = r_{NF}\lambda_E - \frac{\partial H^{NF}}{\partial E}, \quad \lambda_E(T) = 0. && (86)
\end{aligned}$$

[51]In order to simplify the presentation we do not explicitly introduce the Lagrangian function and the restrictions on the state variables, but we check a posteriori that all these restrictions are satisfied. The time argument is also eliminated when no confusion can arise.

Since the Hamiltonian is linear in v, condition (83) and $\frac{\partial H^{NF}}{\partial v} = \lambda_E E$, lead to the following optimal bang-bang solution:

$$v^*(t) = \begin{cases} v_{\min} & \text{if} \quad \lambda_E(t)E(t) < 0, \\ \widetilde{v} \in [v_{\min}, v_{\max}] & \text{if} \quad \lambda_E(t)E(t) = 0, \\ v_{\max} & \text{if} \quad \lambda_E(t)E(t) > 0. \end{cases}$$

Equations (84), (85) and (86) can be written as:

$$\dot{\lambda}_F = (r_{_{NF}} - \eta)\lambda_F + \varphi\lambda_S, \quad \lambda_F(T) = 0,$$

$$\dot{\lambda}_S = r_{_{NF}}\lambda_S + 2c(S - \underline{S}), \quad \lambda_S(T) = 2c\phi[\underline{S} - S(T)], \tag{87}$$

$$\dot{\lambda}_E = (r_{_{NF}} - v)\lambda_E - a + bE - \lambda_S, \quad \lambda_E(T) = 0, \tag{88}$$

where

$$\phi = \frac{1}{r_{_{NF}}}\left(1 - e^{-r_{_{NF}}T}\right).$$

Let us assume $v(t) = v^*$ constant over the planning horizon, where v^* denotes either $v_{\min}, v_{\max}$ or $\widetilde{v}$. Solving the differential equation in (83) we can characterize the optimal trajectory of emissions, $E(t)$, which is given by (65).

From the problem of forest owners, the optimal path of the forest stock is known and given by equation (63). Take equations (63) and (65), and plug them in (83). Integration of the resulting expression gives the expression in (66).

Given our parameter domain, it can be shown that both the optimal paths of emissions and stock of greenhouse gases are always greater than zero.

Using the expressions for the optimal paths of the state variables $F(t)$, $E(t)$ and $S(t)$ (expressions (63), (65) and (66) respectively), we can retrieve the optimal paths of the three co-state variables.

From the integration of the differential equation of the shadow price of the pollution stock, λ_S, in (87), we get:

$$\begin{aligned}\lambda_S(t)=&K_S e^{r_{NF}t}-\frac{2c}{r_{NF}}\left[S_0-\underline{S}-\frac{E_0}{v^*}+\frac{\varphi}{\eta}\left(F_0-\frac{D_{\max}}{\eta}\right)\right]\\&+2c\left[\frac{1}{r_{NF}}\left(t+\frac{1}{r_{NF}}\right)\left(W+\frac{\varphi}{\eta}D_{\max}\right)-\frac{\varphi}{\eta}\left(F_0-\frac{D_{\max}}{\eta}\right)\frac{e^{\eta t}}{\eta-r_{NF}}+\frac{E_0}{v^*}\frac{e^{v^*t}}{v^*-r_{NF}}\right],\end{aligned}$$

where K_S denotes the constant of integration. This constant can be easily determined using the transversality condition $\lambda_S(T)=2c\phi[\underline{S}-S(T)]$. After replacing this constant on λ_S the optimal path of the shadow price of the pollution stock reads:

$$\lambda_S(t)=\Lambda_1+\Lambda_2 e^{-r_{NF}(T-t)}+\Lambda_3 t+\Lambda_4 e^{v^*t}+\Lambda_5 e^{\eta t},$$

where:

