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CVA Calculation with Implied Recovery

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Abstract

This Master's thesis proposes an approach to calculate CVA (credit value adjustment) with the implied recovery rates. We use the methodology introduced by Das and Hanouna (2009) to extract the implied recovery rates and probabilities of default from CDS spreads. We compare CVA calculated with implied recovery rates to CVA calculated with constant recovery rates. The results show that constant recovery CVA generally underestimates the credit risk exposure, compared to the model-implied CVA. Using CVA with non-constant recovery appears to be more prudent to protect against credit risk.

Keywords: CVA; Implied recovery rate; Credit risk; OTC derivatives

Résumé

Ce mémoire propose une approche pour calculer CVA avec les taux de recouvrement implicites et endogènes. Nous utilisons la méthode proposée par Das et Hanouna (2009), qui permet d'extraire les taux de recouvrement et les probabilités de default implicites des écarts de CDS. Nous comparons CVA calculé avec les taux de recouvrement constants et CVA calculé avec les taux de recouvrement implicites. Les résultats montrent que CVA avec taux de recouvrement constants sous-estime l'exposition au risque de crédit, en comparaison de CVA calculé avec taux de recouvrement implicites. CVA calculé avec taux de recouvrement non constants apparait plus prudent pour se protéger contre le risque de crédit.

Mots-clés : CVA; taux de recouvrement implicite, risque de crédit; Produits dérivés

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Contents

Introduction

Like the Russia/LTCM crisis in 1998, the collapse of financial system in 2008 showed that credit risk was an important source of systematic risk. The modern financial system is "an interwoven network of financial obligations" and any counterparty default can lead to an important "gridlock" due to the network effects (Brunnermerier, 2009). This interconnectedness was largely built by the OTC derivative markets, and the failure to capture the credit risk on these off-balance sheet transactions was the central reason of the amplified financial crisis.

CVA (credit value adjustment) is the most recent methodology to measure the market value of counterparty credit risk for OTC derivatives. In fact, the concept of "credit charges" (what we call CVA now) dates back to more than the mid-1990s. After the Russian bond default and LTCM crisis in 1998, large dealers started calculating CVA to assess the cost of the counterparty credit risk. In 2006, new accountancy regulations (FAS 157, IAS 39) required that the value of derivatives positions be corrected for counterparty risk and banks start calculating CVA on a monthly basis. Nevertheless, it is from the 2008 financial crisis that financial institutions really began managing and hedging counterparty credit risk actively. Some large dealers have set up CVA desks to adjust the price of OTC derivatives and reduced the variation of CVA charge. In this context, regulators (Basel III) have also devised a new regulatory framework by better addressing counterparty credit risk and CVA.

As a credit risk estimator, CVA is composed of three major components: exposure at default, probability of default and recovery rate at default. We find an extensive literature on the first two components with much less on the third one. Recovery rate has often been assumed as a constant (commonly 40%) for CVA calculation.

However, much evidence suggests that historical recovery rate possesses large variation according to different seniorities, sectors and issuers. For example, the historical recovery rate seems to be volatile and to coincide somewhat with macroeconomic cycles: during the crisis, the aggregate recovery rate tends to decrease compared with economic boom periods. (Figure 1) Altman & al. (2005) show that variance of recoveries on corporate bonds can be explained substantially by default rates. Moody's reports on "Corporate Default and Recovery Rate" project the different distributions of recovery rate across all seniority and rating levels. Archarya & al. (2007) point out that creditor recoveries are heavily influenced by industry conditions at the time of default.

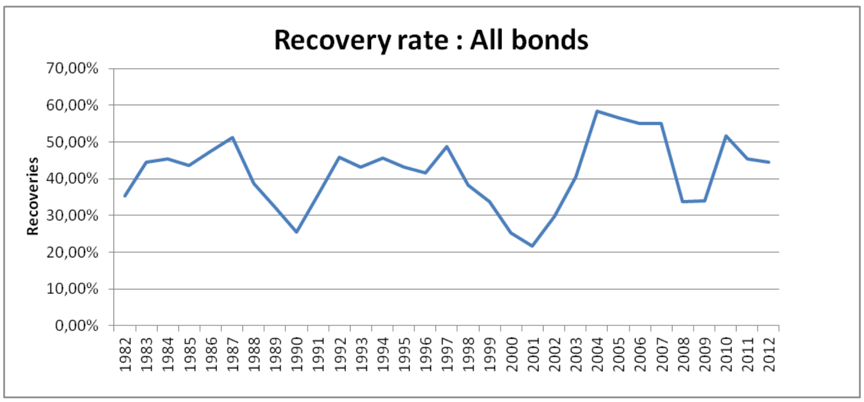

Figure 1: Historical Average Recovery rate[1] for all bonds (from Moody's 1920-2012)

Due to these characteristics of historical data, it is inaccurate and unrealistic to treat recovery rate as certain when modeling credit risk. In this work, we aim to improve the methodology of CVA calculation with forward term structure of implied recovery rate.

This Master's thesis is divided into 4 sections. In the first part, we review the literature on extracting default probability and recovery rate term structures. We also study the methodology of CVA calculation. In the next section, we develop the methodology of CVA calculation using implied recovery. The empirical results will be presented and

[1] The recovery rate is measured by the market price of defaulted bond as a percentage of par, observed 30 days after default.

analysed in Section 4. Finally, we resume our findings and provide some possible directions for further research.

Section II Literature Review

In this part, we review at first CVA calculation and then the literature on implied recovery rate.

A. Credit value adjustment (CVA) calculation

Credit value adjustment (CVA) by definition is the difference between the risk-free value of a financial contract and its risky value accounting the default probability of the counterparty before the maturity. In other words, CVA represents the market price of the counterparty credit risk. Two classes of financial products are subject to counterparty risk: OTC derivatives and securities financing transactions, such as repo, security borrowing and lending. They are privately negotiated and not supervised and guaranteed by the organized market. As the recent financial crisis has shown, no counterparty is risk-free, and consequently counterparty credit risk must be taken into the valuation of OTC derivatives.

A.1. Assumptions and general formula

Here we introduce the standard way to calculate CVA based on the methodology of Gregory (2012). Suppose that an institution (for example, an investment bank) gets into a derivative position with its counterparty (for example, an orange juice producer). We calculate the unilateral CVA from the perspective of the institution.

The main hypotheses are:

1. The institution itself cannot default.
2. Risk-free valuation is straightforward.
3. The credit exposure and default probability are independent.

We define the risk-free value of an OTC derivative as $V(t,T)$ and the true risky value as $\tilde{V}(t,T)$, where T is the maturity of the contract. We set τ as the time that the counterparty defaults. $\phi(\tau,T)$ is the recovery fraction of the risk-free value of the derivative at default.[2]

There are two possible scenarios:

1. $\tau > T$, counterparty does not default before maturity. And $I(\tau > T)$ takes the value of 1 when default occurs after or at T and takes the value of 0 otherwise. The payoff at t is $I(\tau > T) * V(t,T)$.
2. $\tau \leq T$, counterparty does default before maturity. And $I(\tau \leq T)$ takes the value of 1 when default occurs before or at T and takes the value of 0 otherwise.

   In this case, Gregory (2012) separates the payoff into two parts: cash flows paid up to the default time $I(\tau \leq T) * V(\tau,T)$ and payoff at default time τ. $V(\tau,T)$ is the market risk-free value at default. If $V(\tau,T) > 0$, the institution gets $\phi(\tau,T) * V(\tau,T)$ from the counterparty, where $\phi(\tau,T)$ indexes the recovery rate. If at τ, $V(\tau,T) \leq 0$, the institution still needs to settle the contract and the payoff is $V(\tau,T)$. Hence, the payoff at τ is $I(\tau \leq T) * (\phi(\tau,T) * V(\tau,T)^{+} + V(\tau,T)^{-})$, Where $V(\tau,T)^{+} = \max(V(\tau,T),0)$ and $V(\tau,T)^{-} = \min(V(\tau,T),0)$.

   Combining these two terms, the total payoff in the case of the counterparty's default before maturity is:

$$I(\tau \leq T) * V(\tau,T) + I(\tau \leq T) * (\phi(\tau,T) * V(\tau,T)^{+} + V(\tau,T)^{-})$$

We now can write the risky value of the contract under the risk-neutral measure at t as:

$$\tilde{V}(t,T) = E^{Q}\begin{bmatrix} I(\tau > T) * V(t,T) + \\ I(\tau \leq T) * V(t,\tau) + \\ I(\tau \leq T) * (\phi(\tau,T) * V(\tau,T)^{+} + V(\tau,T)^{-}) \end{bmatrix}$$

Replacing $V(\tau,T)^{-}$ by $V(\tau,T) - V(\tau,T)^{+}$, then we get

[2] $\phi(\tau,T)$ refers to recovery of market value (RMV) in the next section.

$$\tilde{V}(t,T) = E^Q \begin{bmatrix} I(\tau > T) * V(t,T) + \\ I(\tau \le T) * V(t,\tau) + \\ I(\tau \le T) * (\phi(\tau,T) * V(\tau,T)^+ + V(\tau,T) - V(\tau,T)^+) \end{bmatrix}$$

$$\tilde{V}(t,T) = E^Q \begin{bmatrix} I(\tau > T) * V(t,T) + \\ I(\tau \le T) * V(t,\tau) + \\ I(\tau \le T) * ((\phi(\tau,T) - 1) * V(\tau,T)^+ + V(\tau,T)) \end{bmatrix}$$

Because $V(t,\tau) + V(\tau,T) = V(t,T)$, then we get

$$\tilde{V}(t,T) = E^Q \begin{bmatrix} I(\tau > T) * V(t,T) + \\ I(\tau \le T) * V(t,T) + \\ I(\tau \le T) * ((\phi(\tau,T) - 1) * V(\tau,T)^+) \end{bmatrix}$$

Since $I(\tau > T) + I(\tau \le T) = 1$, then we have

$$\tilde{V}(t,T) = E^Q[V(t,T) + I(\tau \le T) * ((\phi(\tau,T) - 1) * V(\tau,T)^+)]$$

$$\tilde{V}(t,T) = V(t,T) - E^Q[I(\tau \le T) * ((1 - \phi(\tau,T)) * V(\tau,T)^+)]$$

The subtrahend term is the credit value adjustment (CVA), the difference between the risk-free value and the true risky value.

$$CVA(t,T) = E^Q[I(\tau \le T) * ((1 - \phi(\tau,T)) * V(\tau,T)^+)]$$

The derivation shows that credit risk of the counterparty can be priced independently from the traditional risk-free valuation of the derivative. This result is consistent with the third assumption, which supposes independence between the exposure and default risk.

As the expectation value covers all the time from t to T, we can rewrite the formula before as the result of integration.

$$CVA(t,T) = E^Q\left[\int_t^T B(t,u)V(u,T)^+ \big(1 - \phi(t,u)\big)dF(t,u)\right]$$

Where $B(t,u)$ is the discount factor and $F(t,u)$ is the cumulative default probability.

For the discrete time formula, suppose that there are N periods from t to T : $\{t_0, t, t_1, t_2, \ldots, t_N\}$, where $t_0 = t$ and $t_N = T$. We then denote the expected exposure and the default probability for each period as: $EE(t_{i-1}, t_i) = E^Q[B(t_{i-1}, t_i)V(t_i)^+]$ and $\lambda_i = F(t, t_i) - F(t, t_{i-1})$.

$$CVA(t,T) \approx \sum_{i=1}^{N} EE(t_{i-1}, t_i) * \left(1 - \phi(t_{i-1}, t_i)\right) * \lambda(t_{i-1}, t_i)$$

In the case of an option, CVA formula should be simple because the long position is always positive. We set $V_{option}(t,T)$ as the upfront premium of the option and CVA should be written as below:

$$CVA_{option} \approx V_{option}(t,T) * \sum_{i=1}^{N} \left(1 - \phi(t_{i-1}, t_i)\right) * \lambda(t_{i-1}, t_i)$$

Sorensen and Bollier (1994) show that CVA can be written as a series of (reserve) European swaptions for an interest rate swap. In fact, the position of the interest rate swap could be positive or negative so that CVA could be 0 or positive depending the position value.

$$CVA_{swap} \approx \sum_{i=1}^{m} \left(1 - \phi(t_{i-1}, t_i)\right) * \lambda(t_{i-1}, t_i) * V_{swaption}(t; t_i, T)$$

“The intuition is that the counterparty might default at any time in the future and, hence, effectively cancel the non-recovered value of the swap, economically equivalent to exercising the reverse swaption.” (Gregory, 2010, page 248)

A.2. BCVA and DVA (Hypothesis 1)

The conception of DVA (debit value adjustment) has arisen because the institution itself can default, just like its counterparty. In other words, we drop down the first assumption. Obviously, the contingent component for DVA is the negative expected exposure (NEE) of the instrument (from the perspective of the institution). Let's denote $\lambda_I(t_{i-1}, t_i)$ and $\phi_I(t_{i-1}, t_i)$ as the default probability and the recovery rate of the institution for the period (t_{i-1}, t_i). DVA exists only when the institution itself defaults whereas the counterparty hasn't defaulted yet.

$$DVA(t,T) = \sum_{i=1}^{N} NEE(t_{i-1}, t_i) * \left(1 - \phi_I(t_{i-1}, t_i)\right) * \lambda_I(t_{i-1}, t_i) * (1 - \lambda_C(0, t_{i-1}))$$

At the same time, we should also adjust CVA. CVA should exist only when both the counterparty defaults and the institution hasn't defaulted yet. We denote $\lambda_C(t_{i-1}, t_i)$ and $\phi_C(t_{i-1}, t_i)$ as the default probability and the recovery rate of the counterparty for the period (t_{i-1}, t_i).

$$CVA(t,T) = \sum_{i=1}^{N} EE(t_{i-1}, t_i) * \left(1 - \phi_C(t_{i-1}, t_i)\right) * \lambda_C(t_{i-1}, t_i) * (1 - \lambda_I(0, t_{i-1}))$$

BCVA (bilateral CVA) is defined as: $BCVA = CVA + DVA$. And the price of the OTC derivative should be: $\tilde{V} = V(t,T) - BCVA = V(t,T) - CVA - DVA$.

The paradox here is that if the institution becomes riskier, the absolute value of DVA will increase, which leads also to an increase in $\tilde{V}$. Institutions might make profits due to the decline of their own credit quality.

A.3. OIS and Funding (Hypothesis 2)

The second hypothesis implies that Libor is risk-free and that the institution can easily borrow and lend at Libor. However, the financial turmoil in 2008 has told us that the two conditions do not hold.

"OIS discounting" means that instead of using Libor as the discounting rate, we employ OIS as the discount factor. In fact, Libor includes credit risk while OIS is now viewed as the risk-free rate.

Funding value adjustment (FVA) includes the funding cost of the transaction (for the institution) in the derivative pricing. This cost exists only when neither the institution nor the counterparty has defaulted. We denote $FS(t_{i-1}, t_i)$ as the funding spread (both for lending and borrowing) between t_{i-1} and t_i. The formula for FVA is as below:

$$FVA = \sum_{i=1}^{N} EE(t_{i-1}, t_i) * (1 - \lambda_C(0, t_{i-1})) * \left(1 - \lambda_I(0, t_{i-1})\right) * FS(t_{i-1}, t_i) * (t_i - t_{i-1}) + \sum_{i=1}^{N} NEE(t_{i-1}, t_i) * (1 - \lambda_C(0, t_{i-1})) * \left(1 - \lambda_I(0, t_{i-1})\right) * FS(t_{i-1}, t_i) * (t_i - t_{i-1})$$

The risky value of the derivative should be written as:

$$\widetilde{V} = V(t, T) - BCVA - FVA$$

A.4. Wrong-way risk (Hypothesis 3)

Wrong-way risk is to identify the unfavourable correlation between the exposure and the counterparty credit quality. (Gregory, 2012, page 307) The counterparty credit quality falls when the exposure is higher and vice versa. Wrong-way risk will surely increase CVA and make it more complex to calculate.

To price CVA in this case, Gregory (2012) replaces the unconditional expected exposure by the expected exposure conditional on the default of the counterparty. We denote τ_c as the counterparty default time and CVA can be expressed as below:

$$CVA(t,T) \approx \sum_{i=1}^{N} EE(t_i | t_i = \tau_c) * \left(1 - \phi(t_{i-1}, t_i)\right) * \lambda(t_{i-1}, t_i)$$

Further, Hull and White (2012) propose another way to calculate CVA with wrong-way risk. Instead of fixing the default time, they set the hazard rate as a function of the future value of the derivative: $h(t) = exp[a(t) + bw(t) + \chi\varepsilon]$, where $a(t)$ is a function of time, b is a constant parameter, $w(t)$ is the value of the derivative, χ measures the amount of noise in the relationship and ε is normally distributed with zero mean and unit variance.[3]

To conclude, CVA is relatively a complex subject covering a vast array of recent topics. Indeed, it is difficult to calculate precisely CVA. The literature mostly focuses on the expected exposure and default probability measurement. In this Master's thesis, we try to improve CVA calculation with market implied recovery rate.

[3] Recovery rate is treated as a constant in this case.

B. Recovery rate

In the case of default, losses occur mainly in three ways: losses from the principal, carrying costs of non-performing loans (e.g. interest income foregone) and trivial workout expenses (e.g. legal, administrative and accounting fees). Moreover, the default process and the ultimate payouts to the bondholders might not be realized within a short delay. It typically takes around two years from the default date to the emergency from Chapter 11. Due to these physical traits, the measurement of recovery rate becomes susceptible: the fractional recovery of which value and at which moment? In this section, we review at first the different recovery processes in the structural model and in the reduced form model; then the different definitions of recovery rate, and at the end, the literature on extracting implied recovery rates from market data.

B.1. Recovery rate process: structural models VS reduced form models

Based on Jarrow and Protter (2004), we compare the recovery rate process in the structural model versus in the reduced form model.

We suppose that the face value of the firm's debt is one dollar with maturity T. Denote r_t as the risk-free rate and τ as the time that default occurs. The indicator $I(\tau \leq T)$ takes the value 1 if default occurs before T and takes 0 otherwise. For the structural model, we determine the default barrier and in the case of default, the bondholders receive a fraction δ_τ of the firm's debt. In the reduced form model, ϕ_τ is assumed to be the recovery rate at default.

The formula from the structural model to evaluate the firm's debt at time 0 is:

$$v(0,T) = E([\, I(\tau \leq T) * \delta_\tau + I(\tau > T) * 1]e^{-\int_0^T r_s ds})$$

The formula from the reduced form model to evaluate the firm's debt at time 0 is:

$$v(0,T) = E([\,I(\tau \le T) * \phi_\tau + I(\tau > T) * 1]e^{-\int_0^T r_s ds})$$

The distinction is small but crucial between the two formulas. "The recovery rate process is prespecified by a knowledge of the liability structure in the structural approach, while (in the reduced form models) it is exogenously supplied." (Jarrow and Protter, 2004, page 5)

In the same article, the authors argue that the difference between the two classes of model is due to the information assumed known by the modeler. In the structural model, the modeler has complete knowledge of all the firm's assets and liabilities, while in the reduced form model, the modeler has incomplete knowledge of the firm's condition but the same information as the market. In this sense, they conclude that in a pricing context, reduced form models are preferred.

B.2. Definitions of recovery rate

As our methodology for pricing credit risk is mostly related to the reduced form models, we review the different definitions of recovery rate in this context. We trace back to the framework of Duffie and Singleton (1999) for pricing defaultable bonds.

