HEC MONTREAL

GLOBAL SUPPLY CHAIN MANAGEMENT

"NETWORK DESIGN FOR FOOD AID DISTRIBUTION WITH A SERVICE PERSPECTIVE"

Fernando Ronald Díaz Abarca

A Thesis submitted to the Department of Global Supply Chain Management in partial fulfilment of the requirements for the degree of

M.Sc. in Global Supply Chain Management

SUPERVISORS:

Ph.D. Ola Jabali
Ph.D. Marie-Ève Rancourt

June, 2015

Statement of original authorship

The work contained in this thesis has not been previously submitted to meet requirements for an award at this or any other higher education institution. To the best of my knowledge and belief, the thesis contains no material previously published or written by another person except where due authorization, acknowledgment and reference have been made.

Signature: 

Date: 18 - June - 2015

Dedicatory

This work is dedicated to our Almighty Lord, who gave me the opportunity to contribute in a research project that can be useful to improve the well-being of people suffering conditions of food insecurity, and who in the most difficult moments inspired me to find the answers and who gave me the strength to pursue my goals.

This work is also dedicated to those, whose effort contributed to mitigate the difficult situation that many people face around the world. They are volunteers, people working in dangerous or difficult situations, researchers, employees of humanitarian organizations and surely many more.

Finally this work is dedicated to the people facing situations of food insecurity, they are a source of inspiration and motivation in the present study and they are also part of my heart.

Acknowledgements

I owe deep gratitude to my supervisors Dr. Ola Jabali and Dr. Marie-Eve Rancourt for their support and continuous guidance during the present work and even in previous stages. Their human and professional qualities motivated me to further explore this interesting field related with social and humanitarian concerns. I am also grateful for the invaluable information they provided me, which represents the strong fundament over which the present work has been conducted. In addition to invaluable remarks and advices, they also allowed me to explore some particular aspects and gave me the liberty to conduct the research following my personal interests and skills. Without their help, this work would not have been possible.

I am grateful to HEC Montreal for the high quality education and the strong values they inculcated me through all my Master studies. I also express my sincere gratitude to all the personal of the Interuniversity Research Centre on Enterprise Networks, Logistics and Transportation (CIRRELT), their organizational support and resources were fundamental to develop the present work. I am also grateful to HEC Montreal and to Université de Quebec à Montreal for helping to fund my research. I would also like to extend my gratitude to IBM Corporation, they generously allowed me the use of specialized software through their Academic Initiative.

I owe deep gratitude to my family as well, for their understanding and constant support. I am grateful with my love, Ingrid, thanks for your patience and your suggestions. I am also grateful with my parents, who always supported and encouraged me to develop as a person and as a professional and who raised me with strong values.

Finally, I am grateful to all those who helped me, directly or indirectly. Among there are have many researchers, authors, organizations, editorials and former teachers. Whereas they are not mentioned here, their contribution was also very important to successfully finish this work.

Keywords

Equality functions

Facility location

Food aid

Food distribution

Garissa district

Humanitarian logistics

Kenya

Network design

Relief network

Service considerations

Service perspective

Supply chain

Time line response

Trade-offs

Table of abbreviations

AD	Aggregated data
AWD	Average Walking Distance
CC	Cost Constraint
DCs	Distribution Centers
DT	Distance Threshold
FAB	Food Assessment Baseline
FAO	Food and Agriculture Organization of the United Nations
GK	Government of Kenya
HL	Humanitarian logistic
IAT	Inhabitants Aggregation Threshold
ICT	Inhabitants Coverage Threshold
Inh.	Inhabitants
KRC	Kenyan Red Cross
KSh	Kenyan Shillings
MAD	Mean absolute deviation
MRD	Mid-Radius Distance
MSC	Minimum Stakeholder Cost
NAD	Non-aggregated data
NGO	Non-Governmental Organization
RC	Red Cross
SMR	Service below the Mid-Radius threshold
TWD	Total Walking Distance
UA	Urban Area
UFM	Uncapacitated Facility Location Model
US	United States
WFP	World Food Programme
WI	Walking Inhabitants

Abstract

Network design for food distribution in the humanitarian context is an issue of increasing importance. International organizations deploy efforts to alleviate food insecurity in many countries. A particular attention has been given to the costs incurred by stakeholders such as the World Food Programme, while less attention has been given to service considerations for the beneficiaries. Integrating a service perspective into the design of food aid distribution networks can have a substantial impact on the welfare of the beneficiaries. This research will test equality and service functions within the formulation of the objective function of a network design model to achieve a higher service level for the beneficiaries. The problem will be formulated as a mixed integer linear program, which will be solved by a state-of-the-art mixed integer programming solver.

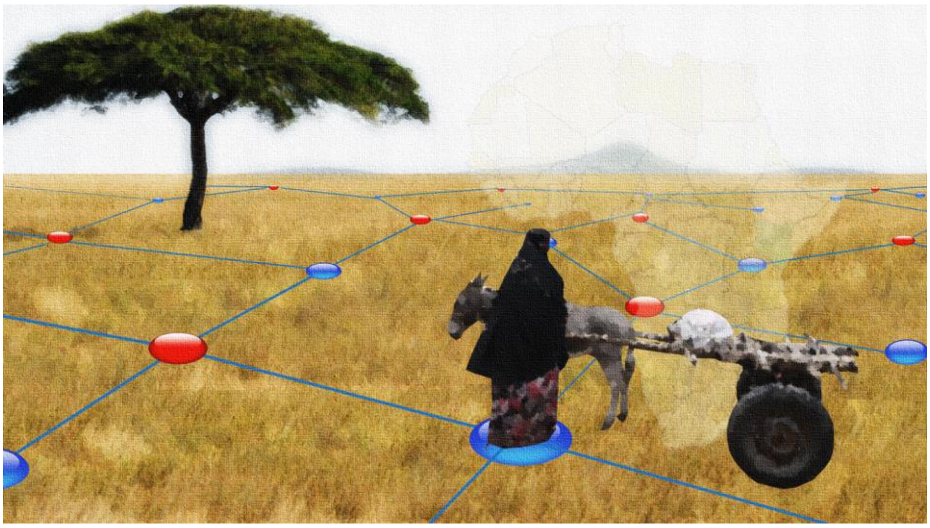


Figure 1. Compound picture representing the geography of Garissa District

In Figure 1 we represent the network and the donkey transportation for the food supplies; the image of the donkey transportation comes from an original photo of Marie-Eve Rancourt that can be found in Rancourt et al. (2015).

Table of contents

List of figures

List of tables

Chapter 1: Introduction

In this chapter we present a general overview of our problem and of the scope of our research, including the hypothesis and objectives of the studied problem. A brief overview of the methodology used to solve the problem is also presented at the end of this chapter.

General overview

Food security is a very complex issue; it affects many people around the world. According to the Food and Agriculture Organization of the United Nations (FAO), Kenya and some other countries in Africa such as Ethiopia, Somalia, Niger, and Uganda are regions severely affected by chronic episodes of famine (FAO, 2014). To fight against hunger, humanitarian efforts are deployed, and food distribution networks are established. Humanitarian logistics (HL), even if sharing some general principles with commercial applications of distribution, have some particularities that have to be taken into account. A particular attention must be given to the access of the beneficiaries, who face vulnerable situations. Food distribution logistics chains reach mainly populations that are close to main urban areas and roads, whereas people with little access to markets and who are living in remote areas have a more difficult access to food supplies, see FAO (2010). The most affected populations are those who live in arid and semi-arid regions, where pastoralism struggles to survive during the drought season. In situations of food insecurity, these people will suffer a more acute crisis.

Garissa is a district located in the north-east of Kenya, close to Somalia. This district is affected by seasonal droughts that create periodic episodes of food insecurity, this especially impact rural and pastoralist populations. The problem of network design for food aid distribution in the district of Garissa (Figure 2), was studied by Rancourt et al. (2015). The authors deal with the problem of locating distribution centers in order to reduce not only the distribution cost, but also the beneficiaries’ access cost. The cost, represented by the travel time of the beneficiaries, was previously misrepresented among other stakeholders of food aid distribution, such as the World Food Programme (WFP) and Kenyan Red Cross (KRC).

The network structure can have an important influence on the beneficiaries. Travelled distance, time and access cost, as well as the coverage are in direct relation with the location of distribution centers and main warehouses. Different levels of service can be offered to the beneficiaries depending on where we decide to locate the distribution centers. Service level in this context could be defined as the ability of food distribution agencies to appropriately cover the food needs of the populations, ensuring a fair travel distance, time, and access cost for the beneficiaries.

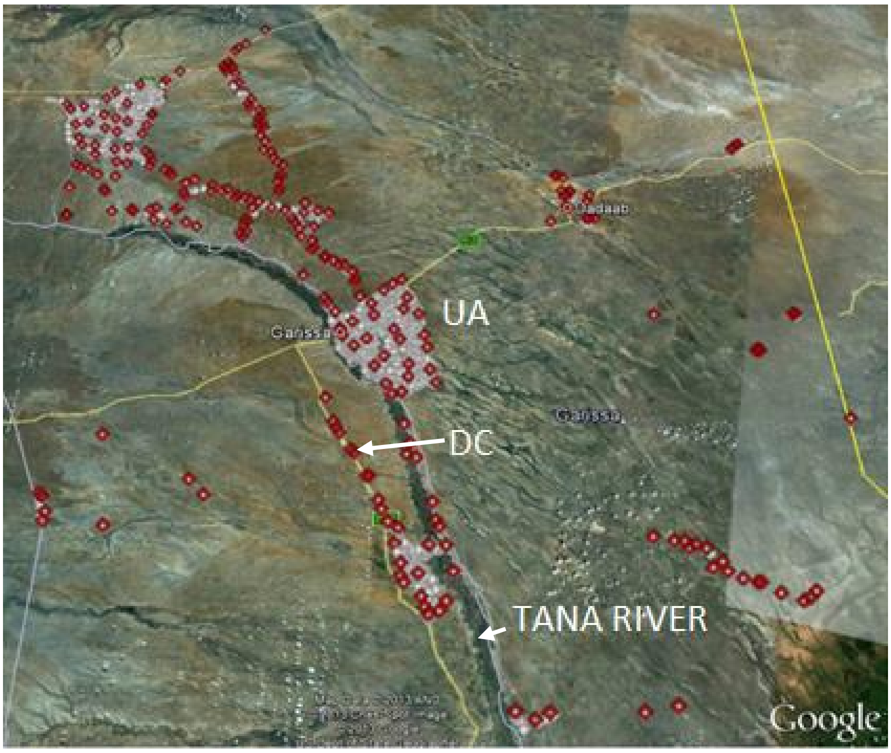


Figure 2. Food distribution network for the district of Garissa in Kenya
The red points represent food distribution centers (DCs) for a suggested solution with 55 km coverage radius, the grey points represent urban areas (UA).
Source: Rancourt et al. (2015).

1.1 Research scope

To our knowledge, one particular aspect that has not been studied yet is the use of equality functions in deciding the network configuration for food distribution in humanitarian logistics. Appropriately locating distribution centers and warehouses can help establish a fair displacement for the beneficiaries, especially for the more affected populations that are mainly located in arid and remote areas.

This project builds upon the work of Rancourt et al. (2015). The information about the Garissa district, provided by this research will be the basis for the present study. This case was selected because of the availability of detailed information on Garissa and considering that it is a representative case for food distribution in the humanitarian context. We will explore and compare the results obtained with different forms of equality and service functions used when designing a food distribution network in the humanitarian context. Thus, the purpose of the present master thesis is to design a logistic network that encompasses equality and service considerations to appropriately locate food distribution centers.

Based on the literature survey, advances in the facility location methodologies and economic theory we think that in the case of the Garissa district in Kenya it is possible to promote a more equalitarian service in terms of coverage, travel distance, time and cost for the beneficiaries. Those objectives can be accomplished by using equality and service functions in the network design.

The main objective of the present thesis are to analyse the particularities and trade-offs derived from the use of equality and service functions; determine the impact of equality and service functions on the level of service for the beneficiaries of food aid distribution. Based on our results we expect to suggest a more equalitarian access in term of coverage, travel distance, time and cost for the beneficiaries of food aid distribution. The previous objectives could help to ensure the sustainability of the service in the long term.

1.2 Methodology

The methodology developed for this research is based on quantitative modelling. A mathematical model will be developed, which will be solved by a state-of-the-art general purpose mixed integer programming solver. The data will come from the original study of Rancourt et al. (2015), which includes different information sources such as the WFP, KRC, and the Government of Kenya. Statistical analysis has also been performed.

Chapter 2: Literature review

Humanitarian response activities can benefit from supply chain management methodologies such as facility location problems that can be solved to support humanitarian efforts such as in the case of food aid distribution. Section 2.1 presents some antecedent studies of location problems in the context of humanitarian logistics. We present two main classifications of location problems. The first classification presented in Section 2.1.1 addresses discrete location problems and the second one, presented in Section 2.1.2, is based on the humanitarian response cycle management. The interest of analysing equality functions within the optimization process of humanitarian logistics problems motivates the inclusion of a section focussing on equality functions in location problems. (Section 2.2). The main criteria to analyse the equality functions are presented in Section 2.2.1. Additionally, non-normalized and normalized functions that could be applied to facility location are discussed in Sections 2.2.2 and 2.2.3; many of them are derived from the economic field. Section 2.3 addresses service considerations. A brief analysis, including conclusions and implications of the chapter's content, are presented in Section 2.4.

2.1 Location problems

Facility location dealing with the optimal placement of facilities is an important field of operations research. Some of the pursued objectives are cost reduction while achieving a suitable proximity to customers. This field was coined by Weber in 1909 by minimizing the total travel distance of customers to a facility in a continuous setting. Since then, many algorithms have been developed in this area. A particular attention has been given to the private sector, where commercial needs have fostered the development of applications and solutions. Traditionally, models have been created to support the distribution industry, but the efforts have spread to other fields such as the location of hospitals, schools and other public facilities. Solution approaches used for facility location can be used in the humanitarian context since the latter is a natural extension of traditional commercial location problems. The main difference between commercial and non-commercial application being the social perspective, which influence the objective function. Network design problems are mainly solved by exact algorithms and heuristic algorithms.

Exact Algorithms guarantee an optimal solution for the problem, these algorithms can be implemented in optimization software packages and deal with problems including thousands variables and constraints. The solution time varies according to the complexity and size of the problem. As explained by ReVelle & Eiselt (2005), for integer solutions in linear programming, it is expected to increase the solution time because of the need of branch and bound. For the present study, the network is represented by 24,453 population nodes and 1,460 potential distribution nodes, refer to Rancourt et al. (2015) for more details. Considering the complexity of food distribution in the humanitarian context and the involvement of important stakeholders (e.g., WFP, KRC and beneficiaries), it is desirable to find an optimal or close to optimal solution. To do so, the present work will make use of a state-of-the- art solver (CPLEX).

Heuristic algorithms do not guarantee an optimal solution, but some of them can give very good solutions and they are also very useful to solve nonlinear problems, which sometimes can represent the reality more accurately than linear models. Because of heuristics' flexibility to represent the reality and model complex scenarios, many researches used them to implement non-linear functions, which is a characteristic of equality and welfare objectives. Among the main heuristics, there are *Greedy Algorithms.* For example, the facilities are added one by one and the algorithm choses the option that has the greatest impact on the objective. *Improvement Algorithms,* where the process starts with a feasible solution and the algorithm tries to improve it, there are in this group a number of neighborhood search structures. Finally, there are *Meta-Heuristics,* which perform a thorough research and are less likely to be trapped in local optimal solutions. In this group, there are Genetic Algorithms, Greedy Randomized Adaptive Search, Adaptive Large Neighborhood Search, and Tabu Search.

The classifications of facility location problems in the present master thesis are based on previous studies developed by Owen & Daskin (1998), ReVelle & Eiselt (2005) and Daskin (2008). We re-examine these studies, focusing on the areas related to our context and we have produced an additional survey of location problems for humanitarian logistics. Some, but not all of the discrete problems found in the literature survey, are mentioned in the main document. However, a summarized list containing additional models and information can be found in Appendix 1. Since our study is based on a deterministic static scenario, we will mainly focus on static models and will give less attention to dynamic models. Because our study is more related to discrete models, we will elaborate on those models and briefly comment the network space and continuous models.

The location problems can be divided in continuous (facilities may be located anywhere on the plane), network and discrete space (facilities may be located on a limited set of points). The continuous models include the traditional Weber problem, where the objective is to minimize the total distance between the facility and demand points. A simplest way to solve this problem is with the Weiszfeld algorithm (Weiszfeld, 1936). We will not dig into this type of problems since in our case the potential locations of the facilities to be located is known (discrete). In humanitarian logistics, authors that explored the continuous space were: McAllister (1977), Berman et al. (2009a) Campbell & Jones (2011).

In the network space, the demand points and distribution facilities are represented by nodes connected to each other by links or arcs. "The objective is to minimize the demand-weighted total distance between the facility and the nodes." Daskin (2008). Typically the shortest path in the network requires a preprocessing to determine the shortest arcs connecting nodes that will be used as the candidate's references to locate the facilities. In HL, network-based problems are among the most explored, some of the authors in this field are: Mete & Zabinsky (2010), Xu et al. (2010), Döyen et al. (2012).

Based on the dimensional space, where d_{ij}^{l} represent the distance between two points *i* and *j* (e.g., between distribution centers and customers); and *x* and *y* represent coordinates in the cartesian plane, ReVelle & Eiselt (2005) mentioned that the literature focuses mainly on the following three norms:

Manhattan Norm (rectangular or ℓ_1) $$d_{ij}^{1} = |x_i - x_j| + |y_i - y_j|$$

Euclidean Norm (Straight line or ℓ_2) $$d_{ij}^{2} = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}$$

Chevischev Norm (Max or ℓ_∞) $$d_{ij}^{\infty} = Max\,\{|x_i - x_j|; |y_i - y_j|\}$$

2.1.1 Discrete location models

The discrete models can be divided into covering-based models, median-based models and *p*-dispersion-based models. They are further explained below.

2.1.1.1. Covering based models

Covering-based models have important applications in the design of emergency services. A demand point is covered if it had been located within a pre specified time or distance from a facility (Drezner & Hamacher, 2004). The goal is to cover as much as demand points as possible with a given number of facilities. These models are very useful when distance-type objectives are not appropriate because the main decision is to serve customers with a given number of facilities.

Location set covering model

The objective of Location Set Covering Models (LSCM) is to locate a finite set of facilities while minimizing the distances and maximizing the coverage of demand points. Costs are incurred for opening facilities, such as fixed, operating and transportation costs. In order to reduce costs, sometimes an increase of coverage distance is allowed. A draw-back of these models is that they do not distinguish small and large demand points. This field was studied by Hale & Moberg (2005), who used CPLEX to implement a integer linear program tested by using random generated data, including minimum and maximum distances as constraints, to locate reliable facilities for disaster response. Dekle (2005) used aggregated data to solve an integer linear problem to minimize the number of disaster recovery facilities in Florida, United Sates (US), using an Excel spreadsheet. A similar model was also studied by Rancourt et al. (2015) in regards to last mile food distribution for the Garissa District in Kenya. The authors used CPLEX, as general mixed integer programming solver, for a network of 24,453 population nodes and 1,460 distributions centers.

Maximal covering model

The goal is to locate a specific number of facilities while maximizing the number of demand points served within an acceptable distance. In contrast with the set covering model, this model differentiates big and small demands and allows some nodes to be uncovered (Daskin, 2008). In HL this model was explored by Current & O'Kelly (1992), these authors formulated an integer linear program, using MPSX software, to locate warning systems in a Midwestern city of the US, and Murray et al. (2008) developed an integer linear problem to locate warning systems in Ohio using CPLEX, heuristics and geographic information systems.

Cooperative covering model

In Cooperative Covering Models (CCM), a demand point can be served by different facilities emitting signals, where the strength received by demand points depend on the distance. A demand point is covered if the aggregate signal from different facilities is superior to a given threshold. Berman et al. (2010) summarized this concept with the terms “signal decay function” and explored discrete cooperative models to locate early warning systems in contamination scenarios.

r-Interdiction covering model

r-Interdiction Covering Model addresses the problem of disruption and tries to minimize the effect of lost facilities in cases such as human disruption, natural disasters or terrorist attacks. This model is also related with stochastic models mentioned by Farahani et al. (2012). In HL this problem was developed by Berman et al. (2007) and Berman et al. (2009b), who used CPLEX and heuristics to solve an integer linear problem using information about the Toronto hospital system and random data to minimize the effect of facility failure considering the significant degree of centralization of hospitals in Toronto's downtown area. Church (2004) used CPLEX to solve an integer linear problem to maximize the distance of impacted facilities using data from Los Angeles, California. O'Hanley & Church (2011) used a bi-level mixed integer problem formulation applied to American and European cities to maximize initial coverage and to ensure a minimum coverage level after disruption of facilities. The r-Interdiction problems are also related to the median-based problems.

p-Center model

p-Center models take a given number of facilities, and tries to find the number of facilities that minimizes the maximum distance (minimax problem) that separates demand nodes from its nearest facility with a complete demand coverage. Owen & Daskin (1998) state that when facility locations are restricted to the nodes of the network, the problem is called the Vertex Center problem, and if they are located anywhere on the network, the problem is called Absolute Center problems. Yan & Shih (2009) explored the minimax problem to minimize the time required for emergency roadway repair and relief distribution for earthquakes scenarios in Taiwan. They used CPLEX combined with heuristics to solve a multi-objective, mixed integer program formulation.

2.1.1.2 Median based models

Median-based models minimize the demand-weighted average distance between demand points and the facility. In the following, we discuss the main models in this category.

p-Median model

The p-Median model focuses on aggregate costs, calculating the total weighted travel distance between the demand points and the facilities. ReVelle & Eiselt (2005) pointed out the similitude between the uncapacitated facility location model (UFM) with the *p*-median model. In the UFM the number of facilities to be located is endogenous to the problem. "The P-median problem fails to account for the fixed costs associated with locating facilities" (Hall, 2003). This is one of the most explored models in HL. Among the authors that have studied this problem, Xu et al. (2010) suggested an integer linear program solved by heuristics to locate early warning systems. Rawls & Turnquist (2012) used CPLEX to solve a mixed integer programming model to minimize cost of shelter location in North Carolina (US).

Medi-center problem

The Medi-center problem minimizes the average distance while ensuring travel distances to be inferior to a specified bound. It allows the evaluation of trade-offs between minimum average distance and maximum distance. It is also considered a combination of a minisum ("median") and a minimax ("center") problems (Handler, 1985). The literature has given little attention to this problem.

2.1.1.3. p-Dispersion-based models

p-Dispersion-based models maximize the minimum distance between facilities and demand points. Such models have practical application in locating facilities when it is desirable to minimize the competition between facilities. In HL the concept of competition is replaced by complementarity or collaboration. Taking this into consideration, these models do not appear to be suited for the humanitarian context.

2.1.2 Classification based on the humanitarian response cycle management

In HL, a non-traditional and more appropriate classification of locations models is based on the humanitarian response cycle management (Figure 3). This classification was developed by Çelik et al. (2012), where HL activities are classified according to their response time. Two main groups of activities can be recognized: *disaster management activities,* which encompasse activities of mitigation, preparedness, response and recovery (these activities are mainly developed in the short-term), and *long-term development activities*. This classification will be very useful to clearly position the solution method that will be applied to the problem we face in the present study.

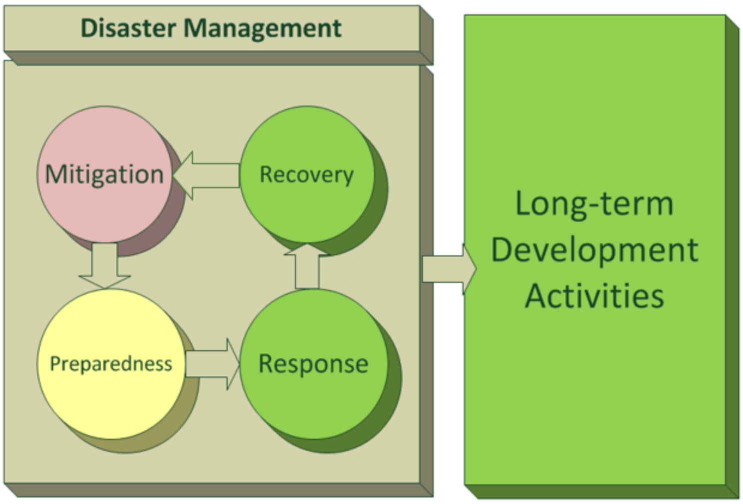


Figure 3. The humanitarian response cycle management adapted according to the framework of Çelik et al. (2012).

This classification is also in accordance with the ideas of Kovács & Spens (2007) who mention that "Humanitarian logistics encompasses very different operations at different times". These authors adopted a framework based on the phases of relief operations: preparation, immediate response and reconstruction. Maxwell et al. (2008) consider the phases of mitigation, emergency response, early recovery, disaster risk reduction and preparedness. Because Humanitarian logistics in general is more reactive than proactive, this classification can also give insight into the speed of reaction of relief networks. Immediate responses are essential to deal with ongoing crisis, but preventive activities and efforts in the long term would have a better outcome. These phases will also ensure a straightforward classification, and selection of specific tools to deal with particular scenarios in humanitarian aid.

The following classification is based on the previously mentioned classifications, as well as some additional research. Some models are listed below, but for a more detailed list of models and problems, see Appendix 1 or refer to Kovács & Spens (2007) and Çelik et al. (2012).

2.1.2.1. Disaster management activities

Çelik et al. (2012) classified the disaster activities according to the life cycle of the disaster: mitigation, preparedness, response and recovery. For additional information (e.g., type of problem and objective function) on the models presented below, refer to Appendix 1.

Mitigation phase

The mitigation phase encompasses measures such as proactive prevention and quick response, which prevent a high impact that could be derived from an ongoing emergency situation. Mitigation activities impact short and long-term humanitarian efforts. In this phase the real needs might not be exactly known, because of the sudden occurrence of a crisis, supply pre-positioning strategies are also limited, see Balcik et al. (2010). The research in that context is related to the facility location with uncertainty.

Some of the specific problems covered in this phase are: location of early warning systems in contamination scenarios (Current & O'Kelly, 1992; Berry et al., 2006; Murray & Tong, 2007; Murray et al., 2008; Berman et al., 2009a; Xu et al., 2010); unreliable facilities (Lee, 2001; Berman et al., 2007); disruption of facilities (Shen et al., 2011); man-made disruption of facilities (Church, 2004; O'Hanley & Church, 2011).

Preparedness phase

The preparedness phase encompasses strategic decisions aiming to facilitate the response and recovery activities, disaster prevention and risk management activities. Planning activities such as identification of candidate suppliers, partners or facility locations can also be included in the preparation phase.

In this phase the problems are: flood, hurricane and shelter location: (Sherali et al., 1991; Chang et al., 2007; Li et al., 2011; Rawls & Turnquist, 2012); humanitarian relief distribution (McCall, 2006; Campbell & Jones, 2011; Döyen et al., 2012; Tzeng et al., 2007; Noyan, 2012); storage of medical supplies (Mete & Zabinsky, 2010); disaster scenarios (Balcik & Beamon, 2008); unreliable facilities and man-made disruptions (Hale & Moberg, 2005); disruption of facilities, route failure and corruption risks (Yushimito & Ukkusuri, 2007).

Response phase

"The response stage starts while the disaster is in progress with the objective of managing the available resources efficiently so as to minimize the suffering of the impacted community" (Çelik et al., 2012). In this phase limited information and uncertainties make it difficult to effectively plan the activities.

In this phase the main studied problems are: routing problem and location of temporary facilities (Afshar & Haghani, 2012); transportation of humanitarian aid (Berkoune et al., 2012); disease outbreaks and food distribution (Ekici et al., 2009); evacuation operations (Yi & Özdamar, 2007); humanitarian relief distribution (Duran et al., 2011; Salmerón & Apte, 2010); disease outbreaks (Carr & Roberts, 2010).

Recovery phase

"Recovery involves the long-term actions taken to stabilize the community and to restore normalcy after the disaster's immediate impact has passed" (Berkoune et al., 2012). This phase encompasses strategic decisions following the response phase. Kovács & Spens (2007) point out the fact that, the management of the relief network can also be affected in this phase, e.g., reduction in staff and different types of help aid. In the long term, we can also see a reduction of the uncertainties, which makes it possible to better plan some activities.

In this phase the main studied problems are: service restoring after hurricanes (Nurre et al., 2012); humanitarian relief distribution (Dekle, 2005; Huang et al., 2012); route failure and corruption risk (Nolz et al., 2011); recycling (Fetter & Rakes, 2012).

2.1.2.2. Long-term humanitarian development

Traditionally the study of long-term activities has been left out of scope in the field of HL, even though there are many important efforts being invested in this area. Activities in the long term can have a durable impact on the populations, ensure that populations will effectively recover from past crises and provide them with the basis of self-sustainable coverage of their needs in the future. It is important to note that problems dealing with long-term activities are especially suitable for equality objectives in humanitarian logistics.

In this phase the main studied problems are: humanitarian food distribution (Rancourt et al., 2015); allocation of resources for health promotion (Rahman & Smith, 2000; Malvankar-Mehta & Xie, 2012); location of public facilities (McAllister, 1977; Mandell, 1991; Kalfakakou 2005).

2.2 Equality functions in location problems

Facility location and routing decisions have traditionally focused on reducing cost. Humanitarian logistics is a field where the cost factor needs to be observed with special attention, but the proposed solutions must also take into account important social considerations. Equality concerns, sometimes referred to as equity, started as soon as in 1977 when Morrill & Symons (1977) were worried about the efficiency and equality in the public sector. They verified that the minimization of cost using measures that focus on the central part of the distribution, present nevertheless highly skewed distributions and extreme variability. The authors also noticed some weaknesses of equality measures, sometimes represented by the balance of those who are extremely well-off with those who are bad-off.

McAllister (1977) considers equity as the degree of equality in the distribution of services among the population, and efficiency as the quantity of services consumed. He mentions that "location decisions typically require that the two criteria be compromised; this necessitates a subjective judgment". The author developed a model applied to public facilities that provides non-delivery services and without charge for the service, the author takes into account the travel cost paid by consumers to go to a center in order to obtain the service. The main decisions in this model were related to the spatial location of centers and the sizes of facilities. In the context of food distribution for humanitarian logistics, Rancourt et al. (2015) developed a model that considers the cost incurred by stakeholders such as the WFP, KRC and beneficiaries. This model accounts for the displacement effort incurred by the beneficiaries, and it constitutes an important step in taking social and equality considerations beyond the traditional scope that only gives importance to monetary values.

In HL, multiple objectives make it difficult to conciliate traditional economic concerns with those given by the necessities of affected populations. Huang et al. (2012) developed a model for humanitarian relief routing in the last-mile distribution, which incorporates alternative objectives based on efficiency, efficacy and equity metrics. The authors mention that "we focus on efficacy (i.e., the extent to which the goals of quick and sufficient distribution are met) and equity (i.e., the extent to which all recipients receive comparable service)". The authors found important differences between solutions that take into account efficacy and equity concerns and solutions that take into account only efficiency concerns. The same authors found that compared to a fleet of large vehicles, a fleet composed of small vehicles results in a more effective and equitable form of aid distribution. This result was achieved by a modest increase in travelling and operating cost, but required additional coordination and implementation of specialized software to support operational decisions. In the same line, Kalfakakou (2005) suggests that “equity in facility location can be interpreted as the attempt to equalize the quality of service for all demands”, the author developed a model to minimize the cost of locating facilities under a budged constraint while taking into account a maximum accepted delivery time to achieve a near-uniform travel time.

Mandell (1991) developed a bi-criteria objective to identify the trade-offs between the overall output (effectiveness) and the equality as measured by the Gini index. The model satisfied the principle of transfers, which requires transfers from a well-off group towards a worse-off group to improve the measure. An efficient frontier was obtained by repeatedly solving the problem with different levels of the Gini Index.

In addition to the previously mentioned equality criteria, there are a number of equality objectives and indexes in the literature, many of them are widely recognized and applied. Medina (2014) define an equality index as a measurement that summarizes the way a variable is distributed in a population. Network design optimization usually uses metrics developed in the field of economics. However we need to take into consideration that general economic measures are used as ex-post indicators that measure a static situation, which is the result of previous events and over which the measurer has no control but only observes the outcome. The context can vary radically when we need to evaluate an ex-ante situation, such as in the case of the optimization process where the final outcome is not defined yet and where the variables can lead to a desirable outcome.

Another particularity of economic measures, is that they are mainly focused on economic growth and the distribution of income resulting in policies aiming to increase the size of the economy (CASSE, 2014). These indicators are expressed often in monetary terms and the main objective usually is to maximize the wealth. Economic equality functions can be applied to network design and facility location, but the monetary input (income) needs to be substituted with some other measure, such as distance or travel time. Furthermore, one must be aware that economic objectives, aiming to maximize the welfare, can have an antagonistic objective in a network design with a minimization objective. A higher income or monetary value can have a positive impact in economic terms, but a longer distance or travel time can have a negative impact in network design. Evidently, when we pursue a minimization objective, some metrics needs to be re-evaluated, possibly interpreted in an inverse way, and constraints might have to be used to avoid undesirable outcomes.

The metrics used as equality objectives have limitations, and understanding them would be very useful to develop better measures. As stated by Stiglitz et al. (2001), “The metrics are generally suited to the decisions we make, depend on what we measure, how we do our measurements, and how we interpret them”. We should focus on increasing well-being; sometimes standard measures fail to capture important aspects of social well-being. We need to prioritize objectives or evaluate the trade-offs of different approaches. A series of indicators could be used as a dashboard to see specificities and also as a basis of trade-offs evaluation. As mentioned by Mandell (1991), “The appropriateness of a particular index is a context-specific issue that depends upon the user’s value and judgments”.

To conclude this subsection, we can add a quotation from Morrill & Symons (1977): “If society requires some people to live far from a center or service, then it is equitable, optimal and efficient for closer customers to subsidize more distant customers”. Some authors argue that people living far away have other benefits and then subsidizing them is not fair. That could be true in a context of the well-being of society, but for this study we make the assumption that in humanitarian logistics and especially for poor countries, those who live far-way are indeed in a naturally worse condition.

2.2.1 Criteria for balancing objectives

In the particular case of humanitarian logistics with equality considerations, where the equality of access can be considered as ideal, we must recognize that the evaluated objectives must comply with a series of requirements to be able to describe a social reality. This is more complex than only evaluating economic indicators based on monetary value. In distribution, this monetary value can be replaced by some other notion such as distance, and be measured by equality indexes. The measures must allow a deeper comparison to understand and improve the social perspective. As mentioned by Stiglitz et al. (2001), “No single measure, or even a limited set of measures, can provide all the information required to asses and manage an economy”. The appropriate information could be seen properly combining two or more indicators with appropriate features. A good indicator could have some of the eleven characteristics described in the following:

1. Scale invariance or proportionality

Drezner (1995) mentions that the degree of equality should not change with the type of measure applied to the problem. Sanchez-Perez et al. (2012) add that the inequality index must be independent of any characteristic of the individual other than the income. In our case, income is replaced by distance. "Mathematically this property is knows as Homogeneity of grade 0, indexes that satisfy this property are very useful to make non-temporal and international comparisons of inequality" (Medina, 2014).

2. Principle of transfers (principle of Pigou-Dalton)

"A society will be better off when a unit of income is transferred from a richer to a poorer individual" (Fishburn & Willig, 1984). This principle allows a low level of income concentration, in our case a low level of concentration in walking distances by a few individuals will be desirable. However, this principle assumes the preservation of the attribute (income or distance). As quoted by Bosmans et al. (2009), "The idea of a transfer that preserves the total amount of the attribute in society may not be meaningful or desirable for each attribute". The same authors recognize that there is no direct link between the level of some single attribute and the level of well-being. Following this idea, Cowell (2009) called this principle as the weak principle of transfers, because, as he mentions, that given a specified transfer, inequality should decrease, but it does not say how much it decreases.

3. Principle of proportional additions

The principle of proportional additions can be stated as "Proportionate additions to all incomes diminish inequality, and that proportionate subtractions increase it" (Dalton, 1920) This principle applies to indexes such as the Gini Index, where an increase of a variable will diminish the index. In facility location problems the variable will be the distance travelled. In an economic context, where higher incomes allow higher standards of well-being, an increase in the variable is appropriate, but in a minimization context, higher values of the variable, e.g., distance, will have the opposite effect. We can obtain a low equality index by choosing long distances traveled for everyone, but this is of course not an appropriate solution because of poor level of service.

4. Principle of growth/contraction

In economics, the principle of growth or economic expansion is considered by many of the most fundamental measurement used to evaluate success in allocating resources, and as a consequence for many economic measures, growth reflects an increase in wealth. However, in a minimization context, it could be necessary to reduce the measured attribute (distance), and that is why the term contraction was added. Daly (1990) expresses that we usually refer to the term "economic growth while we were below the optimum scale" and it becomes "anti economic growth once that optimum has been passed". When additional increases in the attribute are non-desirable, the contraction or reduction can be obtained with special policies or constraints. In economics, an important mean to reduce expenses is through taxation, for our case, we can limit the travelled distances through supply policies. Establishing distribution centers close to customers can prevent them from going too far. In the case of facility location problems, cost constraints can help to limit the travel distance by customers. Adequate travel distance can also help to ensure the sustainability of the service, ensuring that beneficiaries will repeatedly use the services of food distribution agencies to complement their food distribution requirements for the time where this assistance is needed. A measure that allows for a good reduction in inequality will therefore ensure sustainability of service.

5. Decomposability

"This property implies that there should be a coherent relationship of inequality in the whole society and inequality in its constituent parts" (Cowell, 2009). An index that could be calculated for subgroups will comply with this property. The total inequality will be calculated by accumulating the inequalities of individual groups. As mentioned by Foster & Shneyerov (2000), the comparisons between subgroups is applied to standardized distributions, and the distribution needs to be rescaled into income levels that represent each subgroup.

6. Analytic tractability

Analytical tractability may enable the use of exact methods. Wayne et al. (2013) mention that some polynomial algorithms have high exponents and are useless in practice. The same concern can be raised for some indexes that use exponential functions. Sometimes it is suggested to artificially put an upper bound to the exponent, but this solution implies that the index would not show its whole potential.

7. Normalization and standardization of measures

Adjusting values measured on different scales to a notionally common scale, thus allowing a clear comparison of values and data consistency, bringing all the variables into proportion with one another. A common practice is to normalize the values into a zero to one scale. Some models collapse in the extreme values and a very small value must be chosen (e.g., 0.001). The importance of normalization is mentioned by Etzkorn (2012), the author mentions that non-normalized coefficients reflect the positive/negative contribution towards the objective, but the interpretation is not straightforward regarding the relative impact on the objective function.

8. Impartiality, anonymity

Decisions that affect particular nodes or populations must be independent, based on objective criteria rather than bias ones.

9. Sensitivity

Sensitivity deals with the coefficient's changes with respect to a small change in any of the problem parameters. This property is also directly related to the robustness of the solution. Very high or low sensitivities are generally undesirable (Drezner, 1995).

10. Pareto-optimality

The work of Pardalos & Du (2008) mentions that "the main idea behind this concept is that no one can be made better off without making someone else worse off". This seems evident in an ex-post scenario were the disposition of the variables is already established and we can only interchange values within an established distribution. In an optimization process the variables have not yet been assigned to a final value; we are in an ex-ante situation, and the objective will be to find the best configuration of variables for the whole population. However, a final solution must be Pareto optimal in the sense that we need to satisfy multiple objectives, e.g., satisfy efficiency requirements, as well as equality concerns. In this situation we are in a context of multi-objective optimization or Pareto optimizations. The aim would be to identify a set of Pareto optimal solutions.

11. Convexity/concavity

Sen (1997) explains that concavity of a welfare function implies that the weighted average of a social welfare level from two income distributions, needs to be less than or equal to the social welfare of the weighted average of the two combined distributions.

2.2.2 Non normalized equity objectives

In this group, we have measures where the values do not share a common scale, they can take any value. Common notations for the following measures are:

n: total number items in the population (e.g., people or demand points);
x_i: represents a sub-set of a population i;
y_i: is the value of the observation for a population i sub-set (e.g., income in economics or distance for facility location);
log: logarithm.

u: mean or average (e.g., mean distance);
Specific notation is presented accompanying each particular measure.

Minimum standard service

Morrill & Symons (1977) consider that a pattern of facilities is equitable if no more than an acceptable small proportion of people are at a critical distance from a good or service provided by a facility. A representation of minimum standards under different distributions can be seen in Figure 4.

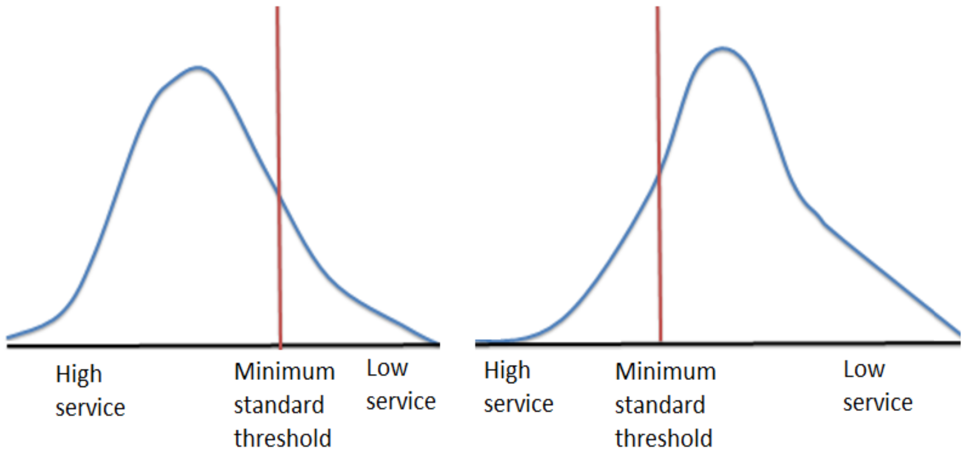


Figure 4. Distributions with equal minimum standard thresholds and unequal skewness, adapted from Morrill & Symons (1977).

Mean (u)

The average distance travelled and average time travelled for an individual are measures directly related and derived from the mean, $u = \frac{\sum_{i=1}^{n} y_i}{n}$. Being close (distance or travel time) is one of the fundamental aspects of location analysis (Drezner & Hamacher, 2004). Kovács & Spens (2012) analysed alternative supply models for humanitarian logistics, and explained lead-time in terms of the ability to provide the required goods to beneficiaries in a timely manner. The authors also suggest how strategically pre-positioning stocks would permit rapid response on the onset of a disaster.

Range (R)

A common way to study the dispersion of a variable is comparing the extreme values presented by the range, $R = \frac{1}{u}(y_{max} - y_{\min})$, where y_{max} and $y_{\min}$ represent the maximum and minimum observations of the variable, (e.g., distance). "The main weakness of measures based on the range is that they are based on extremes and ignore relevant information of the additional data", (Medina, 2014).

Mean absolute deviation (MAD)

The $MAD = \frac{\sum_{i=1}^{n}|y_i - u|}{n}$ represents the mean absolute deviation from the average distance. This measure assumes that inequality is proportional to distances from the mean. A disadvantage of this measure is that if the transfer of the attribute (income or distance) is done between individuals that are in the same extreme of the distribution, the mean absolute deviation will register no change. In economics, transfers have a better effect if they are done from richer to poorer individuals and not between the rich.

Variance (s^2)

The variance, $s^2 = \frac{\sum_{i=1}^{n}(y_i - u)^2}{n}$, measures the dispersion around the mean. The measure squares the values of the deviations and thus accentuate the differences. Therefore, reducing the variance will reduce the inequality, and will make it possible to have less variability in the travelled distance. The output can give a fairer solution especially for those who are worse-off.

Standard deviation (s)

The standard deviation is the squared root of the variance, $s = \sqrt{\frac{\sum_{i=1}^{n}(y_i - u)^2}{n}}$. McAllister (1977) measured the standard deviation of distance to the nearest service center. The author considers that "if equity is defined as the inverse of the standard deviation, then doubling of the space cuts equity in a half". The scale McAllister used, goes from zero (one service center yielding maximum inequality) and infinity, where everyone has a service center next door. To avoid extreme measures, a budget constraint can be used. Morrill & Symons (1977) quote: "if the variability measured by the standard deviation about the mean becomes smaller, that location becomes more equitable". Even if the objective is not to minimize the standard deviation, it can be an intuitive measure to see the distribution obtained from any optimization process.

Variance of logarithms (Sl^2)

The variance of logarithms, $Sl^2 = \frac{\sum_{i=1}^{n}(log\,u - log\,y_i)^2}{n}$, gives more weight to the transfers of the attribute (income or distance) in the lower area of the distribution. Alternatively, it is possible to calculate the squared root of Sl^2 and use the standard deviations of logarithms, $Sl = \sqrt{\frac{\sum_{i=1}^{n}(log\,u - log\,y_i)^2}{n}}$. The problem with these measures is that the logarithm function makes it difficult model with standard solvers.

Coefficient of variation (C_v)

The coefficient of variation, $C_v = \frac{1}{u}\sqrt{\frac{\sum_{i=1}^{n}(y_i - u)^2}{n}}$ or $C_v = \frac{\sqrt{s^2}}{u}$, represents the standard deviation (sum of squared deviations from the mean) divided by the mean, giving weight to the deviations that are far from the mean. This measure is sensible to transfers in the distribution. "This index can be used to differentiate between two distributions when their Gini Index are the same" (Gonzalez Abril & Morente, 2010). In a unimodal distribution, it can be seen as the peak similar to fourth moment from the mean "Kurtosis", $SQ^2 = \frac{\sum_{i=1}^{n}(y_i - u)^4}{n}$, see two examples in Figure 5. The slope of a curve is high for data close to the mean, and low for data dispersed from the mean. This measure satisfies anonymity, scale invariance, population independence and the principle of transfers. The disadvantage for this measure is that it can take any value from zero to infinity.

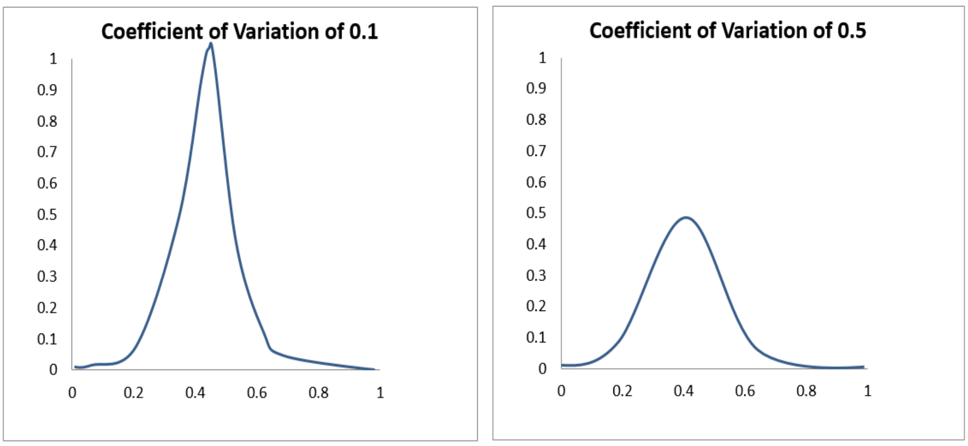


Figure 5. Different values for the coefficient of variation, adapted from Hale (2014).

Theil's coefficient (T)

The Theil's index, $T = \sum_{i=1}^{n} \left\{ \left(\frac{1}{n}\right) \cdot \left(\frac{y_i}{u}\right) \cdot log\left(\frac{y_i}{u}\right) \right\}$, is part of the measures known as Generalized Entropy, it is also one of the most popular measures of inequality. It measures how distant the population is from the ideal equality. The advantage of this measure is that it allows strong transfers while conserving the additive decomposability property. This enables assessing inequality within specific groups. A value of zero for this index equals perfect equality. However, this index can go to infinity. Because of this particularity, Rohde (2007) mentions that: "In this form the Theil measure's information content interpretation is diminished."

2.2.3 Normalized equity objectives

In this group, we have measures whose values can share a common scale, e.g., zero to one. Sometimes they require a normalization factor to accommodate the scale. Another characteristic of this group is that the values in the distribution are ordered according to a rank; complex equality measures require a pre-ordering of groups inside the distribution in the form of hierarchical data sets. In the view of Lambert & Yitzhaki (1995), normalized equality measures can easily be visualized with the Lorenz curve.

The Lorenz curve relies on the histogram of frequencies, which is an intuitive way of ordering the observations, grouping individuals according to an interval and observing the concentration of observations inside each interval. The Lorenz curve represents the cumulative percentage of a variable, for example income or distance, ordered in an ascendant form. If everyone has the same income, then the Lorenz curve is a 45^{o} line (equality line). This curve also allows a geometric construction of the Gini index and similar indexes. An example of the Lorenz curve and the equality line, both encircling the area representing the Gini index and the Robin Hood Index, can be seen in Figure 6.

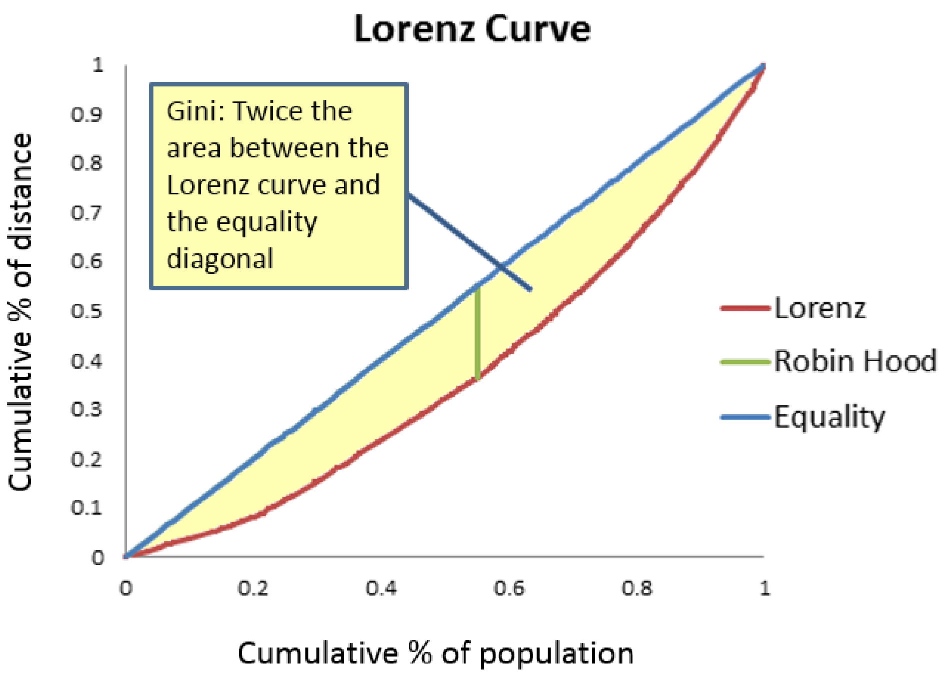


Figure 6. Lorenz curve, equality diagonal, Gini Index and Robin Hood Index.

Among the most important normalized equality measures we have the ones listed in the following.

Gini coefficient (G)

The Gini coefficient or index is maybe one of the most used indicators of social and economic welfare. In economics, this measure represents the dispersion of income distribution in a population. This index has also been applied in many areas beyond the economics. One of the most popular formulations is given by Brown (1994), $G = 1 - \sum_{i=0}^{n-1}(d_{i+1} + d_i)(p_{i+1} - p_i)$, where p_i represents the cumulative proportion of population i with the attribute (income or distance), and d_i represents the cumulative proportion of income or distance for population i. It has been mainly used to compare income distribution between countries, and allows comparisons of two populations regardless of the size of n.

"This index satisfies the properties of scale invariance, symmetry or anonymity, proportionality and convexity." (Sanchez-Perez et al., 2012). The Gini index can be observed geometrically as the ratio of the area that lies between the line of perfect equality and the Lorenz curve (Figure 2). It takes a value of zero (equality) with minimum concentration of the attribute (income or distance) and one with maximum concentration (maximum inequality).

The major problem with this index is that “two very different distributions can have the same value of this index and, therefore, it is not possible to declare which distribution is more equitable” (Gonzalez Abril & Morente, 2010). This index is more sensitive to inequalities in the middle part of the distribution. “It is not clear that such weighting would necessarily accord with social values” (Atkinson, 1970). In some cases the attention needs to be put on the top or bottom of the distribution. Giving little attention to the extremes of the distribution could mean not improving the service for those populations.

Dalton index (D)

The Dalton index was the first index to measure the inequality, $D = 1 - \sum_{i=1}^{n} \frac{U(y_i)}{nU(u)}$, based on the utility *U*, which represents the preference over an attribute (e.g., travelled distance). The index compares actual levels of aggregate utility and the total utility that would be obtained if income or distance were equally divided (Sen, 1997). This index has been criticized since it is varies with changes in the scale.

Hoover index (H)

The Hoover index, $H = \frac{1}{2}\sum_{i=1}^{n}\left(\frac{y_i}{\Sigma_y} - \frac{x_i}{\Sigma_x}\right)$, is often called the Robin Hood Index because it represents the portion of income that needs to be redistributed from the area above the mean to the area below the mean in order to achieve perfect equality. This index can be graphically seen as a vertical arrow that shows the maximum vertical distance between the line of perfect equality (45° line of equal incomes), and the Lorenz curve; henceforth, it is also related to the Gini Index (area below the Lorenz curve), see Figure 6.

Sen index ($s_{(y,z)}$)

Amartya Sen, Nobel prize in economics 1998, proposed an index, $s_{(y,z)} = H_C \cdot [I + (1 - I) \cdot G^p]$, to measure not the central but the inequality in the tails of a distribution, Sen (1997). This index is based on three components: 1) H_c: headcount ratio (poor/entire population); 2) *I*: income gap ratio (mean distance of the incomes of the poor from the poverty line); and 3) G^p: representing the Gini coefficient computed over the incomes of the poor, also called the poverty gap. This decomposed equation is noted as follows: $\rho_{(y,z)} = \sum_{i=1}^{n}(z - y_i) \cdot w_i = A\sum_{i=1}^{n}(z - y_i)(P + 1 - i)$, where w_i represents the decreasing weight assigned to the poorest persons, A is a normalization factor, i is the position of the individual in the income ordered distribution, y_i is the income of population i, z is the poverty line, P represents the incomes of the poor (Harvard Magazine, 2014). This index is interesting since the weights assigned can target the non-desirable income (distances) levels. "Sen index is said to include the three I's of poverty (Incidence, Intensity and Inequality)" (Bellu & Liberati, 2014). In addition to the complex calculation, it will be necessary to re-define the segment of population over and under the poverty line since in economics, poverty equals a low measure (income), which is the opposite for facility location in humanitarian logistics, where a low distance is desirable in contraposition to a longer distance that is non-desirable.

Kawkani coefficient (K)

Traditionally used in taxation, the Kawkani index, $K = G_x - G_n = \frac{t^x P}{1-t^x}$, is based on the principle of vertical and horizontal equality, see Kakwani & Lambert (1998). Taxation measures allow the Lorenz curve to be pushed towards the line of equality. Vertical equality requires that people with higher incomes to pay higher taxes, and horizontal equality implies that people with equal income pay equal taxes. G_x represent a pre-fiscal Gini and G_n a post-fiscal Gini, t^x represent the tax paid, and P the proportion of incomes of the poor people. Graphically it is represented as twice the area between the concentration curve of taxes and the Lorenz curve of before-tax income. The values go from -2 to 1 and they approach the upper limit when there is no inequality (Gini = 0) and the tax burden fall on the richest groups.

Thon index (TH)

The Thon index, $TH = \frac{2}{(n+1)n_z}\sum_{i=1}^{n}(z - y_i)(n + 1 - i)$, follows the same logic of Kakwani, but in this index the weight of poverty gap is measured considering the total number of individuals n and not just the poor P, see Bellu & Liberati (2014). For this index, y_i represents the income or distance, i represents the position of the individual in the rank-ordered distribution, z the poverty line that can be reinterpreted as the maximum travel distance allowed in facility location problems and n_z that could also be reinterpreted as individual travelling a distance inferior to z.

Reynolds-Smolensky index (Rs)

The Reynolds-Smolensky index, $Rs = G_x - G_{x-t}$, is also known as the redistributive effect. G_x, represents the index Gini before taxes, and *t* represents the tax applied. The range goes from -1 to 1, "a negative value indicating regressivity and redistribution towards the better-off and opposite values pointing to the opposite (progressivity)." (Murray et al., 2014). This index is directly related to the Kawkani Index (*K*), where: $Rs = \frac{t}{1-t}K$.

2.3 Additional service considerations

Due to the complexity of social evaluations, especially in the context of facility location for humanitarian logistics, it is important to take into account some additional criteria. Some measures can help better analyse the trade-offs associated to equality functions. The amount that needs to be given or paid by the stakeholders (WFP and Red Cross) and the population (represented by the symbolic cost of travel) would be an interesting additional indicator to measure, because it is preferable to obtain equalitarian distributions at the lowest possible cost. "Social welfare has both equity and efficiency components (cost)" (McAllister, 1977).

Another important service consideration useful to observe the symmetry of distributions is represented by the Skewness, $Sk^2 = \frac{\sum_{i=1}^{n}(y_i - u)^3}{n}$. This measure can be computed as the third moment around the mean. There are as well some other skew measures, such as the first and second coefficient of Pearson. The first coefficient of Pearson is based in the relation between the mean and the mode, whereas the second is based on the relation of the mean with the median, and this coefficient is useful when we have multimodal distributions. The expression for the Pearson's 2nd skewness coefficient is $SP_2 = \frac{3\,(u-m\,)}{S}$, where m represents the median (median distance) and *S* the standard deviation.

Morrill & Symons (1977) have analysed the minimum standard service (see Section 2.2.2) and they have showed how two accessibility distributions with equal minimum standards have unequal skewness. Cowell (2009) mentions that typical income distributions are positively skewed with a right-hand tail that is more noticeable in the case of the distribution of wealth. We can clearly visualize the concept of skewness in the graphic below (Figure 7). Doane & Seward (2011) point out that "(a) "symmetric" need not imply a "bell-shaped" distribution; (b) extreme data values in one tail are not unusual in real data; and (c) real samples may not resemble any simple histogram prototype". They show how this sometimes forgotten measure of statistics is very useful to show the variability of distributions, feature that can be very interesting when applying measures of inequality.

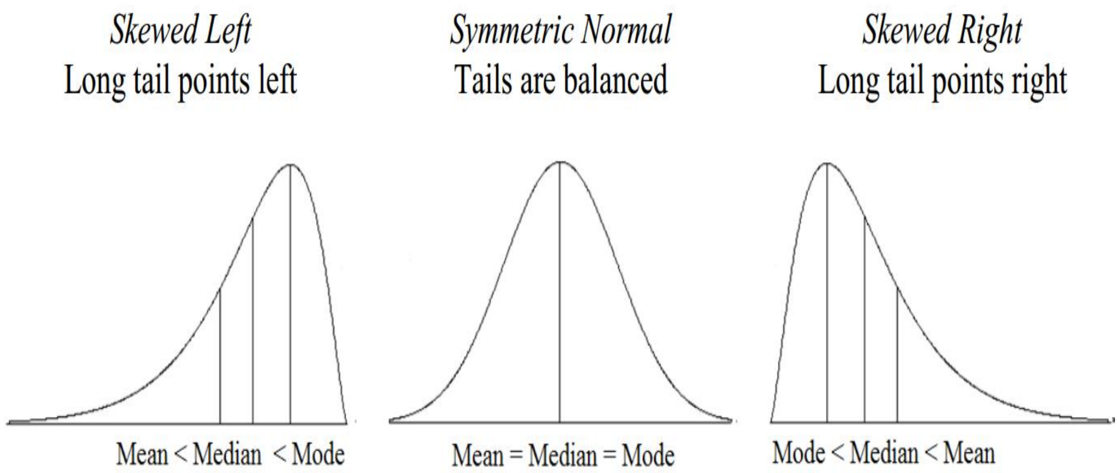


Figure 7. Distributions with different skews.
Source: Doane & Seward (2011).

We must also have a comprehension of how efficiently we are utilizing our resources, e.g., what is the value of our output for every unit of input? "Equity objectives should be used in conjunction with an efficiency objective rather than as stand-alone objectives" (ReVelle & Eiselt, 2005). "As average travel distance increases, facility accessibility decreases, and thus the location effectiveness decreases." (Owen & Daskin, 1998). Efficiency for facility location problems can be interpreted as the extend on which travel time or distance are accomplished without incurring unnecessary cost. We can also consider the cost incurred as input. The output is represented by a satisfactory coverage and adequate travel distance and time for the beneficiaries of food distribution.

2.4 Conclusions and implications

Different facility location models in the context of humanitarian logistics were examined, as well as different equality functions. Many functions, traditionally used in the economic field, can be applicable for optimizing network designs problems by changing the variable income for another more appropriate measure such as distance. We need to take into account that the economic functions need to be re-interpreted, because a higher variable (income), that could be desirable in economics, translates into a higher variable (distance) that could not be desirable for facility location problems.

From the literature review, we can conclude that it is important to respect some criteria when deciding to use one or to develop another measure of inequality reduction. Facility location for HL shares many characteristics with location problems in the public sector. "Social cost minimization, universality of service, efficiency and equity are the goals of the public service" (Drezner & Hamacher, 2004). Additionally, regardless of the chosen equality measure, service indicators can constitute a good way to intuitively determine differences and evaluate some others important aspects of the obtained distribution.

From our analysis, we can conclude that an equality function should respect some criteria to balance the objectives. It is also possible to conclude that normalized equity functions allow a more straightforward comparison of indexes through different distributions, and also because they can be compared time independently. Exponential functions represent an obstacle to compare distribution and measures and because they are not naturally bounded, it is not possible to agree on a decision about the appropriate value of the exponent. New approaches to be applied to facility location in HL could be given by measures that target primarily the tails of the distribution where the inequality tends to be extreme, in contrast with traditional measures that target mainly the central part of the distribution.

Another important point observed is that disaster response activities in general do not encompass equality considerations. This could be because quick response associated with ongoing crises requires the use of all accessible resources, where planning activities are not the main focus. The fact that, uncertainties are highly present is this phase, makes it difficult to know who the affected populations are. If we do not know where the populations and their exact numbers are, it will be impossible to target them with a fair or uniform aid quantity. On the other hand, recovery and long-term development activities allow more detailed planning activities. In this phase we have more certainty about the affected populations, their location, number and exact requirements. In this context, planning development activities, that encompass equality considerations, are feasible.

Considering that the present project is based on the work of Rancourt et al. (2015), and in the same line of the previous authors, a mixed integer linear problem will be developed and presented in Chapter 4. In addition, considering the size of the problem over which we will develop our research and the complexity of some equality measures, where many of them are not suitable to develop linear optimization problems, we decided to explore some specific equality measures such as measures based on the Minimum Standard Service, Average, Mean Absolute Deviation and the Variance. These measures will be coupled with cost objective functions. This approach will allow us to evaluate in our research both, equality and efficiency concerns.

Chapter 3: Research design

This chapter introduces the reader to our research design. We will first describe the problem, we then present the main stakeholders involved in food distribution and the network used in this context. Finally we will describe a mathematical model conceived to deal with the problem at hand.

3.1 Detailed problem description

In this section, we analyse the main issues related to food aid distribution in the context of the region and country. The case of Kenya is examined with a special attention to the Garissa district in the present work.

3.2 Food aid distribution in the context of the region and country

Each year, thousands of lives are lost and many millions more are being affected by natural and man-made disasters (Disasterium, 2014). Many regions around the world suffer from chronic episodes of food insecurity; some parts of Africa are among the regions severely affected. According to the WFP (2014a), Kenya, situated in the area known as the horn of Africa, see Figure 8 (a), is a country deeply affected by food insecurity. It is estimated that 1.5 million people in Kenya are currently in this situation and need assistance. This country, where 80% of the land is arid or semi-arid, depends on rain-fed agriculture for its food requirements. The country has two main rainy seasons: the long rains from March to May and the short rains from October to December. In rural areas, where pastoralism is a common way of life, the deterioration in food security is driven by poor rains during the harvesting season and floods over the last quarter of the year, as it happened in 2013, see FAO (2013). Besides the rural areas, urban areas are also affected by food shortages, and this creates a competition for assistance, which is often answered with political considerations, resulting in fewer resources for distant areas (Battersby, 2013). The author signals how, in the context of food insecurity, it is important to answer to the requirements of urban as well as rural populations to avoid competition for resources. The challenges of rapid urbanization in Kenya are examined by Gallaher et al. (2013). The authors point out the fact that currently between a third and a half of Kenya’s population lives in urban areas, but it is expected that half of the population will live in urban areas by the year 2020.

Garissa, a district situated in east of Kenya (Figure 8(b)), is predominantly a rural area. According to Softkenya (2014), it has a population of approximately 623,060 inhabitants, and an area of about 45,720 km$^{2.}$ The main water way is the Tana River, and the urban areas and populations follow its route. According to the Government of Kenya (GK), the main urban area is Garissa town, with a population of 119,696 inhabitants (GK, 2014). The district is located 350 km east of Nairobi (Kenya's Capital). The population of the district is not uniformly distributed. In the central region, agglomerations of population and small urban areas are located close to food distributions centers, see Figure 1 from Rancourt et al. (2015) and also Figure 8 (d), which results in an uneven competition with populations located far-away, generating inequality between beneficiaries of food aid.

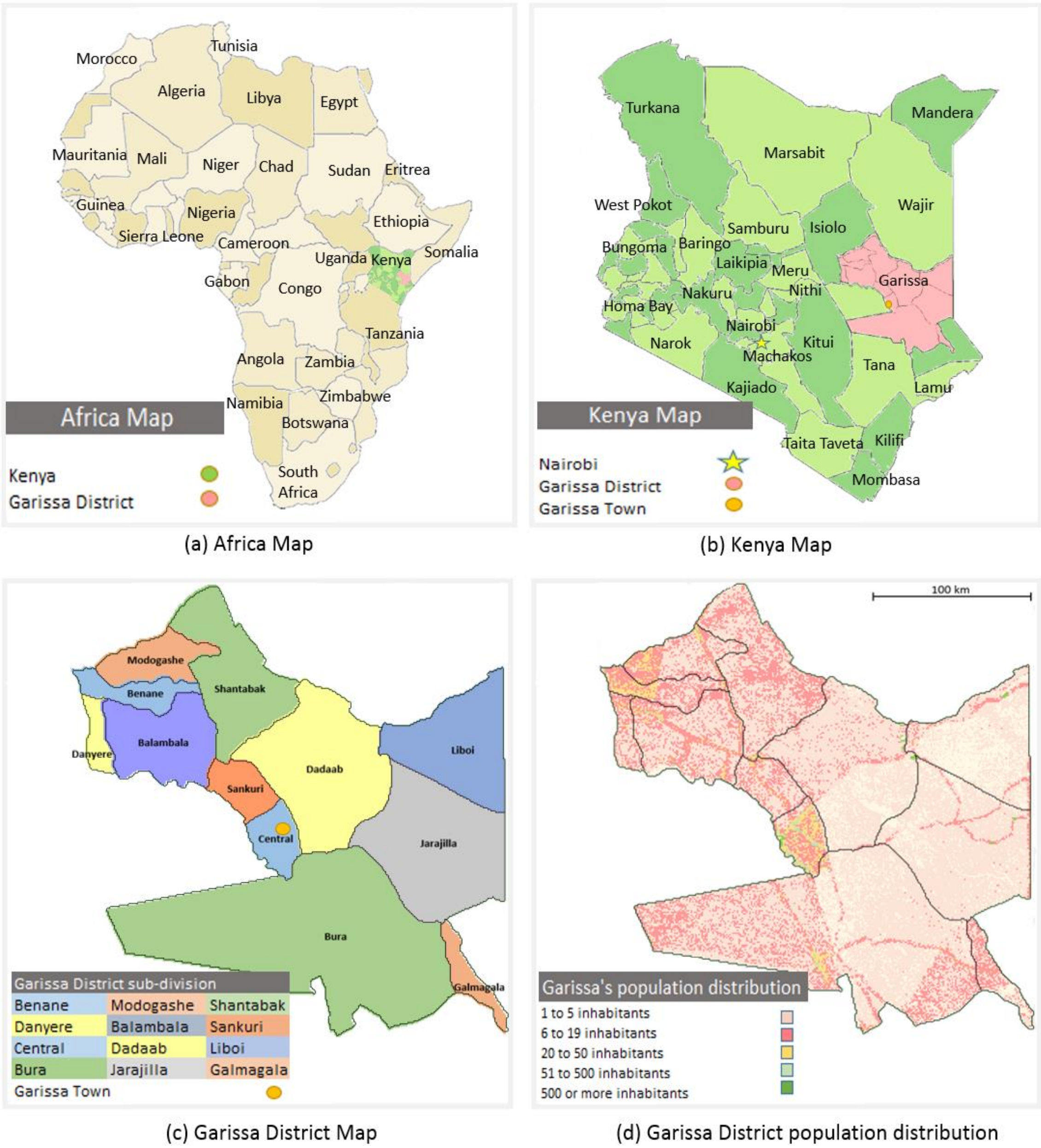


Figure 8. Africa, Kenya and Garissa District maps.

International organisations deploy much effort to cope with the food insecurity in rural and urban areas. The WFP, in partnership with the GK, manages a food programs for 750,000 school children and they also support more than 500,000 refugees in provisory camps. However, social and political tensions in the neighbouring countries, such as Sudan, increase the number of refugees by more than 300 on a daily basis (WFP, 2014b). Rancourt et al. (2015) mention that in the Garissa district, 83,483 people have received food aid considering the 2012 short rain assessment. The author also mentions that an average of 35% of population has been receiving aid. Kovács & Spens (2012) mention that over 90% of products supplied in Africa for humanitarian purposes come from outside the continent primarily from Europe, North America and South East Asia.

3.2.1 Main stakeholders

The logistic activities required to ensure an adequate function of food distribution networks necessitate the participation and collaboration of different entities. A resource-constrained environment must satisfy the interest of multiple stakeholders (donors, WFP, KRC, NGOs, GK, beneficiaries, etc.). These particularities also make this an interesting field. Thus, the development of appropriate methodologies and techniques to support effective aid distribution is gaining increasing attention. In this context, humanitarian logistics and relief supply chains are key factors to face both disasters and long-term humanitarian efforts. We will now describe the main stakeholders considered for the present study, see Figure 9. It is also possible to find detailed and additional information about the stakeholders in the original study of Rancourt et al. (2015).

Figure 9. Pictures of the main stakeholders.
Source: Google images of WFP, KRC and beneficiaries of food aid distribution.

The World Food Programme

The WFP is an organization of the United Nations in charge of fighting hunger under different circumstances. The WFP is in partnership with governments and other institutions, together they identify food insecure populations for which detailed Food Assessment Baselines (FAB) are elaborated in order to implement complex plans to satisfy basic food requirements (WFP, 2014c). As observed by Rancourt et al. (2015), for the case of Kenya, in order to deliver the food from the main warehouses to the distribution centers, the WFP contracts with trucking companies fixing the transportation rates based on bounds, delimiting specific distances. The transportation cost is then calculated by multiplying the weight in tonnes of food delivered by the cost function applicable for the distance over which the food is being transported.

The Kenyan Red Cross Society

This institution acts as a local cooperating lead partner for the WFP, as observed by Rancourt et al. (2015), this institution is a link between the food beneficiaries and the food providers, represented by the WFP. The authors also identified some other stakeholders such as NGOs taking part in specific feeding programs and the participation of a Community Relief Committee (CRC), which consists of representatives of the food beneficiaries whose mission is to ensure a proper and transparent food distribution for all members of the community. Most of the work of the CRC is voluntary and mainly ensures transparency for the food distribution process. The training and monitoring of the CRC is the responsibility of the KRC. Additional information about the participation of these stakeholders can be found in the work of Rancourt et al. (2015).

Beneficiaries

The beneficiaries are the people who suffer from food insecurity and are identified in the FAB of the WFP. They live in urban and rural areas. Those living in more populated areas are in general close to distribution centers, whereas people living in more distant areas have to walk longer distances in order to access food supplies. Rancourt et al. (2015) considered the displacement of these beneficiaries as opportunity costs, since the longer the distance they need to walk, the more time they have to spend; time that could be dedicated to perform productive activities, such as harvesting, pastoralism or activities dedicated to generate revenue that can be used to purchase food supplies. Considering the uneven condition generated for different distances and walking times, we decided to explore objective functions able to reduce or minimize these differences.

Many challenges arise when dealing with food distribution for humanitarian purposes. Many times, the efforts need to be deployed in a context of insufficient infrastructure and political turmoil. The information can be limited or can change suddenly; uncertainties regarding the location and number of affected populations and beneficiaries have a big influence, especially in cases of ongoing crisis. These situations increase the complexity of the decisions to be taken.

The main objective of the present work is to develop mathematical models that could help reduce the uneven conditions generated by different factors, e.g., scarcity of harvest due to short rains or floodings, competition for humanitarian aid between rural and urban areas, social and political problems, configuration of the relief network, resources constraints, etc. A well-designed food distribution network and a good location of distribution centers could be a way to promote a more equalitarian and fair access for the beneficiaries of food assistance. Figure 10 represents how different location of population points in relation with distribution centers impact the accessibility for food distribution. A more equilibrated location of distribution centers can certainly improve the beneficiaries' access as observed on the right of the figure.

Figure 10. An illustration of accessibility differences in food aid distribution.

3.2.2 Descriptive statistics

Particularities of the food distribution network for the Garissa District can be seen in Table 1. The Garissa District has 12 sub-divisions, see also Figure 8 (c). Some of the sub-divisions are highly populated as it is the case of Bura, Central and Dadaab, accounting for higher proportions of population; and some others have very small populations as it is the case of Danyere, Galgamala and Sankuri. It is possible to see the geographical distribution of the population in Figure 8 (d). For the Garissa District, the food distribution network has one main warehouse located at the Central sub-division. There are also 1,459 potential distribution centers, they represent the population points with a population of at least 20 inhabitants located close to roads, see Rancourt et al. (2015).

In Table 1, we can also observe how the potential distribution centers are mainly concentrated in a few sub-divisions, principally in the Central sub-division with a number of 502, practically a third of the total potential DCs. Whereas some other areas have a few potential DCs, representing mainly rural areas and areas where the population is highly dispersed, as it is the case of Galgamala with only four potential DCs. These differences in population and potential distribution centers concentration account for high levels of inequality when the inhabitants need to walk in order to obtain the food supplies distributed by the WFP.

