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Three Essays in Quantitative Marketing Models

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Three Essays in Quantitative Marketing Models

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RÉSUMÉ

L'application de la modélisation quantitative a révolutionné la recherche en marketing. En effet, l'alliance entre les données et la théorie a permis de produire une panoplie d'outils polyvalents pour assister les gestionnaires dans leur prise de décision. Cette thèse se compose de trois essais mettant en évidence l'application de la modélisation quantitative dans la résolution de différentes problématiques d'actualité en marketing.

Le premier essai porte sur l'analyse du comportement des enchérisseurs tardifs dans les enchères en ligne. Nous revisitons la question portant sur les raisons qui motivent les participants à des enchères en ligne à placer leur première enchère juste avant la clôture de la vente. En utilisant des données d'eBay relatives à deux catégories de produits, à savoir les antiquités et les iPods, nous développons deux modèles de comptage afin de modéliser à la fois la présence et l'intensité du phénomène étudié.

Le deuxième essai s'intéresse à la diffusion des services d'abonnement dans une perspective de gestion de la relation client. Nous proposons un nouveau modèle qui incorpore les dépenses en acquisition et en rétention dans le processus de diffusion du service. La croissance du nombre d'abonnés est le résultat de deux dynamiques distinctes : une dynamique d'acquisition et une autre dynamique de rétention. En utilisant la programmation dynamique, nous établissons une nouvelle approche de modélisation dans le but de déterminer les politiques optimales en acquisition et en rétention maximisant le capital client.

Finalement, nous présentons dans le troisième essai une analyse de l'interaction des canaux de distribution en présence de la marque privée. Nous examinons les conséquences d'adoption d'une stratégie parapluie par le détaillant sur sa performance. Cette stratégie consiste à utiliser un même nom pour la commercialisation de différents produits, pouvant être liés ou dépendants. Cette politique offre au détaillant une opportunité de renforcer une position déjà confortable, induite par le lancement préalable de sa marque privée. Notre analyse considère les interactions stratégiques entre manufacturiers et détaillants, ainsi que les effets d'entrainement positifs générés par les ventes de la marque privée dans différentes catégories. Nous adoptons une méthodologie basée sur la théorie des jeux afin de déterminer les stratégies optimales de fixation de prix dans plusieurs cas de figures.

Mots clés : modélisation quantitative, recherche marketing, enchères en ligne, enchérissement tardif, modèles de diffusion, services d'abonnement, relation client, rétention des clients, acquisition des clients, marque privée, stratégie parapluie, canaux de distribution, modèles de comptage, programmation dynamique, théorie des jeux.

ABSTRACT

Quantitative methods and models have produced major practical and scientific value in marketing. By combining data and theory, quantitative modeling marketing provides a versatile set of tools to aid decision makers in a variety setting. This thesis is composed of three articles which apply quantitative modeling to address some topical marketing issues. The first essay focuses on bidder's behavior in online auctions. We revisit the question of why some participants in online auctions place their bids right before the time of closing. Using e-Bay data for two product categories, antiques and iPods, we propose count-data models to look at both the presence of the late-bidding phenomenon and its intensity.

The second essay focuses on proposing the diffusion of subscription services under a customer relationship management perspective. We propose a new diffusion model that incorporates acquisition and retention expenditures. The service growth is characterized by two processes: customer acquisition process and customer attrition process. By using dynamic programming, we introduce an innovative approach to calculate optimal acquisition and retention spending in order to maximize the customer equity.

Finally, the third essay concentrates on marketing channels interactions in the presence of private label. We look into the impact of adopting an umbrella branding strategy on the retailer's performance. This strategy consists in using the same name to market different products which may, or may not, be related. The latter is a way of reinforcing their position through an already established private label in order to benefit from its position. The analysis takes into account the strategic interactions between the manufacturers and the retailer, as well as the positive spillover between sales of the private label in different categories. We adopt a game theoretic methodology to determine optimal pricing strategies under different settings.

Keywords: quantitative modeling, marketing research, online auctions, late-bidding, diffusion models, subscription services, customer relationship, customer retention, customer acquisition, private label, umbrella branding, marketing channel, count data models, dynamic programming, game theory.

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AUTHOR CONTRIBUTIONS

The first essay entitled «An Empirical Investigation of Late Bidding in Online Auctions» is co-authored with Professor Georges Zaccour and published in «Economics Letters». The second essay «Optimal CRM Expenditures for the Diffusion of Subscription Services» is also co-authored with Professor Georges Zaccour and being prepared for submission. Finally, the third essay «Branding Decisions for Retailer's Private Labels» is co-authored with Professor Georges Zaccour and Professor Nawel Amrouche and published in «Journal of Marketing Channels». Authors have equal contributions.

General Introduction

Quantitative methods and models have produced major practical and scientific value in marketing. By combining data and theory, quantitative marketing provides a versatile set of tools to aid decision makers in a variety setting. In fact, managers seek first to understand customer behavior in order to influence it through different marketing strategies. Some strategies could be product-oriented (quality, price, channels, etc.) and others could be customer-centric oriented (customer satisfaction, customer experience, customer relationship management, etc.). In this process, optimization theory plays a crucial role to aid managers determining optimal marketing strategies. The scope of quantitative marketing methods is very wide. Applications go beyond traditional marketing topics to address several new issues related to technological advances such online auctions, social networks, and others. From market structure perspective, some applications have focused on situations restricting to one decision maker, while others have incorporated the interaction between several players. This research work presents new applications of quantitative models in three topical marketing issues, namely, bidders' behavior in online auctions, customer relationship management in the diffusion of subscription services and marketing channel in the presence of private labels.

The first essay focuses on bidders' behavior in online auctions. The easy access and low transaction costs have allowed online auctions to become an increasingly popular and efficient market form, giving everyone the opportunity to sell or buy variety of products. Presently, there are hundreds of websites dedicated to online auctions. In addition, Pew Internet and American Life Project reported that 22% of Internet users have participated in online auctions as of December 2002[1]. This reflected an 85% growth from 13 million users who participated in online auctions as of March 2000, to 24 million users as of December 2002. This huge

1. http://www.pewinternet.org/Reports/2003/Americas-Online-Pursuits

quantity of transactions provides a new source of data to analyse the impact of the game rules (market design) on the behavior of the participants. For this reason, several studies have been carried out to understand different bidders' behaviors in online auctions. The late-bidding or sniping is the most popular behavior observed in eBay auctions. It describes the bidder that places a high bid in the closing seconds of an auction. Some authors argue that late-bidding affects the expected revenue of the seller and suggest eBay managers to change the ending rules of the auction. In this sense, a better understanding of this phenomenon is needed to assess whether late-bidders really represent a threat for sellers and auctioneers revenue.

The first essay revisits the question of why some participants in online auctions place their bids right before the time of closing. The proposed concept differs with respect to published studies in the definition of a late bidder. Whereas the literature considers all bids placed within the late-bidding time window as late bids, we only retain here those bids made by participants who did not before reaching this time window. We believe that our approach is more in line with the provided rationale for late bidding. Using e-Bay data for two product categories, antiques and iPods, we propose count-data models to look at both the presence of the late-bidding phenomenon and its intensity. Our results reveal a notable difference when we compare extremely late bidders or snipers to moderately late bidders. Snipers are the most experienced and object-value enlightened members. They are less-sensitive toward entry-deterrence factors and are motivated by their desire to win the auction.

The modeling framework of the second essay combines two research streams in marketing: diffusion models and customer relationship management. Since the 1980s, the concept of relationship marketing has gained an increased interest in the field of general marketing, and particularly that of direct marketing. The core of relationship marketing is the development and maintenance of long-term relationship with customers. This task requires an efficient allocation of marketing resources that takes into account the benefits and the costs of marketing, sales and customer interactions. In a recent survey of Forbes (2011), more than half of marketing executives surveyed consider customer retention as their top current priority (52%), followed by customer acquisition (38%), and customer profitability (29%). For this, about four in ten executives (39%) allocate the largest part of their marketing budget to customer retention; customer acquisition occupies the second position (36%). Here, the fundamental marketing question is how effectively manage marketing expenditures related

to customer acquisition and customer retention. Prior research has examined parts of this issue, but to date, there has not been a comprehensive examination of optimal CRM strategies, and particularly in services sector. It is well known that services sector has experienced strong growth during the last century. In 2013, the International Telecommunications Union estimates the number of mobile-cellular subscriptions worldwide to 6.8 billion, corresponding to a global penetration of 96%. Beyond their traditional use (home phones, cellular phones), subscription services have revolutionized the film, TV, and digital media sector by introducing many new services such as YouView, iTunes, Apple TV, Netflix and other Internet streaming media services. Despite the growing role of subscription services in the modern economy, published research related to the diffusion of these services remains modest compared to the marketing literature on new product diffusion. Moreover, the need for a deeper understanding of the role CRM tools in the diffusion of subscription services represents a major concern for academics and practitioners.
In this essay, we seek to fill this important gap by developing a new framework for modeling subscription services diffusion. The evolution over time of number of subscribers is governed by a differential equation combining two processes, namely, a customer acquisition process and a customer attrition process. Each of these processes is influenced by internal incentives provided by the firm and external incentives related to all other factors not related to marketing expenditures. We seek to determine the optimal investment level a provider should make to attract new customers and retain existing ones throughout the lifecycle of the service. A model relying on dynamic programming solution techniques has been developed to determine optimal strategies of customer acquisition and retention. Results show that the optimal customer equity represents the sum of the value of existing customers and the value of the remaining market. Moreover, we find that optimal acquisition and retention policies are constant throughout the service growth and does not depend on the penetration rate. A sensitivity analysis is performed to assess the impact of model parameters on the results. Finally, we illustrate our findings in two cases concerning companies in the telecommunications sector.

The last essay focuses on marketing channel interactions in the presence of a private label. This product category reports annually about 370 US billion dollars on the international grocery market and witnessed a sustained growth over 11% for the period 2002-2009, providing the retailers' brand a very comfortable position on their chain shelves. Compared to 2004, global private label sales grew by 5% in 2005, outpacing manufacturer brands (growing by only 2%) in every region except for Latin America. During the economic downturn of 2008-2009, 61% consumers surveyed by Nielsen Company declared purchasing more private label brands, fully 91% said they will continue to do so when the economy improves. Therefore, retailers should have more interest toward private label for the next years. This keen interest is motivated by various reasons. When introducing their own labels, retailers aim to benefit from higher retail margins on private labels than on national brands, to improve their bargaining position, to increase store traffic, to build store loyalty, and thus to enhances their chain profitability. In our work, we look into the impact of adopting an umbrella branding strategy on the retailer's performance. This strategy consists in using the same name to market different products which may, or may not, be related. The latter is a way of reinforcing their position through an already established private label in order to benefit from its position. The analysis takes into account the strategic interactions between the manufacturers and the retailer, as well as the positive spillover between sales of the private label in different categories. Surprisingly, our results show that umbrella branding strategy may lead to lower profits for the retailer. Actually, the results depend ultimately on the power of the core PL compared to the NB, the cross-price competition between the PLs and the NBs and the level of spillover. From manufacturers' perspective, we find that the retailer succeeds in lowering the wholesale price of the NBs and consequently it is never interesting for NBs' manufacturers to see their retailer implementing an umbrella strategy.

Chapter 1

An Empirical Investigation of Late Bidding in Online Auctions

1.1 Abstract

Why some participants in online auctions place their bids right before the time of closing? Using e-Bay data, we propose count-data models to look at both the presence of the late-bidding phenomenon and its intensity. Our results reveal significant differences between extremely late-bidders (snipers) and moderately late-bidders.

Key Words: Late Bidding, Internet Auctions, eBay, Count Models.

1.2 Introduction

The literature has offered three explanations for late-bidding in online auctions, namely: (i) to delay the release of private information in common value auctions; (ii) to avoid price wars with like-minded bidders and with naïve bidders; and (iii) to attempt to generate collusive gains (Roth and Ockenfels, 2002a,b, 2006; Bajari and Hortascu, 2003; Ariely et al., 2005; Nekipelov, 2007; Wintr, 2008; Ely and Hossain, 2009).

We use count-data models to look at both the presence and intensity of the late-bidding phenomenon. We include variables that have not yet been considered (initial price, auction duration, and seniority of bidders) and conduct a sensitivity analysis to highlight the difference between moderately and extremely late-bidders. We obtain that the significance, sign and magnitude of the determinants of late-bidding depend on time remaining before the auction deadline. We consider two product categories, namely, antiques and iPods. We chose iPods because iPod auctions usually end up with a lower price than the threshold at which eBay must hide the bidders' identity, including the creation date of their account. We believe that this data provide interesting information on bidder seniority.

Our database includes 3,527 closed auctions that took place on eBay between January 3 and February 2, 2008. They involved 13,085 distinct bidders who submitted 43,798 bids.

1.3 Model

Late-bidding is measured by the number of bidders during a remaining time T before the closing of the auction. We let T have different

The dependent variable is the number of late-bidders in an auction. This is a count variable, i.e., it only takes nonnegative integer values. It is well known that linear and multinomial models are ill-suited to deal with such variables. Count-data models are a natural choice here because they capture more thoroughly both the presence and intensity of the phenomenon. Within this family, we retained the Poisson and the negative-binomial models.

Poisson model	Negative – binomial model
$P(NLB(T)_i = k_i/X_i) = \frac{\lambda_i^{k_i} e^{-\lambda_i}}{k_i!}$	$P(NLB(T)_i = k_i/X_i, \varepsilon_i) = \frac{\lambda_i^{k_i} e^{-\lambda_i}}{k_i!}$
where	where
$\ln(\lambda_i) = X_i\beta$	$\ln(\lambda_i) = X_i\beta + \varepsilon_i$, and
	$\exp(\varepsilon_i) \sim \text{Gamma}(\alpha^2, 1/\alpha^2)$

where

$NLB(T)_i$: number of late-bidders during T

λ_i: conditional mean of this number given the vector of exogenous variables X_i

ε_i: specification error

α: parameter of gamma distribution

β: vector of coefficients to estimate.

1.3.1 Hypothesis and Variables

Implicit collusion: Roth and Ockenfels (2002a) argue that late-bidding strategy is an implicit collusion among bidders to capture the seller's surplus. As a counter-strategy, many sellers use *a secret-reserve price* to protect themselves from selling at an unsatisfactory price. If the late-bidders' goal is to win the item at the lowest possible price, then their number should decrease for reserve-met auctions.

H_1: Late-bidders entry is less prevalent in reserve-met auctions.

Entry deterrence: The bidder's decision to participate in an auction may depend on several factors. Bajari and Hortacsu (2003) showed that a higher starting price affects the attractiveness of entering the auction.

H_2: There is a negative relationship between the starting price and the number of late-bidders.

The level of competition, reflected in the pre-T bidding activity, may also deter a bidder from joining the auction. To account for the previous bidding activity, we retain two variables, namely, the number of early [1] bidders and the number of early multiple bidders. Indeed, a high number of early bidders signals an aggressive competition, and this may discourage potential bidders from participating. Now, many bidders submit multiple bids in the early period of an auction. While Roth and Ockenfels (2006) consider multiple bidding as a naïve behavior, Nekipelov (2007) argue that some bidders start submitting early multiple bids in order to deter the entry of other rivals.

H_3: The number of late-bidders decreases with the number of early bidders.

1. Early-bidding period corresponds to the first 80% of auction's duration (Nekipelov, 2007).

H_4: The number of late-bidders decreases with the number of early multiple bidders.

Protection of private information: Roth and Ockenfels (2002a, 2006) found that late bids are more numerous in the antiques category, where personal information plays a more crucial role in the item's assessment, than in the computers category. Late-bidding prevents learning by less-informed rivals.

H_5: *Late-bidders* are more numerous in antiques than in iPods auctions.

Bidder's experience: Bidding late is the best strategy to avoid bidding wars with naive participants (Roth and Ockenfels, 2002b; Ely and Hossain, 2009), in the sense that they will not have enough time to resubmit a bid in the auction.

H_6: Late-bidders are the most experienced members of eBay.

Bidder expertise has been measured by eBay's ***feedback score***. We adopt this variable as an indicator of the bidder's activity level, and complement the experience description with the ***seniority of bidder***. This variable, considered for the first time, is measured by the bidder's eBay age (difference between auction and subscription dates). This variable is no longer provided by eBay, and therefore, we have a unique opportunity to assess its importance.

1.4 Results

Table 1.1 provides some descriptive statistics. The specification (likelihood-ratio [2]) tests showed that the Poisson model is the most parsimonious and fits best the data. Table 1.2 reports the maximum-likelihood Poisson-coefficient estimates. The regression of the total number of bidders shows that iPods auctions attract more participants than antiques, and this number is negatively affected by the starting price and the reserve-price-met variable. Our results reveal that the sign, the value and/or the significance of the coefficients vary considerably with T. In particular, we observe a notable difference between extremely late-bidders (1m, 30s and 15s) and moderately late-bidders (15m, 10m and 5m). This holds true for the bidders' average seniority, the product category, entry deterrence variables, and the reserve price.

2. The two models are nested for $\alpha = 0$.

Table 1.1 Descriptive statistics

	T	Antiques (1,696 auctions) Mean	Std	Min	Max	iPods (1,831 auctions) Mean	Std	Min	Max
Number of late-bidders	15s	0.42	0.62	0	4	0.23	0.49	0	4
	30s	0.52	0.67	0	4	0.38	0.62	0	5
	1m	0.60	0.72	0	4	0.58	0.78	0	5
	5m	0.72	0.80	0	4	1.15	1.14	0	6
	10m	0.78	0.84	0	4	1.50	1.30	0	8
	15m	0.83	0.86	0	5	1.76	1.45	0	8
Reserve price met		6%				3%			
Total number of bidders		4.18	2.12	1	16	8.23	3.37	1	21
Starting price		18.75	21.97	0.01	166.00	16.77	31.23	0.01	165.00
Number of early bidders		1.82	1.56	0	10	2.35	2.04	0	11
Number of early multiple bidders		0.37	0.70	0	4	0.85	1.16	0	6
Average-bidder seniority (in years)		5.10	1.65	0.36	9.91	3.31	1.16	0	8.74
Average-bidder-feedback score (divided by 100)		5.03	5.11	0.03	64.72	0.88	0.94	0.01	14.25

Table 1.2 Poisson estimates for different T

Variables	Time window T 15s	30s	1m	5m	10m	15m	Total number of bidders
Constant	-1.210***	-0.923***	-0.594***	-0.222**	-0.001	0.094	1.830***
	(0.000)	(0.000)	(0.000)	(0.014)	(0.994)	(0.229)	(0.000)
Reserve price met	-0.125	-0.201	-0.277**	-0.283***	-0.329***	-0.308***	-0.073**
	(0.39)	(0.123)	(0.022)	(0.004)	(0.000)	(0.000)	(0.037)
Log (Starting price)	-0.007	-0.009	-0.029	-0.041***	-0.057***	-0.069***	-0.158
	(0.801)	(0.689)	(0.12)	(0.005)	(0.000)	(0.000)	(0.000)
Log (Number of early) bidders + 1)	-0.063	-0.046	-0.063	-0.091**	-0.105***	-0.112***	
	(0.361)	(0.433)	(0.217)	(0.026)	(0.004)	(0.001)	
Log (Number of early) multiple bidders + 1)	-0.058	-0.105	-0.109*	-0.097*	-0.095**	-0.116***	
	(0.515)	(0.159)	(0.087)	(0.052)	(0.035)	(0.006)	
iPods	-0.369***	-0.121*	0.118**	0.532***	0.672***	0.776***	0.523***
	(0.000)	(0.064)	(0.039)	(0.000)	(0.000)	(0.000)	(0.000)
Average-bidder seniority (in years)	0.052***	0.048***	0.025	0.004	-0.011	-0.012	-0.006
	(0.01)	(0.008)	(0.12)	(0.758)	(0.376)	(0.308)	(0.283)
Average-bidder-feedback score (divided by 100)	0.029***	0.023***	0.021***	0.018***	0.016***	0.016***	0.004
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.112)
Log-likelihood	-2336.6	-2806.2	-3156.4	-3410.6	-3252.1	-3020.0	20221.2
R-squared (Deviance residuals)	15.90%	21.80%	27.40%	33.10%	38.00%	42.30%	41.7%

*/**/*** indicate significance at the 10/5/1/ percent level, respectively.

