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Service Quality Measures in Ambulance Location Problem

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Abstract

The emergency medical service system (EMS) refers to the provision of personnel and equipment in the appropriate area in an emergency case, to ensure a prompt and synergistic health service. Thus locations of ambulances have become a crucial decision.

Ambulance facility problem is closely related to everybody's life. The rationality of its location and the fairness of distribution are directly related to the efficiency and quality of public service supply. It plays a fundamental role in promoting social construction and improving people's quality of life.

In this paper, we present a literature review of the overall facility location problem, as well as the emergency medical service. Then we propose optimization models based on both deterministic and stochastic performance measures to optimize the ambulance location problem and introduce a new measure—service quality. In addition, we present a computational comparison for these models based on simulations of different indicators and analyse the performance of the approaches with descriptive statistics analysis and data envelopment analysis (DEA). From both aspects, models based on stochastic measures, i.e. expected demand coverage, expected service quality, expected survival rate are considered as better solutions based on the results of the simulations.

The new measure—service quality in the ambulance location problem and our extended models can be applied to a more general supply chain network design. The indicators we proposed can efficiently help to measure the performance at each demand-supply pair.

Key Words: Ambulance location problem; Stochastic; Optimization; Data Envelopment Analysis (DEA)

Sommaire

Le système de services médicaux d'urgence fait référence à la fourniture de personnel et d'équipement dans la zone appropriée dans un cas d'urgence, afin d'assurer un service de santé rapide et synergique. Ainsi, les emplacements des ambulances sont devenus une décision cruciale.

La rationalité de sa localisation et l'équité de la distribution sont directement liées à l'efficacité et à la qualité de l'offre de service public et jouent un rôle fondamental dans la promotion de la construction sociale et l'amélioration de la qualité de vie des populations.

Dans ce mémoire, nous présentons une revue de la littérature sur le problème global de l'emplacement de l'installation, ainsi que sur le service médical d'urgence. Nous proposons ensuite des modèles d'optimisation basés sur des mesures de performance déterministes et stochastiques afin d'optimiser le problème de localisation des ambulances et introduisons une nouvelle mesure -- qualité de service. En outre, nous présentons une comparaison computationnelle pour ces modèles basés sur des simulations de différents indicateurs et analysons la performance des approches avec analyse statistique descriptive et analyse d'enveloppement de données (DEA). Sous ces deux aspects, les modèles basés sur des mesures stochastiques, c'est-à-dire la couverture de la demande prévue, la qualité de service prévue et le taux de survie prévu sont considérés comme de meilleures solutions basées sur les résultats des simulations.

La nouvelle mesure -- qualité de service dans le problème de localisation d'ambulance et nos modèles étendus peuvent être appliqués à une conception de réseau de chaîne d'approvisionnement plus générale. Les indicateurs que nous avons proposés peuvent efficacement aider à mesurer la performance de chaque paire offre-demande.

Mots Clés: Problème de localisation d'ambulance; Stochastique; Optimisation; Data Envelopment Analysis (DEA)

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Chapter 1-Introduction

1.1 Overview

The study of facility location is a long-awaited subject. In ancient times, location decision-making is often based on experience, emotional intuition and even superstition, which lacked scientificity. Modern facility location research originated in 1909, the German scholar Weber's first published paper marked the facility location problem to enter the era of scientific research.

The facility location problem has a long history. Most of the scholars and experts in the field of logistics have studied the location problem from the aspects of the creation and revision of the model, the design of efficient or feasible algorithms, and the consideration of the complexity of the algorithm. Optimization of facility location usually has the following important features. First, the strategic aspects of the selection are significant because management decisions often affect long-term economic benefits and corporate development trends, and their investment in decision-making is generally relatively large; secondly, there are so many factors that need to be weighed in the problem, and to find a satisfactory solution to meet the constraints of reality. Although the model is created and solved with many similar steps or representations, the models for different environments or conditional constraints are not generic in themselves, so the structure of the model depends on a particular problem or case. Furthermore, for the solution of the model algorithm, most of the facilities location problems are NP-hard for the current situation of the study, so it is not easy to obtain the exact solution or the optimal solution for some site applications.

No matter what kind of social environment and historical conditions, people are inseparable from facility location problem. Facility location as an important branch of management science and engineering research, is often related to the strategic level of management decision-making and logistics optimization. Because of its relatively large investment and a long period of time to maintain stability, investment decision-makers have to make rational and careful consideration of facility location decision.

From the point of logistics optimization, the facility location problem is always attached to a certain environmental background, and the corresponding services are provided by the number of facilities and the optimal location of the facilities is under the constraints of existing resources. Facility location generally requires consideration of : the number of facilities required; the location of the facility needs to be established; the size of each facility needs to be determined; the allocation of each facility's customer demand, etc. The decision-making factors of these facilities directly affect the input-output ratio, economic efficiency, social utility and social value of enterprises or organizations. As an important research area in management, economics, computational science, transportation science and other fields, facility location problem has been widely used in many fields such as logistics chain, public facilities, transportation and communication, regional and urban planning.

Public service facilities location selection, such as fire station, ambulance station, is the central issue of urban planning and urban development. The rationality and fairness of distribution are directly related to the quality in urban construction and development, as well as the quality of life.

With the development of economy, the growth of population and the progress of medical undertakings, the process of urbanization is accelerating. As one of the basic elements of public service facilities, ambulance stations play a positive role in promoting the improvement of health conditions in the region. They are also necessary conditions for guaranteeing the orderly operation of emergency medical systems. The location of ambulance stations is crucial for the optimal allocation of medical resources in the region. Reasonably optimizing ambulance station location can effectively save costs and ensure the rational distribution of medical resources among residents to make up for the deficiencies of the existing medical system and promote the balanced and rapid development of the emergency medical system.

1.2 Definition of the Problem

The facility location problem has become the focus of the decision-making content for many years because of its wide range of applications. The covering problem is one of the three basic models in facility location problems (P-median problems, P-center problems, covering problems), which can be divided into set covering location problems (SCLP) and maximal covering location problems (MCLP). The SCLP requires that under the premise of full coverage, how to decrease the number of facilities or decrease the investment cost. The MCLP is based on the practical limitation of investment, within a fixed number of facilities, to cover the demand as far as possible. The problem of gradual coverage is extended on the basic coverage problems. Since there are natural defects in the practical application of the basic coverage problem (the breakpoints of covering radius make the coverage of the demand unreasonable), the concept of the gradual coverage makes up for the lack of basic models. Gradual coverage model is particularly applicable to consumer satisfaction as the goal of the location problem, radio, television, mobile phone signal transmission problems. The gradual covering presents a more comprehensive and systematic extension of the basic location problem model. In the coverage model, the coverage radius and the coverage level in the function can be more flexible to transform the coverage problem into the basic MCLP or the P-median problem, which provides a simplified method and idea for solving the complex gradual coverage model.

In this paper, we intend to answer two research questions:

1. What facility location models can be applied to optimize the ambulance station location problem?

2. What indicators can be incorporated to measure the performance based on the models we apply?

3. Based on the proposed indicators, how well do these different models perform based on simulation?

1.3 Outline and Contributions

The first chapter of this paper has already introduced the subject and the research questions of this work. The second chapter will provide a literature review of the overall facility location problem, and then introduce basic covering models and gradual covering models, as well as the emergency medical service introduction. The third chapter will propose twelve mathematical models. Chapter Four will present a comparison based on different indicators between the results of the models and analyse the performance of the approaches with descriptive statistics analysis and Data Envelopment Analysis. The final chapter will conclude this paper, highlight the contribution and present certain limitations.

Our contribution will be:

1. We incorporate a new measure—service quality and add to our extended optimization models from both deterministic and stochastic sides.
2. We have the indicators—expected response time, expected survival rate, expected service quality and expected demand coverage criteria at the same time, to measure the performance of each model.
3. We apply the Data Envelopment Analysis (DEA) to evaluate the performance of different models.
4. The models we have for the ambulance location problem can actually be extended to a general supply chain network design.

Chapter 2-Literature Review

In this section, we first generally review the introduction of facility location problem. Then two types of models will be presented: basic covering models and the gradual covering models. Next, we provide a brief overview of the emergency medical service (EMS) from different aspects, such as: performance assessment, management structure and service indicators.

2.1 Introduction of Facility Location Problem

The study of the facility location theory has a history of nearly a hundred years. The earliest model of facility location originated from the famous Weber problem. In 1909, Alfred Weber established a 1-median problem model in Euclidean space (Altinel, 2009). Then in 1929, Hotelling introduced the competitive factors into the facility location problem; Weiszfeld then proposed the famous Euclidean space median algorithm in 1937. While the above theoretical research just took the planar location situation into account, the actual situation is much different. The facility location problem is heavily concerned with the study of the network location by Hakimi (1964, 1965), whose combination of guidance and innovative ideas and location issues is widely applied to the realities of factories, facilities and service industries. The facility location problem is divided into P-median problem, P-center problem, coverage problem, closure problem, Hub location problem, dynamic and uncertain location problem, competitive location problem, facility allocation, etc. We generally refer to the three models of P-median problem, P-center problem and coverage problem as basic location problem. On this basis, the various types of problems are collectively referred to as the location expansion problem. For different research background and angle, the subdivision of the location problem is very complicated.

Daskin (1998) summarized and collated the facility location problem from the following five perspectives, as shown in Table 1.

Research Level	From personal decision-making in daily life, to the company or government facility decision-making
Research Perspectives	From the point of the enterprise, to maximize corporate profits, the correct facility location is related to the enterprise profit and market competitiveness; from the perspective of the government and the public sector, the location will affect the efficiency of public facilities and public concern about the price and fairness.
Involving Fields	Involving the logistics industry such as the transportation, communications, logistics chain, urban planning, public facilities, and other aspects.
Common Issue	Since most of the facility location problems are NP-hard, it is difficult to obtain accurate solutions. Most of the practical applications are turned to seek suboptimal solutions or satisfactory solutions.
Model Features	The facility location problem is based on different realistic background and subject to certain constraints; different problems need to be solved by separate models

Table 1- Division of the facility location problem

Daskin (1995) also classified the facility location problem, as shown in Table 2.

Criteria	**Classification**
Topology	Planar, discrete, network
Network type	Tree, general graph
Facility numbers	Single, multiple
Travel time	Static, dynamic
Certainty	Deterministic, probabilistic
Product differences	Single, multiple
Organization Nature	Public, private
Target number	Single, multiple
Demand type	Elastic, inelastic

Facility capacity	Capacitated, uncapacitated
Demand allocation	Nearest, general demand allocation
Hierarchical structure	Single, multiple
Facility attraction	Desirable, indesirable

Table 2- Classification of facility location problem

ReVelle et al. (2008) provided a more succinct classification for facility location problem, dividing it into two branches: the median and the planar location model, the center and the coverage problem. With this standard, the location model can be divided into the following four categories, presented in Table 3.

Analytical models	Based on large-scale simple assumptions (fixed-cost facility location, demand consistency distribution, etc.) and ignore the special cases; can not be applied to the actual decision-making guidance.
Continuous models	Assume that the facility can be placed anywhere in the service area and the demand is usually considered to occur at some discrete points.
Network models	The topology of such sites is a network of points and lines. Most of the relevant literature in this field seeks to find a special structure to make the algorithm solvable in polynomial time.
Discrete location	The demand and candidate points for facility location are discrete, and such problems can often be described and expressed in integer programming or MIP. Most of these models are NP-hard.

Table 3- Model categories

Depending on the classification attributes of the location problem, there will be more detailed categories. The focus of this paper is to consider one of the three basic problems of facility location ---coverage problem, and its application, with discrete location models.

2.2 Basic Covering Models

In the covering problem, the service is not provided by the closest facility, but within certain distance, where it is assumed that there is no difference between customers in the coverage area and the distance is the coverage radius.

Given network $G(V, A)$, V is the set of vertices, $|V| = m$, A is the set of edges. I is the set of the demand points and J is the set of the potential facility locations. Each facility, once established, can cover a predetermined set of the demand points or, in other words, provide the service to the patients in these demand points. This condition can also be represented by the parameter a_{ij} as follows:

$$a_{ij} = \begin{cases} 1, \text{if demand point } i \text{ can be covered by facility at location } j \\ 0, \text{otherwise} \end{cases}$$

In general, whether the demand point can be covered by the facilities is based on the distance between the two is less than or equal to the coverage radius. Within the coverage radius, the demand point is completely covered, while beyond this value, the demand point is not covered by the corresponding facilities, that is, the facility can not provide services for the relevant demand points or customers will not come to this facility. In the specific problem, the basic covering model only consider one coverage radius, and the settings of radius are often associated with the status and importance between the two. The coverage problem is one of the traditional models of network facility location, which can be divided into two types: set covering location model and maximal covering location model.

2.2.1 Set Covering Location Problem

The main content of the set covering location problem is to minimize the total cost of investment or the number of facilities that need to be built on the premise that the needs of customers at all demand points are covered by the facilities. From the perspective of optimization, finding a subset of the required investment costs where all demand can be covered by candidate facilities. The problem is to find a minimum total cost on facilities under a full coverage.

Assume f_j is the fixed cost of candidate facility j, and variable X_j:

$$X_j = \begin{cases} 1, \text{facility built at location } j, \\ 0, \text{otherwise} \end{cases}$$

The set covering location model will be:

$$\text{Min} \sum_{1}^{j} f_j * X_j \qquad (2-1)$$

Subject to:

$$\sum_{1}^{j} a_{ij} * X_j \geq 1, i \, \forall I \qquad (2-2)$$

$$X_j \in \{0,1\}, \; j \forall J \qquad (2-3)$$

The objective function (2-1) is to find the minimum total fixed cost. Constraint (2-2) means that all demand should be covered. Constraint (2-3) is the binary constraint.

The set covering location model was established by Roth and Toregas, and then Garey and Johnson proved that the problem was NP-hard. In the set covering location models, the distance between the demand point and the assigned corresponding facility is known as the criteria for good or bad judgment. If the facility that customers select does not exceed a given standard value, the facility is considered to be able to meet the requirements, and the demand at this point can be covered by the facility, otherwise it is not covered. When all the demand points are covered by at least one facility, how to minimize the total cost, that is, to build the fewest facilities to meet all the needs, has become the initial idea of the set covering location problem.

2.2.2 Maximal Covering Location Problem

Considering the fairness of the demand point service, the set covering location model itself does not take the unevenness of the demand point distribution into account.

While covering all the demand points in real life tend to exceed the budget, from the perspective of efficiency, the solution will lead to a contradiction between capital budget and real cost.

Therefore, another research direction is: under the premise of a given fee, how to establish limited facilities to enlarge the coverage, resulting in the maximal covering location problem (MCLP). The maximal covering model takes into account the fact that the total investment limits of the candidate facilities and the covering radius of the facilities are known, to maximize the total demand what can be covered. The MCLP was raised by Church and ReVelle in 1974, taking the layout of the facilities in the network into account, and the MCLP problem was also proved to be NP-hard.

