The One-Warehouse Multi-Retailer Problem with an Emission Constraint: Formulations, Analyses and Heuristics

by

Qihua Zhong

A Thesis Submitted in Partial Fulfilment
of the Requirements for the Degree of
Master of Science

in

HEC Montréal

November 2014

HEC MONTRĒAL

Retrait d’une ou des pages pouvant contenir des renseignements personnels

Acknowledgements

I would like to express my deepest gratitude to my supervisors of this thesis.

My sincere thanks to Ola Jabali, who is considerate enough to understand my doubts and answer patiently even when I don’t speak complete sentences in English.

My cordial thanks to Raf Jans, who is always enthusiastic about new ideas and encourages me to explore more.

Every meeting with them has been my source of inspiration and motivation. Without their continuous encouragement and patient guidance, this work would not have been made possible.

Abstract

To address the environmental issues in supply chain management at the operational level, this thesis considers the One-Warehouse Multi-Retailer problem with a global Emission Constraint, which we denote as OWMR-EC. We validate three known formulations for the classical One-Warehouse Multi-Retailer (OWMR) problem. We propose formulations for the OWMR-EC problem by incorporating a global emission constraint. Computational results are presented for both OWMR and OWMR-EC. Analyses are given to the maximum potential emission reduction and to the trade-off between costs and emissions. Experiments with different levels of emission cap show piecewise convex trade-off curves between costs and emissions, which indicate that the marginal cost of emission reduction tends to increase as reduced amount increases. We also discuss several heuristics for the OWMR problem, among which the best heuristic is chosen and integrated into an iterative penalized relaxation method to search for feasible solutions for the OWMR-EC problem. Computational results for all heuristics discussed are presented. The final heuristic that we propose for the OWMR-EC problem consumes less than one second on average and gives an average gap of 0.53%. However, it is not always able to find feasible solutions, especially for the highly constrained problems.

Table of Contents

List of Figures

List of Tables

1. Introduction

It was predicted by Shrivastava (1995) that the natural environment would be an important battle field for economic competition in the 21st century. Shrivastava believes that companies could gain competitive advantages through environmental technology improvements. This statement has been supported later on by the fact that more and more firms are explicitly addressing environmental issues as part of their corporate social responsibility.

Among the environmental issues, common concerns are given to global warming and Greenhouse Gas (GHG) emissions. Voiland (2009) from NASA's Earth Science News Team reported that a clear trend of temperature increase has been observed since modern human civilization: the average global temperature has increased by about 0.8 ℃ since 1880. The American Association for the Advancement of Science (2009), on behalf of 18 scientific associations, expressed the consensus view that such a temperature increase is due to the GHG emissions caused by human activities.

As an international effort to reduce GHG emissions, most industrialized countries voluntarily participate in the Kyoto Protocol. The Kyoto Protocol issued by the United Nations (1998) set an GHG emission reduction goal for the first commitment period from 2008 to 2012: participating parties committed to reduce the GHG emission amount by 5% against the 1990 level. Later in 2012, the Doha Amendment by the United Nations (2012) extended the protocol to 2020. The conference in Doha also agreed that the Kyoto Protocol will be replaced by a new treaty by 2015, which will involve both developed and developing countries. In order to achieve the emission reduction goal, many countries have their emission projects implemented or scheduled. According to a report by the World Bank (2014), most European countries have implemented emission trading schemes while some other countries such as Sweden, Mexico, South Africa and Japan have implemented carbon tax systems. The price for emitting one ton of carbon-dioxide (CO_2) differs greatly between different countries. The World Bank (2014) reported that the price ranges from as low as 1 US$/ton in Mexico and New Zealand to as high as 168 US$/ton in Sweden.

As more countries are participating in the Kyoto Protocol and carbon pricing systems are being more widely implemented, it can be foreseen that GHG emission reduction will be of interest to more and more enterprises. For example, in its corporate sustainability report, UPS (2014) states the plan to reduce its carbon emission intensity from transportation by 20 percent by 2020 compared to the baseline of 2007. Such a goal will be achieved through redesigns of the logistics network and upgrades of its fleets with vehicles that consume Liquefied Natural Gas (LNG).

Conventionally, network redesign, advanced technology and alternative clean energy are effective approaches to improve environmental performance. However, these measures usually require a great amount of initial capital investment. Some researchers indicate that a better environmental performance can also be achieved by better planning at the production and distribution stages. This issue has also become an emerging topic in the lot sizing literature.

Solutions to lot sizing problems usually try to minimize the economic cost while the carbon emission factors are ignored. Like economic costs, carbon emissions can be incurred at various activities. Carbon is emitted while the products are produced in the factories, transported to the service outlets or kept as inventory in warehouses. If we take into account these emission factors in lot sizing problems, it is possible to better balance emissions and costs. Costs will generally increase when we address the emission factors in lot sizing problems. However, the benefit of such an approach to reduce emission is that it does not require a major change in the supply chain network or a large amount of capital investment, giving companies more flexibility towards their emission reduction targets.

To explore the possibilities of emission reduction through production and transportation planning, we study a lot sizing problem in a two-level supply chain with a distribution structure, otherwise known as a One-Warehouse Multi-Retailer (OWMR) lot sizing problem and extend it to an emission-constrained context. We propose several formulations based on known formulations for the standard OWMR problem and test how they perform. CPLEX 12.6.0.1 is called to solve the problems using a JAVA coding environment. Data sets of instances from a previous study by Solyalı and Süral (2012) are used as instances input. Due to the lack of available data sets for the problem with emission parameters, we adapt the standard OWMR instances. Computational tests are presented, both for the standard OWMR problem and for the OWMR with an Emission Constraint (OWMR-EC)

Compared to the vast literature base for lot sizing problems, studies focusing on the impact of emission constraints on lot sizing models are relatively sparse. Within the studies on lot sizing problems with emission constraints, most of them deal with classical and general models, such as EOQ, newsvendor problem and single level lot sizing. Very little attention is given to more complicated lot sizing models and very few studies examine heuristic methods for the lot sizing models with emission constraints. Therefore, this thesis will supplement the existing literature by studying the specific OWMR-EC problem.

The contribution of this thesis is fourfold. (1) We validate three known formulations of the OWMR problem using a standard data set and compare our computational results to those of a previous study. (2) We propose formulations for the OWMR-EC problem and, compare them in a computational experiment using an adapted standard data set. (3) Results from the experiments allow us to analyze the trade-off between costs and carbon emissions and to provide managerial insights. (4) We develop and compare different heuristic methods to expedite the computation process for both the OWMR problem and the OWMR-EC problem.

The rest of this thesis is organized as follows. Section 2 reviews existing literature. Section 3 presents and validates three known formulations for the OWMR problem and compare the computational results to a previous study. In section 4, we propose formulations for OWMR-EC problem and present computational results. Section 5 analyzes the trade-off between costs and emissions in the OWMR-EC problem and provides managerial insights. Section 6 proposes several heuristic methods and compares their quality and speed. Section 7 concludes the thesis.

2. Literature review

In this section, we first generally review supply chain decision models that take into account carbon emission factors and discuss different types of emission constraints implied by various studies. Next, we provide a brief overview of the general literature on lot sizing. We then focus on lot sizing with an emission constraint, and discuss the modeling of different policies and criteria.

2.1 Impact of Emissions in Supply Chain Management

Recently there is a growing concern about the environmental impact in supply chain management. Researchers model the impact of carbon emission and examine how it can possibly affect decision making in supply chain management. Emissions can have effects on decisions making from the strategic level like network design, to the operational level like production planning.

At the strategic level, Cachon (2014) considers a retail store density problem and uses the model to decide the size, location and number of retailer stores with consideration of carbon emissions. His model extends the emission responsibility down to consumer's fuel consumption. He assumes that retailers' trucks carry goods more efficiently than consumers' cars. As a denser retailer network reduces the distance traveled by consumers, it results in less carbon emissions from transportation. On the other hand, a dense retailer network incurs higher operating cost and higher emissions from electricity consumption at retail space. Analyses are given on the potential of emission reduction in the long term network redesign.

At the tactical level, Hoen et al. (2014) propose a transportation mode selection model that considers emission costs among other cost such as inventory and transport. Their model selects a transportation mode for an organization which produces a single product and faces stochastic demand in a single period. Exactly one mode out of air, road, rail, and water will be selected by the model. They also conduct a numerical study under three types of carbon emission regulations: (1) a carbon tax, (2) emission trading scheme, and (3) a strict constraint on emissions. The study reveals that a large amount of carbon emissions can be reduced by switching to a different transportation mode and the decision is largely dependent on the type of carbon emission regulation chosen. Another study by Hoen et al. (2013) investigates a different transportation mode selection problem under a self-imposed emission target. The producer decides the transportation mode and product price, which will affect the demand. The scope of the problem is extended to multiple products and multiple customers. They conclude that when the emission reduction target is relatively small (up to 20%), switching transportation modes can be an effective measure to reduce carbon emission.

Demir, Bektaş, and Laporte (2014) review vehicle routing models with fuel consumption components. They first focus on methods to estimate carbon-dioxide emissions in vehicle transportation and identify 13 types of macroscopic models and 12 types of microscopic models. They then categorize different vehicle routing and scheduling models according to the emission estimation methods used. These models consider decisions at both tactical and operational levels. Some of the studies also conduct numerical experiments and report the potential emission reduction amount by minimizing emission instead of transportation time.

At the operational level, Jaber, Glock, and El Saadany (2013) develop a mathematical model for a two-level supply chain with a serial structure in which a carbon emission tax and emission penalties are imposed. This model takes into account the emission amount as a function of the production rate (i.e. the number of units produced in a given time). Decisions on the production rate are made based on setup cost, holding cost and emission cost together. Song and Leng (2012) analyze the newsvendor problem with stochastic demand and a perishable item under different types of carbon emission constraints and examine the impact on the optimal quantity decision. Zhang and Xu (2013) extend the newsvendor problem with cap-and-trade mechanism to a multi-item scenario. Arıkan and Jammernegg (2014) study the newsvendor problem under dual sourcing options, which allow the vendor to fulfill excessive orders at short notice but with higher cost and higher emissions. Their numerical study shows that significant emission reduction can possibly be achieved with a slight increase in economic cost.

All these models indicate that emissions can be an important factor in decision making at all levels of the supply chain. However, emission factors can be incorporated in the models in different ways. Most of the vehicle routing models reviewed by Demir et al. (2014) consider emissions as an objective and compare the results to the situation where time or cost is the objective. Arıkan and Jammernegg (2014) also consider minimizing emissions as an objective. Other models introduced in this section consider emissions as a constraint to the problem where penalties are imposed on the carbon emissions. The Congressional Budget Office of the Congress of the United States (2008) proposes four policy options for limiting carbon-dioxide emission: (1) Mandatory carbon emission capacity: the emission amount of the firm cannot exceed an strict emission cap; (2) Carbon emission tax: a tax is imposed on the firm for each unit of carbon emitted; (3) Cap-and-trade system: there is a carbon emission cap, but firms are allowed to buy or sell emission credits; (4) Investment in the carbon offsets: there is a carbon cap, however if the emission exceeds the cap, firms are allowed to invest in carbon reduction projects to offset their own emission.

Song and Leng (2012) suggest that policy (4) is essentially the same as policy (3) when the selling price of excessive emission credits is 0. Therefore policy (3) and (4) can be considered as one type of mechanism. Following this categorization, Table 1 summarizes the types of emission mechanism analyzed by different studies.

	Strict Cap	Carbon Tax	Cap-and-Trade
Cachon (2014)		√	
Hoen et al. (2014)	√	√	√
Hoen et al. (2013)	√		
Jaber et al. (2013)		√	
Song and Leng (2012)	√	√	√
Zhang and Xu (2013)			√

Table 1. Types of Emission Mechanism Analyzed

For a more comprehensive review on the impacts of environmental issues at different decision phases in the supply chain, see Dekker, Bloemhof, and Mallidis (2012). They systematically discuss environmental factors in the design, planning and control in a supply chain for transportation, inventory of products and facility decisions. The rest of the literature review focuses on a specific domain in supply chain management: the lot sizing problems.

2.2 Lot Sizing

Lot sizing in general deals with the problem of when to order (or produce) and how much to order. The models aim to minimize the total cost by balancing the trade-off between various costs while satisfying demands. There are a lot of different variations in the domain of lot sizing. We will briefly review the lot sizing problem and its extensions. We do not intend to give a comprehensive review of lot sizing problems but we discuss the most important concepts so as to better understand studies on lot sizing problems with emission constraints.

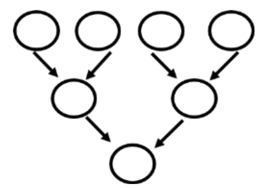
The classical Economic Order Quantity (EOQ) model proposed by Harris (1913) is an example of a lot sizing problem with a relatively simple setting. The EOQ formula minimizes the total cost of ordering and inventory holding while facing stationary demands and no capacity constraint. It involves decisions made by one facility in the supply chain and thus can be considered as a single level lot sizing problem. This model can help decision makers yield the optimal replenishment plan if a series of assumptions are satisfied. These assumptions include: (1) demands are stationary over time, (2) there is no production capacity constraint and (3) the company produces only one type of product.

However, planning in reality usually faces different conditions where some of these assumptions are violated. Therefore, the problem has been extended in many dimensions by researchers to more closely resemble the reality and accommodate different conditions. For example, instead of being stationary, the demand may be dynamic through the planning horizon, which means that the demands are known in advance but vary from period to period. In this case, the problem is denoted as the dynamic lot sizing studied by Wagner and Whitin (1958). Instead of producing a single type of product, a company may also produce multiple items. Instead of having unlimited production capacity, a plant usually has resource constraints and can only produce a limited amount of products in a given time period. A review on modeling single-level dynamic lot sizing problems with various extensions can be found in Jans and Degraeve (2008).

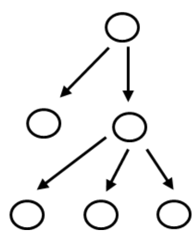
Extending the scope also increases the complexity of the problem. Instead of minimizing the cost of only one facility, a multi-level lot sizing problem deals with multiple parties in the supply chain and minimizes the global total cost.

In multi-level lot sizing problems, the structure of the supply chain can have different features. Arkin (1989) discusses four types of supply chain structure: (1) a *serial system* in which the product passes through a linear sequence of subsequent stages; (2) an *assembly system* in which several subassemblies are combined together at each stage; (3) a *distribution system* in which a product from one facility is distributed to multiple facilities, and finally, (4) a *general system* which can be a combination of the three other systems. Figure 1 shows examples of different types of supply chain structures.

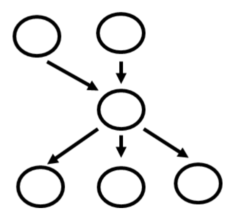
(1) Serial system



(2) Assembly system



(3) Distribution system



(4) General system

Figure 1. Types of Supply Chain Structures

The special case of a two-level distribution system is also known as the One-Warehouse Multi-Retailer (OWMR) problem. The product goes from one warehouse or production plant, to multiple retailers. Figure 2 depicts the structural outline of the OWMR problem.

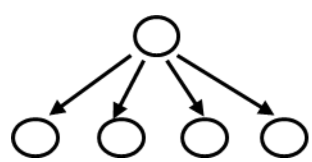


Figure 2. Outline of One-Warehouse Multi-Retailer Structure

The complexity of the OWMR problem has drawn the interest of researchers to propose strong formulations and heuristic methods. Solyalı and Süral (2012) propose a new reformulation to the problem which is called a combined transportation and shortest path based formulation. Their computational results show that the new formulation is stronger than the previously known ones. Federgruen and Tzur (1999) propose time-partitioning heuristics for a multi-item OWMR problem. The heuristics solve the problem by decomposing the planning horizon into smaller intervals.

2.3 Lot Sizing with Emission Constraints

As introduced in Section 2.1, carbon emissions have been considered an important factor in many decisions at different levels of the supply chain. Lot sizing decisions are not an exception. One of the new extensions to lot sizing problems is to add an emission constraint to the problems. This is explained next in more detail.

Benjaafar, Li, and Daskin (2013) add emission constraints to a single-level lot sizing model with dynamic demand under different carbon emission policies and derive insights on the impacts of these different policies. Retel Helmrich et al. (2015) study a similar model under a global emission constraint and apply several heuristic algorithms to solve the problem. Extensive computational tests are presented in their study. Romeijn, Morales, and Van den Heuvel (2014) study biobjective lot-sizing models in which the second objective is to minimize the maximum carbon emission amount. They discuss the computational complexity when emission coefficients take different forms: zero, time-invariant or time-variant. Pareto frontiers are also presented for a 15-period instance. Hua, Cheng, and Wang (2011) adopt the emission constraints into the classical EOQ model and examine the impacts of carbon trade, carbon price and carbon cap on order decisions. Similar models concerning EOQ with emission constraints are studied by Chen, Benjaafar, and Elomri (2013) and an analysis on the trade-off between costs and emissions is presented.

