An Application of Stochastic Programming to Logistics Network Design Under Demand, Cost and Capacity Constraints

By
Ehsanallah Naseri

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ABSTRACT

Supply chain management has grown tremendously in recent years and has become a crucial and competitive tool for companies in today’s fast-paced global economy. Companies try to improve their supply chain performance by employing new technologies and methods, and considering unpredictable behaviors and disruptions in order to survive in the competitive market. One of the most effective ways to tackle uncertainty in supply chain is to have a well-structured resilient network, which can encompass the fluctuations and uncertainties through design parameters and respond effectively to upcoming disruptions.

This thesis addresses the problem of logistics network design in the case of uncertainty in design parameters. Demand, supply, transportation and operating costs, and capacity are among the main parameters that carry a stochastic behavior by nature and fluctuate for various reasons over the planning horizon. In this work, in order to measure the effect of uncertainty and variability in the design parameters, a two-stage stochastic optimization approach is applied to formulate the stochastic logistics network design problem. A heuristic scenario-based solution approach called sample average approximation is used in order to solve the stochastic model.

The results from the stochastic approach and the classical deterministic approach are compared and analyzed through various sample instances in order to demonstrate the effect of the stochastic design parameters and their variability levels on the network design process.
“Here’s to the crazy ones. The misfits. The rebels. The troublemakers. The round pegs in the square holes. The ones who see things differently. They’re not fond of rules. And they have no respect for the status quo. You can quote them, disagree with them, glorify or vilify them. About the only thing you can’t do is ignore them. Because they change things. They push the human race forward. And while some may see them as the crazy ones, we see genius. Because the people who are crazy enough to think they can change the world, are the ones who do.”

(Steve Jobs)
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CHAPTER 1

Introduction

1.1 General Overview

A supply chain is a connected network of suppliers (S), manufacturing plants (P), distributors (D), and customers (C), in which materials move from suppliers to customers, and information flows in both directions (Figure 1.1). Logistics refers to the part of supply chain that plans, implements, and controls the effective and efficient flow of materials and information between points in order to satisfy customers’ demand. Different types of facilities in the supply chain are organized to acquire raw materials, convert them to finished products, and distribute these products to customers (Geoffrion and Powers, 1995).

![Diagram of a basic supply chain network]

Figure 1.1: An example of a basic supply chain network
Logistics network design refers to building an efficient supply chain network in order to satisfy customer needs in a cost-wise approach; this involves making decisions regarding:

- the number, location, capacity and technology of manufacturing plants and warehouses
- the selection of suppliers
- the assignment of products and raw materials to plants and warehouses
- the selection of distribution channels and transportation modes
- the flow of materials through the network: raw materials, semi-finished goods, and finished goods.

These decisions can be classified into three levels of planning due to their importance, impacts, and the planning horizon. The strategic level supply chain planning involves deciding the configuration of the network, i.e., the number, locations, capacity and technology of plants and warehouses with a long-term planning horizon. The tactical level planning of supply chain operations involves selecting suppliers, assigning materials to facilities, as well as selecting distribution channels and transportation modes, in a medium-term planning horizon that can be revised every few months. Finally, the operational level planning of supply chain involves short-term planning of flows of materials through the network (Cordeau et al., 2006).

The physical structure of the supply chain network has a significant impact on the performance of operational activities, such as production and distribution of materials and products, and it is very important to design an efficient supply chain to facilitate the movements of goods. Therefore, the total cost of the finished product, which incorporates the procurement, production, warehousing and distribution cost, is affected by supply chain network architecture.

Today’s competitive fast-moving market forces companies to constantly emphasize productivity gains and customer satisfaction, which leaves no room for mistakes. Companies focus on satisfying customer demands while challenging on time, quality and price as the three important elements in the competitive market. They try to serve customers more rapidly with a higher level of quality while minimizing their total cost and the price
of the product. To this end, they need to precisely concentrate on supply chain design and planning process as a key factor in delivering products to customers competently. The globalization of economic activities together with fast developments in information technologies have led to shorter product life cycles, smaller lot sizes and very dynamic customer behaviour in terms of their preferences. These aspects result in growing customer demand uncertainty (Melo et al., 2009).

Supply chain operations are inevitably faced with a wide range of uncertainties and variations in critical parameters such as customer demand, supply, inventory and transportation costs, and resource capacity. In fact, uncertainty rules the supply chain. Sales deviate from the forecast. Components are damaged in transit. Fabrication yields fail to meet the plan. Shipments are held up in customs. Therefore, these uncertainties may lead to delays and bottlenecks, and hamper the performance output of the supply chain. Hence, if the designed supply chain is not robust enough with respect to all these uncertainties and variations, the impact of operational inefficiencies such as delays and disruptions would be larger than necessary (Santoso et al., 2005).

1.2 Problem Definition

In order to take into account the effects of the uncertainty in supply chain design, this thesis addresses the Logistics Network Design Problem (LNDP) of a manufacturing firm in the presence of uncertainty. We consider uncertainty in certain design parameters such as customer demands, supplies, production and transportation costs, and capacities of plants and warehouses to design a reliable and responsive supply chain network. More precisely, the LNDP consists in answering the following:
How many plants do we need? And where should we locate them?
How many warehouses and distribution centers do we need? And where to locate them?
How much should be the capacity of each plant and warehouse for each product?
From which supplier should we get each raw material?
In which plant and which warehouse should we produce and keep each product?
How to send materials and products through the network? With which mode?
How much each plant should produce and each warehouse should keep?
How much materials and product should be transported in the network to satisfy all customer demands?

The objective of the problem is to minimize the total fixed and variable costs associated with configuration design, procurement, production, warehousing, and transportation, while satisfying customer demands and capacity constraints, and considering uncertainty in design parameters. The main purpose of our work is to analyze the impact of different types of uncertainty on the network design and quantify the benefits that can be achieved by taking uncertainty into account.

1.3  Solution Methodology

The problem we face in this study is a logistics network design with uncertainty in design parameters. In order to tackle this problem, a stochastic optimization approach is employed. A two-stage stochastic programming model with a recourse function is proposed, in which the first stage consists of deciding the configuration decision with binary variables, and the second stage consists of processing and transporting products from suppliers to customers in an optimal way based on the configuration and the realized uncertain scenario. The objective function of the problem is to minimize the total fixed cost of current investment and the expected future variable costs.
The exact approaches to tackle the problem of stochastic programming for supply chain design under uncertainty are suited for a very small number of scenarios. Consider a logistics network with only 50 facilities, each with an uncertain parameter. Now assume that the operating level for a facility can be one of only three possibilities and these are independent across facilities. Hence, the total number of scenarios would be $3^{50}$ scenarios for the realization of the uncertainties, which is far more than what can be handled by existing exact methods to solve the problem. Therefore, heuristic approaches are often applied to tackle large-scale problems. In this study, we use a sampling strategy in order to repeatedly generate a moderate number of realized scenarios and solve the equivalent deterministic optimization problems. This approach is called Sample Average Approximation (SAA), and it can produce high quality solutions to stochastic problems (Verweij et al., 2003). In order to assess the performance of the proposed approach, an experimental study is performed to solve a realistic supply chain design problem. Several test problems and scenarios are generated and solved with CPLEX 12.5.1. The stochastic solution is obtained by applying the SAA method on the designed test problems. Furthermore, the deterministic solutions are obtained by taking average of the objective function values of the deterministic model for the designed test problems. Finally, the solutions of two approaches are compared and discussed.

1.4 Thesis Outline

This work is organized as follows. An introduction to supply chain planning, network design, and solution approaches to logistics network design is given in Chapter 1. A review of the relevant literature is then given in Chapter 2. Problem description, modeling and solution approaches are presented in Chapter 3. An experimental design and data analysis and results are presented in Chapter 4. Finally, conclusions and future research directions are presented in Chapter 5.
CHAPTER 2

Literature Review

This chapter provides a brief overview of the current state of the literature on several aspects of supply chain network modeling and optimization within the following research streams:

- Supply chain management
- Facility location models
- Distribution network design
- Logistics network design
- Logistics network design in the presence of uncertainty

2.1 Supply Chain Management

A supply chain is a group of organizations performing various processes required to make a finished product. The chain begins with raw materials and ends with the finished product which is delivered to the customers. This chain includes suppliers, manufacturers, distributors, warehouses, distribution centers, retailers, and customers. Within each organization, the supply chain includes all functions involved in satisfying customer demand. Supply chain management (SCM) is defined as the set of approaches utilized to efficiently integrate suppliers, manufacturers, warehouses, and retailers so that the merchandize is produced in the right quantities, distributed to the right locations, and at the right time in order to minimize the total cost of the system while satisfying service level requirements and quality issues (Simchi-Levi et al., 2008). The Council of Supply Chain Management Professionals (2008) (CSCMP) defines supply chain management as follows:
“Supply chain management encompasses the planning and management of all activities involved in sourcing and procurement, conversion, and all logistics management activities. Importantly, it also includes coordination and collaboration with channel partners, which can be suppliers, intermediaries, third party service providers, and customers. In essence, supply chain management integrates supply and demand management within and across companies.”

Generally speaking, SCM is the collection of practices that enable the efficient flow of materials, products, and information to meet customer demand.

Logistics is a key component of supply chain management. It is defined by the Council of Logistics Management (2004) (CLM) as follows:

“That part of the supply chain process that plans, implements, and controls the efficient, effective flow and storage of goods, services, and related information from point-of-origin to point-of-consumption in order to meet customers' requirements.”

For a given supply chain structure, most of studies focused on identifying the best approaches to make the flows efficient. However, one essential criterion for these approaches to be successful is that there exists a well-designed supply chain structure. This underlying structure typically is called the logistics network and can be represented as a directed connected graph where each edge has a capacity and receives a flow. If a logistics network is not carefully designed, even the best SCM approaches will have limited efficiency.

Supply chain network design is a critical and important decision. It comprises which products to manufacture, which markets to serve, which suppliers to select, which technologies to acquire, which transportation modes to utilize, which distribution channels to use, and where to locate plants, warehouses, distribution centers, and retailers in order to serve customers more efficiently while minimizing the total cost. The nature of these decisions may vary from very long term and strategic decisions which require large
investments, to tactical and operational decisions which involve medium and short term horizons, respectively, and require small investments.

Simchi-Levi et al. (2008) classified supply chain decision phases into three categories based on the frequency with which they are made and the length of the planning horizon over which they have an impact: Strategic, tactical, and operational. Strategic decisions, which include decisions regarding location, capacity, and technology, would have effects on a firm over the next few years or even decades. They also involve huge investments, such as the cost of building a factory or acquiring a certain technology. Tactical decisions, which include supplier selection, product range assignment, inventory policies, distribution channel and transportation mode selection, may have an impact on a firm over the next three to twelve months. Finally, operational decisions such as routing materials and products through the network, and scheduling would affect very short term operations of a firm, and are usually made on a daily basis. Table 2.1 shows more examples of decisions in SCM in each category. This table is based on Ballou (2004).