$$\begin{aligned}\Lambda_1&=-\frac{2c}{r_{NF}}\left[S_0-\underline{S}-\frac{E_0}{v^*}+\frac{\varphi}{\eta}\left(F_0-\frac{D_{\max}}{\eta}\right)-\frac{1}{r_{NF}}\left(W+\frac{\varphi}{\eta}D_{\max}\right)\right],\\ \Lambda_2&=-\Lambda_1-2c\left[\left(\frac{W}{r_{NF}}+\frac{\varphi}{\eta}\frac{D_{\max}}{r_{NF}}\right)T-\frac{\varphi}{\eta}\left(F_0-\frac{D_{\max}}{\eta}\right)\frac{1}{\eta-r_{NF}}e^{\eta T}+\frac{E_0}{v^*}\frac{e^{v^*T}}{v^*-r_{NF}}\right]\\&\quad+2c\phi[\underline{S}-S(T)],\\ \Lambda_3&=\frac{2c}{r_{NF}}\left(W+\frac{\varphi}{\eta}D_{\max}\right),\\ \Lambda_4&=2c\frac{E_0}{v^*(v^*-r_{NF})},\\ \Lambda_5&=-2c\frac{\varphi}{\eta}\left(F_0-\frac{D_{\max}}{\eta}\right)\frac{1}{\eta-r_{NF}},\\ S(T)&=S_0-WT-\frac{\varphi}{\eta}\left(D_{\max}T-\left(F_0-\frac{D_{\max}}{\eta}\right)(1-e^{\eta T})\right)-\frac{E_0}{v^*}(1-e^{v^*T}).\end{aligned}$$

Once we have λ_S we can plug it in expression (88) to obtain λ_E. Integrating the resulting expression gives:

$$\lambda_E(t) = \frac{a}{r_{NF} - v^*} - \frac{bE_0}{r_{NF} - 2v^*}e^{v^*t} + \frac{\Lambda_1}{r_{NF} - v^*} + \frac{\Lambda_2}{v^*}e^{-r_{NF}(T-t)}e^{-2v^*t}$$
$$+ \frac{\Lambda_3}{r_{NF} - v^*}\left(t + \frac{1}{r_{NF} - v^*}\right) + \frac{\Lambda_4}{r_{NF} - 2v^*}e^{v^*t} + \frac{\Lambda_5}{r_{NF} - v^* - \eta}e^{\eta t} + K_E e^{(r_{NF} - v^*)t},$$

where K_E denotes the constant of integration. To determine K_E we use the transversality condition $\lambda_E(T) = 0$, and substitute its value in the expression above. The co-state variable $\lambda_E(t)$ then reads as in expression (89).

$$\begin{aligned}\lambda_E(t) = {} & (\Lambda_1 + a)\frac{1}{r_{NF} - v^*}(1 - \Gamma(t)) - \frac{bE_0}{r_{NF} - 2v^*}\left(e^{v^*t} - e^{v^*T}\Gamma(t)\right) \\ & + \frac{\Lambda_2}{v^*}[e^{-2v^*t}e^{-r_{NF}(T-t)} - e^{-2v^*T}\Gamma(t)] \\ & + \frac{\Lambda_3}{r_{NF} - v^*}\left(t + \frac{1}{r_{NF} - v^*} - \left(T + \frac{1}{r_{NF} - v^*}\right)\Gamma(t)\right) \\ & + \frac{\Lambda_4}{r_{NF} - 2v^*}(e^{v^*t} - e^{v^*T}\Gamma(t)) + \frac{\Lambda_5}{r_{NF} - v^* - \eta}(e^{\eta t} - e^{\eta T}\Gamma(t)).\end{aligned} \tag{89}$$

where $\Gamma(t) = e^{-(r_n - v^*)(T-t)}$.