Suppose that we price a defaultable bond with the final payment $\$\,X$. As before, we denote r_t as the short-term risk-free rate and τ indexes the time that default occurs. $B(t,T) = E^Q\left[exp\left(-\int_t^T r_s\, ds\right)\right]$ is the time t price of Treasury zero-coupon bond (face value of \$ 1 and maturity T). In the case of default, the recovery amount at time τ (in \$) is $\varphi(\tau,T)$. The value of the defaultable bond at t is:

$$\tilde{V}(t,T) = B(t,\tau) * \varphi(\tau,T) * I(\tau \le T) + B(t,T) * X * I(\tau > T)$$

And the recovery rate ϕ_τ can be expressed as:

$$\phi_\tau = \frac{\varphi(\tau, T)}{denominator\ value}$$

According to the different assumptions for the denominator, Duffie and Singleton (1999) present three ways of recovery rate modeling: 1) recovery of par (face value), 2) recovery of Treasury and 3) recovery of market value.

- **Recovery of par (RFV)**

Recovery rate here is a fraction of face value, which is generally consistent with the bond covenants and CDS contracts. Recovery of face value is also conformable with liquidation at default and absolute priority rules since it "implies equal recovery for bonds of equal seniority of the same issuer" (Duffie and Singleton, 1999).

In reality, it is often adopted by the rating agencies. For example, Moody's uses the ratio of the market bid price observed roughly 30 days after default to its face value. This measurement is not only direct and objective but also reflects the market expectation for the ultimate recovery soon after the credit event for the final recovery rate.

In the literature of reduced form models, recovery of par refers to the "RFV" model in Duffie and Singleton (1999). The recovery amount $\varphi(\tau, T)$ can be written as a function of recovery of par:

$$\varphi(\tau, T) = \phi_\tau * X$$

The value of the defaultable bond at t is:

$$\tilde{V}(t, T) = B(t, \tau) * \phi_\tau * X * I(\tau \le T) + B(t, T) * X * I(\tau > T)$$

When $\phi_\tau = \phi_0$, the recovery rate becomes a constant fraction of the face value, as in the model of Duffie (1999) and Lando (1998). Recovery rate of par captures only the loss from the principle and implies zero recovery for the coupons.

- **Recovery of Treasury (RT)**

Recovery of Treasury is the fractional recovery of the present value of the face value, which is equivalent to the price of a Treasury bond with the same face value and maturity.

In this case, the recovery amount $\varphi(\tau, T)$ can be expressed as below:

$$\varphi(\tau, T) = \phi_\tau * B(\tau, T) * X$$

And the value of the defaultable bond at t is:

$$\tilde{V}(t, T) = B(t, T) * \phi_\tau * X * I(\tau \leq T) + B(t, T) * X * I(\tau > T)$$

This method is employed by Longstaff and Schwartz (1995) and Jarrow and Turnbull (1995).

- **Recovery of market value (RMV)**

Recovery rate here is the fraction of the pre-default market value of the defaultable bond. In this case, the recovery amount $\varphi(\tau, T)$can be expressed as below:

$$\varphi(\tau, T) = \phi_\tau * \tilde{V}(\tau^-, T)$$

And the value of the defaultable bond at t is:

$$\tilde{V}(t, T) = B(t, \tau) * \phi_\tau * \tilde{V}(\tau^-, T) * I(\tau \leq T) + B(t, T) * X * I(\tau > T)$$

This RMV model is proposed by Duffie and Singleton (1999) and it assumes the same recovery rate for coupons and principle. With this definition of recovery rate, they price the defaultable bond as the final payment X discounted at the risk-free rate plus a default premium. The equation is as below:

$$\tilde{V}_0 = E_0^Q\left[exp\left(-\int_0^T R_t \, dt\right)X\right], \quad R_t = r_t + h_t(1 - \phi_t)$$

Where h_t is the hazard rate and ϕ_t is the fractional recovery of market value if default occurs at time t. As we can see, the pricing of credit risk is subsumed within the discount factor R_t. In this way, defaultable bonds can be priced by any default-free term structure modeling techniques.

B.3. Implied recovery rate

The literature of extracting implied recovery rate from market data is relatively new and limited. Due to the market context, most of the papers extract the fractional recovery of face value. Here, we review several representative research papers and compare them.

B.3.1. Related Literatures

Bakshi, Madan and Zhang (2006) assume that default probability and recovery rate are functions of the risk-free rate and allow dependence between the two variables. Using the market data of 25 BBB-rated unsecured US corporate bonds from 1989 to 1998, the authors extract distinct implied default probability and recovery rate in time series by a calibration model. Further, they show that compared with RFV and RMV, RT provides better fitting models.

Similarly, Gaspar and Slinko (2008) suppose that hazard rate and recovery rate are related to S&P 500 and that the two rates are negatively correlated. They assume the constant recovery rate for all firms in the data and finally obtain an average recovery rate of 30 % of US investment grade bonds from 2004 to 2007.

Güntay, Madan and Unal (2003) propose a completely different framework. They apply a pure recovery model which is free of default timing consideration. This ARS (adjust relative spread), which should only be related to recovery level and variance, is defined as the proportion of senior debt times the ratio of the difference between the prices of the senior debts and junior debts to the difference between default-free debts and junior debts. After transformation, the formula for ARS is as below:

$$ARS = \frac{E(y) - E(y^J)}{1 - E(y^J)}$$

Where y is the aggregate recovery rate to all outstanding debt and y^J is the average recovery rate of junior debt. Recovery rates are supposed to be a transformed normal distribution and related to the risk-free rate and the firm's asset tangibility. Using the data of 28 US corporate junior and senior bonds from 1990 to 1997, their results show that the risk-neutral recovery level and variance are generally much lower than the industrial historical recovery.

Another category of models exploits the information carried in the CDS term structures.

Zhang (2003) aims to separate default probability from a constant expected recovery rate throughout the period of Argentinean sovereign CDS data and across all the maturities. The model allows the default probability to be the function of three state variables related to the risk free rate. The result of Zhang (2003) suggests an expected recovery rate of 27.5% for Argentina in 2001. Pan and Singleton (2008) adopts a similar methodology. They allow recovery to change in time series but stay constant across all the CDS maturities. They extract recovery rates of face value from the sovereign CDS spreads of three countries (Mexico, Turkey and Korea) and suggest average recovery rates of 76.9%, 76.4% and 16.7% respectively, which is quite different from Zhang (2003). Schneider, Sögner and Veza (2009) apply the same methodology of Pan and Singleton (2008) to CDS on senior unsecured bonds. They find an overall implied recovery of 79 % for 282 US obligors from 2004 to 2006.

Our methodology to extract implies recovery rates is based mainly on Das and Hanouna (2009). They suppose that recovery rate and default probability are correlated and are functions of firm-specific stock price and its volatility. The authors have developed a calibration model to extract the forward term structure of implied recovery and default probability from CDS spreads of different maturities. Using data of *CreditMetrics* from January 2000 to July 2002, they arrive with declining recovery rate term structures for all the quintiles. However, the average is quite different: from 90% implied recovery for the best quality credit quintile to 30% for the worst quintile.

B.3.2. Comparison of the methodologies

We can differentiate these models in three ways: 1) the extracted information sources, 2) dependence (independence) between the default probability and recovery rate and 3) market available information, to which hazard rate and recovery rate are related.

Obviously, there are two extracted information sources: corporate bonds and CDS term structures. Which one is better? First of all, they are both problematic since wealth of evidence showing nondefault components is significant in bond prices and CDS spreads. Elton & al. (2001) point out that the rate spreads between corporate and government bonds include three main components: expected default loss, taxes and systematic risk premium. By adding the dependence between default probability and recovery rate, Dionne & al. (2008) has proved that the proportion of default risk can reach 76% for Canadian Baa bonds during the 1987-1991 period.

There are also recent studies indicating liquidity premium in bond and CDS markets. For corporate bonds, Longstaff & al. (2005) explain that the significant nondefault component is highly related to the illiquidity measures, such as bid-ask spreads, outstanding principle amount and Treasury richness. In the CDS markets, Guo and Newton (2001) show that liquidity determinant is significant and time-varying both in dummy-variable pooling regression and Markov regime-switching model. Arora & al. (2012) also demonstrate that the counterparty risk of the seller is priced in the CDS spreads.

It might be difficult to conclude which source is better for extracting implied default measure, although some research has shown that CDS market has a lead over the bond market for corporate (Coudert and Gex, 2010, Alexopoulou & al. 2009).
It is interesting to note that the pure recovery model of Güntay, Madan and Unal (2003) could be an efficient way to eliminate the irrelevant components to recovery rate, such as liquidity risk. This would be done by taking the difference between the senior and junior bonds. However, it requires two classes of bonds, which limits the applicable objectives.

Furthermore, Bakshi, Madan and Zhang (2001), Gaspar and Slinko (2008) and Das and Hanouna (2009) allow the dependence between default probability and recovery rate. The others assume the independence between the two variables. Obviously, a more precise model should include the latter correlation.

Finally, according to related variables, we can also separate these models into three categories: hazard and recovery rate related to systematic variables, to industrial variables and to firm-specific variables. Bakshi, Madan and Zhang (2001) and Gaspar and Slinko (2008) relate hazard and recovery to systematic variables (risk-free rate and S&P 500) while Schneider, Sögner and Veza (2009) study the recovery rates for different sectors. In the case of Güntay, Madan and Unal (2003) and Das and Hanouna (2009), they both include systematic (risk-free rate) and firm-specific variables (asset tangibility, stock price and volatility).

We have chosen the Das and Hanouna (2009) model for further research. It allows the correlation between default probability and recovery rate. Hazard rate is related to both systematic and firm-specific risks. This methodology can be applied for public firms who are the reference entities of CDS contracts.

Section III Methodology

This section explains the methodology of combining CVA calculation with the implied recovery rate. There are two main steps: Firstly, based on the model of Das and Hanouna (2009), we find the term structure of implied recovery rate of the studied firm. Secondly, we adopt the results to the CVA calculation.

A. Extracting implied recovery rate

In this part, we discuss 1) the main assumptions of our methodology and 2) the model to extract the CDS implied recovery rate term structure.

A.1. Assumptions

Our methodology is mainly based on the model of Das and Hanouna (2009). At each point in time, by calibrating the term structure of CDS spreads, the authors extract the term structure of default probabilities and recovery rates. In their model, the default probability is driven by the stock price (the only state variable) and the recovery rate is linked to the default probability by a parametric function.

One of the main changes we make is the state variable. Instead of using the stock price, we write the default probability as a function of market capitalization. From a mathematical viewpoint, they are not so different: market capitalisation is the product of stock price and number of shares. However, since default probability is at the firm level, market capitalization should be more adequate than stock price as the state variable.

Market capitalization refers to the equity value of a firm. We denote the capital structure of the firm as $VF = E + D$, where VF is the firm value, E is firm equity value and D is its debt value. Equity value E and debt value D should have the same default probability (the firm's default probability). The difference between these securities lies in the recovery rates. In our model, in the case of default, we suppose that the recovery rate on equity is zero while debt[4] has no-zero recovery rates, which are dependent on default probability.

We are aware that the zero recovery rate for equity is unrealistic. Even after the firm files for bankruptcy, the equity price will not immediately fall to zero. However, compared to the other stakeholders, the shareholders have the lowest priority to recover their loss. Further, this assumption can be easily relaxed by setting a certain amount of equity recovery rate in the model.

From a discrete time approach, at each time interval, the default probability of the firm is driven by its market value at the beginning of the period. The recovery rate on debt D becomes also contingent upon market capitalization through its dependence on the default probability. In the view of Jarrow and Protter (2004), our reduced model assumes that market capitalization is the only available information on the market.

Under the forward risk-neutral measure, the absence of arbitrage opportunities implies that the market capitalization grows at forward risk-free rates. Knowing the market value evolution, we can write the corresponding endogenous default probabilities and recovery rates. Later, these two variables will be used to calibrate the CDS spread curve.

Another reasoning of Das and Hanouna's methodology is that the information carried in CDS spread curve on the "reference loan" is comprised of its probability of default and recovery rate. The accurate forward-looking default and recovery rate term structure can be found by calibrating the CDS spread curve. It infers that only the credit risk of the

[4] We study the senior unsecured bonds in the empirical part, since the recovery rate on unsecured obligations is considered to be near to the OTC derivatives.

underlying firm is priced in the CDS spreads. There should be almost no liquidity risk or counterparty credit risk of the underwriter.[5]

The last important hypothesis is the dependence between recovery rate and default probability. As we mentioned before, this correlation has already been examined by some research, such as Altman & al. (2004) and Dionne & al. (2008). Our model imposes the negative correlation between the two variables and supposes that default probability is the only variable to explain the recovery rate. In this way, our methodology successfully separates the default probability and the recovery rate from the CDS spreads.

A.2. Methodology of extracting implied recovery

This part is divided into four sections. At first, we introduce the CDS pricing methodology. Secondly, we model the stochastic behavior of the state variable, the market capitalization. Thirdly, we define how to obtain state-dependent default probabilities and recovery rates from the calibration model. Finally, we will discuss the constraints of the model.

A.2.1. CDS pricing methodology

We use the CDS pricing model that is proposed in the Das and Hanouna (2009).

We assume N periods in the model, $j = 1, \dots, N$ and h as the time interval of each step. T_j is the end of the jth period.

The discount function is a function of risk-free forward rates:

[5] As we discussed in the literature review, this assumption might be tricky since in some situations, the two additional risks could be significantly priced in the CDS spread curves. But theoretically, the price of CDS should reflect the credit risk of the underlying entity.

$$D(T_j) = exp\left\{-\sum_{k=1}^{j} f_k h\right\}$$

The survival function is defined as a function of default intensity of the firm $\varepsilon_j \equiv \varepsilon(T_{j-1}, T_j)$:

$$Q(T_j) = exp\left\{-\sum_{k=1}^{j} \varepsilon_k h\right\}$$

Suppose that a CDS is contingent on a defaultable bond or loan, whose nominal value is \$ 1. We define the CDS spread for N periods is C_N, and assume that default occurs at the end of N periods. The expected present value of the premiums paid is:

$$A_N = C_N h \sum_{j=1}^{N} Q(T_{j-1}) D(T_j)$$

If the default occurs during the CDS contract, the seller needs to pay the buyer 1 minus recovery rate. We denote the recovery rate $\phi_j \equiv \phi(T_{j-1}, T_j)$. Therefore, if the default occurs in period j, the loss payment will be \$ 1 $*$ $(1 - \phi_j)$. It implies that our model uses the "recovery of par" definition for pricing the CDS.

The possibility that the firm survives until period $(j-1)$ and defaults at period j is $Q(T_{j-1})(1 - e^{-\varepsilon_j h})$. The expected present value of loss payments is:

$$B_N = \sum_{j=1}^{N} Q(T_{j-1})(1 - e^{-\varepsilon_j h}) D(T_j)(1 - \phi_j)$$

The accurate C_N should make $A_N = B_N$:

$$C_N = \frac{h \sum_{j=1}^{N} Q(T_{j-1}) D(T_j)}{\sum_{j=1}^{N} Q(T_{j-1})(1 - e^{-\varepsilon_j h}) D(T_j)(1 - \phi_j)}$$

A.2.2. Market capitalization modeling

As discussed before, at time t, market capitalization is the only variable to explain the default intensity (default probability) of the firm and that default probability is the only variable to explain the recovery rate. As a result, both default probability and recovery rate are dependent on market capitalization.

Identical to Das and Hanouna's model, we model the stochastic behavior of market capitalization M with the Cox et al. (1979) binomial tree by adding a state of jumping to default. Below is a one period example:

$$M \longrightarrow \begin{cases} Mu = Me^{\sigma\sqrt{h}} & with\ probability\ q(1-\lambda) \\ Md = Me^{-\sigma\sqrt{h}} & with\ probability\ (1-q)(1-\lambda) \\ 0 & with\ probability\ \lambda \end{cases}$$

Where λ is the probability of jumping to default and q is the probability of an up move if the firm survives.

As mentioned before, under the forward risk-neutral measurement, the condition of absence of arbitrage opportunities implies that market capitalization grows at the forward risk-free rate. Otherwise, this asset will dominate or be dominated by other assets since there is no risk preference. The expected return on M can be written as below:

$$E\left[\frac{M_j - M_{j-1}}{M_{j-1}}\right] = e^{f_j h}$$

Since $E[M_j] = M_{j-1} * [q(1-\lambda) * u + (1-q)(1-\lambda) * d + \lambda * 0]$,

$$E\left[\frac{M_j - M_{j-1}}{M_{j-1}}\right] = q(1-\lambda) * u + (1-q)(1-\lambda) * d = e^{f_j h}$$

After the transformation, we get the risk-neutral probability of the up move: $q = \frac{\frac{R}{1-\lambda} - d}{u-d}$ and $R = e^{fh}$.

Compared to the risk-neutral probability for an original Cox et al. (1979) binomial tree, the result of adding the state of jumping to default is the change of the drift of market capitalization. We rewrite the drift as below:

$$\frac{R}{1-\lambda} = \frac{e^{fh}}{1-(1-e^{-\varepsilon h})} = e^{(f+\varepsilon)*h}$$

We notice that it is consistent with Duffie and Singleton (1999). Their "RMV" pricing model shows that the drift of a defaultable asset under risk neutral measurement should be the risk-free rate plus the loss rate: $R_t = r_t + \varepsilon_t(1 - \phi_t)$. Applying it to our model, the drift of the market capitalization should be $f_j + \varepsilon_j * \left(1 - \phi_j\right)$. Since we set the zero recovery rate[6] for market capitalization ($\phi_j = 0$), the risk-neutral drift is equal to $f_j + \varepsilon_j$, which is identical to the drift derived from the risk-neutral probability.

We need to mention that volatility here should be the implied volatility of the return on market capitalization. Instead of using historical volatility as do Das and Hanouna (2009), we define implied volatility σ as a parameter to estimate in the calibration model.

A.2.3. Term structure of default probability and recovery rate

As we discussed in *A.2.1*, to price CDS, we need the default intensity ε and recovery rate ϕ.[7] The key step of this calibration model is to define the default probability and recovery rate function.

After modeling the evolution of market capitalization, we can write the state-dependent default intensity and recovery rate. We denote each node on the tree with the index $[i, j]$, where j indexes time (from 0 to N) and i indexes the level of the node at time j. In the jth

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[6] In our model, we use the "recovery of par" definition. However, because it is 0 "recovery of par", it will also be 0 "recovery of market value".

[7] For now, we suppose that the risk-free forward rates are available. Later, in the data description, we will introduce the way to extract the forward rates term structure from US Treasury yields.

period, i takes value 0 at the top and value j at the bottom. Das and Hanouna define the probability of default and recovery rate as below:

$$\varepsilon[i,j] = \frac{1}{M[i,j]^b}$$

$$\lambda[i,j] = 1 - e^{-\varepsilon[i,j]h}$$

$$\phi[i,j] = \frac{1}{1 + exp(a_0 + a_1\lambda[i,j])}$$

$$M[i,j] = M[0,0]u^{j-i}d^i = M[0,0]\exp(\sigma\sqrt{h}(j - 2i))$$

The calibration model contains four parameters $\{a_0, a_1, b, \sigma\}$. Parameter b captures the relation between the market capitalization and the default intensity. Parameter a_1 captures the dependence between the default probability and recovery rate. Parameter σ is implied volatility of the return on market capitalization.