<table>
<thead>
<tr>
<th>Decisions/Phases</th>
<th>Strategic</th>
<th>Tactical</th>
<th>Operational</th>
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<tbody>
<tr>
<td>Location</td>
<td>Number of facilities,</td>
<td>Inventory level</td>
<td>Routing, dispatching,</td>
</tr>
<tr>
<td></td>
<td>capacity, location</td>
<td></td>
<td>expediting</td>
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<tr>
<td>Transportation</td>
<td>Mode selection</td>
<td>Seasonal service mix</td>
<td>Replenishment quantity and</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>timing</td>
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<tr>
<td>Order Processing</td>
<td>Selecting and designing</td>
<td>Priority rules for customer</td>
<td>Expediting orders</td>
</tr>
<tr>
<td></td>
<td>ordering system</td>
<td>service</td>
<td></td>
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<tr>
<td>Customer service</td>
<td>Standard setting</td>
<td>Pre-transaction and transaction</td>
<td>Providing an appropriate service</td>
</tr>
<tr>
<td></td>
<td></td>
<td>setting</td>
<td>level</td>
</tr>
<tr>
<td>Warehousing</td>
<td>Layout and site selection</td>
<td>Seasonal space choice</td>
<td>Order fulfilment</td>
</tr>
<tr>
<td>Purchasing</td>
<td>Policies</td>
<td>Contracting and supplier</td>
<td>Order releasing</td>
</tr>
<tr>
<td></td>
<td></td>
<td>selection</td>
<td></td>
</tr>
</tbody>
</table>

Table 2.1 Examples of decisions in SCM

Based on the definition of logistics given above and the examples of decision phases in Table 2.1, it is clear that two major issues in designing an efficient logistics network are
facility location and distribution. Management should decide how many plants and distribution centers to operate, where to locate them, which set of retailers to assign to each distribution center and which set of suppliers to feed each plant, and how often, in what quantities, by which mode, and in which path to deliver the material and product to all layers and units so that a satisfactory level of customer service is achieved at minimum total cost. These two decision areas (location and distribution) are interrelated in the sense that a change in the location or number of plants and distribution centers affects the routing and distribution procedures. Likewise, a change in distribution and routing plan affects assignment decisions and thereby may affect location decisions as well. Moreover, changes in either area would affect the customer service level of the company.

In order to achieve a high performance supply chain, decisions in different phases must be optimized in an integrated manner. Most supply chain optimization models in the literature have thoroughly considered facility location problems and distribution problems separately as these two decisions are not dimensionally compatible. Decisions related to facility location are strategic decisions whereas those related to distribution and routing are tactical and operational (Peidro et al., 2009). Hence, the classical approach of tackling these problems separately leads to substantial excess costs because the supply chain is run sub-optimally. Recently, different models proposed in the literature have integrated these two decision areas. Developing an integrated location-distribution supply chain network design model is the main motivation of this work.

Another motivation of this research is that real world supply chain network problems are generally subject to uncertainty and disruptions. However, the majority of the research assumes that the operational characteristics of, and hence the design of parameters for, the supply chain are deterministic. Unfortunately, critical parameters such as customer demands, transportation costs, prices, and resource capacity are quite uncertain (Santoso et al., 2005). In this work, we study the design of an integrated supply chain network in an uncertain environment, in which demand, transportation costs, supply, and capacity are subject to change and may fluctuate over time. To this end, a stochastic optimization method is applied to model and solve the problem. The remainder of this chapter will present various models of locations problems, distributions problems, integrated network
design models, and finally the logistics network design models in the presence of uncertainty in the literature, and for each case, solution methodologies and related approaches will be discussed.

2.2 Facility Location Models

Francis et al. (1983) defined location problems as locating one or more new facilities, which may include fixed costs, transportation costs, constraints on the number of new facilities, upper bounds on distances between new and existing facilities, as well as determining the amounts to be shipped between new and existing facilities.

There exists a wide literature related to location models, and since it is not possible to survey all of it, this review first categorizes these models and then explores the more relevant models to this work.

2.2.1 Continuous Models

Location models can be categorized into discrete and continuous forms based on the feasible solution space and decision variables. Continuous location models are characterized by two essential attributes: the solution space is continuous, which means it is feasible to locate facilities in every point in the plane, and distance is measured with a suitable metric. Typically, the Manhattan (right-angle distance) metric, or the Euclidian (straight line distance) metric, is employed to solve these problems (Klose and Drexl, 2005).

2.2.2 Discrete Models

Discrete models are perhaps the most relevant ones in the context of logistics due to the incorporation of fixed costs and distribution costs (Francis et al., 1983). Discrete facility location problems cover a wide range of problems in which the goal is to locate a set of facilities in a distribution network while meeting several requirements. Unlike the continuous models, the solution space for discrete models consists of a finite set of sites
with nonnegative cost of opening corresponding to each. The need for a new facility location can arise from many sources. As an example, a newly formed organization must decide the location of its operating facilities. As another example, a power provider company must find the best possible location for a new power generating plant. The rest of this review will focus on discrete facility location models.

### 2.2.2.1 Deterministic and Static Models

One of the simplest location problems relates to a static and deterministic model in which $p$ facilities are to be located in order to minimize the total distances or cost for supplying customer demands. This is the so-called $p$-median problem which was introduced by Hakimi (1964), and in which all candidate sites are equivalent in terms of fixed costs for locating new facilities. This problem minimizes the total weighted distance between the customers and the $p$ located facilities.

For some problems, selecting locations to minimize the average distance travelled would not be quite appropriate. For example in the problem of locating ambulances or fire stations, the key issue is maximizing the overall coverage. The demand is said to be covered if it can be served within a specified time or distance. The literature on covering problems is divided into two major segments; the location set covering problem and the maximal covering problem (Owen and Daskin, 1998). In the first case, the objective is to minimize the cost of facility location such that a specified coverage level is obtained. This allows us to examine how many facilities are needed to reach to a certain coverage level to all customers. The second case seeks to maximize the amount of demand covered within the acceptable service distance by locating a fixed number of facilities.

The two problems mentioned above provide a strong foundation for much of the location theory but they do not consider the fixed cost of facility locations. In order to explicitly incorporate the location costs, the family of fixed charge facility location problems has been developed. One model in this set is the uncapacitated fixed charge facility location problem (UFLP) in which the objective function of the $p$-median problem is extended with a term for fixed facility location costs and as a result, the number of facilities to be
established typically becomes an endogenous decision variable (Melo et al., 2009). Dearing (1985) called this problem the problem of simple plant location problem (SPLP) which can be formulated as follows:

Let the potential plant locations be indexed by \( i = 1, \ldots, p \), and the customer demand locations by \( j = 1, \ldots, d \). If a plant is located at \( i \), the variable \( y_i = 1 \) and a fixed cost \( f_i \) is incurred. Otherwise, \( y_i = 0 \). The variable \( x_{ij} \) denotes the fraction of customer \( j \)’s demand shipped from plant \( i \). The total cost to serve demand location \( j \) from \( i \) is \( c_{ij} \).

The UFLP or SPLP can be written as follows:

Minimize

\[
\sum_{i=1}^{p} \sum_{j=1}^{d} c_{ij} x_{ij} + \sum_{i=1}^{p} f_i y_i
\]  \hspace{1cm} (2.1)

subject to:

\[
\sum_{i=1}^{p} x_{ij} = 1 \hspace{1cm}, \hspace{1cm} \forall \ j
\]  \hspace{1cm} (2.2)

\[
0 \leq x_{ij} \leq y_i \hspace{1cm}, \hspace{1cm} \forall \ i, j
\]  \hspace{1cm} (2.3)

\[
y_i \in \{0,1\} \hspace{1cm}, \hspace{1cm} \forall \ i
\]  \hspace{1cm} (2.4)

The objective function (2.1) represents the total fixed and variable costs of locating \( p \) facilities, whereas constraints (2.2) ensure that the demand at each customer zone is satisfied. Constraints (2.3) guarantee that customer demand can be produced and shipped only from locations where a facility is established, i.e., if \( y_i = 1 \), and in such a case the firm incurs the associated fixed cost. The weak formulation of UFLP uses a more compact formulation of these constraints by aggregating constraints (2.5) into a single set of constraints for each facility location \( i \) as follows:

\[
\sum_{j} x_{ij} \leq d y_i \hspace{1cm}, \hspace{1cm} \forall \ i
\]  \hspace{1cm} (2.5)
Krarup and Pruzan (1983) extended this model and included the per unit cost of operations in facility $i$, by adding a $p_i$ parameter to $c_{ij}$ in the objective function which considers production and administrative costs. The authors gave an excellent survey of SPLP and its relationships with packing, set covering, and set partitioning problems. According to Guignard and Spielberg (1977):

“The SPLP is one of the simplest mixed integer problems which exhibit all the typical combinatorial difficulties of mixed (0,1) programming and at the same time has a structure that invites the application of various specialized techniques.”

This problem was shown by Cornuéjols et al. (1991) to be NP-hard. There exist many exact and approximate methods in the literature to solve this problem. The very first exact approach has been proposed by Balinski and Wolfe (1963, 1988) who used a Benders decomposition technique. Efroymson and Ray (1966) solved the problem by using a branch-and-bound algorithm. Their study proposed simplifications to the branch-and-bound algorithm that reduces the number of branches that have to be considered; thereby they reduced the necessary time to find solutions for problems with up to 50 potential warehouse locations and 200 demand nodes (retailers).

Generally speaking, branch-and-bound algorithms for the UFLP use the fact that it is not necessary to require the variables $x_{ij}$ to be integer. It is in fact sufficient to branch on the binary variables $y_i$. But these approaches are quite inappropriate even for medium-size UFLP instances. Hence, due to this complexity, most approaches to solve UFLP are heuristics. Feldman et al. (1966) proposed a greedy heuristic algorithm, DROP, which initially opens all the facilities and eventually closes them one by one. Manne (1964) proposed a local search procedure that moves from one solution to a neighbour which gives the greatest decrease in the total cost. Erlenkotter (1978) developed a method DUALLOC based on a linear programming dual formulation to solve UFLP, which obtained and verified optimal solutions quickly with no branching required. Daskin (1995) proposed the most common Lagrangian relaxation method for UFLP which involves relaxing the
assignments constraints (2.2) in order to decompose the original problem into easier sub-problems.

The UFLP can be extended further in order to incorporate facility capacities. In this model which is called the \textit{capacitated plant location problem} (CPLP), capacities are the total number of units each facility can serve. By adding a set of constraints to the UFLP formulation, we require that the sum of demands ($D_j$) assigned to each facility not exceed the capacity ($S_i$).

The CPLP model can be written as follows:

$$\text{Minimize} \quad \sum_{i=1}^{p} \sum_{j=1}^{d} c_{ij} x_{ij} + \sum_{i=1}^{p} f_i y_i$$

subject to:

$$\sum_{i=1}^{p} x_{ij} = 1 \quad , \quad \forall j \quad (2.7)$$

$$0 \leq x_{ij} \leq y_i \quad , \quad \forall i, j \quad (2.8)$$

$$\sum_{j=1}^{d} D_j x_{ij} \leq S_i y_i \quad , \quad \forall i \quad (2.9)$$

$$y_i = 0,1 \quad , \quad \forall i \quad (2.10)$$

In this formulation, constraints (2.9) are the capacity constraints which indicate that no facility can supply more than its capacity.

The applications of the CPLP are not limited to plant location. For example, the same mathematical model is quite appropriate for making optimal lot sizing decisions in production planning (Cornuéjols et al., 1991). Sankaran and Raghavan (1997) extended the classical CPLP model to incorporate the endogenous selection of facility sizes. Mukundan and Daskin (1991) considered a similar approach in modeling a profit maximization content. The CPLP is also NP-hard and it is difficult to solve for large size problems. Albareda-Sambola et al. (2011) studied the problem of \textit{capacity and distance constrained plant location problem} (CDCPPLP), which is an extension of the discrete
capacitated plant location problem, where the customers assigned to each plant have to be assigned to vehicles. This is a difficult combinatorial problem combining the CPLP and the bin packing problem. The authors presented several formulations for the problem and proposed different families of valid inequalities, and analyzed the results in order to find a general-purpose solver. Cornuéjols et al. (1991) applied several relaxation techniques to generate heuristic feasible solutions which are better than the classical greedy or interchangeable heuristics, both in computational time and the quality of the obtained solutions. Another interesting extension of CPLP is to consider several products in the model. This problem is called the multiproduct capacitated facility location (MPCFL) problem in which demand for a number of different product families must be supplied from a set of facility sites, and each site offers a choice of facility types having different capacities. Mazzola and Neebe (1999) modeled this problem as follows:

Minimize $\sum_{f \in F} \left[ \sum_{i \in I} \sum_{j \in J} c_{ijf}x_{ijf} + \sum_{i \in I} e_{if}y_{if} \right] + \sum_{k \in K} \sum_{i \in I} f_{ik}z_{ik}$ \quad (2.11)

subject to:

$\sum_{i \in I} x_{ijf} = 1 , \forall j \in J , f \in F$ \quad (2.12)

$x_{ijf} - y_{if} \leq 0 , \forall i \in I , j \in J , f \in F$ \quad (2.13)

$\sum_{f \in F} y_{if} D_f - \sum_{k \in K} z_{ik} S_k \leq 0 , \forall i \in I$ \quad (2.14)

$\sum_{k \in K} z_{ik} \leq 1 , \forall i \in I$ \quad (2.15)

$x_{ijf} \geq 0 , \forall i \in I , j \in J , f \in F$ \quad (2.16)

$y_{if} \in \{0,1\} , \forall i \in I , f \in F$ \quad (2.17)

$z_{ik} \in \{0,1\} , \forall i \in I , k \in K$ \quad (2.18)
This model contains three sets of decision variables. For $i \in I$, $j \in J$, and $f \in F$, let $x_{ijf}$ be the fraction of customer $j$’s demand for product $f$ that is shipped from a facility located at site $i$; also, let $y_{if}$ be a binary variable which indicates if the facility at site $i$ is supplying the product $f$, and $z_{ik}$ is another binary variable which indicates if a facility of type $k$ is opened at site $i$.