The optimal path for the shadow price of the forest stock $\lambda_F(t)$ can be obtained analogously and is given by expression (90).

$$\begin{aligned}\lambda_F(t) = \varphi \Bigg[& \left(-\frac{\Lambda_1}{r_n - \eta} + \frac{\Lambda_2}{\eta}\Psi(t)\right)(1 - \Psi(t)) - \frac{\Lambda_4}{r_n - \eta - v^*}(e^{v^*t} - \Psi(t)) \\ & - \frac{\Lambda_3}{r_n - \eta}\left(t + \frac{1}{r_n - \eta} - \left(T + \frac{1}{r_n - \eta}\right)\Psi(t)\right) - \frac{\Lambda_5}{r_n - 2\eta}(e^{\eta t} - \Psi(t)e^{-\eta T})\Bigg],\end{aligned} \tag{90}$$

where $\Psi(t) = e^{-(r_n - \eta)(T-t)}$.

Appendix D: Switching time

If there is only one switch in the optimal policy (switch at time $\tilde{t}_v$) then the jump should be of the following type: First apply $v_{\max}$ $\forall t \in [0, \tilde{t}_v]$ and then apply $v_{\min}$ $\forall t \in [\tilde{t}_v, T]$. Applying $v_{\max}$ always brings greater yields in the short run than it does $v_{\min}$ and if one optimizes using a positive discount rate it is better to allocate emissions at the beginning of the planning horizon.

Recall that in absence of switching time we have that $S(t)$ is given by (66). Whereas, when there is a switch, the optimal expression for $S(t)$ changes. We now have a two-part expression, one before the switch and another afterwards.

$$\begin{aligned}
S(t) &= S_0 - tW - \frac{\varphi}{\eta} D_{\max} t - \frac{E_0}{v_{\max}}(1 - e^{v_{\max} t}) + \frac{\varphi}{\eta}\left(F_0 - \frac{D_{\max}}{\eta}\right)(1 - e^{\eta t}), \forall t \in [0, \tilde{t}_v), \\
S(t) &= S(\tilde{t}_v) - \left(t - \tilde{t}_v\right)\left(W + \frac{\varphi}{\eta} D_{\max}\right) - \frac{E_0}{v_{\min}} e^{v_{\max}\tilde{t}_v}(1 - e^{v_{\min}\left(t - \tilde{t}_v\right)}) \\
&\quad + \frac{\varphi}{\eta}\left(F(\tilde{t}_v) - \frac{D_{\max}}{\eta}\right) e^{\eta \tilde{t}_v}(1 - e^{\eta\left(t - \tilde{t}_v\right)}), \quad \forall t \in [\tilde{t}_v, T],
\end{aligned}$$

where

$$S(\tilde{t}_v) = S_0 - \tilde{t}_v W - \frac{\varphi}{\eta} D_{\max}\tilde{t}_v - \frac{E_0}{v_{\max}}(1 - e^{v_{\max}\tilde{t}_v}) + \frac{\varphi}{\eta}\left(F_0 - \frac{D_{\max}}{\eta}\right)(1 - e^{\eta \tilde{t}_v}).$$

These expressions are straightforward to obtain considering that:

$$E(t) = \begin{cases} E_0 e^{v_{\max} t}, & \forall t \in [0, \tilde{t}_v] \\ E_0 e^{v_{\max}\tilde{t}_v} e^{v_{\min}(t - \tilde{t}_v)}, & \forall t \in [\tilde{t}_v, T]. \end{cases}$$

Once we have $S(t)$, $\lambda_S(t)$ can be computed using the transversality condition from the salvage value function. Proceeding similarly as we did to obtain $\lambda_S(t)$ in the case without switch, the following expression for the interval $[\tilde{t}_v, T]$ is obtained:

$$\lambda_S(t) = \Upsilon_1 + \Upsilon_2 e^{-r_{NF}(T-t)} + \Upsilon_3 t + \Upsilon_4 e^{v_{\min}\left(t - \tilde{t}_v\right)} + \Upsilon_5 e^{\eta t}, \tag{91}$$

where:

