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THE EVOLUTION OF THE PORTFOLIO THEORY: AN APPLICATION OF RISK PARITY TO EQUITY PORTFOLIO CONSTRUCTION

By

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Summary

In this dissertation we implement the approach of Anderson, Bianchi and Goldberg (2012) and test the benefits of applying the Risk Parity approach to equity portfolio construction. We evaluate the performance of five strategies - unlevered RP, levered RP, levered RP adjusted for trading costs, 1/n strategy and value weighted market portfolio- over a long sample period and three different subperiods. We consider several hypotheses for borrowing costs, transaction costs and volatility measures. Our main findings suggest that the Risk Parity approach to asset allocation, generates higher returns compared to the other strategies. In addition, transaction costs can negate the outperformance of the levered Risk Parity portfolio. Our results support the previous literature on the outperformance of the Risk Parity approach.

Keywords: Portfolio Selection, CAPM, Fundamental Indexing, Risk Parity Portfolio

Résumé

Le présent mémoire porte sur l'implémentation de l'approche des risques à parité pour construire un portefeuille d'investissement avec actions, tel que décrit par Anderson, Bianchi et Goldberg (2012). Nous avons évalué la performance de cinq stratégies d'investissement : parité de risque sans levier, parité de risque avec levier, parité de risque avec levier et coûts de transaction, ainsi que la performance d'un portefeuille de marché et d'un portefeuille à pondération égale. Nous avons estimé la performance sur une longue période (1934-2015), et sur trois sous-périodes (1934-1970, 1971-1999, 2000-2015). Nous avons considéré l'impact des plusieurs hypothèses concernant les coûts de transaction, les coûts d'emprunt ainsi que différents niveaux de volatilité. Nos principaux résultats suggèrent que l'approche de la parité de risque à l'allocation du capital, génère des rendements plus élevés par rapport aux autres stratégies. Toutefois, les coûts de transaction et d'emprunt, peuvent avoir un impact négatif sur la performance de l'approche des risques à parité avec levier. En conclusion, nos résultats confirment les études antérieures sur la performance positive de l'approche de la parité des risques.

Mot clés : Sélection de Portefeuille, MEDAF, Indexation Fondamentale, Approche des Risques à Parité.

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Introduction

In 1952, Harry Markowitz revolutionized the field of finance with his ground-breaking article entitled *Portfolio Selection*. Markowitz argued that the objective of a rational investor is to find the set of efficient portfolios that have the greatest expected return for a given level of risk. However, subsequent research has shown that Markowitz model is sensitive to input parameters. Furthermore, estimation errors in the forecasts significantly impact the resulting portfolio weights. Sharpe (1964) and Lintner (1965) develop the CAPM by adding two additional assumption to the Markowitz model. In the CAPM framework, all investors choices are aggregated into market portfolio that must contain all the marketable assets that are provided in equilibrium. As a consequence, the optimal investment strategy is to hold the market portfolio, which corresponds to a cap-weighted index. Cap-weighted indexes play an important role in the investment industry due to the benefits that they provide, such as representativeness, efficiency and track record. These indexes represent the benchmark to be beaten by active managers. However, cap-weighted indexes face also several criticisms such as momentum bias, growth bias and lack of diversification. It is in this context that alternative weighted indexes, such as the fundamental indexation, have gained a great interest. The fundamental indexation refers to the construction of indexes in which assets are weighted by price-insensitive fundamental measures such as book-value, income, gross dividends, revenues, sales and total company employment. The empirical research has shown that the fundamental indexation generated superior returns compared to cap-weighted market portfolios for the US and European market (Arnott, 2005 and Hemminki and Puttonen, 2008). Estrada (2008) extends the research to 16 global countries and concludes that the fundamental indexation delivers higher returns and has lower volatility than cap-weighted indexes.

The financial crises of 2008 profoundly changed the industry of asset management, by putting risk management at the heart of most investment processes. With the lower risk tolerance of investors, a new risk-based investment style emerged: the Risk Parity (RP) approach to asset allocation.

The concept of RP [1] was pioneered over twenty years ago, when Bridgewater Associates introduced the All Weather asset allocation strategy in 1996. The principles for their asset allocation strategy have been published in an article by Ray Dalio (2004), founder of Bridgewater Associates, in which he describes how a portfolio needs to be structured to approach a 10% return with an annual standard deviation of 10 - 12%.

RP refers to an investment strategy that consists in equalizing or carefully targeting the risk contribution of each asset in the portfolio. The unlevered RP is a fully invested strategy so that the ex-post risk contributions from the asset classes are equal. However, in order to achieve a better risk-adjusted performance, we lever this strategy to match the ex-post volatility of the value weighted portfolio.

The underlying idea of RP is to build a portfolio in which the investor specifies the risk allocation between assets of the portfolio, without any consideration of expected returns. However, in order to successfully apply this strategy, it is necessary to have some stability within the risk component [2]. The constraint of having a predictable systematic risk behaviour has lead the academic research to test the performance of RP on an asset class level, essentially bond/equities (Qian 2005, Chaves, Hsu, Li, Shakernia, 2011, Scherer 2012).

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[1] RP is one of the three budgeting techniques in asset allocation. The other two are: the weight budgeting and the performance budgeting approaches. In the weight budgeting strategy, we directly define the portfolio weights. For instance, in a 30/70 policy rule, the weight of asset one is 30% and the weight of asset two is 70%. The performance budgeting approach consists in calibrating the assets weights' in order to achieve some performance contribution. For example, if we target a portfolio return of 10%, in accordance with our previous example, we would like the performance contribution of the assets to be 3% and 7%, respectively.

[2] Contrary to the Markowitz approach, we only consider the risk dimension, due to the consideration that the performance dimension is too complicated to encompass because of the non-robustness of the forecasting process. This is the reason why we focus on the patterns of the portfolio risk. In accordance, with the empirical research on RP, the volatility of returns is considered as the risk measure.

The empirical research has shown that the RP approach generates higher risk-adjusted returns than a 60/40 strategy (Asness, Frazzini and Pederson, 2012) and provides better diversification in terms of risk allocation (Qian, 2005 and Chaves, Hsu, Li, Shakernia, 2011).

There is substantial body of literature confirming the outperformance of the RP approach. However, one difficulty when comparing these different studies is the heterogeneity of portfolio construction. For instance, most studies use an annual frequency, while others use semi-annual or monthly portfolio rebalancing. Other differences concern the asset class used in portfolio construction, or the different assumptions concerning borrowing and trading costs. In this context, it is difficult to draw conclusions as of the superiority of one methodology with respect to the other portfolio construction approaches.

The previous empirical research focuses on evaluating the performance of the RP approach on an asset class level, such as equities and bonds, however, the purpose of this dissertation is to evaluate the performance of RP on a more granular portfolio construction. In fact, the main objective of this dissertation is to investigate the performance of RP on the Fama and French US equity sector data. The need to have a predictive systematic behaviour within the risk component, has lead us to consider US equity sector data and not single stock returns. In order to investigate the possible value in risk parity, we use two sets of data, the 10 and 49 industry portfolios with monthly returns from February 1934 to December 2015, encompassing over eight decades of US equity sector data.

This dissertation is interested in assessing the performance of an unlevered and levered Risk Parity strategy relative to a value-weighted market portfolio and to a 1/n investment strategy. More specifically, this paper will evaluate the performance of these strategies under three sets of assumptions about transaction costs, borrowing costs and asset class inclusions. We evaluate the performance of these strategies over a long sample period, February 1934 to December 2015, and three different sub-periods (1934-1970, 1971-1999, 2000-2015). The first sub-period, which includes years after the Great Depression and World War II, was characterized by periods of inflationary spikes and uneven equity markets.

The second sub-period, begins with the first energy crisis, is characterized by high interest rates, inflationary spikes and the emergence of the technology. The third subperiod, starts with the burst of the dotcom bubble and comprises also the last financial crises.

We simulate portfolio returns of five monthly-rebalanced strategies – unlevered Risk Parity, levered Risk Parity, levered Risk Parity adjusted for trading cost, 1/n strategy and value-weighted market portfolio. Following Anderson, Bianchi and Goldberg (2012), the levered RP is rebalanced so that its ex post volatility over a three-year window matches the ex post volatility of the value weighted strategy at monthly rebalancing date. For each strategy, we report monthly compounded returns, annualized excess returns, annualized volatility and Sharpe ratio. In addition, we also report standard statistics such as skewness, excess kurtosis, and as well as the Jarque-Bera test.

Furthermore, we investigate whether the returns generated by these strategies are explained by their exposure to known risk factors. For each of the strategies, we examine estimates of alphas after regressing the returns on the three Fama and French (1992) risk factors in addition to the momentum factor suggested by Carhart (1997).

For the strategies constructed using 49 industry portfolios, we provide two additional performance measures such as the Sortino and Price ratio (1994) and Smetters and Zhang (2013) statistic. These two additional performance statistics allow us to evaluate strategy returns by taking into consideration higher moments of returns and providing a connection to the utility function of investors.

Our analysis establishes the main following result, that the Risk Parity approach to asset allocation generates higher returns than the value-weighted market portfolio, for the whole sample period and three different sub-periods. However, borrowing and trading costs, can negate the outperformance of the levered RP.

The remainder of this paper is structured as follows: Section 2 reviews the evolution of the portfolio theory. Section 3 describes the data, variables and the hypothesis used to construct the strategies. Section 4 gives an overview of the Risk Parity approach to asset allocation, as well as the methodology used to evaluate portfolio performance.

Section 5 presents and interprets the results, while Section 6 outlines the robustness tests and their results. Finally, section 7 concludes.

2 Literature Review

Harry Markowitz, in his 1952 article *Portfolio Selection* proposes a mathematical procedure to determine the optimal investment portfolio. The paper is acclaimed for generating a revolution in the theory of finance and laying the foundations for modern capital market theory. The model is derived under the assumptions that investors are efficient, risk averse and single-period utility maximizers. Portfolio decisions are made on the basis of mean and standard deviation of returns. Indeed, the objective of the portfolio analysis is to find the set of efficient portfolios which have the greatest expected return for a given level of risk. As a consequence, the Markowitz model is called mean variance model.

The collection of all efficient portfolios composes a curve in the risk-return space called the efficient frontier. The variance of expected returns (coherent with Fisher,1965) is adopted as a measure of economic risk.

Central to the Markowitz's model is the idea of portfolio diversification. Through diversification risk can be reduced but not completely eliminated. Indeed, for an investor is not important the security's own risk but rather is important the contribution that the security makes to the variance of the entire portfolio. As a consequence, securities can be properly evaluated only as a group and not standalone.

Tobin (1958) showed that in the presence of a risk-free asset, the efficient frontier becomes a straight line and the optimal portfolio (the tangency portfolio) is a combination of a risk - free asset and an efficient portfolio.

After Markowitz developed the Mean-Variance model, researchers began investigating the implications of this model on financial markets. These empirical investigations took two different directions.

On one hand, many researchers began criticizing the Mean-Variance approach. For instance, the model is very sensitive to input parameters (Chopra and Ziemba, 1993) and the first moments are much harder to estimate with considerable accuracy than the second moments (Merton, 1980).

In addition, Green and Hollifield (1992) show that mean-variance portfolios are not necessarily well diversified, while Michaud (1998) shows that portfolio optimizers are often error maximizers. Furthermore, Black and Litterman (1991, 1992) established that mean variance optimization can produce extreme or non - intuitive weights for some of the assets in the portfolio.

On the other hand, Sharpe (1964) and Lintner (1965) developed the Capital Asset Pricing Model (CAPM) by adding two additional assumptions to the MV approach. The first assumption consists in borrowing and lending at the risk-free rate, which is assumed to be the same for all investors and does not depend on the amount borrowed or lent. The second assumption refers to homogeneous expectations which results in a total agreement on the probability distribution of asset returns. So the efficient frontier, which consists of linear combinations of the market portfolio and the risk free rate is the same for all investors due to homogenous expectations regarding each securities ex-ante expected return, variance and covariance during the universal planning horizon. These uniform expectations are called sometimes idealized uncertainty.

In the CAPM, all investors portfolio choices are aggregated into a market portfolio that must contain all marketable assets in the proportions that they are supplied in equilibrium. However, investors have different utility functions and as a consequence they will select different portfolios which will involve a long or short position in the risk-free asset. According to Tobin's separation theorem, all investors should hold the same mix of risky securities in their optimum portfolio, and then use borrowing and lending at the risk- free rate to attain the desired level of risk class.

The Sharpe-Lintner CAPM states that the expected return on any asset is the risk-free rate plus a risk premium, which is the asset's market beta times the premium per unit of beta risk (Fama and French, 2004). The market beta of an asset measures the sensitivity of the asset's return to variations in the market return. Beta is mathematically measured by the covariance of the asset return with the market return divided by the variance of the market returns. According to the CAPM, the variability in the security and portfolio expected returns will be explained only by changes in the market beta and other variables should have no explanatory power. Essentially, in the CAPM framework, the best investment strategy is to hold the market portfolio, which corresponds to capitalization-weighted indexes, since it is impossible to beat the market without taking on more risk. This conclusion is strengthened by the efficient market hypothesis, developed by Fama (1965), which asserts that prices reflect all the available information. Subsequently, one cannot beat the market and consistently achieve excess returns with respect to average market returns. Thus, the problem of optimal portfolio construction is reduced to simply buying and holding a capitalization-weighted index. It was John McQuown who developed the first institutional cap-weighted index fund at Wells Fargo in 1970 (Bernstein, 2007). The cap-weighted indexes offer several advantages such as the representativeness, the efficiency, and the track record of such a portfolio construction (Roncalli, 2011).

However, many empirical studies have rejected the mean-variance efficiency of the CAPM portfolio and as a consequence the cap-weighted indexation is not the optimal portfolio. In fact, the empirical evidence shows that the CAPM is poor enough to invalidate the way it is used in empirical applications (Fama and French, 2004). On one hand, recent research has brought into question the usefulness of the CAPM as an indicator of expected returns. Several variables have been associated with abnormal returns and ratios involving stock prices which have information about expected returns that are missed by market betas.

Basu (1977) showed that, during the sample period of April 1957 to March 1971, the low price-earning portfolios have, on average, earned higher absolute and risk-adjusted rates of returns than the high price-earning portfolios. Banz (1981) documents the size effect for the period 1936-1975. In fact, stocks with lower market capitalization have, on average, higher risk-adjusted returns than stocks with higher market capitalization. Furthermore, Brown, Kleidon and Marsh (1983) also report a size effect on the NYSE for the period 1969-1979. While Bauman, Conover and Miller (1998) assess the size effects in countries such as, Europe, Australasia and Canada over the examination period 1986 to 1996.

Bhandari (1988) observed that leveraged firms, such as firms with high debt-to- equity ratio are associated with returns that are too high relative to their market beta.

The second category of criticism that the CAPM faces involves the portfolio construction. Since the cap-weighted index is a trend-following strategy, it incorporates momentum bias which leads to bubble exposure risk as the weight of the best performers increases and the weight of the worst performers decreases (Roncalli, 2011). As a consequence, the cap-weighted index overweights overvalued stocks and underweights undervalued stock that can lead to a lack of risk diversification.

Jegadeesh and Titman (1993) documented the profitability of momentum strategies. In fact, they show that strategies consisting in buying stocks that have performed well in the past and sell stocks that have performed poorly, generated positive returns over the 1965-1989 period. Moreover, Jegadeesh and Titman (2001) demonstrate that these momentum strategies generated profits also in the 1990s.

Treynor (2005) asserts that market-valuation indexes are better than cap-weighted indexes because they avoid the problem of overweighting and underweighting implicit to cap-weighted.

Mayers (1976) states that the mean-variance efficient portfolio should include all risky assets and not just stocks. Moreover, Markowitz (2005) observes that once the real-world constraints are taken into consideration in the construction of the market portfolio, such a portfolio is no more mean-variance optimal.

Fama and French (1992) update and synthesize the evidence on the empirical failures of the CAPM. Using a cross-section regression approach, they identify a value premium in the US stock market for the period 1963-1990. They document that stocks with high ratios of the book value of equity to the market value of equity (value stocks) have higher average returns than stocks with low book-to-market ratios (growth stocks). Fama and French (2006) extend the results of their paper of 1992. In fact, they reject CAPM portfolios formed on size, book-to-market and Beta for the period 1928-1963 and for the period 1963-2004. They conclude that it is size and book-to-market ratio, or risk related to them, and not Beta, that are rewarded in average returns.

Research has also been conducted in order to show that the CAPM anomalies are not sample specific but they are present in many capital markets other than the USA. Chan, Hamao and Lakonishok (1991) document a strong relation between book-to-market equity (B/M) and average return in Japanese market. Capaul, Rowley and Sharpe (1993) observe a similar effect in four European stock markets and in Japan. Fama and French (2012) find value premiums in average returns in all four regions they examine (North America, Europe, Japan, Asia and Pacific), and there are strong momentum returns in all regions except Japan.

In addition, Siegel (2006) and Hsu (2006) assert that if prices are noisy and subject to unpredictable and temporary shocks the cap-weighted portfolio is sub-optimal.

In conclusion, the CAPM faces three main empirical challenges (Bornholt, 2013). The challenges are the beta anomaly, the value anomaly and the momentum anomaly. The synthesis on evidence and empirical problems provided by Fama and French (1992) marks the point where is generally acknowledged that the CAPM has fatal problems. This suggests that it should be possible to construct stock market indexes that are more mean-variance efficient than those based on market capitalization (Arnott et al., 2005).

Arnott et al. (2005) propose a new way in constructing indexes by weighting assets by measures other than the market capitalization. They propose the fundamental indexation (F.I.) which refers to the construction of indexes in which assets are weighted by price-insensitive fundamental measures such as book value, income, gross dividends, revenues, sales and total company employment. The Fundamental Indexation has its roots in the noisy market hypothesis developed by Siegel (2006). This paradigm states that prices do not always reflect a firm's true value since they are subject to unpredictable shocks or noise. These shocks obscure a securities value and are originated from speculators and momentum traders, insiders, institutions which trade for different reasons rather than a securities fundamental value.