The MCLP can be modelled as:

d_i: demand at location i

p: number of facilities that will be built

$$Z_i = \begin{cases} 1, \text{demand point } i \text{ can be covered,} \\ 0, \text{otherwise} \end{cases}$$

$$X_j = \begin{cases} 1, \text{facility built at location } j, \\ 0, \text{otherwise} \end{cases}$$

$$a_{ij} = \begin{cases} 1, \text{if demand point } i \text{ can be covered by facility at location } j \\ 0, \text{otherwise} \end{cases}$$

Objective function:

$$\max \sum_{1}^{i} d_i * Z_i \qquad (2-4)$$

Subject to:

$$\sum_{1}^{j} a_{ij} * X_j \geq Z_i, i \; \forall \; I \qquad (2-5)$$

$$\sum_{1}^{j} X_j = p \, , j \; \forall \; J \qquad (2-6)$$

$$Z_i \in \{0,1\}, \; i \forall I \qquad (2-7)$$

$$X_j \in \{0,1\}, \ j \forall J \tag{2-8}$$

Objective function (2-4) is to maximize the demand coverage; constraint (2-5) means that if demand *i* is covered, there must be at least one facility providing service to it; (2-6) means that the quantity of facilities must be equal to; constraints (2-7) and (2-8) are binary constraints.

2.3 Gradual Covering Problem

From the recent study of facility location, the assumption of basic coverage problem shows limitations in the practical application and operation. The basic covering problem defines strictly for the distance and assumes that the demand points are completely covered within a given distance, and beyond that distance, the demand is completely unfulfilled. The distance assumed by the theory becomes the breakpoint of the application of the actual covering problem. The expression of the absolute mathematic distance is not practical in many real cases. Like we can not determine that being at the edge of the radius, the demand point is totally unacceptable, and even if it exists within the coverage radius, all the points that are theoretically fully covered may also differ in the degree of service acceptance. Therefore, the concept of gradual covering comes into sight.

The earliest concept of gradual coverage can be traced back to 1998, when Drezner used the Logit model for the division of market share in the context of the study of a facility location model under practical application, involving the purchase of competitiveness and full market shares, to express the purchasing power of the demand point more intuitively. This model shows that the purchasing power of the demand point within a certain range will decrease, and beyond that range, the purchasing power will be 0. The literature initially reflects the concept of gradual covering. Then Berman (2002) introduced a case where there may be a partial coverage in the maximal coverage problem by setting the coverage as a descending segment function of the distance.

The basic coverage problem holds that whether the demand point is covered depends entirely on the distance to the nearest facility, and that the coverage radius *D* in the theoretical model implies a strict assumption that the demand point is fully covered within the facility coverage radius *D*, otherwise the demand point is completely not covered, so the distance *D* has become a breakpoint of the coverage model, which is unrealistic in many practical applications. With the reality of the study of the coverage problem, the researchers have put forward different function of coverage distance and coverage level in practical application for the more accurate description of the coverage, so that the problem of gradual covering becomes a branch of the facility location study.

Drezner (1998) has proposed a concept of gradual coverage, taking into account the customer purchasing power and market share in the competitive environment in the use of mathematical models. The purchasing power of demand points is gradually decreased by distance, reflecting the gradual coverage of the meaning. Drezner (2002) clearly raised the problem of gradual coverage at the meeting of the Institute of Decision Science, described in detail the coverage level at the coverage radius, and described the practical requirements with incremental coverage. There are four applications for the gradual covering model: (1) to meet consumer satisfaction as the goal of the location problem; (2) emergency facilities service; 3) the competitive location problem; (4) the broadcast, television, mobile phone signal transmission problems.

Berman and Krass (2002) proposed a generalized large coverage location model (GMCLP), which refine the concept of coverage and extend the coverage hypothesis of [0,1] to a multiple hypothesis, and introduce the idea of "partial coverage". They consider the distance between the demand points and facility as a non-increasing segment function, considering that there are several facilities near each demand point *i*, and that the different facilities have a different coverage for *i*, so each demand point corresponds to a multiple coverage level set, to maximize the weight of the total coverage over the difference in segment coverage. Berman (2003) has improved the theory of gradual coverage, and discussed the problem from two perspectives: P-median model and maximal coverage model. Eiselt et al. (2008) added the minimum tolerance to the setting of the gradual coverage problem, analyzed the coverage problem considering the quality of service, merged the different service levels into the target by different description of the horizontal fading function in the solution, and expanded the traditional set coverage problems. Berman and Drezner (2009) studied the coverage problem with variable radius, and linked the cost of construction to the coverage distance that the corresponding coverage radius and the number of facilities were determined by the limitation of investment cost, and presented the heuristic algorithm, and the model of discrete problem is discussed at the same time.

Mestre (2006), Kalcsics (2010) and others scholars have studied and validated the problem of gradual coverage or variable radius coverage from different areas. The above studies on the coverage problem are classified or segmented from different aspects of the covering function, but the non-continuity problem is encountered, whether it is based on the plane Euclidean distance or the network distance. Drezner (2010) further considers the problem of random coverage in continuous state, and describes the linear attenuation by means of the continuous uniform distribution function curve. At the same time, the BTST (Big Triangle Small Triangle) method for the random coverage problem of plane single facility is given. Berman, Drezner and Krass (2010) analyzed the situation of three types of facility location models, such as the gradual coverage problem, the combined coverage problem and the variable radius problem in the location model.

In the facility location problem, different researches are described by the objective function from different angles of cost or benefit, and many examples in real life just show that not only from the cost and benefit point of view can accurately measure the rationality of its location, and sometimes more emphasis on utility, fair and other factors. Such as signal coverage, car rental, ambulance station, fire station, post office, park and other convenient public service facilities location decisions have similar characteristics.

Karasakal and Karasakal (2008) studied the problem of gradual covering based on quality of service. The concept of minimal tolerance service is introduced, taking into account the different stages of the attenuation function based on distance, and the different levels of service into the objective function to solve. Berman and Kalcsics (2009) combined with the sequential median model and the gradual decline coverage model, the progressive decline model of the network is proposed, as well as the three recessive functions of linear recession, segmentation recession and linear segmentation regression are given. On the basis of linear recession, the single facility and multi-facility gradual decline coverage location problem have been studied respectively. Berman and Drezner (2009) studied the gradual covering problem with variable radius. The article assumes that the cost of the facility is a monotonically increased with distance. The decision maker determines the radius of the facility and the number of facilities according to the budget. Drezner and Drezner (2010) further consider the random coverage problem in continuous state. By the continuous uniform distribution function curve, the linear attenuation is described and proved. At the same time, the BTST solution of the random coverage problem of single facility is given. Berman, Drezner, Krass (2010) reviewed the research on the three types of facility location models, such as the gradual coverage problem, the combination coverage problem and the variable radius problem in the location model, and look ahead the progress of the next step.

The gradual covering problem presents a more comprehensive and systematic extension of the basic location problem models. Interestingly, in the gradual covering model, the coverage radius and the level of coverage involved in the coverage function can be more flexible to transform into MCLP or P-median problems. The extension of gradual coverage is limited to the plane or Euclidean distance. The quantitative description of the gradual covering function is limited to the fact that the segmentation function is used to describe the actual situation, and there is still the bottleneck of discontinuity. Second, from the actual needs and other factors, the randomness of gradual coverage is not studied clearly with the practical application. For example, there are a large number of subjective or objective uncertainties like cost, demand, travel time, in many fields such as management science, engineering technology, military decision-making, etc.

2.4 Emergency Medical Service

The original concept of the emergency medical service system (EMS) originated in 1793, by the chief military officer Dominique-Jean Larrey from the Napoleon army. He found that if the injured soldiers could be quickly sent from the front to the base and got treated, then not only the humanitarianism could be shown but also further maintain the combat capability of the army. So he invented the system of flying ambulance, transported the wounded in accordance with the severity of the injury (triage). This system was later adopted by the armies of Europe and the United States, on which a modern emergency medical service system is based.

But until 1966, the emergency medical ambulance system was really established. At that time, the US National Academy of Sciences published the "accidental death and disability" white paper. After that, the United States set up the National Highway Authority to take responsibilities for the development of EMS.

Nowadays, the emergency medical service system refers to the provision of personnel and equipment in the appropriate area in an emergency case, to ensure a synergistic health service. Its main task is to treat patients with traumatic and acute care, as well as the pre-hospital care and the treatment during transition provided by the ambulance staff. At present, the development of emergency medical system in countries and regions around the world is unbalanced. We will review the current status of EMS.

First, the main steps of emergency medical service and the measurement of service performance are shown in Figure 1:

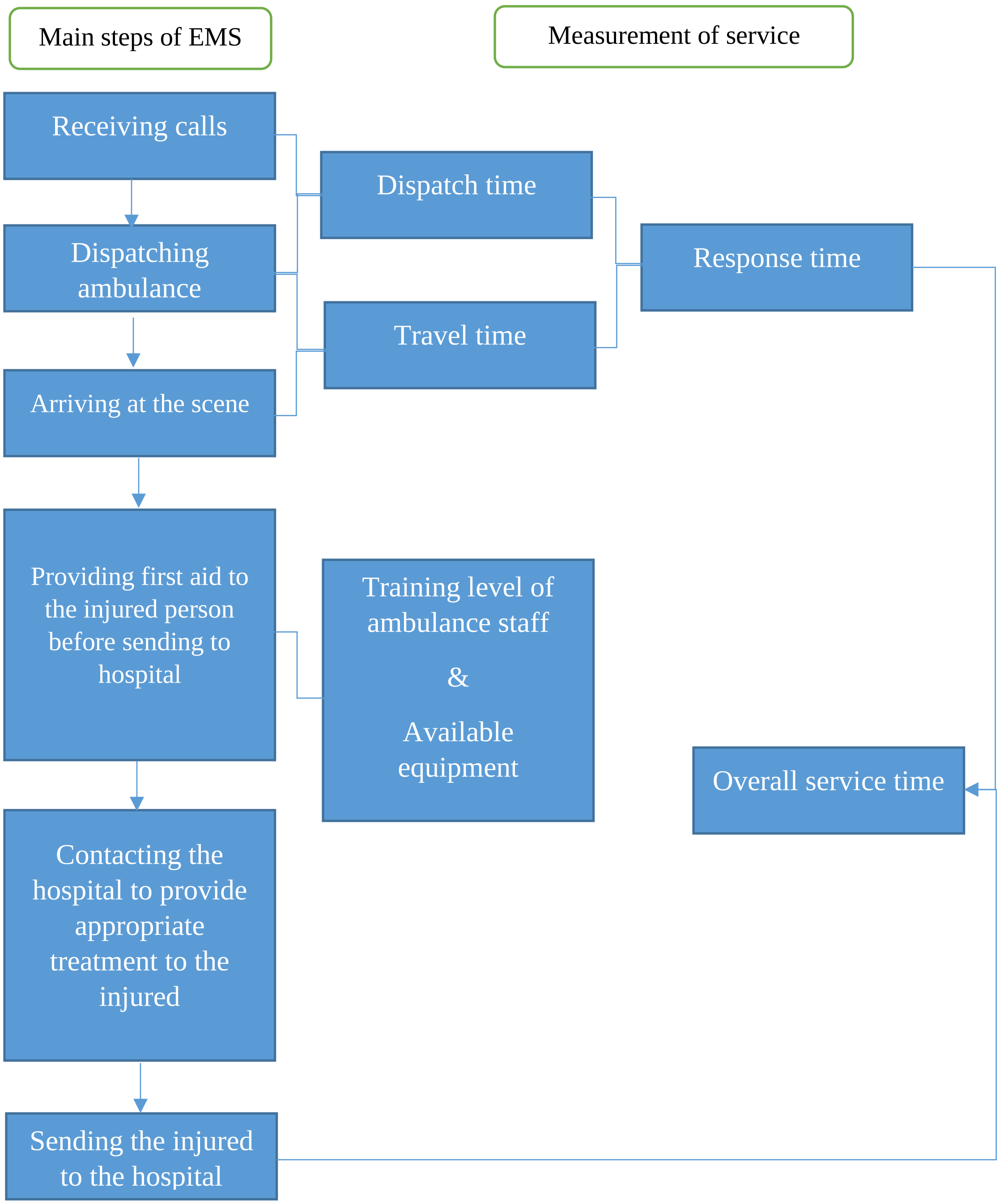

Figure 1- Main steps of EMS and measurement of service

(Source: Emergency Ambulance Service, Legislative Council of the Hong Kong Special Administrative region of the People's Republic of China, 1998)

In summary, there are several viable quantitative criteria for assessing the performance of emergency medical services:

1. Dispatch time: from the time of receiving the call to mobilize an ambulance.

Depending on the start-up time, it is possible to measure whether the ambulance is available for mobility, whether the program is effective or not, and whether the person responsible for the transfer is working.

2. Travel time: by moving an ambulance to the scene of the accident.

According to the travel time, it is possible to measure the actual time on route and to see whether ambulance coverage network is efficient.

3. Response time: from the time of receiving the call to the ambulance arriving at the scene of the accident.

According to the response time, we can measure how long it takes for ambulances to provide emergency medical services.

4. Overall service time: from the time of receiving the call to sending the injured to the hospital.

According to the service time, we can measure how long it takes for an ambulance to complete an emergency medical service.

2.4.1 Management Structure

In several areas, ambulance services are considered one of several emergency services, and emergencies are usually handled by the Fire Service Department, so the ambulance service is under the Fire Service Department. In addition, some areas (such as Queensland, Australia) have State Emergency Service (SES), ambulance service is one of the operations, and other operations like fire services, disaster relief services.

In other areas, however, ambulance services are under the Health Department to be closely monitored, since ambulance services not only provide basic life-sustaining services, but also provide treatment services or advanced life-sustaining services. Table 4 shows the examples of management structure in certain areas.

	EMS belongs to Fire Department	**EMS belongs to Health Department**
Hong Kong, China	√	
Tokyo, Japan	√	
London, UK		√
British Columbia, Canada		√
Alberta, Canada		√
San Francisco, USA	All public ambulances	All private ambulances
New South Wales, Australia		√
Queensland, Australia	SES	

Table 4- Management structure of EMS in certain areas

(Source: Emergency Ambulance Service, Legislative Council of the Hong Kong Special Administrative region of the People's Republic of China; EMS in the Calgary Zone, Alberta Health Services)

2.4.2 Service Indicators

The medical community around the world is deepening the view that there is a need to refer to more relevant clinical criteria when calculating service time indicators. For example, if the patient suddenly has hypoxia, brain attack or heart disease, they are required to be treated within a few minutes, so as to avoid irreparable trauma.

In the case of life-threatening illnesses, if the ambulance personnel can shorten the call response time, then more people's lives can be saved. In most areas, call response time is used as a service time indicator. Table 5 sets out the service time indicators used in a number of selected areas.

Area			**Service Indicator**	
			Response time	**Travel time**
Asia	China	Beijing	√	
		Shanghai	√	
		Hong Kong		√
	Japan	Tokyo	√	
	Singapore		√	
Oceania	Australia	Canberra	√	
		New South Wales	√	
Europe	UK		√	
North America	Canada	British Columbia	√	
		Alberta	√	
	USA	San Francisco	√	
		Houston	√	
		Honolulu	√	

Table 5- Service indicators in certain areas

(Source: Emergency Ambulance Service, Legislative Council of the Hong Kong Special Administrative region of the People's Republic of China; EMS in the Calgary Zone, Alberta Health Services)

Table 6 shows the performance of emergency ambulance services in selected areas. It can be seen from this data that not all regions are able to meet their service targets.