The objective function (2.11) minimizes the sum of variable costs which contains production and shipping costs, and the fixed costs. Constraints (2.12) ensure that all of customer $j$’s demand for each product is satisfied. Constraints (2.13) require that each facility site $i$ first be equipped to produce product $f$ in order for any of product $f$ to be shipped from it. Constraints (2.14) establish the finite capacity of the facility types at each site. Specifically, it is assumed that once a facility at location $i$ is equipped to supply product $f$, then all of the demand, $D_f$, for that product can be supplied from the site. These constraints then ensure that the total demand required to produce the corresponding families at each site $i$ does not exceed the total capacity of the facility types opened at that site. Constraints (2.15) provide for the selection of at most one facility type from among the set $K$ of facility types at each potential site. Constraints (2.16) allow for the inclusion of additional system configuration constraints involving the $y$ variables. Finally, constraints (2.17) and (2.18) provide for non-negativity and integrality of decision variables.

2.2.2 Dynamic Models

The strategic nature of facility location problems brings the idea of optimization over time, which involves the problem of facility locations over an extended planning horizon. Indeed, decision makers must not only select robust locations, which will effectively serve changing demand over time, but also they must consider the timing of facility expansions and relocations over the long term. Ballou (1968) was the first to attempt locating a single warehouse so as to maximize the profits over a discrete but finite planning horizon. Wesolowsky (1975) examined another unconstrained version of single facility location over a finite planning horizon with explicit facility location costs. Drezner and
Wesolowsky (1991) studied locating a facility in a growing city with predictable population shifts, which means demand would change over time but in a deterministic manner.

2.2.2.3 Stochastic Models

The models mentioned and discussed above all assume that input parameters have known values or they vary deterministically over time. In this section, several works are presented which address the stochastic nature of real world problems. Owen and Daskin (1998) categorized stochastic location problems into two main families: the probabilistic approach and the scenario planning approach.

The probabilistic approach considers probability distributions associated with modeled random quantities. Some authors incorporate these distributions into standard mathematical programming, while others use them within a queuing framework (Owen and Daskin, 1998). Uncertainty in demand, supply, facility availability, vehicle availability, traveling time, and capacity of facilities are probabilistic parameters that have been considered in the literature.

Scenario planning is an approach in which decision makers capture uncertainty by introducing a number of possible future states. The objective is to find solutions that satisfy the scenarios and perform well under all scenarios (Mobasheri et al., 1989). In other words, this approach provides a set of scenarios which represent all possible realizations of unknown parameters. Owen and Daskin (1998) presented three approaches to incorporate scenario planning into location modeling as follows:

1. Optimizing the expected performance over all scenarios,
2. Optimizing the worst-case performance, and
3. Minimizing the expected or worst-case regret across all scenarios.

The regret in this case can be calculated by comparing the performance of optimal locations for the scenario with the performance of the compromise locations when the scenario is
realized. Ghosh and McLafferty (1982) applied scenario planning concepts to make retail location decisions in an uncertain environment.

2.3 Distribution Network Design

As defined earlier, SCM is the delivery of enhanced customer and economic value through synchronized management of flow of physical goods and information from sourcing to consumption. A further extension of the location problem is the two-echelon facility location problem. Here, deliveries are made from first-echelon facilities (such as plants) to second-echelon facilities (such as warehouses and distribution centres), and from there to customers (Jayaraman and Pirkul, 2001). The standard distribution problem as stated by Geoffrion and Powers (1995) is to find a minimal cost configuration of a company’s production and distribution network that satisfies product demands at specified customer service levels. Inputs for this problem consist of product lists and customer demands, facility locations and capacities, available transportation modes and their corresponding costs, and various possible policies such as shipment rules and customer service requirements. Desired outputs of this model include answering to such questions as follows:

- How many distribution centers (DC) or consolidation centers are required, and where should they be located?
- Should all DCs carry all products or should they be specialized by certain products?
- Which customer should be served by each DC for each product?
- What should each plant produce and how much?
- Which supplier should be selected and at which level?
- What should be the annual transportation flows through the system?

Geoffrion and Graves (1974) proposed one of the first models for designing a distribution network that provides the optimal location of intermediate distribution facilities between plants and facilities and assignments of products and customers to these centers in a way to minimize the total cost of the system. They assumed there exist several commodities produced at several plants with given production capacities. There is also a known demand
for each commodity at each of a number of customer zones. This demand is satisfied by shipping the required goods and products via regional DCs, with each customer zone being assigned exclusively to a single DC. Each DC has a lower bound and an upper bound on the allowable total annual throughput which represents its capacity. The possible locations of DC are given, but the particular sites to be used are to be selected in a way that minimizes the total distribution costs. Hence, the problem is to determine which DC sites to select, what capacity to have at each selected site, what customer zone should be served by each DC, and what should be the pattern for flowing commodities through the network.

The mathematical formulation of the problem uses the following notation:

- $p$ index for plants,
- $f$ index for commodities,
- $i$ index for possible DC sites,
- $j$ index for customer demand zones,
- $S_{fp}$ production capacity for commodity $f$ at plant $p$,
- $D_{fj}$ demand for commodity $f$ in customer zone $j$,
- $V_i, \overline{V}_i$ minimum and maximum allowed total annual throughput for a DC at site $i$,
- $f_i$ fixed portion of the annual possession and operating costs for a DC at site $i$,
- $v_i$ variable unit cost of throughput for a DC at site $i$,
- $c_{ijfp}$ average unit cost of producing and shipping commodity $f$ from plant $p$ through DC $i$ to customer zone $j$,
- $x_{ijfp}$ a decision variable that identifies the amount of commodity $f$ shipped from plant $p$ through DC $i$ to customer zone $j$,
- $y_{ij}$ a binary variable that identifies if DC $i$ serves customer zone $j$,
- $z_i$ a binary variable that identifies if a DC is acquired at site $i$.

The problem can be formulated as the following mixed-integer linear program:
Minimize

\[
\sum_{f \in F} \sum_{p \in P} \sum_{i \in I} \sum_{j \in J} c_{ijfp} x_{fpij} + \sum_{i \in I} \left[ f_i z_i + v_i \sum_{f \in F} \sum_{j \in J} D_{fj} y_{ij} \right]
\]  \hspace{1cm} (2.19)

subject to:

\[
\sum_{i \in I} \sum_{j \in J} x_{fpij} \leq S_{fp}, \quad \forall f, p
\]  \hspace{1cm} (2.20)

\[
\sum_{p \in P} x_{fpij} = D_{fj} y_{ij}, \quad \forall i, j, f
\]  \hspace{1cm} (2.21)

\[
\sum_{i \in I} y_{ij} = 1, \quad \forall j
\]  \hspace{1cm} (2.22)

\[
V_i z_i \leq \sum_{f \in F} \sum_{j \in J} D_{fj} y_{ij} \leq \overline{V_i} z_i, \quad \forall i
\]  \hspace{1cm} (2.23)

\[
x_{fpij} \geq 0, \quad \forall f, p, i, j
\]  \hspace{1cm} (2.24)

\[
y_{ij} \in \{0,1\}, \quad \forall i, j
\]  \hspace{1cm} (2.25)

\[
z_i \in \{0,1\}, \quad \forall i
\]  \hspace{1cm} (2.26)

The objective (2.19) is to minimize the total cost of shipping and producing the commodities plus the total cost of having and running DCs. Constraints (2.20) are the supply constraints, and constraints (2.21) are set to satisfy all the demands. Constraints (2.22) specify that each customer zone must be served by a single DC. Constraints (2.23) imply that the total annual throughput for a DC when it is open should be between its upper bound and lower bound. Moreover these constraints enforce the correct logical relationship between the binary variables \(y\) and \(z\) (i.e., \(z_i = 1 \Leftrightarrow y_{ij} = 1\) for some \(j\)). Constraints (2.24) - (2.26) are non-negativity and binary conditions for decision variables in the model.
This problem can be solved by a Benders decomposition approach, a well-known partitioning procedure: in this case, the multi-commodity LP sub-problem decomposes into as many independent classical transportation problems as there are commodities. In other words, the decomposition separates the problem at each iteration into several easily solved LPs (one for each commodity). Thomas and Griffin (1996) remarked that the computational results show that the Benders decomposition approach of Geoffrion and Graves (1974) performs remarkably well on this class of problems.

The model presented above (Geoffrion and Graves, 1974) does not choose among alternative plant sites and also does not incorporate any fixed cost for annual possession and operating cost for plants. Pirkul and Jayaraman (1996) extended this model in a way to minimize the sum of the fixed costs of establishing and operating the plants and DCs plus the variable cost of transporting units of commodities through the network, and costs for distributing multiple products from the DCs to the customers in order to satisfy the multiple demands of the customers. Both plants and warehouses are capacitated in their model. Hence, two key decisions have to be made; first, choosing \( W \) out of \( M \) possible DCs and assigning of customer demands for multiple products from the set of open DCs, and second, choosing \( U \) out of \( N \) possible location sites. Therefore, the constraints that these decisions must satisfy can be formulated as follows:

\[
\sum_{i=1}^{M} z_i = W ,
\]

\[
\sum_{k=1}^{N} u_k = U,
\]

where:

\( u_k \) a binary variable which indicates if site \( k \) is open,

\( U \) the number of required plants to be located,

\( W \) the number of required DCs to be located.
In this model, the number of open DCs and plants is fixed in advance. However, the model can be made more general by incorporating the number of open DCs and plants as decision variables. Pirkul and Jayaraman (1996) applied Lagrangian relaxation to the model, and also presented a heuristic to produce efficient feasible solutions for the problem. Jayaraman and Ross (2003) studied a class of distribution network design problems called PLOT (Production, Logistics, Outbound Transportation), which is characterized by multiproduct, a central manufacturing plant site, multiple distribution centers, and the customers zones which demand multiple units of several commodities. Therefore, they proposed two key stages in decision making; strategic, which incorporates selecting the best DCs to operate, and operational, which decides the required quantity of commodities need to be transhipped through the network. They used a simulated annealing (SA) methodology which performed well for solving strategic and operational planning problems.

2.4 Logistics Network Design

The two-echelon facility location problem can be further extended to incorporate the procurement process and supplier selection. One of the first efforts to integrate procurement, production and distribution decisions belongs to Cohen and Lee (1988) who presented a framework for linking decisions and performance through the supply chain. Goetschalckx et al. (2002) defined the logistics network design problem (LNDP) as follows: given a set of potential suppliers, potential manufacturing facilities, and distribution centers with multiple possible configurations, and customers with given demands, the process of determining the configuration of the production–distribution system such that customer demands and service requirements are met and the profit of the corporation is maximized. In other words, this approach is an integrated location, production, and distribution problem. A multi-period model for LNDP in a global context is proposed by Arntzen et al. (1995). Their proposed global supply chain model (GSCM) is a large mixed integer linear program that incorporates a global, multi-product bill of materials for supply chains with arbitrary echelon structure and comprehensive model of manufacturing and distribution decisions. A sophisticated solution methodology based on elastic constraints, row factorization, cascaded problem solution and constraint-branching
enumeration has been applied to solve the problem faced by Digital Equipment Corporation. Dogan and Goetschalckx (1999) studied the integrated design of strategic supply chain networks, and determination of tactical production-distribution allocation in the case of seasonal demand for customers. Their model is multi-period and integrates issues such as facility location and sizing, along with tactical decisions concerning production, inventory, and customer allocation. They used Benders decomposition approach in which the sub-problem separates into a set of network flow problems.