Since we now have the default and recovery rate of every node, we can find the expected premiums and expected losses of CDS by recursion.

$$A[i,j] = \frac{C_N}{R} + \frac{1}{R}\left[q[i,j](1-\lambda[i,j])A[i,j+1] + (1-q[i,j])(1-\lambda[i,j])A[i+1,j+1]\right]$$

$$B[i,j] = \lambda[i,j](1-\phi[i,j]) + \frac{1}{R}\left[q[i,j](1-\lambda[i,j])B[i,j+1] + (1-q[i,j])(1-\lambda[i,j])B[i+1,j+1]\right]$$

We need to fix the payments of the two legs at maturity T. For the premiums, $A[i,N] = C_N$, since the last coupon payment should be at maturity for the majority of CDS contracts. For the expected loss payment, $B[i,N] = \lambda[i,N](1 - \phi[i,N])$, because the last period should be recovered by the CDS contract.

Fair C_N should make the two expected values at time 0 the same. That is $A[0,0]$ should equal to $B[0,0]$.

We calibrate the CDS spread curve by minimising the error term[8]:

$$\min_{a0,a1,b,\sigma} \frac{1}{N}\sum_{j=1}^{N}\left[C_j(a0,a1,b,\sigma) - C_j^{observed}\right]^2$$

Where $C_j^{observed}$ is the observed CDS spread in the market.

After calibration, we get the four estimated parameters $\{a_0, a_1, b,\ \sigma\}$. We must return to the jump-to-default tree to calculate the term structure of default probabilities and recovery rates on the defaultable "reference loan". We denote $p(i,j)$ as the probability of reaching node (i,j), and the term structure of default probabilities and recovery rates can be calculated as below:

$$\lambda_j = \sum_{i=0}^{j} p[i,j]\lambda[i,j], \qquad \forall j$$

$$\phi_j = \sum_{i=0}^{j} p[i,j]\phi[i,j], \qquad \forall j$$

Here, the difficulty is to get the risk-neutral probability to reach each node $p(i,j)$. The risk-neutral probability of an up move $q(i,j)$ is inconstant and depends on default probabilities $\lambda[i-1, j-1]$ and $\lambda[i, j-1]$ of the last step. Therefore, we cannot apply the general formula for node probabilities of a regular binomial tree. Instead, we cumulate the node probabilities step by step. Below is a three-period example:

$p[0,0] = 1;$

$p[0,1] = p[0,0] * (1 - \lambda[0,0]) * q[0,0];$

$p[1,1] = p[0,0] * (1 - \lambda[0,0]) * (1 - q[0,0]);$

$p[0,2] = p[0,1] * (1 - \lambda[0,1]) * q[0,1];$

$p[1,2] = p[0,1] * (1 - \lambda[0,1]) * (1 - q[0,1]) + p[1,1] * (1 - \lambda[1,1]) * q[1,1];$

$p[2,2] = p[1,1] * (1 - \lambda[1,1]) * (1 - q[1,1]);$

$p[0,3] = p[0,2] * (1 - \lambda[0,2]) * q[0,2];$

[8] We will discuss the constraints in the next part *A.2.4.*

$p[1,3] = p[0,2] * (1 - \lambda[0,2]) * (1 - q[0,2]) + p[1,2] * (1 - \lambda[1,2]) * q[1,2];$
$p[2,3] = p[1,2] * (1 - \lambda[1,2]) * (1 - q[1,2]) + p[2,2] * (1 - \lambda[2,2]) * q[2,2];$
$p[3,3] = p[2,2] * (1 - \lambda[2,2]) * (1 - q[2,2]);$

By step-by-step accumulation, we can write $p[i,j]$ at each node in the tree. $\lambda[i,j]$ and $\phi[i,j]$ are easy to calculate with the estimated 4 parameters. Therefore, we can get the term structure of default probabilities λ_j and recovery rates ϕ_j.

Until now, we have explained how to obtain the term structure of default probability and recovery rate step by step. To sum up, the inputs of our model are the term structure of CDS spreads $C_j, j = 1, \dots, N$, risk-free forward rates $f_j, j = 1, \dots, N$ and market capitalization M. The outputs are the implied function for default probability $\lambda[i,j] = f(S, b, \sigma)$ and recovery rates $\phi[i,j] = f(a0, a1, \lambda)$, and their term structures λ_j and ϕ_j. Figure 2 below shows the general process of our methodology:

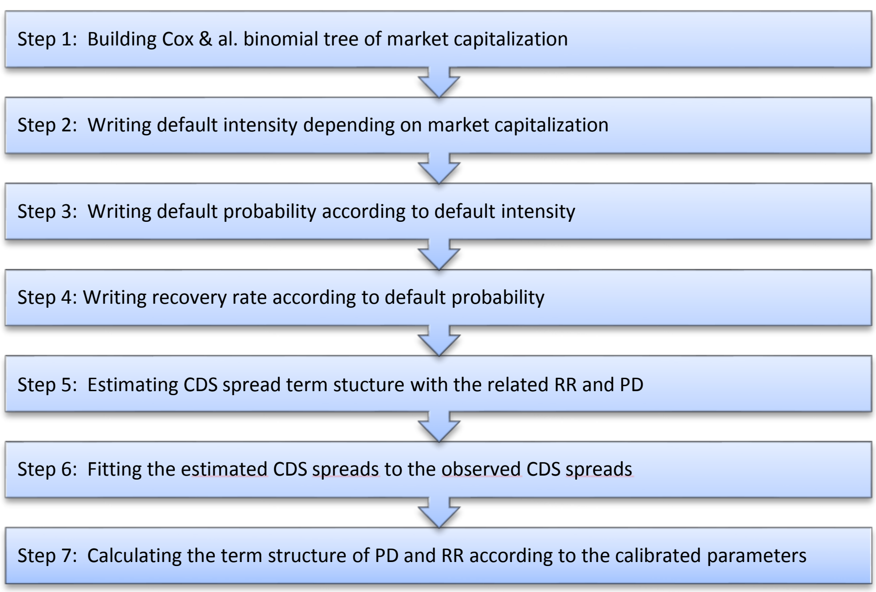


Figure 2: Process of the methodology

A.2.4. Constraints

In this section, we discuss about the parameters' constraints, especially for $a1$, b and σ. Firstly, we impose negative correlation between the default probability and recovery rate, parameter $a1 < 0$ (*Constraint 1*).

Secondly, the implied volatility σ must be bigger than 0, $\sigma > 0$ (*Constraint 2*).

Thirdly, to make sure the negative correlation between the market capitalization and default intensity ($\varepsilon = \frac{1}{M^b}$), parameter b should also be bigger than 0, $b > 0$ (*Constraint 3*).

Finally, as $q[i,j] = \frac{\frac{R}{1-\lambda[i,j]}-d}{u-d}$ is a probability, $q[i,j]$ must be between 0 and 1 for $\forall i,j$. Substituting the $u = e^{\sigma\sqrt{h}}$, $d = e^{-\sigma\sqrt{h}}$, $R = e^{fh}$ and $1-\lambda = e^{-\varepsilon h} = e^{-\frac{1}{M^b}h}$, we have $\sigma \geq \left(f + \frac{1}{M^b}\right)\sqrt{h} \geq -\sigma$, where $\sigma > 0$. Since $\left(f + \frac{1}{M^b}\right)\sqrt{h}$ is certainly bigger than 0, it is certainly bigger than $-\sigma$. Hence, we only need to deal with $\left(f + \frac{1}{M^b}\right)\sqrt{h} \leq \sigma$, which can be transferred to $\frac{1}{M^b} \leq \frac{\sigma}{\sqrt{h}} - f$ (*Inequality 1*).

Since $\frac{1}{M^b}$ is always positive, it means that $\frac{\sigma}{\sqrt{h}} - f > 0$ and $\sigma > \sqrt{h} * f$ (*Constraint 4*). As *Constraint 4* is stricter than *Constraint 2*, we now drop *Constraint 2*.

As the two sides of *Inequality 1* are positive numbers, we impose ln in front of them. The inequation becomes $-b * ln(M) \leq \ln(\frac{\sigma}{\sqrt{h}} - f)$. Three situations are possible:

1) If $M > 1$, then $ln(M) > 0$ and $b \geq \frac{-ln(\frac{\sigma}{\sqrt{h}}-f)}{lnM}$. Adding *Constraint 3*, parameter b has two constraints.

2) If $M < 1$, then $ln(M) < 0$ and $b \leq \frac{-ln(\frac{\sigma}{\sqrt{h}}-f)}{lnM}$. As $b > 0$, $\frac{-ln(\frac{\sigma}{\sqrt{h}}-f)}{lnM}$ has to be also bigger than 0. As the numerator $-ln(\frac{\sigma}{\sqrt{h}} - f)$ should be smaller than 0, we need to impose $\sigma \geq \sqrt{h} * (f + 1)$, which is stricter than *Constraint 4*. For parameter b, $0 < b \leq \frac{-ln(\frac{\sigma}{\sqrt{h}}-f)}{lnM}$.

3) If $M = 1$, then $ln(M) = 0$ and $\sigma \geq \sqrt{h} * (f + 1)$.

To conclude, in the stock price Cox & al. (1979) binominal tree, the constraints are as below:

When all $M > 1$, the constraints are: $\sigma > \sqrt{h} * f$, $b > 0$ and $b \geq \frac{-ln(\frac{\sigma}{\sqrt{h}}-f)}{lnM}$.

When all $M < 1$, the constraints are: $\sigma \geq \sqrt{h} * (f + 1)$ and $0 < b \leq \frac{-ln(\frac{\sigma}{\sqrt{h}}-f)}{lnM}$.

When all $M = 1$, the constraints are: $\sigma \geq \sqrt{h} * (f + 1)$ and $b > 0$.

Now we discuss the mixed situation. Obviously, σ must be bigger than $\sqrt{h} * (f + 1)$. In this case, for the stock prices bigger than 1, we can keep only $b > 0$ and drop $b \geq \frac{-ln(\frac{\sigma}{\sqrt{h}}-f)}{lnM}$, since $\frac{-ln(\frac{\sigma}{\sqrt{h}}-f)}{lnM}$ is always inferior to 0. For stock prices less than 1, the constraint for b stays the same. Therefore, in a mixed situation, the constraints are: $\sigma \geq \sqrt{h} * (f + 1)$ and $0 < b \leq \frac{-ln(\frac{\sigma}{\sqrt{h}}-f)}{lnM}$.

B. CVA with implied default probability and recovery rate

In this section, we explain how we implant the implied term structure of the default probabilities and recovery rates into CVA calculation.

B.1. Assumptions

The essential condition to implant the results from ***A.2*** is that the default probability and recovery rate on OTC derivatives should be identical to the "reference loan" of CDS.

For default probability, it is the same intuition as in **Part A**. If the firm files for bankruptcy, all its securities default, including OTC derivatives. In other words, its OTC derivatives get the same default probability as its equity and debt.

The recovery rate is more complicated since we know that the difference among those securities is due to their recoveries. "Normally, OTC derivatives would rank *pari passu* with senior unsecured debt, which in turn is the reference in most CDS contracts." (Gregory J, 2012, page 210) It implies that to implant the results from ***A.2*** to CVA calculation, we need to choose CDS contracts whose "reference loan" is senior unsecured debt. In fact, the common practice in the industry is to assume a constant recovery of 40% on OTC derivatives, which is nearly the historical average recovery rate of senior unsecured debt.[9]

B.2. CVA with implied default probability and recovery rate

As we deduced before, CVA can be priced separately from the traditional risk free valuation. The formula for CVA calculation in discrete time with the stochastic recovery rate is as below:

[9] According to Annual Default Study of Moody's (2013), the historical average recovery rate is nearly 43% in 2012 for senior unsecured debt.

$$CVA(t,T) \approx \sum_{j=1}^{N} EE(t_{j-1},t_j) * \left(1-\phi(t_{j-1},t_j)\right) * \lambda(t_{j-1},t_j)$$

Where $EE(t_{j-1},t_j) = E^Q\left[B(t_{j-1},t_j)V(t_{j-1},t_j)^+\right]$.

Replacing $\lambda(t_{i-1},t_i)$ and $\phi(t_{i-1},t_i)$ by the term structure of default probabilities and recovery rates (λ_j and ϕ_j) that we calculated in **Part A**, we can rewrite the CVA formula:

$$CVA(t,T) = \sum_{j=1}^{N} EE(t_j) * (1-\phi_j) * \lambda_j$$

Where $EE(t_j) = E^Q[\exp(-f(j)*h)*V(t_i)^+]$, $V(j)^+ = \max(V(j),0)$ and $V(j)$ is the value of the OTC derivative at period j.

B.3. Comparison with the constant recovery rate

Gregory (2012) shows the standard equation for CVA with constant recovery rate as below:

$$CVA = (1-\phi)\sum_{j=1}^{N} EE(t_j) * \lambda_j$$

The term structure of default probability can also be linked to the CDS spreads:

$$\lambda_j = \lambda_j{}^{cumulative} - \lambda_{j-1}{}^{cumulative}$$

$$\lambda_j{}^{cumulative} = 1 - e^{-\varepsilon_j * h} \; and \; \varepsilon_j = \frac{CDS\ Spread_j}{1-\phi}$$

Where ε is the default intensity and h is the related time interval.

In the following part, we compare these two methodologies empirically, in order to see the impact of the non-constant recovery rate assumption on the CVA calculation.

Section IV Analysis and results

In this section, we apply the methodology to extract the implied recovery rate and default probability from our sample. Then we compute CVA according to the implied recovery and compare it with the CVA calculated with constant recovery.

A. Data Description

In this section, we describe the three input data series of our model: CDS spread term structures, market capitalizations of the underlying firms and forward risk free rate term structures.

A.1. CDS spread term structures and market capitalizations

We use DATASTREAM of Thomson Reuters to download CDS spread term structures and market capitalizations. Both of them are monthly data from 01/01/2007 to 01/09/2010[10], covering the recent financial crisis. Hence, for each underlying firm, we have 45 (months) CDS spread terms structures and 45 market capitalizations in time series.

In DATASTREAM, we choose the single-name CDS traded on the US market in USD and exclude all the CDS XR, CR, MR or MM, which refer to different restructuring clauses as (or not as) a credit event. Each term structure contains 10 CDS spreads for 10 maturities, from 1-year to 10-year. The selected spread is the average of bid and ask (midspread).

[10] Our data is provided by CMA via DATASTREAM and the agreement between CMA and Thomson Reuters has been discontinued since 1st September 2010.

The output of Thomson Reuters database shows a sample of CDS on 503 "reference loans" with data for 10 maturities.[11] We take off 66 "reference loans" with missing data during the studied period. After all these operations, there are 437 "reference loans" with the complete term structures for the studied period.

It is important to mention that among these 66 incomplete reference loans, there are only three firms (Ambac Assurance Corp, CIT Group and Energy Future Holding) which have default records, according to FISD (Fixed investment security data base) and Moody's. In other words, the survivorship bias is very limited in our data selection.

The next step is to clean the sample so that all the CDS contracts are on senior unsecured debts and that all the "reference obligors" are public companies. Senior unsecured CDS is selected to match the right recovery rate with CVA calculation on derivatives. We delete seven "subordinated reference loans" and four "reference loans" issued by civil and provincial governments. We also take off one "reference loan" because it is written on a subsidiary and keep the "reference loan" on its parent company.

When looking for the market capitalization of underlying issuers, we lose 28 "reference loans (or obligors)" due to incomplete information: 2 obligors are private companies; 19 firms do not have available market capitalization from the Thomson Reuters database and 7 firms do not have complete market capitalization data for the tested period. In the case of CDS written on a subsidiary (3 cases), we use the market capitalization of its parent company.

Finally, our data is comprised of 397 CDS spreads curves of 10 maturities on 397 distinct issuers from 01/01/2007 to 01/09/2010. During this interval, there are 10 default records (six firms) on senior unsecured debt during the sample period (Appendix D). Regardless of these defaults, there is no suspension of CDS quote so that the term structures of all the

[11] We notice that senior unsecured CDS quotes on Lehman Brothers Holding Inc. are absent from the Thomson Reuters sample.

issuers remain complete for the studied period. This is another proof of limited survivorship bias in our research.

Below is an example, with the monthly market capitalizations and the monthly CDS term structures from 01/01/2007 to 01/09/2010 on the senior unsecured debt of a "reference obligor (Goldman Sachs)":

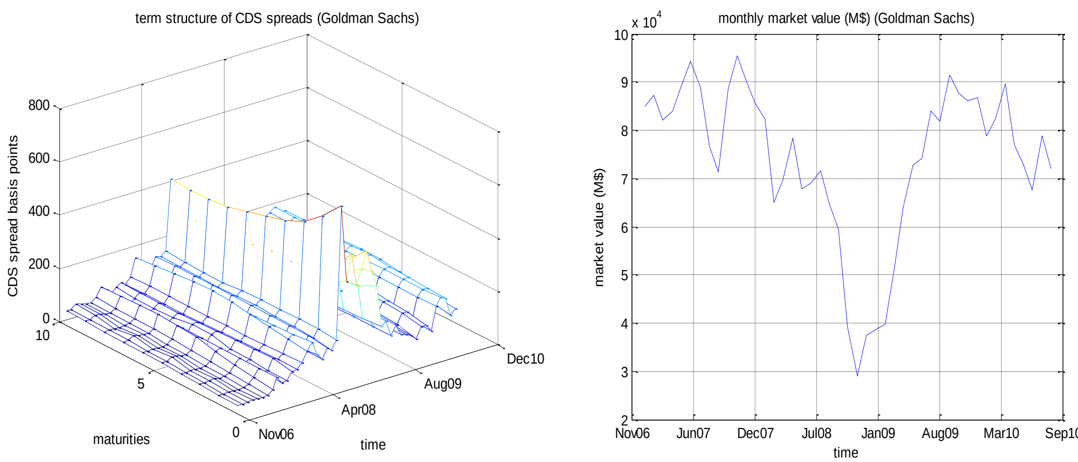


Figure 3: Data example of a "reference obligor (Goldman Sachs)"

A.1.1 CDS spreads term structure

We now take a bird's eye view on the total sample. We calculate the average of CDS spreads of the sample according to different maturities and obtain the figure below, showing the evolution of CDS spread term structure in time series. The CDS spread spikes around September 2008 and then drops around March 2009. We observe humped and even reverse term structures from September 2008 to October 2009.

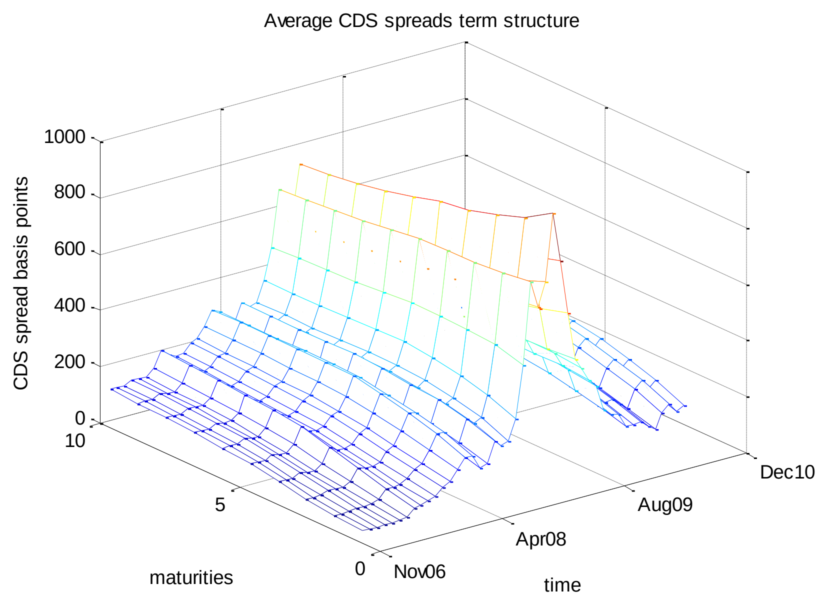


Figure 4: Average CDS spread term structures

A.1.2 5-year CDS spreads and stock returns

Here, we focus on the 5-year spreads of our sample, since it is the most liquid. We see their evolutions in timeline versus stock return, their distributions and compare them according to different sectors.