Area		**Service Indicator**	**Service performance**
Asia			
China	Beijing	N/A	Response time: ≤10 min: 27.66%

			10-15 min: 24.14% 15-20 min: 24.05% 20-30 min: 17.11% >30 min: 7.04%
	Shanghai	Urban areas response time: 8 min Rural areas response time: 30 min	N/A
	Hong Kong	Travel time < 10 min: 95%	89.53%
Japan	Tokyo	N/A	Response time < 5 min: 42%
Singapore		Response time ≤ 11 min	N/A
Oceania			
Australia	Canberra	Response time < 8 min: 90%	Response time <8 min: 50% Response time < 14.5 min: 90%
	New South Wales	Urban areas response time ≤ 14 min: 95% Rural areas response time ≤ 19 min: 95%	Dispatch time ≤ 3 min: 95% Urban areas response time ≤ 8 min: 50% Rural areas response time ≤ 9 min: 50%
Europe			
UK		Urban areas response time ≤ 14 min: 95% Rural areas response time ≤ 17 min: 95%	Urban areas response time ≤ 14 min: 84.1% Rural areas attendance time ≤ 17 min: 96,2%
North America			

Canada	British Columbia	Response time: 8 min	Response time < 6.91 min: 50% Response time < 12 min: 90%
	Alberta	Response time: 12 min	Response time < 15 min: 90%
USA	San Francisco	Urban areas response time: 8-10 min Suburb areas response time: 10-15 min Rural areas response time: 15-20 min	Urban areas response time 8-10 min: 80% Other areas response time: 75-80%
	Houston	Response time: 6min	N/A
	Honolulu	Response time 8-10 min: 90%	Response time ≤ 10 min: 92%

Table 6- Service performance of EMS in certain areas

(Source: Emergency Ambulance Service, Legislative Council of the Hong Kong Special Administrative region of the People's Republic of China; EMS in the Calgary Zone, Alberta Health Services)

From the tables above, we can see that the most used indicators in EMS is response time. However, measuring the service of EMS only with response time is not enough, we need a more holistic approach to optimizing the operation of the ambulance station. In the following chapters, we will discuss different ambulance station location optimization models and measure the corresponding performance with more indicators.

Chapter 3-Mathematical Models

In this chapter, we will first introduce two measures into our models—survival probability and service quality. Then we will discuss four known facility location models (MCLP, MSLP, MCLP+PR and MSLP+PR) for ambulance station location problem with objectives of maximizing demand coverage and maximizing expected survivors based on the paper from Erhan Erkut et al. (2007), and eight extended models which incorporate new parameters and further standards based on the models we have. Table 7 below shows the summary of the models.

Problem	**Certainty**	**Objective**	**Location**	**Assignment**
MCLP	Deterministic	Max. demand coverage	√	
MSLP	Deterministic	Max. expected survivors	√	√
MQLP	Deterministic	Max. service quality	√	√
MCLP+PR	Stochastic	Max. demand coverage	√	√
MSLP+PR	Stochastic	Max. expected survivors	√	√
MQLP+PR	Stochastic	Max. service quality	√	√
MCLP+PR+S	Stochastic	Max. demand coverage	√	√
MCLP+PR+Q	Stochastic	Max. demand coverage	√	√
MSLP+PR+C	Stochastic	Max. expected survivors	√	√
MSLP+PR+Q	Stochastic	Max. expected survivors	√	√

MQLP+PR+C	Stochastic	Max. service quality	√	√
MQLP+PR+S	Stochastic	Max. service quality	√	√

Table 7-Summary of models

3.1 Introduction of Parameters

3.1.1 Survival Probability

Most published research paper relevant to survival rates to EMS response time focuses on cardiac arrest.

A total of 29 articles concerning the survival rates of out-of-hospital cardiac arrest (OHCA) were reviewed and analyzed by Eisenberg et al. (1990). As a result, various factors were involved in affecting survival rates, such as response periods, demographic as well as physiological variations among areas, system design (the exact ways of training EMS staff, as well as the precise procedures of performance), diverse definitions of terms, including “response time” as well as “cardiac arrest”, and the exact accordance of applied procedures with standards.

In order to reduce the possibility of death in critically ill patients, whether to reach the scene as soon as possible to give the patient first aid, becomes the key to emergency medical system configuration. According to Eisenberg’s study (1993), patients who were OHCA were able to be given Basic Life Support (BLS) within 4 minutes of cardiopulmonary function, or be given Advanced Cardiac Life Support (ACLS) within 8 minutes, the survival rate of patients is about 43%; but if the BLS for more than 8 minutes or ACLS more than 16 minutes did not implement, the patient's survival rate close to zero, as shown in Figure 2.

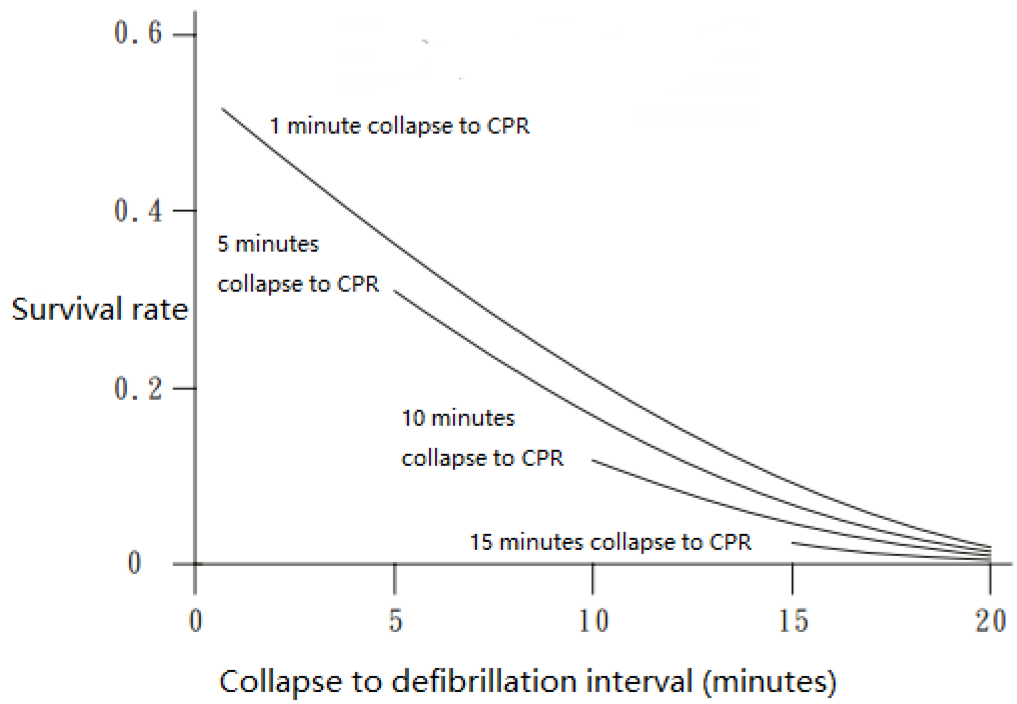

Figure 2- Survival rate of OHCA patients (Source: Valenzuela et al, 1997)

In the above-described study, assumptive survival curves from collapse time point were exhibited for five types of EMS system: paramedic, EMS vehicles solely equipped with emergency medical technicians (EMT), EMT equipped with defibrillation capacity (EMT-D), EMT-D accompanied by paramedic, as well as EMT accompanied by paramedic. It is assumed that the survival rate starts with 100% at collapse, while declines linearly to zero within 10 minutes without intervention in assumptive survival curves. After the arrival of EMTs as well as administration of cardiopulmonary resuscitation (CPR), a decreased but negative slope is speculated in the survival curve, which is speculated to decline profoundly in case of defibrillation application. The stabilization of the curve (slope = zero) is reached in case of the arrival of paramedics with the application of medication as well as intubation, or admission to hospital if EMS systems are not equipped with paramedics.

The benchmark survival rates over stabilization varied from 35% of EMT-D/paramedic systems to 10% of EMT systems, which are consistent with the findings in King County, WA, where evolution of EMS system has initiated from EMT, EMT-D, EMT/Paramedic, finally to EMT-D/Paramedic.

The research performed in casinos (Valenzuela et al. 2000) is likely to provide the most persuasive support that survival rates of cardiac arrest is enhanced by short response periods, of which, security guards were well-trained executors of defibrillation as well as CPR. The precise time of collapse was calculated from videos, and intervals from collapse to CPR were generally less than 3 minutes. The accurate time period of the study distinguishes it from others, the majority of which are either neglected or estimated by bystanders, and are burdened with longer intervals from collapse to CPR. Here, survival rate of 74% was observed in patients undergoing defibrillation within 3 minutes after collapse, however, survival rate at only 49% was detected in those not receiving defibrillation in time.

Part of the study enrolled by Eisenberg et al. lacked response times, or averages or proportions were documented, while response time distributions as well as estimated survival speculated from response periods were presented in several studies. Hence, four correlated articles with survival analyses are further discussed in this study. We should notice that all these studies defined “survival” as “survival until discharge from hospital”.

Firstly, using data of King County, Washington, US, Larsen et al. (1993) applied linear regression for survival rate based on it’s system of cardiac arrest surveillance. The equation is:

$$P\left(I_{CPR}, I_{Defib}, I_{ACLS}\right) = 0.67 - 0.023 I_{CPR} - 0.011 I_{Defib} - 0.021 I_{ACLS} \qquad \text{(3-1-1)}$$

where $I_{CPR}, I_{Defib}, I_{ACLS}$ denote the minutes from collapse to CPR, defibrillation and ACLS separately. There was a huge difference between expected and observed survival probability when the response time was quite long. For example, due to the different factors embedded in the system, the expected survival rate was supposed to be 0%, while the actual rates varied from 3% to 20%.

In the second study, conducted by Valenzuela et al. (1997), data from King County (Washington, US) as well as Tucson (Arizona, US) were utilized to analyze survival function by using logistic regression. Of the above studies, a number of factors were enrolled, including age, time period from collapse to CPR, manual CPR performed by bystanders/collapse to CPR interval interaction, time period from collapse to defibrillation, as well as manual CPR conducted by bystanders. Remarkably, no significant effect was detected in location (Tucson or King County) in terms of survival after balance of the above-described indicators, for example, the same survival function could be applied in both urban regions. Afterwards, another survival function was presented, which, however, only enrolled the time period from collapse to defibrillation as well as time period from collapse to CPR. The second function approximates more accurately and precisely than the first one:

$$P(I_{CPR}, I_{Defib}) = 1/(1 + \mathrm{e}^{-0.260+0.106 I_{CPR}+0.139 I_{Defib}}) \quad (3\text{-}1\text{-}2)$$

In this study, the authors pointed out that the survival probability is overestimated when there was long response time, which was in contrast with the first research.

We refer to the third study by Waaelwijn et al. (2001), which collected statistics from Netherlands, Amsterdam, as well as the surrounding areas. Consequently, three diverse survival functions were determined by use of logistic regression, including the paramedic, the first responder, as well as the perceptions of the bystander. The first and second functions were affected by multiple factors, including the essential role of advanced CPR as well as initial diagnosis of heart rhythm. The last function was influenced by three indicators, including the witness of collapse by EMS staff, time periods from collapse to elementary CPR, as well as time periods from CPR to the application of EMS vehicle. The function is:

$$P(X_{EMS}, I_{CPR}, I_{Response}) = 1/\left(1 + e^{0.04+0.7X_{EMS}+0.3I_{CPR}+0.14\left(I_{Response}-I_{CPR}\right)}\right) \quad (3\text{-}1\text{-}3)$$

where X_{EMS} equals to 1 if the cardiac arrest was witnessed by an EMS staff and 0 otherwise, and $I_{Response}$ presents the response time in minutes.

The last study we discuss here was performed by De Maio et al. (2003), in which statistics from different municipalities in Ontario, Canada were employed, followed by stepwise logistic regression in estimation of survival possibility. As a result, age, witness of collapse, EMS response time as well as CPR applied either by bystander or by police or fire were indicators of the final model. After that, an ad-hoc procedure was applied to balance the influences of all explanatory factors, except response time, leading to a predictive function of survival possibility only relying on EMS response time among the studying population. The function is:

$$P\left(I_{Response}\right) = 1/(1 + e^{0.679+0.262 I_{Response}}) \qquad (3\text{-}1\text{-}4)$$

3.1.2 Service Quality

Most of the basic covering problems are modeled with a certain coverage radius and scope from the view of enterprises. Based on this, the facility location model and decision are taken. From the point of service quality, the reference point of the facility location is based on the view of users, which presents a difference between effectiveness and efficiency.

In the past, the coverage problem involved the binary of variable selection, that is, the demand point is either covered or not covered in the decision result. On the expansion of the binary coverage problem, the expression of the covering function is mostly a segmented or ladder transition, there are still non-continuous bottlenecks, so the performance is still not closer to reality. Commonly used coverage functions are shown in Figure 3.

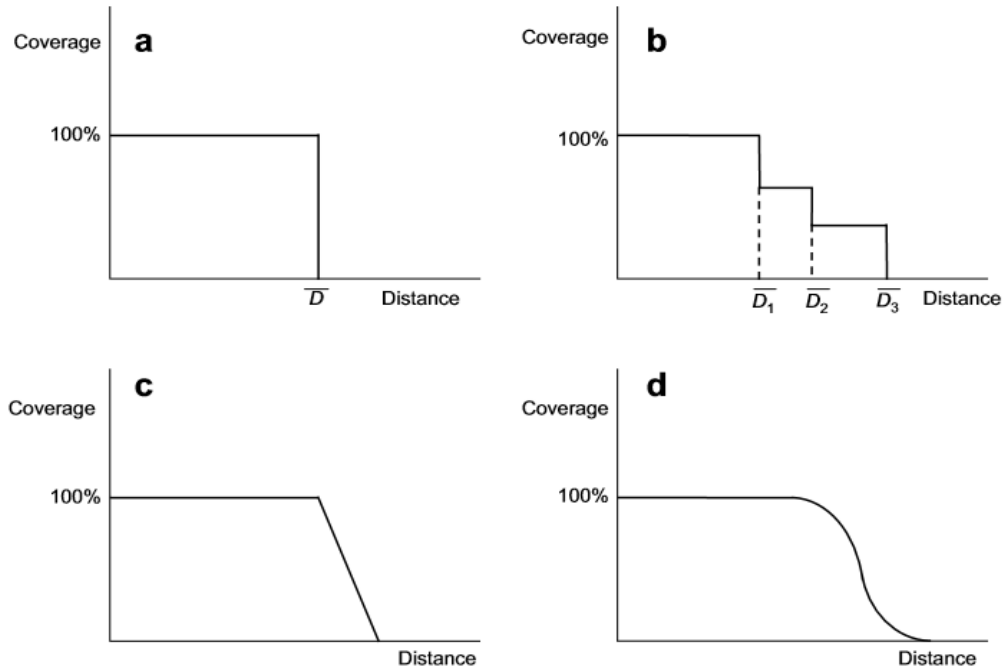

Figure 3- Coverage Functions (Source: Eiselt et al, 2008)

Figure 3(a) is a basic binary coverage function; Figure 3(b) shows a stepwise diminishing coverage; Figure 3(c) shows a linear decreasing function; Figure 3(d) is neither concave nor convex.

In Figure 3(a), the demand point is completely covered within the coverage radius $\overline{D}$, and if it exceeds this radius, it is not covered at all. This strict dichotomy is relatively simple, but most of the time the actual situation is much different.

Based on this, when the distance exceeds the coverage radius, the scholars put forward a variety of coverage functions. Church and Roberts (1983) proposed a concept of gradual coverage, describing the coverage level as a segmentation function, that is, when the distance between the demand point and the facility exceeds the coverage radius, the coverage level will be a stepped downward trend like in Figure 3(b).

In Figure 3(b), the function can be $f_i(t) = a_i^k$, t ∈ (r_i^{k-1}, r_i^k), $k = 1,2, \dots,$ K, $0 = a_i^K < \cdots < a_i^2 < 1$, $l_i = r_i^0 < r_i^1 < \cdots < r_i^K = U_i$. In the function, a_i^k is the coverage standard, r_i^k is the coverage radius. Apparently, when the coverage standard and the coverage radius are both equal to 1, it can be converted to the Maximal Covering Location Problem (MCLP), like Figure 3(a).