$$\begin{aligned}
\Upsilon_1 &= -\frac{2c}{r_{NF}}\left[S(\tilde{t}_v) - \underline{S} - \frac{E_0 e^{v_{\max}\tilde{t}_v}}{v_{min}} + \frac{\varphi}{\eta}\left(F(\tilde{t}_v) - \frac{D_{\max}}{\eta}\right)\right] \\
&\quad - \frac{2c}{r_{NF}}\left(W + \frac{\varphi D_{\max}}{\eta}\right)\left(\tilde{t}_v - \frac{1}{r_{NF}}\right), \\
\Upsilon_2 &= -\Upsilon_1 - 2c\left[\frac{W}{r_{NF}}T + \frac{\varphi}{\eta}\left(\frac{D_{\max}}{r_{NF}}T - \left(F(\tilde{t}_v) - \frac{D_{\max}}{\eta}\right)\frac{1}{\eta - r_{NF}}e^{\eta(T-\tilde{t}_v)}\right)\right. \\
&\quad \left. + \frac{E_0}{v_{\min}}e^{v_{\max}\tilde{t}_v}\frac{e^{v_{\min}(T-\tilde{t}_v)}}{v_{\min} - r_{NF}}\right] + 2c\phi[\underline{S} - S(T)], \\
\Upsilon_3 &= \frac{2c}{r_{NF}}\left(W + \frac{\varphi}{\eta}D_{\max}\right), \\
\Upsilon_4 &= 2c\frac{E_0 e^{v_{\max}\tilde{t}_v}}{v_{\min}\left(v_{\min} - r_{NF}\right)}, \\
\Upsilon_5 &= -2c\frac{\varphi}{\eta}\left(F(\tilde{t}_v) - \frac{D_{\max}}{\eta}\right)\frac{1}{\eta - r_{NF}}.
\end{aligned}$$

Once $\lambda_S(t)\ \forall t \in [\tilde{t}_v, T]$ is known, $\lambda_S(t)\ \forall t \in [0, \tilde{t}_v]$ can be computed analogously and can be written in a compact manner as follows:

$$\lambda_S(t) = \Sigma_1 + \Sigma_2 e^{r_{NF}(t-\tilde{t}_v)} + \Sigma_3 t + \Sigma_4 e^{v_{\max}t} + \Sigma_5 e^{\eta t},$$

where

$$\begin{aligned}
\Sigma_1 &= -\frac{2c}{r_{NF}}\left[S_0 - \underline{S} - \frac{E_0}{v_{max}} + \frac{\varphi}{\eta}\left(F_0 - \frac{D_{\max}}{\eta}\right) - \left(\frac{W}{r_{NF}} + \frac{\varphi}{\eta}\frac{D_{\max}}{r_{NF}}\right)\right], \\
\Sigma_2 &= -\Sigma_1 - 2c\left[\left(\frac{W}{r_{NF}} + \frac{\varphi}{\eta}\frac{D_{\max}}{r_{NF}}\right)\tilde{t}_v - \frac{\varphi}{\eta}\left(F_0 - \frac{D_{\max}}{\eta}\right)\frac{e^{\eta\tilde{t}_v}}{\eta - r_{NF}} + \frac{E_0}{v_{\max}}\frac{e^{v_{\max}\tilde{t}_v}}{v_{\max} - r_{NF}}\right] \\
&\quad + \lambda_S(\tilde{t}_v), \\
\Sigma_3 &= \frac{2c}{r_{NF}}\left(W + \frac{\varphi}{\eta}D_{\max}\right), \\
\Sigma_4 &= 2c\frac{E_0}{v_{\max}\left(v_{\max} - r_{NF}\right)}, \\
\Sigma_5 &= -2c\frac{\varphi}{\eta}\left(F_0 - \frac{D_{\max}}{\eta}\right)\frac{1}{\eta - r_{NF}},
\end{aligned}$$

and where $\lambda_S(\tilde{t}_v)$ is the boundary condition to this problem that can be obtained by substituting for time $t = \tilde{t}_v$ in equation (91).