Goetschalckx et al. (2002) introduced two models for the LNDP and presented associated solution methodologies. Their first model maximizes the total after tax profit of an international corporation by focusing on transfer pricing. They proposed an efficient heuristic iterative solution algorithm, which alternates between the optimization of the transfer prices and the material flows. In their second model, they focused on production and distribution allocation in a single country considering seasonal demand for customers. They developed an integrated design methodology based on primal decomposition methods for the mixed integer programming formulation. The primal decomposition allows a natural split of the production and transportation and provides a very efficient solution algorithm for this general class of large mixed integer programming models.

Eskigun et al. (2005) considered the design of an outbound supply chain network considering lead times, location of distribution facilities and choice of transportation mode. They presented a Lagrangian heuristic which provided and excellent solution quality in reasonable computational time.

Cordeau et al. (2006) presented a general integrated formulation of the LNDP for the deterministic, single-country, single period context. The formulation is flexible and can easily be adapted to handle multiple technology and capacity alternatives at any given location. The authors presented two solution approaches for the problem: a simplex-based branch-and-bound approach and a Benders decomposition approach. Moreover, the authors proposed valid inequalities to strengthen the LP relaxation of the model and improve both algorithms. The computational experiments show that their methods are competitive and that the Benders decomposition approach is slightly more advantageous on the more difficult problems.
2.5 Logistics Network Design in the Presence of Uncertainty

Rosenhead et al. (1972) divided decision making environments into three categories: certainty, risk, and uncertainty. In certainty situations, all parameters are deterministic and known, whereas in risk situations, there are uncertain parameters whose values are ruled by probability distributions that are known by decision makers. In uncertainty situations, parameters are uncertain, and furthermore, no information about probabilities is known. Problems in risk situations are known as stochastic optimization problems in which a common goal is to optimize the expected value of the objective function. Problems under uncertainty are known as robust optimization problems and often attempt to optimize the worse-case performance of the system. The difference between uncertainty and risk may relate to the type of outcome that might be expected. According to Simangunsong et al. (2012), some researchers suggest that risk is only associated with issues that may lead to negative outcomes, whilst issues of uncertainty can have both negative and positive outcomes. For example, the risk associated with a natural disaster can only lead to supply chain problems; while uncertainty regarding customer demand can result in estimated demand being either better or worse. Van der Vorst and Beulens (2002) define supply chain uncertainty as follows:

“Supply chain uncertainty refers to decision making situations in the supply chain in which the decision maker does not know definitely what to decide as he [or she] is indistinct about the objectives; lacks information about (or understanding of) the supply chain or its environment; lacks information on processing capacities; is unable to accurately predict the impact of possible control actions on supply chain behaviour; or, lacks effective control actions (non-controllability).”

In supply chain planning, critical parameters such as customer demands, supplies, and process/manufacturing are quite uncertain. Uncertainty in supply is caused by the variability brought by how the suppliers operate, as a result of the faults or delays in the suppliers’ deliveries. Uncertainty in the process is a result of the poorly reliable production process, for example a machine breakdown. Finally, uncertainty in demand, which is the most important factor according to Davis (1993), is defined as instability of demand or as
inexact forecasting of demands (Peidro et al., 2009). Therefore, unless the supply chain is
designed to be robust with respect to uncertain operating conditions, the impact of
operational inefficiencies such as delays and disruptions would be larger than necessary.
Santoso et al. (2005) emphasized the importance of considering risk and uncertainty into
SCM by giving a report about a company that announced a supply chain disruption, such
as a production or shipment delay; it is shown that its stock prices can decrease significantly
with an average decrease of 8.6% on the day of the announcement, and as much as 20%
over the next six months.

The significance of uncertainty has prompted a number of researchers to address stochastic
parameters in their supply chain design and planning. Peidro et al. (2009) categorized four
modeling approaches in the literature for supply chain planning problems as follows:

- Analytical models: stochastic programming, robust optimization, game theory,
  linear programming, and parametric programming.
- Artificial intelligence models: multi-agent systems, fuzzy multi-objective
  programming, fuzzy goal programming, reinforcement learning, evolutionary
  programming, and genetic algorithm.
- Simulation models: discrete event simulation, and system dynamics.
- Hybrid models: linear programming and simulation, model predictive control
  (MPC), stochastic dynamic programming and discrete event simulations, genetic
  algorithm and simulation and MILP and system dynamics.

The rest of this chapter presents several recent studies on supply chain network design
problems considering uncertain parameters, and for each case, the corresponding modeling
approach and solution methodology are discussed.

MirHassani et al. (2000) formulated a LNDP as a stochastic program with fixed recourse;
the model contains a set of binary first-stage variables and continuous second-stage
variables. The objective function coefficients are deterministic; uncertainty is only
considered for the right-hand side of the recourse constraints, such as demands or
capacities. The authors especially focused on parallel implementation issues for the
Benders decomposition algorithm. Sabri and Beamon (2000) presented an integrated multi-
objective supply chain model which facilitates simultaneous strategic and operational SC planning. Their model incorporates production, delivery, and demand uncertainty, and provides an appropriate performance measure by using a multi-objective analysis for the entire SC network. Tsiakis et al. (2001) studied a multi-product, multi-echelon supply chain under scenario-based demand uncertainty. The goal is to choose middle-echelon facility locations and their corresponding capacities, transportation links, and flows to minimize the total expected cost. Transportation costs are assumed to be piecewise linear concave. The model is formulated as a large-scale MIP and solved using CPLEX. The authors presented a case study using a European supply chain network involving 14 products, 18 customer locations, 6 distribution center locations, and 3 demand scenarios.

Santoso et al. (2005) proposed a stochastic programming model and solution approach for solving supply chain network design problems of a realistic scale. The authors presented a two-stage stochastic program considering uncertainty in processing and transportation costs, demands, supplies, and capacities, in which they have known joint distribution. The design objective is to minimize the sum of current investment costs and expected future processing and transportation costs. Furthermore, an additional cost term has been considered in the model to penalize shortfall in satisfying demands. The authors formulated the two step stochastic optimization problem in a compact, matrix-based form as follows:

\[
\min_y \{ f(y) := c^T y + E[Q(y, \varepsilon)] \} \quad (2.42)
\]

\[
y \in Y \subseteq \{0, 1\}^{|P|} \quad (2.43)
\]

Where \( Q(y, \varepsilon) \) is the optimal value of the following problem:

\[
\min_{xz} \quad q^T x + h^T z \quad (2.44)
\]

\[
Nx = 0 \quad (2.45)
\]

\[
Dx + z \geq d \quad (2.46)
\]

\[
Sx \leq s \quad (2.47)
\]
\[ R x \leq M y \]  

\[ x \in \mathbb{R}_+^{|A| \times |K|} \]  

The above vectors \( c, q, d, s, \) and \( M \) correspond to investment costs, processing and transportation costs, demands, supplies, and capacity respectively. The matrices \( N, D, S, \) and \( R \) correspond to the summations on the left-hand-side of the constraints (2.45) – (2.48), respectively which are used for balancing, demand, supply, and capacity. Note that \( \varepsilon \) in (2.42) is a random vector corresponding to the uncertain processing or transportation costs, demands, supplies, and capacities, and the optimal value \( Q(y, \varepsilon) \) of the second-stage problem (2.44) – (2.49) is a function of the first stage decision variable \( y \) and a realization (or a scenario) \( \varepsilon = (q, d, s, M) \) of the uncertain parameters. The expectation in (2.42) is taken with respect to the probability distribution of \( \varepsilon \) which is supposed to be known. The variable \( z \) in constraint (2.46) and the cost component \( h^T z \) in (2.44) correspond to the penalty incurred for failing to satisfy demand.

In order to solve the above problem with continuous distributions for uncertain parameters, and hence an infinite number of scenarios, the authors integrated the sample average approximation (SAA) scheme with an accelerated Benders decomposition algorithm. They provided empirical results on a domestic and an international case for the design of realistic supply chain networks.

Azaron et al. (2008) developed a multi-objective stochastic programming approach for supply chain design under uncertainty. The authors considered demand, supply, processing, transportation, storage and capacity expansion costs as the uncertain parameters. In order to develop a robust model, the authors added two additional objective functions into the traditional comprehensive LNDP objective function. This multi-objective model includes: (i) the minimization of the sum of current investment costs and the expected future processing, transportation, shortage, and capacity expansion costs, (ii) the minimization of the variance of the total cost, and (iii) the minimization of the financial risk or the probability of not meeting a certain budget. They applied a goal programming approach in order to obtain the Pareto-optimal solutions.
A key difficulty in solving the stochastic problem (2.42) - (2.43) is in evaluating the expectation in the objective function (Santoso et al., 2005). The sample average approximation (SAA) method is an approach for solving stochastic optimization problem by using Monte Carlo simulation. In this technique, the expected objective function of the stochastic problem is approximated by a sample average estimate derived from a random sample. Then, the resulting sample average approximating problem is solved by deterministic optimization techniques. This process is repeated with different samples in order to obtain candidate solutions along with statistical estimates of their optimality gaps (Verweij et al., 2003). Kleywegt et al. (2001) gave a comprehensive study on the SAA and discussed convergence rates, stopping rules, and computational complexity of this approach. The authors presented a numerical study of the SAA for the stochastic knapsack problem.

This approach has been widely applied in solving stochastic variants of location, routing, distribution, and network design problem. Verweij et al. (2003) presented a computational study of the application of SAA to solve stochastic routing problems, which involved an extremely large number of scenarios. The authors used decomposition and branch-and-cut to solve the approximation problem within the SAA scheme. Their results revealed the efficiency of the approximation method in handling up to $2^{1694}$ scenarios within an estimated 1% optimality gap.

Vila et al. (2009) studied the design of production-distribution networks for the lumber industry. The authors modeled the problem as a two-stage stochastic program with recourse, and proposed the SAA method based on Monte Carlo sampling techniques to solve the problem. It is shown in their study that this approach outperforms comparable deterministic models based on averages.

Schutz et al. (2009) studied a stochastic, multi-commodity LNDP with a detailed description of operational consequences from the strategic decisions for the Norwegian meat industry. The authors formulated the problem as a two-stage stochastic program, in which the first stage decisions are strategic location decisions, while the second stage consists of operational decisions. In particular, their model emphasizes the importance of
operational flexibility when making strategic decisions. To this end, short-term uncertainty (within 1-2 weeks) is considered as well as long-term uncertainty (5-10 years). For solving the model, the authors combined SAA approach with dual decomposition.

Bidhandi and Yusuff (2011) studied an integrated supply chain network design with uncertainty in operational costs, the customer demands, and capacity of the facilities. The authors proposed a modified solution method for solving the LNDP under uncertainty. In this approach, the SAA is integrated with an accelerated Benders decomposition approach to improve the mixed integer linear programming solution phase. An improved algorithm based on surrogate constraints is generated. The generated problem with the surrogate constraints is a valid relaxation of the main problem with binary variables.