In table 1, we observe different food assessment per beneficiary according to the Garissa sub-divisions. For the case of Central, we have the lowest food assessment per beneficiary (0.0040 tonnes). However, this sub-division accounts for the highest proportion of potential DCs. On the contrary some areas as Danyere, Sankuri and Galgamala have higher food requirements (0.1145, 0.0837 and 0.0693 tonnes, respectively), but there are only a few potential DCs available in those areas. We also observe a column showing the number of walking inhabitants for each sub-division, they are calculated by dividing the population of one km^2 by six and rounding it to the upper bound, since a beneficiary accounts for a household composed of six people.

Table 1. Particularities of the food aid distribution network for the Garissa District.

Garissa sub-division	Population points	Potential DCs	Warehouse	Inhabitants	Walking Inhabitants	Food assessment (tonnes)	Food assessment per beneficiary (tonnes)
Balambala	3,075	157	-	26,313	5,709	768.25	0.0292
Benane	1,192	222	-	14,493	2,888	336.92	0.0232
Bura	8,726	218	-	67,855	13,679	580.34	0.0086
Central	1073	502	1	89,672	15,392	355.11	0.0040
Dadaab	2,186	44	-	126,743	21,008	546.05	0.0043
Danyere	619	30	-	4,937	1,080	565.26	0.1145
Galgamala	686	4	-	4,145	871	287.35	0.0693
Jarajilla	1,014	12	-	34,544	5,000	579.16	0.0168
Liboi	771	20	-	39,708	6,458	421.06	0.0106
Modogashe	1,650	181	-	16,459	3,430	659.69	0.0401
Sankuri	1,064	49	-	8,622	1,848	721.33	0.0837
Shantabak	2,397	20	-	18727	3,535	435.16	0.0232
Total	24,453	1,459	1	452,218	80,898	6,255.66	

Based on the previous information we can understand how a network design based only in economic factors can mainly benefit highly populated areas, these areas account for the highest proportion of potential DCs but with lower food requirements. Considering that in highly populated areas, many inhabitants live very close to potential DCs; the displacement costs for these inhabitants will be very low and a cost minimization objective in the optimization process could give more weight to those areas while limiting the number of open DCs in remote and scarcely populated areas.

3.2.3 The food distribution network

In this section, we describe the food distribution network in a similar way of Rancourt el al. (2015). The food distribution network for the Garissa District can be represented on a graph $G = (V,A)$, where V is a set of nodes and A is a set of arcs. "The set V can be partitioned into $\{\{0\}, V_1, V_2\}$, where 0 is the MW supplying a specific affected region, V_1 is a set of population points, and V_2 is a set of potential locations for DCs. The quantity of food, expressed in tonnes (t), required by the population located at node i during the planning horizon is equal to q_i. We define $W_i(r) \subseteq V_2$ as the set of potential DCs located within a coverage radius of r km from i, i.e., $W_i(r) = \{j \in V_2 \mid d_{ij}^{g} \leq r\}$, where d_{ij}^{g} is the geographical distance from i to j. We also define a set $V_1(r) = \{i \in V_1 \mid \exists\, j \text{ with } d_{ij}^{g} \leq r\}$ of population points with at least one potential DC located within r km. Our models do not consider remote population points i for which $W_i(r) = \emptyset$ for a given r, as is often the case in the location of emergency services such as fire stations in rural areas", (Rancourt et al., 2015).

The arc $(0,j)$ represents the route taken by a transporter to go from 0 to j, and the arc (i,j) represents the path taken by a beneficiary from population point i to distribution center j. Each population point i is inhabited by p_i inhabitants, representing the total number of inhabitants living in one km^2 and whose centroid is representing the particular population point. In p_i, we have some inhabitants that are in charge of collecting the food supplies for the people living in population i.

Population points with more than 20 inhabitants, within a distance of 0.2 km to roads can be considered as potential DCs. Among them some will be selected to become open DCs under a particular six-month food assessment period for the Garissa District. The distances were obtained using a PostGIS application, see Rancourt et al. (2015) for additional information. In this study we used the data for the short rain assessment of 2012, see Rancourt et al. (2013a). The insights gained in this study can be easily applied into future food assessment periods in similar contexts. Each population point i has a food necessity of q_i. The total food, in tonnes, transported from the warehouse to the DCs, represented by z_{0j}, must be equal to the food requirements in tonnes of the population points assigned to open DCs j and cannot be superior to the total food assessment F_g for the Garissa District in the corresponding period.

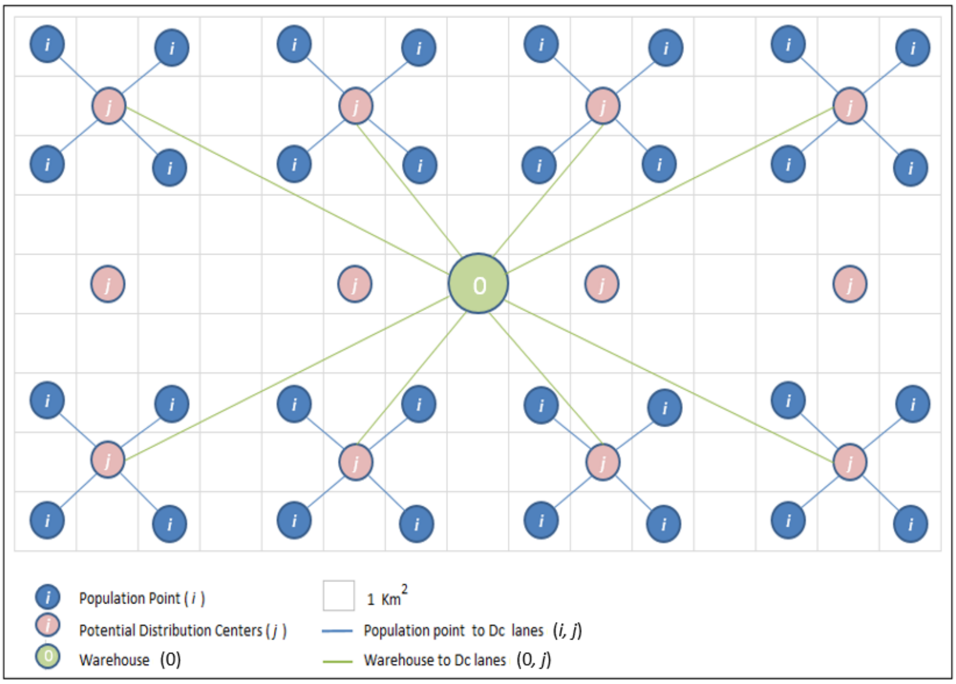


Figure 11. An example of a distribution network for the Garissa District.

Table 2 below, presents descriptive statistics on the distribution network in the Garissa District. The average number of inhabitants for a population point is 17.36, however we observe a high standard deviation of 221.15. As it can be observed in Table 1, a minimum and maximum values for the number of inhabitants also reflect the high concentration of population in certain areas. The indicator food assessment, for the beneficiaries (q_i), show that food insecurity is higher in certain areas. Looking at the indicator for the distance from a population point to its closest potential DC (d_{ij}^{g}), we can also notice that there are high levels of inequality when walking to have access to the food supplies for some inhabitants. There are inhabitants that need to walk only a few meters, whereas some others, who do not have an open distribution center close to their dwellings, have to walk very long distances to reach the food supplies, as is the case of inhabitants walking 54.48 km. This distance can also be considered as the maximum distance necessary to cover all inhabitants in the Garissa District. The indicator for the road distance from a potential DC to the warehouse (d_{oj}^{r}) also reflects the relationship between the concentration of population points and distribution centers, where for some areas the trucks may travel only 0.05 km and in some other cases they may travel up to 268.93 km to reach distribution centers .

Table 2. Summary of food aid distribution network for the Garissa District.

Source: Rancourt et al. (2015).

Description	Parameter	n	Mean	Std.	Median	Min	Max
Population nodes	V_1						
Inhabitants	p_i	24,453	17.36	221.15	5	3	13,793
Six-month food need per beneficiary (tonnes)	q_i	24,453	0.02438	0.02422	0.01136	0.0396	0.11534
Distance to closest route (km)	d_i^{r*}	24,453	11.03	9.34	8.49	0	50.34
Grographical distance to closest potential DC (km)	d_{ij}^{g}	24,453	11.85	10.91	8.71	0	54.48
Potential DC nodes	V_2						
Road distance to Warehouse	d_{oj}^{r}	1,459	106.32	71.41	107.16	0.05	268.93

3.3 Model description

We now present the components of the model developed for a given radius *r* for the present study. We first describe the following variables and parameters.

Variables

y_{ij} : a binary variable equal to 1 if and only if population i is covered by DC located at j during the planning horizon, with $i \in V_1$ (r) and $j \in W_i$ (r).

x_j : a binary variable equal to 1 if and only if DC located at j is in operation during the planning horizon, with $j \in V_2$.

z_{0j} : Variable representing the food delivered from Warehouse 0 to DC j in operation during the planing horizon, with $j \in V_2$.

Parameters

For the present model we have seven scalars (S_1 to S_7). These scalars activate certain terms (see Section 3.3.1) of the objective function when their value is set to one. If the value of the scalar is set to zero its corresponding term will not be optimized. However, the value of the term will be obtained as a secondary output based on the results of the optimized terms. These scalars allow yield valuable additional information and give flexibility to optimize different objectives. It is also possible to use values higher than one for the scalars and in this way we can generate solutions based on weighted values. Nevertheless, the different weights assigned to each objective function require a consensus among decision makers, considering time limitations and the infinite spectrum of weights for the scalars, this option was left out from our scope, but this option could be used for future analysis and research.

S_1 : The scalar that activates the term associated with the beneficiaries' acces cost.

S_2 : The scalar that activates the term asscoaited with the WFP's cost.

S_3 : The scalar that activates the term associated with KRC' cost.

S_4 : The scalar that activates the term associated with the beneficiaries' average walking distance.

S_5 : The scalar that activates the term associated with the beneficiaries' mean absolute deviation of walking distance.

S_6 : The scalar that activates the term associated with the beneficiaries' variance of walking distance.

S_7 : The scalar that activates the term associated with the percentage of walking inhabitants above a predefined threshold

t : The distance threshold in km below which the walking inhabitants are considered to receive high service distance levels.

3.3.1 Description of the objective function

The objective function is the summation of seven terms. The first three terms, relate to the main stakeholder costs, as in the original study of Rancourt et al. (2013a). These three terms, have small differences compared with the original model. These small modifications were necessary in order to better suit the model with the data files generated for the present study; however, the results obtained in both cases are the same.

The first term (T_1) is given by the beneficiaries' access cost, multiplied by its correspondent objective scalar.

$$T_1 = S_1 . \sum_{i \in V_1(r)} \sum_{j \in W_i(r)} \alpha_{ij} . y_{ij},$$

where α_{ij} represents the transportation cost, from population point *i* to distribution center *j*. This expression can be decomposed into the following:

$$\alpha_{ij} = (2 . d_{ij}^{g} . c_{km} . w_i) + \left(\{20 \text{ KSh} + 2.5 \text{ KSh/km} . d_{ij}^{g}\} . w_i \right),$$

where $2d_{ij}^{g} \cdot c_{km} \cdot w_i$ represents the beneficiaries walking opportunity cost, with $2d_{ij}^{g}$ is the return trip from a population point *i* to distribution center *j (*the distance is measured by the euclidean norm). This information was obtained from the original study of Rancourt et al. (2015), where it is possible to find additional information regarding how this distances were calculated. c_{km} represents the beneficiaries cost for walking one km. In the original study of Rancourt et al. (2015), it was established that an adequate minimum hourly wage rate for unskilled labour is given by 22.25 KSh/hour. Assuming a walking speed of 4 km/hour, we obtain 22.25 KSh/hour / 4 km/hour = 5.5625 KSh/km. This value is used in the calculation of the present study. The cost expressed as a function of the distance being equivalent to the cost expressed as a function of the time. Moreover, w_i is the number of walking inhabitants from a population point *i.* Because of integrality reasons, this value is calculated by dividing the number of inhabitants p_i by six, and rounding the result to its upper bound, as it was also calculated by Rancourt et al. (2015). $\{20 \text{ KSh} + 2.5 \text{ KSh/km} \cdot d_{ij}^{g}\} \cdot w_i$ is the linear regresion function to calculate the donkey transportation cost as it was determined by Rancourt et al. (2015).

The second term (T_2) is given by the WFP's cost, multiplied as well by its correspondent objective scalar.

$$T_2 = S_2 \cdot \sum_{j \in V_2} \beta_{0j} \cdot z_{0j} ,$$

where β_{0j} represents the cost per tonne of food transported from warehouse 0 to distribution center *j*.

$$\beta_{0j} = \begin{cases} c_0, & if\ d_{0j}^{r} \in [0, \bar{d}_0] \\ c_1 d_{0j}^{r}, & if\ d_{0j}^{r} \in [\bar{d}_0, \bar{d}_1] \\ c_2 d_{0j}^{r}, & if\ d_{0j}^{r} > \bar{d}_1. \end{cases}$$

Here, c_0, c_1, c_2 represent the WFP cost per tonne per km for three different bounds of distance. Also, d_{0j}^{r} represent the road distances from a potential distribution center j to the warehouse 0, and $\bar{d}_0, \bar{d}_1$ represent specific bound distances delimiting the cost function. As mentioned in Rancourt et al. (2015), it is not possible to give details about specific costs, as this is a confidential information.

The third term (T_3) represents the KRC's cost multiplied with its correspondent objective scalar.

$$T_3 = S_3 . \sum_{j \in V_2} \gamma . x_j ,$$

where γ represents the fixed cost paid for opening distribution center j, this cost was approximately KSh 23,453 for each distribution center.

The last four terms are equality and service functions. The costs and equality objectives will be used to compare results obtained using different data sets: non-aggregated data (in Section 5.1), aggregated data (Section 5.2) and generated data (Section 5.3). Terms T_4, T_5 and T_6 are well known measeares representing the beneficiaries' average walking distance, mean absolute deviation of walking distance and variance of walking distance. These functions are also multiplied by their corresponding scalars.

The term (T_4) is the beneficiaries' average walking distance:

$$T_4 = S_4 . \frac{\sum_{i \in V_1(r)} \sum_{j \in W_i(r)} d_{ij}^g . w_i . y_{ij}}{\sum_{i \in V_1(r)} w_i} .$$

The term (T_5) is the beneficiaries' mean absolute deviation of walking distance:

$$T_5 = s_5 . \frac{\left| \sum_{i \in V_1(r)} \sum_{j \in W_i(r)} d_{ij}^g . w_i . y_{ij} - \frac{\sum_{i \in V_1(r)} \sum_{j \in W_i(r)} d_{ij}^g . w_i . y_{ij}}{\sum_{i \in V_1(r)} w_i} \right|}{\sum_{i \in V_1(r)} w_i} .$$

The term (T_6) is the beneficiaries' variance of walking distance:

$$T_6 = S_6 . \frac{\left(\sum_{i \in V_1(r)} \sum_{j \in W_i(r)} d_{ij}^{g} . w_i . y_{ij} - \frac{\sum_{i \in V_1(r)} \sum_{j \in W_i(r)} d_{ij}^{g} . w_i . y_{ij}}{\sum_{i \in V_1(r)} w_i} \right)^2}{\sum_{i \in V_1(r)} w_i} .$$

The term (T_7) is an equality and service function based on Minimum Standard Services (see Chapter 2), and this function is also multiplied by its correspondent objective scalar:

$$T_7 = S_7 . \frac{\sum_{i \in V_1(r)} \sum_{j \in W_i(r):\, d_{ij}^{g} \geq t} d_{ij}^{g} . w_i . y_{ij}}{\sum_{i \in V_1(r)} w_i} . 100 .$$

This particular term represents the percentage of walking inhabitants that travel distances superior to t. We can then consider that inhabitants travelling a maximum distance lower than t are covered with a high service levels. On the contrary, inhabitants walking distances larger than this threshold can be considered covered with low service distance levels. Hence, while minimizing the percentage of walking inhabitants above the distance threshold, we maximize the percentage of walking inhabitants above the distance threshold.

The previous objectives were selected because of the possibility to optimize them using CPLEX. However, CPLEX can encounter difficulties dealing with big sized problems, as in the case of the present study, such difficulties encountered during the project are described in Chapter 5.

3.3.2 Description of constraints

The model includes constraints limiting the maximum value that each objective can reach (C_1 to C_5). We can remove these limits by setting these constraints to a value sufficiently high. On the contrary, if we want to activate a constraint we need to set a limiting value in the right hand side of the constraint. We can for example limit the maximum cost for the stakeholders while minimizing an equality function. The potential combinations of using scalars and objective functions constraints are many, for the present study we decided to explore solutions limiting the stakeholder cost to four different percentages of costs increases above the minimum costs obtained when uniquely minimizing the stakeholder costs.

$$(C_1)\quad S_1 . \sum_{i \in V_1(r)} \sum_{j \in W_i(r)} \alpha_{ij} . y_{ij} \; + \; S_2 . \sum_{j \in V_2} \beta_j . z_{0j} + S_3 . \sum_{j \in V_2} \gamma_j . x_j \leq \text{Max stakeholder costs allowed}$$

$$(C_2)\quad S_4 . \frac{\sum_{i \in V_1(r)} \sum_{j \in W_i(r)} d^g_{ij} . w_i . y_{ij}}{\sum_{i \in V_1(r)} w_i} \leq \text{Max average walking distance allowed}$$

$$(C_3)\quad S_5 . \frac{\left| \sum_{i \in V_1(r)} \sum_{j \in W_i(r)} d^g_{ij} . w_i . y_{ij} - \frac{\sum_{i \in V_1(r)} \sum_{j \in W_i(r)} d^g_{ij} . w_i . y_{ij}}{\sum_{i \in V_1(r)} w_i} \right|}{\sum_{i \in V_1(r)} w_i}$$

$$\leq \text{Max mean absolute deviation of walking distance allowed}$$

$$(C_4)\quad S_6 . \frac{\left(\sum_{i \in V_1(r)} \sum_{j \in W_i(r)} d^g_{ij} . w_i . y_{ij} - \frac{\sum_{i \in V_1(r)} \sum_{j \in W_i(r)} d^g_{ij} . w_i . y_{ij}}{\sum_{i \in V_1(r)} w_i} \right)^2}{\sum_{i \in V_1(r)} w_i}$$

$$\leq \text{Max variance of walking distance allowed}$$

$$(C_5)\quad S_7 . \frac{\sum_{i \in V_1(r)} \sum_{j \in W_i(r): d^g_{ij} \geq t} d^g_{ij} . w_i . y_{ij}}{\sum_{i \in V_1(r)} w_i} .100$$

$$\leq \text{Max \% beneficiaries in low service distances allowed}$$

Constraint C_6 sets the percentage of beneficiaries with a high service distances to a minimum value:

$$(C_6)\ 1-\frac{\sum_{i\in V_1(r)}\sum_{j\in W_i(r)\, d_{ij}^g\geq t} d_{ij}^g . w_i . y_{ij}}{\sum_{i\in V_1(r)} w_i} .100 \geq \% \text{ beneficiaries with a high service levels .}$$

Constraints (C_7) are demand satisfaction constraints:

$$(C_7)\sum_{j\in W_i(r)} y_{ij}=1\ ;\ \forall\, i\in V_1(r)\ .$$

These constraints specify that the food requirements of every population point i must be satisfied. In the present study we use an Inhabitants Coverage Threshold (ICT) for the non-aggregated data and an Inhabitants Aggregation Threshold (IAT) for the aggregated data. These impose that a demand point must be served by a facility within the ICT for non-aggregated data and within the IAT for the aggregated data case. We therefore use tuples, representing "subset of valid combinations" as proposed by Dong & IBM (2009), which eliminate arcs that do not adhere to the ICT and IAT, for the non-aggregated and aggregated case, respectively.

Constraints (C_8) ensure that a population points i can only be assigned to an open distribution center j:

$$(C_8)\ y_{ij}\leq x_j\ ;\ \forall\, i\in V_1(r),\ \forall\, j\in W_i(r)\ .$$

Constraints (C_9) represent the flow conservation constraint:

$$(C_9)\ z_{0j}=\sum_{i\in V_1(r)} q_i y_{ij}\ ;\ \forall\, i\in V_2\ .$$

These constraints ensure that the food in tonnes, delivered from Warehouse 0 to distribution center j, must be equal to the food requirements in tonnes of all population points i assigned to the distribution center j.

Constraints (C_{10}) maintain the warehouse food availability constraint:

$$(C_{10})\sum_{j\in V_2} Z_{0j}\leq F_g\ .$$

This constraint ensures that the total amount of food delivered from warehouse 0 to all distribution centers must be less or equal to the total food assessment in tonnes for the Garissa District in the correspondent period (F_g). Considering the 2012 short rain assessment for Garissa District, the food requirements were 6,255.66 tonnes, see Table 1 and Rancourt et al. (2015). For the present case only one warehouse is being used, but the model can be extended to use multiples warehouses.

Constraints (C_{11}) are non-negativity constraints:

$$(C_{11})\ z_{0j} \geq 0 \ ;\ \forall j \in V_2 .$$

Constraints (C_{12}) and (C_{13}) are integrality constraints:

$$(C_{12})\ y_{ij} \in \{0,1\} \ ;\ \forall\, i \in V_1(r),\ \forall\, j \in W_i(r) \ ;$$

$$(C_{13})\ x_j \in \{0, 1\} \ ;\ \forall j \in V_2 .$$

3.3.3 Presentation of the model

We will now present the complete model. This model is partially based on the model of Rancourt et al. (2015). It allows for a variety of problem configurations considering the multiple combinations of parameters and constraints. Time and resource limitations impose us to explore some particular combinations and scenarios. However, the model can be used to perform additional optimization problems when new questions and future researches arise. In the following chapter, we will explore the data used in combination with the model in the next page.

Minimize

$$S_1 \,.\, \sum_{i \in V_1(r)} \sum_{j \in W_i(r)} \alpha_{ij} \,.\, y_{ij}$$

$$+ \; S_2 \,.\, \sum_{j \in V_2} \beta_j \,.\, z_{0j}$$

$$+ \; S_3 \,.\, \sum_{j \in V_2} \gamma_j \,.\, x_j$$

$$+ \; S_4 \,.\, \frac{\sum_{i \in V_1(r)} \sum_{j \in W_i(r)} d_{ij}^{g} \,.\, w_i \,.\, y_{ij}}{\sum_{i \in V_1(r)} w_i}$$

$$+ S_5 \,.\, \frac{\left| \sum_{i \in V_1(r)} \sum_{j \in W_i(r)} d_{ij}^{g} \,.\, w_i \,.\, y_{ij} - \frac{\sum_{i \in V_1(r)} \sum_{j \in W_i(r)} d_{ij}^{g} \,.\, w_i \,.\, y_{ij}}{\sum_{i \in V_1(r)} w_i} \right|}{\sum_{i \in V_1(r)} w_i}$$

$$+ S_6 . \frac{\left(\sum_{i \in V_1(r)} \sum_{j \in W_i(r)]} d_{ij}^{g} \,.\, w_i \,.\, y_{ij} - \frac{\sum_{i \in V_1(r)} \sum_{j \in W_i(r)} d_{ij}^{g} \,.\, w_i \,.\, y_{ij}}{\sum_{i \in V_1(r)} w_i} \right)^2}{\sum_{i \in V_1(r)} w_i}$$

$$+ S_7 . \frac{\sum_{i \in V_1(r)} \sum_{j \in W_i(r): d_{ij}^{g} \geq t} d_{ij}^{g} \,.\, w_i \,.\, y_{ij}}{\sum_{i \in V_1(r)} w_i} \;.100$$

Subject to:

(C_1) - (C_{13})

Chapter 4: Data description

In this chapter, the particularities of the data provided by Rancourt et al. (2015) are analysed. It consists of a set of large CSV files, the biggest one containing around five gigabytes of information and more than 68 million pairs of origin destination arcs or variables (i, j), each one representing an origin population point i and a corresponding distribution center destination j. The data for the Garissa District contains 47,242 population points i, from which 24,353 represent covered population points considering an Inhabitants Coverage Threshold (ICT) of three inhabitants. Those population point close to roads and with more than 20 people are potential distribution centers j, the number of potential DCs is 1,459 and there is also only one warehouse for the Garissa District. The present study uses the OPL programming language and CPLEX 12.6 as a solver.

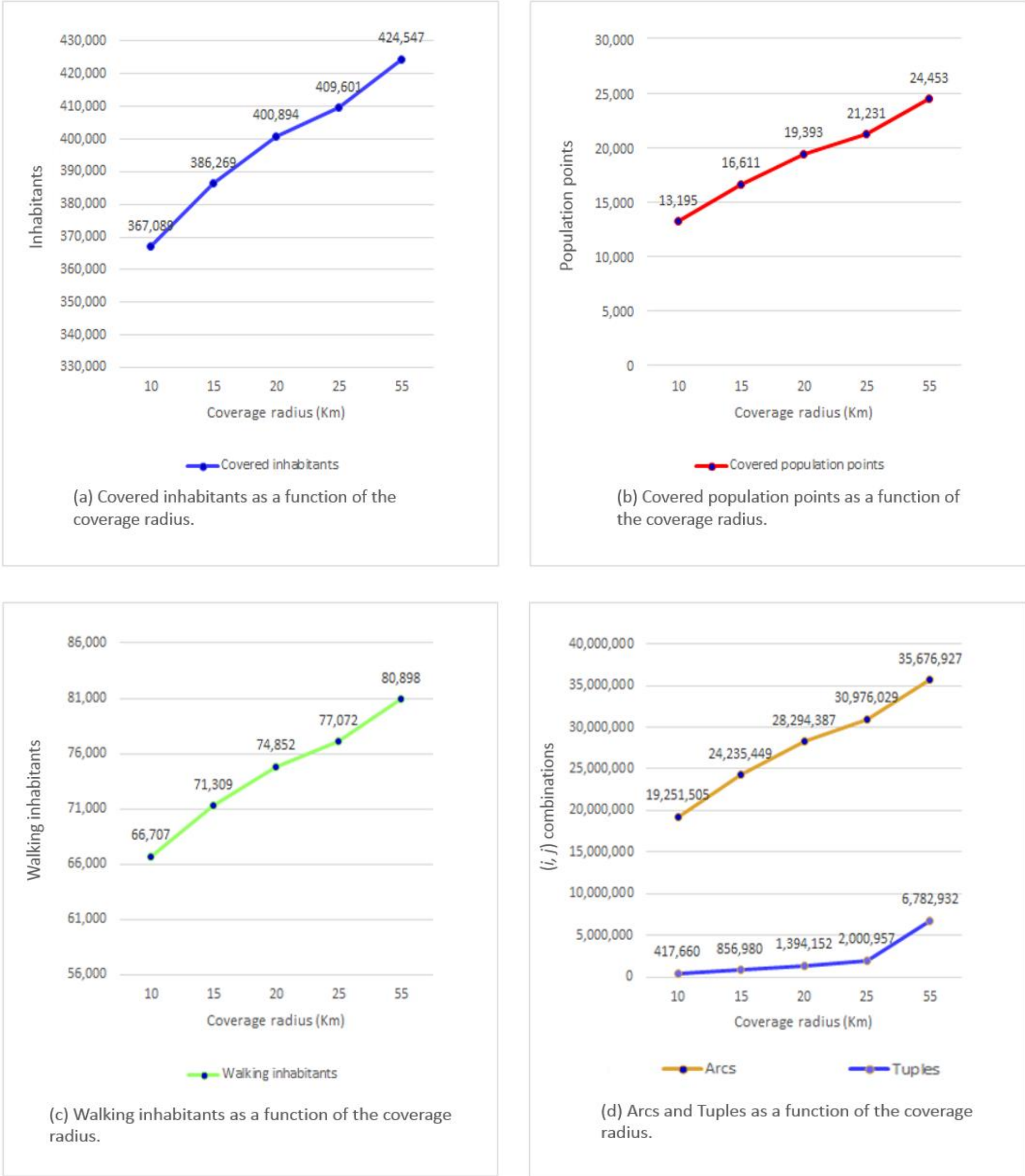
At the beginning of this project many tests were performed with generated data. Since then, the issue of efficiently using the processing time and memory arose. For some problems and for some functions it was possible to consider all variables in CPLEX and then let the solver select the valid combinations. Nevertheless, this approach can yield more memory and time requirements, and in some cases CPLEX was not be able to solve the problem. At this point, we realized that, in order to deal with large problems, we have three main strategies available. The first one consists in limiting the number of analysed scenarios. For example, studying some representative coverage radius for the problem, so we could make more efforts in analysing special variations and functions for the selected radius. The second strategy consists of aggregating the data; with this second strategy we could use more complex objective functions considering the reduced number of variables. It was also possible to solve some problems for which a solution was not available using non-aggregated data and to analyse more in depth variations of the selected scenarios. The main inconvenience of this strategy was that it could not represent the non-aggregated model and thus we lose accuracy with the new model. The third strategy consists of using tuples, and thus reducing the load of looking for solutions with non-valid data on CPLEX. According to IBM (2009), the tuples are important in obtaining more readable and efficient models. Watson & Cacioppi (2014) mention that the tuples are useful for creating sparse data structures and the multiple elements of the tuple provide flexibility. In Section 4.1 we describe the non-aggregated data. The aggregation process is presented in Section 4.2. The aggregated data and a small set of generated data that was used to test some functions and algorithms are presented in Sections 4.3 and 4.4, respectively.

4.1 Non-aggregated data (NAD)

The present study benefits of the data already gathered by Rancourt et al. (2015) and the results of their research. Based on the provided information, at the beginning of the project it was already known that the whole set of population points can be served within a coverage radius of 55 km. Based on this information, we use Excel spreadsheets to filter the available data.

The main data file originally contains 68 million combinations of distances and was partitioned in multiple small CSV files. We have retracted from these files the combinations corresponding to the 24,453 covered population points. It was thus possible to reduce the data into approximately 35.67 million combinations. We further reduced this number to 6.78 million combinations by selecting only the distances less or equal to 55 km. In Figure 12 (d), we can see the numbers of covered population points as a function of five representative coverage radii. The number of valid tuples can then be submitted to CPLEX. Using tuples can efficiently reduce the input load for the CPLEX solver, but it has the inconvenience that a specific set of tuples needs to be prepared for each specific coverage radius examined.

The data filtering process and tuples generation allow us to know important demographic information before the optimization process. In Figure 12, we can see the information for five coverage radii, 55 km represents the coverage radius allowing a complete coverage for the 24,453 population points.



(a) Covered inhabitants as a function of the coverage radius.

(b) Covered population points as a function of the coverage radius.

(c) Walking inhabitants as a function of the coverage radius.

(d) Arcs and Tuples as a function of the coverage radius.

Figure 12. Graphs showing characteristics of the non-aggregated data.

The original data contains the geographical coordinates for latitude and longitude. Combining this information with the information obtained through the filtering process, it was possible to generate maps for the different examined coverage radius. The maps were generated using complex Excel spreadsheets combining different functions and conditional formats for the colours. In Figure 13, some representative coverage radii are presented: 55 km, 35 km, 25 km, 20 km and 15 km coverage radius. The map for the coverage radius of 35 km, even if not analysed in the present study, was included for visual comparison reasons. The maps show for each case in different colours the covered and non-covered population points (the total number is 24,453), the potential distribution centers (the total number is 1,459), and the warehouse (only one).

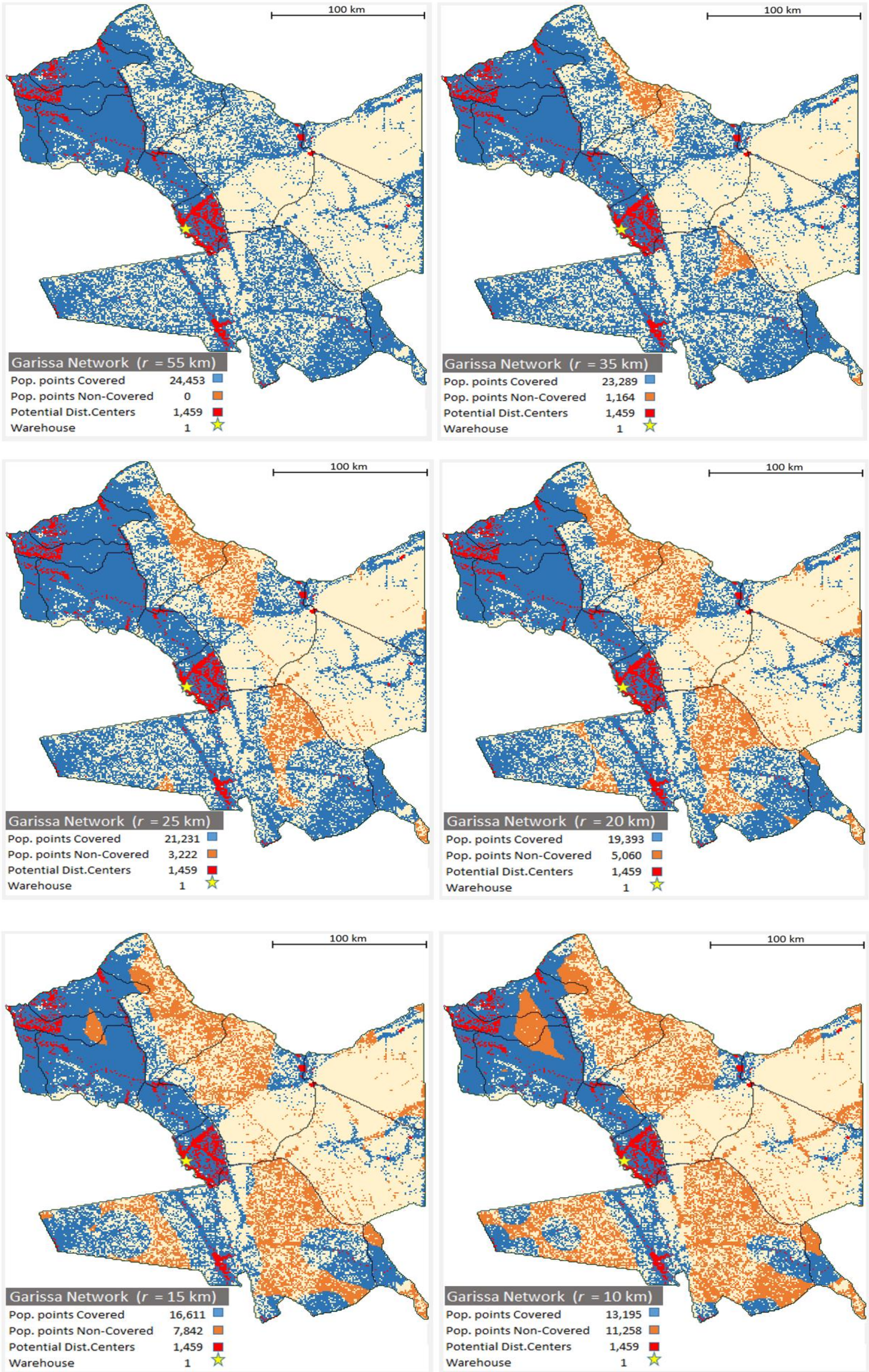


Figure 13. Maps of non-aggregated data for six coverage radii.

4.2 Data aggregation

The necessity to use aggregated data arouse based on the desire of exploring in further details some scenarios, where their solutions would be very difficult to obtain using non-aggregated data. It is also important to evaluate the differences between the results obtained using aggregated and non-aggregated data. We must mention that we only aggregated the population points, whereas the number of potential distribution centers was not reduced since their number was relatively low compared with the number of population points.

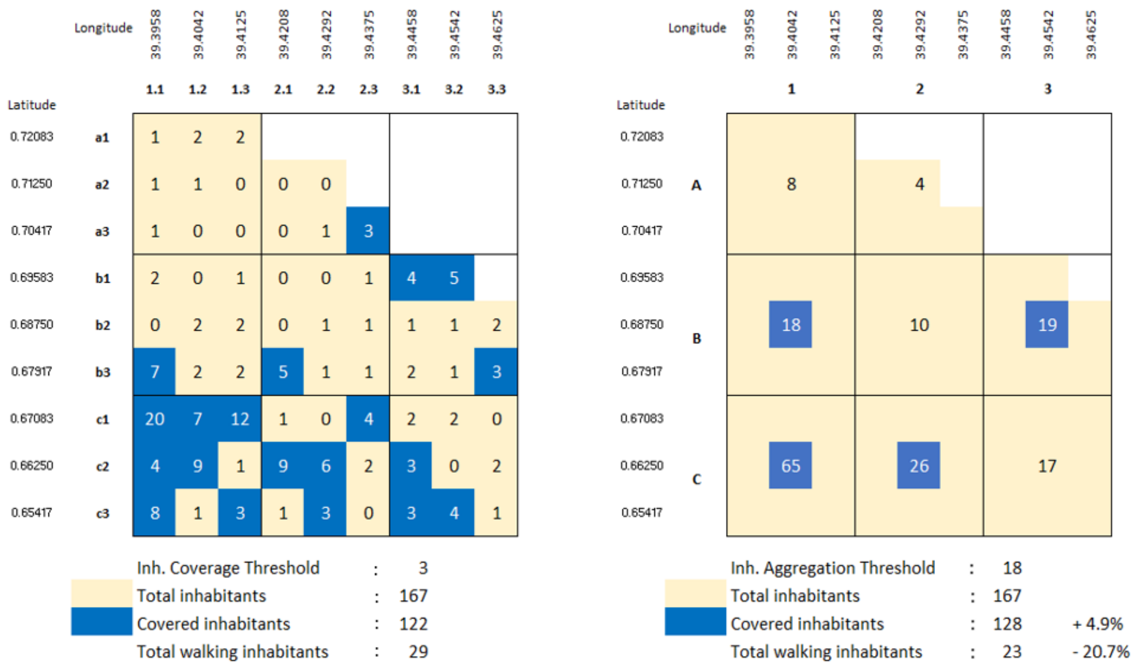
In order to aggregate the data, it was imperative to find a good aggregation strategy. The main CSV file contains 68.92 million lines records including the coordinates of the origin and destination for each arc. To aggregate the data it was necessary to see the spatial relationship between the population points. As an initial stage, the CSV file was transformed from a column based data set into a more compact matrix data set. This new file contains only 47,242 lines for the population points and 1,459 columns for the distribution centers with an additional column and row for the latitude and longitude coordinates. The file obtained has, however, approximately 600 megabytes of information, but could be contained in a single spreadsheet that can further be manipulated. We combined the features of the spreadsheet that generates the maps with the information contained within this new file and the smaller files with the project information. With all this combined information, we were able to graphically see the exact location of each population point, the number of inhabitants at each population point and some additional information, such as the food requirements and other statistics.

The generated map of coordinates has 301 rows and 284 columns representing the latitude and longitude coordinates. Within the big square, generated by the coordinates, we have 85,484 small squares, each one representing one km^2. Among these 85,484 squares of one km^2, we have 47,242 squares corresponding to population points with more than one inhabitant. In Figure 14 (a), we can see a representation of a small portion of the map with the coordinates. In this case we show the number of inhabitants conforming each population point. For example, the point (a.1; 1.1) located at the coordinates (0.72083; 39.3958) represents one km^2 and has one inhabitant. Therefore, not reaching the inhabitants' coverage threshold of three inhabitants, this threshold is required for population points to be considered covered in the solutions using the non-aggregated data. The point (c.1; 1.1) has 20 inhabitants, and it can be considered as a population point needing coverage and belonging to the 24,453 population points, which surpasses the threshold of 3 inhabitants. If this population point is close to a road, it can be considered a potential distribution center, as in the original study of Rancourt et al. (2015).

For the data aggregation, we decided to cluster nine squares of one km^2 into a cluster representing nine km^2; we have then reduced the big square contained by "381 rows × 284 columns" into a smaller one contained by "101 rows × 95 columns", generating a total of 9,595 squares, with each one representing an area of nine km^2. Among these 9,595 squares, we had 5,496 squares with more than one inhabitant. The inhabitants presented at each population point for the non-aggregated data were aggregated to the center of the 9 km^2 for the aggregated data. For example, in Figure 14 (a), the nine non-aggregated points starting from the point (a.1; 1.1) up to the point (a.3; 1.3), each one representing an area of one km^2, were aggregated into a bigger square of nine km^2 represented by the point A1 in Figure 14 (b). The center of this square is the point (a.2; 1.2) in Figure 14 (a), at this location we added the inhabitants corresponding to the nine population points. From this point the distances between the clustered population points and the 1,459 potential distribution centers are then computed.

We developed an Excel spreadsheet such that the data aggregation can automatically detect the nine squares of one km^2 and aggregate the data into the center of the clustered square of nine km^2. We also have to add to this, the complexity to automatically calculate the food requirements for the inhabitants in each population point. The Garissa District has 12 subdivisions and the inhabitants in each subdivision have a different food requirement. The data aggregation process took into account this fact, and even if two or more population points belonging to different subdivisions were aggregated into a single population cluster, the food requirement was calculated by considering where the inhabitants originally come from. It was also necessary to calculate the food requirement for each combination of aggregation threshold and coverage radius selected, since the total food assessment have to be distributed among the covered population points, as is the case in the original study of Rancourt et al. (2015), and these covered populations change according to the radius and threshold used. We, therefore, have at this point a complex spreadsheet that can aggregate the data according to the threshold and radius selected, calculate the covered populations points and inhabitants, calculate the number of walking inhabitants, calculate the adjusted food requirements for inhabitants and for population points, calculate the walking and distribution distances. With all these information, it was also possible to generate the tuples to be used for CPLEX. However, all this information calculated by the spreadsheets was time consuming since anytime we want to generate a new scenario, the spreadsheets would take a few minutes to perform all the calculations. It is of course possible to generate a single and faster spreadsheet for each aggregation scenario, it can reduce the size of the spreadsheet by approximately 40 to 50%, but considering the multiple radius and thresholds explored, we would have ended up with many smaller spreadsheets with a total size about 20 times bigger to store. The file we have used to aggregate the data and calculate the other parameters has approximately a size of 400 megabytes. Another consideration in order to use as less Excel files as possible is to reduce the number of modifications on the spreadsheets; if the information is contained in one spreadsheet, we only need to do modifications once and then by modifying parameters we can calculate the results for the different radius and thresholds. On the contrary, dealing with multiple spreadsheets can lead to the need of modifying all of them one by one and for us this is not an efficient process.

Returning to the distances used for the aggregated population points or clusters, we must clarify that it is also possible to consider a weighted distance between *i* and *j* for the walking inhabitants presented in this new cluster, but we already had spreadsheets dealing with millions of records and using complex functions that were already very big to be efficiently manipulated, and the weighting process could have added extra processing requirements possibly resulting in a failure of the spreadsheets. For the non-aggregated data the inhabitants' coverage threshold to serve a population is three inhabitants. For the aggregated data, we observe that with different aggregation thresholds, we have different service levels. In the example shown in Figure 14 (a), we see how some populations not originally covered using non-aggregated data become covered when using aggregated data in Figure 14 (b). There are also variation in the number of walking inhabitants when using non-aggregated and aggregated data. Note that the white data points represent areas outside the geographical area of the Garissa District.

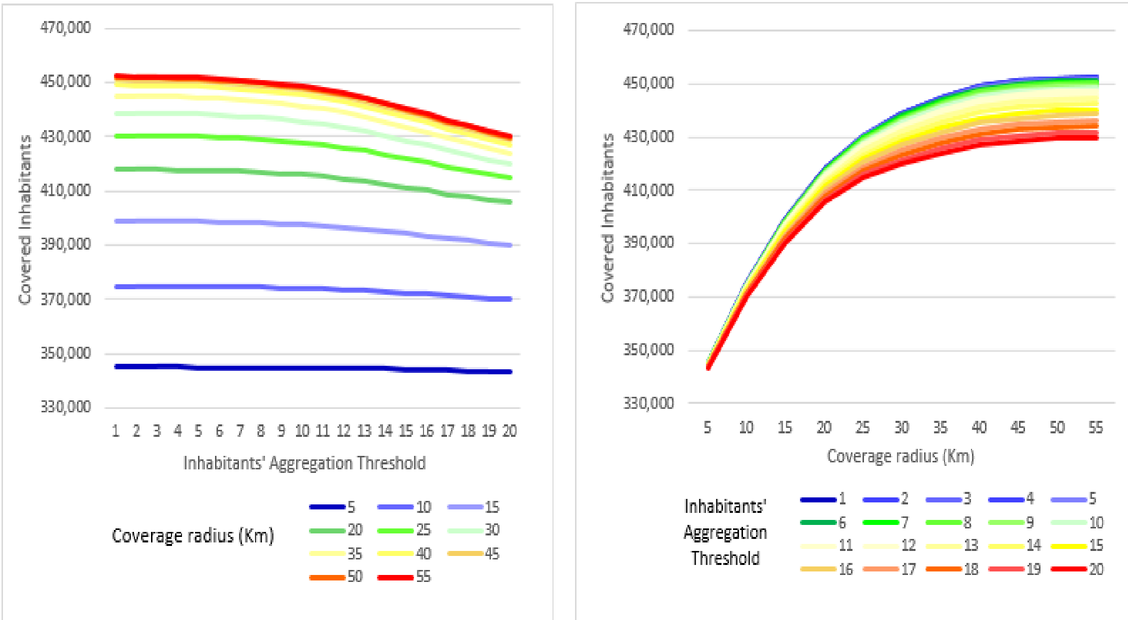


(a) Non-Aggregated data (b) Aggregated data

Figure 14. Data aggregation example.

4.2.1 Aggregated data (AD)

With the spreadsheets we have developed, it was also possible to perform data filtering and see the effects of different aggregation threshold levels and coverage radii on the covered inhabitants. We have explored, using this filtering process, 20 different inhabitants' aggregation threshold ranging from one to 20, and 11 different coverage radii, from 5 km up to 55 km coverage radius using an increment of 5 km. We have selected these thresholds and coverage radii because they can give us a clear picture of the service variations. We must again mention that we have chosen a maximum limit of 55 km coverage radius because we are benefiting from the original study of Rancourt et al. (2015). In Figure 15, we see the effect of different levels of aggregation thresholds and coverage radii on the number of covered inhabitants.



(a) Covered inhabitants as a function of aggregation threshold and coverage radius

(b) Covered inhabitants as a function of coverage radius and aggregation threshold

Figure 15. Different levels of covered inhabitants for the aggregated data.

Figure 15 (a) shows that by increasing the inhabitants' aggregation threshold, the number of inhabitants covered decreases. This effect is mainly present for the higher coverage radii, e.g., 45 to 55 (red lines) and it is less evident for lower radii, e.g., 5 to 15 (blue lines). Figure 15 (b) shows the same results, but as a function of the coverage radius in this case. We note that for high coverage radii, e.g., 30 to 55 km, the covering curve is almost flat. The patterns are very similar for low aggregation thresholds (blue lines) and for high aggregation thresholds (red lines), but with higher levels of coverage for the lower aggregation thresholds. In Figure 15 (b), we also observe how from 25 km to 5 km coverage radii the curve steeply descends. The same patterns can be observed in Figure 15 for the inhabitants and in Figure 16 for the clustered population points.

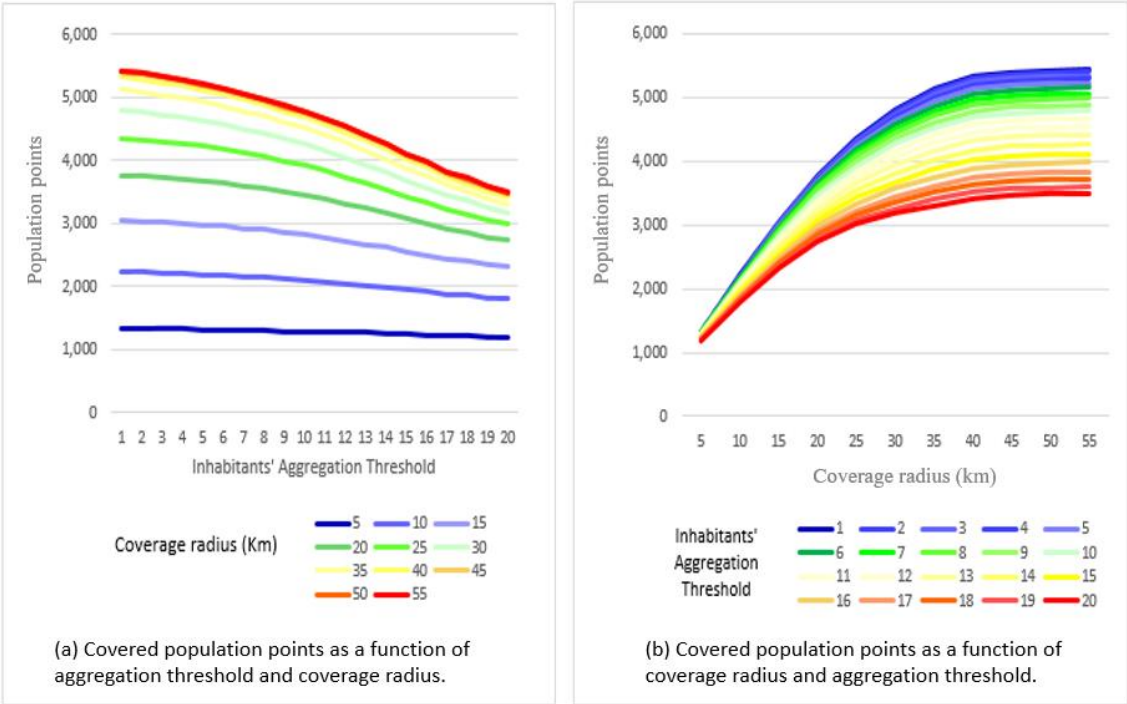
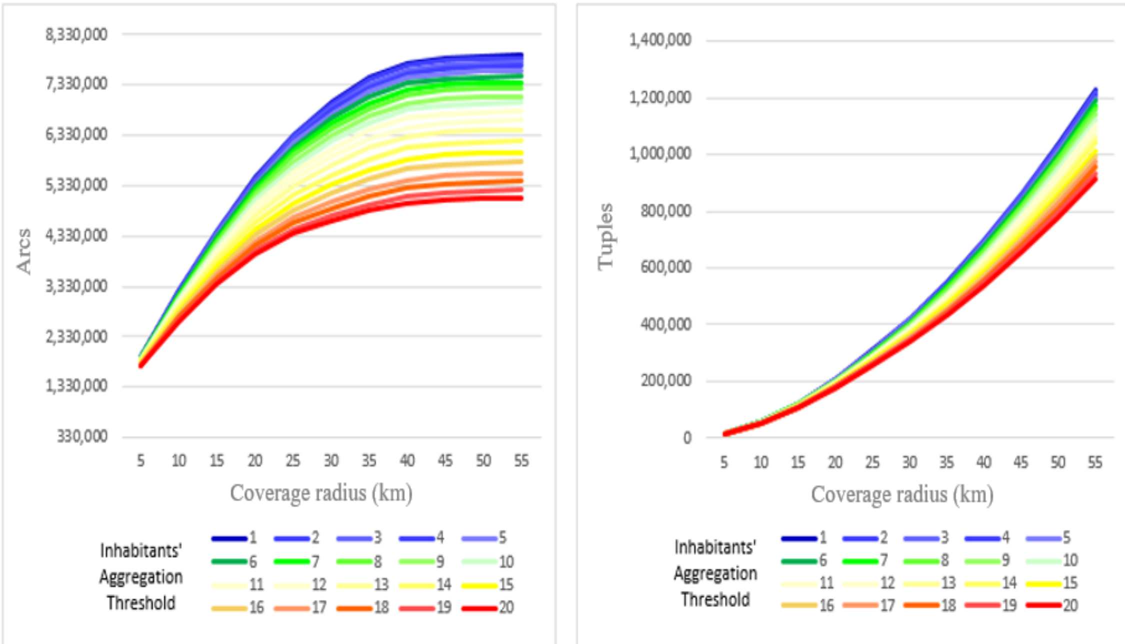


Figure 16. Different levels of covered population points for the aggregated data.

Considering time and resources limitations, we cannot explore all the possible scenarios shown in these figures. We then have to choose some representative scenarios. Based on the results in Figures 15 and 16 and based on the information provided by Rancourt et al. (2015), we have identified five areas of special interest for the coverage radius. They correspond to coverage radii of 10 km, 15 km, 20 km, 25 km and 55 km coverage radius, which is the service radius that ensures complete coverage. Indeed, these coverage radiuses can represent the ascending curves shown in Figures 15 (b) and 16 (b). We did not explore coverage radii lower than 10 km since they can lead to poor service levels, considering the non-covered inhabitants living far away from potential distribution centers; this could be in opposition with the equality and service considerations taken into account in the present study. Regarding the aggregation threshold to consider a population point as one that should be served within a particular coverage radius, we have selected 10, 12, 14, 16 and 18 inhabitants as thresholds. We consider this spectrum to be appropriate, since thresholds smaller than 10 will lead to serve very small populations, resulting in an increase in the size of the problem for CPLEX, thus losing the advantages of aggregation. Another consideration is that it is necessary to explore the results that could be obtained with different levels of covered population points. In real networks, as it is the case of the present study, there are other considerations independent of the network design that could affect the number of covered population points or inhabitants, e.g., politics, war, civil unrest, etc. The approach of exploring different levels of covered population points can be useful in anticipating non-strict network considerations. The selection will also be very useful since reducing the number of scenarios allows us to explore more deeply different variants for each scenario. This situation will be impossible if we analyse every single possible scenario. As we can see with Figures 15 and 16, there are flat areas in the curves that represent scenarios with little change and they will probably not contribute to a deeper understanding of our problem.

In Figure 17, we compare the number of arcs and valid tuples that need to be generated for CPLEX in order to solve the problem. It was possible to filter the valid tuples using the spreadsheet previously described. For example, in Figure 17 (a), we see that to find a solution to cover all the population points presented in a radius of 55 km and with one inhabitant aggregation threshold, CPLEX needs to analyse approximately 8 million arcs for the aggregated data. On the other hand, using only the valid arcs, CPLEX needs to analyse approximately 1.2 million of valid tuples. The tuples generated in the filtering process need to be further tailored for the special requirements of the OPL programming language in order to be used in CPLEX. We did this task only for the previously mentioned combination of coverage radius and threshold.



(a) Arcs as a function of the coverage radius and Inhabitants' Aggregation Threshold.

(b) Tuples as a function of the coverage radius and Inhabitants' Aggregation Threshold.

Figure 17. Size of the problem: arcs and tuples.

4.2.2 Aggregated data for 18 inhabitants' aggregation threshold

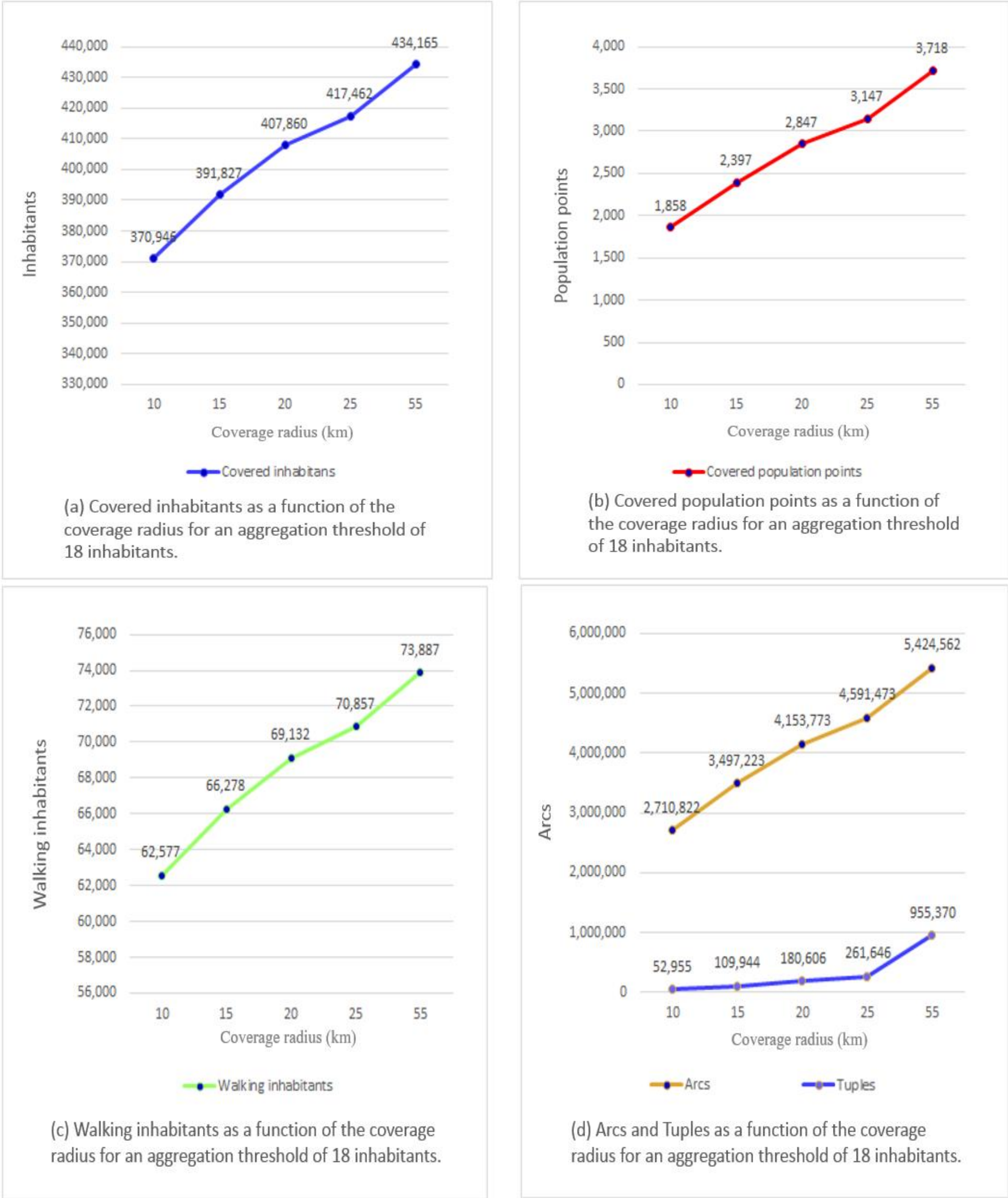
The present section describes the aggregated data for the coverage radii of interest (10, 15, 20, 25 and 55 km) and for an inhabitants' aggregation threshold of 18. The information contained in the graphs below is also presented in Figures 15 to 17, but we decided to present the particular case of an inhabitants' aggregation threshold of 18 in order to make some comparisons with the non-aggregated data, see Figure 12. The threshold of 18 inhabitants has been chosen because this threshold presents similarities with the non-aggregated-data. For example, considering the inhabitants' coverage threshold of three inhabitants for the non-aggregated data and a coverage radius of 55 km, we have 424,547 covered inhabitants out of a population of 452,418 inhabitants, representing the 93.88% of the total population, as shown in Figure 12 (a). For the aggregated data considering an inhabitants' aggregation threshold of 18 inhabitants and coverage radius 55 km, we have 434,165 inhabitants covered, representing the 96.81% of the total population, as shown in Figure 18 (a). If we compare these results with the 10 inhabitants' aggregation threshold and the same coverage radius of 55 km, we have 448,538 covered inhabitants or 99.19% of the total population. Detailed information about coverage levels can be found in Appendices 2, 3 and 4.

At this point, we observed that the non-aggregated data and aggregated data may present completely different coverage levels and hence they are not directly comparable. However, the aggregation threshold of 18 inhabitants has the closest similarities with the non-aggregated data, and will be used to compare how the different objective functions and indicators behave. We also need to take into account that the objective of the present study is not necessarily to obtain similar results with non-aggregated and aggregated data. The objective is to observe and understand the differences, examine more scenarios and mainly evaluate the objective functions and the behaviour of the main indicators for large data sets. The understanding gained in this process can be of general applicability for similar situations and can have more value than a particular result of an optimization problem.

In Figure 18 (b), we can observe the population points covered with an aggregation threshold of 18 inhabitants and for different coverage radii. The maximum number of population points for the aggregated data is 5,426 considering all the nine km^2 squares with at least one inhabitant; the 3,718 population points for the 55 km coverage radius represent 68.52% of all the clustered population points, but they represent the 96.01% of the total population. In contrast the 24,453 population points with at least three inhabitants in the case of the non-aggregated data represent the 51.76% out of a total of 47,242 population points with more than one inhabitant. It also represent 93.88% of the total population. With this information we can conclude that the percentage of covered inhabitants is a better indicator than the number of covered population points when analysing the performance of the food distribution network. We can also see how the data in the case of the non-aggregated is sparser.

Considering the non-covered population, for the non-aggregated data, 22,798 population points represent 48.24% of the population points and only 6.12% of the total population. Whereas, for the case of the aggregated, 1,708 clustered population points represent 31.48% of the total clustered population points and only 3.99% of the total population. These are indicators that, optimization problems for the same radius using aggregated and non-aggregated data, are not entirely comparable since they represent different covering scenarios. Either way, we can observe the differences in the results obtained and infer from them general conclusions.

With the comparisons previously mentioned, we can also elucidate how it is better to leave some small population points out of the optimization process, since their inclusion does not have a big impact on the coverage levels, and on the contrary could double the size of the optimization problem. The same argument is also valid for smaller coverage radius (e.g., 10 to 20 km radii), since these account for the majority of the population. For small theoretical problems with small data sets, these coverage considerations could possibly not represent an obstacle, but for a real network design problem dealing with millions of variables, these small changes can represent the difference between a network that can be optimized, and one for which it is not possible to find an optimal solution. This is particularly true in the case of the problems using the non-aggregated data, and even to some extend for the problems using aggregated data. In Figure 18 (c), we see the number of walking inhabitants and in Figure 18 (d), we see the effects on the size of the problem using the whole set of distances as opposed to only the valid ones for some particular scenarios.



(a) Covered inhabitants as a function of the coverage radius for an aggregation threshold of 18 inhabitants.

(b) Covered population points as a function of the coverage radius for an aggregation threshold of 18 inhabitants.

(c) Walking inhabitants as a function of the coverage radius for an aggregation threshold of 18 inhabitants.

(d) Arcs and Tuples as a function of the coverage radius for an aggregation threshold of 18 inhabitants.

Figure 18. Aggregated data characteristics for a threshold of 18 inhabitants.

This study follows the same methodology used by Rancourt et al. (2015) to calculate the number of walking inhabitants; because the number of inhabitants for each population point needs necessarily to be an integer number, the number of inhabitants in each population point is divided by six and then rounded to the nearest integer. An example of this process can be observed in Figure 14. Because the beneficiaries' cost is calculated as a function of the number of walking inhabitants, which represent the mothers of the beneficiary household, these differences will have an impact on the costs observed using non-aggregated and aggregated data for the same coverage radius. For example, in the case of the 55 km coverage radius (Figure 18 (c)), we have 73,887 walking inhabitants. This number is relatively lower than the corresponding figure for the non-aggregated data, with a total number of 80,898 walking inhabitants (Figure 12 (c)), even if the total number of covered inhabitants is higher for the aggregated data than for the non-aggregated data (434,165 vs. 424,547 inhabitants). This fact can be explained by the effect of the data sparsity reduction observed in the case of aggregated data, and mainly on how the number of walking inhabitants is calculated. Figure 18 (d) shows how the size of the problem can also be reduced for the aggregated data when using the tuples in the optimization process. For example, in this case we have 5.4 million arcs for a radius of 55 km and approximately only one million valid arcs for the same radius.

In Figure 19, we see the representation of the network obtained using an aggregation threshold of 18 inhabitants for two coverage radii: 55 km and 15 km. We can see the blue dots as covered population points and the orange dots as not covered population points. As explained above, even if those orange dots occupy extended areas of the map, especially for the lower coverage radii, they represent only a small proportion of the population of the Garissa District.

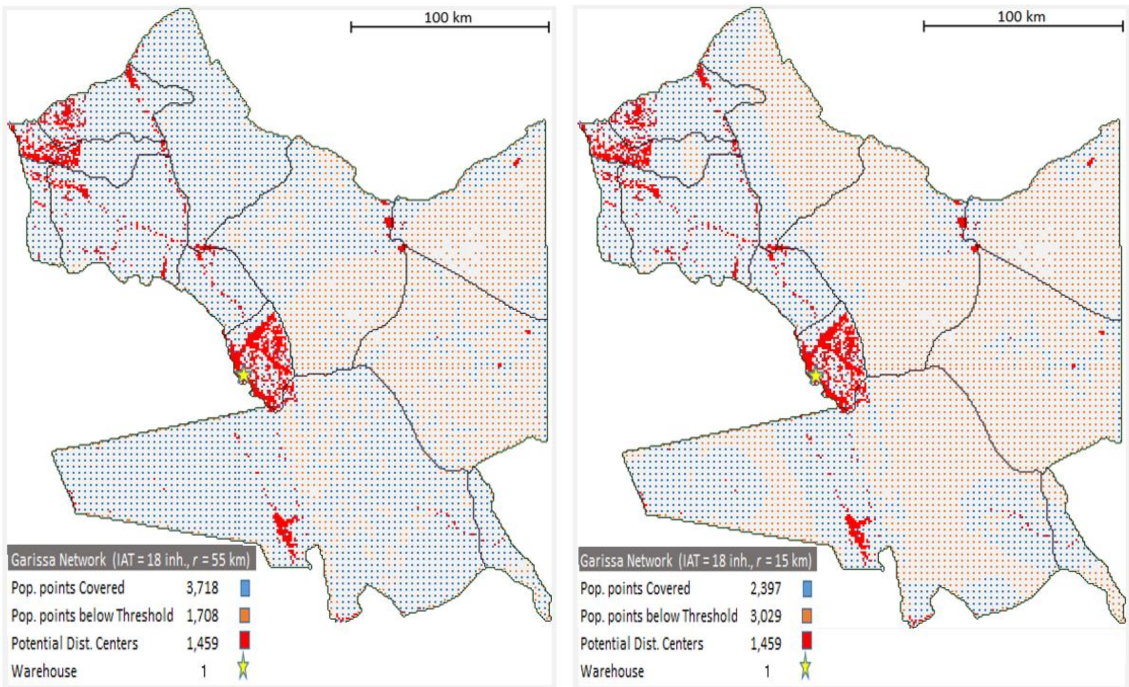


Figure 19. Maps of aggregated data, IAT = 18 inh. for r = 55 km and r = 15 km.

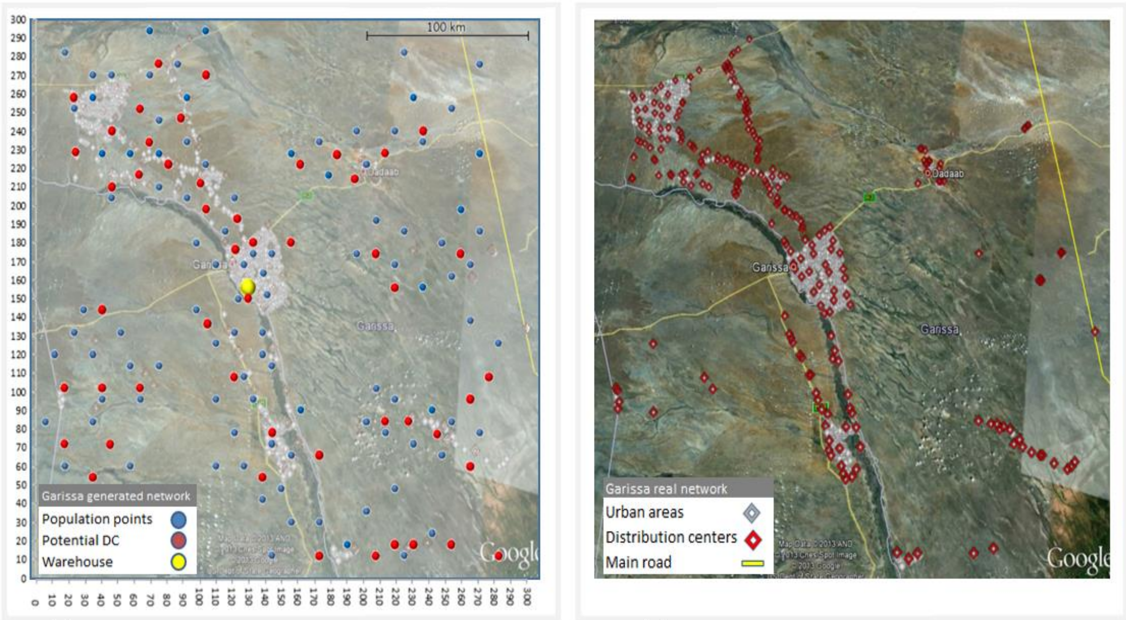
4.3 Small model with generated data

We now present an example with a small data set with 50 potential distribution centers and 100 population points. The purpose of this small model was to test some functions, and to have a better picture of possible difficulties that we can encounter in the later project stages.

4.3.1 Data generation

A representation of the network optimized by Rancourt et al. (2015), see Figure 20 (b) was used to guide the generation of the data. Based on this figure we located the distribution centers, population points and the warehouse in a square representing 9,000 km^2 (300 km x 300 km), see Figure 20 (a). This area is approximately the same as the area where the real map of the Garissa District and has 303 latitude points and 285 longitude points with a total area of 86,355 km^2. The number of population points for this network is the same as in the real problem for the non-aggregated data, which is 452,218 inhabitants. Considering the small number of total variables and arcs that we have for this small model, the optimizations were performed for a maximum coverage radius of 55 km, covering in this case 100% of the population. Regarding the number of walking inhabitants, for the generated data, the same calculation method for the non-aggregated and aggregated data was used, and in this case the similar effects related to the reduction in data sparsity were observed, with a reduction in the total number of walking inhabitants to 75,406 compared with 80,898 walking inhabitants for the coverage radius of 55 km and non-aggregated data.

Considering differences with the real problem using non-aggregated data, it is important to mention that in the real problem we have a high percentage of inhabitants living at the same point or km^2 where many potential distribution centers are located, this fact accounts for a reduction in the required total walking distances. In the case of the generated network no single population point was located in the same spatial point of a potential distribution center, this can account for an increase in costs and walking distances as well as variations in other indicators. The distribution of the population and some parameters have; however, been fine-tuned with multiple tests to reflect approximately the same general results that can be obtained with the real network. In Figure 20 (a) we see the network with the generated data and in Figure 20 (b) a solution for a 55 km coverage radius from Rancourt et al. (2015).



(a) Generated network for the Garissa District.

(b) Real network for the Garissa District. Source: Rancourt et al. (2015).

Figure 20. Generated vs. real network for the Garissa District.

Figure 20 (a) uses the background of the geographical area shown in Figure 20 (b) only for referential purposes. The purpose of developing the generated data set, was not to represent the real network with 100% accuracy, but to obtain a small version useful to understand the real problem and as an initial data set to test functions and for the model development.

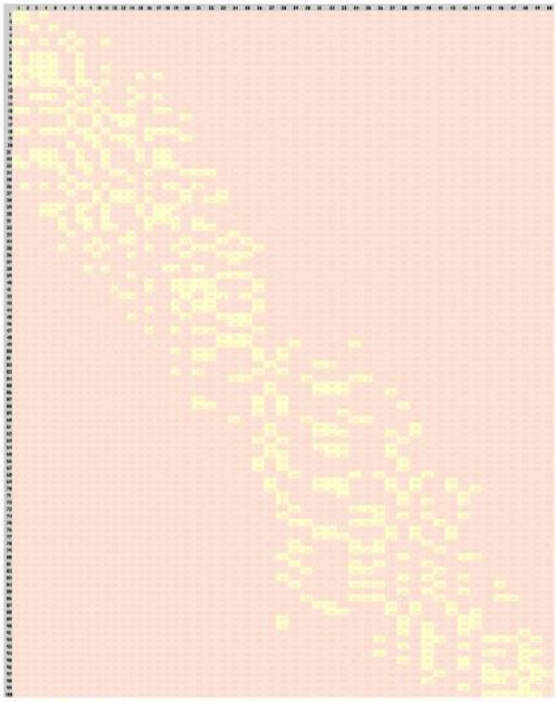
To calculate the distances between the population points and the distribution center nodes, the Euclidean norm was used. In Figure 21 (a), we can see a small section of the calculation table containing the distances d_{ij}^{e} between the population points *i* and distribution center *j*. The population points *i* were numbered from 1 to 100 and the distribution centers *j* were numbered 1 to 50. Figure 21 (b) shows the complete calculation table. The yellow squares represent the distances below the maximum coverage radius necessary to cover all the population points and the pink squares represent the distances above this coverage radius. In Figure 21 (b), it is also possible to see a diagonal pattern showing the walking distances lower than 55 km and we can clearly see how the yellow squares are comparatively less in number compared with the total number of squares. In total, we have 5,000 arcs (100 x 50) from which only 596 are yellow squares, representing in this case the valid arcs or tuples. For small instances, as it is the case of the generated data set, we can submit to CPLEX the whole set of arcs or only the valid tuples. However, there will be some differences regarding the solution time and computational effort. In a small data set, these differences can be irrelevant, but for complicated and large problems they need to be seriously taken into account.

	j 1	j 2	j 3	j4	j 5	j 6	j 7	j 8	j 9	j 10
i 1	19	43.3	60	42.4	59.1	72.2	182	60	85.3	164
i 2	35	24	91.4	59.4	80.7	55.3	148	70	68.4	132
i 3	60.3	90.8	24.7	56.6	51.6	110	232	72.2	121	211
i 4	156	127	211	171	191	121	43.7	169	121	55.3
i 5	12	19	68.4	33.9	55.3	46.9	160	45.7	60	140
i 6	204	174	259	217	237	166	50.9	214	163	76.8
i 7	42.4	72	17	35	32.3	89.2	212	50.9	99.3	191
i 8	30.6	60	26.8	25.5	30	78	200	43.3	88.6	179
i 9	8.49	36	49.5	19	38.4	56.6	177	36	68.4	156
i 10	45.7	73	12	30.6	21.6	85.9	211	43.3	94.9	188
i 11	25.5	17	72	30.6	51.3	30	151	33.9	42.4	130
i 12	163	133	216	174	193	121	19	170	118	35
i 13	59.1	85.9	6	42	26.8	96.7	222	51.3	105	199
i 14	188	157	240	198	216	144	21.6	193	140	48.4
i 15	30	38.4	55.3	13.4	30.6	42.4	168	13.4	51.3	145
i 16	131	101	181	139	156	84	42	132	78.9	21.6
i 17	154	124	205	162	180	108	18	156	103	13.4
i 18	45.7	37.9	75.9	35	48.4	24.7	150	24	30.6	126
i 19	110	80.5	158	115	132	60.3	66.3	108	54.3	42.4
i 20	173	143	223	181	198	126	6	174	120	24.7

distance lower than radius coverage of 55 km

distance higher than radius coverage of 55 km

(a) Section of the distance calculation table.



distance lower than radius coverage of 55 km

distance higher than radius coverage of 55 km

(b) Picture of the distance calculation table.