One is tempted to consider emission constraints the same as the traditional production capacity constraint. However, Benjaafar et al. (2013) point out that an emission constraint is different from a production capacity constraint in two aspects: 1) the scope of emission activities and 2) the time horizon of emissions.

Several studies indicate that carbon emissions can come from various activities.

Carbon is emitted at various stages through the supply chain. It occurs while a machine is being set up for production, while products are being produced, transported and even being kept in inventory. In other words, the emissions have to be taken into account in a joint capacity shared by activities at different stages. Changes in the decisions related to one activity (e.g. transportation) could affect other activities (e.g. inventory keeping) due to the joint carbon capacity limit.

Apart from being shared by different activities, carbon emission capacity can be shared by multiple periods or even the entire planning horizon. Production capacities, on the other hand, are modeled for each period. Absi et al. (2013) further suggest that the emission constraint can be imposed on different types of time intervals. They study four types of carbon emission constraints, namely 1) *periodic* carbon emission constraint, 2) *cumulative* carbon emission constraint, 3) *global* carbon emission constraint and 4) *rolling* carbon emission constraint. Figure 3 illustrates how emissions are evaluated under these four types of constraints.

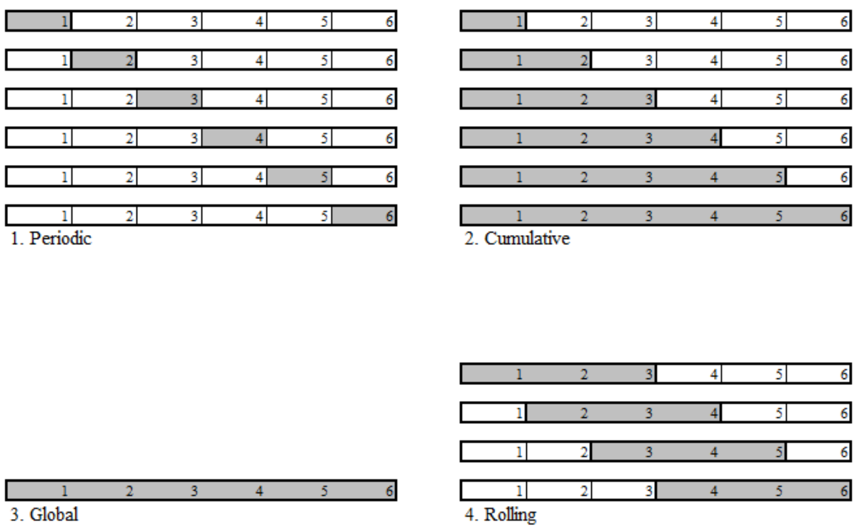


Figure 3. Different Types of Time Intervals

2.4 Modeling Emissions under Different Policies

Following the same categories of emission policies as introduced in section 2.1, namely a strict emission cap, a carbon tax, and a cap-and-trade scheme, we will review how different types of emission policies are modeled in lot sizing problems.

A strict emission cap limits the total amount of carbon that can be emitted during the planning horizon. It does not affect the objective function in models, but serves as a hard constraint. If we denote the total emissions from all activities as ET, and the total emission cap as EC, a strict emission cap generally takes the form of:

$$ET \le EC$$

The second policy, a carbon tax, imposes an extra cost to firms for each unit of carbon emitted. Such a carbon tax mechanism affects the objective cost function but not the constraints, as opposed to a strict emission cap which serves as a constraint. By adding such an extra cost, only the cost coefficients of each activities (e.g. production setup cost, inventory holding cost and variable production cost) are changed, but not the structure of the objective function. Therefore, Benjaaafar et al. (2013) conclude that under this mechanism, “the problem reduces to one of pure cost minimization”.

The third policy, a cap-and-trade mechanism, is considered as a combination of a strict cap and a carbon tax (see Benjaafar et al. (2013)). It alters both the objective function and the constraints. The additional cost (or benefit) in the objective function is modeled as:

$$p(E^+ - E^-)$$

where *p* is the price of buying or selling one unit of carbon emission credit, E^+ is the amount of emission credit bought by the company and E^- is the amount of emission credit sold. A constraint on the total emission is also imposed:

$$ET \le EC + E^+ - E^-$$

However, some studies have a different approach to represent the cap-and-trade mechanism. Chen et al. (2013) suggest that when the selling price of emission credit equals the buying price, the problem becomes similar to the one under the carbon tax mechanism. Hoen et al. (2014) also conclude that a cap-and-trade scheme resembles a carbon tax scheme but a certain amount of emission is exempted from tax.

2.5 Total Emission and Average Emission

As an alternative to limiting the *total amount of emission*, constraints can also be imposed to limit the *average amount of emission* for producing each unit of product, emphasizing the environmental efficiency of the firm.

Absi et al. (2013) apply the average emission constraint when analyzing an uncapacitated multi-sourcing single-item problem. They define a *mode* as a combination of a production facility and a transportation method. Each mode is characterized by its unitary cost and unitary emission. A mode is called *ecological* when the unitary emission is less than the imposed average cap. They then derive that at most two modes are needed for obtaining an optimal solution, one being more economical and possibly another one being more ecological.

Benjaafar et al. (2013) also present a formulation to accommodate this type of average emission constraint. They change the limit of the constraint from a *total emission cap* to an *average emission cap multiplied by total demand.* Since, in their model, the total demand is known in advance and all demand has to be satisfied without shortage, the result of the *average cap multiplied by total demand* is a deterministic number. In that case, it does not make a difference whether the constraint is imposed on average emissions or total emissions.

However, in some other settings, the problem may change under an average emission constraint. For example, if a shortage is allowed with a penalty, a firm can produce less to satisfy the total emission constraint, whereas under the average emission constraint, producing a lower quantity of products does not necessarily reduce the average emission.

	Strict cap	carbon tax	cap-and-trade	Total Emission	Average Emission	Global	Periodic, cumulative and rolling
Benjaafar et al. (2013)	√	√	√	√	√	√	√
Helmrich et al. (2011)	√			√		√	
Hua et al. (2011)		√	√	√		√	
Chen et al. (2013)	√	√	√	√		√	
Absi et al. (2013)	√				√	√	√

Table 2. Summary of Types of Emission Constraints

3. One-Warehouse Multi-Retailer Problem: Formulations and Computational Results

In this chapter, we discuss three known formulations for the OWMR problem with a single product: the standard basic formulation, the four index facility location formulation and a combined transportation and shortest path formulation. Computational results are presented at the end of the chapter.

3.1 Basic Formulation

General notations for the formulations are as follows:

Parameters

T	Set of time periods $\{1, \dots, m\}$
N_c	Set of retailers $\{1, \dots, n\}$
N	Set of facilities which include both the plant and the retailers. $N = N_c \cup \{0\}$ where 0 represents the plant
CS	The setup cost of production at plant
CD_{it}	The setup cost of delivery which occurs when products are delivered to retailer i in period t, $\forall i \in N_c, \forall t \in T$
CU	The unit production cost
CT_i	The unit transportation cost from the plant to retailer i, $\forall i \in N_c$
d_{it}	The demand of retailer i at time period t, $\forall t \in T, \forall i \in N_c$
d_{0t}	The aggregate demand of all retailers in period t, $d_{0t} = \sum_{i \in N_c} d_{it}$, $\forall t \in T$
d_{itu}	The sum of the demand at facility i from period t to u, $d_{itu} = \sum_{q=t}^{u} d_{iq}$, $\forall i \in N, 0 \leq t \leq u \leq m$
h_i	The holding cost at facility i, $\forall i \in N$
I_{i0}	The initial inventory at facility i, $\forall i \in N$

Decisions variables

I_{it}	The inventory at facility i at the end of period t, $\forall i \in N, \forall t \in T$
r_{it}	The quantity delivered to retailer i in period t, $\forall i \in N_c, \forall t \in T$
p_t	Production quantity in period t, $\forall t \in T$
y_{0t}	Binary variable that takes the value of 1 if a production setup is done at

the plant in period t, and the value of 0 otherwise, $\forall t \in T$

y_{it} Binary variable that takes the value of 1 if a delivery to retailer i in period t is made, and takes the value of 0 otherwise, $\forall t \in T$, $\forall i \in N$

The basic formulation is a natural representation of the OWMR problem. It is intuitive and can naturally deal with instances with initial inventory. The objective function (1) minimizes the sum of the inventory cost, production setup cost, delivery setup cost, the variable production cost, and the variable delivery cost. Constraints (2) ensure the inventory balance at the retailers. Constraints (3) ensure the inventory balance at the plant. Constraints (4) and (5) are the setup forcing constraints for production and delivery respectively. Constraints (6) - (9) are the binary and non-negativity constraints.

$$\text{Min} \quad \sum_{t\in T}\sum_{i\in N} h_i I_{it} + \sum_{t\in T} CS\, y_{0t} + \sum_{i\in N_c}\sum_{t\in T} CD_{it}\, y_{it} + \sum_{t\in T} CUp_t + \sum_{t\in T}\sum_{i\in N_c} CT_i r_{it} \tag{1}$$

$$\text{S.T.} \quad I_{i,t-1} + r_{it} = d_{it} + I_{i,t}, \qquad \forall i \in N_c, \forall t \in T \tag{2}$$

$$I_{0,t-1} + p_t = \sum_{i\in N_c} r_{it} + I_{0t} \qquad \forall t \in T \tag{3}$$

$$p_t \le d_{0tm} y_{0t} \qquad \forall t \in T \tag{4}$$

$$r_{it} \le d_{itm} y_{it} \qquad \forall i \in N_c, \forall t \in T \tag{5}$$

$$y_{it} \in \{0,1\} \qquad \forall i \in N, \forall t \in T \tag{6}$$

$$I_{it} \ge 0, \qquad \forall i \in N, \forall t \in T \tag{7}$$

$$r_{it} \ge 0 \qquad \forall i \in N_c, \forall t \in T \tag{8}$$

$$p_t \ge 0 \qquad \forall t \in T \tag{9}$$

Note that in our problem, all demands have to be satisfied. Therefore the total amount of products produced $\sum_{t\in T} p_t$ equals the difference between the total demand and the initial inventory, which is a constant. The same is also true for the total amount of products delivered to the retailers $\sum_{t\in T}\sum_{i\in N_c} r_{it}$. Furthermore, we assume that the unit production and the unit transportation costs are time-invariant. As a result, the sum of the last two items of objective function (1)

$$\sum_{t\in T} CUp_t + \sum_{t\in T}\sum_{i\in N_c} CT_i r_{it}$$

is a constant. For the sake of simplicity of the formulation, we take out these fixed components from the objective function and replace (1) with (10). The Basic Formulation (BF) then is defined as follows:

$$\textbf{BF:}\quad \text{Min}\quad \sum_{t\in T}\sum_{i\in N} h_i I_{it} + \sum_{t\in T} CS\, y_{0t} + \sum_{i\in N_c}\sum_{t\in T} CD_{it}\, y_{it} \tag{10}$$

S.T. (2) - (9)

3.2 Four Index Facility Location Formulation

The Four Index Facility Location Formulation is proposed and discussed by Levi et al. (2008). Solyalı and Süral (2012) extend the formulation to incorporate initial inventory and analyze its impact on the computation complexity. Ruokokoski et al. (2010) adapt the formulation and apply it to the Production-Routing Problem. In the four index facility location formulation, a new set of variables with four indices is introduced. The new variable f_{iuqt} represents the amount of products produced in period u that is delivered to retailer i ($\forall i \in N_c$) in period q to satisfy the demand of period t ($1 \le u \le q \le t \le m$). The following relationship exists between the original variables p_u and r_{it} and the new variables f_{iuqt}:

$$p_u = \sum_{i\in N_c}\sum_{q=u}^{m}\sum_{t=q}^{m} f_{iuqt}$$

$$r_{it} = \sum_{u=1}^{t} \sum_{q=t}^{m} f_{iutq}$$

The resulting formulation is as follows:

FIFL-WI: Min $\sum_{t \in T} \sum_{i \in N} h_i I_{it} + \sum_{t \in T} CS y_{0t} + \sum_{i \in N_c} \sum_{t \in T} CD_{it}\, y_{it}$ (11)

S.T. $\sum_{u=1}^{t} \sum_{q=u}^{t} f_{iuqt} = d_{it}$ $\quad \forall i \in N_c, \forall t \in T$ (12)

$I_{i,t-1} + \sum_{u=1}^{t} \sum_{q=t}^{m} f_{iutq} = d_{it} + I_{i,t}$ $\quad \forall i \in N_c, \forall t \in T$ (13)

$I_{0,t-1} + \sum_{i \in N_c} \sum_{q=t}^{m} \sum_{u=q}^{m} f_{itqu} = \sum_{i \in N_c} \sum_{u=1}^{t} \sum_{q=t}^{m} f_{iutq} + I_{0t}$ $\quad \forall t \in T$ (14)

$\sum_{q=u}^{t} f_{iuqt} \le d_{it} y_{0u}$ $\quad \forall i \in N_c,$ $1 \le u \le t \le m$ (15)

$\sum_{u=1}^{q} f_{iuqt} \le d_{it} y_{iq}$ $\quad \forall i \in N_c,$ $1 \le q \le t \le m$ (16)

$y_{0t} \in \{0,1\}$ $\quad \forall t \in T$ (17)

$y_{it} \ge 0$ $\quad \forall i \in N_c, \forall t \in T$ (18)

$I_{it} \ge 0,$ $\quad \forall i \in N, \forall t \in T$ (19)

$f_{iuqt} \ge 0$ $\quad \forall i \in N_c, 1 \le u$ $\le q \le t \le m$ (20)

The objective function (11) minimizes the sum of the inventory cost, production setup cost, and delivery setup cost. As we have discussed in the Section 3.1, we exclude the variable production cost and the variable delivery cost in the objective function for the sake of simplicity:

$$\sum_{i\in N_c}\sum_{u=1}^{m}\sum_{q=u}^{m}\sum_{t=q}^{m} CUf_{iuqt} + \sum_{i\in N_c}\sum_{u=1}^{m}\sum_{t=u}^{m}\sum_{q=t}^{m} CT_i f_{iutq}$$

Constraints (12) state that all the demand must be satisfied. Constraints (13) and (14) ensure the inventory balance at the retailers and the plant. Constraints (15) and (16) enforce a setup if any production or delivery is made. Constraints (17) - (20) are the non-negativity and binary constraints. Note that in constraint (17), although y_{it} ($\forall i \in N_c, \forall t \in T$) is defined as a continuous variable, it naturally takes integral value of 0 or 1 in the optimal solution. The proof of this proposition can be found in Solyalı and Süral (2012).

Reformulation without inventory variables

In the four index facility location formulation, the four index decision variable f_{iuqt} itself contains the information of when the product is produced, delivered and consumed. Therefore, inventory quantities at the plant and retailers at every period are fixed and computable once the values of f_{iuqt} have been determined. Let H_{iqt} ($\forall i \in N, 1 \le q \le t \le m$) be the holding cost of keeping one unit of products at facility *i* through periods *q* to *t*. H_{iqt} can be calculated as:

$$H_{iqt} = (q - t)h_i$$

For example, $H_{0,1,5}$ is the total holding cost *at the plant* for one unit of product that is produced in period 1 and delivered to the retailer in period 5, while $H_{1,1,5}$ is the total holding cost *at retailer 1* for keeping one unit of product that is delivered in period 1 and consumed in period 5. When *q* equals *t*, it indicates that this unit arrives at the facility (either by production or delivery) and leaves it in the same period and thus no inventory is incurred. With this new parameter, the inventory decision variables and the inventory balance constraints can be taken out to simplify the formulation. Constraints (13), (14) and (19) are removed and the objective function is replaced by (21). The formulation then becomes:

FIFL: Min $$\sum_{i\in N_c}\sum_{u=1}^{m}\sum_{q=u}^{m}\sum_{t=q}^{m}(H_{iqt} + H_{0uq})f_{iuqt} + \sum_{t\in T} CSy_{0t} + \sum_{i\in N_c}\sum_{t\in T} CD_{it}\, y_{it} \quad (21)$$

S.T. (12), (15) - (18) and (20)

Note that after the reformulation without inventory variable, the FIFL formulation can no longer accommodate initial inventory. In order to deal with initial inventory at the plant, we need to introduce a new set of decision variable. Let f_{i0qt} ($\forall i \in N_c, 1 \leq q \leq t \leq m$) be the amount of initial inventory that is delivered to retailer i in period q to satisfy the demand of period *t*.