In this work, we study the problem of supply chain network design in an uncertain environment in which we have uncertainty in customer demands, supplies, capacities, and costs. This problem is modeled as a two-stage stochastic optimization problem, and the SAA technique is applied to solve the model. An experimental study is designed to verify the quality of solutions and compare them with the results from the deterministic case.
CHAPTER 3
Mathematical Modeling

This chapter presents a mathematical model to design logistics networks in the presence of uncertain parameters. The proposed model is a multi-product, multi-echelon supply chain which consists of suppliers, manufacturing facilities, finishing facilities, distribution centers, and customers as depicted in Figure 3.1. This model considers uncertainty in design parameters such as demand, supply, operating cost, and capacity. Thus, for different types of uncertainty and variability levels of parameters, the proposed model will be compared with classical deterministic models. In this chapter, first, the mathematical model related to the deterministic case is presented. Then the two-stage stochastic model is explained, and the application of this model on the problem of logistics network design is further studied.

Figure 3.1. Supply chain network architecture
3.1 Mathematical Modeling: Deterministic Case

The deterministic model in this study is based on the models proposed by Santoso et al. (2005) and Cordeau et al. (2006). The problem is to design a multi-echelon, multi-product, single-period integrated supply chain model. The manufacturing plants receive raw materials from the suppliers, and finishing facilities receive the intermediate products from the manufacturing plants. The finishing facilities send finished products to the warehouses, and the warehouses distribute them to the customers.

The main assumptions of this model are the following:

(A1) All alternatives for the location of the facilities and suppliers are already defined and known. Hence, we have a fixed number of potential facility locations, and the fixed costs corresponding to each location of the facilities are known.

(A2) The structure of the supply chain is fixed.

(A3) There is a single period, a single mode of transportation, and multiple products.

(A4) The production processes are simple manufacturing processes with no assembly involved.

(A5) The production process is a linear production technology, where one input unit is transformed into one output unit.

(A6) The operational costs, capacities, supplies, and customer demands are known with certainty.

(A7) There is no backorder or order-on-hold if customer demands cannot be met, as this is a single period planning problem.

(A8) There are penalty costs of outsourcing in the model in case of insufficient internal supply or capacity.

Let $G = (V, A)$ be a connected graph, where $V$ is the set of vertices or nodes and $A$ is the set of arcs in the model. The set $V$ consists of the set of actual and potential locations for suppliers $B$, processing facilities $P$, and customers $C$, i.e., $V = B \cup P \cup C$. The processing facilities include manufacturing centers $H$, and finishing facilities $F$, and warehouses and
distribution centers $W$, i.e., $P = H \cup F \cup W$. Let $K$ be the set of commodities flowing through the supply chain network. For notational convenience, $O$ is defined as the set of origins i.e., $O = B \cup H \cup F \cup W$, and $D$ is defined as the set of destinations i.e., $D = H \cup F \cup W \cup C$.

The logistics network design problem consists of deciding which of the supplier to select, which of the processing centers to build (major configuration decisions), and which processing and finishing machines to procure (minor configuration decisions). A binary variable $Y_o$ is associated to these decisions: $Y_o = 1$ if supplier $o$ is selected, or processing facility $o$ is built, or machine $o$ is procured, and 0 otherwise. There are two types of artificial variables in the model, $v^k_o$ and $g_d$, which represent the amount of outsourcing material for supply and capacity respectively. Furthermore, the unit costs associated to the artificial variables of supply and capacity are defined by $A_s$ and $A_m$ respectively. The operational decisions consist of routing the flow of product $k \in K$ from the suppliers to the customers. Let $x^k_{od}$ be the flow of product $k$ from the origin $o$ to the destination $d$ in the network where $(o, d) \in E$. The mathematical model for the deterministic logistics network design problem can be presented as follows:

\[
\text{Minimize} \quad \sum_{o \in O} c_o Y_o + \sum_{k \in K} \sum_{(o,d) \in A} q^k_{od} x^k_{od} + A_s v^k_o + A_m g_d \quad (3.1)
\]

subject to:

\[
\sum_{o \in \{B,H,F\}} x^k_{od} - \sum_{l \in \{F,W,C\}} x^k_{dl} = 0 \quad \forall \ k \in K \ ; \ d \in P \quad (3.2)
\]

\[
\sum_{o \in W} x^k_{od} = a^k_d \quad , \quad \forall \ k \in K \ ; \ d \in C \quad (3.3)
\]

\[
\sum_{d \in H} x^k_{od} \leq s^k_o Y_o + v^k_o \quad , \quad \forall \ o \in S \ ; \ k \in K \quad (3.4)
\]

\[
\sum_{k \in K} r^d_k \left( \sum_{o \in \{B,H,F\}} x^k_{od} \right) \leq m_d Y_d + g_d \quad , \quad \forall \ d \in P \quad (3.5)
\]
In the above model, \( c_o \) denotes the fixed cost of investment for selecting supplier \( o \), or building facility \( o \), or procuring machine \( o \). The coefficient \( q_{od}^k \) denotes the total per-unit cost of procurement, production, processing and transhipment of commodity \( k \) from the origin \( o \) to the destination \( d \). The coefficients \( A_s \) and \( A_m \) are the cost of outsourcing of material from an external supplier, and within the external capacity respectively. The objective function (3.1) consists of minimizing total investment, operational and outsourcing costs. Constraints (3.2) ensure that the total amount of commodity \( k \) shipped to the facility \( d \) should also leave that facility. Constraints (3.3) require that the total flow of product \( k \) from all the available warehouses to the customer \( c \), should meet the demand \( a^k_c \) of that customer. Hence, it is assumed in this model that the demand of customers should only be satisfied from warehouses, and there is no direct link among customers and manufacturers. Constraints (3.4) require that the total flow of the raw material \( k \) from the supplier \( s \) should be less than the supply \( s_o^k \) of that supplier if the supplier is selected (\( Y_o = 1 \)) and the amount from an external supplier. Indeed, the artificial variable \( v_o^k \) is added to the model to make it be feasible for all the values of supply. Constraints (3.5) enforce capacity constraints of the processing facilities. Let \( r_d^k \) be the per-unit processing requirement for product \( k \) at destination center \( d \), then the equations (3.5) enforce that the total processing requirement of all products flowing into a processing center \( d \in P \) should be smaller than the capacity \( m_d \) of facility \( d \) if it is built (\( Y_d = 1 \)) and the external capacity. If facility \( d \) is not built (\( Y_d = 0 \)) the constraints will force all flow variables \( x_{od}^k = 0 \) for all \( o \in \{ S, M, F \} \). Constraints (3.6) enforce the non-negativity of the flow variables from the origin \( o \) to the destination \( d \) for the commodity \( k \) through the network. Finally, constraints (3.7) enforce the binary nature of the configuration and selection decisions for the facility \( o \). The notations are summarized in Table (3.1) and (3.2).

Model (3.1) – (3.7) can be extended in several ways to handle various additional realistic situations. It is worth mentioning that by adding a ratio of the amount of required raw
material for the semi-finished and finished product in constraints (3.2), the model can embrace production and assembly operations comprehensively. By reversing the inequality sign of constraints (3.4) and (3.5), lower limits on acquisition, production, storage, and transportation activities can be imposed. Such constraints can be used, for example, when a minimum amount of raw materials must be purchased from a supplier to obtain a quantity discount. They can also be used to model situations where a minimum amount of finished product must be manufactured for a plant to be economically viable.

The model assumes a double manufacturing stage and a single distribution stage. It is also assumed that commodities move through the network from upstream (supplier) to downstream (customer), and level by level. Nevertheless, the model can easily be modified for the case of moving commodities backward and among the non-adjacent echelons, such as the case of failure or a rework needed, and even more to encompass the reverse logistics situation. These assumptions could be easily relaxed by extending the network structure and modifying constraints (3.2) - (3.4) accordingly.

Different transportation modes can be considered in the model by defining variable $x_{od}^{km}$, where $m$ denotes the mode of transportation applied for transshipping commodity $k$ from the origin $o$ to the destination $d$. Moreover, in the case of seasonal demand, several planning periods can be incorporated into the model by defining $x_{od}^{kmt}$, where $t$ denotes the period number, and introducing additional end-of-period inventory variables.

Demand in this model is considered to be met with the equality constraints (3.3). It is worth mentioning that by adding a planning horizon for serving the customers, service level, penalty cost of stockout, and holding cost of inventory, the model can encompass the inventory routing problem along with the logistics network design in order to minimize the total cost of location, distribution, inventory and stockout during the planning period to satisfy the certain amount of customers’ orders.

Moreover, the model can be extended in order to consider the assignment of the commodity $k$ to the origin $o$ by adding binary variables $V_{o}^{k}$ into the objective function, and updating the constraints (3.4) accordingly.

It will be more convenient to work with the compact form for the model (3.1) – (3.7):
Minimize \[ c^T y + q^T x + A_s v + A_m g \] (3.8)

subject to: \[ Nx = 0 \] (3.9)
\[ Dx = d \] (3.10)
\[ Sx \leq sy + v \] (3.11)
\[ Rx \leq My + g \] (3.12)

\[ x, v \in \mathbb{R}^{|A| \times |\mathcal{K}|}_+, \quad g \in \mathbb{R}^{|A|}_+ \] (3.13)

\[ y \in \{0,1\}^{|\mathcal{O}|} \] (3.14)

Above, the vectors \( c, q, d \) and \( s \) correspond to investment costs, processing and transportation costs, demands, and supplies respectively. The matrixes \( N, D \) and \( S \) are appropriate matrices that correspond to summations of the left-hand-side of the constraints (3.2), (3.3), and (3.4) respectively. The notation \( R \) corresponds to a matrix of \( r^k_d \) and the notation \( M \) corresponds to a matrix with \( m_d \) along the diagonal.

Following tables present a summary of notations used in the deterministic model.


<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C$</td>
<td>Set of customers</td>
</tr>
<tr>
<td>$D$</td>
<td>Set of destinations</td>
</tr>
<tr>
<td>$F$</td>
<td>Set of finishing facility locations</td>
</tr>
<tr>
<td>$K$</td>
<td>Set of commodities</td>
</tr>
<tr>
<td>$H$</td>
<td>Set of potential manufacturing facility locations</td>
</tr>
<tr>
<td>$O$</td>
<td>Set of origins</td>
</tr>
<tr>
<td>$B$</td>
<td>Set of potential suppliers</td>
</tr>
<tr>
<td>$W$</td>
<td>Set of potential warehouse locations</td>
</tr>
</tbody>
</table>

Table 3.1. Summary of notation for the sets in the deterministic model

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a^k_d$</td>
<td>Demand of customer $d$ for product $k$</td>
</tr>
<tr>
<td>$c_o$</td>
<td>Fixed cost of selecting origin $o$</td>
</tr>
<tr>
<td>$m_d$</td>
<td>Total capacity of facility at destination $d$ in equivalent units</td>
</tr>
<tr>
<td>$q^{k}_{od}$</td>
<td>Unit cost for providing commodity $k$ from the origin $o$ to the destination $d$</td>
</tr>
<tr>
<td>$s^k_o$</td>
<td>Supply of supplier $o$ for commodity $k$</td>
</tr>
<tr>
<td>$r^k_d$</td>
<td>Per-unit processing requirement for product $k$ at destination center $d$</td>
</tr>
<tr>
<td>$A_s$</td>
<td>Unit cost of outsourcing of supply</td>
</tr>
<tr>
<td>$A_m$</td>
<td>Unit cost of outsourcing of capacity</td>
</tr>
</tbody>
</table>

Table 3.2. Summary of notation for the parameters in the deterministic model

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x^{k}_{od}$</td>
<td>Amount of commodity $k$ provided by the origin $o$ to the destination $d$</td>
</tr>
<tr>
<td>$Y_o$</td>
<td>=1 if origin $o$ is selected, 0 otherwise</td>
</tr>
<tr>
<td>$v^k_o$</td>
<td>Amount of external supply for the origin $o$ for product $k$</td>
</tr>
<tr>
<td>$g_d$</td>
<td>Amount of external capacity for the destination $d$</td>
</tr>
</tbody>
</table>

Table 3.3. Summary of notation for the variables in the deterministic model
3.2 Mathematical Modeling: Stochastic Case

The model presented in the previous section considers that all the parameters are deterministic and assumed to be known beforehand. However, in real-life situations, most operating parameters in supply chain design such as customer demands, supplies, prices, and resource capacities are often not known with complete certainty, or they might change along the planning horizon. The uncertainties are mostly found in the operational stage as most operational parameters are not fully known when the strategic decisions have to be made. These parameters will become known with more certainty after the supply chain is in operation. Since we deal with two stages of decisions, strategic and operational, a two-stage stochastic programming approach can be applied to incorporate these uncertainties into the planning process. Next, a short introduction to two-stage stochastic programming is given, based on the work of Birge and Louveaux (1997), and then the logistics network design problem modeled by the stochastic programming approach is presented.