We take the average of the 5-year spreads and market capitalisation in time series for each firm. The histogram (Figure 5) and the main statistics (Table 1) of the sample are as below.

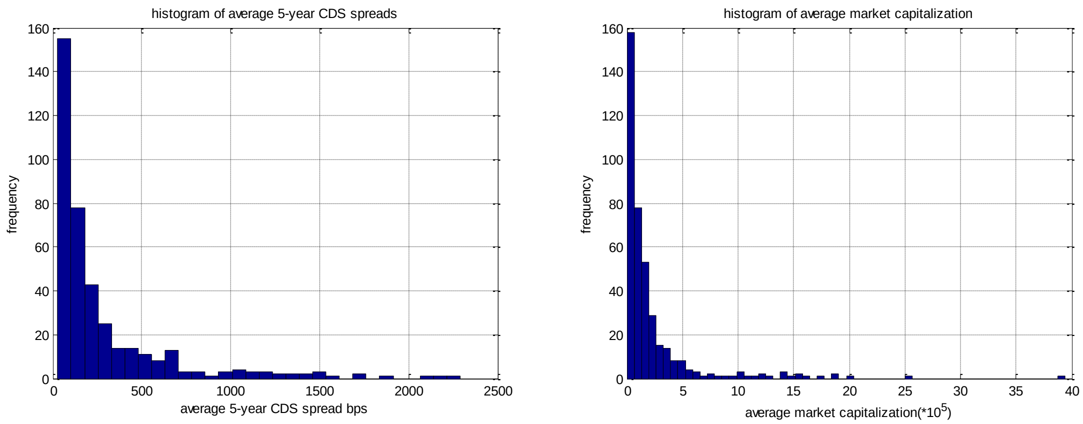


Figure 5: Histogram of average market capitalization and 5-year CDS spread

	CDS 5 years maturity spreads (basis points)	**Market Capitalization (B$)**
Mean	283.90	21.69
Median	111.93	9.03
Standard deviation	564.15	40.52
Maximum	3	0.00034
Minimum	14627	513.31

Table 1: Summary data statistics on CDS spreads (5 years) and market capitalization

We then take the average of 5-year spreads and market capitalisations of all the firms for each month. Figure 6 shows the evolution of 5-year CDS spreads against monthly stock return in time series. The stock price is downloaded at the same time with market capitalization from Thomson Reuters. The correlation between the average CDS spread and stock return during all sample period is -0.19.

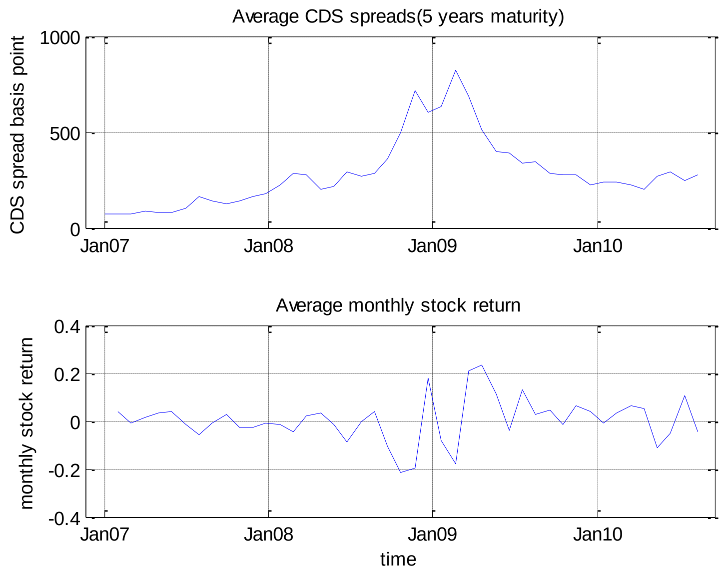


Figure 6: Evolution of 5-year CDS spreads against monthly stock return

A.1.3 CDS spreads and market capitalizations by sector and by rating class

As we discussed in ***Literature Review***, recovery rate is proven to be significantly related to sector and rating. We now sort CDS spreads (5-years maturity) data and market capitalizations by sector and by rating.

We classify 397 firms by their SIC code and the data is divided into 8 categories (Table 2). Almost 40% of our sample comes from the manufacturing sector, and it also represents the largest average market capitalization. The construction sector gets the highest average CDS spread while the wholesale trade industry seems to be the lowest.

Industry	Number of firms	CDS spread (5 years) (basis points)		Market capitalization (B$)	
		Average	standard deviation	Average	standard deviation
Construction	15	514.95	247.35	1.84	7.03
Finance	60	321.15	234.86	25.93	7.41
Manufacturing	149	225.26	157.03	26.67	4.96
Mining	22	225.32	116.77	16.20	3.24
Retail trade	34	268.41	162.26	20.33	2.12
Services	31	432.23	289.03	18.88	3.28
Transportation[12]	78	317.44	185.07	17.07	2.65
Wholesale Trade	8	77.61	39.97	11.09	1.20

Table 2: Data by industry

We then classify our firms by rating. Rating classification is downloaded from *Moody's Credit risk calculator*. Since the firms are not rated at the same time, we take the last rating results before October 2010 (the end of our sample period). We mention that there are 20 firms which do not get any rating.

Moody's provides 20 different rating scales and we regroup them into 5 categories (Table 2). The total sample counts 249 firms who are rated below (including) Baa and 131 firms are rated under Baa. We notice that average CDS spread increases and market capitalization decreases when rating goes downwards.

[12] The complete specification of the sector is: Transportation, Communications, Electric, Gas, And Sanitary Services.

Category	notation	Number of firms	CDS spread (5 years) (basis points)		Market capitalization (B$)	
			Average	**standard deviation**	**Average**	**standard deviation**
Category 1	$Aaa - Aa3$	15	71.60	59.40	138.02	20.61
Category 2	$A1 - A3$	83	98.77	100.44	43.38	9.36
Category 3	$Baa1 - Baa3$	148	145.53	184.23	13.96	4.25
Category 4	$Ba1 - B3$	106	478.53	385.31	6.28	0.70
Category 5	$Caa1 - C$	25	949.99	662.31	3.60	0.72
Category 6	No rating	20	371.25	420.13	5.88	1.00

Table 3: Data by rating category

A.2. Forward rate term structures

Another series we need is the forward interest rate term structures, which are extracted from US Treasury yields. From the Federal Reserve website, we got historical monthly Treasury yields for 9 maturities (1 month, 3 months, 6 months, 1 year, 2 years, 3 years, 5 years, 7 years and 10 years). We observe an abrupt decrease of 1-month Treasury yields from mid-2007 to mid-2008.

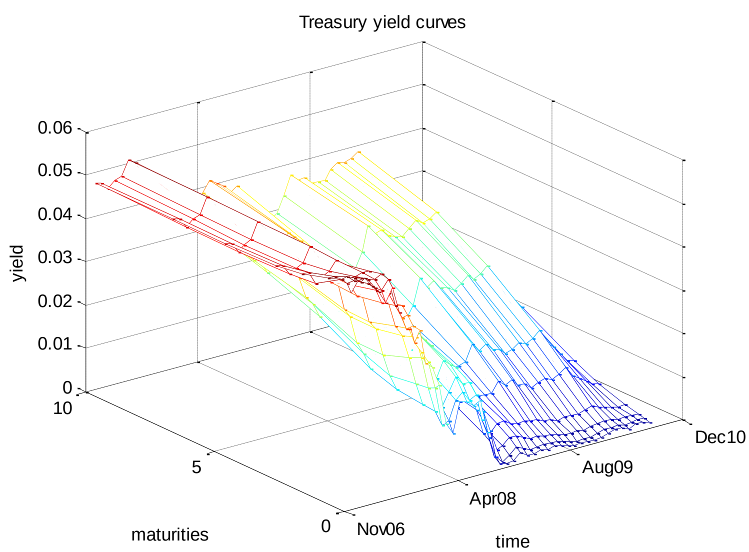


Figure 7: Treasury yield term structure (source: U.S. FED)

To calculate the forward interest rate term structures, we first do Nelson-Siegle curve fitting to get the spot rate curves. We compute the forward rate term structures (Figure 6) from the fitted spot rate curves. The forward rate can be expressed as $f(t_1, t_2, t_2 + 0.5)$, where t_1 varies from the first month (Jan. 2007) to the last (45^{th}) month (Sep. 2010), and for each t_1, t_2 takes value from 0 to 9.5, at 0.5 year interval. The size of forward rates matrix is [20 intervals *45 months]. We notice that the shape of the forward rate term structures changes around 2008: at the beginning of the sample period, the slope tends to be much flatter than the later period.

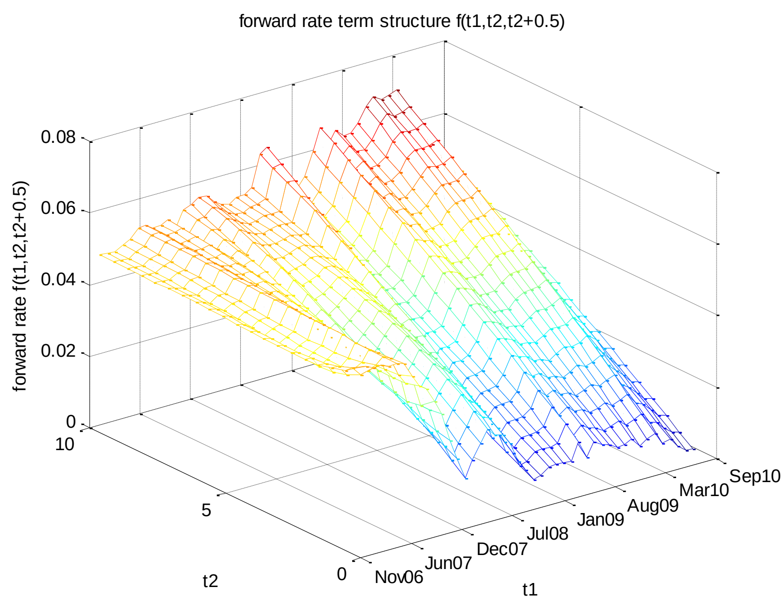


Figure 8: Monthly forward rate term structures $f(t_1, t_2, t_2 + 0.5)$

B. CDS term structure calibration

We apply our calibration model to the data. Since we have 17 865 term structures ($397\ firms\ *\ 45\ months$), the results contain 17 865 sets of parameters, 17 865 default probability term structures and 17 865 recovery rate term structures.

Firstly, we study the RMSEs (root-mean-square error) to see the calibration quality. Then we show the results of 4 calibrated parameters. Finally, we calculate the term structures of default probability/recovery rate based on the parameters. We display the results (parameters, default probability term structures and recovery rate term structures) at four levels: 1) results for the total sample, 2) results based on the phases of business cycle, 3) results sorted by industry (SIC) and 4) results sorted by rating (Moody's).

B.1 RMSE

We calculate RMSE as below for each term structure. RMSE is calculated as below:

$$RMSE = \sqrt{0.1 * \sum_{j=1}^{10} \left[C_j^{estimated} - C_j^{observed}\right]^2}$$

We report RMSE information for the total sample in Table 4.

Table 4: RMSE distribution

In Table 4, RMSE is calculated by $RMSE = \sqrt{0.1 * \sum_{j=1}^{10} \left[C_j^{estimated} - C_j^{observed}\right]^2}$. We have in total 17 865 RMSEs. The RMSEs are expressed in bps/1000.

	RMSE (bps/10 000)	RMSE/$\mathbf{0.1} * \sum_{j=1}^{10} C_j^{observed}$
Mean	0.0786	0.1331
Median	0.0027	0.1003
Standard deviation	0.4444	0.1112
Maximum	14.6366	0.9631
Minimum	0.0000	0.0307
95% Quintile	0.2153	0.3141

In order to improve the precision, we clean our results by throwing out all $RMSE/0.1 * \sum_{j=1}^{10} C_j^{observed}$ > 0.3141 (95% Quintile). It eliminates 893 term structures and the results now contain 16 972 sets of data. Table 5 reports the RMSE information for the cleaned results.

Table 5: RMSE distribution: the cleaned results

In Table 5, RMSE is calculated by $RMSE = \sqrt{0.1 * \sum_{j=1}^{10} \left[C_j^{estimated} - C_j^{observed}\right]^2}$. We have in total 16 972 RMSEs. The RMSEs are expressed in bps/1000.

	RMSE (bps/10 000)	**RMSE/$\mathbf{0.1} * \sum_{j=1}^{10} C_j^{observed}$**
Mean	0.0167	0.1127
Median	0.0024	0.0976
Standard deviation	0.0408	0.0548
Minimum	0.0000	0.0307
Maximum	0.3949	0.3140
95% Quintile	0.0855	0.2359

B.2 Results

As in ***Data Description***, the results (parameters, default probability term structures and recovery rate term structures) will be exhibited at four levels: 1) results for the total sample, 2) results based on the phases of business cycle, 3) results sorted by industry (SIC) and 4) results sorted by rating (Moody's).

B.2.1 Parameters

We show the results of parameters $\{a0, a1, b, \sigma\}$ for the total sample in Table 6. The distributions of $\{a0, a1\}$ and σ, which measure the dependence between the recovery rate to default probability and the implied volatility of market capitalization changes, seem to be asymmetric and fat tailed. As we noticed in the previous part, one of the main reasons might be that the distributions of the two main inputs (CDS spreads and market capitalizations) are also asymmetric and fat tailed. The magnitude of implied volatility is relatively big compared to what we know (e.g. the implied volatility calculated by *OptionMetrics*). We will discuss this point later in **Appendix A**. Parameter b, which measures the dependence of hazard rate to the market capitalization, is more centralized.

Table 6: Parameters for the truncated sample

Table 6 shows the information of estimated parameters for the total sample, after taking off the outliers. The sample now contains 16 972 sets of parameters $\{a0, a1, b, \sigma\}$. We report the average, median, standard deviation, minimum and maximum of each parameter across all the firms and all the sample period. Parameter $a1$ measures the dependence between the default probability and recovery rate. $\phi[i.j] = \frac{1}{1+exp(a_0+a_1\lambda[i.j])}$. $\{b\}$ estimates the correlation between the market capitalization and hazard rate, $\varepsilon[i.j] = \frac{1}{M[i.j]^b}$. $\{\sigma\}$ is the implied volatility of market capitalization changes.

	Mean	**Median**	**Std.**	**Min**	**Max**
$\boldsymbol{a0}$	-380.8420	-23.2132	8175.0274	-120386.26	558531.03
$\boldsymbol{a1}$	4208.7738	219.4769	27224.02	0.0001	819324.89
$\boldsymbol{b}$	0.0658	0.0630	0.0420	0.0000	0.2325
$\boldsymbol{\sigma}$	5.1585	2.0066	10.4732	0.06461	57.2428

Let us now look at the variation in timeline. According to NBER, we separate the sample period into three parts: before the crisis (before December 2007), during the crisis (between December 2007 to June 2009) and after the crisis (after June 2009). The 45-month sample period is divided into three under-periods: the first 12 months, from 13th month to 30th month (17 months), and from 31st month to the 45th month (16 months).

The parameters according to different phases of the business cycle are exhibited below (Table 7). Parameter a_1, which measures the dependence between default probability and recovery rate, suggests that during and after the crisis, this dependence seems to be higher than in the period before the crisis. Volatility σ is also higher during and after the crisis than the period before the crisis.

Parameter b however is higher before the crisis. Parameter b estimates the correlation between the market capitalization and the hazard rate, $\varepsilon[i.j] = \frac{1}{M[i.j]^b}$. The default probability increases when b gets smaller. Therefore, it is reasonable that b is higher before the crisis.

We notice that it is difficult to separate the period during and after the crisis. The credit risk seems to be persistent even after the economic crisis ends. Actually, some research (ex: Bruche and González-Aguado, 2010) has proven that the credit cycle distinguishes itself from the NBER business cycle. The credit cycle starts much earlier and ends later than the business cycle. Chun, Dionne and François (2013) also show that the high-level regime of credit spreads is long-lived and “often outlasts NBER economic recessions”.

We then compare these parameters for different industries. Table 8 shows the wholesale trade sector gets the smallest parameter b and the biggest parameter $a1$. The construction sector reveals the highest parameter b. The result of parameter b is consistent with the average CDS spread (5 years maturity): the wholesale trade sector is the lowest CDS spread, while the construction sector represents the highest spread. The mining industry seems to have the smallest dependence between default probability and recovery rate.

Table 7: Parameters by phases of business cycle

Table 7 shows the information of estimated parameters for different phases of business cycle. Before the crisis, there are 12 months and 5 790 term structures in all. During the crisis, there are 18 months and 6 520 term structures in all. After the crisis, there are 15 months and 5 709 term structures. Each term structure matches one set of parameters $\{a0. a1. b. \sigma\}$. $\{a1\}$ measures the dependence between default probability and recovery rate. $\phi[i.j] = \frac{1}{1+exp(a_0+a_1\lambda[i.j])}$. $\{b\}$ estimates the correlation between the market capitalization and hazard rate. $\varepsilon[i.j] = \frac{1}{M[i.j]^b}$. $\{\sigma\}$ is the implied volatility of market capitalization changes.

		Phases of Business Cycle		
		Before the crisis	**During the crisis**	**After the crisis**
$\boldsymbol{a0}$	**Mean**	-58.5483	-480.4497	-523.7096
	Median	-9.2220	-29.9377	-45.4671
	Std.	643.6287	3653.6664	13444.8197
$\boldsymbol{a1}$	**Mean**	747.2101	3733.0206	7509.6444
	Median	116.4347	234.2299	444.9751
	Std.	11803.1616	28182.28	33754.42
$\boldsymbol{b}$	**Mean**	0.0925	0.0524	0.0600
	Median	0.0893	0.0462	0.05746
	Std.	0.0376	0.0386	0.0395
$\boldsymbol{\sigma}$	**Mean**	2.4977	6.4963	5.7489
	Median	2.1286	1.9139	1.9071
	Std.	2.9052	12.6455	11.1774

Table 8: Parameters by sectors

Table 8 shows the information of estimated parameters for different sectors. Category 1 is the construction sector (594 term structures); category 2 is the finance sector (2 502 term structures); category 3 is the manufacturing sector (6 449 term structures); category 4 is the mining sector (1 013 term structures); category 5 is the retail trade sector (1 473 term structures); category 6 is the services sector (1 297 term structures); category 7 is the transportation, communications, electric, gas and sanitary services sector (3 239 term structures); category 8 is the wholesale trade sector (405 term structures). Each term structure matches one set of parameters $\{a0, a1, b, \sigma\}$. $\{a1\}$ measures the dependence between default probability and recovery rate, $\phi[i,j] = \frac{1}{1+exp(a_0+a_1\lambda[i,j])}$. $\{b\}$ estimates the correlation between the market capitalization and hazard rate, $\varepsilon[i,j] = \frac{1}{M[i,j]^b}$. $\{\sigma\}$ is the implied volatility of market capitalization changes.