Arnott et al. (2005) construct fundamental weighted indexes of 1000 US companies for the examining period 1962-2004. They found that the fundamental index has superior returns compared to the S&P 500 Index and to the cap-weighted benchmark constructed from the same sample over the 43-year period. They calculate also a composite index which outperforms the S&P 500 Index and the cap-weighted benchmark by 1.94% and 2.12% per annum respectively. In addition, Arnott et al. (2005) find that the fundamental index is more liquid and offers higher risk-adjusted returns when compared to the cap-weighted benchmark. Furthermore, the authors assess the performance of the fundamental indexes in different sub-periods. The authors find that the fundamental index outperforms by 6.73% per annum in bear markets and 0.34% per annum in bull markets. They conclude that the performance of fundamental index is robust across the phases of the business cycle, across bear and bull markets, across raising and falling interest rate regimes.

Hemminki and Puttonen (2008) examine the benefits of fundamental indexation using European data for the sample period January 1996 to December 2006. The firm metrics considered in the construction of these indexes are book value of equity, total employment, sales, cash flow and dividends. The cap-weighted market portfolio used to be compared to the fundamental portfolios was the DJ Euro Stoxx 50 Index. The research paper concludes that the fundamental index approach is able to produce consistently higher returns and risk-adjusted returns. Moreover, the results suggest that an investor should use the book-value of equity or the dividend amount as metrics in the construction of the fundamental index. Furthermore, the paper concludes that when managing fees and transaction costs are expected to be the same, the fundamental index should consistently outperform the cap-weighted index with reference to net returns.

Estrada (2008) extends the research to international capital markets and addresses the relationship between fundamental indexation and international diversification. In fact, his main objective was to evaluate whether capitalization, price-insensitive fundamentals or other measures are the best way to weight country index funds. In the study, 16 countries are taken into examination over the sample period 1973-2005 and the price insensitive fundamental variable considered is the dividend per share. The main results of the study indicate that the Dividend Weighted Index has higher return, lower volatility and higher risk-adjusted return than the cap-weighted index. In addition, Estrada (2008) concludes that while the fundamental index approach appears to be more mean-variance efficient than the cap-weighted index, no additional international diversification benefits can be explored through this method.

Walkhausl and Lobe (2011) provide a worldwide assessment of fundamental weighted portfolios on a global and country level for the sample period 1982-2008. By applying a bootstrap procedure for robust performance testing and data snooping control on the data sample of 50 developed and emerging markets, they confirm the previous literature results on the superiority of fundamental index.

However, the fundamental index approach has many drawbacks. First of all, it requires periodic rebalancing which triggers taxes and transaction costs. Moreover, these indexes do not represent properly the investable opportunity set, neither reflect returns of the average investor nor are market-clearing portfolios. Asness (2006) sustains that the fundamental indexation is a value investing strategy in a different guise, while Perold (2007) challenges the noisy market hypothesis and as a consequence undermines the logic in which this approach is based. Blitz and Swinkles (2008) argue that fundamental indexes are essentially a new breed of value indexes. In addition, they sustain that this approach resembles more to an active investment strategy than to a traditional passive investment.

Kaplan (2008) sustains that the F.I. is flawed because ignores risk and expected returns implicit in the market prices. In addition, he sustains that the F.I. is an alternative way to introduce value bias in the portfolio and since value-biased portfolios have outperformed unbiased portfolios it is no surprise that a fundamentally index outperforms a market-cap index of the same stock.

An important principle at the core of most portfolio construction techniques, such as mean-variance, CAPM and fundamental indexing, is the insight that diversification pays. In fact, it is well known that diversification is the only free lunch in financial markets. Through diversification risk can be reduced but not generally eliminated while constituting a portfolio with uncorrelated assets.

There are different views on the meaning of portfolio diversification. For instance, Woerheide and Persson (1993) focus on portfolio weights while Frahm and Wiechiers (2013) focus on portfolio return variance. Portfolio diversification is at the core of most traditional investment strategies. Nevertheless, in distressed times, different market segments can become increasingly correlated and as a consequence the risk contribution of certain assets in the portfolio increases. For instance, with the recent financial crisis, the risk contribution of equities to the total risk of the portfolio in a 60/40 equity/bond strategy, exceeded the forecasted limits due to the increased correlations with other asset classes.

Recognizing this lack of diversification that clearly emerged with the 2008 financial crisis, investors and portfolio managers have moved toward strategies that are more risk balanced.

One alternate approach that has gained a lot of attention is the Risk Parity approach, first introduced on an academic level by Qian (2005) and whose properties have been studied by Maillard, Roncalli and Teiletche (2010).

The term Risk Parity (RP) refers to an investment strategy that consists in equalizing or carefully targeting the risk contribution of each asset class in the portfolio. In order to implement this strategy, there is no need to estimate expected returns. On the other hand, RP requires estimates of volatility and other measures of risk, which can be estimated more accurately than expected returns based on historical data (Merton, 1980).

The RP is an investment strategy that addresses the equity risk concentration inherent to most asset allocation methodologies by suggesting to diversify by risk and not by dollars. In order to avoid the dominance of an asset class in driving the portfolio volatility, the RP advocates a greater investment in low-risk assets rather than in high-risk assets. Furthermore, in order to increase the expected return and risk to the desired levels, investors can apply leverage to this risk-balanced portfolio. Therefore, leverage is an essential element that distinguishes the RP in two categories: the unlevered RP and the levered RP.

Frazzini and Pedersen (2010) elaborate the theory of leverage aversion as the theoretical underpinning for the RP approach. According to this theory, no one holds the market portfolio as suggested by the CAPM, and equilibrium is achieved because some investors overweight safer assets while other overweight riskier assets. Some investors choose to overweight riskier assets in order to avoid leverage and as a result the return on those assets is reduced. In contrast, safer assets that are underweighted by these investors offer higher returns since they trade at low prices. Hence, investors that can apply leverage can obtain higher risk-adjusted returns by overweighting safer assets. Frazzini and Pedersen (2010) find consistent empirical evidence from 20 global stock markets, Treasury bond markets, credit markets and futures markets. Their findings show that the theory of leverage aversion holds up in a variety of tests across and within many asset classes. Asness, Frazzini and Pedersen (2012) further tested the theory of leverage aversion by examining a portfolio of US and global stocks, bonds, corporate bonds and commodities over the examination period 1926-2010. They find that the RP portfolio provides statistically significant Sharpe ratios in excess of Sharpe ratios for market portfolio.

Scherer (2012) investigates a risk parity strategy by considering futures data on US equities and government bonds. Consistent with Asness, Frazzini and Pedersen (2012), the author finds that the RP strategy delivers higher Sharpe ratios than a 60/40 equity/bond strategy or than the market portfolio. He also concludes that risk parity performance arises from higher returns for leveraged low-risk assets, as predicted by the leverage aversion theory.

However, Anderson, Bianchi and Goldberg (2012) don't find empirical support for the consistent dominance of the RP strategy but their results suggest that RP may be a preferred strategy under certain market conditions. They evaluated four strategies – value weighted, 60/40 equity/bond, unlevered RP and levered RP – and two asset classes of US equities and government bonds, they found that the performance depends on the backtesting period. In fact, they assessed the four strategies over a long sample period (1926-2010) and four sub-periods. They found that for the long sample the RP had the highest cumulative return, while, when considering the other sub-periods, for instance 1946-1982, the 60/40 and value weighted strategy had the highest cumulative return. Furthermore, they found that the performance depends on the assumptions about market frictions and on the measure. For instance, over the long sample Anderson, Bianchi and Goldberg (2012) found that the unlevered RP had the highest Sharpe ratio and lowest expected return. However, when the RP was levered to have the same volatility as the market portfolio, transaction costs reduced the Sharpe ratio and the cumulative returns. Chaves, Hsu. Li and Shakernia (2011) also do not confirm consistent dominance of the RP portfolio. By comparing a representative RP strategy with other asset allocation strategies – equal weighting, minimum variance, mean variance optimization and 60/40 strategy, they find that RP does not consistently outperform the equal weighting or 60/40 portfolio construction on a risk-adjusted basis. Using nine asset classes and data over the examination period 1980-2010, they find that RP outperforms minimum variance and mean variance strategies. Chaves, Hsu, Li and Shakernia (2011) evaluated the RP strategy also in terms of portfolio diversification. They concluded that the RP provides a better diversification in terms of risk allocation, confirming the findings of Qian (2005).

The authors raise several questions about the definition of an asset class for the inclusion in the portfolio, the time period chosen and the metric used to evaluate the performance. RP has additional pitfalls that needs to be addressed during portfolio construction.

Inker (2010) raises questions about the use of standard deviation as a measure of volatility and the inclusion of certain risk premia in the portfolio that may not have a positive return associated with them. Ruban and Melas (2011) discuss the impact of trading costs when introducing leverage to lower the portfolio volatility and study the conditions for this to occur. They establish that adding leverage can reduce portfolio volatility only if the correlation between asset classes is sufficiently negative.

Levell (2010) and Foresti and Rush (2010) also show that leverage introduces questions about the availability of financing, financing costs and the impact of liquidity crisis on the investors portfolio.

3 Data and Variables

The dataset comes from Kenneth French's data library and contains the value-weighted returns of 10 US industry portfolios. The 10 industries considered in the construction of the portfolio are: non-durables, durables, manufacturing, energy, high-tech, telecom, retail, health, utilities and others. The sample period ranges from February 1934 to December 2015. The goal of this paper is to evaluate the performance of an unlevered and levered Risk Parity strategy relative to a value-weighted market portfolio and to a 1/n investment strategy. The secondary goal is to evaluate the performance of these strategies under three sets of assumptions about transaction costs, borrowing costs, volatility levels for three different subperiods. The final goal of this dissertation is to evaluate the performance of RP approach considering 49 US industry portfolios. There are several approaches to compute the optimal RP portfolio, however, in this dissertation we adopt the approach proposed by Anderson, Bianchi and Goldberg (2012).

The proxy for the risk-free rate is the one-month U.S. Treasury bill rate. As a proxy for borrowing costs, for the period 1971-2015, the 3-months Eurodollar deposit rate was considered, while for the pre-1971 period, a constant of 60 bps were added to the risk free rate. For this hypothesis, the previous literature on the subject was taken into consideration. In fact, Naranjo (2009) shows that Libor has been more accurate as an estimate of the implicit rate of interest at which investors can lever by using futures. While Anderson et al. (2012) show that the three-month LIBOR and the three-month Eurodollar deposit rate track one another closely over the period of overlap. In addition, since the data for LIBOR are available from 1987 while for the Eurodollar deposit rate the data availability starts in 1971, we opted for the Eurodollar three-months deposit rate. Furthermore, Anderson et al. (2012) show that the average spread of the Eurodollar deposit rate over the risk-free rate is 100 bps. We follow the approach of Anderson et al. (2012) and we add 60 bps to risk free rate to obtain the cost of borrowing for the pre 1971 period. Moreover, transaction costs were considered when implementing the strategies. Following the approach of Anderson, Bianchi and Goldberg (2012), we formulated three different cost hypothesis for the whole sample period. For the examination period 1934 - 1955 transaction costs were assumed to be 1%, while for the subsequent period 1956-1970 transaction costs were assumed to be 0.5% and for the rest of the sample period (1971-2015) transaction costs were assumed to be 0.1%. The turnover-induced trading costs were incorporated into the returns of the risk parity strategies. The strategies are rebalanced monthly.

The performance statistics for the value weighted returns of the 10 industries are presented in the following table. Table 1 reports descriptive statistics for the sample estimates of volatility, skewness, kurtosis and Jarque-Bera test of normality. We employ monthly returns to compute the descriptive statistics and the results displayed in table 1 show that average returns are positive for the whole sample period and the three sub-periods. For the whole sample period, average monthly returns range from 0.87% to 1.11%, with telecommunications having the lowest return and health having the highest return. Furthermore, monthly returns are negatively skewed for 9 industries, and in general, this skewness persists over the subperiods. The negative skewness indicates that the mass of returns is concentrated to the right of the mean and as such, the portfolio has a tail of returns that is lower than the mean. Moreover, the negative skewness implies that the portfolio returns are not normally distributed.

However, the most interesting insight can be drawn when examining the volatility in conjunction with the time series weights of the RP strategy that are shown in page 64 of this dissertation. For instance, when examining telecommunications, we notice that in the first sub-period the volatility estimate has the lowest level, a value of 3.49 compared to 4.27 and 5.46 for the second and third subperiod. In accordance, with the RP approach at a high level of volatility the relative weight of this industry in the whole portfolio should decrease. As reported in figure 10 in page 56 of this dissertation, telecommunications have a higher weight in the first subperiod, while in the last two the relative weight decreases as the volatility increases. This characteristic can be observed also for other industries such as health. The volatility ranges from 3.95 in the third subperiod to 4.75 and 5.39 for the second and third subperiod. These results suggest a strong relationship between volatility and portfolio weights. In fact, with higher volatility levels for a certain sector, the corresponding weight in the portfolio decreases and *vice versa*. In addition, the returns of all industries, except durables, are negatively skewed over the long sample period.

Table 1. Descriptive statistics of the dataset

	NoDur	Durbl	Manuf	Enrgy	HiTec	Telcm	Shops	Hlth	Utils	Other
A. Descriptive Statistics, 1934-2015										
Average	1.01	1.03	1.00	1.06	1.08	0.87	1.05	1.11	0.88	0.94
Volatility	4.13	6.44	5.20	5.50	6.22	4.24	5.07	4.85	4.52	5.41
Skewness	-0.43	0.09	-0.25	-0.02	-0.25	-0.21	-0.34	-0.01	-0.10	-0.46
Kurtosis	2.60	4.31	4.07	2.02	1.78	2.05	3.10	2.18	2.50	3.12
Jarque-Bera test	8.35*	1.10	7.05*	0.02	1.39	1.32	7.60*	0.00	0.44	14.08*
B. Descriptive Statistics, 1934-1970										
Average	0.86	1.20	1.00	1.09	1.21	0.75	1.04	1.15	0.86	0.94
Volatility	3.86	6.39	5.40	5.37	5.66	3.49	4.82	4.75	5.03	5.56
Skewness	-0.70	0.07	0.00	-0.14	-0.32	-0.17	-0.43	-0.12	-0.06	-0.43
Kurtosis	3.09	3.49	5.27	3.16	2.43	3.30	3.76	1.41	2.81	4.06
Jarque-Bera test	14.49*	0.17	0.00	0.63	1.91	1.00	8.10*	0.09	0.09	9.50*
C. Descriptive Statistics, 1971-1999										
Average	1.25	1.04	1.14	1.19	1.33	1.41	1.25	1.27	0.94	1.19
Volatility	4.74	5.62	5.01	5.41	6.22	4.27	5.67	5.39	3.94	5.26
Skewness	-0.26	-0.24	-0.48	0.24	0.05	-0.13	-0.31	0.12	0.25	-0.47
Kurtosis	2.15	2.34	3.34	1.82	1.11	0.67	2.80	2.69	1.36	2.38
Jarque-Bera test	0.76	0.76	6.24*	0.47	0.01	0.02	1.79	0.24	0.28	2.99
D. Descriptive Statistics, 2000-2015										
Average	0.90	0.59	0.76	0.79	0.33	0.19	0.70	0.74	0.80	0.49
Volatility	3.47	7.78	5.08	5.93	7.32	5.46	4.42	3.95	4.26	5.30
Skewness	-0.51	0.41	-0.53	-0.16	-0.41	-0.18	-0.33	-0.37	-0.73	-0.52
Kurtosis	0.62	5.36	1.90	0.44	1.29	1.33	0.86	0.32	0.99	1.99
Jarque-Bera test	0.13	6.39*	1.36	0.01	0.38	0.08	0.11	0.02	0.69	1.45

Notes: this table reports the descriptive statistics of the ten industry sectors for the long sample period and the three different subperiods. The ten industries are: non-durables, durables, manufacturing, energy, hi-technology, telecommunications, shops, health, utilities and others.
*Not significant at the 5 percent level.

4 Methodology

4.1 Strategies

This section outlines the strategies implemented in this thesis and presents a detailed overview of the construction technique for the Risk Parity portfolio. Furthermore, we present the criteria used to evaluate strategies performance.

We examined the following strategies:

4.1.1 Value-weighted strategy

This fully invested strategy value weights the 10 U.S. industry portfolios. The performance of this strategy is considered as the benchmark to be beaten by the risk parity strategies and the 1/n strategy.

4.1.2 Equal-weighted strategy

The Equal - Weighted Strategy is defined as one of the most naive portfolio heuristics as there is no need for any assumptions or estimations of covariances or returns. The structure of the portfolio depends only on the number of assets because the weights are equal and uniform. It is a mean variance optimal strategy only if the expected returns and covariances among asset classes are the same. Demiguel, Garlappi and Uppall (2009) and Chow, Hsu, Kalesnik and Little (2000) show the ability of this naïve strategy to provide superior returns when applied to US and Global Equity Portfolio Construction. We implement the 1/N strategy which rebalances monthly and the portfolio weight for asset *i* is given by:

$$w_i = \frac{1}{N},$$

while the portfolio return is computed as:

$$r_p = \sum_{i=1}^{N} r_i * w_i.$$

4.1.3 Unlevered Risk Parity

The volatility of each industry portfolio is estimated at the end of month by considering a window of 36-month trailing returns. As a consequence, the time *t* volatility for industry portfolio *i* is given by the standard deviation measured over a 36-months rolling window of returns.

$$\hat{\sigma}_{i,t} = \sqrt{\sum_{i=1}^{10} \sum_{t=1}^{36} \frac{(r_{i,t} - M_r)^2}{T-1}},$$

where:

$r_{i,t} =$ is the return for asset class *i* at time *t*,

$M_r =$ is the average return over the 36 - month rolling window of observations.