FIFL-II: Min
$$\sum_{i \in N_c} \sum_{u=1}^{m} \sum_{q=u}^{m} \sum_{t=q}^{m} (H_{iqt} + H_{0uq}) f_{iuqt} + \sum_{i \in N_c} \sum_{q=1}^{m} \sum_{t=q}^{m} H_{iqt} f_{i0qt} + H_{0,1,m+1} I_{00} - \sum_{i \in N_c} \sum_{q=1}^{m} \sum_{t=q}^{m} H_{0q,m+1} f_{i0qt} + \sum_{t \in T} CS y_{0t} + \sum_{i \in N_c} \sum_{t \in T} CD_{it}\, y_{it} \qquad (22)$$

S.T. (6), (15), (20) and

$$\sum_{q=1}^{t} \sum_{u=0}^{q} f_{iuqt} = d_{it} \qquad \forall i \in N_c, \forall t \in T \qquad (23)$$

$$\sum_{i \in N_c} \sum_{q=1}^{m} \sum_{t=q}^{m} f_{i0qt} \leq I_{00} \qquad \forall i \in N_c, 1 \leq q \leq t \leq m \qquad (24)$$

$$\sum_{u=0}^{q} f_{iuqt} \leq d_{it} y_{iq} \qquad \forall i \in N_c, 1 \leq q \leq t \leq m \qquad (25)$$

$$f_{i0qt} \geq 0 \qquad \forall i \in N_c, 1 \leq q \leq t \leq m \qquad (26)$$

The objective function (22) minimizes the total cost. As initial inventory is used, this potentially incurs inventory costs at the plant, inventory cost at the retailers and setup cost of delivery. The third term imposes the total inventory costs of initial inventory at the plant throughout the planning horizon assuming the initial inventory never leaves the plant.

When initial inventory is actually used and leaves the plant, the fourth term deducts the corresponding inventory cost at the plant. By doing so, inventory cost at the plant is properly accounted for even when part or the whole of initial inventory is not used.

Constraints (12) and (16) are replaced by (23) and (25) respectively to accommodate the new variables. Constraint (24) is added to ensure that not more initial inventory is used than available. Since initial inventory is introduced, the integer setup variables of delivery can no longer be relaxed since initial inventory serves as an alternative source of limited capacity. Therefore y_{it} ($\forall i \in N_c$) no longer naturally takes integral values. Constraints (17) and (18) are replaced by (6) to ensure setup variables take binary values.

3.3 A Combined Transportation and Shortest Path Formulation

This formulation was introduced by Solyalı and Süral (2012) and is denoted as SP. A new set of parameters G_{itk} is introduced. Let $G_{itk} = \sum_{l=t}^{k-1} h_i \times d_{i,l+1,k}$ be the total holding cost of satisfying the demand of retailer i ($\forall i \in N_c$) from period t to k by delivering in period t ($1 \leq t \leq k \leq m$). The cost G_{itk} can be viewed as the distance of an arc that connects point *t* and point *k*. In the optimal solution, a series of arcs are chosen to connect the starting point (period 1) to the end point (period *m*) with the shortest distance. Therefore, the lot sizing problem at the retailer level can be considered to be represented by a shortest path problem.

On the other hand, instead of assigning a binary variable to G_{itk} to indicate whether an arc is chosen or not, new set of decision variables are introduced. Let u_{ikqt} represent the fraction of the total demand at retailer i ($\forall i \in N_c$) from period q to *t* that is satisfied by products produced in period k and delivered in period q ($1 \leq k \leq q \leq t \leq m$). The sum $\sum_{k=1}^{q} u_{ikqt}$ represents the fraction of the total demand at retailer i ($\forall i \in N_c$) from period q to t that is satisfied by products delivered in period q ($1 \leq k \leq q \leq t \leq m$). Therefore, the variables still keep track of the products from its production and delivery at the plant level.

The complete formulation uses a shortest path-based formulation to represent the lot sizing problems at the retailer level, and facility location type representation for the plant level. The SP formulation is as follows:

$$\textbf{SP}: \text{Min} \quad \sum_{i\in N_c}\sum_{q=1}^{m}\sum_{t=q}^{m}\sum_{k=1}^{q} H_{0kq} d_{iqt} u_{ikqt} + \sum_{i\in N_c}\sum_{q=1}^{m}\sum_{t=q}^{m}\sum_{k=1}^{q} G_{iqt} u_{ikqt} + \sum_{t\in T} CS y_{0t} + \sum_{i\in N_c}\sum_{t\in T} CD_{it}\, y_{it} \tag{27}$$

S.T. (17), (18) and

$$\sum_{t=1}^{m} u_{i11t} = 1 \qquad \forall i \in N_c \tag{28}$$

$$\sum_{k=t}^{m}\sum_{q=1}^{t} u_{iqtk} - \sum_{k=1}^{t-1}\sum_{q=1}^{k} u_{iqk,t-1} = 0 \qquad \forall i \in N_c,\ 2 \le t \le m \tag{29}$$

$$\sum_{k=q}^{t}\sum_{r=t}^{m} a_{ikr} u_{iqkr} \le y_{0q} \qquad \forall i \in N_c,\ 1 \le q \le t \le m \tag{30}$$

$$\sum_{t=q}^{m}\sum_{k=1}^{q} a_{iqt} u_{ikqt} \le y_{iq} \qquad \forall i \in N_c,\ 1 \le q \le m \tag{31}$$

$$u_{iuqt} \ge 0 \qquad \forall i \in N_c,\ 1 \le u \le q \le t \le m \tag{32}$$

The objective function (27) minimizes inventory cost, production setup cost and delivery setup cost. Similar to the previous formulations, we take out the following redundant parts of the objective function:

$$\sum_{i\in N_c}\sum_{q=1}^{m}\sum_{t=q}^{m}\sum_{k=1}^{q} CU d_{iqt} u_{ikqt} + \sum_{i\in N_c}\sum_{q=1}^{m}\sum_{t=q}^{m}\sum_{k=1}^{q} CT_i d_{iqt} u_{ikqt}$$

Constraints (28) and (29) are the shortest path representation of the lot sizing problem at retailer level. Constraints (30) and (31) impose setup cost for production and delivery respectively. Constraint (32) ensures non negativity.

Extension to initial inventory at the plant:

$$\textbf{SP-II}\text{: Min} \quad \sum_{i\in N_c}\sum_{q=1}^{m}\sum_{t=q}^{m}\sum_{k=0}^{q} G_{iqt}u_{ikqt} + \sum_{i\in N_c}\sum_{q=1}^{m}\sum_{t=q}^{m}\sum_{k=1}^{q} H_{0kq}d_{iqt}u_{ikqt}$$

$$+H_{01,m+1}I_{00} - \sum_{i\in N_c}\sum_{q=1}^{m}\sum_{t=q}^{m} H_{0q,m+1}d_{iqt}u_{i0qt} + \sum_{t\in T} CSy_{0t} \tag{33}$$

$$+\sum_{i\in N_c}\sum_{t\in T} CD_{it}\, y_{it}$$

S.T. (6), (30), (32)

$$\sum_{t=1}^{m} U_{i11t} + \sum_{t=1}^{m} U_{i01t} = 1 \qquad \forall i \in N_c \tag{34}$$

$$\sum_{k=t}^{m}\sum_{q=0}^{t} U_{iqtk} - \sum_{k=1}^{t-1}\sum_{q=0}^{k} U_{iqk,t-1} = 0 \qquad \forall i \in N_c, 2 \le t \le m \tag{35}$$

$$\sum_{i\in N_c}\sum_{q=1}^{m}\sum_{t=q}^{m} U_{i0qt}d_{iqt} \le I_{00} \tag{36}$$

$$\sum_{t=q}^{m}\sum_{k=0}^{q} a_{iqt}U_{ikqt} \le y_{iq} \qquad \forall i \in N_c, 1 \le q \le m \tag{37}$$

$$U_{i0qt} \ge 0 \qquad \forall i \in N_c, 1 \le q \le t \le m \tag{38}$$

In order to incorporate initial inventory, similar modifications are needed as those are made for the four index facility location formulation. Objective function (33) minimizes the total cost. The objective function first imposes the inventory cost of initial inventory at the plant throughout the planning horizon and then deducts the inventory cost of the units that are delivered to the retailers.

Constraints (28) and (29) are replaced by (34) and (35) respectively to accommodate the new initial inventory variables. Constraint (36) ensures that the total initial inventory used does not exceed the available amount. Setup variables for delivery y_{it} ($i \in N_c$) no longer naturally takes integral values. Constraints (17) and (18) are replaced by constraint (6) to ensure setup variables take binary values.

3.4 Computational Results

The data sets we used for our experiments are taken from the test instances used by Solyalı and Süral (2012). The data sets include production setup cost, delivery setup cost, holding cost at the plant, holding cost at the retailers. The production setup cost is static throughout the time horizon for each instance. In different instances, the production setup cost can be different and follows a uniform distribution in the range between 1500 and 4500. The delivery setup cost was randomly generated following a uniform distribution in the range between 5 and 100. The delivery setup cost differs between retailers and is dynamic through the time horizon. The holding cost at the plant is constant at 0.5 per unit per period. The holding cost at the retailers is randomly generated following a uniform distribution in the range between 0.5 and 1. It differs from retailer to retailer, but for each retailer it remains static throughout the time horizon.

In this thesis, we assume that all demands have to be satisfied. Therefore the total number of products that will be produced and transported to the retailers is fixed and independent of production plans and delivery plans. For this reason, we have removed the constant production cost and transportation cost from the formulations in the experiments. Table 3 summarizes the cost parameters that are used by Solyalı and Süral (2012).

	Cost	**Time (In)Variance**
Production Setup	1500-4500	Static
Delivery Setup	5-100	Time-Variant
Holding one unit at the Plant	0.5	Static
Holding one unit at Retailers	0.5-1.0	Static
Producing one unit	-	-
Delivering one unit	-	-

Table 3. Characteristics of the Cost Parameters

Instances can have 50, 100 or 150 retailers. The time horizon consists of 15 or 30 periods. Every combination of number of retailers and time horizon consists of 10 instances without initial inventory and 10 instances with initial inventory. In total there are 120 different instances and we test them with each of the previously discussed formulations.

The formulations were coded in a JAVA environment with CPLEX 12.6.0.1. Each instance was solved on one out of two Intel(R) Xeon(R) CPU X5675 3.07 GHz processors of a machine with 96 GB of RAM. The optimality tolerance was set to 10^{-6} using a single thread. The computation is stopped when the time used exceeds 7200 seconds.

We first test two versions of the four index facility location formulation, the one with inventory variables (FIFL-WI) and the reformulated one without inventory variables (FIFL) discussed in Section 3.2. The faster one out of the two is then chosen to be compared with the basic formulation and the combined transportation and shortest path formulation. Table 4 shows the average CPU time (in seconds) needed to solve the OWMR problem using these two formulations. The reformulation without inventory variables is significantly faster than the FIFL-WI.

Retailers	**Periods**	**FIFL-WI**	**FIFL**
50	15	2.2	1.1
100	15	4.1	2.8
150	15	6.5	3.7
50	30	26.6	8.6
100	30	92.9	24.1
150	30	166.0	38.0
Average		49.7	13.1

Table 4. CPU Times of FIFL-WI and FIFL of Instances without Initial Inventory

Table 5 summarizes the computational results for BF, SP and FIFL on instances without initial inventory and Table 6 summarizes the results on instances with initial inventory. Instances with 15 periods are solved to optimality by all formulations within 7200 seconds. Instances with 30 periods are solved to optimality by SP, SP-II, FIFL and FIFL-II but not BF. The column MILP Time indicates the average time used to solve an instance in seconds. LP Time indicates time needed to solve the problem when the binary variables are relaxed as continuous variables. The last column shows the average gap when the binary constraints are relaxed. All values are the average results over 10 instances with the same size.

Retailers	Period	I_{00}	BF MILP Time	BF LP Time	BF GAP	SP MILP Time	SP LP Time	SP GAP	FIFL MILP Time	FIFL LP Time	FIFL GAP
50	15	0	30.8	0.1	68.17%	1.2	0.9	0.00%	1.1	0.7	0.00%
100	15	0	161.4	0.1	71.34%	2.8	1.3	0.02%	2.8	1.8	0.02%
150	15	0	402.2	0.2	72.29%	4.1	1.9	0.02%	3.7	2.6	0.02%
50	30	0	>7200	-	-	12.8	4.4	0.03%	8.6	3.5	0.03%
100	30	0	>7200	-	-	33.9	9.6	0.05%	24.1	9.5	0.05%
150	30	0	>7200	-	-	42.3	45.5	0.03%	38.0	21.3	0.03%
Average			-	-	-	**16.2**	**10.6**	**0.03%**	**13.1**	**6.6**	**0.03%**

Table 5. Computational results for BF, SP and FIFL without initial inventory

Retailers	Period	I_{00}	BF MILP Time	BF LP Time	BF GAP	SP-II MILP Time	SP-II LP Time	SP-II GAP	FIFL-II MILP Time	FIFL-II LP Time	FIFL-II GAP
50	15	>0	33.4	0.1	68.21%	5.3	0.9	1.62%	2.6	0.9	1.63%
100	15	>0	291.7	0.1	71.35%	14.0	2.9	1.19%	8.0	2.1	1.19%
150	15	>0	371.4	0.2	72.26%	28.0	3.9	1.01%	18.7	2.7	1.01%
50	30	>0	>7200	-	-	30.3	6.6	1.02%	19.7	4.6	1.02%
100	30	>0	>7200	-	-	103.4	15.3	0.61%	57.6	9.7	0.61%
150	30	>0	>7200	-	-	337.7	33.9	0.57%	72.2	27.0	0.57%
Average			-	-	-	**86.5**	**10.6**	**1.00%**	**29.8**	**7.8**	**1.01%**

Table 6. Computational Results for BF, SP-II and FIFL-II with Initial Inventory

The results show that both the number of retailers and the number of periods have a considerable impact on the computation time. Increasing these two numbers, especially the number of periods, significantly increases the computation time. In all instances with 30 periods, the basic formulation fails to solve the problem to optimality within 7200 seconds.

The BF appears to be inferior to the other formulations not only with respect to the solution time, but also with respect to the LP relaxation gap. The BF has gaps of around 70%, while SP and FIFL have an average initial gap of as little as 0.03% for instances without initial inventory, and SP-II and FIFL-II have an average gap of around 1% for instances with initial inventory. The average initial gaps of SP, SP-II, FIFL and FIFL-II obtained from our experiments are exactly the same as those reported by Solyalı and Süral (2012). The shortest path formulations and the four index facility location formulations have the same size of gap in almost all instances. However, in certain instances, the shortest path formulations provide a slightly better LP gap than the four index facility location formulations. This result is aligned with the theorems proved by Solyalı and Süral (2012) that SP has an equal or better LP gap compared to FIFL, and SP-II has an equal or better gap compared to FIFL-II.

With respect to speed, our results shows that FIFL and FIFL-II are faster in almost all instances. However in results presented by Solyalı and Süral (2012) experiments, SP and SP-II are slightly faster than FIFL and FIFL-II, which is contradictory to our results. This may be caused by the difference in CPLEX version or hardware.

4. Incorporating a Global Emission Constraint: Formulations and Computational Results

Based on the three known formulations for the OWMR problem, we propose three formulations with a global emission constraint. Most studies assume that carbon emission amount is correlated to energy consumption. For example, Girod et al. (2013) assume that when fossil fuel is used, 71.5 tons of CO_2 are emitted while consuming one terajoule energy. This emission factor, together with other factors such as transportation mode, vehicle types and fuel types, are used to calculate the final emission amount. Palak, Ekşioğlu and Geunes (2014) divide transportation related emission into a variable part and a fixed part, in which the fixed part comes mainly from the loading and unloading process. They also consider inventory related emissions due to the heating or cooling system at the facility. To properly account for all sources of emission, our model uses a similar approach and takes into account emissions from all activities, including production setups, delivery setups, inventory holding at the plant and at the retailers. For the sake of brevity, we only discuss here the formulations with initial inventory. The notation for emission parameters is as follows:

EC	The global emission cap over the entire planning horizon
ET_i	Emission of transporting one unit of product from the plant to retailer i, $\forall i \in N_c$
EI_i	Emission of keeping one unit of inventory at facility i, $\forall i \in N$
ES	Emission of production setup
ED_{it}	Emission of delivery setup which occurs when products are delivered to retailer i in period t, , $\forall i \in N_c, \forall t \in T$
EP	Emission of producing one unit of product

4.1 Basic Formulation with Emission

This formulation is the same as the basic formulation, except that a global emission constraint (39) is imposed to limit the total emission. The constraint is similar to the objective function (10) in its structure, and requires that the total emission cannot exceed a global emission cap.