Implicit collusion: Exceeding the reserve price significantly affects the number of moderately late-bidders, but not the number of extremely late-bidders. This means that, while moderately late-bidders do indeed attempt to win the item at the lowest price, extremely late-bidders are not that price-sensitive and are motivated by their desire to win the auction. This is conveyed by the fact that the reserve-price elasticity [3] of the number of late-bidders is decreasing with the remaining time. Therefore, accepting or rejecting the implicit collusion hypothesis depends on the choice of T.

Entry deterrence: Expectedly, we obtain a negative relationship between the opening-price and the number of late-bidders. However, the opening-price elasticity value for the last-15-minutes (-0.07) is ten times that of its last-15-seconds counterpart (-0.007). Furthermore, the effect of this variable is not significant for extremely late-bidders. A higher opening price discourages moderately late-bidders from entering in the auction but not extremely late-bidders. We found the same results for variables describing the previous bidding activity. A higher number of early bidders decreases significantly only the number of moderately late-bidders. In the final seconds of the auction, the prevalence of late-bidders seems not affected by the early bidding activity and even by the presence of multiple bidders.

Bidder's experience: The feedback-score coefficient is positive and significant for all T. Further, the lower the remaining time, the higher the impact of the feedback scores. This suggests that late-bidders are the most active members on eBay. For bidder seniority, we obtain a significantly positive correlation with the number of extremely late-bidders and we find that the impact of seniority is strictly increasing when the auction is coming to an end. These results state that extremely late-bidders are the oldest eBay members. As postulated, more-experienced bidders bid later than less-experienced ones.

Protection of private information: Contrary to expectations, iPods attract more late-bidders for the four highest values of T than do antiques (Wintr, 2008). In the final 15 minutes, the average number of late-bidders for iPods is double ($e^{0.776}$) that for antiques. As iPod prices are highly visible on the Internet, one assumes that it does not require much expertise to evaluate them, and that all participants have similar valuations. This gives all bidders incentive to bid late, to avoid bidding wars with like-minded bidders. However, when we consider the results for T less-or-equal-to-30-seconds, the story changes, and late-bidders

3. The elasticity of $E(NLB(T))$ with respect to x_j is equal to β_j when x_j is discrete or x_j is continuous and is in logarithmic form (Winkelmann, 2008).

are more numerous in the antiques auctions. In the last 15 seconds, the average number of late-bidders is 1.5 times higher for antiques than for iPods. Therefore, the hypothesis on the protection of private information is only confirmed for extremely late-bidders. This would make sense because, unlike extreme late bidders, moderately late ones act when there is still time for their opponents to use their private information to formulate a counter-bid.

To conclude, our results clearly show that the effects of some variables are different for moderately late-bidders compared to extremely late-bidders. Extremely late-bidders are : *(i)* the oldest and most experienced members, *(ii)* more present in antiques auctions in which personal information plays a crucial role; and *(iii)* less-sensitive toward entry-deterrence factors.

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Chapter 2

Optimal CRM Expenditures for the Diffusion of Subscription Services

2.1 Abstract

In this paper, we propose a new model to deal with the diffusion of subscription services. The evolution over time of number of subscribers is governed by a differential equation combining two processes, namely, a customer acquisition process and a customer attrition process. Assuming profit-maximization behavior of the firm, we use dynamic programming to maximize the customer equity and determine optimal customer relationship marketing expenditures, taking into account the presence of some external incentives to join and leave the service. A sensitivity analysis is performed to assess the impact of model's parameters on the results. Finally, we illustrate our findings in two cases concerning companies in the telecommunications sector.

Key Words: Diffusion Models, Subscription Service, Customer Retention, Customer Acquisition, Optimal Spending, Dynamic programming

2.2 Introduction

With the sustained improvement in the Information and Communication Technologies, subscription-based services have witnessed a rapid growth in recent years. In 2013, the International Telecommunications Union estimates the number of mobile-cellular subscriptions worldwide to 6.8 billion, corresponding to a global penetration of 96%. Beyond their traditional use (home phones, cellular phones), subscription services have revolutionized the film, TV, and digital media sector by introducing many new services such as YouView, iTunes, Apple TV, Netflix and other Internet streaming media services. Despite the growing role of subscription services in the modern economy, published research related to the diffusion of these services remains modest compared to the marketing literature on new product diffusion. The few studies that focused on this issue have omitted a fundamental aspect: the relationship between customers and service providers. Indeed, the service growth is not restricted to the number of adopters at each period but depends also on the number of subscribers who stayed with the service. The development and maintenance of long-term relationship with customers represent the core of customer relationship management (CRM). This task requires an efficient management of marketing expenditures that takes into account the benefits and the costs of marketing, sales and customer interactions. In a recent review, Peres, Muller, and Mahajan (2010) highlighted the need to incorporate CRM concepts into the diffusion framework in the service sector. They consider that modeling should be directed more toward tying diffusion and CRM concepts to describe the influence of relationship measures on the growth and profitability of customers and firms.

This paper attempts to fill the gap by developping a new framework for modeling subscription services diffusion. We propose a model where the service growth is described by two processes: customer acquisition process and customer retention process. Each of these processes is influenced by internal incentives provided by the firm and external incentives related to all other factors not related to marketing expenditures. Using a dynamic programming approach, we determine the optimal acquisition and retention spendings to maximize the customer equity. The results provide a better understanding of the relationship between customer lifetime value and prospect lifetime value to identify optimal CRM strategies. We conduct a sensitivity analysis to assess how marketing effectiveness, external incentives, margin and discount rate influence the optimal acquisition and retention strategies. We also report the empirical results obtained for two companies operating in the TV sector. Our results reveal a significant impact of internal and external incentives on the acquisition and retention processes. A recurrent question of interest to CRM managers is the allocation of spendings between acquisition and retention. Our results show that each firm has its own reality, that is, there is no clear cut answer to whether acquisition is more critical than retention or the other way around.

The paper is organized as follows: In Section 2, we give an overview of the literature on diffusion of services and optimal CRM spending. In Section 3, we develop a diffusion model that links CRM expenditures to customer acquisition and retention. In Section 4, we determine the optimal acquisition and retention spendings, and in Section 5 we proceed with an empirical illustration. Section 6 briefly concludes.

2.3 Literature Background

Our approach draws on two literatures, namely, the diffusion of new products (or services) in marketing and on customer-relationship marketing. In the next two subsections, we briefly report on most relevant papers to ours.

2.3.1 Diffusion of Services

The new product diffusion literature is undoubtly one of the most active one in marketing science. Since the seminal paper by Bass (1969), hundreds of papers have been published and addressed a large variety of issues and contexts pertaining the diffusion of durable products (see the surveys in Mahajan et al., 1990 and Peres et al. 2010). Surprisingly, little effort has been dedicated to model and forecast the growth of service markets despite its considerable evolution during the past few decades.

The sparse literature in services that used the Bass-type (or S-shaped) model includes empirical papers that aimed at forecasting the sevice growth in some sectors such as telecommunications (see, e.g., Botelho and Pinto,2004; Jongsu and Minkyu, 2009; Michalakelis et al, 2008). Other papers added some features to basic Bass model. For instance, Krishnan et al (2000) studied brand-level diffusion model in the cellular telephone industry, and analysed the impact of new entrant on the diffusion dynamics of the category and on existing brands. Jain et al. (1991) and Islam and Fiebig (2001) extended the modeling framework to incorporate supply restrictions, and evaluate their impact on the growth of a new service in telecommunications markets.

In the above references, the models did not account for the role marketing variables in the diffusion process. Mesak and Darrat (2002) examined the impact of interdependence between the adoption processes of consumers and retailers on the optimal pricing policy of new subscriber services. Fruchter and Rao (2001) studied the pricing decision in the presence of two components: service access pricing and usage pricing. They show that the adoption rate could be boosted by keeping the membership fee low. These studies have a common shortcoming, namely, they modeled the diffusion of services as if they were durable goods. However, services diffusion differ from durable goods diffusion by the presence of two processes, that is, the adoption process, and the retention process. The influence of customer retention on the service growth has generally been neglected. Indeed, ignoring customer attrition will likely distort forecasts and leads to underestimate the price sensitivity (Danaher, 2002). Libai et al. (2009) were probably the first to incorporate customer attrition into the Bass diffusion model. They showed that customer attrition affects considerably the market growth of a new service as well as the stock market value of firms. However, their modeling framework did not incorporate marketing variables neither in the acquisition process nor in the retention process. Therefore, the adoption and attrition rates were assumed to be constant over time. All these papers have used the aggregate approach to model the services diffusion. In many marketing applications, especially for subscription services, it is important to anticipate adoption timing and defection timing at the individual level. Therefore, alternative models have been proposed that disaggregate the diffusion process to customer's level. These models allow to introduce heterogeneity of customers into the diffusion process by incorporating explanatory variables. Numerous studies have attempted to examine services adoption drivers (Wareham et al. ,2004; Prins and Verhoef , 2007; Nam et al, 2010; Landsman and Givon, 2010; Katona et al, 2011; Nitzan and Libai, 2011) and services defection drivers (Li, 1995; Bolton, 1998; Bolton and Lemon, 1999; Reinartz and Kumar, 2003; Verhoef, 2003). The two main influences leading to adoption and retention decisions include those under firm's control (price, advertising, direct mailing, service's quality, loyalty program, etc.) and influences related to the individual (demographic profile,economic profile, satisfaction, usage patterns, personnal social network/word-of-mouth, etc.).

2.3.2 Optimal CRM Spending

It is well established that the duration of the relationship with their customers has an important financial implications for firms (Bolton, 1998; Reinartz and Kumar, 2003, Nagar and Rajan, 2005; Rust and Chung, 2006). Consequently, the optimal management of customer relationship marketing expenditures has been the subject of considerable interest by both practitioners and researchers in recent years. Blattberg and Deighton (1996), along with others, used a decision calculus approach to determine separately the optimal acquisition spending and the optimal retention spending maximizing the individual customer equity. Several extensions have been proposed for their model. For instance, Berger and Bechwati (2001) dealt with budget allocation between acquisition and retention, under several different market situations. Pfeifer (2005) examined the context in which the cost to acquire a new customer is five times the cost of retaining an existing one. He demonstrated that the optimal allocation depends on the concept of costs (average costs vs marginal costs). Calciu (2008) focused on the comparison of the Blattberg and Deighton's model and its extensions. He also proposed an alternative to the 'lost-for-good' assumption, stating that a lost customer leaves the service for a limited time and then may return.

Reinartz et al. (2005) propose a conceptual framework that established the link between customer acquisition, relationship duration and customer profitability. They studied the case in which the firm can invest on customer acquisition and retention through several communication channels (telephone, face-to-face, and E-mail). Using the parameter estimates, the authors simulate the allocation of resources between customer acquisition and customer retention, under several scenarios. Reinartz et al. (2005) find that a suboptimal allocation to retention spending generates greater impact on long-term customer profitability than suboptimal acquisition spending. Furthermore, they show that the optimal choice of communication channels depends on the maximizing-criteria.

In a competitive context, Musalem and Joshi (2009) consider two firms competing for customer relationship in two periods by investing in acquisition and retention efforts. Their model is based on utility functions that depend on the intrinsic customer preference and the effectiveness of CRM efforts. They suggest that retention efforts should be focused on the moderately responsive customers, while acquisition efforts should be most aggressively targeted towards moderately profitable competitor's customers.

Surprisingly, all these papers did not address the question of optimal CRM spending in a dynamic diffusion context. This omission is particularly striking for subscription services.

2.4 The Model

Denote by m the market potential for a service, and by $N(t)$ the number of subscribers at time $t \in [0, \infty)$. Therefore, the remaining market potential at time t is given by $m - N(t)$. Let $a(t)$ be the conditional probability that a new customer subscribes to this service at time t, given that he is not an actual user. This conditional probability represents the customer acquisition rate at time t. Denote by $r(t)$ the retention rate of customers, which measures the conditional probability that a current client will not unsubscribe at time t. The evolution of subscribers' number is then governed by the following differential equation:

$$\dot{N}(t) = \frac{dN(t)}{dt} = a(t)[m - N(t)] - [1 - r(t)]N(t), \quad N(0) = N_0. \tag{2.1}$$

Equation (2.1) presents the balance between new and lost customers at each time t. It is useful, from the outset, to make the following three comments:

1. Our market dynamics assume that a customer who stops his subscription is not necessary lost forever, but joins the untapped market $m - N(t)$. Put differently, we consider that the alternative "lost-for-good" customer assumption made in, e.g., Berger and Nasr (1998), may not be empirically supported in some sectors, where the switching cost to another sevice provider is relatively low. In particular, in the telecommunications sector, a customer could leave a company for a while and comes back later on because of a special offer or an improvement in service quality (Libai et al., 2009).
2. Our dynamics do not distinguish between new (never-have-been) customers and won-back ones in terms of their reaction to marketing effort. This simplifying assumption is common in the relationship-marketing literature (see, e.g., Libai et al., 2009 and Calciu, 2008).

3. Contrary to the case of durable products, which have been heavily studied in the literature, the number of "adopters "$N(t)$ is not necessarily monotonically increasing overtime.

Denote by $A(t)$ the expenditure to acquire a new subscriber and by $R(t)$ the expenditure to retain an existing one. The acquisition spending includes the cost of different marketing actions to attract new subscribers, e.g., incentives (premiums, special saving), advertising. The retention spending refers to marketing expenditures in terms of, e.g., loyalty programs, direct-mail campaigns. Following Berger and Nasr-Bechwati (2001), Pfeifer (2005) and Calciu (2008), we assume that acquisition and retention rates are related to marketing expenditures through the following exponential functions:

$$a(t) = \gamma_a \left(1 - e^{-f_1 A(t) - f_0}\right), \quad (2.2)$$

$$r(t) = \gamma_r \left(1 - e^{-h_1 R(t) - h_0}\right), \quad (2.3)$$

where γ_a and γ_r are ceiling parameters belonging to $(0, 1]$.

The acquisition ceiling rate γ_a is the maximum proportion of targeted prospects who would be acquired if there were no limit to spending. The positive parameters f_1 and h_1 measure the effectiveness of marketing efforts. They could also be interpreted as the customer's sensitivity toward CRM spending. Floor rates are determined by the positive constants f_0 and h_0, and are equal to $\gamma_a \left(1 - e^{-f_0}\right)$ and $\gamma_r \left(1 - e^{-h_0}\right)$ for acquisition and retention, respectively. Here, the novelty of our model lies in the introduction of two different dimensions influencing the customer's decision, namely, the internal incentives (f_1, h_1) and the external incentives (f_0, h_0). The external part represents incentives that are not provided by the firm. In this sense, switching costs have a significant role in customer retention (Jones et al., 2000). They make it more costly for consumers to churn. Switching costs include learning cost, transaction cost, and all efforts resulting from changing service provider. Other social barriers may also reduce the customer defection (Woisetschläger et al., 2011). On the acquisition side, some customers join the firm as a result of their own initiative and based on their intrinsic motivation, that is, not as a result of marketing efforts. These self-determined customers perceive their act of adoption service as self-instigated (Dholakia, 2006). In contrast, firm-determined customers adopt a new product/service in response to firm's incentives.

The specifications in (2.2) and (2.3) have two important properties. First, for all non-negative $A(t)$ and $R(t)$, the resulting values of $a(t)$ and $r(t)$ are in the interval $[0,1]$. This is consistent with our definition of $a(t)$ and $r(t)$ as conditional probabilities.

In the new-product diffusion literature, this property has been often neglected. Second, they exhibit strictly diminishing returns to acquisition (retention) spending. Indeed, we clearly have

$$
\begin{aligned}
\frac{da(t)}{dA(t)} &= \gamma_a f_1 e^{-f_1 A(t)-f_0} > 0, \quad \frac{d^2 a(t)}{d\left(A(t)\right)^2} = -\gamma_a f_1^2 e^{-f_1 A(t)-f_0} < 0, \\
\frac{dr(t)}{dR(t)} &= \gamma_r h_1 e^{-h(t)R(t)-h_0} > 0, \quad \frac{d^2 r(t)}{d\left(R(t)\right)^2} = -\gamma_r h_1^2 e^{-h_1 R(t)-h_0} < 0.
\end{aligned}
$$

Following the literature, see, e.g., Gupta et al.,2004; Libai et al., 2009, we assume that the service provider selects the acquisition and retention strategies that maximize the customer equity (CE). According to Wiesel et al (2008), the calculation of this value combines three customer metrics, namely, the net present value of customer revenue, the net present value of customer acquisition expenditures, and the net present value of customer retention expenditures. Formally, the optimization problem is stated as follows:

$$
\begin{aligned}
J &= \max_{R,A} CE = \max_{R,A} \int_0^{\infty} \left[N\left(t\right)g - N\left(t\right)R(t) - \left(m - N\left(t\right)\right)A(t)\right]e^{-\rho t}dt, && (2.4) \\
\text{subject to} &: && (2.5) \\
\dot{N}(t) &= a(t)[m - N(t)] - [1 - r(t)]N(t), \quad N\left(0\right) = N_0, && (2.6) \\
a(t) &= \gamma_a \left(1 - e^{-f_1 A(t) - f_0}\right), && (2.7) \\
r(t) &= \gamma_r \left(1 - e^{-h_1 R(t) - h_0}\right) && (2.8)
\end{aligned}
$$

where constant g is the net revenue per subscriber and ρ is the discount rate. $A\left(t\right)$ and $R\left(t\right)$ are the control variables and $N\left(t\right)$ is the state variable. We shall from now on omit the time argument when no confusion may arise. The last two equations in the above problem can be written equivalently as follows:

$$A(a) = -\frac{1}{f_1}\left[\ln(1-\frac{a}{\gamma_a})+f_0\right], \tag{2.9}$$

$$R(r) = -\frac{1}{h_1}\left[\ln(1-\frac{r}{\gamma_r})+h_0\right], \tag{2.10}$$

Substituting for $A(a)$ and $R(r)$ in the objective function (2.4), the optimal-control problem of the service provider becomes

$$\begin{aligned} J &= \max_{a,r\in[0,1]} CE = \max_{a,r\in[0,1]} \int_0^\infty \Bigg(Ng + \frac{N}{h_1}\left[\ln(1-\frac{r}{\gamma_r})+h_0\right] \\ &\quad + \frac{(m-N)}{f_1}\left[\ln(1-\frac{a}{\gamma_a})+f_0\right]\Bigg)e^{-\rho t}dt, \end{aligned} \tag{2.11}$$

$$\dot{N} = a\left[m-N\right]-\left[1-r\right]N, \quad N(0)=N_0, \tag{2.12}$$

where the acquisition rate $a(t)$ and the retention rate $r(t)$ are the new control variables and $N(t)$ is the state variable. Optimal CRM expenditures $A(t)$ and $R(t)$ are easily calculated based on equations (2.9) and (2.10).

2.5 Optimal Customer Acquisition and Retention Policies

To determine the optimal acquisition and retention policies, we need to solve the standard dynamic programming problem in (2.11)-(2.28). Denote by $V(N)$ the value function, that is, the maximal customer equity value that can be achieved when the number of subscribers is equal to N, assuming that the service provider implements the optimal acquisition and retention policies. The following proposition characterizes these policies.