The time *t* portfolio weight for asset class *i* is given by:

$$w_{i,t}^{u} = \delta_t * \hat{\sigma}_{i,t}^{-1},$$

where

$$\delta_t = \frac{1}{\Sigma_i \hat{\sigma}_{i,t}^{-1}},$$

For this strategy, we set the portfolio weights to be non-negative:

$$\sum_{i=1}^{N} w_{i,t}^{u} = 1,$$

$$0 \leq w_{i,t}^{u} \leq 1.$$

4.1.4 Levered Risk Parity

Similar to the unlevered RP, the levered RP equalizes ex-ante volatilities across all asset classes. The RP levered is rebalanced such that the ex-post volatility over a 36-months rolling window of returns, matches the ex-post volatility of the value-weighted market portfolio at each rebalancing date. The time *t* estimate of volatility for a strategy *s* is given by the following formula:

$$\hat{\sigma}_{i,t} = \sqrt{\sum_{i=1}^{10} \sum_{t=1}^{36} \frac{(r_{i,t} - M_r)^2}{T-1}},$$

where:

$r_{i,t} =$ is the return for asset class *i* at time *t*,

$M_r =$ is the average return over the 36 - month rolling window of observations.

The leverage level (l_t) required to match the trailing 36-month realized volatility of the value-weighted benchmark, is the quotient of the volatility estimate for the value-weighted portfolio ($\hat{\sigma}_{v,t}$), and the estimate for the volatility for the unlevered RP portfolio ($\hat{\sigma}_{u,t}$):

$$l_t = \frac{\hat{\sigma}_{v,t}}{\hat{\sigma}_{u,t}}.$$

The time *t* portfolio weight for asset class *i* in the levered RP is given by:

$$w_{l,i,t} = l_t * w_{u,i,t}.$$

The return of the levered RP strategy at time *t* is given by:

$$r_{l,t} = \Sigma_i w_{u,i,t} r_{i,t} + \Sigma_i (w_{l,i,t} - w_{u,i,t})(r_{i,t} - r_{b,t}),$$

where $r_{b,t}$ is the borrowing rate at time *t.*

Furthermore, in order to quantify trading costs arising from turnover, we need to express the change in portfolio weights that derives from price movements over a single period. Therefore, the time *t* modified return-modified weight to asset *i* is given by:

$$\acute{\omega}_{i,t} = \frac{(1+r_{i,t})w_{i,t-1}}{\Sigma_i (1+r_{j,t}) w_{j,t-1}},$$

and the turnover required to balance the strategy is given by:

$$x_t = \Sigma_i \left| \acute{\omega}_{j,t} - w_{j,t} \right|.$$

However, in order to show the impact of leverage in the RP strategy, the previous expression is modified as follows:

$$x_t = \Sigma_i \left| \acute{\omega}_{u,j,t} * l_{t-1} - w_{u,j,t} * l_t \right|.$$

Therefore, trading costs at time *t* are then computed as:

$$c_t = x_t * z_t,$$

where z_t refers to the different assumptions for trading costs. Consistent with Anderson, Bianchi and Goldberg (2012), z_t is assumed to be 1% for the period 1934-1970, 0.5% for the examination period 1971-1999, and 0.1% for 2000-2015.

In conclusion, trading cost-adjusted returns for the levered RP are given by:

$$r_{l,t}{}' = r_{l,t} - c_t.$$

The excess returns ($er_{s,t}$) are calculated as the difference between the returns of a given strategy *s* at time *t* ($r_{s,t}$) and the risk-free rate at time *t* ($r_{f,t}$):

$$ER_{s,t} = R_{s,t} - R_{f,t}.$$

4.2 Portfolio performance evaluation

The performance of the unlevered and levered Risk Parity portfolio is compared to a value-weighted market portfolio and to a 1/n investment strategy. In addition of the Sharpe ratio, for each strategy, the annualized excess returns and volatility over the 82 years long sample and three sub-periods, are provided. With the aim of evaluating the normality of portfolio returns, the Jarque - Bera Test as well as skewness and excess kurtosis are reported. For each of the strategies, we examine estimates of alphas after regressing the returns from the strategies on a set of four risk factors. The first three factors are those of Fama and French (1993). The first factor represents the excess return on the US equity market, the second factor (SMB) is designed to capture the risk of holding small stocks versus the risk of holding large stocks and the third factor (HML) captures the value premium. To these three factors we add the momentum factor of Carhart (1997) that captures the effect of buying winners and selling losers. A revision of the Sharpe ratio, the Jarque – Bera statistic and of the two models of performance measurements, is provided.

4.2.1 Sharpe ratio

As stated by Scholz and Wilkens (2005), in the financial literature there are almost no scientifically established conclusions regarding the performance measure that an investor should adopt in order to evaluate the investment decisions. However, in order to make a comparison between strategies on a risk-adjusted basis, Sharpe ratios, as introduced by Sharpe (1996), are computed.

Let R_t be the return of the portfolio at time t and $R_{f,t}$ be the return on the risk-free rate at time t, then the differential return is given by:

$$D_t = R_{s,t} - R_{f,t}.$$

By defining $\overline{D}$ as the average value of D_t over the historic period from t=1 through T:

$$\bar{D} = \frac{1}{T}\sum_{t=1}^{T} D_t,$$

and σ_D the standard deviation over the period t=1 through T:

$$\sigma_D = \sqrt{\frac{\left(\frac{1}{T}\sum_{t=1}^{T}(D_t - \bar{D}\,\right)^2}{T-1}}.$$

The ex post Sharpe Ratio (S_h), which indicates the historic average differential return per unit of historic variability of the differential return is given by:

$$S_h = \frac{\bar{D}}{\sigma_D}.$$

4.2.2 Jarque-Bera test

The assumption that stock returns are normally distributed is widely used in finance and it originates from the random walk theory which states that when stock prices follow a random walk then stock returns are i.i.d. The Central Limit Theorem states that if enough i.i.d. returns are collected then their limit distribution should follow a Normal one. However, there is strong empirical evidence against this hypothesis, pioneered by the following papers: Mandelbrot (1963), Fama (1963) and Clark (1963). The supposition that market returns follow a normal distribution is debatable when information does not arrive linearly to the market, or, when it does, investors do not react linearly to it. As a consequence, a leptokurtic distribution should be observed. As a matter of fact, when information does not arrive linearly, investors are forced to react similarly and as a result a leptokurtic distribution is observed. However, information may arrive promptly to the market but is ignored by investors until a certain trend is established, and even in this case a leptokurtic distribution of market returns is observed (Peters, 1991).

Therefore, in order to test the normality of returns, the Jarque-Bera (1980) statistic is examined in addition to Skewness and Kurtosis.

The null hypothesis considered in the Jarque-Bera test is that the returns follow a normal distribution against the alternative hypothesis of non-normal distribution of returns.

The Jarque-Bera test is:

$$JB = n\left[\frac{S^2}{6} x \frac{(EK)^2}{24}\right]$$

where:

$S =$ skewness

$EK =$ excess kurtosis

The test statistic is compared with a (χ^2) chi-square distribution with 2 degrees of freedom. The null hypothesis of normality of returns is rejected whenever the test statistic exceeds the critical value obtained from the $\chi^2_{(2)}$ distribution.

4.3 Models of Performance Measurement

This section briefly describes the Fama and French three factor model and the Carhart model.

4.3.1 Fama-French three-factor model

Fama and French (1993) propose a three-factor model in order to explain the cross-section portfolio returns. In their model, they consider size and book-to-market equity in addition to the market premium. They claim that, even though size and book-to-market equity are not state variables themselves, the higher average returns on small stocks and high book-to-market stocks reflect unidentified state variables that produce undiversifiable risks in returns that are not captured by market betas. In support of their statement, they show that the returns on the stocks of small firms covary more with one another than with returns on the stocks of large firms, and returns on value stocks covary more with one another than with returns on growth stock. Based on their empirical evidence, the authors propose the following model to capture patterns in returns:

$$R_{i,t} - R_{f,t} = a_i + b_{iM}[\, R_{m,t} - R_{f,t}] + b_{iS}SMB_t + b_{iH}HML_t + e_t,$$

where:

$R_{i,t}$ = the return on asset i for month t,

$R_{f,t}$= the risk-free rate at time t,

$R_{m,t}$ = the market return,

SMB_t = difference between the returns on diversified portfolios of small stocks and big stocks,

HML_t = difference between the returns on diversified portfolios of high book-to-market stocks and low book-to-market stocks.

4.3.2 Carhart Four Factor Model

The Carhart model is an extension of the Fama and French (1993) three factor model and it includes an additional factor that captures Jegadeesh and Titman's (1993) momentum anomaly. This model is motivated by the inability of the three-factor model to explain cross sectional variation in momentum-sorted portfolio returns (Fama and French, 1996). Chan, Jegadeesh and Lakonishok (1996) suggest that the momentum anomaly can be attributed to the slow reaction in cases of new information. Carhart extended the Fama and French model with a momentum factor constructed by simulating returns of a monthly strategy that consists in buying the best performing stocks by trailing 12-months returns, excluding the most recent months, and short selling the worst performing stocks. The four- factor model can be interpreted as an attribution performance model, where the coefficients and premia on the factor mimicking portfolios indicate the proportion of mean return attributable to four elementary strategies: high versus low beta stocks, large versus small market capitalization stocks, value versus growth stocks, and one year return momentum versus contrarian stocks (Carhart, 1997).

We estimate the performance relative to the four-factor model as:

$$R_{i,t} - R_{f,t} = a_i + b_{iM}[\, R_{m,t} - R_{f,t}] \; + b_{iS}SMB_t + b_{iH}HML_t + b_{im}MoM_t + e_t,$$

where:

$R_{i,t}$ = the return on asset i for month t,

$R_{f,t}$= the risk-free rate at time t,

$R_{m,t}$ = the market return,

SMB_t = is the difference between the returns on diversified portfolios of small stocks and big stocks,

HML_t = is the difference between the returns on diversified portfolios of high book-to-market stocks and low book-to-market stocks.

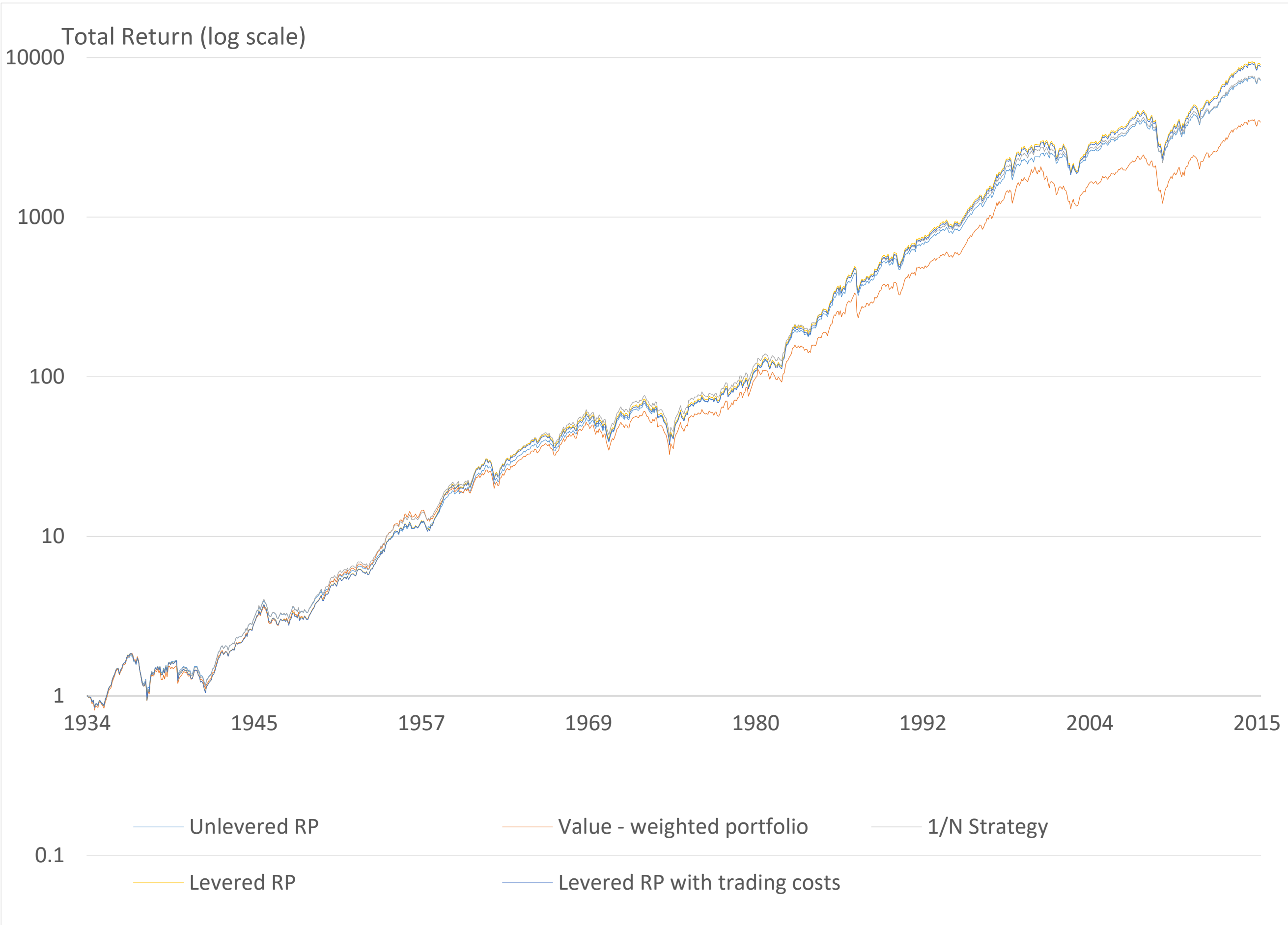
MoM_t = the momentum factor [(3).]

[(3)] Data on four factors are obtained from Kenneth's French data library.

5 Discussion of findings

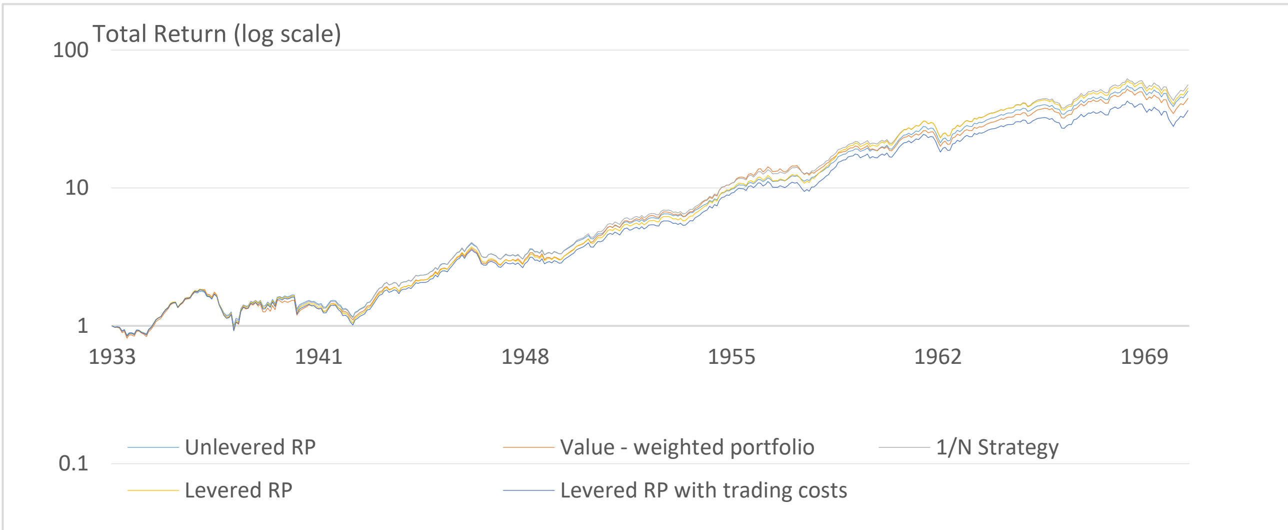
The figures presented below, show monthly compounded returns for the five strategies over the sample period (1934-2015) and the three sub-periods (1934-1970, 1971-1999 and 2000-2015). As displayed in Figure 1, the levered RP had the highest cumulative return followed by the unlevered RP, 1/n strategy and levered RP adjusted for trading costs. The performance, on the other hand, is uneven when the three different sub-periods are examined, as it can be seen in figure 2, figure 3 and figure 4. The levered RP prevailed for both the two most recent subperiods, while for the first subperiod, the levered RP is closely followed by the 1/n strategy. Furthermore, all the examined strategies are able to generate higher returns compared to the value-weighted market portfolio, over the whole examination period and for the two most recent subperiods. During the first sub-period, the value-weighted market portfolio is able to beat only the levered RP adjusted for trading costs. Furthermore, figure 2 illustrates a visible difference between the return of the unlevered RP and the levered RP adjusted for trading costs. This can be attributed to the fact that borrowing and trading costs can negate the outperformance of the levered RP. As it is depicted in figure 5 panel A and panel B, the leverage exacerbates turnover, and consequently, trading costs have a negative impact on the performance of the levered RP. In fact, panel A of figure 5 displays the leverage required for the estimated volatility of the RP strategy to match the estimated volatility of the market at each rebalancing date. The RP strategies require additional rebalancing compared to a value-weighted market portfolio or to an equal weighted strategy and thus have higher turnover rates. Following Anderson, Bianchi and Goldberg (2012), we measured turnover as the resulting from price changes. The overall conclusion drawn from figure 5 is that leverage and consequential trading costs can be a substantial drag on the performance of the levered RP.