Although this type of functions are subdivided on the basis of the basic covering function, they are still discrete functions and can not describe the continuous change in real life.

Pirkul and Schilling (1982) defined coverage level as a linear function with distance, and when the distance increases, the coverage level is linearly decreasing.

In Figure 3(c), the function can be $f_i(t) = 1 - \frac{1}{a}t$, $i \in V$. $a = max_i$, $d_i(j)$ is a parameter. For $i \in V$, $u_i = a$, and $l_i = 0$, so:

$$\sum_i C_i\,(S) = \sum_i w_i f_i\left(d_i(S)\right) = \sum_i w_i - \frac{1}{a}\sum_i d_i\,(S) w_i \qquad (3\text{-}1\text{-}5)$$

And the coverage function can be transformed into the traditional P-center problem. In this regard, this kind of function can describe the situation of continuous change, but this linear relationship is too simple, it is difficult to describe the complex changes in real life situation.

Berman and Krass defined the coverage function as a non - convex non - concave function with distance, like in Figure 3(d). The function can be:

$$\mathrm{f}(d_{ij}) = \begin{cases} 1, d_{ij} < \overline{D_1} \\ \frac{1}{2} + \frac{1}{2}\cos(\frac{\pi}{\overline{D_2} - \overline{D_1}}\left(d_{ij} - \frac{\overline{D_2} + \overline{D_1}}{2}\right) + \frac{\pi}{2}),\, d_{ij} \in [\overline{D_1}, \overline{D_2}\,] \\ 0, d_{ij} > \overline{D_2} \end{cases} \qquad (3\text{-}1\text{-}6)$$

This kind of function is closer to the relationship between distance and coverage in real life, and can simulate the real situation well.

The quantitative representation of the coverage functions is to explain the relevance between the basic coverage problem and the gradual coverage model. By describing and defining the service quality function, the understanding of the concept of "service quality" is more clearly expressed.

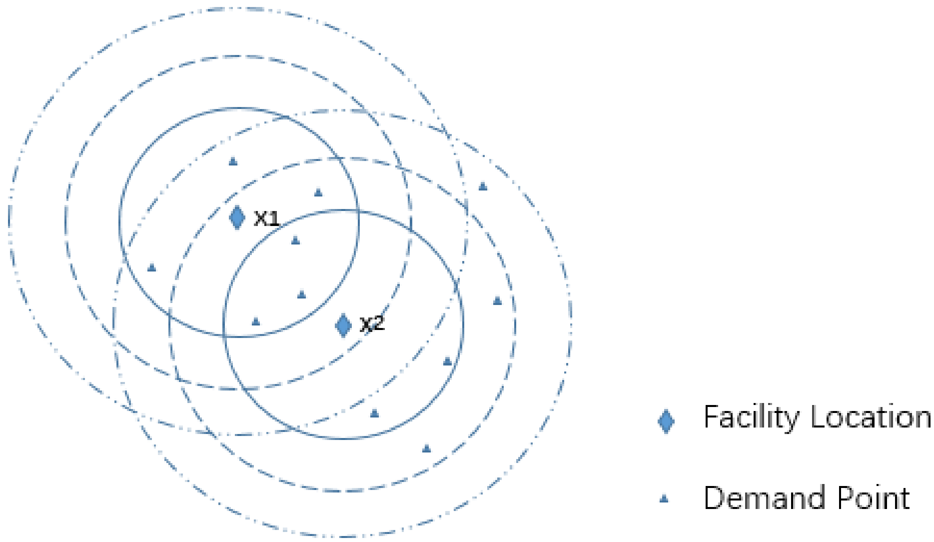

Figure 4- Difference between MCLP and gradual covering models based on service quality

In the MCLP model, there is only one coverage radius, while in the gradual covering models, there will be various radius. The difference between MCLP and gradual covering models based on service quality is illustrated in Figure 4.

If we define the service quality with the three levels of high, median and low as circles shown in Figure 4 from small to large circle, which are coverage radius of the facility, and assume that there are 11 customer demand points and two candidate facilities (*X1* and *X2*, respectively) in the location problem, the facility *X1* will be selected as the service facility location based on the traditional MCLP method. Six demand points will have the high service quality, but will cause the other five demand points are not satisfied or worse; if we accord to the idea proposed by this research, then *X2* will be chose for the service facility, although only five demand points of service quality are high, there will also be three points of demand for median and low service quality, which means that the overall service quality will be higher and help the operation of EMS system.

3.1.3 Models with Deterministic and Stochastic Measures

We will discuss models with both deterministic and stochastic measures in the following section. For the deterministic measures, we apply the average response time to them. While for the stochastic measures, we obtain the expected response time based on the probabilistic distribution and put them in the simulation so that we will get different scenarios of the response times. For each scenario of the response time, the resulting service measure, which is a function of the response time, is calculated. The expected values of service measure parameters are then computed based on the results of this simulation. The model with stochastic measures are indicated by PR.

3.2 Models with Deterministic Measures

In this section, we will present three models with deterministic measures: MCLP, MSLP and MQLP.

3.2.1 The Maximal Coverage Location Problem (MCLP)

The MCLP is one of the most basic models when it comes to facility location problems. As we mentioned in Chapter 2, the MCLP aims to establish limited facilities to enlarge the coverage. The maximal covering model takes into account the fact that the total investment limits of the candidate facilities and the covering radius of the facilities are known, to maximize the total demand that can be covered.

General notations for the formulations are as follows:

Parameters:

m: the number of demand nodes,

n: the number of candidate locations,

q: the maximum number of stations,

d_i : the demand of node i,

t_c : the coverage radius of a station in time units,

t_{ji} : the travel time from candidate location j to demand node i,

t_d : the pre-travel delay,

$$a_{ij} = \begin{cases} 1, \text{ if demand node } i \text{ is covered by candidate location } j, \text{i.e.}\, t_{ji} + t_d \le t_c \\ 0, \text{otherwise} \end{cases}$$

Variables:

$$x_j = \begin{cases} 1, \text{ if candidate location } j \text{ is selected} \\ 0, \text{otherwise} \end{cases}$$

$$y_i = \begin{cases} 1, \text{ if demand node } i \text{ is covered} \\ 0, \text{otherwise} \end{cases}$$

Objective function:

$$\max \sum_{i=1}^{m} d_i * y_i \qquad (3-1)$$

Subject to:

$$\sum_{j=1}^{n} a_{ij} * x_j \ge y_i \,, i = 1,2, \dots, m \qquad (3-2)$$

$$\sum_{j=1}^{n} x_j \le q \qquad (3-3)$$

$$x_j \in \{0,1\}, j = 1,2, \dots, n \qquad (3-4)$$

$$y_j \in \{0,1\}, i = 1,2, \dots, m \qquad (3-5)$$

The objective function (3-1) maximizes total demand points that can be covered. Constraints (3-2) state that demand point *i* can only be covered if at least one candidate location that covers i is selected; (3-3) limits the number of facilities to q; (3-4) and (3-5) are binary constraints.

3.2.2 The Maximal Survival Location Problem (MSLP)

In the MSLP, we incorporate a new parameter—survival probability. We assume that each demand point is served by the closest ambulance station and based on the function (3-1-2), so we will get the survival probability for deterministic response times of each demand—supply pair.

Additional notation for this formulation is as follows:

Parameters:

p_{ij} : probability that a patient at demand node i survives and is served by an EMS vehicle from station j based on the function (3-1-2)

Variables:

$$x_j = \begin{cases} 1, \text{if candidate location } j \text{ is selected} \\ 0, \text{otherwise} \end{cases}$$

$$y_{ij} = \begin{cases} 1, \text{if demand node } i \text{ is served by an EMS vehicle at location } j \\ 0, \text{otherwise} \end{cases}$$

Objective function:

$$\max \sum_{i=1}^{m} d_i \sum_{j=1}^{n} p_{ij} * y_{ij} \qquad (3-6)$$

Subject to:

$$\sum_{i=1}^{m} y_{ij} \leq m x_j , j = 1,2, \dots , n \qquad (3-7)$$

$$\sum_{j=1}^{n} y_{ij} = 1, i = 1,2, \dots , m \qquad (3-8)$$

$$\sum_{j=1}^{n} x_j \leq q \qquad (3-9)$$

$$x_j \in \{0,1\}, j = 1,2, \dots, n \qquad (3-10)$$

$$y_{ij} \in \{0,1\}, i = 1,2, \dots, m, j = 1,2, \dots, n \qquad (3-11)$$

The objective function (3-6) maximizes the expected number of patients who survive. Constraints (3-7) and (3-8) ensure that a demand node is assigned to only one open EMS facility. (3-9) limits the number of facilities to q. (3-10) and (3-11) are binary constraints.

3.2.3 The Maximal Quality Location Problem (MQLP)

In the MQLP, we incorporate another new parameter—service quality. Based on the function (3-1-6), we use the deterministic response time as the distance measure and obtain the service quality for deterministic response times of each demand—supply pair. More specifically, the service quality is calculated as follows:

$$s_{ij} = \begin{cases} 1, t_{ij} < \overline{T_1} \\ \frac{1}{2} + \frac{1}{2}\cos(\frac{\pi}{\overline{T_2} - \overline{T_1}}\left(t_{ij} - \frac{\overline{T_2} + \overline{T_1}}{2}\right) + \frac{\pi}{2}), t_{ij} \in [\overline{T_1}, \overline{T_2}] \\ 0, t_{ij} > \overline{T_2} \end{cases}$$

Where $\overline{T_1}$ and $\overline{T_2}$ are the break points for the best and worst acceptable response time.

Additional notation for this formulation is as follows:

Parameters:

s_{ij} : service quality that a patient at demand node i served by an EMS vehicle from station j

Objective function:

$$\max \sum_{i=1}^{m} d_i \sum_{j=1}^{n} s_{ij} * y_{ij} \qquad (3-12)$$

Subject to:

(3-7) – (3-11)

The objective function (3-12) maximizes the total service quality. Constraints (3-7) and (3-8) ensure that a demand node is assigned to only one open EMS facility. (3-9) limits the number of facilities to q. (3-10) and (3-11) are binary constraints.

3.3 Models with Stochastic Measures

In this section, we will present nine models with stochastic measures: MCLP+PR, MSLP+PR, MQLP+PR, MCLP+PR+S, MCLP+PR+Q, MSLP+PR+C, MSLP+PR+Q, MQLP+PR+C and MQLP+PR+S.

3.3.1 The Maximal Coverage Location Problem with Probabilistic Response Time (MCLP+PR)

In real life, the response time will not be deterministic and remains uncertain. So based on the MCLP, we place the probabilistic response time to calculate the probability of arrival within the coverage time threshold so we will get a new model MCLP+PR.

Additional notation for this formulation is as follows:

Parameters:

$\tilde{R}_{ij}$: expected value of probability that an ambulance at station *j* can reach demand node *i* within the coverage time standard

Objective function:

$$\max \sum_{i=1}^{m} d_i \sum_{j=1}^{n} \tilde{R}_{ij} * y_{ij} \qquad (3-13)$$

Subject to:

(3-7) – (3-11)

The Objective function (3-12) maximizes the total expected demand covered from each demand point, taking into account the coverage probabilities. Constraints (3-7) and (3-8) ensure that a demand node is assigned to only one open EMS facility. Constraint (3-9) limits the number of facilities to q. (3-10) and (3-11) are binary constraints.

3.3.2 The Maximal Survival Location Problem with Probabilistic Response Time (MSLP+PR)

Same as the MCLP and MCLP+PR, we place probabilistic response time, also based on function (3-1-2) to the MSLP to be closer to the real situation, so we have the MSLP+PR.

Additional notation for this formulation is as follows:

Parameters:

$\tilde{P}_{ij}$: expected value of probability that a patient at demand node *i* survives and is served by an EMS vehicle form station *j*

Objective function:

$$\max \sum_{i=1}^{m} d_i \sum_{j=1}^{n} \tilde{P}_{ij} * y_{ij} \qquad (3-14)$$

Subject to:

(3-7)— (3-11)

The objective function (3-14) maximizes the expected number of patients who survive. Constraints (3-7) and (3-8) ensure that a demand node is assigned to only one open EMS facility. (3-9) limits the number of facilities to q. (3-10) and (3-11) are binary constraints.

3.3.3 The Maximal Quality Location Problem with Probabilistic Response Time (MQLP+PR)

Same as the MCLP and MCLP+PR, MSLP and MSLP+PR, we place probabilistic response time, also based on the adjusted function of (3-1-6) to the MQLP to be closer to the real situation, so we have the MQLP+PR.

Additional notation for this formulation is as follows:

Parameters:

$\tilde{S}_{ij}$: excepted value of service quality that a patient at demand node i served by an EMS vehicle from station *j* based on probabilistic response time

Objective function:

$$\max \sum_{i=1}^{m} d_i \sum_{j=1}^{n} \tilde{S}_{ij} * y_{ij} \qquad (3-15)$$

Subject to:

(3-7) – (3-11)

The objective function (3-15) maximizes the service quality at all demand points. Constraints (3-7) and (3-8) ensure that a demand node is assigned to only one open EMS facility. (3-9) limits the number of facilities to q. (3-10) and (3-11) are binary constraints.

3.3.4 The Maximal Coverage Location Problem with Probabilistic Response Time and Survival (MCLP+PR+S)

Based on the MCLP+PR, we incorporate a standard of survival probability into the model, so we will get MCLP+PR+S. When maximizing the demand coverage, we also make sure to maintain the survival probability at a certain level by allowing only the acceptable level of survival probability to be considered in the objective function. We denote by b_{ij} the survival probability from demand node *i* to ambulance station *j* and by $b_{standard}$ the minimum acceptable survival probability.

Additional notation for this formulation is as follows:

Parameters:

$$\hat{b}_{ij} = \begin{cases} b_{ij}, \text{if } b_{ij} \geq b_{standard} \\ 0, \text{otherwise} \end{cases}$$

Objective function:

$$\max \sum_{i=1}^{m} d_i \sum_{j=1}^{n} (\tilde{R}_{ij} * \hat{b}_{ij})\, y_{ij} \qquad (3-16)$$

Subject to:

(3-7)— (3-11)

The Objective function (3-16) maximizes the total expected demand covered, taking into account the survival probabilities. Constraints (3-7) and (3-8) ensure that a demand node is assigned to only one open EMS facility. Constraint (3-9) limits the number of facilities to *q*. (3-10) and (3-11) are binary constraints.

3.3.5 The Maximal Coverage Location Problem with Probabilistic Response Time and Quality (MCLP+PR+Q)

Based on the MCLP+PR, we incorporate a standard of service quality into the model, so we will get MCLP+PR+Q. When maximizing the demand coverage, we also make sure to maintain the service quality at a certain level by allowing only the acceptable level of service quality to be considered in the objective function. We denote by c_{ij} the service quality from demand node *i* to ambulance station *j* and by $c_{standard}$ the minimum acceptable service quality.

General notations for the formulations are as follows:

Parameters:

$$\hat{c}_{ij} = \begin{cases} c_{ij}, \text{if } c_{ij} \geq c_{standard} \\ 0, \text{otherwise} \end{cases}$$

Objective function:

$$\max \sum_{i=1}^{m} d_i \sum_{j=1}^{n} (\tilde{R}_{ij} * \hat{c}_{ij})\, y_{ij} \qquad (3-17)$$

Subject to:

(3-7)— (3-11)

The Objective function (3-17) maximizes the total expected demand covered, taking into account the service quality. Constraints (3-7) and (3-8) ensure that a demand node is assigned to only one open EMS facility. Constraint (3-9) limits the number of facilities to q. (3-10) and (3-11) are binary constraints.