		Industry category							
		1.CON	**2.FIN**	**3.MAN**	**4.MIN**	**5.RET**	**6.SER**	**7.TRA**	**8.WHO**
$a0$	**Mean**	-581.36	-666.25	-444.21	-144.53	-784.71	1045.38	-443.69	-389.11
	Med.	-106.29	-25.518	-18.345	-16.308	-20.832	-54.837	-25.752	-18.482
	Std.	1753.40	2592.70	4634.02	688.95	2715.56	27140.0	2739.9	1655.31
$a1$	**Mean**	3679.5	4728.6	3667.7	1425.8	9851.7	1987.5	3281.9	11169.4
	Med.	741.80	232.86	186.14	202.16	204.77	332.12	243.18	162.59
	Std.	11485.8	23148.4	33295.8	8977.0	35545.4	8198.73	16159.2	50638.1
b	**Mean**	0.0488	0.0604	0.0700	0.0705	0.0689	0.0557	0.0645	0.0791
	Med.	0.0425	0.0575	0.0671	0.0711	0.0667	0.0478	0.0609	0.0810
	Std.	0.0361	0.0411	0.0420	0.0422	0.0400	0.0419	0.0420	0.0464
σ	**Mean**	7.0040	4.5200	4.8313	6.3544	4.7019	7.1319	5.1176	4.4180
	Med.	2.3807	1.6349	1.9674	2.2677	2.1279	2.9914	1.9794	1.7396
	Std.	13.326	10.729	10.019	11.982	9.118	11.037	10.271	10.260

Finally, we compare the results of parameters based on the rating classes provided by Moody's. Table 9 suggests that it is the rating class Baa1-Baa2 which has the highest dependence between default probability and recovery rate. Parameter b decreases when the rating gets worse. Implied volatility increases when the credit quality deteriorates.

Table 9: Parameters by rating

Table 9 shows the information of estimated parameters for different rating according to Moody's. Category 1 is the firms which get rating Aaa-Aa3 (674 term structures); category 2 is the firms which get rating A1-A3 (3 704 term structures); category 3 is the firms which get rating Baa1-Baa3 (6 603 term structures); category 4 is the firms which get rating Ba1-B3 (4 337 term structures); category 5 is the firms which get rating Caa1-C (817 term structures); category 6 is the firms which do not get rating (837 term structures). Each term structure matches one set of parameters $\{a0, a1, b, \sigma\}$. $\{a1\}$ measures the dependence between default probability and recovery rate, $\phi[i,j] = \frac{1}{1+exp(a_0+a_1\lambda[i,j])}$. $\{b\}$ estimates the correlation between the market capitalization and hazard rate, $\varepsilon[i,j] = \frac{1}{M[i,j]^b}$. $\{\sigma\}$ is the implied volatility of market capitalization changes.

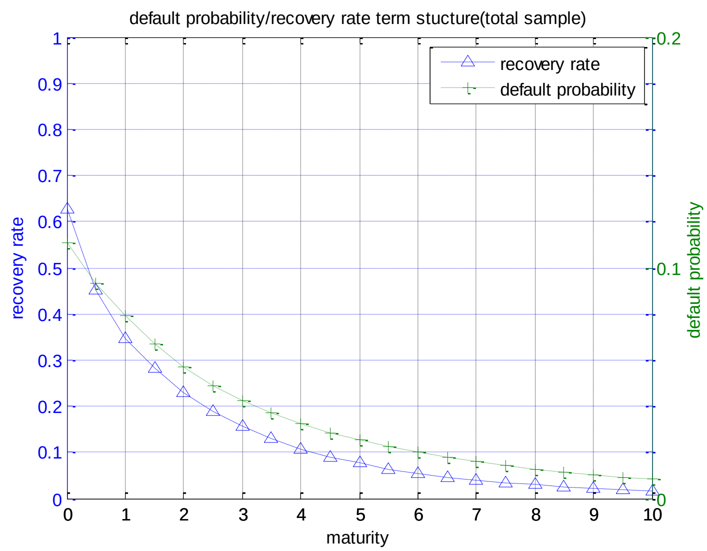
		Rating category					
		1. Aaa-Aa3	**2. A1-A3**	**3.Baa1-Baa3**	**4.Ba1-B3**	**5.Caa1-C**	**6. No rating**
	Mean	-157.4628	-333.4116	-626.7857	-77.5548	-163.0859	-615.837
$a0$	**Med.**	-18.5111	-20.0315	-13.6682	-56.7794	-42.2903	-57.6416
	Std.	919.0857	1605.5195	4866.7293	14877.67	1436.4852	2466.203
	Mean	1053.7688	4873.9202	5143.5745	3335.341	1059.458	4088.937
$a1$	**Med.**	260.8665	258.9691	147.9025	302.3869	198.7494	521.0052
	Std.	3264.5423	25184.349	35439.8562	19816.98	6921.194	14243.09
	Mean	0.0756	0.0756	0.0744	0.0496	0.0350	0.0621
b	**Med.**	0.0702	0.0704	0.0733	0.0441	0.0265	0.0568
	Std.	0.0458	0.0428	0.0419	0.0338	0.0328	0.0426
	Mean	2.6829	3.4506	3.8679	7.5544	12.5734	5.1101
σ	**Med.**	1.3492	1.3885	1.8382	3.2439	5.5899	2.2216
	Std.	6.5449	8.4968	8.8475	12.2674	16.0312	10.2513

B.2.2 Default probability and recovery rate term structures

Displayed below are the average default probability and recovery rate term structures for the total sample and over all the sample period in Figure 9. They are both downwards sloping term structures. The short term default probability is very high, but declines rapidly conditional on survival. For the declining recovery rate term structure, Das and Hanouna (2009) argue that "forward recovery rate is lower when the firm defaults later rather than sooner […] firms that migrate slowly into default suffer greater dissipation of assets over time, whereas the firm that has a short-term surprise default may be able to obtain greater resale values for its assets".

Figure 9: Total sample default probability and recovery rate

Figure 9 plots the default probability and recovery rate term structure for the total sample (16 972 term structures). We take the average default probability and recovery rate across all the firms (397 firms) and the sample period (45 months). The default probability at 0.5 year maturity is 11.1% and at 10 years maturity is 0.8%; the recovery rate at 0.5 year maturity is 62.81% and at the 10 years maturity is 1.75%.



The time evolution is exhibited in Figure 10 below. We compare the term structures of default probability/recovery rate for three phases of the business cycle. Figure 9 shows that all term structures are declining. However, the level and the steepness of the slope are different for each term structure.

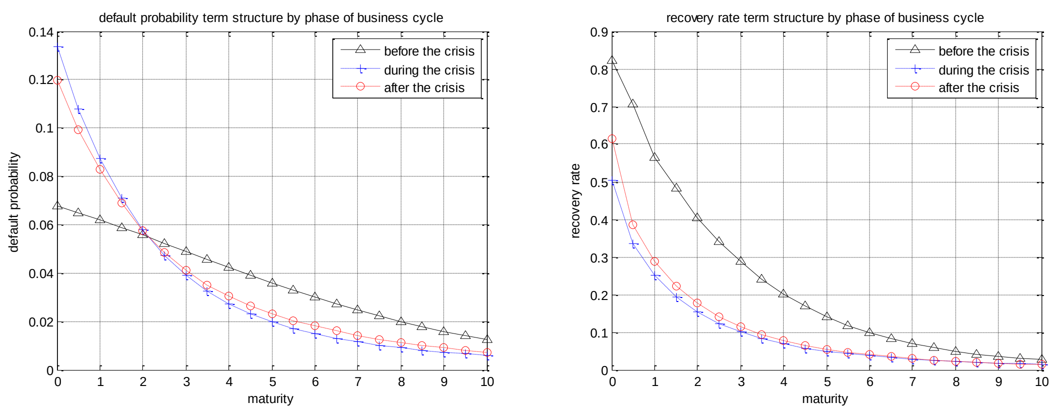
Before the crisis, the default probability term structure is the most gradual and the recovery rate is at the highest level, compared with the two other periods. This is consistent to the previous research. For the firms which are riskier, the short-run default probability is higher and the long-run default probability declines very fast, conditional on survival.

We might ask why the default probability term structure is not up-sloping before the crisis. It could refer again to the literature that points to longer credit cycles than NBER business cycles. Since the credit cycle seems to start before the economic recession, it might affect the results on the first 12 months of the sample. This should be one of the reasons why we also obtain a downward sloping default probability term structure for the period before the crisis.

We notice that the difference between the period during the crisis and after the crisis is not obvious. As discussed for the parameter results, it might be due to the persistence of the credit cycle.

Figure 10: default probability (recovery rate) term structure according to business cycle

Figure 10 plots the default probability and recovery rate term structures according to different phases of business cycle. Before the crisis (before December 2007), there are 12 months and 5 790 term structures in all. During the crisis (between December 2007 to June 2009), there are 18 months and 6 520 term structures in all. After the crisis (after June 2009), there are 15 months and 5 709 term structures. We take the average default probability and recovery rate across all firms (397 firms).



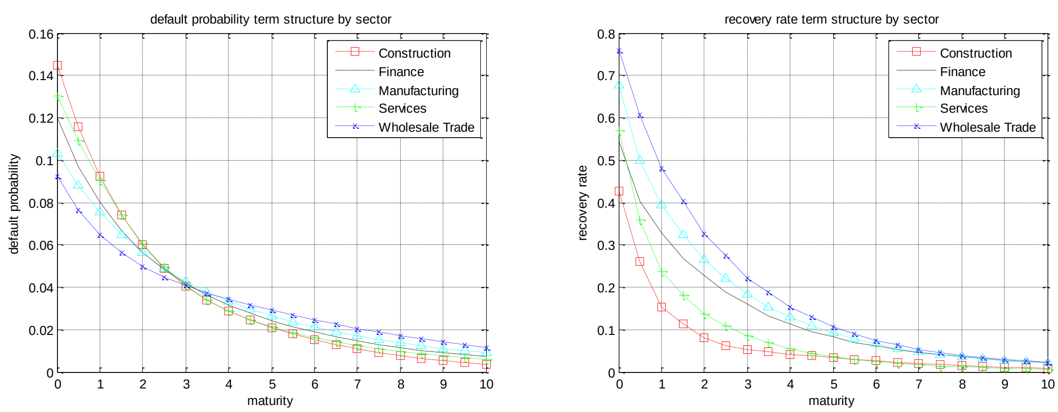
We now compare the default probability and recovery rate across the sectors. We choose five sectors (construction, finance, manufacturing, services and wholesale trade). From ***Data Description***, we know that the wholesale trade sector has the lowest CDS spreads and the construction sector is the highest one. As for the manufacturing sector, it represents most firms in the sample (37%).

Figure 11 shows that the wholesale trade sector is on average the least likely to default in the short term but the most likely to default at long term. It also seems to represent a higher recovery rate than all the other sectors. The construction sector, whose average CDS spread is the highest, is the opposite of the wholesale trade sector.

Another sector that has a very low recovery is the service industry. In fact, the recovery rate is related to the firm's resale value in the case of default. The service industry has obviously very low resale value since these firms might possess fewer assets than those from other industries.

Figure 11: default probability (recovery rate) term structure according to sectors

Figure 11 plots the default probability and recovery rate term structures according to sectors. The construction sector counts 594 term structures; the finance sector counts 2 502 term structures; the manufacturing sector counts 6 449 term structures; the services sector counts 1 297 term structures; the wholesale trade sector counts 405 term structures. We do not plot the mining sector, the retail trade sector and the transportation, communications, electric, gas, and sanitary services sector, since their results are almost the same as the finance sector. We take the average default probability and recovery rate across all the firms (397 firms) and the sample period (45 months).

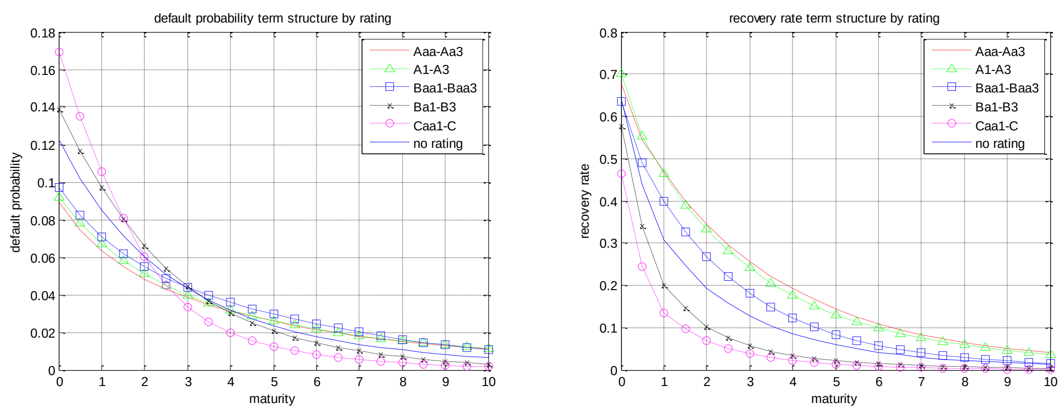


We now compare the results of default probability/recovery rate term structures across all the rating classes (Figure 12). Caa1-C rating class reveals the steepest default probability term structure and the lowest recovery rate for all maturities.

Figure 12 suggests that it is between the investment grade and under-investment grade (between Baa1-Baa3 and Ba1-B3 rating classes) where displays the biggest change. For the first two rating classes (Ass-Aa3 and A1-A3), the results show very little difference.

Figure 12: default probability (recovery rate) term structure according to rating

Figure 12 plots the default probability and recovery rate term structures according to sectors. Class 1 is the firms who get rating Aaa-Aa3 (674 term structures); class 2 is the firms who get rating A1-A3 (3 704 term structures); class 3 is the firms who get rating Baa1-Baa3 (6 603 term structures); class 4 is the firms who get rating Ba1-B3 (4 337 term structures); class 5 is the firms who get rating Caa1-C (817 term structures); class 6 is the firms who do not get rating (837 term structures). We take the average default probability and recovery rate across all the firms (397 firms) and all the sample period (45 months).



Despite some model risk, the final outputs of the term structures seem to be meaningful and reasonable. In the next step, we use these results to compute CVA and compare it to the CVA calculation with constant recovery rate.

C. CVA comparison

In this section, we compare CVA calculation with implied recovery rate to CVA with constant recovery rate. We have choosen 17 "representative obligors" from the sample, and for each obligor, we compare the CVA calculations by the two methodologies.

C.1 "Representative obligors"

To compare the two CVA calculations, we choose 17 representative obligors from the sample. We take the median[13] of the CDS term structures and market capitalisations 1) at the total sample level; 2) for each business cycle; 3) for each sector and 4) for each rating class. Below is the list of 17 representative obligors:

Total sample	**Phases of business cycle**	**Sectors**	**Rating**	**Total**
1 Representative obligor	3 Representative obligors	8 Representative obligors	5 Representative obligors	**17 Representative obligors**

As discussed in **Section 3 Methodology**, the formula for CVA calculation with implied recovery rate is as below:

$$CVA_{ir}(t,T) = \sum_{j=1}^{N} EE(t_j) * (1 - \phi_j{}^{ir}) * \lambda_j{}^{ir}$$

Where $\phi_j{}^{ir}$ and $\lambda_j{}^{ir}$ is the implied default probability/recovery rate term structures extracted from the CDS spread term structure.

[13] Since our sample is asymmetric, we take the median instead of the mean.

And the formula for CVA calculation with constant recovery is as below:

$$CVA_{cr}(t,T) = (1-\phi^{cr})\sum_{j=1}^{N} EE(t_j) * \lambda_j^{\ cr}$$

Where $\lambda_j^{\ cr} = \lambda_j^{\ cumulative} - \lambda_{j-1}^{\ cumulative}$, and $\lambda_j^{\ cumulative} = 1 - e^{-\varepsilon_j * h}$, $\varepsilon_j = \frac{CDS\ Spread_j}{1-\phi^{cr}}$. $\phi_j^{\ cr}$ is the constant recovery and $\lambda_j^{\ cr}$ is the corresponding default probability term structures. We calculate constant recovery CVAs when $\phi_j^{\ cr} = \{60\%, 40\%, 20\%, 0\%\}$, among which 40% constant recovery rate is commonly used in the industry.

Since the expected exposure is the same in both cases, we set $EE(t_j) = 1\$$ for $\forall j$. In this way, we do not take into account of the correlation between expected exposure and default risk. This might reduce the precision of CVA calculation. However, the objective of our Master's thesis is to study the recovery effect on CVA. Hence, setting expected exposure as a fixed amount can eliminate influences from other factors, such as wrong-way or right-way risk. We can be sure that the difference between the two CVAs is only due to the recovery assumption.

C.2 CVA comparison for each "representative obligor"

This part shows the results of the two CVA calculations in the case of each obligor. It is divided into 4 parts: 1) for the "representative obligor" from the total sample level; 2) for the "representative obligor" from each business cycle; 3) for the "representative obligor" from each sector and 4) for the "representative obligor" from each rating class.

For CVA with constant recovery, we tested 4 recovery rates, 60%, 40%, 20% and 0%. 40% is the common assumption in the industry for CVA calculation.

C.2.1 "Representative obligor" of the total sample:

We take the median of CDS term structures and the median of market capitalisation for the total sample to get the first "representative obligor". Then we apply the methodology to get the implied recovery rate/default probability term structures and adopt the results to compute CVA (Figure 13). We also compute CVA with the methodology of constant recovery rate. We tested the constant recovery rate for 4 different values, 60%, 40%, 20% and 0%.

By the methodology of implied recovery, we can extract two downward sloping term structures. However, by the methodology of constant recovery, we get a flat recovery term structure and a humped default probability term structure, which reaches the maximum at 4 years maturity. In the figure, we show the example of 40% constant recovery. It is the same shape of default probability term structure for the other constant recovery rates.

This is intuitive. When we allow the recovery rate to vary, the evolution in CDS spread term structure can be absorbed by both recovery and default probability. In the case of constant recovery rate, all the changes in CDS term structure have to be transferred completely to the default probability. The methodology of constant recovery shows that the marginal default probability term structure is mainly responsible for absorbing the changes in CDS term structure between every maturity. No matter the constant recovery, the shape of default probability term structures will stay similar since they are extracted from the same CDS term structure.

In the short run, constant recovery seems to overestimate loss given default (1-recovery rate), and the corresponding default probability term structure seems to underestimate the real probability of default. For the longer term, the dynamic between the two pairs of term structure becomes the opposite: the methodology with the constant recovery underestimates the loss given default and overestimates the default probability.

The figure of CVA comparison is very informative. Firstly, in the short run, compared to implied recovery methodology, CVA with constant recovery seems to underestimate the exposure to credit risk, even with the most conservative assumption (recovery equals to 0%). This could be due partly to the negative correlation between the default probability and recovery rate, which generally contributes to a higher credit risk valuation. Constant recovery CVA does not capture this effect while we impose negative correlation when extracting the implied recovery rate.

Secondly, we notice that before 3 years maturity, constant recovery CVAs seem to be the same value, no matter the constant recovery levels. This is another proof that constant recovery CVA is quite naive. In the short run, since increases in recovery rate lead automatically to the increase in default probability, CVAs are very likely to stay at the same level. But in the long run, the default probability of smaller recovery rate decreases faster than that of the higher recovery rate. In other words, the default probability of 40% recovery rate is higher than the default probability 0% recovery rate at longer maturities. This contributes to the increasing of CVA and explains the dispersion of constant recovery CVAs at long term.