Figure 1. Cumulative Returns for All Strategies, 1934-2015



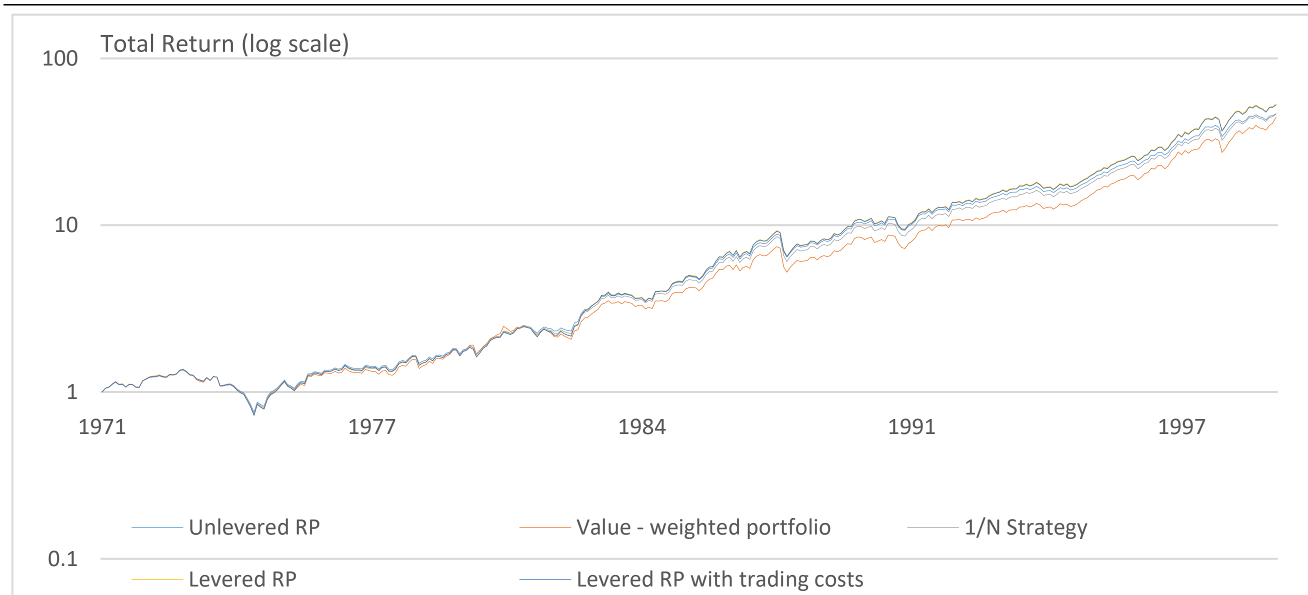
Notes: This figure shows monthly compounded returns for the five strategies, over the long sample period (1934-2015). For the levered RP, borrowing was at the three-month Eurodollar deposit rate starting in 1971 and was equal to the risk-free rate plus 60 bps before. Turnover-induced trading costs were 1% over 1926-1955, 0.5% over 1956-1970, and 0.1% over 1971-2010. The ranking of the five strategies at the end of 2015 was: levered RP, levered RP adjusted for trading costs, 1/n strategy, unlevered RP and market portfolio.

Figure 2. Cumulative Returns for All Strategies, 1934-1970



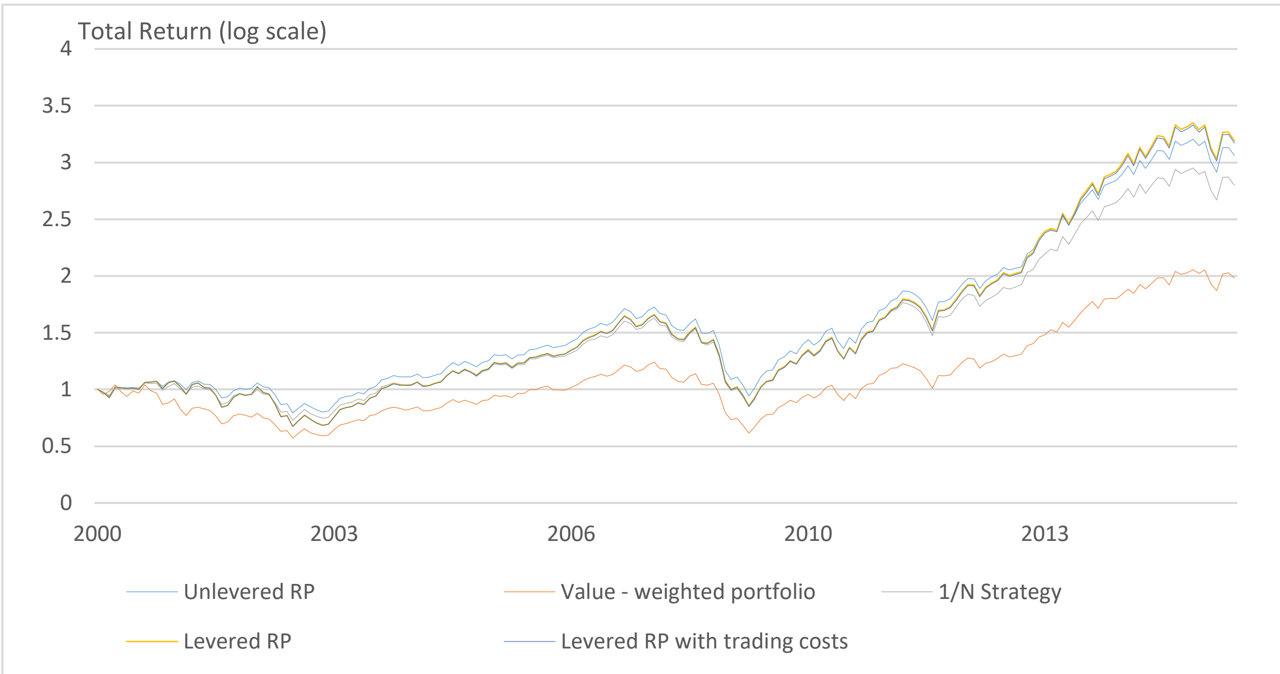
Notes: This figure shows monthly compounded returns for the five strategies over the sample period 1934-1970. The ranking of the strategies at the end of 1970 was the following: 1/n strategy, levered RP, unlevered RP, value – weighted portfolio and levered RP adjusted for trading costs.

Figure 3. Cumulative Returns for All Strategies, 1971-1999



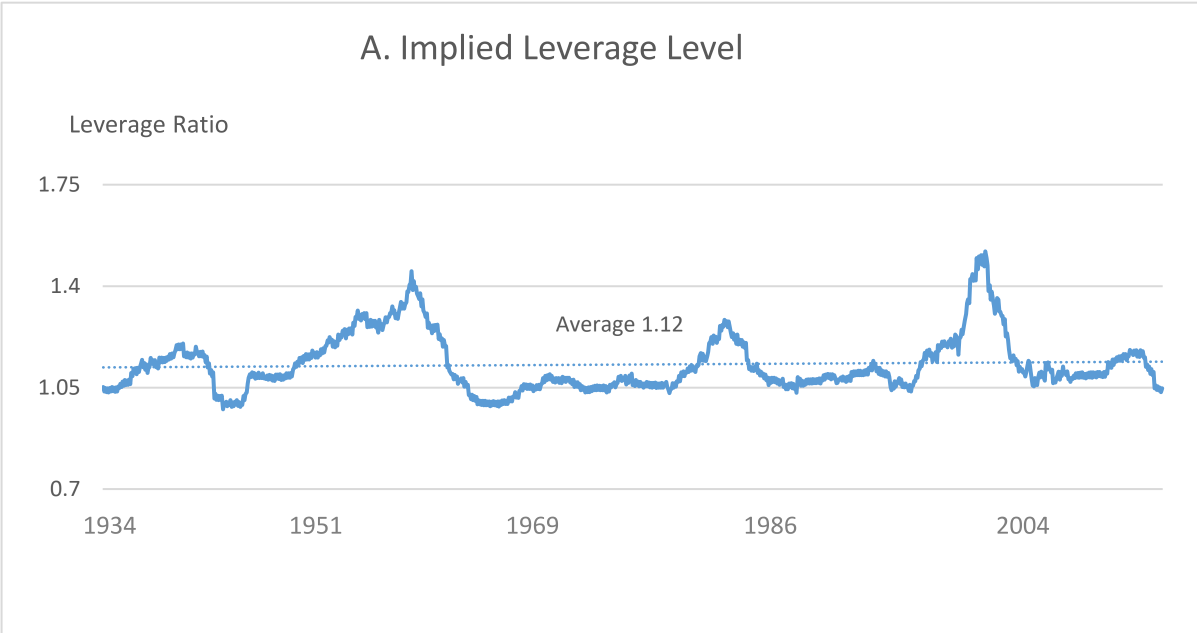
Notes: This figure shows monthly compounded returns for the five strategies over the sample period 1971-1999.The ranking of the strategies at the end of 1999 was the following: levered RP, levered RP adjusted for trading costs, unlevered RP, 1/n strategy and value – weighted portfolio.

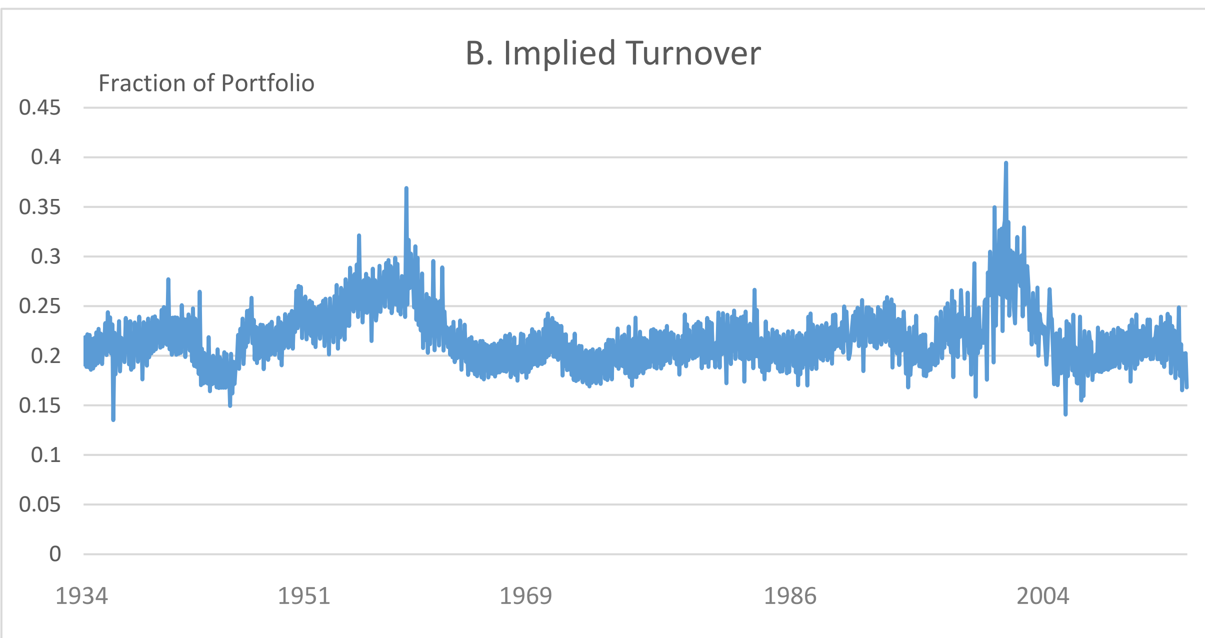
Figure 4. Cumulative Returns for All Strategies, 2000-2015



Notes: This figure shows monthly compounded returns for the five strategies over the sample period 2000-2015.The ranking of the strategies at the end of 2015 was the following: levered RP, levered RP adjusted for trading costs, unlevered RP, 1/n strategy and value – weighted portfolio.

Figure 5. Implied Leverage Ratio and Implied Turnover, 1934-2015





Notes: Panel A plots the leverage required for the estimated volatility of the risk parity strategy to match the estimated volatility of the market at each rebalancing. The average over the entire sample period is 1.12. The spikes in leverage occurred in three periods where markets were characterized by low volatility. Panel B shows the turnover of the risk parity at each rebalancing.

In the following table 2, panel A reports the performance statistics for the whole sample period, while, in panel B, panel C, and panel D, we take a closer look at the robustness of the examined strategies, by computing the performance statistics for the three sub-periods. The levered RP generated the highest annualized excess return for the whole sample period. However, the dominance of the levered RP isn't absolute across the three sub-periods. Specifically, we observe that for the first sub-period, the 1/n strategy obtained the best performance in terms of excess returns. Table 2, reports also the associated volatility to portfolio excess returns. The unlevered RP has the lowest volatility amongst the other portfolio strategies. The levered RP and levered RP adjusted for trading costs have nearly the same volatility, 15.90% and 15.89%, respectively. We also observe that the 1/n strategy has lower volatility compared to the value-weighted market portfolio. In addition, the volatility results for the three subperiods considered, mirror those presented for the whole sample period. Column 3 of Table 2, displays the Sharpe ratio for each strategy over a long sample period and three different sub-periods. We observe that risk-adjusted returns are higher for the risk parity strategies compared to the value-weighted market portfolio. In fact, Sharpe ratios range between 0.43 and 0.54 for the whole sample period, with the market portfolio having the lowest risk-adjusted performance and the unlevered RP having the highest Sharpe ratio. For the first sub-period, we notice an improvement in risk-adjusted performance for the whole strategies. The unlevered RP presents the highest Sharpe ratio with a value of 0.66. This compares with a Sharpe ratio of 0.53 for the levered RP adjusted for trading costs. The main conclusion that can be drawn is that turnover rates can have a negative impact on the risk-adjusted performance of RP strategies. In fact, compared to an unlevered RP strategy, that requires rebalancing only in response to a limited set of events, the need to lever the low-risk portfolio assets introduces trading costs that can negate the performance.

Table 2 reports additional statistics for the five strategies considered in this dissertation. Preliminary evidence on the normality distribution of returns for the strategies under consideration can be gathered by considering the third and fourth moments of the distribution. The results for excess kurtosis show that monthly returns follow a distribution that features leptokurtosis. Leptokurtosis arises in financial markets when periods of high volatility are followed by periods of relative stability. In general, positive skewness and high kurtosis is desirable.

However, all strategies exhibit negative skewness and high kurtosis which can lead to deleveraging costs.

Moreover, Table 2 displays also the Jarque-Bera statistic as a test of normality of returns and the associated p-value. The first insight consists in the fact that, at 5% significance level, we can reject the null hypothesis of a normal distribution of portfolio returns, for the whole sample period and for the first subperiod. However, for the last two subperiods, at a 5% significance level, the null hypothesis of a normality distribution of returns cannot be rejected. However, the non-rejection of the null hypothesis can also be due to the fact the Jarque - Bera test doesn’t perform well in smaller samples.

Table 2. Historical Performance, Monthly Rebalancing

	Excess Return	Volatility	Sharpe ratio	Skewness	Excess Kurtosis	Jarque-Bera Test
A. Historical Performance, 1934-2015						
Unlevered RP	7.67%	14.13%	0.54	-0.54	3.15	20.03*
Value-weighted portfolio	6.87%	15.81%	0.43	-0.52	3.04	16.69*
1/n Strategy	7.68%	14.96%	0.51	-0.51	3.12	17.07*
Levered RP	7.96%	15.90%	0.50	-0.55	3.14	20.41*
Levered RP with trading costs	7.44%	15.89%	0.47	-0.56	3.16	21.69*
B. Historical Performance, 1934-1970						
Unlevered RP	9.44%	14.25%	0.66	-0.63	4.18	21.37*
Value-weighted portfolio	9.07%	15.90%	0.57	-0.51	4.20	14.39*
1/n Strategy	9.76%	15.16%	0.64	-0.55	4.07	15.48*
Levered RP	9.59%	15.94%	0.60	-0.64	4.46	25.41*
Levered RP with trading costs	8.45%	15.94%	0.53	-0.68	4.54	29.01*
C. Historical Performance, 1971-1999						
Unlevered RP	7.01%	14.28%	0.49	-0.40	2.59	2.60
Value-weighted portfolio	6.82%	15.76%	0.43	-0.50	2.78	4.73
1/n Strategy	6.99%	14.86%	0.47	-0.44	2.68	3.31
Levered RP	7.50%	15.78%	0.48	-0.38	2.51	2.18
Levered RP with trading costs	7.46%	15.78%	0.47	-0.38	2.51	2.18
D. Historical Performance, 2000-2015						
Unlevered RP	5.80%	13.52%	0.43	-0.65	1.57	1.40
Value-weighted portfolio	2.77%	15.62%	0.18	-0.57	0.81	0.28
1/n Strategy	5.16%	14.63%	0.35	-0.55	1.53	0.96
Levered RP	6.09%	15.97%	0.38	-0.64	1.26	0.86
Levered RP with trading costs	6.04%	15.97%	0.38	-0.64	1.26	0.86

Notes: This table reports performance statistics for the five strategies over the long sample period, and three different subperiods. Excess returns, volatilities and Sharpe ratios are annualized. Excess kurtosis is equal to the kurtosis of monthly excess returns minus 3.

*Not significant at the 5 percent level.

The Fama-French-Carhart model has become a benchmark in performance evaluation as it tries to capture the cross-section of expected returns without specifying the underlying economic model. In attempt to understand what drives the returns of the unlevered RP, levered RP, and levered RP adjusted for trading costs, we run a four-factor model regression, for the whole sample period and the three subperiods. The first three risk factors are those of Fama and French (1993): the equity premium designed to capture the excess returns on the US equity market, the size factor designed to capture the return differential between the average small cap and the average big cap portfolio and the value premium designed to measure the return differential between the average value and the average growth portfolios. To these three factors, we add the momentum factor suggested by Carhart (1997), designed to capture the momentum effect of buying "winners" and selling "losers".