Figure 21. Distance calculation table for the model with generated data.

For the non-aggregated and aggregated data, similar patterns to those in Figure 21 have been observed. However, in those cases, we have distance matrices comprised of thousands of rows and columns and millions of arcs. A reduced picture of those tables will lose all the details and is not suitable for didactic purposes. Figure 21 and the graphical representations of the variables and tuples presented in the previous sections (Figures 12, 17 and 18) for both, non-aggregated and aggregated data, support the use of tuples for the present study. This suggestion can be extended for problems dealing with big data sets and complex functions. For small problems it could possibly generate an unnecessary extra burden in data filtering, but the model generated this way can be easily extended for bigger data sets.

Chapter 5: Results and analysis

In this chapter, we present the optimization results for the non-aggregated data, aggregated data and the small model with generated data. The statistics and results come from two sources: first the CPLEX results and second, Excel spreadsheets able to reconstruct all the solutions and providing additional information obtained from CPLEX results.ne

Considering CPLEX results, it is necessary to mention that we have developed a flexible and integrated model that is able to compute the values of different objectives. The objectives and their corresponding constraints can be activated by modifying some parameters and scalars. The advantage of a model generated in this way, is that if we minimize the average walking distance, for example, we can also obtain the results on costs, service levels and some other important indicators. However, while testing the small model with generated data, we have observed that for any additional information obtained from CPLEX, extra effort and time were incurred. We observed that the objective function requiring more time and effort was the one with the variance and we were able to obtain results for this function only for the small model with the generated data. We also observed that including this function for larger problems adds an important extra effort for CPLEX. For these reasons, we decided to remove this function from the algorithm used with the aggregate and non-aggregated data. We must mention that even if we have invested much effort in developing a flexible model, there are possibly better and more efficient ways of obtaining the same solutions. We have developed our model using the OPL programming language, which is a very useful tool to quickly develop models for optimization problems.

Regarding the objective functions used for the larger sized problems, such as the stakeholder average costs, the mean absolute deviation and the standard service, we must add that each extra objective, requires extra computational effort from CPLEX. If we run each function separately, we can obtain results in an efficient manner, but we will lose the extra valuable information about the interaction between functions obtained with an integrated model. Considering this for the present study, we have used tuples that effectively reduced the size of the problems to implement a flexible and integrated model. Another reason to maintain a unified model was that the tuple generation for each scenario generated big amount of data and information, which was difficult to manage and organize. Managing different models for each scenario could have created an extra burden, where the time and resources were already limited.

The second source for our results comes from Excel spreadsheets. At the time of processing the results, we had previously developed complex models able to generate tuples for the different scenarios. By modifying these Excel sheets, we created spreadsheets able to reconstruct any solution with a 100% accuracy and obtain additional information not directly provided by CPLEX. We were able to calculate and evaluate the behaviour of indexes such as the Gini Index and the Robin Hood Index. The details of these results can be found in Appendices 2, 3 and 4.

The Gini Index is a complex index, whose calculation requires much more technical effort than the calculation of the variance for example. The Gini index and some other non-linear functions are often impossible to optimize in CPLEX. To optimize such indexes, using genetic algorithms and heuristics might be more efficient, but implementing such approaches was out of the scope of the present project. These indexes are nonetheless considered important references for equality measures, and are also taken into account for welfare considerations. It is possible to calculate these indexes using Excel spreadsheets. Nonetheless, such calculation can be challenging considering that we need to have an ordered distribution using the measured attribute (e.g., income or distance). This ranking process requires the use of complex functions and considering that we are dealing with millions of records and thousands of rows of information, we needed to push Excel and the computer capabilities to their limits. Another limitation for the kind of calculations we executed, is the Excel maximum number of rows, which is limited to 1,048,576. This sometimes required to split files, which necessitate extra time and effort to integrate the results.

The Excel calculations were conducted on a Dell Studio PC with an Intel Core 2 Quad processor running at 2.7 GHz and 16 GB of RAM under a windows environment. The optimization experiments were conducted on an Intel(R) Xeon(R) CPU X5675 with 12-Core 3.07 GHz and 96 GB of RAM (by using a single thread) under a Linux environment.

The results presented below compare the different objectives evaluated based on different scenarios. We will give a particular importance to the cost based evaluation of the results. There are two reasons for this kind of evaluation: first, to compare the performance of using different objective functions with the results obtained with the stakeholder costs minimization objectives; second, the cost factor is extremely important and the resources are limited in humanitarian networks, they depend in many cases on international cooperation and donations. Saving resources in those cases can be an important factor in determining the sustainability of the network.

The results will not only present solutions using different coverage radii, but also different levels of covered inhabitants depending on the data set used. We will also compare different solutions.

Because of space limitations, we will present some representative scenarios; detailed information about additional solutions not presented in this chapter is available in Appendices 2, 3 and 4. The Excel spreadsheets that reconstruct the solutions also gave a deep insights regarding the distributions obtained using the different objective functions; for these reasons we will present a detailed analysis about a representative solution for each evaluated objective function. This analysis will be very useful to understand how different costs, service and equality objectives affect the solutions. This level of details is not possible to obtain when only doing comparisons between different scenarios and we believe that based on the nature of our study, dealing with service and equality concerns, a deep understanding of the situation is of upmost importance. We will thus present such additional detailed analysis mainly for the non-aggregated data, for two reasons: space limitations and higher accuracy in the case of using the original information.

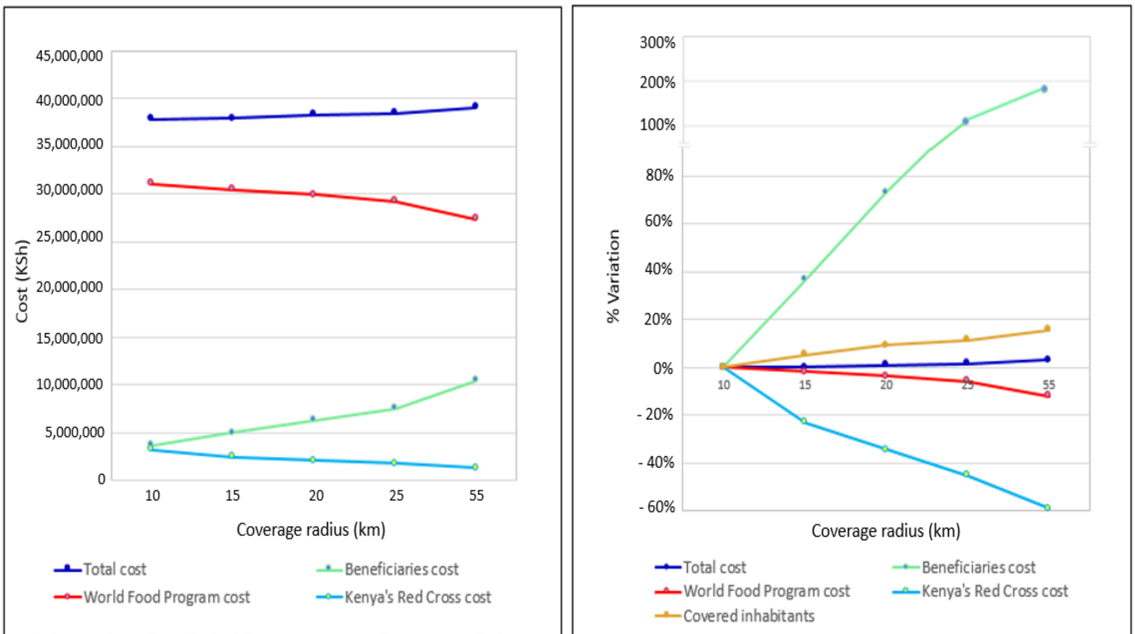
5.1 Results obtained with non-aggregated data

The main equality and service functions evaluated, are beneficiaries' average walking distance (T_4), beneficiaries' mean absolute deviation of walking distance (T_5) and beneficiaries in low service distances allowed (T_7). For T_7, most of the cases were evaluated with the mid- radius for the coverage distance. The stakeholder costs, beneficiaries' access cost (T_1), WFP's cost (T_2) and KRC's cost (T_3), were mainly evaluated for comparative reasons, since a more detailed study including a wide range of coverage radius and scenarios was performed in the original study of Rancourt et al. (2015). The cost objectives evaluated in their study are a natural and important reference when evaluating additional objective functions considering the social aspect. The previous study and its conclusions constitute the basis for our study and gave us guidance for the present project. It was important for us to evaluate how the different equality and service functions behave compared with cost minimization objectives. Understanding these differences will be very useful to plan distribution networks taking into account some additional social concerns and their repercussions on the cost factor. Humanitarian distribution networks depending on donations must give special attention to this factor since limited resources must reach as many people as possible.

It was possible to solve all the scenarios to optimality. The scenarios were solved between 74.99 seconds and 141.17 hours of computational time, see Appendix 2.1 for specific details. We believe that the use of tuples greatly facilitated the solution process, even though the addition of all the objectives in a unique integrated model generated extra load for CPLEX, especially for the largest coverage radius. We first present the results for the stakeholder costs minimization because this objective function will be the basis of comparison for the following objective functions.

5.1.1 Results obtained when minimizing the stakeholder costs

Cost is a fundamental factor in distribution network. This factor, is extremely important in the case of humanitarian networks. The stakeholder costs must be regarded with special attention considering that the success and sustainability of the network lay in this factor. Rancourt et al. (2015), included beneficiaries' access cost (T_1); this element was previously misrepresented among humanitarian network designers. It is natural to think that beneficiaries receiving food aid at high costs, in terms of opportunity cost, would probably opt for not receiving it. This assumption particularly suits situations when the food distribution network is scattered upon a large geographical area, where some beneficiaries need to walk long distances in order to reach the food supplies. Figure 22 (a) presents the results for the sum of T_1, T_2 and T_3 for five different coverage radiuses. We must remind the reader that more detailed information can be found in Rancourt et al. (2015). Figure 22 (b) presents a percentage based comparison of the evolution of the costs for the coverage radius analysed, when compared to a coverage radius of 10 km. For the stakeholder cost minimization objectives, the running time for CPLEX were 154.93, 229.57, 447.63, 746.09 and 4,285.41 seconds for the coverage radii 10, 15, 20, 25 and 55 km, respectively. Aditional information can be found in Appendix 2.1.



(a) Total and stakeholder costs as a function of the coverage radius obtained when minimizing the total cost

(b) Variation in total, stakeholder costs and covered inhabitants as a function of the coverage radius when minimizing the total cost

Figure 22. Costs, percentage of variation in costs and covered inhabitants for different coverage radii when minimizing the stakeholder costs using NAD.

In Figure 22 (a), we see how the WFP and KRC’s costs decrease as we increase the coverage radius; on the contrary, the total cost increases due to a bigger increase in the beneficiaries’ acces costs. In Figure 22 (b), we see how for higher coverage radius the total cost has almost a flat curve, and the cost of the WFP has a small decreasing tendency for the higher coverage radius. We can see nevertheless, a big increase in the beneficiaries’ acces cost, up by approximately 200% for a coverage radius of 55 km; this fact is explained by long walking distances incurred by the inhabitants living far-away from distribution centers mainly located in urban areas and close to roads in more densely populated areas. On the contrary, the percentual weight of the KRC’s cost dimishes by aproximately 60% for the same coverage radius; this reduction is explained by the fact that, for longer coverage radius, inhabitants are dispersed and need to be served by only a few available distribution centers. Whereas for small coverage radius, the lower cost solutions are found when inhabitants are very close to distributions centers, forcing many more distribution center to be open in those cases.

In Figure 22 (b), we can also observe that the increase in beneficiaries’ acces costs (200%) observed for a coverage radius of 55 km, is much higher than the increase in inhabitants coverage (20%). In this case it will be preferable to choose a small coverage radius since the trade-off cost-coverage for the beneficiaries is too high. Similar conclussions were found by Rancourt et al. (2015), where they concluded that for costs minimization objectives, the best solutions are in the range of 10 and 17 km, and that beneficiaires living far-away can walk and register in the food distribution program. Considering these facts we choose a coverage radius *r* of 15 km to analyze more in details the solutions obtained with the costs objectives and with the equality and service objectives.

5.1.2 Results obtained when minimizing the stakeholder costs for *r* = 15 km

In this section we describe a particular solution obtained when minimizing the sum of the costs objectives (T_1, T_2 and T_3) for a coverage radius of 15 km. For this particular case, as well as for the subsequent cases, we will first present the map of the solution, which gives us a clear picture of the obtained network. We then present a table describing the characteristics of the solution. We then present a table showing service levels based on three distance thresholds, and finally we present a figure showing some graphics for the obtained distribution.

The data and information for all cases comes from the results obtained directly with the CPLEX outputs, all cases were solved to optimality. We will analyse the different elements and relevant information in each case. There are some elements in the tables and graphics below that will become clearer when compared with the same information presented for the additional objectives in their corresponding sections. For this first case, in order to explain properly the different elements, we will give some extra information and perform a more detailed analysis than for the other objectives. Once the main elements will be clear, the comprehension of similar analysis will be simplified.

Figure 23, shows the map of the obtained network; the blue points represent population points covered within a radius of 15 km, whereas the orange points represent non-covered population points within this particular radius. The red points are the potential distribution centers, which are mainly concentrated in densely populated areas. From these red points, the open distribution centers are represented as triangular green shapes. The main warehouse is represented as a yellow star. The blue, red and orange points together represent the 24,453 population points with more than three inhabitants that are considered for the solutions with the non-aggregated data. We also observe in the map light yellow areas, which represent non-populated areas or population points with one or two inhabitants. These are 22,789 points, which together with the 24,453 population represent the 47,242 km^2 of the Garissa District. The black lines represent the subdivisions of the Garissa District. The map, including the white non-conveying information areas, represents 86,355 km^2 (303 latitude points x 285 longitude points).

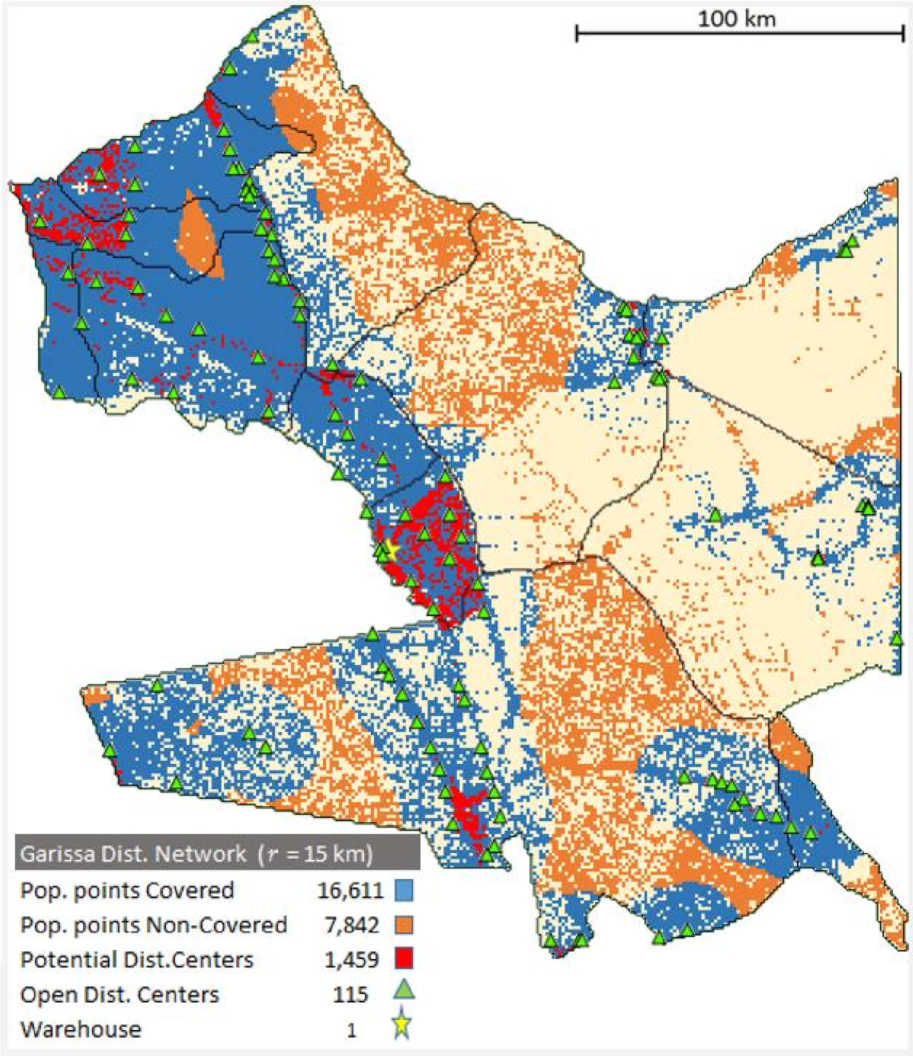


Figure 23. Map for the solution obtained when minimizing the stakeholder cost using NAD for r =15 km.

In the map we can observe how the blue areas are comparatively smaller than the orange and light yellow areas. However, the blue areas contain the majority of the population (86.82%), as shown in Table 3. There are 16,611 blue points representing 63.97% of the total population (24,453 points) considered for the non-aggregated data. The blue points also represent 35.16% of the total number of population points, i.e., 47,242 points. The previous analysis confirms the high concentration of population points and inhabitants around urban areas and main roads. This also confirms that humanitarian networks dealing with a highly scattered networks in remote areas face difficulties to reach all the potential beneficiaries.

Table 3. Characteristics of the solution obtained when minimizing the stakeholder costs using NAD for r = 15 km.

Demographic indicators		%	Walking inhabitants' service indicators	
Garissa inhabitants	452,218	100.00%	Time measure:	
Covered inhabitants	386,269	85.42%	Average walking time (hrs)	1.82
Walking inhabitants (covered inh.)	71,309	18.46%		
Pop. points (including 1 -2 Inhabitants)	47,242		Distance measures:	
Population points (> 3 Inhabitants)	24,453	100.00%	Average walking distance (km)	3.63
Covered population points	16,611	67.93%	Standard deviation of distance (km)	4.34
Potential distribution centers	1,459	100.00%	Variance of distance (km)	18.83
Open distribution centers	115	7.88%	Mean absolute deviation of distance (km)	1.87
Warehouse	1		Median walking distance (km)	1.31
Economic indicators		%	Pearson's 2° skewness coefficient	1.61
Total cost (KSh)	37,867,430	100.00%	Distribution left tail	0.16
Beneficiaries acces cost (KSh)	4,956,553	13.09%	Distribution right tail	0.84
World Food Program cost (kSh)	30,468,967	80.46%	Gini Index	0.611
Kenya's Red Cross cost (kSh)	2,441,910	6.45%	Hoover Index (Robin Hood)	0.492

In Table 3, in addition to the demographic indicators previously mentioned we can also notice the economic indicators, showing the total and stakeholder costs in Kenyan Shillings (KSh). We can see that the WFP's cost accounts for 80.46% of the total cost while the beneficiaries' access cost accounts for 13.09% of the total costs. We can also notice some important service indicators such as the average walking time that is in this case 1.82 hours, or 3.63 km, the standard deviation and the variance with values of 4.34 km and 18.83 km, respectively. The skew based on the Pearson's 2º coefficient (SP_2) is 1.61, see Figure 24 (e), this is based on the relation between the mean (u), the median (m) and the standard deviation (s) and in this case accounts for a bigger weight on the right hand side of the distribution. We can confirm this value with other indicators, such as the distribution left tail that accounts for 16% of the walking distances and in the distribution right tail that accounts for 84% of the walking distances. If we use the skew as a measure of service, we can consider the people located at the right hand side of the average walking distance, and especially those in the extreme of this tail, as receiving low levels of service and travelling very long distances, whereas on the other side of the distribution, the interpretation is inverse. It is possible to find additional information and indicators in Appendix 2.1.

In Table 4, we provide the service level measures for three different distance thresholds (DTs) for the present solution. This table provides a more detailed view on how the population points, inhabitants, walking inhabitants and total walking distances are concentrated in relation with these service thresholds. This table also reinforces the interpretation of the skew measure that we have previously presented. We can see for example that below the average walking distance, we have 70.54% of the inhabitants representing 16.85% of the population points and 65.32% of the walking inhabitants. These inhabitants walk distances lower than the average walking distance to reach their food supplies in their assigned distribution centers. The difference between the 70.54% and 65.32% is due to the calculation process for the walking inhabitants, explained in the data aggregation process. The threshold of 10 km was chosen as an additional standard to compare solutions obtained with different objective functions. We also use the mid-radius of the coverage distance as a threshold, i.e., 7.5 km. We note that the standard service objective (T_7) uses this particular threshold and its inclusion will facilitate comparisons.

Table 4. Service levels based on three distance thresholds for the solution obtained when minimizing the stakeholder costs using NAD for r =15 km.

Service level measure \ Distance Threshold (DT)		Average walking distance 3.63 km	10 km distance threshold 10.00 km	Mid-radius 7.5 km
Number of population points	16,611	100.00%	100.00%	100.00%
% below DT		16.85%	63.43%	47.92%
% above DT		83.15%	36.57%	52.08%
Number of inhabitants	386,269	100.00%	100.00%	100.00%
% below DT		70.54%	88.81%	83.76%
% above DT		29.46%	11.19%	16.24%
Number of walking inhabitants	71,309	100.00%	100.00%	100.00%
% below DT		65.32%	86.50%	80.39%
% above DT		34.68%	13.50%	19.61%
Total walking distances (km)	259,110	100.00%	100.00%	100.00%
% below DT		16.13%	53.20%	38.38%
% above DT		83.87%	46.80%	61.62%

In addition to Tables 3 and 4, we see the correspondin graphics in Figure 24. These graphics give us a clear picture about the distribution of the solution. The graphic construction is based on ranked distributions, from the population with the smallest walking distance up to the population with the highest walking distance. In Figure 24 (a), the blue line represents walking distances lower than the average walking distance and the red line represents walking distances higher than the average walking distance. We also indicate the population point whose inhabitants travel a distance aproximately equal to the average walking distance (AWD). In Figure 24 (b) we see a bar chart with green and orange vertical lines representing the number of inhabitants walking a distance lower or higher than the average walking distance. This graphic representation can be corroborated with the information presented in Table 4. However, Figure 24 (b) allows us to see how the inhabitants of the population points with more inhabitants walk very short distances while a large number of populations points with few inhabitants walk very long distances. Moreover, we see that the orange vertical lines represent the majority of inhabitants and the shape of this distribution is also reflecting a skewed distribution with the long tail pointing to the right.

In Figure 24 (c), we see cumulative percentage values for the walking inhabitants (WI) and the total walking distances (TWD) travelled by the inhabitants. The TWD are calculated by multiplying the walking distances and the number of walking inhabitants for each population point, for example, we see approximately 65% of the WI account for approximately 16% of the TWD. The corresponding values are also observed in Table 4 for the average walking distance threshold. As in Figure 24 (a), the blue and red lines represent values for the number of walking inhabitants, but in this case for cumulative percentages, and "p" points at the division between cumulative values that are below or above the AWD when compared with the distance travelled by these inhabitants. In Figure 24 (c), we have a green and an orange line, the colors are similar to those in Figure 24 (b), but in this case they represent the total walking distances, where "d" points at the division between the TWD that are below or above the AWD.

In Figure 24 (d), we present the Gini Index. The Gini Index is represented by the yellow area; encircling this area we have a blue line that is traditionally called the equality line, the orange line representing the Lorenz curve and the green line representing the Robin Hood or Hoover Index. The Robin Hood Index signals the area of the distribution where the differences in the studied attribute (distance) are the highest compared with the rest of the distribution. This graphic is constructed using two axis, the horizontal axis showing the cumulative values for the walking inhabitants. The vertical axis represents the cumulative value of the attribute analysed. In this case we are interested in distance measures. However, in economics, the atribute traditionally used, is income. Point "e" signals the projection of the Robin Hood Index on the equality line and point "f" in Figure 24 (d) contains the same information as points "p" and "d" in Figure 24 (c), and it represents on the vertical axis the cumulative percentage of distance and on the horizontal axis the cumulative percentage of inhabitants.

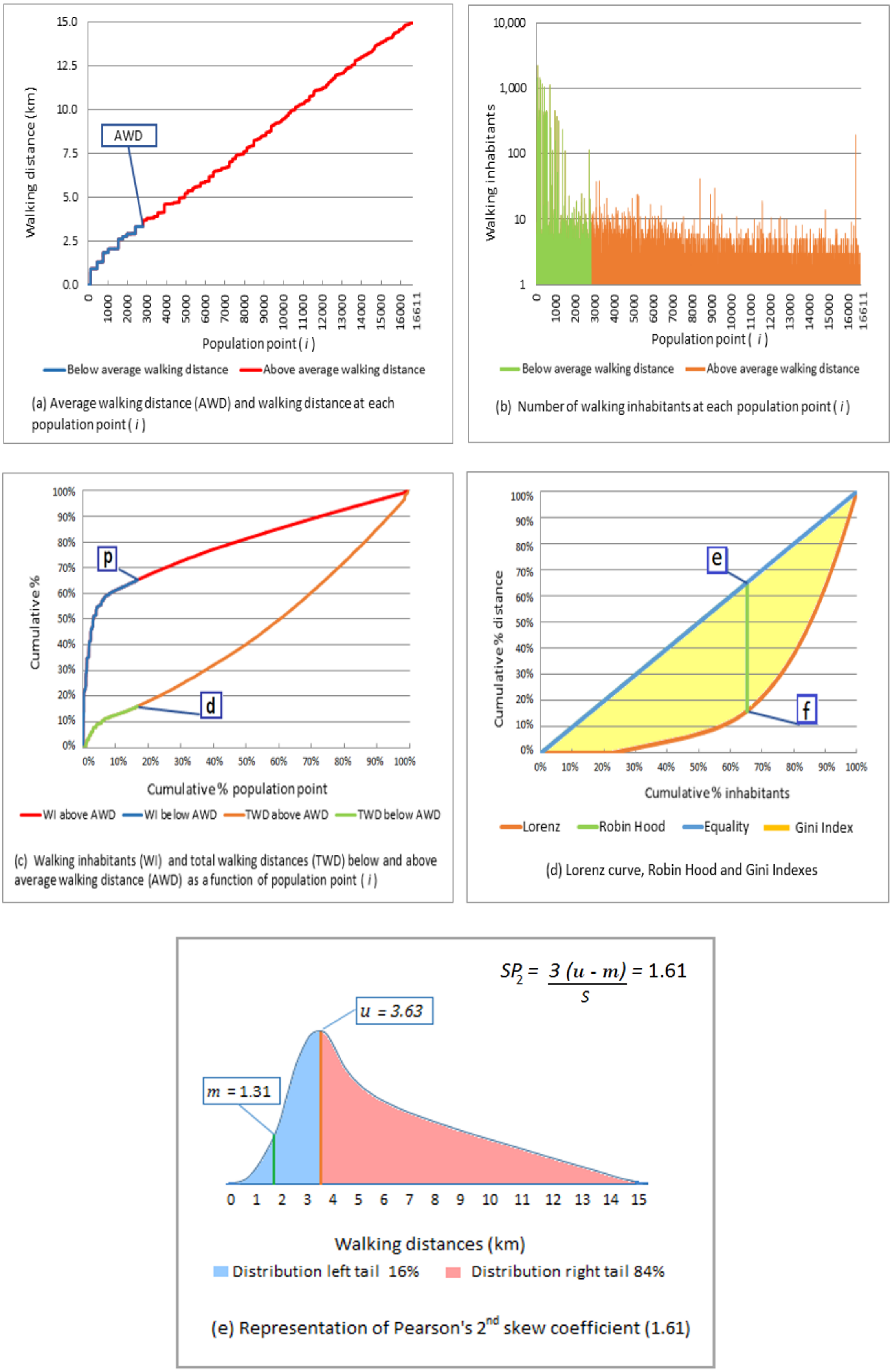


Figure 24. Characteristics of the distribution obtained when minimizing the stakeholder costs using NAD for r = 15 km.

First we have to mention that the Gini Index and its graphic representation based on the Lorenz curve, are based on two dimensions of the distribution; they are, the cumulative values for the number of inhabitants and the cumulative values for the atribute (distance) analysed. This graphic is intended to show specific equality measures. The graphic we have developed in Figure 24 (c) was mainly intended to see the relationship between the AWD and three dimmensions of the distribution, which are the cumulative values for the walking inhabitants, the cumulative values for the atribute (distance) and the cumulative values for the population points, each population point being conformed by the inhabitants living in a 1 km^2 area.

After we analyzed many solutions comparing graphics for different distributions, with the help of the different colors that show the different measures, a charateristic pattern was observed, which motivates us to include in the graphics the “p” and “d” in Figures 24 (c) and the “e” and “f” in Figure 24 (d). In these figures, the distance between point “p” and point “d” and the distance between point “e” and point “f” are exactly the same, and they represent the Robin Hood Index that in economics represents the maximum deviation from the line of perfect equality. In both Figures 24 (c) and 24 (d), we use ordered distributions and show cumulative values for the number of walking inhabitants and the total walking distances. In Figure 24 (d), for the case of the Gini and Lorenz curve representation, the cumulative values for the walking inhabitants are projected from the horizontal axis, and the cumulative values for the distances are projected from the vertical axis. In Figure 24 (c), both measures are projected from the vertical axis, since in our case we add a third dimension for the horizontal axis represeting the cumulative values for the population points. This additional level of information reinforces and clarifies the information we previously presented in Tables 3 and 4, and in Figures 24 (a) and 24 (b). Figure 24 (c) also allows us to see how a low number of population points contains the majority of inhabitants but inversely a lower cumulative value for the total walking distances.

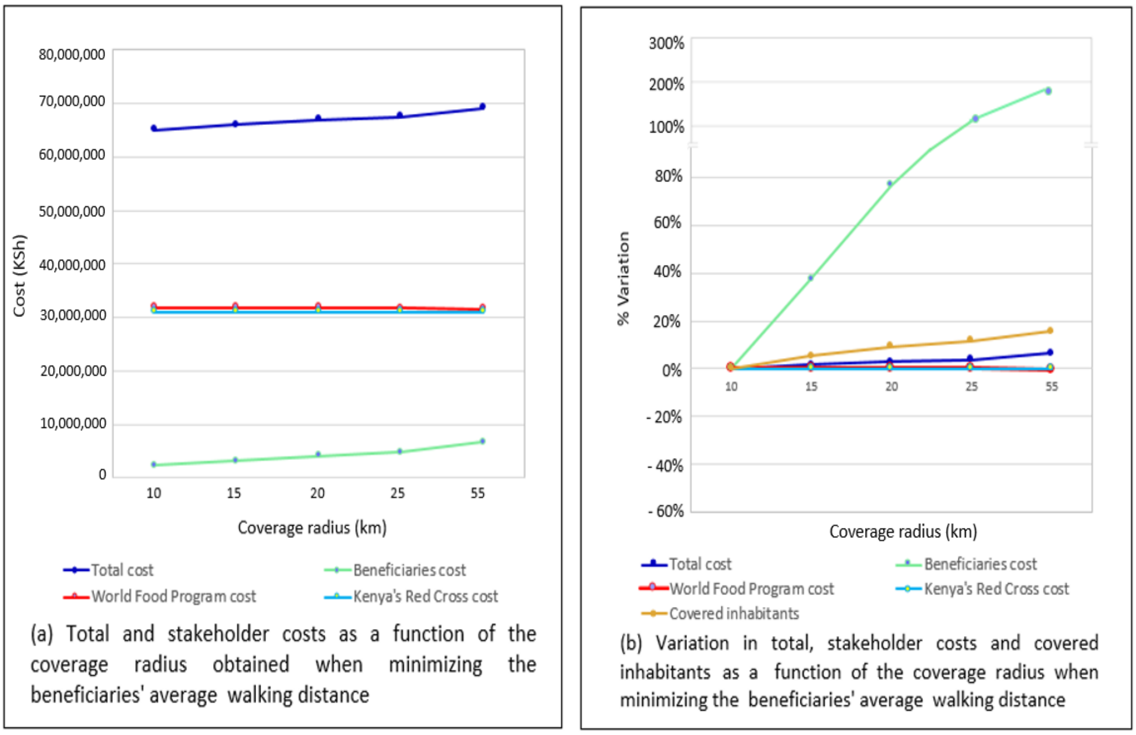
Following the same logic of the equality diagonal line (blue line) presented in Figure 24 (d), for the case of Figure 24 (c), the cumulative values for an equilibrate distribution must follow a diagonal pattern, this fact implies that thse cumulative values for the total walking distances and the total walking inhabitants have the same proportion. Henceforth, representing a line of perfect equality. With the information from our Excel spreadsheets, we also noticed that the area between the upper and lower curves in Figure 24 (c) represents the same area as the Gini Index in Figure 24 (d). We have therefore, with Figure 24 (c), an alternative way of representing equality measures such as the Gini and the Robin Hood Index with the advantage of the extralayer of information it offers, such as the relationship of the average (mean) with the three components of the studied distribution: inhabitants, attribute (distance) and population points. However, we present the tradittional Gini and Robin Hood Indexes since they are widely used, and also because they are useful for comparative reasons.

It is also important to mention that Figure 24 (c) show the points “p” and “d”, which show the WI and TWD below and above the AWD. These points also show the two extremes of the Robin Hood index and highlight the segment of the distribution where the inequality is higher, thus showing the strong relationship between the average (mean) and equality measures such as the Gini Index. Based on the previous relationship, we can infer that the use of the average as a meassure of equality and service is highly justified and its evaluation must be considered in any meassure of equality. We must add however, that additional analysis would be important in order to fully understand the relationship between the mean, the Robin Hood index and the equality measures based on ranked distributions and also to confirm these observations with completely different distributions sets.

For the particular case of the solution obtained shown in Figure 24 (c), when minimizing the stakeholder costs for a coverage radius of 15 km, we see how the cumulative distances below the average walking distance have a lower weight and that the cumulative values for the walking inhabitants have a high weight in the left of the distribution. An inverse pattern is observed for the right of the distribution, accounting in this case for high levels of inequality. The separation of both from the hypotetical diagonal equality line reflects the high levels of the Gini and Robin Hood Indexes as also shown in Figure 24 (d). This fact can be explained because we have travel distances with a large range of variability; we have an standard deviation of 4.34 km and a variance of 18.83 km (Table 3).

5.1.3 Results obtained when minimizing the beneficiaries’ average walking distance

In this section, we minimize the AWD (T_4) which can be considered as an important service and equality measure. For this objective function, we have obtained AWD of 1.09, 1.82, 2.56, 3.13 and 4.57 km. The solution times in CPLEX were 128.66, 297.43, 918.02, 3921.49 and 6,233.35 seconds for the coverage radii of 10, 15, 20, 25 and 55 km, respectively. Additional information can be found in Appendix 2.1. These different values have economic implications for which we will perform our analysis. In Figure 25 (a), we present the costs observed when minimizing the average walking distance and in Figure 25 (b) a percent variation analysis is presented.



(a) Total and stakeholder costs as a function of the coverage radius obtained when minimizing the beneficiaries' average walking distance

(b) Variation in total, stakeholder costs and covered inhabitants as a function of the coverage radius when minimizing the beneficiaries' average walking distance

Figure 25. Costs, percentage of variation in costs and covered inhabitants for different coverage radius when minimizing the beneficiaries’ average walking distance using NAD.

In Figure 25 (a), we first observe the high costs related to minimizing the average walking distance when compared with the solutions obtained when minimizing the stakeholder costs (T_1, T_2 and T_3), see Figure 22. The main component of this additional cost is related to the increase in the KRC’s cost, on average 1,345% higher considering the five presented cases. This is explained by the use of practically all the distribution centers for all the coverage radii examined. The WFP’s cost follows a flat curve and not a descending curve, as observed for the solutions using the cost objective fucntion, on average this cost only increases of 7% for the five cases presented. The costs curve for the beneficiaries is however, slighlty lower compared with the solutions when minimizing the cost objective function presented in Figure 22, with an average reduction for the five cases presented of 36%. Specific details about these costs can be found in Appendix 2.1. In Figure 25 (b), we observe that KRC’s cost and the WFP’s cost reach their maximum with smaller coverage radii. We see however, a big increase for the beneficiaries’ costs for higher coverage radii, up to almost 200% in the case of a coverage radius of 55 km; this high increase in cost is much higher compared to the additional increase in covered inhabitants. A similar situation was observed with the cost minimization objective. In the following section, we describe a particular solution for this objective funcion that will help us better understand some implications and relations with other indicators. The coverage radius chosen is again 15 km; using a standard coverage radius will allow a clear comparison between solutions.

5.1.4 Results obtained when minimizing the beneficiaries' average walking distance for r = 15 km

In Figure 26, we observe the map for the present solution. The blue and orange points are the same as in the previous solution. The main difference that we can observe concerns the distribution centers. In this case, we do not have red points, because in this solution all the potential distribution centers are open, the open DCs are represented by green points in the map, not as triangular green shapes as is the case of the preceding map. In this case, the large number of DCs open are clearly visible and they do not need to be graphically highlighted to be seen properly.

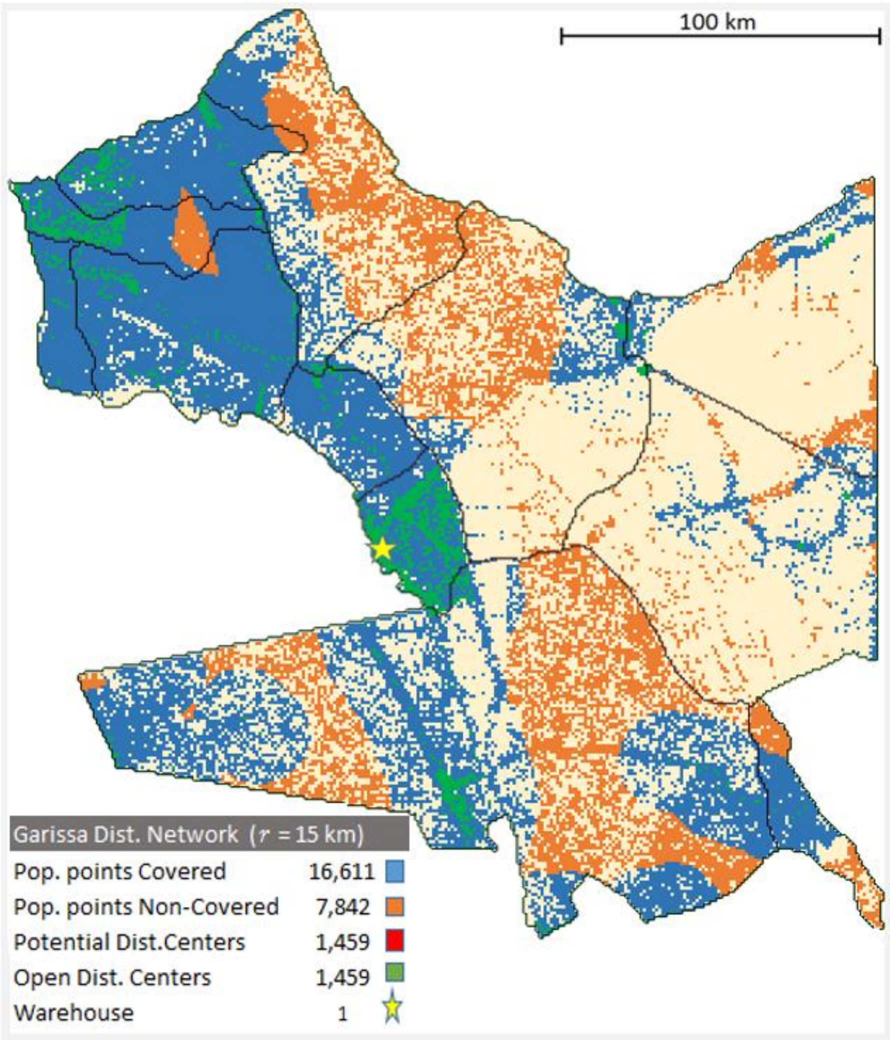


Figure 26. Map for the solution obtained when minimizing the beneficiaries' average walking distance using NAD for r =15 km.

Table 5 presents important indicators for this solution. We can see that all 1,459 potential distribution centers are open, and the high cost associated with the minimization of the average walking distance. The cost for both, WFP and KRC are clearly very high; nonetheless, a very low beneficiaries' access costs. We also observe the short average walking time that in this case is less than an hour and only 1.82 km. The standard deviation, variance and skew are also slightly lower than in the case of the stakeholder costs minimization (Table 3). A fact that calls out attention is the high Gini and Hoover indexes, with values of 0.813 and 0.709, respectively.

Table 5. Characteristics of the solution obtained when minimizing the beneficiaries' average walking distance using NAD for r = 15 km.

Demographic indicators		%
Garissa inhabitants	452,218	100.00%
Covered inhabitants	386,269	85.42%
Walking inhabitants (covered inh.)	71,309	18.46%
Pop. points (including 1 -2 Inhabitants)	47,242	
Population points (> 3 Inhabitants)	24,453	100.00%
Covered population points	16,611	67.93%
Potential distribution centers	1,459	100.00%
Open distribution centers	1,459	100.00%
Warehouse	1	
Economic indicators		**%**
Total cost (KSh)	66,050,194	100.00%
Beneficiaries acces cost (KSh)	3,192,667	4.83%
World Food Program cost (kSh)	31,877,121	48.26%
Kenya's Red Cross cost (kSh)	30,980,406	46.90%

Walking inhabitants' service indicators	
Time measure:	
Average walking time (hrs)	0.91
Distance measures:	
Average walking distance (km)	1.82
Standard deviation of distance (km)	3.57
Variance of distance (km)	12.77
Mean absolute deviation of distance (km)	1.30
Median walking distance (km)	0.00
Pearson's 2° skewness coefficient	1.53
Distribution left tail	0.04
Distribution right tail	0.96
Gini Index	0.813
Hoover Index (Robin Hood)	0.709

In Table 6, we observe the average walking distance threshold with a value of 1.82 km, and we observe better indicators below this threshold when compared with similar indicators for the threshold representing the AWD for the solution obtained when minimizing the costs objective functions in Table 4; in that case the AWD distance threshold has a value of 3.63 km. In the present case, we observe how below the AWD threshold, we have higher percentages for population points (24.17%), inhabitants (80.34%) and walking inhabitants (74.93%). However, we have a lower percentage of total walking distances (3.97%). The corresponding values in Table 4 were 16.85%, 70.54%, 65.32% and 16.13%. For the present case, this means that 74.93% of walking inhabitants walk distances inferior to 1.82 km and only 3.97% of the total walking distances, indicating that most of the people walk very low distances, this is highly desirable to ensure easy access for the food supplies. We also see a substantial improvement in most indicators for the 10 km distance threshold and also for the mid-radius threshold compared with the solution using the costs objective functions.

Table 6. Service levels based on three distance thresholds for the solution obtained when minimizing the beneficiaries' average walking distance using NAD for r = 15 km.

Distance Threshold (DT) / Service level measure		Average walking distance 1.82 km	10 km distance threshold 10.00 km	Mid-radius 7.5 km
Number of population points	16,611	100.00%	100.00%	100.00%
% below DT		24.17%	79.44%	68.35%
% above DT		75.83%	20.56%	31.65%
Number of inhabitants	386,269	100.00%	100.00%	100.00%
% below DT		80.34%	95.03%	92.20%
% above DT		19.66%	4.97%	7.80%
Number of walking inhabitants	71,309	100.00%	100.00%	100.00%
% below DT		74.93%	93.55%	89.93%
% above DT		25.07%	6.45%	10.07%
Total walking distances (km)	129,650	100.00%	100.00%	100.00%
% below DT		3.97%	55.85%	38.36%
% above DT		96.03%	44.15%	61.64%

In Figure 27 (a), we see how the AWD is close to the extreme of the left tail of the distribution. In Figure 27 (b), we see how the population points with most inhabitants walk very short distances and how the rest of the distribution, which includes populations with less than 5 inhabitants follows a flat pattern, indicating a direct relationship between the walking distance and the number of inhabitants. In Figure 27 (c), we cleary see how a very low percentage of the total walking distance are bellow the AWD. In Figures 27 (c) and 27 (d), the upper and lower line encircle a wide area representing the high values observed for the Gini and Robin Hood Indexes.

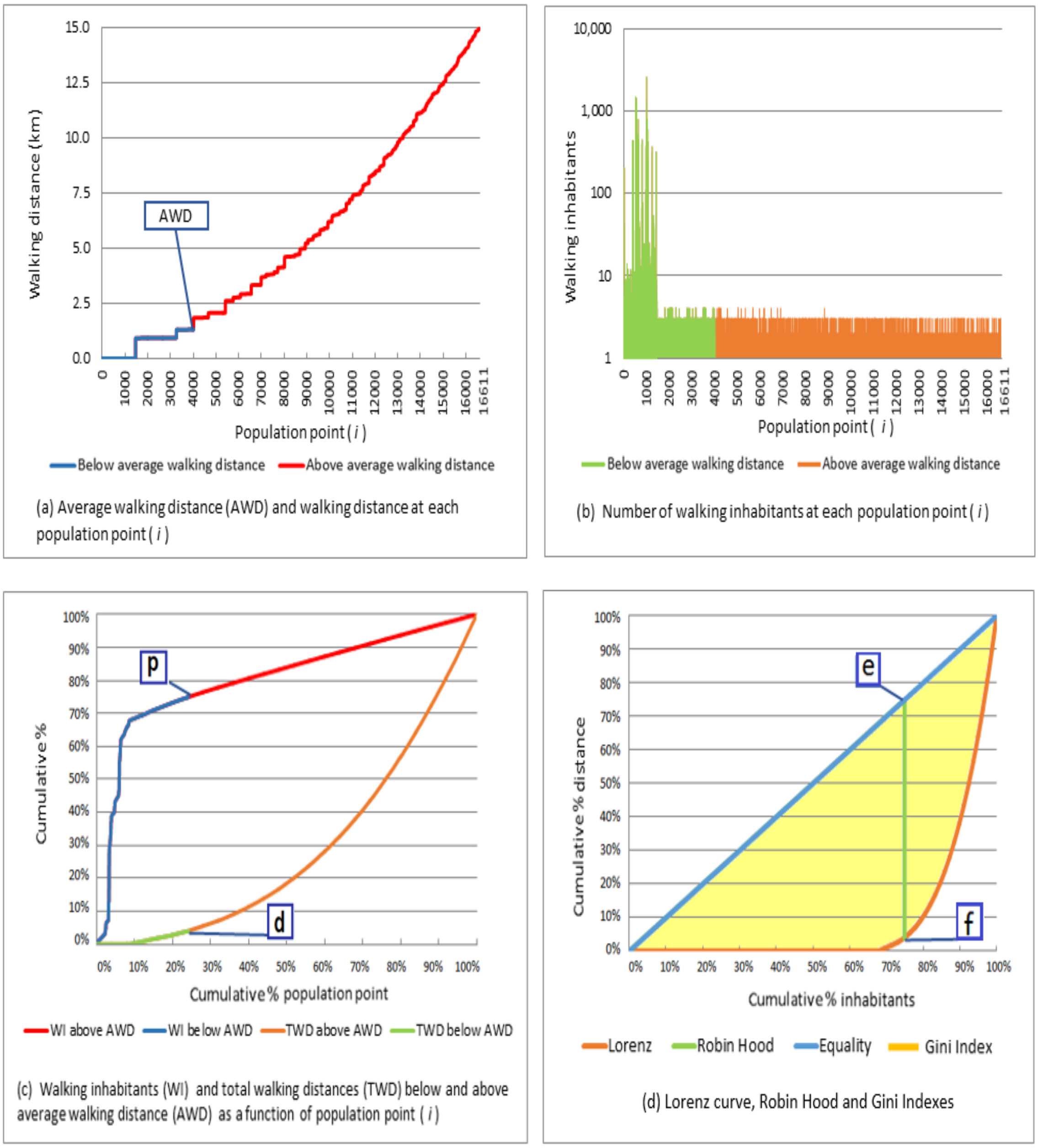


Figure 27. Characteristics of the distribution obtained when minimizing the beneficiaries' average walking distance using NAD for r = 15 km.

The shape for the Gini and Robin Hood Indexes is peculiar, despite the fact that this setting provides a very accesible network for the food beneficiaries, Figure 27(d) shows high levels of inequality.

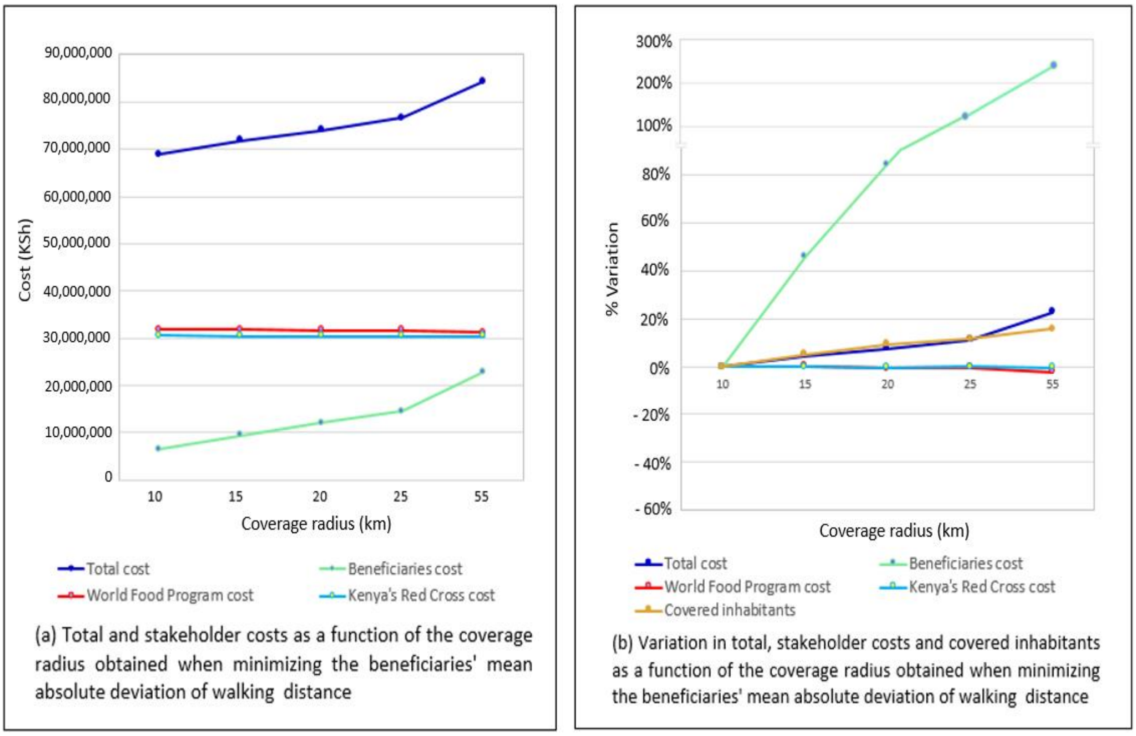
We note that if all the beneficiaries have a food distribution center next door, this situation will of course put every one in a situation of perfect equality, and that implies that they together will be travelling practically a close to zero total walking distances. If we observe the graphic representation of the Gini Index, that situation is only possible when the Lorez Curve perfectly is aligned with the horizontal axis, and with the yellow area completely covering half of the square, below the obliquous blue line.

Second, in economics the traditional attribute measured is income, not distance, and the objective is that everyone has a high income, as we can corroborate from the government efforts to increase the Gross Domestic Product (GDP). The Government is therefore dealing with a maximization objective. In our case, we are dealing with a minimization objective, corresponding to the minimization of the walking distances. For us, it it makes no sense to have everyone walk very large distances only to obtain a more equalitarian distribution. In this case, distance is a negative attribute and does not increase the wellbeing as a positive attribute.

At this point it was clear for us, that when dealing with a minimization objective, an atribute that can be considered negative, such as the distance, requires the reinterpretation and adaptation of classical equality measures, such as the Gini or Robin Hood Index. We have also noticed that we were dealing with a particular distribution network in a particular situation. It is possible that for other scenarios, our observation cannot be applied. Then, further studies and analysis will be required. Taking into account the observation mentioned in previous paragraphs, we decided to perform some tests, in this case we changed the minimization objective for a maximization objective for the AWD. For all the cases the results showed a very low and close to zero Gini and Robin Hood Indexes. It is possible to see the detailed results in Appendix 2.1.

5.1.5 Results obtained when minimizing the beneficiaries' mean absolute deviation of walking distance

For the third main analysed function (T_5), the values for the mean absolute deviation of walking distance that we have obtained were 0.69, 1.22, 1.80, 2.26 and 3.44 km. The solution times in CPLEX were 74.99, 221.25, 472.79, 913.47 and 12,572.60 seconds for the coverage radius of 10, 15, 20, 25 and 55 km, for additional information see Appendix 2.1. As in the preceding objective functions, we are also concerned with the economic implications. In Figure 28 (a), we observe the high cost associated with this objective function. We observe a total cost even higher than the ones obtained when minimizing the average walking distance objective function. It almost doubles the cost observed when using the stakeholder costs minimization objective function. These high costs are explained by higher costs for the walking inhabitants, on average 99.35% higher for the five cases presented when comparing with the solutions in Figure 22. These costs are also on average 209.84% higher than the results shown in Figure 25. The cost for the KRC shows an average increase of 13.19% compared with the solutions shown in Figure 22 and practically 0% difference with the solution shown in figure 25. We also observe that the costs for the WFP are practically similar compared with the solutions in Figures 22 and 25. In Figure 28 (b), we also observe the same patterns as in the previous solution, but with a higher variation for the beneficiaries' access cost, especially for the coverage radius of 55 km, where the increase in costs reaches approximately 250% vs. 200% in the previous case (Figure 25). More details about this objective function are presented in the next solution, that again follows the same structure and consider the same coverage radius of 15 km as in the preceding cases.



(a) Total and stakeholder costs as a function of the coverage radius obtained when minimizing the beneficiaries' mean absolute deviation of walking distance

(b) Variation in total, stakeholder costs and covered inhabitants as a function of the coverage radius obtained when minimizing the beneficiaries' mean absolute deviation of walking distance

Figure 28. Costs, percentage of variation in costs and covered inhabitants for different coverage radius when minimizing the beneficiaries' mean absolute deviation of walking distance using NAD.

5.1.6 Results obtained when minimizing the beneficiaries' mean absolute deviation of walking distance for r = 15 km

The map of the solution shown in Figure 29, presents a high number of open distribution centers, leaving only a few of them closed. This is almost the same network used as in the preceding solution obtained when minimizing the average walking distance.

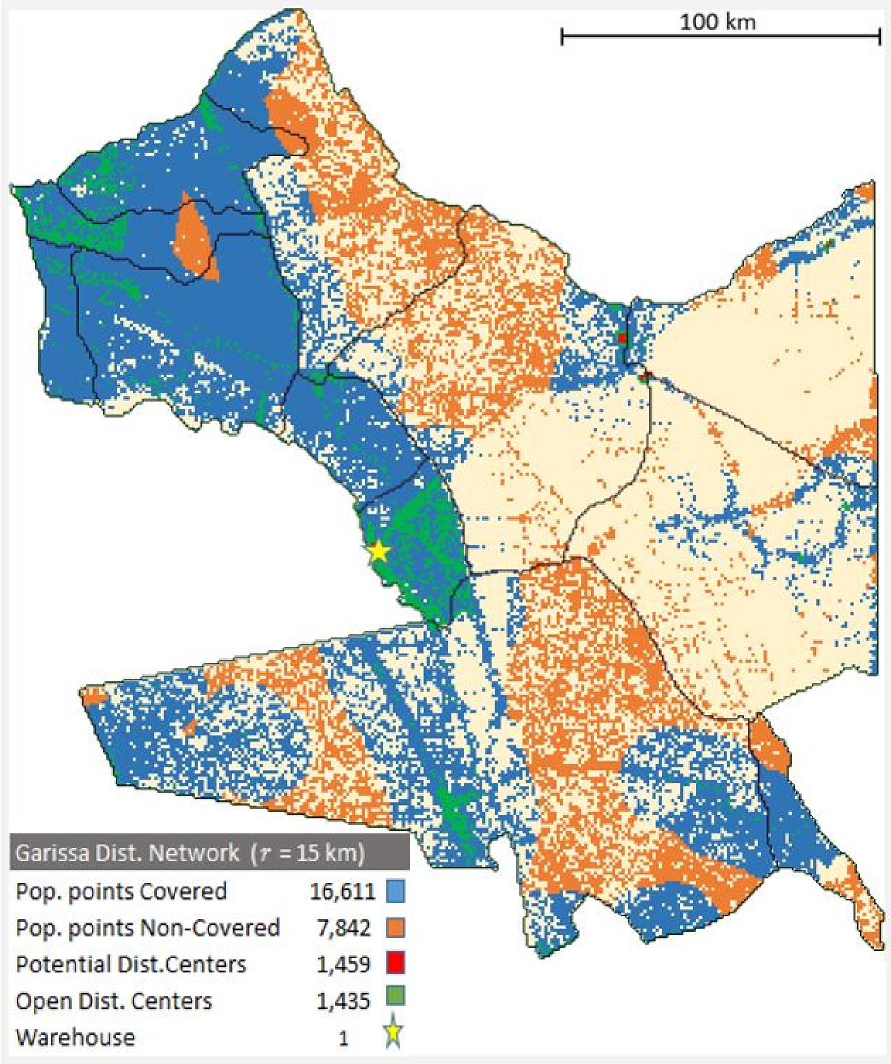


Figure 29. Map for the solution obtained when minimizing the beneficiaries' mean absolute deviation of walking distance using NAD for r =15 km.

Observing the map, one may think that the minimization of the average and the mean absolute deviation of walking distance, are very similar. However, we see some indicators that give us more clarity about the particularities of this solution in Table 7. First, we see that the costs are comparatively much higher than in the preceding solutions, this is explained by higher walking distances. We see for this case an average walking time of 4.15 hours and an average walking distance of 8.30 km, which are very high when compared with preceding solutions. Regarding the value of the mean absolute deviation of walking distance, it has a value of 1.22. However, the average walking distance minimization with a lower total cost had a value close to 1.30 for this coefficient. We also see a negative skew with a value of 0.64, showing a completely different shape for the distribution.

Table 7. Characteristics of the solution obtained when minimizing the beneficiaries' mean absolute deviation of walking distance using NAD for $r = 15$ km.

Demographic indicators		%
Garissa inhabitants	452,218	100.00%
Covered inhabitants	386,269	85.42%
Walking inhabitants (covered inh.)	71,309	18.46%
Pop. points (including 1 -2 Inhabitants)	47,242	
Population points (> 3 Inhabitants)	24,453	100.00%
Covered population points	16,611	67.93%
Potential distribution centers	1,459	100.00%
Open distribution centers	1,435	98.36%
Warehouse	1	
Economic indicators		**%**
Total cost (KSh)	71,821,018	100.00%
Beneficiaries acces cost (KSh)	9,492,674	13.22%
World Food Program cost (kSh)	31,857,554	44.36%
Kenya's Red Cross cost (kSh)	30,470,790	42.43%

Walking inhabitants' service indicators	
Time measure:	
Average walking time (hrs)	4.15
Distance measures:	
Average walking distance (km)	8.30
Standard deviation of distance (km)	6.41
Variance of distance (km)	41.04
Mean absolute deviation of distance (km)	1.22
Median walking distance (km)	9.68
Pearson's 2° skewness coefficient	(0.64)
Distribution left tail	0.11
Distribution right tail	0.89
Gini Index	0.420
Hoover Index (Robin Hood)	0.369

In Table 8, we observe that the majority of population points are below the AWD threshold. However, we see how only 45.33% of the inhabitants are below this threshold. The same can be observed for the 10 km distance threshold were we have 50.48% of the inhabitants vs. 93.55% for the same threshold in the case of the solution obtained when minimizing the AWD. This means that even if the distribution centers are practically the same, the allocation decisions are completely different. When we minimize the AWD, the inhabitants are located as close as possible to distribution centers, whereas for this solution, many inhabitants walk to distribution centers located far away, even if a closer distribution center is available. Minimizing the absolute value does not take into account service considerations for specific population points; the minimum value for the coefficient is reached by balancing the allocation of inhabitants to distribution centers.

Table 8. Service levels based on three distance thresholds for the solution obtained when minimizing the beneficiaries' mean absolute deviation of walking distance using NAD for $r = 15$ km.

Service level measure \ Distance Threshold (DT)		Average walking distance 8.3 km	10 km distance threshold 10.00 km	Mid-radius 7.5 km
Number of population points	16,611	100.00%	100.00%	100.00%
% below DT		71.24%	79.18%	68.10%
% above DT		28.76%	20.82%	31.90%
Number of inhabitants	386,269	100.00%	100.00%	100.00%
% below DT		45.33%	47.36%	44.52%
% above DT		54.67%	52.64%	55.48%
Number of walking inhabitants	71,309	100.00%	100.00%	100.00%
% below DT		47.89%	50.48%	46.86%
% above DT		52.11%	49.52%	53.14%
Total walking distances (km)	592,036	100.00%	100.00%	100.00%
% below DT		11.02%	13.87%	10.04%
% above DT		88.98%	86.13%	89.96%

In Figure 30 (a), we see how most population points are below the AWD and in Figure 30 (b), we see how the inhabitants are distributed between the two sides of the AWD, and how in the extreme of the the right tail we have population points with many inhabitants that will travel long distances. In Figure 30 (c), we see significant inequality levels for different segments of population points; however, we see a Gini and Robin Hood Indexes with lower values than in the preceding cases. This fact is explained because these indexes give more attention to the central part of the distribution. Figure 30 (c) allows us to see especific inequality differences not possible to see in the graphic in Figure 30 (d). We also obsereve an increase in walking distances when minimizing negative attributes (distance), which can reduce the Gini and Robin Hood Indexes, as explained in preceding sections.

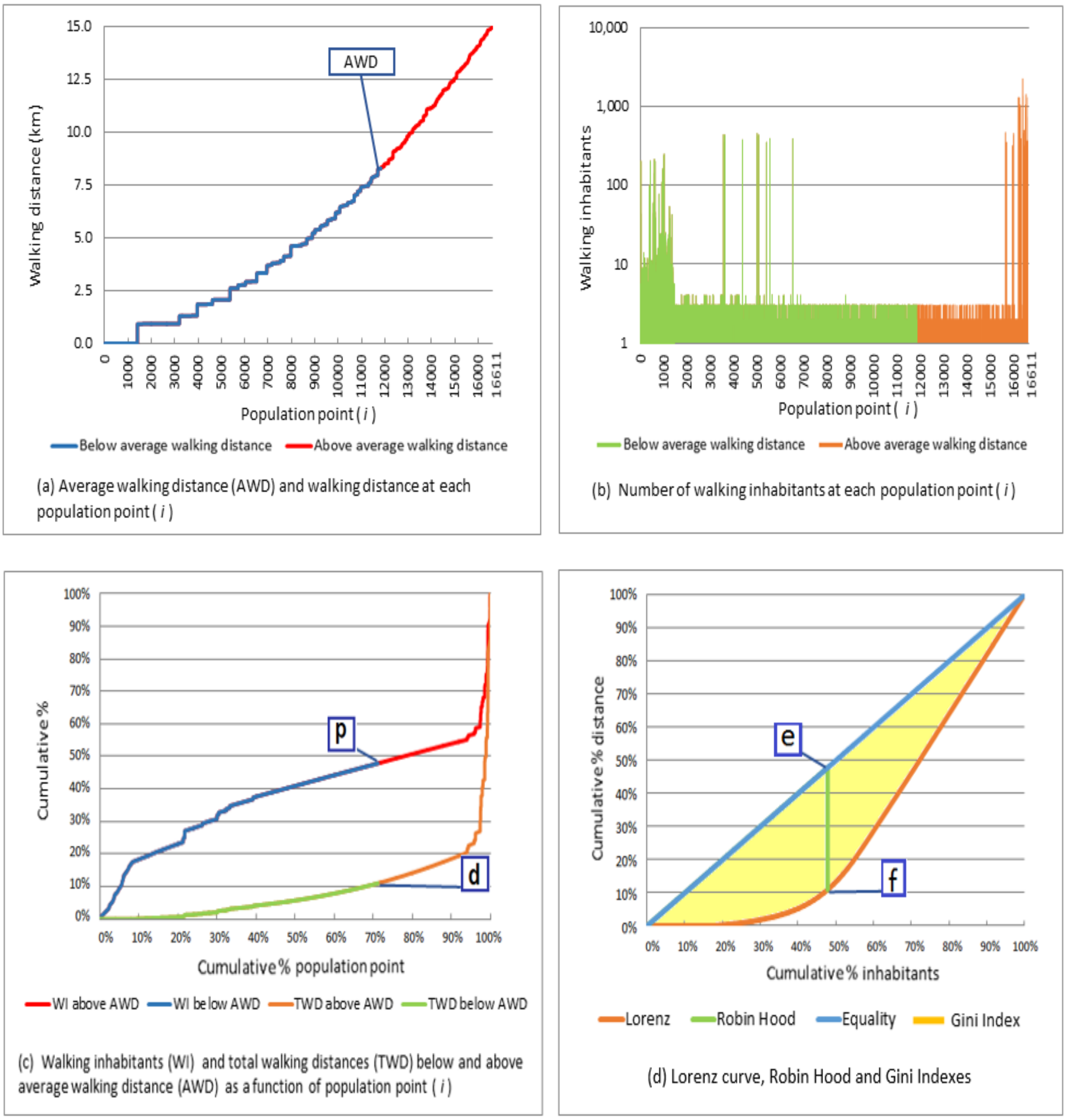
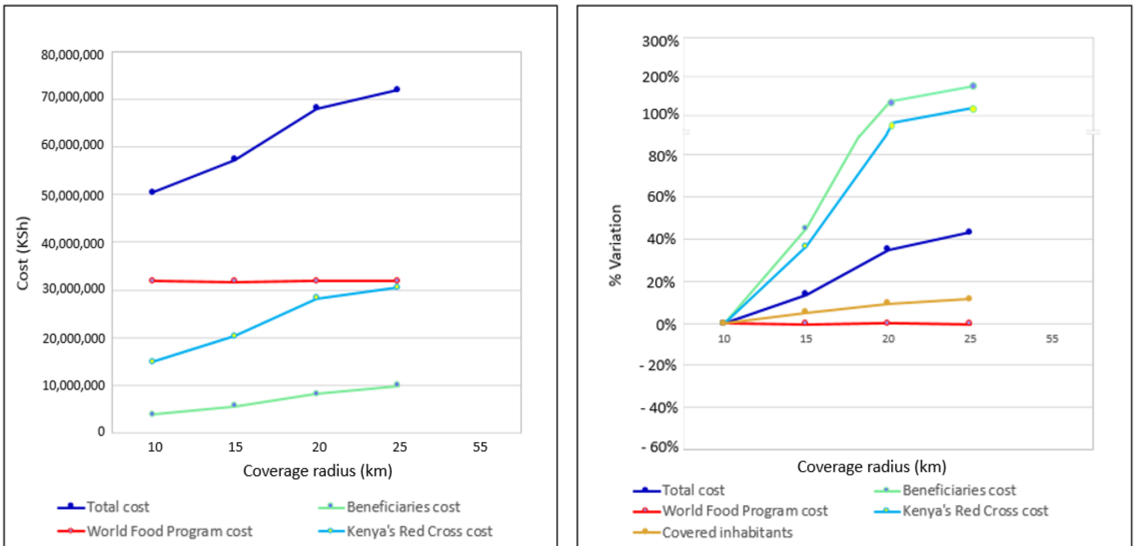


Figure 30. Characteristics of the distribution obtained when minimizing the beneficiaries' mean absolute deviation of walking distance using NAD for $r = 15$ km.

5.1.7 Results obtained when maximizing the beneficiaries' standard service below the mid-radius distance threshold

For the present objective function (T_7), the percentage of walking inhabitants below the mid-radius threshold that were obtained were 91.05%, 89.93%, 89.12% and 89.74%. The solution times in CPLEX were 248.90, 422.68, 2,036.82 and 4,532.19 seconds for the coverage radii of 10, 15, 20 and 25 km respectively; whereas, for the coverage radius of 55 km, we were not able to obtain a solution. Additional information can be found in Appendix 2.1. Considering economic implications, we observe in Figure 31 (a) how for the smaller coverage radii, the costs are relatively lower than in the two preceding cases, and that for large coverage radii the costs are very high as in the other two cases. Even though, there is no solution for the coverage radius of 55 km, the tendency is clearly observed in the graphic. For the smaller coverage radii, we observe a lower cost for the KRC; this cost was always high using the two preceding objective functions. The WFP and beneficiaries' access cost follow a very similar pattern to those on the preceding objective functions. Observing the Figure 31 (b), we see the high percentage of variation in the KRC and beneficiaries' access costs, especially for the higher coverage radii.



(a) Total and stakeholder costs as a function of the coverage radius obtained when maximizing the beneficiaries' standard service using the MRD threshold

(b) Variation in total, stakeholder costs and covered inhabitants as a function of the coverage radius when maximizing the beneficiaries' standard service using the MRD threshold

Figure 31. Costs, percentage of variation in costs and number of covered inhabitants for different coverage radii obtained when maximizing the beneficiaries' standard service below the MRD using NAD.

It is possible to find detailed information regarding the solutions by using this objective function in Appendix 2.1. Specific details about the behaviour of this objective function are presented in the following section for a coverage radius of 15 km.

5.1.8 Results obtained when maximizing the beneficiaries' standard service below the mid-radius distance threshold for *r* = 15 km.

In Figure 32, we see the map for the solution obtained when maximizing the beneficiaries' standard service below the mid-radius distance (MRD) threshold, and we can clearly see that for this objective function and a low coverage radius (15 km), the opening of a very large number of distribution centers is not required, as it was the case with the two precedings objective functions.

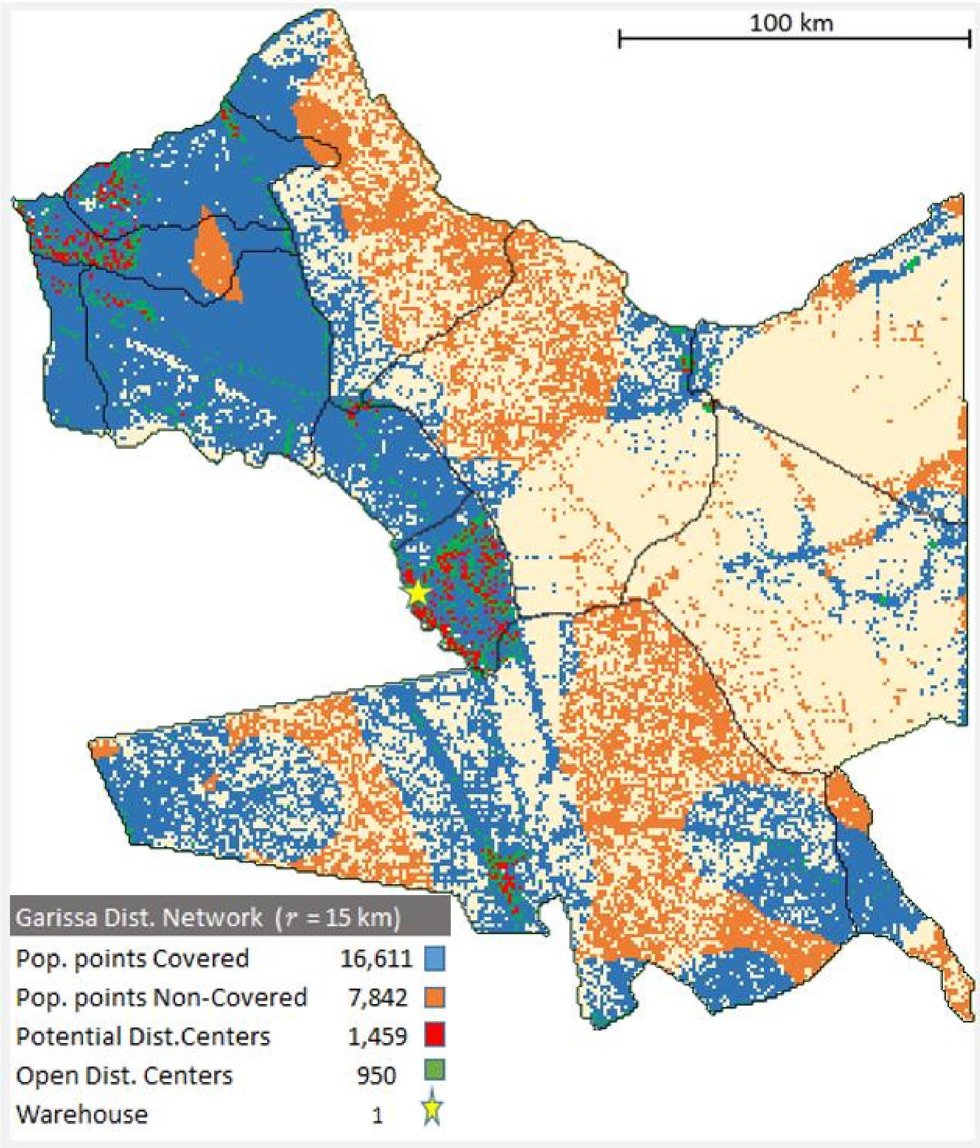


Figure 32. Map for the solution obtained when maximizing the beneficiaries' standard service below the MRD using NAD for r =15 km.

In Table 9, we see some importants indicators. For example, the average walking time of 2.07 hours or 4.14 km. These values are slighlty higher than when minimizing the stakeholder costs, but approximately these values are half of those obtained when minimizing the beneficiaries mean absolute deviation of walking distance. For the present solution, we have a positive skew of 1.03 and a Gini and Robin Hood Indexes of 0.448 and 0.333, respectively. These values are very close to those obtained with the mean absolute deviation minimization, with correspondent values of 0.420 and 0.369, but with a negative skew of 0.64. This observation confirms that two completely different distributions can nevertheless present close Gini and Robin Hood Indexes, and also confirms the fact that these indexes have more value in the central part of the distribution, but loose important information in the extremes of the distribution.