The opposite is true for the model-implied CVA. It grows faster at short run than at long run. Its CVA curve suggests that for short term, the effect of decreasing in recovery rate dominates the effect of decreasing in default probability. But for long term, the market anticipates stable CVA since the effect of decreasing in default probability starts to compensate decreasing in recovery rate. In fact, the implied recovery model is forward-looking which it is not the case for the constant recovery methodology.

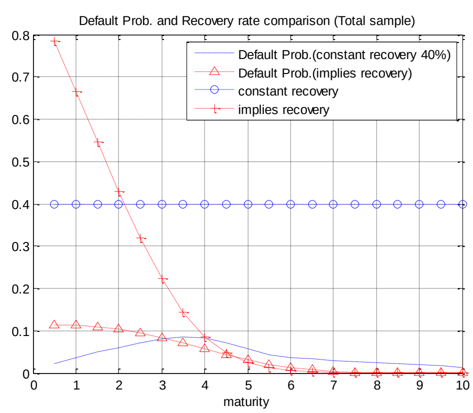
We find that for 20% and 0% constant recovery, their CVAs intersect with implied recovery CVA. And for 40% constant recovery, the difference with model-implied CVA grows until 5 years maturity and shrinks for longer maturities. It suggests that for the short term, the effect of under estimating default probability by the constant recovery methodology seems to dominate the effect of over estimating the loss given default.

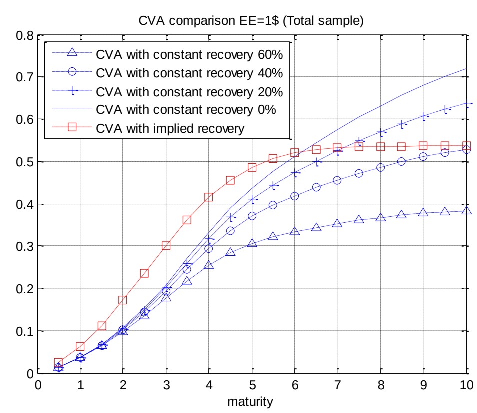
Therefore, for the short term, CVA with constant recovery rate is much smaller than CVA with implied recovery. For the longer term, the effect of under estimating loss given default by the constant recovery methodology starts to dominate the default probability overestimation. Hence, constant recovery CVA gradually overtakes implied recovery CVA.

In other words, CVA is somewhat the combination of the recovery rate and default probability. The effect is from both sides. From the same CDS spread, increasing recovery rate (decreasing loss given default) is related to increased default probability. In this sense, high constant recovery could overestimate CVA in the case of bad credit quality, because it overestimates more default probability than underestimates loss given default. And low constant recovery could also underestimate CVA in the case of good credit quality, since it underestimates more default probability than overestimates loss given default.

Figure 13: CVA comparison: "representative obligor" of total sample

Figure 13 (1) plots the results for default probability/recovery rate term structures (inconstant recovery and constant recovery 40%) for the representative "median" obligor of the total sample. $RMSE/mean(CDS\ spreads)$ for the implied recovery calibration is 1.7%. Figure 13 (2) plots the results of CVA calculation according to the term structures in Figure 13 (1). We set the expected exposure at \$ 1 for all maturities.





C.2.2 “Representative obligor” for each business cycle:

- **Before the crisis**

We take the median of CDS term structures and the median of market capitalization for the sample before the crisis. We compute CVA by the two methodologies and compare them in Figure 14.

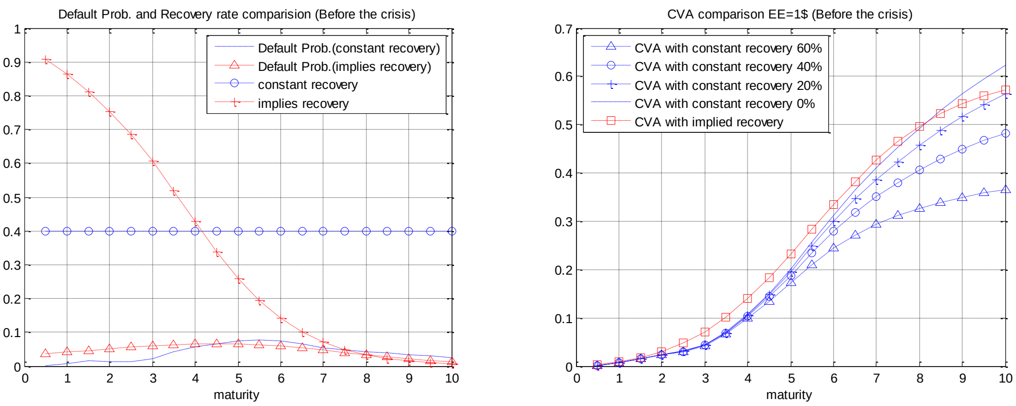
For the term structures, the implied recovery is declining while the implied default probability term structure seems to be humped. For constant recovery, we also get a humped default probability term structure. The level and the shape suggest that the obligor before the crisis is less risky than the obligor of the total sample.

The dynamics between the two pairs of term structures are quite similar as in the case of the total sample, although the difference between the two CVAs seems to be smaller. CVA with implied recovery is bigger than constant recovery CVAs in general. Even 0% constant recovery rate CVA seems to underestimate the exposure to credit risk, compared with model-implied recovery. Until 5 years maturity, all constant recovery CVAs are almost identical.

Implied CVA curve here is clearly increasing from 0.5 to 10 years, which is different from the total sample CVA that is flat after 7 years maturity. It justifies the forward-looking characteristic of our model. For less risky firms, the investors anticipate smaller credit risk in short term but an increasing credit risk for the future.

Figure 14: CVA comparison: "representative obligor" before the crisis

Figure 14 (1) plots the results for default probability/recovery rate term structures (inconstant recovery and constant recovery 40%) for the representative "median" obligor for the sample before the crisis. $RMSE/mean(CDS\ spreads)$ is 8.8%. Figure 14 (2) plots the results of CVA calculation according to term structures in Figure 14 (1). We set the expected exposure at \$ 1 for all maturities.



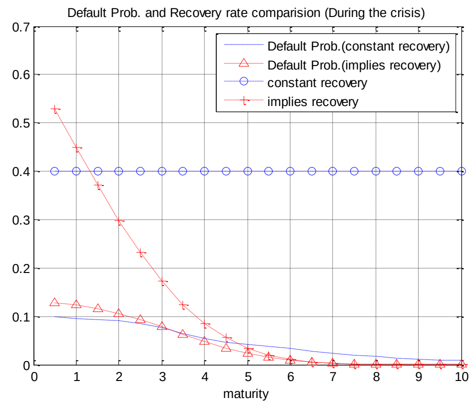
- **During the crisis**

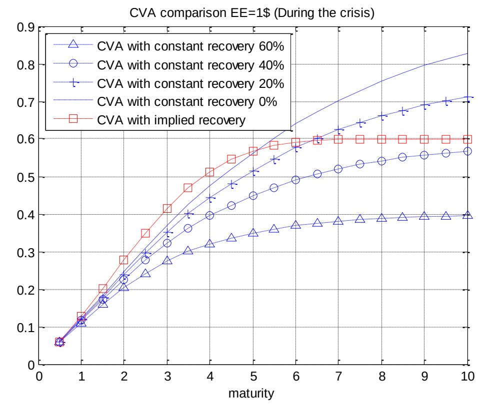
Figure 15 shows the results of default probability/recovery rate term structures and CVA calculation for the "representative obligor" during the crisis. The implied term structures are decreasing. For constant recovery, we also get a declining default probability term structure.

The dynamics between the term structures and CVAs are both quite similar as the total sample obligor. For the short term, 0% recovery rate CVA seems to be quite the equivalent to implied recovery CVA. They insect each other sooner than the case of the total sample.

Figure 15: CVA comparison: "representative obligor" during the crisis

Figure 15 (1) plots the results for default probability/recovery rate term structures (inconstant recovery and constant recovery 40%) for the representative "median" obligor for the sample during the crisis. $RMSE/mean(CDS\ spreads)$ for the implied recovery calibration is 3.2%. Figure 15 (2) plots the results of CVA calculation according to the term structures in Figure 15 (1). We set the expected exposure at \$ 1 for all maturities.





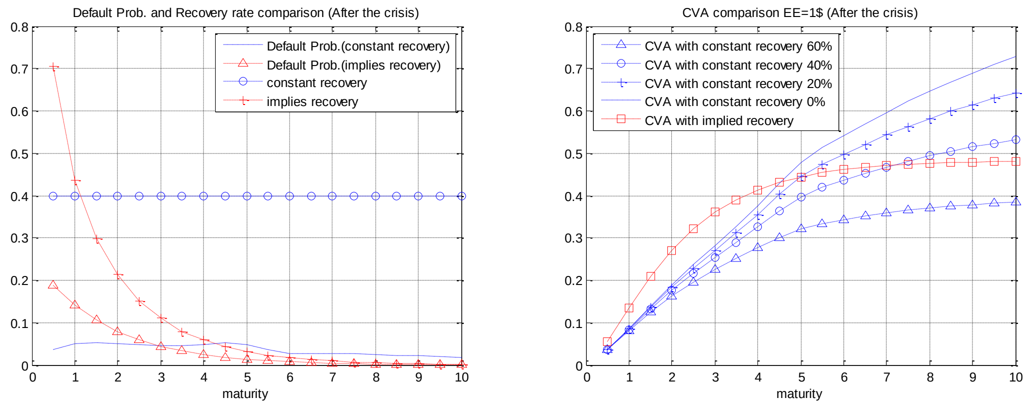
- **After the crisis**

Figure 16 shows the results of default probability/recovery rate term structures and CVA calculation for the "representative obligor" of the period after the crisis.

The dynamics between term structures and CVAs are both quite similar as the total sample obligor. The effect of under estimating loss given default by the constant recovery methodology starts to dominate the default probability overestimation at sooner maturity than in the case of the total sample obligor.

Figure 16: CVA comparison: “representative obligor” after the crisis

Figure 16 (1) plots the results for default probability/recovery rate term structures (inconstant recovery and constant recovery 40%) for the representative “median” obligor for the sample after the crisis. RMSE/mean(CDS spreads) is 8.3%. Figure 16 (2) plots the results for CVA calculation according to the term structures in Figure 16 (1). We set the expected exposure at $ 1 for all maturities.



C.2.3. “Representative obligors” for industry categories:

We display the results for three sectors: construction, manufacturing and wholesale trade. Construction is the sector with the highest average CDS spreads (5 years maturity) and the wholesale trade is the lowest. Manufacturing represents the most firms in the sample. For the other sectors, we observe similar results of CVA comparison.

- **Construction**

We have choosen the median of CDS term structure and the median of market capitalisation for the sample firms from the construction sector.

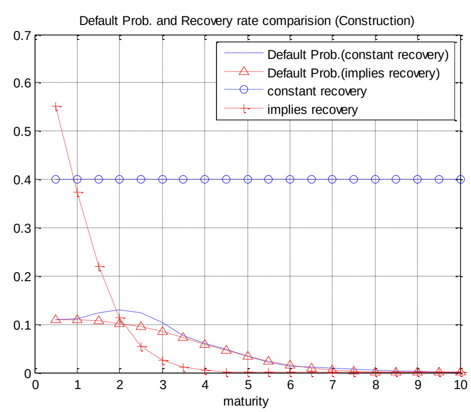
For the term structures, the implied recovery is declining while the implied default probability is decreasing. For the constant recovery, we also get a generally declining default probability term structure. It is coherent with the fact that construction represents the highest average CDS 5-year spread among all the sectors.

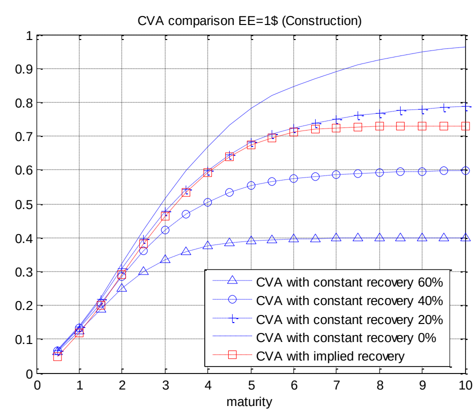
From the CVA comparison, we observe that a 20% constant recovery CVA seems to match quite well the curve of implied CVA. At very short term, all the constant recovery CVAs seem to be slightly bigger than implied CVA.

Later, we find that this is the case for the service sector and the last two rating classes. Firstly, this is might due to some model risk, which we discuss in the next section. The model tends to be less performant in the high default probability cases. Secondly, due to the high CDS spread, constant recovery rate methodology can capture some "forward-looking" information since the marginal default probabilities fall to 0 at the long run. Hence, the curve shape of constant recovery CDS is similar to the model-implied CVA.

Figure 17: CVA comparison: "representative obligor" from construction

Figure 17 (1) plots the results for default probability/recovery rate term structures (inconstant recovery and constant recovery 40%) for the representative "median" obligor for the sample firms from the construction sector. RMSE/mean(CDS spreads) is 6.3%. Figure 17 (2) plots the results of CVA calculation according to the two pairs of term structures in Figure 17 (1). We set the expected exposure at $ 1 for all maturities.



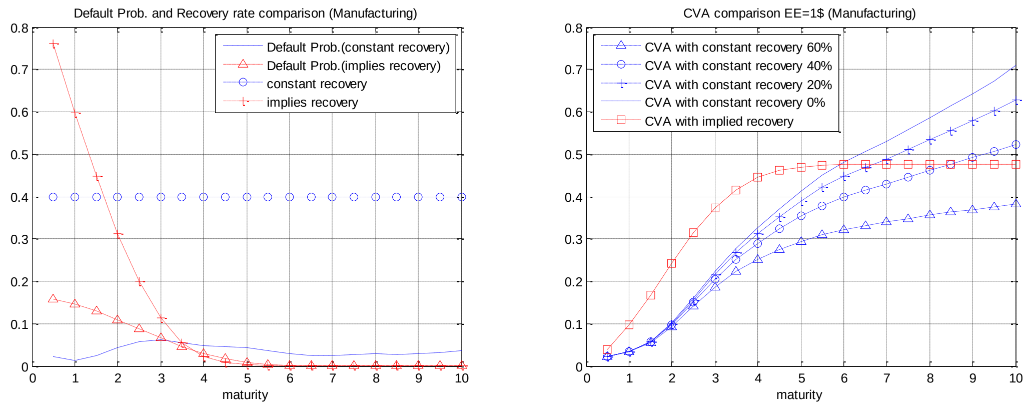


- **Manufacturing**

We have choosen the median of CDS term structure and the median of market capitalisation for the sample firms from the manufacturing sector. The dynamics between the term structures and CVA calculations are very alike to the "representative obligor" after the crisis. We also observe similar results for the four other sectors: finance, mining, retail trade and transportation[14], although the magnitude of the difference between the two CVAs is not the same. They are both "middle" risky cases. (Figure 18)

Figure 18: CVA comparison: "representative obligor" from manufacturing

Figure 18 (1) plots the results for default probability/recovery rate term structures (inconstant recovery and constant recovery 40%) for the representative "median" obligor for the sample firms from the manufacturing sector. RMSE/mean(CDS spreads) is 11.2%. Figure 18 (2) plots the results of CVA calculation according to the term structures in Figure 18 (1). We set the expected exposure at $ 1 for all maturities.



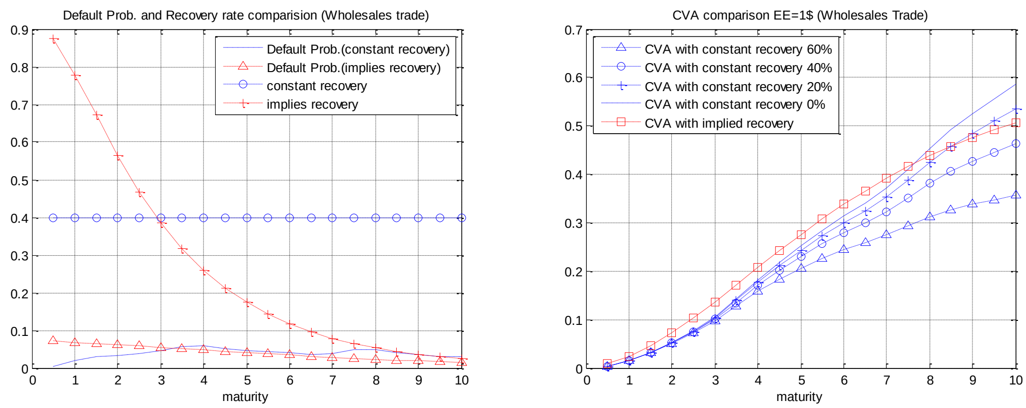
- **Wholesale trade**

We have choosen the median of CDS term structure and the median of market capitalisation for the sample firms from the construction sector. Both implied recovery and default probability term structures are declining. For constant recovery, we get a humped default probability term structure. (Figure 19) CVA comparison looks quite the equivalent to the "representative obligor" before the crisis. They are both less risky cases.

[14] The complete specification of the sector is Transportation, Communications, Electric, Gas, And Sanitary Services.

Figure 19: CVA comparison: "representative obligor" from wholesale trade

Figure 19 (1) plots the results for default probability/recovery rate term structures (inconstant recovery and constant recovery 40%) for the representative "median" obligor for the sample firms from wholesale trade. RMSE/mean (CDS spreads) is 4.9%. Figure 19 (2) plots the results of CVA calculation according to the term structures in Figure 19 (1). We set the expected exposure at $ 1 for all maturities.



C.2.4. "Representative obligor" of each rating class:

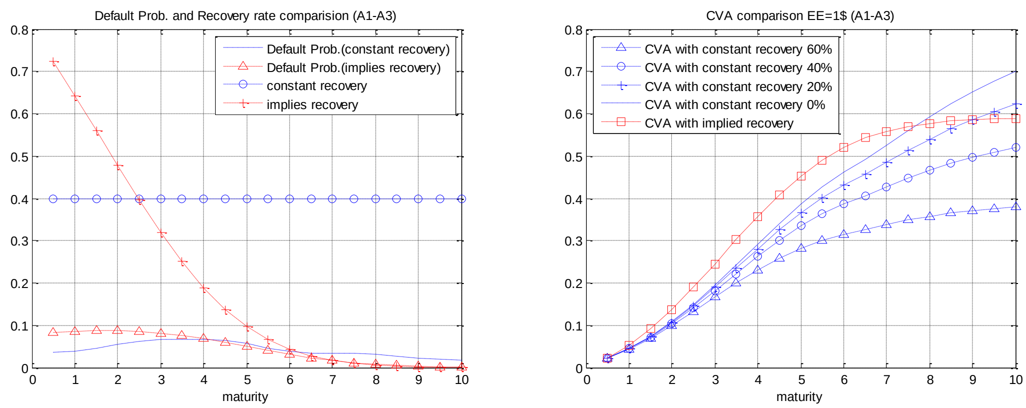
We display the results for 2 rating classes: rated between A1 and A3 and rated between Ba1 and B3. They are chosen to represent accordingly the good credit quality obligors and bad credit quality obligors. We observe similar results in CVA comparison for the other rating classes.

- **Rated between A1 and A3**

Figure 20 shows the results of default probability/recovery rate term structures and CVA calculation for the representative obligor of the rating category A1 to A3. The dynamics between the term structures and CVA calculations are very alike to the "representative obligors" after the crisis and from manufacturing sector. We observe almost the same results for the rating class Baa1 to Baa3. Although the difference between the two CVAs is less obvious, rating class Aaa to Aa3 also shows similar dynamics for CVA comparison.