Table 3. Descriptive Statistics, February 1934 to December 2015

Factor	Average Monthly	Std	t-stat	Cross-Correlations			
Portfolio	Return	Dev	for Mean = 0	Mkt-RF	SMB	HML	MoM
Mkt-RF	0.66	4.56	4.55	1.00			
SMB	0.22	2.95	2.29	0.35	1.00		
HML	0.36	2.94	3.79	-0.02	0.01	1.00	
MoM	0.67	4.06	5.20	-0.17	-0.11	-0.21	1.00

Note: This table reports descriptive statistics for the four factors over February 1934 to December 2015: time series mean, standard deviation, t-stat for the bilateral test of the time series mean equal to 0 as well as cross correlations between factors. Mkt-Rf is the excess return on Fama and French's market proxy.
SMB and HML are Fama and French's factor mimicking portfolios for size and book to market equity. MoM is a factor that reflects the returns differential between the highest and the lowest prior-return portfolio.

Table 3 reports descriptive statistics for size, book-to-market, and momentum premiums over the period February 1934 to December 2015. All the four factors displayed positive returns over the period under examination. The equity premium and the momentum factor displayed the highest returns, with a value of 0.66 and 0.67 respectively. However, the equity premium proved to be more volatile with a standard deviation of 4.55.

Furthermore, the positive value of 0.22 for the size or SMB factor, indicates that companies with a small market capitalization outperform companies with large market capitalization. Furthermore, the positive value for HML indicates that companies with high BE/ME-ratio (value stocks) outperform companies with low BE/ME-ratio (growth stocks), over the whole sample period. It is also interesting to note that the value for the momentum factor is also positive, meaning that a momentum effect is present over this period. The high mean returns on SMB, HML and MoM suggest that these factors could account for much cross-sectional variation in the mean returns on equity portfolios. In addition, the relatively high variance of SMB, HML and MoM and their low correlations with each other, indicate that the four- factor model can explain sizeable time series variation.

The following Table 4 reports the results for the coefficients estimates after regressing the returns of a levered RP, unlevered RP, levered RP adjusted for trading costs and 1/n strategy on the four risk factors. The table reports the results for the whole sample period and the three sub-periods and the t-statistic is shown in parenthesis. A comparison of the estimated alphas from the different strategies show that alphas remain large and significantly different from zero. The estimated monthly alpha for the unlevered RP strategy is 0.41 over the whole sample period. The equally weighted strategy, the levered RP and the levered RP adjusted for trading costs display positive and statistically significant alphas (0.39, 0.40 and 0.36 respectively) over the whole sample period. These results are also robust with respect to the three different subsamples.

Furthermore, equity premiums over the whole sample period are large and statistically significant for all the strategies, ranging from 0.88 per month for the unlevered RP to 0.99 per month for the levered RP adjusted for trading costs. These results suggest that the aforementioned strategies can generate positive alphas with less exposure to the market risk.

However, all the strategies have a negative exposure to the size factor. As a result of the negative SMB Beta the model loads more heavily than the market factor on large caps.

Table 4 reports estimates for the adjusted R-square that measures the performance of the asset pricing model in explaining variations of portfolio returns. The unlevered RP strategy displays an adjusted R-square of 0,96. It follows that 96% of monthly returns of the unlevered RP strategy are explained by the four risk factors. The adjusted R-Square improves when the two first subperiods are taken into consideration. The goodness of fit of the model is confirmed when the other strategies are considered, however, we notice that the 1/n strategy is the only one that has the highest adjusted R-square. The lowest Adjusted R-Square is obtained by the two levered risk parity strategies for the third subperiod. Overall, the results from the four-factor model show that the monthly alphas generated by all the strategies are highly significant and positive. These results are also robust with respect to the three different subsamples. In addition, the momentum factor is statistically significant for all the strategies

Overall, the main conclusion of the four-factor model, for all the strategies considered in this thesis, is that their performance was almost entirely due to the exposure to known risk factors.

Table 4. Fama-French Regression Results, Monthly Rebalancing

	1934-2015		1934-1970		1971-1999		2000-2015	
A. Unlevered RP								
Adjusted R-Square	0.96		0.98		0.97		0.95	
Coefficients:								
Intercept	0.41	*(15.12)*	0.28	*(8.71)*	0.62	*(16.06)*	0.34	*(5.34)*
Mkt-Rf	0.88	*(143.98)*	0.89	*(117.18)*	0.92	*(101.60)*	0.86	*(54.94)*
SMB	-0.06	*(-5.97)*	0.02	*(1.90)*	-0.04	*(-3.39)*	-0.09	*(-4.47)*
HML	0.06	*(6.91)*	-0.07	*(-5.86)*	0.10	*(6.60)*	0.22	*(10.98)*
Mom	-0.02	*(-2.54)*	-0.01	*(-1.08)*	-0.06	*(-5.79)*	0.00	*(0.25)*
B. 1/N strategy								
Adjusted R-Square	0.97		0.99		0.96		0.96	
Coefficients:								
Intercept	0.39	*(17.1)*	0.26	*(10.55)*	0.62	*(18.78)*	0.29	*(4.90)*
Mkt-RF	0.94	*(180.1)*	0.95	*(160.69)*	0.95	*(123.48)*	0.92	*(63.26)*
SMB	-0.04	*(-5.2)*	0.02	*(1.98)*	-0.03	*(-2.40)*	-0.06	*(-3.05)*
HML	0.05	*(6.9)*	-0.06	*(-6.18)*	0.06	*(5.01)*	0.20	*(10.66)*
MOM	-0.03	*(-4.9)*	-0.01	*(-1.73)*	-0.06	*(-6.78)*	-0.02	*(-1.93)*
C. Levered RP								
Adjusted R-Square		0.96		0.98		0.94		0.94
Coefficients:								
Intercept	0.40	*(13.46)*	0.23	*(6.90)*	0.78	*(12.85)*	0.36	*(4.49)*
Mkt-RF	0.99	*(148.29)*	1.00	*(123.96)*	1.01	*(72.22)*	1.01	*(50.04)*
SMB	-0.08	*(-7.47)*	0.00	*(0.16)*	-0.03	*(-1.29)*	-0.12	*(-4.69)*
HML	0.06	*(5.63)*	-0.07	*(-6.09)*	0.04	*(1.89)*	0.21	*(8.11)*
MOM	-0.03	*(-4.65)*	-0.02	*(-2.07)*	-0.06	*(-4.03)*	-0.02	*(-1.22)*
D. Levered RP adjusted for trading costs								
Adjusted R-Square		0.96		0.97		0.97		0.94
Coefficients:								
Intercept	0.36	*(12.09)*	0.15	*(4.50)*	0.62	*(14.80)*	0.36	*(4.45)*
Mkt-RF	0.99	*(148.08)*	1.00	*(127.82)*	1.02	*(103.98)*	1.01	*(50.04)*
SMB	-0.08	*(-7.49)*	0.00	*(0.10)*	-0.05	*(-3.64)*	-0.12	*(-4.69)*
HML	0.06	*(5.64)*	-0.07	*(-6.22)*	0.09	*(5.88)*	0.21	*(8.11)*
MOM	-0.03	*(-4.67)*	-0.02	*(-2.19)*	-0.06	*-(5.54)*	-0.02	*(-1.22)*

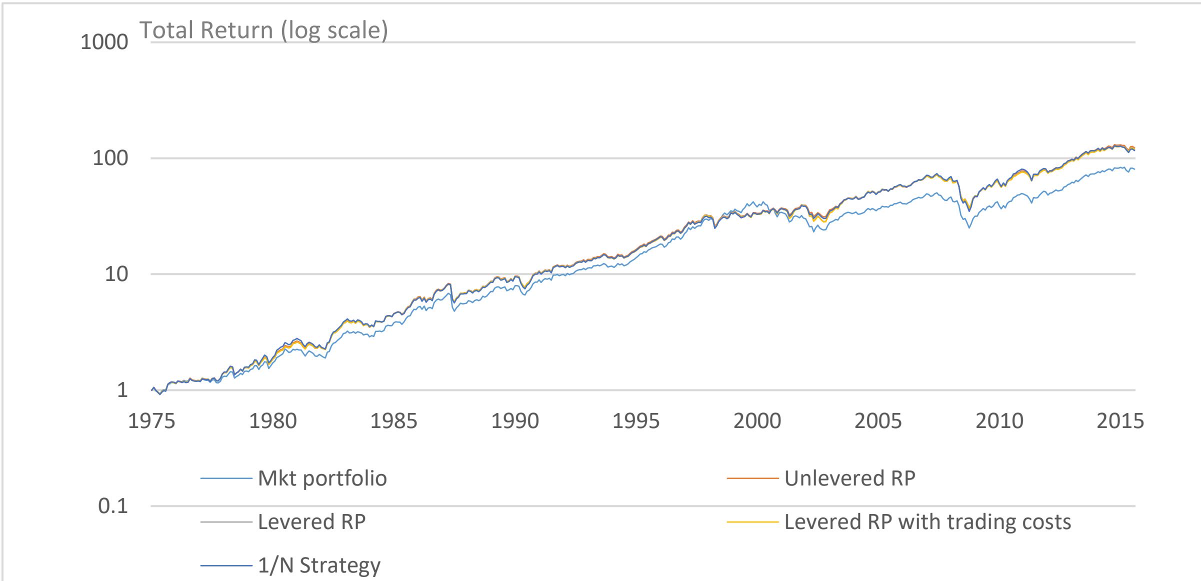
Note: This table reports the results of regressing the returns of the unlevered RP, levered RP, levered RP adjusted for trading costs and equal-weighted strategy against the Fama-French three factors in addition to the Carhart momentum factor. The t-statistic is indicated in parenthesis.

6 Robustness tests

6.1 Asset class inclusion – 49 industry portfolios.

As a first robustness test, we examine the performance of five strategies – unlevered RP, levered RP, equal-weighted strategy, levered RP adjusted for trading costs and the value-weighted market portfolio - based on 49 US industry portfolios. We evaluated the five strategies over the sample period 1975 – 2015. The unlevered RP is a fully invested strategy such as the ex - post risk contributions from the asset classes are equal. However, to obtain a higher risk-adjusted performance we lever this strategy to match the ex-post volatility of the value weighted portfolio. The volatility is estimated at month end by using a 36-month rolling window of returns. As in the base set up of this dissertation, the risk-free rate is the one-month U.S. Treasury bill rate. Following the approach of Anderson, Bianchi and Goldberg (2012) we also evaluate the impact of borrowing and transaction costs on the performance of the RP strategies. In accordance with the aforementioned authors, we considered the U.S. three-month Eurodollar deposit rate as an estimate of the cost of borrowing, while the transaction costs were assumed to be 0.1% over the whole sample period.

Figure 6. Cumulative Returns, Monthly Rebalancing, 1975-2015



Note: this figure displays monthly compounded returns for the five strategies, over the sample period 1975-2015. For the levered RP borrowing was at the three-month Eurodollar deposit rate. Turnover induced trading costs were 0.1% for the sample period. The ranking of the five monthly rebalanced strategies at the end of 2015 is: unlevered RP, levered RP, RP adjusted for trading costs, 1/n and market portfolio.

Figure 6 displays cumulative returns for the five strategies over 1975 - 2015. On the basis of cumulative return the unlevered RP prevailed over the sample period. However, all the strategies closely track each other, and at the end of 2015 the ranking was the following one: unlevered RP, levered RP, RP adjusted for trading costs, equal weighted strategy and the value weighted market portfolio. The performance statistics displayed in Table 5 confirm the previous ranking based on cumulative returns. Our results show that the unlevered RP realized the highest annualized excess return and has the highest level of volatility among the three RP strategies. The level of excess return and volatility for the levered RP is similar to that of RP adjusted for trading costs. As a consequence, in terms of risk-adjusted performance all the three risk parity strategies have the same Sharpe ratio equal to 0,49. The value weighted market portfolio has the lowest Sharpe ratio with a value of 0.43 while the equal weighted strategy realizes a Sharpe ratio of 0.45. The results displayed in Table 5 demonstrate that RP strategies outperform an equal-weighted approach and the value weighted portfolio both in terms of annualized returns and in Sharpe ratios.

However, the main drawback of the Sharpe ratio is that it doesn't provide a suitable metric for comparison in presence of skewness and kurtosis. As such we will first evaluate strategy returns considering the Sortino and Price (1994) ratio. The Sortino ratio is given by the ratio of average excess returns divided by the standard deviation of negative returns. It offers an insight on the size of returns relative to the downside risk which is relevant in strategies with negative skewness. We assess the ranking of the strategies as we would do with the Sharpe ratio. The values for the Sortino index range from 0.189 to 0.209. The ranking of the strategies considering the Sortino ratio confirms the previous ranking obtained with the Sharpe ratio. In fact, the levered RP and Levered RP adjusted for trading costs realized the highest Sortino ratio. However, one of the drawbacks of the Sortino ratio is that it doesn`t take in consideration the investor preferences. As a consequence, as an additional ranking measure that we take into consideration is the statistic proposed by Smetters and Zhang (2013). They demonstrate that in presence of non-normal distribution of returns, a performance ranking measure cannot be independent of investor preferences.

The version that we report is based on the power utility function where the utility function is written as:

$$U_r = \frac{(1+r)^{1-\gamma}}{1-\gamma},$$

Where γ is the constant coefficient of relative risk-aversion. Following Barroso and Santa-Clara (2015), we opted for this utility function as it sensitive to higher order of moments in returns.

The Smetters and Zhang (2013) statistic when considering the four moments of the distribution of returns (SR_4) is computed as following:

$$SR\_4 = \frac{(1+r)^{-y}}{y(1+r)^{-(1+y)}} \left(\frac{SR^2}{2} + \frac{y(1+y)}{6} SR^3\ Skew - \frac{y(1+y)(2+y)}{24} SR^4\ (Kurt - 3) \right),$$

Where SR is the Sharpe ratio, Skew is Skewness and Kurt is kurtosis for the time series of strategy returns. Following Bliss and Panigirzogly (2004) we set the coefficient of risk-aversion equal to 4.

We also compute the Smetters and Zhang (2013) statistics that takes into consideration the first three moments of the distribution of returns (SR_3), and as such we exclude kurtosis.

We interpret the ranking and the relative size of the performance as we would do with the Sharpe ratio. The most striking insight is the improved performance of the value-weighted portfolio, changing the previous performance ranking. In fact, the market portfolio has a value of 0.007 that compares to -0.025 for the two levered RP strategies. The difference in values between the SR_4 and SR_3 are originated by the fact that kurtosis isn't considered when computing the SR_3 statistic. The most striking insight is that the performance ranking of the strategies changes and this demonstrates that the market portfolio should be favoured by risk-averse investors.

Table 5. Performance Statistics, Monthly Rebalancing

	RP Unlevered	RP Levered	Levered RP with trading costs	1/N Strategy	Mkt Portfolio
Annualized Excess Return	7.55	7.48	7.47	7.40	6.41
Std Deviation	15.42	15.27	15.27	16.40	15.39
Sharpe ratio	0.49	0.49	0.49	0.45	0.42
Sortino ratio	0.208	0.209	0.209	0.200	0.189
SR_4	-0.046	-0.025	-0.025	-0.027	0.007
SR_3	-0.023	-0.022	-0.022	-0.014	-0.011
Skewness	-0.76	-0.74	-0.74	-0.69	-0.67
Kurtosis	3.60	3.07	3.08	3.48	2.21
JB test	25.14	17.41	17.49	19.33	7.41
p-value	0.00	0.00	0.00	0.00	0.02

Note: this table presents performance ranking measures for the five investment strategies. SR_4 and SR_3 are the generalized measures proposed by Smetters and Zhang (2013), for up to four and three moments for each return strategy.

The following Table 6 reports the values for the SR_4 and SR_3 statistic when considering different values for the coefficient of risk aversion. We provide results for risk aversion equal to 5 and 10 in addition to the original set up for y=4. We can conclude that the results for SR_4 and SR_3 are not sensitive to the variations of this parameter.

Table 6. Risk - Aversion coefficient, Robustness test

	RP Unlevered	RP Levered	Levered RP with trading costs	1/N Strategy	Value-weighted portfolio
SR_4					
y=4	-0.046	-0.025	-0.025	-0.027	0.007
y=5	-0.057	-0.032	-0.032	-0.035	0.006
y=10	-0.074	-0.037	-0.038	-0.048	0.014
SR_3					
y=4	-0.023	-0.022	-0.022	-0.014	-0.011
y=5	-0.029	-0.029	-0.029	-0.019	-0.016
y=10	-0.033	-0.032	-0.033	-0.023	-0.022

Note: this table displays the results for SR_4 and SR_3 when the risk aversion coefficient is hypothesized to be equal to 4, 5 and 10.

Finally, we examined if the returns generated by the strategies could be explained by their exposure to known risk factors. We regress the returns of the following strategies – unlevered RP, levered RP, levered RP adjusted for trading costs and unlevered RP - on four risk factors. The first three risk factors are: MKT designed to capture the equity premium, SMB designed to capture size premium and HML designed to capture value premium. In addition, to these three Fama and French (1992) three risk factors, we consider the Carhart (1997) momentum factor.

The results for the 1975 - 2015 regressions for the monthly rebalance strategies are displayed in the following Table 7. Examining the regression estimates for alphas, all the strategies display monthly significant alphas, as confirmed by the t-statistic in parenthesis. The two RP strategies realized the highest monthly alpha (0.44) that compares to 0.39 realized by the equal-weighted strategy. Furthermore, all the risk parity strategies realized higher returns and with less exposure to the market risk. In fact, the beta coefficients range from 0.97 to 0.98. In contrast to the RP strategies the 1/n strategy has a coefficient of 1.03 for the exposure to market risk.

Furthermore, as predicted by the academic literature, all the strategies have positive exposure to the size and value premium. The momentum factor, designed to capture the effect of buying winners and selling losers, is negative and statistically significant for all the strategies. Furthermore, when examining the goodness of fit of the model, an adjusted R – Square of 94% is obtained. It follows that 94% of monthly returns are explained by the four risk factors. Moreover, the adjusted R – Square is the same for all the four strategies.