3.2.1 Overview of Stochastic Programming

A stochastic linear program is a linear program in which some parameters and data may be considered uncertain. These uncertain parameters can be represented in the form of random variables. Hence, an accurate probabilistic description of the random variables is assumed to be available, under the form of the probability distributions, densities or, more generally, probability measures. Furthermore, particular values of random variables are only known after realization of random sampling experiments. Uncertainty is represented in terms of random experiments with outcomes called as a realization and denoted by \( \omega \). The set of all outcomes is represented by \( \Omega \). Therefore, the value of a random vector \( \xi \) for a certain realization \( \omega \) is denoted by \( \xi(\omega) \).

A recourse program is a program in which some decisions or recourse actions can be taken after uncertainty is realized. Therefore, the set of decisions in a stochastic program is divided into two stages:
A number of decisions have to be taken before the experiments, which are called as the first-stage decisions. The period when these decisions are taken is called the first stage.

A number of decisions can be taken after the experiments, which are called as the second-stage decisions. The corresponding period is called the second stage.

The first-stage decisions are represented by the vector $y$, while the second-stage decisions are represented by the vector $x$. The sequence of events and decisions can be summarized as follows:

$$y \rightarrow \xi(\omega) \rightarrow x(\omega, y)$$

These two-stage decisions form a two-stage stochastic linear program with a fixed recourse. This program can be formulated as follows:

Minimize \quad z = c^T y + E_\xi[\min q(\omega)^T x(\omega)] \tag{3.15}

subject to: \quad U y = b \tag{3.16}

$$T(\omega)y + Wx(\omega) = h(\omega) \tag{3.17}$$

$$y \geq 0 \tag{3.18}$$

$$x(\omega) \geq 0 \tag{3.19}$$

The first-stage decisions are represented by the $n_1 \times 1$ vector $y$. The first-stage vectors and matrices $c,b$ and $U$ of the sizes $n_1 \times 1, m_1 \times 1, \text{and } m_1 \times n_1$ respectively are associated to $y$. In the second stage, a number of random events $\omega \in \Omega$ may realize. For a given realization $\omega$, the second-stage problem data $q(\omega), h(\omega)$ and $T(\omega)$ become known, where $q(\omega)$ is $n_2 \times 1$, $h(\omega)$ is $m_2 \times 1$, and $T(\omega)$ is $m_2 \times n_1$.

Each component of $q,T$, and $h$ is a possible random variable. Let $T_1(\omega)$ be the $i$th row of $T(\omega)$. Piecing together the stochastic components of the second-stage data, we obtain a
vector $\xi^T(\omega) = (q(\omega)^T, h(\omega)^T, T_1(\omega), \cdots, T_{m2}(\omega))$, with potentially up to $N = n2 + m2 + (m2 \times n1)$ components. As indicated before, a single random event $\omega$ influences several random variables, here, all components of $\xi$.

When the random event $\omega$ is realized, the second-stage problem data $q, h$, and $T$ become known. Then, the second-stage decision $x(\omega)$ or $(x(\omega, y))$ must be taken. The objective function (3.15) contains a deterministic term $c^T y$ and the expectation of the second-stage objective $q(\omega)^T x(\omega)$ taken over all realizations of the random event $\omega$. This second-stage term, $E_\xi [\min q(\omega)^T x(\omega)]$, is the more difficult one because, for each $\omega$, the value $x(\omega)$ is the solution of a linear program. To properly present this fact, a deterministic equivalent program is often used. For a given realization $\omega$, let

$$Q(y, \xi(\omega)) = \min_x \{q(\omega)^T x | Wx = h(\omega) - T(\omega)y, x \geq 0\}$$

be the second-stage value function. Then, define the second stage value function as follows:

$$Q(y) = E_\xi[Q(y, \xi(\omega))],$$

and the deterministic equivalent program (DEP) can be written as follows:

Minimize $\quad z = c^T y + Q(y)$

subject to: $\quad Uy = b$

$\quad y \geq 0$

This representation of a stochastic program clearly illustrates that the major difference from a deterministic formulation is in the second-stage value function. If that function is given, then a stochastic program is just an ordinary nonlinear program.
3.2.2 Stochastic Modeling of Logistics Network Design Problem

This section presents the mathematical model for the stochastic logistics network design problem considering uncertainty in demands, supplies, capacities, and operational costs which is based on the work of Santoso et al. (2005). The main assumptions of this model are quite similar to the deterministic case and are presented as follows:

(A1) All suppliers can supply their products to all plants and they are reliable during the planning horizon.

(A2) All transportation channels are ready to use and there is no failure during the planning horizon.

(A3) Production lead-times are fixed.

(A4) Operational costs, production costs, procurement costs, as well as transportation costs are stochastic with known probability distribution functions.

(A5) Customer demand, supply of suppliers, and capacity of the facilities are uncertain and it is assumed that their probability distribution functions are exactly known.

(A6) All stochastic parameters are normally distributed with mean equal to their deterministic parameters and standard deviation equal to the fraction of their mean.

(A7) The production process is fixed, simple and linear.

(A8) There are penalty costs of outsourcing in the model in case of insufficient amount of internal supply and capacity.

In a stochastic logistics network design problem, we deal with several decisions which must be made through two stages. The first stage consists of decisions that need to be taken before the uncertainty is resolved. So, the decisions in the first-stage will be as follow:

- The number and location of production facilities, finishing facilities, and distribution centers (variables $Y_o$)
• Supplier selection (variables \( Y_o \))

Thereafter, the decisions of the second stage are taken when the realization of uncertain parameters are known. Each realization of the uncertain parameters is called a scenario, and \( N \) is the set all possible scenarios. It is assumed that operational costs, demands, supplies, and capacities are stochastic parameters with known joint distribution. We denote by \( \xi = (q, d, s, M) \) a random variable (random vector) of the random parameters respectively, and by \( \xi = (q, d, s, M) \) a realization of the random vector for each scenario. Henceforth, we denote \( x_{odn}^k \) the number of units of commodity \( k \) transported from the origin \( o \) to the destination \( d \) under scenario \( n \).

The objective function of the stochastic problem is then to minimize the sum of current investment costs, expected future operational costs and the total cost of outsourcing. In order to meet the demand which may vary due to its uncertain nature, and lack of sufficient internal supply and capacity, we may need to use external suppliers or external sources of capacity. In order to incorporate this outsourcing into the model, we consider two artificial variables in the supply and capacity constraints, and their corresponding cost of outsourcing in the objective function. The formulation for the total fixed cost does not change with respect to the deterministic case as all the fixed costs are deterministic. However, the total variable costs change from a simple summation in the deterministic case to a calculation of the expected value.

The mathematical formulation for the model in the compact format is as follows:

\[
\begin{align*}
\min_y \ f(y) := c^T y + \mathbb{E}[Q(y, \xi)] \\
\text{s. t.} \quad y \in Y \subseteq \{0,1\}^{[O]}
\end{align*}
\]  

(3.25)

(3.26)

where \( Q(y, \xi) \) is the optimal value of the following problem:
Minimize  
\[ q^T x + A_s v + A_m g \]  
subject to:  
\[ N x = 0 \]  
\[ D x = d \]  
\[ S x \leq s y + v \]  
\[ R x \leq M y + g \]  
\[ x, v \in \mathbb{R}^{[A] \times [K]}, \quad g \in \mathbb{R}^{|A|} \]  

The random vector \( \xi \) in (3.25) corresponds to the uncertain operational costs, demands, supplies, and capacities. The optimal value \( \mathcal{Q}(y, \xi) \) of the second-stage problem (3.27) – (3.32) is a function of the first-stage decision variable \( y \) and a realization (or a scenario) \( \xi = (q, d, s, M) \) of the uncertain parameters. The expectation function in (3.25) is taken with respect to the probability distribution of \( \xi \), which is supposed to be known. The variables \( v \) and \( g \) in constraints (3.29) and (3.30) and the cost component \( A_s \) and \( A_m \) in (3.27) correspond to the cost of outsourcing of supply and capacity.

Model (3.25) – (3.32) is a two-stage stochastic program. The first-stage consists of the deciding the configuration decisions \( y \), and the second-stage consists of processing and transporting commodities from suppliers to customers in an optimal way based upon the configuration and the realized uncertain scenario. The objective is then to minimize current investment costs \( c^T y \), and the expected future operating costs \( \mathbb{E}[\mathcal{Q}(y, \xi)] \). The outsourcing penalty cost \( A_s v + A_m g \) guarantees that the model would be always feasible and \( \mathcal{Q}(y, \xi) < \infty \) for all \( y \) and \( \xi \). Furthermore, we assume that possible realizations of the operating costs \( q \), and the penalty costs \( l \) are sufficiently high such that \( \mathcal{Q}(y, \xi) > -\infty \) for all \( y \) and \( \xi \), and hence \( \mathcal{Q}(y, \xi) \) is finite valued for all \( y \in Y \) and the possible realizations of the random data. We further assume that the expected value \( \mathbb{E}[\mathcal{Q}(y, \xi)] \) is a well defined and finite value for the considered distribution of \( \xi \). Consequently, problem (3.25) – (3.26) has a well-defined objective function \( f(y) \) and possesses an optimal solution, since the set \( Y \) is non-empty and finite.
The notation used in the stochastic model is presented in the following tables.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C$</td>
<td>Set of customers</td>
</tr>
<tr>
<td>$D$</td>
<td>Set of destinations</td>
</tr>
<tr>
<td>$F$</td>
<td>Set of finishing facility locations</td>
</tr>
<tr>
<td>$K$</td>
<td>Set of commodities</td>
</tr>
<tr>
<td>$H$</td>
<td>Set of potential manufacturing facility locations</td>
</tr>
<tr>
<td>$N$</td>
<td>Set of scenarios, indexed by $n$</td>
</tr>
<tr>
<td>$O$</td>
<td>Set of origins</td>
</tr>
<tr>
<td>$B$</td>
<td>Set of potential suppliers</td>
</tr>
<tr>
<td>$W$</td>
<td>Set of potential warehouse locations</td>
</tr>
</tbody>
</table>

Table 3.4. Summary of notation for the sets in the stochastic model

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_{dn}^k$</td>
<td>Demand of customer $d$ for product $k$ under scenario $n$</td>
</tr>
<tr>
<td>$c_o$</td>
<td>Fixed cost of selecting origin $o$</td>
</tr>
<tr>
<td>$m_{dn}$</td>
<td>Total capacity of facility at destination $d$ in equivalent units under scenario $n$</td>
</tr>
<tr>
<td>$p_n$</td>
<td>Probability of scenario $n$ happening</td>
</tr>
<tr>
<td>$q_{odn}^k$</td>
<td>Unit cost for providing commodity $k$ from the origin $o$ to the destination $d$ under scenario $n$</td>
</tr>
<tr>
<td>$s_{on}^k$</td>
<td>Supply of supplier $o$ for commodity $k$ under scenario $n$</td>
</tr>
<tr>
<td>$r_{dn}^k$</td>
<td>Per-unit processing requirement for product $k$ at destination center $d$ under scenario $n$</td>
</tr>
<tr>
<td>$A_s$</td>
<td>Unit cost of outsourcing of supply</td>
</tr>
<tr>
<td>$A_m$</td>
<td>Unit cost of outsourcing of capacity</td>
</tr>
</tbody>
</table>