BF-E: Min (10)

S.T. (2) - (9) and

$$\sum_{t\in T}\sum_{i\in N} EI_i\, I_{it} + \sum_{t\in T} ES y_{0t} + \sum_{i\in N_c}\sum_{t\in T} ED_{it} y_{it} \le EC \tag{39}$$

The total emission accounts for the emission of keeping inventory both at the plant and the retailers, the emission of production setup and the emission of delivery setup. Note that in general, emission also comes from the unit production and delivery. The sum of their variable emission of production and delivery is:

$$\sum_{t\in T} EP p_t + \sum_{t\in T}\sum_{i\in N_c} ET_i r_{it}$$

However, similar to the variable production cost and variable delivery cost, we assume that these two emission parameters are time-invariant. Since we have a global emission constraint, the total production and delivery quantities are constant, and their effect on emissions is hence also constant. To simplify the emission constraint, we leave out these two terms.

4.2 Four Index Facility Location Formulation with Emission

To calculate the emission amount, we introduce a new parameter that is similar to H_{iut}. We denote by $EH_{iut} = (t-u)EI_i$ the emission of holding one unit of inventory at facility i ($\forall i \in N$) from period u to t ($1 \le u \le m, u \le t \le m+1$). This formulation is the same as FIFL-II, except that a global emission constraint (40) is imposed.

FIFL-E: Min (11)

S.T. (12) - (20) and

$$\sum_{i\in N_c}\sum_{u=1}^{m}\sum_{q=u}^{m}\sum_{t=q}^{m}\left(EH_{iqt}+EH_{0uq}\right)f_{iuqt}+\sum_{i\in N_c}\sum_{q=1}^{m}\sum_{t=q}^{m}EH_{iqt}f_{i0qt}$$

$$+EH_{01,m+1}I_{00}-\sum_{i\in N_c}\sum_{q=1}^{m}\sum_{t=q}^{m}EH_{0q,m+1}f_{i0qt}+\sum_{t\in T}ESy_{0t} \tag{40}$$

$$+\sum_{i\in N_c}\sum_{t\in T}ED_{it}y_{it}\leq EC$$

Constraint (40) limits the total emission under the global emission cap. The structure of the constraint is similar to the objective function (11). The first term of the objective function accounts for the emission of keeping inventory at the retailers and at the plant. The last two terms account for the emission incurred by production setups and delivery setups. The remaining three terms in the middle account for the inventory holding emission for the initial inventory. For the inventory holding emission at the plant, we apply the same technique to calculate the emission amount: The third term in constraint (40) imposes the total inventory emissions for the initial inventory at the plant throughout the planning horizon, assuming the initial inventory never leaves the plant. When the products leave the plant, the fourth term deducts the corresponding inventory holding emission at the plant.

4.3 A Combined Transportation and Shortest Path Formulation with Emission

A new parameter similar to G_{itk} is introduced to record the emission amount. $EG_{itk}=\sum_{l=t}^{k-1}EI_i*d_{i,l+1,k}$ is defined as the total holding emission of satisfying demand of retailer i ($\forall i\in N_c$) from period t to k by delivering products in period t ($1\leq t\leq k\leq m$).

SP-E: Min (27)

S.T. (17), (18), (28) - (32) and

$$\sum_{i\in N_c}\sum_{q=1}^{m}\sum_{t=q}^{m}\sum_{k=0}^{q} EG_{iqt}U_{ikqt} + \sum_{i\in N_c}\sum_{q=1}^{m}\sum_{t=q}^{m}\sum_{k=1}^{q} EH_{0kq}d_{iqt}U_{ikqt}$$

$$+EH_{01,m+1}I_{00} - \sum_{i\in N_c}\sum_{q=1}^{m}\sum_{t=q}^{m} EH_{0q,m+1}d_{iqt}U_{i0qt} + \sum_{t\in T} ESy_{0t} \tag{41}$$

$$+\sum_{i\in N_c}\sum_{t\in T} ED_{it}y_{it} + \sum_{i\in N_c}\sum_{t\in T} CD_{it}\, y_{it} \le EC$$

The global emission constraint limits the total amount of emissions of various activities. The first term accounts for the emission from inventory holding at the retailers. The second, third and fourth terms together account for the emission from inventory holding at the plant. The last two terms account for the emission of delivery setup and production setup respectively.

4.4 Introduction of the Data Set with Emission Parameters

Due to the lack of existing data set with emission parameters for the OWMR, we adapt the instances of Solyalı and Süral (2012) by generating new sets of emission parameters. The generated emission parameters are combined with the cost parameters in the data set provided by Solyalı and Süral (2012) to form a data set with both costs and emissions for the following four activities: production setup, delivery setup, inventory holding at the plant, inventory holding at the retailers.

Since the emission intensity is usually related to energy consumptions, which is also a major component of the various costs, we assume that the amount of emission is correlated with the cost of the corresponding activity with a certain level of deviation. We consider three levels for the correlation between the cost and emission parameters. For the first level of correlation, we allow a 50% positive or negative deviation for the emission parameters, with respect to the cost of the corresponding parameter. This means that for each activity (i.e. inventory holding, production setup and delivery setup), the emission amount can be within [50%, 150%] of the corresponding cost. The exact deviation is taken from a uniform distribution within this range. For example, for a plant with production setup cost of 2000, the production setup emission is generated within the range [1000, 3000] following a uniform distribution. The two other levels of correlation are obtained by allowing a maximum deviation of 20% and 100%. We consider instances with a 50% maximum deviation for the purpose of comparing between the formulations. The other two data sets are tested for the purpose of an analysis on the emission amount in Section 5.

We tailor the emission cap parameters for each instance instead of applying a random generation. When the emission cap is too tight, there will be no feasible solution for the problem. On the other hand, when the emission cap is more than ample, it becomes non-binding and will not have any effect on the solution cost. Therefore, in order to have a fair comparison between the instances, we calculate the upper bounds and lower bound for the emission cap, within which the emission cap is binding for the problem and at the same time allows feasible solutions. Then we run a series of experiments in which the emission cap is gradually tightened, from the upper bound to the lower bound.

An upper bound on the emission cap is defined as the minimum emission amount needed in a cost-optimal solution. Any solution that needs an emission amount higher than the upper bound is inefficient because such a solution can be improved by either reducing the cost without increasing the emission amount, or by reducing the emission amount without increasing cost.

An upper bound on the emission cap can be obtained through two steps. First, we minimize the total cost regardless of the emission constraint and record the optimal cost. Second, we minimize the emission amount while constraining the cost to be the optimal cost. The second step is necessary because there may exist alternative solutions which give the same optimal cost but have different emission amounts. This also suggests that companies can possibly reduce their emission amount slightly while keeping their costs at the optimal level.

A lower bound on the emission cap is defined as the minimum emission amount needed to provide a feasible solution. Imposing an emission cap lower than this lower bound will make the problem infeasible. A lower bound on the emission can be obtained by minimizing the emission amount while satisfying all demand, regardless of the costs.

Once we know the emission upper bound and lower bound, we can calculate the

Maximum Potential Reduction (MPR) on emissions using the following formula:

$$MPR = \frac{Emission\ Upper\ Bound - Emission\ Lower\ Bound}{Emission\ Upper\ Bound}$$

The larger the MPR is, the greater emission reduction we can potentially achieve compared to the base case where we only optimize the cost without taking into account an emission constraint.

4.5 Computational Results with an Emission Constraint

For each instance, we run 20 experiments with different levels for the emission cap. In the first experiment, the emission cap is set to be:

$$Emission\ Cap = Emission\ Upper\ Bound - MPR \times 5\%$$

When solving such an instance, we obtain a solution for which the emission reduction corresponds to at least 5% of the maximum achievable reduction. Then in the next experiments, a tighter emission cap corresponding to a 10% MPR reduction is imposed. The emission cap is gradually tightened to 100% MPR reduction in the 20th experiment.

$$Emission\ Cap = Emission\ Upper\ Bound - MPR \times 100\%$$

When the global emission constraint is imposed, BF-E takes more than 2 hours on average to solve even the smallest instance with 50 retailers and 15 periods. Therefore only results using FIFL-E and SP-E are presented. In order to have complete results on computation times, no time limit is set for FIFL-E and SP-E. All instances are solved to optimality. Table 7 and Table 8 show the CPU times for SP-E and FIFL-E respectively.

retailers	**period**	I_{00}	**5**	**10**	**15**	**20**	**25**	**30**	**35**	**40**	**45**	**50**	**55**	**60**	**65**	**70**	**75**	**80**	**85**	**90**	**95**	**100**	**Average**
50	15	0	3	2	4	5	5	7	6	9	9	9	10	11	12	13	14	17	17	11	16	3	9
100	15	0	9	14	19	16	17	19	33	27	25	37	26	55	44	47	55	44	59	77	79	9	35
150	15	0	18	44	32	44	28	37	39	60	83	126	110	70	70	83	106	113	162	140	163	12	77
50	30	0	35	88	68	186	210	150	169	276	210	264	290	250	383	407	277	439	377	353	440	41	246
100	30	0	284	267	494	551	700	869	781	1243	1440	867	970	1118	1396	1200	1567	1573	1904	2088	1542	173	1051
150	30	0	358	392	783	630	911	1858	1747	1759	1687	1558	1868	2442	2341	2938	3505	2320	3471	2629	3005	248	1822
50	15	>0	10	11	10	21	14	18	15	19	20	19	21	25	22	27	27	29	26	30	28	11	20
100	15	>0	43	41	61	32	54	54	54	81	63	78	74	107	112	128	130	142	101	134	143	44	84
150	15	>0	91	151	98	126	99	113	125	130	146	171	155	158	162	151	162	196	162	176	173	39	139
50	30	>0	110	309	165	321	316	437	275	366	495	396	508	337	383	450	396	489	587	507	584	217	383
100	30	>0	497	775	1169	967	1476	1579	1590	2006	2619	2025	1756	2155	2028	2922	2566	2156	2178	2914	2143	873	1820
150	30	>0	549	1187	1415	1221	1508	2817	2561	3258	2746	3111	4268	3233	4049	4070	4244	3626	3675	3825	4711	1673	2887
Average			217	412	486	448	578	836	770	977	1015	967	1130	1003	1126	1291	1254	1106	1122	1264	1297	476	889

Table 7. CPU Times of FIFL-E

retailers	**period**	I_{00}	**5**	**10**	**15**	**20**	**25**	**30**	**35**	**40**	**45**	**50**	**55**	**60**	**65**	**70**	**75**	**80**	**85**	**90**	**95**	**100**	**Average**
50	15	0	5	4	5	6	9	14	8	11	8	9	12	12	13	18	19	21	19	15	30	3	12
100	15	0	10	17	19	15	23	28	41	27	40	70	50	54	69	86	79	104	110	96	90	8	52
150	15	0	23	30	37	48	32	51	85	66	93	137	126	123	127	136	173	205	185	223	211	12	106
50	30	0	41	86	86	184	225	233	264	350	410	282	378	286	501	420	559	496	489	462	659	32	322
100	30	0	185	405	720	566	1035	1058	1303	1331	1601	1383	1705	1786	2268	2480	2354	3068	3008	2648	2625	155	1584
150	30	0	883	1205	895	1395	1393	1564	3073	1768	2375	1065	2282	2324	2958	3003	3580	3310	3740	3689	4008	192	2235
50	15	>0	12	14	15	22	19	26	28	20	19	19	26	27	30	26	31	26	29	35	36	11	24
100	15	>0	60	69	71	76	80	90	88	125	95	116	109	138	118	122	167	166	147	204	191	54	114
150	15	>0	170	207	176	187	228	187	180	194	286	353	196	274	299	209	277	244	273	382	323	55	235
50	30	>0	175	234	235	367	515	382	484	419	661	519	523	552	476	622	580	732	848	734	671	247	499
100	30	>0	683	719	1151	953	1159	2088	1765	2089	2512	2334	2462	2814	2634	3407	3040	2712	3177	4171	3793	1201	2243
150	30	>0	841	1180	1900	2040	2136	2993	3912	2779	3882	4059	5713	4804	4229	5042	5331	6388	11137	14525	5947	3899	4637
Average			257	348	442	488	571	726	936	765	998	862	1132	1099	1144	1298	1349	1456	1930	2265	1549	489	1005

Table 8. CPU Times of SP-E

Overall, FIFL-E performs consistently faster than SP-E. Results also reveal that as the emission cap gradually becomes tighter, the computation time tends to increase dramatically for both FIFL-E and SP-E, as shown by Figure 4. However, in the last experiment when the emission cap is set exactly to the lower bound, the computation time drops significantly.

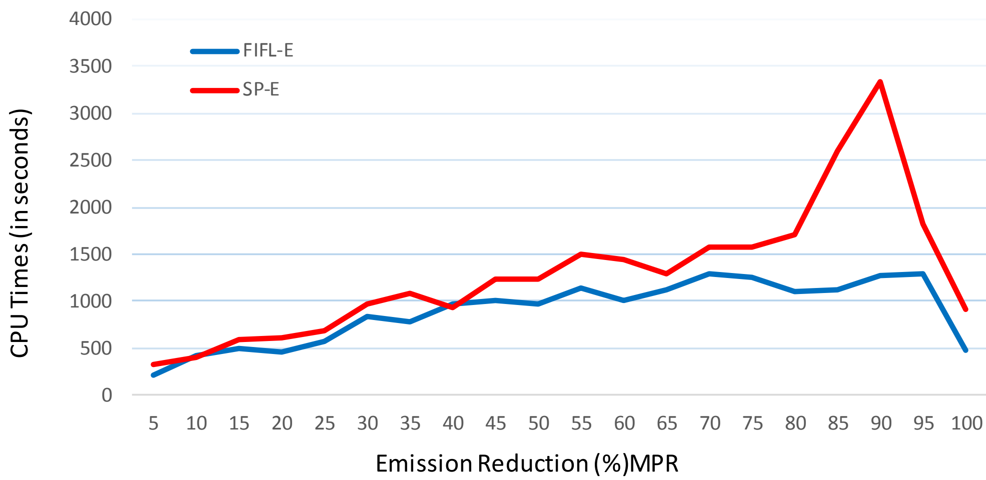


Figure 4. Comparison of Computation Time between FIFL-E and SP-E

Table 9 shows the computation time of the OWMR and the OWMR-EC problem. The column OWMR indicates the average time consumed to solve an OWMR problem in seconds. Each number corresponds the average value over ten instances with the same size. The column OWMR-EC indicates the average time consumed to solve an OWMR-EC problem in seconds. Each instance is tested with twenty different emission caps shown in Table 7 and Table 8. Ten instances in each size are tested and therefore each value in the column OWMR-EC is the average time over 200 experiments. The values of third column $^{\mathrm{OWMR-EC}}/_{\mathrm{OWMR}}$ is calculated as the value the average time needed to solve the OWMR-EC problems divided by the average time needed to solve the OWMR problems. As the size of the instances increases, the impact of the emission constraint on the computation time also increases. In the largest instances without initial inventory, the global emission constraint increases the computation time by up to 50 times approximately for both SP and FIFL.

Retailers	Period	I_{00}	SP (SP-II) OWMR	SP-E OWMR-EC	$^{OWMR\ -\ EC}/_{OWMR}$	FIFL (FIFL-II) OWRM	FIFL-E OWMR-EC	$^{OWMR\ -\ EC}/_{OWMR}$
50	15	0	1.2	12.0	1001%	1.1	9.2	836%
100	15	0	2.8	51.8	1849%	2.8	35.4	1264%
150	15	0	4.1	106.1	2589%	3.7	77.2	2085%
50	30	0	12.8	322.2	2517%	8.6	245.7	2857%
100	30	0	33.9	1584.2	4673%	24.1	1051.3	4362%
150	30	0	42.3	2235.1	5284%	38.0	1822.4	4796%
Average			**16.2**	**718.6**	**2985%**	**13.1**	**540.2**	**2700%**
50	15	>0	5.3	23.6	445%	2.6	20.1	772%
100	15	>0	14.0	114.3	817%	8.0	83.8	1047%
150	15	>0	28.0	235.1	839%	18.7	139.3	745%
50	30	>0	30.3	498.8	1646%	19.7	378.7	1923%
100	30	>0	103.4	2243.3	2170%	57.6	1819.8	3159%
150	30	>0	337.7	4636.9	1373%	72.2	2887.4	3999%
Average			**86.5**	**1292.0**	**1215%**	**29.8**	**888.2**	**1941%**

Table 9. Average CPU Time of SP, SP-E, FIFL and FIFL-E

5. Analysis of Emissions

As previously discussed in Section 4.5, we assume that emission amounts are proportional to the cost of the corresponding activities with a 50% maximum deviation. In reality, the emission amount may be more strongly correlated to the cost of the corresponding activities. Such circumstance can be found in energy intensive industries where energy usage incurs both cost and emission, and thus these two factors are strongly correlated. In some other industries, the opposite situation can be true where these two factors have a weak correlation (e.g. in industries where the production cost is mainly determined by the cost of labour). Therefore, in addition to the emission parameters with a 50% maximum deviation, we also generate and test emission parameters with a 20% maximum deviation and a 100% maximum deviation. These correspond to the high and low correlation case respectively. Our base case with the 50% maximum deviation corresponds to the medium correlation case. Using these different data sets, we analyze the costs and the emissions under different scenarios and examine their trade-off.