Figure 20: CVA comparison: “representative obligor” between A1 and A3

Figure 20 (1) plots the results for default probability/recovery rate term structures (inconstant recovery and constant recovery 40%) for the representative “median” obligor for the sample rated between A1 and A3 (including A1 and A3) by Moody’s. RMSE/mean (CDS spreads) is 2.9%. Figure 20 (2) plots the results of CVA calculation according to the term structures in Figure 20 (1). We set the expected exposure at $ 1 for all maturities.



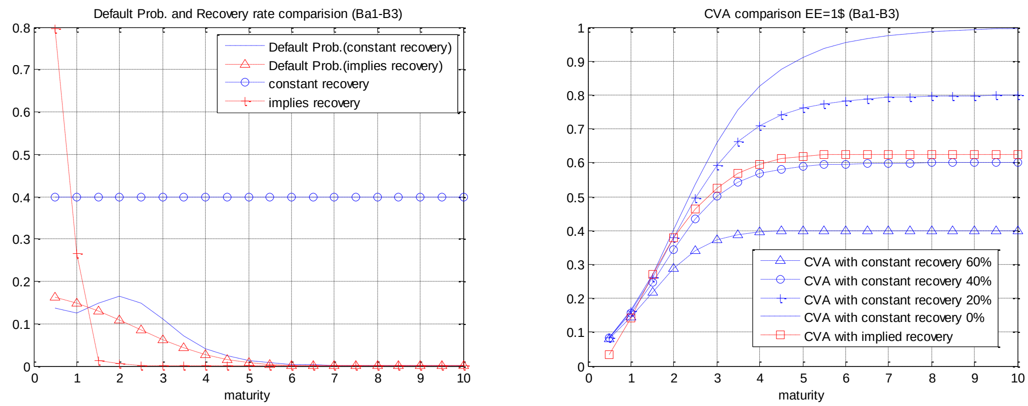
- **Rated between Ba1 and B3**

The “representative obligor” for this class is quite similar to the construction sector. (Figure 21) We observe that a 40% constant recovery CVA seems to matches quite well the curve of implied CVA. For the very short term, all the constant recovery CVAs also seem to slightly be higher than implied CVA. For the rating class Caa1 to C, the results seem to be nearly the same.

The result suggests a higher constant recovery rate then the construction sector, although the CDS spread level is higher for the Ba1-B3 obligor. As discussed at the beginning of the section, CVA is the combined effect from both recovery rate and default probability. Bad credit quality firms can be associated with high constant recovery, because it overestimates more default probability than underestimates loss given default. Good credit quality firms can be associated with low constant recovery, because it underestimates more default probability than overestimates loss given default.

Figure 21: CVA comparison: “representative obligor” between Ba1 and B3

Figure 21 (1) plots the results for default probability/recovery rate term structures (inconstant recovery and constant recovery 40%) for the representative “median” obligor for the sample rated above between Ba1 and B3 (including Ba1 and B3) by Moody’s. RMSE/mean (CDS spreads) is 24.9%. Figure 21 (2) plots the results of CVA calculation according to the two pairs of term structures in Figure 21 (1). We set the expected exposure at $ 1 for all maturities.



It is obvious that CVA comparison results vary from case to case. However, from all these “represntative obligors”, we can classify the results for three categories: “very safe obligors”, “middle risky obligors” and “very risky obligors”. The results suggest that it is the “middle risky obligors” who represents the biggest difference between model-implied CVA and constant recovery CVA. For less risky firms, even 0% constant recovery assumption underestimates the credit risk exposure compared to implied recovery CVA, especially for the short term. For more risky firms, 40% recovery CVA seems to be very close to model-implied CVA. It might suggest that in the two extreme cases, CVA is driven more by default probabilities than by recovery rates.

Conclusion

In this Master’s thesis, we have examined mainly the impact of model-implied recovery rate on CVA calculation. Based on the methodology of Das and Hanouna (2009), we have extracted implied recovery rate and default probability term structures from CDS spread term structure. Although this model might present some risks, and the tests are conducted on the recent financial crisis period, the outputs are quite meaningful and demonstrate timeline variations, industry characteristics and rating class differences.

Our main finding shows that CVA calculation is the combined product of recovery rate and default probability. In the case of very high credit quality and very low credit quality firms, CVA seems to be driven more by default probability. And the difference between constant recovery CVA and implied recovery CVA is relatively small. However, for the middle risky obligors, non-constant recovery rate is very important for CVA calculation. Compared with model-implied CVA, constant recovery CVA seems to underestimate the exposure to credit risk. In other words, people using constant recovery CVA might not be protected enough against credit risk whereas those using non-constant recovery rate CVA appear to be more prudent.

A natural criticism of the work might be that these "representative obligors" cannot "represent" well the total sample. David (2008) argues that "credit spreads are convex functions of firms' solvency ratio, and hence the average spreads are larger than the spread evaluated at the average solvency ratio." In other words, taking the median of CDS spreads and market capitalization does not mean that we get the median level of credit risk of the sample. The alternative way is firstly to calculate CVA for each term structure and then take the mean or median of all these CVAs. This should be much more informative than calculating CVA from one median term structure. Currently, we are pursuing the investigation on this point and trying to extract more information from the sample on the comparison of the two CVAs.

Appendix A. Calibrated Parameter σ

The magnitude of parameter σ from the calibration exercise is relatively wide, compared to the implied volatility calculated by *OptionMetrics*. To fix this problem, we have tried other means for calibration: fixing the implied volatility as provided by *OptionMetrics* and normalizing the state-variable (market capitalization) by the firm's debt. However, after these changes, fitting quality suffers a lot. Hence, we have decided to give the priority to the fitting and conserve the results of parameter σ.

By construction, our model might imply a high volatility parameter. From **Section 3 Methodology**, default probability at each node is a function of market capitalization: $\lambda[i,j] = 1 - e^{-\frac{1}{M[i,j]^b}h}$. We notice that under the condition $b > 0$, if $M[i,j] > 1$, the maximum value of $\lambda[i,j]$ is $1 - exp(-h)$. When $M[i,j] < 1$, the minimum value of $\lambda[i,j]$ is $1 - exp(-h)$. In our case, the time interval is 0.5, and $1 - exp(-h)$ equals 0.39. In other words, if fitting requires default probability higher than 39% for some nodes in the binomial tree, market capitalization must be smaller than 1.

Our sample period covers the recent financial crisis. Hence, it should be common that the lower nodes in the binomial tree need to give high default probabilities. The market capitalization must be smaller than 1 for these lower nodes. As a result, implied volatility must be high enough to allow market capitalization become smaller than 1 in the tree.

Figure 22 plots the mean of parameter σ at each month. Despite the "wrong" magnitude, the calibrated volatility captures well the evolution in timeline. In fact, the results of sigma are much related to the level of CDS spread term structure and its shape (increasing, humped or decreasing). Actually, the riskier the obligor, the sooner its market capitalization needs to be smaller than 1 in the tree. As a consequence, implied volatility has to be bigger.

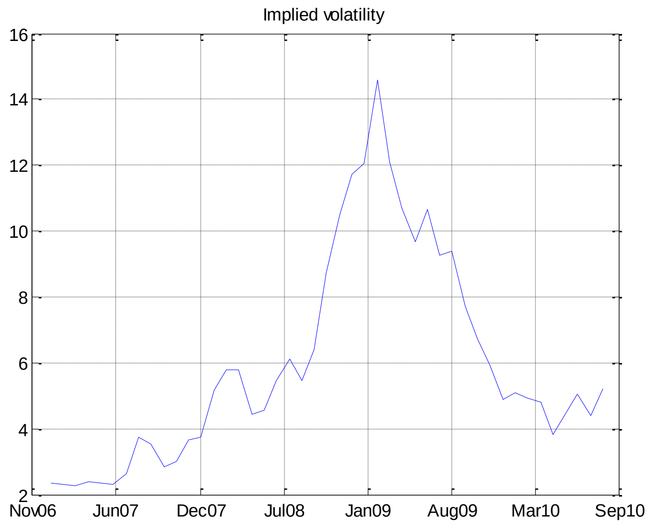


Figure 22: Evolution of calibrated parameter σ

Appendix B. Default probability bound

By the model construction, the default probability might be bounded. From **Section 3 Methodology**, we can compute the probability that the market capitalization goes up at each node as $q[i,j] = \frac{\frac{R}{1-\lambda[i,j]}-d}{u-d}$. Since the probability should stay between $[0,1]$, we get $0 \leq \frac{\frac{R}{1-\lambda[i,j]}-d}{u-d} \leq 1 (Inequation\ 2)$.

By replacing $u = e^{\sigma\sqrt{h}}$, $d = e^{-\sigma\sqrt{h}}$ and $R = e^{fh}$, *Inequation 2* can be transformed as below:

$$1 - \exp(f * h + \sigma\sqrt{h}) \leq \lambda \leq 1 - \exp(f * h - \sigma\sqrt{h})$$

The left part is always satisfied since $1 - \exp(f * h + \sigma\sqrt{h}) < 0$. We need to focus on the right side $\lambda \leq 1 - \exp(f * h - \sigma\sqrt{h})$. Set $y = 1 - \exp(x), x \in (-\infty, 0]$, y varies between $[0,1]$. However, since the nature of parameters f, h and σ (interest rate, time interval and return volatility), $(f * h - \sigma\sqrt{h})$ cannot vary between $(-\infty, 0]$.

If we take the minimum value of the forward rates and maximum value of volatility, we estimate that λ will not be higher than 30%. Actuality, after calibration, the maximum default probability (λ) we get is 23.82%, before taking out the term structures whose $RMSE/mean\ (CDS\ spreads)$ is bigger than 0.31. Further, all these "outliers" that we reject attain the maximum level of default probabilities (Figure 23). In other words, we are eliminating the cases that the firms might be the most likely to default. Our default probability for the total sample estimated before might be biased downwards. Through the dependence, the recovery rate at the total sample level might be biased upwards.

However, the objective of this Master's thesis is to compare constant recovery and implied recovery for CVA calculation. Removing those high RMSEs makes the estimation of implied recovery rates more precise. Further, the cleaned results are still adequate for comparison by industry and rating.

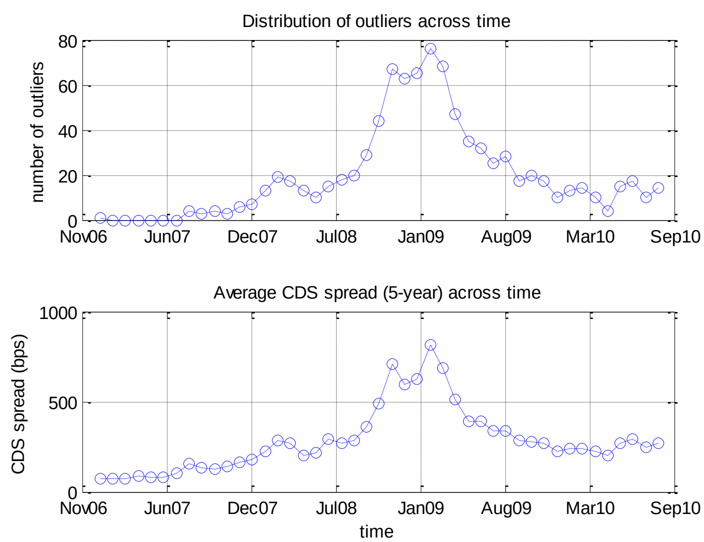


Figure 23: Number of rejected term structures/average CDS spread

Appendix C. List of 397 issuers

Name	Sector	Rating
ALCOA INC	Mining	Baa1-Baa3
AMERISOURCEBERGEN CORP	Wholesale Trade	Baa1-Baa3
ALBERTSON'S INC	Retail Trade	Ba1-B3
ABBOTT LABORATORIES	Manufacturing	A1-A3
ACE LTD	Finance[15]	A1-A3
ARCHER-DANIELS-MIDL. CO	Manufacturing	A1-A3
AMERICAN ELEC PWR CO INC	Transportation[16]	Baa1-Baa3
AES CORP	Transportation	Ba1-B3
AETNA INC	Finance	Baa1-Baa3
ALLERGAN INC	Manufacturing	A1-A3
HESS CORP	Manufacturing	Baa1-Baa3
AMERICAN INTL.GROUP	Finance	A1-A3
AK STEEL CORP	Manufacturing	No rating
ALLSTATE CORP	Finance	A1-A3
ADVANCED MICRO DEVC INC	Manufacturing	Ba1-B3
AMGEN INC	Finance	No rating
AMKOR TECHNOLOGY INC	Manufacturing	Ba1-B3
AMERICAN TOWER CORP	Finance	Baa1-Baa3
AON CORP	Finance	Baa1-Baa3
APACHE CORP	Mining	A1-A3
ANADARKO PETROLEUM CORP	Mining	Ba1-B3
AIR PRODUCTS & CHEMS INC	Manufacturing	A1-A3
ARVINMERITOR INC	Manufacturing	Baa1-Baa3
ARROW ELECTRONICS INC	Wholesale Trade	Baa1-Baa3
ASHLAND INC	Manufacturing	Ba1-B3
ARAMARK CORPORATION	Retail Trade	Ba1-B3
ALLTEL CORP	Transportation	Baa1-Baa3
AVALONBAY COMMNS. INC	Finance	Baa1-Baa3
AVIS BUDGET CAR RENT LLC	Services	No rating
AVON PRODUCTS	Manufacturing	A1-A3
AVNET INC	Wholesale Trade	Baa1-Baa3
ALLIED WASTE NA INC	Transportation	Ba1-B3
AMERICAN AXLE & MNFG INC	Manufacturing	Ba1-B3
AMERICAN EXPRESS CO	Finance	A1-A3
ALLEGHENY EN SUPP CO LLC	Manufacturing	Baa1-Baa3

[15] The complete specification of the sector is: Finance, Insurance, And Real Estate.
[16] The complete specification of the sector is: Transportation, Communications, Electric, Gas, And Sanitary Services.

AUTOZONE INC	Retail Trade	Baa1-Baa3
BANK OF AMERICA CORP	Finance	A1-A3
BAXTER INTERNATIONAL INC	Manufacturing	A1-A3
BRUNSWICK CORP	Manufacturing	Caa1-C
BLACK & DECKER CORP	Manufacturing	Baa1-Baa3
BAKER HUGHES INC	Manufacturing	A1-A3
BELO CORP	Transportation	Ba1-B3
BELLSOUTH CORP	Transportation	A1-A3
BURLINGTON NTHN SNT FE	Finance	Baa1-Baa3
BOEING CO	Manufacturing	A1-A3
BAUSCH & LOMB INC	Manufacturing	Ba1-B3
BERKSHIRE HATH.INC	Finance	Aaa-Aa3
BOSTON SCIENTIFIC CORP	Manufacturing	Ba1-B3
PEABODY ENERGY	Mining	Ba1-B3
BORGWARNER INC	Manufacturing	Baa1-Baa3
BOSTON PROPERTIES LP	Finance	No rating
BOYD GAMING CORP.	Services	Caa1-C
BEAZER HOMES USA INC	Construction	Caa1-C
CITIGROUP INC	Finance	A1-A3
CONAGRA FOODS INC	Manufacturing	Baa1-Baa3
CARDINAL HEALTH INC	Wholesale Trade	Baa1-Baa3
CONTINENTAL AIRLINES INC	Transportation	Ba1-B3
CATERPILLAR INC	Manufacturing	A1-A3
CATERPILLAR FINL.SVS CORP	Manufacturing	A1-A3
CHUBB CORP	Finance	A1-A3
CONSTELLATION BRAND	Services	No rating
CBS CORPORATION	Transportation	Baa1-Baa3
COMCAST CABLE COMMS LLC	Manufacturing	No rating
CARNIVAL CORP	Transportation	A1-A3
AVIS BUDGET GRP INC	Services	Baa1-Baa3
CONSTELLATION EN.GP. INC	Mining	Baa1-Baa3
CHESAPEAKE ENERGY CORP	Mining	Ba1-B3
CIGNA CORP	Finance	Baa1-Baa3
COLGATE-PALMOLIVE CO	Manufacturing	Aaa-Aa3
MACK-CALI REALTY L.P.	Finance	No rating
CLOROX COMPANY	Manufacturing	Baa1-Baa3
COMMERCIAL METALS CO	Manufacturing	Baa1-Baa3
COMCAST CORP	Manufacturing	Baa1-Baa3
CUMMINS INC	Manufacturing	Baa1-Baa3
CMS ENERGY CORP	Transportation	Ba1-B3
CNA FINANCIAL CORP	Finance	Baa1-Baa3

CENTERPOINT ENERGY INC	Transportation	Ba1-B3
CENTERPOINT EN.RES. CORP	Transportation	A1-A3
CAPITAL ONE BANK	Finance	Baa1-Baa3
CAPITAL ONE FINL. CORP	Finance	Baa1-Baa3
MOLSON COORS BREWING COM	Manufacturing	Ba1-B3
CONOCOPHILLIPS	Manufacturing	A1-A3
COSTCO WHOLESALE CORP	Retail Trade	A1-A3
COX COMMUNICATIONS INC	Transportation	Baa1-Baa3
CAMPBELL SOUP CO	Manufacturing	A1-A3
CREDIT SUISSE USA INC	Manufacturing	Baa1-Baa3
COMPUTER SCIENCES CORP	Services	Baa1-Baa3
CISCO SYSTEMS INC	Services	A1-A3
CSX CORP	Transportation	Baa1-Baa3
COOPER TIRE & RUBBER	Manufacturing	Ba1-B3
CENTURYTEL INC	Transportation	Baa1-Baa3
CENTEX CORP	Construction	Ba1-B3
CABLEVISION SYS CORP	Transportation	Ba1-B3
COVENTRY HLTH. CARE	Finance	Ba1-B3
CVS CAREMARK CORP	Retail Trade	Baa1-Baa3
CHEVRONTEXACO CAP.CO	Manufacturing	Aaa-Aa3
CYTEC INDUSTRIES INC	Manufacturing	Baa1-Baa3
CITIZENS COMMS.CO	Transportation	Ba1-B3
DOMINION RESOURCES INC	Transportation	Baa1-Baa3
DU PONT E.I. DE NEMO URS	Manufacturing	A1-A3
DEVELOPERS DIVR.REAL	Finance	Baa1-Baa3
DILLARDS INC	Retail Trade	Ba1-B3
DEERE & CO	Manufacturing	A1-A3
JOHN DEERE CAPITAL CORP	Manufacturing	A1-A3
DELL INC	Manufacturing	No rating
DEAN FOODS CO	Manufacturing	A1-A3
DR HORTON INC	Construction	Ba1-B3
DANAHER CORP	Manufacturing	A1-A3
WALT DISNEY CO/THE	Transportation	A1-A3
DELHAIZE AMERICA INC	Retail Trade	Baa1-Baa3
DELUXE CORP	Manufacturing	Ba1-B3
DIAMOND OFFSHORE DRL	Mining	Baa1-Baa3
DOVER CORP	Manufacturing	A1-A3
DOW CHEMICAL CO	Manufacturing	Baa1-Baa3
DPL INC	Transportation	Baa1-Baa3
DUKE REALTY LP	Finance	Baa1-Baa3
DARDEN RESTAURANTS INC	Retail Trade	Baa1-Baa3