Proposition 1 *The optimal value of CE at N subscribers is equal to*

$$V(N) = \eta_1 \times N + \eta_0,$$

where η_0 and η_1 are constant, with η_1 being the solution of the equation

$$H(\eta) = B\eta + C\ln(\eta) + D = 0,$$

where

$$\begin{aligned} B &= \rho + 1 + \gamma_a - \gamma_r, \quad C = \frac{1}{h_1} - \frac{1}{f_1}, \\ D &= \frac{1}{f_1}\left[f_0 - \ln\left(\gamma_a\right) - \ln\left(f_1\right) - 1\right] - \frac{1}{h_1}\left[h_0 - \ln\left(\gamma_r\right) - \ln\left(h_1\right) - 1\right] - g. \end{aligned}$$

Proof. See Appendix. ■

Proposition 1 indicates that the optimal customer equity value (value function or payoff-to-go in the language of dynamic optimization) is a linear function of the total number of subscribers. In particular, if the service provider chooses optimally the expenditure rates in acquisition and retention at initial date throughout the planning horizon, then its total payoff would be given by

$$J^* = V\left(N_0\right) = \eta_1 \times N_0 + \eta_0.$$

From the proof of Proposition 1, it can be easily seen that the value function can be rewritten as

$$V\left(N\right) = \eta_1 \times N + \eta_2 m,$$

or equivalently as

$$V\left(N\right) = \left(\eta_1 + \eta_2\right) \times N + \eta_2 \times \left(m - N\right), \tag{2.13}$$

where

$$\eta_2 = \frac{1}{\rho\left(f_1 - h_1\right)}\left[\eta_1\left(\gamma_a f_1 + \left(\rho + 1 - \gamma_r\right) h_1\right) + f_0 - h_0 + \ln\left(\frac{h_1 \gamma_r}{f_1 \gamma_a}\right) - h_1 g\right].$$

Equation (2.13) has an interesting marketing interpretation: it states that the optimal customer equity is composed of two parts, namely, the value of actual customers given by $\left(\eta_1 + \eta_2\right) N$, and the value of untapped market potential given by $\eta_2\left(m - N\right)$. In this sense, the sum of coefficients $\left(\eta_1 + \eta_2\right)$ can be interpreted as the customer life-time value (CLV), that is, the present value of all future profits generated by an existing customer (Kamakura et al., 2005). Similarly, we interpret η_2 as the prospect lifetime value (PLV), that is, the value of a potential customer. This last measure provides a valuable information to current and potential investors, creditors, managers and other possible stakeholders, in terms of assessement of the financial performance of future prospects or potential customers. The few papers (Pfeifer and Farris, 2004; Calciu, 2008) that focused on PLV concept argue that the firm should invest to convince prospects with high PLV to become customers. Other studies (Gupta and Lehmann, 2003 ; Gupta and Zeithaml, 2006) claim that customer acquisition decisions should be based on CLV. Through our model, we show that the CRM policy of the service provider is based, among others, on the difference between CLV and PLV. This difference is given by the coefficient η_1 and represents the marginal customer equity $V'(N)$. We will get back to this interesting finding. To obtain the optimal acquisition and retention policies, it suffices to substitute for $V'(N)$ in (2.29) and (2.30) (see Appendix 2.9.1) to get

$$a^* = \gamma_a - \frac{1}{f_1\eta_1}, \tag{2.14}$$

$$r^* = \gamma_r - \frac{1}{h_1\eta_1}. \tag{2.15}$$

Unfortunately, the coefficient η_1, which is the solution of the following equation

$$H(\eta) = B\eta + C\ln(\eta) + D = 0, \tag{2.16}$$

cannot be determined analytically. In that follows, we discuss the existence of η_1.

Deriving (2.16), we obtain

$$H'(\eta) = B + \frac{C}{\eta}, \quad H''(\eta) = -\frac{C}{\eta^2}.$$

Therefore, $H(\eta)$ can be characterized as follows:

1. if $C > 0$, that is, the acquisition effectiveness is greater than the retention effectiveness $(f_1 > h_1)$, then $H(\eta)$ is strictly concave and increasing from $-\infty$. There exists one η_1 satisfying (2.16).

2. if $C = 0$, that is, $f_1 = h_1$, then we would have $\eta_1 = -\frac{D}{B}$. As B is strictly positive for all parameter values, we must have $D < 0$ for η_1 to be positive, which is intuitively and conceptually appealing; otherwise the acquisition and retention rates (2.14 and 2.18) become greater than the ceiling rates.

3. if $C < 0$, that is, $f_1 < h_1$, then $H(\eta)$ is strictly convex and reachs its minumum at $-\dfrac{B}{C} > 0$. To obtain at least one solution for (2.16), we have then to assume that $H\left(-\dfrac{B}{C}\right) \leq 0$.

Inserting (2.14)-(2.15) in (2.17)- (2.18) yields the following optimal CRM expenditures:

$$A^* = \frac{1}{f_1}\left(\ln(\gamma_a f_1 \eta_1) - f_0\right), \tag{2.17}$$

$$R^* = \frac{1}{h_1}\left(\ln(\gamma_r h_1 \eta_1) - h_0\right). \tag{2.18}$$

The above equations show that the optimal acquisition and retention rates and expenditures are independent of the number of subscribers, which is consistent with our model assumptions, namely, the revenue per subscriber, as well as the individual acquisition and retention costs do not vary with the number of subscribers. We will get back to this point later on. Equations (2.17)-(2.18) also show that the optimal acquisition and retention expenditures depend through η_1 on all model's parameters. Furthermore, the optimal expenditures are increasing with η_1. Similarly, optimal rates, given by (2.14) and (2.15), become closer to ceiling rates for high values of η_1 and closer to floor rates for low values of η_1. This means, when the marginal customer equity is high, the provider should invest more on customers' acquisition and retention.

For the above expenditures to be non negative, η_1 must satisfy the following inequality:

$$\eta_1 \geq \eta_m = \min\left(\frac{e^{f_0}}{\gamma_a f_1}, \frac{e^{h_0}}{\gamma_r h_1}\right), \tag{2.19}$$

otherwise, the firm will not invest in CRM and the acquisition and retention rates would only depend on ceiling and external parameters, that is,

$$a = \gamma_a \left(1 - e^{-f_0}\right), \qquad r = \gamma_r \left(1 - e^{-h_0}\right).$$

Note that in the absence of external factors leading to subscription and retention ($f_0 = h_0 = 0$), the service provider will be then out of business. Interestingly, as we will see in the next section, we observe that empirically the parameters f_0 and h_0 are significantly different from zero. If $\eta_1 < \frac{e^{f_0}}{\gamma_a f_1}$, then the service provider should not invest in acquisition. Similarly, if $\eta_1 < \frac{e^{h_0}}{\gamma_r h_1}$, then it is optimal not to invest in retention.

A recurrent question in CRM is whether the firm should invest more in acquisition or retention. In our case, the answer depends on the value of η_1, that is, on all model's parameters. Indeed, from (2.16), we have

$$A^* - R^* = \left(\rho + 1 + \gamma_a - \gamma_r\right) \eta_1 - g + \frac{1}{h_1} - \frac{1}{f_1},$$

Let

$$\eta^* = \frac{g + \frac{1}{f_1} - \frac{1}{h_1}}{\left(\rho + 1 + \gamma_a - \gamma_r\right)}$$

When $\eta_m \leq \eta_1 \leq \eta^*$, the service provider should invest more in acquisition. When the marginal customer equity η_1 exceeds η^*, retention expenditures should be higher. We will provide deeper insights in the empirical part. Finally, the steady state of the dynamics is

given by

$$N_{ss} = \frac{a^* m}{1 + a^* - r^*}, \tag{2.20}$$

a level at which the number of new subscribers is equal to the number of lost subscribers. As $a(t)$ and $r(t)$ are constant over time, it is easy to verify that the number of subscribers at time t is given by

$$N(t) = \left(N_0 - N_{ss}\right) e^{-(1 + a^* - r^*)t} + N_{ss}, \tag{2.21}$$

where a^* and r^* are given by (2.14)-(2.15).

2.5.1 Sensitivity Analysis

Table 2.1 summarizes the effect of varying the model parameters on acquisition and retention policies (+ positive, − negative, ? depends on parameter values). The computations of derivatives are provided in the Appendix.

Table 2.1 Optimal strategies sensitivity toward models parameters

	a^*	A^*	r^*	R^*
g	+	+	+	+
ρ	−	−	−	−
γ_a	?	?	−	−
γ_r	+	+	+	+
h_0	+	+	+	?
f_0	−	−	−	−
h_1	+	+	+	?
f_1	?	?	−	−

Impact of f_1: A higher acquisition effectiveness leads to lower retention expenditures, and consequently to lower retention rate. The intuition behind this result is that when acquisition effectiveness increases, retention becomes less an issue and retention expenditures could be safely reduced. Put differently, for a firm that is highly effective in attracting customers, it would be less costly to acquire (possibly lost) customers than to invest in retaining them. We note that this result is a by-product of our "not-lost-for-good" assumption, meaning that lost customers join the pool of untapped market potential, and therefore remain candidates for a come back.

Impact of h_1: When retention effectiveness increases, acquisition expenditures (and consequently acquisition rate) and retention rate increase. A firm that is efficient in having a long-term relationship with its customers, this is indeed what retaining a customer is about, then it is worth to heavily invest to acquire these customers. Taking into account the above result, we see that increasing retention effectiveness and acquisition effectiveness do not yield the same impacts.

Impact of g **and** ρ : A higher margin per subscriber leads to an increase in customer relationship marketing expenditures and rates. This result is intuitive as an increase in the margin means a more profitable customer relationship, and therefore the service provider has an incentive to increase its investment in acquisition and retention. A higher discount rate leads to the opposite effect because future cash flows are less valued, and consequently the firm is better off reducing its current investment in acquisition and retention.

Impact of f_0 and h_0 : When the external incentives f_0 to subscribe to the service go up, then the optimal acquisition and retention expenditures decline, and consequently the corresponding rates. This result is intuitive: if the service provider benefits from freely positive marketing circumstances, e.g., higher switching cost, customer intrinsic motivation, then there is less need to invest in marketing effort to reach the same outcome. The impact of the external incentives to keep the service h_0 is simply the opposite of f_0.

2.6 Empirical Study

To illustrate the role of CRM in the diffusion process in a real business context, we apply our model to two well known service providers, namely, **Sky Deutschland AG** and **DIRECTV**. Sky Deutschland AG is a leading pay-TV company that operates in Germany and Austria, and offers a collection of basic and premium digital subscription television channels of different categories via satellite and cable television. DIRECTV is also a leading provider of digital television entertainment services.

This company operates in the United States, Brazil, Mexico and other countries in Latin America. To avoid problems of data comparability and the use of different monetary units, we only retain the U.S. market. We use quarterly customer and financial data that are available in annual reports, namely, the number of subscribers, the number of new subscribers, the number of lost subscribers, revenues and operating expenses, and total acquisition and retention spendings. The following transformations were performed to fit our needs in terms of estimation of the model's parameters (see Table 2.2 for some descriptive statistics):

Margin: The margin per subscriber is given by the difference between total revenues and operating costs (excluding acquisition and retention expenditures) during a quarter divided by the total number of subscribers. To smooth the computation, we follow Gupta et al. (2004) and Libai (2009) and take the average over the preceding four quarters.

Acquisition spending per prospect: The acquisition cost is the ratio of total acquisition expenditures to the number of newly acquired customers during a given period. As we need the acquisition spending per prospect, we divide the total acquisition expenditures by the size of the remaining (untapped) market at the beginning of a quarter.

Retention spending per customer: It is obtained by dividing total retention expenditures by the current number of subscribers at the beginning of each quarter.

Acquisition rate: It is the number of newly acquired customers during a given quarter divided by the size of the remaining market.

Retention rate: It is given by the ratio of the number of retained subscribers to the total number of subscribers at the beginning of each time period.

Finally, we proceeded as follows to estimate the market size and the discount rate:

Market size: As subscription to TV services is household based, we set the market size equal to the total number of households (Sudhir, 2001).

Discount rate: Researchers and practioners suggest a range of 8% to 16% for the annual discount rate (Gupta et al., 2004). Following Gupta et al. (2004), we estimated the annual discount rate using the weighted average of cost of capital. For both Sky and DIRECTV, this rate is equal to 9%,[1] which translates into a quarter discount rate of 2.2%.

1. http://www.wikiwealth.com/

Table 2.2 Descriptive Statistics

DIRECTV				
Data period: From March 2007 To September 2012				
Quarterly Statistics	**Mean**	**St. deviation**	**Min**	**Max**
Number of new subscribers ('000)	1,020.27	101.62	863	1280
Acquisition rate (a)	1.06%	0.11%	0.91%	1.34%
Acquisition spending per prospect (CA \$)	6.39	0.96	4.52	8.3
Number of lost subscribers ('000)	844.64	91.57	689	1,040
Retention rate (r)	95.39%	0.33%	94.78%	95.94%
Retention spending per subscriber (CR \$)	15.02	1.5	12.17	17.49
Revenue per subscriber (g \$)	101.15			
Total number of subscribers at beginning of quarter ('000)	16,188			
Total number of subscribers at end of quarter ('000)	19,981			
Number of Households (Millions)	115			

Sky Deutschland AG				
Data period: From June 2007 To September 2012				
Quarterly Statistics	**Mean**	**St. deviation**	**Min**	**Max**
Number of new subscribers ('000)	145.14	44.72	58	246
Acquisition rate (a)	0.32%	0.10%	0.13%	0.54%
Acquisition spending per prospect (CA e)	0.98	0.32	0.42	1.57
Number of lost subscribers ('000)	110.90	29.53	65	171
Retention rate (r)	95.64%	1.38%	93.16%	97.62%
Retention spending per subscriber (CR e)	6.17	0.82	4.48	7.54
Revenue per subscriber (g e)	30.67			
Total number of subscribers at beginning of quarter ('000)	2,495			
Total number of subscribers at end of quarter ('000)	3,212			
Number of Households (Millions)[2]	48			

2. Germany and Austria.

2.6.1 Estimation and Results

Our diffusion model has two equations, namely, acquisition and retention equations (see (2.2)-(2.3)). Three parameters have to be estimated for each equation, that is, the ceiling rate (γ_a or γ_r) , the spending effectiveness (f_1 or h_1) and the external icentives (f_0 or h_0). As these equations are nonlinear in their parameters, we started by using two nonlinear methods implemented in SAS: least-squares estimation (proc nlin) and seemingly unrelated regression (proc model). However, as the results turned out to be unreliable due to very high parameter estimate correlations, we adopted the linearization approach proposed by Horsky and Simon (1983) in also a context of new-product diffusion model, i.e.,

$$\begin{aligned}\ln\left(1-\frac{a(t)}{\gamma_a^k}\right) &= -f_1A(t)-f_0,\\ \ln\left(1-\frac{r(t)}{\gamma_r^k}\right) &= -h_1R(t)-h_0,\end{aligned}$$

where γ_a^k and γ_r^k are the values at iteration k, with $\gamma_a^k \in [\gamma_a^{\max}, 1]$ and $\gamma_r^k \in [\gamma_r^{\max}, 1]$. This procedure deserves some explanations. The lower bound in each of the interval, that is, $\gamma_a^{\max}$ and $\gamma_r^{\max}$, is the highest value observed in the data set. The reason for this choice is twofold. First, the ceiling rate is the maximun rate can be reached when there is no limit to spending. Therefore, this rate cannot logically be less than an already empirically observed one. Second, a ceiling rate below this maximum leads to a technical problem, namely, a logarithm of a negative number for all values exceeding this rate. Adopting a mesh size of 0.01%, we run regressions for all admissible values of γ_a^k and γ_r^k, and select the result exhibiting the highest goodness of fit (R^2) for each equation. Finally, we note that although the linearization does not allow to test for the null hypothesis of γ_a and γ_r, our estimation approach is still attractive because obviously the ceiling acquisition and retention rates are larger than zero for an existing service. Actually, what is important here is the assessment of the significance of marketing expenditures and external incentives.

Parameter estimates and fit statistic for each of the two companies are provided in Table 2.3. These results call for the following comments:

1. The R^2 for the acquisition equations is much higher than for the retention equations, that is, 73% versus 45% for Sky and 56% versus 24% for DIRECTV. One possible explanation is that the variable we used to measure retention does not fully capture all expenditures made by the companies.
2. The marketing efficiency parameters (f_0, f_1, h_0, h_1) have all the expected (positive) sign, which confirms the positive impact of CRM expenditures on the service growth.
3. Without any acquisition spending, DIRECTV can acquire 0.44% of potential customers in each period. This percentage represents 13.2% of the maximum rate that DIRECTV

can reach (0.44/3.34). For Sky, only 0.05% of prospects can be acquired without acquisition effort, which represents only 1.25% of the ceiling rate. This implies that the external acquistion incentives is more than ten times higher in DIRECTV market than in Sky market.

4. The external factors influencing a subscriber to stay with the service are (again) stronger in the case DIRECTV than for Sky. Indeed, whereas 94.41% of DIRECTV subscribers keep the service for the next period without retention spending, Sky would lose almost a quarter of its subscribers if it makes no retention effort.

A first general conclusion is that our model performs well empirically, and that the external factors are market specific.

Table 2.3 Parameter estimates

	Sky Deutschland AG		**DIRECTV**	
	Parameters	s.e	Parameters	s.e
Acquisition Equation				
Ceiling acquisition rate (γ_a)	3.99%		3.34%	
Floor acquisition rate	0.05%		0.44%	
Acquisition spending effectiveness (f_1)	0.0719***	0.0100	0.0385***	0.0077
External acquisition incentives (f_0)	0.0135	0.0103	0.1404***	0.0495
Goodness of fit (R^2)	73.00%		55.87%	
Retention Equation				
Ceiling retention rate (γ_r)	100.00%		99.90%	
Floor retention rate	77.38%		94.41%	
Retention spending effectiveness (h_1)	0.2753***	0.0694	0.0159**	0.0076
External retention incentives (h_0)	1.4865***	0.4315	2.9012***	0.1110
Goodness of fit (R^2)	45.32%		24.05%	

(***) Significant at 1%

(**) Significant at 5%

2.6.2 Impact of CRM Expenditures on Service Growth

We can use the estimated coefficients to assess the impact of CRM spendings on the growth of the service, that is, the evolution overtime of the percentage of subscribers for each of the two companies. To do so, we use the general form of the equation (2.21)

$$N(t) = \frac{am}{1+a-r}\left(1 - e^{-(1+a-r)t}\right), N_0 = 0$$

with

$$a = \hat{\gamma}_a \left(1 - e^{-\hat{f}_1 A(t) - \hat{f}_0}\right), \tag{2.22}$$

$$r = \hat{\gamma}_r \left(1 - e^{-\hat{h}_1 R(t) - \hat{h}_0}\right), \tag{2.23}$$

where a hat refers to the estimated value of the coefficient in Table 2.3. Four scenarios are compared, namely: (1) the firm does not invest in marketing efforts, i.e., $A(t) = R(t) = 0$ and acquisition and retention rates are at their floor values); (2) the firm only invests one unit in acquisition efforts, and retention level is at its floor rate; (3) the firm only invests one unit in retention efforts, and acquisition level is at its floor rate; and (4) the firm invests in acquisition and retention efforts. Figure 3.1 (resp. Figure 3.2) shows the penetration rate growth (the number of subscribers divided by the market potential) over time for the four scenarios for DIRECTV (resp. for Sky). The main takeaways are the following: (i) As theory predicts, the penetration rate exhibits marginal decreasing returns to scale for both companies, with the steady state being reached more rapidly in the case of Sky than for DIRECTV; (ii) The importance of the external incentives varies considerably between Sky and DIRECTV. Indeed, roughly speaking, in the no-marketing-spending scenario, the steady state represents 0.2% of the market potential for Sky and 7.3% for DIRECTV; (iii) Acquisition spendings have a higher impact, in terms of upward shift with respect to the no-marketing-spendings benchmark than retention efforts for both companies; and finally (iv) Retention spendings are more crucial for Sky. Indeed, compared to scenario 2, investing the same amount in customer retention allows to reach 212,414 more subscribers for Sky (0.4% of the market potential) and 191,607 more subscribers for DIRECTV (0.1% of the market potential).