5.1 Maximum Potential Reduction and Its Cost

We have discussed the Maximum Potential Reduction (MPR) of emission in Section 4.4, but the cost of achieving such reduction has not been discussed. As the emission amount in a cost-optimal solution is denoted as 100% emission budget, we also denote the cost in a cost-optimal solution as 100% cost, so that fair comparison can be made between instances. The Cost of Maximum Emission Reduction (CMER) is defined as the additional cost of the minimum emission solution compared to the minimum cost solution.

$$CMER = \frac{Cost\ in\ the\ minimal\ emission\ solution - Cost\ in\ the\ minimal\ cost\ solution}{Cost\ in\ the\ minimal\ cost\ solution}$$

Table 10 summarizes the MPR and CMER in different scenarios. A 20% deviation means that the emission parameters are highly correlated to the cost and 100% means that the data is weakly correlated to each other. It is clear from the table that, in scenarios where the emission parameters are weakly correlated to the cost parameters higher maximum emission reduction can potentially be achieved.

Emission Deviation			20%		50%		100%	
Retailers	Periods	I_{00}	MPR	CMER	MPR	CMER	MPR	CMER
50	15	0	0.53%	0.46%	3.04%	3.82%	18.56%	32.59%
100	15	0	0.50%	0.46%	2.84%	3.65%	12.23%	21.06%
150	15	0	0.52%	0.53%	4.29%	5.03%	16.65%	24.20%
50	30	0	0.52%	0.52%	3.00%	3.19%	15.12%	20.74%
100	30	0	0.61%	0.63%	3.05%	3.47%	14.80%	24.45%
150	30	0	0.54%	0.53%	4.69%	4.21%	14.00%	24.54%
50	15	>0	0.43%	0.56%	3.05%	3.68%	18.11%	31.47%
100	15	>0	0.44%	0.43%	2.79%	3.51%	12.03%	20.39%
150	15	>0	0.54%	0.51%	4.31%	5.20%	16.94%	24.68%
50	30	>0	0.54%	0.54%	3.06%	3.18%	15.10%	20.50%
100	30	>0	0.58%	0.64%	2.97%	3.43%	14.68%	23.97%
150	30	>0	0.53%	0.54%	4.68%	4.41%	13.94%	24.23%
Average			**0.52%**	**0.53%**	**3.48%**	**3.90%**	**15.18%**	**24.40%**

Table 10. Average Maximum Potential Reduction with Different Maximum Emission Deviations

As the emission parameters deviate farther from the corresponding costs, not only the MPR increases, the cost of achieving such a maximum reduction also increases. MCER increases even at a faster rate compared to MPR, indicating that the marginal cost of emission reduction is increasing.

5.2 Cost Emission Trade-Off Curve

In a series of experiments, we gradually tighten the emission cap from the upper bound to the lower bound, and analyze how the cost increases.

Figure 5 displays an example of the aggregated trade-off curve over twenty instances (ten with initial inventory and ten without initial inventory) with 50 retailers and 15 periods and a medium correlation level. As we reduce the emission amount, the cost increases at an increasing speed. The first 1% emission reduction only incurs about 0.13% extra cost while reducing emission by 2% results in about 0.83% of additional cost. The average maximum amount of emission reduction that can be achieved is 3.05% at a price of 3.75% cost on average.

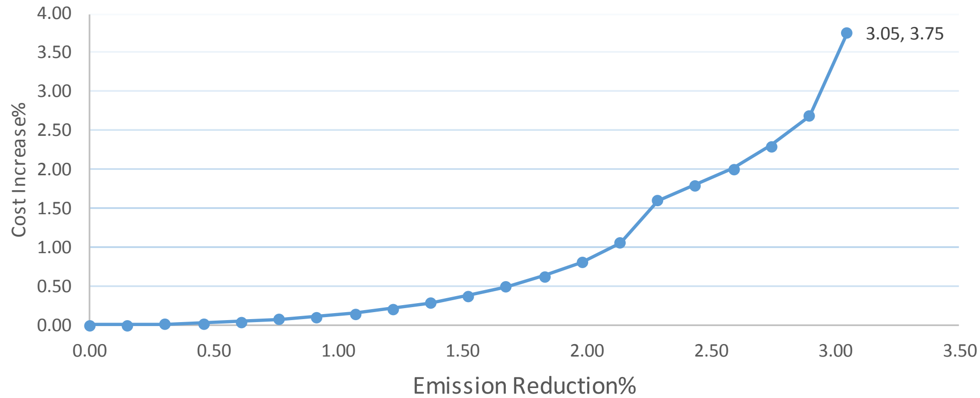


Figure 5. Average Trade-Off Curve over Twenty Instances with 50 Retailers and 15 Periods (Medium Correlation)

Figure 6 and Figure 7 show the aggregated trade-off curves of larger size instances with 100 retailers and 150 retailers respectively. These figures show similar results: the marginal cost of emission reduction tends to increase as higher levels of emission reduction needs to be achieved.

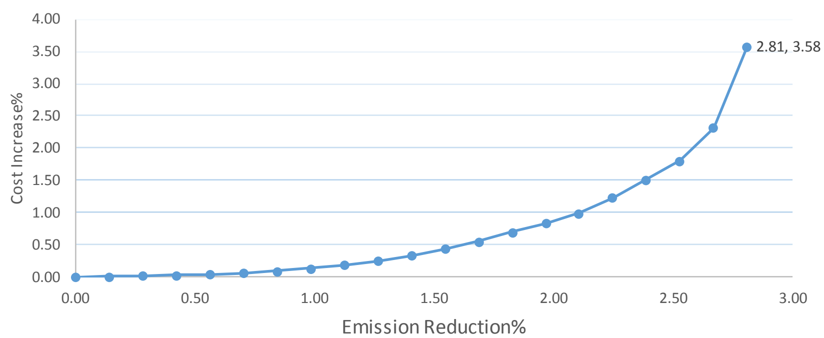


Figure 6. Average Trade-Off Curve over Twenty Instances with 100 Retailers and 15 Periods (Medium Correlation)

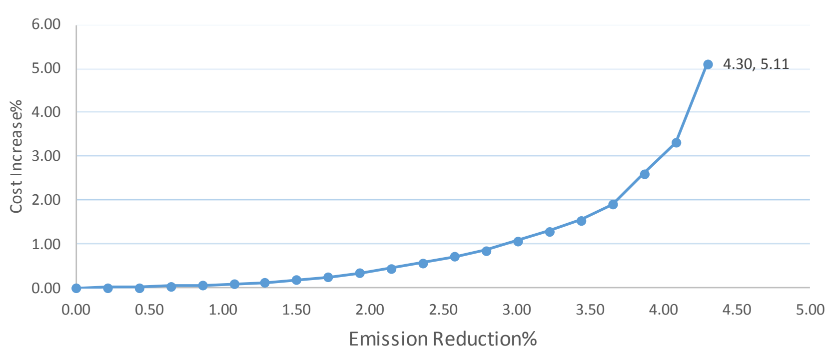


Figure 7. Average Trade-Off Curve over Twenty Instances with 150 Retailers and 15 Periods (Medium Correlation)

Note that in Figure 5, the curve is piecewise convex. To avoid the aggregation effect, we look at a specific instance in Figure 8. It displays the trade-off curve of one of the instances with 50 retailers and 15 periods. As the emission cap is gradually tightened from 100% to 96.6%, the cost rises in an increasing pace. Note that the curve is piecewise convex in two sections. The first section starts from 100% emission and ends at 97.3, and the second section covers the rest. Such a curve indicates that within each section, the marginal cost of reducing emission increases. In this case, reducing the emission by 2.4%, from 100% to 97.6% incurs 1% extra cost. However, if we continue to tighten the emission, only 0.3% reduction incurs almost another 1% extra cost. A similar pattern repeats within the second convex section.

When we look at the solution of each point on the graph, we can see that each convex section corresponds to one specific production setup plan at the plant. The light blue bars indicate in how many periods production setups are made out of 15 periods. All solutions in the first section have 5 production set-ups while the second have 6 production setups.

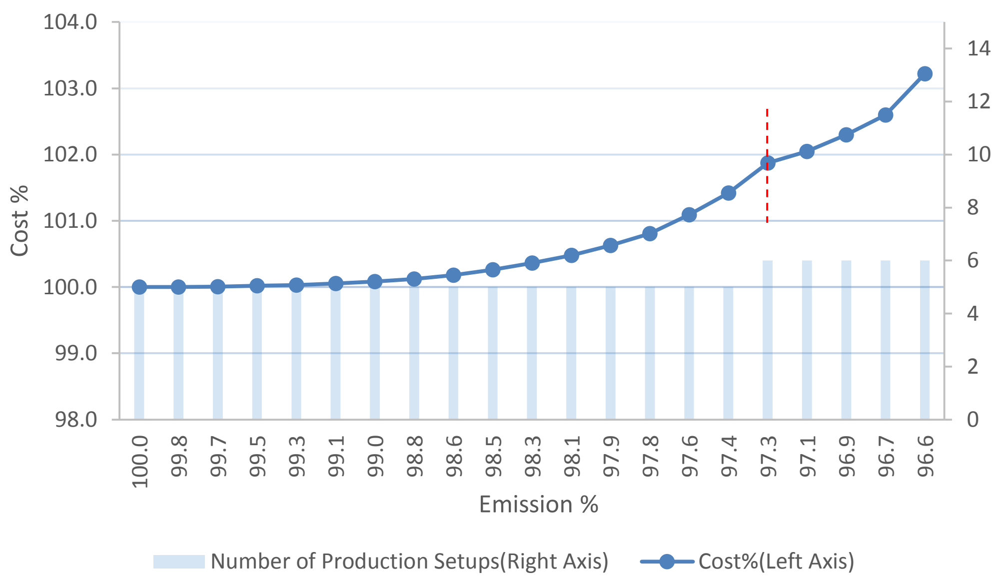


Figure 8. Trade-off Curve: An Instance with 50 Retailers and 15 Periods

6. Heuristics

As shown by the results presented in the previous section, a global emission constraint significantly increases the computation time. In some cases, even the fastest formulation proposed in Section 4 cannot give an optimal solution in two hours. As an attempt to expedite the computation process, we propose several heuristics. We decompose the OWMR problem into two stages and apply different heuristics to each stage. We also extend the best heuristics to the solve OWMR-EC problem.

At stage 1, we develop a production plan at the plant level. The production setup decisions are passed to stage 2 as inputs while the decisions on production quantity are discarded. In other words, stage 1 decides when to produce at the plant. We use two methods, namely the Simple Retailers Aggregation (SRA) and a Modified Silver-Meal approach (MSM) to produce the production setup plan at the plant. At stage 2, we take the decisions made at stage 1 as input, and develop the delivery plan for the retailers. Once the delivery plan for all retailers is produced, the production quantity is determined based on the delivery quantity and a solution to the OWMR problem is provided by the end of stage 2. We test three approaches at stage 2: optimization, optimization with Local Branching (LB) and a Time Partitioning Fix-and-Optimize (TP) method. The emission constraint has been ignored so far. At stage 3, we use an iterative Penalized Relaxation (PR) heuristic to find a feasible solution for the problem under the emission constraint. In the iteration process of PR, a series of OWMR problems are solved using the heuristics at stage 1 and stage 2. Figure 9 summarizes all the methods used in these three stages.

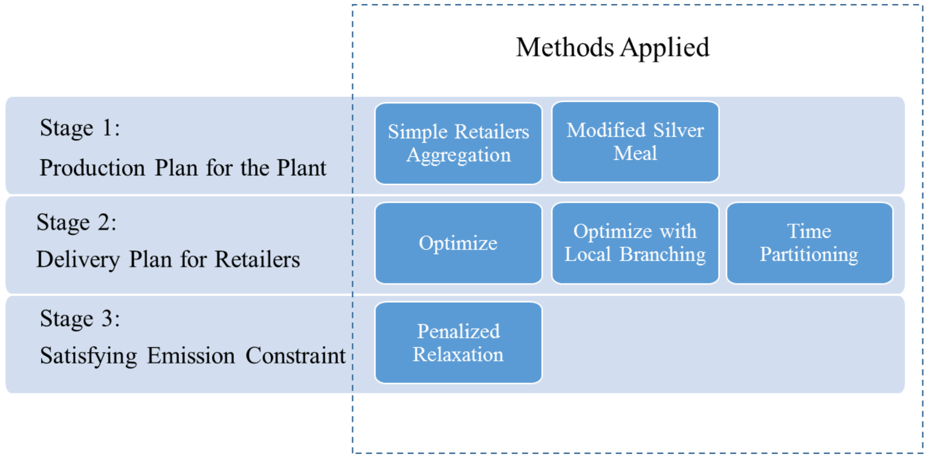


Figure 9. Summary of Methods Applied to Solve OWMR-EC at Various Stages

6.1 Production Plan for the Plant

At this stage, the goal is to develop a production plan at the plant as an input for stage 2. At the end of stage 1, only the decisions on the production setups are passed on to stage 2 while the decisions on production quantity are discarded. In other words, stage 1 aims to decide when to produce. The SRA method and MSM are applied and results are presented in Section 6.1.3.

6.1.1 Simple Retailers Aggregation

To simplify the complexity of the OWMR problem, we aggregate all the retailers and treat them as one. The demand of this aggregate retailer d_{0t} ($\forall t \in T$) is the sum of demands from all retailers, i.e., $\sum_{i=1}^{n} d_{it}$. The delivery setup cost for this retailer CD_{0t} ($\forall t \in T$) is the sum of delivery setup costs for all retailers, i.e., $\sum_{i=1}^{n} CD_{it}$. After aggregating all the retailers, the structure of the supply chain becomes a two-level serial system. Nonetheless, it can also be viewed as a special case of the OWMR problem but with only one retailer. Therefore, we apply the FIFL formulation to solve this aggregated problem. The SRA works as follow:

Step 1. Aggregate all retailers into one.

Step 2. Optimize with FIFL.

Step 3. Record the production setup decisions at the plant $\hat{y}_{0t}((t \in T))$.

6.1.2 Modified Silver-Meal Heuristic

Instead of simply adding up demands from all retailers, an alternative to aggregating the retailers is to apply the Silver-Meal model to each retailer separately and then aggregate all the orders from the retailers as the demand for the plant.

The original Silver-Meal heuristic was proposed by Silver and Meal (1973) to deal with the single-level dynamic lot sizing problem. This forward algorithm addresses the average cost per period and balance the trade-off between ordering cost (or setup cost) and inventory holding cost. Define AC_{tk} as the average cost per period of placing an order at period t to cover the demands of the next k periods (including period t). AC_{tk} can be derived as:

$$AC_{tk} = (S + \sum_{q=1}^{k} (q-1) d_{t+\mathrm{q}-1} h)/k$$

where S is the ordering cost, d_t is the demand at period t and h is the holding cost per period. When $k = 1$, an order is made to satisfy the demand in period t only. Since zero lead time is assumed, products arrive and are consumed in the same period and thus no inventory holding cost incurs. The procedure of Silver-Meal heuristic is as follows:

Step 1. Set $t = 1$.

Step 2. Calculate the average cost AC_{tk} for ascending integer values of k (starting at 1) until the average cost increases at time $k^* + 1$ ($AC_{t,k^*} \leq AC_{t,k^*+1}$).

Step 3. The decision is made to place an order at period t to cover the demands of the next k^* periods.

Step 4. If the end of the planning horizon is reached, stop. Otherwise, set t equal to $t + k^*$ and continue with Step 2.

The Silver-Meal heuristic cannot be applied directly to each retailer because the delivery setup cost is dynamic while the Silver-Meal heuristic assumes that the ordering cost is constant. In the Silver-Meal heuristic, making a decision to order in period t to cover the next k periods also implies that an order will be placed at period $t + k$. Therefore, when the setup cost is dynamic, the setup cost of the next order should also be considered at the present planning. To accommodate the dynamic setup cost situation, we modify the method of calculating the average cost per period and denote it as AC_{tk}^{α}:

$$AC_{tk}^{\alpha} = (\alpha CD_t + (1-\alpha)CD_{t+k} + \sum_{q=1}^{k}(q-1)d_{t+\mathrm{q}-1}h)/k$$

where the ordering cost S is replaced by the weighted delivery setup cost $\alpha CD_t + (1-\alpha)CD_{t+k}$ and the weight α can be any decimal number between 0 and 1.

After applying this weighted Silver-Meal heuristic to all retailers, we are able to aggregate orders from retailers and treat it as the demand for the plant. Then the classical Silver-Meal heuristic is applied to the plant since the setup costs at the plant are stationary. The complete process of the Modified Silver-Meal heuristic (MSM) is as follows:

Step 1. Set $\alpha = 1$. For each retailer $i \in N_c$, use AC_{tk}^{α} as the average cost and apply the Silver-Meal heuristic. Record the orders for all retailers.