Table 9. Characteristics of the solution obtained when maximizing the beneficiaries' standard service below the MRD for r = of 15 km.

Demographic indicators		%	Walking inhabitants' service indicators	
Garissa inhabitants	452,218	100.00%	Time measure:	
Covered inhabitants	386,269	85.42%	Average walking time (hrs)	2.07
Walking inhabitants (covered inh.)	71,309	18.46%		
Pop. points (including 1 -2 Inhabitants)	47,242		Distance measures:	
Population points (> 3 Inhabitants)	24,453	100.00%	Average walking distance (km)	4.14
Covered population points	16,611	67.93%	Standard deviation of distance (km)	3.52
Potential distribution centers	1,459	100.00%	Variance of distance (km)	12.41
Open distribution centers	950	65.11%	Mean absolute deviation of distance (km)	1.61
Warehouse	1		Median walking distance (km)	2.93
Economic indicators		%	Pearson's 2° skewness coefficient	1.03
Total cost (KSh)	57,336,921	100.00%	Distribution left tail	0.28
Beneficiaries acces cost (KSh)	5,446,186	9.50%	Distribution right tail	0.72
World Food Program cost (kSh)	31,718,435	55.32%	Gini Index	0.448
Kenya's Red Cross cost (kSh)	20,172,300	35.18%	Hoover Index (Robin Hood)	0.333

In Table 10, we see how most of the total walking distances are below the mid-radius threshold, with 69.95%, and also for the 10 km threshold we have a high percentage of 73.40%. These values are highly superior to the percentages obtained when minimizing the AWD with a 38.36% and 55.85%, respectively in Table 6. These differences are even more accentuated compared with the same percentages obtained when minimizing the mean absolute deviation of distance, with 10.04 % and 13.87%, respectively in Table 8.

Table 10. Service levels based on three distance thresholds for the solution obtained when maximizing the beneficiaries' standard service below the MRD for r = 15 km.

Service level measure \ Distance Threshold (DT)		Average walking distance 4.14 km	10 km distance threshold 10.00 km	Mid-radius 7.5 km
Number of population points	16,611	100.00%	100.00%	100.00%
% below DT		29.31%	73.17%	68.35%
% above DT		70.69%	26.83%	31.65%
Number of inhabitants	386,269	100.00%	100.00%	100.00%
% below DT		64.18%	93.43%	92.20%
% above DT		35.82%	6.57%	7.80%
Number of walking inhabitants	71,309	100.00%	100.00%	100.00%
% below DT		60.78%	91.51%	89.93%
% above DT		39.22%	8.49%	10.07%
Total walking distances (km)	295,046	100.00%	100.00%	100.00%
% below DT		27.50%	73.40%	69.95%
% above DT		72.50%	26.60%	30.05%

In Figure 33 (a), we can see the AWD that approximately corresponds to the walking distance of the population point 4,868. We also see an additional square pointing to the walking distance of the last population point below the MRD threshold and this corresponds to the population point 11,354. We see at this point a variation that shows how the algorithm tried to limit the walking distances below this distance threshold without giving much attention to the distances above the same threshold.

In Figure 33 (b), we see how the population points with more walking inhabitants are concentrated below the MRD threshold and how population points with a few inhabitants, in all cases with less than five inhabitants, are above the MRD threshold. The concentration of population points with a small number of inhabitants in this extreme also contributes to control the costs for the present solution, since these inhabitants have to walk the largest distances. In Figure 33 (c) and 33 (d), we see the corresponding cumulative values and we can obsereve completely different shapes obtained with this distribution, compared with the distribution obtained using the previous objective function in Figure 30, even though presenting very close Gini and Robin Hood Indexes.

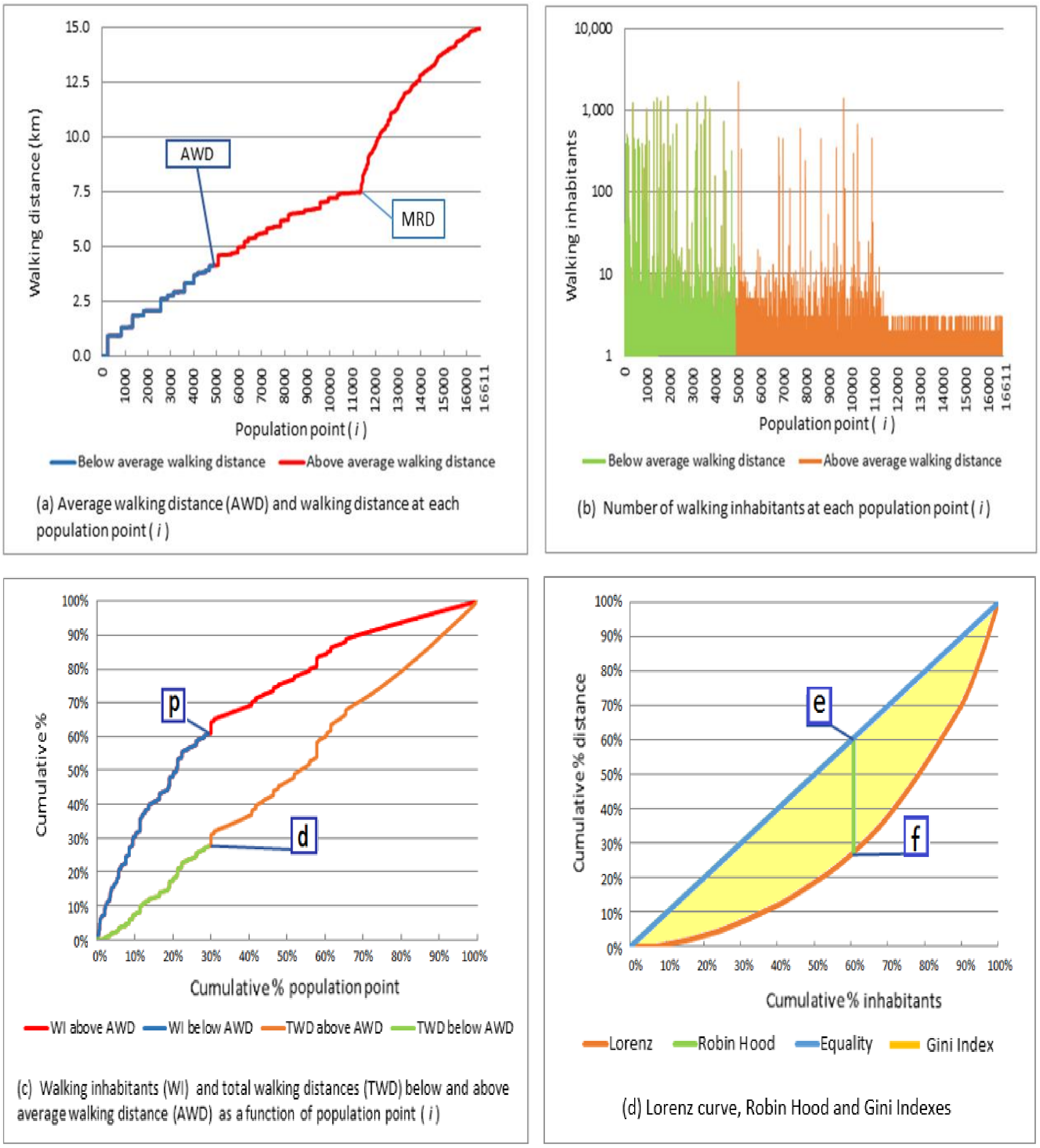


Figure 33. Characteristics of the distribution obtained when maximizing the beneficiaries' standard service below the MRD for r = 15 km.

5.2 Results obtained with aggregated data

In the previous section we described solutions for five different coverage radii and also presented a detailed analysis for representative solutions with a coverage radius of 15 km. For the present section, we will describe solutions for the following radii: 10, 15, 20, 25 and 55 km and with the same objective functions as those used with the non-aggregated data, but considering space restrictions, we will not present the detailed analysis for specific solutions. The examples presented in this section use an aggregation threshold of 18 inhabitants since, as discussed early, this threshold has more resemblances with the models with non-aggregated data. Nevertheless, it is possible to find additional information and more scenarios in Appendix 3.

We need to clarify that the solutions obtained using aggregated data and non-aggregated data are only comparable for referential purposes because, as explained in previous sections, the aggregation process and the calculation for the walking inhabitants gave in some cases very different scenarios and different coverage levels. However, the analysis of how the aggregated-data affects the obtained solution, will give us some important insights. Furthermore, we need to consider that obtaining particular solutions for specific situations is as important as to understand the general behaviour of functions and models under different scenarios and contexts.

Before introducing specific results, it is important to briefly discuss some important points of the solutions that use the non-aggregated data, these will help us understand some differences with the solutions obtained with the aggregated data. For the non-aggregated data case, when minimizing beneficiaries' AWD, the beneficiaries' mean absolute deviation of walking distance and the beneficiaries' standard service objective functions, we do not have a limitation of the number of DCs that can be opened, and that is why for these objective functions, we had all or almost all the DCs open. However, for the case of the solutions with aggregated data, we observed consistently that the KRC's cost were relatively lower than in the solutions with the non-aggregated data; because KRC's cost and open DCs are in direct relationship, it implies that many less distribution centers are open. Many intricate interrelationships in the model elements can influence these differences, we however believe that an important reason is explained by a simple but non-evident fact.

Consider the example of the two networks presented in Figure 34. In Figure 34 (a), we see a small part of a distribution network for an area of 54 km^2. The population points, representing an area of one km^2, are allocated to their closest distribution center. In this case, the minimization of costs can be obtained when the population points walk the less, this solution can be obtained when all the DCs are open. On the other hand, in Figure 34 (b), we aggregate the population points located in an area of nine km^2 to the center of the nine km^2 square, as is also the aggregation process for our food distribution network; we observe that these new aggregated population points need half of the distribution centers used in the non-aggregate case. A similar situation can be observed for the solutions obtained using aggregated data, where the new network resulting from the aggregation process created a natural restriction for the total number of DCs that can be opened, and yielded reductions on the KRC's cost.

Considering the objective function minimizing the stakeholder costs, we are minimizing the KRC cost; hence, this objective function presented more similitudes with the solutions obtained using non-aggregated data. Additional elements could have influenced the above observations. It is also possible that our observations are only valid for the particular problem we are studying. Additional observations would be interesting using different networks for different problems.

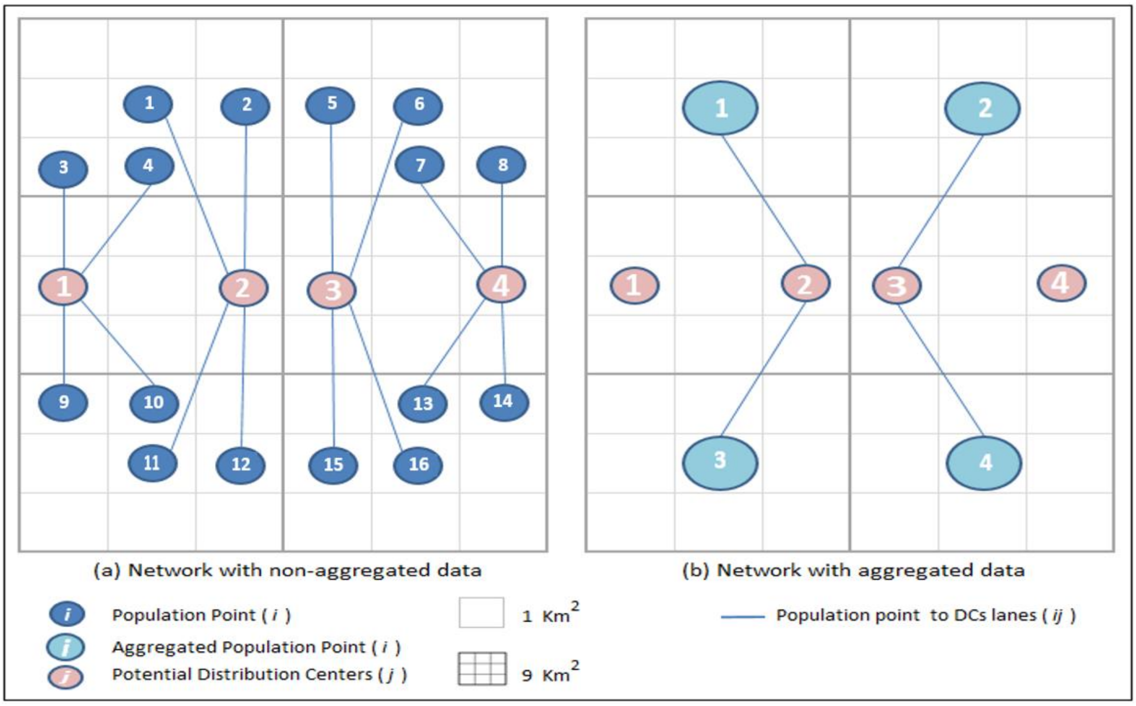


Figure 34. Representation of data-aggregation effects on the network of distribution centers used.

5.2.1 Results obtained when minimizing the stakeholder costs for an aggregation threshold of 18 inhabitants.

In Figure 35 (a), we observe that the costs are lower, but very close to those obtained using the non-aggregated data, see Figure 22. The shape of the curves for the different cost components also follows a similar pattern. The small differences in costs can be explained by the different covering levels obtained when using non-aggregated and aggregated data and for the reduction in the number of walking inhabitants for the aggregated data. In Figure 35 (b), we also observe the same patterns in the solution with non-aggregated data, we have in this case a small reduction in the percent variation of costs, especially for the largest coverage radii. These differences are nevertheless non-significant. For the present solutions, CEPLEX running times were 5.3, 6.16, 14.65, 33.96 and 215.95 seconds for the coverage radiuses of 10, 15, 20, 25 and 55 km respectively. Additional information as well as additional solutions using the stakeholder cost as an objective function can be found in Appendix 3.1.

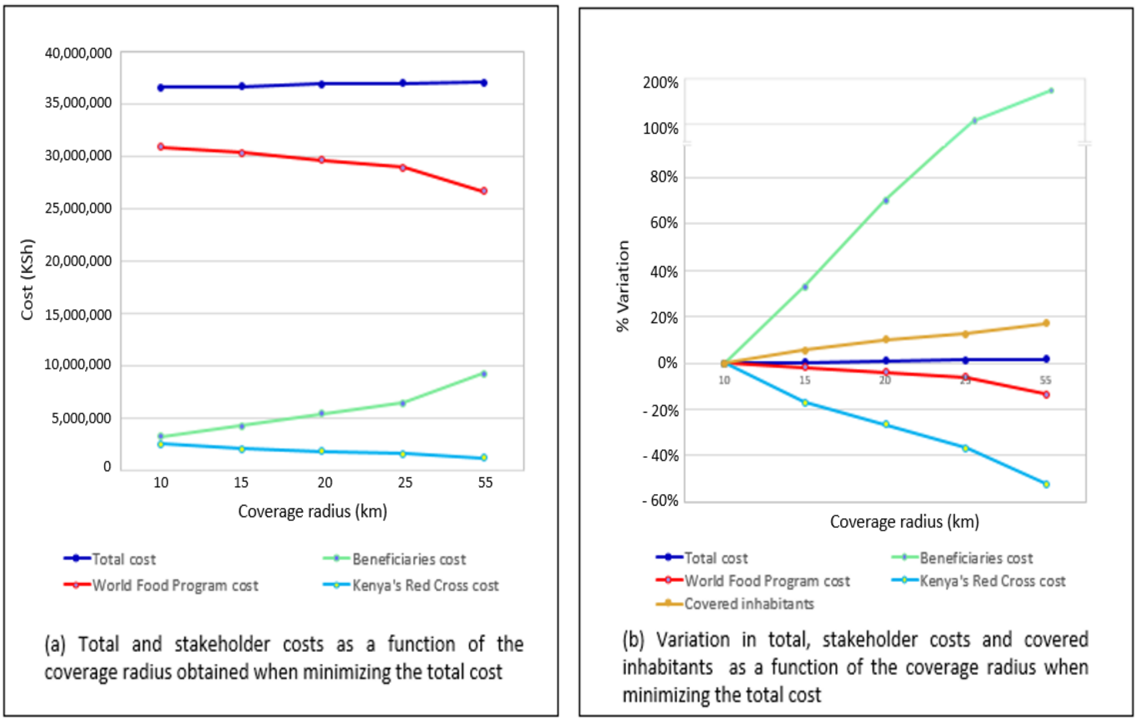
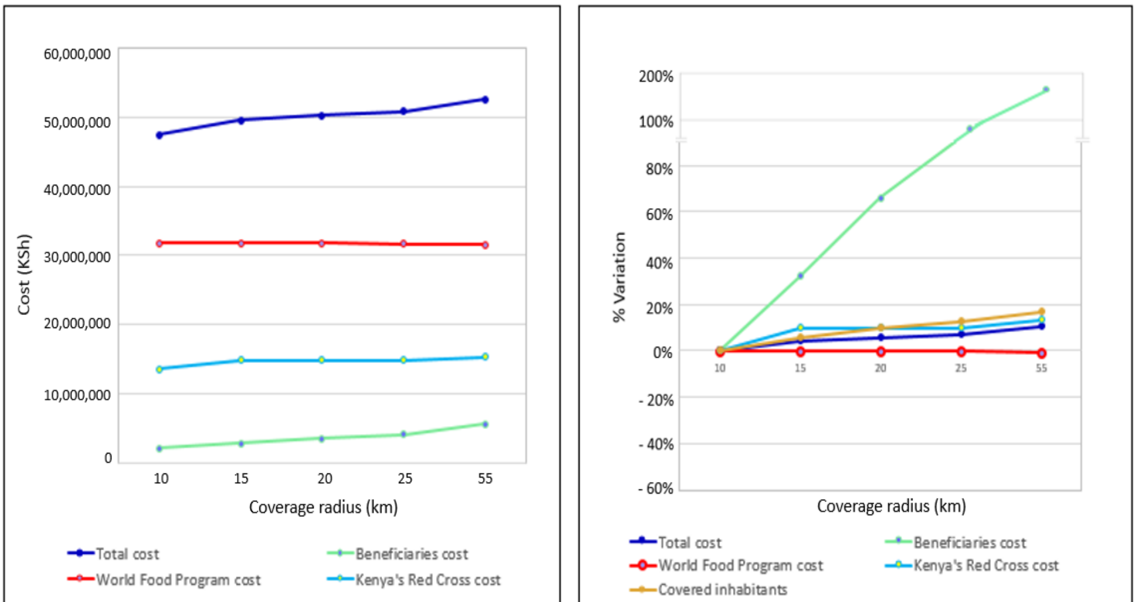


Figure 35. Costs, percentage of variation in costs and covered inhabitants for different coverage radius when minimizing the stakeholder costs using AD for IAT = 18 inh.

Based on the comparative analysis between solutions obtained from non-aggregated data and aggregated data, we conclude that aggregating data for this case gives in general close results. However, it is possible that by using an aggregation strategy that gives a closer scenario compared with the original one, these differences can become even smaller. For example, it is possible to aggregate only the inhabitants covered in the original model, but for our aggregation strategy, the aggregation process allows us to improve the coverage levels, as it was possible to serve small population points that were not covered in the non-aggregated case. The actual method of calculation favours the reduction of walking inhabitants for aggregated scenarios; this situation was explained in Section 4.2. We can also add, based on the small differences observed, that neither the solutions with non-aggregated data nor the solutions with aggregated data are inexact; there are simply differences in model generation and in general assumptions.

5.2.2 Results obtained when minimizing the beneficiaries' average walking distance for an aggregation threshold of 18 inhabitants

For the average walking distance minimization objective function, the values obtained for the solutions were 1.07, 1.71, 2.35, 2.84 and 4.11 km. The CPLEX running times were 11.55, 68.49, 182.85, 1,103.26 and 14,016.60 seconds for the coverage radii of 10, 15, 20, 25 and 55 km, respectively. It is also possible to find the results for additional scenarios in Appendix 3.2. In Figure 36 (a), we see the curves representing the costs obtained when using aggregated data. The main difference we observe compared with similar solutions for non-aggregated data presented in Figure 25, is much lower total costs explained by the lower costs for the KRC. However, when we observe the WFP and beneficiaries' cost curves, we see that these curves represent costs very close to those obtained with the non-aggregated data. The reduction in the KRC's cost is explained by the reduction in the number of open DCs. This reduction is due to the particularities of the aggregated network that resulted in a restriction for the needed number of DCs. In Figure 36 (b), we observe similar results for the variation in costs as in the solutions obtained using non-aggregated data. The main difference being again an increase in the variability for the KRC's cost curve, that is slightly higher for the largest coverage radius, and it can be explained because for larger coverage radiuses we have more scattered populations. In such situations, the aggregation process is not as effective as it is for densely populated areas, generating a low reduction in the natural restriction effect on the open DCs observed for the aggregated data.



(a) Total and stakeholder costs as a function of the coverage radius obtained when minimizing the beneficiaries' average walking distance

(b) Variation in total, stakeholder costs and covered inhabitants as a function of the coverage radius when minimizing the beneficiaries' average walking distance

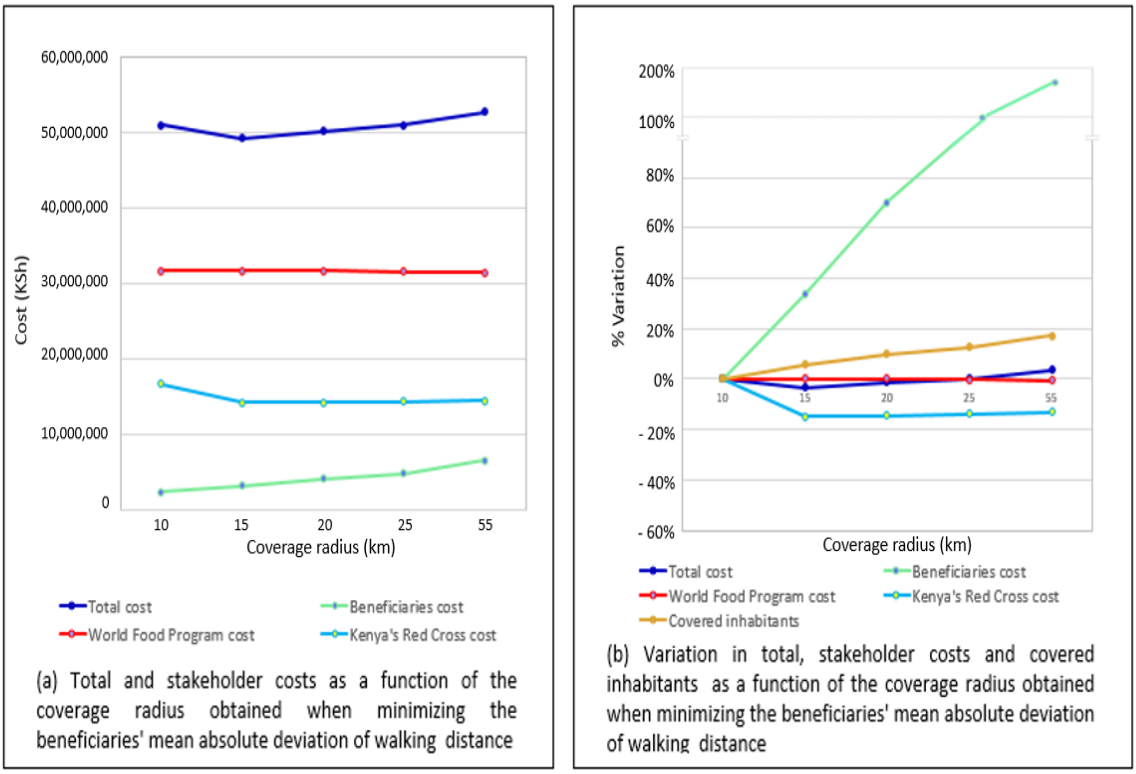
Figure 36. Costs, percentage of variation in costs and covered inhabitants for different coverage radius when minimizing the beneficiaries' average walking distance using AD for IAT = 18 inh.

5.2.3 Results obtained when minimizing the beneficiaries' mean absolute deviation of walking distance for an aggregation threshold of 18 inhabitants.

For the present objective function, the values obtained were 0.10, 0.18, 0.29, 0.39 and 0.63 km. The CPLEX running times were 3.2, 12.29, 24.62, 39.30 and 281.15 seconds for the coverage radii of 10, 15, 20, 25 and 55 km, respectively. Additional information and scenarios can be found in Appendix 3.4.

In Figure 37 (a), we also observe lower costs compared with equivalent solutions using non-aggregated data in Figure 28. These cost reductions, observed when minimizing the average walking distance, are due to the same restriction on the required DCs by the data aggregation. In the case of the KRC's cost, the cost reduction effects are slightly amplified for larger coverage radii. The WFP's cost follows a similar pattern as observed in the solution with non-aggregated data. The beneficiaries' costs are relatively lower compared with the solutions with non-aggregated data, but they also follow the same increasing tendency observed for the larger coverage radius.

In Figure 37 (b), we observe the percent variation in costs for different coverage radii. The main difference with the solutions using non-aggregated data is that the KRC's cost curve shows a greater reduction in cost for the larger coverage radius, compared with equivalent solutions in Figure 28 (b). For the walking inhabitants, in the case of the 55 km coverage radius, we have costs increases lower than 200%; whereas in Figure 28 (b), the values are much higher than 200%. The reductions in cost for the KRC can be explained because of the limited number of open DCs in solutions using the aggregated data, as observed in the example presented in Figure 34. The reduction in cost for the walking inhabitants can be explained by the calculation process that favours the reduction in walking inhabitants for aggregated scenarios.



(a) Total and stakeholder costs as a function of the coverage radius obtained when minimizing the beneficiaries' mean absolute deviation of walking distance

(b) Variation in total, stakeholder costs and covered inhabitants as a function of the coverage radius obtained when minimizing the beneficiaries' mean absolute deviation of walking distance

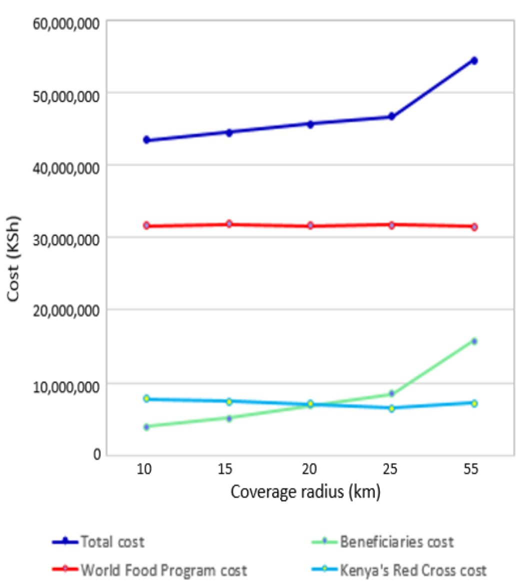
Figure 37. Costs, percentage of variation in costs and covered inhabitants for different coverage radiuses when minimizing the beneficiaries' mean absolute deviation of walking distance using AD for IAT = 18 inh.

5.2.4 Results obtained when maximizing the beneficiaries' standard service below the mid-radius distance threshold for an aggregation threshold of 18 inhabitants.

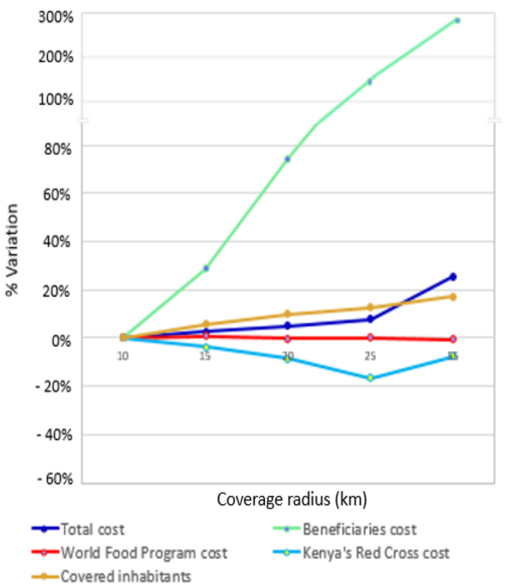
For the maximization of service measured in number of walking inhabitants below the mid-radius threshold, the results obtained using aggregated data were 92.35%, 91.24%, 90.52%, 91.10% and 96.73%. The CPLEX running times were 3.68, 10.65, 23.51, 41.15 and 520.16 seconds for the coverage radii of 10, 15, 20, 25 and 55 km, respectively. Additional information and scenarios can be found in Appendix 3.6. Compared with the solutions using non-aggregated data we have the solution for the 55 km coverage radius; hence, we have a more complete view of the costs curves.

In Figure 38 (a), we observe lower total costs curves for the solutions using aggregated data compared with solutions using non-aggregated data in Figure 31. The patterns also show an increase in total cost for the larger coverage radii. However, the total cost for small coverage radii is not too high compared with the costs obtained with the functions previously analysed and also compared with the corresponding solutions using non-aggregated data. Regarding the KRC's cost, we observe the influence of the aggregated data on the number of open DCs, as also observed with the previous objective functions.

In Figure 38 (b) we observe the percentage of variation in costs. For the aggregated data, we obtained the solutions for the 10, 15, 20, 25 and 55 km coverage radii, whereas for the non-aggregated data, the solution for the 55 km coverage radius was not obtained. For the radius that can be compared, we observe a reduction in the percentage of increase for the total cost and for the beneficiaries' cost and also a reduction in the KRC's cost compared with equivalent information shown in Figure 31 (b). We also observe that the cost component has less variation in costs across different coverage radii for the WFP's cost, this observation is also valid for the two previous objective functions.



(a) Total and stakeholder costs as a function of the coverage radius obtained when maximizing the beneficiaries' standard service using the MRD threshold



(b) Variation in total, stakeholder costs and covered inhabitants as a function of the coverage radius when maximizing the beneficiaries' standard service using the MRD threshold

Figure 38. Costs, percentage of variation in costs and covered inhabitants for different coverage radius when maximizing the beneficiaries' standard service below the MRD using AD for IAT = 18 inh.

5.3 Results obtained with generated data

In this section, we present results for the model using the generated data set. Considering space limitations, we will not present the results for some objective functions for which we already have detailed analysis and results using both non-aggregated and aggregated data. However, results for all the objective functions using this data set can be found in Appendix 4. The main differences between this data set and the two preceding ones are related to the size of the problems and in case of the generated data, the inhabitants were grouped into a small number of population points as explained in the chapter describing the generated data. Nonetheless, the results obtained for these problems share many similarities with the results obtained using the non-aggregated data set.

Considering the size of the problem and the fact that this particular data set was mainly used to understand the problem, we only explored solutions covering all the inhabitants for a coverage radius of 55 km. In this section we are mainly interested in describing the objective function for which it was not possible to find solutions using the non-aggregated and aggregated data sets, i.e., the solution that minimizes the variance of the walking distance objective function.

For the generated data set, we will first describe a solution with the objective functions that minimize the stakeholder costs in order to have a comparative basis for the solution using the variance as an objective function. We note that these solutions are not directly comparable with solutions obtained with the other two data sets. In this case, we have a particular scenario and the comparisons are mainly useful to understand the general behaviour of problem and to gain some important insights using the variance as an objective for the general context over which we develop our study.

For the solutions presented below, we will follow the same structure as the detailed analysis presented for the non-aggregated data. We will, however, omit the presentation of maps because in this case we have a small and non-real network.

5.3.1 Results obtained when minimizing the stakeholder costs using generated data.

In Table 11, we see exactly the same number of inhabitants for the Garissa District as in the real model, the difference being that because we have a small number of population points, the inhabitants are aggregated and we do not have small population points. Therefore, we have 100% of covered inhabitants and 100% of covered population points; whereas, for the non-aggregated model, we have percentages of 93.88% and 51.76%, respectively, considering a coverage radius of 55 km. It is possible to find detailed information for the solution with non-aggregated data and coverage radius of 55 km in Appendix 2.1. For this solution, the CPLEX running time was only 0.020 seconds. It is also possible to see additional information regarding additional objective function components not presented here for space limitations in Appendix 4.

Regarding the number of walking inhabitants, we see that the number is lower than in the case of the real network using non-aggregated data for the same coverage radius (75,406 vs. 80,898), despite covering more inhabitants in the generated network. This difference can also be explained by the effects of aggregated population on the calculation of the walking inhabitants; similar effects were also observed for the solutions using aggregated data.

Regarding the cost observed for this solution, we see a total cost of 42.19 million KSh, and for the case of the real network and the same coverage radius we have a cost of 39.04 million KSh. Analysing the composition of the costs, in case of the real network, we have 26.6% for the beneficiaries' access costs, 70% for the WFP cost and 3.3% for the KRC cost. These percentages are relatively close and follow the same pattern as the corresponding costs and percentages observed in the Table 11, with 35.5%, 60.8% and 3.65%, respectively.

For the present solution we have a higher average walking time and distance than in the solution obtained with the real network because, for the generated network, we do not have population points located in the same geographical points as the potential distribution centers. This factor limited costs and walking distances for the solutions obtained using the non-aggregated and the aggregated data. Even though there are some differences related to the model generation and size of the problem, we can see that the small generated network has in general a similar behaviour compared with the real one. The results are not directly comparable, but can be used to understand how some objective functions behave as we did in initial stages of the project.

Table 11. Characteristics of the solution obtained when minimizing the stakeholder costs using generated data for r = 55 km.

Demographic indicators		%	Walking inhabitants' service indicators	
Garissa inhabitants	452,218	100.00%	Time measure:	
Covered inhabitants	452,218	100.00%	Average walking time (hrs)	6.56
Walking inhabitants (covered inh.)	75,406	16.67%		
			Distance measures:	
Population points	100	100.00%	Average walking distance (km)	13.12
Covered population points	100	100.00%	Standard deviation of distance (km)	9.45
Potential distribution centers	50	100.00%	Variance of distance (km)	89.33
Open distribution centers	28	56.00%	Mean abs. deviation of distance (km)	12.65
Warehouse	1		Median walking distance (km)	8.49
Economic indicators		**%**	Pearson's 2° skewness coefficient	1.47
Total cost (KSh)	42,182,205	100.00%	Distribution left tail	0.28
Beneficiaries acces cost (KSh)	14,988,345	35.53%	Distribution right tail	0.72
World Food Program cost (kSh)	25,653,861	60.82%	Gini Index	0.356
Kenya's Red Cross cost (kSh)	1,540,000	3.65%	Hoover Index (Robin Hood)	0.278

In Table 11, we also observe the variance of walking distance for the present solution, with a value of 89.44 km. We also present some other important indicators such as the Gini and Robin Hood Indexes that will be useful to compare with the solution obtained when minimizing the variance in the following section. In Table 12, in the column of the MRD threshold, we observe that the majority of the population points, inhabitants, walking inhabitants and total walking distances are below this threshold. Therefore, we have beneficiaries walking relatively short distances. This observation is also confirmed by observing the other two distance thresholds presented in Table 12.

Table 12. Service levels based on three distance thresholds for the solution obtained when minimizing the stakeholder costs using generated data for r = 55 km.

Service level measure \ Distance Threshold (DT)		Average walking distance 13.12 km	10 km Threshold norm 10.00 km	Mid-radius 27.5 km
Total population points	100	100.00%	100.00%	100.00%
% below DT		21.00%	61.00%	79.00%
% above DT		79.00%	39.00%	21.00%
Total inhabitants	452,218	100.00%	100.00%	100.00%
% below DT		56.22%	81.25%	92.86%
% above DT		43.78%	18.75%	7.14%
Total walking inhabitants	75,406	100.00%	100.00%	100.00%
% below DT		56.20%	81.24%	92.85%
% above DT		43.80%	18.76%	7.15%
Total walking distances (Km)	989,374	100.00%	100.00%	100.00%
% below DT		28.42%	58.54%	79.25%
% above DT		71.58%	41.46%	20.75%

In Figure 39, we can see the particularities of the distribution for the present solution. In this case, in Figure 39 (a), we have an ascending curve, a pattern very similar to the one observed in Figure

24 (a) for a similar solution using the non-aggregated data. The curves for the solutions with non-aggregated data are smoother, because in that case we are using a large number of population points. For the present case the value for the AWD is higher, which can be explained by the higher coverage radius for the present solution. In Figure 39 (b), we also see how the population points with more inhabitants are situated on the left of the distribution, similar to the observed in Figure 24 (b). The shapes of Figures 39 (c) and 39 (d) are also similar to those observed in Figures 24 (c) and 24 (d) and can also be interpreted in a similar way, but in this case the graphics show a lower Gini and Robin Hood Indexes with values of 0.356 and 0.278, respectively.

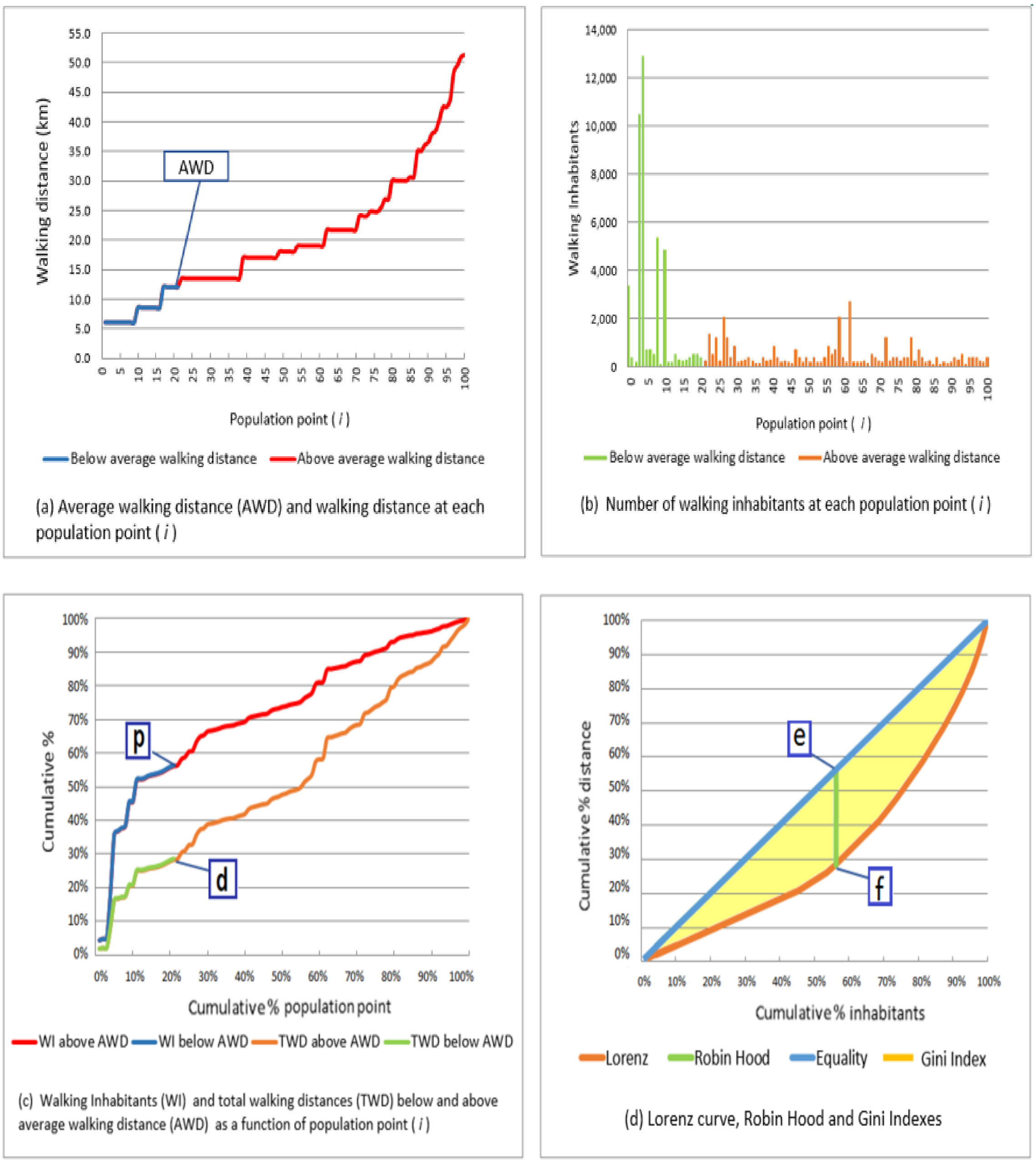


Figure 39. Characteristics of the distribution obtained when minimizing the stakeholder costs using generated data for r = 55 km.

5.3.2 Results obtained when minimizing the beneficiaries' variance of walking distance using generated data.

The use of the variance as objective function engender more difficulties to obtain solutions. Indeed, we were only able to obtain results with an optimality gap of 1% after running CPLEX for 1,412,374 seconds. We also observed that the CPLEX deployed much effort as compared with obtaining other solutions. Specific details regarding the CPLEX effort expressed in number of ticks and solution times, can be found in Appendices 2, 3 and 4 for this solution and the other ones. The results presented below are valid for referential purposes only and provide a general understanding when using this objective function for a particular scenario as well as in the context for which we develop our study.

In Table 13, we can see the demographic indicators, which are the same as those presented for the solution minimizing the stakeholders' costs using the generated data set. The differences that can be observed is that we have 42 distribution centers open in this case, and the number was 28 in the previous case. Regarding the costs, we see a substantial increase in the total costs, and they are almost twice the preceding solution. We also see that this increase in cost is mainly generated by a high increase in the beneficiaries' access cost, up to 48.95 million KSh from 14.98 million KSh in the previous case. The other cost components present comparatively small cost variations.

Regarding the service indicators, we observe a high average walking time and distance with values of 23.09 hours and 46.18 km, respectively; in the previous solution these values were only 6.56 hours and 13.12 km, respectively. Among other indicators, we observe for this case the reduction in the variance of walking distance with a value of 19.39 km, for the previous solution we had a high value of 89.33 km. It is also interesting to notice the very low and close to zero Gini and Robin Hood Indexes with values of 0.048 and 0.034, respectively.

Table 13. Characteristics of the solution obtained when minimizing the beneficiaries' variance of walking distance using generated data for r = 55 km.

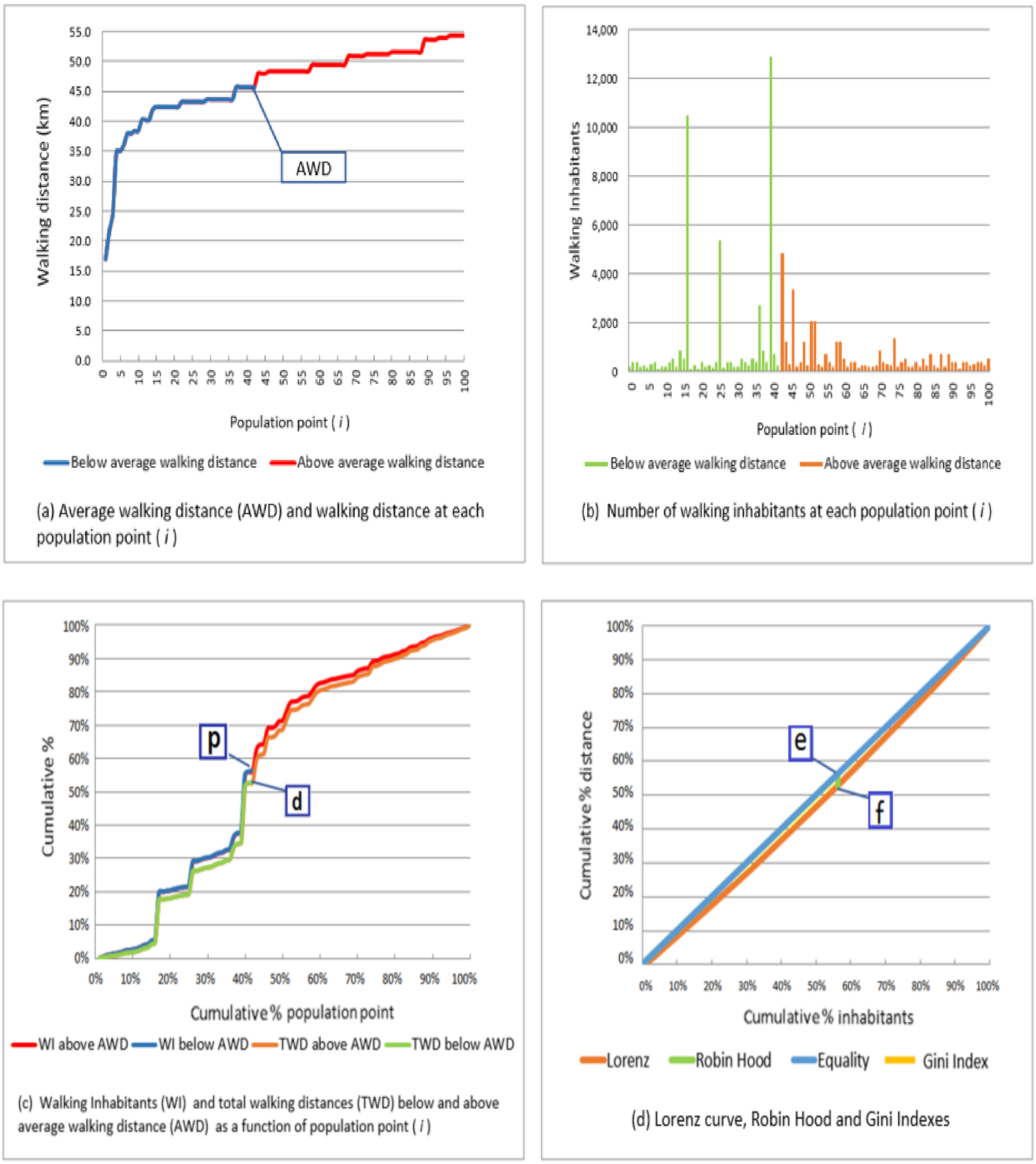
Demographic indicators		%	Walking inhabitants' service indicators	
Garissa inhabitants	452,218	100.00%	Time measure:	
Covered inhabitants	452,218	100.00%	Average walking time (hrs)	23.09
Walking inhabitants (covered inh.)	75,406	16.67%		
			Distance measures:	
Population points	100	100.00%	Average walking distance (km)	46.18
Covered population points	100	100.00%	Standard deviation of distance (km)	4.40
Potential distribution centers	50	100.00%	Variance of distance (km)	19.39
Open distribution centers	42	84.00%	Mean abs. deviation of distance (km)	52.29
Warehouse	1		Median walking distance (km)	45.69
Economic indicators		%	Pearson's 2° skewness coefficient	0.33
Total cost (KSh)	82,399,114	100.00%	Distribution left tail	0.53
Beneficiaries acces cost (KSh)	48,951,054	59.41%	Distribution right tail	0.47
World Food Program cost (kSh)	31,138,060	37.79%	Gini Index	0.048
Kenya's Red Cross cost (kSh)	2,310,000	2.80%	Hoover Index (Robin Hood)	0.034

In Table 14, observing the MRD threshold, we see that we have most population points, inhabitants, walking inhabitants and total walking distances above this threshold. The inverse was observed in the case of the previous solution. This observation combined with the low values for the Gini and Robin Hood Indexes confirm a previous observation regarding the use of the average walking distance as an objective function (see specific details in Section 5.1.4). This observation shows how some equality functions, in a minimization context, have to be reinterpreted and how low levels of service for the inhabitants present low levels of equality measures as well. In our study the distance can be considered a negative attribute that must be reduced in order to provide higher service levels for the beneficiaries.

Table 14. Service levels based on three distance thresholds for the solution obtained when minimizing the beneficiaries' variance of walking distance using generated data for r = 55 km.

Service level measure \ Distance Threshold (DT)		Average walking distance 46.18 km	10 km Threshold norm 10.00 km	Mid-radius 27.5 km
Total population points	100	100.00%	100.00%	100.00%
% below DT		42.00%	1.00%	3.00%
% above DT		58.00%	99.00%	97.00%
Total inhabitants	452,218	100.00%	100.00%	100.00%
% below DT		56.28%	0.22%	1.11%
% above DT		43.72%	99.78%	98.89%
Total walking inhabitants	75,406	100.00%	100.00%	100.00%
% below DT		56.28%	0.22%	1.11%
% above DT		43.72%	99.78%	98.89%
Total walking distances (Km)	3,482,050	100.00%	100.00%	100.00%
% below DT		52.85%	0.08%	0.53%
% above DT		47.15%	99.92%	99.47%

In Figure 40, we see the particularities of the obtained distribution. In Figure 40 (a), we observe how the average walking distance has been displaced to the right of the distribution compared with a similar representation in Figure 39 (a). The information in Figures 40 (a) and (b) explain how for this solution we have low level of service for the beneficiaries and higher values for the total walking distances. In Figure 40 (c), we observe the cumulative values for the inhabitants and for the total walking distances. We can see how these two lines are very close to each other, leaving a very small area between them and approximating the shape of an oblique line. The graphic representations of the Lorenz curve, Gini and Robin Hood Indexes shown in Figure 40 (d) also confirm the results shown in Figure 40 (c). The yellow area representing the Gini Index is very small and even difficult to be observed. It is also possible to see the points "p" and "d", representing the cumulative values for the inhabitants and cumulative values for the total walking distances, showing again the relationship of the average (mean) with other equality measures.



(a) Average walking distance (AWD) and walking distance at each population point (i)

(b) Number of walking inhabitants at each population point (i)

(c) Walking Inhabitants (WI) and total walking distances (TWD) below and above average walking distance (AWD) as a function of population point (i)

(d) Lorenz curve, Robin Hood and Gini Indexes

Figure 40. Characteristics of the distribution obtained when minimizing the beneficiaries' variance of walking distance using generated data for r = 55 km.

Chapter 6: Inclusion of cost constraints

In the present chapter, we will explore the inclusion of cost constraints (CC). In the previous chapter we have analysed the objective functions without using cost restrictions. For the solutions discussed in this chapter, we decided to constraint the different equality functions using four different percentage increases over the minimum stakeholder costs; these solutions represent a comparative standard. We think that these four percentages can give us a clear picture of the behaviour of the equality objectives under cost restrictions. We did not examined higher percentage increases in costs, as in humanitarian food distribution networks the costs are a relevant factor that needs to be taken into account with special care.

We also need to consider that in addition to the four percentages analysed, we have the solutions representing the two extreme solutions that can be used to have a complete picture about each analysed measure. On the extreme left side of the figures, (e.g., Figure 41(a)), we have the minimum cost for the corresponding coverage radius and on the extreme right side we have the solution obtained using the objective function without cost constraints. These solutions were presented in the previous chapter. The costs for the solution on the right side represent the maximum costs over which each objective function can be constrained in order to obtain a minimum or a maximum value for the corresponding objective function.

We analysed the different percentage increases in cost for each of the five coverage radii. However, in this section, we will present the results comparing only two coverage radii. After presenting the first analysis for these radii, we will present the effects of cost constraints on different coverage radii for the cost increase considered the best in the previous analysis. In Appendices 2, 3 and 4, it is possible to find detailed information regarding the values obtained using the different objective functions, as well as additional indicators for the solutions presented in this chapter.

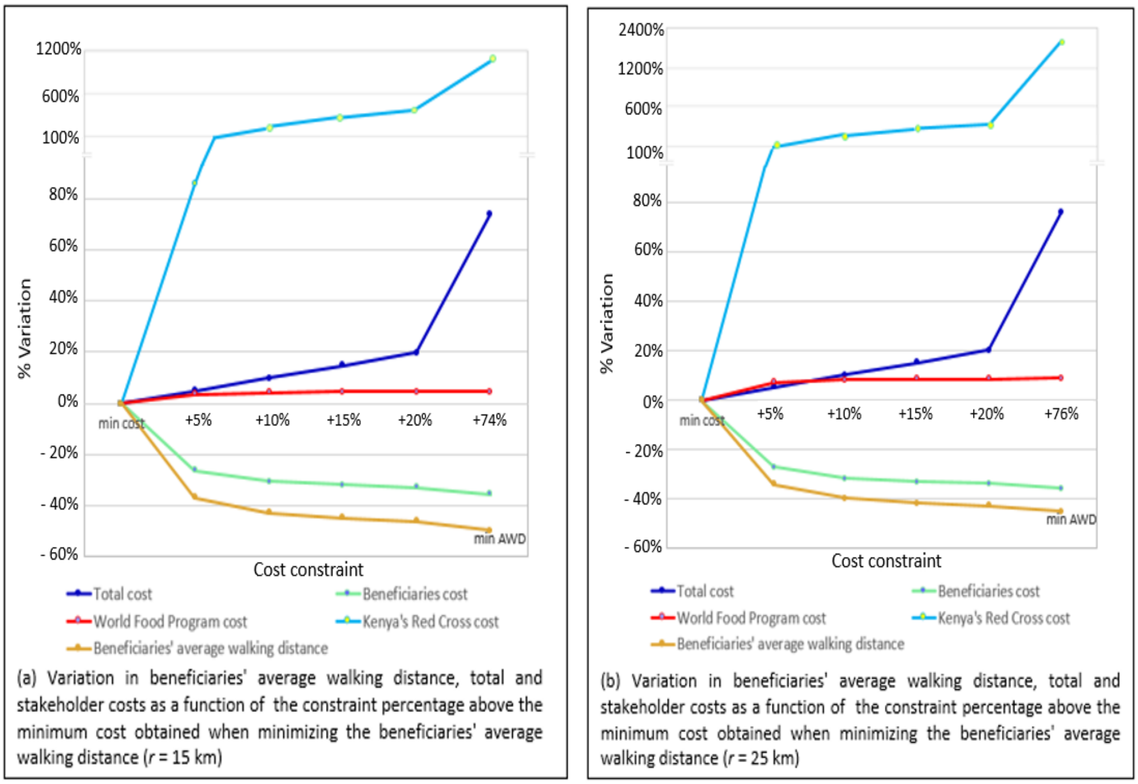
The graphics presented in this chapter have the same structure of the corresponding graphics in the previous chapter. Following these analyses, and in the case of the non-aggregated data, we will present a detailed analysis for the coverage radius of 15 km subject to the cost constraint, a scenario consider as the best case for each particular objective function. This analysis will be useful to have a better understanding of specific solutions using different equality functions. Similar analyses were presented in the previous chapter regarding the solution obtained using the non-aggregated data. We will also present this detailed analysis for the solution considering the variance as objective function and using the generated data.

6.1 Results obtained with non-aggregated data and cost constraints

In the present section, we present analysis comparing different percentages of cost increases and also compare the effects on different coverage radii. Additionally, as it was the case in the preceding chapter, we will present more detailed analyses for specific solutions using non-aggregated data, considering again the space limitations and the highest accuracy of the non-aggregated data. The logic and structure is the same as in the previous chapter.

6.1.1 Results obtained when minimizing the beneficiaries' average walking distance

In Figure 41, we observe an increase in the percentage variation in cost as we increase the cost constraint above the minimum cost. We see that in both solutions, the minimum AWD can be obtained with high increases in cost, 74% for a radius of 15 km and 77% for a radius of 25 km. We also observe less variability for the WFP's cost for a coverage radius of 15 km. Observing the curves representing the percent variation on the average walking distance, we see that the highest reduction in the average walking distance (orange line), approximately 40% (vertical axis), is obtained when we increase the costs between 5% and 10% (horizontal axis), when compared with the solution obtained when minimizing the stakeholder costs. Increases higher than 10% have only a marginal effect on the value of the AWD; we can then conclude that higher increases in costs are non-desirable for this objective function, since with low increases in cost (5% to 10%) we obtain solutions with a value close to the minimum walking distance. Therefore, it is possible to improve the service for the beneficiaries and taking into account the cost factor. Additional information and more scenarios can be found in Appendix 2.2.



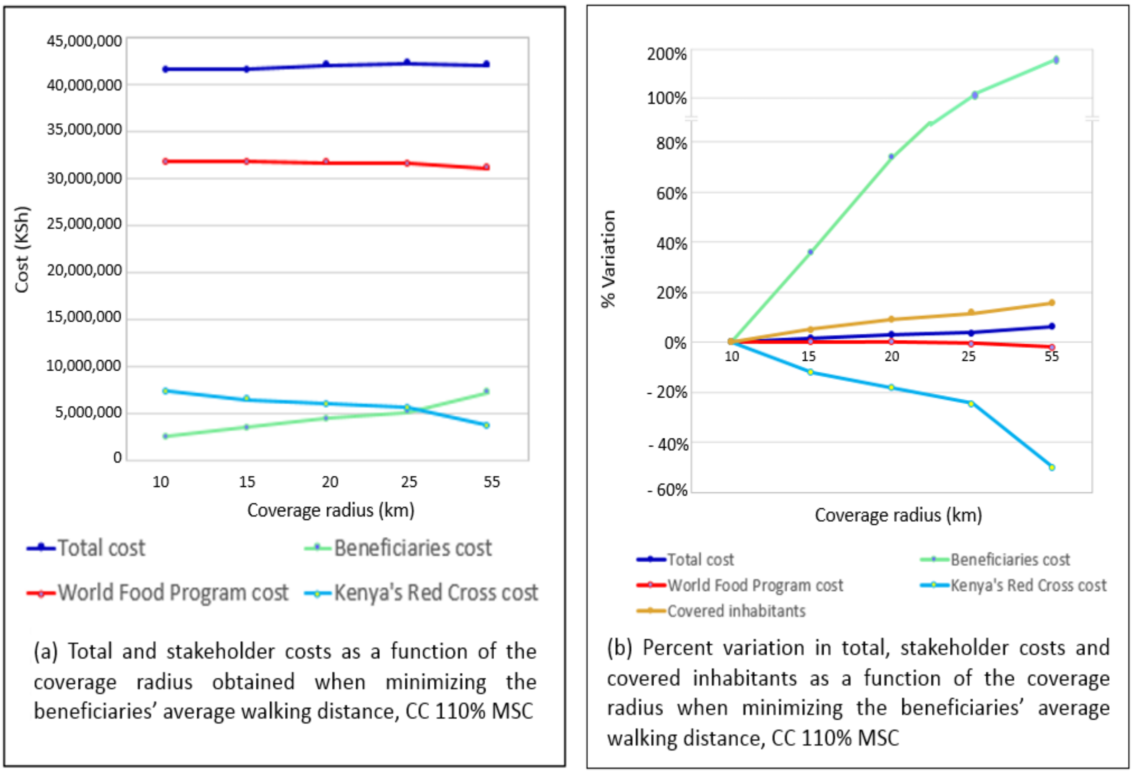
(a) Variation in beneficiaries' average walking distance, total and stakeholder costs as a function of the constraint percentage above the minimum cost obtained when minimizing the beneficiaries' average walking distance (*r* = 15 km)

(b) Variation in beneficiaries' average walking distance, total and stakeholder costs as a function of the constraint percentage above the minimum cost obtained when minimizing the beneficiaries' average walking distance (*r* = 25 km)

Figure 41. Variation in beneficiaries' average walking distance, total and stakeholder costs as a function of the CC above the MSC when minimizing the beneficiaries' average walking distance using NAD for r = 15 km and r = 25 km.

In Figure 42 (a), we present the costs obtained for different coverage radii a cost constraint of 10% above the minimum stakeholder cost, as this percentage was identified as suitable for this particular objective function. We observe that the KRC's cost component, which generated the higher increases in cost when using the same objective function without cost constraint, has been effectively controlled and reduced compared with the results minimizing the beneficiaries' average walking distance, as shown in Figure 25 (a). We also observe how the total costs have been controlled and are not far from the results obtained when minimizing the stakeholder costs, as observed in Figure 22 (a). The slightly low values for the beneficiaries' access cost observed in Figure 42 (a), compared with Figure 22 (a), show that lower travel distances are used. The higher values for the KRC cost in Figure 42 (a), compared with the corresponding values in Figure 22 (a), show that the origin of the improvement in service levels for the beneficiaries is due to the higher number of open distribution centers. Figure 42 (b), shows the percent variation in costs and how for large coverage radii we have substantial increases in the beneficiaries' access cost. However, the decrease in the KRC's cost compensates the increase in the beneficiaries' access cost, resulting in a controlled total cost shown by small percent increases for the total costs curve in the case of the larger coverage radii, as seen in Figure 42 (a). In the solution obtained using the average walking distance as objective function, without cost constraint (Figure 25), the KRC's cost follows a flat curve; the increase in the Beneficiaries' cost is not compensated by an opposite reduction in the KRC's cost, resulting in higher total costs observed in Figure 25. Hence, we can conclude that cost constraints effectively improve the service levels when using the beneficiaries' walking distance as an objective function; the improvement is explained by the additional distribution centers open.

For the solutions shown in Figure 42, the average walking distances were 1.31, 2.07, 2.82, 3.42 and 5.15 km. The CPLEX running times were 2,399.02, 9,299.01, 8,454.14 42,300.90 and 219,062 seconds for the coverage radiuses of 10, 15, 20, 25 and 55 km, respectively. Additional information as well as additional solutions using the beneficiaries' average of walking distance as objective function can be found in Appendix 2.2.



(a) Total and stakeholder costs as a function of the coverage radius obtained when minimizing the beneficiaries' average walking distance, CC 110% MSC

(b) Percent variation in total, stakeholder costs and covered inhabitants as a function of the coverage radius when minimizing the beneficiaries' average walking distance, CC 110% MSC

Figure 42. Costs, percentage of variation in costs and covered inhabitants for different coverage radii when minimizing the beneficiaries' average walking distance using NAD, CC 110 % MSC.

6.1.2 Results obtained when minimizing the beneficiaries' average walking distance, CC 110% MSC for *r* = 15 km

In Figure 43, we observe the map of the network for a coverage radius of 15 km and a cost constraint of 10% above the minimum cost. In this case, we observe the reduction in the number of open distribution centers, compared with the similar solution without cost constraints in Figure 26. For the present case, the distribution centers were highlighted by triangular green shapes. Comparing this solution with the solution using the minimization objective for the stakeholder costs in Figure 23, where it is possible to see only 115 open DCs, we have 303 open DCs in the present solution. In the case of the minimization of the average walking distance without cost constraint, in Figure 26, we had all 1,459 DCs open.

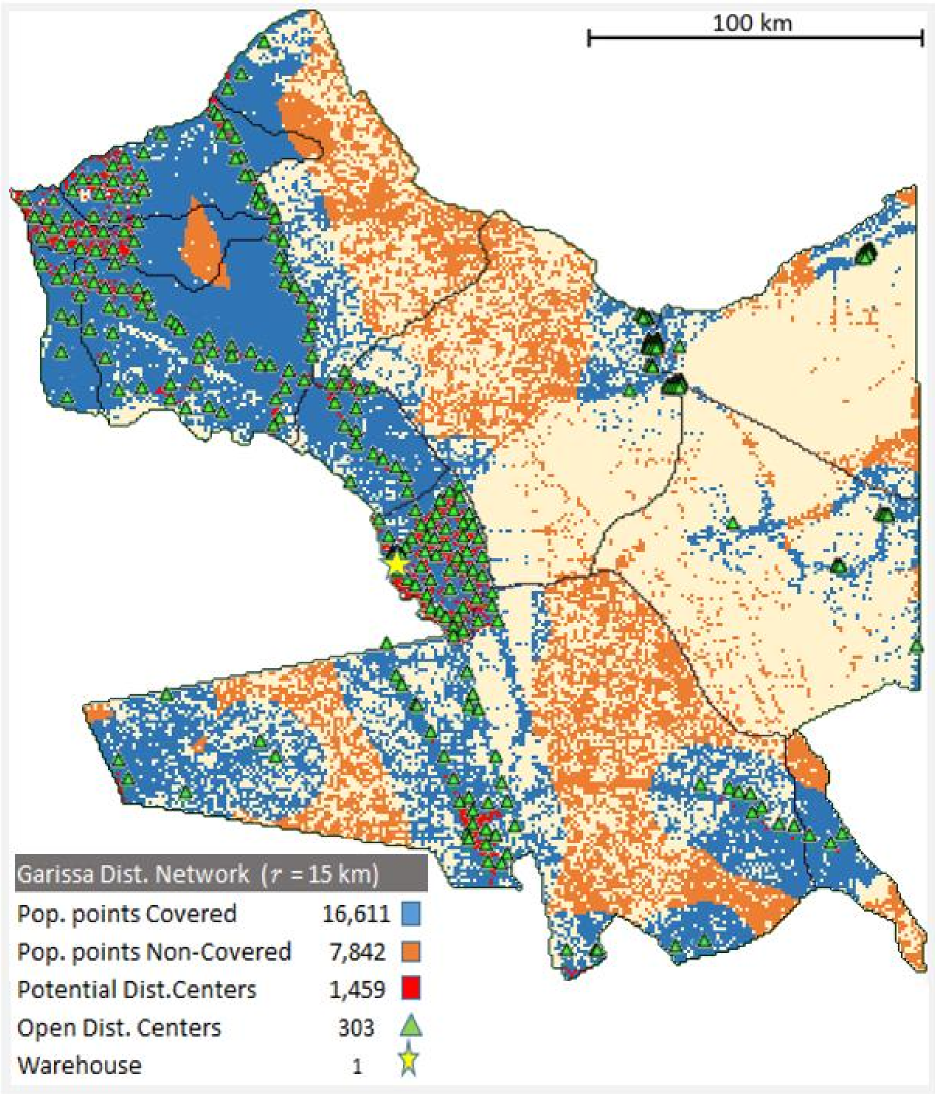


Figure 43. Map for the solution obtained when minimizing the beneficiaries' average walking distance using NAD, CC 110% MSC for *r* =15 km.

In Table 15, we observe indicators for the present solution. Regarding the economic indicators, we observe how the beneficiaries' access cost and the WFP's cost have very close values compared with the values obtained with the solution obtained when minimizing the average walking distance discussed in the previous chapter, see Table 5. For the KRC's cost, the value for the present solution is approximately five times lower (6.4 million KSh vs. 30.98 million KSh), showing how the cost constraints particularly limit this cost component. We also observe low values for the average walking time and for the average walking distance, with 1.04 hours and 2.07 km, respectively. These values are very close to those obtained with the corresponding solutions without costs restrictions in the previous chapter and with values of 0.91 hours and 1.82 km.

In addition, among other indicators, we also observe slightly lower values for the Pearson skewness coefficient and for the Gini and Robin Hood Indexes, compared with similar values in the corresponding solution obtained when minimizing the average walking distance without costs constraints. To interpret these values we need to take into account that we are minimizing a negative attribute (distance) and we obtained a satisfactory solution in terms of service level and costs, even when considering the small reduction in the equality indicators.

Table 15. Characteristics of the solution obtained when minimizing the beneficiaries' average walking distance using NAD, CC 110% MSC for r = 15 km.

Demographic indicators		%	Walking inhabitants' service indicators	
Garissa inhabitants	452,218	100.00%	Time measure:	
Covered inhabitants	386,269	85.42%	Average walking time (hrs)	1.04
Walking inhabitants (covered inh.)	71,309	18.46%		
Pop. points (including 1 -2 Inhabitants)	47,242		Distance measures:	
Population points (> 3 Inhabitants)	24,453	100.00%	Average walking distance (km)	2.07
Covered population points	16,611	67.93%	Standard deviation of distance (km)	3.54
Potential distribution centers	1,459	100.00%	Variance of distance (km)	12.56
Open distribution centers	303	20.77%	Mean absolute deviation of distance (km)	1.37
Warehouse	1		Median walking distance (km)	0.00
Economic indicators		%	Pearson's 2° skewness coefficient	1.75
Total cost (KSh)	41,653,639	100.00%	Distribution left tail	0.11
Beneficiaries acces cost (KSh)	3,438,419	8.25%	Distribution right tail	0.89
World Food Program cost (kSh)	31,781,317	76.30%	Gini Index	0.761
Kenya's Red Cross cost (kSh)	6,433,902	15.45%	Hoover Index (Robin Hood)	0.619

In Table 16, we observe service indicators and we particularly see the improvement in the total walking distances below the three distance thresholds. Compared with the indicators in Table 6 for the corresponding solution for the average walking distance minimization without cost constraints, the improvements in these indicators account for the substantial cost reductions observed in the present solution. We also observe small reductions in the number of inhabitants and walking inhabitants that are below the average walking distance threshold compared with similar values in Table 6. These small changes account for small additional walking distances, but only for people that are really well served considering the small distances they travel. If we consider the improvements in average walking distance, we can clearly observe how the principle of transfers studied in chapter 2 is applied in this case. This principle implies the transfer of a positive attribute (income) from a richer to a poorer individual in economics. However, in our case it will be more accurate to say: the subtraction of a negative attribute, from which the addition does not generate wellbeing (distance in our case), from those who are worse-off to those who are better-off. Indeed, in our case, this transfer does not affect those who are better-off, but allowed improvements in service levels for those who are worse-off.

Table 16. Service levels based on three distance thresholds for the solution obtained when minimizing the beneficiaries' average walking distance using NAD, CC 110% MSC for $r = 15$ km.

Service level measure \ Distance Threshold (DT)		Average walking distance 2.07 km	10 km distance threshold 10.00 km	Mid-radius 7.5 km
Number of population points	16,611	100.00%	100.00%	100.00%
% below DT		22.79%	79.26%	67.97%
% above DT		77.21%	20.74%	32.03%
Number of inhabitants	386,269	100.00%	100.00%	100.00%
% below DT		78.02%	94.98%	92.11%
% above DT		21.98%	5.02%	7.89%
Number of walking inhabitants	71,309	100.00%	100.00%	100.00%
% below DT		72.66%	93.48%	89.81%
% above DT		27.34%	6.52%	10.19%
Total walking distances (km)	147,687	100.00%	100.00%	100.00%
% below DT		10.76%	60.84%	45.25%
% above DT		89.24%	39.16%	54.75%

In Figure 44, we observe the characteristics of the distribution for the present solution. In Figure 44 (a), we observe almost the same AWD compared with the AWD in Figure 27 (a). Regarding the number of walking inhabitants in Figure 44 (b), we observe how, in this case, we have more walking inhabitants above the AWD and that the pattern on the right hand side of the distribution is not strictly flat as it was the case for the corresponding solution but without costs constraints, see Figure 27 (a). This pattern also indicates a slight increase in total walking distance for some populations in order to compensate for the reduction in the number of distribution centers open in the present solution.

In Figures 44 (c) and 44 (d), we can also see very similar patterns as those observed in Figures 27 (c) and 27 (d), but in this case the area encircled by the two lines representing the cumulative inhabitants and cumulative walking distances shown in Figure 27 (c), as well as the area showing the Gini Index in Figure 44 (d), are slightly smaller. These small areas show small reductions in the equality measures, result also in small variations in terms of service for the whole population as shown by the very small increases in the AWD. The increase in AWD is non-significative, but at the same time it yields costs reductions, as verified by the reductions of the KRC's cost, compared with the solution obtained without costs restrictions. We can also observe how the corresponding points "p" and "d" of Figure 44 (c) and points "e" and "f" of Figure 44 (d) are separated by the same distance, confirming again the value of Figure 44 (c) to represent equality measures; in this case under cost restrictions.

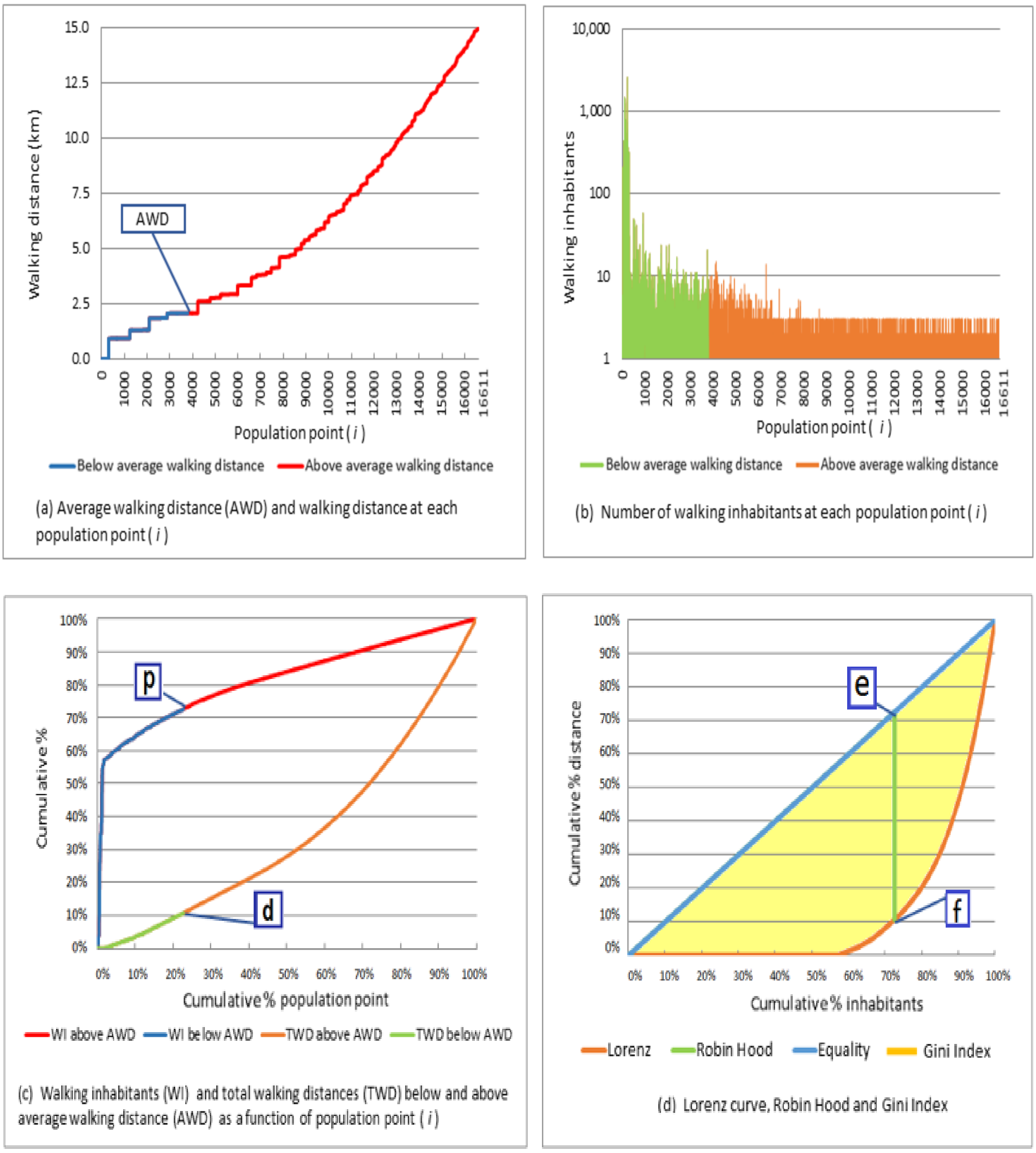
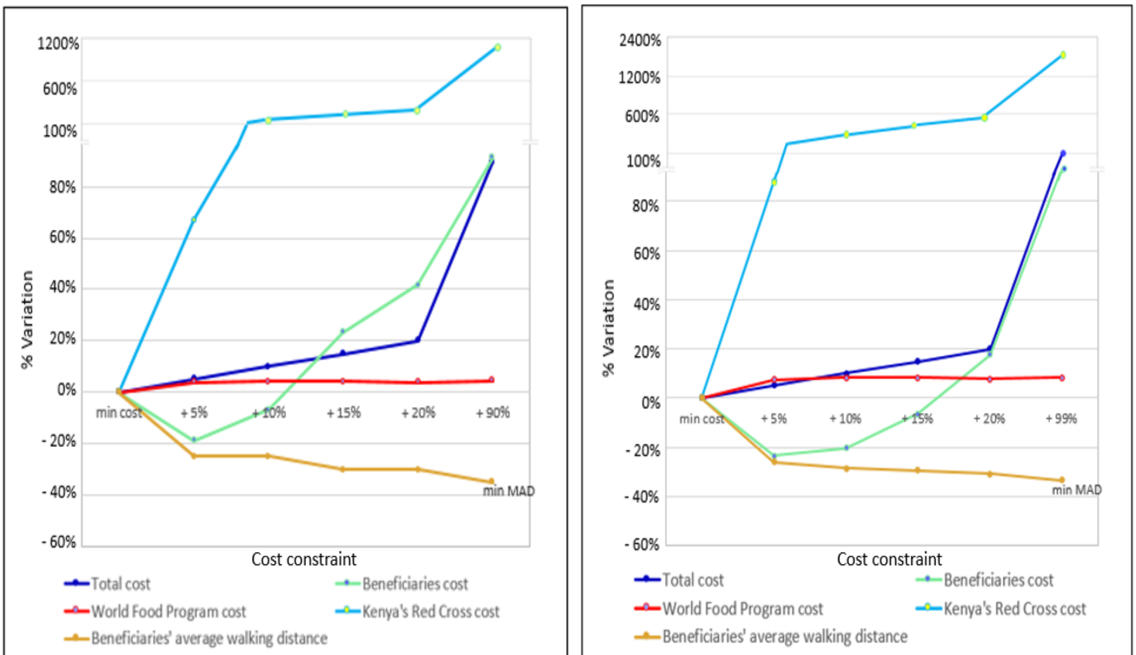


Figure 44. Characteristics of the distribution obtained when minimizing the beneficiaries' average walking distance using NAD, CC 110% MSC for r = 15 km.