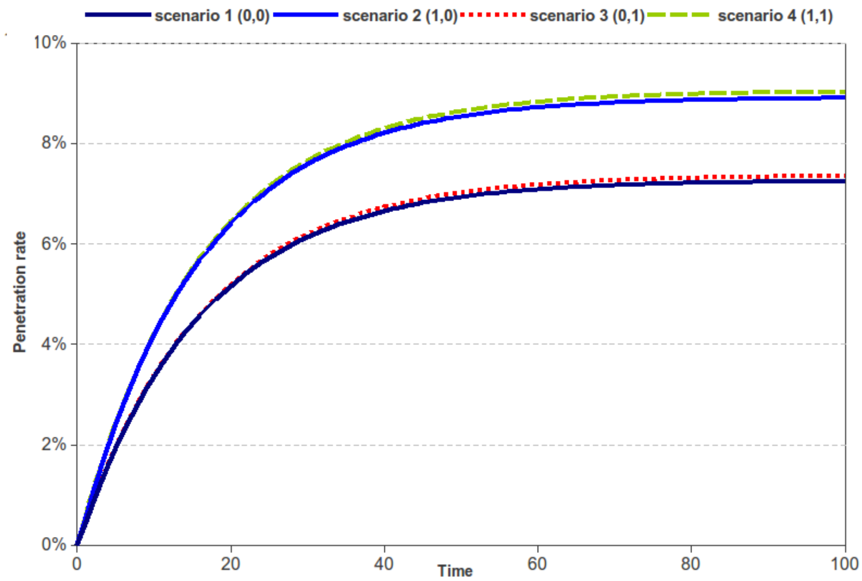

Figure 2.1 Impact of CRM spending on DIRECTV service growth

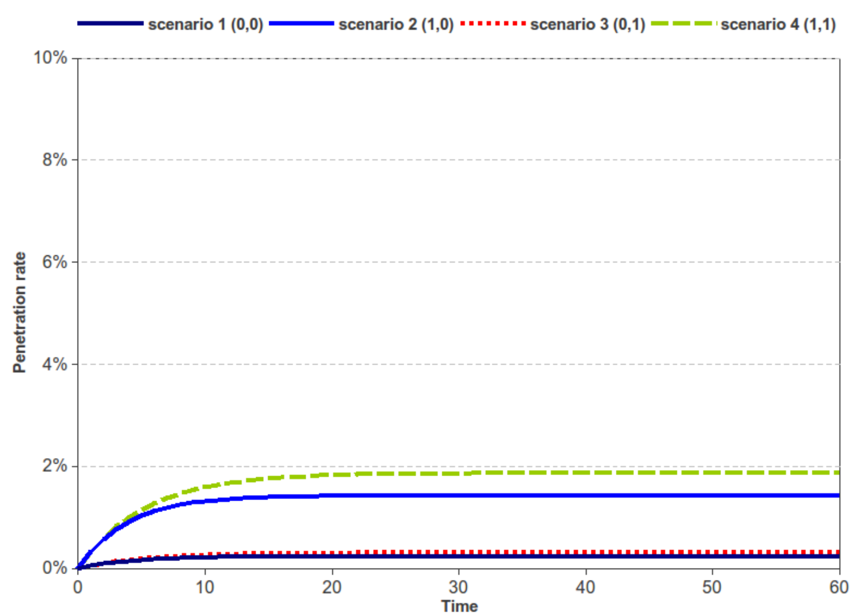

Figure 2.2 Impact of CRM spending on Sky service growth

2.6.3 Optimal Spendings

We used the estimation results to determine optimal acquisition and retention expenditures based on (2.17) and (2.18). Table 2.4 reports the optimal spending strategies and customer metrics for each company.

$$A^* = \frac{1}{\hat{f}_1}\left(\ln(\hat{\gamma}_a \hat{f}_1 \eta_1) - \hat{f}_0\right), \tag{2.24}$$

$$R^* = \frac{1}{\hat{h}_1}\left(\ln(\hat{\gamma}_r \hat{h}_1 \eta_1) - \hat{h}_0\right). \tag{2.25}$$

$$a^* = \hat{\gamma}_a\left(1 - e^{-\hat{f}_1 A^* - \hat{f}_0}\right), \tag{2.26}$$

$$r^* = \hat{\gamma}_r\left(1 - e^{-\hat{h}_1 R^* - \hat{h}_0}\right), \tag{2.27}$$

Our results show that the arbitration between acquisition spending and retention spending is not obvious. While Sky should invest on retention twice the amount spent on acquisition, DIRECTV have to manage its CRM expenditures oppositely. We can therefore conclude that the optimal strategies is closely determined by the market structure shown through the model parameters. In accordance, we make the following comments:

1. As the customer retention due to external incentives is considerably closer to the ceiling rate, DIRECTV has no interest to invest on retention, contrary to the acquisition dimension for which marketing efforts could improve the performance.
2. For Sky, the external incentives are low for both acquisition and retention leading to a higher ratio of the CLV to the PLV (7 for Sky versus 5 for DIRECTV). In this case, marketing expenditures play a crucial role to acquire new customers and retain existing ones. Sky should invest 42% of the margin generated by a subscriber to retain him. In contrast, this percentage is equal to 4% for DIRECTV.

Table 2.4 Optimal spending strategies

	Sky Deutschland AG	**DIRECTV**
Optimal CE at September 2012 (*Billion*)	6.05 *e*	59.07 $
Optimal CLV	644.10 *e*	1517.07 $
Optimal PLV	88.93 *e*	302.64 $
Optimal acquisition spending	6.28 *e*	7.94 $
Optimal acquisition rate	1.48%	1.20% $
Optimal retention spending	12.87 *e*	3.75 $
Optimal retention rate	99.35%	94.78%

2.6.4 Which Is More Critical: Underspending or Overspending?

We follow the same approach applied by Reinartz et al. (2005), which focuses on the impact of not optimizing acquisition expenditures versus not optimizing retention expenditures. We analyze the effect of expenditure change on customer equity. In Table 2.5, we report how the customer equity deviates from the optimal state at different levels. The results for Sky and DIRECTV confirmed the findings in Reinartz et al. (2005), namely:

1. Under the assumption that retention spending is optimal, underspending on acquisition is worse than overspending by the same amount.
2. Under the assumption that acquisition spending is optimal, overspending on retention is better than underspending by the same amount.[2]

An interesting result appears when we analyse situations in which the expenditures are not optimal. When the acquisition spending is not at its optimum value, overspending on retention remains the best strategy for both Sky and DIRECTV. On the other hand, when the retention spending is not at its optimum value, the best acquisition strategy is not the same for the two compagnies. While overspending on acquisition is better for DIRECTV, underspending generates a higher customer equity for Sky. This result suggests that even the conclusions mentioned above (Reinartz et al., 2005) could not be generalizable to all markets,

2. For DIRECTV, as a result of rounding, increasing and decreasing the retention expenditure appears to give the same result. However, we find that decreasing the retention spending results a slightly lower change.

Table 2.5 Percentage Change in CE from Optimal CE for Sky and DIRECTV

DIRECTV	Retention Expenditures				
Acquisiation Expenditures	$0.75 \times Optimal$	$0.9 \times Optimal$	*Optimal*	$1.1 \times Optimal$	$1.25 \times Optimal$
$0.75 \times Optimal$	−1.040%	−1.029%	−1.026%	−1.028%	−1.037%
$0.9 \times Optimal$	−0.174%	−0.162%	−0.160%	−0.162%	−0.173%
Optimal	−0.014%	−0.002%	–	−0.002%	−0.014%
$1.1 \times Optimal$	−0.170%	−0.157%	−0.154%	−0.157%	−0.169%
$1.25 \times Optimal$	−0.957%	−0.942%	−0.939%	−0.941%	−0.954%
Sky Deutschland AG	Retention Expenditures				
Acquisiation Expenditures	$0.75 \times Optimal$	$0.9 \times Optimal$	*Optimal*	$1.1 \times Optimal$	$1.25 \times Optimal$
$0.75 \times Optimal$	−27.44%	−7.03%	−3.33%	−6.46%	−21.13%
$0.9 \times Optimal$	−27.11%	−4.51%	−0.50%	−4.02%	−20.21%
Optimal	−28.18%	−4.23%	–	−3.72%	−20.78%
$1.1 \times Optimal$	−30.17%	−4.91%	−0.46%	−4.34%	−22.16%
$1.25 \times Optimal$	−34.64%	−7.52%	−2.71%	−6.76%	−25.53%

all companies and all services. In what follows, we present an example that confirms our latter statement.

The parameters of this example were partially compiled data from Netflix data, an American provider of on-demand Internet streaming media. As Netflix does not publish its retention expenditures, we have chosen parameter values of the retention equation (γ_r, h_0, h_1). The rest of the parameters were estimated from data. Of course, the purpose of this example is not to estimate the model parameters, thing already done with DIRECTV and Sky data, but rather to give a context in which overspending can be riskier.

Table 2.6 provides the percentage change in customer equity from its Optima. Unlike the previous two cases (DIRECTV and Sky), and under the assumption that acquisition spending is optimal, overspending on retention is worse than underspending by the same amount. Thus, overspending can be more critical in some economic contexts and less critical in some others.

Table 2.6 Percentage Change in CE from Optimal CE for Netflix

Netflix	Retention Expenditures				
Acquisiation Expenditures	$0.75 \times Optimal$	$0.9 \times Optimal$	$Optimal$	$1.1 \times Optimal$	$1.25 \times Optimal$
$0.75 \times Optimal$	-14.16%	-6.01%	-4.24%	-6.12%	-17.06%
$0.9 \times Optimal$	-11.73%	-2.58%	-0.65%	-2.80%	-15.14%
$Optimal$	-11.82%	-2.06%	$-$	-2.30%	-15.44%
$1.1 \times Optimal$	-13.15%	-2.80%	-0.61%	-3.03%	-16.88%
$1.25 \times Optimal$	-17.26%	-6.05%	-3.65%	-6.20%	-20.93%

$m = 115$ millions, $g = 18.18\$, \rho = 2.2\%, \gamma_a = 3.4\%, f_0 = 0.015, f_1 = 0.844, \gamma_r = 100\%, h_0 = 0.925, h_1 = 0.147$

2.7 Concluding Remarks

Our research aimed to directly tie the concepts and processes of customer relationship marketing to the diffusion literature. Indeed, customer relationship management has become a central consideration for most firms. Though research in this area is burgeoning, little work has addressed the issue of customer acquisition and retention efforts in the context of diffusion of subscription services. For these services, customer relationship duration affects hugely the service growth and the customer profitability. We proposed a new diffusion model that incorporates acquisition and retention expenditures. The service growth is characterized by two processes: customer acquisition process and customer attrition process. By using dynamic programming, we introduced an innovative approach to calculate optimal acquisition and retention spending in order to maximize the customer equity. We showed that the optimal customer equity represents the sum of the value of existing customers and the value of the remaining market. Moreover, we found that optimal acquisition and retention policies are constant throughout the service growth and does not depend on the penetration rate. Through our results, the marginal customer equity, given by the difference between the customer lifetime-value and the prospect lifetime-value, plays a crucial role in the determination of the optimal CRM expenditures. The sensitivity analysis provided the consequences of varying the effectiveness of internal incentives as well as the magnitude of external incentives on optimal policies.

For empirical evidence regarding the impact of acquisition and retention efforts on the services growth, we estimated the model parameters by using real data of two pay-TV companies. The results confirmed the positive impact of CRM expenditures on the service growth and the significant presence of external factors that influenced acquisition and retention processes. The external factors are market specific. We showed that underspending on CRM may be less critical than overspending in some market context.

Finally, we mention two possible extensions to our work. First, we assumed that individual acquisition and retention costs do not vary with the penetration rate. This could explain why optimal spending policies were constant. Several researchers have shown that the innovators are less price sensitive than other consumers (Goldsmith, 1996; Goldsmith and Newell, 1997; Goldsmith et al., 2005; Park et al., 2010). The acquisition cost of early adaptors should be low compared to laggards who are more resistant to change. Likewise, the retention behaviour could change with the penetration rate (Fader and Schmittlein, 1992). Based on these remarks, an interesting extention to this work would to be to assume that marketing effectiveness varies with the penetration rate. Second, our model does not distinguish between new customers and won-back ones in terms of their acquisition cost. In reality, it is less expensive to acquire a new subscriber than a lost one who has probably a negative perception toward the service. In this sense, the model might be extended by treating won-back customers differently. Finally, our study is developed in a non-competitive market. Though the external incentives parameters could capture the effect of marketing efforts of other firms, competitive framework with strategic interactions might yield additional insights on the role of CRM expenditures in the diffusion of subscription services.

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2.9 Appendix:

2.9.1 Proof of Proposition 1

The value function must satisfy the following Hamilton-Jacobi-Bellman (HJB):

$$\rho V\left(N\right)=\max_{a,\, r\in[0,1]}\left\{\begin{array}{c} Ng+\dfrac{N}{h_1}\left[\ln(1-\frac{r}{\gamma_r})+h_0\right]+\dfrac{(m-N)}{f_1}\left[\ln(1-\frac{a}{\gamma_a})+f_0\right] \\ +\dfrac{\partial V}{\partial N}[a(m-N)-(1-r)N] \end{array}\right\}. \tag{2.28}$$

Assuming an interior solution, the maximizatioin of the right-hand side of (2.28) yields the following first-order optimiality conditions:

$$a = \gamma_a-\frac{1}{f_1 V'\left(N\right)}, \tag{2.29}$$

$$r = \gamma_r-\frac{1}{h_1 V'\left(N\right)}. \tag{2.30}$$

Substituting in (2.28), leads to

$$\begin{aligned}\rho V &= \frac{m\left(f_0 - \ln\left(\gamma_a\right)\right)}{f_1} + N\left(g + \frac{h_{0-}\ln\left(\gamma_r\right)}{h_1} - \frac{f_{0-}\ln\left(\gamma_a\right)}{f_1}\right) \\ &\quad - \frac{N}{h_1}\ln\left(h_1 V'\left(N\right)\right) - \frac{(m-N)}{f_1}\ln\left(f_1 V'\left(N\right)\right) \\ &\quad + V_N\left[\left(\gamma_a - \frac{1}{f_1 V'\left(N\right)}\right)(m-N) - \left(1 - \left(\gamma_r - \frac{1}{h_1 V'\left(N\right)}\right)\right)N\right]\end{aligned} \tag{2.31}$$

We conjecture that $V\left(N\right)$ is linear in N, i.e.,

$$V(N) = \eta_1 N + \eta_0.$$

Substituting for $V\left(N\right)$ and $V'\left(N\right)$ in (2.32) yields

$$\begin{aligned}\rho\left(\eta_1 N + \eta_2\right) &= \frac{m\left(f_0 - \ln\left(\gamma_a\right)\right)}{f_1} + N\left(g + \frac{h_{0-}\ln\left(\gamma_r\right)}{h_1} - \frac{f_{0-}\ln\left(\gamma_a\right)}{f_1}\right) - \frac{N}{h_1}\ln\left(h_1\eta_1\right) \\ &\quad - \frac{(m-N)}{f_1}\ln\left(f_1\eta_1\right) + \eta_1\left[\left(\gamma_a - \frac{1}{f_1\eta_1}\right)(m-N) - \left(1 - \left(\gamma_r - \frac{1}{h_1\eta_1}\right)\right)N\right].\end{aligned}$$

By Identification of coefficients in order of N, we obtain

$$\begin{aligned}\rho\eta_1 &= -\left(\frac{1}{h_1} - \frac{1}{f_1}\right)\ln\left(\eta_1\right) - \eta_1\left(1 + \gamma_a - \gamma_r\right) + \frac{1}{h_1}\left[h_0 - \ln\left(\gamma_r\right) - \ln\left(h_1\right) - 1\right] \\ &\quad - \frac{1}{f_1}\left[f_0 - \ln\left(\gamma_a\right) - \ln\left(f_1\right) - 1\right] + g\end{aligned} \tag{2.32}$$

$$\rho\eta_0 = m\left(\gamma_a\eta_1 - \frac{1}{f_1}\ln\left(\eta_1\right) + \frac{f_0 - \ln\left(\gamma_a\right) - \ln\left(f_1\right) - 1}{f_1}\right). \tag{2.33}$$

The coefficient η_1 is therefore the solution of the following equation:

$$H\left(\eta\right) = B\eta + C\ln(\eta) + D = 0, \tag{2.34}$$

where

$$\begin{aligned} B &= \rho + 1 + \gamma_a - \gamma_r, \quad C = \frac{1}{h_1} - \frac{1}{f_1}, \\ D &= \frac{1}{f_1}\left[f_0 - \ln\left(\gamma_a\right) - \ln\left(f_1\right) - 1\right] - \frac{1}{h_1}\left[h_0 - \ln\left(\gamma_r\right) - \ln\left(h_1\right) - 1\right] - g. \end{aligned}$$

From (2.34), we get

$$\ln(\eta_1) = \frac{-B\eta_1 - D}{C}.$$

Substituting in (2.33), gives

$$\rho\eta_0 = \left[\eta_1\left(\gamma_a + \frac{B}{f_1 C}\right) + \frac{f_0 - \ln\left(\gamma_a\right) - \ln\left(f_1\right) - 1}{f_1} + \frac{D}{C f_1}\right] m,$$

or equivalently

$$\eta_0 = \left[\frac{\eta_1}{\rho}\left(\gamma_a + \frac{\left(\rho + 1 + \gamma_a - \gamma_r\right) h_1}{\left(f_1 - h_1\right)}\right) + \frac{1}{\rho\left(f_1 - h_1\right)}\left(f_0 - h_0 + \ln\left(\frac{h_1\gamma_r}{f_1\gamma_a}\right) - h_1 g\right)\right] m.$$

2.9.2 Sensitivity Analysis

Let

$$\begin{aligned} H\left(\rho, \gamma_a, \gamma_r, g, h_0, f_0, h_1, f_1, \eta_1\right) = \left(\rho + 1 + \gamma_a - \gamma_r\right)\eta_1 + \left(\frac{1}{h_1} - \frac{1}{f_1}\right)\ln(\eta_1) \\ + \frac{1}{f_1}\left[f_0 - \ln\left(\gamma_a\right) - \ln\left(f_1\right) - 1\right] - \frac{1}{h_1}\left[h_0 - \ln\left(\gamma_r\right) - \ln\left(h_1\right) - 1\right] - g = 0, \end{aligned}$$

We use the implicit function theorem to determine the effect of a model parameter on η_1. Let x_i be a parameter, then we have

$$\frac{\partial \eta_1}{\partial x_i} = -\frac{\frac{\partial H}{\partial x_i}}{\frac{\partial H}{\partial \eta_1}},\ x_i$$

We first determine the value of $\frac{\partial H}{\partial \eta_1}$ and its sign, that is,

$$\begin{aligned}\frac{\partial H}{\partial \eta_1} &= (\rho + 1 + \gamma_a - \gamma_r) + \left(\frac{1}{h_1} - \frac{1}{f_1}\right)\frac{1}{\eta_1} \\ &= \rho + 1 + a^* - r^* > 0\end{aligned}$$

The derivatives of H with respect to the different parameters are given by

$$\begin{aligned}\frac{\partial H}{\partial f_0} &= \frac{1}{f_1},\ \frac{\partial H}{\partial h_0} = -\frac{1}{h_1},\ \frac{\partial H}{\partial \gamma_a} = \eta_1,\ \frac{\partial H}{\partial \gamma_r} = -\eta_1\ \frac{\partial H}{\partial g} = -1 \\ \frac{\partial H}{\partial \rho} &= \eta_1,\ \frac{\partial H}{\partial f_1} = \left(\frac{1}{f_1}\right)^2 [\ln(\eta_1 \gamma_a f_1) - f_0]\ \frac{\partial H}{\partial h_1} = -\left(\frac{1}{h_1}\right)^2 [\ln(\eta_1 \gamma_r h_1) - h_0].\end{aligned}$$

The derivatives of η_1 with respect to the different parameters are given by

$$\begin{aligned}\frac{\partial \eta_1}{\partial f_0} &= -\frac{1}{f_1 \frac{\partial H}{\partial \eta_1}} < 0,\ \frac{\partial \eta_1}{\partial h_0} = \frac{1}{h_1 \frac{\partial H}{\partial \eta_1}} > 0,\ \frac{\partial \eta_1}{\partial \gamma_a} = -\frac{\eta_1}{\frac{\partial H}{\partial \eta_1}} < 0, \\ \frac{\partial \eta_1}{\partial \gamma_r} &= \frac{\eta_1}{\frac{\partial H}{\partial \eta_1}} > 0,\ \frac{\partial \eta_1}{\partial g} = \frac{1}{\frac{\partial H}{\partial \eta_1}} > 0,\ \frac{\partial \eta_1}{\partial \rho} = -\frac{\eta_1}{\frac{\partial H}{\partial \eta_1}} < 0 \\ \frac{\partial \eta_1}{\partial f_1} &= -\frac{1}{\frac{\partial H}{\partial \eta_1}} \left(\frac{1}{f_1}\right)^2 [\ln(\eta_1 \gamma_a f_1) - f_0] < 0, \\ \frac{\partial \eta_1}{\partial h_1} &= \frac{1}{\frac{\partial H}{\partial \eta_1}} \left(\frac{1}{h_1}\right)^2 [\ln(\eta_1 \gamma_r h_1) - h_0] \quad > 0 \\ \text{Since}, \ln(\eta_1 \gamma_a f_1) - f_0 &\geq 0 \text{ and } \ln(\eta_1 \gamma_r h_1) - h_0 \geq 0 \text{ (floor rates constraints)}\end{aligned}$$