Table 7. Regression Results, Monthly data, 1975-2015

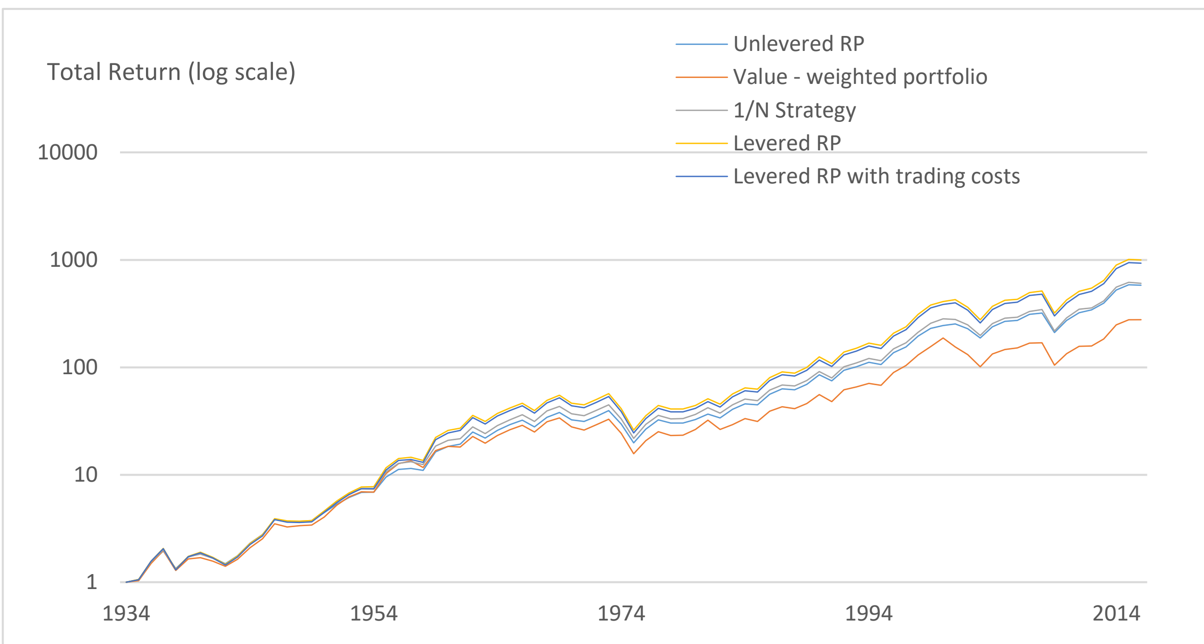
	Unlevered RP		*Levered RP*		*Levered RP with trading costs*		*1/N Strategy*	
Adjusted R Square	0.94		0.94		0.94		0.94	
Coefficients								
Intercept	0.41	*(8.11)*	0.44	*(8.86)*	0.44	*(8.84)*	0.39	*(7.09)*
Mkt-RF	0.98	*(82.57)*	0.97	*(82.61)*	0.97	*(82.61)*	1.03	*(81.11)*
SMB	0.11	*(6.68)*	0.11	*(6.40)*	0.11	*(6.40)*	0.16	*(8.78)*
HML	0.22	*(12.18)*	0.2	*(10.83)*	0.20	*(10.83)*	0.22	*(11.45)*
MoM	-0.02	*(-1.93)*	-0.05	*(-4.41)*	-0.05	*(-4.40)*	-0.04	*(-3.30)*

Note: This table reports the results of regressing the returns of the unlevered RP, Levered RP, 1/n strategy and Levered RP adjusted for trading costs against the Fama-French three factors in addition to the Carhart factors. The t-stat is indicated in parenthesis.

6.2 Rebalancing period.

All the results presented so far were based on the monthly rebalancing of the strategies. However, in order to show the impact that a different rebalancing period can have on portfolio performance, the strategies are rebalanced annually for the whole sample period. Figure 7 illustrates the cumulative returns of the strategies. First of all, we notice that the levered RP generated slightly higher returns than the levered RP adjusted for trading costs, over the studied period. Further, we observe that the value-weighted market portfolio realized the lowest return, while the unlevered RP and 1/n strategy tied for the third place. However, a closer and more detailed analysis of strategies` performance over the examination period is provided in the following table 8.

Figure 7. Cumulative Returns for All Strategies, Annual Rebalancing, 1934-2015



Notes: This figure shows monthly compounded returns for the five strategies, over the long sample period (1934-2015), with annual rebalancing. For the levered RP, borrowing was at the three-month Eurodollar deposit rate starting in 1971 and was equal to the risk-free rate plus 60 bps before. Turnover-induced trading costs were 1% over 1926-1955, 0.5% over 1956-1970, and 0.1% over 1971-2010. The ranking of the five strategies at the end of 2015 was: Levered RP, levered RP adjusted for trading costs, 1/n strategy, unlevered RP and market portfolio.

Table 8, provides the performance statistics for the strategies examined over the whole sample period 1934-2015. The table provides mixed results because the outperformance of one strategy over another depends on the performance measure considered. For instance, when annualized excess returns are considered as the performance measure, the levered RP parity generated the highest return. In second places comes the levered RP adjusted for trading costs, while the unlevered RP realized the worst performance. However, when the performance is evaluated on a risk-adjusted basis the 1/n strategy obtained the highest Sharpe ratio, followed by the two levered RP strategies.

Table 8. Historical Performance, Annual Rebalancing, 1934-2015

	Excess Returns	Volatility	Sharpe Ratio	Skewness	Kurtosis	Jarque-Bera test
Unlevered RP	8.07%	16.92%	0.44	-0.42	-0.03	14.52*
Value – weighted portfolio	7.10%	18.54%	0.42	-0.44	0.28	17.21*
1/n Strategy	8.12%	17.84%	0.46	-0.43	0.22	16.18*
Levered RP	8.78%	19.34%	0.45	-0.31	0.42	11.71*
Levered RP with trading costs	8.69%	19.31%	0.45	-0.32	0.42	11.78*

Notes: This table reports performance statistics for the five strategies over the long sample period (1934-2015), with annual rebalancing.
*Not significant at the 5 percent level.

Next, we examine whether the returns generated by the strategies could be explained by their exposure to known risk factors. As such, we examine estimates of alphas after regressing the returns from the strategies on the three Fama French risk factors and on the Carhart momentum factor. The first three factors summarize the excess return on the market (Rm-Rf), the performance of small stocks relative to big stocks (SMB), and the performance of value stocks relative to growth stocks (HML). Additionally, we consider the Carhart momentum factor designed to capture the effect of buying "winners" and selling "losers".

The regression results based on monthly data are presented in the following Table 9. When examining the goodness of fit of the model all the strategies display high adjusted R - Squares. In fact, it can be noticed that 95% of the returns are explained by the four risk-factors for the following strategies: unlevered RP, levered RP and levered RP adjusted for transaction costs. However, the 1/n strategy has the highest Adjusted R-square (97%). When investigating the portion of the strategies' performance that is not explained by the three factors, the alpha is positive for all the strategies. Furthermore, the unlevered RP and the equal – weighted strategy generate higher returns with less exposure to the market risk.

Table 9. Fama – French – Carhart Regression Results, Annual Rebalancing, 1934 – 2015

	Unlevered RP		*Levered RP*		*RP adjusted for trading costs*		*1/n strategy*	
Adjusted R Square	0.95		0.95		0.95		0.97	
Coefficients								
Intercept	1.69	*(2.93)*	1.82	*(2.72)*	1.75	*(2.61)*	1.49	*(3.22)*
Mkt-RF	0.87	*(35.51)*	1.01	*(35.26)*	1.01	*(35.17)*	0.93	*(47.5)*
SMB	0.02	*(0.48)*	-0.01	*(-0.31)*	-0.01	*(-0.30)*	0.02	*(0.70)*
HML	0.09	*(2.75)*	0.09	*(2.42)*	0.09	*(2.40)*	0.07	*(2.65)*
MoM	-0.05	*(-1.64)*	-0.06	*(-1.69)*	-0.06	*(-1.70)*	-0.05	*(-2.12)*

Note: This table shows the regression's results for the whole sample period. The strategies are rebalanced annually. The t-stat for the coefficients estimates is indicated in parenthesis.

6.3 Borrowing cost assumptions

As a further robustness test, we consider the impact on portfolio performance of different hypothesis for the cost of borrowing for the pre-1971 period. Similar to Anderson, Bianchi and Goldberg (2012), we considered the three-months Eurodollar deposit rate (3M EDR) as a proxy for the cost of borrowing for the examination period 1971-2015. Since for the sample period 1934-1970, the 3M EDR was unavailable, in accordance with the aforementioned authors, we add 60 basis points to the one-month Treasury bill rate. The authors have shown that the average spread between the Treasury bill and the 3M EDR is 100 basis points (bps) for the period 1971-2010. The spreads considered in the thesis are 25 bps, 50 bps, 60 bps, 75 bps, 100 bps and 125 bps. Table 10, panel A, shows the impact on the performance of different spreads for the pre-1971 period on the levered RP, while panel B shows the impact of the different assumptions about borrowing costs on the levered RP adjusted for trading costs. Furthermore, Table 11 reports the results of different hypothesis of borrowing costs for the monthly rebalanced strategies. The regression results reported in Table 10 and Table 11, indicate that, increasing the extrapolated spread over the one-month T-bill, has a negative impact on the annualized excess returns and on the Sharpe ratio of each strategy.

Table 10. Effect of Alternative Borrowing Costs Assumptions for the Pre-1971 Period, Annual Rebalancing

Pre-1971 Borrowing Cost Spread over T-Bill	Excess Return	Volatility	Sharpe Ratio	Skewness	Kurtosis	Jarque-Bera test
A. RP Levered						
60 BPS	8.78%	19.34%	0.45	-0.31	0.42	11.71*
50 BPS	8.79%	19.34%	0.45	-0.31	0.42	11.71*
25 BPS	8.81%	19.35%	0.46	-0.31	0.43	11.72*
75 BPS	8.77%	19.34%	0.45	-0.31	0.42	11.71*
100 BPS	8.76%	19.34%	0.45	-0.32	0.42	11.70*
125 BPS	8.74%	19.34%	0.45	-0.32	0.41	11.69*
B. RP Levered with trading costs						
60 BPS	8.69%	19.31%	0.45	-0.32	0.41	11.78*
50 BPS	8.70%	19.32%	0.45	-0.32	0.42	11.78*
25 BPS	8.71%	19.32%	0.45	-0.32	0.42	11.78*
75 BPS	8.68%	19.31%	0.45	-0.32	0.41	11.77*
100 BPS	8.67%	19.31%	0.45	-0.32	0.41	11.76*
125 BPS	8.65%	19.31%	0.45	-0.32	0.41	11.76*

Notes: This table reports the impact of the borrowing cost extrapolation on the risk premium of the levered risk parity over the sample period 1934-2015. The strategies are rebalanced annually.

*Not significant at the 5 percent level.

Table 11. Effect of Alternative Borrowing Cost Assumptions for the Pre-1971 period, Monthly Rebalancing

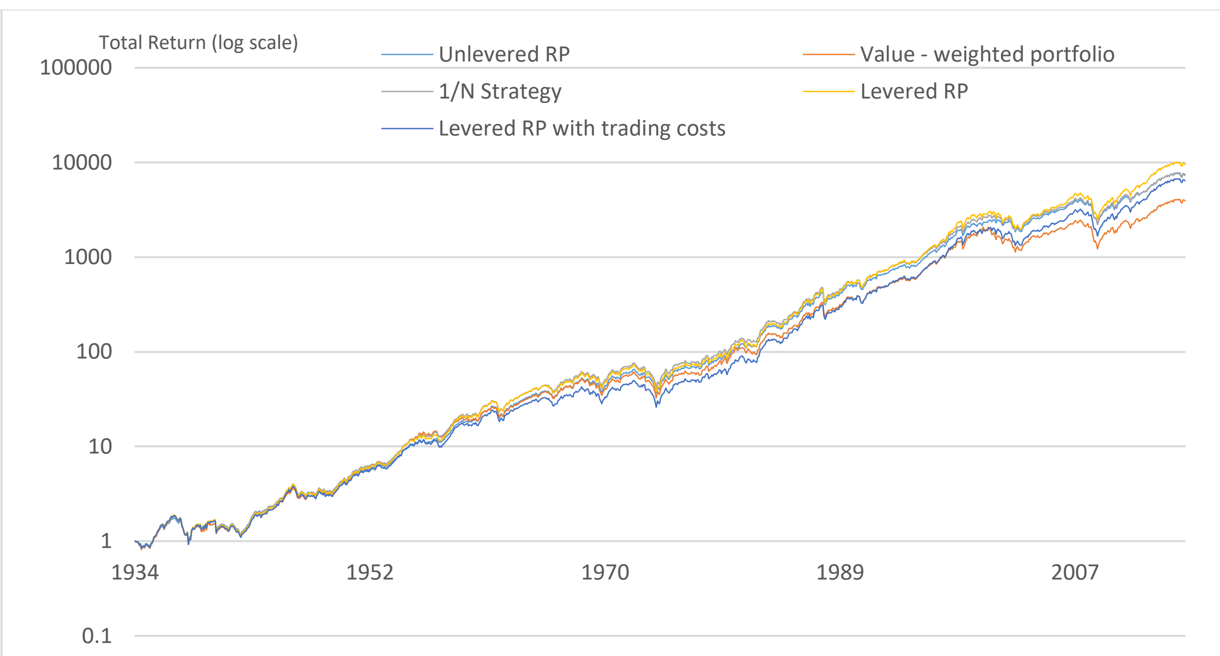
Pre-1971 Borrowing Cost Spread over T-Bill	Excess Return	Volatility	Sharpe Ratio	Skewness	Kurtosis	Jarque-Bera test
A. RP Levered						
60 BPS	7.96%	15.90%	0.501	-0.55	3.14	20.41*
50 BPS	8.03%	15.90%	0.505	-0.55	3.14	20.40*
25 BPS	8.22%	15.90%	0.517	-0.55	3.14	20.39*
75 BPS	7.84%	15.90%	0.494	-0.55	3.14	20.41*
100 BPS	7.66%	15.89%	0.482	-0.55	3.15	20.40*
125 BPS	7.47%	15.89%	0.470	-0.55	3.15	20.39*
B. RP Levered adjusted for Trading Costs						
60 BPS	7.44%	15.89%	0.468	-0.56	3.16	21.69*
50 BPS	7.52%	15.89%	0.473	-0.56	3.16	21.69*
25 BPS	7.71%	15.90%	0.485	-0.56	3.16	21.69*
75 BPS	7.33%	15.89%	0.461	-0.56	3.16	21.68*
100 BPS	7.14%	15.89%	0.449	-0.56	3.16	21.66*
125 BPS	6.95%	15.89%	0.437	-0.56	3.16	21.63*

Notes: This table reports the impact of the borrowing cost extrapolation on the risk parity premium of the levered risk parity over the sample period 1934-2015. The strategies are rebalanced monthly.
*Not significant at the 5 percent level

6.4 Volatility Measure.

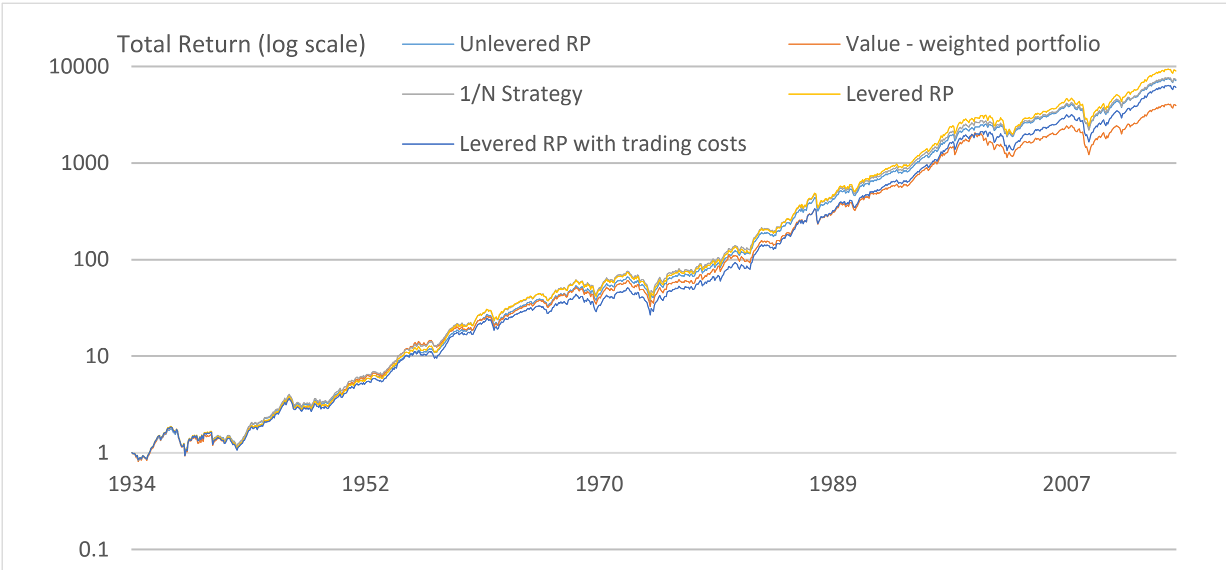
As an additional robustness test for the risk parity approach, we considered the impact on portfolio performance of two different measures of volatility. In the first case, the volatility of each industry portfolio is estimated at the end of month by considering a rolling window of 12-month returns. Whereas in the second case, we considered a 24-month rolling window of returns in order to estimate the portfolio volatility. The following figures 8 and 9, report the cumulative performance for all the strategies with monthly rebalancing, for the whole sample period. The levered RP realized the highest cumulative return. Furthermore, it can be noticed that borrowing and trading costs have a negative impact on performance. In fact, when the unlevered RP was levered to have the same volatility as the value-weighted benchmark, the turnover costs negated the performance.

Figure 8. Cumulative Performance, 12-months volatility



Note: This figure shows monthly compounded returns for the five strategies over the sample period 1934-2015. The strategies are monthly rebalanced. The volatility of a strategy is measured at the end of month, by using a 12-months rolling window of returns. At the end of 2015, the ranking of the strategies is the following: levered RP, unlevered RP, 1/n strategy, levered RP adjusted for trading costs and value – weighted portfolio.