3.3.6 The Maximal Survival Location Problem with Probabilistic Response Time and Coverage (MSLP+PR+C)

Based on the MSLP+PR, we incorporate a standard of demand coverage into the model, so we will get MSLP+PR+C. When maximizing the survival rate, we also make sure to limit the response time at a certain level by allowing only the acceptable response time to be considered in the objective function.

Additional notation for this formulation is as follows:

Parameters:

$$a_{ij} = \begin{cases} 1, \text{if demand node } i \text{ is covered be candidate location } j, \text{i.e.}\, t_{ji} + t_d \leq t_c \\ 0, \text{otherwise} \end{cases}$$

Objective function:

$$\max \sum_{i=1}^{m} d_i \sum_{j=1}^{n} (\tilde{P}_{ij} * a_{ij})\, y_{ij} \qquad (3-18)$$

Subject to:

(3-7)— (3-11)

The Objective function (3-18) maximizes the total survivors, taking into account the demand coverage. Constraints (3-7) and (3-8) ensure that a demand node is assigned to only one open EMS facility. Constraint (3-9) limits the number of facilities to q. (3-10) and (3-11) are binary constraints.

3.3.7 The Maximal Survival Location Problem with Probabilistic Response Time and Quality (MSLP+PR+Q)

Based on the MSLP+PR, we incorporate a standard of service quality into the model, so we will get MSLP+PR+Q. When maximizing the survival rate, we also make sure to maintain the service quality at a certain level by allowing only the acceptable level of service quality to be considered in the objective function.

Objective function:

$$\max \sum_{i=1}^{m} d_i \sum_{j=1}^{n} (\tilde{P}_{ij} * \hat{c}_{ij}) \, y_{ij} \qquad (3-19)$$

Subject to:

(3-7)— (3-11)

The objective function (3-19) maximizes the total survivors, taking into account the service quality. Constraints (3-7) and (3-8) ensure that a demand node is assigned to only one open EMS facility. Constraint (3-9) limits the number of facilities to q. (3-10) and (3-11) are binary constraints.

3.3.8 The Maximal Quality Location Problem with Probabilistic Response Time and Coverage (MQLP+PR+C)

Based on the MQLP+PR, we incorporate a standard of demand coverage into the model, so we will get MQLP+PR+C. When maximizing the service quality, we also make sure to maintain the demand coverage at a certain level by allowing only the acceptable response time to be considered in the objective function.

Objective function:

$$\max \sum_{i=1}^{m} d_i \sum_{j=1}^{n} \left(\tilde{S}_{ij} * a_{ij}\right) y_{ij} \qquad (3-20)$$

Subject to:

(3-7)— (3-11)

The objective function (3-20) maximizes the total service quality, taking into account the demand coverage. Constraints (3-7) and (3-8) ensure that a demand node is assigned to only one open EMS facility. Constraint (3-9) limits the number of facilities to q. (3-10) and (3-11) are binary constraints.

3.3.9 The Maximal Quality Location Problem with Probabilistic Response Time and Survival (MQLP+PR+S)

Based on the MQLP+PR, we incorporate a standard of survival rate into the model, so we will get MQLP+PR+S. When maximizing the service quality, we also make sure to maintain the survival rate at a certain level by allowing only the acceptable level of survival probability to be considered in the objective function.

Objective function:

$$\max \sum_{i=1}^{m} d_i \sum_{j=1}^{n} \left(\tilde{S}_{ij} * \hat{b}_{ij}\right) y_{ij} \qquad (3-21)$$

Subject to:

(3-7)— (3-11)

The objective function (3-21) maximizes the total service quality, taking into account the survival rate. Constraints (3-7) and (3-8) ensure that a demand node is assigned to only one open EMS facility. Constraint (3-9) limits the number of facilities to q. (3-10) and (3-11) are binary constraints.

Based on all the optimization models we have, in the next chapter, we will compare and analyze the results with four indicators using both the descriptive statistic analysis and the data envelopment analysis.

Chapter 4-Computational Comparison and Analysis

In this section, we will analyze and compare the twelve optimization models with four indicators: expected response time, expected service quality, expected survival probability and expected demand coverage criteria, to measure the optimization models separately. All models will be presented in descriptive statistics analysis and data envelopment analysis (DEA).

4.1 Methodology

In this chapter, we will test all the models based on a public data from Calgary, Alberta, EMS system, 2004. The data are available at https://sites.ualberta.ca/~aingolfs/Data.htm.

Alberta Health Services (AHS) is the first and largest province-wide health system in Canada, which is responsible to provide fully-integrated health services to the over four million people living in Alberta. Being one of the biggest region in Alberta, the Emergency Medical Services of Calgary Zone respond to over 160,000 events annually. Figure 5 below presents the EMS station of Calgary zone in the year 2014.

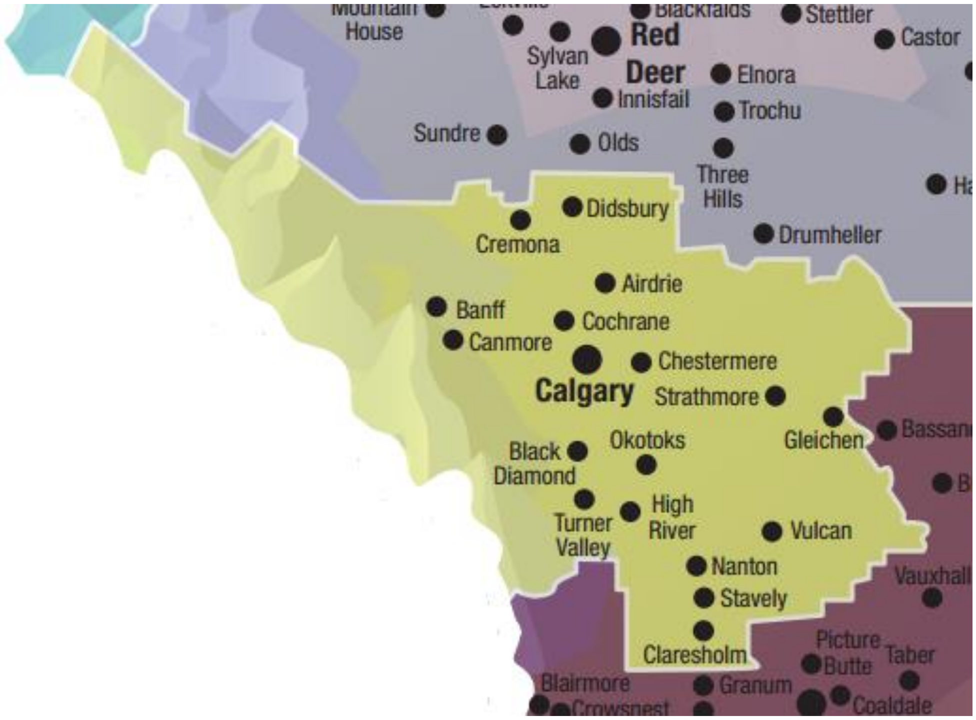

Figure 5- EMS station map of Calgary Zone

(Source: Alberta Health Services, 2014)

The total number of calls of the year 2004 was 45,294 and the number of urgent calls was 13,203, which we considered in our research. There were also 16 ambulance stations and 180 demand points. From the data set, we can find the fraction of calls at each demand point, average response times in seconds, standard deviation of response times in seconds, deterministic coverage for an 8-minute threshold (0 or 1), survival probability for deterministic response times, survival probability for probabilistic response times, etc.

We will focus on situations where the number of ambulance stations equal to 4, 8, 12 and 16. It is chosen so that the step-size is equal. The results will be further analyzed in the context of DEA. We assume that each ambulance station has perfect capacities, which means that there is no upper limit for the number of ambulances as long as we need one. For the new measure service quality, we get the value based on the mean and standard deviation of the response time from the dataset with a probabilistic distribution, and calculate it based on the function (3-1-6) and then run the simulation for 500 scenarios to get the expected service quality based on the probabilistic distribution. The models are solved on a Dell Inspiration with 2.58 GHz CPU clock and 2GB of RAM, and the average run time is around 300 seconds, with the maximal run time of 321 seconds. Codes are written in Python 3.4.2 using Jupyter Notebook and solved by Gurobi 7.0.2. Table 8 compares the sizes and characteristics of the twelve optimization models.

Problem	Binary variables	Constraints
MCLP	196	181
MSLP	2896	197
MQLP	2896	197
MCLP+PR	2896	197
MSLP+PR	2896	197
MQLP+PR	2896	197
MCLP+PR+S	2896	197
MCLP+PR+Q	2896	197
MSLP+PR+C	2896	197
MSLP+PR+Q	2896	197
MQLP+PR+C	2896	197
MQLP+PR+S	2896	197

Table 8- Sizes of 12 optimization models solved

4.2 Descriptive Statistics Analysis

In this part, we will focus on the descriptive statistics analysis of the four indicators: expected response time, expected service quality, expected survival probability and expected demand coverage criteria. From the public data set, we know that the number of urgent calls in Calgary, Alberta, in the year of 2004 was 13,203, so it would be approximately 36 urgent calls per day. With the fraction of calls at each demand point, we can get the expected urgent call per day for each demand point.

Basically, we calculate the values of indicators based on the result of assignment we have from the optimization models. With the assignment, we can get the response time, service quality, survival probability and demand coverage criteria at each demand point. With the sum product of these indicators and the fraction call of each demand separately, we can get the expected response time, expected service quality, expected survival probability and expected demand coverage criteria (we set a binary criteria of 480 seconds) per day. For models with deterministic and stochastic measures, we run the simulation for 500 scenarios based on the average and the standard deviation of responses times in seconds from the public data set, to get the performance of the four indicators. Table 9 to 12 present the descriptive statistics, which are the average value of these indicators and the coefficient of variable, within the 12 models under 4, 8, 12 and 16 ambulance stations and the best three performances for each indicator are marked in bold. Although when q=16, all ambulance stations will be open, the assignment of each model is different, so we still analyze the performance of it.

Also here is a note that the survival probability is calculated by the number provided in the data set, since the parameters used in the function are not available, we are not able to simulate it, while the results of the rest three indicators are based on the simulation.

q=4		Exp. Response time	Exp. Service Quality	Exp. Survival Probability	Exp. Demand Coverage Criteria
Mean	MCLP	**296.9279**	**0.9800**	0.0578	**0.6995**
	MSLP	311.7705	0.9508	0.0630	0.6039
	MQLP	**310.7406**	0.9684	0.0615	0.6163
	MCLPPR	313.1400	**0.9703**	**0.0672**	**0.6363**
	MSLPPR	311.8986	0.9501	**0.0673**	0.6021
	MQLPPR	311.2900	**0.9703**	**0.0672**	**0.6360**
	MCLPPRS	321.8863	0.8805	0.0640	0.5787
	MCLPPRQ	313.2410	**0.9692**	**0.0672**	0.6319
	MSLPPRC	362.9193	0.8279	0.0601	0.5592
	MSLPPRQ	**310.1179**	**0.9689**	**0.0672**	0.6355
	MQLPPRC	338.9319	0.8586	0.0604	0.5628
	MQLPPRS	333.5891	0.8790	0.0626	0.6015

CV	MCLP	12.17%	0.40%	N/A	7.52%
	MSLP	12.41%	0.78%	N/A	7.88%
	MQLP	11.97%	0.59%	N/A	7.40%
	MCLPPR	12.13%	0.54%	N/A	8.02%
	MSLPPR	12.66%	0.78%	N/A	8.44%
	MQLPPR	12.54%	0.57%	N/A	8.41%
	MCLPPRS	11.27%	1.19%	N/A	8.29%
	MCLPPRQ	12.19%	0.54%	N/A	8.45%
	MSLPPRC	10.53%	1.72%	N/A	9.23%
	MSLPPRQ	12.52%	0.54%	N/A	8.42%
	MQLPPRC	11.25%	1.42%	N/A	8.90%
	MQLPPRS	11.94%	1.22%	N/A	8.30%

Table 9- Descriptive statistics of the twelve models under 4 ambulance stations

q=8		**Exp. Response time**	**Exp. Service Quality**	**Exp. Survival Probability**	**Exp. Demand Coverage Criteria**
Mean	MCLP	**252.1995**	**0.9973**	0.0679	**0.8930**
	MSLP	284.5390	**0.9920**	0.0744	0.7920
	MQLP	303.5978	0.9912	0.0728	0.7621
	MCLPPR	**282.4585**	**0.9920**	**0.0788**	**0.7952**
	MSLPPR	284.8084	**0.9920**	**0.0788**	0.7912
	MQLPPR	288.1348	**0.9920**	**0.0788**	0.7900
	MCLPPRS	288.4825	0.9645	0.0775	0.7798
	MCLPPRQ	285.0797	**0.9916**	**0.0788**	0.7921
	MSLPPRC	295.2048	0.9542	0.0768	0.7667
	MSLPPRQ	**282.7854**	0.9915	**0.0788**	**0.7946**
	MQLPPRC	296.3099	0.9567	0.0767	0.7690
	MQLPPRS	333.5891	0.9724	0.0626	0.6015
CV	MCLP	13.13%	0.08%	N/A	3.33%
	MSLP	13.30%	0.22%	N/A	5.69%
	MQLP	13.30%	0.29%	N/A	7.47%
	MCLPPR	12.78%	0.23%	N/A	5.56%
	MSLPPR	13.08%	0.21%	N/A	6.12%
	MQLPPR	14.02%	0.23%	N/A	6.27%
	MCLPPRS	13.84%	0.43%	N/A	6.17%
	MCLPPRQ	13.16%	0.21%	N/A	6.15%
	MSLPPRC	12.94%	0.67%	N/A	6.42%
	MSLPPRQ	13.60%	0.22%	N/A	6.47%
	MQLPPRC	12.74%	0.67%	N/A	5.98%

	MQLPPRS	11.94%	0.36%	N/A	8.30%

Table 10- Descriptive statistics of the twelve models under 8 ambulance stations

q=12		Exp. Response time	Exp. Service Quality	Exp. Survival Probability	Exp. Demand Coverage Criteria
Mean	MCLP	**253.5882**	**0.9994**	0.0670	**0.9521**
	MSLP	279.4903	0.9957	0.0793	0.8478
	MQLP	293.3907	0.9947	0.0727	0.7950
	MCLPPR	277.8134	**0.9963**	**0.0834**	0.8531
	MSLPPR	**274.5099**	0.9955	**0.0839**	0.8467
	MQLPPR	274.9171	**0.9961**	0.0833	**0.8554**
	MCLPPRS	281.7596	0.9848	**0.0834**	0.8383
	MCLPPRQ	275.5277	**0.9961**	**0.0834**	**0.8543**
	MSLPPRC	278.3587	0.9860	0.0832	0.8428
	MSLPPRQ	**272.9058**	0.9957	**0.0839**	0.8527
	MQLPPRC	286.5083	0.9857	0.0800	0.8300
	MQLPPRS	278.9222	0.9893	0.0833	0.8425
CV	MCLP	13.15%	0.03%	N/A	2.79%
	MSLP	13.09%	0.14%	N/A	5.43%
	MQLP	13.73%	0.19%	N/A	6.53%
	MCLPPR	14.77%	0.14%	N/A	5.45%
	MSLPPR	13.83%	0.15%	N/A	5.34%
	MQLPPR	14.21%	0.16%	N/A	5.12%
	MCLPPRS	13.35%	0.17%	N/A	5.61%
	MCLPPRQ	13.59%	0.15%	N/A	4.83%
	MSLPPRC	13.25%	0.19%	N/A	5.35%
	MSLPPRQ	14.03%	0.14%	N/A	5.12%
	MQLPPRC	13.46%	0.21%	N/A	6.18%
	MQLPPRS	12.96%	0.18%	N/A	5.52%