The derivatives of CA are given by

$$
\begin{aligned}
\frac{\partial CA}{\partial f_0} &= -\frac{1}{f_1}\left(1+\frac{1}{f_1\eta_1\frac{\partial H}{\partial \eta_1}}\right)<0,\ \frac{\partial CA}{\partial h_0}=\frac{1}{h_1 f_1\eta_1\frac{\partial H}{\partial \eta_1}}>0 \\
\frac{\partial CA}{\partial f_1} &= -\left(\frac{1}{f_1}\right)^2[\ln(\gamma_a f_1\eta_1)-f_0]\left[1+\frac{1}{f_1\eta_1\frac{\partial H}{\partial \eta_1}}\right]+\left(\frac{1}{f_1}\right)^2 \text{ could be } + \text{ or } - \\
\frac{\partial CA}{\partial h_1} &= \frac{1}{f_1\eta_1\frac{\partial H}{\partial \eta_1}}\left(\frac{1}{h_1}\right)^2[\ln(\eta_1\gamma_r h_1)-h_0]\geq 0 \\
\frac{\partial CA}{\partial \gamma_a} &= \frac{1}{f_1}\left(\frac{1}{\gamma_a}-\frac{1}{\frac{\partial H}{\partial \eta_1}}\right) \text{ could be } + \text{ or } -,\ \frac{\partial CA}{\partial \gamma_r}=\frac{1}{f_1\frac{\partial H}{\partial \eta_1}}>0 \\
\frac{\partial CA}{\partial g} &= \frac{1}{f_1\eta_1\frac{\partial H}{\partial \eta_1}}>0,\ \frac{\partial CA}{\partial \rho}=-\frac{1}{f_1\frac{\partial H}{\partial \eta_1}}<0
\end{aligned}
$$

The derivatives of CR are given by

$$
\begin{aligned}
\frac{\partial CR}{\partial f_0} &= -\frac{1}{f_1 h_1\eta_1\frac{\partial H}{\partial \eta_1}}<0,\ \frac{\partial CR}{\partial h_0}=\frac{1}{h_1}\left(\frac{1}{h_1\eta_1\frac{\partial H}{\partial \eta_1}}-1\right) \text{ could be } + \text{ or } - \\
\frac{\partial CR}{\partial f_1} &= -\frac{1}{h_1\eta_1\frac{\partial H}{\partial \eta_1}}\left(\frac{1}{f_1}\right)^2[\ln(\gamma_r h_1\eta_1)-f_0]\leq 0; \\
\frac{\partial CR}{\partial h_1} &= \left(\frac{1}{h_1}\right)^2[\ln(\eta_1\gamma_r h_1)-h_0]\left(\frac{1}{h_1\eta_1\frac{\partial H}{\partial \eta_1}}-1\right)+\left(\frac{1}{h_1}\right)^2 \text{ could be } + \text{ or } - \\
\frac{\partial CR}{\partial \gamma_r} &= \frac{1}{h_1}\left(\frac{1}{\gamma_r}+\frac{1}{\frac{\partial H}{\partial \eta_1}}\right)>0,\quad \frac{\partial CR}{\partial \gamma_a}=-\frac{1}{h_1\frac{\partial H}{\partial \eta_1}}<0 \\
\frac{\partial CR}{\partial g} &= \frac{1}{h_1\eta_1\frac{\partial H}{\partial \eta_1}}>0,\quad \frac{\partial CR}{\partial \rho}=-\frac{1}{h_1\frac{\partial H}{\partial \eta_1}}<0
\end{aligned}
$$

The derivatives of a are given by

$$
\begin{aligned}
\frac{\partial a}{\partial f_0} &= -\frac{1}{(f_1\eta_1)^2\,\frac{\partial H}{\partial \eta_1}} < 0,\ \frac{\partial a}{\partial h_0} = \frac{1}{f_1 h_1 (\eta_1)^2\,\frac{\partial H}{\partial \eta_1}} > 0 \\
\frac{\partial a}{\partial f_1} &= \frac{1}{(f_1\eta_1)^2}\left(\eta_1 - \frac{1}{f_1\frac{\partial H}{\partial \eta_1}}\left[\ln(\eta_1\gamma_a f_1) - f_0\right]\right) \text{ could be } + \text{ or } - \\
\frac{\partial a}{\partial h_1} &= \frac{1}{f_1(\eta_1)^2}\frac{1}{\frac{\partial H}{\partial \eta_1}}\left(\frac{1}{h_1}\right)^2\left[\ln(\eta_1\gamma_r h_1) - h_0\right] > 0 \\
\frac{\partial a}{\partial \gamma_a} &= 1 - \frac{1}{f_1\eta_1\frac{\partial H}{\partial \eta_1}} \text{ could be } + \text{ or } -,\ \frac{\partial a}{\partial \gamma_r} = \frac{1}{f_1\eta_1\frac{\partial H}{\partial \eta_1}} > 0 \\
\frac{\partial a}{\partial g} &= \frac{1}{f_1(\eta_1)^2}\frac{1}{\frac{\partial H}{\partial \eta_1}} > 0, \frac{\partial a}{\partial \rho} = -\frac{1}{f_1\eta_1\frac{\partial H}{\partial \eta_1}} < 0
\end{aligned}
$$

The derivatives of r are given by

$$
\begin{aligned}
\frac{\partial r}{\partial f_0} &= -\frac{1}{f_1 h_1 (\eta_1)^2\,\frac{\partial H}{\partial \eta_1}} < 0,\quad \frac{\partial r}{\partial h_0} = \frac{1}{(h_1\eta_1)^2\,\frac{\partial H}{\partial \eta_1}} > 0 \\
\frac{\partial r}{\partial f_1} &= -\frac{1}{h_1(\eta_1)^2}\frac{1}{\frac{\partial H}{\partial \eta_1}}\left(\frac{1}{f_1}\right)^2\left[\ln(\eta_1\gamma_a f_1) - f_0\right] < 0 \\
\frac{\partial r}{\partial h_1} &= \frac{1}{(h_1\eta_1)^2}\left(\eta_1 + \frac{1}{h_1\frac{\partial H}{\partial \eta_1}}\left[\ln(\eta_1\gamma_r h_1) - h_0\right]\right) > 0 \\
\frac{\partial r}{\partial \gamma_a} &= -\frac{1}{h_1\eta_1\frac{\partial H}{\partial \eta_1}} < 0, \frac{\partial r}{\partial \gamma_r} = 1 + \frac{1}{h_1\eta_1\frac{\partial H}{\partial \eta_1}} > 0 \\
\frac{\partial r}{\partial g} &= \frac{1}{h_1(\eta_1)^2}\frac{1}{\frac{\partial H}{\partial \eta_1}} > 0,\ \frac{\partial r}{\partial \rho} = -\frac{1}{h_1\eta_1\frac{\partial H}{\partial \eta_1}} < 0
\end{aligned}
$$

Chapter 3

Branding Decisions for Retailer's Private Labels

3.1 Abstract

Umbrella branding strategies for manufacturers' products have received considerable attention in the literature. Not much is known about the use of this strategy for private labels. Using a game-theoretic approach, this paper intends to reassess the benefits of introducing a private label in a distinct category, and provide favorable conditions for the retailer to implement umbrella or individual branding for his private labels. We find that (i) by implementing umbrella branding, the retailer succeeds in lowering the wholesale price and the retail prices of both national brands; (ii) the national brands' manufacturers prefer individual branding over umbrella branding for private labels (iii) the profitability of umbrella branding is not always guaranteed for the retailer though it produces a positive spillover effect between both private labels.

Keywords: Umbrella branding, Private labels, Game theory, Pricing strategies.

3.2 Introduction

Consider a retailer who offers his own private label in a given product category and who is considering launching a new private label in another category. Should he use umbrella-branding strategy, that is, adopt the same name, or should he use a different name for his new product? This is essentially the research question we wish to answer in this paper. We shall characterize the profitability of both possible strategies, and assess their impact on the manufacturers whose brands are already on retailer's shelves.

The competition between a manufacturer's brand (also know as a national brand) and a retailer's brand (also called a private label or store brand) has long been a prominent topic in the literature. When private labels (PLs) were first launched, over fifty years ago, retailers were smaller than manufacturers and consumers had greater trust for national brands (NBs), which were seen as a symbol of quality and innovation. During the 1970s, retailers such as Wal-Mart, Ikea and Carrefour started to consolidate, expand globally, and improve the quality of their PLs (Kumar and Steenkamp 2007). This created a power shift benefiting retailers who became the manufacturers' competitors rather than simply distributors of their products. Apart from their increasing bargaining power (Ailawadi and Harlam 2004), retailers have also total control over the assortment and positioning of brands on their shelves, and these are key drivers of their PLs' success (Morton and Zettelmeyer 2004). Also, PLs help increase store traffic and loyalty to the store (Ailawadi et al. 2008).

Over the last decade, annual sales of PL products increased by 40% in supermarkets and 96% in drug chains according to the Private Label Manufacturers Association (PLMA 2011).[1] A GFK Roper study (2011) reports that 8 out of 10 consumers in the US say that PLs are as good as or better than NBs; more than 50% frequently purchase PLs; and, more than 50% are aware of PLs.[2] Their popularity and success are not only due to their competitive prices but also to their constantly improving quality (see, e.g., Hock and Banerji 1993; Wilensky 1994), lack of perceived difference due to premium PLs (e.g., President's Choice from Loblaws), the good value they offer, their packaging design and shelf placement (Mintel study 2011).[3] Retailers are introducing a variety of innovative tools to boost PL sales and power such as new flavors, creative packaging design and super-premium PLs.

The literature has studied a variety of topics related to PLs including: (i) determining factors in PL success (e.g., Dhar and Hock 1997); (ii) purchasing behavior of consumers and positioning of PLs (e.g., Erdem et al. 2004); (iii) price competition between PLs and NBs (e.g., Sethuraman et al. 1999); (iv) advertising strategies for NBs confronting PLs (e.g.,

1. "2011 private label yearbook". Retrieved from www.plma.com.
2. "Store brand perception and shopping behavior". Retrieved from www.plma.com
3. "Store brands: As good as or better than national brands". Retrieved from www.plma.com.

Abe 1995); (v) effectiveness of promotion and brand switching (Putsis and Dhar 2001); and (vi) impact of introducing a PL on the pricing strategies and performance of supply-chain members (e.g., Mills 1995, 1999; Narasimhan and Wilcox 1998; Raju et al. 1995; Morton and Zettelmeyer 2004; Chintagunta et al. 2002; Ailawadi and Keller 2004). In this paper, we reassess the issue of PL introduction and extend the analysis to a new topic, namely, the effectiveness of using umbrella branding for PLs.

Previous articles dealing with umbrella branding (UB) focused on different issues. Montgomery and Wernerfelt (1992) investigated the risk-reducing effect of this strategy and showed that this effect is stronger for expensive products. Wernerfelt (1988) studied the quality-guarantee function of UB and its signaling effect. The author showed that a firm should not use umbrella branding when the old product (already available on the market) or the new product are of poor quality. In the first case, customers will perceive the new product as being as low-quality as the old one and in the second case, the new product will benefit from the good quality of the old one but will then hurt sales of the old one. Sullivan (1990) and Balachander and Ghose (2003) proposed empirical models to measure spillover effects. Sullivan (1990) showed that umbrella branding should be used in markets characterized by low uncertainty and little perceived substitutability between the established product and the new one. The author proposed a decomposition of the spillover effect into brand-image and intrabrand substitution effects. Both occur and can be measured. Balachander and Ghose (2003) found that advertising a brand extension has a significant reciprocal spillover effect on the choice of the parent brand. Hakenes and Peitz (2008) and Erdem (1998) examined the conditions of using and the incentives to use umbrella branding. Hakenes and Peitz (2008) gave insight into the role of asymmetric costs, consumer valuations and quality-detection probabilities[4] on UB profitability. Erdem (1998) explored the influence of marketing mix strategies (e.g., free samples) in one product category on learning, quality perceptions and consumer-risk reduction, which ultimately affect the consumer-choice process in a different category. Contrary to some previous studies insisting on ensuring the strength of the parent brand and its good fit with the new same-name product, the author highlights that those conditions do not ensure UB success if the quality of the extension does not match consumer expectations.

4. The quality-detection probability varies between 0 and 1. If it is equal to 0, a product is a pure credence good. If it is equal to 1, a product is a pure experience good.

Several studies (e.g., Aaker and Keller 1990, 1992; Moorthy 2012) have investigated the sources of failure and success of UB, while others analyzed the case of line extensions under the same name (e.g., Nijssen and Agustin 2005; Alexander and Colgate 2005; Martinez and Pina 2003; Laforet 2008). However, few papers considered umbrella branding in the context of PLs. Wang et al. (2007) proposes a Bayesian multivariate Poisson-regression model to highlight the benefits of using that strategy across categories and to uncover the factors that increase the likelihood of buying PLs. The analysis is conducted at the retailer level and does not include possible interactions with manufacturers. In a descriptive paper, Thompson (1999) highlights the perceptions of different managers concerning the challenges facing PLs. Erdem and Chang (2012) extended the previous work by Erdem (1998) to study the learning spillover effects of umbrella brands across five categories in three countries (United States, United Kingdom, and Spain) for NBs and PLs. Their results revealed the presence of cross-category learning effects for both PLs and umbrella NBs. They also found that PLs in the United States provide less consistent consumer experiences compared to NBs, which is not the case in the UK and Spain. On the other hand, the literature dealing with UB for NBs (e.g., Wernerfelt 1988; Erdem 1998; Hakenes and Peitz 2008) skips over the retailer's role and focuses on the manufacturer's or the consumer's role in the decision process. Previous research about UB has also used different approaches such as experiments (Martinez and Pina 2003); conjoint analysis (Nijssen and Agustin 2005); surveys (Laforet 2008); Mutivariate Multinominal Probit model (Erdem and Chang 2012); and regression analysis (Wang et al. 2007). It seems, however, that no paper has yet studied the profitability of UB for PLs competing against NBs, by taking into account the interaction between supply-chain members in shaping their respective decisions. This paper aims at filling this gap by using a game-theoretic approach and evaluates how and to which level such interaction impacts optimal decisions.

To summarize, we reassess the favorable circumstances for introducing a new PL in a second product category. Further, we investigate whether the new PL should be under the same name as the core PL (already available on the market) or whether the new PL should be under a different name. More specifically, we wish to address the following questions:

1. What are the conditions for introducing a profitable new PL in a second product category?

2. What are the equilibrium wholesale and retail-pricing strategies when the retailer implements an umbrella-branding strategy (UB scenario) versus when he does not (NO UB scenario)?
3. Under which conditions is UB detrimental to NB manufacturers, profit-wise?
4. Assuming that UB creates positive spillovers between PL sales in different product categories, are there still conditions under which UB is a bad strategy for the retailer?

To answer these questions, we propose a parsimonious model where the retailer carries two NBs, in two independent product categories. The retailer also offers a PL in the first category, which enjoys a level of power relative to the NB in this first category. The retailer is, then, interested in introducing a new PL in the second category, where a NB is already available on the market. We determine the favorable conditions for the retailer to launch the new PL, and whether or not he should use UB. We characterize and contrast the equilibrium outcomes of two Stackelberg games, with and without UB, in which the manufacturers move first by announcing their wholesale prices, and then the retailer determines the retail prices. The main results are the following: (i) by implementing UB, the retailer succeeds in lowering the wholesale price of the NBs and consequently their retail prices; (ii) it is never interesting for NBs' manufacturers to see their retailer implementing an umbrella strategy; and lastly (iii) there are indeed instances where the retailer is better off not implementing UB (though it has a positive spillover effect) and should rather use a distinct name for his new PL. Different parameters must be examined to make the optimal decision, namely, the relative power of the core PL compared to the NB, the expected relative power of the new PL in the second category, the cross-price competition between the PLs and the NBs and the level of spillover expected to occur if a new PL is introduced.

The remainder of the paper is organized as follows. In Section 2, we introduce the model. In Section 3, we characterize the equilibrium strategies for the two scenarios and compare them. In Section 4, we deal with the profitability of UB. We briefly conclude in Section 5.

3.3 The Model

We consider a retailer carrying two product categories and offering one NB in each. In the first category, the retailer also markets a PL, which has a certain level of power relative to the NB, e.g., low, medium or high. The retailer is, then, interested in introducing a new

PL in the second category where a NB is already available on the market. The key question is then should the retailer use the same name as the core PL or a different name?

To isolate the effect of umbrella branding on strategies and payoffs, we assume that the two categories are independent. In other words, there is no apparent complementarity or substitutability in the demand for the two products (e.g., detergent and frozen juice). We say that the retailer is using UB if the PL products in both categories bear the same name. We would then expect the consumer to make an association between the retailer's two items in terms of factors such as quality, value for the money, environmentally friendly packaging, etc. We shall capture this spillover impact by linking the baseline demand for the retailer's two items. In the NO UB scenario, the retailer is using different names for his two products and the consumer treats them as two different brands. For instance, Sears sells appliances and hardware under two different names, i.e., Kenmore and Craftsman. Nabisco went with a whole new brand with Snackwells instead of linking it to Oreo cookies or Ritz crackers (The Gale Group Inc. 2013). [5]

Let $c = 1, 2$ refer to the product category and $b = n, s$ to the brand (n for NB and s for PL). Denote by p_{bc} the retail price of brand b in category c, and by w_{bc} the wholesale price paid by the retailer to the manufacturer. We follow the literature on PLs (e.g., Raju et al. 1995) and assume that the retailer buys the store-brand items from non-strategic manufacturers at given wholesale prices denoted $\tilde{w}_{sc}$. Put differently, we assume that the retailer already has a long-term contract with PL manufacturers, which allows him to get the PL at a price close to the marginal cost of production. This is consistent with the industry practice (McMaster 1987). As in Raju et al. (1995), we assume that the production costs of the NB and the PL are equal and set them equal to 0 for simplicity. These assumptions, made for tractability, will allow us to focus on the strategic issues of pricing and branding rather than on costs. That being said, in principle, there would be no conceptual difficulty in adding these costs.

3.3.1 Demand Structure Prior to New PL Introduction

We assume that the demand for a brand in the first category depends on the retail price of both brands. In the second category, the demand for the NB depends only on its retail

5. The Gale Group Inc. (2013). Cookies and crakers. Retrieved from http://business.highbeam.com/industry-reports/food/cookies-crackers.

price, i.e.,

$$\begin{aligned} D_{b1} &= f(p_{n1}, p_{s1}), \quad b = n, s, \\ D_{n2} &= g(p_{n2}) \end{aligned}$$

with

$$\frac{\partial D_{kc}}{\partial p_{kc}} < 0, \quad \frac{\partial D_{kc}}{\partial p_{lc}} \geq 0, \quad k, l = n, s, \quad k \neq l.$$

That is, each brand's demand is decreasing in its own price and, when relevant, increasing in the competing brand's price. These are standard assumptions in economics (e.g., Cotterill et al. 2000). We specify the demand functions in the two categories as follows:

$$\begin{aligned} D_{n1} &= S_{n1} - p_{n1} + \theta_1 (p_{s1} - p_{n1}), \\ D_{s1} &= S_{s1} - p_{s1} + \theta_1 (p_{n1} - p_{s1}), \\ D_{n2} &= 1 - p_{s2}, \end{aligned}$$

where θ_1, $0 \leq \theta_1 \leq 1$, represents the consumer's sensitivity to the difference in price between the NB and the PL in category 1. In particular, $\theta_1 = 0$ means that the two products in a category are independent. We adopt the same assumption as in Raju et al. (1995) using equal cross-price sensitivity in the NB and PL's demand. $S_{b1}, 0 < S_{b1} < 1$, is a parameter representing the baseline demand of the NB ($b = n$) and the PL ($b = s$) in the first category. Our specification assumes that the demand for each brand is linear in both prices and that the market potential (i.e., the total category demand when prices tend toward zero) is equal to 1 independently of the retailer's branding strategy ($S_{n1} + S_{s1} = 1$). The linearity assumption is very common in the economics and marketing literature (see the same demand structure in Raju et al. (1995)) and can be easily justified on the grounds that a linear demand is derivable from the maximization of the consumer's utility function. Further, a linear demand is tractable and is often a very good local approximation (i.e., in a certain price range) of possible non linearities. The normalization of the market potential to 1 in each category ensures that the difference in results between the different scenarios (UB versus NO UB) can be safely attributed to, and only to, the different branding strategies used by the retailer and not to market expansion or shrinking.