Figure 9. Cumulative Performance, 24-months volatility



Notes: This figure shows monthly compounded returns for the five strategies over the sample period 1934-2015. The strategies are monthly rebalanced. The volatility of a strategy is measured at the end of each month, by using a 24-months rolling window of returns. At the end of 2015, the ranking of the strategies is the following: levered RP, 1/n strategy, unlevered RP, levered RP adjusted for trading costs and value – weighted portfolio.

Table 12. Historical Performance, Monthly Rebalancing, 1934-2015

	Excess Return	Volatility	Sharpe Ratio	Skewness	Excess Kurtosis	Jarque-Bera Test
A. Historical Performance, 12 months- volatility						
Unlevered RP	7.71%	13.85%	0.557	-0.58	3.30	25.26*
Benchmark	6.87%	15.81%	0.435	-0.53	3.06	17.93*
1/N strategy	7.68%	14.96%	0.514	-0.53	3.13	18.96*
Levered RP	8.04%	15.83%	0.508	-0.61	3.43	29.95*
Levered RP with trading costs	7.51%	15.83%	0.475	-0.61	3.43	29.37*
B. Historical Performance, 24 months-volatility						
Unlevered RP	7.66%	14.04%	0.55	-0.57	3.21	23.06*
Benchmark	6.87%	15.81%	0.43	-0.53	3.06	17.93*
1/N strategy	7.68%	14.96%	0.51	-0.53	3.13	18.96*
Levered RP	7.95%	15.80%	0.50	-0.59	3.27	25.49*
Levered RP with trading costs	7.44%	15.80%	0.47	-0.59	3.27	24.98*

Notes: This table reports performance statistics for the five strategies over the long sample period. The strategies are monthly rebalanced. Panel A. reports the results when the volatility of a strategy is measured at the end of month by using a 12-months rolling window of returns, while on panel B, the volatility is measured on a 24-months rolling window of returns.

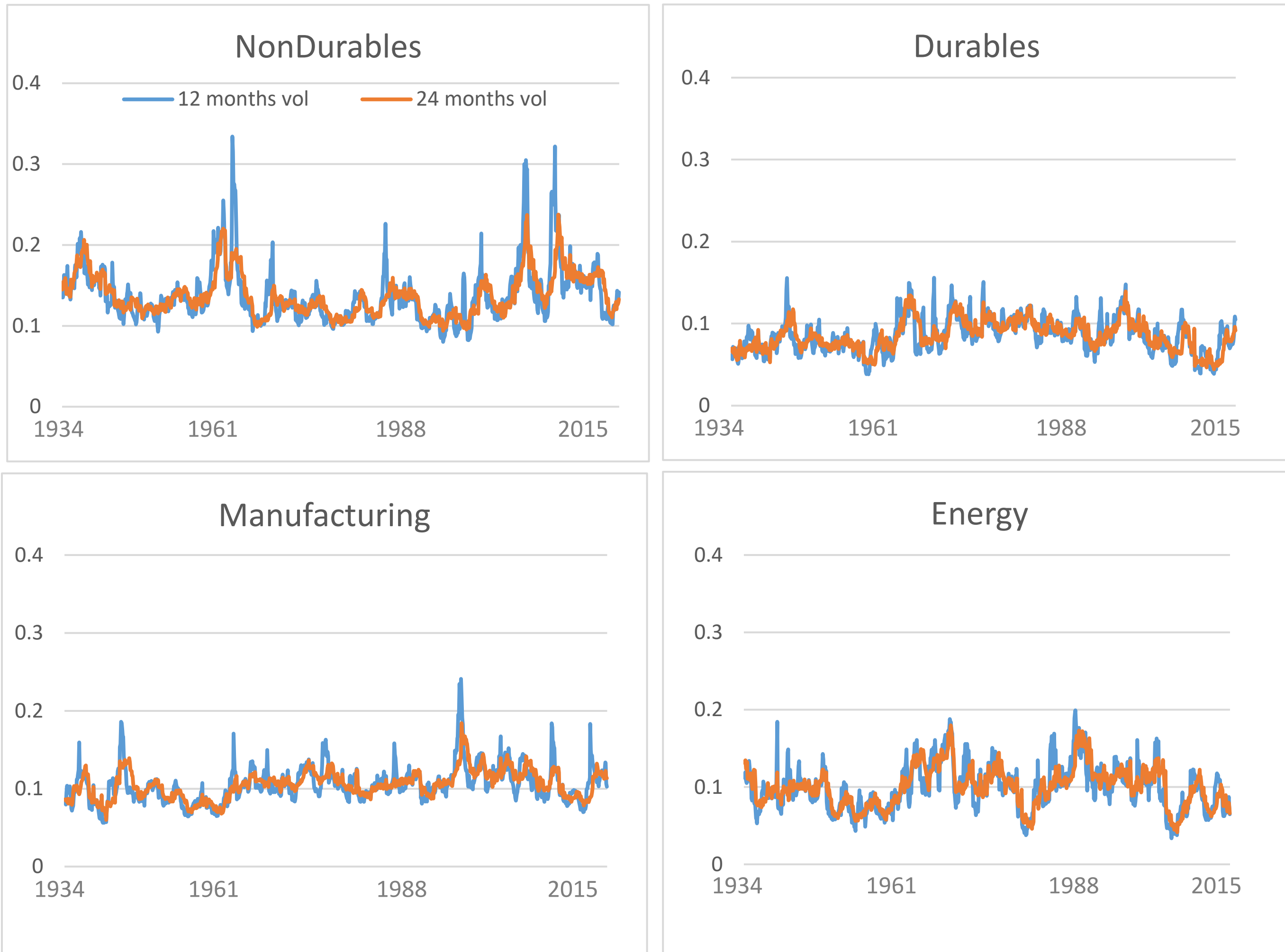
*Not significant at the 5 percent level.

The above table 12, panel A, reports the performance statistics for a 12-months volatility measure, while panel B, reports the performance statistics for a 24-months volatility measure. This table provides for mixed results because the dominance of a strategy depends on the performance measure. For instance, the levered RP realized the highest excess return in both cases. On the other hand, when the performance is evaluated on a risk-adjusted basis, the unlevered RP realized the highest Sharpe ratio. Moreover, in terms of volatility, the unlevered RP had the lowest volatility over the 82-years sample period. Overall, the results are robust to changes in volatility measures.

Table 12, also displays the standard statistics for the five strategies over the long sample period. Preliminary evidence on the normality distribution of returns is obtained from the examination of skewness and excess kurtosis. The coefficient estimate for excess kurtosis is greater than zero for all the strategies. As a consequence, the monthly returns follow a distribution that features leptokurtosis. Moreover, table 12 reports the coefficient estimate for the Jarque-Bera test of normality. At a five percent significance level, there is no evidence to accept the null hypothesis of a normal distribution of portfolio returns.

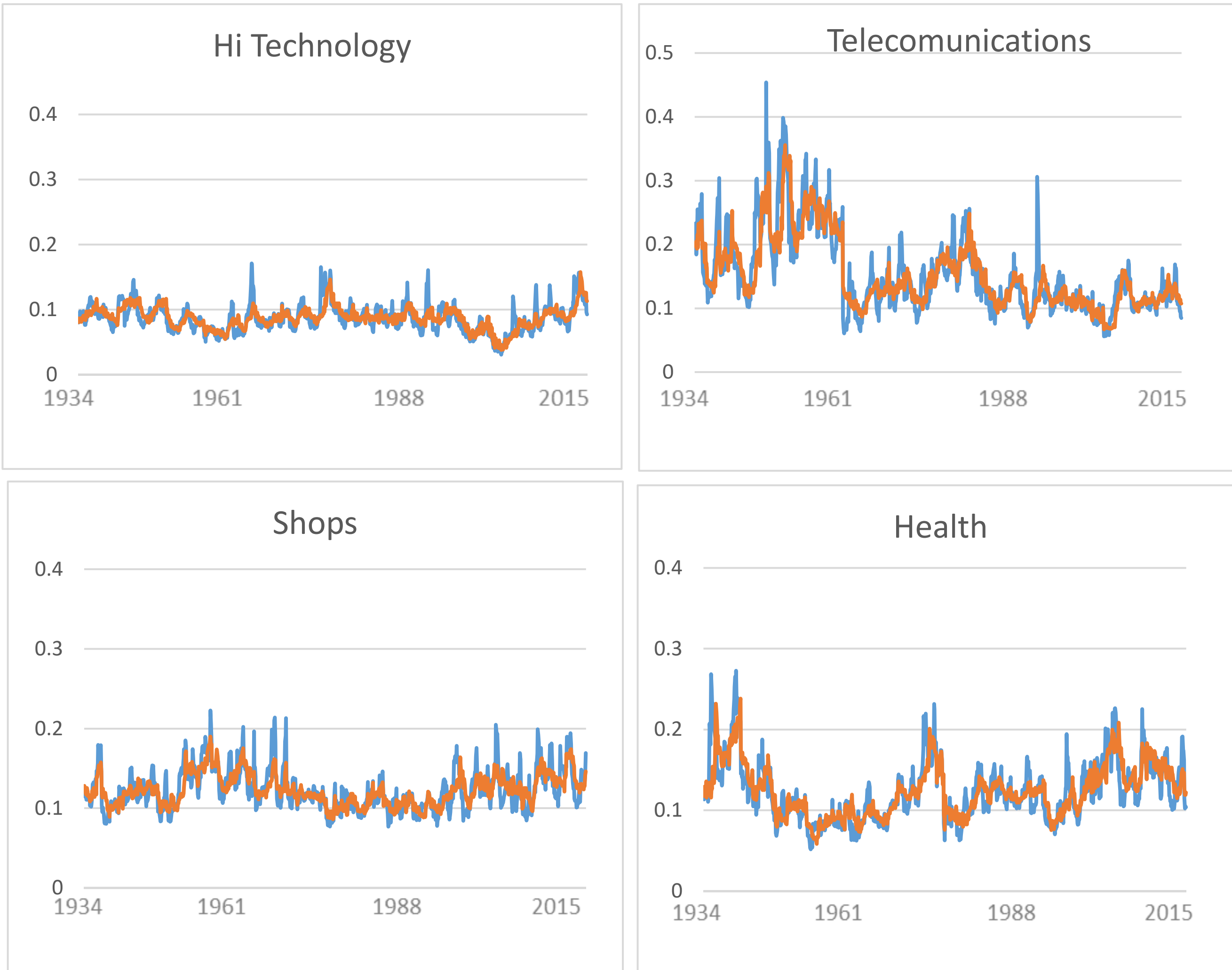
In the figure below, we investigate the time series of the portfolio weights for each industry considered for the construction of the levered Risk Parity portfolio. Figure 10 displays the times series weights under two sets of assumptions concerning the volatility measure. In a first case, the volatility was measured considering a 12-month rolling window of returns, while, in a second case, the volatility was measured considering a 24-month rolling window of returns. The main conclusion is that the increased volatility, negatively affects the weights of each industry sector. For instance, if we consider the oil crisis of 1970-1980 and 2008, it can be noticed that the weight of the energy sector has declined. Similarly, when we consider the dot-com bubble, the weight of the hi-technology sector has decreased because the risk parity approach tends to put more weight in lower risk assets.

Figure 10. Time Series of Portfolio Weights, 1934-2015, Monthly Rebalancing



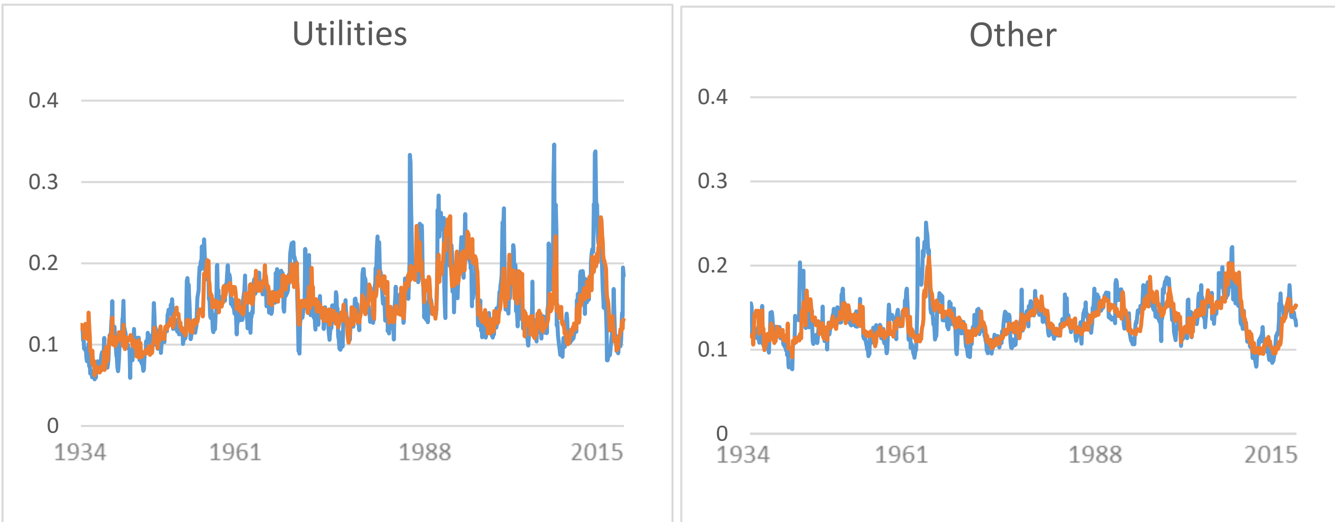
Notes: These figures display the time series of portfolio weights for each industry sector considered in the construction of the Risk Parity portfolio. The time series of portfolio weights refer to the long sample period, 1934-2015. The strategy is monthly rebalanced.

Figure 10. Time Series of Portfolio Weights, 1934-2015, Monthly Rebalancing (continued)



Notes: These figures display the time series of portfolio weights for each industry sector considered in the construction of the Risk Parity portfolio. The time series of portfolio weights refer to the long sample period, 1934-2015. The strategy is monthly rebalanced.

Figure 10. Time Series of Portfolio Weights, 1934-2015, Monthly Rebalancing (continued)



Notes: These figures display the time series of portfolio weights for each industry sector considered in the construction of the Risk Parity portfolio. The time series of portfolio weights refer to the long sample period, 1934-2015. The strategy is monthly rebalanced.

7 Conclusion

The aim of this dissertation was to further the understanding of the Risk Parity (RP) approach to asset allocation. This was done by reviewing the evolution of the portfolio theory and back-testing the RP approach relative to a value-weighted market portfolio and to a 1/n strategy. Applying the theoretical framework of Anderson, Bianchi and Goldberg (2012), we considered three different RP strategies: unlevered RP, levered RP and levered RP adjusted for trading costs. We evaluated the performance of these strategies over a long sample period (1934-2015) and in three different subperiods (1934-1970, 1971-1999, 2000-2015). The base setup throughout the dissertation was a monthly rebalancing of the strategies, with the volatility estimated on a 36-months rolling window of returns. In addition, we considered three set of assumptions for the cost of borrowing and trading costs based on the previous literature on the subject. Our analysis established that the levered RP realized the highest annualized excess return over the long sample period. However, we found that the performance depends on the back-testing period. For instance, for the first subperiod (1934-1970), the 1/n strategy generated the highest excess return.

Furthermore, we also found that the performance depends on the assumptions about market frictions. In fact, we notice that they constitute a substantial drag on the performance of the levered RP. For instance, after adjusting for transaction costs, both the unlevered RP and the 1/n strategy, yielded higher excess returns.

Moreover, the outperformance of one strategy over another depends on the performance measure. For example, when evaluating the performance on a risk-adjusted basis, the unlevered RP yields the highest Sharpe ratio, with a value of 0.54. When the unlevered RP was levered to have the same volatility as the value-weighted benchmark, transactions costs reduced the performance, and the realized Sharpe ratio is equal to 0.47.

Our results are robust to several modifications of the portfolio construction methodology. As such, we considered the impact on portfolio performance of annual rebalancing of the strategies, and alternative hypothesis for the cost of borrowing and transaction costs. Additionally, we evaluated the performance of the aforementioned strategies on the 49 US industry portfolios. In addition to the standard performance evaluation measures, we assessed the ranking of these strategies using measures that take in account the impact of higher moments in returns. Specifically we employ the Sortino ratio and the Smetters and Zhang statistic to take in account the preferences of risk averse investors.

The base case assumed borrowing at the three-months Eurodollar deposit rate starting in 1971. Before 1971, we assumed borrowing at the risk-free rate plus 60 basis points.

In addition, to the borrowing assumptions, we added the turnover-induced trading costs of 1% over the 1934-1955, 0.5% over 1956-1970, and 0.1% over 1971-2015. As a further test, we considered the impact of varying the extrapolated spread on the strategy performance, for the pre-1971 period. We considered spreads, over the one-month T-bill, ranging between 25 and 125 basis points.

As a final robustness test, we evaluated the performance of the strategies under two sets of hypothesis for the volatility measure. More specifically, in a first case, we measured the volatility as the standard deviation on a 12-month rolling window of returns, and, in a second case, the standard deviation was measured on a 24-months rolling window of returns.

Our findings confirm the previous literature on the benefits of the Risk Parity approach, and as a consequence, this approach may represent a valuable addition to traditional investment management techniques. However, to execute on Risk Parity successfully, careful selection of assets is very critical, and “for the time being, that remains an art rather a science”. (Chaves, Hsu, Li and Shakernia, 2010).

Bibliography

Anderson, R., Bianchi, S. and Goldberg, L. (2012), Will my Risk Parity Strategy Outperform? Financial Analysts Journal, 75-93.

Arnott, R. D. (2005), Criticism on Market-Cap Indexation, Editor's Corner, Financial Analysts Journal, 10-14.

Arnott, R. D, Hsu, J. and Moore, P. (2005), Fundamental Indexation, Financial Analysts Journal, vol. 61, no 2, pp. 83-99.