Step 2. Sum up the orders from all retailers resulting from this plan to calculate the demands for the plant.

Step 3. Use the production setup cost at the plant as the ordering cost *S* and apply the original Silver-Meal heuristic to the plant, with the aggregate demands as calculated in Step 2. Calculate the total cost including production setup cost, delivery setup cost, holding cost at the plant and holding cost at retailers.

Step 4. Repeat *Step 1* to *Step 3* with $\alpha \in \{0.9, 0.8, \ldots, 0.1, 0\}$.

Step 5. Pick the plan with the value of α that renders the lowest total cost.

Step 6. Record the production setup decisions at the plant $\hat{y}_{0t}$ $(t \in T)$.

When there is initial inventory at the plant, a minor modification to the Silver-Meal heuristic implemented at *step 5* is needed. Before implementing the Silver-Meal heuristic at *step 5*, we add an initiation process:

We first compare the initial inventory quantity to the aggregated demand over all retailers from period 1 to q, e.g. $d_{0,1,u} = \sum_{q=1}^{u}\sum_{i\in N_c} d_{iq}$. Find u^* such that $d_{0,1,u^*} \leq I_{00} \leq d_{01,u^*+1}$. We fix the production setup to be 0 for periods 1 to u^* and start the original Silver-Meal heuristic from period $u^* + 1$.

6.1.3 Computational Results at Stage 1

Both heuristics yield the production setup decisions at the plant $\hat{y}_{0t}$ as the output. To examine the quality of this output, we define two indices to measure solutions' deviation from the production setup decisions in the optimal solution of the original OWMR problem denoted with y_{0t}^{OPT}. The first index measures the Difference in the Total Setups (DTS) between the heuristic solution and the optimal solution. It is defined as:

$$DTS = \left| \sum_{t \in T} y_{0t}^{OPT} - \sum_{t \in T} \hat{y}_{0t} \right|$$

The second index measures the Difference in the Individual Setups (DIS). It is defined as:

$$DIS = \sum_{t \in T} |y_{0t}^{OPT} - \hat{y}_{0t}|$$

For both indices, a smaller value indicates that the solution given by the heuristic more closely resembles the optimal solution. Table 11 summarizes the average computation time, DTS and DIS of these two methods for all instances. On instances with 15 periods, the MSM method only takes about one third of the time needed for the SRA method to give a solution to the production planning problem at stage 1. On instances with 30 periods, MSM's advantage in computation time is even larger. Yet, both methods are very fast since they consume less than 0.1 second on average.

			Time		DIS		DTS	
retailers	period	I_{00}	SRA	MSM	SRA	MSM	SRA	MSM
50	15	0	0.017	0.007	1.4	3.8	0.3	0.2
100	15	0	0.021	0.004	1.9	6.3	0.1	0.1
150	15	0	0.019	0.010	3.4	3.5	2.6	0.3
50	15	>0	0.022	0.005	1.4	2.4	0.2	0.2
100	15	>0	0.024	0.008	1.9	3.8	0.1	0.2
150	15	>0	0.024	0.009	3.4	2	2.6	0.2
Average			0.021	0.007	2.2	3.6	1.0	0.2
50	30	0	0.082	0.008	3.5	8.1	0.5	0.9
100	30	0	0.079	0.013	5.4	10.2	0.4	0.6
150	30	0	0.086	0.029	7.9	2.4	5.6	0.4
50	30	>0	0.101	0.012	3.5	9.4	0.5	1
100	30	>0	0.093	0.016	5.4	8.9	0.4	0.7
150	30	>0	0.089	0.025	7.9	2.5	5.5	0.3
Average			0.088	0.017	5.6	6.9	2.2	0.7

Table 11. DTS and DIS and Time of Two Heuristics for All Instances

Over the 60 instances with the planning horizon of 15 periods, the SRA method has an average DTS of 1.0, while the MSM method has an average DTS of only 0.20, which means the MSM method provide a better solution in terms of the total number of production setups. On the other hand, SRA has an average DIS of 2.2, which is slightly smaller than 2.6, the average DIS of MSM.

In other 60 instances with a longer planning horizon of 30 periods the SRA method has an average DTS of 2.2, while the MSM method has an average DTS of only 0.7. With respect to DIS, SRA has a smaller average 5.6, compared to 6.9 of MSM.

When the length of the planning horizon is doubled, DTS for both methods more than doubled. This indicates that the solution quality in terms of total production setups for both methods deteriorates when the length of the planning horizon increases.

With respect to the impact of the number of retailers on the solution quality, we can observe that the SRA method does not have a stable performance in the instances with a large number of retailers. On the contrary, the MSM method is more stable and even has small values for both DIS and DTS in large instances with 150 retailers.

6.2 Delivery Plan for Retailers

At stage 2, we take the production setup decisions obtained from stage 1 as input and proceed to make the delivery plan for all retailers. There are a few options to produce the delivery plan. We can resort to the FIFL formulation and use CPLEX to optimize the stage 2 problem while fixing all the production setup variables. Alternatively, we can fix the production setup variables but allow for certain flexibility at the same time through a Local Branching method. Finally, we also experiment a Time Partitioning Fix-and-Optimize approach, which divides the planning horizon into a series of overlapping intervals and optimizes the sub-problem separately. Each of these approaches is combined with the SRA and the MSM, yielding a specific heuristic for the OWMR problem. Computational results for each of these combinations are presented at Section 6.2.3.

6.2.1 Optimize and Local Branching

Given the output $\hat{y}_{0t}$ obtained from stage 1, we can now optimize the delivery problem at stage 2 by applying the four index facility location formulations while fixing the production setup variables as follows:

$$y_{0t} = \hat{y}_{0t} \qquad \forall t \in T \tag{42}$$

Instead of strictly fixing all the production setup variables, we can also relax this constraint and allow the program to revoke a small part of production setup decisions. Such a relaxation provides the opportunity for the program to search for solutions that have similar, but not necessarily the same, production setups as those obtained at stage 1. Pochet and Wolsey (2006) discussed this Local Branching method using the following constraint:

$$\sum_{t \in T, \hat{y}_{0t}=0} y_{0t} + \sum_{t \in T, \hat{y}_{0t}=1} (1 - y_{0t}) \leq \gamma \tag{43}$$

The left side of the constraint measures the DIS between a potential solution and the solution provided at stage 1 and the integer γ is the maximum difference allowed. When γ takes the value of 0, which means no difference is tolerated, the constraint has the same function as that of constraint (42). As the value of γ increases, we allow more decisions made at the stage 1 to be revoked, and thus possibly providing a better result.

However, more branches are searched and therefore an impact on the computation time is also expected.

6.2.2 Time Partitioning Fix-and-Optimize

The third approach aims to reduce the computation time and disaggregates the OWMR problem into many smaller problems. We treat each retailer separately and divide the planning horizon into a series of shorter time intervals with overlaps. If we note ρ as the length of the optimization periods and κ as the length of the overlapping periods, the procedure of TP is as follows:

Step 1. For each retailer $i \in N_c$, do step 2 to step 6.

Step 2. Starting from the first period $u = 1$, optimize the total cost for time interval u to $u + \rho - 1$. Record the optimal delivery setup decisions as $\hat{y}_{it}$ $(u \leq t \leq u + \rho - 1)$.

Step 3. Fix $y_{it} = \hat{y}_{it}$ for period *t* when $1 \leq t \leq u + \rho - 1 - \kappa$.

Step 4. If $u + \rho - 1 < m$, update $u = u + \rho - \kappa$, optimize the total cost for time interval u to $u + \rho - 1$, while ignoring the periods after $u + \rho - 1$. Record the optimal delivery setup decisions as $\hat{y}_{it}$ $(u \leq t \leq u + \rho - 1)$.

Step 5. Repeat Step 3 and step 4 until $u + \rho - 1$ reaches *m*, the entire planning horizon is optimized.

Step 6. Fix $y_{it} = \hat{y}_{it}$ for period *t* when $u \leq t \leq u + \rho - 1$.

Figure 10 illustrates the procedure of step 2 to step 6 for a 30-period instance with the optimizing interval $\rho = 10$ and the overlapping interval $\kappa = 5$. We first optimize the total cost for periods 1 to 10, the remaining periods are left out of the problem. In the next optimization, we fix the setup decisions for periods that have been optimized except the overlapping part 6 to 10. Therefore, we fix periods 1 to 5 and optimize periods 6 to 15. Then periods 1 to 10 are fixed and periods 10 to 20 are optimized. The same procedure continues until all 30 periods are optimized.

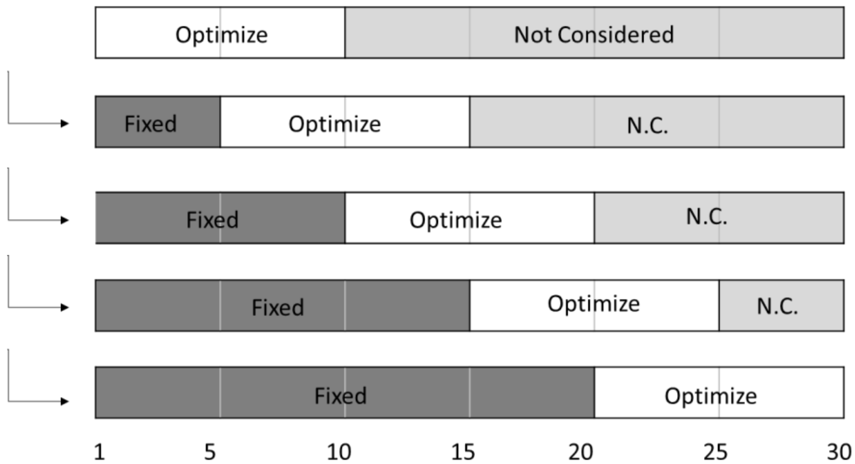


Figure 10. An Example of TP Procedure for a 30-Period Instance

The goal of the TP method is to disaggregate the OWMR problem into many small problems, which can be solved in an extremely short time. Following the same example, when $\rho = 10$ and $\kappa = 5$, for each retailer five small MILP need to be solved in an instance with 30 periods. If there are 150 retailers, then 750 small problems are optimized instead one big OWMR problem. To solve these small problems, we consider two options. We can either employ the four index facility location formulation with CPLEX, or enumerate all possible delivery setup decisions. The second option is also known as Complete Enumeration (CE). Again using the same example, when $\rho = 10$, there are two possible scenarios (to deliver or not to deliver) at each period and in total there are 2^{10}, or 1024, different possible delivery plans. The CE method evaluates all these plans and chooses the one with the lowest cost. Depending on the optimization method used, we denote the Time-Partition Fix-and-Optimize process as TP (CPLEX) and TP (CE) respectively.

6.2.3 Computational Results at Stage 2

Each of the approaches at the stage 2 is combined with either SRA or MSM and forms a unique heuristic method for the OWMR problem. All combinations are tested and the computational results are compared to the exact solutions of FIFL and FIFL-II, the fastest exact method for the OWMR problem discussed in Section 3.

The Local Branching method is tested when the maximum allowed DIS, or the parameter k, takes the values: 1, 2 and 3. For the Time Partitioning Fix-and-Optimize method, the length of the optimizing periods ρ is set to be 10, and the length of the overlapping periods κ is set to be 5. It is tested using Complete Enumeration and CPLEX to solve the sub-problems. Applying TP with CE or CPLEX gives exactly the same solutions, but the time consumed may be different.

As the benchmark, the time needed to solve an instance by FIFL or FIFL-II is considered 100% time. The Comparative Time (C. Time) used by the heuristic method to solve the problem is then calculated as:

$$C.Time = \frac{Time\ Consumed\ by\ the\ Heuristic}{Time\ Consumed\ by\ FIFL} \times 100\%$$

Table 12 and Table 13 show the results of the approaches at stage 2 combined with the SRA method. With respect to the solution quality, the overall average gap for all heuristics is less than 1%. When there is no initial inventory, SRA-TP has the same gaps as SRA-OPTIMIZE, which suggests that in all instances without an initial inventory, the TP method finds the optimal solution to the problems at stage 2 when given a set of fixed production setup decisions from stage 1. However, this is only an observation limited to the instances we have tested. In other instances, TP may not guarantee an optimal solution. The Local Branching method improves the solution quality and has smaller gaps. As the maximum DIS allowed increases, the solution quality keeps improving, from 0.49% to 0.34%. Although SRA-LB has better gaps than other heuristics, it consumes even more time than the exact method. SRA-OPTIMIZE reduces the time to about a quarter of the time needed by the exact method. SRA-TP (CE) only takes 2.1% of the time consumed by the exact method, which is significantly faster than other methods.

			SRA - OPTIMIZE		SRA - LB						SRA - TP			
					γ =1		γ =2		γ =3		CE		CPLEX	
Retailers	**Period**	I_{00}	**Gap**	**C.Time**	**Gap**	**C.Time**	**Gap**	**C.Time**	**Gap**	**C.Time**	**Gap**	**C.Time**	**Gap**	**C.Time**
50	15	0	0.19%	33.3%	0.19%	130.9%	0.18%	137.0%	0.15%	177.6%	0.19%	4.6%	0.19%	67.0%
100	15	0	0.22%	25.3%	0.16%	132.3%	0.07%	107.8%	0.07%	182.8%	0.22%	2.7%	0.22%	48.9%
150	15	0	1.44%	30.1%	0.99%	150.3%	0.78%	158.0%	0.57%	201.0%	1.44%	2.3%	1.44%	48.0%
50	30	0	0.09%	24.2%	0.08%	140.2%	0.07%	133.4%	0.02%	184.2%	0.09%	1.7%	0.09%	40.9%
100	30	0	0.29%	20.6%	0.27%	140.1%	0.23%	134.6%	0.22%	180.8%	0.29%	0.8%	0.29%	29.3%
150	30	0	1.37%	20.8%	1.24%	165.2%	1.13%	154.6%	1.03%	206.4%	1.37%	0.7%	1.37%	30.3%
Average			0.60%	25.7%	0.49%	143.2%	0.41%	137.6%	0.34%	188.8%	0.60%	2.1%	0.60%	44.1%

Table 12. Computational Results for Approaches at stage 2 Combined with SRA on Instances without Initial Inventory

			SRA - OPTIMIZE		SRA - LB						SRA - TP			
					γ =1		γ =2		γ =3		CE		CPLEX	
Retailers	**Period**	I_{00}	**Gap**	**C.Time**	**Gap**	**C.Time**	**Gap**	**C.Time**	**Gap**	**C.Time**	**Gap**	**C.Time**	**Gap**	**C.Time**
50	15	>0	0.15%	29.8%	0.15%	138.6%	0.15%	153.6%	0.15%	200.8%	0.24%	8.2%	0.24%	50.5%
100	15	>0	0.14%	20.8%	0.08%	103.5%	0.02%	106.3%	0.02%	164.7%	0.26%	3.1%	0.26%	35.1%
150	15	>0	1.24%	17.6%	1.01%	116.1%	0.78%	138.7%	0.54%	151.2%	1.49%	1.7%	1.49%	24.6%
50	30	>0	0.08%	19.4%	0.08%	132.3%	0.07%	127.2%	0.04%	177.4%	0.13%	1.3%	0.13%	24.8%
100	30	>0	0.23%	17.4%	0.23%	143.7%	0.16%	126.8%	0.13%	191.2%	0.31%	1.1%	0.31%	19.9%
150	30	>0	1.37%	27.0%	1.17%	167.6%	1.06%	167.9%	0.96%	217.5%	1.40%	0.9%	1.40%	27.4%
Average			0.54%	22.0%	0.45%	133.6%	0.37%	136.8%	0.31%	183.8%	0.64%	2.7%	0.64%	30.4%

Table 13. Computational Results for Approaches at stage 2 Combined with SRA on Instances with Initial Inventory

When an initial inventory is considered, TP no longer provides the optimal plan at stage 2. The average gap of TP increases slightly to 0.64%, compared to the average gap of SRA-OPTIMIZE 0.54%.

Table 14 and Table 15 show the results of the approaches at stage 2 combined with the MSM method. When we have a horizontal comparison between OPTIMIZE, LB and TP, the overall average results are similar to those discussed earlier when these heuristics are combined with SRA. When there is no initial inventory, gaps of MSM-TP are the same as those of MSM-OPIMIZE. The Local Branching method has the best gaps and continues to improve as the maximum allowed DIS increases. In terms of computation time, The Local Branching method exceeds 100% of the time consumed by the exact method. MSM-TP (CE) again is faster than other heuristics.