Denote by η_{s1} the power of the NB and by η_{n1} the power of the PL in category 1. Raju et al. (1995) found that the PLs' share is higher when their baseline demand (a measure of their strength compared to NBs) is higher. Following these authors, we link the baseline demand of each brand to their relative power. More specifically, we assume that

$$\begin{aligned} S_{n1} &= \frac{\eta_{n1}}{\eta_{s1}+\eta_{n1}} = \frac{1}{1+\alpha_1} \\ S_{s1} &= \frac{\eta_{s1}}{\eta_{s1}+\eta_{n1}} = \frac{\alpha_1}{1+\alpha_1} \end{aligned}$$

where $\alpha_1 = \frac{\eta_{s1}}{\eta_{n1}}$ is the power ratio of the PL with respect to the NB. Hence, the higher the power of PL1 compared to NB1 (high α_1), the lower is NB1's baseline demand and the higher is PL1's baseline demand. Further, Reisenbeck and Perrey (2009) argue that a brand's sales potential is not only due to the strength of the brand but is also related to marketing mix strategies. They illustrate this with the example of the Volkswagen Passat's high rate of conversion from the stage of awareness to that of familiarity, compared to the Mercedes C-Class (52% versus 39% according to a McKinsey survey (2002)). They assert that this result is due to the price differential and to the Passat's greater presence on the road, rather than simply to brand power. Our model accounts for both the brand's power and the price differential between the two brands.

3.3.2 Demand Structure After New PL Introduction

The retailer now introduces the PL in category 2. Two options are available to him, namely, adopting an umbrella-branding strategy or adopting a distinct branding strategy. From now on, we use the superscript N in the NO UB scenario and U in the UB scenario. The demand for each brand in the NO UB scenario is giving by

$$\begin{aligned} D^N_{nc} &= \frac{1}{1+\alpha_c} - p^N_{nc} + \theta_c\left(p^N_{sc} - p^N_{nc}\right), \quad c = 1, 2, \\ D^N_{sc} &= \frac{\alpha_c}{1+\alpha_c} - p^N_{sc} + \theta_c\left(p^N_{nc} - p^N_{sc}\right), \quad c = 1, 2. \end{aligned}$$

When the retailer offers both PLs under the same name (UB), then, as mentioned before, we expect some spillover effects between the two categories. This spillover is assumed to be proportional to the power ratio in the other category. More specifically, under the UB scenario, the demands are assumed to be as follows:

$$\begin{aligned} D_{nc}^{U} &= \frac{1}{1+(\alpha_c+\delta\alpha_{3-c})} - p_{nc}^{U} + \theta_c\left(p_{sc}^{U} - p_{nc}^{U}\right), \quad c=1,2, \\ D_{sc}^{U} &= \frac{(\alpha_c+\delta\alpha_{3-c})}{1+(\alpha_c+\delta\alpha_{3-c})} - p_{sc}^{U} + \theta_c\left(p_{nc}^{U} - p_{sc}^{U}\right), \quad c=1,2, \end{aligned}$$

where δ is the spillover parameter satisfying $0 \leq \delta \leq 1$. Clearly, the higher the power of the PL in category $3-c$ $(c=1,2)$, the higher the spillover and the higher the PL's baseline demand in category c. Therefore, when implementing an umbrella-branding strategy, the PL's baseline demand in category 1 increases by

$$\Delta S_{s1} = \frac{\delta\alpha_2}{(1+\alpha_1+\delta\alpha_2)(1+\alpha_1)}.$$

Using our example of the two independent categories of detergent and frozen juice, our spillover assumption is simply stating that the higher the PL's baseline demand (a proxy measure of the brand's attractiveness, quality, popularity, etc.) in the detergent category, the higher is its attractiveness in the orange-juice category. The rationale for this relationship is that consumers use the information acquired about the brand in one category in their perceptual assessment of the same brand in the other category. Hence, umbrella branding confers onto new products using the same name a certain awareness, goodwill and association that are already established in the market. More specifically, the higher is the PL1 power with respect to NB1, the lower is the variation of the PL1 baseline due to UB. However, a higher PL2 power with respect to NB2 boosts the impact of UB on the PL1 baseline. Indeed, we have the following derivatives:

$$\begin{aligned} \frac{\partial\Delta S_{s1}}{\partial\alpha_1} &= -\frac{\delta\alpha_2(2\alpha_1+\delta\alpha_2+2)}{(\alpha_1+1)^2(\alpha_1+\delta\alpha_2+1)^2} < 0 \\ \frac{\partial\Delta S_{s1}}{\partial\alpha_2} &= \frac{\delta}{(\alpha_1+\delta\alpha_2+1)^2} > 0. \end{aligned}$$

We notice that the two demand functions are nested. Indeed, setting $\delta = 0$ in the UB demand functions leads us to the demands in the NO UB scenario. Although our model could also easily allow for negative spillover, we only consider the situation where the spillover effect is non negative. The situation where there is a negative effect is excluded because the retailer would obviously not implement UB or would stop using it after realizing that it is harmful. Actually, an empirical observation of positive spillover is provided in Wang et al. (2007), where they find all spillover effects to be positive across the five retained categories.

3.3.3 Profit-Maximization Problems

Assuming profit-maximization behavior by channel members, the optimization problems of the retailer and two manufacturers before the PL's introduction into category 2 read as follows:

$$\begin{aligned}
\max_{p_{n1},p_{n2},p_{s1}} \pi_R &= [(p_{n1}-w_{n1})\,D_{n1}+p_{s1}D_{s1}]+(p_{n2}-w_{n2})\,D_{n2}-F^N \\
\max_{w_{n1}} \pi_{M_1} &= w_{n1}D_{n1}, \\
\max_{w_{n2}} \pi_{M_2} &= w_{n2}D_{n2},
\end{aligned}$$

where F^N represents the retailer's merchandising cost, which is assumed to be fixed. In the UB scenario, this cost is denoted F^U, with $F^U \leq F^N$. DeGraba and Sullivan (1995) explained that extending a brand name to other categories provides a stock of information about the product's quality and reduces the need for advertising. Also, using the same brand name lowers the promotional costs and facilitates the brand trade.

After the PL's introduction into category 2, the optimization problems of the retailer and the two manufacturers in the NO UB scenario are given by

$$\begin{aligned}
\max_{p^N_{n1},p^N_{n2},p^N_{s1},p^N_{s2}} \pi^N_R &= \left[\left(p^N_{n1}-w^N_{n1}\right)D^N_{n1}+p^N_{s1}D^N_{s1}\right]+\left[\left(p^N_{n2}-w^N_{n2}\right)D^N_{n2}+p^N_{s2}D^N_{s2}\right]-F^N \\
\max_{w^N_{n1}} \pi^N_{M_1} &= w^N_{n1}D^N_{n1}, \\
\max_{w^N_{n2}} \pi^N_{M_2} &= w^N_{n2}D^N_{n2}.
\end{aligned}$$

After the PL's introduction into category 2, the optimization problems of the retailer and the two manufacturers in the UB scenario are given by

$$\begin{aligned}
\max_{p_{n1}^U, p_{n2}^U, p_{s1}^U, p_{s2}^U} \pi_R^U &= \left(p_{n1}^U - w_{n1}^U\right) D_{n1}^U + p_{s1}^U D_{s1}^U + \left(p_{n2}^U - w_{n2}^U\right) D_{n2}^U + p_{s2}^U D_{s2}^U - F^U, \\
\max_{w_{n1}^U} \pi_{M_1}^U &= w_{n1}^U D_{n1}^U, \\
\max_{w_{n2}^U} \pi_{M_2}^U &= w_{n2}^U D_{n2}^U.
\end{aligned}$$

3.3.4 Private-Label and National-Brand Types

The parameters α and θ allow us to consider different types of PLs and NBs. Before precisely specifying these links, let us recall the possible positioning of PLs (see, e.g., Burt 2000; Kumar and Steenkamp 2007).

Generic PL: Low-quality PLs for which retailers cannot establish loyalties (Parker and Kim 1997). Examples are the A&P Saving Plus line and Great Value from Wal-Mart. Another group of PLs related to generics are **second-tier** store brands whose image could be improved but which are aimed at consumers who are highly price-sensitive and do not care much about brand, e.g., Carrefour Group's One brand (Fernandez and Gomez 2005). These PLs are the cheapest among all types of PLs, their objective being to expand the store's customer base, and they offer large discounts of 20% to 50% below leading NBs. They have poor quality and are less visible on the shelves (Kumar and Steenkamp 2007).

Copycats: Also called me-too PLs. They compete with NBs by targeting the same segments and offer similar attributes to mimic leading NBs. Examples are ChipMates from Kroger, which imitate Chips Ahoy! in the cookies category. Their objective is to increase retailer's negotiating power and the PL's share in the category profits. The discount on these brands is moderate (up to 25%) and they are positioned very close to NBs on the shelves (Kumar and Steenkamp 2007).

Premiums: These brands are differentiated from NBs by offering high quality PLs and targeting separate segments from those targeted by leading NBs. Examples include President's Choice from Loblaws or Sam's Choice from Wal-Mart. Regular premiums are called "premium lite" brands and super-premiums, which may have a higher quality and even a higher price than NBs are called "premium price" brands (Kumar and Steenkamp 2007). Examples of super-premiums are the Eating Right and O Organics brands offered by Safeway (Amrouche and Yan 2012).

Recalling that α is a measure of the PL's strength compared to the NB and that θ is the differential-price sensitivity parameter, our model represents all the above concepts except super-premiums, as we assume that the baseline NB demand in category 1 is higher than the PL demand in the same category ($\frac{1}{1+\alpha_1} > \frac{\alpha_1}{1+\alpha_1}$). Indeed, we have the following (qualitative) configuration:

	α	θ
Generics	*low*	*low*
Nonsense	*low*	*high*
Premiums	*high*	*low*
Copycats	*high*	*high*

Kumar and Steenkamp (2007) and Mullik-Kanwar (2004)[6] explain the evolution of each PL concept and help categorize the level of (α, θ) as low or high. For instance, though generics were more successful to compete against NBs during the 80's and early 90's, by offering a cheaper choice and forcing NBs to lower their prices (Mullik-Kanwar 2004), they have lost nowadays shelf space and importance to copycats and premiums (Kumar and Steenkamp 2007). Unfortunately, the old strategy led to missed opportunities since they were not considering untapped needs (Mullik-Kanwar 2004). Nowadays, however, retailers are offering more diversification and a clear positioning of each concept. This classification will be helpful in interpreting the results in the next section.

6. Mullik-Kanwar, M. (2004). The evolution of private label branding. Retrieved from www.brandchannel.com.

3.4 Equilibrium Pricing Strategies

We solve a Stackelberg game between the manufacturers and the retailer. The manufacturers move first and simultaneously announce their transfer prices (w_{n1}, w_{n2}). Knowing this, the retailer selects the price-to-consumer for three products when the PL is not introduced in category 2 $(p_{n1},\ p_{n2},\ p_{s1})$ and the price-to-consumer for four products when the PL is implemented in category 2 $(p_{n1}, p_{n2}, p_{s1}, p_{s2})$. To determine a subgame-perfect equilibrium, we solve as usual in the reverse order to obtain the follower (retailer) reaction functions to the manufacturers' announcements and then solve a Nash game between the manufacturers.

3.4.1 Benchmark Equilibrium

Proposition 2 *Assuming an interior solution, the unique subgame-perfect Stackelberg equilibrium is given by*

$$\begin{aligned} p_{n1} &= \frac{2\theta_1\left[2+(1+\theta_1)(1+\alpha_1)\right]+3}{4(1+\alpha_1)(1+\theta_1)(1+2\theta_1)} \\ p_{s1} &= \frac{\theta_1(1+\alpha_1)+\alpha_1}{2(1+\alpha_1)(1+2\theta_1)} \\ w_{n1} &= \frac{1}{2(1+\alpha_1)(1+\theta_1)} \\ p_{n2} &= \frac{3}{4}, \quad w_{n2}=\frac{1}{2}. \end{aligned}$$

Proof. See Appendix. ■

These results call for the following observations: (i) All prices are strictly positive, and therefore the solution is indeed interior. (ii) The retailer sells the NB in category 1 at a higher price than his PL. This result is not surprising given our assumption that the NB enjoys a higher baseline market potential than do the PLs. (iii) The difference in both retail prices is negatively related to the relative power of the PL to that of the NB. Indeed, we have

$$\frac{\partial(p_{n1}-p_{s1})}{\partial\alpha_1} = -\frac{(6\theta_1+5)}{4(1+\alpha_1)^2\left(2\theta_1^2+3\theta_1+1\right)} < 0.$$

The result implies that the higher the PL's power, the lower is the price differential. Hence, the closer the positioning of the PL to the NB in terms of popularity, attractiveness or image, then the closer is the PL's price to the NB's. In the terminology of PL, this means that offering a me-too PL justifies asking for a price that's close to the NB, compared to offering a generic PL, where the price would be expected to be much lower than the NB price. (iv) The derivative of the price differential with respect to θ_1 is given by

$$\frac{\partial\left(p_{n1}-p_{s1}\right)}{\partial\theta_1}=\frac{4\alpha_1\left(1+\theta_1\right)^2-12\theta_1-8\theta_1^2-5}{4\left(1+\alpha_1\right)\left(2\theta_1^2+3\theta_1+1\right)^2}<0.$$

This result states that the higher the θ_1, i.e., the higher the degree of consumer sensitivity to the difference in price between the NB and PL, then the closer are these prices. To reinterpret the result in the context of competition between NBs and PLs, we recall that Pauwels and Srinivasan (2004) showed that second-tier NBs experience a higher long-term sensitivity to PLs and that the reverse occurs for leading NBs. Combining our result with theirs, we can conclude that it is more likely that the retailer will position a generic PL (e.g., Great Value cereals from Wal-Mart) closer to a NB having the characteristics of a second-tier NB (e.g., Quaker cereals) rather than a leading NB (e.g., Kellogg's cereals) when price sensitivity is expected to be very high for a long time. However, Sayman et al. (2002) found empirically that, only for categories with high-quality PLs, the PL competes more aggressively (in terms of price) against leading NBs rather than secondary (or second-tier) NBs. Combining our result with theirs, we can conjecture that if the retailer proposes a high-quality PL, then the PL should be a me-too brand (e.g., ChipMates cookies) competing strongly in terms of price with a leading NB (e.g., Chips Ahoy! cookies) rather than with a premium PL. The rationale is that a premium PL is positioned in a distinct region of the perceptual map from a leading NB, as they target separate segments of the market.

To conclude, the results are rather intuitive and the main value of this scenario is derived from its role as a benchmark for the two others.

3.4.2 Equilibria with a Private Label in Both Categories

As we are considering a new-product launch, our results should be interpreted while keeping in mind that α_1 is the **observed** power ratio in category 1, while α_2 is the **expected** power ratio in category 2. Similarly, the cross-price parameter θ_1 can be estimated using historical data, whereas θ_2 is hypothetical. Alternatively, we could assume that the results were generated by two experiments, which consist of launching a new PL under the same name and another one under UB in two identical retail stores located in two areas with the same characteristics.

The next proposition characterizes the equilibrium prices when the retailer introduces his new product in category 2 under a different name than the one used in category 1.

Proposition 3 *Assuming an interior solution, the unique subgame-perfect Stackelberg equilibrium in the NO UB scenario is given by*

$$\begin{aligned} p_{nc}^{N} &= \frac{2\theta_c\left[2+(1+\theta_c)(1+\alpha_c)\right]+3}{4(1+\alpha_c)(1+\theta_c)(2\theta_c+1)}, \quad c=1,2, \\ p_{sc}^{N} &= \frac{\theta_c(1+\alpha_c)+\alpha_c}{2(1+\alpha_c)(1+2\theta_c)}, \quad c=1,2, \\ w_{nc}^{N} &= \frac{1}{2(1+\alpha_c)(1+\theta_c)}, \quad c=1,2. \end{aligned}$$

Proof. See Appendix. ■

The equilibrium prices are positive and vary as follows with respect to the PL's relative power and the cross-price parameter:

$$\begin{aligned} \frac{\partial p_{nc}^{N}}{\partial \alpha_c} &< 0, \quad \frac{\partial p_{sc}^{N}}{\partial \alpha_c} > 0, \quad \frac{\partial w_{nc}^{N}}{\partial \alpha_c} < 0, \quad c=1,2, \\ \frac{\partial p_{nc}^{N}}{\partial \theta_c} &< 0, \quad \frac{\partial p_{sc}^{N}}{\partial \theta_c} > 0, \quad \frac{\partial w_{nc}^{N}}{\partial \theta_c} < 0, \quad c=1,2. \end{aligned}$$

The manufacturers' wholesale prices are decreasing in both the PLs' relative power and cross-price competition. In other words, offering a premium PL that enjoys a high reputation will be used as a negotiating tool against the NB's manufacturer. Further, the PL's relative power and cross-price competition have a positive effect on the PLs' retail prices and a negative effect on the NBs' retail prices. This means that both assets give the retailer an opportunity to ask a premium price for his PL and a closer price positioning to the NB.

Proposition 4 *Assuming an interior solution, the unique subgame-perfect Stackelberg equilibrium in the UB scenario is given by*

$$
\begin{aligned}
p_{nc}^{U} &= \frac{2\theta_c\left[2+\left(1+\theta_c\right)\left(1+\alpha_c+\delta\alpha_{3-c}\right)\right]+3}{4\left(1+\alpha_c+\delta\alpha_{3-c}\right)\left(1+\theta_c\right)\left(1+2\theta_c\right)}, \quad c=1,2,\\
p_{sc}^{U} &= \frac{\theta_c\left(1+\alpha_c+\delta\alpha_{3-c}\right)+\left(\alpha_c+\delta\alpha_{3-c}\right)}{2\left(1+\alpha_c+\delta\alpha_{3-c}\right)\left(1+2\theta_c\right)}, \quad c=1,2,\\
w_{nc}^{U} &= \frac{1}{2\left(1+\alpha_c+\delta\alpha_{3-c}\right)\left(1+\theta_c\right)}, \quad c=1,2.
\end{aligned}
$$

Proof. See Appendix. ■

The main interesting message from the above proposition is that the prices in each category now also depend on the brands' strength in the other category. Umbrella branding induces strategic interactions between the two categories, which are a priori fully independent. The derivatives with respect to the PL's relative power and to the cross-price parameter are given by

$$
\begin{aligned}
\frac{\partial p_{nc}^{U}}{\partial\alpha_c} &< 0; \quad sign\left(\frac{\partial p_{nc}^{U}}{\partial\theta_c}\right)=sign\left(2\left(1+\theta_c\right)^2\left(\alpha_c+\delta\alpha_{3-c}\right)-2\theta_c\left(3\theta_c+4\right)-3\right),\\
\frac{\partial p_{sc}^{U}}{\partial\alpha_c} &> 0; \quad sign\left(\frac{\partial p_{sc}^{U}}{\partial\theta_c}\right)=sign\left(1-\alpha_c-\delta\alpha_{3-c}\right),\\
\frac{\partial w_{nc}^{U}}{\partial\alpha_c} &< 0; \quad \frac{\partial w_{nc}^{U}}{\partial\theta_c}<0.
\end{aligned}
$$

The difference with the results obtained in the NO UB case is that the direction of the variation in retail price, with respect to substitution parameter θ_c, is no longer clear-cut. Qualitatively speaking, $\frac{\partial p_{nc}^{U}}{\partial\theta_c}$ would be negative, as it is in the NO UB scenario, if the spillover parameter δ is small enough, i.e.,

$$
\delta<\frac{2\theta_c\left(3\theta_c+4\right)+3-2\alpha_c\left(1+\theta_c\right)^2}{2\left(1+\theta_c\right)^2\alpha_{3-c}}.
$$

The relationships between the prices and δ are as follows:

$$
\frac{\partial p_{nc}^{U}}{\partial\delta}<0, \quad \frac{\partial p_{sc}^{U}}{\partial\delta}>0, \quad \frac{\partial w_{nc}^{U}}{\partial\delta}<0, \quad c=1,2,
$$

that is, a higher spillover allows the retailer to negotiate lower wholesale prices in both categories and to price his PLs higher.