Table 11- Descriptive statistics of the twelve models under 12 ambulance stations

q=16		Exp. Response time	Exp. Service Quality	Exp. Survival Probability	Exp. Demand Coverage Criteria
Mean	MCLP	**225.4382**	**0.9997**	0.0703	**0.9798**
	MSLP	272.3165	**0.9972**	0.0819	**0.8777**
	MQLP	292.1154	0.9946	0.0729	0.7964
	MCLPPR	**271.5101**	0.9970	**0.0865**	**0.8761**

	MSLPPR	272.0710	**0.9971**	**0.0865**	**0.8760**
	MQLPPR	274.8080	**0.9971**	**0.0865**	0.8731
	MCLPPRS	274.0697	0.9906	0.0862	0.8703
	MCLPPRQ	274.5319	**0.9971**	**0.0865**	0.8752
	MSLPPRC	272.4142	0.9903	0.0862	0.8732
	MSLPPRQ	**268.2299**	**0.9971**	**0.0865**	0.8759
	MQLPPRC	276.7716	0.9902	0.0861	0.8678
	MQLPPRS	272.6788	0.9907	0.0862	0.8711
CV	MCLP	13.37%	0.02%	N/A	1.23%
	MSLP	13.00%	0.10%	N/A	4.63%
	MQLP	12.69%	0.18%	N/A	5.98%
	MCLPPR	14.09%	0.13%	N/A	4.81%
	MSLPPR	13.72%	0.12%	N/A	5.02%
	MQLPPR	13.74%	0.11%	N/A	5.22%
	MCLPPRS	13.81%	0.15%	N/A	5.06%
	MCLPPRQ	13.73%	0.11%	N/A	5.04%
	MSLPPRC	14.26%	0.14%	N/A	5.08%
	MSLPPRQ	14.76%	0.13%	N/A	4.89%
	MQLPPRC	13.87%	0.15%	N/A	5.38%
	MQLPPRS	14.70%	0.15%	N/A	5.51%

Table 12- Descriptive statistics of the twelve models under 16 ambulance stations

From the tables above, we can see that with the increase of ambulance stations, the expected response time are decreasing and expected service quality, expected survival probability and expected demand coverage criteria are raising.

With ambulance station value from 4, 8, 12 and 16, models MCLP and MSLP+PR+Q always perform better than other models in terms of the expected response time; models MQLP, MQLP+PR and MCLP+PR always perform better than other models in terms of the expected service quality; models MCLP+PR, MSLP+PR, MQLP+PR, MCLP+PR+Q and MSLP+PR+Q always perform better than other models in terms of the expected survival rate; models MCLP and MCLP+PR always perform better than other models in terms of the expected demand coverage criteria.

Additionally, we also find that there is certain linear correlation between the four indicators. For example, a shorter response time will lead to a higher demand coverage, like MCLP and MCLP+PR; also a higher quality of service will lead to a higher probability of survival, like MCLP+PR and MQLP+PR. We calculate the correlation coefficient with the models in Table 13, and we can see that there is a moderate downhill linear relationship between expected response time and expected service quality and expected survival probability as well; a nearly perfect negative linear relationship between expected response time and expected demand coverage; a moderate uphill linear relationship between expected service quality as well as expected demand coverage; a moderate uphill linear relationship between expected survival rate and expected demand coverage.

	Exp. Response time	**Exp. Service Quality**	**Exp. Survival Probability**	**Exp. Demand Coverage Criteria**
Exp. Response time	1			
Exp. Service Quality	-0.80	1		
Exp. Survival Probability	-0.66	0.61	1	
Exp. Demand Coverage Criteria	-0.94	0.75	0.78	1

Table 13- Correlation coefficient for exp. response time, exp. service quality, exp. survival probability and exp. demand coverage

With all the four indicators and the overall performances with different numbers of ambulance station, we find that models MCLP, MCLP+PR, MQLP+PR, MCLP+PR+Q and MSLP+PR+Q perform better.

But we should also notice that the models with deterministic measures-- MCLP, MSLP and MQLP have weaknesses because of ignoring the discrimination between time or distance that are within or outside the coverage standard, and in reality, the actual travel times between a demand point and an ambulance station are highly variable, so the models with stochastic measures are much closer to real life.

In this regard, the optimisation models MCLP+PR, MQLP+PR and MSLP+PR+Q will be considered as the best solutions. To see the difference more intuitively, Figure 6 to 9 will show the performance of these three models with models which are slightly underperformed—MQLP+PR+C and MQLP+PR+S.

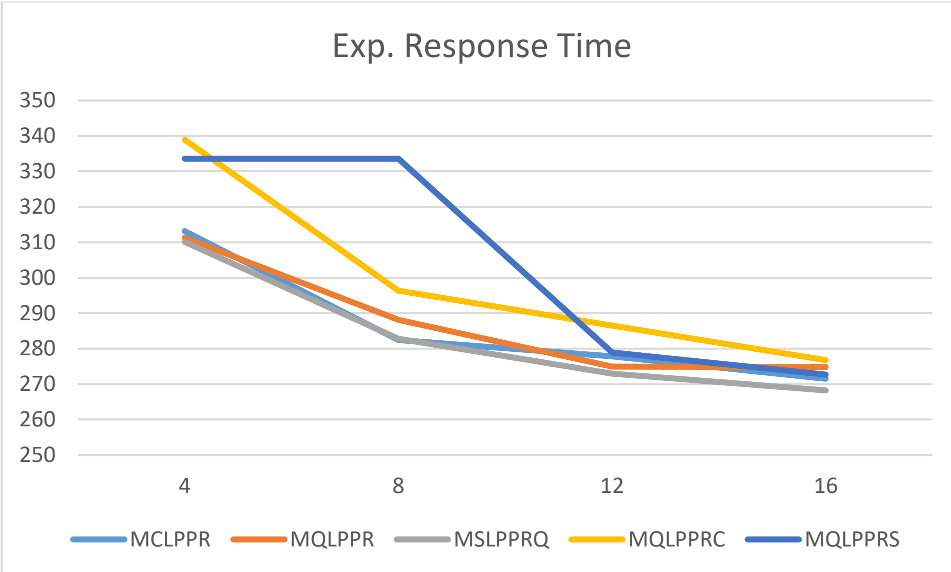

Figure 6- Differences between models on the performance of expected response time

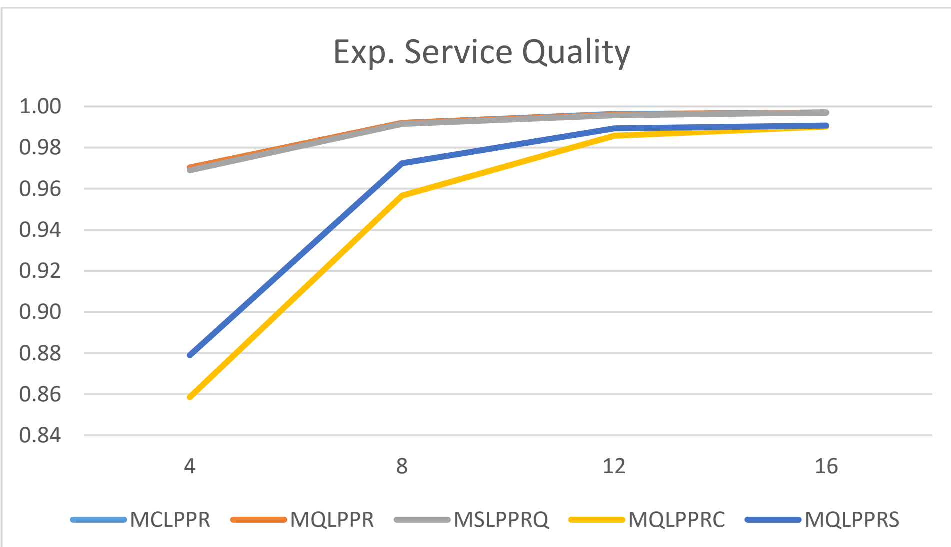

Figure 7- Differences between models on the performance of expected service quality

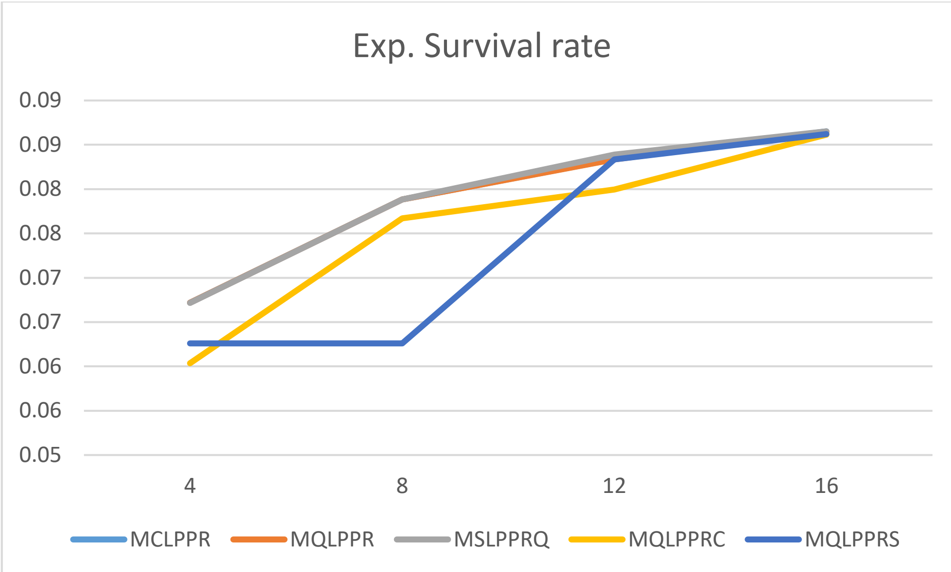

Figure 8- Differences between models on the performance of expected survival rate

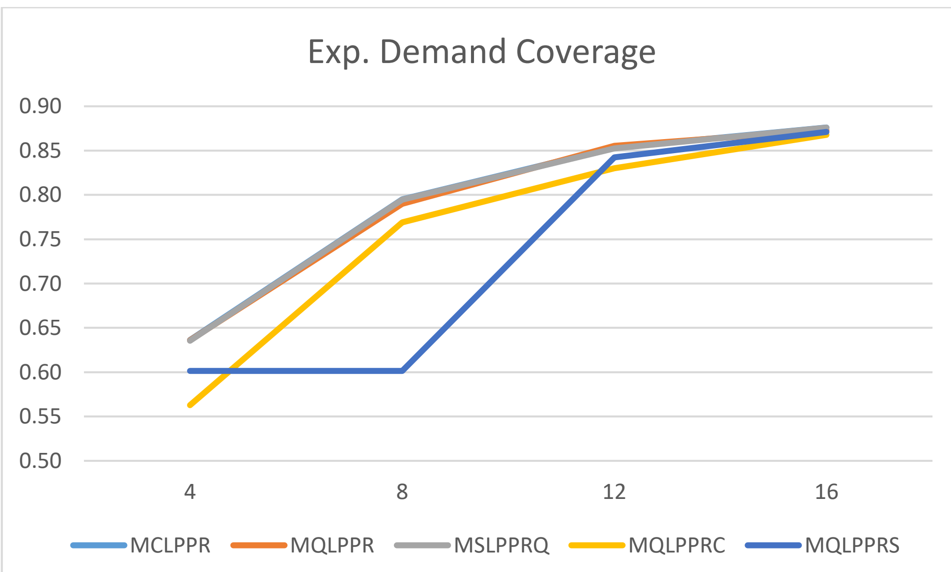

Figure 9- Differences between models on the performance of expected demand coverage

4.3 Data Envelopment Analysis

Other than comparing all the indicators separately, we use DEA in this part to analyze the indicators as a whole. Since we have multiple indicators here to measure the performance of each model under different numbers of ambulance station, DEA is a good method to evaluate the efficiency of multi-input and multi-output decision making. Since in the DEA, the efficiency is measured as the weighted sum of the outputs divided by the weighted sum of the inputs, the technique is generally used based on the assumption that the return to scale is constant. In the context of our analysis, we do not attempt to make a direct comparison among the different solutions as in the conventional DEA application but instead provide an evidence (as shown in the results) that the return to scale of this ambulance location application is in fact decreasing. The implication of the results suggests that the investment in this area yields a decreasing return to scale and thus the decision maker can convey a reasonable expectation for the investment for the service quality improvement.

In the DEA, first, we set all 12 models under 4 different numbers of ambulance stations (q=4, 8, 12 and 16) as decision making units (DMU), so there will be 48 DMUs. Then we adjust the expected response time into expected improvement of response time, because in DEA, it is assumed that the output should be increasing with input. In this case, the expected response time is decreasing with the increase of ambulance station, so we set the result of model MSLP under one ambulance station as a standard, and generate the expected improvement of response time, which will be raising with the increase of ambulance station.

Then, we set the number of station as input, expected improvement of response time, expected service quality, expected survival rate and expected demand coverage as output. To describe the DEA model, we let:

Parameter:

u_k: the input quantity (number of stations)for DMU(K)

v_{1k}: first output quantity (exp. improvememt of response time) for DMU(K)

v_{2k}: second output quantity (exp. service quality) for DMU(K)

v_{3k}: third output quantity (exp. survival probability) for DMU(K)

v_{4k}: fourth output quantity (exp. demand coverage) for DMU(K)

Variables:

i_1: weight for the input (number of stations)

o_1: weight for the first output (exp. improvememt of response time)

o_2: weight for the second output (exp. service quality)

o_3: weight for the third output (exp. survival probability)

o_4: weight for the fourth output (exp. demand coverage)

Objective function:

$$\max(\sum_{1}^{m} v_{1k} * o_1 + \sum_{1}^{m} v_{2k} * o_2 + \sum_{1}^{m} v_{3k} * o_3 + \sum_{1}^{m} v_{4k} * o_4) \qquad (4-1)$$

Subject to:

$$\sum_{1}^{m} u_k * i_i = 1, i = 1,2, \dots, m \qquad (4-2)$$

$$\sum_{1}^{m} v_{1k} * o_1 + \sum_{1}^{m} v_{2k} * o_2 + \sum_{1}^{m} v_{3k} * o_3 + \sum_{1}^{m} v_{4k} * o_4 - \sum_{1}^{m} u_k * i_i \leq 0, i = 1,2, \dots, m \qquad (4-3)$$

$$i_1, o_1, o_2, o_3, o_4 \geq 0 \qquad (4-4)$$

The objective function (4-1) is to maximize the efficiency of the output (expected improvement of response time, expected service quality, expected survival rate and expected demand coverage). Constraint (4-2) means that the weighted input value is constrained to 1. We assume that the efficiency of a decision-making unit (DMU) will not exceed 1. Constraint (4-3) means that the value of output will never be greater than the value of input (number of stations) for each DMU. Constraint (4-4) means that all weights should be non-negative.