Now $\lambda_S(t)$ $\forall t \in [\tilde{t}_v, T]$ and $\lambda_S(t)$ $\forall t \in [0, \tilde{t}_v]$ are known; $\lambda_E(t)$ $\forall t \in [\tilde{t}_v, T]$ and $\lambda_E(t)$ $\forall t \in [0, \tilde{t}_v]$ can be obtained analogously. In this case it is easier since the boundary condition for λ_E (i.e., $\lambda_E(T)$) is equal to zero. The expression of $\lambda_E(t)$ $\forall t \in [\tilde{t}_v, T]$ reads:

$$\begin{aligned}\lambda_E(t) = {} & \frac{a}{r_{NF} - v_{min}} - \frac{bE_0 e^{v_{\max}\tilde{t}_v}}{r_{NF} - 2v_{min}} e^{v_{min}\left(t-\tilde{t}_v\right)} + \frac{\Upsilon_1}{r_{NF} - v_{min}} - \frac{\Upsilon_2}{v_{min}} e^{-r_{NF}(T-t)} \\ & + \frac{\Upsilon_3}{r_{NF} - v_{min}} \left(t + \frac{1}{r_{NF} - v_{min}}\right) + \frac{\Upsilon_4}{r_{NF} - 2v_{min}} e^{v_{min}\left(t-\tilde{t}_v\right)} \\ & + \frac{\Upsilon_5}{r_{NF} - v_{min} - \eta} e^{\eta\left(t-\tilde{t}_v\right)} + K_E e^{(r_{NF} - v_{min})t}.\end{aligned}$$

The constant of integration K_E can be obtained using the transversality condition $\lambda_E(T) = 0$. Denote $\Pi(t) = e^{(r_{NF} - v_{min})(t-T)}$, then the value of $\lambda_E(t)$ $\forall\, t \in [\tilde{t}_v, T]$ can be written as follows:

$$\begin{aligned}\lambda_E(t) = {} & (a + \Upsilon_1) \frac{1}{r_{NF} - v_{min}} (1 - \Pi(t)) - \frac{\Upsilon_2}{v_{min}} \left(e^{-r_{NF}(T-t)} - \Pi(t)\right) \\ & - \frac{bE_0 e^{v_{\max}\tilde{t}_v}}{r_{NF} - 2v_{min}} \left(e^{v_{min}\left(t-\tilde{t}_v\right)} - e^{v_{min}\left(T-\tilde{t}_v\right)} \Pi(t)\right) \\ & + \frac{\Upsilon_3}{r_{NF} - v_{min}} \left[\left(t + \frac{1}{r_{NF} - v_{min}}\right) - \left(T + \frac{1}{r_{NF} - v_{min}}\right) \Pi(t)\right] \\ & + \frac{\Upsilon_4}{r_{NF} - 2v_{min}} \left(e^{v_{min}\left(t-\tilde{t}_v\right)} - e^{v_{min}\left(T-\tilde{t}_v\right)} \Pi(t)\right) \\ & + \frac{\Upsilon_5}{r_{NF} - v_{min} - \eta} \left(e^{\eta\left(t-\tilde{t}_v\right)} - e^{\eta\left(T\tilde{t}_v\right)} \Pi(t)\right). \end{aligned} \tag{92}$$