DTE ENERGY COMPANY	Transportation	Baa1-Baa3
DIRECTV HOLDINGS LLC	Transportation	Ba1-B3
DUKE ENRGY CAROLINAS	Transportation	Baa1-Baa3
SPECTRA ENERGY CAP.	Transportation	No rating
DEVON ENERGY CORP	Mining	Baa1-Baa3
CONSOLIDATED EDISON INC	Transportation	A1-A3
ELECTRONIC DATA SYS. CORP	Manufacturing	A1-A3
ENBRIDGE ENERGY LTD PSHP	Transportation	Baa1-Baa3
EASTMAN CHEMICAL CO	Manufacturing	Baa1-Baa3
EMERSON ELECTRIC CO	Manufacturing	A1-A3
EQUITY OFFICE PROPS. TST	Finance	No rating
EL PASO CORP	Transportation	Ba1-B3
ENTERPRISE PRDS.PTNS LP	Transportation	No rating
EMBARQ CORP	Transportation	Baa1-Baa3
EATON CORP	Manufacturing	A1-A3
ENTERGY CORP	Transportation	Baa1-Baa3
EXELON CORP	Transportation	Baa1-Baa3
EXPEDIA INC	Transportation	Ba1-B3
EXELON GENERATION CO LLC	Transportation	Baa1-Baa3
FORD MOTOR CO	Manufacturing	Ba1-B3
FELCOR LODGING LP	Finance	Caa1-C
MACY'S INCORPORATED	Retail Trade	Ba1-B3
FIRST DATA CORP	Services	A1-A3
FEDEX CORP	Transportation	Baa1-Baa3
FIRSTENERGY CORP	Transportation	Baa1-Baa3
FORD MOTOR CREDIT CO LLC	Manufacturing	Ba1-B3
FIRST INDUSTRIAL LP	Finance	No rating
FINANCIAL SCTY ASR INC	Finance	A1-A3
FREESCALE SEMICON INC	Manufacturing	Ba1-B3
FOREST OIL CORP	Mining	Ba1-B3
GANNETT CO INC	Manufacturing	Baa1-Baa3
GENERAL DYNAMICS CORP	Manufacturing	A1-A3
GENERAL ELEC.CAPITAL CORP	Manufacturing	Aaa-Aa3
GENERAL MILLS INC	Manufacturing	Baa1-Baa3
CORNING INC	Manufacturing	Baa1-Baa3
GMAC LLC	Finance	Ba1-B3
GENWORTH FINANCIAL INC	Finance	Baa1-Baa3
GEORGIA-PACIFIC GP.	Manufacturing	Ba1-B3
GRAPHIC PACK INTL INC	Manufacturing	Ba1-B3
GAP INC	Retail Trade	Ba1-B3
GOODRICH CORP	Manufacturing	Baa1-Baa3

GOLDMAN SACHS GP INC	Finance	A1-A3
GLOBALSANTAFE CORP	Mining	Baa1-Baa3
GOODYEAR TIRE & RUB. CO	Manufacturing	Ba1-B3
HALLIBURTON CO	Mining	A1-A3
HASBRO INC	Manufacturing	Baa1-Baa3
HCA INC	Services	Ba1-B3
HEALTH CRE.PR.INVRS. INC	Finance	Baa1-Baa3
MANOR CARE INC	Services	Baa1-Baa3
HOME DEPOT INC	Retail Trade	Baa1-Baa3
HSBC Finance CORP	Finance	A1-A3
HARTFORD FINL.SVS.GP	Finance	Baa1-Baa3
HILTON HOTELS CORP	Services	Caa1-C
HEALTH MAN.ASSOCS. INC	Services	Ba1-B3
HOST HOTELS&RESORTS LP	Finance	Ba1-B3
HEINZ (HJ) CO	Manufacturing	Baa1-Baa3
HONEYWELL INTL.INC	Manufacturing	A1-A3
STARWOOD HTLS.& RSTS WWD	Services	Ba1-B3
K HOVNANIAN ENTS INC	Construction	Caa1-C
BLOCK FINANCIAL CORP	Services	No rating
HERSHEY FOODS CORP	Manufacturing	A1-A3
HUMANA INC	Finance	Ba1-B3
HUNTSMAN INTL LLC	Manufacturing	Ba1-B3
HEWLETT-PACKARD CO	Manufacturing	A1-A3
INTL.BUS.MCHS.CORP	Services	Aaa-Aa3
INTERNATIONAL GAME TECH	Manufacturing	Baa1-Baa3
IKON OFFICE SLTN.INC	Services	A1-A3
INTERNATIONAL PAPER CO	Manufacturing	Baa1-Baa3
INTERPUBLIC GP.COS. INC	Services	Ba1-B3
IRON MOUNTAIN	Services	Ba1-B3
ILLINOIS TOOL WORKS INC	Manufacturing	A1-A3
JETBLUE AIRWAYS CORP	Transportation	Caa1-C
JOHNSON CONTROLS INC	Manufacturing	Baa1-Baa3
JC PENNEY CO	Retail Trade	Ba1-B3
JOHNSON & JOHNSON	Manufacturing	Aaa-Aa3
JONES APPAREL GP. INC	Manufacturing	Ba1-B3
JPMORGAN CHASE & CO	Finance	A1-A3
NORDSTROM INC	Retail Trade	Baa1-Baa3
KELLOGG CO	Manufacturing	A1-A3
KB HOME	Construction	Ba1-B3
KIMCO REALTY CORP	Finance	Baa1-Baa3
KIMBERLY-CLARK CORP	Manufacturing	A1-A3

KERR-MCGEE CORP	Manufacturing	Ba1-B3
KINDER MORGAN ENERGY PRNS	Transportation	Baa1-Baa3
COCA-COLA CO	Manufacturing	Aaa-Aa3
KROGER CO	Retail Trade	Baa1-Baa3
MCCLATCHY CO.	Manufacturing	Caa1-C
KEYSPAN CORPORATION	Transportation	Baa1-Baa3
KOHLS CORP	Retail Trade	Baa1-Baa3
LENNAR CORP	Construction	Ba1-B3
LABORATORY CORP.OF AMER	Services	Baa1-Baa3
LIZ CLAIBORNE INC	Manufacturing	Caa1-C
L-3 COMMUNICATIONS CORP	Manufacturing	Ba1-B3
ELI LILLY & CO	Manufacturing	A1-A3
LOCKHEED MARTIN CORP	Manufacturing	Baa1-Baa3
LOUISIANA-PACIFIC CORP	Manufacturing	Ba1-B3
LTD BRANDS	Retail Trade	Ba1-B3
LOEWS CORP	Finance	A1-A3
ALCATEL-LUCENT USA INC	Manufacturing	No rating
SOUTHWEST AIRLINES	Transportation	Baa1-Baa3
LEVEL 3COMMS.INC	Transportation	Caa1-C
LOWES COMPANIES INC	Retail Trade	A1-A3
LUBRIZOL CORP	Manufacturing	Baa1-Baa3
MARRIOTT INTL.INC	Services	Baa1-Baa3
MASCO CORP	Manufacturing	Ba1-B3
MATTEL INC	Manufacturing	Baa1-Baa3
MACY'S RET HDG INCO	Retail Trade	Ba1-B3
MBIA INC	Finance	Ba1-B3
MBIA INSURANCE CORP	Finance	Ba1-B3
MCDONALD'S CORP	Retail Trade	A1-A3
MCKESSON CORP	Wholesale Trade	Baa1-Baa3
MDC HOLDINGS INC	Construction	Baa1-Baa3
MEDIACOM LLC	Construction	Caa1-C
MEDTRONIC INC	Manufacturing	A1-A3
MASSEY ENERGY CO	Mining	Ba1-B3
MERRILL LYNCH & CO. INC	Finance	A1-A3
METLIFE INC	Finance	A1-A3
MGM MIRAGE INC	Services	Caa1-C
MOHAWK INDUSTRIES INC	Manufacturing	Ba1-B3
MEDCO HEALTH SLTN. INC	Retail Trade	Baa1-Baa3
MIRANT NORTH AMERICA INC	Transportation	Caa1-C
MARTIN MARIETTA MATS INC	Mining	Baa1-Baa3
MARSH & MCLENNAN COS INC	Finance	Baa1-Baa3

3M COMPANY	Manufacturing	Aaa-Aa3
MAGELLAN MDSTM PTNS LP	Transportation	Baa1-Baa3
ALTRIA GROUP INC	Manufacturing	Baa1-Baa3
MOSAIC GLOBAL HLDGS	Manufacturing	Baa1-Baa3
MOTOROLA INC	Manufacturing	Baa1-Baa3
MERCK & CO INC	Manufacturing	A1-A3
MARATHON OIL CORP	Manufacturing	Baa1-Baa3
MONSANTO CO	Manufacturing	A1-A3
MGIC INVESTMENT CORP	Finance	Ba1-B3
MERITAGE HOMES CORP	Construction	Ba1-B3
MURPHY OIL CORP	Manufacturing	Baa1-Baa3
MORGAN STANLEY GP. INC	Finance	Aaa-Aa3
MEADWESTVACO CORP	Manufacturing	Ba1-B3
NAVISTAR INTL.CORP	Manufacturing	Ba1-B3
ONEOK PARTNERS L.P.	Transportation	Baa1-Baa3
NABORS INDUSTRIES INC	Mining	Baa1-Baa3
NEWMONT MINING CORP	Mining	Baa1-Baa3
NISOURCE Finance CORP	Finance	Baa1-Baa3
NALCO CO	Manufacturing	No rating
NEIMAN-MARCUS GROUP INC	Retail Trade	Ba1-B3
NORTHROP GRUMMAN CORP	Manufacturing	Baa1-Baa3
NRG ENERGY INC	Transportation	Caa1-C
NORFOLK SOUTHERN CORP	Transportation	Baa1-Baa3
NORTHEAST UTILITIES	Transportation	Baa1-Baa3
NUCOR CORP	Manufacturing	A1-A3
NVR INCORPORATED	Construction	Baa1-Baa3
NEWELL RUBBERMAID INC	Manufacturing	Baa1-Baa3
THE NEW YORK TIMES CO.	Manufacturing	Ba1-B3
OMNICARE INC	Retail Trade	Ba1-B3
OFFICE DEPOT INC	Retail Trade	Caa1-C
OWENS-ILLINOIS INC	Manufacturing	Ba1-B3
ONEOK INC	Transportation	Baa1-Baa3
OLIN CORP	Manufacturing	Ba1-B3
OMNICOM GROUP	Services	Baa1-Baa3
ORACLE CORP	Finance	A1-A3
OCCIDENTAL PETROLEUM CORP	Mining	A1-A3
PEPSI BOTTLING GROUP INC	Manufacturing	A1-A3
PITNEY BOWES INC	Manufacturing	A1-A3
PPACIFIC GAS & ELEC CO	Transportation	Baa1-Baa3
PHELPS DODGE CORP	Manufacturing	Baa1-Baa3
PRIDE INTERNATIONAL INC	Mining	Ba1-B3

PEPSICO INC	Manufacturing	Aaa-Aa3
PFIZER INC	Manufacturing	A1-A3
PRUDENTIAL FINANCIAL INC	Finance	No rating
PROCTER & GAMBLE CO	Manufacturing	Aaa-Aa3
PROGRESS ENERGY INC	Transportation	Baa1-Baa3
PULTE HOMES INC	Construction	Ba1-B3
PARKER DRILLING CO	Mining	Ba1-B3
PACK CORP OF AMERICA	Manufacturing	Baa1-Baa3
PROLOGIS TRUST	Finance	Baa1-Baa3
POLYONE CORP	Manufacturing	Ba1-B3
PEPCO HOLDINGS INC	Transportation	Baa1-Baa3
PPG INDUSTRIES INC	Manufacturing	Baa1-Baa3
PPL ENERGY SUPP.LLC	Transportation	Baa1-Baa3
PRIMEDIA INCO	Manufacturing	Ba1-B3
PACTIV CORPORATION	Manufacturing	Caa1-C
PRAXAIR INC	Manufacturing	A1-A3
PIONEER NATURAL RESC CO	Mining	Ba1-B3
QWEST COMMS INTL INC	Services	Ba1-B3
QWEST CAPITAL FDG. INC	Transportation	Ba1-B3
QWEST CORP	Transportation	Ba1-B3
RYDER SYSTEM INC	Services	Baa1-Baa3
RITE AID CORP	Retail Trade	Caa1-C
REALOGY CORPORATION	Transportation	No rating
ROYAL CRBN.CRUISES LTD	Transportation	Ba1-B3
RADIAN GROUP INC	Finance	Ba1-B3
TRANSOCEAN INC	Mining	Baa1-Baa3
ROCK-TENN CO	Manufacturing	Ba1-B3
RELIANT ENERGY	Transportation	Caa1-C
ROHM & HAAS CO	Manufacturing	Baa1-Baa3
RPM INTERNATIONAL INC	Manufacturing	Baa1-Baa3
RR DONNELLEY & SONS	Manufacturing	Baa1-Baa3
REPUBLIC SERVICES INC	Transportation	Baa1-Baa3
RADIOSHACK CORP	Retail Trade	Ba1-B3
RAYTHEON CO	Manufacturing	Baa1-Baa3
REYNOLDS AMERICAN INC	Manufacturing	Baa1-Baa3
RYLAND GROUP INC	Construction	Ba1-B3
AT&T INC.	Transportation	A1-A3
SINCLAIR BRDCT GP	Transportation	Ba1-B3
SCANA CORP	Transportation	Baa1-Baa3
SUNGARD DATA SYSTEMS	Services	Ba1-B3
SMITHFIELD FOODS INC	Manufacturing	Caa1-C

ISTAR FINANCIAL INC	Finance	Caa1-C
SCHERING-PLOUGH CORP	Manufacturing	A1-A3
SHERWIN-WILLIAMS CO	Wholesale Trade	A1-A3
SAKS INC	Retail Trade	Ba1-B3
SARA LEE CORP	Retail Trade	No rating
SOLECTRON CORP	Manufacturing	Ba1-B3
SOUTHERN CALI EDISON CO	Transportation	Baa1-Baa3
STANDARD PACIFIC CORP	Construction	Ba1-B3
SIMON PROPERTY GROUP INC	Finance	A1-A3
STAPLES INC	Wholesale Trade	Baa1-Baa3
SEMPRA ENERGY	Transportation	Baa1-Baa3
SERVICE CORP INTL	Finance	Ba1-B3
SOUTHERN COMPANY	Finance	Baa1-Baa3
SEAGATE TECH HDD HDG	Manufacturing	No rating
SOUTHERN UN CO	Transportation	Baa1-Baa3
SUNOCO INC	Manufacturing	Baa1-Baa3
SUN MICROSYSTEMS INC	Manufacturing	Ba1-B3
SERVICEMASTER CO.	Mining	Caa1-C
SUPERVALU INC	Retail Trade	Ba1-B3
STANLEY WORKS	Manufacturing	Baa1-Baa3
SAFEWAY INC	Retail Trade	Baa1-Baa3
SYSCO CORPORATION	Wholesale Trade	A1-A3
TECO ENERGY INC	Transportation	Baa1-Baa3
TARGET CORP	Retail Trade	A1-A3
TENET HEALTHCARE CORP	Services	Caa1-C
TEMPLE-INLAND INC	Manufacturing	Ba1-B3
TJX COMPANIES INC/ THE	Retail Trade	A1-A3
TIME WARNER ENTM CO LP	Services	Baa1-Baa3
TOLL BROTHERS INC	Construction	Ba1-B3
SUNCOM WIRELESS INC	Construction	No rating
TRW AUTOMOTIVE INC	Manufacturing	Ba1-B3
SABRE HOLDINGS CORP	Transportation	Caa1-C
TYSON FOODS INC	Manufacturing	Ba1-B3
TIME WARNER INC	Services	Baa1-Baa3
TEXAS INSTRUMENT INC	Manufacturing	A1-A3
TEXTRON INC	Manufacturing	Baa1-Baa3
TEXTRON FINANCIAL CORP	Manufacturing	Baa1-Baa3
TXU CORP	Transportation	Ba1-B3
TXU ENERGY CO LLC	Transportation	Ba1-B3
UNIVERSAL HEALTH SVS INC	Services	Ba1-B3
UNISYS CORP	Services	Ba1-B3

UNITEDHEALTH GROUP INC	Finance	Baa1-Baa3
UNUM GROUP	Finance	A1-A3
UNION PACIFIC CORP	Transportation	Baa1-Baa3
UNITED PARCEL SER. INC	Transportation	Aaa-Aa3
UNITED RENTALS INC	Services	Ba1-B3
UST INC	Finance	A1-A3
UNITED TECHNOLOGIES CORP	Manufacturing	A1-A3
UNVL.CORP/RICHMND. VA	Manufacturing	Ba1-B3
UNIVISION COMM INC.	Transportation	Ba1-B3
VF CORP	Manufacturing	A1-A3
VIACOM INC	Transportation	Baa1-Baa3
VALERO ENERGY CORP	Manufacturing	Baa1-Baa3
VORNADO REALTY LP	Finance	Baa1-Baa3
VERIZON COMMS INC	Transportation	A1-A3
WISCONSIN ENERGY CORP	Transportation	A1-A3
WENDY'S INTL.INC	Retail Trade	Caa1-C
WELLS FARGO & CO	Finance	A1-A3
WEATHERFORD INTL.INC	Mining	Baa1-Baa3
WHIRLPOOL CORP	Manufacturing	Baa1-Baa3
WINDSTREAM COMMS	Transportation	Ba1-B3
WELLPOINT INC	Finance	Baa1-Baa3
WILLIAMS COS INC/THE	Transportation	Baa1-Baa3
WASTE MANAGEMENT INC	Transportation	Baa1-Baa3
WAL-MART STORES INC	Retail Trade	Aaa-Aa3
WESTERN UNION CO/THE	Services	A1-A3
WEYERHAEUSER CO	Manufacturing	Ba1-B3
WYETH	Manufacturing	A1-A3
UNITED STATES STEEL	Manufacturing	Ba1-B3
XCEL ENERGY INC	Transportation	Baa1-Baa3
EXXON MOBIL CORP	Manufacturing	Aaa-Aa3
XEROX CORP	Manufacturing	Baa1-Baa3
XTO ENERGY INC	Mining	Aaa-Aa3
YRC WORLDWIDE INC	Transportation	Caa1-C
YUM! BRANDS INC	Retail Trade	Baa1-Baa3

Appendix D. List of defaults

1. On December 31, 2008, GMAC LLC ("GMAC") announced that it has completed a debt restructuring in which cash, notes and preferred shares were exchanged for approximately $17.5 billion of old GMAC LLC notes and $3.7 billion of old Residential Capital, LLC .
2. Beginning on December 3, 2008, Hovnanian Enterprises, Inc. ("Hovnanian") had been completing open market debt repurchases of its senior unsecured and senior subordinated notes at substantial discounts to par. On December 3, 2008, K. Hovnanian Enterprises.
3. On April 6, 2009, Ford Motor Company (“Ford”) announced the results of its debt restructuring and exchange offers in which approximately $9.9 billion of outstanding debt was reduced. The transactions were funded with $2.4 billion in cash and 468 million.
4. On June 25, 2009, The McClatchy Company (“McClatchy”) accepted for purchase at discount to par value about $103 million of its outstanding Senior Unsecured bonds of various maturities. Moody's believes the completion of the exchange offer to retire debt.
5. In the second quarter of 2009, Beazer Homes USA, Inc. (“Beazer”) made a repurchase of $115.5 million face value of senior unsecured notes for an aggregate purchase price of $58.2 million.
6. On December 30th 2009, YRC Worldwide Inc (“YRCW” or the company) completed a debt-for-equity exchange on substantial majority of the Senior Notes and Convertible notes issued by YRCW and its subsidiary - USF Corp.

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