			MSM - OPTIMIZE		MSM - LB						MSM - TP			
					$\gamma=1$		$\gamma=2$		$\gamma=3$		CE		CPLEX	
Retailers	Period	I_{00}	Gap	C.Time	Gap	C.Time	Gap	C.Time	Gap	C.Time	Gap	C.Time	Gap	C.Time
50	15	0	1.40%	34.1%	1.06%	149.5%	0.86%	161.7%	0.51%	198.5%	1.40%	6.0%	1.40%	61.2%
100	15	0	0.86%	25.4%	0.82%	162.0%	0.43%	124.9%	0.40%	197.1%	0.86%	3.6%	0.86%	48.0%
150	15	0	0.51%	30.0%	0.49%	156.8%	0.18%	113.6%	0.18%	202.7%	0.51%	3.1%	0.51%	53.4%
50	30	0	0.98%	22.5%	0.78%	126.8%	0.65%	158.1%	0.51%	201.0%	0.98%	1.3%	0.98%	36.3%
100	30	0	0.53%	20.4%	0.51%	170.3%	0.42%	140.3%	0.36%	204.4%	0.53%	0.7%	0.53%	30.3%
150	30	0	0.12%	21.9%	0.09%	127.4%	0.04%	131.7%	0.04%	172.6%	0.12%	0.7%	0.12%	29.7%
Average			0.73%	25.7%	0.63%	148.8%	0.43%	138.4%	0.33%	196.0%	0.73%	2.6%	0.73%	43.1%

Table 14. Computational Results for Approaches at stage 2 Combined with MSM on Instances without Initial Inventory

			MSM - OPTIMIZE		MSM - LB						MSM - TP			
					$\gamma=1$		$\gamma=2$		$\gamma=3$		CE		CPLEX	
Retailers	Period	I_{00}	Gap	C.Time	Gap	C.Time	Gap	C.Time	Gap	C.Time	Gap	C.Time	Gap	C.Time
50	15	>0	1.22%	28.4%	0.89%	144.9%	0.75%	177.6%	0.35%	199.6%	1.32%	3.8%	1.32%	45.8%
100	15	>0	0.67%	19.9%	0.65%	131.7%	0.26%	94.9%	0.25%	171.8%	0.80%	1.5%	0.80%	27.5%
150	15	>0	0.29%	18.5%	0.26%	129.4%	0.10%	104.6%	0.10%	149.7%	0.59%	1.5%	0.59%	22.5%
50	30	>0	1.07%	18.9%	0.88%	101.5%	0.72%	143.8%	0.58%	225.3%	1.15%	1.0%	1.15%	25.0%
100	30	>0	0.47%	16.6%	0.44%	186.0%	0.35%	178.2%	0.29%	216.9%	0.57%	0.4%	0.57%	19.2%
150	30	>0	0.23%	26.1%	0.20%	167.6%	0.07%	135.7%	0.06%	185.1%	0.26%	0.6%	0.26%	32.6%
Average			0.66%	21.4%	0.55%	143.5%	0.38%	139.1%	0.27%	191.4%	0.78%	1.5%	0.78%	28.8%

Table 15. Computational Results for Approaches at stage 2 Combined with MSM on Instances with Initial Inventory

When we make a vertical comparison between SRA and MSM, the difference in computation time is insignificant. However, there are a few interesting observations about the solution quality. When combined with different approaches at stage 2, SRA almost always has better gaps compared to MSM. The only exception is LB when γ takes the value of 3. As the maximum DIS allowed increases, MSM-LB improves its solution quality at a faster pace than SRA-LB and finally surpass SRA-LB when γ equals 3.

As the size of the instances increases, either in the number of retailers or the length of the planning horizon, the quality of solution is affected. When the number of retailers increases, the gaps of SRA tend to deteriorate. However the opposite is true for MSM. Its solution quality improves as the number of retailers increases. As the length of the planning horizon increases, the solution quality of MSM also improves.

Figure 11 summarizes the overall gaps and Comparative Time of all heuristics discussed. Since all these heuristics have an overall average gap of less than 1%, we also include the results of applying FIFL with CPLEX while setting the optimal gap tolerance as 1%, to validate these heuristics' effectiveness on computation time. The heuristics that fall in the shaded area are dominated by the exact method, since they perform worse than the exact method in terms of both computation time and solution quality. Considering the heuristics that are not dominated by the exact method, SRA-TP (CE) has the second best average gap and its computation time is significantly faster than others. Therefore, we choose SRA-TP (CE) and integrate it into the Penalized Relaxation method at stage 3.

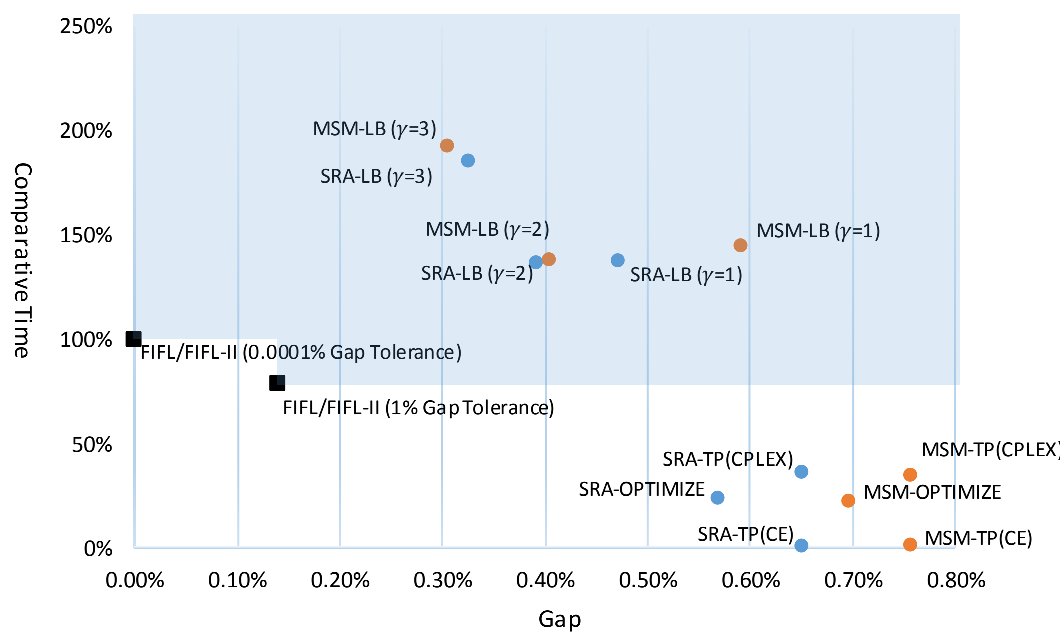


Figure 11. Summary of Average Gaps and Comparative Times of Different Heuristics

6.3 Accommodating the Emission Constraint

To satisfy the global emission constraint, we introduce a Penalized Relaxation (PR) method inspired bi-objective optimization. In this method, the global emission constraint is relaxed and all the emission factors are taken into account in the objective function as penalties. The penalties are added to the objective function through the adjustments of all cost coefficients. Let β $(0 \leq \beta \leq 1)$ be the penalty factor, the cost coefficients are adjusted through:

$$Adjusted\ Cost\ Coefficient = (1-\beta) \times Cost\ of\ an\ Activity + \beta \times Emission\ of\ the\ Coresponding\ Activity$$

The costs that need to be adjusted include all activities: the setup cost of production, the setup cost of delivery, the inventory holding cost at the plant and the inventory holding cost at the retailers. When β takes the value of 0, there is no penalty enforced, and minimizing the objective function will lead to purely minimizing the total cost regardless of the emissions. On the other hand, if β takes the value of 1, full penalty is implemented and minimizing the objective function will lead to minimizing the total amount of emissions regardless of the costs.

The Penalized Relaxation method iteratively solves a series of OWMR problems with different values of β, and thus different coefficients of costs. The solutions obtained are then verified with the true cost coefficients and emission coefficients. The solutions that do not satisfy the emission constraint are discarded. If solutions that satisfy the emission constraint are found, the one with the least cost is selected as the final solution. If all solutions are discarded, the heuristic fails to provide a feasible solution.

In the iterative process, we solve an OWMR problem and update the penalty factor before solving the next OWMR problem. In order to quickly approach the appropriate penalty factor that can balance well the costs and emissions, we apply the bisection method. We start with $\beta = 0.5$, or $(\frac{1}{2})^1$, and solve the OWMR problem with adjusted cost coefficients. If the resulting total emission amount exceeds the allowed emission cap, we increase the penalty factor $\beta = (\frac{1}{2})^1 + (\frac{1}{2})^2$ and try to satisfy the emission constraint in the next iteration. On the contrary, if the resulting total emission amount already satisfy the emission constraint, we decrease the penalty factor $\beta = (\frac{1}{2})^1 - (\frac{1}{2})^2$ and try to lower the total cost in the next iteration. Let l be the iteration index. The complete process of the Penalized Relaxation is described as follows:

Step 1. Start the first iteration, $l = 1$ and $\beta = (\frac{1}{2})^l$, update all adjusted costs:

$$CS' = (1-\beta) \times CS + \alpha \times ES$$

$$CD_{it}' = (1-\beta) \times CD_{it} + \alpha \times ED_{it}, \forall i \in N_c, \forall t \in T$$

$$h_i' = (1-\beta) \times h_i + \alpha \times EI_i, \forall i \in N$$

Step 2. Solve the OWMR problem with adjusted costs. Verify the solution with original costs and emission coefficients. Record the actual total cost and the total emissions.

Step 3. Update iteration index $l = l + 1$. If the total emissions is larger than the emission cap, update $\beta = \beta + (\frac{1}{2})^l$. Otherwise, update $\alpha = \beta - (\frac{1}{2})^l$.

Step 4. Update all adjusted costs. Solve the OWMR problem and verify the solution with original costs and emission coefficients. Record the actual total cost and the total emissions.

Step 5. Repeat *Step 3* and *Step 4* until l reaches 10, ten iterations have been performed.

Step 6. Discard all solutions that do not satisfy the emission constraint and pick the one with the least cost out of the remaining solutions.

At *Step 2* and *Step 4* of this Penalized Relaxation method, either an exact method or a heuristic can be applied to solve the OWMR problem. However, the iteration process at *Step 5* solves the OWMR problem 10 times with different parameters. This requires that the method that is chosen to solve the OWMR problem should be fast enough. Therefore, we choose SRA-TP (CE), which delivers a good solution quality within a short time, and integrate it into the PR heuristic.

The PR heuristic is tested in the same manner as FIFL-E and SP-E are tested in Section 4.5. For each of the 120 instances, 20 different levels of emission cap are applied. The emission cap is gradually tightened from the emission upper bound to the emission lower bound. Every step corresponds to 5% of MPR and at the 20th step, 100% of MPR need to be reduced.

Table 16 summarizes the average gap between the solution given by the heuristic and the optimal solution under 20 different levels of emission cap. There are a few cells with no gap. This indicates that at that level of emission cap, the PR heuristic fails to give a feasible solution for all of the 10 instances with the same size.

The rightmost column of Table 16 shows the average gap over different levels of emission cap of instances with the same size. As the number of retailers increases, the average gap tends to increase. However the trend is not very clear and there are some exceptions. With respect to the length of planning horizon, for instances with 30 periods the average gap over all experiments is 0.31%, which is lower than the average gap of 0.61% over all instances with 15 periods. Again, the trend is not clear and there exists some exceptions. The bottom row shows the average gap over instances with different sizes when imposing a certain level of emission cap.

Reduction (%MPR)			5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80	85	90	95	100	Average
retailers	**period**	I_{00}																					
50	15	0	0.00%	0.00%	0.00%	0.00%	0.00%	0.01%	0.00%	0.00%	0.06%	0.01%	0.00%	0.09%	1.32%	1.04%	0.71%	0.26%	0.09%	0.04%	0.02%	0.00%	**0.18%**
100	15	0	0.30%	0.32%	0.32%	0.34%	0.35%	0.37%	0.40%	0.43%	0.49%	0.80%	0.53%	0.55%	0.65%	0.94%	0.51%	0.45%	0.43%	0.36%	0.11%	0.00%	**0.43%**
150	15	0	0.46%	0.47%	0.48%	0.50%	0.65%	0.69%	0.75%	0.89%	0.88%	0.98%	1.14%	1.46%	1.51%	1.53%	1.53%	1.58%	2.22%	2.44%	-	-	**1.12%**
50	30	0	0.10%	0.10%	0.09%	0.07%	0.07%	0.06%	0.07%	0.09%	0.12%	0.15%	0.31%	0.30%	0.26%	0.22%	0.56%	0.57%	0.50%	0.49%	0.32%	0.00%	**0.22%**
100	30	0	0.31%	0.30%	0.30%	0.30%	0.30%	0.32%	0.40%	0.24%	0.24%	0.24%	0.27%	0.27%	0.31%	0.44%	0.45%	0.41%	0.45%	0.60%	0.20%	-	**0.33%**
150	30	0	0.43%	0.42%	0.41%	0.32%	0.31%	0.30%	0.29%	0.28%	0.27%	0.27%	0.26%	0.25%	0.24%	0.24%	0.27%	0.33%	0.59%	0.23%	0.28%	0.00%	**0.30%**
50	15	>0	0.25%	0.25%	0.25%	0.26%	0.26%	0.55%	0.59%	0.38%	0.39%	0.36%	1.03%	0.99%	1.08%	0.94%	0.57%	0.69%	0.72%	0.59%	0.18%	0.00%	**0.52%**
100	15	>0	0.31%	0.31%	0.32%	0.33%	0.35%	0.38%	0.41%	0.46%	0.55%	0.74%	0.65%	0.69%	0.75%	1.05%	0.72%	0.77%	0.66%	0.59%	0.15%	0.00%	**0.51%**
150	15	>0	0.33%	0.34%	0.36%	0.38%	0.53%	0.59%	0.65%	0.81%	0.79%	0.81%	0.90%	1.16%	1.18%	1.25%	1.32%	1.45%	1.98%	-	-	-	**0.87%**
50	30	>0	0.15%	0.15%	0.14%	0.13%	0.12%	0.12%	0.13%	0.15%	0.17%	0.21%	0.38%	0.49%	0.46%	0.42%	0.70%	0.72%	0.60%	0.64%	0.53%	0.00%	**0.32%**
100	30	>0	0.33%	0.35%	0.37%	0.39%	0.43%	0.52%	0.36%	0.37%	0.39%	0.41%	0.46%	0.47%	0.53%	0.73%	0.74%	0.78%	0.89%	1.13%	0.47%	-	**0.53%**
150	30	>0	0.45%	0.46%	0.48%	0.44%	0.44%	0.44%	0.43%	0.42%	0.41%	0.41%	0.40%	0.39%	0.37%	0.38%	0.41%	0.47%	0.74%	0.52%	0.42%	0.00%	**0.42%**
Average			**0.28%**	**0.29%**	**0.29%**	**0.29%**	**0.32%**	**0.36%**	**0.37%**	**0.38%**	**0.40%**	**0.45%**	**0.53%**	**0.59%**	**0.72%**	**0.77%**	**0.71%**	**0.71%**	**0.82%**	**0.69%**	**0.27%**	**0.00%**	**0.53%**

Table 16. Average Gaps of PR under Different Levels of Emission Cap

Reduction (%MPR)			5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80	85	90	95	100	Average
retailers	**period**	I_{00}																					
50	15	0	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	8	6	6	5	2	**8.9**
100	15	0	10	10	10	10	10	10	10	10	10	10	8	8	7	6	5	5	5	5	5	2	**7.8**
150	15	0	6	6	6	6	6	6	6	6	5	5	4	3	3	3	3	3	2	1	0	0	**4.0**
50	30	0	10	10	10	10	10	10	10	10	10	10	10	10	10	10	9	9	9	8	8	2	**9.3**
100	30	0	10	10	10	10	10	10	10	9	9	9	9	9	9	9	9	8	8	8	5	0	**8.6**
150	30	0	7	7	7	6	6	6	6	6	6	6	6	6	6	6	6	6	6	5	5	1	**5.8**
50	15	>0	10	10	10	10	10	10	10	9	9	9	9	9	9	9	9	7	7	5	4	2	**8.4**
100	15	>0	10	10	10	10	10	10	10	10	10	9	8	8	8	7	6	6	5	4	3	2	**7.8**
150	15	>0	6	6	6	6	6	6	6	6	5	5	5	4	2	2	2	2	2	0	0	0	**3.9**
50	30	>0	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	8	8	1	**9.4**
100	30	>0	10	10	10	10	10	10	9	9	9	9	9	9	9	9	8	7	7	6	2	0	**8.1**
150	30	>0	7	7	7	6	6	6	6	6	6	6	6	6	6	6	6	6	6	5	4	1	**5.8**
Average			**8.8**	**8.8**	**8.8**	**8.7**	**8.7**	**8.7**	**8.6**	**8.4**	**8.3**	**8.2**	**7.8**	**7.7**	**7.4**	**7.3**	**6.9**	**6.4**	**6.1**	**5.1**	**4.1**	**1.1**	7.3

Table 17. Number of Instances (out of 1 0) that PR Gives a Feasible Solution

Figure 12 depicts the shape of average gaps at different levels of emission cap. As the emission cap tightens, the average gap increases until an emission reduction of 70% MPR is imposed. Then the average gap remains at a relatively high level before it drops dramatically when the emission cap is very tight. It seems that the drop at the end can be explained by having less instances with a feasible solution, i.e., we are left with the somewhat easier instances.