An important question in supply chain is how the pricing strategies of the manufacturers and retailers interact. Their decisions are said to be strategic complements (substitutes) if, when one increases, the other increases (decreases). The interest in this question lies in the fact that the actual pricing decisions and the payoffs depend on the type of strategic interactions between the players (see, e.g., Bulow et al. 1985; Moorthy 1988). In our case, these relationships are given by the following derivatives:

$$\begin{aligned}
\frac{\partial p_{n1}(w_{n1},w_{n2})}{\partial w_{n1}} &= \frac{\partial p_{n2}(w_{n1},w_{n2})}{dw_{n2}} = \frac{1}{2} > 0; \frac{\partial p_{n1}(w_{n1},w_{n2})}{\partial w_{n2}} = \frac{\partial p_{n2}(w_{n1},w_{n2})}{\partial w_{n1}} = 0, \\
\frac{\partial p_{s1}(w_{n1},w_{n2})}{\partial w_{n1}} &= \frac{\partial p_{s1}(w_{n1},w_{n2})}{\partial w_{n2}} = \frac{\partial p_{s2}(w_{n1},w_{n2})}{\partial w_{n1}} = \frac{\partial p_{s2}(w_{n1},w_{n2})}{\partial w_{n2}} = 0.
\end{aligned}$$

These results show that the retail price of the NB in category c is increasing in the wholesale price of that brand and is independent of all other wholesale prices. This result is not surprising given our assumption of independent categories. Also note that the PLs' retail prices are independent of the NBs' wholesale prices.

3.4.3 Comparison of the Two Branding Strategies

Comparing the results in Propositions 2 and 3 leads to the following interesting observations. In the NO UB scenario, the wholesale and retail prices in category c are independent of the brands' relative power in category $3-c$, that is, the two categories can be managed separately. As seen from Proposition 3, the equilibrium UB wholesale and retail prices in category c include the term $\alpha_{3-c}, c=1,2$. By using the same name for his two products, the retailer is altering the competitive map by strategically relating the pricing of the two NBs (versus PLs), which are *a priori* independent.

The next proposition compares the equilibrium prices in the different scenarios.

Proposition 5 *The equilibrium wholesale and retail prices in the different scenarios compare as follows:*

$$\text{Category 1: } \left\{ \begin{array}{c} w_{n1}^{U} < w_{n1}^{N} = w_{n1}, \\ \\ p_{n1} = p_{n1}^{N} > p_{n1}^{U}, \\ \\ p_{s1} = p_{s1}^{N} < p_{s1}^{U}, \end{array} \right\} \qquad \text{Category 2: } \left\{ \begin{array}{c} w_{n2}^{U} < w_{n2}^{N} < w_{n2}, \\ \\ p_{n2} > p_{n2}^{N} > p_{n2}^{U}, \\ \\ p_{s2}^{N} < p_{s2}^{U}. \end{array} \right\}$$

Proof. See Appendix. ■

The introduction of a PL product in category 2, leads to a lower NB wholesale prices in this category. This result is in line with Narasimhan and Wilcox (1998), Mills (1995) and Bontems et al. (1999) who found that the retailer is always getting better deals when he introduces a PL, and ultimately decreases the NBs' prices. Note that the decrease in the wholesale price is more pronounced when the retailer uses UB. The new insight obtained here is that UB also leads to a lower NB wholesale price in the other category, i.e., $w_{n1}^{U} < w_{n1}^{N}$. Put differently, UB increases the bargaining power of the retailer against a manufacturer whose product has a priori nothing to do with the products offered in a different and independent category. This is the main strategic feature of umbrella branding. The direct implication of lower NB wholesale prices is the decrease in the retail prices. This is expected, given the strategic complementarity between retail and wholesale prices. In an empirical study of the pasta and oats categories, Chintagunta et al. (2002) found the opposite, that is, that the NB's price is higher when a PL is introduced. Indeed, the authors found that price elasticities increase in magnitude after a PL introduction. The retail price of NBs drops for all brands under study except for the Floresta brand. This finding could be attributed to the manufacturer's ability to enhance the value of its products by offering more varieties to consumers. In general, Chintagunta et al. (2002) found results that are consistent with previous studies, where an increase in price sensitivity due to a PL introduction puts downward pressure on retail prices and ultimately on wholesale prices. Exceptionally, Floresta's manufacturer seemed to "behave in a less accommodating fashion" after Dominick's stores launched their PL. Indeed, the PL introduction led to an increase in retail prices and to a higher increase in wholesale prices.

Regarding the PLs' prices, we obtain that the retailer charges a higher price under UB in both categories. One would then expect the demand for the PLs to be lower in the umbrella-branding scenario than in the NO UB one. However, we obtain

$$D_{sc}^{U} - D_{sc}^{N} = \frac{\delta\alpha_{3-c}\left(2+\theta_c\right)}{4\left(1+\alpha_c+\delta\alpha_{3-c}\right)\left(1+\alpha_c\right)\left(1+\theta_c\right)} \geq 0, \quad c=1,2.$$

This means that the potential loss in demand due to a higher price more than compensated for the increase in the market potential baseline of the PLs due to UB. This result is consistent with Raju et al. (1995), who found that the PLs can gain greater sales without lowering their prices. This increase in the demand for PLs is borrowed from the NBs, whose demands in the different scenarios are related as follows:

$$\begin{aligned} D_{n2}^{U} &= \frac{\theta_2+2\left(\alpha_2+\delta\alpha_1\right)\left(1+\theta_2\right)}{4\left(1+\alpha_2+\delta\alpha_1\right)\left(1+\theta_2\right)} \leq D_{n2}^{N} = \frac{1}{4\left(1+\alpha_2\right)} \leq D_{n2} = \frac{1}{4}, \\ D_{n1}^{U} &= \frac{\theta_1+2\left(\alpha_1+\delta\alpha_2\right)\left(1+\theta_1\right)}{4\left(1+\alpha_1+\delta\alpha_2\right)\left(1+\theta_1\right)} \leq D_{n1}^{N} = D_{n1} = \frac{1}{4\left(1+\alpha_1\right)}. \end{aligned}$$

Recalling that we assumed the purchasing cost of the PLs to be zero, the margins in the different scenarios compare as follows: [7]

$$\begin{aligned} p_{s1} &= p_{s1}^{N} < p_{s1}^{U}, \quad p_{s2}^{N} < p_{s2}^{U}, \\ \left(p_{nc}^{U} - w_{nc}^{U}\right) &\leq \left(p_{nc}^{N} - w_{nc}^{N}\right) \leq \left(p_{n2} - w_{n2}\right). \end{aligned}$$

A first conclusion is that choosing to market both PLs under the same name is a double-facet strategy and the retailer needs to carefully trade-off between the two opposite effects demonstrated above. More specifically, the UB increases the appeal (the baseline demand) for the PL in both categories, which allows the retailer to enjoy higher demands for his brands with higher margins, at the expense of the demands and margins for NBs. However, what remains to be seen is how the profits compare under the two strategies.

7. Indeed, we have

$$\begin{aligned} \left(p_{n2}^{N} - w_{n2}^{N}\right) - \left(p_{n2} - w_{n2}\right) &= -\frac{\left(\theta_2+\alpha_2\left(1+\theta_2\right)\right)}{4\left(1+\alpha_2\right)\left(1+\theta_2\right)\left(2\theta_2+1\right)} \leq 0 \\ \left(p_{nc}^{U} - w_{nc}^{U}\right) - \left(p_{nc}^{N} - w_{nc}^{N}\right) &= -\frac{\alpha_1\delta}{4\left(1+\alpha_2\right)\left(1+\alpha_2+\delta\alpha_1\right)\left(1+\theta_2\right)\left(1+2\theta_2\right)} \leq 0 \end{aligned}$$

3.5 Profitability of Umbrella Branding

The NB manufacturers' profits in the different scenarios are given by

$$\begin{aligned}
\text{PL in only category 1} \quad &: \quad \pi_{M_1} = \frac{1}{8\left(1+\theta_1\right)\left(1+\alpha_1\right)^2}, \quad \pi_{M2} = \frac{1}{8}, \\
\text{PL in both categories (NO UB)} \quad &: \quad \pi^N_{M_c} = \frac{1}{8\left(1+\theta_1\right)\left(1+\alpha_1\right)^2}, \quad c = 1, 2, \\
\text{PL in both categories (UB)} \quad &: \quad \pi^U_{Mc} = \frac{1}{8\left(1+\theta_c\right)\left(1+\alpha_c+\delta\alpha_{3-c}\right)^2}, \quad c = 1, 2.
\end{aligned}$$

For the manufacturers of NBs, the introduction of a PL under a different name is detrimental to their profits, and it is even worse when the retailer adopts an umbrella-branding strategy. Indeed, straightforward computations lead to the following ranking of profits: [8]

$$\pi^U_{M_c} \leq \pi^N_{M_c} \leq \pi_{M_c}, \quad c = 1, 2.$$

The above result is a direct consequence of the previous ones, namely, that UB implies lower retail prices and a lower demand for NBs. Faced with such outcome, the manufacturers should then find suitable counterstrategies to the introduction of store brands, and particularly, to UB strategies.

We now turn to the crucial issue of the retailer's performance. Recall that the retailer is the player who actually has the option of implementing an umbrella strategy or not. The

8. Indeed, the difference in profits is given by

$$\begin{aligned}
\pi_{M_1} &= \pi^N_{M_1}, \\
\pi^N_{M_2} - \pi_{M_2} &= -\frac{\alpha_2\left(2+\alpha_2\right)\left(1+\theta_2\right)+\theta_2}{8\left(1+\theta_2\right)\left(1+\alpha_2\right)^2} \leq 0, \\
\pi^U_{M_c} - \pi^N_{M_c} &= -\frac{\delta\alpha_{3-c}\left(2\left(1+\alpha_c\right)+\delta\alpha_{3-c}\right)}{8\left(1+\theta_c\right)\left(1+\alpha_c\right)^2\left(1+\alpha_c+\delta\alpha_{3-c}\right)^2} \leq 0, \quad c = 1, 2.
\end{aligned}$$

differences in profit with respect to the benchmark scenario are given by

$$
\begin{aligned}
\pi_R - \pi_R^N &= \frac{\theta_2 + 2\alpha_2(1-\theta_2) - 2\theta_2^2(1+2\alpha_2) - \alpha_2^2\left(3 + 2\theta_2^2 + 5\theta_2\right)}{16(1+\alpha_2)^2(1+\theta_2)(1+2\theta_2)}, \\
\pi_R - \pi_R^U &= \frac{4(\theta_1+\alpha_1)^2 + 2\theta_1 + 4\theta_1\alpha_1(\theta_1\alpha_1 + 2\alpha_1 + 2\theta_1) + 1}{16(1+\alpha_1)^2(1+\theta_1)(1+2\theta_1)} + \frac{1}{16} \\
&\quad - \frac{2\theta_1(1+2\theta_1) + 4(\alpha_1+\delta\alpha_2)^2(1+\theta_1)^2 + 8\theta_1(\alpha_1+\delta\alpha_2)(1+\theta_1) + 1}{16(1+\alpha_1+\delta\alpha_2)^2(1+\theta_1)(1+2\theta_1)} \\
&\quad - \frac{2\theta_2(1+2\theta_2) + 4(\alpha_2+\delta\alpha_1)^2(1+\theta_2)^2 + 8\theta_2(\alpha_2+\delta\alpha_1)(1+\theta_2) + 1}{16(1+\alpha_2+\delta\alpha_1)^2(1+\theta_2)(1+2\theta_2)}.
\end{aligned}
$$

These highly nonlinear expressions involve all the model's parameters, namely, the cross-price coefficients (θ_1, θ_2), the relative power of PLs (α_1, α_2) and the spillover parameter δ. Unfortunately, the signs of the above expressions cannot be analytically determined. Therefore, we resort to numerical analysis to get some insight into our only pending research question, namely, when UB is in the best interest of the retailer. Schematically, the differences in profit are the result of the trade-offs between three items, namely: (i) the incremental profits realized on PL sales; (ii) the savings on the fixed cost; and (iii) the losses on the NBs. Let us disregard the fixed-cost differential term, i.e., $F^N - F^U$, and focus on the other elements that are more strategic in nature. Actually, if we find that the umbrella-branding strategy is profit-improving without accounting for this cost term, then there is no need to consider it. Otherwise, it will provide a lower bound for the profitability of UB.

Recall that, by construction of the model, the bounds on the parameters are given by

$$
0 \leq \alpha_c \leq 1, \quad 0 \leq \theta_c \leq 1, \quad 0 \leq \delta \leq 1, \quad c = 1, 2.
$$

Our numerical simulations are conducted as follows: We fix the values of δ and $\theta_c, c = 1, 2$, and discretize the interval of admissible values of the parameters $\alpha_c, c = 1, 2$, using a mesh size of 0.001. Therefore, for each vector $(\delta, \theta_1, \theta_2, \alpha_1, \alpha_2)$, we have a grid of one million points. For each point, we compute $\pi_R - \pi_R^N$ and $\pi_R - \pi_R^U$. The result of the comparison of these two differences lies in one of the following regions in the (α_1, α_2)-space:

$$\begin{aligned} \text{No PL introduction (Region 1)} &: \quad \pi_R \geq \pi_R^N \quad \text{and} \quad \pi_R \geq \pi_R^U, \\ \text{PL introduction under NO UB (Region 2)} &: \quad \pi_R^N \geq \pi_R \quad \text{and} \quad \pi_R^N \geq \pi_R^U, \\ \text{PL introduction under UB (Region 3)} &: \quad \pi_R^U \geq \pi_R \quad \text{and} \quad \pi_R^U \geq \pi_R^N. \end{aligned}$$

We conducted a high number of runs, and in all cases, we obtained a pattern that can generically be represented by Figure 3.1, which is drawn in the (α_1, α_2)-space for fixed values $\delta = 0.1$ and $\theta_1 = \theta_2 = 0.1$, that is, the upper-right side is region 3, the lower-right side is region 2 and the left side is region 1. Figures 3.2 and 3.3 illustrate how the three regions shown in Figure 3.1 vary when the values of the spillover parameter and the cross-price coefficients are changed.[9] More specifically, the panels in the first row of Figure 2 correspond to a low spillover value ($\delta = 0.1$), whereas the second row's panels are drawn for a high spillover value ($\delta = 0.3$). The panels in Figure 3 show the changes in the three regions for different competitive-structure configurations in the two categories. For illustration purpose, we retain three values for θ_1 and θ_2, namely, 0.1, 0.3 and 0.6. A high value of θ_c corresponds to the case where the PL is of the copycat variety. A low value can designate either a generic, if α is low, or a premium, if α is high. Note that the panels on the diagonal in Figure 3.3 correspond to a situation where the competitive intensity between the NB and the PL is the same in both categories.

Our numerical results allow for the following general observations:

1. In all simulations, the (α_1, α_2)-space is divided into **continuous** parts, with the right-hand-side part corresponding to the union of regions 2 and 3 (i.e., launching a PL is profitable), and the left-hand side corresponding to the region where it is not profitable to add a PL in category 2. This result tends to imply that there is a minimal value for α_2, say $\tilde{\alpha}_2$, below which launching a new PL is not profitable. Clearly, this threshold depends on the other parameters, namely, the spillover δ and the cross-price coefficient θ_2 in category 2. Interestingly, the competition intensity in category 1, measured by θ_1, does not seem to play an important role in determining $\tilde{\alpha}_2$. This can be seen by inspecting the three panels in each column in Figure 3, where θ_1 is varied, while

9. Results for any parameter values can be provided by the authors upon request.

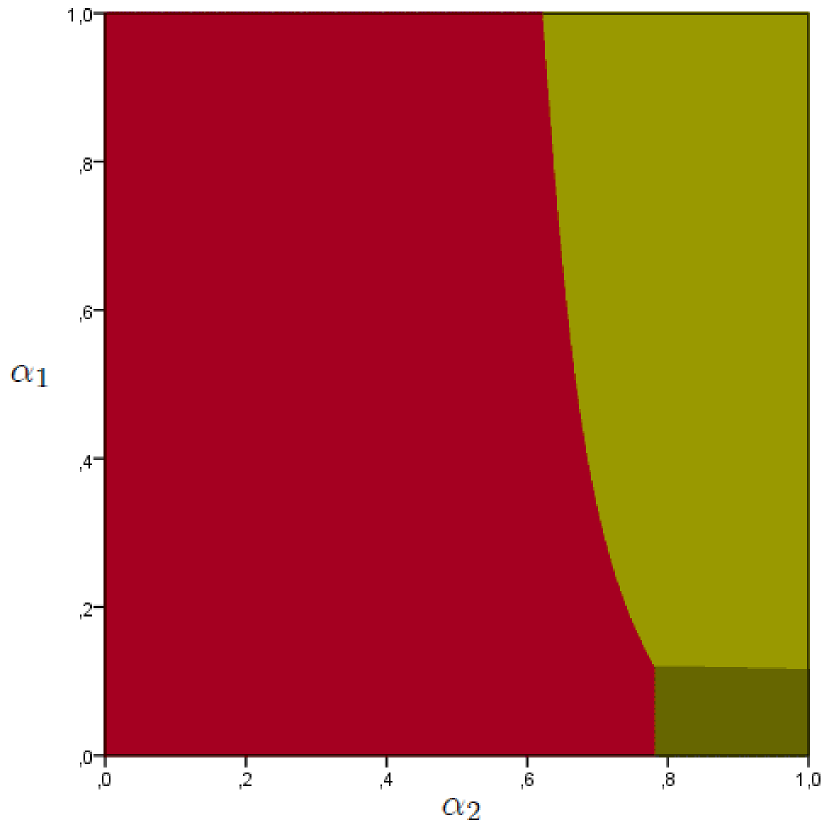

Figure 3.1 Definition of regions

θ_2 and δ are kept constant. The same variation in cross-price parameters leads to a completely different impact on the location of the three regions. A higher θ_1 promotes the implementation of the UB strategy without significantly expanding the regions where it is beneficial for the retailer to introduce the PL. In contrast, a higher θ_2 leads to a significant reduction in the region where the PL's introduction is not profitable. To wrap up, our first conclusion is that it takes a minimum market potential for a new PL launch to be successful and profit-improving, and that this minimum is lower for a higher spillover δ and when the retailer launches a copycat. Note that the **lowest** value for $\tilde{\alpha}_2$ that we obtained in our simulations is slightly larger than 0.20. Such a minimal level may still be quite demanding if the NB has an excellent reputation and is fairly priced.

2. Even when the UB strategy induces only positive spillovers between the categories, there is still a region in the (α_1, α_2)-space, albeit a small one, where distinct branding is preferred to umbrella branding. Recall that our assessment was carried out under the assumption that UB does not involve a cost saving for the retailer. If UB involves a cost reduction (e.g., in merchandising or advertising), then we would expect the regions where UB is optimal to expand accordingly.

In the following claims based on Figures 3.2–3.3,[10] we refine our interpretations regarding UB versus NO UB strategies.

Claim 6 *Implementing a distinct-branding strategy can be optimal only if the existing PL is generic.*

To "prove" this claim, we first mention that in none of our simulations did we observe an instance where (i) distinct-branding strategy dominates the two other strategies (no PL launch and UB), and (ii) α_1 is relatively high (say above 0.15). Second, the only cases where we observed a small zone where NO UB is optimal are when θ_1 is low (0.1) or medium (0.3). In this last case, the region in question is very small. As a generic PL is characterized by both a low α and a low θ, we arrive at the conclusion stated in the claim. The main contribution of this result is in providing the **necessary** conditions for a NO UB strategy to be potentially optimal.

10. We call these results claims instead of propositions, because they are based on numerical simulations and not formal proofs.