6.1.3 Results obtained when minimizing the beneficiaries' mean absolute deviation of walking distance

In Figure 45, we observe an increase in the percentage of variation in cost as we increase the cost constraint above the minimum stakeholder costs. We see that in both solutions, for the walking distance, the minimum MAD is obtained when we increase the costs by 90% and 99% for the radii of 15 km and 25 km, respectively (solutions in the extreme left). However with cost increases of only 5% (horizontal axis), it is possible to obtain significative reductions in the MAD compared with the solution obtained while minimizing the stakeholder cost. For increases higher than 5%, we mainly observe marginal effects on the value of the MAD, but we also observe how the curve for the beneficiaries' access cost raises consistently after the 5% cost increase, and how, for the largest coverage radius this cost becomes particularly very high.

We can conclude that increases in cost higher than 5% are non-desirable for this objective function, since we obtain most of the possible reduction of the MAD with a 5% cost increase. This low percent increase in costs will also improve the service for the beneficiaries. However, in this case, the costs need to be controlled with special precaution, because cost increases of more than the 5%, particularly affect the beneficiaries. In the case of the average walking distance minimization, we observed that higher increases in cost do not have a negative effect on the beneficiaries. We also observed low gains in the reduction of the corresponding objective function. It is possible to find additional information and more scenarios in Appendix 2.3.



(a) Variation in beneficiaries' mean absolute deviation of walking distance, total and stakeholder costs as a function of the constraint percentage above the minimum cost obtained when minimizing the beneficiaries' mean absolute deviation of walking distance (r = 15 km)

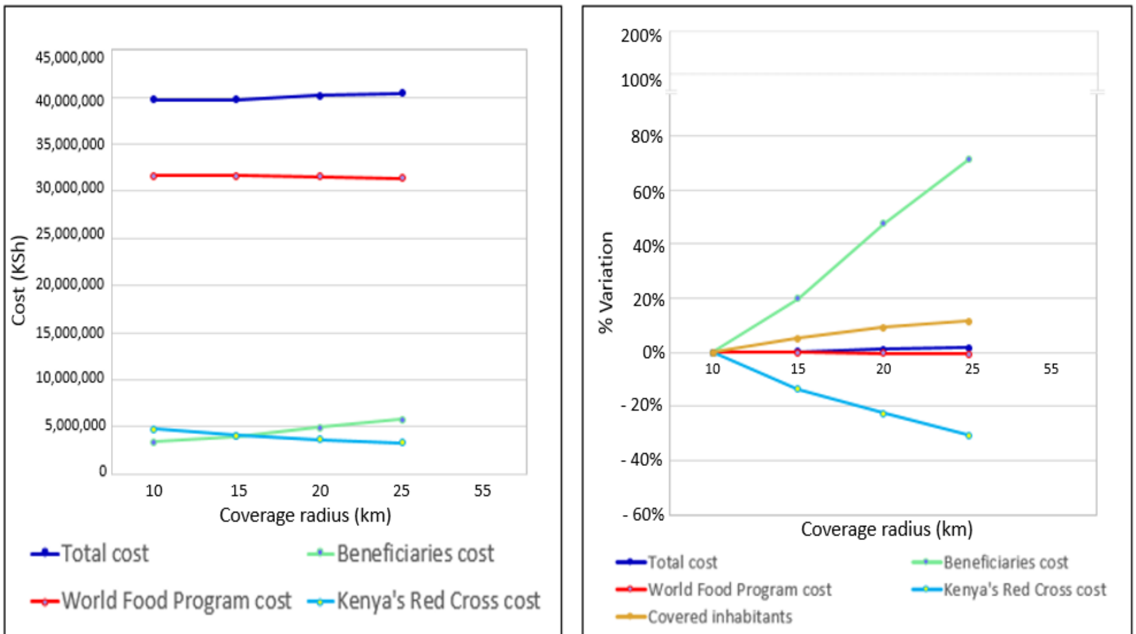
(b) Variation in beneficiaries' mean absolute deviation of walking distance, total and stakeholder costs as a function of the constraint percentage above the minimum cost obtained when minimizing the beneficiaries' mean absolute deviation of walking distance (r = 25km)

Figure 45. Variation in beneficiaries' mean absolute deviation of walking distance, total and stakeholder costs as a function of the CC above the MSC obtained when minimizing the beneficiaries' mean absolute deviation of walking distance using NAD for r = 15 km and r = 25 km.

In Figure 46 (a), we observe the costs obtained for different coverage radii, except for the coverage radius of 55 km, for which we were not able to obtain a solution. In this case we are using a cost constraint of 5% above the minimum stakeholder cost, as this percentage was identified as a suitable increase for this particular objective function. As in the previous case, we also observe how the total costs have been controlled and are not far from the results obtained when minimizing the stakeholder costs, as shown in Figure 22 (a). In the case of KRC's cost and beneficiaries costs, we also observe very close patterns to those obtained when minimizing the average walking distance using a cost constraint, see Figure 41; showing again how costs constraints can effectively control the KRC's cost comparing with the equivalent solution without costs constraints in Figure 28 (a).

Figure 46 (b) shows the percent variation in cost and we see that for large coverage radii, we have significant increases in beneficiaries' access cost. However, the decrease in KRC's cost compensates the previous tendency as it happens for the previous objective functions, resulting in a controlled total cost. For the solutions obtained when minimizing the MAD without cost constraints, this compensation is not observed because we did not impose limits on the number of distribution centers that can be open, as can be seen by the flat curve representing the percent variation of KRC's cost in Figure 28 (b). For the present function, we can conclude that a small increase in costs, can improve service levels for the beneficiaries, and this particularly when using small coverage radii, as is the case of 10 km or 15 km coverage radius.

For the solutions shown below, the values for the MAD were 0.83, 1.40, 2.01 and 2.51 km. The CPLEX running times were 22,251.90, 28,523.80, 29,921.80 and 91,398.70 seconds for the coverage radii of 10, 15, 20 and 25 km, respectively. Additional information as well as additional solutions using the mean absolute deviation of walking distance as an objective function can be found in Appendix 2.3.



(a) Total and stakeholder costs as a function of the coverage radius obtained when minimizing the beneficiaries' mean absolute deviation of walking distance, CC 105% MSC

(b) Variation in total, stakeholder costs and covered inhabitants as a function of the coverage radius when minimizing the beneficiaries' mean absolute deviation of walking distance, CC 105% MSC

Figure 46. Costs, percent variation in costs and covered inhabitants for different coverage radius when minimizing the beneficiaries' mean absolute deviation of walking distance using NAD, CC 105% MSC.

6.1.4 Results obtained when minimizing beneficiaries' mean absolute deviation of walking distance, CC 105% MSC for *r* = 15 km.

In Figure 47, we observe the map for the solution obtained when minimizing the mean absolute deviation of walking distance by using a cost constraint of 5% above the minimum stakeholder cost. In this case, we have 192 open DCs, compared with 1,435 for the corresponding solution without costs constraints shown in Figure 29. We, however, have less open DCs than in the previous solution obtained when minimizing the average walking distance with a cost constraint of 10% (Figure 43), with 303 DCs open. The difference in this case is explained by a 5% additional allowed cost compared with the previous case, the 10% allowed cost increase is less restrictive for the total number of DCs that can be opened.

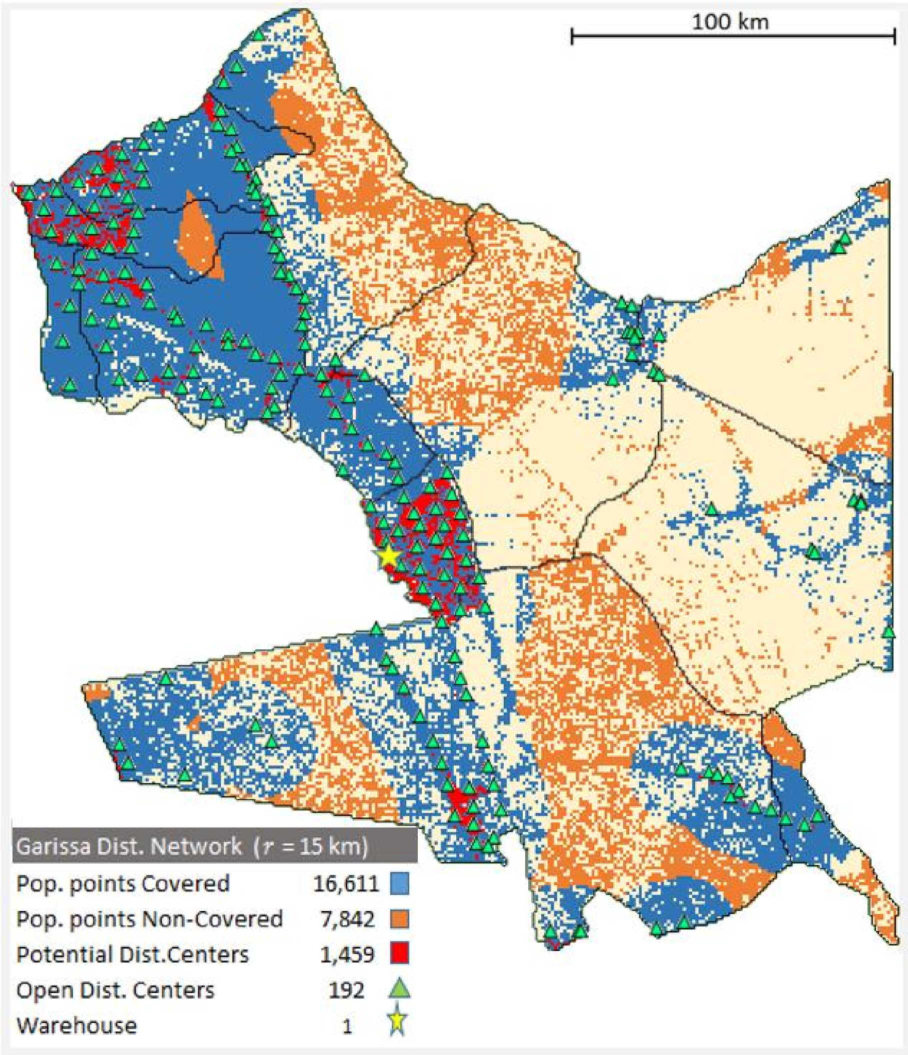


Figure 47. Map for the solution obtained when minimizing beneficiaries' mean absolute deviation of walking distance using NAD, CC 105% MSC for *r* = 15 km.

In Table 17, we see how the main cost are borne by the WFP with approximately 31.6 million KSh; this amount is very close to the value obtained when minimizing the stakeholder cost that is 30.4 million KSh (Table 3). Observing this and other solutions, we see the stability of the WFP cost in most of the solutions. In this case, the beneficiaries' access and the KRC's costs are very close; 4.01 million KSh and 4.06 million KSh, respectively. Observing Table 3, the corresponding values are 4.95 and 2.44 million KSh. We see how the increase in the KRC cost allows a small reduction in the total beneficiaries' access costs. However, the small beneficiaries' cost reduction combined with a higher number of open DCs allow for shorter distances for the walking inhabitants, especially those living far away and resulting in improved service indicators for the walking inhabitants, as we can see when comparing the values in Table 17 with the corresponding values in Table 3.

Comparing this solution with the solution obtained when minimizing the beneficiaries' average walking distance using a costs constraint (Table 15), we observe a higher contribution of the beneficiaries' access cost, representing in this case 10.10% of the total coat (Table 17) and 8.25% previously (Table 15). KRC's cost for the present case is 10.25%, whereas in Table 15 this value is 15.45%.

Table 17. Characteristics of the solution obtained when minimizing the beneficiaries' mean absolute deviation of walking distance using NAD, CC 105% MSC for r = 15 km.

Demographic indicators		%	Walking inhabitants' service indicators	
Garissa inhabitants	452,218	100.00%	Time measure:	
Covered inhabitants	386,269	85.42%	Average walking time (hrs)	1.33
Walking inhabitants (covered inh.)	71,309	18.46%		
Pop. points (including 1 -2 Inhabitants)	47,242		Distance measures:	
Population points (> 3 Inhabitants)	24,453	100.00%	Average walking distance (km)	2.66
Covered population points	16,611	67.93%	Standard deviation of distance (km)	3.34
Potential distribution centers	1,459	100.00%	Variance of distance (km)	11.12
Open distribution centers	192	13.16%	Mean absolute deviation of distance (km)	1.40
Warehouse	1		Median walking distance (km)	1.31
Economic indicators		%	Pearson's 2° skewness coefficient	1.22
Total cost (KSh)	39,757,584	100.00%	Distribution left tail	0.25
Beneficiaries acces cost (KSh)	4,014,202	10.10%	Distribution right tail	0.75
World Food Program cost (kSh)	31,666,454	79.65%	Gini Index	0.585
Kenya's Red Cross cost (kSh)	4,076,928	10.25%	Hoover Index (Robin Hood)	0.448

Results shown in Table 18 also confirm the improvement of service indicators. For example, we can see higher percentages below the average walking distance or the mid-radius threshold distance compared with the values of Table 4, corresponding to the solution for the stakeholder costs minimization using a similar coverage radius.

Table 18. Service levels based on three distance thresholds for the solution obtained when minimizing the beneficiaries' mean absolute deviation of walking distance using NAD, CC 105% MSC for r = 15 km.

Distance Threshold (DT) / Service level measure		Average walking distance 2.66 km	10 km distance threshold 10.00 km	Mid-radius 7.5 km
Number of population points	16,611	100.00%	100.00%	100.00%
% below DT		22.95%	79.15%	67.73%
% above DT		77.05%	20.85%	32.27%
Number of inhabitants	386,269	100.00%	100.00%	100.00%
% below DT		75.01%	94.92%	92.01%
% above DT		24.99%	5.08%	7.99%
Number of walking inhabitants	71,309	100.00%	100.00%	100.00%
% below DT		69.94%	93.41%	89.69%
% above DT		30.06%	6.59%	10.31%
Total walking distances (km)	189,947	100.00%	100.00%	100.00%
% below DT		25.15%	69.22%	56.98%
% above DT		74.85%	30.78%	43.02%

As we compare Figure 48 (a), with that of the solution obtained when minimizing the MAD in Figure 44 (a), we observe a slight increase in the AWD. On the other hand, when observing Figure 48 (b), we see that the population points with more inhabitants are less concentrated towards the left side of the distribution, compared with the solution obtained when minimizing the AWD distance using a cost constraint, as can be seen in Figure 44 (b). Combining the information of walking inhabitants in Figure 48 (b) with the walking distances in Figure 48 (a), we observe an increase in total walking distances. Figures 48 (c) and 48 (d) show close patterns to those observed in the previous case and can be interpreted in a similar way. Based on the previous and additional information presented in Appendices 2.2 and 2.3 we can see that, when using costs constraints, the AWD minimization promotes higher reductions in beneficiaries' access costs and the mean absolute deviation of walking distance has more impact on KRC's cost, even if different cost constraints are considered.

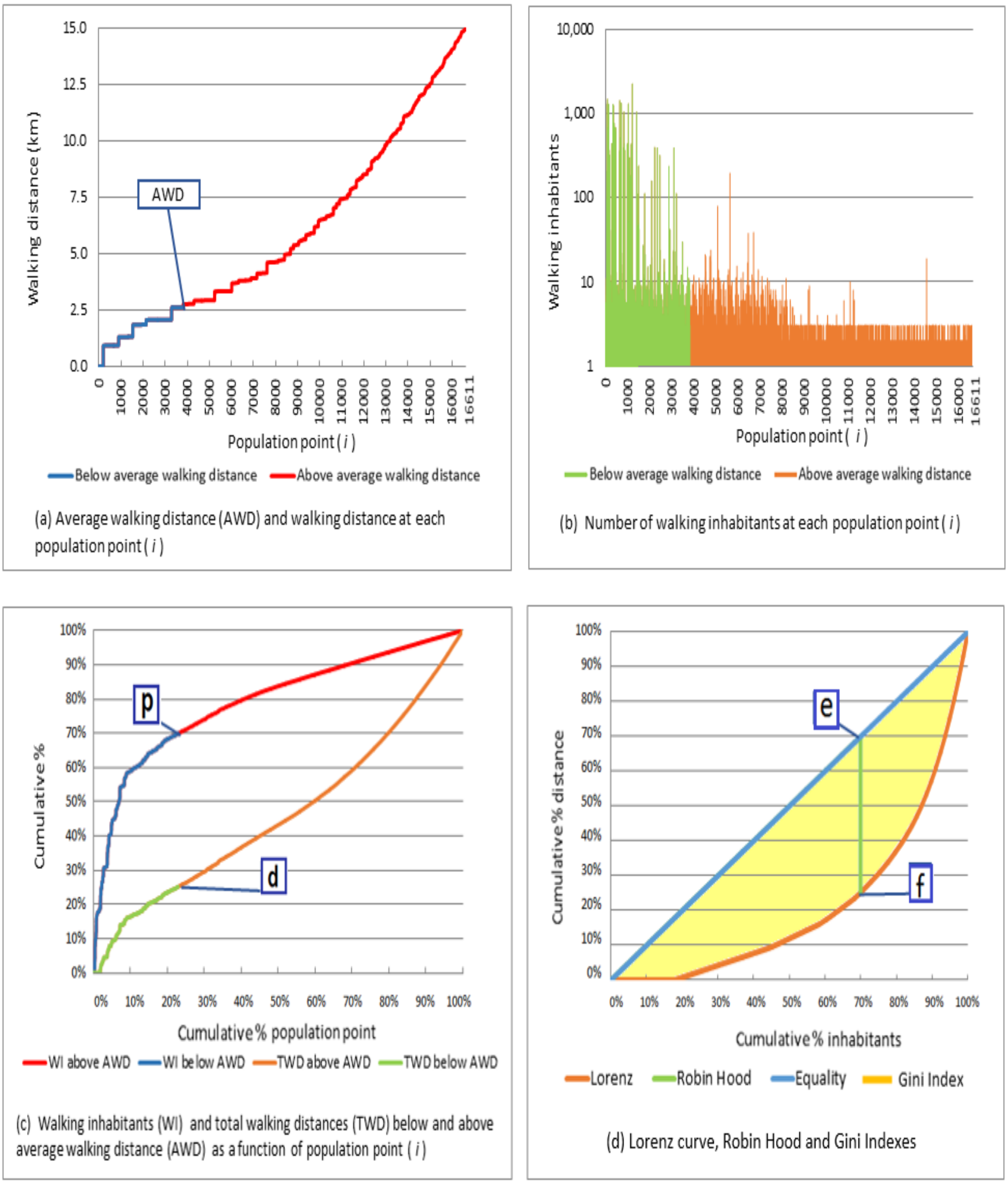
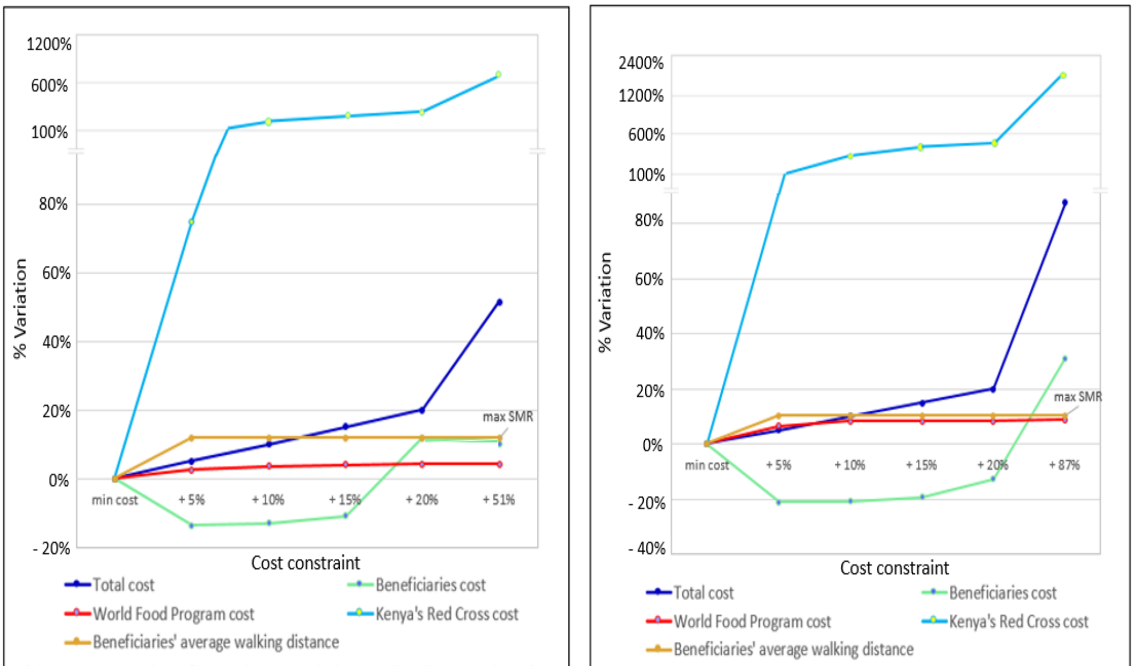


Figure 48. Characteristics of the distribution obtained when minimizing the beneficiaries' mean absolute deviation of walking distance using NAD, CC 105% MSC for r = 15 km.

6.1.5 Results obtained when maximizing the beneficiaries' standard service below the mid-radius distance threshold.

In Figure 49, we observe the percentage of variation in costs as we increase the cost constraint above the minimum stakeholder costs. For both solutions, increases in cost of 5% allow to obtain maximum service levels below the mid-radius distance threshold (SMR). For increases in cost higher than 5%, the curve is practically flat, showing no additional improvements on this indicator. We can conclude that the marginal gains are quite small and not worthy considering the highest costs incurred. The previous cost can reach 51% and 87% for the coverage radii of 15 km and 25 km, respectively. Additional scenarios and information can be found in Appendix 2.4. We also observe the high variation in cost for the KRC's and the low variability for the curve representing the WFP's cost. For the case of the beneficiaries' access cost we observe how this cost raises considerably after the allowed 5% cost increase. Based on these figures and additional information that can be found in Appendix 2.4, we conclude that high increases in cost are not desirable for this objective function, since we obtain solutions very close to the optimal maximum service below the mid-radius threshold with a 5% cost increase over the minimum stakeholder cost. Thus, similarly as it was observed for the solutions when minimizing the mean absolute deviation of walking distance using cost constraints, we need to mention that it is very important to allow only small percentage cost increases in order to avoid negative effects on the beneficiaries.



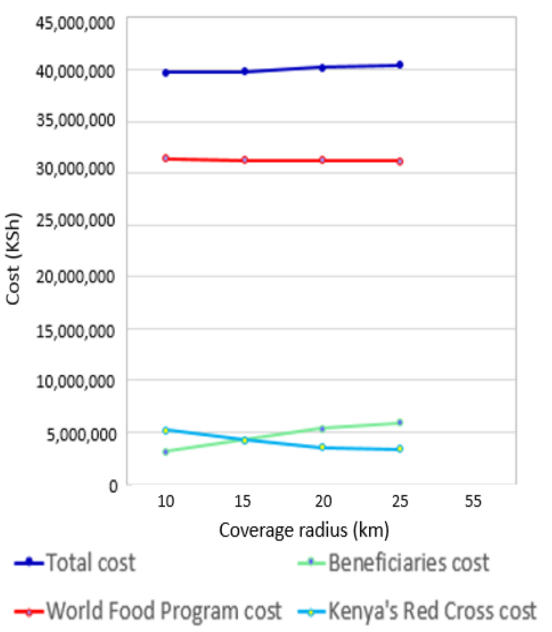
(a) Variation in beneficiaries' service below mid-radius, total and stakeholder costs as a function of the constraint percentage above the minimum cost obtained when maximizing the beneficiaries' standard service using the mid radius threshold (r = 15 km)

(b) Variation in beneficiaries' service below mid-radius, total and stakeholder costs as a function of the constraint percentage above the minimum cost obtained when maximizing the beneficiaries' standard service using the mid radius threshold (r = 25 km)

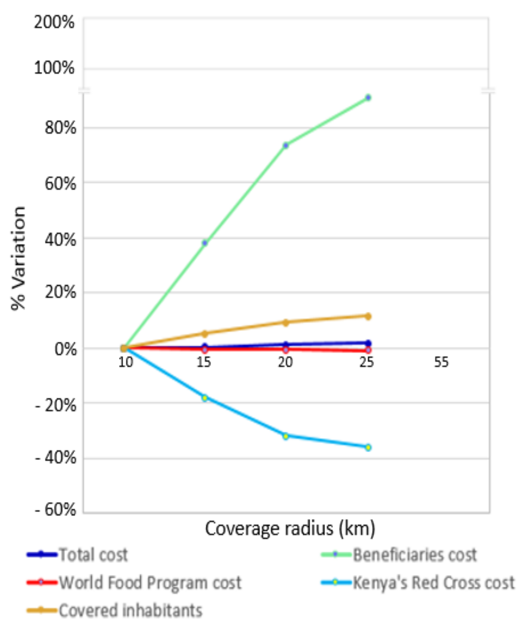
Figure 49. Variation in beneficiaries' standard service below the MRD, total and stakeholder costs as a function of the CC above the MSC when maximizing the beneficiaries' service below the MRD using NAD for r = 15 km and r = 25 km.

In Figure 50 (a), we observe the cost obtained for the studied coverage radii, except for the coverage radius of 55 km, in which case we were not able to obtain a solution. We observe how the patterns for the curves are very similar to those obtained when minimizing the mean absolute deviation of walking distance using a cost constraint, see Figure 46 (a). In Figure 50 (b), observing the percent variation in costs and comparing it with similar values in Figure 46 (b), it is also possible to see similar patterns for all the cost components, but in this case we have higher increases for the beneficiaries' access cost as we also increase the coverage radius and correspondingly, we observe higher reductions for the KRC's cost. The patterns observed for this solution and for the solutions with the two previous objective functions using cost constraints, show how the total costs are controlled by an increase in beneficiaries' access cost and a corresponding decrease in KRC's cost. This variation in cost also increases as we increase the coverage radius. In addition, it is also possible to confirm the stability of the costs for the WFP independently of the coverage radius used or of the objective function considered. We can also conclude that for the beneficiaries the best solutions are found using small coverage radii and for the KRC the best solutions are obtained using the larger coverage radius. For the present objective function and under a cost constraint of 5% above the minimum stakeholder costs, an equilibrium for both the beneficiaries' access cost and the KRC's cost, can be represented by the solution obtained with a coverage radius of 15 km, as we can see on the intersection of both curves in Figure 50 (a).

For the solutions shown below, the values for the inhabitants below the MRD threshold were 91.04, 89.93, 89.74 and 89.12 %. The CPLEX running times were 2,268.96, 22,722.90, 12,529.39 and 198,251 seconds for the coverage radii of 10, 15, 20 and 25 km, respectively. Additional information as well as additional solutions using the beneficiaries' standard service as objective function can be found in Appendix 2.4.



(a) Total and stakeholder costs as a function of the coverage radius obtained when maximizing the beneficiaries' standard service using the MRD threshold, CC 105% MSC



(b) Variation in total, stakeholder costs and covered inhabitants as a function of the coverage radius when maximizing the beneficiaries' standard service using the MRD threshold, CC 105% MSC

Figure 50. Costs, percent variation in costs and covered inhabitants for different coverage radius when maximizing the beneficiaries' standard service below the MRD using NAD, CC 105% MSC.

6.1.6 Results obtained when maximizing the beneficiaries' standard service below the mid-radius distance threshold, CC 105% MSC for *r* = 15 km.

In Figure 51, we observe the map for the present solution. In this case we also observe how the number of open DCs are 201 versus 950 open DCs for the case of the corresponding solution without costs constraints, see Figure 32. We can also see the similarities in terms of open DCs between this network and the network generated in the previous solution obtained when minimizing the MAD in Figure 47, where the open DCs were 192. This is an indication that by using different objective functions for the same coverage radius and using the same costs constraints, we obtain very similar networks regarding the number of open DCs.

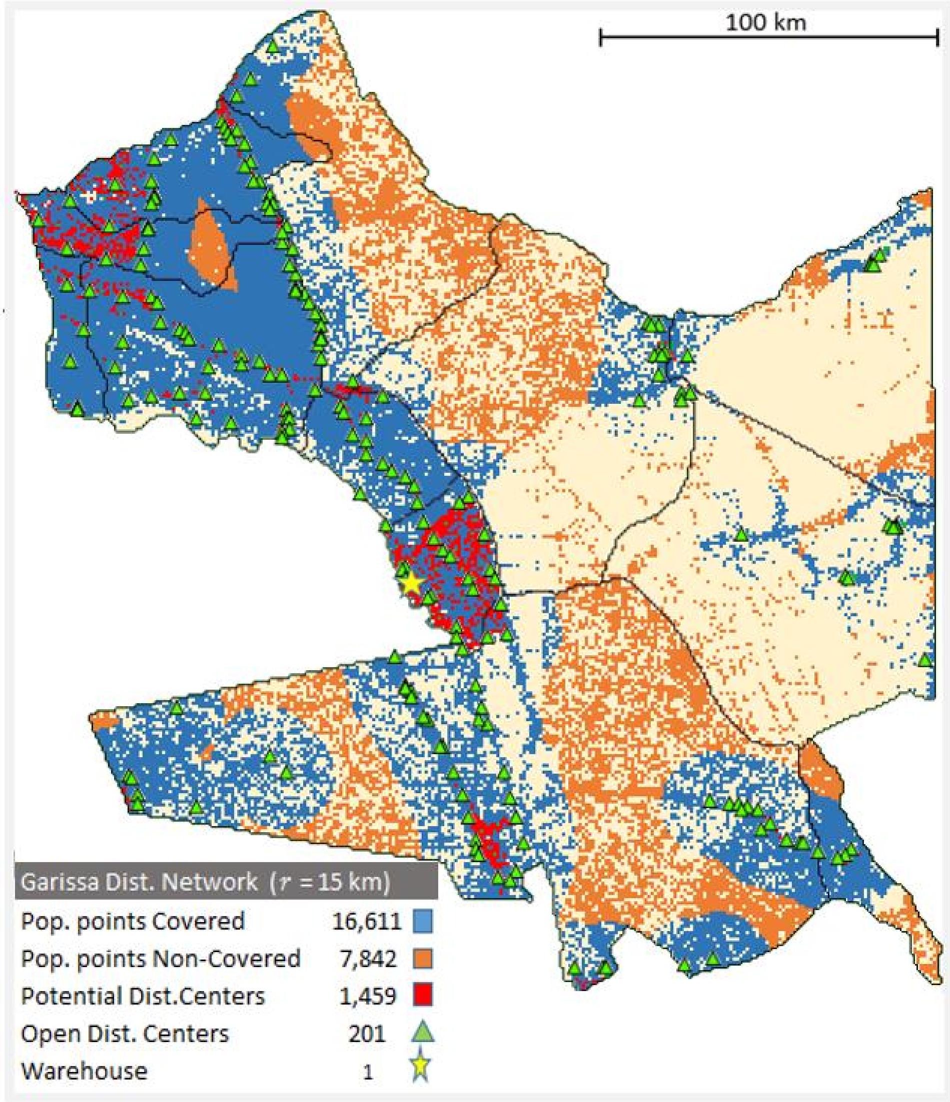


Figure 51. Map for the solution obtained when maximizing the beneficiaries' standard service below the MRD using NAD, CC 105% MSC for r = 15 km.

In Table 19, we observe significant costs reductions compared with the corresponding solution obtained without the cost constraint. In this case, we have 39.7 million KSh of total costs compared with 57.3 million KSh as shown in Table 9. We again observe the high stability in cost for the WFP, and we can see how the cost reductions are due to reductions in beneficiaries' access cost, but mainly for significant reductions in KRC's cost (20.17 million KSh shown in Table 9 vs. 4.26 million KSh shown in Table 19). These cost reductions are also accompanied by improvements in service indicators; in this case we have 1.47 hours of average walking time and 2.95 km of average walking distance compared with values of 2.07 hours and 4.14 km for the solution without costs constraints shown in Table 9. By comparing equality indicators for the present case we observe a higher Gini and Robin Hood Indexes with values of 0.614 and 0.482, compared with values of 0.448 and 0.333 shown in Table 9. As also noted for similar analyses, the increase in these values accounts for lower total walking distances, as shown by the reduction in cost for the beneficiaries.

Table 19. Characteristics of the solution obtained when maximizing the beneficiaries' standard service below the MRD using NAD, CC 105% MSC for r = 15 km.

Demographic indicators		%	Walking inhabitants' service indicators	
Garissa inhabitants	452,218	100.00%	Time measure:	
Covered inhabitants	386,269	85.42%	Average walking time (hrs)	1.47
Walking inhabitants (covered inh.)	71,309	18.46%		
Pop. points (including 1 -2 Inhabitants)	47,242		Distance measures:	
Population points (> 3 Inhabitants)	24,453	100.00%	Average walking distance (km)	2.94
Covered population points	16,611	67.93%	Standard deviation of distance (km)	3.66
Potential distribution centers	1,459	100.00%	Variance of distance (km)	13.42
Open distribution centers	201	13.78%	Mean absolute deviation of distance (km)	1.56
Warehouse	1		Median walking distance (km)	1.31
Economic indicators		%	Pearson's 2° skewness coefficient	1.34
Total cost (KSh)	39,760,799	100.00%	Distribution left tail	0.20
Beneficiaries acces cost (KSh)	4,278,759	10.76%	Distribution right tail	0.80
World Food Program cost (kSh)	31,214,006	78.50%	Gini Index	0.614
Kenya's Red Cross cost (kSh)	4,268,034	10.73%	Hoover Index (Robin Hood)	0.482

In Table 20, we see that in this case the total walking distances are 209,364 km, whereas for the corresponding solution without cost constraints shown in Table 10, we have 295,046 km, representing a substantial reduction of about 41%. Observing the total walking distances that are below and above the MRD threshold, we see 59.34% and 40.66%, respectively. In Table 10, the values are 69.95% and 30.05%. The previous percentages represent, for the solution in Table 20: 124,236 km below the MRD threshold and 85,128 km above the MRD threshold. For the solution in Table 10, the percentages represent 206,394 km below the MRD threshold and 88,652 km above the MRD threshold, showing clearly, for the present solution, shorter travel distances, especially for the inhabitants that are below the MRD threshold.

Table 20. Service levels based on three distance thresholds for the solution obtained when maximizing the beneficiaries' standard service below the MRD using NAD, CC 105% MSC for r = 15 km.

Service level measure \ Distance Threshold (DT)		Average walking distance 2.94 km	10 km distance threshold 10.00 km	Mid-radius 7.5 km
Number of population points	16,611	100.00%	100.00%	100.00%
% below DT		21.44%	75.87%	68.35%
% above DT		78.56%	24.13%	31.65%
Number of inhabitants	386,269	100.00%	100.00%	100.00%
% below DT		72.95%	94.04%	92.20%
% above DT		27.05%	5.96%	7.80%
Number of walking inhabitants	71,309	100.00%	100.00%	100.00%
% below DT		67.95%	92.35%	89.93%
% above DT		32.05%	7.65%	10.07%
Total walking distances (km)	209,364	100.00%	100.00%	100.00%
% below DT		19.78%	66.65%	59.34%
% above DT		80.22%	33.35%	40.66%

In Figure 52 (a), when observing the curve representing the distances, we see a very close shape with the curve representing the distances in Figure 33 (a) obtained for the corresponding solution without cost constraints. The difference is that in this case, we have a lower value for the AWD. We also notice how, for this objective function, the algorithm gives much more importance to the populations below the MRD threshold and gives very little attention to the populations above this threshold. Observing Figure 52 (b), we notice that for the present solution, the population points with more inhabitants are mainly concentrated in the left side of the distribution; whereas in Figure 33 (b), the population points with a large number of inhabitants are scattered in a broad area of the distribution. If we consider the distances that the inhabitants need to walk in each solution, we can clearly understand why in this solution the total walking distances are lower compared with the solution obtained when maximizing the service below the mid-radius distance threshold without costs constraints.

In Figure 52 (c), we observe the representation of the cumulative values for the inhabitants and cumulative values for the total walking distances. Where 68% of the inhabitants walk 32% of the total distance (blue and green lines). On the contrary, 20% of the inhabitants walk 80% of the total distance (red and orange lines). In Figure 52 (d), we see the representation for the Lorenz curve, Gini Index and Robin Hood indexes. The increase in the yellow area representing the Gini Index, when compared with the corresponding area in Figure 32 (d) for the equivalent solution without costs constraints, shows the reduction in total walking distances as the Lorenz curve is being pushed towards the horizontal axis. Hence, we have more walking inhabitants cumulating less walking distances. These lower walking distances improve other indicators such as the average walking time and distance observed in Table 19.

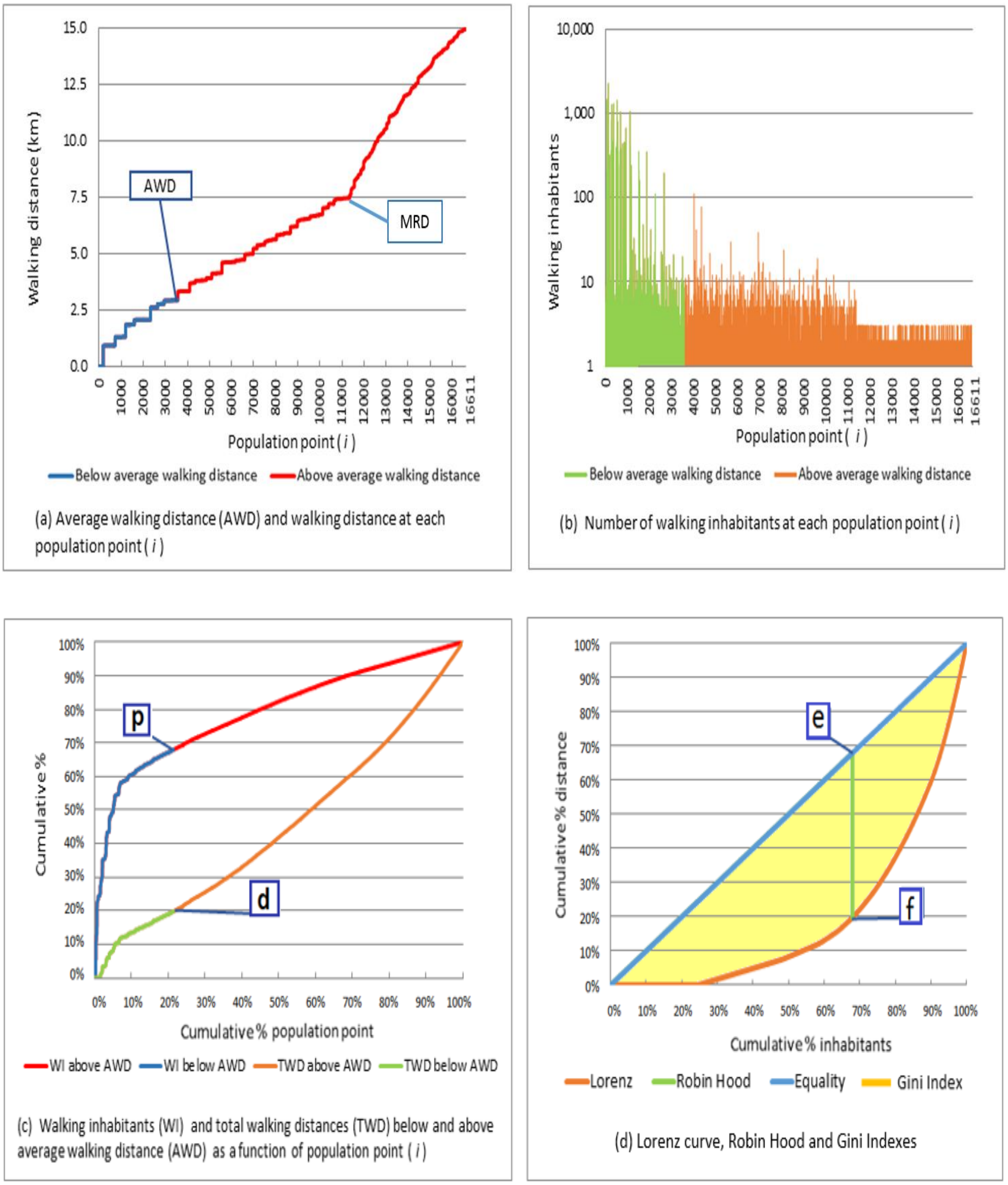


Figure 52. Characteristics of the distribution obtained when maximizing the beneficiaries' standard service below the MRD using NAD, CC 105% MSC for $r = 15$ km.

It is also important to note that the shapes for the representation of the distributions observed in Figure 52 (b), 52 (c) and 52 (d) are more similar to the corresponding figures for the solution obtained when minimizing the MAD of walking distance with a cost constraint shown in Figure 47 than with the shapes observed in the corresponding solution obtained when maximizing the SMR without the cost constraints, see Figure 33. There are also some similarities with the shapes of the distribution obtained when minimizing the AWD (Figure 44). However, for that solution we are using a constraint of 10% above the minimum stakeholder cost, whereas for the other two cases the percentage is 5%.

The similarities observed using the different objective functions also show that the use of cost constraints reduces the differences between the solutions obtained using different objective functions. This means that for solutions using different objective functions, the population points are allocated to farther or closer distribution centers. It was also observed that the number and location of the distribution centers, considering similar coverage radii and costs constraints, stay more or less the same. In Figure 53, we observe the map for the Garissa District where we represent three different solutions for the same coverage radius (15 km) and a cost constraint of 5% above the minimum stakeholder cost. In this Figure, the green round shapes represent the selected DCs obtained when using the function that minimizes the beneficiaries' AWD. The yellow diamond shapes represent the selected DCs obtained when using the function that minimizes the MAD of walking distance. The red squares represent the selected DCs obtained when using the function that maximizes the SMR. We can observe that in many cases the representation of the distribution centers are super-imposed, meaning that the same DCs were selected for different solutions. We can observe more similarities for the networks representing the solution that optimize the MAD and SMR. Consistently, we also observed significant similarities in the shapes of the distribution obtained in Figure 52 and Figure 47, corresponding to the same objective functions.

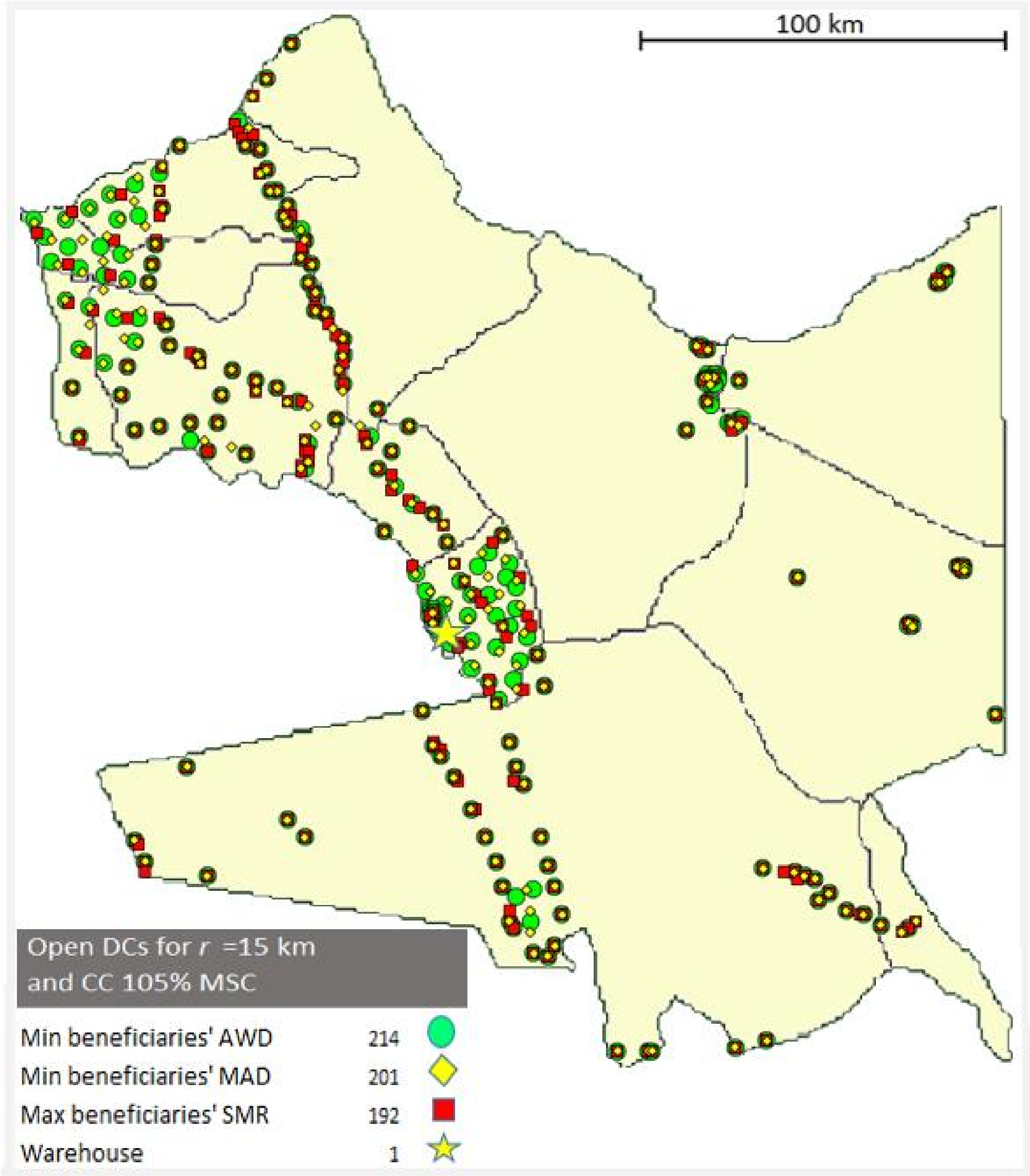


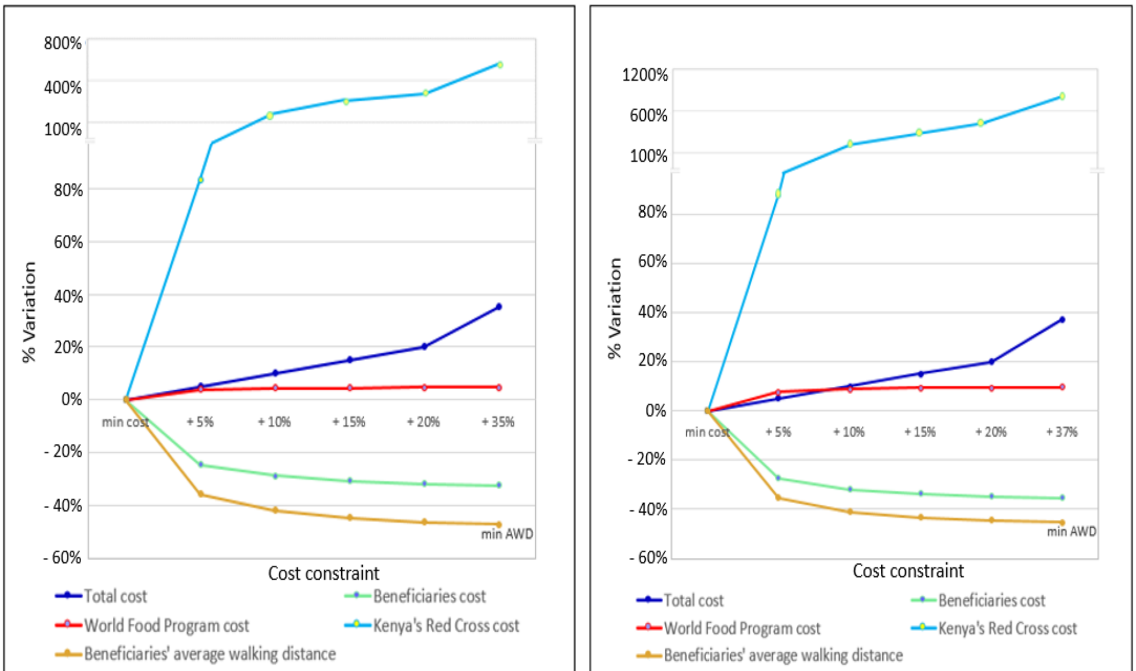
Figure 53. Representation of distribution centers networks using different objective functions, NAD, CC 105% MSC for r = 15 km.

6.2 Results obtained with aggregated data and cost constraints

In this section, we present comparisons between solutions obtained using different percentage increases of cost constraints above the minimum stakeholder costs, and we also present comparisons among different coverage radiuses for the percentage increase considered to be the most effective. As in the preceding chapter, for the aggregated data, we will not present detailed analyses considering specific solutions since similar analyses with more accurate data were presented for the solutions using non-aggregated data and cost constraints. However, the behaviour of the objective functions and shapes of the curves represented in the graphics are very similar. It is possible to find additional scenarios and more detailed information in Appendix 3.

6.2.1 Results obtained when minimizing the beneficiaries' average walking distance for an aggregation threshold of 18 inhabitants

In Figure 54, we observe the increase in the percentage of variation in costs, as we also increase the cost constraint above the minimum stakeholder costs. The minimum AWD can be obtained by increasing the cost by 35% for a radius of 15 km and by 37% for a radius of 25 km. For the solutions obtained for aggregated data, we observe less variation in cost. For example, in the case of the KRC, it requires increases in cost of approximately 600% and 850% for the coverage radius of 15 km and 25 km, respectively; whereas the corresponding percentages for the solutions obtained for non-aggregated data and costs constraints are approximately 1200% and 1700%, see Figure 41. Even considering the reduction in variability observed for the present solutions, we can clearly see how the patterns are very similar with the solutions using non-aggregated data. Observing the curve representing the variation of the average walking distance, we see that the highest reductions in the AWD are obtained when we increase the costs between 5% and 10% when compared with the solution obtained when minimizing the stakeholders' costs. The same was observed for the solution obtained for non-aggregated data. We can then conclude that higher increases in cost are not necessary for this objective function since with low increases in cost (5% to 10%), we obtain values for the AWD close to the optimal; therefore, improving service for the beneficiaries without spending unnecessary resources.



(a) Variation in beneficiaries' average walking distance, total and stakeholder costs as a function of the constraint percentage above the minimum cost obtained when minimizing the beneficiaries' average walking distance (r = 15 km)

(b) Variation in beneficiaries' average walking distance, total and stakeholder costs as a function of the constraint percentage above the minimum cost obtained when minimizing the beneficiaries' average walking distance (r = 25 km)

Figure 54. Variation in beneficiaries' average walking distance, total and stakeholder costs as a function of the CC above the MSC obtained when minimizing the beneficiaries' average walking distance using AD for r = 15 km and r = 25 km.

In Figure 55, we present the costs and percentage of variation in costs obtained for different coverages radii using a cost constraint of 10% above the minimum stakeholder cost. We observe similarities with the equivalent solutions obtained using non-aggregated data (Figure 42). Hence, a similar analysis and interpretation can be applied. We can conclude that by using costs constraints, this objective function behaves in a similar way and gives very similar results to those with the aggregated and non-aggregated data. The small differences can be attributed to the aggregation process, but this element can even be improved with a different aggregation strategy. We can, for example, fix the number of walking inhabitants instead of calculating them based on rounded values, as explained in previous sections. For the solutions shown in Figure 55, the values for average walking distance were 1.22, 1.88, 2.55, 3.07 and 4.46 km. The CPLEX running times were 153.12, 204.64, 194.47, 361.83 and 18,966.80 seconds for coverage radii of 10, 15, 20, 25 and 55 km, respectively. Additional information as well as additional solutions using the AWD as an objective function can be found in Appendix 3.3.

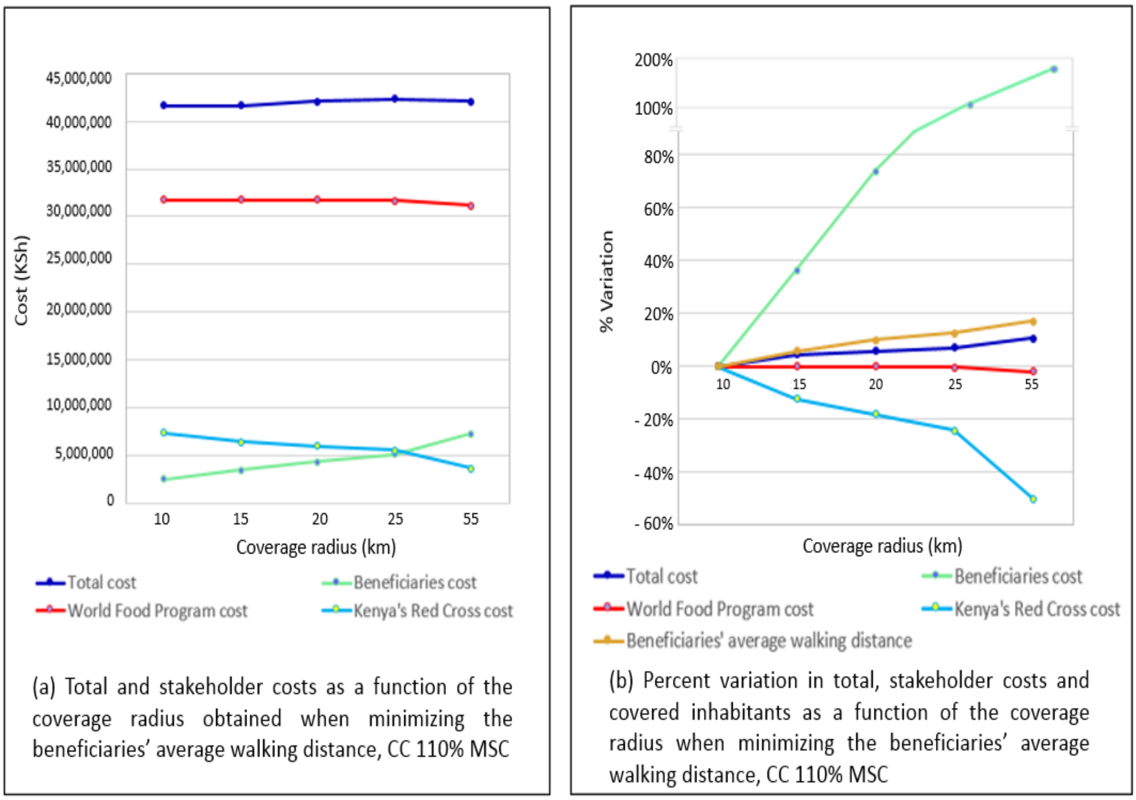
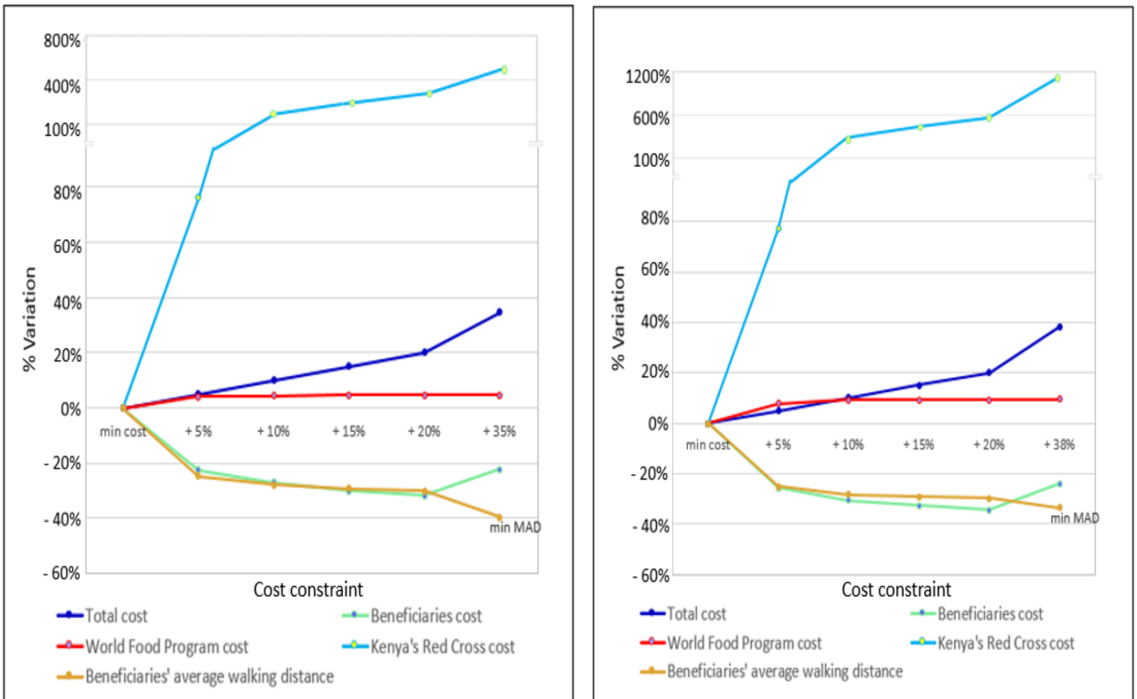


Figure 55. Costs, percentage of variation in costs and covered inhabitants for different coverage radii obtained when minimizing the beneficiaries' average walking distance using AD, CC 110% MSC.

6.2.2 Results obtained when minimizing the beneficiaries' mean absolute deviation of walking distance for an aggregation threshold of 18 inhabitants

In Figure 56, we observe the increase in the percentage of variation in costs as we increases the cost constraint above the minimum stakeholder costs. For this case, we observe that the minimum beneficiaries' MAD, are obtained with cost increases of 35% and 38% for the coverage radii of 15 km and 25 km. These values are lower than the equivalent values obtained when using non-aggregated data in Figure 45, with values of 90% and 99%, respectively. There are some other differences, for example in the model using non-aggregated data, the curves showing the beneficiaries' cost raise for a 5% cost constraint Figure 44; whereas for the present solution, the increases in the beneficiaries cost raise after a 20% cost constraint above the minimum stakeholder cost. Even though we have a large range of cost constraints favourable for the beneficiaries using the aggregated data, the marginal gains in reducing in the MAD after the 5% increase on the cost constraints are also as small as in the original model using non-aggregated data. Comparing the information in Figure 56 with the previous solution obtained when minimizing the AWD using aggregated data and costs constraints in Figure 54, we see that the minimum values obtained with the previous objective function are obtained with cost increases of 35% and 37%, and for the present case we have costs increases of 35% and 38%. We also observe similitudes in the shapes of the curves representing the different cost components. This shows that reductions in the size of the model resulting from the data aggregation, coupled with cost constraints, reduce the differences obtained when using different objective functions.

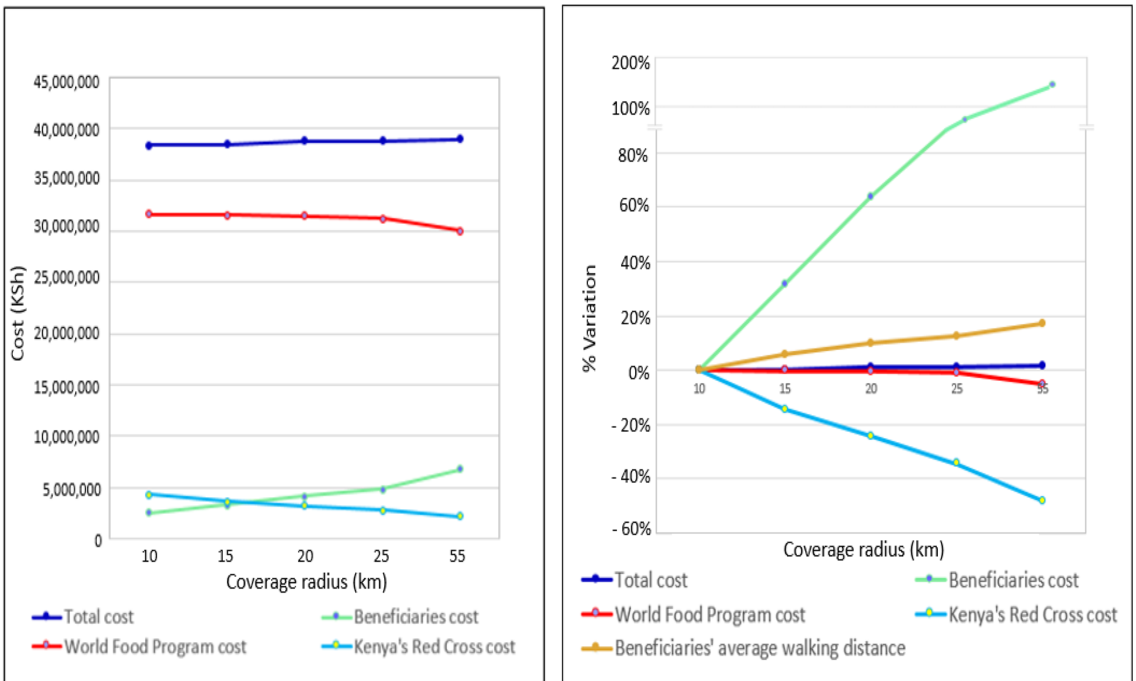


(a) Variation in beneficiaries' mean absolute deviation of walking distance, total and stakeholder costs as a function of the constraint percentage above the minimum cost obtained when minimizing the beneficiaries' mean absolute deviation of walking distance (*r* = 15 km)

(b) Variation in beneficiaries' mean absolute deviation of walking distance, total and stakeholder costs as a function of the constraint percentage above the minimum cost obtained when minimizing the beneficiaries' mean absolute deviation of walking distance (*r* = 25 km)

Figure 56. Variation in beneficiaries' average walking distance, total and stakeholder costs as a function of the CC above the MSC obtained when minimizing the beneficiaries' mean absolute deviation of walking distance using AD for r = 15 km and r = 25 km.

In Figure 57 (a), we observe the costs curves for different coverage radii obtained with a cost constraint of 5% above the minimum stakeholder costs. In Figure 57 (a), we can see that the values are slightly lower than the corresponding values for the equivalent solutions using non-aggregated data in Figure 46 (a). The differences are explained by a low number of walking inhabitants and an increase in covered inhabitants as a result of the aggregation process. However, we can see that the patterns in the curves are very similar and it is also possible to obtain an equilibrate solution for the beneficiaries and the KRC using a coverage radius of 15 km, as it was also observed for the solution using non-aggregated data. In Figure 57 (b), we observe the percentage of variation in costs as we increase the coverage radius. We see close patterns to those presented in the equivalent solutions using non-aggregated data in Figure 46 (b). It is also important to notice that in Figures 57 (a) and 57 (b), we show the results for the coverage radius of 55 km; for this particular radius we were not able to obtain the solution using the non-aggregated data. The values presented here allow us to have a complete picture of the behaviour of the different cost components. However, we can also say that the costs curves follow clear patterns, this is useful to understand how the cost evolves as we increase the coverage radius. For the solutions shown in Figure 57, the values for the MAD were 0.13, 0.23, 0.34, 0.43 and 0.65 km. The CPLEX running times were 50.55, 88.89, 1,229.93, 3,926.90 and 1,823.92 seconds for the coverage radii of 10, 15, 20, 25 and 55 km, respectively. Additional information as well as additional solutions using the mean absolute deviation of walking distance as objective function can be found in Appendix 3.5.



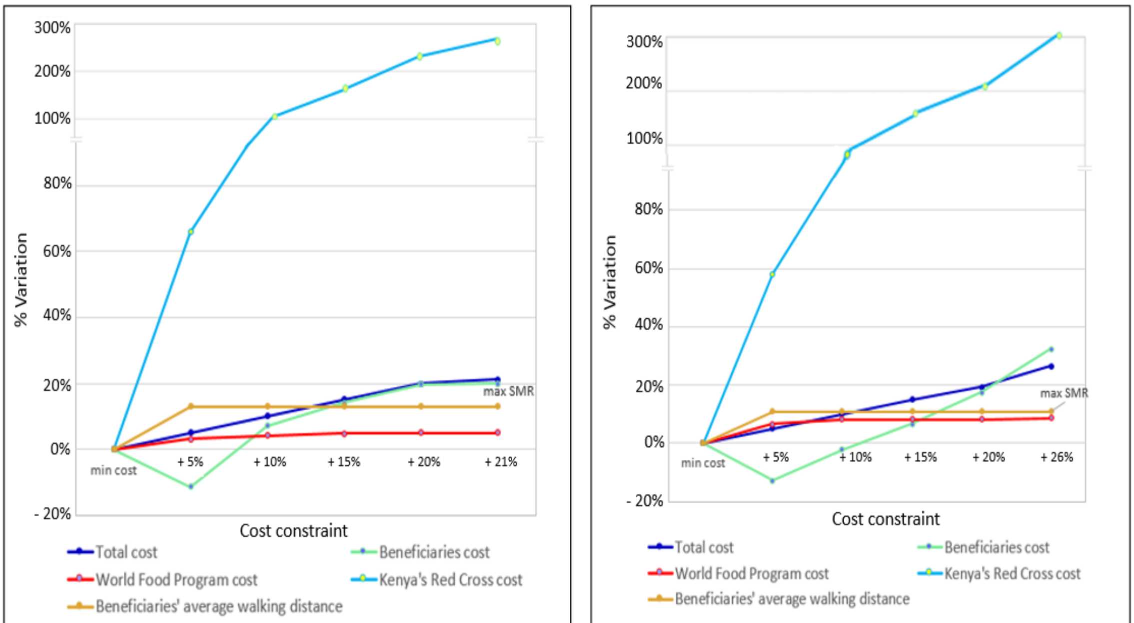
(a) Total and stakeholder costs as a function of the coverage radius obtained when minimizing the beneficiaries' mean absolute deviation of walking distance, CC 105% MSC

(b) Variation in total, stakeholder costs and covered inhabitants as a function of the coverage radius when maximizing the beneficiaries' standard service using the MRD threshold, CC 105% MSC

Figure 57. Costs, percentage of variation in costs and covered inhabitants for different coverage radii obtained when minimizing the beneficiaries' MAD using AD, CC 105% MSC.

6.2.3 Results obtained when maximizing the beneficiaries' standard service below the mid-radius distance threshold for an aggregation threshold of 18 inhabitants

In Figure 58, we observe the percentage of variation in cost obtained as we increase the cost constraint above the minimum stakeholder costs. As it was observed for the solutions using non-aggregated data, we see that a very close value to the maximum SMR can be obtained with a 5% cost increase. In the case of the beneficiaries' access cost curve, the lower costs are obtained at 5% cost increase, this is explained by shorter travel distances due to the additional open DCs. The increase in the KRC cost favours a reduction for the beneficiaries' cost, but as it increases beyond 5%, lower total costs are obtained by increasing the beneficiaries' access cost as well. For the present case, the variation in cost for the KRC can reach approximately 300% for the coverage radii of 15 km and 25 km, whereas in the solutions using non-aggregated data the corresponding values where higher than 600% and 1200%, in both cases (Figure 49). This lower variability can also be explained for the effects of the required data aggregation on the network of DCs as explained in Figure 34. We can conclude that for this objective function, only small cost increases of about 5% must be allowed, in order to avoid increases in the beneficiaries' access costs.



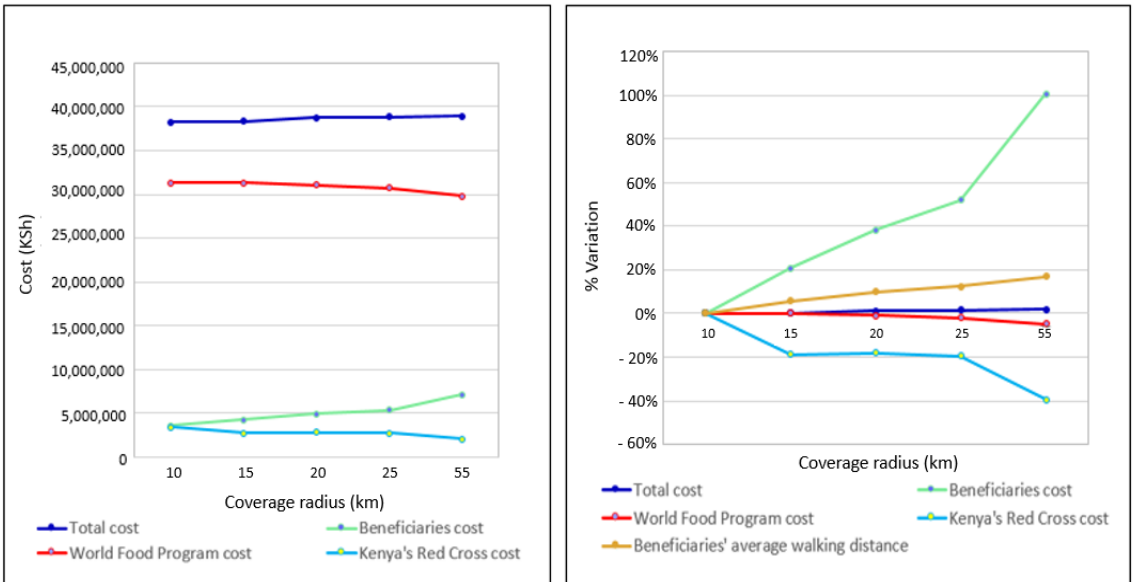
(a) Variation in beneficiaries' service below mid-radius, total and stakeholder cost as a function of the constraint percentage above the minimum cost obtained when maximizing the beneficiaries' standard service using the MRD threshold (*r* = 15 km)

(b) Variation in beneficiaries' service below mid-radius, total and stakeholder cost as a function of the constraint percentage above the minimum cost obtained when maximizing the beneficiaries' standard service using the MRD threshold (*r* = 25 km)

Figure 58. Variation in beneficiaries' service below the MRD, total and stakeholder costs as a function of the CC above the MSC obtained when maximizing the beneficiaries' standard service below the MRD using AD for *r* = 15 km and *r* = 25 km.

In Figure 59 (a), we observe the cost obtained for different coverage radii. In this case we were also able to obtain the solution for the coverage radius of 55 km. A solution for this radius and using this objective function were not possible to obtain using the non-aggregated data, see Figure 50 (a). For the present case we observe small increases in the beneficiaries' access cost compared with the results obtained using non-aggregated data, and correspondingly we observe small reductions in the KRC's cost compared with the results shown in Figure 50 (a).

The percentage of variation in costs is presented in Figure 59 (b) in which we observe the reduction in the percent variation for the different cost components compared with the corresponding values shown in Figure 50 (b). In Figure 59 (b), for example, it is possible to see that the percent increase in cost for the KRC is approximately 100% for the coverage radius of 55 km, whereas using non-aggregated data a similar percent increase was obtained with a coverage radius of 25 km. It is also possible to conclude that when using this particular objective function more equilibrated solutions, in term of costs for the beneficiaries and KRC, can be obtained using small coverage radii (e.g., 10 km or 15 km). For the solutions shown below, the values for the walking inhabitants below the MRD threshold were 92.35%, 91.24%, 90.52%, 91.10% and 96.73%. The CPLEX running times were 24.12, 64.94, 112.69, 397.81, and 3,508.43 seconds for the coverage radii of 10, 15, 20, 25 and 55 km, respectively. Additional information can be found in Appendix 3.7.



(a) Total and stakeholder costs as a function of the coverage radius obtained when maximizing the beneficiaries' standard service using the MRD threshold, CC 105% MSC

(b) Variation in total, stakeholder costs and covered inhabitants as a function of the coverage radius when maximizing the beneficiaries' standard service using the MRD threshold, CC 105% MSC

Figure 59. Costs, percentage of variation in costs and covered inhabitants for different coverage radius when maximizing the beneficiaries' standard service below the MRD threshold using AD, CC 105% MSC.