Similarly, for $\lambda_E(t)\,\forall\, t \in [0, \tilde{t}_v]$ we obtain the following expression:

$$\begin{aligned}\lambda_E(t) = {} & (a+\Sigma_1)\,\frac{1}{r_{NF}-v_{max}}\,(1-\Delta(t)) - \frac{bE_0}{r_{NF}-2v_{max}}\left(e^{v_{max}t}-e^{v_{max}\tilde{t}_v}\Delta(t)\right)\\ & -\frac{\Sigma_2}{v_{max}}\left(e^{-r_{NF}\left(\tilde{t}_v-t\right)}-\Delta(t)\right)+\frac{\Sigma_4}{r_{NF}-2v_{max}}\left(e^{v_{max}t}-e^{v_{max}\tilde{t}_v}\Delta(t)\right)\\ & -\frac{\Sigma_3}{r_{NF}-v_{max}}\left[\left(t+\frac{1}{r_{NF}-v_{max}}\right)-\left(\tilde{t}_v+\frac{1}{r_{NF}-v_{max}}\right)\Delta(t)\right]\\ & +\frac{\Sigma_5}{r_{NF}-v_{max}-\eta}\left(e^{\eta t}-e^{\eta\tilde{t}_v}\Delta(t)\right)+\lambda_E(\tilde{t}_v)\Delta(t), \end{aligned} \tag{93}$$

where $\lambda_E(\tilde{t}_v)$ in (93) can be obtained from (92) and $\Delta(t) = e^{(r_{NF}-v_{max})\left(t-\tilde{t}_v\right)}$.

With the two equations for $\lambda_E(t)$ (before and after the switch) the switching time can be obtained. The switching time (provided it is unique) has to satisfy the following first-order condition:

$$\lambda_E(t)E(t) > 0 \quad \forall t \in [0, \tilde{t}_v),$$
$$\lambda_E(t)E(t) < 0 \quad \forall t \in (\tilde{t}_v, T].$$

Bibliography

[1] Andrés-Domenech, P., Saint-Pierre P., & Zaccour, G. (2011). Forest conservation and CO_2 emissions: A viable approach. *Environmental Modeling & Assessment, 16*(6), 519-539.

[2] Bahn, O., Haurie, A., & Malhamé, R. (2008). A stochastic control model for optimal timing of climate policies. *Automatica, 44*, 1545-1558.

[3] Barbier, E. B., & Burgess, J. C. (2001). The economics of tropical deforestation. *Journal of Economic Surveys, 15*(3), 413-433.

[4] Barbier, E. B., & Rauscher, M. (1994). Trade, tropical deforestation and policy intervention. *Environmental and Resource Economics, 4*(1), 75-90.

[5] Benchekroun, H., & Long, N. V. (2002). On the multiplicity of efficiency-inducing tax rules. *Economic Letters, 76*, 331-336.

[6] Bernstein, P. M., Montgomery, W. D., & Tuladhar, S. D. (2006). Potential for reducing carbon emissions from non-Annex B countries through changes in technology, *Energy Economics, 28*(5-6), 742-762.

[7] Breton, M., Martín-Herrán, G., & Zaccour, G. (2006). Equilibrium investment strategies in foreign environmental projects. *Journal of Optimization Theory and Applications, 130*(1), 23-40.

[8] Breton, M., Zaccour, G. & Zahaf. M. (2005). A differential game of joint implementation of environmental projects. *Automatica, 41*, 1737-1749.

[9] Chen, Y. (1988). Early holocene population expansion of some rainforest tree at Lake Barrine basin, Queensland. *Australian Journal of Ecology, 13*, 225-233.

[10] Dockner, E. J., & Long, N. V. (1993). International pollution control: Cooperative versus noncooperative strategies. *Journal of Environmental Economics and Management, 24*, 13-29.

[11] Energy Information Administration (EIA) (2008). *International Energy Outlook 2008.*

[12] Eswaran, H., Lal, R., & Reich, P. F. (2001). Land degradation: an overview. In E. M. Bridges, I. D. Hannam, L.R. Oldeman, F. W. T. Penning de Vries, S. J. Scherr & S. Sombatpanit (Eds.), *Response to land degradation* (pp. 20-35). Enfield, NH, USA: Science Publishers Inc.