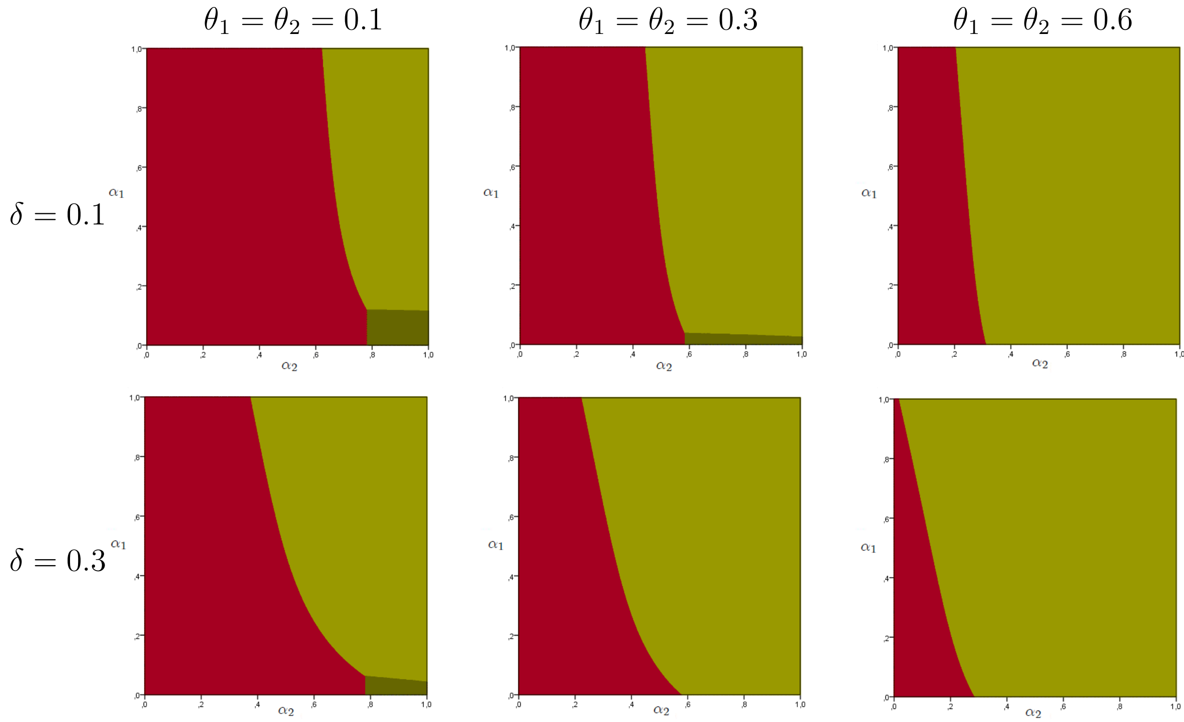

Figure 3.2 Varying the spillover parameter in a symmetric competitive structure

Claim 7 *If the two categories exhibit the same competitive structure, i.e., $\theta_1 = \theta_2 = \theta$, then, when optimal, distinct branding involves a generic PL in category 1 and a new premium PL in category 2.*

From Claim 1, we know that distinct branding necessarily involves a generic in category 1. All cases for which we obtained a (small) zone where distinct branding is optimal involve a high α_2 and a low θ, that is, a premium PL according to the characterization of the different PL types provided above. The managerial implication is clear. A retailer targeting the generic segment in one category and the premium one in the other, is advised to use different names. This is common sense. Indeed, as mentioned in the introduction, the rationale for umbrella branding is to simplify advertising and to transfer value from one category to another. This cannot be done when the retailer is differently positioning his brands in the different categories.

The next claim straightforwardly extends the analysis to the case where the two categories differ in terms of product substitutions.

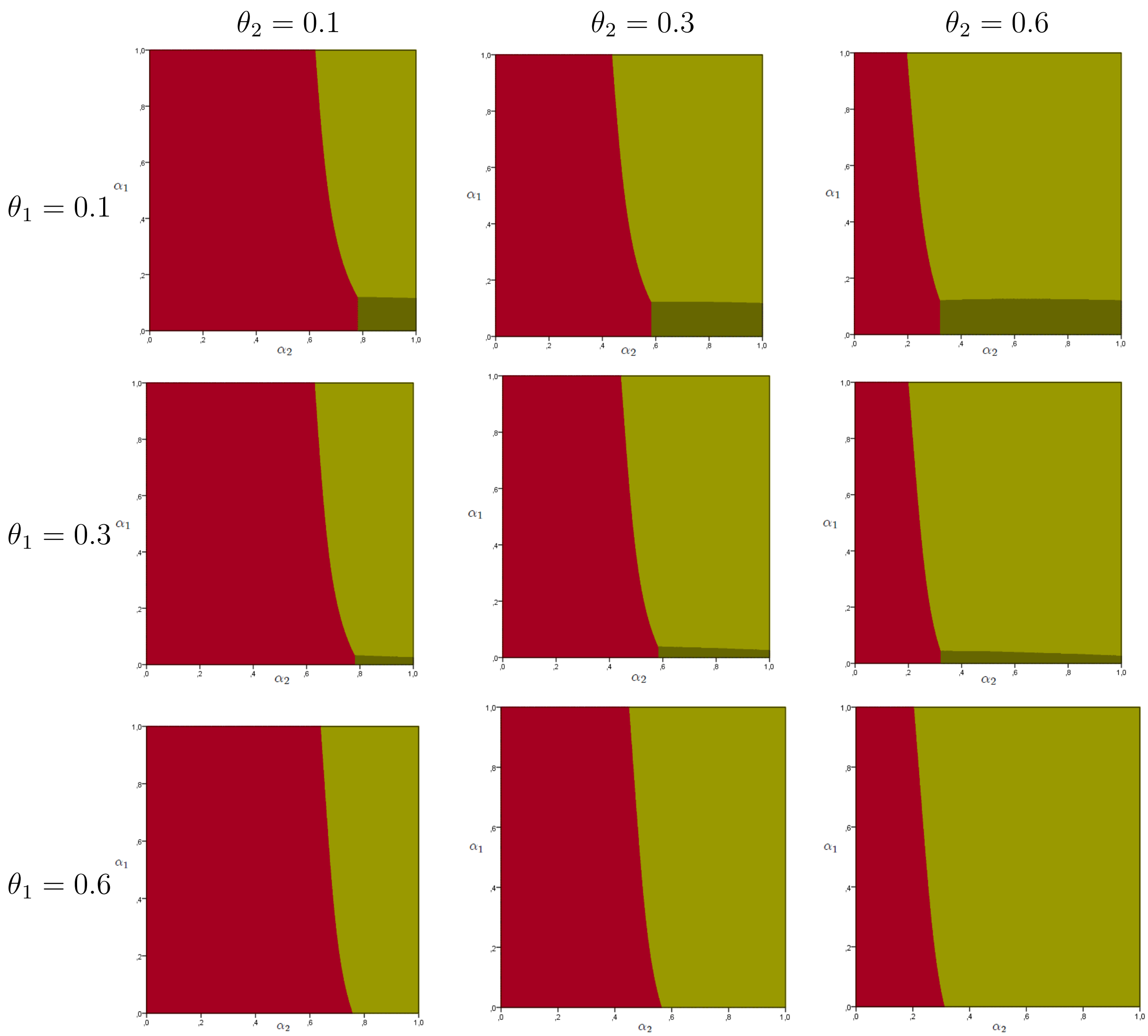

Figure 3.3 Varying the competitive structure ($\delta = 0.1$)

Claim 8 *When the competitive structure is different in the two categories, Region 2 increases in size with θ_2 and shrinks when θ_1 is higher. See Figure 3.3.*

Analyzing Figure 3.3, it seems that a high cross-price competition in category 1 and low cross-price competition in category 2 lead to no chances of introducing PL2 under a different name and the smallest region to implement UB. However, under opposite circumstances, the retailer has room for more situations to introduce a PL in category 2 under a distinct name. This result is in line with Claim 1, stating that a necessary condition to use NO UB is first offer a generic PL that requires a low θ_1.

Claim 9 *Increasing the value of the spillover parameter δ:*

(i) significantly enlarges the region where launching a new PL is profitable;

(ii) significantly shrinks the region where NO UB is optimal.

The impact of the spillover parameter can easily be seen in Figure 3.2 for symmetric categories.[11] These results are related to those obtained earlier. Indeed, we found that UB allows the retailer to extract lower wholesale prices from NB manufacturers in **both** categories and to increase the price of his PLs without decreasing the PLs' demands. As the benefits of UB increase with δ, the reverse is expected to hold for a NO UB strategy. Given that an increase of the spillover has a strategic impact on the retailer's decision, research should investigate the determining factors that could boost such a spillover (e.g., PL positioning, product characteristics, retailer format, etc.).

Claim 10 *Very high θ_1 in the symmetric and asymmetric competitive structure gives exclusive use of the UB strategy.*

In particular (see Figure 3.3) for me-too PLs (high level of cross-price effects combined with high level of power ratios-observed in category 1 and expected in category 2), UB is always a winning strategy for the retailer. Indeed, me-too PLs have dual benefits: (i) the close positioning to the NB in terms of quality helps them take advantage of the positive image association to the NB and ultimately results in more sales for PLs (Lassar et al. 1995); and (ii) at the same time, they enjoy a positive spillover between PLs from different categories under the same name.

3.6 Conclusion

Previous research has focused on the umbrella strategy, using the following: experiments (see e.g., Martinez and Pina 2003; Aaker and Keller 1992); conjoint analysis (see e.g., Nijssen and Agustin 2005); surveys (see e.g., Laforet 2008); modeling at the firm level (see e.g., Hakenes and Peitz 2008; Montgomery and Wernerfelt 1992; Degraba and Sullivan 1995); regression analysis (see e.g., Wang et al. 2007). But, to our knowledge, no study has used a game-theory setting involving the retailer and the manufacturers. Additionally, few papers have studied this strategy when the retailer offers NBs along with PLs. This paper therefore

11. The results are very similar in the asymmetric case and are omitted to save on space.

makes a number of contributions. We investigate the use of an umbrella strategy in the context of NBs competing against PLs. We also take into account the interaction of the NB manufacturers with the retailer to shape their decisions in terms of: (1) introducing or not a PL in a new category; (2) implementing an umbrella strategy or using a separate name for the new PL; and, (3) choosing the optimal pricing strategies under different settings. More specifically, the paper explores whether choosing the same brand name for PLs could be detrimental for the retailer even if there is a positive-spillover effect between the products in the two different categories. As pointed out above, the answer is yes in some instances. The findings of this paper can therefore be used as a strategic dashboard for the retailers, to make more effective branding decisions.

This research could be extended in different directions. One could study the effectiveness of joint promotions along with the choice of umbrella strategy for PLs. Indeed, Wang et al. (2007) explained that retailers should take advantage of the positive correlation between products under the same name and maximize their profits by capitalizing on joint promotions. Also, one could consider a dynamic model where the PL's reputation evolves over time due to an investment in quality, for instance. Further, it may be of interest for NB manufacturers to look at potential channel-coordination solutions to avoid the losses when the umbrella strategy is used by the retailer. Finally, it is of interest to NB manufacturers to investigate what type of counterstrategy they can implement to minimize the damage done by an umbrella strategy.

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3.8 Appendix: Proofs of Propositions

3.8.1 Proof of Proposition 1

Assuming an interior solution, we first determine the retailer's reaction functions from the first-order-optimality, that is,

$$\begin{aligned}
p_{n1}(w_{n1}, w_{n2}) &= \frac{w_1(2\theta_1+1)(1+\alpha_1)+\theta_1(1+\alpha_1)+1}{2(1+\alpha_1)(2\theta_1+1)}, \\
p_{s1}(w_{n1}, w_{n2}) &= \frac{\theta_1(1+\alpha_1)+\alpha_1}{2(1+\alpha_1)(2\theta_1+1)}, \\
p_{n2}(w_{n1}, w_{n2}) &= \frac{1}{2}w_3+\frac{1}{2}.
\end{aligned}$$

Substituting in the manufacturers' problems and optimizing leads to, after some straightforward calculations to,

$$\begin{aligned}
w_1 &= \frac{1}{2(1+\alpha_1)(1+\theta_1)}, \\
w_3 &= \frac{1}{2}.
\end{aligned}$$

Substituting for the transfer prices in the retailer's reaction functions gives the retail prices in the Proposition. Note that all prices are strictly positive, and therefore, the solution is indeed interior.

3.8.2 Proof of Proposition 2

The retailer's reaction functions from the first-order-optimality are:

$$\begin{aligned} p_{nc}^{N}\left(w_{nc}^{N}, w_{nc}^{N}\right) &= \frac{w_c\left(1+2\theta_c\right)\left(1+\alpha_c\right)+\theta_c\left(1+\alpha_c\right)+1}{2\left(1+\alpha_c\right)\left(1+2\theta_c\right)}, c=1,2, \\ p_{s1}^{N}\left(w_{n1}^{N}, w_{n2}^{N}\right) &= \frac{\theta_c\left(1+\alpha_c\right)+\alpha_c}{2\left(1+\alpha_c\right)\left(1+2\theta_c\right)}, c=1,2. \end{aligned}$$

Substituting in the manufacturers' problems and optimizing leads to, after some straightforward calculations to,

$$w_{nc}^{N*} = \frac{1}{2\left(1+\alpha_c\right)\left(1+\theta_c\right)}, \quad c=1,2.$$

Substituting for the transfer prices in the retailer's reaction functions gives the retail prices in the Proposition.

3.8.3 Proof of Proposition 3

Follows the same steps as Proposition 4, while replacing α_c by $\alpha_c + \delta\alpha_{3-c}$.

3.8.4 Proof of Proposition 4

Straightforward computations lead to the following differences:

$$\begin{aligned} w_{n1}^{N} &= w_{n1}, \quad w_{n2}^{N} - w_{n2} = -\frac{\alpha_2+\theta_2\left(1+\alpha_2\right)}{2\left(1+\alpha_2\right)\left(1+\theta_2\right)} \leq 0, \\ w_{nc}^{U} - w_{nc}^{N} &= -\frac{\delta\alpha_{3-c}}{2\left(\alpha_c+\delta\alpha_{3-c}+1\right)\left(1+\alpha_c\right)\left(1+\theta_c\right)} \leq 0, \quad c=1,2, \\ p_{n2}^{N} - p_{n2} &= -\frac{\left[\alpha_2+\theta_2\left(1+\alpha_2\right)\right]\left(4\theta_2+3\right)}{4\left(1+\alpha_2\right)\left(1+\theta_2\right)\left(2\theta_2+1\right)} \leq 0, \\ p_{nc}^{U*} - p_{nc}^{N*} &= -\frac{\delta\alpha_{3-c}\left(4\theta_c+3\right)}{4\left(\alpha_c+\delta\alpha_{3-c}+1\right)\left(1+\alpha_c\right)\left(1+\theta_c\right)\left(1+2\theta_c\right)} \leq 0, \quad c=1,2, \\ p_{sc}^{U*} - p_{sc}^{N*} &= \frac{\delta\alpha_{3-c}}{2\left(\alpha_c+\delta\alpha_{3-c}+1\right)\left(1+\alpha_c\right)\left(1+2\theta_c\right)} \geq 0, \quad c=1,2. \end{aligned}$$

General Conclusion

The progress achieved in statistical software, data collection and storage techniques has increased the importance of quantitative modeling in marketing research. The wide variety of techniques and modeling approaches has provided practitioners and researchers with the opportunity to a deeper understanding of various phenomena and to overcome renewable marketing challenges. This thesis contributes to extend quantitative marketing literature by developing new models applied to three topical issues, namely, bidders' behavior in online auctions, customer relationship management in the diffusion of subscription services, and marketing channel in presence of private labels. The modeling framework of this research incorporates three modeling approaches: response modeling, dynamic programming and game theory.

In the first paper, we investigated the late-bidding behavior in online auctions. The results showed a notable difference between extremely late-bidders or snipers and moderately late-bidders. Snipers are the oldest and the most active members. The prevalence of these bidders is not affected by entry-deterrence factors (reserve price, opening price and level of competition). These results provide some interesting managerial implications for sellers and auctioneers. Since snipers do not aim essentially to cut selling price, eBay managers do not have interest to change the ending rules of the auction. Indeed, switching from fixed end-time to automatically extended end-time (such in iGavel and TradeMe) could eliminate late-bidding but will increase operation costs [12] without guaranteeing better revenue. Likely, sellers can use a secret-reserve price to impose a minimum price for their objects. However, we show that this strategy do not allow to deter snipers. On the contrary, a secret reserve-price reduces significantly the number of other bidders and therefore the auction income.

12. An automatic extension leads a high increase on the number of ongoing auctions.

To conclude, it is clear that late-bidding became a common practice in online auctions, but this does not represent any serious threat for sellers and auctioneers. The success and sustainability of eBay confirm our statement.

This study can be extended in several directions. First, it would be interesting to use panel data to investigate whether some bidders are late for all the auctions in which they participate, or if they are only occasional late-bidders, i.e., whether their strategy is auction specific. By using panel data, we would be also able to model the interdependence between bidders' behavior. In other words, the first bid timing of a specific bidder could depend on the first bid timing of the other bidders. Second, studying more product categories and different auction ending rules would surely be a welcome move to check the generalizability of our findings. Finally, form a technical perspective, models used (Poisson and negative-binomial models) dismiss the underdispersion hypothesis of data. It would be interesting to consider other count-data models such zero-inflated models, hurdle models, etc.

The second paper focused on the role of CRM expenditures in the diffusion of subscription services. In this sector, customer relationship duration affects hugely the service growth and the customer profitability. We proposed a new diffusion model that incorporates acquisition and retention expenditures. By using dynamic programming, we introduced an innovative approach to calculate optimal acquisition and retention spending in order to maximize the customer equity. Our results reveal that the optimal customer equity represents the sum of the value of existing customers and the value of the remaining market. Further, we show that the marginal customer equity, given by the difference between the customer lifetime-value and the prospect lifetime-value, plays a crucial role in the determination of the optimal CRM expenditures.

Through our empirical results, we confirm the positive impact of CRM expenditures on the service growth and the significant presence of external factors influencing acquisition and retention processes. These external factors are market specific. Moreover, we show that underspending on CRM may be less critical than overspending in some market context.

The main contribution of this paper is fourfold: (i) incorporating CRM considerations in the growth of subscription services, (ii) considering the impact of external incentives in the determination of optimal CRM spending, (iii) illustrating optimal expenditures sensitivity toward marketing effectiveness, margin, discount rate, and depth of external factors, (iiii) analysing situations in which customer retention spending is more critical than customer acquisition spending and vice versa.

We mention three possible extensions for our work. First, the present research assumed that individual acquisition and retention costs do not vary with the penetration rate. As several researchers have shown that innovators are less price sensitive than other consumers, the acquisition cost of early adaptors should be low compared to laggards who are more resistant to change. Likewise, the retention behavior could change with the penetration rate. Based on these remarks, an interesting extension to this work would be to assume that marketing effectiveness varies with the penetration rate. Second, our model does not distinguish between new customers and won-back ones in terms of their acquisition cost. In reality, it comes less expensive to acquire a new subscriber than a lost one having probably a negative perception toward the service. In this sense, the model might be extended by treating won-back customers differently. Finally, our study is developed in a non-competitive market. Though the external incentives parameters could capture the effect of marketing efforts of other firms, competitive framework with strategic interactions might yield additional insights on the role of CRM expenditures in the diffusion of subscription services.

Finally, in the third essay, we focused on marketing channel interactions in the presence of private labels. We looked into the impact of adopting an umbrella branding strategy on the retailer's performance. This strategy consists in using the same name to market different products which may, or may not, be related. We take into account the interaction of the NB manufacturers with the retailer to shape their decisions in terms of: (1) introducing or not a PL in a new category; (2) implementing an umbrella strategy or using a separate name for the new PL; and, (3) choosing the optimal pricing strategies under different settings. The main finding are the following: (i) by implementing UBS, the retailer succeeds in lowering the wholesale price of the NBs and consequently their retail prices; (ii) there is no interesting for NBs' manufacturers to see their retailer implementing an umbrella strategy; and lastly (iii) there are indeed instances where the retailer is better off not implementing UBS and should rather use a distinct name for his new PL.

This research could be extended in different directions. One attempt is to study the effectiveness of joint promotions along with the choice of umbrella strategy for PLs. Indeed, retailers should take advantage of the positive correlation between products under the same name and maximize their profits by capitalizing on joint promotions. Also, we could consider a dynamic model where the PL's reputation evolves over time due to an investment in quality, for instance. Further, it may be of interest for NB manufacturers to look at potential channel-coordination solutions to avoid the losses when the umbrella strategy is used by the retailer. Finally, it is of interest to NB manufacturers to investigate what type of counterstrategy they might implement to minimize the damage done by an umbrella strategy.