6.3 Results obtained with generated data and cost constraints

For the present case we explored a 10% cost increase above the minimum cost. We have not performed further analysis considering additional cost increases because in this case we are using non-real data, and the information obtained with this analysis will serve only for referential purposes to gain a better understanding about the use of the variance as equality function with a cost constraint. This solution also presented the major computational requirements. We allow to run CPLEX for 1,417,542 seconds. It is possible to find detailed information in Appendix 4.

6.3.1 Results obtained when minimizing the beneficiaries' variance of walking distance using generated data

In Table 21, we can see that for the present solution we have 37 open DCs, whereas in the equivalent solution without costs constraints (Table 13), we have 42 open DCs. In this case, we have a total cost of 44.79 million KSh. For the beneficiaries' access cost, we have a cost of 14.81 million KSh, which is very close to the value of the solution obtained when minimizing the stakeholder costs (Table 11) with 14.98 million KSh. Among the service indicators we see an average walking distance of 12.95 km, which is very close to the 13.12 km in Table 11. However, in this case, we have a lower variance of walking distance, with a value of 59.01 km compared with 89.33 km obtained when minimizing the stakeholder costs, but higher than the 19.39 km obtained when minimizing the variance of walking distance without costs constraints, see Table 13. The Gini and Robin Hood Indexes are much closer to the corresponding values obtained for the solution when minimizing the stakeholder costs, than with the values of the solution obtained when minimizing the variance without costs constraints, where the values are very close to 0.

Table 21. Characteristics of the solution obtained when minimizing the beneficiaries' variance of walking distance using generated data, CC 110% MSC for r = 55 km.

Demographic indicators		%	Walking inhabitants' service indicators	
Garissa inhabitants	452,218	100.00%	Time measure:	
Covered inhabitants	452,218	100.00%	Average walking time (hrs)	6.48
Walking inhabitants (covered inh.)	75,406	16.67%		
			Distance measures:	
Population points	100	100.00%	Average walking distance (km)	12.95
Covered population points	100	100.00%	Standard deviation of distance (km)	7.68
Potential distribution centers	50	100.00%	Variance of distance (km)	59.01
Open distribution centers	37	74.00%	Mean abs. deviation of distance (km)	11.35
Warehouse	1		Median walking distance (km)	12.00
Economic indicators		%	Pearson's 2° skewness coefficient	0.37
Total cost (KSh)	44,795,745	100.00%	Distribution left tail	0.29
Beneficiaries acces cost (KSh)	14,814,038	33.07%	Distribution right tail	0.71
World Food Program cost (kSh)	27,946,707	62.39%	Gini Index	0.310
Kenya's Red Cross cost (kSh)	2,035,000	4.54%	Hoover Index (Robin Hood)	0.238

In Table 22, we observe additional service indicators. When observing the indicators for the different distance thresholds, we see improvements for all the service measures compared with the solution obtained when minimizing the stakeholder costs shown in Table 12. For example, for the mid-radius distance threshold, the percentage of population points below the threshold is 91%, the percentage of covered inhabitants it is 96.02%, percentage of walking inhabitants is 96.01% and for the total walking distances is 89.12%. The corresponding percentages for the solution obtained when minimizing the stakeholder costs are 79%, 92.86%, 92.85% and 79.25% (Table 12). The preceding information shows that using limited costs constraints when minimizing the variance of walking distance improve the service for the beneficiaries. On the contrary, observing the same percentages for the solution obtained when minimizing the variance without costs constraint in Table 14, the values are 3%, 1.11%, 1.11% and 0.53%.

The previous information shows that unrestricted cost increases for this particular objective function have a negative effect on the beneficiaries; the poor services levels obtained in that case are due to the fact that a solution with minimum variance require the inhabitants to walk to distribution centers located far away. In this way, almost all the inhabitants travel similar distances. This situation also explains the high cost for the solution without cost constraints and it shows that equality functions, when used in a minimization context, behave in a different way than when they are used in a maximization context, as it is the case of the maximization of welfare in economics.

Table 22. Service levels based on three distance thresholds for the solution obtained when minimizing the beneficiaries' variance of walking distance using generated data, CC 110% for $r = 55$ km.

Service level measure / Distance Threshold (DT)		Average walking distance 12.95 km	10 km Threshold norm 10.00 km	Mid-radius 27.5 km
Total population points	100	100.00%	100.00%	100.00%
% below DT		20.00%	70.00%	91.00%
% above DT		80.00%	30.00%	9.00%
Total inhabitants	452,218	100.00%	100.00%	100.00%
% below DT		52.46%	87.59%	96.02%
% above DT		47.54%	12.41%	3.98%
Total walking inhabitants	75,406	100.00%	100.00%	100.00%
% below DT		52.44%	87.59%	96.01%
% above DT		47.56%	12.41%	3.99%
Total walking distances (Km)	976,581	100.00%	100.00%	100.00%
% below DT		28.65%	73.56%	89.12%
% above DT		71.35%	26.44%	10.88%

In Figure 60, we see the particularities of the obtained distribution. In Figures 60 (a) and 60 (b), we observe very close patterns to those of the solution obtained when minimizing the stakeholder costs in Figure 38. For Figures 59 (c) and 59 (d), we observe how the area encircled by the upper and lower curves, representing the Gini Index, is slightly smaller compared with the corresponding areas of Figure 39. We observe smaller Gini and Robin Hood Indexes in the present solution, with values of 0.310 and 0.238, respectively. The equivalent values for the solution obtained when minimizing the stakeholder costs shown in Table 11, are 0.356 and 0.278; whereas for the solution when minimizing the variance without cost constraints (Table 13), we have quite low values of 0.048 and 0.034, respectively. These results indicate that very small equality measures obtained using the variance as an objective function in a minimization do not necessarily mean good service levels, as it also has been observed using other objective functions. It is possible to obtain equal travel distances for the beneficiaries, but this means that inhabitants living close to potential DCs would need to walk very long distances to reach DCs located far-away, as it is also the case for inhabitants located in remote areas who do not have another option but to walk very long distances in order to reach the food supplies. It is possible to prevent those illogical solutions by using limited costs constraints above the minimum costs and minimizing the stakeholder costs.

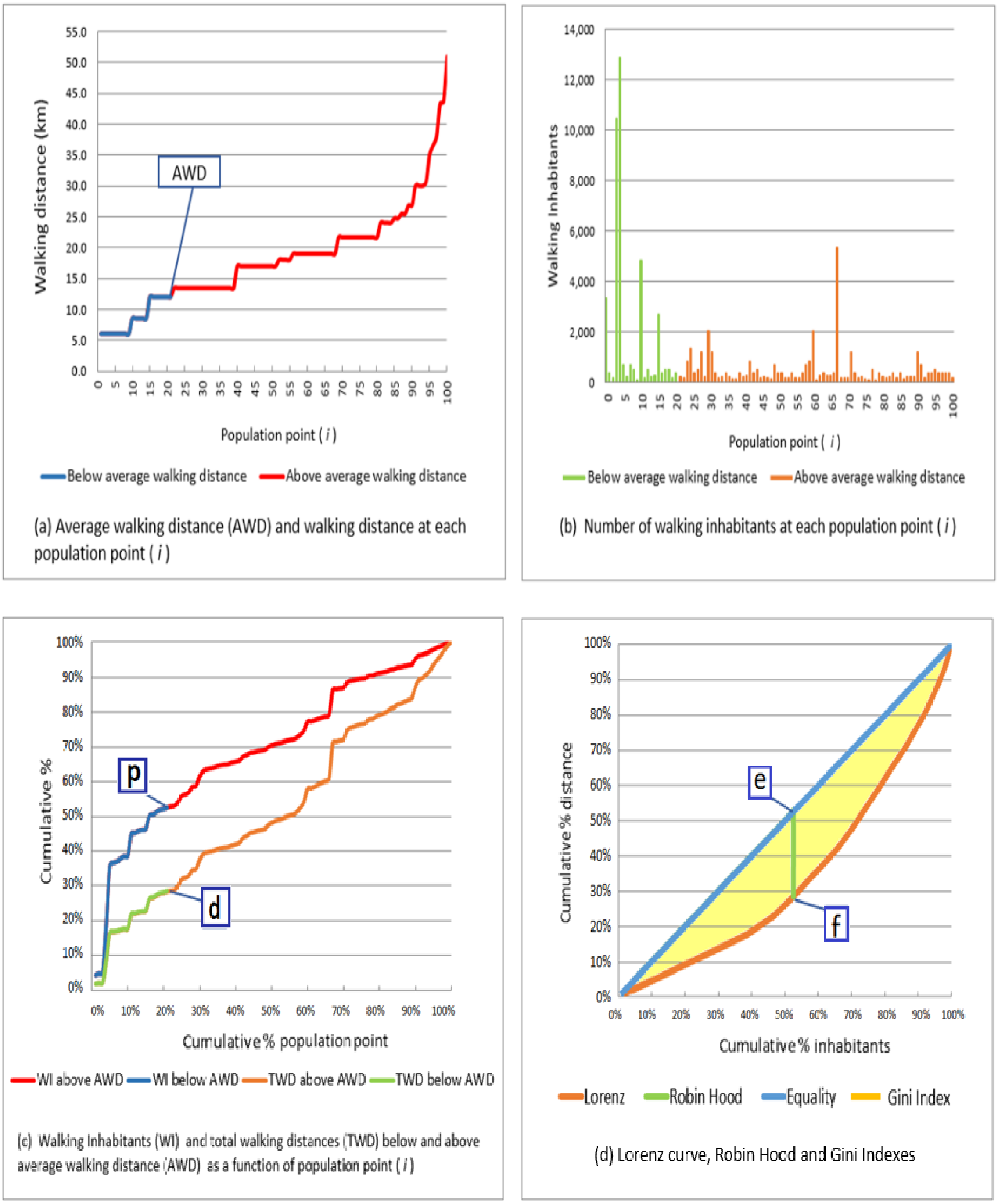


Figure 60. Characteristics of the distribution obtained when minimizing the beneficiaries' variance of walking distance using generated data, CC 110% MSC for r = 55 km.

Chapter 7: Conclusions

Through the present study we have observed some important results obtained by conducting several analyses, many of them were elaborated in detail. However, in this chapter we present main findings.

Benefits of data aggregation

Considering the aggregated data, we observed that the data aggregation or data generation strategy influenced the coverage level. In addition, we need to add the differences obtained during the calculation process for the walking inhabitants. Hence, for the aggregated data, we have obtained scenarios that are not completely equivalent with the scenarios generated using non-aggregated data. The previous observation does not imply that the results using aggregated or generated data are not valid, but it implies that different assumptions and data generation processes may result in different networks. It would also be possible to reduce these differences by using different aggregation strategies. Considering the beneficiaries, we can aggregate the number of walking inhabitants instead of calculating them based on the covered inhabitants. It is also possible to use weighted walking distances on one hand, regarding the KRC's cost for the solutions obtained without costs constraints, the aggregation process restricted the open distributions centers, which in turn limited the costs for this stakeholder. On the other hand, for the solutions obtained when using costs constraints, we have an imposed restriction to the number of distribution centers that can be opened and the solutions using non-aggregated data and aggregated data followed very similar patterns. We can then conclude that the solutions using aggregated data, subject to costs constraints, can closely represent the solution obtained with non-aggregated data. Therefore, simplifying the process of obtaining solutions and the exploration of different networks for the case of big networks or big data sets.

Relevance of the average value

By analysing the results using different data sets (non-aggregated data, aggregated data and generated data) we observe a strong relationship between the average and equality measures for which the calculation requieres ordered or ranked distributions, as it is the case for the Gini Index and Robin Hood Index. We can then conclude that using the average as an equality and service measure is highly justified and its evaluation must be considered in situations dealing with equality issues. We must add, however, that additional analysis should be conducted in order to fully undertand this relationships and to generalize these observations for different contexts. It would also be important to evaluate this relationship using equality measures not considered in the present study and to compare the results obtained using the average as an objective function with the results obtained using complex equality measures as objective functions. Considering that the average can be easily modeled and optimized with a mixed integer linear solvers such as CPLEX, and that complex equality measures with quadratic coefficients are difficult to optimize, it is often preferable to obtain only approximations to the optimal value using heuristics algorithms.

From the analysis of different equality and service functions without cost constraint and for different data sets, we can conclude that the use of these objective functions without the influence of additional objectives or constraints, results in very high total costs mainly due to large walking distances for the beneficiaries or large numbers of open distribution centers. It is also important to clearly distinguish equality and service. It is possible to obtain more equalitarian travel distances for the beneficiaries, but that implies for inhabitants originally close to potential distribution centers to walk to distribution centers located further away, as it is also the case for inhabitants located in remote areas who are compelled to walk very long distances. This type of allocation will produce more or less equal walking distances, but clearly poor service levels.

Previous observations shows how some equality functions and measures had to be reinterpreted when they have to be minimized, and how low levels of service for inhabitants can present low levels of equality measures as well. In our study, the distance can be considered as a negative attribute because well-being does not improve when it increases. In economics, the main attribute used is income and well-being improves when it increases. In this case, we are in a maximization context, where the interpretation of equality is more traditional and implies that the inhabitants are better-off with more income.

Efficiency of adding a cost constraint

The present study explored the use of cost constraints, which imposes a maximum on the total cost evaluated from the minimum stakeholder cost, when optimizing objective functions such as the average, mean absolute deviation of walking distance, standard service and variance. The cost constraints allowed to obtain values close to the optimum obtained when optimizing the objective functions without using costs constraints. Furthermore, these values were obtained without incurring unnecessary cost. Substantial improvements in the beneficiaries' average walking distance can be obtained by constraining the costs to be at most 5% to 10% more than the MSC. Increases in cost beyond these percentages only allow small reductions on the average walking distance. Substantial improvements were obtained by increasing the cost by 5% over the MSC for the mean absolute deviation of walking distance and for the number of walking inhabitants within high service distances. In the case of the generated data, a 10% cost increase allowed significant improvements for variance of walking distance. This small cost increase considerably improves the service for the beneficiaries, it also preserves limited resources for a network depending greatly on donations and voluntary work. It would be interesting to explore small cost increases in addition to those explored in the present study, e.g., additional increases of 1% in order to delimit more accurately the permissible cost increases over the MSC to consider beneficiaries' concerns.

Cost repartition among stakeholders

Considering different costs components, the most stable cost is the WFP's cost. This cost mainly depends on the tonnes of transported food for which there are small variations in the solutions obtained with different objective functions. On the other hand, beneficiaries' access cost and KRC's cost showed high variability, especially for larger coverage radii and for larger increases of the cost constraints. These two costs tend to compensate each other in the settings with cost constraints; hence reducing beneficiaries' access costs is possible by increasing the KRC's cost. We also observed that the network of distribution centers is relatively stable in terms of number of open DCs and in the geographical locations for these open DCs when using different objective functions for the same coverage radius, inhabitants covered and used cost constraint. The differences observed in costs and indicators are mainly due to different allocation patterns of inhabitants to distribution centers. This can also mean that the implementation of different solutions for different objectives functions could probably not present many complications and can even be based on the actual network.

Recommended model to support decision making process

We can also conclude that the different analysed equality and service objective functions can improve the service of the beneficiaries when they are subject to costs constraints. The concerns of the other stakeholders are also taken into account with some variations depending on the coverage radius and on the cost constraints. There are as well variations and particularities for each objective function, which were analysed in their respective sections. As a general observation, the objective function that minimizes beneficiaries' average walking distance promotes higher reductions in beneficiaries' access costs; the objective function that minimizes the mean absolute deviation of walking distance and the objective function that maximizes the service levels below the mid-radius distance threshold give more or less equal costs for the beneficiaries and the KRC. Finally, the objective function that minimizes the variance of walking distance has more impact on the KRC cost; close results to those obtained using this equality function are also obtained by maximizing beneficiaries' average walking distance.

Based on the previous information and on the results obtained in Chapter 5, we suggest to minimize the average walking distance and using small coverage radii if the main concerns are related to beneficiaries' accessibility for their food supplies. Strategic decisions must take into account multiple stakeholder interests. The final decision on the network implementation would nevertheless fall upon the decision makers and the priorities at the moment of taking the decisions.

Limits of the results

Finally we need to mention, that the conclusions of the present study are valid for our specific data, scenarios and the context over which we have developed our research. It is probable that some of our conclusions could be generalized, but to confirm, deny or limit the extent of possible generalizations, further analyses and research would be required. Considering that the present study analysed in depth some equality and service functions, future research could use our observations, findings, suggestions and conclusions to explore additional equality and social concerns.

Chapter 8: Future research

From the results, observations and conclusions of the present study, it is possible to identify new and interesting fields of research. The study of equality and service functions, dealing with humanitarian concerns, is not strictly circumscribed to network design and future research can follow a multidisciplinary perspectives, such as economics, politics, sociology, international relations, etc. Similar studies considering the same and also additional objective functions could be very interesting in the field of economics. The inclusion of additional attributes could also help to better understand the implications of the use of equality and service functions for different contexts, considering maximization and minimization objectives as well. In the present section, we will also discuss some specific concerns that arouse during the present study, as well as reflections about the humanitarian problem we are dealing with.

An aspect that worth exploring is developing algorithms to explore the use of additional equality objectives, e.g., Gini Index and Hoover Index. Considering the relationship between some equality and service measures such as the average with these more complex equality measures, it would be interesting to analyse these relationships further in detail, in order to confirm or deny the validity of the observations of the present study. Considering the non-linear nature of such complex equality measures and the large size of the problems studied in humanitarian logistic, as it is the case for the network design in the Garissa District, it is possible that heuristics algorithms could be developed for such studies. Such algorithms may be used not only for humanitarian network design but also for more general settings.

Another concern that arouse while developing the present project is related to the large distances some inhabitants need to walk in order to reach their food supplies. Considering the beneficiaries' access cost, the best solutions were obtained using smaller coverage radii. From the visual analysis of the maps, we observed that the solutions with small coverage radii had wide non-covered geographical areas. For the present study, there are potential distribution centers if there are population points with more than 20 inhabitants located close to roads. Considering the natural demographic evolution, the migratory patterns and the constant influx of refugees from neighbouring countries into Kenya, we expect that many population points, which previously had less than 20 inhabitants, would reach or surpass this limit in the current time. Thus, it is possible to slightly lower the population threshold to consider the establishment of a distribution center.

Considering the second condition, that is the proximity of roads to those populations, we need to mention that the Kenya's road network is not precise and probably there is missing information that could be updated, and new potential distribution centers could be identified. Identifying these new potential distribution centers is especially important for the non-covered areas and could help reduce the necessary coverage radius of 55 km to serve all the population points. This analysis would require, if available, updated information concerning the Kenya's road network, and probably precise research to such identify roads. Considering the high cost associated to food distribution networks, the investment and efforts necessary to gather the missing information could be worth the price.

In Figure 60, we present a hypothetical network for the distribution centers. In this case we see the large number of open distribution centers (triangular green shapes), some hypothetical new distribution centers (triangular purple shapes) and the area delimiting the coverage of the hypothetical distribution centers (purple circles). Twenty hypothetical DCs are located in specific areas in order to maximize the coverage of population points. These DCs have been, of course, deliberately positioned, but this solution can help us understand how strategically positioning some new distribution centers to help increase the levels of service, and probably, reduce the costs for the food distribution network. The number of new distribution centers could be even smaller if we consider slightly larger coverage radii (e.g., 25 km), in which case the required number of new distribution centers could probably be less than 10.

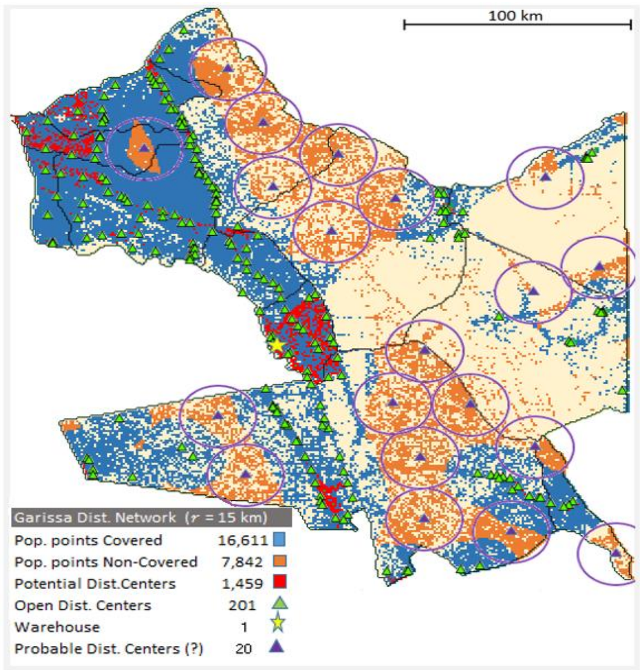


Figure 61. Representation of coverage increases by hypothetically positioning new distribution centers for r = 15 km.

Another question that might be worth exploring is the use of alternative methods for humanitarian help. Different strategies have been adopted to deal with the problem of food insecurity, where food distribution is one of the most implemented. However, there are also other forms of interventions, although somewhat controversial, such as cash grants and cash for work programs.

Many authors suggest that food distribution is preferable in situations of acute crisis, when the markets are destabilized because of food shortfall and when populations have limited access to markets. On the other hand, Maxwell et al. (2008) state that "There are occasions when markets themselves may be better mechanisms for delivering goods and services than normal humanitarian programs". The same author also suggests that predictable and stable transfers allow better planning and investment by the beneficiaries and give a better cost-benefit ratio. OXFAM (2014) found that small regular payments were more likely to be used to buy food. The previous comments can be applied when the problem of food insecurity is relatively predictable and when there are markets already in place.

Some concerns emerged considering these types of aid. There is risk of inflation when injecting money in markets and also a surplus in food aid can drive local prices down, affecting the local production and the capacity of beneficiaries to auto-recover from the crises. Decisions regarding the best approach are very difficult to take; it can depend on the duration of the crisis, the severity and the capacity of local markets to satisfy the population's demand for food. Non-food interventions have gained attention in recent years, particularly after the Tsunami in Indonesia in 2004, where cash grants were extensively used. Regardless of this experience, "the degree to which non-food interventions improve food security in emergency settings remains relatively under-researched and poorly understood" (Maxwell et al., 2008). Meanwhile, there are voices that want to explore new alternatives for help. A member of the WFP said: "The variety of contexts in which we work means we need to explore new tools, WFP receives half of its donations in cash rather than food, which gives us more flexibility in how we distribute our aid" (IRIN, 2008).

Reducing the cost associated with humanitarian help can improve the use of resources and generate a better response. It is then necessary to explore new ways of reducing the logistic costs of humanitarian logistic, improving at the same time the access for the beneficiaries and trying to establish a fair displacement, especially for the more affected populations. A project considering those aspects could probably be called: Network design for food and cash distribution in the context of humanitarian logistics.

To our knowledge, one particular aspect that has not been studied yet is the effect of combining food aid distribution and cash grants in the network design of humanitarian logistics. The situation of Kenya can especially be suited for this approach since we have extensive segments of population suffering from food insecurity, markets in place, an open economy and rural population with difficult access to markets. Exploring this particular humanitarian intervention could help understand how combining food and cash grants can reduce the cost for the stakeholders. One of the most important consideration of such project could be that populations located close to functioning markets could be better served by cash grants in the form of donations or through programs of food for work; whereas, for people located in remote areas where the access to market is poor, could be better served by food distribution centers. There must be a balance between these two approaches, giving cash grants only if inflationary risks are minimal and when local markets have the resources to satisfy the generated demand. In remote areas, where markets are not in place and where food is a scarce item, food allocation will probably not generate price increases; instead it might help balance the high prices for food in those remote areas.

Among the objectives that can be considered in a future study, we could analyse the particularities and trade-offs of combining food distribution and cash grants in situations of food insecurity, evaluate the capacity of local food markets to satisfy the demand, determine the impact of food or cash help requirements for specific areas, and determine the best combinations of food and monetary help for specific situations in order to serve populations that are distant or close to main markets.

Appendix

Appendix 1.1 Location models based on the humanitarian response cycle management, response, recovery and long-term humanitarian development phase.

Phase / Author	Setting	Area	Problem	Objective	Objective function
Mitigation					
Church (2004)	man-made disruption of facilities	Los Angeles, CA, USA	facility-interdiction problem	single-objective	Maximize the resulted weighted distance impact
Berman et al. (2009)	locate early warning systems/contamination	North Orange, CA, USA	Cooperative maximal covering problem	single-objective	Min number facilities to cover max demand points
Berman et al. (2010)	locate early warning systems/contamination	Random data	Cooperative maximal covering problem	single-objective	Min number facilities to cover max demand points
Current et O'Kelly (1992)	locate early warning systems/contamination	Midwestern, USA	Maximal covering problem	single-objective	Min total cost
Berry et al. (2006)	locate early warning systems/contamination	International	p-median problem	single-objective	Min impact of contamination
Murray & Tong (2007)	locate early warning systems/contamination	Ohio, USA	Maximal covering problem	single-objective	Max coverage of warning systems
Murray et al. (2008)	locate early warning systems/contamination	Ohio, USA	Set covering and p-center problem	single-objective	Min number of warning systems
Xu et al. (2010)	locate early warning systems/contamination	International	p-median problem	single-objective	Min Maximum of contamination
Xu et al. (2010)	locate early warning systems/contamination	International	Set covering problem	single-objective	Min Maximum of uncovered nodes
Xu et al. (2010)	locate early warning systems/contamination	International	Hybrid p-median and set covering problem	bi-objective	Min Max of contamination and Min Max uncovered nodes
Lee (2001)	unreliable facilities	Random data	p-median problem	single-objective	Min travel time
Berman et al. (2007)	unreliable facilities	Random data, Canada	p-median problem	single-objective	Min travel time
Shen et al. (2011)	disruption of facilities	Random data	p-median problem	single-objective	Min Costs
O'Hanley & Church (2011)	man-made disruption of facilities	Europe and American cities	facility-interdiction problem	bi-objective	2 steps: Maximize initial coverage and ensure minimum coverage level after disruption
Preparedness					
Chang et al. (2007)	Flood emergencies	Taiwan	p-median problem	single-objective	Min facility construction costs
Sherali et al. (1991)	Flood, hurricane threats and shelter location	Virginia, USA	set covering problem	Multi objective	Min evacuation time
Li et al. (2011)	Flood, hurricane threats and shelter location	Gulf Coast, USA	p-median problem	Multi objective	Min transportation, shortage and surplus cost
Rawls & Turnquist (2012)	Hurricane threats and shelter location	North Carolina, USA	p-median multi comodity problem	Multi objective	Min facility operating costs, flow cost, salvage and shortage costs
McCall (2006)	humanitarian relief distribution	Random data	p-median problem	bi-objective	Min trasnportation cost and shortages
Campbell & Jones (2011)	humanitarian relief distribution	Random data	p-median problem	bi-objective	Min facility operating and inventory costs
Döyen et al. (2012)	humanitarian relief distribution	Random data	Hybrid p-median and set covering problem	Multi objective	Min facility stablishing, inventory holging and shortage cost
Tzeng et al. (2007)	humanitarian relief distribution	Taichung, Nantou, Taiwan	p-median problem	Multi objective	Min total cost, travel time and Max the minimal satisfaction
Noyan (2012)	humanitarian relief distribution	North Carolina, USA	p-median multi comodity problem	Multi objective	Min facility operating costs, flow cost, salvage and shortage costs
Mete & Zabinsky (2010)	storage of medical supplies	Seatle, USA	p-median location-routing problem	bi-objective	Min warehouse operating cost and routing costs
Balcik & Beamon (2008)	disasters scenarios	National, Ramdom data	Maximal covering problem	single-objective	Max the total gain of satisfied demands
Hale & Moberg (2005)	unreliable facilities and man-made disruptions	Random data	set covering problem	bi-objective	Min number of storage facilities and min cost
Yushimito (2007)	disruption of facilities, route failure and corruption risk	Sioux Falls, USA	p-median location-routing problem	single-objective	Maximize reliability of the relief network
Alsalloum & Rand (2006)	Probability of covering a demand point	Saudi Arabia	Maximal covering problem	Multi objective	Max coverage of demand and reduce the spare capacities of ambulances

NA= Non Available

Appendix 1.1 (continuation) Location models based on the humanitarian response cycle management, response, recovery and long-term humanitarian development phase.

Phase / Author	Algorithm and Method	Problem Size	Model				Result			Time	Term		Constraints																Space		Equity consideration							
			MIP (mixed integer programming)	ILP (integer linear programming)	Multi stage / stochastic	Game theory	Optimal	Near optimal	Paretto solutions		Short term	Long term	Facility capacity	Number of facilities	adapted technology	vehicle capacity	Fleet size	demand satisfaction	Service levels	travel time	allowable distance	Budget constraint	Resources constraints	Shortage and salvage	Inventory	schedule	pre-positioning	Stochastic	Discrete	Continuos	Standard deviation or deviation	Gini Index	Ratio closest/longest travel time	Utility functions/proportional allocation	average distance/time	Proportion of people satisfied	Minimum service level	Maximal service distance/time
Mitigation																																						
Church (2004)	CPLEX	5 to 9 facilities		*			*			<3.92"	*			*				*			*							*										
Berman et al. (2009)	Heuristic et Exact	10000 nodes; 100 source points		*				*		<21558"	*							*											*	*								
Berman et al. (2010)	CPLEX,tabu search, gen. algorithms	100-900 nodes, 5-200 source points		*			*	*		<130'	*							*											*									
Current et O'Kelly (1992)	MPSX (ILP Software)	5-29 omn. Sirens, 5-17 rot sirens		*			*			<1.2'	*				*						*	*							*									
Berry et al. (2006)	Simulation, CPLEX	400-12000 junctions, 4500-14000 pipes	*					*		<13.5'	*											*						*	*									
Murray & Tong (2007)	Heuristics	255 potential facilities	*				*			188"	*																			*								
Murray et al. (2008)	CPLEX et Heuristic	555 to 11,246 polygons;213 to 5297 sites		*				*		<74750"	*																		*									
Xu et al. (2010)	Heuristic	129 nodes		*			*			2"	*																	*	*									
Xu et al. (2010)	Heuristic	130 nodes		*			*			1"	*																	*	*									
Xu et al. (2010)	Heuristic	131 nodes		*			*		*	NA	*																	*	*									
Lee (2001)	Heuristic	20-200 demand points	*					*		90"	*																	*	*									
Berman et al. (2007)	CPLEX, Heuristics	10-150 facilities		*				*		NA	*																	*	*									
Shen et al. (2011)	Heuristic	50-100 nodes			*			*		<300"	*																	*	*									
O'Hanley & Church (2011)	Decomposition algorithm, C++ and CPLEX	25-70 potential facilities, 30-70 demand points	*		*		*			<1535"	*			*				*																				
Preparedness																																						
Chang et al. (2007)	Sample average approximation	35-159 rescue demand points	*					*		NA	*		*		*									*			*	*	*									
Sherali et al. (1991)	Heuristics, Exact implicit enumeration algorithm	74 nodes, 118 arcs	*				*	*		<6.44"	*		*	*																								
Li et al. (2011)	L-shaped algorithm, C++, CPLEX	83 potential facilities, 423520 max population	*				*			3509"	*		*														*	*	*									
Rawls & Turnquist (2012)	CPLEX	16 potential storage facilities, 50 shelter sites	*				*			10'-47h	*		*											*				*	*									
McCall (2006)	CPLEX	7-10 potential storage facilities	*				*			NA	*		*									*		*			*		*									
Campbell & Jones (2011)	Heuristic-Improvement	10 x 10 grid	*					*		<30"	*																*	*		*								
Döyen et al. (2012)	Heuristic-Lagrangian relaxation	50 potential facilities	*					*		1800"	*		*							*							*	*	*									
Tzeng et al. (2007)	Fuzzy programming, Lingo	8 demand points, 5 supply points, 4 transfer points	*					*		NA	*					*											*		*									
Noyan (2012)	CPLEX	50 nodes, up to 712 links	*				*			<1"	*		*											*				*	*									
Mete & Zabinsky (2010)	CPLEX	5 supply points, 10 demand points	*				*			NA	*		*										*				*	*	*									
Balcik & Beamon (2008)	GAMSD, CPLEX	45 potential facilities	*				*			NA	*		*									*					*	*	*									
Hale & Moberg (2005)	CPLEX	7 facilities		*			*			NA	*			*							*								*									
Yushimito (2007)	CPLEX	5 potential facilities		*			*			NA	*												*					*	*									
Alsalloum & Rand (2006)	Goal Programming	17 potential facilities		*				*		NA	*																	*	*									

NA= Non Available. ' (min), '' (sec)

Appendix 1.2 Location models based on the humanitarian response cycle management, mitigation and preparedness phase.

Phase / Author	Setting	Area	Problem	Objective	Objective function
Response					
Afshar & Haghani (2012)	Routing, multicommodity delivery and location of temporary facilities	USA	Transportation problem in disaster response-operations	Multi objective	Min wighted unsatisfied demand
Berkoune et al. (2012)	Transportation of humanitarian aid	Ramdom Data, Qc, Canada	fgnf	Multi objective	Min transportation, docking and loading/unloading times
Yan et Shih (2009)	Transportation of humanitarian aid and roadway repair/earthquake	Nantou, Taiwan	p-center problem	Multi objective	Min roadway repairng time and min time for relief distribution
Ekici et al. (2009)	humanitarian food distribution and disease outbreaks	Georgia, USA	p-medianlocation-routing problem	bi-objective	Min Facility and transportation costs
Sherali et al. (1991)	hurricane and flood	Istambul, Turkey	p-median problem	single-objective	Min Average distance to the population and number of facilities
Görmez et al. (2011)	locate disaster response facilities after eartquake	Turkey	p-center problem	bi-objective	Min demand weighted distance and open facilities
Kongsomsaksakul et al. (2005)	flood evacuation and shelter location	Utah, USA	p-median location-routing problem	bi-objective	Min number of shelters and handling cost
Yi & Özdamar (2007)	evacuation operations in disasters	Istambul, Turkey	p.median routing problem	bi-objective	Min Weighted sum of unsatisfied demand and wounded people
Duran et al. (2011)	humanitarian relief distribution?	International	p.median problem	single-objective	Min average response time
Salmerón & Apte (2010)	humanitarian relief distribution?	Public, International	p.median problem	bi-objective	Min expected casualties and unmet demand
Carr & Roberts (2010)	disease outbreaks	North Carolina, USA	Maximal set covering problem	single-objective	Max Coverage
Recovery and reconstruction					
Nurre et al. (2012)	restoration of services/Hurricane	United Sates	Network design and scheduling problem	Multi objective	Max cumulative weighted flow arriving at demand nodes
Dekle (2005)	humanitarian relief distribution?	Alachua, Florida, USA	Set covering problem	single-objective	Min number of disaster recovery facilities
Huang et al (2012)	humanitarian relief distribution	Random data	VRP-LMDP Last mile delivery problem	alternative onjective	Equity: Miin disparity in efficay among nodes
Nolz et al. (2011)	route failure and corruption risk	Ecuador	Set covering routing problem	Multi objective	Min risk corruption, Min travel time, coverage objective
Fetter & Rakes (2012)	Hurricane	Chesapeake< Virginia, USA	p.median problem	Multi objective	Min Fixed and variable costs max debris collection, max recycling income
Long-term humanitarian development					
Rancourt et al. (2013a)	humanitarian food distribution	Kenya	Hybrid p-median and set covering problem	Multi objective	Min weighted costs of the stakeholders
Rahman & Smith (2000)	Location-allocation of resources for health promotion	International	Revision of the p-median problem and the maximal covering problem		various
Malvankar-Mehta & Xie (2012)	Allocation of resources for health promotion	California, New York	Multi stage problem	Multi objective	Max the number of infections averted
McAllister (1977)	Urban Public facilities	Los Angles, California	Space between public facilities	bi-objective	Max average per capita service
Mandell (1991)	Public facilities	Random data	alternative allocation of service resources	bi-objective	Max overall output (effectiveness) and Min equity index
Kalfakakou (2005)	location of public facilities	Random data	Set covering problem	single-objective	Min facilities instalation cost

NA= Non Available

Appendix 1.2 (continuation) Location models based on the humanitarian response cycle management, mitigation and preparedness phase.

Phase / Author	Algorithm and Method	Problem Size	Model				Result			Time	Term		Constraints																Space		Equity consideration							
			MIP (mixed integer programming)	ILP (integer linear programming)	Multi stage / stochastic	Game theory	Optimal	Near optimal	Paretto solutions		Short term	Long term	Facility capacity	Number of facilities	adapted technology	vehicle capacity	Fleet size	demand satisfaction	Service levels	travel time	allowable distance	Budget constraint	Resources constraints	Shortage and salvage	Inventory	schedule	pre-positioning	Stochastic	Discrete	Continuos	Standard deviation or deviation	Gini Index	Ratio closest/longest travel time	Utility functions/proportional allocation	average distance/time	Proportion of people satisfied	Minimum service level	Maximal service distance/time
Response																																						
Afshar & Haghani (2012)	CPLEX	2-10 facilities, up to 265,995 variables	*				*			<231,035"	*		*				*	*					*			*			*							*	*	
Berkoune et al. (2012)	CPLEX, heuristic, Genetic Algorithms	3 -7 vehicles, 20-60 demand points	*				*	*		30"-600"		*		*		*	*			*	*		*		*		*	*	*									
Yan et Shih (2009)	CPLEX and Heuristics assisted by CPLEX	30, 574 nodes, 112, 837 arcs	*							3 days	*							*		*			*						*									
Ekici et al. (2009)	Add-Drop Heuristic	20 supply points, 603 demand nodes	*		*		*	*		NA		*	*	*							*								*									
Sherali et al. (1991)	Benders decomposition	5-10 potential shelters	*					*		NA	*																		*									
Görmez et al. (2011)	no available	40 potential facility location, 964 demand points	*						*	NA	*		*	*						*	*						*								*	*		
Kongsomsaksakul et al. (2005)	Genetic Algorithm	10 potential shelters, 126 nodes, 360 links	*					*		NA		*	*																*									
Yi & Özdamar (2007)	CPLEX	20 to 60 nodes, 3-8 potential shelter location	*					*		15.9'	*					*	*		*										*									
Duran et al. (2011)	CPLEX	22-100 demand points, 12 potential warehouse loc.	*				*			<4h	*														*		*	*	*									
Salmerón & Apte (2010)	CPLEX	6 potential facilities	*				*			NA	*		*														*	*	*									
Carr & Roberts (2010)	Simulation, C++, CPLEX	105 potential facilities	*					*		<5'		*										*	*					*	*									
Recovery and reconstruction																																						
Nurre et al. (2012)	CPLEX, Heuristics	377-2621 arcs, 386-1810 nodes	*							<21600"	*		*					*					*			*			*									
Dekle (2005)	Excel IP Solver	162-198 potential facilities		*			*			NA	*																	*	*									
Huang et al (2012)	Grasp Metaheuristic, Concert, CPLEX	8-10 nodes for small settings	*				*	*		1'-3631'		*				*	*	*	*							*					*							*
Nolz et al. (2011)	Memetic Algorithm	12 potential facilities, 79-138 nodes	*						*	Few Min	*																	*	*									
Fetter & Rakes (2012)	Excel, Frontline Solvers	7 processing facilites, 49 demand points	*				*			<5'	*			*	*													*	*									
Long-term humanitarian development																																						
Rancourt et al. (2013a)	CPLEX	1460 potential facilities, 24453 demand points	*					*		<20346"		*									*								*						*			
Rahman & Smith (2000)	no available	NA	*				*	*		NA		*																										*
Malvankar-Mehta & Xie (2012)	no available	3 risk grupos, 18,804 to 134,054 population				*	*			NA		*										*						*	*					*				
McAllister (1977)	no available	NA	*				*		*	NA		*	*	*					*	*		*								*	*							
Mandell (1991)	Multiobjective Programming	NA	*						*	NA		*		*																		*				*		
Kalfakakou (2005)	Backtracking, Fortran 77	15-25 potential supply points, 50 -100 demand loc.	*				*			<9"		*								*		*											*					*

NA= Non Available. ' (min), " (sec)

Appendix 2.1 Characteristics of the solutions obtained using non-aggregated data and different objective functions.

Solution identifiers					Demographic indicators										Economic indicators				Additional service and equality indicators for the WI				
					Inhabitants				Population points			Distribution centers			Costs								
Data set	Objective function	Coverage radius	Cost constraint	ICT	Total	Covered	Covered	Walking	Total	Covered	Covered	Total	open DCs	open DCs	Total	Beneficiaries	WFP	KRC	High service threshold	WI in high service distances	WI in low service distances	Gini Index	Hood Index
		km	%		No.	No.	%	No.	No.	No.	%	No.	No.	%	KSh.	KSh.	KSh.	KSh.	km	%	%		
NAD	Min SC	10	-	3	452,218	367,089	81.18%	66,707	47,242	13,195	27.93%	1,459	149	10.21%	37,826,646	3,630,223	31,032,556	3,163,866	5.0000	0.7976	0.2024	0.5823	0.4658
NAD	Min SC	15	-	3	452,218	386,269	85.42%	71,309	47,242	16,611	35.16%	1,459	115	7.88%	37,867,430	4,956,553	30,468,967	2,441,910	7.5000	0.8039	0.1961	0.6108	0.4918
NAD	Min SC	20	-	3	452,218	400,894	88.65%	74,852	47,242	19,393	41.05%	1,459	98	6.72%	38,261,566	6,285,938	29,894,697	2,080,932	10.0000	0.8057	0.1943	0.6215	0.5012
NAD	Min SC	25	-	3	452,218	409,601	90.58%	77,072	47,242	21,231	44.94%	1,459	82	5.62%	38,481,845	7,499,265	29,241,392	1,741,188	12.5000	0.8141	0.1859	0.6306	0.5085
NAD	Min SC	55	-	3	452,218	424,547	93.88%	80,898	47,242	24,453	51.76%	1,459	61	4.18%	39,048,556	10,409,833	27,343,449	1,295,274	10.0000	0.7236	0.2764	0.6664	0.5287
NAD	Min AWD	10	-	3	452,218	367,089	81.18%	66,707	47,242	13,195	27.93%	1,459	1,459	100.00%	65,105,958	2,320,644	31,804,909	30,980,406	5.0000	0.9105	0.0895	0.8319	0.7332
NAD	Min AWD	15	-	3	452,218	386,269	85.42%	71,309	47,242	16,611	35.16%	1,459	1,459	100.00%	66,050,194	3,192,667	31,877,121	30,980,406	7.5000	0.8993	0.1007	0.8135	0.7091
NAD	Min AWD	20	-	3	452,218	400,894	88.65%	74,852	47,242	19,393	41.05%	1,459	1,459	100.00%	66,913,250	4,104,247	31,828,597	30,980,406	10.0000	0.8912	0.1088	0.8009	0.6935
NAD	Min AWD	25	-	3	452,218	409,601	90.58%	77,072	47,242	21,231	44.94%	1,459	1,459	100.00%	67,583,934	4,823,694	31,779,834	30,980,406	12.5000	0.8974	0.1026	0.7949	0.6847
NAD	Min AWD	55	-	3	452,218	424,547	93.88%	80,898	47,242	24,453	51.76%	1,459	1,459	100.00%	69,219,324	6,650,155	31,588,763	30,980,406	27.5000	0.9624	0.0376	0.7962	0.6748
NAD	Max AWD	10	-	3	452,218	367,089	81.18%	66,707	47,242	13,195	27.93%	1,459	1,387	95.07%	70,582,418	9,424,282	31,706,578	29,451,558	5.0000	0.0984	0.9016	0.0994	0.0830
NAD	Max AWD	15	-	3	452,218	386,269	85.42%	71,309	47,242	16,611	35.16%	1,459	1,458	99.93%	77,045,064	14,442,520	31,643,372	30,959,172	7.5000	0.0958	0.9042	0.0983	0.0862
NAD	Max AWD	20	-	3	452,218	400,894	88.65%	74,852	47,242	19,393	41.05%	1,460	1,458	99.86%	81,544,995	19,251,328	31,334,496	30,959,172	10.0000	0.0558	0.9442	0.1020	0.0818
NAD	Max AWD	25	-	3	452,218	409,601	90.58%	77,072	47,242	21,231	44.94%	1,461	1,456	99.66%	86,497,233	24,233,579	31,346,949	30,916,704	12.5000	0.0425	0.9575	0.1106	0.0904
NAD	Min MAD	10	-	3	452,218	367,089	81.18%	66,707	47,242	13,195	27.93%	1,459	1,437	98.49%	68,845,378	6,512,370	31,819,750	30,513,258	5.0000	0.4501	0.5499	0.4022	0.3508
NAD	Min MAD	15	-	3	452,218	386,269	85.42%	71,309	47,242	16,611	35.16%	1,459	1,435	98.36%	71,821,018	9,492,674	31,857,554	30,470,790	7.5000	0.4686	0.5314	0.4201	0.3686
NAD	Min MAD	20	-	3	452,218	400,894	88.65%	74,852	47,242	19,393	41.05%	1,459	1,431	98.08%	74,051,099	12,023,730	31,641,514	30,385,854	10.0000	0.4430	0.5570	0.4128	0.3424
NAD	Min MAD	25	-	3	452,218	409,601	90.58%	77,072	47,242	21,231	44.94%	1,459	1,435	98.36%	76,582,050	14,447,335	31,663,925	30,470,790	12.5000	0.4528	0.5472	0.4306	0.3427
NAD	Min MAD	55	-	3	452,218	424,547	93.88%	80,898	47,242	24,453	51.76%	1,459	1,431	98.08%	84,407,508	22,875,001	31,146,653	30,385,854	27.5000	0.7329	0.2671	0.5309	0.4051
NAD	Max SMR	10	-	3	452,218	367,089	81.18%	66,707	47,242	13,195	27.93%	1,459	696	47.70%	50,411,914	3,758,178	31,874,872	14,778,864	5.0000	0.9105	0.0895	0.3968	0.2856
NAD	Max SMR	15	-	3	452,218	386,269	85.42%	71,309	47,242	16,611	35.16%	1,459	950	65.11%	57,336,921	5,446,186	31,718,435	20,172,300	7.5000	0.8993	0.1007	0.4480	0.3327
NAD	Max SMR	20	-	3	452,218	400,894	88.65%	74,852	47,242	19,393	41.05%	1,459	1,328	91.02%	68,103,072	8,029,842	31,874,478	28,198,752	10.0000	0.8912	0.1088	0.4113	0.3183
NAD	Max SMR	25	-	3	452,218	409,601	90.58%	77,072	47,242	21,231	44.94%	1,459	1,437	98.49%	72,119,901	9,832,152	31,774,491	30,513,258	12.5000	0.8974	0.1026	0.4225	0.3373
NAD	Max S10kmT	10	-	3	452,218	367,089	81.18%	66,707	47,242	13,195	27.93%	1,459	1,283	87.94%	66,414,760	8,222,891	30,948,647	27,243,222	10.0000	1.0000	-	0.2082	0.1749
NAD	Max S10kmT	15	-	3	452,218	386,269	85.42%	71,309	47,242	16,611	35.16%	1,459	815	55.86%	56,139,530	6,930,091	31,903,728	17,305,710	10.0000	0.9355	0.0645	0.4096	0.3359
NAD	Max S10kmT	20	-	3	452,218	400,894	88.65%	74,852	47,242	19,393	41.05%	1,459	1,328	91.02%	68,103,072	8,029,842	31,874,478	28,198,752	10.0000	0.8912	0.1088	0.4113	0.3183
NAD	Max S10kmT	25	-	3	452,218	409,601	90.58%	77,072	47,242	21,231	44.94%	1,459	1,437	98.49%	72,119,901	9,832,152	31,774,491	30,513,258	12.5000	0.8974	0.1026	0.4225	0.3373

NAD = non-aggregated data
ICT = inhabitants' coverage threshold
WI = walking inhabitants

SC = stakeholder costs
AWD = beneficiaries' average walking distance
MAD = beneficiaries' Mean absolute deviation of walking distance
SMR = service below the mid-radius distance threshold
S10kmT = service below the 10 km distance threshold

Appendix 2.1 (continuation) Characteristics of the solutions obtained using non-aggregated data and different objective functions.

Solution identifiers					Indicators for the variables (i, j) beneficiaries walking distance from pop. point to DCs						Indicators for the variables (i, j) beneficiaries walking time from pop. point to DCs					Indicators for the variables (0, j) road distance from DCs to Warehouse					Problem characteristics variables (i, j)		Problem characteristics algorithm effort	
Data set	Objective function	Coverage radius	Cost constraint	ICT	Mean absolute deviation	Average	Minimum	Maximum	Variance	Standard deviation	Average	Minimum	Maximum	Variance	Standard deviation	Average	Minimum	Maximum	Variance	Standard deviation	Total	Tuples	CPU time	CPLEX ticks
		km	%		km	km	km	km	km	km	hours	hours	hours	hours	hours	km	km	km	km	km	No.	No.	Seconds	No.
NAD	Min SC	10	-	3	1.11	2.53	-	9.99	8.05	2.84	1.2631	-	4.9944	4.0270	1.4190	131.78	1.33	268.93	3,675.77	60.63	19,251,505	417,660	154.93	120,685
NAD	Min SC	15	-	3	1.87	3.63	-	14.97	18.83	4.34	1.8168	-	7.4864	9.4150	2.1697	130.14	1.33	268.93	3,890.26	62.37	24,235,449	856,980	229.57	222,470
NAD	Min SC	20	-	3	2.68	4.70	-	19.98	32.78	5.73	2.3478	-	9.9877	16.3903	2.8627	126.10	1.33	268.93	4,223.18	64.99	28,294,387	1,394,152	447.63	391,415
NAD	Min SC	25	-	3	3.40	5.67	-	25.00	49.77	7.05	2.8368	-	12.4988	24.8864	3.5275	121.28	1.33	268.93	4,199.62	64.80	30,976,029	2,000,957	746.09	607,695
NAD	Min SC	55	-	3	4.17	7.98	-	54.97	124.06	11.14	3.9882	-	27.4829	62.0319	5.5692	115.93	1.33	243.19	3,763.05	61.34	35,676,927	6,782,932	4,285.41	2,799,316
NAD	Min AWD	10	-	3	0.75	1.09	-	9.99	5.19	2.28	0.5427	-	4.9944	2.5944	1.1389	106.39	0.49	268.93	5,091.51	71.35	19,251,505	417,660	128.66	115,642
NAD	Min AWD	15	-	3	1.30	1.82	-	14.97	12.77	3.57	0.9091	-	7.4864	6.3854	1.7868	106.39	0.49	268.93	5,091.51	71.35	24,235,449	856,980	297.43	229,762
NAD	Min AWD	20	-	3	1.88	2.56	-	19.97	23.24	4.82	1.2782	-	9.9836	11.6176	2.4101	106.39	0.49	268.93	5,091.51	71.35	28,294,387	1,394,152	918.02	583,424
NAD	Min AWD	25	-	3	2.36	3.13	-	25.00	33.55	5.79	1.5628	-	12.4985	16.7751	2.8961	106.39	0.49	268.93	5,091.51	71.35	30,976,029	2,000,957	3,921.49	1,933,600
NAD	Min AWD	55	-	3	3.58	4.57	-	54.48	75.72	8.70	2.2827	-	27.2380	37.8600	4.3509	106.39	0.49	268.93	5,091.51	71.35	35,676,927	6,782,932	6,233.35	3,422,190
NAD	Max AWD	10	-	3	1.75	8.90	-	9.99	5.46	2.34	4.4506	-	4.9944	2.7285	1.1680	108.89	0.49	268.93	4,983.57	70.59	19,251,505	10,931	9,775.57	3,856,867
NAD	Max AWD	15	-	3	3.14	13.40	-	14.97	13.68	3.70	6.6985	-	7.4865	6.8392	1.8492	106.68	0.49	268.93	5,086.92	71.32	24,235,449	10,931	35,657.00	11,654,737
NAD	Max AWD	20	-	3	4.71	17.41	-	19.98	15.97	4.00	8.7043	-	9.9877	7.9867	1.9983	106.85	0.49	268.93	5,087.96	71.33	28,294,387	10,931	142,699.00	38,406,336
NAD	Max AWD	25	-	3	6.32	21.61	-	25.00	26.25	5.12	10.8047	-	12.4988	13.1232	2.5616	106.68	0.49	268.93	5,084.53	71.31	30,976,029	10,931	508,247.00	130,788,355
NAD	Min MAD	10	-	3	0.69	5.70	-	9.99	17.62	4.20	2.8487	-	4.9944	8.8093	2.0987	106.67	1.70	268.93	5,084.56	71.31	19,251,505	417,660	74.99	74,698
NAD	Min MAD	15	-	3	1.22	8.30	-	14.97	41.04	6.41	4.1512	-	7.4864	20.5207	3.2032	106.68	1.70	268.93	5,084.77	71.31	24,235,449	856,980	221.25	184,753
NAD	Min MAD	20	-	3	1.80	10.32	-	19.97	57.70	7.60	5.1609	-	9.9836	28.8500	3.7980	106.64	1.70	268.93	5,090.67	71.35	28,294,387	1,394,152	472.79	345,371
NAD	Min MAD	25	-	3	2.26	12.29	-	25.00	86.65	9.31	6.1450	-	12.4985	43.3250	4.6543	106.57	1.70	268.93	5,094.11	71.37	30,976,029	2,000,957	913.47	578,039
NAD	Min MAD	55	-	3	3.44	19.29	-	54.48	351.05	18.74	9.6427	-	27.2380	175.5273	9.3682	106.64	1.70	268.93	5,090.67	71.35	35,676,927	6,782,932	12,572.60	6,794,429
NAD	Max SMR	10	-	3	0.86	2.67	-	9.99	4.23	2.06	1.3335	-	4.9944	2.1153	1.0284	124.07	3.30	268.93	4,371.41	66.12	19,251,505	417,660	248.90	254,995
NAD	Max SMR	15	-	3	1.61	4.14	-	14.97	12.41	3.52	2.0688	-	7.4864	6.2030	1.7611	111.98	0.49	268.93	4,659.34	68.26	24,235,449	856,980	422.68	474,746
NAD	Max SMR	20	-	3	1.89	6.41	-	19.97	22.68	4.76	3.2028	-	9.9836	11.3395	2.3811	106.71	2.06	268.93	5,006.55	70.76	28,294,387	1,394,152	2,036.82	1,923,187
NAD	Max SMR	25	-	3	2.31	7.90	-	25.00	36.28	6.02	3.9476	-	12.4985	18.1414	3.0118	106.85	1.70	268.93	5,059.52	71.13	30,976,029	2,000,957	4,532.19	3,803,645
NAD	Max S10kmT	10	-	3	1.54	7.58	-	9.99	10.69	3.27	3.7897	-	4.9944	5.3429	1.6345	98.31	0.49	268.93	4,689.63	68.48	19,251,505	417,660	193.25	245,049
NAD	Max S10kmT	15	-	3	1.29	5.66	-	14.97	17.04	4.13	2.8324	-	7.4864	8.5212	2.0641	114.68	2.21	268.93	4,841.48	69.58	24,235,449	856,980	818.46	1,051,270
NAD	Max S10kmT	20	-	3	1.89	6.41	-	19.97	22.68	4.76	3.2028	-	9.9836	11.3395	2.3811	106.71	2.06	268.93	5,006.55	70.76	28,294,387	1,394,152	1,918.12	1,923,187
NAD	Max S10kmT	25	-	3	2.31	7.90	-	25.00	36.28	6.02	3.9476	-	12.4985	18.1414	3.0118	106.85	1.70	268.93	5,059.52	71.13	30,976,029	2,000,957	4,532.19	3,803,645

Appendix 2.2 Characteristics of the solutions obtained using non-aggregated data and costs constraints above the MSC for the beneficiaries' average walking distance objective function.

Solution identifiers					Demographic indicators: Inhabitants				Demographic indicators: Population points			Demographic indicators: Distribution centers			Economic indicators: Costs				Additional service and equality indicators for the WI				
Data set	Objective function	Coverage radius	Cost constraint	ICT	Total	Covered	Covered	Walking	Total	Covered	Covered	Total	open DCs	open DCs	Total	Beneficiaries	WFP	KRC	High service threshold	WI in high service distances	WI in low service distances	Gini Index	Hood Index
		km	%		No.	No.	%	No.	No.	No.	%	No.	No.	%	KSh.	KSh.	KSh.	KSh.	km	%	%		
NAD	Min AWD	10	5%	3	452,218	367,089	81.18%	66,707	47,242	13,195	27.93%	1,459	254	17.41%	39,716,812	2,665,144	31,658,232	5,393,436	5.0000	0.9026	0.0974	0.7424	0.6140
NAD	Min AWD	10	10%	3	452,218	367,089	81.18%	66,707	47,242	13,195	27.93%	1,459	345	23.65%	41,603,617	2,521,653	31,756,234	7,325,730	5.0000	0.9086	0.0914	0.7690	0.6348
NAD	Min AWD	10	15%	3	452,218	367,089	81.18%	66,707	47,242	13,195	27.93%	1,459	436	29.88%	43,491,303	2,461,893	31,771,386	9,258,024	5.0000	0.9098	0.0902	0.7829	0.6477
NAD	Min AWD	10	20%	3	452,218	367,089	81.18%	66,707	47,242	13,195	27.93%	1,459	526	36.05%	45,384,840	2,430,384	31,785,372	11,169,084	5.0000	0.9095	0.0905	0.7930	0.6599
NAD	Min AWD	15	5%	3	452,218	386,269	85.42%	71,309	47,242	16,611	35.16%	1,459	214	14.67%	39,765,795	3,642,357	31,579,362	4,544,076	7.5000	0.8937	0.1063	0.7319	0.5916
NAD	Min AWD	15	10%	3	452,218	386,269	85.42%	71,309	47,242	16,611	35.16%	1,459	303	20.77%	41,653,639	3,438,419	31,781,317	6,433,902	7.5000	0.8981	0.1019	0.7608	0.6187
NAD	Min AWD	15	15%	3	452,218	386,269	85.42%	71,309	47,242	16,611	35.16%	1,459	392	26.87%	43,529,781	3,367,754	31,838,299	8,323,728	7.5000	0.8981	0.1019	0.7740	0.6374
NAD	Min AWD	15	20%	3	452,218	386,269	85.42%	71,309	47,242	16,611	35.16%	1,459	483	33.10%	45,419,012	3,317,943	31,845,047	10,256,022	7.5000	0.8987	0.1013	0.7833	0.6531
NAD	Min AWD	20	5%	3	452,218	400,894	88.65%	74,852	47,242	19,393	41.05%	1,459	192	13.16%	40,164,749	4,643,372	31,444,449	4,076,928	10.0000	0.8873	0.1127	0.7239	0.5786
NAD	Min AWD	20	10%	3	452,218	400,894	88.65%	74,852	47,242	19,393	41.05%	1,459	282	19.33%	42,087,092	4,377,876	31,721,227	5,987,988	10.0000	0.8904	0.1096	0.7556	0.6160
NAD	Min AWD	20	15%	3	452,218	400,894	88.65%	74,852	47,242	19,393	41.05%	1,459	373	25.57%	43,997,020	4,300,490	31,776,248	7,920,282	10.0000	0.8900	0.1100	0.7682	0.6370
NAD	Min AWD	20	20%	3	452,218	400,894	88.65%	74,852	47,242	19,393	41.05%	1,459	465	31.87%	45,912,629	4,245,812	31,793,007	9,873,810	10.0000	0.8904	0.1096	0.7765	0.6510
NAD	Min AWD	25	5%	3	452,218	409,601	90.58%	77,072	47,242	21,231	44.94%	1,459	170	11.65%	40,404,744	5,472,048	31,322,916	3,609,780	12.5000	0.8940	0.1060	0.7164	0.5692
NAD	Min AWD	25	10%	3	452,218	409,601	90.58%	77,072	47,242	21,231	44.94%	1,459	261	17.89%	42,319,292	5,131,404	31,645,814	5,542,074	12.5000	0.8968	0.1032	0.7516	0.6134
NAD	Min AWD	25	15%	3	452,218	409,601	90.58%	77,072	47,242	21,231	44.94%	1,459	354	24.26%	44,253,907	5,023,931	31,713,141	7,516,836	12.5000	0.8973	0.1027	0.7648	0.6341
NAD	Min AWD	25	20%	3	452,218	409,601	90.58%	77,072	47,242	21,231	44.94%	1,459	446	30.57%	46,174,190	4,960,703	31,743,123	9,470,364	12.5000	0.8974	0.1026	0.7730	0.6483
NAD	Min AWD	55	5%	3	452,218	424,547	93.88%	80,898	47,242	24,453	51.76%	1,459	111	7.61%	40,174,325	8,253,214	29,564,137	2,356,974	27.5000	0.9517	0.0483	0.7048	0.5584
NAD	Min AWD	55	10%	3	452,218	424,547	93.88%	80,898	47,242	24,453	51.76%	1,459	173	11.86%	42,087,701	7,256,100	31,158,119	3,673,482	27.5000	0.9620	0.0380	0.7391	0.5901
NAD	Min AWD	55	15%	3	452,218	424,547	93.88%	80,898	47,242	24,453	51.76%	1,459	263	18.03%	43,994,547	6,954,293	31,455,712	5,584,542	27.5000	0.9623	0.0377	0.7639	0.6251
NAD	Min AWD	55	20%	3	452,218	424,547	93.88%	80,898	47,242	24,453	51.76%	1,459	355	24.33%	45,913,821	6,842,766	31,532,985	7,538,070	27.5000	0.9624	0.0376	0.7744	0.6415

NAD = non-aggregated data
ICT = inhabitants' coverage threshold
WI = walking inhabitants
MSC = minimum stakeholder costs

AWD = beneficiaries' average walking distance

Appendix 2.2 (continuation) Characteristics of the solutions obtained using non-aggregated data and costs constraints above the MSC for the beneficiaries' average walking distance objective function.

Solution identifiers					Indicators for the variables (i,j) beneficiaries walking distance from pop. point to DCs						Indicators for the variables (i,j) beneficiaries walking time from pop. point to DCs					Indicators for the variables $(0,j)$ road distance from DCs to Warehouse					Problem characteristics variables (i,j)		Problem characteristics algorithm effort	
Data set	Objective function	Coverage radius	Cost constraint	ICT	Mean absolute deviation	Average	Minimum	Maximum	Variance	Standard deviation	Average	Minimum	Maximum	Variance	Standard deviation	Average	Minimum	Maximum	Variance	Standard deviation	Total	Tuples	CPU time	CPLEX ticks
		km	%		km	km	km	km	km	km	hours	hours	hours	hours	hours	km	km	km	km	km	No.	No.	Seconds	No.
NAD	Min AWD	10	5%	3	0.86	1.46	-	9.99	5.57	2.36	0.7322	-	4.9944	2.7827	1.1796	124.64	0.49	268.93	4,389.43	66.25	19,251,505	417,660	4,107.65	1,957,622
NAD	Min AWD	10	10%	3	0.81	1.31	-	9.99	5.22	2.28	0.6533	-	4.9944	2.6090	1.1421	119.06	0.49	268.93	4,571.51	67.61	19,251,505	417,660	2,399.02	1,168,705
NAD	Min AWD	10	15%	3	0.79	1.24	-	9.99	5.13	2.27	0.6204	-	4.9944	2.5672	1.1330	116.44	0.49	268.93	4,600.86	67.83	19,251,505	417,660	1,608.85	797,512
NAD	Min AWD	10	20%	3	0.78	1.21	-	9.99	5.15	2.27	0.6031	-	4.9944	2.5774	1.1352	114.57	0.49	268.93	4,681.64	68.42	19,251,505	417,660	2,114.33	1,108,815
NAD	Min AWD	15	5%	3	1.44	2.28	-	14.97	12.95	3.60	1.1405	-	7.4864	6.4737	1.7991	124.74	0.49	268.93	4,370.20	66.11	24,235,449	856,980	30,004.30	13,856,885
NAD	Min AWD	15	10%	3	1.37	2.07	-	14.97	12.56	3.54	1.0355	-	7.4864	6.2824	1.7723	118.36	0.49	268.93	4,662.00	68.28	24,235,449	856,980	9,299.01	3,314,648
NAD	Min AWD	15	15%	3	1.34	2.00	-	14.97	12.59	3.55	0.9992	-	7.4864	6.2955	1.7742	117.79	0.49	268.93	4,639.21	68.11	24,235,449	856,980	2,757.33	1,352,216
NAD	Min AWD	15	20%	3	1.33	1.95	-	14.97	12.58	3.55	0.9735	-	7.4864	6.2890	1.7733	115.57	0.49	268.93	4,635.68	68.09	24,235,449	856,980	3,727.65	1,757,621
NAD	Min AWD	20	5%	3	2.05	3.09	-	19.97	22.97	4.79	1.5425	-	9.9836	11.4852	2.3964	123.27	0.49	268.93	4,293.33	65.52	28,294,387	1,394,152	24,301.80	9,317,277
NAD	Min AWD	20	10%	3	1.96	2.82	-	19.98	22.71	4.77	1.4124	-	9.9877	11.3538	2.3826	117.18	0.49	268.93	4,690.87	68.49	28,294,387	1,394,152	8,454.14	3,385,909
NAD	Min AWD	20	15%	3	1.94	2.75	-	19.97	22.92	4.79	1.3744	-	9.9836	11.4617	2.3939	117.84	0.49	268.93	4,612.74	67.92	28,294,387	1,394,152	5,675.61	2,229,993
NAD	Min AWD	20	20%	3	1.92	2.70	-	19.97	23.02	4.80	1.3476	-	9.9836	11.5107	2.3990	115.71	0.49	268.93	4,637.46	68.10	28,294,387	1,394,152	4,328.15	1,580,552
NAD	Min AWD	25	5%	3	2.57	3.74	-	25.00	32.96	5.74	1.8715	-	12.4985	16.4780	2.8704	123.05	0.49	268.93	4,304.31	65.61	30,976,029	2,000,957	43,900.80	16,181,206
NAD	Min AWD	25	10%	3	2.45	3.42	-	25.00	32.72	5.72	1.7093	-	12.4986	16.3620	2.8602	120.56	0.49	268.93	4,633.19	68.07	30,976,029	2,000,957	42,300.90	15,039,976
NAD	Min AWD	25	15%	3	2.42	3.32	-	25.00	32.85	5.73	1.6582	-	12.4986	16.4273	2.8659	117.51	0.49	268.93	4,624.86	68.01	30,976,029	2,000,957	90,878.30	33,268,712
NAD	Min AWD	25	20%	3	2.39	3.26	-	25.00	32.99	5.74	1.6281	-	12.4986	16.4946	2.8718	115.76	0.49	268.93	4,614.70	67.93	30,976,029	2,000,957	10,588.30	3,917,598
NAD	Min AWD	55	5%	3	4.17	6.02	-	54.81	86.13	9.28	3.0099	-	27.4049	43.0642	4.6403	120.98	0.49	268.93	4,204.49	64.84	35,676,927	6,782,932	268,415.00	71,542,419
NAD	Min AWD	55	10%	3	3.77	5.12	-	54.70	73.99	8.60	2.5576	-	27.3504	36.9952	4.3009	122.24	0.49	268.93	4,342.69	65.90	35,676,927	6,782,932	219,062.00	64,941,403
NAD	Min AWD	55	15%	3	3.67	4.84	-	54.48	74.21	8.61	2.4207	-	27.2380	37.1049	4.3073	120.81	0.49	268.93	4,579.53	67.67	35,676,927	6,782,932	189,728.00	20,924,961
NAD	Min AWD	55	20%	3	3.63	4.74	-	54.48	74.57	8.64	2.3701	-	27.2380	37.2853	4.3177	117.04	0.49	268.93	4,628.17	68.03	35,676,927	6,782,932	79,275.80	13,583,469

Appendix 2.3 Characteristics of the solutions obtained using non-aggregated data and costs constraints above the MSC for the beneficiaries' mean absolute deviation of walking distance objective function.

Solution identifiers					Demographic indicators										Economic indicators				Additional service and equality indicators for the WI				
					Inhabitants				Population points			Distribution centers			Costs								
Data set	Objective function	Coverage radius	Cost constraint	ICT	Total	Covered	Covered	Walking	Total	Covered	Covered	Total	open DCs	open DCs	Total	Beneficiaries	WFP	KRC	High service threshold	WI in high service distances	WI in low service distances	Gini Index	Hood Index
		km	%		No.	No.	%	No.	No.	No.	%	No.	No.	%	KSh.	KSh.	KSh.	KSh.	km	%	%		
NAD	Min MAD	10	5%	3	452,218	367,089	81.18%	66,707	47,242	13,195	27.93%	1,459	222	15.22%	39,717,227	3,351,324	31,651,955	4,713,948	5.0000	0.8727	0.1273	0.5415	0.4072
NAD	Min MAD	10	10%	3	452,218	367,089	81.18%	66,707	47,242	13,195	27.93%	1,459	271	18.57%	41,605,009	4,234,035	31,616,560	5,754,414	5.0000	0.7523	0.2477	0.5051	0.3953
NAD	Min MAD	10	15%	3	452,218	367,089	81.18%	66,707	47,242	13,195	27.93%	1,459	322	22.07%	43,494,094	4,992,707	31,664,039	6,837,348	5.0000	0.6526	0.3474	0.4920	0.4127
NAD	Min MAD	10	20%	3	452,218	367,089	81.18%	66,707	47,242	13,195	27.93%	1,459	385	26.39%	45,379,565	5,504,272	31,700,203	8,175,090	5.0000	0.5854	0.4146	0.4680	0.4011
NAD	Min MAD	15	5%	3	452,218	386,269	85.42%	71,309	47,242	16,611	35.16%	1,459	192	13.16%	39,757,584	4,014,202	31,666,454	4,076,928	7.5000	0.8969	0.1031	0.5849	0.4477
NAD	Min MAD	15	10%	3	452,218	386,269	85.42%	71,309	47,242	16,611	35.16%	1,459	250	17.14%	41,649,896	4,617,360	31,724,036	5,308,500	7.5000	0.8663	0.1337	0.5783	0.4511
NAD	Min MAD	15	15%	3	452,218	386,269	85.42%	71,309	47,242	16,611	35.16%	1,459	270	18.51%	43,526,885	6,105,431	31,688,274	5,733,180	7.5000	0.7339	0.2661	0.5579	0.4590
NAD	Min MAD	15	20%	3	452,218	386,269	85.42%	71,309	47,242	16,611	35.16%	1,459	317	21.73%	45,432,915	7,033,736	31,668,001	6,731,178	7.5000	0.6584	0.3416	0.5333	0.4568
NAD	Min MAD	20	5%	3	452,218	400,894	88.65%	74,852	47,242	19,393	41.05%	1,459	172	11.79%	40,167,117	4,939,646	31,575,223	3,652,248	10.0000	0.8891	0.1109	0.6271	0.4883
NAD	Min MAD	20	10%	3	452,218	400,894	88.65%	74,852	47,242	19,393	41.05%	1,459	234	16.04%	42,063,626	5,395,545	31,699,325	4,968,756	10.0000	0.8601	0.1399	0.6130	0.4905
NAD	Min MAD	20	15%	3	452,218	400,894	88.65%	74,852	47,242	19,393	41.05%	1,459	257	17.61%	43,753,914	6,661,571	31,635,205	5,457,138	10.0000	0.7860	0.2140	0.5830	0.4775
NAD	Min MAD	20	20%	3	452,218	400,894	88.65%	74,852	47,242	19,393	41.05%	1,459	291	19.95%	45,908,385	8,214,303	31,514,988	6,179,094	10.0000	0.6760	0.3240	0.5486	0.4665
NAD	Min MAD	25	5%	3	452,218	409,601	90.58%	77,072	47,242	21,231	44.94%	1,459	154	10.56%	40,405,927	5,733,608	31,402,283	3,270,036	12.5000	0.8953	0.1047	0.6411	0.5026
NAD	Min MAD	25	10%	3	452,218	409,601	90.58%	77,072	47,242	21,231	44.94%	1,459	222	15.22%	42,329,854	5,964,880	31,651,026	4,713,948	12.5000	0.8673	0.1327	0.6294	0.5088
NAD	Min MAD	25	15%	3	452,218	409,601	90.58%	77,072	47,242	21,231	44.94%	1,459	256	17.55%	44,114,310	7,017,636	31,660,769	5,435,904	12.5000	0.8298	0.1702	0.6184	0.5027
NAD	Min MAD	25	20%	3	452,218	409,601	90.58%	77,072	47,242	21,231	44.94%	1,459	273	18.71%	46,106,336	8,821,800	31,487,654	5,796,882	12.5000	0.7173	0.2827	0.5776	0.4799

NAD = non-aggregated data
ICT = inhabitants' coverage threshold
WI = walking inhabitants
MSC = minimum stakeholder costs

MAD = beneficiaries' mean absolute deviation of walking distance

Appendix 2.3 (continuation) Characteristics of the solutions obtained using non-aggregated data and costs constraints above the MSC for the beneficiaries' mean absolute deviation of walking distance objective function.