[13] Food and Agriculture Organization of the United Nations (FAO) (1997). *Asia-Pacific Forestry Sector Outlook Study: Forest Industry Structure and the Evolution of Trade Flows in the Asia-Pacific Region - Scenarios to 2010.*

[14] Food and Agriculture Organization of the United Nations (FAO) (2004). *The state of food and agriculture 2003-2004.*

[15] Food and Agriculture Organization of the United Nations (FAO) (2006). *Forest Resources Assessment 2005.*

[16] Fredj, K., Martín-Herrán, G., & Zaccour, G. (2004). Slowing deforestation rate through subsidies: A differential game, *Automatica, 40*(2), 301-309.

[17] Fredj, K., Martín-Herrán, G., & Zaccour, G. (2006). Incentive mechanisms to enforce sustainable forest exploitation. *Environmental Modeling & Assessment, 11*(2),145-156.

[18] Intergovernmental Panel on Climate Change (IPCC) (2000). *Land use, land-use change and forestry. Special report.* Cambridge: Cambridge University Press.

[19] Intergovernmental Panel on Climate Change (IPCC) (2007). *Climate change 2007: Impacts, adaptation and vulnerability, fourth assessment report.* Cambridge: Cambridge University Press.

[20] Jørgensen, S., Martín-Herrán, G., & Zaccour, G. (2010). Dynamic games in the economics and management of pollution. *Environmental Modeling & Assessment, 15*(6), 433-467.

[21] Le Quéré, C., Raupach, M. R., Canadell, J. G., Marland, G., et al. (2009). *Trends in the sources and sinks of carbon dioxide. Nature Geoscience, 2*, 831-836.

[22] Martín-Herrán, G., Cartigny, P., Motte, E., & Tidball, M. (2006). Deforestation and foreign transfers: A Stackelberg differential game approach. *Computers & Operations Research, 33*(2), 386-400.

[23] Martín-Herrán, & G., Tidball, M. (2005). Transfer mechanisms inducing a sustainable forest exploitation. In C. Deissenberg & R. Hartl (Eds.) *Optimal Control and Dynamic Games: Applications in Finance* (pp. 85-103). *Management Science and Economics.* The Netherlands: Springer.

[24] National Institute for Occupational Safety and Health (NIOSH) (2007). *Carbon dioxide: IDLH Documentation.*

[25] National Oceanic & Atmospheric Administration (NOAA) (2007). *Trends in atmospheric carbon dioxide.*

[26] Van Soest, D., & Lensink, R. (2000). Foreign transfers and tropical deforestation: what terms of conditionality? *American Journal of Agricultural Economics, 82*(2), 389-399.

[27] Van der Ploeg, F., & De Zeeuw, A. (1992). International aspects of pollution control. *Environmental & Resource Economics, 2*(2),117-139.

General Conclusion

The over exploitation of world forests can be seen as a global *tragedy of the commons* and addressing the problem is quite challenging for it involves different economic actors with conflicting economic objectives. In this dissertation we have accounted for many of these conflicts and proposed new forest management strategies that are compatible with sustainable forest development. These strategies, however, need not be in the best interest of all the parties involved and, in absence of a regulatory agent, they will not be applicable unless all the actors have a strong incentive to cooperate.

We have analyzed the role of economic transfers as a key mechanism to align all actors' objectives. In this sense, transfers can be seen as monetary compensations from those who gain with the application of *greener* forest policies towards those who may be harmed by them.

We have modelled the forestry sector using real data. Our results replicate some very well known stylized facts, i.e., forest depletion is a direct result of rural poverty, ill defined property rights, population and agricultural expansion, and timber demand.

In the first essay we analyze the problem of forest depletion from a microeconomic perspective. We have modelled the productive system of a forest-dependent traditional society -The Tandroy- as well as some of the causes explaining the large deforestation rates in the region. We have showed that sustainable economic development necessarily requires a more or less stable population size, a less cattle-intensive productive system, better defined property rights over forest land, and a reduction on the net deforestation rate to at least half of its current level.