In this regard, we generate the 48 DMUs based on different models with different number of stations, and the result is shown in Table 14:

Results	DMUs	Solution	Weighted Input (in %)	Weighted Output (in %)				Efficiency (in %)
			Number of station	Exp. Improvement of Response time	Exp. Service Quality	Exp. Survival Probability	Exp. Demand Coverage	
q=4	1	MCLP	**100.00%**	0.00%	73.40%	0.00%	26.60%	**100.00%**
	2	MSLP	**100.00%**	0.00%	84.45%	14.36%	0.00%	98.81%
	3	MQLP	**100.00%**	100.00%	0.00%	0.00%	0.00%	**100.00%**
	4	MCLPPR	**100.00%**	0.00%	84.69%	15.31%	0.00%	**100.00%**
	5	MSLPPR	**100.00%**	0.00%	8.58%	91.42%	0.00%	**100.00%**
	6	MQLPPR	**100.00%**	0.47%	82.26%	17.28%	0.00%	**100.00%**
	7	MCLPPRS	**100.00%**	0.00%	0.00%	43.39%	53.18%	96.57%
	8	MCLPPRQ	**100.00%**	0.00%	0.00%	45.54%	54.44%	**99.98%**
	9	MSLPPRC	**100.00%**	0.00%	0.00%	40.72%	48.67%	89.39%
	10	MSLPPRQ	**100.00%**	0.00%	0.00%	45.54%	54.44%	**99.98%**
	11	MQLPPRC	**100.00%**	0.00%	0.00%	40.93%	49.29%	90.21%
	12	MQLPPRS	**100.00%**	0.00%	0.00%	42.45%	51.49%	93.94%
q=8	13	MCLP	**100.00%**	0.00%	0.00%	0.00%	71.22%	**71.22%**
	14	MSLP	**100.00%**	0.00%	0.00%	25.22%	36.43%	61.64%
	15	MQLP	**100.00%**	0.00%	0.00%	0.00%	68.93%	**68.93%**
	16	MCLPPR	**100.00%**	0.00%	0.00%	26.73%	36.43%	**63.16%**
	17	MSLPPR	**100.00%**	0.00%	0.00%	26.73%	36.43%	**63.16%**
	18	MQLPPR	**100.00%**	0.00%	0.00%	26.73%	36.43%	**63.16%**
	19	MCLPPRS	**100.00%**	0.00%	0.00%	26.27%	35.99%	62.26%
	20	MCLPPRQ	**100.00%**	0.00%	0.00%	26.73%	36.43%	**63.15%**
	21	MSLPPRC	**100.00%**	0.00%	0.00%	26.03%	35.77%	61.79%
	22	MSLPPRQ	**100.00%**	0.00%	0.00%	26.73%	36.43%	**63.15%**

	23	MQLPPRC	**100.00%**	0.00%	0.00%	26.00%	35.61%	61.61%
	24	MQLPPRS	**100.00%**	0.00%	0.00%	26.38%	36.09%	62.47%
q=12	25	MCLP	**100.00%**	0.00%	0.00%	0.00%	53.75%	**53.75%**
	26	MSLP	**100.00%**	4.75%	0.00%	17.54%	22.06%	44.35%
	27	MQLP	**100.00%**	0.00%	0.00%	0.00%	41.70%	41.70%
	28	MCLPPR	**100.00%**	0.26%	0.00%	18.83%	26.37%	**45.46%**
	29	MSLPPR	**100.00%**	0.00%	0.00%	18.96%	26.41%	45.37%
	30	MQLPPR	**100.00%**	0.00%	0.00%	18.84%	27.10%	**45.94%**
	31	MCLPPRS	**100.00%**	0.00%	0.00%	18.85%	26.15%	45.00%
	32	MCLPPRQ	**100.00%**	0.26%	0.00%	18.83%	26.37%	**45.46%**
	33	MSLPPRC	**100.00%**	0.00%	0.00%	18.80%	26.49%	45.29%
	34	MSLPPRQ	**100.00%**	0.00%	0.00%	18.96%	26.41%	**45.37%**
	35	MQLPPRC	**100.00%**	0.00%	0.00%	18.07%	26.00%	44.08%
	36	MQLPPRS	**100.00%**	0.00%	0.00%	18.84%	26.49%	45.33%
q=16	37	MCLP	**100.00%**	0.00%	0.00%	0.00%	40.82%	**40.82%**
	38	MSLP	**100.00%**	0.00%	0.00%	13.88%	20.48%	34.36%
	39	MQLP	**100.00%**	8.85%	0.00%	0.00%	23.21%	32.06%
	40	MCLPPR	**100.00%**	0.00%	0.00%	14.67%	20.48%	35.15%
	41	MSLPPR	**100.00%**	0.00%	0.00%	14.67%	20.48%	35.15%
	42	MQLPPR	**100.00%**	0.00%	0.00%	14.66%	20.85%	**35.51%**
	43	MCLPPRS	**100.00%**	0.00%	0.00%	14.62%	20.35%	34.97%
	44	MCLPPRQ	**100.00%**	0.00%	0.00%	14.67%	20.48%	35.15%
	45	MSLPPRC	**100.00%**	0.00%	0.00%	14.61%	20.34%	34.95%
	46	MSLPPRQ	**100.00%**	0.00%	0.00%	14.67%	20.48%	35.15%
	47	MQLPPRC	**100.00%**	0.00%	0.00%	14.60%	20.71%	**35.31%**
	48	MQLPPRS	**100.00%**	0.00%	0.00%	14.62%	20.35%	34.97%

Table 14- Results (weighted input and output, efficiency) of DEA

From the table above, we can see that the smaller the input is, the higher the DEA efficiency (which is measured as the ratio of the weighted output over the weighted input) will be. When q=4, the DEA efficiency of all twelve models are more than 90%, and when the number of stations increase to 16, the DEA efficiency gradually drops to around 35%. This results demonstrate that adding ambulance locations yields a decreasing return to scale in terms of improvement in the four performance indicators. The results also show that the Pareto frontier of the DEA is generally characterized by the expected survival probability and expected demand coverage.

While under the same number of ambulance station, although most of them are not fully efficient, models MCLP, MCLP+PR, MQLP+PR, MCLP+PR+Q and MSLP+PR+Q always have a better performance than others. But we should also notice that the models with deterministic measures--MCLP, MSLP and MQLP have weaknesses because of ignoring the discrimination between time or distance that are within or outside the coverage standard, and in reality, the actual travel times between a demand point and an ambulance station are highly variable, so the models with stochastic measures are much closer to real life.

In this regard, we can consider that the optimisation models MCLP+PR, MQLP+PR, MCLP+PR+Q and MSLP+PR+Q will be the best solutions.

Although from Table 9 to 12, the expected service quality, expected survival rate and expected demand coverage are raising and the expected response time is decreasing according to the increase of ambulance stations, the DEA efficiency is gradually reducing. The reason is that with the increase of input (number of stations), the outputs (expected improvement of response time, expected service quality, expected survival rate and expected demand coverage) are not increasing with the same portion. We can see that with the number of ambulance station changes from 4, 8, 12 and 16, input has been doubled, tripled and quadrupled, while the outputs (expected improvement of response time, expected service quality, expected survival rate and expected demand coverage) are increased much slower than the input. In DEA, we assume that the inputs and outputs are weighted linear functions so if the outputs do not increase linearly, the efficiency diminishes. We take the model MQLP+PR as an example, which performs better under all the different numbers of station, shown in Figure 10 to 13, we can see that the more the ambulance stations open, the larger difference between input and output.

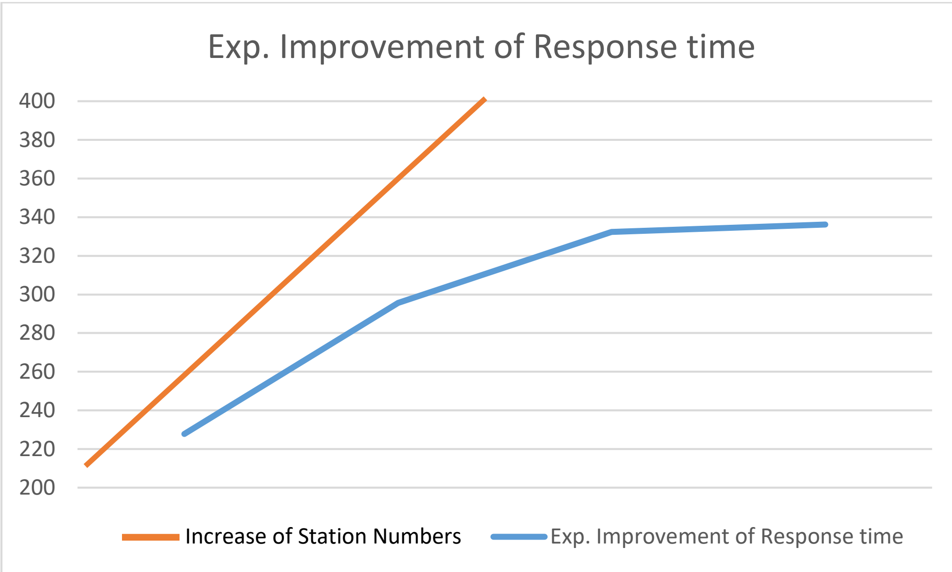

Figure 10- Increase of ambulance stations Vs. Increase of expected improvement of response time

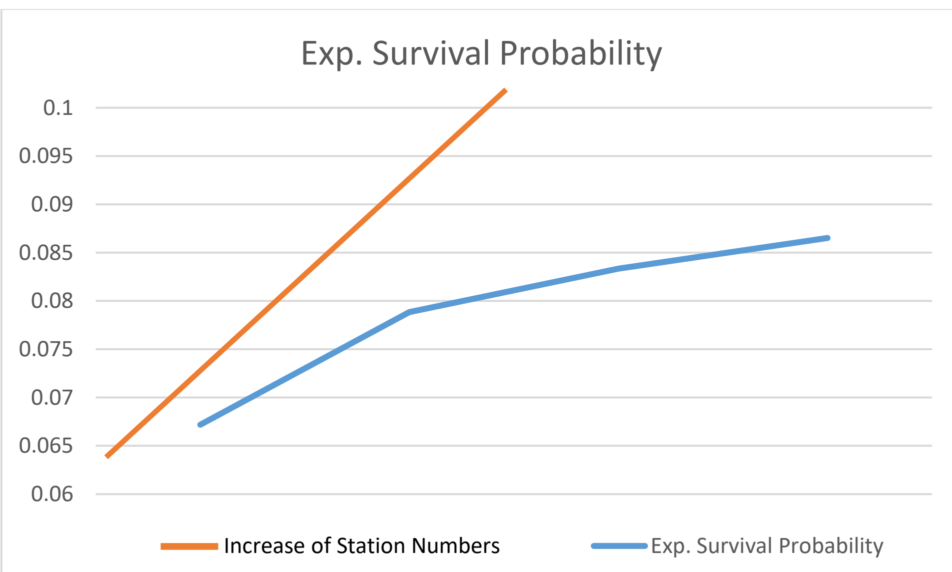

Figure 11- Increase of ambulance stations Vs. Increase of expected survival probability

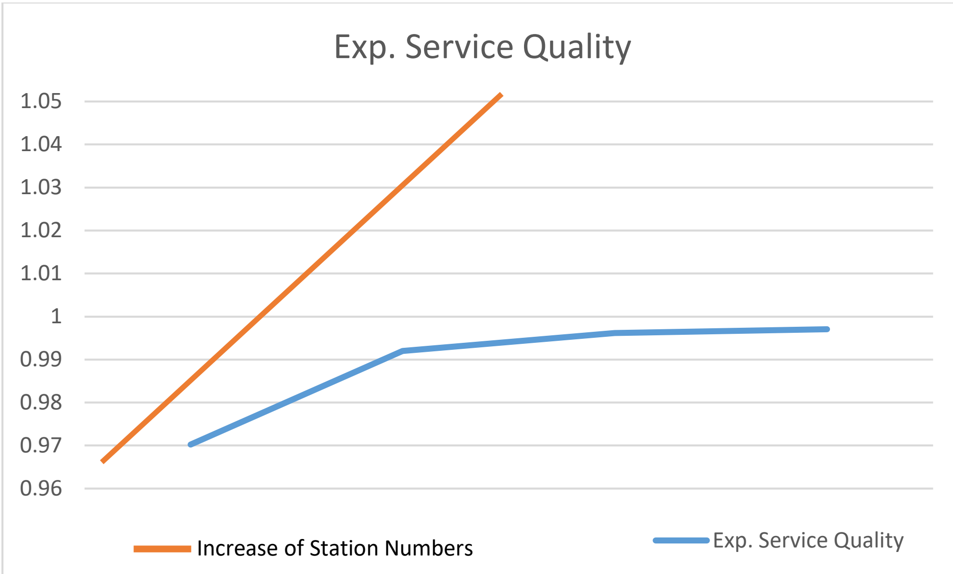

Figure 12- Increase of ambulance stations Vs. Increase of expected service quality

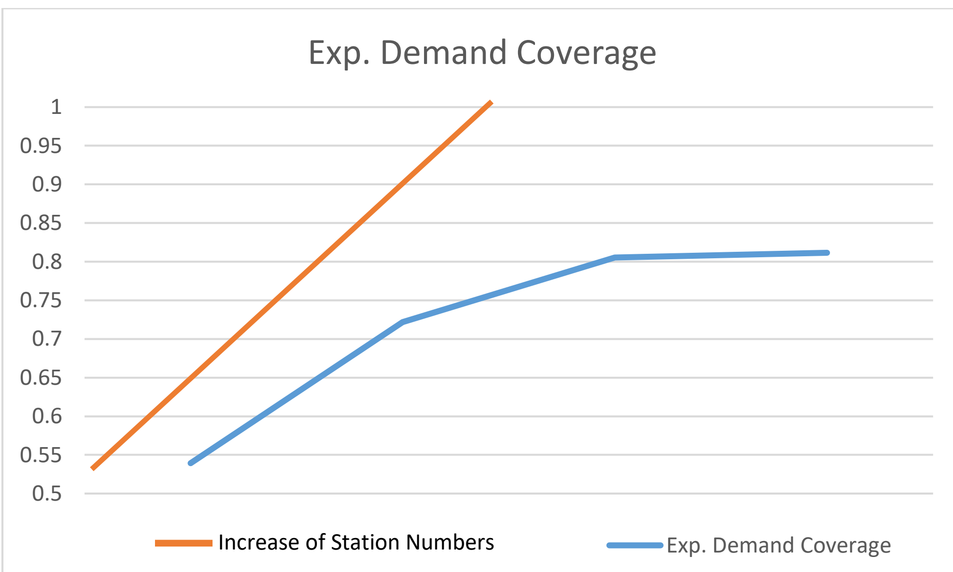

Figure 13- Increase of ambulance stations Vs. Increase of expected demand coverage

Although the number of stations is increasing, some of the resource may be wasted and the performance of expected improvement of response time, expected service quality, expected survival rate and expected demand coverage could be better. From the implication aspect, the reason may be that when opening a new ambulance station, more staff need to be hired and trained, which takes longer time and also their performance may be uneven; also more staff and ambulances will lead to a more complex dispatching system, which can have an impact on the operations if conducted inefficiently; with the increase of ambulance stations, the fixed cost and operation cost will be higher. If the decision makers intend to control the cost unduly, the performance of operations will also be influenced. There is no universal standard to measure the service quality, survival probability and demand coverage criteria, so if other functions are applied in the same situation, the results may be different.

While in this context, we employ the DEA as a method to analyze the overall performance of each models, the main objective is still to be able to save as many people as possible with certain standards, instead of having efficiency equals to 100%.

4.4 Further Application

In our design, the models and the measurement indicators in this paper are not only suitable for ambulance location problem, they can be extended to the supply chain network design, where service quality measures come into play.