Solution identifiers					Indicators for the variables (i, j) beneficiaries walking distance from pop. point to DCs						beneficiaries walking time from pop. point to DCs					Indicators for the variables (0, j) road distance from DCs to Warehouse					Problem characteristics variables (i, j)		algorithm effort	
Data set	Objective function	Coverage radius	Cost constraint	ICT	Mean absolute deviation	Average	Minimum	Maximum	Variance	Standard deviation	Average	Minimum	Maximum	Variance	Standard deviation	Average	Minimum	Maximum	Variance	Standard deviation	Total	Tuples	CPU time	CPLEX ticks
		km	%		km	km	km	km	km	km	hours	hours	hours	hours	hours	km	km	km	km	km	No.	No.	Seconds	No.
NAD	Min MAD	10	5%	3	0.83	2.22	-	9.99	6.11	2.47	1.1097	-	4.9944	3.0570	1.2363	126.00	1.70	268.93	3,924.70	62.65	19,251,505	417,660	22,251.90	6,684,195
NAD	Min MAD	10	10%	3	0.78	3.19	-	9.99	9.21	3.04	1.5953	-	4.9944	4.6060	1.5176	123.55	1.70	268.93	4,027.91	63.47	19,251,505	417,660	2,323.98	1,187,086
NAD	Min MAD	10	15%	3	0.75	4.03	-	9.99	13.33	3.65	2.0127	-	4.9944	6.6651	1.8255	123.87	1.70	268.93	4,019.60	63.40	19,251,505	417,660	2,122.79	1,063,413
NAD	Min MAD	10	20%	3	0.73	4.59	-	9.99	15.27	3.91	2.2941	-	4.9944	7.6348	1.9538	120.93	2.06	268.93	4,149.83	64.42	19,251,505	417,660	1,439.85	709,171
NAD	Min MAD	15	5%	3	1.40	2.66	-	14.97	11.12	3.34	1.3319	-	7.4864	5.5619	1.6676	126.10	1.33	268.93	3,964.77	62.97	24,235,449	856,980	28,523.80	7,294,187
NAD	Min MAD	15	10%	3	1.40	3.28	-	14.97	15.28	3.91	1.6423	-	7.4864	7.6407	1.9546	124.48	1.70	268.93	3,941.49	62.78	24,235,449	856,980	28,523.80	7,294,187
NAD	Min MAD	15	15%	3	1.31	4.82	-	14.97	26.64	5.16	2.4080	-	7.4864	13.3204	2.5807	123.74	1.70	268.93	3,963.89	62.96	24,235,449	856,980	5,467.25	1,896,633
NAD	Min MAD	15	20%	3	1.31	5.77	-	14.97	33.28	5.77	2.8858	-	7.4864	16.6381	2.8843	122.48	1.70	268.93	4,045.37	63.60	24,235,449	856,980	5,467.25	1,896,633
NAD	Min MAD	20	5%	3	2.01	3.38	-	19.97	20.82	4.56	1.6878	-	9.9836	10.4076	2.2812	124.71	0.49	268.93	4,000.38	63.25	28,294,387	1,394,152	29,921.80	9,343,535
NAD	Min MAD	20	10%	3	1.94	3.82	-	19.97	25.07	5.01	1.9113	-	9.9836	12.5362	2.5036	124.15	1.70	268.93	3,880.03	62.29	28,294,387	1,394,152	20,900.10	7,745,030
NAD	Min MAD	20	15%	3	1.91	5.06	-	19.97	34.61	5.88	2.5320	-	9.9836	17.3026	2.9413	121.94	1.70	268.93	3,817.39	61.79	28,294,387	1,394,152	7,791.90	2,621,269
NAD	Min MAD	20	20%	3	1.87	6.59	-	19.97	45.77	6.77	3.2932	-	9.9836	22.8836	3.3826	120.31	1.70	268.93	4,008.44	63.31	28,294,387	1,394,152	13,854.60	2,871,449
NAD	Min MAD	25	5%	3	2.51	3.99	-	25.00	30.54	5.53	1.9961	-	12.4986	15.2703	2.7632	124.12	0.49	268.93	3,934.99	62.73	30,976,029	2,000,957	91,398.70	15,303,546
NAD	Min MAD	25	10%	3	2.42	4.21	-	25.00	32.01	5.66	2.1062	-	12.4986	16.0043	2.8288	123.53	1.70	268.93	3,944.59	62.81	30,976,029	2,000,957	109,533.00	41,824,972
NAD	Min MAD	25	15%	3	2.40	5.21	-	25.00	44.66	6.68	2.6074	-	12.4985	22.3275	3.3412	125.77	1.70	268.93	3,880.73	62.30	30,976,029	2,000,957	12,975.60	4,508,909
NAD	Min MAD	25	20%	3	2.35	6.93	-	25.00	58.20	7.63	3.4665	-	12.4986	29.0988	3.8144	121.99	1.70	268.93	4,010.69	63.33	30,976,029	2,000,957	15,391.80	5,407,283

Appendix 2.4 Characteristics of the solutions obtained using non-aggregated data and costs constraints above the MSC for the beneficiaries' standard service objective function.

Solution identifiers					Demographic indicators: Inhabitants				Demographic indicators: Population points			Demographic indicators: Distribution centers			Economic indicators: Costs				Additional service and equality indicators for the WI				
Data set	Objective function	Coverage radius	Cost constraint	ICT	Total	Covered	Covered	Walking	Total	Covered	Covered	Total	open DCs	open DCs	Total	Beneficiaries	WFP	KRC	High service threshold	WI in high service distances	WI in low service distances	Gini Index	Hood Index
		km	%		No.	No.	%	No.	No.	No.	%	No.	No.	%	KSh.	KSh.	KSh.	KSh.	km	%	%		
NAD	Max SMR	10	5%	3	452,218	367,089	81.18%	66,707	47,242	13,195	27.93%	1,459	245	16.79%	39,713,388	3,099,891	31,411,167	5,202,330	5.0000	0.9104	0.0896	0.5990	0.4583
NAD	Max SMR	10	10%	3	452,218	367,089	81.18%	66,707	47,242	13,195	27.93%	1,459	316	21.66%	41,608,970	3,164,174	31,734,852	6,709,944	5.0000	0.9105	0.0895	0.5580	0.4224
NAD	Max SMR	10	15%	3	452,218	367,089	81.18%	66,707	47,242	13,195	27.93%	1,459	394	27.00%	43,498,221	3,358,064	31,773,961	8,366,196	5.0000	0.9105	0.0895	0.4980	0.3684
NAD	Max SMR	10	20%	3	452,218	367,089	81.18%	66,707	47,242	13,195	27.93%	1,459	484	33.17%	45,382,647	3,319,590	31,785,802	10,277,256	5.0000	0.9105	0.0895	0.5042	0.3756
NAD	Max SMR	15	5%	3	452,218	386,269	85.42%	71,309	47,242	16,611	35.16%	1,459	201	13.78%	39,760,799	4,278,759	31,214,006	4,268,034	7.5000	0.8993	0.1007	0.6142	0.4815
NAD	Max SMR	15	10%	3	452,218	386,269	85.42%	71,309	47,242	16,611	35.16%	1,459	272	18.64%	41,654,106	4,310,050	31,568,409	5,775,648	7.5000	0.8993	0.1007	0.6282	0.4960
NAD	Max SMR	15	15%	3	452,218	386,269	85.42%	71,309	47,242	16,611	35.16%	1,459	351	24.06%	43,545,906	4,419,609	31,673,163	7,453,134	7.5000	0.8993	0.1007	0.5992	0.4730
NAD	Max SMR	15	20%	3	452,218	386,269	85.42%	71,309	47,242	16,611	35.16%	1,459	384	26.32%	45,423,365	5,538,507	31,731,002	8,153,856	7.5000	0.8993	0.1007	0.4429	0.3430
NAD	Max SMR	20	5%	3	452,218	400,894	88.65%	74,852	47,242	19,393	41.05%	1,459	167	11.45%	40,174,463	5,379,301	31,249,084	3,546,078	10.0000	0.8912	0.1088	0.6278	0.4950
NAD	Max SMR	20	10%	3	452,218	400,894	88.65%	74,852	47,242	19,393	41.05%	1,459	210	14.39%	42,046,835	5,997,344	31,590,351	4,459,140	10.0000	0.8912	0.1088	0.5584	0.4328
NAD	Max SMR	20	15%	3	452,218	400,894	88.65%	74,852	47,242	19,393	41.05%	1,459	286	19.60%	43,898,027	6,157,389	31,667,714	6,072,924	10.0000	0.8912	0.1088	0.5520	0.4277
NAD	Max SMR	20	20%	3	452,218	400,894	88.65%	74,852	47,242	19,393	41.05%	1,459	378	25.91%	45,854,145	6,091,618	31,736,076	8,026,452	10.0000	0.8912	0.1088	0.5400	0.4177
NAD	Max SMR	25	5%	3	452,218	409,601	90.58%	77,072	47,242	21,231	44.94%	1,459	157	10.76%	40,405,502	5,906,545	31,165,219	3,333,738	12.5000	0.8974	0.1026	0.6376	0.4995
NAD	Max SMR	25	10%	3	452,218	409,601	90.58%	77,072	47,242	21,231	44.94%	1,459	222	15.22%	42,320,249	5,940,968	31,665,333	4,713,948	12.5000	0.8974	0.1026	0.6348	0.5121
NAD	Max SMR	25	15%	3	452,218	409,601	90.58%	77,072	47,242	21,231	44.94%	1,459	307	21.04%	44,218,736	6,055,088	31,644,810	6,518,838	12.5000	0.8974	0.1026	0.5958	0.4679
NAD	Max SMR	25	20%	3	452,218	409,601	90.58%	77,072	47,242	21,231	44.94%	1,459	374	25.63%	46,122,468	6,535,081	31,645,871	7,941,516	12.5000	0.8974	0.1026	0.5838	0.4660

NAD = non-aggregated data
ICT = inhabitants' coverage threshold
WI = walking inhabitants
SMR = service below the mid-radius distance threshold

Appendix 2.4 (Continuation) Characteristics of the solutions obtained using Non-Aggregated Data and Costs Constraints above the MSCfor the Beneficiaries' Standard Service Objective Function

Solution Identifiers: Data Set	Objective Function	Coverage Radius (km)	Cost Constraint (%)	ICT	Indicators for the allocated variables ij — beneficiaries walking distance from Pop. Point to DCs): Mean Absolute Deviation (km)	Average (km)	Minimum (km)	Maximum (km)	Variance (km)	Standard Deviation (km)	beneficiaries walking time from Pop. Point to DCs): Average (hours)	Minimum (hours)	Maximum (hours)	Variance (hours)	Standard Deviation (hours)	Indicators for the allocated variables oj — Road distance from DCs to Warehouse: Average (km)	Minimum (km)	Maximum (km)	Variance (km)	Standard Deviation (km)	Problem Characteristics — Variables ij: Total (No.)	Valid (Tuples) (No.)	Algorithm effort: CPU Time (Seconds)	Cplex Ticks (No.)
NAD	Max SMR	10	5%	3	0.91	1.94	-	9.99	5.52	2.35	0.9714	-	4.9944	2.7575	1.1742	125.82	1.33	268.93	3,713.64	60.94	19,251,505	417,660	2,268.96	1,328,551
NAD	Max SMR	10	10%	3	0.88	2.01	-	9.99	5.10	2.26	1.0067	-	4.9944	2.5522	1.1296	125.83	1.33	268.93	3,918.91	62.60	19,251,505	417,660	756.57	516,165
NAD	Max SMR	10	15%	3	0.88	2.23	-	9.99	4.79	2.19	1.1134	-	4.9944	2.3966	1.0947	129.55	0.49	268.93	3,811.24	61.74	19,251,505	417,660	669.71	477,182
NAD	Max SMR	10	20%	3	0.87	2.18	-	9.99	4.81	2.19	1.0922	-	4.9944	2.4058	1.0968	131.74	1.33	268.93	3,922.51	62.63	19,251,505	417,660	877.94	637,369
NAD	Max SMR	15	5%	3	1.56	2.94	-	14.97	13.42	3.66	1.4680	-	7.4864	6.7080	1.8314	126.39	0.49	268.93	3,753.61	61.27	24,235,449	856,980	22,722.90	8,679,868
NAD	Max SMR	15	10%	3	1.62	2.97	-	14.97	14.31	3.78	1.4841	-	7.4864	7.1572	1.8917	127.29	2.86	268.93	3,625.15	60.21	24,235,449	856,980	16,840.00	5,148,296
NAD	Max SMR	15	15%	3	1.62	3.08	-	14.97	14.04	3.75	1.5405	-	7.4864	7.0177	1.8732	127.20	2.86	268.93	3,522.26	59.35	24,235,449	856,980	13,127.30	3,805,825
NAD	Max SMR	15	20%	3	1.34	4.23	-	14.97	11.75	3.43	2.1163	-	7.4864	5.8726	1.7136	121.26	3.88	268.93	4,049.65	63.64	24,235,449	856,980	31,431.10	7,894,578
NAD	Max SMR	20	5%	3	2.20	3.81	-	19.98	23.41	4.84	1.9033	-	9.9877	11.7047	2.4192	129.96	1.33	268.93	3,530.59	59.42	28,294,387	1,394,152	12,529.30	6,801,172
NAD	Max SMR	20	10%	3	2.16	4.41	-	19.98	22.50	4.74	2.2063	-	9.9877	11.2520	2.3719	132.20	5.03	268.93	3,509.87	59.24	28,294,387	1,394,152	8,726.87	3,787,556
NAD	Max SMR	20	15%	3	2.14	4.57	-	19.98	22.91	4.79	2.2848	-	9.9877	11.4541	2.3931	129.95	3.88	268.93	3,573.17	59.78	28,294,387	1,394,152	4,605.07	2,819,901
NAD	Max SMR	20	20%	3	2.11	4.51	-	19.98	21.97	4.69	2.2526	-	9.9876	10.9863	2.3437	130.42	3.88	268.93	3,748.96	61.23	28,294,387	1,394,152	7,334.55	4,256,560
NAD	Max SMR	25	5%	3	2.58	4.16	-	25.00	31.20	5.59	2.0784	-	12.4986	15.6022	2.7930	130.27	1.33	268.93	3,909.60	62.53	30,976,029	2,000,957	198,251.00	75,718,962
NAD	Max SMR	25	10%	3	2.44	4.19	-	25.00	31.71	5.63	2.0948	-	12.4986	15.8561	2.8157	124.68	1.33	268.93	3,874.77	62.25	30,976,029	2,000,957	26,769.50	10,871,712
NAD	Max SMR	25	15%	3	2.40	4.30	-	25.00	29.36	5.42	2.1491	-	12.4986	14.6791	2.7092	119.47	1.70	268.93	4,265.42	65.31	30,976,029	2,000,957	12,361.30	6,889,391
NAD	Max SMR	25	20%	3	2.38	4.76	-	25.00	31.20	5.59	2.3777	-	12.4986	15.6024	2.7931	115.51	1.70	268.93	4,406.79	66.38	30,976,029	2,000,957	12,198.50	5,876,129

Appendix 2.4 (continuation) Characteristics of the solutions obtained using non-aggregated data and costs constraints above the MSCfor the beneficiaries' standard service objective function.

Solution identifiers					Indicators for the variables (i, j) beneficiaries walking distance from pop. point to DCs						Indicators for the variables (i, j) beneficiaries walking time from pop. point to DCs					Indicators for the variables $(0, j)$ road distance from DCs to Warehouse					Problem characteristics variables (i, j)		Problem characteristics algorithm effort	
Data set	Objective function	Coverage radius	Cost constraint	ICT	Mean absolute deviation	Average	Minimum	Maximum	Variance	Standard deviation	Average	Minimum	Maximum	Variance	Standard deviation	Average	Minimum	Maximum	Variance	Standard deviation	Total	Tuples	CPU time	CPLEX ticks
		km	%		km	km	km	km	km	km	hours	hours	hours	hours	hours	km	km	km	km	km	No.	No.	Seconds	No.
NAD	Max SMR	10	5%	3	0.91	1.94	-	9.99	5.52	2.35	0.9714	-	4.9944	2.7575	1.1742	125.82	1.33	268.93	3,713.64	60.94	19,251,505	417,660	2,268.96	1,328,551
NAD	Max SMR	10	10%	3	0.88	2.01	-	9.99	5.10	2.26	1.0067	-	4.9944	2.5522	1.1296	125.83	1.33	268.93	3,918.91	62.60	19,251,505	417,660	756.57	516,165
NAD	Max SMR	10	15%	3	0.88	2.23	-	9.99	4.79	2.19	1.1134	-	4.9944	2.3966	1.0947	129.55	0.49	268.93	3,811.24	61.74	19,251,505	417,660	669.71	477,182
NAD	Max SMR	10	20%	3	0.87	2.18	-	9.99	4.81	2.19	1.0922	-	4.9944	2.4058	1.0968	131.74	1.33	268.93	3,922.51	62.63	19,251,505	417,660	877.94	637,369
NAD	Max SMR	15	5%	3	1.56	2.94	-	14.97	13.42	3.66	1.4680	-	7.4864	6.7080	1.8314	126.39	0.49	268.93	3,753.61	61.27	24,235,449	856,980	22,722.90	8,679,868
NAD	Max SMR	15	10%	3	1.62	2.97	-	14.97	14.31	3.78	1.4841	-	7.4864	7.1572	1.8917	127.29	2.86	268.93	3,625.15	60.21	24,235,449	856,980	16,840.00	5,148,296
NAD	Max SMR	15	15%	3	1.62	3.08	-	14.97	14.04	3.75	1.5405	-	7.4864	7.0177	1.8732	127.20	2.86	268.93	3,522.26	59.35	24,235,449	856,980	13,127.30	3,805,825
NAD	Max SMR	15	20%	3	1.34	4.23	-	14.97	11.75	3.43	2.1163	-	7.4864	5.8726	1.7136	121.26	3.88	268.93	4,049.65	63.64	24,235,449	856,980	31,431.10	7,894,578
NAD	Max SMR	20	5%	3	2.20	3.81	-	19.98	23.41	4.84	1.9033	-	9.9877	11.7047	2.4192	129.96	1.33	268.93	3,530.59	59.42	28,294,387	1,394,152	12,529.30	6,801,172
NAD	Max SMR	20	10%	3	2.16	4.41	-	19.98	22.50	4.74	2.2063	-	9.9877	11.2520	2.3719	132.20	5.03	268.93	3,509.87	59.24	28,294,387	1,394,152	8,726.87	3,787,556
NAD	Max SMR	20	15%	3	2.14	4.57	-	19.98	22.91	4.79	2.2848	-	9.9877	11.4541	2.3931	129.95	3.88	268.93	3,573.17	59.78	28,294,387	1,394,152	4,605.07	2,819,901
NAD	Max SMR	20	20%	3	2.11	4.51	-	19.98	21.97	4.69	2.2526	-	9.9876	10.9863	2.3437	130.42	3.88	268.93	3,748.96	61.23	28,294,387	1,394,152	7,334.55	4,256,560
NAD	Max SMR	25	5%	3	2.58	4.16	-	25.00	31.20	5.59	2.0784	-	12.4986	15.6022	2.7930	130.27	1.33	268.93	3,909.60	62.53	30,976,029	2,000,957	198,251.00	75,718,962
NAD	Max SMR	25	10%	3	2.44	4.19	-	25.00	31.71	5.63	2.0948	-	12.4986	15.8561	2.8157	124.68	1.33	268.93	3,874.77	62.25	30,976,029	2,000,957	26,769.50	10,871,712
NAD	Max SMR	25	15%	3	2.40	4.30	-	25.00	29.36	5.42	2.1491	-	12.4986	14.6791	2.7092	119.47	1.70	268.93	4,265.42	65.31	30,976,029	2,000,957	12,361.30	6,889,391
NAD	Max SMR	25	20%	3	2.38	4.76	-	25.00	31.20	5.59	2.3777	-	12.4986	15.6024	2.7931	115.51	1.70	268.93	4,406.79	66.38	30,976,029	2,000,957	12,198.50	5,876,129

Appendix 3.1 Characteristics of the solutions obtained using aggregated data and different inhabitants' aggregation threshold for the stakeholder costs objective functions.

Solution identifiers					Demographic indicators: Inhabitants				Demographic indicators: Population points			Distribution centers			Economic indicators: Costs				Additional service and equality indicators for the WI				
Data set	Objective function	Coverage radius	Cost constraint	IAT	Total	Covered	Covered	Walking	Total	Covered	Covered	Total	open DCs	open DCs	Total	Beneficiaries	WFP	KRC	High service threshold	WI in high service distances	WI in low service distances	Gini Index	Hood Index
		km	%		No.	No.	%	No.	No.	No.	%	No.	No.	%	KSh.	KSh.	KSh.	KSh.	km	%	%		
AD	Min SC	5	-	18	452,218	343,777	76.02%	57,791	5,426	1,209	22.28%	1,459	178	12.20%	37,209,889	2,170,201	31,260,036	3,779,652	2.5000	0.7665	0.2335	0.6400	0.5026
AD	Min SC	10	-	10	452,218	374,166	82.74%	63,208	5,426	2,096	38.63%	1,459	121	8.29%	36,719,864	3,267,748	30,882,802	2,569,314	5.0000	0.7906	0.2094	0.6674	0.5480
AD	Min SC	10	-	12	452,218	373,577	82.61%	63,096	5,426	2,040	37.60%	1,459	119	8.16%	36,669,487	3,259,072	30,883,569	2,526,846	5.0000	0.7906	0.2094	0.6679	0.5487
AD	Min SC	10	-	14	452,218	372,915	82.46%	62,964	5,426	1,987	36.62%	1,459	117	8.02%	36,611,252	3,245,510	30,881,364	2,484,378	5.0000	0.7911	0.2089	0.6697	0.5498
AD	Min SC	10	-	16	452,218	371,875	82.23%	62,748	5,426	1,915	35.29%	1,459	117	8.02%	36,571,288	3,215,808	30,871,102	2,484,378	5.0000	0.7926	0.2074	0.6718	0.5508
AD	Min SC	10	-	18	452,218	370,946	82.03%	62,577	5,426	1,858	34.24%	1,459	117	8.02%	36,555,770	3,194,365	30,877,027	2,484,378	5.0000	0.7940	0.2060	0.6726	0.5516
AD	Min SC	15	-	10	452,218	397,503	87.90%	67,391	5,426	2,817	51.92%	1,459	103	7.06%	36,900,659	4,400,585	30,312,972	2,187,102	7.5000	0.8034	0.1966	0.6828	0.5602
AD	Min SC	15	-	12	452,218	396,543	87.69%	67,207	5,426	2,725	50.22%	1,459	102	6.99%	36,842,602	4,361,670	30,315,064	2,165,868	7.5000	0.8052	0.1948	0.6855	0.5627
AD	Min SC	15	-	14	452,218	395,220	87.40%	66,944	5,426	2,619	48.27%	1,459	101	6.92%	36,793,244	4,325,070	30,323,540	2,144,634	7.5000	0.8061	0.1939	0.6869	0.5639
AD	Min SC	15	-	16	452,218	393,381	86.99%	66,563	5,426	2,492	45.93%	1,459	99	6.79%	36,704,602	4,275,939	30,326,497	2,102,166	7.5000	0.8074	0.1926	0.6891	0.5663
AD	Min SC	15	-	18	452,218	391,827	86.65%	66,278	5,426	2,397	44.18%	1,459	97	6.65%	36,639,838	4,245,321	30,334,819	2,059,698	7.5000	0.8079	0.1921	0.6906	0.5682
AD	Min SC	20	-	10	452,218	416,041	92.00%	70,735	5,426	3,453	63.64%	1,459	86	5.89%	37,206,750	5,786,488	29,594,139	1,826,124	10.0000	0.7981	0.2019	0.6714	0.5485
AD	Min SC	20	-	12	452,218	414,569	91.67%	70,455	5,426	3,313	61.06%	1,459	86	5.89%	37,157,548	5,730,755	29,600,669	1,826,124	10.0000	0.8002	0.1998	0.6734	0.5503
AD	Min SC	20	-	14	452,218	412,629	91.25%	70,065	5,426	3,158	58.20%	1,459	85	5.83%	37,072,005	5,655,750	29,611,364	1,804,890	10.0000	0.8038	0.1962	0.6759	0.5515
AD	Min SC	20	-	16	452,218	410,170	90.70%	69,555	5,426	2,988	55.07%	1,459	86	5.89%	36,981,295	5,508,850	29,646,322	1,826,124	10.0000	0.8065	0.1935	0.6876	0.5642
AD	Min SC	20	-	18	452,218	407,860	90.19%	69,132	5,426	2,847	52.47%	1,459	86	5.89%	36,913,640	5,430,323	29,657,193	1,826,124	10.0000	0.8092	0.1908	0.6905	0.5667
AD	Min SC	25	-	10	452,218	427,892	94.62%	72,901	5,426	3,920	72.24%	1,459	76	5.21%	37,378,594	6,871,870	28,892,940	1,613,784	12.5000	0.8095	0.1905	0.6784	0.5535
AD	Min SC	25	-	12	452,218	426,018	94.21%	72,545	5,426	3,742	68.96%	1,459	76	5.21%	37,311,262	6,791,706	28,905,771	1,613,784	12.5000	0.8116	0.1884	0.6803	0.5552
AD	Min SC	25	-	14	452,218	423,437	93.64%	72,024	5,426	3,536	65.17%	1,459	76	5.21%	37,205,577	6,668,124	28,923,670	1,613,784	12.5000	0.8151	0.1849	0.6838	0.5577
AD	Min SC	25	-	16	452,218	420,445	92.97%	71,403	5,426	3,329	61.35%	1,459	75	5.14%	37,084,611	6,547,202	28,944,859	1,592,550	12.5000	0.8183	0.1817	0.6867	0.5603
AD	Min SC	25	-	18	452,218	417,462	92.31%	70,857	5,426	3,147	58.00%	1,459	74	5.07%	36,970,659	6,440,887	28,958,456	1,571,316	12.5000	0.8217	0.1783	0.6894	0.5625
AD	Min SC	55	-	10	452,218	448,538	99.19%	76,705	5,426	4,780	88.09%	1,459	58	3.98%	37,816,107	10,017,475	26,567,060	1,231,572	27.5000	0.9043	0.0957	0.7059	0.5694
AD	Min SC	55	-	12	452,218	446,027	98.63%	76,227	5,426	4,541	83.69%	1,459	57	3.91%	37,688,660	9,909,954	26,568,368	1,210,338	27.5000	0.9041	0.0959	0.7081	0.5718
AD	Min SC	55	-	14	452,218	442,516	97.85%	75,516	5,426	4,261	78.53%	1,459	57	3.91%	37,505,577	9,687,130	26,608,109	1,210,338	27.5000	0.9063	0.0937	0.7119	0.5751
AD	Min SC	55	-	16	452,218	438,445	96.95%	74,670	5,426	3,979	73.33%	1,459	56	3.84%	37,291,327	9,490,915	26,611,308	1,189,104	27.5000	0.9070	0.0930	0.7156	0.5786
AD	Min SC	55	-	18	452,218	434,165	96.01%	73,887	5,426	3,718	68.52%	1,459	56	3.84%	37,096,152	9,239,665	26,667,382	1,189,104	27.5000	0.9092	0.0908	0.7197	0.5822

NAD = non-aggregated data
IAT = inhabitants' aggregation threshold
WI = walking inhabitants
SC = stakeholder costs

Appendix 3.1 (continuation) Characteristics of the solutions obtained using aggregated data and different inhabitants' aggregation threshold for the stakeholder costs objective functions.

Solution identifiers					Indicators for the variables (i,j)											Indicators for the variables $(0,j)$					Problem characteristics			
					beneficiaries walking distance from pop. point to DCs						beneficiaries walking time from pop. point to DCs					road distance from DCs to Warehouse					variables (i,j)		algorithm effort	
Data set	Objective function	Coverage radius	Cost constraint	IAT	Mean absolute deviation	Average	Minimum	Maximum	Variance	Standard deviation	Average	Minimum	Maximum	Variance	Standard deviation	Average	Minimum	Maximum	Variance	Standard deviation	Total	Tuples	CPU time	CPLEX ticks
		km	%		km	km	km	km	km	km	hours	hours	hours	hours	hours	km	km	km	km	km	No.	No.	Seconds	No.
AD	Min SC	5	-	18	1.30	1.29	-	4.99	2.50	1.58	0.6441	-	2.4972	1.2501	0.7906	126.17	1.33	268.93	3,685.61	60.71	1,763,931	14,988	2.48	2,181
AD	Min SC	10	-	10	0.20	2.33	-	9.99	9.18	3.03	1.1632	-	4.9945	4.5883	1.5146	126.84	1.33	268.93	3,979.31	63.08	3,058,064	55,026	4.29	3,279
AD	Min SC	10	-	12	0.20	2.32	-	9.99	9.17	3.03	1.1616	-	4.9945	4.5856	1.5142	127.41	1.33	268.93	3,990.29	63.17	2,976,360	54,552	4.50	3,759
AD	Min SC	10	-	14	0.19	2.32	-	9.99	9.19	3.03	1.1576	-	4.9945	4.5961	1.5159	127.87	1.33	268.93	4,033.41	63.51	2,899,033	54,155	4.39	3,733
AD	Min SC	10	-	16	0.18	2.29	-	9.99	9.12	3.02	1.1468	-	4.9945	4.5609	1.5101	127.76	1.33	268.93	4,049.47	63.64	2,793,985	53,287	4.52	3,244
AD	Min SC	10	-	18	0.18	2.28	-	9.99	9.04	3.01	1.1393	-	4.9945	4.5194	1.5032	127.73	1.33	268.93	4,054.15	63.67	2,710,822	52,955	5.30	3,642
AD	Min SC	15	-	10	0.36	3.32	-	14.97	20.21	4.50	1.6624	-	7.4865	10.1069	2.2480	128.43	1.33	268.93	4,308.46	65.64	4,110,003	116,431	6.67	6,488
AD	Min SC	15	-	12	0.35	3.30	-	14.97	20.14	4.49	1.6477	-	7.4865	10.0712	2.2440	127.31	1.33	268.93	4,278.39	65.41	3,975,775	114,821	9.36	7,115
AD	Min SC	15	-	14	0.33	3.27	-	14.97	20.02	4.47	1.6370	-	7.4865	10.0106	2.2373	128.08	1.33	268.93	4,261.43	65.28	3,821,121	113,462	6.71	5,907
AD	Min SC	15	-	16	0.32	3.25	-	14.97	19.91	4.46	1.6234	-	7.4865	9.9539	2.2309	127.96	1.33	268.93	4,364.07	66.06	3,635,828	110,939	6.45	5,702
AD	Min SC	15	-	18	0.31	3.23	-	14.97	19.90	4.46	1.6166	-	7.4865	9.9522	2.2307	127.57	1.33	268.93	4,255.39	65.23	3,497,223	109,944	6.16	5,661
AD	Min SC	20	-	10	0.55	4.54	-	19.98	35.96	6.00	2.2681	-	9.9877	17.9807	2.9984	124.28	1.33	268.93	4,311.69	65.66	5,037,927	194,940	18.41	12,886
AD	Min SC	20	-	12	0.52	4.50	-	19.98	35.79	5.98	2.2510	-	9.9877	17.8965	2.9914	124.14	1.33	268.93	4,293.99	65.53	4,833,667	191,273	15.52	12,398
AD	Min SC	20	-	14	0.50	4.46	-	19.98	35.51	5.96	2.2283	-	9.9877	17.7566	2.9796	124.00	1.33	268.93	4,352.86	65.98	4,607,522	187,986	14.04	12,129
AD	Min SC	20	-	16	0.47	4.35	-	19.98	35.57	5.96	2.1725	-	9.9877	17.7859	2.9821	122.96	1.33	268.93	4,394.62	66.29	4,359,492	183,006	14.78	11,341
AD	Min SC	20	-	18	0.45	4.30	-	19.98	35.31	5.94	2.1486	-	9.9877	17.6569	2.9713	122.95	1.33	268.93	4,397.62	66.31	4,153,773	180,606	14.65	11,091
AD	Min SC	25	-	10	0.72	5.45	-	25.00	54.04	7.35	2.7252	-	12.4989	27.0179	3.6754	117.35	1.33	268.93	4,225.70	65.01	5,719,280	287,763	45.10	29,876
AD	Min SC	25	-	12	0.69	5.40	-	25.00	53.67	7.33	2.7017	-	12.4989	26.8351	3.6630	117.35	1.33	268.93	4,225.70	65.01	5,459,578	280,940	53.32	27,773
AD	Min SC	25	-	14	0.65	5.33	-	25.00	53.10	7.29	2.6636	-	12.4989	26.5497	3.6435	117.34	1.33	268.93	4,223.77	64.99	5,159,024	274,631	52.32	27,767
AD	Min SC	25	-	16	0.61	5.26	-	25.00	52.59	7.25	2.6310	-	12.4989	26.2966	3.6261	118.47	1.33	268.93	4,179.45	64.65	4,857,011	266,239	38.88	26,916
AD	Min SC	25	-	18	0.58	5.20	-	25.00	52.17	7.22	2.6018	-	12.4989	26.0868	3.6116	119.40	1.33	268.93	4,153.36	64.45	4,591,473	261,646	33.96	24,705
AD	Min SC	55	-	10	1.20	8.12	-	54.97	143.27	11.97	4.0586	-	27.4829	71.6360	5.9848	113.33	1.33	268.93	4,357.79	66.01	6,974,020	1,125,757	237.97	117,958
AD	Min SC	55	-	12	1.15	8.07	-	54.97	143.28	11.97	4.0369	-	27.4829	71.6379	5.9849	110.56	1.33	245.59	4,006.85	63.30	6,625,319	1,085,432	234.50	108,818
AD	Min SC	55	-	14	1.07	7.95	-	54.97	141.88	11.91	3.9735	-	27.4829	70.9397	5.9557	110.47	1.33	245.59	4,021.08	63.41	6,216,799	1,042,672	169.37	100,296
AD	Min SC	55	-	16	1.02	7.86	-	54.97	141.76	11.91	3.9305	-	27.4863	70.8816	5.9532	109.84	1.33	245.59	4,066.71	63.77	5,805,361	995,997	211.14	91,965
AD	Min SC	55	-	18	0.95	7.71	-	54.97	139.81	11.82	3.8551	-	27.4863	69.9029	5.9120	109.84	1.33	245.59	4,066.71	63.77	5,424,562	955,370	215.95	84,391

Appendix 3.2 Characteristics of the solutions obtained using aggregated data and different inhabitants' aggregation threshold for the beneficiaries' average walking distance objective function.

Solution identifiers: Data set	Solution identifiers: Objective function	Solution identifiers: Coverage radius (km)	Solution identifiers: Cost constraint (%)	Solution identifiers: IAT	Demographic indicators – Inhabitants: Total (No.)	Inhabitants: Covered (No.)	Inhabitants: Covered (%)	Inhabitants: Walking (No.)	Population points: Total (No.)	Population points: Covered (No.)	Population points: Covered (%)	Distribution centers: Total (No.)	Distribution centers: open DCs (No.)	Distribution centers: open DCs (%)	Economic indicators – Costs: Total (KSh.)	Costs: Beneficiaries (KSh.)	Costs: WFP (KSh.)	Costs: KRC (KSh.)	Additional service and equality indicators for the WI: High service threshold (km)	WI in high service distances (%)	WI in low service distances (%)	Gini Index	Hood Index
AD	Min AWD	10	-	10	452,218	374,166	82.74%	63,208	5,426	2,096	38.63%	1,459	668	45.78%	48,155,686	2,235,223	31,736,151	14,184,312	5.0000	0.9177	0.0823	0.7898	0.6407
AD	Min AWD	10	-	12	452,218	373,577	82.61%	63,096	5,426	2,040	37.60%	1,459	667	45.72%	48,124,841	2,222,590	31,739,173	14,163,078	5.0000	0.9189	0.0811	0.7904	0.6406
AD	Min AWD	10	-	14	452,218	372,915	82.46%	62,964	5,426	1,987	36.62%	1,459	673	46.13%	48,243,725	2,207,644	31,745,599	14,290,482	5.0000	0.9204	0.0796	0.7910	0.6404
AD	Min AWD	10	-	16	452,218	371,875	82.23%	62,748	5,426	1,915	35.29%	1,459	657	45.03%	47,884,034	2,186,332	31,746,963	13,950,738	5.0000	0.9219	0.0781	0.7922	0.6405
AD	Min AWD	10	-	18	452,218	370,946	82.03%	62,577	5,426	1,858	34.24%	1,459	638	43.73%	47,467,230	2,167,991	31,751,947	13,547,292	5.0000	0.9235	0.0765	0.7930	0.6403
AD	Min AWD	15	-	10	452,218	397,503	87.90%	67,391	5,426	2,817	51.92%	1,459	685	46.95%	49,316,726	3,026,035	31,745,400	14,545,290	7.5000	0.9033	0.0967	0.7897	0.6613
AD	Min AWD	15	-	12	452,218	396,543	87.69%	67,207	5,426	2,725	50.22%	1,459	705	48.32%	49,719,874	2,999,629	31,750,275	14,969,970	7.5000	0.9048	0.0952	0.7909	0.8474
AD	Min AWD	15	-	14	452,218	395,220	87.40%	66,944	5,426	2,619	48.27%	1,459	684	46.88%	49,244,565	2,959,095	31,761,413	14,524,056	7.5000	0.9072	0.0928	0.7923	0.9947
AD	Min AWD	15	-	16	452,218	393,381	86.99%	66,563	5,426	2,492	45.93%	1,459	685	46.95%	49,222,341	2,906,283	31,770,768	14,545,290	7.5000	0.9102	0.0898	0.7945	0.9994
AD	Min AWD	15	-	18	452,218	391,827	86.65%	66,278	5,426	2,397	44.18%	1,459	702	48.12%	49,550,138	2,865,526	31,778,345	14,906,268	7.5000	0.9124	0.0876	0.7962	0.6646
AD	Min AWD	20	-	10	452,218	416,041	92.00%	70,735	5,426	3,453	63.64%	1,459	689	47.22%	50,208,109	3,885,625	31,692,259	14,630,226	10.0000	0.8936	0.1064	0.7853	0.6659
AD	Min AWD	20	-	12	452,218	414,569	91.67%	70,455	5,426	3,313	61.06%	1,459	720	49.35%	50,823,608	3,834,382	31,700,747	15,288,480	10.0000	0.8956	0.1044	0.7870	0.6671
AD	Min AWD	20	-	14	452,218	412,629	91.25%	70,065	5,426	3,158	58.20%	1,459	715	49.01%	50,659,868	3,761,110	31,716,448	15,182,310	10.0000	0.8987	0.1013	0.7893	0.6687
AD	Min AWD	20	-	16	452,218	410,170	90.70%	69,555	5,426	2,988	55.07%	1,459	693	47.50%	50,125,400	3,675,128	31,735,110	14,715,162	10.0000	0.9021	0.0979	0.7924	0.6710
AD	Min AWD	20	-	18	452,218	407,860	90.19%	69,132	5,426	2,847	52.47%	1,459	702	48.12%	50,254,781	3,599,100	31,749,414	14,906,268	10.0000	0.9052	0.0948	0.7949	0.6726
AD	Min AWD	25	-	10	452,218	427,892	94.62%	72,901	5,426	3,920	72.24%	1,459	693	47.50%	50,932,464	4,588,824	31,628,478	14,715,162	12.5000	0.8973	0.1027	0.7826	0.6653
AD	Min AWD	25	-	12	452,218	426,018	94.21%	72,545	5,426	3,742	68.96%	1,459	709	48.59%	51,208,317	4,513,202	31,640,208	15,054,906	12.5000	0.8996	0.1004	0.7847	0.6670
AD	Min AWD	25	-	14	452,218	423,437	93.64%	72,024	5,426	3,536	65.17%	1,459	697	47.77%	50,858,273	4,397,228	31,660,947	14,800,098	12.5000	0.9036	0.0964	0.7877	0.6691
AD	Min AWD	25	-	16	452,218	420,445	92.97%	71,403	5,426	3,329	61.35%	1,459	708	48.53%	50,992,451	4,274,826	31,683,954	15,033,672	12.5000	0.9072	0.0928	0.7913	0.6719
AD	Min AWD	25	-	18	452,218	417,462	92.31%	70,857	5,426	3,147	58.00%	1,459	702	48.12%	50,770,488	4,158,270	31,705,950	14,906,268	12.5000	0.9110	0.0890	0.7943	0.6743
AD	Min AWD	55	-	10	452,218	448,538	99.19%	76,705	5,426	4,780	88.09%	1,459	688	47.16%	52,414,173	6,387,242	31,417,938	14,608,992	27.5000	0.9607	0.0393	0.7870	0.6645
AD	Min AWD	55	-	12	452,218	446,027	98.63%	76,227	5,426	4,541	83.69%	1,459	723	49.55%	53,038,292	6,259,046	31,427,064	15,352,182	27.5000	0.9614	0.0386	0.7897	0.6671
AD	Min AWD	55	-	14	452,218	442,516	97.85%	75,516	5,426	4,261	78.53%	1,459	702	48.12%	52,419,952	6,059,368	31,454,316	14,906,268	27.5000	0.9630	0.0370	0.7938	0.6708
AD	Min AWD	55	-	16	452,218	438,445	96.95%	74,670	5,426	3,979	73.33%	1,459	733	50.24%	52,893,502	5,836,336	31,492,643	15,564,522	27.5000	0.9650	0.0350	0.7983	0.6752
AD	Min AWD	55	-	18	452,218	434,165	96.01%	73,887	5,426	3,718	68.52%	1,459	723	49.55%	52,494,842	5,613,009	31,529,651	15,352,182	27.5000	0.9673	0.0327	0.8026	0.6789

NAD = non-aggregated data
IAT = inhabitants' aggregation threshold
WI = walking inhabitants
AWD = beneficiaries' average walking distance

Appendix 3.2 (continuation) Characteristics of the solutions obtained using aggregated data and different inhabitants' aggregation threshold for the beneficiaries' average walking distance objective function.

Solution identifiers					Indicators for the variables (i, j) beneficiaries walking distance from pop. point to DCs						Indicators for the variables (i, j) beneficiaries walking time from pop. point to DCs					Indicators for the variables (0, j) road distance from DCs to Warehouse					Problem characteristics variables (i, j)		Problem characteristics algorithm effort	
Data set	Objective function	Coverage radius	Cost constraint	IAT	Mean absolute deviation	Average	Minimum	Maximum	Variance	Standard deviation	Average	Minimum	Maximum	Variance	Standard deviation	Average	Minimum	Maximum	Variance	Standard deviation	Total	Tuples	CPU time	CPLEX ticks
		km	%		km	km	km	km	km	km	hours	hours	hours	hours	hours	km	km	km	km	km	No.	No.	Seconds	No.
AD	Min AWD	10	-	10	0.14	1.13	-	9.99	4.68	2.16	0.5638	-	4.9939	2.3420	1.5304	118.20	1.33	268.93	4,469.65	66.86	3,058,064	55,026	32.04	13,895
AD	Min AWD	10	-	12	0.14	1.12	-	9.99	4.63	2.15	0.5587	-	4.9939	2.3137	1.5211	118.18	1.33	268.93	4,475.18	66.90	2,976,360	54,552	20.22	12,055
AD	Min AWD	10	-	14	0.13	1.11	-	9.99	4.56	2.14	0.5527	-	4.9939	2.2794	1.5098	118.27	1.33	268.93	4,481.30	66.94	2,899,033	54,155	19.93	13,343
AD	Min AWD	10	-	16	0.12	1.09	-	9.99	4.48	2.12	0.5447	-	4.9939	2.2381	1.4960	118.41	1.33	268.93	4,460.21	66.78	2,793,985	53,287	15.36	12,634
AD	Min AWD	10	-	18	0.12	1.07	-	9.99	4.40	2.10	0.5374	-	4.9939	2.1986	1.4828	118.18	1.33	268.93	4,464.62	66.82	2,710,822	52,955	11.55	10,951
AD	Min AWD	15	-	10	0.27	1.83	-	14.97	11.93	3.45	0.9139	-	7.4863	5.9655	2.4424	118.20	1.33	268.93	4,470.70	66.86	4,110,003	116,431	76.17	51,606
AD	Min AWD	15	-	12	0.25	1.81	-	14.97	11.79	3.43	0.9040	-	7.4863	5.8942	2.4278	118.18	1.33	268.93	4,475.17	66.90	3,975,775	114,821	80.41	52,346
AD	Min AWD	15	-	14	0.24	1.78	-	14.97	11.53	3.40	0.8882	-	7.4863	5.7658	2.4012	118.26	1.33	268.93	4,483.19	66.96	3,821,121	113,462	84.64	58,242
AD	Min AWD	15	-	16	0.22	1.74	-	14.97	11.23	3.35	0.8683	-	7.4863	5.6148	2.3696	118.61	1.33	268.93	4,480.38	66.94	3,635,828	110,939	74.72	48,927
AD	Min AWD	15	-	18	0.21	1.71	-	14.97	10.99	3.31	0.8527	-	7.4863	5.4938	2.3439	118.41	1.33	268.93	4,482.41	66.95	3,497,223	109,944	68.49	49,162
AD	Min AWD	20	-	10	0.41	2.56	-	19.97	22.38	4.73	1.2819	-	9.9874	11.1923	3.3455	118.18	1.33	268.93	4,471.17	66.87	5,037,927	194,940	255.51	144,669
AD	Min AWD	20	-	12	0.39	2.53	-	19.97	22.02	4.69	1.2632	-	9.9874	11.0120	3.3184	118.18	1.33	268.93	4,474.99	66.90	4,833,667	191,273	191.10	124,922
AD	Min AWD	20	-	14	0.36	2.47	-	19.97	21.49	4.64	1.2360	-	9.9874	10.7445	3.2779	118.16	1.33	268.93	4,482.07	66.95	4,607,522	187,986	190.20	124,940
AD	Min AWD	20	-	16	0.34	2.41	-	19.97	20.93	4.57	1.2051	-	9.9874	10.4626	3.2346	118.36	1.33	268.93	4,482.82	66.95	4,359,492	183,006	184.52	120,680
AD	Min AWD	20	-	18	0.32	2.35	-	19.97	20.37	4.51	1.1766	-	9.9874	10.1830	3.1911	118.19	1.33	268.93	4,477.70	66.92	4,153,773	180,606	182.85	121,501
AD	Min AWD	25	-	10	0.54	3.15	-	25.00	33.08	5.75	1.5760	-	12.4989	16.5414	4.0671	118.19	1.33	268.93	4,469.90	66.86	5,719,280	287,763	457.39	273,525
AD	Min AWD	25	-	12	0.51	3.10	-	25.00	32.47	5.70	1.5491	-	12.4989	16.2355	4.0293	118.18	1.33	268.93	4,476.04	66.90	5,459,578	280,940	422.59	263,747
AD	Min AWD	25	-	14	0.47	3.01	-	25.00	31.44	5.61	1.5065	-	12.4989	15.7176	3.9645	118.14	1.33	268.93	4,483.70	66.96	5,159,024	274,631	406.39	248,163
AD	Min AWD	25	-	16	0.44	2.93	-	25.00	30.46	5.52	1.4631	-	12.4989	15.2321	3.9028	118.37	1.33	268.93	4,481.55	66.94	4,857,011	266,239	322.05	185,712
AD	Min AWD	25	-	18	0.41	2.84	-	25.00	29.40	5.42	1.4196	-	12.4984	14.6983	3.8338	118.19	1.33	268.93	4,476.99	66.91	4,591,473	261,646	1,103.26	505,054
AD	Min AWD	55	-	10	0.88	4.64	-	54.34	76.03	8.72	2.3218	-	27.1676	38.0141	6.1656	118.20	1.33	268.93	4,469.91	66.86	6,974,020	1,125,757	13,767.50	5,986,682
AD	Min AWD	55	-	12	0.83	4.56	-	54.34	74.84	8.65	2.2793	-	27.1676	37.4175	6.1170	118.17	1.33	268.93	4,476.18	66.90	6,625,319	1,085,432	22,488.20	7,798,591
AD	Min AWD	55	-	14	0.77	4.42	-	54.34	72.76	8.53	2.2106	-	27.1676	36.3789	6.0315	118.14	1.33	268.93	4,483.56	66.96	6,216,799	1,042,672	21,478.10	7,891,266
AD	Min AWD	55	-	16	0.71	4.27	-	54.34	70.36	8.39	2.1344	-	27.1676	35.1782	5.9311	118.40	1.33	268.93	4,480.35	66.94	5,805,361	995,997	11,336.40	5,250,029
AD	Min AWD	55	-	18	0.65	4.11	-	54.34	67.59	8.22	2.0539	-	27.1676	33.7974	5.8136	118.20	1.33	268.93	4,476.66	66.91	5,424,562	955,370	14,016.60	5,927,028

Appendix 3.3 Characteristics of the solutions obtained using aggregated data, costs constraints above the MSC and IAT = 18 inh. for the beneficiaries' average walking distance objective function.

Solution identifiers					Demographic indicators: Inhabitants				Demographic indicators: Population points			Demographic indicators: Distribution centers			Economic indicators: Costs				Additional service and equality indicators for the WI				
Data set	Objective function	Coverage radius	Cost constraint	IAT	Total	Covered	Covered	Walking	Total	Covered	Covered	Total	open DCs	open DCs	Total	Beneficiaries	WFP	KRC	High service threshold	WI in high service distances	WI in low service distances	Gini Index	Hood Index
		km	%		No.	No.	%	No.	No.	No.	%	No.	No.	%	KSh.	KSh.	KSh.	KSh.	km	%	%		
AD	Min AWD	10	5%	18	452,218	370,946	82.03%	62,577	5,426	1,858	34.24%	1,459	207	14.19%	38,383,550	2,423,683	31,564,429	4,395,438	5.0000	0.9153	0.0847	0.7399	0.6032
AD	Min AWD	10	10%	18	452,218	370,946	82.03%	62,577	5,426	1,858	34.24%	1,459	295	20.22%	40,211,337	2,288,739	31,658,568	6,264,030	5.0000	0.9192	0.0808	0.7704	0.6300
AD	Min AWD	10	15%	18	452,218	370,946	82.03%	62,577	5,426	1,858	34.24%	1,459	382	26.18%	42,039,112	2,218,423	31,709,301	8,111,388	5.0000	0.9219	0.0781	0.7840	0.6373
AD	Min AWD	10	20%	18	452,218	370,946	82.03%	62,577	5,426	1,858	34.24%	1,459	469	32.15%	43,866,825	2,183,022	31,725,057	9,958,746	5.0000	0.9231	0.0769	0.7909	0.6402
AD	Min AWD	15	5%	18	452,218	391,827	86.65%	66,278	5,426	2,397	44.18%	1,459	178	12.20%	38,471,827	3,197,862	31,494,313	3,779,652	7.5000	0.9069	0.0931	0.7398	0.5957
AD	Min AWD	15	10%	18	452,218	391,827	86.65%	66,278	5,426	2,397	44.18%	1,459	264	18.09%	40,303,808	3,022,962	31,675,070	5,605,776	7.5000	0.9102	0.0898	0.7688	0.6278
AD	Min AWD	15	15%	18	452,218	391,827	86.65%	66,278	5,426	2,397	44.18%	1,459	353	24.19%	42,135,798	2,937,223	31,702,973	7,495,602	7.5000	0.9113	0.0887	0.7847	0.6476
AD	Min AWD	15	20%	18	452,218	391,827	86.65%	66,278	5,426	2,397	44.18%	1,459	439	30.09%	43,967,777	2,890,135	31,755,916	9,321,726	7.5000	0.9121	0.0879	0.7925	0.6595
AD	Min AWD	20	5%	18	452,218	407,860	90.19%	69,132	5,426	2,847	52.47%	1,459	158	10.83%	38,759,310	4,003,583	31,400,755	3,354,972	10.0000	0.9014	0.0986	0.7386	0.5872
AD	Min AWD	20	10%	18	452,218	407,860	90.19%	69,132	5,426	2,847	52.47%	1,459	246	16.86%	40,605,001	3,786,285	31,595,152	5,223,564	10.0000	0.9040	0.0960	0.7671	0.6249
AD	Min AWD	20	15%	18	452,218	407,860	90.19%	69,132	5,426	2,847	52.47%	1,459	334	22.89%	42,450,666	3,686,487	31,672,022	7,092,156	10.0000	0.9049	0.0951	0.7826	0.6495
AD	Min AWD	20	20%	18	452,218	407,860	90.19%	69,132	5,426	2,847	52.47%	1,459	422	28.92%	44,296,352	3,631,340	31,704,264	8,960,748	10.0000	0.9051	0.0949	0.7906	0.6642
AD	Min AWD	25	5%	18	452,218	417,462	92.31%	70,857	5,426	3,147	58.00%	1,459	139	9.53%	38,819,186	4,668,986	31,198,674	2,951,526	12.5000	0.9072	0.0928	0.7363	0.5871
AD	Min AWD	25	10%	18	452,218	417,462	92.31%	70,857	5,426	3,147	58.00%	1,459	224	15.35%	40,667,718	4,383,288	31,528,014	4,756,416	12.5000	0.9106	0.0894	0.7640	0.6209
AD	Min AWD	25	15%	18	452,218	417,462	92.31%	70,857	5,426	3,147	58.00%	1,459	312	21.38%	42,516,241	4,265,526	31,625,707	6,625,008	12.5000	0.9110	0.0890	0.7806	0.6468
AD	Min AWD	25	20%	18	452,218	417,462	92.31%	70,857	5,426	3,147	58.00%	1,459	400	27.42%	44,364,771	4,201,451	31,669,720	8,493,600	12.5000	0.9110	0.0890	0.7888	0.6628
AD	Min AWD	55	5%	18	452,218	434,165	96.01%	73,887	5,426	3,718	68.52%	1,459	107	7.33%	38,950,958	6,556,691	30,122,229	2,272,038	27.5000	0.9626	0.0374	0.7450	0.5968
AD	Min AWD	55	10%	18	452,218	434,165	96.01%	73,887	5,426	3,718	68.52%	1,459	171	11.72%	40,805,765	5,972,142	31,202,609	3,631,014	27.5000	0.9671	0.0329	0.7659	0.6189
AD	Min AWD	55	15%	18	452,218	434,165	96.01%	73,887	5,426	3,718	68.52%	1,459	258	17.68%	42,660,568	5,781,994	31,400,202	5,478,372	27.5000	0.9672	0.0328	0.7851	0.6480
AD	Min AWD	55	20%	18	452,218	434,165	96.01%	73,887	5,426	3,718	68.52%	1,459	347	23.78%	44,515,378	5,690,391	31,456,789	7,368,198	27.5000	0.9673	0.0327	0.7948	0.6644

NAD = non-aggregated data
IAT = inhabitants' aggregation threshold
WI = walking inhabitants
MSC = minimum stakeholder costs

AWD = beneficiaries' average walking distance

Appendix 3.3 (continuation) Characteristics of the solutions obtained using aggregated data, costs constraints above the MSC and IAT = 18 inh. for the beneficiaries' average walking distance objective function.

Solution identifiers					Indicators for the variables (i,j) beneficiaries walking distance from pop. point to DCs						Indicators for the variables (i,j) beneficiaries walking time from pop. point to DCs					Indicators for the variables $(0,j)$ road distance from DCs to Warehouse					Problem characteristics variables (i,j)		Problem characteristics algorithm effort	
Data set	Objective function	Coverage radius	Cost constraint	IAT	Mean absolute deviation	Average	Minimum	Maximum	Variance	Standard deviation	Average	Minimum	Maximum	Variance	Standard deviation	Average	Minimum	Maximum	Variance	Standard deviation	Total	Tuples	CPU time	CPLEX ticks
		km	%		km	km	km	km	km	km	hours	hours	hours	hours	hours	km	km	km	km	km	No.	No.	Seconds	No.
AD	Min AWD	10	5%	18	0.14	1.37	-	9.99	4.96	2.23	0.6874	-	4.9939	2.4812	1.5752	118.24	1.33	268.93	4,598.32	67.81	2,710,822	52,955	69.44	43,453
AD	Min AWD	10	10%	18	0.13	1.22	-	9.99	4.68	2.16	0.6082	-	4.9939	2.3406	1.5299	113.31	1.33	268.93	4,572.08	67.62	2,710,822	52,955	153.12	101,355
AD	Min AWD	10	15%	18	0.12	1.13	-	9.99	4.52	2.13	0.5670	-	4.9939	2.2579	1.5026	114.61	1.33	268.93	4,600.68	67.83	2,710,822	52,955	31.66	27,019
AD	Min AWD	10	20%	18	0.12	1.09	-	9.99	4.44	2.11	0.5463	-	4.9939	2.2215	1.4905	117.09	1.33	268.93	4,571.60	67.61	2,710,822	52,955	19.23	13,300
AD	Min AWD	15	5%	18	0.23	2.07	-	14.97	11.53	3.40	1.0367	-	7.4864	5.7654	2.4011	119.57	1.33	268.93	4,397.43	66.31	3,497,223	109,944	259.91	175,637
AD	Min AWD	15	10%	18	0.22	1.88	-	14.97	11.18	3.34	0.9398	-	7.4864	5.5894	2.3642	113.10	1.33	268.93	4,516.01	67.20	3,497,223	109,944	204.64	107,434
AD	Min AWD	15	15%	18	0.22	1.78	-	14.97	11.09	3.33	0.8924	-	7.4863	5.5460	2.3550	115.14	1.33	268.93	4,543.76	67.41	3,497,223	109,944	166.69	103,315
AD	Min AWD	15	20%	18	0.21	1.73	-	14.97	11.03	3.32	0.8663	-	7.4863	5.5153	2.3485	116.40	1.33	268.93	4,600.63	67.83	3,497,223	109,944	89.16	54,355
AD	Min AWD	20	5%	18	0.34	2.78	-	19.97	20.70	4.55	1.3913	-	9.9874	10.3509	3.2173	120.01	1.33	268.93	4,451.74	66.72	4,153,773	180,606	289.71	143,850
AD	Min AWD	20	10%	18	0.33	2.55	-	19.97	20.40	4.52	1.2759	-	9.9874	10.1975	3.1934	112.50	1.33	268.93	4,428.08	66.54	4,153,773	180,606	194.47	104,636
AD	Min AWD	20	15%	18	0.32	2.45	-	19.97	20.38	4.51	1.2229	-	9.9874	10.1892	3.1921	115.46	1.33	268.93	4,605.08	67.86	4,153,773	180,606	354.44	200,129
AD	Min AWD	20	20%	18	0.32	2.39	-	19.97	20.37	4.51	1.1937	-	9.9874	10.1867	3.1917	115.73	1.33	268.93	4,611.04	67.90	4,153,773	180,606	318.76	197,600
AD	Min AWD	25	5%	18	0.44	3.37	-	25.00	30.05	5.48	1.6841	-	12.4984	15.0261	3.8763	121.80	1.33	268.93	4,333.35	65.83	4,591,473	261,646	732.16	380,679
AD	Min AWD	25	10%	18	0.42	3.07	-	25.00	29.25	5.41	1.5362	-	12.4984	14.6259	3.8244	114.53	1.33	268.93	4,478.00	66.92	4,591,473	261,646	361.83	207,679
AD	Min AWD	25	15%	18	0.41	2.95	-	25.00	29.30	5.41	1.4752	-	12.4984	14.6495	3.8275	114.55	1.33	268.93	4,659.88	68.26	4,591,473	261,646	475.45	274,621
AD	Min AWD	25	20%	18	0.41	2.88	-	25.00	29.35	5.42	1.4420	-	12.4984	14.6764	3.8310	115.57	1.33	268.93	4,579.75	67.67	4,591,473	261,646	414.51	253,705
AD	Min AWD	55	5%	18	0.72	5.05	-	54.34	73.99	8.60	2.5225	-	27.1676	36.9974	6.0825	118.71	1.33	268.93	4,377.27	66.16	5,424,562	955,370	25,262.60	9,063,018
AD	Min AWD	55	10%	18	0.67	4.46	-	54.34	66.91	8.18	2.2322	-	27.1676	33.4548	5.7840	117.63	1.33	268.93	4,463.98	66.81	5,424,562	955,370	18,966.80	7,670,541
AD	Min AWD	55	15%	18	0.66	4.28	-	54.34	67.12	8.19	2.1378	-	27.1676	33.5616	5.7932	113.22	1.33	268.93	4,527.99	67.29	5,424,562	955,370	17,737.60	6,743,872
AD	Min AWD	55	20%	18	0.65	4.18	-	54.34	67.37	8.21	2.0923	-	27.1676	33.6836	5.8038	115.54	1.33	268.93	4,603.46	67.85	5,424,562	955,370	11,455.20	5,176,810

Appendix 3.4 Characteristics of the solutions obtained using aggregated data and different inhabitants' aggregation threshold for the beneficiaries' mean absolute deviation of walking distance objective function.

Solution identifiers					Demographic indicators: Inhabitants				Population points			Distribution centers			Economic indicators: Costs				Additional service and equality indicators for the WI				
Data set	Objective function	Coverage radius	Cost constraint	IAT	Total	Covered	Covered	Walking	Total	Covered	Covered	Total	open DCs	open DCs	Total	Beneficiaries	WFP	KRC	High service threshold	WI in high service distances	WI in low service distances	Gini Index	Hood Index
		km	%		No.	No.	%	No.	No.	No.	%	No.	No.	%	KSh.	KSh.	KSh.	KSh.	km	%	%		
AD	Min MAD	10	-	10	452,218	374,166	82.74%	63,208	5,426	2,096	38.63%	1,459	762	52.23%	50,314,502	2,411,036	31,723,158	16,180,308	5.0000	0.9143	0.0857	0.7886	0.6404
AD	Min MAD	10	-	12	452,218	373,577	82.61%	63,096	5,426	2,040	37.60%	1,459	726	49.76%	49,651,144	2,498,824	31,736,436	15,415,884	5.0000	0.9129	0.0871	0.7896	0.6403
AD	Min MAD	10	-	14	452,218	372,915	82.46%	62,964	5,426	1,987	36.62%	1,459	726	49.76%	49,655,628	2,496,309	31,743,434	15,415,884	5.0000	0.9163	0.0837	0.7905	0.6402
AD	Min MAD	10	-	16	452,218	371,875	82.23%	62,748	5,426	1,915	35.29%	1,459	633	43.39%	47,662,328	2,474,897	31,746,308	13,441,122	5.0000	0.9202	0.0798	0.7908	0.6399
AD	Min MAD	10	-	18	452,218	370,946	82.03%	62,577	5,426	1,858	34.24%	1,459	792	54.28%	51,035,136	2,468,598	31,749,210	16,817,328	5.0000	0.9234	0.0766	0.7911	0.6394
AD	Min MAD	15	-	10	452,218	397,503	87.90%	67,391	5,426	2,817	51.92%	1,459	650	44.55%	48,989,973	3,443,580	31,744,293	13,802,100	7.5000	0.8950	0.1050	0.7840	0.5811
AD	Min MAD	15	-	12	452,218	396,543	87.69%	67,207	5,426	2,725	50.22%	1,459	640	43.87%	48,770,563	3,433,489	31,747,314	13,589,760	7.5000	0.9024	0.0976	0.7899	0.7403
AD	Min MAD	15	-	14	452,218	395,220	87.40%	66,944	5,426	2,619	48.27%	1,459	684	46.88%	48,990,113	3,386,229	31,759,316	13,844,568	7.5000	0.9044	0.0956	0.7902	0.8692
AD	Min MAD	15	-	16	452,218	393,381	86.99%	66,563	5,426	2,492	45.93%	1,459	640	43.87%	48,702,820	3,343,903	31,769,157	13,589,760	7.5000	0.9085	0.0915	0.7905	0.8686
AD	Min MAD	15	-	18	452,218	391,827	86.65%	66,278	5,426	2,397	44.18%	1,459	673	46.13%	49,362,717	3,296,684	31,775,551	14,290,482	7.5000	0.9123	0.0877	0.7920	0.6613
AD	Min MAD	20	-	10	452,218	416,041	92.00%	70,735	5,426	3,453	63.64%	1,459	693	47.50%	50,901,226	4,494,758	31,691,305	14,715,162	10.0000	0.8906	0.1094	0.7788	0.6636
AD	Min MAD	20	-	12	452,218	414,569	91.67%	70,455	5,426	3,313	61.06%	1,459	688	47.16%	50,745,062	4,435,981	31,700,089	14,608,992	10.0000	0.8938	0.1062	0.7802	0.6656
AD	Min MAD	20	-	14	452,218	412,629	91.25%	70,065	5,426	3,158	58.20%	1,459	682	46.74%	50,566,459	4,369,902	31,714,969	14,481,588	10.0000	0.8962	0.1038	0.7834	0.6665
AD	Min MAD	20	-	16	452,218	410,170	90.70%	69,555	5,426	2,988	55.07%	1,459	691	47.36%	50,694,954	4,288,496	31,733,764	14,672,694	10.0000	0.9029	0.0971	0.7903	0.6684
AD	Min MAD	20	-	18	452,218	407,860	90.19%	69,132	5,426	2,847	52.47%	1,459	677	46.40%	50,321,961	4,198,620	31,747,923	14,375,418	10.0000	0.9052	0.0948	0.7917	0.6712
AD	Min MAD	25	-	10	452,218	427,892	94.62%	72,901	5,426	3,920	72.24%	1,459	688	47.16%	51,589,693	5,352,565	31,628,136	14,608,992	12.5000	0.8895	0.1105	0.7781	0.6637
AD	Min MAD	25	-	12	452,218	426,018	94.21%	72,545	5,426	3,742	68.96%	1,459	688	47.16%	51,542,834	5,293,878	31,639,964	14,608,992	12.5000	0.8922	0.1078	0.7816	0.6654
AD	Min MAD	25	-	14	452,218	423,437	93.64%	72,024	5,426	3,536	65.17%	1,459	701	48.05%	51,693,420	5,150,680	31,657,706	14,885,034	12.5000	0.8954	0.1046	0.7842	0.6678
AD	Min MAD	25	-	16	452,218	420,445	92.97%	71,403	5,426	3,329	61.35%	1,459	683	46.81%	51,221,034	5,038,160	31,680,053	14,502,822	12.5000	0.8977	0.1023	0.7871	0.6703
AD	Min MAD	25	-	18	452,218	417,462	92.31%	70,857	5,426	3,147	58.00%	1,459	682	46.74%	51,090,655	4,904,858	31,704,209	14,481,588	12.5000	0.9111	0.0889	0.7909	0.6716
AD	Min MAD	55	-	10	452,218	448,538	99.19%	76,705	5,426	4,780	88.09%	1,459	644	44.14%	52,651,478	7,560,304	31,416,478	13,674,696	27.5000	0.9597	0.0403	0.7835	0.6630
AD	Min MAD	55	-	12	452,218	446,027	98.63%	76,227	5,426	4,541	83.69%	1,459	666	45.65%	53,013,098	7,446,231	31,425,023	14,141,844	27.5000	0.9605	0.0395	0.7858	0.6650
AD	Min MAD	55	-	14	452,218	442,516	97.85%	75,516	5,426	4,261	78.53%	1,459	665	45.58%	52,764,183	7,190,507	31,453,066	14,120,610	27.5000	0.9616	0.0384	0.7920	0.6668
AD	Min MAD	55	-	16	452,218	438,445	96.95%	74,670	5,426	3,979	73.33%	1,459	673	46.13%	52,743,782	6,965,930	31,487,370	14,290,482	27.5000	0.9643	0.0357	0.7949	0.6675
AD	Min MAD	55	-	18	452,218	434,165	96.01%	73,887	5,426	3,718	68.52%	1,459	687	47.09%	52,819,374	6,705,603	31,526,013	14,587,758	27.5000	0.9674	0.0326	0.7976	0.6683

NAD = non-aggregated data
IAT = inhabitants' aggregation threshold
WI = walking inhabitants

MAD = beneficiaries' mean absolute deviation of walking distance

Appendix 3.4 (continuation) Characteristics of the solutions obtained using aggregated data and different inhabitants' aggregation threshold for the beneficiaries' mean absolute deviation of walking distance objective function.

Solution identifiers					Indicators for the variables (i, j)											Indicators for the variables (0, j)					Problem characteristics			
					beneficiaries walking distance from pop. point to DCs						beneficiaries walking time from pop. point to DCs					road distance from DCs to Warehouse					variables (i, j)		algorithm effort	
Data set	Objective function	Coverage radius	Cost constraint	IAT	Mean absolute deviation	Average	Minimum	Maximum	Variance	Standard deviation	Average	Minimum	Maximum	Variance	Standard deviation	Average	Minimum	Maximum	Variance	Standard deviation	Total	Tuples	CPU time	CPLEX ticks
		km	%		km	km	km	km	km	km	hours	hours	hours	hours	hours	km	km	km	km	km	No.	No.	Seconds	No.
AD	Min MAD	10	-	10	0.13	1.24	-	9.99	4.57	2.14	0.6209	-	4.9939	2.2850	1.5116	118.04	1.33	268.93	4,494.89	67.04	3,058,064	55,026	4.71	2,878
AD	Min MAD	10	-	12	0.12	1.19	-	9.99	4.50	2.12	0.5927	-	4.9939	2.2491	1.4997	118.22	1.33	268.93	4,477.68	66.92	2,976,360	54,552	2.98	2,927
AD	Min MAD	10	-	14	0.11	1.16	-	9.99	4.35	2.09	0.5800	-	4.9939	2.1774	1.4756	118.26	1.33	268.93	4,486.37	66.98	2,899,033	54,155	2.80	2,811
AD	Min MAD	10	-	16	0.11	1.12	-	9.99	4.32	2.08	0.5617	-	4.9939	2.1585	1.4692	118.28	1.33	268.93	4,458.03	66.77	2,793,985	53,287	2.66	2,714
AD	Min MAD	10	-	18	0.10	1.09	-	9.99	4.27	2.07	0.5434	-	4.9939	2.1337	1.4607	118.29	1.33	268.93	4,462.05	66.80	2,710,822	52,955	3.20	2,770
AD	Min MAD	15	-	10	0.23	2.07	-	14.97	11.58	3.40	1.0350	-	7.4863	5.7886	2.4060	118.11	1.33	268.93	4,471.02	66.87	4,110,003	116,431	13.60	11,919
AD	Min MAD	15	-	12	0.22	1.95	-	14.97	11.49	3.39	0.9735	-	7.4863	5.7468	2.3972	118.12	1.33	268.93	4,482.82	66.95	3,975,775	114,821	11.11	11,585
AD	Min MAD	15	-	14	0.21	1.84	-	14.97	11.20	3.35	0.9214	-	7.4863	5.5981	2.3660	118.26	1.33	268.93	4,483.19	66.96	3,821,121	113,462	84.64	58,242
AD	Min MAD	15	-	16	0.20	1.77	-	14.97	11.12	3.33	0.8841	-	7.4863	5.5602	2.3580	118.48	1.33	268.93	4,477.03	66.91	3,635,828	110,939	12.34	13,126
AD	Min MAD	15	-	18	0.18	1.73	-	14.97	10.84	3.29	0.8641	-	7.4863	5.4204	2.3282	118.39	1.33	268.93	4,481.11	66.94	3,497,223	109,944	12.29	12,704
AD	Min MAD	20	-	10	0.35	2.87	-	19.97	22.29	4.72	1.4329	-	9.9874	11.1469	3.3387	118.08	1.33	268.93	4,466.19	66.83	5,037,927	194,940	27.19	28,455
AD	Min MAD	20	-	12	0.34	2.72	-	19.97	22.08	4.70	1.3614	-	9.9874	11.0398	3.3226	118.25	1.33	268.93	4,479.28	66.93	4,833,667	191,273	25.54	27,304
AD	Min MAD	20	-	14	0.31	2.57	-	19.97	21.37	4.62	1.2860	-	9.9874	10.6836	3.2686	118.13	1.33	268.93	4,483.90	66.96	4,607,522	187,986	27.09	26,614
AD	Min MAD	20	-	16	0.29	2.41	-	19.97	20.87	4.57	1.2062	-	9.9874	10.4338	3.2301	118.48	1.33	268.93	4,481.93	66.95	4,359,492	183,006	23.91	24,977
AD	Min MAD	20	-	18	0.29	2.39	-	19.97	20.20	4.49	1.1937	-	9.9874	10.1013	3.1783	118.06	1.33	268.93	4,474.98	66.90	4,153,773	180,606	24.62	24,439
AD	Min MAD	25	-	10	0.48	3.58	-	25.00	32.59	5.71	1.7883	-	12.4989	16.2945	4.0366	118.10	1.33	268.93	4,473.10	66.88	5,719,280	287,763	47.51	48,558
AD	Min MAD	25	-	12	0.44	3.33	-	25.00	31.09	5.58	1.6670	-	12.4989	15.5439	3.9426	118.04	1.33	268.93	4,474.22	66.89	5,459,578	280,940	47.19	46,593
AD	Min MAD	25	-	14	0.41	3.13	-	25.00	30.82	5.55	1.5646	-	12.4989	15.4108	3.9257	118.14	1.33	268.93	4,479.75	66.93	5,159,024	274,631	45.22	44,532
AD	Min MAD	25	-	16	0.39	2.95	-	25.00	29.64	5.44	1.4743	-	12.4989	14.8220	3.8499	118.32	1.33	268.93	4,482.44	66.95	4,857,011	266,239	41.74	41,102
AD	Min MAD	25	-	18	0.39	2.87	-	25.00	29.09	5.39	1.4342	-	12.4984	14.5455	3.8139	118.14	1.33	268.93	4,484.46	66.97	4,591,473	261,646	39.30	39,643
AD	Min MAD	55	-	10	0.74	5.50	-	54.34	74.99	8.66	2.7483	-	27.1676	37.4956	6.1234	118.10	1.33	268.93	4,468.78	66.85	6,974,020	1,125,757	601.09	349,611
AD	Min MAD	55	-	12	0.70	5.12	-	54.34	73.03	8.55	2.5616	-	27.1676	36.5147	6.0427	118.16	1.33	268.93	4,475.94	66.90	6,625,319	1,085,432	555.87	335,644
AD	Min MAD	55	-	14	0.68	4.85	-	54.34	71.34	8.45	2.4233	-	27.1676	35.6700	5.9724	118.31	1.33	268.93	4,489.62	67.00	6,216,799	1,042,672	507.78	310,553
AD	Min MAD	55	-	16	0.65	4.31	-	54.34	69.97	8.37	2.1575	-	27.1676	34.9868	5.9150	118.48	1.33	268.93	4,480.10	66.93	5,805,361	995,997	305.03	217,664
AD	Min MAD	55	-	18	0.63	4.15	-	54.34	64.64	8.04	2.0736	-	27.1676	32.3195	5.6850	118.20	1.33	268.93	4,479.87	66.93	5,424,562	955,370	281.15	205,640

Appendix 3.5 Characteristics of the solutions obtained using aggregated data, costs constraints above the MSC and IAT = 18 inh. for the beneficiaries' mean absolute deviation of walking distance objective function.