Current population sets great pressure on the resource and current consumption levels are not sustainable in the long run. Sustainability of the forest can be achieved at the price of a 40% reduction in the current consumption rate of the region. Since poverty is the main factor explaining forest depletion in Androy, it is quite unlikely that further consumption reductions will lead to a stabilization of the forest area. We analyze the role that economic transfers can play as a means to reconcile the need of the Tandroy to subsist with the conservation of their valuable dry forest. According to our estimates, as much as 31 million 2011 US$ are needed on a yearly basis to compatibilize current consumption rates with the conservation of the forest.

In this dissertation we also account for the need to treat world forest depletion as a global problem. We analyze the causes and consequences of deforestation and provide a novel modelling framework at the conceptual level by linking forest depletion to climate change issues.

In the second essay, the consequences of deforestation are analyzed. We enlarge our scope and consider the global negative externality that forest depletion creates on the accumulation of greenhouse gases in the atmosphere. Trees sequester carbon as they grow and forest depletion reduces carbon sequestration through the so called *reduced-carbon-sequestration effect*. We show that current net deforestation and emissions rates are far from being sustainable and lead to forest depletion and excessive greenhouse gas accumulation respectively. Our results suggest the need for a concerted effort: First, the current net deforestation rate must greatly decrease. If reducing the deforestation rate is too costly, then afforestation needs to more than triple in order to compensate for it. Second, emissions must decrease below 1990 emissions levels in all cases. Monetary transfers can pay for an increase in world forest surface area and contribute significantly. The greater the forest area the lower the emissions decrease needed. This being said, the carbon sequestration potential of forests is limited and we cannot simply rely on them to achieve our environmental targets. That is, monetary transfers can partly ease the problem, but large emissions reductions are still needed.

In the third essay we follow a dynamic game-theoretical approach where two types of agents (forest owners and non-forest owners) are clearly identified. By doing so it is possible to capture the existence of colliding objectives between them: Forest owners obtain economic revenues by exploiting the forest. This leads to forest depletion and creates a negative economic externality on non-forest owners who suffer the damage from the accumulation of greenhouse gases in the atmosphere.

Cooperation may achieve *greener* and economically more efficient outcomes. It may help to reduce net deforestation and thus partly or fully internalize this negative externality. However, this will be possible only if the long term damages are accounted for.

The results obtained in this work call for a number of comments. In the first place, we have accounted for the economic value that forests have as a carbon sink. Other sources of value, such as the biodiversity, aesthetic and recreational values have been neglected due to the lack of data availability and extreme difficulty to estimate them. We leave for future investigations the task to compute the value of forests in a more accurate and holistic way.

Some of the parameters used in this work are still rough approximations and in some cases the data did not exist or had to be estimated using other primary or secondary sources. As the sources of these data become increasingly available and reliable, so will the conclusions derived from them.

Also, the emissions-deforestation model proposed can be expanded to include other state variables, such as population and/or temperature dynamics. Further, we did not account here for technological progress which may help in reducing emissions by unit of output or may lead to a more productive agriculture. These aspects need to be integrated in future investigations.

In the present work we have acknowledged for the existence of environmental thresholds triggering irreversible damages or feedback changes. There is yet much uncertainty and the value of both thresholds and damages is still subject to debate. Empirical observation and the accumulation of scientific evidence will partly reduce this uncertainty in the next years to come. In the mean time, a natural way to extend the present work is by modelling this uncertainty in a more explicit manner.

Finally, the present work has called for the need for joint cooperation by all countries. Sustainable forest management requires much larger afforestation rates and lower deforestation rates at the global level. During the last few years, some countries have become increasingly aware and intensified their efforts towards a more sustainable development. These efforts, however, still remain local and insufficient, and more countries are expected to join in the next years to come. As this process evolves new issues related to the formation, implementation and time consistency of international forestry agreements will be raised.