Table 3.5. Summary of notation for the parameters in the stochastic model

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_{odn}^k$</td>
<td>Amount of commodity $k$ provided by the origin $o$ to the destination $d$ under scenario $n$</td>
</tr>
<tr>
<td>$Y_o$</td>
<td>$=1$ if origin $o$ is selected, $0$ otherwise</td>
</tr>
<tr>
<td>$v_{on}^k$</td>
<td>Amount of external supply for the origin $o$ for product $k$ under scenario $n$</td>
</tr>
<tr>
<td>$g_{dn}$</td>
<td>Amount of external capacity for the destination $d$ under scenario $n$</td>
</tr>
</tbody>
</table>

Table 3.6. Summary of notation for the variables in the stochastic model
The problem can then be modeled in an extensive form as follows:

Minimize \[
\sum_{o \in O} c_o Y_o
\]
\[
\quad + \sum_{n \in N} p_n \left[ \sum_{k \in K} \sum_{(o,d) \in A} q_{odn} x_{odn}^k + \sum_{c \in C} \sum_{k \in K} A_c v_{on}^k + \sum_{d \in P} A_m g_{dn} \right]
\]

subject to:

\[
\sum_{o \in \{B,H,F\}} x_{odn}^k - \sum_{l \in \{F,W,C\}} x_{dln}^k = 0 , \, \forall \, k \in K ; \, d \in P ; n \in N
\]

\[
\sum_{o \in W} x_{odn}^k = a_{dn}^k , \quad \forall \, k \in K ; \, d \in C ; n \in N
\]

\[
\sum_{d \in H} x_{odn}^k \leq s_{on}^k Y_o + v_{on}^k , \quad \forall \, o \in S ; k \in K ; n \in N
\]

\[
\sum_{k \in K} r_{dn}^k \left( \sum_{o \in \{B,H,F\}} x_{odn}^k \right) \leq m_{dn} Y_d + g_{dn} , \quad \forall \, d \in P ; k \in K ; n \in N
\]

\[
x_{odn}^k \geq 0 , \quad \forall \, k \in K ; o \in O ; d \in D ; n \in N
\]

\[
Y_o \in \{0,1\} , \quad \forall \, o \in O
\]

\[
v_{on}^k \geq 0 , \quad g_{dn} \geq 0 \quad \forall \, o \in O ; d \in D ; n \in N
\]
CHAPTER 4

Solution Methodology

This chapter presents details of the proposed algorithmic strategy for solving the stochastic logistics network design problem (3.25) – (3.32). The proposed methodology in this study is a scenario-based approximation method for solving the two-stage stochastic programming called Sample Average Approximation (SAA), Kleywegt et al. (2001). In the two-stage programming method, the first-stage decisions are made prior to the realization of the stochastic variables, and the second-stage decisions which are affected by the first-stage decisions will be taken afterward. In this chapter, we give details about the algorithm in general based on the works of Verweij et al. (2003) and Lacasse-Guay (2003), and its application to the proposed problem.

4.1 Overview of Sample Average Approximation

The SAA method is a heuristics-based solution approach for stochastic optimization problems with a large number of scenarios. This approach gives an approximation of the expected objective function value using Monte Carlo simulation. The idea is to generate samples of the stochastic parameters to construct and solve the approximation instead of the exact values. In other words, this approach aims to solve the same stochastic problem for a number of different realized scenarios. This method reduces the dimension of the original problem and thus the problem becomes much easier to solve. Because of the statistical requirements, this process needs to be repeated many times with independent sample sets. At each replication, the candidate solution and the estimates of the sample upper bound and sample lower bound of the solution value are recorded. This heuristic
consists of two steps; sampling and solving stage, and evaluation stage, which can be explained as follows.

In the SAA scheme, $|N|$ random sample scenarios $\xi^1, ..., \xi^{|N|}$ are generated according to a probability distribution function $P$, and then the expected value function of total variable cost is approximated according to the realized scenario $\xi^n$ by the sample average function as follows.

$$
\mathbb{E}[Q(y, \xi)] = \frac{1}{|N|} \sum_{n=1}^{|N|} Q(y, \xi^n)
$$

(4.1)

Where $Q(y, \xi)$ is the total variable cost given decision $y$ of the first-stage, and $|N|$ is the total number of scenarios. Consequently, the stochastic problem (3.25) – (3.26) approximated by the above approach can be written as follows.

$$
\min_{y \in Y} \left\{ \tilde{f}_n(y) := c^T y + \frac{1}{|N|} \sum_{n=1}^{|N|} Q(y, \xi^n) \right\},
$$

(4.2)

where $c^T y$ is the total fixed cost and $\tilde{f}_n(y)$ is the expected total cost.

4.2 The Proposed Algorithm

The proposed algorithm which consists of two stages, sampling and evaluation can be described as follows.

4.2.1 Sampling and Solving Stage

In the SAA procedure, we first generate a random sample $\xi^1, ..., \xi^{|N|}$ of $|N|$ realization (scenarios) of the random vector $\xi$, and then solve the stochastic problems with $|N|$ scenarios for $M$ times, i.e., $(\xi_j^1, ..., \xi_j^N)$ for $j = 1, ..., M$, to obtain a set of $M$ candidate solutions $y^1, ..., y^M_{|N|}$, and $M$ corresponding optimal objective function values $v^1, ..., v^M_{|N|}$. 
The value for $|N|$ and $|M|$ should be statistically large enough while maintaining the trackability of the problems.

In order to find the best solution out of the $|M|$ candidate solutions, the following evaluation approach is utilized.

4.2.2 Evaluation Stage

Let $\xi_1, \xi_2, \ldots, \xi_S$ be a sample of size $S$, where $S \gg |N|$. We select the candidate solution which minimizes the corresponding estimated objective function $v_S(y)$ where:

$$v_S(y) = c^T y + \frac{1}{S} \sum_{n=1}^{S} Q(y, \xi^n)$$ (4.3)

We denote $x^S$ as the candidate solution with the least objective value of $v_S(y)$, which means:

$$x^S \in \arg\min \left\{ v_S(y) \big| y \in \{ y^1_{|N|}, \ldots, y^M_{|N|} \} \right\} \quad (4.4)$$

Hence, for each of the $M$ candidate solutions obtained in the first step, we solve (4.3) and find the corresponding value of $v_S^M(y)$. Then the best solution of the stochastic problem as it is shown by the Equation (4.4) corresponds to the $x^S$ which has the least objective value of (4.3).

Kleywegt et al. (2001) showed that the quality of the final solution increases with the sample size $|N|$. They found that the convergence rate depends on the conditioning of the problem, which in turn tends to become poorer with an increase in the number of decision variables. Furthermore, the SAA method has the advantage of ease of use in combination with existing techniques for solving deterministic optimization problems. Nevertheless, the main drawback of this method is the high amount of computation time necessary to solve $M$ stochastic problems each with $|N|$ scenarios.
CHAPTER 5

Results and Analysis

In this chapter we describe computational experiments using the proposed methodology in order to solve the deterministic and stochastic LNDP (3.1) – (3.7) and (3.3) – (3.40), respectively. To this end, we first describe the characteristics of the test problems and data generation process in Section 5.1. Then, we present a summary of the computational results for the deterministic and stochastic problems for each instance based on the proposed methodology in Section 5.2, and finally we give an analysis and comments on the quality of the stochastic programming solutions compared to those obtained by the deterministic approach in Section 5.3.

5.1 Description of Data

The data and instances generated to test the proposed model are adapted from the works of Cordeau et al. (2006) and Santoso et al. (2005). The details of this process are as follows.

We randomly generated a set of 75 instances according to assumptions that strike a balance between realism and ease of generation, solvability and reproducibility. Instances vary according to two main dimensions: scenarios of stochastic parameters and variability index in the stochastic parameters. The design parameters in this study, as introduced in Section 3.2.2, are operating cost, demand, supply, and capacity which are defined in the form of a stochastic vector $\xi = (q, d, s, M)$, and the realized value for this vector is represented by $\xi = (q, d, s, M)$. In this study, we aim to measure the effect of uncertainty for each of these design parameters in the objective function value. To this end, we define instances
which encompass all the possible combination of one, two, three, and four of these stochastic parameters as shown in the Table 5.1.

<table>
<thead>
<tr>
<th>Number of stochastic parameters in each scenario</th>
<th>Stochastic parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>q</td>
</tr>
<tr>
<td></td>
<td>d</td>
</tr>
<tr>
<td></td>
<td>s</td>
</tr>
<tr>
<td></td>
<td>M</td>
</tr>
<tr>
<td>2</td>
<td>sM</td>
</tr>
<tr>
<td></td>
<td>qd</td>
</tr>
<tr>
<td></td>
<td>ds</td>
</tr>
<tr>
<td></td>
<td>qs</td>
</tr>
<tr>
<td></td>
<td>qM</td>
</tr>
<tr>
<td></td>
<td>dM</td>
</tr>
<tr>
<td>3</td>
<td>qds</td>
</tr>
<tr>
<td></td>
<td>dsM</td>
</tr>
<tr>
<td></td>
<td>qdM</td>
</tr>
<tr>
<td></td>
<td>qsM</td>
</tr>
<tr>
<td>4</td>
<td>qdsM</td>
</tr>
</tbody>
</table>

Table 5.1. Problem instances for scenarios of stochastic parameters

The variability of instances is defined in the form of a relative standard deviation (RSD), which is expressed as the percentage of dispersion of a stochastic variable around its mean and can be calculated as the ratio of the standard deviation to the mean. In this problem, five levels of variability are considered for each of the 15 instances described above in order to identify the level of variability of the stochastic parameters. These variability levels are 10%, 20%, 30%, 40%, and 50%.

The size of the problem is considered to be fixed for all the 75 instances and it is given by the number of suppliers ($|B|$), the number of potential manufacturing plant locations ($|H|$), the number of potential finishing facility locations ($|F|$), the number of potential warehouse locations ($|W|$), the number of customers ($|C|$), the number of raw materials and finished products ($|K|$). For an instance with $|C| = 30$, we have set $|B| = |H| = |F| = |W| = 5$ and $|K| = 10$. 
The cost structure is determined as follows. For each supplier $b \in B$, a fixed cost of selection $c_b$ is chosen randomly in the interval $[10^3, 10^4]$ according to a uniform distribution. For each manufacturing plant $h \in H$, the fixed cost $c_h$ is chosen randomly in the interval $[10^5, 10^6]$ according to a uniform distribution. And finally, for each finishing facility and warehouse, the fixed cost $c_f$ and $c_w$ is chosen randomly in the interval $[10^4, 10^5]$ according to a uniform distribution. The penalty costs of outsourcing for supply $A_s$ and capacity $A_m$ are considered to be a randomly chosen from the intervals $[2 \times 10^4, 3 \times 10^4]$ and $[6 \times 10^4, 8 \times 10^4]$ respectively, which are technically large enough to satisfy the nature of artificial variables and big-M method.

For every variable $x_{od}^k$, the variable cost $q_{od}^k$ is a stochastic variable which is composed of two distinct terms: the unit transportation cost of commodity $k$ from the origin $o$ to the destination $d$, and the unit cost of procurement, production or warehousing of commodity $k$ at the origin $o$. For every commodity $k$, every origin $o \in O$ and every destination $d \in D$, an average unit transportation cost $\bar{t}_{od}^k$ is first generated by multiplying the Euclidean distance between $o$ and $d$ by a random number chosen according to a normal distribution with a mean of 10. For every location, Euclidean coordinates are themselves chosen randomly in the unit square $[0, 1] \times [0, 1]$. Then, $t_{od}^k$ is chosen from the interval $[\alpha \bar{t}_{od}^k, \beta \bar{t}_{od}^k]$, where $\alpha = 0.75$ and $\beta = 1.25$. Next, for every commodity $k \in K$ and every origin $o \in O$ the unit cost of procurement, purchase, production, finishing or warehousing cost $a_o^k$ is chosen randomly according to a normal distribution with a mean of 10. Finally, the cost $q_{od}^k$ is obtained by setting $q_{od}^k = t_{od}^k + a_o^k$.