Solution identifiers					Demographic indicators: Inhabitants				Demographic indicators: Population points			Demographic indicators: Distribution centers			Economic indicators: Costs				Additional service and equality indicators for the WI				
Data set	Objective function	Coverage radius	Cost constraint	IAT	Total	Covered	Covered	Walking	Total	Covered	Covered	Total	open DCs	open DCs	Total	Beneficiaries	WFP	KRC	High service threshold	WI in high service distances	WI in low service distances	Gini Index	Hood Index
		km	%		No.	No.	%	No.	No.	No.	%	No.	No.	%	KSh.	KSh.	KSh.	KSh.	km	%	%		
AD	Min MAD	10	5%	18	452,218	370,946	82.03%	62,577	5,426	1,858	34.24%	1,459	200	13.71%	38,383,547	2,497,241	31,639,505	4,246,800	5.0000	0.9194	0.0806	0.7072	0.5676
AD	Min MAD	10	10%	18	452,218	370,946	82.03%	62,577	5,426	1,858	34.24%	1,459	290	19.88%	40,210,977	2,360,278	31,692,839	6,157,860	5.0000	0.9230	0.0770	0.7304	0.5711
AD	Min MAD	10	15%	18	452,218	370,946	82.03%	62,577	5,426	1,858	34.24%	1,459	381	26.11%	42,039,092	2,238,524	31,710,414	8,090,154	5.0000	0.9233	0.0767	0.7755	0.6270
AD	Min MAD	10	20%	18	452,218	370,946	82.03%	62,577	5,426	1,858	34.24%	1,459	468	32.08%	43,866,591	2,191,492	31,737,587	9,937,512	5.0000	0.9234	0.0766	0.7869	0.6352
AD	Min MAD	15	5%	18	452,218	391,827	86.65%	66,278	5,426	2,397	44.18%	1,459	171	11.72%	38,471,822	3,289,915	31,550,893	3,631,014	7.5000	0.9102	0.0898	0.7134	0.5646
AD	Min MAD	15	10%	18	452,218	391,827	86.65%	66,278	5,426	2,397	44.18%	1,459	259	17.75%	40,303,615	3,093,066	31,710,943	5,499,606	7.5000	0.9121	0.0879	0.7400	0.5817
AD	Min MAD	15	15%	18	452,218	391,827	86.65%	66,278	5,426	2,397	44.18%	1,459	350	23.99%	42,135,676	2,968,898	31,734,879	7,431,900	7.5000	0.9122	0.0878	0.7723	0.6256
AD	Min MAD	15	20%	18	452,218	391,827	86.65%	66,278	5,426	2,397	44.18%	1,459	438	30.02%	43,967,799	2,902,217	31,765,089	9,300,492	7.5000	0.9123	0.0877	0.7881	0.6502
AD	Min MAD	20	5%	18	452,218	407,860	90.19%	69,132	5,426	2,847	52.47%	1,459	151	10.35%	38,753,319	4,091,346	31,461,640	3,206,334	10.0000	0.9027	0.0973	0.7177	0.5591
AD	Min MAD	20	10%	18	452,218	407,860	90.19%	69,132	5,426	2,847	52.47%	1,459	238	16.31%	40,605,003	3,882,241	31,669,071	5,053,692	10.0000	0.9051	0.0949	0.7385	0.5817
AD	Min MAD	20	15%	18	452,218	407,860	90.19%	69,132	5,426	2,847	52.47%	1,459	330	22.62%	42,450,342	3,744,587	31,698,535	7,007,220	10.0000	0.9052	0.0948	0.7644	0.6217
AD	Min MAD	20	20%	18	452,218	407,860	90.19%	69,132	5,426	2,847	52.47%	1,459	421	28.86%	44,296,221	3,644,549	31,712,158	8,939,514	10.0000	0.9052	0.0948	0.7871	0.6571
AD	Min MAD	25	5%	18	452,218	417,462	92.31%	70,857	5,426	3,147	58.00%	1,459	131	8.98%	38,819,186	4,788,838	31,248,694	2,781,654	12.5000	0.9087	0.0913	0.7129	0.5565
AD	Min MAD	25	10%	18	452,218	417,462	92.31%	70,857	5,426	3,147	58.00%	1,459	215	14.74%	40,667,631	4,471,016	31,631,304	4,565,310	12.5000	0.9110	0.0890	0.7426	0.5895
AD	Min MAD	25	15%	18	452,218	417,462	92.31%	70,857	5,426	3,147	58.00%	1,459	307	21.04%	42,516,023	4,334,475	31,662,710	6,518,838	12.5000	0.9110	0.0890	0.7613	0.6213
AD	Min MAD	25	20%	18	452,218	417,462	92.31%	70,857	5,426	3,147	58.00%	1,459	399	27.35%	44,364,502	4,219,237	31,672,899	8,472,366	12.5000	0.9110	0.0890	0.7849	0.6555
AD	Min MAD	55	5%	18	452,218	434,165	96.01%	73,887	5,426	3,718	68.52%	1,459	103	7.06%	38,950,844	6,782,803	29,980,938	2,187,102	27.5000	0.9602	0.0398	0.7304	0.5736
AD	Min MAD	55	10%	18	452,218	434,165	96.01%	73,887	5,426	3,718	68.52%	1,459	163	11.17%	40,805,753	6,066,229	31,278,382	3,461,142	27.5000	0.9673	0.0327	0.7506	0.5955
AD	Min MAD	55	15%	18	452,218	434,165	96.01%	73,887	5,426	3,718	68.52%	1,459	252	17.27%	42,660,568	5,856,284	31,453,315	5,350,968	27.5000	0.9673	0.0327	0.7704	0.6294
AD	Min MAD	55	20%	18	452,218	434,165	96.01%	73,887	5,426	3,718	68.52%	1,459	344	23.58%	44,515,353	5,729,254	31,481,603	7,304,496	27.5000	0.9673	0.0327	0.7876	0.6545

NAD = non-aggregated data
IAT = inhabitants' aggregation threshold
WI = walking inhabitants
MSC = minimum stakeholder costs

MAD = beneficiaries' mean absolute deviation of walking distance

Appendix 3.5 (continuation) Characteristics of the solutions obtained using aggregated data, costs constraints above the MSC and IAT = 18 inh. for the beneficiaries' mean absolute deviation of walking distance objective function.

Solution identifiers					Indicators for the variables (i, j)											Indicators for the variables $(0, j)$					Problem characteristics			
					beneficiaries walking distance from pop. point to DCs						beneficiaries walking time from pop. point to DCs					road distance from DCs to Warehouse					variables (i, j)		algorithm effort	
Data set	Objective function	Coverage radius	Cost constraint	IAT	Mean absolute deviation	Average	Minimum	Maximum	Variance	Standard deviation	Average	Minimum	Maximum	Variance	Standard deviation	Average	Minimum	Maximum	Variance	Standard deviation	Total	Tuples	CPU time	CPLEX ticks
		km	%		km	km	km	km	km	km	hours	hours	hours	hours	hours	km	km	km	km	km	No.	No.	Seconds	No.
AD	Min MAD	10	5%	18	0.13	1.46	-	9.99	4.78	2.19	0.7305	-	4.9939	2.3892	1.5457	123.27	1.33	268.93	4,069.91	63.80	2,710,822	52,955	50.55	34,080
AD	Min MAD	10	10%	18	0.13	1.30	-	9.99	4.47	2.11	0.6502	-	4.9939	2.2334	1.4945	120.33	1.33	268.93	4,064.05	63.75	2,710,822	52,955	20.97	19,181
AD	Min MAD	10	15%	18	0.12	1.16	-	9.99	4.48	2.12	0.5788	-	4.9939	2.2386	1.4962	117.88	1.33	268.93	4,399.10	66.33	2,710,822	52,955	22.25	18,491
AD	Min MAD	10	20%	18	0.12	1.10	-	9.99	4.42	2.10	0.5512	-	4.9939	2.2095	1.4865	117.41	1.33	268.93	4,535.07	67.34	2,710,822	52,955	13.56	11,900
AD	Min MAD	15	5%	18	0.23	2.18	-	14.97	11.19	3.34	1.0876	-	7.4864	5.5929	2.3649	125.46	1.33	268.93	4,039.29	63.56	3,497,223	109,944	88.89	59,678
AD	Min MAD	15	10%	18	0.22	1.96	-	14.97	10.80	3.29	0.9786	-	7.4863	5.4012	2.3240	121.82	1.33	268.93	4,085.48	63.92	3,497,223	109,944	260.87	142,675
AD	Min MAD	15	15%	18	0.22	1.82	-	14.97	10.93	3.31	0.9099	-	7.4863	5.4631	2.3373	118.46	1.33	268.93	4,328.78	65.79	3,497,223	109,944	30.29	19,721
AD	Min MAD	15	20%	18	0.21	1.75	-	14.97	10.97	3.31	0.8730	-	7.4863	5.4853	2.3421	117.74	1.33	268.93	4,582.81	67.70	3,497,223	109,944	91.31	75,521
AD	Min MAD	20	5%	18	0.34	2.88	-	19.97	20.20	4.49	1.4379	-	9.9874	10.0988	3.1779	123.90	1.33	268.93	4,000.76	63.25	4,153,773	180,606	1,229.93	507,335
AD	Min MAD	20	10%	18	0.33	2.65	-	19.97	19.79	4.45	1.3269	-	9.9874	9.8965	3.1459	123.08	1.33	268.93	3,982.40	63.11	4,153,773	180,606	733.57	306,651
AD	Min MAD	20	15%	18	0.32	2.51	-	19.97	20.02	4.47	1.2538	-	9.9874	10.0109	3.1640	117.54	1.33	268.93	4,285.47	65.46	4,153,773	180,606	66.34	43,300
AD	Min MAD	20	20%	18	0.32	2.40	-	19.97	20.29	4.50	1.2007	-	9.9874	10.1453	3.1852	116.89	1.33	268.93	4,551.98	67.47	4,153,773	180,606	410.26	241,523
AD	Min MAD	25	5%	18	0.43	3.49	-	25.00	29.14	5.40	1.7462	-	12.4984	14.5710	3.8172	125.04	1.33	268.93	4,042.87	63.58	4,591,473	261,646	3,926.90	1,570,093
AD	Min MAD	25	10%	18	0.42	3.16	-	25.00	28.59	5.35	1.5816	-	12.4984	14.2930	3.7806	123.71	1.33	268.93	4,061.00	63.73	4,591,473	261,646	277.98	150,399
AD	Min MAD	25	15%	18	0.41	3.02	-	25.00	28.79	5.37	1.5109	-	12.4984	14.3958	3.7942	119.86	1.33	268.93	4,167.75	64.56	4,591,473	261,646	109.26	64,921
AD	Min MAD	25	20%	18	0.41	2.90	-	25.00	29.23	5.41	1.4512	-	12.4984	14.6163	3.8231	118.71	1.33	268.93	4,391.27	66.27	4,591,473	261,646	1,103.26	505,054
AD	Min MAD	55	5%	18	0.71	5.27	-	54.97	77.33	8.79	2.6349	-	27.4829	38.6651	6.2181	123.13	1.33	268.93	3,947.89	62.83	5,424,562	955,370	1,823.82	826,334
AD	Min MAD	55	10%	18	0.66	4.56	-	54.34	65.82	8.11	2.2790	-	27.1676	32.9113	5.7368	125.85	1.33	268.93	4,091.04	63.96	5,424,562	955,370	7,573.52	2,997,225
AD	Min MAD	55	15%	18	0.65	4.35	-	54.34	66.30	8.14	2.1747	-	27.1676	33.1477	5.7574	122.43	1.33	268.93	4,106.03	64.08	5,424,562	955,370	1,208.02	599,052
AD	Min MAD	55	20%	18	0.65	4.22	-	54.34	66.96	8.18	2.1116	-	27.1676	33.4822	5.7864	117.66	1.33	268.93	4,268.25	65.33	5,424,562	955,370	617.80	307,719

Appendix 3.6 Characteristics of the solutions obtained using aggregated data and different inhabitants' aggregation threshold for the beneficiaries' standard service objective function

Solution identifiers					Demographic indicators: Inhabitants				Population points			Distribution centers			Economic indicators: Costs				Additional service and equality indicators for the WI				
Data set	Objective function	Coverage radius	Cost constraint	IAT	Total	Covered	Covered	Walking	Total	Covered	Covered	Total	open DCs	open DCs	Total	Beneficiaries	WFP	KRC	High service threshold	WI in high service distances	WI in low service distances	Gini Index	Hood Index
		km	%		No.	No.	%	No.	No.	No.	%	No.	No.	%	KSh.	KSh.	KSh.	KSh.	km	%	%		
AD	Max SMR	10	-	10	452,218	374,166	82.74%	63,208	5,426	2,096	38.63%	1,459	423	28.99%	44,404,573	3,707,042	31,715,549	8,981,982	5.0000	0.9177	0.0823	0.4131	0.3030
AD	Max SMR	10	-	12	452,218	373,577	82.61%	63,096	5,426	2,040	37.60%	1,459	385	26.39%	43,588,878	3,615,542	31,798,246	8,175,090	5.0000	0.9189	0.0811	0.3872	0.2748
AD	Max SMR	10	-	14	452,218	372,915	82.46%	62,964	5,426	1,987	36.62%	1,459	398	27.28%	43,972,246	3,755,215	31,765,899	8,451,132	5.0000	0.9204	0.0796	0.3944	0.2772
AD	Max SMR	10	-	16	452,218	371,875	82.23%	62,748	5,426	1,915	35.29%	1,459	382	26.18%	43,529,700	3,549,254	31,869,058	8,111,388	5.0000	0.9219	0.0781	0.4197	0.3153
AD	Max SMR	10	-	18	452,218	370,946	82.03%	62,577	5,426	1,858	34.24%	1,459	365	25.02%	43,388,873	3,961,081	31,677,382	7,750,410	5.0000	0.9235	0.0765	0.3355	0.2411
AD	Max SMR	15	-	10	452,218	397,503	87.90%	67,391	5,426	2,817	51.92%	1,459	402	27.55%	45,483,716	5,115,340	31,832,308	8,536,068	7.5000	0.9033	0.0967	0.4411	0.3288
AD	Max SMR	15	-	12	452,218	396,543	87.69%	67,207	5,426	2,725	50.22%	1,459	392	26.87%	46,404,944	6,230,072	31,851,145	8,323,728	7.5000	0.9048	0.0952	0.3298	0.3627
AD	Max SMR	15	-	14	452,218	395,220	87.40%	66,944	5,426	2,619	48.27%	1,459	359	24.61%	44,598,233	5,273,724	31,701,503	7,623,006	7.5000	0.9072	0.0928	0.4348	0.9852
AD	Max SMR	15	-	16	452,218	393,381	86.99%	66,563	5,426	2,492	45.93%	1,459	353	24.19%	44,451,209	4,999,686	31,955,921	7,495,602	7.5000	0.9102	0.0898	0.4364	0.9980
AD	Max SMR	15	-	18	452,218	391,827	86.65%	66,278	5,426	2,397	44.18%	1,459	351	24.06%	44,430,823	5,094,883	31,882,805	7,453,134	7.5000	0.9124	0.0876	0.4241	0.3225
AD	Max SMR	20	-	10	452,218	416,041	92.00%	70,735	5,426	3,453	63.64%	1,459	388	26.59%	47,643,207	7,811,848	31,592,567	8,238,792	10.0000	0.8936	0.1064	0.3547	0.2518
AD	Max SMR	20	-	12	452,218	414,569	91.67%	70,455	5,426	3,313	61.06%	1,459	336	23.03%	45,950,072	7,241,368	31,574,081	7,134,624	10.0000	0.8956	0.1044	0.3939	0.2766
AD	Max SMR	20	-	14	452,218	412,629	91.25%	70,065	5,426	3,158	58.20%	1,459	348	23.85%	45,835,819	6,715,157	31,731,229	7,389,432	10.0000	0.8987	0.1013	0.4239	0.3088
AD	Max SMR	20	-	16	452,218	410,170	90.70%	69,555	5,426	2,988	55.07%	1,459	346	23.71%	46,793,677	7,721,708	31,725,005	7,346,964	10.0000	0.9021	0.0979	0.3471	0.2364
AD	Max SMR	20	-	18	452,218	407,860	90.19%	69,132	5,426	2,847	52.47%	1,459	333	22.82%	45,579,109	6,901,239	31,606,948	7,070,922	10.0000	0.9052	0.0948	0.4322	0.3120
AD	Max SMR	25	-	10	452,218	427,892	94.62%	72,901	5,426	3,920	72.24%	1,459	370	25.36%	49,174,284	9,530,189	31,787,515	7,856,580	12.5000	0.8973	0.1027	0.3660	0.2577
AD	Max SMR	25	-	12	452,218	426,018	94.21%	72,545	5,426	3,742	68.96%	1,459	356	24.40%	46,971,065	7,777,691	31,634,071	7,559,304	12.5000	0.8996	0.1004	0.4917	0.3698
AD	Max SMR	25	-	14	452,218	423,437	93.64%	72,024	5,426	3,536	65.17%	1,459	358	24.54%	49,348,084	9,942,800	31,803,513	7,601,772	12.5000	0.9036	0.0964	0.3234	0.2180
AD	Max SMR	25	-	16	452,218	420,445	92.97%	71,403	5,426	3,329	61.35%	1,459	321	22.00%	46,255,690	7,786,704	31,652,872	6,816,114	12.5000	0.9072	0.0928	0.4567	0.3440
AD	Max SMR	25	-	18	452,218	417,462	92.31%	70,857	5,426	3,147	58.00%	1,459	304	20.84%	46,717,770	8,524,155	31,738,479	6,455,136	12.5000	0.9110	0.0890	0.3927	0.2727
AD	Max SMR	55	-	10	452,218	448,538	99.19%	76,705	5,426	4,780	88.09%	1,459	388	26.59%	56,767,267	16,369,083	32,159,392	8,238,792	27.5000	0.9607	0.0393	0.4173	0.3166
AD	Max SMR	55	-	12	452,218	446,027	98.63%	76,227	5,426	4,541	83.69%	1,459	391	26.80%	56,575,401	16,453,796	31,819,111	8,302,494	27.5000	0.9614	0.0386	0.4165	0.3115
AD	Max SMR	55	-	14	452,218	442,516	97.85%	75,516	5,426	4,261	78.53%	1,459	320	21.93%	55,020,321	17,368,926	30,856,515	6,794,880	27.5000	0.9630	0.0370	0.3422	0.2458
AD	Max SMR	55	-	16	452,218	438,445	96.95%	74,670	5,426	3,979	73.33%	1,459	352	24.13%	54,552,231	15,401,434	31,676,429	7,474,368	27.5000	0.9650	0.0350	0.3796	0.2710
AD	Max SMR	55	-	18	452,218	434,165	96.01%	73,887	5,426	3,718	68.52%	1,459	337	23.10%	54,451,089	15,792,086	31,503,146	7,155,858	27.5000	0.9673	0.0327	0.3489	0.2369

NAD = non-aggregated data
IAT = inhabitants' aggregation threshold
WI = walking inhabitants
SMR = service below the mid-radius distance threshold

Appendix 3.6 (continuation) Characteristics of the solutions obtained using aggregated data and different inhabitants' aggregation threshold for the beneficiaries' standard service objective function.

Solution identifiers					Indicators for the variables (i, j) beneficiaries walking distance from pop. point to DCs						Indicators for the variables (i, j) beneficiaries walking time from pop. point to DCs					Indicators for the variables $(0, j)$ road distance from DCs to Warehouse					Problem characteristics variables (i, j)		Problem characteristics algorithm effort	
Data set	Objective function	Coverage radius	Cost constraint	IAT	Mean absolute deviation	Average	Minimum	Maximum	Variance	Standard deviation	Average	Minimum	Maximum	Variance	Standard deviation	Average	Minimum	Maximum	Variance	Standard deviation	Total	Tuples	CPU time	CPLEX ticks
		km	%		km	km	km	km	km	km	hours	hours	hours	hours	hours	km	km	km	km	km	No.	No.	Seconds	No.
AD	Max SMR	10	-	10	0.18	2.84	-	9.99	4.89	2.21	1.4183	-	4.9944	2.4437	1.1054	116.32	2.21	268.93	4,200.14	64.81	3,058,064	55,026	3.95	3,777
AD	Max SMR	10	-	12	0.17	2.74	-	9.99	4.34	2.08	1.3689	-	4.9944	2.1698	1.0416	118.05	2.21	268.93	4,017.98	63.39	2,976,360	54,552	3.54	3,782
AD	Max SMR	10	-	14	0.16	2.91	-	9.99	4.63	2.15	1.4547	-	4.9945	2.3147	1.0758	118.11	4.02	268.93	4,105.27	64.07	2,899,033	54,155	3.47	3,683
AD	Max SMR	10	-	16	0.16	2.68	-	9.99	4.69	2.17	1.3418	-	4.9944	2.3443	1.0827	119.37	2.21	268.93	3,952.11	62.87	2,793,985	53,287	3.56	3,555
AD	Max SMR	10	-	18	0.15	3.18	-	9.99	4.00	2.00	1.5890	-	4.9944	2.0014	1.0003	116.12	2.21	268.93	4,031.69	63.50	2,710,822	52,955	3.68	3,469
AD	Max SMR	15	-	10	0.33	4.10	-	14.97	11.98	3.46	2.0516	-	7.4865	5.9924	1.7310	117.71	4.62	268.93	3,817.29	61.78	4,110,003	116,431	11.63	13,240
AD	Max SMR	15	-	12	0.32	5.34	-	14.97	10.69	3.27	2.6679	-	7.4865	5.3470	1.6351	118.31	6.51	268.93	3,933.03	62.71	3,975,775	114,821	12.59	12,941
AD	Max SMR	15	-	14	0.31	4.31	-	14.97	12.23	3.50	2.1570	-	7.4865	6.1172	1.7489	118.14	5.64	268.93	3,988.60	63.16	3,821,121	113,462	11.99	12,749
AD	Max SMR	15	-	16	0.29	4.04	-	14.97	11.70	3.42	2.0225	-	7.4865	5.8489	1.7101	121.08	7.22	268.93	3,831.90	61.90	3,635,828	110,939	10.87	12,389
AD	Max SMR	15	-	18	0.28	4.17	-	14.97	11.43	3.38	2.0870	-	7.4865	5.7128	1.6901	118.31	4.62	268.93	3,861.84	62.14	3,497,223	109,944	10.65	12,077
AD	Max SMR	20	-	10	0.52	6.64	-	19.98	19.41	4.41	3.3188	-	9.9877	9.7067	2.2030	117.06	2.06	268.93	3,901.34	62.46	5,037,927	194,940	25.39	27,859
AD	Max SMR	20	-	12	0.49	6.08	-	19.98	20.45	4.52	3.0378	-	9.9877	10.2226	2.2608	119.51	2.06	268.93	3,718.69	60.98	4,833,667	191,273	26.09	27,196
AD	Max SMR	20	-	14	0.47	5.57	-	19.98	20.73	4.55	2.7832	-	9.9877	10.3657	2.2766	120.19	2.06	268.93	3,634.78	60.29	4,607,522	187,986	23.49	26,457
AD	Max SMR	20	-	16	0.44	6.68	-	19.98	18.99	4.36	3.3400	-	9.9877	9.4948	2.1789	117.48	2.06	268.93	3,720.79	61.00	4,359,492	183,006	22.42	25,418
AD	Max SMR	20	-	18	0.42	5.86	-	19.98	21.97	4.69	2.9294	-	9.9877	10.9843	2.3435	123.64	2.06	268.93	3,625.66	60.21	4,153,773	180,606	23.51	25,050
AD	Max SMR	25	-	10	0.70	8.13	-	25.00	30.89	5.56	4.0634	-	12.4989	15.4427	2.7787	116.83	6.67	268.93	3,890.84	62.38	5,719,280	287,763	52.53	51,274
AD	Max SMR	25	-	12	0.66	6.40	-	25.00	36.46	6.04	3.2004	-	12.4989	18.2287	3.0190	117.03	6.67	268.93	3,794.59	61.60	5,459,578	280,940	45.80	49,634
AD	Max SMR	25	-	14	0.62	8.66	-	25.00	27.75	5.27	4.3321	-	12.4989	13.8754	2.6340	119.45	7.26	268.93	3,599.17	59.99	5,159,024	274,631	49.74	48,712
AD	Max SMR	25	-	16	0.58	6.54	-	25.00	32.87	5.73	3.2680	-	12.4989	16.4329	2.8664	116.99	6.67	268.93	4,008.13	63.31	4,857,011	266,239	43.58	46,326
AD	Max SMR	25	-	18	0.55	7.36	-	25.00	29.78	5.46	3.6808	-	12.4989	14.8907	2.7286	120.31	6.67	268.93	3,747.16	61.21	4,591,473	261,646	41.15	45,457
AD	Max SMR	55	-	10	1.26	14.19	-	55.00	121.70	11.03	7.0974	-	27.4988	60.8514	5.5159	126.05	7.90	268.93	3,879.92	62.29	6,974,020	1,125,757	1,839.72	1,167,394
AD	Max SMR	55	-	12	1.25	14.37	-	55.00	121.55	11.03	7.1873	-	27.4993	60.7769	5.5126	98.24	2.56	268.93	4,017.82	63.39	6,625,319	1,085,432	1,717.47	1,205,395
AD	Max SMR	55	-	14	1.20	15.41	-	54.99	95.61	9.78	7.7065	-	27.4948	47.8029	4.8889	122.48	18.20	268.93	3,036.10	55.10	6,216,799	1,042,672	711.06	725,397
AD	Max SMR	55	-	16	1.07	13.67	-	54.98	94.33	9.71	6.8352	-	27.4901	47.1642	4.8561	111.24	5.76	268.93	3,964.84	62.97	5,805,361	995,997	542.08	515,343
AD	Max SMR	55	-	18	1.02	14.22	-	54.98	88.57	9.41	7.1095	-	27.4875	44.2826	4.7055	114.71	6.67	268.93	4,090.92	63.96	5,424,562	955,370	520.16	491,695

Appendix 3.7 Characteristics of the solutions obtained using aggregated data, costs constraints above the MSC and IAT = 18 inh. for the beneficiaries' standard service objective function.

Solution identifiers					Demographic indicators: Inhabitants				Demographic indicators: Population points			Demographic indicators: Distribution centers			Economic indicators: Costs				Additional service and equality indicators for the WI				
Data set	Objective function	Coverage radius	Cost constraint	IAT	Total	Covered	Covered	Walking	Total	Covered	Covered	Total	open DCs	open DCs	Total	Beneficiaries	WFP	KRC	High service threshold	WI in high service distances	WI in low service distances	Gini Index	Hood Index
		km	%		No.	No.	%	No.	No.	No.	%	No.	No.	%	KSh.	KSh.	KSh.	KSh.	km	%	%		
AD	Max SMR	10	5%	18	452,218	370,946	82.03%	62,577	5,426	1,858	34.24%	1,459	197	13.50%	38,383,449	2,763,117	31,437,235	4,183,098	5.0000	0.9235	0.0765	0.6640	0.5216
AD	Max SMR	10	10%	18	452,218	370,946	82.03%	62,577	5,426	1,858	34.24%	1,459	245	16.79%	40,193,183	3,398,732	31,592,121	5,202,330	5.0000	0.9235	0.0765	0.4801	0.3461
AD	Max SMR	10	15%	18	452,218	370,946	82.03%	62,577	5,426	1,858	34.24%	1,459	314	21.52%	42,036,170	3,729,100	31,639,594	6,667,476	5.0000	0.9235	0.0765	0.3938	0.2851
AD	Max SMR	10	20%	18	452,218	370,946	82.03%	62,577	5,426	1,858	34.24%	1,459	376	25.77%	43,421,946	3,773,630	31,664,331	7,983,984	5.0000	0.9235	0.0765	0.3892	0.2821
AD	Max SMR	15	5%	18	452,218	391,827	86.65%	66,278	5,426	2,397	44.18%	1,459	161	11.03%	38,449,285	3,763,108	31,267,503	3,418,674	7.5000	0.9124	0.0876	0.6622	0.5171
AD	Max SMR	15	10%	18	452,218	391,827	86.65%	66,278	5,426	2,397	44.18%	1,459	195	13.37%	40,303,620	4,548,725	31,614,264	4,140,630	7.5000	0.9124	0.0876	0.5105	0.3780
AD	Max SMR	15	15%	18	452,218	391,827	86.65%	66,278	5,426	2,397	44.18%	1,459	260	17.82%	42,133,921	4,838,153	31,774,928	5,520,840	7.5000	0.9124	0.0876	0.4369	0.3327
AD	Max SMR	15	20%	18	452,218	391,827	86.65%	66,278	5,426	2,397	44.18%	1,459	330	22.62%	43,967,472	5,086,729	31,873,523	7,007,220	7.5000	0.9124	0.0876	0.4234	0.3218
AD	Max SMR	20	5%	18	452,218	407,860	90.19%	69,132	5,426	2,847	52.47%	1,459	131	8.98%	38,746,008	4,899,822	31,064,532	2,781,654	10.0000	0.9052	0.0948	0.6608	0.5229
AD	Max SMR	20	10%	18	452,218	407,860	90.19%	69,132	5,426	2,847	52.47%	1,459	171	11.72%	40,564,347	5,538,754	31,394,580	3,631,014	10.0000	0.9052	0.0948	0.5819	0.4370
AD	Max SMR	20	15%	18	452,218	407,860	90.19%	69,132	5,426	2,847	52.47%	1,459	223	15.28%	42,235,281	5,886,631	31,613,468	4,735,182	10.0000	0.9052	0.0948	0.5038	0.3923
AD	Max SMR	20	20%	18	452,218	407,860	90.19%	69,132	5,426	2,847	52.47%	1,459	283	19.40%	44,292,938	6,622,579	31,661,137	6,009,222	10.0000	0.9052	0.0948	0.4473	0.3327
AD	Max SMR	25	5%	18	452,218	417,462	92.31%	70,857	5,426	3,147	58.00%	1,459	117	8.02%	38,819,173	5,610,521	30,724,274	2,484,378	12.5000	0.9110	0.0890	0.6777	0.5406
AD	Max SMR	25	10%	18	452,218	417,462	92.31%	70,857	5,426	3,147	58.00%	1,459	145	9.94%	40,638,547	6,306,708	31,252,909	3,078,930	12.5000	0.9110	0.0890	0.5976	0.4671
AD	Max SMR	25	15%	18	452,218	417,462	92.31%	70,857	5,426	3,147	58.00%	1,459	192	13.16%	42,499,228	6,863,785	31,558,516	4,076,928	12.5000	0.9110	0.0890	0.5251	0.4046
AD	Max SMR	25	20%	18	452,218	417,462	92.31%	70,857	5,426	3,147	58.00%	1,459	226	15.49%	44,116,403	7,566,054	31,751,465	4,798,884	12.5000	0.9110	0.0890	0.4620	0.3554
AD	Max SMR	55	5%	18	452,218	434,165	96.01%	73,887	5,426	3,718	68.52%	1,459	66	4.52%	38,940,792	8,841,260	28,698,088	1,401,444	27.5000	0.9673	0.0327	0.6413	0.5182
AD	Max SMR	55	10%	18	452,218	434,165	96.01%	73,887	5,426	3,718	68.52%	1,459	80	5.48%	40,694,892	9,561,572	29,434,600	1,698,720	27.5000	0.9673	0.0327	0.5941	0.4682
AD	Max SMR	55	15%	18	452,218	434,165	96.01%	73,887	5,426	3,718	68.52%	1,459	76	5.21%	42,558,686	10,863,825	30,081,077	1,613,784	27.5000	0.9673	0.0327	0.5236	0.3976
AD	Max SMR	55	20%	18	452,218	434,165	96.01%	73,887	5,426	3,718	68.52%	1,459	177	12.13%	44,425,168	10,599,442	30,067,309	3,758,418	27.5000	0.9673	0.0327	0.6014	0.4795
AD	Max S10kmT	10	5%	18	452,218	370,946	82.03%	62,577	5,426	1,858	34.24%	1,459	160	10.97%	38,228,649	3,540,409	31,290,799	3,397,440	10.0000	1.0000	-	0.5810	0.4458
AD	Max S10kmT	10	10%	18	452,218	370,946	82.03%	62,577	5,426	1,858	34.24%	1,459	188	12.89%	40,031,128	4,573,914	31,465,223	3,991,992	10.0000	1.0000	-	0.4272	0.3258
AD	Max S10kmT	10	15%	18	452,218	370,946	82.03%	62,577	5,426	1,858	34.24%	1,459	236	16.18%	42,037,649	5,287,813	31,738,611	5,011,224	10.0000	1.0000	-	0.4044	0.3264
AD	Max S10kmT	10	20%	18	452,218	370,946	82.03%	62,577	5,426	1,858	34.24%	1,459	306	20.97%	43,866,794	5,528,809	31,840,381	6,497,604	10.0000	1.0000	-	0.3572	0.2869

NAD = non-aggregated data
IAT = inhabitants' aggregation threshold
WI = walking inhabitants
MSC = minimum stakeholder costs

SMR = service below the mid-radius distance threshold
S10kmT = service below the 10 km distance threshold

Appendix 3.7 (continuation) Characteristics of the solutions obtained using aggregated data, costs constraints above the MSC and IAT = 18 inh. for the beneficiaries' standard service objective function.

Solution identifiers					Indicators for the variables (i, j) beneficiaries walking distance from pop. point to DCs						Indicators for the variables (i, j) beneficiaries walking time from pop. point to DCs					Indicators for the variables $(0, j)$ road distance from DCs to Warehouse					Problem characteristics variables (i, j)		Problem characteristics algorithm effort	
Data set	Objective function	Coverage radius	Cost constraint	IAT	Mean absolute deviation	Average	Minimum	Maximum	Variance	Standard deviation	Average	Minimum	Maximum	Variance	Standard deviation	Average	Minimum	Maximum	Variance	Standard deviation	Total	Tuples	CPU time	CPLEX ticks
		km	%		km	km	km	km	km	km	hours	hours	hours	hours	hours	km	km	km	km	km	No.	No.	Seconds	No.
AD	MaxSMR	10	5%	18	0.15	1.77	-	9.99	5.68	2.38	0.8864	-	4.9945	2.8399	1.1916	126.52	1.33	268.93	3,754.59	61.27	2,710,822	52,955	24.12	16,033
AD	MaxSMR	10	10%	18	0.15	2.52	-	9.99	5.15	2.27	1.2592	-	4.9944	2.5758	1.1349	124.68	4.02	268.93	3,643.90	60.36	2,710,822	52,955	13.78	11,098
AD	MaxSMR	10	15%	18	0.15	2.91	-	9.99	4.60	2.15	1.4529	-	4.9944	2.3021	1.0729	119.07	2.21	268.93	3,908.31	62.52	2,710,822	52,955	5.04	4,272
AD	MaxSMR	10	20%	18	0.15	2.96	-	9.99	4.61	2.15	1.4790	-	4.9944	2.3057	1.0737	116.98	2.21	268.93	4,034.54	63.52	2,710,822	52,955	3.92	3,707
AD	MaxSMR	15	5%	18	0.26	2.70	-	14.97	13.22	3.64	1.3496	-	7.4865	6.6082	1.8177	129.77	1.33	268.93	3,583.21	59.86	3,497,223	109,944	64.94	48,186
AD	MaxSMR	15	10%	18	0.27	3.57	-	14.97	12.16	3.49	1.7846	-	7.4865	6.0784	1.7433	124.62	5.36	268.93	3,753.67	61.27	3,497,223	109,944	38.69	27,887
AD	MaxSMR	15	15%	18	0.27	3.89	-	14.97	11.17	3.34	1.9449	-	7.4865	5.5827	1.6707	122.55	5.64	268.93	3,659.16	60.49	3,497,223	109,944	18.45	17,414
AD	MaxSMR	15	20%	18	0.27	4.17	-	14.97	11.31	3.36	2.0825	-	7.4865	5.6562	1.6817	120.00	4.62	268.93	3,856.87	62.10	3,497,223	109,944	16.40	14,671
AD	MaxSMR	20	5%	18	0.39	3.73	-	19.98	24.82	4.98	1.8670	-	9.9877	12.4125	2.4912	131.51	1.33	268.93	3,841.43	61.98	4,153,773	180,606	112.69	70,020
AD	MaxSMR	20	10%	18	0.41	4.41	-	19.98	24.41	4.94	2.2062	-	9.9877	12.2052	2.4703	129.60	5.36	268.93	3,590.07	59.92	4,153,773	180,606	68.56	47,530
AD	MaxSMR	20	15%	18	0.41	4.78	-	19.98	22.61	4.75	2.3908	-	9.9877	11.3047	2.3775	125.37	4.62	268.93	3,508.53	59.23	4,153,773	180,606	38.94	31,526
AD	MaxSMR	20	20%	18	0.41	5.56	-	19.98	21.87	4.68	2.7815	-	9.9877	10.9343	2.3382	125.38	2.06	268.93	3,616.82	60.14	4,153,773	180,606	35.41	32,078
AD	MaxSMR	25	5%	18	0.51	4.34	-	25.00	36.49	6.04	2.1718	-	12.4987	18.2428	3.0202	132.76	1.33	268.93	3,780.15	61.48	4,591,473	261,646	397.81	226,209
AD	MaxSMR	25	10%	18	0.53	5.06	-	25.00	36.45	6.04	2.5323	-	12.4989	18.2233	3.0186	130.34	6.67	268.93	3,647.50	60.39	4,591,473	261,646	131.34	88,551
AD	MaxSMR	25	15%	18	0.54	5.64	-	25.00	34.43	5.87	2.8208	-	12.4989	17.2150	2.9339	128.29	6.67	268.93	3,409.87	58.39	4,591,473	261,646	125.99	126,184
AD	MaxSMR	25	20%	18	0.54	6.37	-	25.00	32.71	5.72	3.1846	-	12.4989	16.3533	2.8595	124.73	6.67	268.93	3,489.47	59.07	4,591,473	261,646	70.89	55,711
AD	MaxSMR	55	5%	18	0.86	7.31	-	54.94	93.39	9.66	3.6572	-	27.4689	46.6946	4.8319	119.61	1.33	268.93	4,327.59	65.78	5,424,562	955,370	3,508.43	1,716,869
AD	MaxSMR	55	10%	18	0.89	8.03	-	54.89	92.80	9.63	4.0150	-	27.4475	46.3975	4.8165	127.93	3.30	268.93	3,851.50	62.06	5,424,562	955,370	10,571.10	5,202,766
AD	MaxSMR	55	15%	18	0.91	9.32	-	54.84	89.08	9.44	4.6618	-	27.4185	44.5390	4.7191	124.98	2.21	268.93	4,129.67	64.26	5,424,562	955,370	13,358.70	5,521,418
AD	MaxSMR	55	20%	18	0.95	9.06	-	54.99	113.00	10.63	4.5304	-	27.4925	56.5003	5.3151	116.09	5.03	268.93	4,154.32	64.45	5,424,562	955,370	2,120.69	944,227
AD	MaxS10kmT	10	5%	18	0.18	2.68	-	9.99	8.54	2.92	1.3423	-	4.9945	4.2680	1.4608	123.27	5.36	268.93	3,863.15	62.15	2,710,822	52,955	35.71	20,178
AD	MaxS10kmT	10	10%	18	0.19	3.90	-	9.99	8.60	2.93	1.9484	-	4.9945	4.3008	1.4664	127.81	4.62	268.93	3,582.63	59.86	2,710,822	52,955	19.99	13,369
AD	MaxS10kmT	10	15%	18	0.19	4.73	-	9.99	11.54	3.40	2.3670	-	4.9945	5.7685	1.6983	122.94	10.16	268.93	3,614.27	60.12	2,710,822	52,955	8.62	6,650
AD	MaxS10kmT	10	20%	18	0.20	5.02	-	9.99	9.96	3.16	2.5083	-	4.9945	4.9775	1.5776	121.71	3.30	268.93	3,901.04	62.46	2,710,822	52,955	8.35	6,081

Appendix 3.8 Characteristics of additional solutions obtained using aggregated data, costs constraints above the MSC and IAT = 18 inh. for the beneficiaries' standard service objective function.

Solution identifiers					Demographic indicators										Economic indicators				Additional service and equality indicators for the WI				
					Inhabitants				Population points			Distribution centers			Costs								
Data set	Objective function	Coverage radius	Cost constraint	IAT	Total	Covered	Covered	Walking	Total	Covered	Covered	Total	open DCs	open DCs	Total	Beneficiaries	WFP	KRC	High service threshold	WI in high service distances	WI in low service distances	Gini Index	Hood Index
		km	%		No.	No.	%	No.	No.	No.	%	No.	No.	%	KSh.	KSh.	KSh.	KSh.	km	%	%		
AD	Max S10kmT	15	5%	18	452,218	391,827	86.65%	66,278	5,426	2,397	44.18%	1,459	130	8.91%	38,349,365	4,276,527	31,312,418	2,760,420	10.0000	0.9442	0.0558	0.6022	0.4674
AD	Max S10kmT	15	10%	18	452,218	391,827	86.65%	66,278	5,426	2,397	44.18%	1,459	173	11.86%	40,208,278	4,890,103	31,644,693	3,673,482	10.0000	0.9442	0.0558	0.4660	0.3671
AD	Max S10kmT	15	15%	18	452,218	391,827	86.65%	66,278	5,426	2,397	44.18%	1,459	231	15.83%	42,088,678	5,335,672	31,847,952	4,905,054	10.0000	0.9442	0.0558	0.4441	0.3520
AD	Max S10kmT	15	20%	18	452,218	391,827	86.65%	66,278	5,426	2,397	44.18%	1,459	304	20.84%	43,962,125	5,576,163	31,930,826	6,455,136	10.0000	0.9442	0.0558	0.4238	0.3300
AD	Max S10kmT	20	5%	18	452,218	407,860	90.19%	69,132	5,426	2,847	52.47%	1,459	131	8.98%	38,746,008	4,899,822	31,064,532	2,781,654	10.0000	0.9052	0.0948	0.6608	0.5229
AD	Max S10kmT	20	10%	18	452,218	407,860	90.19%	69,132	5,426	2,847	52.47%	1,459	171	11.72%	40,564,347	5,538,754	31,394,580	3,631,014	10.0000	0.9052	0.0948	0.5819	0.4370
AD	Max S10kmT	20	15%	18	452,218	407,860	90.19%	69,132	5,426	2,847	52.47%	1,459	223	15.28%	42,235,281	5,886,631	31,613,468	4,735,182	10.0000	0.9052	0.0948	0.5038	0.3923
AD	Max S10kmT	20	20%	18	452,218	407,860	90.19%	69,132	5,426	2,847	52.47%	1,459	283	19.40%	44,292,938	6,622,579	31,661,137	6,009,222	10.0000	0.9052	0.0948	0.4473	0.3327
AD	Max S10kmT	25	5%	18	452,218	417,462	92.31%	70,857	5,426	3,147	58.00%	1,459	129	8.84%	38,818,792	5,373,539	30,706,067	2,739,186	10.0000	0.8831	0.1169	0.6939	0.5467
AD	Max S10kmT	25	10%	18	452,218	417,462	92.31%	70,857	5,426	3,147	58.00%	1,459	151	10.35%	40,576,237	6,303,701	31,066,202	3,206,334	10.0000	0.8831	0.1169	0.5745	0.4353
AD	Max S10kmT	25	15%	18	452,218	417,462	92.31%	70,857	5,426	3,147	58.00%	1,459	199	13.64%	42,506,849	6,879,690	31,401,593	4,225,566	10.0000	0.8831	0.1169	0.5030	0.3763
AD	Max S10kmT	25	20%	18	452,218	417,462	92.31%	70,857	5,426	3,147	58.00%	1,459	267	18.30%	44,358,129	7,093,525	31,595,127	5,669,478	10.0000	0.8831	0.1169	0.4890	0.3648
AD	Max S10kmT	55	5%	18	452,218	434,165	96.01%	73,887	5,426	3,718	68.52%	1,459	97	6.65%	38,950,913	7,099,513	29,791,702	2,059,698	10.0000	0.8412	0.1588	0.7320	0.5728
AD	Max S10kmT	55	10%	18	452,218	434,165	96.01%	73,887	5,426	3,718	68.52%	1,459	136	9.32%	40,805,044	7,720,424	30,196,796	2,887,824	10.0000	0.8469	0.1531	0.7279	0.5679
AD	Max S10kmT	55	15%	18	452,218	434,165	96.01%	73,887	5,426	3,718	68.52%	1,459	150	10.28%	42,649,826	9,069,911	30,394,815	3,185,100	10.0000	0.8469	0.1531	0.7297	0.5784
AD	Max S10kmT	55	20%	18	452,218	434,165	96.01%	73,887	5,426	3,718	68.52%	1,459	164	11.24%	44,403,039	10,131,691	30,788,972	3,482,376	10.0000	0.8469	0.1531	0.6722	0.5304
AD	Max S10kmT	55	-	18	452,218	434,165	96.01%	73,887	5,426	3,718	68.52%	1,459	307	21.04%	50,282,455	12,450,326	31,313,291	6,518,838	10.0000	0.8469	0.1531	0.5467	0.4095

NAD = non-aggregated data
IAT = inhabitants' aggregation threshold
WI = walking inhabitants
MSC = minimum stakeholder costs

S10kmT= service below the 10 km distance threshold

Appendix 3.8 (continuation) Characteristics of additional solutions obtained using aggregated data, costs constraints above the MSC and IAT = 18 inh. for the beneficiaries' standard service objective function.

Solution identifiers					Indicators for the variables (i, j) beneficiaries walking distance from pop. point to DCs						Indicators for the variables (i, j) beneficiaries walking time from pop. point to DCs					Indicators for the variables (0, j) road distance from DCs to Warehouse					Problem characteristics variables (i, j)		Problem characteristics algorithm effort	
Data set	Objective function	Coverage radius	Cost constraint	IAT	Mean absolute deviation	Average	Minimum	Maximum	Variance	Standard deviation	Average	Minimum	Maximum	Variance	Standard deviation	Average	Minimum	Maximum	Variance	Standard deviation	Total	Tuples	CPU time	CPLEX ticks
		km	%		km	km	km	km	km	km	hours	hours	hours	hours	hours	km	km	km	km	km	No.	No.	Seconds	No.
AD	Max S10kmT	15	5%	18	0.28	3.27	-	14.97	14.05	3.75	1.6339	-	7.4865	7.0250	1.8742	130.27	2.21	268.93	3,712.51	60.93	3,497,223	109,944	82.03	53,066
AD	Max S10kmT	15	10%	18	0.29	3.95	-	14.97	12.37	3.52	1.9736	-	7.4865	6.1830	1.7583	126.68	6.67	268.93	3,440.19	58.65	3,497,223	109,944	28.24	21,970
AD	Max S10kmT	15	15%	18	0.29	4.44	-	14.97	13.14	3.62	2.2203	-	7.4865	6.5684	1.8122	123.68	4.62	268.93	3,327.93	57.69	3,497,223	109,944	18.71	16,407
AD	Max S10kmT	15	20%	18	0.29	4.71	-	14.97	12.97	3.60	2.3535	-	7.4865	6.4850	1.8007	120.04	2.06	268.93	3,449.81	58.74	3,497,223	109,944	15.26	12,983
AD	Max S10kmT	20	5%	18	0.39	3.73	-	19.98	24.82	4.98	1.8670	-	9.9877	12.4125	2.4912	131.51	1.33	268.93	3,841.43	61.98	4,153,773	180,606	123.53	70,020
AD	Max S10kmT	20	10%	18	0.41	4.41	-	19.98	24.41	4.94	2.2062	-	9.9877	12.2052	2.4703	129.60	5.36	268.93	3,590.07	59.92	4,153,773	180,606	66.65	47,530
AD	Max S10kmT	20	15%	18	0.41	4.78	-	19.98	22.61	4.75	2.3908	-	9.9877	11.3047	2.3775	125.37	4.62	268.93	3,508.53	59.23	4,153,773	180,606	35.88	31,526
AD	Max S10kmT	20	20%	18	0.41	5.56	-	19.98	21.87	4.68	2.7815	-	9.9877	10.9343	2.3382	125.38	2.06	268.93	3,616.82	60.14	4,153,773	180,606	37.25	32,078
AD	Max S10kmT	25	5%	18	0.49	4.10	-	25.00	35.89	5.99	2.0490	-	12.4987	17.9454	2.9954	128.36	1.33	268.93	3,850.91	62.06	4,591,473	261,646	419.58	245,150
AD	Max S10kmT	25	10%	18	0.51	5.06	-	25.00	35.36	5.95	2.5308	-	12.4988	17.6816	2.9733	127.51	5.36	268.93	3,661.95	60.51	4,591,473	261,646	123.68	88,839
AD	Max S10kmT	25	15%	18	0.53	5.66	-	25.00	34.02	5.83	2.8291	-	12.4988	17.0090	2.9162	123.83	8.08	268.93	3,596.96	59.97	4,591,473	261,646	132.31	114,115
AD	Max S10kmT	25	20%	18	0.53	5.88	-	25.00	34.17	5.85	2.9398	-	12.4989	17.0851	2.9228	121.79	8.08	268.93	3,684.64	60.70	4,591,473	261,646	55.50	45,753
AD	Max S10kmT	55	5%	18	0.77	5.58	-	54.95	87.55	9.36	2.7922	-	27.4729	43.7732	4.6783	121.96	1.33	268.93	4,056.66	63.69	5,424,562	955,370	3,884.75	1,667,640
AD	Max S10kmT	55	10%	18	0.88	6.20	-	54.93	112.41	10.60	3.1005	-	27.4630	56.2055	5.3012	122.97	1.33	268.93	4,109.61	64.11	5,424,562	955,370	1,850.95	862,961
AD	Max S10kmT	55	15%	18	1.08	7.54	-	54.97	171.42	13.09	3.7708	-	27.4827	85.7093	6.5463	125.69	4.36	268.93	3,947.24	62.83	5,424,562	955,370	835.72	434,570
AD	Max S10kmT	55	20%	18	1.14	8.60	-	55.00	186.86	13.67	4.2981	-	27.4993	93.4277	6.8348	123.11	5.58	268.93	3,725.49	61.04	5,424,562	955,370	1,549.14	1,028,070
AD	Max S10kmT	55	-	18	1.19	10.90	-	55.00	184.21	13.57	5.4497	-	27.4993	92.1042	6.7862	121.62	2.06	268.93	3,603.65	60.03	5,424,562	955,370	282.92	158,894

Appendix 4. Characteristics of the solutions obtained using generated data and different objective functions.

Solution identifiers					Demographic indicators										Economic indicators				Additional service and equality indicators for the WI				
					Inhabitants				Population points			Distribution centers			Costs								
Data set	Objective function	Coverage radius	Cost constraint	ICT	Total	Covered	Covered	Walking	Total	Covered	Covered	Total	open DCs	open DCs	Total	Beneficiaries	WFP	KRC	High service threshold	WI in high service distances	WI in low service distances	Gini Index	Hood Index
		km	%		No.	No.	%	No.	No.	No.	%	No.	No.	%	KSh.	KSh.	KSh.	KSh.	km	%	%		
Generated	Min SC	55	-	1	452,218	452,218	100.00%	75,406	100	100	100.00%	50	28	56.00%	42,182,205	14,988,345	25,653,861	1,540,000	27.5000	0.9285	0.0715	0.3562	0.2778
Generated	Min AWD	55	-	1	452,218	452,218	100.00%	75,406	100	100	100.00%	50	38	76.00%	42,954,898	13,639,064	27,225,834	2,090,000	27.5000	0.9601	0.0399	0.3166	0.2479
Generated	Max AWD	55	-	1	452,218	452,218	100.00%	75,406	100	100	100.00%	50	45	90.00%	85,246,411	51,874,477	30,896,934	2,475,000	27.5000	0.0111	0.9889	0.0521	0.0398
Generated	Min MAD	55	-	1	452,218	452,218	100.00%	75,406	100	100	100.00%	50	38	76.00%	43,357,167	13,639,064	27,628,103	2,090,000	27.5000	0.9601	0.0399	0.3166	0.2479
Generated	Max SMR	55	-	1	452,218	452,218	100.00%	75,406	100	100	100.00%	50	35	70.00%	48,736,659	18,704,221	28,107,438	1,925,000	27.5000	0.9601	0.0399	0.3119	0.2377
Generated	Min VAR	55	-	1	452,218	452,218	100.00%	75,406	100	100	100.00%	50	42	84.00%	82,399,114	48,951,054	31,138,060	2,310,000	27.5000	0.0111	0.9889	0.0481	0.0343
Generated	Min VAR	55	10%	1	452,218	452,218	100.00%	75,406	100	100	100.00%	50	36	72.00%	45,248,200	14,921,399	28,346,801	1,980,000	27.5000	0.9535	0.0465	0.3147	0.2407

Appendix 4. (continuation) Characteristics of the solutions obtained using generated data and different objective functions.

Solution identifiers					Indicators for the variables (*i*, *j*)											Indicators for the variables (0, *j*)					Problem characteristics			
					beneficiaries walking distance from pop. point to DCs						beneficiaries walking time from pop. point to DCs					road distance from DCs to Warehouse					variables (*i*, *j*)		algorithm effort	
Data set	Objective function	Coverage radius	Cost constraint	ICT	Mean absolute deviation	Average	Minimum	Maximum	Variance	Standard deviation	Average	Minimum	Maximum	Variance	Standard deviation	Average	Minimum	Maximum	Variance	Standard deviation	Total	Tuples	CPU time	CPLEX ticks
		km	%		km	km	km	km	km	km	hours	hours	hours	hours	hours	km	km	km	km	km	No.	No.	Seconds	No.
Generated	Min SC	55	-	1	12.65	13.12	6.00	51.26	89.33	9.45	6.5603	3.0000	25.6320	44.6626	4.7256	101.06	6.00	164.75	1,396.15	37.37	5,000	516	0.0200	8.9
Generated	Min AWD	55	-	1	11.35	11.81	6.00	50.91	56.78	7.53	5.9037	3.0000	25.4558	28.3879	3.7675	109.90	6.00	164.75	1,435.57	37.89	5,000	516	0.0300	16.5
Generated	Max AWD	55	-	1	45.85	49.02	16.97	54.33	25.48	5.05	24.5114	8.4853	27.1662	12.7411	2.5240	104.33	6.00	203.65	1,529.72	39.11	5,000	516	3.0700	1,960
Generated	Min MAD	55	-	1	11.35	11.81	6.00	50.91	56.78	7.53	5.9037	3.0000	25.4558	28.3879	3.7675	109.90	6.00	164.75	1,435.57	37.89	5,000	516	3.1500	1,924
Generated	Max SMR	55	-	1	16.08	16.74	6.00	54.33	94.91	9.74	8.3687	3.0000	27.1662	47.4564	4.8712	105.00	6.00	162.11	1,416.13	37.63	5,000	516	3.5200	1,914
Generated	Min VAR	55	-	1	3.17	46.18	16.97	54.33	19.39	4.40	23.0887	8.4853	27.1662	9.6938	2.2016	105.84	6.00	203.65	1,605.26	40.07	5,000	516	1,412,374	240,894,739
Generated	Min VAR	55	10%	1	11.35	13.06	6.00	50.91	62.56	7.91	6.5277	3.0000	25.4558	31.2820	3.9549	103.41	6.00	164.75	1,613.36	40.17	5,000	516	1,417,542	882,660,000

ICT = inhabitants' coverage threshold
WI = walking inhabitants

SC = stakeholder costs
AWD = beneficiaries' average walking distance
MAD = beneficiaries' mean absolute deviation of walking distance
SMR = service below the mid-radius distance threshold
VAR = beneficiaries' variance of walking distance

BIBLIOGRAFY

Afshar, A., & Haghani, A. (2012). Modeling integrated supply chain logistics in real-time large-scale disaster relief operations. *Socio-Economic Planning Sciences, 46*(4), 327-338.

Atkinson, A. B. (1970). On the measurement of inequality. *Journal of Economic Theory, 2*(3), 244-263.

Balcik, & Beamon, B. (2008). Facility location in humanitarian relief. *International Journal of Logistics Research and Applications, 11*(2), 101-121.

Balcik, B., Beamon, B. M., Krejci, C. C., Muramatsu, K. M., & Ramirez, M. (2010). Coordination in humanitarian relief chains: Practices, challenges and opportunities. *International Journal of Production Economics, 126*(1), 22-34.

Battersby, J. (2013). Hungry Cities: A Critical Review of Urban Food Security Research in Sub-Saharan African Cities. *Geography Compass, 7*(7), 452-463.

Bellu, L., & Liberati, P. (2006). Policy Impacts on Inequality, The Atkinson Index. *Easypol, Food and Agriculture Organization of the United Nations.* Retrieved 18/4/2014, from http://www.fao.org/docs/up/easypol/451/welfare_measures_inequa_atkinson_050en.pdf

Bellu, L., & Liberati, P. (2014). Impact of Policies on Poverty, Generalized Poverty Gap Measures. *Easypol, Food and Agriculture Organization of the United Nations* Retrieved 12/5/2014, from http://www.fao.org/docs/up/easypol/324/gnrlsed_pvty_gap_msrs_010en.pdf

Berkoune, D., Renaud, J., Rekik, M., & Ruiz, A. (2012). Transportation in disaster response operations. *Socio-Economic Planning Sciences, 46*(1), 23-32.

Berman, O., Drezner, E., & Krass, D. (2009a). Cooperative Location Problems: The planar case. *IIE Transactions, 42*(3), 232-246.

Berman, O., Drezner, Z., & Krass, D. (2010). Discrete cooperative covering problems. *Journal of the Operational Research Society, 62*(11), 2002-2012.

Berman, O., Krass, D., & Menezes, M. B. C. (2007). Facility Reliability Issues in Network p-Median Problems: Strategic Centralization and Co-Location Effects. *Operations Research, 55*(2), 332-350.

Berman, O., Krass, D., & Menezes, M. B. C. (2009b). Locating Facilities in the Presence of Disruptions and Incomplete Information*. *Decision Sciences, 40*(4), 845-868.

Berry, J., Hart, W. E., Phillips, C. A., Uber, J. G., & Watson, J.-P. (2006). Sensor Placement in Municipal Water Networks with Temporal Integer Programming Models. *Journal of Water Resources Planning & Management, 132*(4), 218-224.

Bosmans, K., Lauwers, L., & Ooghe, E. (2009). A consistent multidimensional Pigou–Dalton transfer principle. *Journal of Economic Theory, 144*(3), 1358-1371.

Brown, M. C. (1994). Using Gini-style indices to evaluate the spatial patterns of health practitioners. *Social Science Medicine, 38*(9), 1243-1256.

Campbell, A. M., & Jones, P. C. (2011). Prepositioning supplies in preparation for disasters. *European Journal of Operational Research, 209*(2), 156-165.

Carr, S., & Roberts, S. (2010, 5-8 Dec. 2010). *Planning for infectious disease outbreaks: A geographic disease spread, clinic location, and resource allocation simulation.*

Paper presented at the Simulation Conference (WSC), Proceedings of the 2010 Winter.
CASSE. (2014). Economic Grow FAQs. Retrieved 25/04/2014, from http://steadystate.org/discover/faqs/
Çelik, M., Ergun, Ö., Johnson, B., Keskinocak, P., Lorca, Á., Pekgün, P., & Swann, J. (2012). Humanitarian Logistics, New Directions in Informatics, Optimization, Logistics, and Production. *Hanover*, 18-49.
Chang, M.-S., Tseng, Y.-L., & Chen, J.-W. (2007). A scenario planning approach for the flood emergency logistics preparation problem under uncertainty. *Transportation Research Part E: Logistics and Transportation Review, 43*(6), 737-754.
Chotikapanich, D., & Griffiths, W. (2000). On calculation of the extended Gini Coefficient. *University of New England, School of Economic Studies*, 1-15.
Church, R. L. S. M. P. M. R. S. (2004). Identifying Critical Infrastructure: The Median and Covering Facility Interdiction Problems. *Annals of the Association of American Geographers, 94*(3), 491-502.
Cowell, F. A. (2009). Measuring Inequality. Retrieved 16/04/2014, from http://darp.lse.ac.uk/papersdb/Cowell_measuringinequality3.pdf
Current, J., & O'Kelly, M. (1992). Locating Emergency Warning Sirens. *Decision Sciences, 23*(1), 221.
Dalton, H. (1920). The Measurement of the Inequality of Incomes. *The Economic Journal, 30*(119), 348-361.
Daly, H. E. (1990). TOWARD SOME OPERATIONAL PRINCIPLES OF SUSTAINABLE DEVELOPMENT. *Ecological Economics, Elsevier Science Publishers, 2*, 1-6.
Daskin, M. S. (2008). What you should know about location modeling. *Naval Research Logistics (NRL)*(January), 283-294.
Dekle, J. M. S. M. E.-F. H. R. L. (2005). A Florida County Locates Disaster Recovery Centers. *Interfaces, 35*(2), 133-139.
Disasterium. (2014). Natural and Man Made disasters. Retrieved 29/05/2014, from http://www.disasterium.com/
Doane, D. P., & Seward, L. E. (2011). Measuring Skewness : A Forgotten Statistic? *Journal of Statistics Education, 19*(2), 1-18.
Dong, T., & IBM. (2009). Efficient modeling with the IBM ILO OPL-CPLEX Development Bundles.
Döyen, A., Aras, N., & Barbarosoğlu, G. (2012). A two-echelon stochastic facility location model for humanitarian relief logistics. *Optimization Letters, Springer, 6*(6), 1123-1145.
Drezner, E., & Hamacher, H. W. (2004). *Facility Location, Application an Theory* (1 ed.): Springer-Verlag.
Drezner, Z. (1995). *Facility Location, A Survey of Application and Methods*. New York, NY: Springer New York.
Duran, S., Gutierrez, M. A., & Keskinocak, P. (2011). Pre-Positioning of Emergency Items for CARE International. *Interfaces, 41*(3), 223-237.
Ekici, A., Keskinocak, P., & Swann, J. L. (2009). Modelling Influenza Pandemic, Intervention Strategies, and Food Distribution *Management Science Manuscript* (pp. 1-32): Georgia Institute of Technology.
Etzkorn, B. (2012). Data Standardization, Thoughts. Retrieved 20/5/2014, 2014, from http://www.benetzkorn.com/2011/11/data-normalization-and-standardization/
FAO. (2010). Food Insecurity in the Horn of Africa. Retrieved 28/05/2014, from http://www.fao.org/fileadmin/templates/tc/tce/pdf/eastafrica_foodinsecurityinhornofafrica.pdf

FAO. (2013). Crop Prospects and Food situation. Retrieved 29/05/2014, from http://www.fao.org/docrep/017/al998e/al998e.pdf

FAO. (2014). Food and Agriculture Organization of the United Nations. Retrieved 15/5/2014, from http://www.fao.org/publications/e-book-collection/en/

Farahani, R. Z., Asgari, N., Heidari, N., Hosseininia, M., & Goh, M. (2012). Covering problems in facility location: A review. *Computers & Industrial Engineering, 62*(1), 368-407.

Fetter, G., & Rakes, T. (2012). Incorporating recycling into post-disaster debris disposal. *Socio-Economic Planning Sciences, 46*(1), 14-22.

Fishburn, P. C., & Willig, R. D. (1984). Transfer principles in income redistribution. *Journal of Public Economics, 25*(3), 323-328.

Foster, J. E., & Shneyerov, A. A. (2000). Path Independent Inequality Measures. *Journal of Economic Theory, 91*(2), 199-222.

Gallaher, C. M., Kerr, J. M., Njenga, M., Karanja, N. K., & WinklerPrins, A. M. G. A. (2013). Urban agriculture, social capital, and food security in the Kibera slums of Nairobi, Kenya. *Agriculture and Human Values, 30*(3), 389-404.

GK. (2014). Kenya Open Data Survey 2014, Goverment of Kenya. Retrieved 09/06/2014, from https://www.opendata.go.ke/Population/2009-Census-Vol-1-Table-3-Rural-and-Urban-Populati/e7c7-w67t/1

Gonzalez Abril, L., & Morente, V. (2010). The Similarity between the Square of the Coefficient of Variation and the Gini Index of a General Random Variable. *Revista de Metodos Cuantitativos para la Economia y la Empresa*(10), 5-18.

Hale, T. (2014). The Theoretical Basics of Popular Inequality Measures. *University of Texas Inequality Project.* Retrieved 25/05/2014, from http://utip.gov.utexas.edu/

Hale, T., & Moberg, C. R. (2005). Improving supply chain disaster preparedness: A decision process for secure site location. *International Journal of Physical Distribution & Logistics Management, 35*(3), 195-207.

Hall, R. (2003). *Handbook of Transportation Science* (2 ed.). New York.

Handler, G. Y. (1985). Medi-Centers of a Tree. *Transportation Science, 19*(3), 246.

Harvard_Magazine. (2014). Measuring Poverty. Retrieved 12/5/2014, from harvardmagazine.com/2011/01/who-is-poor

Huang, M., Smilowitz, K., & Balcik, B. (2012). Models for relief routing: Equity, efficiency and efficacy. *Transportation Research Part E: Logistics and Transportation Review, 48*(1), 2-18.

IBM. (2009). IBM ILOG OPL V6.3 IBM ILOG OPL Language Reference Manual. 264.

IRIN. (2008). West Africa: Do high food prices warrant a cash response? Retrieved 07/03/2014, from http://www.irinnews.org/report/80154/west-africa-do-high-food-prices-warrant-a-cash-response

Kakwani, N., & Lambert, P. J. (1998). On measuring inequity in taxation: a new approach. *European Journal of Political Economy, 14*(2), 369-380.

Kalfakakou, G. (2005). Minimum cost facility allocation with an equity restriction. *Journal of Management Sciences and Regional Development*(5), 247-261.

Kovács, G., & Spens, K. M. (2007). Humanitarian logistics in disaster relief operations. *International Journal of Physical Distribution & Logistics Management, 37*(2), 99-114.

Kovács, G., & Spens, K. M. (2012). *Relief Supply Chain Management for Disasters: Humanitarian Aid and Emergency Logistics*: Business Science Reference.

Lambert, P. J., & Yitzhaki, S. (1995). Equity, equality and Welfare. *European Economic Review, 39*, 674-682.

Lee, S.-D. (2001). On solving unreliable planar location problems. *Computers & Operations Research, 28*(4), 329-344.

Lerman, R. I., & Yitzhaki, S. (1984). A note on the calculation and interpretation of the Gini index. *Economics Letters, 15*(3-4), 363-368.

Li, L., Jin, M., & Zhang, L. (2011). Sheltering network planning and management with a case in the Gulf Coast region. *International Journal of Production Economics, 131*(2), 431-440.

Malvankar-Mehta, M. S., & Xie, B. (2012). Optimal incentives for allocating HIV/AIDS prevention resources among multiple populations. *Health Care Manag Sci, 15*(4), 327-338.

Mandell, M. B. (1991). MODELLING EFFECTIVENESS-EQUITY TRADE-OFFS IN PUBLIC SERVICE DELIVERY SYSTEMS. *Management Science, 37*(4), 467-482.

Maxwell, D., Sadler, K., Sim, A., Mutonyi, M., Egan, R., & Webster, M. (2008). Good Practice Review Emergency food security interventions. *Humanitarian Practice Network, 44*(10), 146.

McAllister, D. M. (1977). Equity and Efficiency in Public Facility Location. *Geographical Analysis, 8*, 47-63.

McCall, V. M. (2006). *DESIGNING AND PREPOSITIONING HUMANITARIAN ASSISTANCE PACK-UP KITS (HA PUKs) TO SUPPORT PACIFIC FLEET EMERGENCY RELIEF OPERATIONS,* Naval Postgraduate School.

Medina, F. (2014). Consideraciones sobre el índice de gini para medir la concentración del ingreso. *Economic Commission for Latin America and the Caribbean (ECLAC) United Nations.* march, 2001. Retrieved 12/05/2014, from http://www.eclac.org/publicaciones/xml/0/6570/lcl1493e.pdf

Mete, H. O., & Zabinsky, Z. B. (2010). Stochastic optimization of medical supply location and distribution in disaster management. *International Journal of Production Economics, 126*(1), 76-84.

Morrill, R. L., & Symons, J. (1977). Efficiency and Equity Aspects of Optimum Location. *Geographical Analysis, 9*(3), 215-225.

Murray, A. T., O'Kelly, M. E., & Church, R. L. (2008). Regional service coverage modeling. *Computers & Operations Research, 35*(2), 339-355.

Murray, A. T., & Tong, D. (2007). Coverage optimization in continuous space facility siting. *International Journal of Geographical Information Science, 21*(7), 757-776.

Murray, C., Xu, K., Klavus, J., & WKawabata, K. (2014). Assessing the Distribution of Household Financial Contributions to the Health System: Concepts and Empirical Application (chapter 38). Retrieved 12/5, 2014, from http://www.who.int/health_financing/documents/hspa_chp38-hh_finan_contrib.pdf

Nolz, P. C., Semet, F., & Doerner, K. F. (2011). Risk Approaches for Delivering Disaster Relief Supplies. *OR Spectrum, 33*(3), 543-569.

Noyan, N. (2012). Risk-averse two-stage stochastic programming with an application to disaster management. *Computers & Operations Research, 39*(3), 541-559.

Nurre, S. G., Cavdaroglu, B., Mitchell, J. E., Sharkey, T. C., & Wallace, W. A. (2012). Restoring infrastructure systems: An integrated network design and scheduling (INDS) problem. *European Journal of Operational Research, 223*(3), 794-806.

O'Hanley, J. R., & Church, R. L. (2011). Designing robust coverage networks to hedge against worst-case facility losses. *European Journal of Operational Research, 209*(1), 23-36.

Owen, S. H., & Daskin, M. S. (1998). Strategic facility location: A review. *European Journal of Operational Research, 111*, 423-447.

OXFAM. (2014). Cash-Transfer Programming in Emergencies. Retrieved 05/02/2014, from OXFAM www.sdc-cashprojects.ch/document.php?itemID=925&langID=1

Pardalos, P., & Du, D.-Z. (2008). Pareto Optimality, Game Theory and Equilibria. *Springer Optimization and Its Applications, 17*, 868.

Rahman, S.-u., & Smith, D. K. (2000). Use of location-allocation models in health service development planning in developing nations. *Europen journal of Operational Research, 123*, 437-452.

Rancourt, M.-È., Cordeau, J. F., Laporte, G. & Watkins, B. (2015). Tactical Network Planning for Food Aid Distribution in Kenya. *Computer & Operations Research*, *56*, 68-83.

Rancourt, M.-È., Bellavance, F. & Goentzel, J. (2014). Market Analysis and Transportation Procurement for Food Aid in Ethiopia. *Socio-Economic Planning Sciences, 48*(3), 198-219.

Rawls, C. G., & Turnquist, M. A. (2012). Pre-positioning and dynamic delivery planning for short-term response following a natural disaster. *Socio-Economic Planning Sciences, 46*(1), 46-54.

ReVelle, C. S., & Eiselt, H. A. (2005). Location analysis: A synthesis and survey. *European Journal of Operational Research, 165*(1), 1-19.

Rohde, N. (2007). Derivation of Theil's Inequality Measure from Lorenz Curves. *Schools of Economics, The University of Queensland, Brisbane , Australia*, 1-19.

Sage. (1982). On the location of Industries, by Alfred Weber. *Gae Journals, Progress in Human Geography.* Retrieved 21/02/2014, from http://www.sagepublications.com

Salmerón, J., & Apte, A. (2010). Stochastic Optimization for Natural Disaster Asset Prepositioning. *Production & Operations Management, 19*(5), 561-574.

Sanchez-Perez, J., Plata-Perez, L., & Sanchez, F. (2012). An elementary characterization of the Gini Index. *Munich Personal RepEc Archive.* Retrieved 12/04/2014, from http://mpra.ub.uni-muenchen.de/36328/1/MPRA_paper_36328.pdf

Schechtman, E., Sheva, B., & Yitshaki, S. (1985). A Family of Correlation Coefficients Based on Extended Gini. *Joint Statistical Meetings-Business & Economic Statistics Section*, 3083-3088.

Sen, A. (1997). *On Economic Inequality* (O. University Ed. 1 ed.): Clarendon Press, Oxford.

Shen, Z.-J., Lezhou Zhan, R., & Zhang, J. (2011). The reliable facility location problem: formulations, heuristics, and approximation algorithms. *Informs Journal on computing, 23*(3), 470.

Sherali, H. D., Carter, T. B., & Hobeika, A. G. (1991). A Location-Allocation Model and Algorithm for evacuation planning under Hurricane/Flood conditions. *Pergamon Press plc, 25B*(6), 439-452.

Softkenya. (2014). All About Kenya, Garissa. Retrieved 09/06/2014, from http://softkenya.com/garissa-county

Stiglitz, J. E., Sen, A., & Fitoussi, J.-P. (2001). The Measurement of Economic Performance and Social Progress Revisited, Relections and Overview. Retrieved 4/21/2014, from http://stiglitz-sen-fitoussi.fr/documents/overview-eng.pdf

ThinkEatSave_Organization. (2013). Ministers Dine on Food Grown in Kenya but Rejected by UK Supermarkets for Cosmetics Reasons. Retrieved 05/03/2014, from http://www.thinkeatsave.org/index.php/ministers-dine-on-food-grown-in-kenya-but-rejected-by-uk-supermarkets-for-cosmetic-reasons

Thomson_Reuters_Foundation. (2013). Cutting Food Waste Crucial to Ensuring Food Security. Retrieved 06/12/2014, from http://www.trust.org/item/?map=cutting-food-waste-crucial-to-ensuring-food-security-experts-say

Tzeng, G.-H., Cheng, H.-J., & Huang, T. D. (2007). Multi-objective optimal planning for designing relief delivery systems. *Transportation Research Part E: Logistics and Transportation Review, 43*(6), 673-686.

Watson, M., & Cacioppi, P. (2014). *A Deep Dive Into Strategic Network Design Programming* Amazon (Ed.) (pp. 1386).

Wayne, K., Kleidenberd, J., & Tardos, E. (2013). 2 . Algorithm Analysis. Retrieved 12/4/2014, from http://www.cs.princeton.edu/~wayne/kleinberg-tardos/pdf/02AlgorithmAnalysis-2x2.pdf

Weiszfeld, E. (1936). Sur le point pour lequel la somme des distances de n points donnes est minimum. *Tohokku Mathematics Journal, 43*, 355-386.

Weiszfeld, E., & Plastria, F. (2009). On the point for which the sum of the distances to n given points is minimum. *Annals of Operations Research, 167*(1), 7-41.

WFP. (2014a). World Food Programme , Kenya. Retrieved 5/9/2014, from http://www.wfp.org/countries/kenya/food-security

WFP. (2014b). World Food Programme, Fighting Hunger Worldwide, Kenya overview. Retrieved 14/5/2014, from http://www.wfp.org/countries/kenya/overview

WFP. (2014c). World Food Programme, Food Security Analysis. Retrieved 4/12/2014, from http://www.wfp.org/food-security

World Bank. (2014). Kenya, Country at a Glance. Retrieved 10/04/2014, from www.worldbank.org/en/country/kenya

Xu, J., Johnson, M. P., Fischbeck, P. S., Small, M. J., & VanBriesen, J. M. (2010). Robust placement of sensors in dynamic water distribution systems. *European Journal of Operational Research, 202*(3), 707-716.

Yan, S., & Shih, Y.-L. (2009). Optimal scheduling of emergency roadway repair and subsequent relief distribution. *Computers & Operations Research, 36*(6), 2049-2065.

Yi, W., & Özdamar, L. (2007). A dynamic logistics coordination model for evacuation and support in disaster response activities. *European Journal of Operational Research, 179*(3), 1177-1193.

Yushimito, W. F., & Ukkusuri, S. V. (2007). Location Routing Approach for the Humanitarian Prepositioning Problem. *Transportation Research Record*, 1-23.