More specifically,

- for models like MCLP, MCLP+PR with objective function of maximizing demand coverage, they can be viewed as the most widely used models in network design. How to cover more regions and/or customers is always the concern when it comes to sales;
- for models like MSLP, MSLP+PR with objective function of maximizing survivors, they can be of great significance for the transportation of the relatively time-efficient items such as fresh agricultural products;
- for models like MQLP, MQLP+PR with objective function of maximizing service quality, they will be applied by the companies which take customer satisfaction as priority thus improving the lead time management will help companies to win the time-based competition;

- for models like MCLP+PR+S, MCLP+PR+Q, MSLP+PR+C, MSLP+PR+Q, MQLP+PR+C and MQLP+PR+S, in which we incorporate certain standard in the objective function, can meet a deeper need of companies. A single objective may not be sufficient for the network design, when necessary, these models can satisfy a higher and more complex requirement of the company.

Also, all models except MCLP will give us the assignment of each supply-demand pair, with the indicators we have, we can then monitor the performance at each point to see whether it operates efficiently.

Chapter 5-Conclusion

Ambulance facility problem is closely related to everybody's life. The rationality of its location and the fairness of distribution are directly related to the efficiency and quality of public service supply. It plays a fundamental role in promoting social construction and improving people's quality of life.

In this study, we discussed ambulance location models based on both deterministic and stochastic indicators to maximize the demand coverage, expected survivors and service quality in the ambulance station location problem. Also, we incorporated four indicators—expected response time, expected service quality, expected survival probability and expected demand coverage to measure the performance of our optimization models. These indicators aim to help decision makers to see more deeply and minutely into the ambulance facility location problem, and also help them to monitor the performance at each demand point and ambulance station. When maximizing the demand coverage, expected survivors and service quality, decision makers can evaluate the service at each demand point to see if the expected response time, expected service quality, expected survival probability and expected demand coverage maintain at a satisfying level, and if the operation of the ambulance stations is efficient.

From the paper, we also found that the DEA efficiency of models is reducing with the increase of ambulance stations, which can also be a warning to us that we may not fully deploy the resource we have. The increase of ambulance stations can lead to a longer process to recruit and train EMS staff; a more complex dispatching and scheduling system; a larger fixed and operation cost to operate an ambulance station; a more sophisticated management structure, etc. All these factors can have an impact on the efficiency of the ambulance station. So the decision makers have to find a way to not only focus on external, quantifiable indicators to improve the service level and quality of the ambulance station, but also start from the internal management to improve the operational efficiency of the ambulance station. But still, the main objective of these models is to be able to save as many people as possible with certain standard, instead of an excessive pursuit in efficiency.

The contributions of the paper are: 1) the extended models with deterministic and stochastic measures. We incorporate new indicators in the objective functions of the known models to restrain the certain coverage, survival rate and service quality standard. Some of the models generate a better performance based on the measurement we have, and would be helpful when decision makers do have the need to segment and differentiate demand points. 2) the indicators—expected response time, expected service quality, expected survival probability and expected demand coverage to measure the performance of optimization models. As we can see from Chapter 2 and 4, some of the existing research have studied these indicators separately. But in this paper, we discussed and analyzed the four indicators at the same time. 3) applying Data Envelopment Analysis into ambulance station location problem. None of the studies on facility location problem applied DEA to analyze the performance and measure the efficiency of the station. 4) the extension to supply chain network. In our design, the ambulance facility location is just one epitome of the supply chain network. The response time here can derive to a more general delivery time. With the models, we can have the assignments of each demand and supply pair, and with the indicators, we can evaluate the performance at each location. So the same pattern can not only apply to public facility location, but also to a more commercial case like restaurant delivery, or location of warehouses and production plants.

There are also limitations in the paper. First, all of our models are linear and with single objective function. As in our models, the objective is either to maximize demand coverage, to maximize expected survivors or maximize total service quality, if we could set multi-objective models, we would get a more precise sight on the balance of these factors. Second, when comparing and analyzing the results we have in Chapter 4, we assumed that each ambulance station has sufficient capacities, which means that there is no upper limit for the number of ambulances as long as we need one. But this situation could be unrealistic when the resources or the number of ambulances are very limited. Third, the indicators we chose—expected response time, expected service quality, expected survival probability and expected demand coverage may not be a comprehensive measurement to evaluate the performance of the facility. There could be other indicators and combinations to measure the results of the optimization. In addition, we deployed DEA in Chapter 4 as a method to measure the efficiency of each situation. But DEA itself does have some weaknesses. It's hard to get specific recommendations and suggestions from DEA. Thus, we could only use it in a limited setting to demonstrate that the investment in ambulance locations yield decreasing return to scale. Also, any DEA is based on input and output, but many input and output data are not quantifiable. For those input and output which can not be quantified, DEA simply assumes that all decision-making units are not different. So that may cause an uncompleted comparison and analysis for our models

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Appendix

A1. Python code example for model MSLP+PR+Q

```
import pandas as pd
demand_pd = pd.read_csv("demand1.csv")
survival_prob_pd = pd.read_csv("spforpr.csv", index_col="Demand")
demand = demand_pd['Demand'].values
survival_prob=survival_prob_pd.values
service_quality_pd = pd.read_csv("SQ_STOC.csv", index_col="Demand")
service_quality = service_quality_pd.values

N_DEMAND = len(survival_prob_pd.index)
N_SUPPLY = len(survival_prob_pd.columns)

q_min = 1
q_max = 16

b = []
for i in range(N_DEMAND):
    b.append([service_quality_pd.values[i][j] if service_quality_pd.values[i][j] >= 0.6 else 0.0 for j in
range(N_SUPPLY)])
print(demand_pd['Demand'].values)
print(survival_prob_pd)

#Model
from gurobipy import *
m=Model()

# variables
x = [(None,None,GRB.BINARY)]*N_SUPPLY
y = [[(None,None,GRB.BINARY)]*N_DEMAND]*N_SUPPLY
print(y)

# define variables
m_x = []
for j,val in enumerate(x):
    m_x.append(m.addVar(vtype=val[2],name="x"+str(j)))

m_y = {}
```

```
for j in range(N_SUPPLY):
    for i in range(N_DEMAND):
        m_y[i,j]=m.addVar(vtype=val[2],name="y"+str(i)+str(j))

m.update()

# constraints
for j in range(N_SUPPLY):
    s = [m_y[i,j] for i in range(N_DEMAND)]
    m.addConstr(quicksum(s)<=N_DEMAND*m_x[j])

for i in range(N_DEMAND):
    s = [m_y[i,j] for j in range(N_SUPPLY)]
    m.addConstr(quicksum(s)==1)
    m.setObjective(quicksum(quicksum(survival_prob[i][j]*m_y[i,j]*b[i][j] for j in
range(N_SUPPLY))*demand[i] for  i in range(N_DEMAND)),GRB.MAXIMIZE)


q=q_min
constr_max_q = m.addConstr(quicksum(m_x[j] for j in range(N_SUPPLY))<=q)

for q in range(q_min, q_max+1):
    constr_max_q.setAttr(GRB.Attr.RHS, q)
    m.update()
    print(constr_max_q)
    m.optimize()

    if m.status == GRB.Status.OPTIMAL:
        print('\nobjVal: %g' % m.objVal)
        solx = m.getAttr('x', m_x)
        print(solx)

        soly = m.getAttr('x', m_y)
        print(soly)


        idx = pd.MultiIndex.from_product([range(1,N_DEMAND+1),range(1,N_SUPPLY+1)])
        print(idx[:N_SUPPLY])

        servestation_series = pd.Series([soly[i,j] for i in range(N_DEMAND) for j in range(N_SUPPLY)], index =
idx, name='y')
        amstation_series = pd.Series([solx[j] for j in range(N_SUPPLY)], index = idx[:N_SUPPLY], name='x')
```

```
    sol_df = pd.concat([servestation_series,amstation_series], axis = 1)

    print(sol_df)
    sol_df.to_csv("MSLPPRQ_solution_"+str(q)+".csv")

else:
    print('No solution')
```

A2. Python code example for stochastic service quality calculation

```
import numpy as np
import pandas as pd

avgrestime_pd = pd.read_csv("avgrestime.csv",index_col="Demand")
sdrestime_pd = pd.read_csv("sdrestime.csv",index_col="Demand")

print(avgrestime_pd)
print(sdrestime_pd)

stsq=pd.DataFrame(index=avgrestime_pd.index,columns=avgrestime_pd.columns)
print(stsq)

import math
d1=480
d2=1500

i = 1
j = 2

for i in range(len(avgrestime_pd.index)):
   for j in range(len(avgrestime_pd.columns)):

      #stochastic
      rand_response = np.random.normal(avgrestime_pd.iloc[i,j], sdrestime_pd.iloc[i,j],1000)
      sq_response = [1 if val < d1 else 0 if val > d2            else 0.5+0.5*math.cos(math.pi/(d2-d1)*(val-
(d1+d2)/2)+math.pi/2) for val in rand_response]

      stsq.iloc[i,j] = np.average(sq_response)
print(stsq)
stsq.to_csv("SQ_STOC.csv")
```

A3. Demand at the 180 demand points

1	161	***31***	31	***61***	51	***91***	23	***121***	49	***151***	78
2	427	***32***	76	***62***	235	***92***	156	***122***	17	***152***	59
3	677	***33***	246	***63***	113	***93***	230	***123***	77	***153***	31
4	252	***34***	188	***64***	62	***94***	123	***124***	94	***154***	1
5	1300	***35***	295	***65***	27	***95***	165	***125***	92	***155***	23
6	53	***36***	193	***66***	46	***96***	134	***126***	42	***156***	18
7	92	***37***	167	***67***	28	***97***	178	***127***	14	***157***	1
8	105	***38***	426	***68***	26	***98***	101	***128***	4	***158***	18
9	152	***39***	100	***69***	24	***99***	39	***129***	1	***159***	34
10	178	***40***	119	***70***	14	***100***	113	***130***	8	***160***	11
11	42	***41***	123	***71***	41	***101***	269	***131***	1	***161***	1
12	88	***42***	73	***72***	151	***102***	85	***132***	1	***162***	3
13	138	***43***	5	***73***	127	***103***	24	***133***	0	***163***	4
14	284	***44***	6	***74***	22	***104***	62	***134***	0	***164***	2
15	79	***45***	63	***75***	146	***105***	41	***135***	1	***165***	2
16	38	***46***	38	***76***	32	***106***	126	***136***	15	***166***	5
17	82	***47***	22	***77***	148	***107***	6	***137***	8	***167***	21
18	69	***48***	123	***78***	156	***108***	1	***138***	1	***168***	23
19	7	***49***	57	***79***	169	***109***	14	***139***	6	***169***	4
20	10	***50***	10	***80***	21	***110***	6	***140***	32	***170***	0
21	17	***51***	8	***81***	25	***111***	33	***141***	35	***171***	1
22	7	***52***	35	***82***	13	***112***	0	***142***	3	***172***	14
23	271	***53***	3	***83***	1	***113***	0	***143***	6	***173***	1
24	64	***54***	5	***84***	4	***114***	1	***144***	0	***174***	6
25	23	***55***	60	***85***	3	***115***	122	***145***	28	***175***	2
26	160	***56***	41	***86***	0	***116***	69	***146***	2	***176***	9
27	39	***57***	33	***87***	22	***117***	93	***147***	12	***177***	6
28	66	***58***	21	***88***	86	***118***	67	***148***	9	***178***	1
29	282	***59***	92	***89***	18	***119***	21	***149***	8	***179***	1
30	59	***60***	28	***90***	82	***120***	90	***150***	40	***180***	21

A4. Ambulance location for all models

MCLP	q=4	q=8	q=12	q=16
1			√	√
2		√	√	√
3			√	√
4		√		√
5				√
6		√	√	√
7	√			√
8	√		√	√
9			√	√
10	√	√	√	√
11		√		√
12		√	√	√
13		√	√	√
14			√	√
15			√	√
16	√	√	√	√

MSLP	q=4	q=8	q=12	q=16
1			√	√
2		√	√	√
3				√
4		√	√	√
5				√
6		√	√	√
7				√
8			√	√
9			√	√
10	√	√	√	√
11	√	√	√	√
12		√	√	√
13	√	√	√	√
14				√
15			√	√
16	√	√	√	√

MQLP	q=4	q=8	q=12	q=16
1			√	√
2	√	√	√	√
3			√	√
4				√
5	√			√
6		√	√	√
7				√
8	√		√	√
9			√	√
10	√	√	√	√
11		√		√
12		√	√	√
13		√	√	√
14			√	√
15		√	√	√
16		√	√	√

MCLPPR	q=4	q=8	q=12	q=16
1			√	√
2		√	√	√
3			√	√
4		√		√
5				√
6		√	√	√
7				√
8	√		√	√
9			√	√
10	√	√	√	√
11		√	√	√
12		√	√	√
13	√	√	√	√
14				√
15			√	√
16	√	√	√	√

MSLPPR	q=4	q=8	q=12	q=16
1			√	√
2		√	√	√
3				√
4		√	√	√
5				√
6		√	√	√
7				√
8			√	√
9			√	√
10	√	√	√	√
11	√	√	√	√
12		√	√	√
13	√	√	√	√
14				√
15			√	√
16	√	√	√	√

MQLPPR	q=4	q=8	q=12	q=16
1			√	√
2		√	√	√
3			√	√
4		√		√
5				√
6		√	√	√
7				√
8	√		√	√
9			√	√
10	√	√	√	√
11		√	√	√
12		√	√	√
13	√	√	√	√
14				√
15			√	√
16	√	√	√	√

MCLPPRS	q=4	q=8	q=12	q=16
1			√	√
2		√	√	√
3				√
4	√	√	√	√
5				√
6		√	√	√
7				√
8			√	√
9			√	√
10	√	√	√	√
11	√	√	√	√
12		√	√	√
13		√	√	√
14				√
15			√	√
16	√	√	√	√

MCLPPRQ	q=4	q=8	q=12	q=16
1			√	√
2		√	√	√
3			√	√
4		√		√
5				√
6		√	√	√
7				√
8	√		√	√
9			√	√
10	√	√	√	√
11		√	√	√
12		√	√	√
13	√	√	√	√
14				√
15			√	√
16	√	√	√	√

MSLPPRC	q=4	q=8	q=12	q=16
1				√
2		√	√	√
3			√	√
4		√	√	√
5				√
6		√	√	√
7				√
8	√		√	√
9			√	√
10	√	√	√	√
11		√	√	√
12		√	√	√
13	√	√	√	√
14				√
15			√	√
16	√	√	√	√

MSLPPRQ	q=4	q=8	q=12	q=16
1			√	√
2		√	√	√
3				√
4		√	√	√
5				√
6		√	√	√
7				√
8	√		√	√
9			√	√
10	√	√	√	√
11		√	√	√
12		√	√	√
13	√	√	√	√
14				√
15			√	√
16	√	√	√	√

MQLPPRC	q=4	q=8	q=12	q=16
1			√	√
2		√	√	√
3			√	√
4		√		√
5				√
6		√	√	√
7	√			√
8	√		√	√
9			√	√
10	√	√	√	√
11		√		√
12		√	√	√
13		√	√	√
14			√	√
15			√	√
16	√	√	√	√

MQLPPRS	q=4	q=8	q=12	q=16
1				√
2		√	√	√
3			√	√
4		√	√	√
5	√			√
6		√	√	√
7				√
8	√		√	√
9			√	√
10	√	√	√	√
11		√	√	√
12		√	√	√
13	√	√	√	√
14				√
15			√	√
16		√	√	√