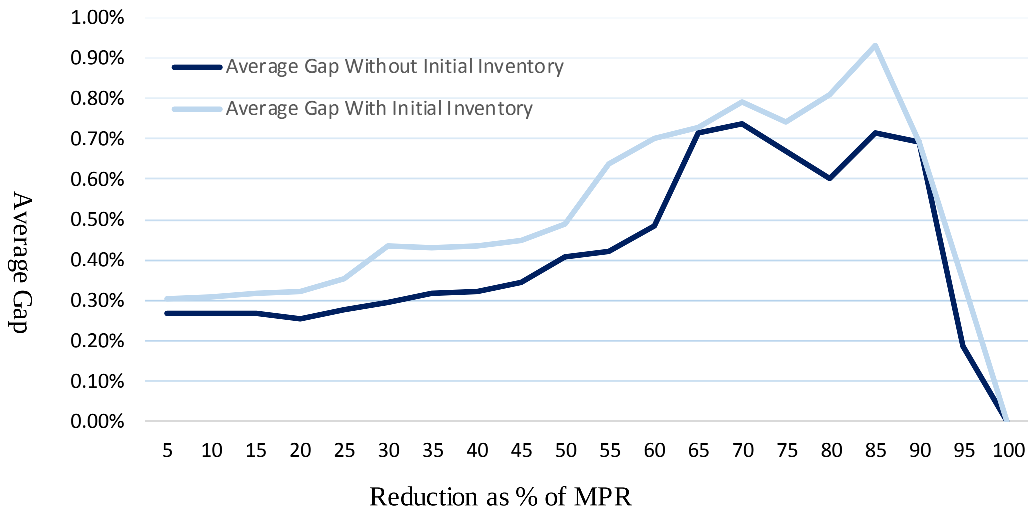


Figure 12. Average Gap of PR for All Instances under Different Levels of Emission Cap

Note that even though the heuristic has small average gaps when the emission cap is very tight (e.g. an emission reduction target of more than 90% MRP), it provides feasible solutions to only a few instances out of ten. In other instances, no solution is provided and therefore these instances are not accounted for the average gaps.

The number of instances solved by the heuristic measures the quality of the heuristic from another perspective. Table 17 gives detailed information on the number of instances solved by the heuristic. As the emission cap tightens, fewer instances can be solved by the heuristic. When we impose an emission cap of 100% MPR, the optimal solution is the only feasible solution, so it becomes difficult for the heuristic to find such a solution.

From this table, the impact of the size of instances on the solution quality can also be observed without ambiguity. When the number of retailers increases, there is less chance that the heuristic can provide a feasible solution. On the other hand, when the length of planning horizon increases, PR has a higher possibility of providing a feasible solution.

With respect to the computation time, on average PR only consumes less than 0.9 second to solve an OWMR-EC problem. This is only 0.69% of the time used by FIFL-E, the fastest formulation discussed in Section 4.5. The advantage in computation time is even more significant as the size of the instances become larger. As the size increases, the average CPU time for PR also increases, but the magnitude of increase in time is much smaller than that of FIFL-E. Table 19 presents the detailed average CPU times for PR to solve all instances under different levels of emission cap. Different from the exact methods, whose computation time is obviously affected by the tightness of the emission cap, PR has average computation times that are insensitive to the tightness of the emission cap.

			OWMR-EC		
Retailers	**Period**	I_{00}	**FIFL-E (s)**	**PR (s)**	**C.Time (%)**
50	15	0	9.2	0.3	2.88%
100	15	0	35.4	0.4	1.23%
150	15	0	77.2	0.6	0.75%
50	30	0	245.7	1.0	0.41%
100	30	0	1051.3	1.3	0.13%
150	30	0	1822.4	1.8	0.10%
50	15	>0	20.1	0.3	1.37%
100	15	>0	83.8	0.5	0.54%
150	15	>0	139.3	0.6	0.42%
50	30	>0	378.7	1.1	0.29%
100	30	>0	1819.8	1.5	0.08%
150	30	>0	2887.4	1.8	0.06%
Average			**714.2**	**0.9**	**0.69%**

Table 18. Summary of Average CPU Times to Solve an OWMR-EC Problem by FIFL-E and PR

Reduction (%MPR)			5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80	85	90	95	100	Average
Retailers	**Period**	I_{00}																					
50	15	0	0.27	0.26	0.27	0.27	0.26	0.27	0.26	0.27	0.27	0.26	0.27	0.26	0.26	0.26	0.26	0.27	0.26	0.27	0.26	0.26	**0.26**
100	15	0	0.42	0.46	0.44	0.45	0.44	0.45	0.44	0.44	0.45	0.44	0.43	0.44	0.44	0.43	0.42	0.43	0.43	0.43	0.43	0.43	**0.44**
150	15	0	0.57	0.58	0.58	0.58	0.60	0.58	0.58	0.58	0.58	0.58	0.58	0.60	0.59	0.58	0.58	0.58	0.58	0.58	-	-	**0.58**
50	30	0	0.99	0.98	0.98	0.97	0.99	0.97	0.99	0.98	0.98	0.98	0.98	0.98	0.98	0.97	1.02	1.03	1.03	1.03	1.04	1.25	**1.01**
100	30	0	1.33	1.32	1.32	1.34	1.32	1.58	1.34	1.36	1.32	1.33	1.32	1.36	1.34	1.32	1.34	1.32	1.32	1.33	1.35	-	**1.35**
150	30	0	1.79	1.77	1.77	1.76	1.76	1.76	1.76	1.77	1.76	1.76	1.76	1.80	1.76	1.77	1.77	1.77	1.77	1.78	1.76	1.77	**1.77**
50	15	>0	0.27	0.28	0.27	0.29	0.27	0.27	0.27	0.27	0.27	0.28	0.27	0.27	0.27	0.27	0.28	0.29	0.28	0.27	0.28	0.26	**0.28**
100	15	>0	0.45	0.46	0.45	0.45	0.45	0.46	0.46	0.45	0.45	0.45	0.45	0.46	0.45	0.45	0.45	0.46	0.45	0.46	0.44	0.43	**0.45**
150	15	>0	0.58	0.58	0.59	0.60	0.57	0.58	0.58	0.58	0.58	0.58	0.59	0.59	0.58	0.58	0.58	0.58	0.58	-	-	-	**0.58**
50	30	>0	1.05	1.05	1.20	1.05	1.04	1.04	1.04	1.04	1.04	1.04	1.04	1.04	1.04	1.04	1.12	1.13	1.12	1.12	1.13	1.67	**1.10**
100	30	>0	1.47	1.48	1.51	1.69	1.48	1.52	1.53	1.46	1.46	1.46	1.46	1.47	1.48	1.46	1.43	1.44	1.44	1.43	1.34	-	**1.47**
150	30	>0	1.82	1.92	1.85	1.81	1.81	1.82	1.85	1.83	1.81	1.86	1.82	1.82	1.82	1.82	1.82	1.82	1.83	1.84	1.82	1.74	**1.83**
Average			**0.92**	**0.93**	**0.94**	**0.94**	**0.92**	**0.94**	**0.92**	**0.92**	**0.91**	**0.92**	**0.91**	**0.92**	**0.92**	**0.91**	**0.92**	**0.92**	**0.92**	**0.96**	**0.99**	**0.98**	**0.93**

Table 19. Detailed Average CPU Times (in seconds) to Solve an OWMR-EC Problem under Different Level of Emission Cap Using PR

7. Conclusion

We have addressed the one-warehouse multi-retailer problem in its classical form as well as with a global emission constraint, which we denote as OWME-EC. Three known formulations for the OWMR problem are validated with a standard data set. Computational results show that the four index facility location formulation is significantly faster than the other two formulations, although the combined transportation and shortest path formulation provides slightly smaller average LP gaps.

To accommodate the production situation where companies need to limit their carbon emissions due to either regulations or their own target on environmental performance, we propose three formulations that incorporate a global emission constraint based on previously known formulations.

Due to the fact that there is no available standard data set that contains both cost factors and emission factors, we generated emission parameters that have a certain level of correlation with the cost parameters. Computational results show that the four index facility location formulation with an emission constraint (FIFL-E) is faster than the other two formulations. Results also show that as the emission cap tightens, the computation time increases dramatically.

Solving a series of OWMR-EC problems with different parameters also allows us to analyze the trade-off between costs and emissions. The (piecewise) convex trade-off curves indicate that the marginal cost of emission reduction tends to increase as the reduced amount increases. In other words, the first 1% emission reduction is cheaper than the second 1% emission reduction and the cost per percentage emission reduction grows when a higher reduction needs to be achieved.

Results with the first set of emission parameters, which are moderately correlated with the costs, shows that on average up to 3.2% of emissions can be reduced. The maximum potential emission reduction is not very significant compared to other methods such as using clean energy or upgrading machine. However, it can be achieved at the operational level, simply by the re-planning of production and delivery. Moreover, experiments with two other sets of emission parameters show that the maximum potential emission reduction is largely dependent on the correlation between cost and emission coefficients. Our experiments indicate that in the case where cost and emission factors are highly correlated, the potential to reduce emissions is very limited. On the other hand, in industries where costs and emissions are weakly correlated (e.g. labor intensive industries), higher emission reduction can potentially be achieved.

As the computation time for the OWMR-EC problem is considerably long, heuristics are proposed and tested to facilitate the computation process. We first design an efficient heuristic for the OWMR and then adapt it to the OWMR-EC. We decompose the OWMR problem into two stages. Different combinations of heuristics for these two stages are tested. Results show that the combination of the Simple Retailers Aggregation (SRA) method and the Time-Partitioning Fix-and-Optimize method using Complete Enumeration (TP (CE)) provides good solutions to the OWMR problem within a very short time. In this combination SRA is applied to decide upon the production setup decisions and TP (CE) is used to decide the delivery plan to the retailers. Finally, a penalized relaxation approach is applied to satisfy the emission constraint by iteratively solving the OWMR problem using SRA and TP (CE). This proposed heuristic solves the OWMR-EC problem in less than one second on average and has an average optimality gap of 0.65%. However, it is not always able to find feasible solutions, especially for the highly constrained problems.

The results of this research are limited to a specific problem where there is one plant (or warehouse) that has unlimited capacity and multiple retailers. A promising research avenue is to apply the emission constraint to other lot sizing problems. Future researches can also implement other types of emission constraints instead of a global strict emission cap.

References

Absi, N., Dauzère-Pérès, S., Kedad-Sidhoum, S., Penz, B., & Rapine, C. (2013). Lot sizing with carbon emission constraints. *European journal of operational research*, *227*(1), 55-61.

Adulyasak, Y., Cordeau, J. F., & Jans, R. (2012). Optimization-based adaptive large neighborhood search for the production routing problem. *Transportation Science*, *48*(1), 20-45.

American Association for the Advancement of Science. (2009). Statement on climate change from 18 scientific associations. Retrieved Agu. 20, 2014, from http://www.aaas.org/sites/default/files/migrate/uploads/1021climate_letter1.pdf

Arıkan, E., & Jammernegg, W. (2014). The single period inventory model under dual sourcing and product carbon footprint constraint. *International Journal of Production Economics*.

Arkin, E. (1989). Computational complexity of uncapacitated multi-echelon production planning problems. *Operations Research Letters, 8*(2), 61-66.

Benjaafar, S., Li, Y., & Daskin, M. (2013). Carbon Footprint and the Management of Supply Chains: Insights From Simple Models. *IEEE Transactions on Automation Science and Engineering, 10*(1), 99-116.

Bogusz, C., & Howlett, C. (2008). *Policy Options for Reducing CO2 Emissions*.

Cachon, G. P. (2014). Retail store density and the cost of greenhouse gas emissions. *Management Science*.

Chen, X., Benjaafar, S., & Elomri, A. (2013). The carbon-constrained EOQ. *Operations Research Letters, 41*(2), 172-179.

Dekker, R., Bloemhof, J., & Mallidis, I. (2012). Operations Research for green logistics – An overview of aspects, issues, contributions and challenges. *European journal of operational research, 219*(3), 671-679.

Demir, E., Bektaş, T., & Laporte, G. (2014). A review of recent research on green road freight transportation. *European journal of operational research, 237*(3), 775-793.

Federgruen, A., & Tzur, M. (1999). Time-partitioning heuristics: application to one warehouse, multiitem, multiretailer lot-sizing problems. *Naval Research Logistics*, *46*(5), 463-486.

Girod, B., van Vuuren, D. P., Grahn, M., Kitous, A., Kim, S. H., & Kyle, P. (2013). Climate impact of transportation A model comparison. *Climatic change, 118*(3-4), 595-608.

Harris, F. W. (1913). How Many Parts to Make at Once. Factory. *The Magazine of Management* 10, 135-136, 152

Hoen, K., Tan, T., Fransoo, J., & Van Houtum, G. (2014). Effect of carbon emission regulations on transport mode selection under stochastic demand. *Flexible Services and Manufacturing Journal, 26*(1-2), 170-195.

Hoen, K. M., Tan, T., Fransoo, J. C., & van Houtum, G.-J. (2013). Switching Transport Modes to Meet Voluntary Carbon Emission Targets. *Transportation Science*.

Hua, G., Cheng, T. C. E., & Wang, S. (2011). Managing carbon footprints in inventory

management. *International Journal of Production Economics, 132*(2), 178-185.
Jaber, M. Y., Glock, C. H., & El Saadany, A. M. (2013). Supply chain coordination with emissions reduction incentives. *International Journal of Production Research, 51*(1), 69-82.
Jans, R., & Degraeve, Z. (2008). Modeling industrial lot sizing problems: a review. *International Journal of Production Research, 46*(6), 1619-1643.
Levi, R., Roundy, R., Shmoys, D., & Sviridenko, M. (2008). A constant approximation algorithm for the one-warehouse multiretailer problem. *Management Science, 54*(4), 763-776.
Palak, G., Ekşioğlu, S. D., & Geunes, J. (2014). Analyzing the impacts of carbon regulatory mechanisms on supplier and mode selection decisions: An application to a biofuel supply chain. *International Journal of Production Economics, 154*, 198-216.
Pochet, Y., & Wolsey, L. A. (2006). *Production planning by mixed integer programming*: Springer.
Retel Helmrich, M. J., Jans, R., van den Heuvel, W., & Wagelmans, A. P. (2015). The economic lot-sizing problem with an emission capacity constraint. *European Journal of Operational Research, 241*(1), 50-62.
Romeijn, H. E., Morales, D. R., & Van den Heuvel, W. (2014). *Computational complexity of finding Pareto efficient outcomes for biobjective lot-sizing models*. Naval Research Logistics (NRL), 61(5), 386-402.
Ruokokoski, M., Solyali, O., Cordeau, J.-F., Jans, R., & Süral, H. (2010). *Efficient formulations and a branch-and-cut algorithm for a production-routing problem*: Groupe d'études et de recherche en analyse des décisions.
Shrivastava, P. (1995). Environmental Technologies and Competitive Advantage. *Strategic Management Journal, 16*(SPECIAL ISSUE), 183-200.
Silver, E. A., & Meal, H. (1973). A heuristic for selecting lot size quantities for the case of a deterministic time-varying demand rate and discrete opportunities for replenishment. *Production and Inventory Management, 14*(2), 64-74.
Solyalı, O., & Süral, H. (2012). The one-warehouse multi-retailer problem: reformulation, classification, and computational results. *Annals of Operations Research, 196*(1), 517-541.
Song, J., & Leng, M. (2012). Analysis of the Single-Period Problem under Carbon Emissions Policies. In T.-M. Choi (Ed.), *Handbook of Newsvendor Problems* (Vol. 176, pp. 297-313): Springer New York.
United Nations (1998). Kyoto Protocol to the United Nations Framework Convention on Climate Change. Retrieved Aug. 20, 2014, from http://unfccc.int/resource/docs/convkp/kpeng.pdf
United Nations (2012). Doha amendment to the Kyoto Protocol. Retrieved Aug. 20, 2014, from http://unfccc.int/files/kyoto_protocol/application/pdf/kp_doha_amendment_english.pdf
UPS. (2014). UPS Corporate Sustainability Report 2013: UPS.
Voiland, A. (2009). 2009: Second Warmest Year on Record; End of Warmest Decade.

Retrieved Aug. 20, 2014, from http://www.nasa.gov/topics/earth/features/temp-analysis-2009.html

Wagner, H. M., & Whitin, T. M. (1958). Dynamic version of the economic lot size model. *Management Science, 5*(1), 89-96.

World Bank. (2014). State and Trends of Carbon Pricing. Washington DC: World Bank.

Zhang, B., & Xu, L. (2013). Multi-item production planning with carbon cap and trade mechanism. *International Journal of Production Economics, 144*(1), 118-127.