Arnott, R. D. and West, J. M. (Fall 2006), Fundamental Indexes: Current and Future Applications, Institutional Investor's 5th Annual Exchange-Traded Funds Review, pp. 111-121.

Asness, C. (2006), The Value of Fundamental Indexing, Investors Journal, 1-3.

Asness, C., Frazzini, A., & Pedersen, L. H. (2012), Leverage aversion and risk parity, Financial Analysts Journal, vol. 68, pp. 47–59.

Banz, R. (1981), The relationship between return and market value of common stocks, Journal of Financial Economics, vol. 9, pp. 3-18.

Barroso, P. and Santa-Clara, P. (2015), Momentum has its Moments, Journal of Financial Economics, pp. 111–120.

Basu, S. (1977), The Investment Performance of Common Stocks in Relation to Their Price to Earnings Ratio: A Test of the Efficient Markets Hypothesis, Journal of Finance, vol. 32, no. 3, pp. 663-682.

Bauman, C., Conover, C. M. and Miller, R. E. (1998), Growth versus Value and Large-Cap versus Small-Cap Stocks in International Markets, Financial Analysts Journal, vol. 54, no 2, pp. 75-89

Bhansali, V. (2011), Beyond Risk Parity, Journal of Investing, vol.20, no. 1, pp. 137–147.

Bernstein,W. (2007), Fundamental Indexing and the Three-Factor Model, http://www.efficientfrontier.com/ef/0adhoc/fi.htm.

Black, F. and Litterman, R. (1991), Global Portfolio Optimization, Journal of Fixed Income, vol. 1, no. 2, pp. 7-18.

Black, F. and Litterman, R. (1992), Global Portfolio Optimization, Financial Analysts Journal, vol 48, no 5, pp. 28–43.

Black.F,. Jensen, M.C. and Scholes, M. (1972), The Capital Asset Pricing Model: Some Empirical Tests, in ed. Michael C. Jensen, Studies in the Theory of Capital Markets. New York: Praeger.

Bliss, R. and Panigirzoglu, N. (2004), Option-Implied Risk Aversion Estimates, Journal of Finance, vol 59, no 1, pp. 407-446.

Blitz, D. and Swinkles, L. (2008), Fundamental Indexation: An Active Value Strategy in Disguise, Working paper.

Bhandari, L. (1988), Debt/Eqity Ratio and Expected Common Stock Returns: Empirical Evidence, Journal of Finance, pp. 507-528.

Bornholt, G. (2013), The failure of the Capital Asset Pricing Model, Abacus, 36-43.

Brown, P., Kleidon, A. W. and Marsh, T. A. (1983), New Evidence on the Nature of Size-Related Anomalies in Stock Prices, Journal of Financial Economics, pp. 33-56.

Bruder B., and Roncalli, T. (2012), Managing Risk Exposures using the Risk Budgeting Approach, Working paper, Lyxor.

Capaul, C., Rowley, I. and Sharpe,W. (1993), International value and growth stock returns. Financial Analysts Journal, pp. 27-36.

Carhart, M. (1997), On Persistence in Mutual Fund Performance, Journal of Finance, vol. 52, no. 1, pp 57-82.

Chan, L. K. C., N. Jegadeesh and Lakonishok,J. (1996), Momentum strategies, Journal of Finance, vol. 51, no. 5,pp. 1681—1713.

Chan,L., Hamao, Y. and Lakonishok, J. (1991), Fundamentals of Stock Returns in Japan, Journal of Finance, pp. 1793-1764.

Chaves D., J. Hsu, F. Li, and Shakernia, O. (2011), Risk Parity Portfolio vs. Other Asset Allocation Heuristic Portfolios, Journal of Investing, vol. 20, no. 1, pp. 108–118.

Chaves D., J. Hsu, F. Li, and Shakernia, O. (2012), A Risk Factor-Based Approach to Asset Allocation, Research Affiliates working paper.

Chaves D., J. Hsu, F. Li, and Shakernia, O. (2012), Efficient Algorithms for Computing Risk Parity Portfolio Weights, Journal of Investing Fall, pp. 150-163.

Chen, A., Fabozzi, F. J., & Huang, D. (2010). Models for portfolio revision with transaction costs in the mean–variance framework. In J. B. Guerard, Jr. (Ed.), The handbook of portfolio construction: Contemporary applications of Markowitz Techniques, New York: Springer.

Chopra, V. K., & Ziemba, W. T. (1993), The effect of errors in means, variances, and covariances on optimal portfolio choice, Journal of Portfolio Management, vol. 19, pp. 6–11.

Chopra, N., Lakonishok, J. and Ritter, J. R. (1992), Measuring Abnormal Performance – Do Stocks Overreact, Journal of Financial Economics, no 31, pp.235-268.

Choueifaty, Y. and Coignard, Y. (2008), Towards maximum diversification, Journal of Portfolio Management vol. 34, no. 4, pp. 40–51.

Choueifaty Y., T. Froidure, and J. Reynier (2013), Properties of the Most Diversifed Portfolio, Journal of Investment Strategies, vol. 2, pp. 49-70.

Chow, T., Hsu, J., Kalesnik, V. and Little, B. (2010), A survey of alternative equity index strategies, Research Affiliates, working paper.

Cochrane, J. H. (2005), Asset Pricing, Revised Edition. Princeton, NJ: Princeton University Press.

Clark, P. (1963). A Subordinated Stochastic Process Model With Finite Variance for

Speculative Prices, Econometrica, vol. 41, pp. 135-155.

Clarke, R., De Silva, H., & Thorley, S. (2002), Portfolio constraints and the fundamental law of active management, Financial Analysts Journal, vol. 58, pp. 48–66.

Clarke R., H. De Silva, and S. Thorley (2011), Minimum-Variance Portfolio Composition, Journal of Portfolio Management vol. 37, pp. 31-45.

Clarke R., H. De Silva, and S. Thorley (2012), Minimum Variance, Maximum Diversification and Risk Parity: An Analytic Perspective, Working Paper, 40 pages.

Dalio, Ray (2004), Engineering Targeted Returns and Risks, Bridgewater Associates.

de Carvalho, R.L., Lu, X. and Moulin, P. (2012) Demystifying equity risk-based strategies: A simple alpha plus beta description, Journal of Portfolio Management, vol. 38, no. 3, pp. 56–70.

DeMiguel, V., Garlappi, L., and Uppal, R. (2009). Optimal versus naive diversification: How inefficient is the 1/N portfolio strategy? Review of Financial Studies, vol. 22, pp. 1915–1953.

DeMiguel, V., and Nogales, F. J. (2009), Portfolio selection with robust estimation, Operations Research, vol. 57, pp. 560–577.

DeMiguel V., L. Garlappi, F. Nogales, and R. Uppal (2009), A Generalized Approach to Portfolio Optimization: Improving Performance by Constraining Portfolio Norms, Management Science vol. 55, pp. 798-812.

Estrada, J. (2008), Fundamental Indexation and International Diversification, Journal of Portfolio Management, vol. 34, no. 3, pp. 93-109.

Fama, E. F. (1963), Mandelbrot and the stable paretian hypothesis, The Journal of Business vol. 36, pp. 420–429.

Fama, E (1965), The behavior of stock market prices, Journal of Business, vol 38, no 1, pp. 34-105.

Fama, E and French, K (1992), The Cross-Section of Expected Stock Returns, Journal of Finance, vol. 47, no. 2, pp. 427-465.

Fama, E and French, K (1993), Common Risk Factors in the Returns on Stocks and Bonds, Journal of Financial Economics vol. 33, no. 1, pp. 3–56.

Fama, E and French, K (2004), The Capital Asset Pricing Model, Theory and Evidence, Journal of Economic Perspectives, pp. 25-46.

Fama, E and French, K (2006), The Value Premium and the CAPM, Journal of Finance, pp. 2163-2185.

Fama, E. and French, K. (2009), Dissecting anomalies, Journal of Finance vol. 63, pp. 1653–1678.

Fama, E. and French, K. (2012), Size, value, and momentum in international stock returns, Journal of Financial Economics vol. 105, pp. 457–472.

Feldstein, M. S. (1969), Mean-Variance Analysis in the Theory of Liquidity Preference and Portfolio Selection, Review of Economic Studies 36, no. 1, pp. 5-12.

Fisher, I. (1965), The Theory of Interest, New York: Augustus M. Kelley.

Foresti, S.J. and Rush, M. (2010), Risk-focused Diversification: Utilizing Leverage within Asset Allocation, Wilshire Consulting Paper.

Frahm, G. und Wiechers, C. (2013), A Diversification Measure for Portfolios of Risky Assets, Advances in Financial Risk Management, pp. 312-330, Palgrave Macmillan.

Frazzini, A. and Pedersen, L.H. (2010), Betting against Beta, AQR Capital Management and NBERWorking Paper 16601.

Friend, I. and Marshall E. Blume, (1970), Measurement of Portfolio Performance Under Uncertainty, American Economic Review vol. 60, pp. 561-575.

Green, R. and Hollifield, B. (1992), When will mean-variance efficient portfolios be well diversified,The Journal of Finance, 1992.

Haugen R., and Baker, N. (1991), The Efficient Market Inefficiency of Capitalization-Weighted Stock portfolios, Journal of Portfolio Management vol. 17, pp. 35-40.

Hemminki, J. and Puttonen, V. (2008) Fundamental indexation in Europe. Journal of Asset Management, vol. 8, pp. 401-405.

Hsu, J. C. (2006), "Cap-Weighted Portfolios Are Sub-Optimal Portfolios, Journal of Investment Management, vol. 4, no 3, pp. 1–10.

Hsu, J. C. and Campollo, C. (2006), New Frontiers in Index Investing: An Examination of Fundamental Indexation, Journal of Indexes, pp. 32-38.

Inker, B. (2010), The hidden risks of risk parity portfolios, GMO white paper.

Jacob, N. (1971), The Measurement of Systematic Risk for Securities and Portfolios: Some Empirical Results, Journal of Financial and Quantitative Analysis vol. 6, no. 1, pp. 815-34.

Jarque,C. and Bera, A. (1980), Efficient tests for normality, homoscedasticity and serial independence of regression results, Economics Letters.

Jegadeesh, N. (1990), Evidence of predictable behavior of security returns, Journal of Finance, vol. 45, no. 3, p.881-898.

Jegadeesh, N. and Titman, S. (1993), Returns to buying winners and selling losers, Journal of Finance, vol 48, pp. 65–91

Jegadeesh, N. and Titman, S. (2001), Momentum, Working paper, university of Illinois

Jensen, M.C. (1968), The Performance of Mutual Funds in the Period 1945-1964, Journal of Finance vol. 23, no. 2, pp. 389-416.

Jurczenko, E., Michel, T., & Teiletche, J. (2013), Generalized Risk-Based Investing, working paper.

Kaplan, P. D. (2008), Why Fundamental Indexation Might- or Might Not- Work, Financial Analysts Journal, vol. 64, no 5, pp. 32-39.

Kaya H., and W. Lee (2012), Demystifying Risk Parity, Working Paper, Neuberger Berman.

Lintner, J. (1965), The Valuation of Risk Assets and the Selection of Risky Investments in Stock Portfolios and Capital Budgets, The Review of Economics and Statistics, pp. 13-37.

Lee, J.H. and Stefek, D. (2008). Do risk factors eat alphas? Journal of Portfolio Management, vol. 34, pp. 12–25.

Lee, W. (2011), Risk-Based Asset Allocation: A New Answer To An Old Question, Journal of Portfolio Management, vol. 37, pp. 11-28.

Levell, C.A. (2010), Risk Parity: In the Spotlight after 50 years, NEPC, March.

Linzmeier, D. (2011), Risk-Balanced Portfolio Construction, Working paper.

Lohre, S., Opfer, H. and Orszag, G. (2012), Diversifying Risk Parity, Working paper.

Lohre S., Neugebauer, U.and Zimmer, C. (2012), Diversifed Risk Parity Strategies for Equity Portfolio Selection, Journal of Investing Fall, pp. 111-128.

Maillard, S., Roncalli, T., and Teïletche, J. (2010), The properties of equally weighted risk contribution portfolios, Journal of Portfolio Management, vol. 36, pp. 60–70.

Mandelbrot, B. (1963). The Variation of Certain Speculative Prices, Journal of Business, vol. 36, pp. 394-419.

Markowitz, H. (1959), Portfolio Selection: Efficient Diversification of Investment, New York: John Wiley & Sons.

Markowitz, H. (1952), Portfolio selection, Journal of Finance, vol. 7, pp. 77–91.

Markowitz, H. (1987), Mean–variance analysis in portfolio choice and capital markets, Cambridge, MA: Basil Blackwell

Markowitz, H. (2005) Market efficiency: a theoretical distinction and so what? Financial Analysts Journal, vol. 61, pp. 17-30.

Mayers, S. (1976), Determinants of Corporate Borrowing, Journal of Financial Economics, pp. 144-175.

Merton, R. C. (1973), "An Analytic Derivation of the Efficient Portfolio Frontier, Journal of Financial and Quantitative Analysis, vol. 7, pp. 1851-1872.

Merton, R. (1980), On Estimating the Expected Return on the Market: An Exploratory Investigation, Journal of Financial Economics vol. 8, pp. 1-39.

Meucci, A. (2009), Managing Diversification, Bloomberg Education and Quantitative Research and Education paper.

Michaud, R. (1989), The Markowitz Optimization Enigma: Is 'Optimized' Optimal? Financial Analysts Journal, vol. 45, no. 1, pp. 31–42.

Michaud, R. (1998), Efficient asset allocation: A practical guide to stock portfolio optimization and asset allocation, Boston: Harvard Business School Press.

Mossin, J. (1966), Equilibrium in a Capital Asset Market, Econometrica, vol. 34, no. 2: pp. 768-83.

Naranjo, L. (2009), Implied Interest rates in a Market with Frictions. Working paper, Essec Business School.

Perold, A. (2007), Fundamentally flawed indexing, Financial Analysts Journal, vol. 63, pp. 31-37.

Peters, E. (1991), Chaos and Order in the Capital Markets, A New View of Cycles, Prices and Market Volatility. John Wiley and Sons.

Peters, E. (2009), Balancing Betas: Essential Risk Diversification, FQ Perspective, vol. 6, no. 2.

Qian, E. (2005), Risk Parity Portfolios: Efficient Portfolios through True Diversification, PanAgora Asset Management White Paper.

Qian E. (2006), On the Financial Interpretation of Risk Contributions: Risk Budgets Do Add Up, Journal of Investment Management, vol. 4, pp. 1-11.

Qian, E. (2009), Risk Parity Portfolios: The Next Generation, PanAgora Asset Management White Paper.

Roll, R. (1969), Bias in Fitting the Sharpe Model to Time Series Data, Journal of Financial and Quantitative Analysis vol. 4, pp. 271-89.

Roncalli, T. (2011), An introduction to Risk Parity and Risk Budgeting Approach, Chapman and Hall, CRC Financial Mathematics Series.

Roncalli T., and Weisang, G. (2012), Risk Parity Portfolios with Risk Factors Approach, Working paper, Lyxor, 32 pages.

Ross, S. (1976), The arbitrage theory of capital asset pricing, Journal of Economic Theory, vol. 13, no.3, pp. 341–360.

Ruban, O. and Melas, D. (2010), The perils of Risk Parity, MSCI Barra Research Insight.

Ruban, O. and Melas, D. (2011), Constructing Risk Parity portfolios, Rebalance, Leverage or Both? Journal of Investing, vol. 20, no. 1.

Rubinstein, M. (2002), Markowitz's ''portfolio selection'': A fifty-year retrospective, Journal of Finance, vol. 57, pp. 1041–1045.

Scherer, B. (2004) Portfolio Construction and Risk Budgeting, 2nd edn. London: Riskwaters Books.

Scherer, B. (2012), Risk Parity in US futures markets, Journal of Asset Management, pp. 155-161.

Scholz, H. and Wilkens, M. (2005), Investor specific performance measurement: A justification of Sharpe ratio and Treynor ratio, International Journal of Finance.

Sebastian, M. (2012), Risk Parity and the Limits of Leverage, Journal of Investing, vol. 21, no. 3.

Sharpe, W (1964), Capital Asset Prices - A Theory of Market Equilibrium under Conditions of Risk, Journal of Finance, vol. 19, no. 3, pp. 425-442,

Sharpe, W. (1966), Mutual Fund Performance. Journal of Business vol. 39, pp. 119-138.

Sharpe, W. (1970), Portfolio Theory and Capital Markets, New York: McGraw-Hill.

Sharpe, W. (1994), The Sharpe ratio. The Journal of Portfolio Management.

Siegel, J. (2006), The noisy market hypothesis, Wall Street Journal.

Smetters, K. and Zhang, X. (2013), A Sharper Ratio: A General Measure for Ranking Non- Normal Investment Risks, Wharton School, University of Pennsylvania.

Sortino, F. and Price, L., (1994), Performance Measurement in a Downside Risk Framework, Journal of Investing, vol. 3, no. 3, pp. 59-65.

Steinbach, M. C. (2001), Markowitz revisited: Mean–variance models in financial portfolio analysis, SIAM Review, vol. 43, pp. 31–85.

Tobin, J. (1958), Liquidity Preferences as Behavior Towards Risk, Review of Economic Studies vol. 25: pp. 65-85.

Treynor, J. (2005), Why market-valuation-indifferent indexing works, Financial Analysts' Journal, vol. 61, no.5, pp.65–69.

Treynor, J. (2008), Fundamentally flawed indexing: comments, Financial Analysts' Journal, vol. 64, no. 2, pp.14.

Vasicek, O. A. 1971. "Capital Asset Pricing Model with No Riskless Borrowing": Wells Fargo Bank. Unpublished manuscript.

Walkhausl, C. and Lobe, S. (2010), Vice versus virtue investing around the world, Working paper.

Woerheide, W. and Person, D. (1993), An index of Portfolio Diversification, Financial Services Revew, pp. 73-85.