For every product $k$ and every customer $d$, the demand $a_d^k$ is considered as a stochastic variable with log-normal distribution with a normal mean of 10. Santoso et al. (2003) argued that the non-negativity of parameter values is preserved by using log-normal distributions.

For every product $k$ and every supplier $o$, the supply $s_o^k$ is considered as a stochastic variable with normal distribution with a mean of 10($\frac{N_2}{N}$). Note that the coefficient ($\frac{N_2}{N}$) is multiplied to balance the overall supply and demand.
For the capacity constraint parameters, for every product \( k \) and every customer \( d \), in order to generate the processing requirement \( r_{d}^{k} \), \( \bar{r}_{d} \) is first chosen randomly from the interval \([0, 1]\). Then, \( r_{d}^{k} \) is chosen from the interval \([\alpha.\bar{r}_{d}, \beta.\bar{r}_{d}]\), where \( \alpha = 0.75 \) and \( \beta = 1.25 \). Furthermore, \( M_{d} \), the total capacity of facility \( d \), is considered to be a normal distribution with a mean of 10.

It should be noted that, for the case of deterministic parameters, the value of each parameter is equal to the mean of the stochastic case. The standard deviations for the distributions are chosen as certain fractions of the mean value as the coefficient of variation in each instance.

Recall from Chapter 4 that the SAA method calls for the solution of \(|M|\) instances of the approximating stochastic program (3.25) – (3.32), each having \(|N|\) sampled scenarios. Validation of candidate solution is then carried out by evaluating the objective function using\(|S|\) sampled scenarios. In this study, we use \(|N| = 5\); \(|M| = 3\); and \(|S| = 50\). The small number of samples and replications is necessary because of the huge number of variables and constraints which make the problem difficult to solve. The size of deterministic and stochastic problems corresponding to the generated instances and data is presented in the Table 5.2.

<table>
<thead>
<tr>
<th></th>
<th>Constraints</th>
<th>Variables</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Equality</td>
<td>Inequality</td>
</tr>
<tr>
<td>Deterministic</td>
<td>450</td>
<td>200</td>
</tr>
<tr>
<td>Stochastic</td>
<td>2250</td>
<td>1000</td>
</tr>
</tbody>
</table>

Table 5.2. Size of the deterministic and the stochastic problems for \(|C| = 30\)

### 5.2 Computational Results

In this study, we aim to measure and illustrate how much better we are doing by solving the stochastic problem instead of just solving the deterministic LNDP. Hence, we need to compare the obtained solutions of the deterministic case with the ones from the stochastic case for each instance. To this end, we need an evaluation mechanism to estimate the true
expected value of the objective function. Recall from Chapter 4 that the solutions obtained by solving the stochastic problem are evaluated on a set of scenarios $|S|=50$, to estimate the expected value of the objective function for each case. These expected values are taken for comparison, and the value of the stochastic solution (VSS) is defined as a measure to identify how well the deterministic model solutions perform relative to solutions from the more complicated stochastic programs (Birge, 1982).

The proposed algorithmic scheme was implemented in MATLAB R2013a with CPLEX 12.5.0.1 for solving mixed-integer and linear programs. All computations were carried out on a 3.07 GHz PC running Linux.

Table 5.3 and Figure 5.1 give a comparison of computation times for solving the problem with deterministic, stochastic and the proposed SAA approaches. The first column represents the average computation time of five instances with different levels of variability in the design parameters when solving a single problem with deterministic approach. The second column relates to the average computation time of solving the problem with stochastic programming approach with $|N| = 5$ and $|M| = 1$, and the last column represents the average computation time of solving the problem with the proposed SAA approach when $|N| = 5$ and $|M| = 3$. The results reveal that the stochastic problem requires significantly more computation time for a certain instance than the deterministic problem, and the SAA approach takes almost three times on average more than the stochastic programming approach (Figure 5.1). This gap is a result of the number of scenarios, variables and constraints between deterministic and stochastic approach, which is shown in Table 5.2. The differences in computation time are also more notable for larger problems in terms of size or number of scenarios, in which the stochastic approach and the SAA will require a long computation time to solve the problem. For example, for an instance with $|C| = 100$, and $|B| = |H| = |F| = |W| = 10$, and $|K| = 20$, when $|N| = 20; |M| = 20$; and $|S| = 100$ the stochastic problem approach took 2064 minutes, while the deterministic problem approach took 347 minutes to reach to the optimal solution. As is shown with the given example, and due to the complexity of the problem, the computation time is highly affected by the size of the problem.
<table>
<thead>
<tr>
<th>Instance</th>
<th>Computation time (seconds)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Deterministic</td>
</tr>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>q</td>
<td>3.265E+1</td>
</tr>
<tr>
<td>d</td>
<td>1.478E+1</td>
</tr>
<tr>
<td>s</td>
<td>1.090E+1</td>
</tr>
<tr>
<td>M</td>
<td>8.740E+0</td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>qd</td>
<td>3.192E+1</td>
</tr>
<tr>
<td>ds</td>
<td>2.520E+1</td>
</tr>
<tr>
<td>sM</td>
<td>2.340E+1</td>
</tr>
<tr>
<td>qs</td>
<td>3.049E+1</td>
</tr>
<tr>
<td>qM</td>
<td>3.701E+1</td>
</tr>
<tr>
<td>dM</td>
<td>1.963E+1</td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>qds</td>
<td>9.120E+0</td>
</tr>
<tr>
<td>dsM</td>
<td>1.870E+1</td>
</tr>
<tr>
<td>qdM</td>
<td>3.190E+1</td>
</tr>
<tr>
<td>qsM</td>
<td>3.265E+1</td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
<tr>
<td>qdsM</td>
<td>3.150E+1</td>
</tr>
<tr>
<td>Average</td>
<td>2.391E+1</td>
</tr>
</tbody>
</table>

Table 5.3. CPU time of deterministic and stochastic instances
Figure 5.1 Average CPU time of deterministic, stochastic, and SAA approaches for each instance

The following tables present the results for the deterministic and the proposed stochastic approaches in solving LNDP, and the corresponding VSS for each instance. We note that, the stochastic solutions in the following tables relate to the results of the proposed stochastic approach, SAA when $|N| = 5$ and $|M| = 3$, which has been explained in Chapter 4. The instances are categorized based on the number of stochastic parameters in each as follows. The notations of $SP$ and $MVDP$ correspond to the solution of the stochastic approach and the solutions of mean value of the deterministic approach respectively.
### 5.2.1 Computational Results of Instance with One Stochastic Parameter

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Variability</th>
<th>Det. Solution</th>
<th>Sto. Solution</th>
<th>VSS</th>
<th>VSS/Sto (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(q) (Cost)</td>
<td>10%</td>
<td>3.493E+6</td>
<td>3.494E+6</td>
<td>-1171</td>
<td>-0.03%</td>
</tr>
<tr>
<td></td>
<td>20%</td>
<td>3.479E+6</td>
<td>3.478E+6</td>
<td>804</td>
<td>0.02%</td>
</tr>
<tr>
<td></td>
<td>30%</td>
<td>3.465E+6</td>
<td>3.462E+6</td>
<td>2398</td>
<td>0.07%</td>
</tr>
<tr>
<td></td>
<td>40%</td>
<td>3.447E+6</td>
<td>3.441E+6</td>
<td>5888</td>
<td>0.17%</td>
</tr>
<tr>
<td></td>
<td>50%</td>
<td>3.436E+6</td>
<td>3.429E+6</td>
<td>6919</td>
<td>0.20%</td>
</tr>
<tr>
<td>(d) (Demand)</td>
<td>10%</td>
<td>6.769E+6</td>
<td>6.711E+6</td>
<td>58327</td>
<td>0.869%</td>
</tr>
<tr>
<td></td>
<td>20%</td>
<td>8.952E+6</td>
<td>8.029E+6</td>
<td>922944</td>
<td>11.496%</td>
</tr>
<tr>
<td></td>
<td>30%</td>
<td>9.640E+6</td>
<td>8.737E+6</td>
<td>902562</td>
<td>10.330%</td>
</tr>
<tr>
<td></td>
<td>40%</td>
<td>1.125E+7</td>
<td>1.009E+7</td>
<td>1156620</td>
<td>11.458%</td>
</tr>
<tr>
<td></td>
<td>50%</td>
<td>1.235E+7</td>
<td>9.936E+6</td>
<td>2411432</td>
<td>24.270%</td>
</tr>
<tr>
<td>(s) (Supply)</td>
<td>10%</td>
<td>3.937E+6</td>
<td>3.957E+6</td>
<td>-20252</td>
<td>-0.512%</td>
</tr>
<tr>
<td></td>
<td>20%</td>
<td>4.464E+6</td>
<td>4.380E+6</td>
<td>83573</td>
<td>1.908%</td>
</tr>
<tr>
<td></td>
<td>30%</td>
<td>4.980E+6</td>
<td>4.834E+6</td>
<td>146609</td>
<td>3.033%</td>
</tr>
<tr>
<td></td>
<td>40%</td>
<td>5.464E+6</td>
<td>5.240E+6</td>
<td>224304</td>
<td>4.281%</td>
</tr>
<tr>
<td></td>
<td>50%</td>
<td>5.866E+6</td>
<td>5.580E+6</td>
<td>286090</td>
<td>5.127%</td>
</tr>
<tr>
<td>(M) (Capacity)</td>
<td>10%</td>
<td>6.109E+6</td>
<td>6.259E+6</td>
<td>-150028</td>
<td>-2.397%</td>
</tr>
<tr>
<td></td>
<td>20%</td>
<td>8.558E+6</td>
<td>8.822E+6</td>
<td>-263446</td>
<td>-2.986%</td>
</tr>
<tr>
<td></td>
<td>30%</td>
<td>1.234E+7</td>
<td>1.181E+7</td>
<td>534370</td>
<td>4.526%</td>
</tr>
<tr>
<td></td>
<td>40%</td>
<td>1.562E+7</td>
<td>1.470E+7</td>
<td>924420</td>
<td>6.289%</td>
</tr>
<tr>
<td></td>
<td>50%</td>
<td>1.857E+7</td>
<td>1.685E+7</td>
<td>1723330</td>
<td>10.230%</td>
</tr>
</tbody>
</table>

Mean of VSS/Sto (%) = 4.422%

Table 5.4. Value of deterministic and stochastic problem for the instances having one stochastic parameter
Figure 5.2 (a,b) Effect of variability on the total mean value of deterministic (MVDP) and stochastic (SP) problems for the instance of single stochastic parameter, demand (d), and capacity (M)
### 5.2.2 Computational Results of Instance with Two Stochastic Parameters

<table>
<thead>
<tr>
<th>Instances Parameter</th>
<th>Variability</th>
<th>Deterministic solution</th>
<th>Stochastic solution</th>
<th>VSS</th>
<th>VSS/Sto (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(q, d)$</td>
<td>10%</td>
<td>6.910E+6</td>
<td>6.308E+6</td>
<td>601585</td>
<td>9.536%</td>
</tr>
<tr>
<td></td>
<td>20%</td>
<td>7.562E+6</td>
<td>7.143E+6</td>
<td>419353</td>
<td>5.871%</td>
</tr>
<tr>
<td></td>
<td>30%</td>
<td>8.529E+6</td>
<td>8.071E+6</td>
<td>457911</td>
<td>5.674%</td>
</tr>
<tr>
<td></td>
<td>40%</td>
<td>9.311E+6</td>
<td>9.090E+6</td>
<td>220496</td>
<td>2.426%</td>
</tr>
<tr>
<td></td>
<td>50%</td>
<td>1.055E+7</td>
<td>1.010E+7</td>
<td>449440</td>
<td>4.448%</td>
</tr>
<tr>
<td>$(d, s)$</td>
<td>10%</td>
<td>6.735E+6</td>
<td>6.785E+6</td>
<td>-50274</td>
<td>-0.741%</td>
</tr>
<tr>
<td></td>
<td>20%</td>
<td>8.268E+6</td>
<td>8.275E+6</td>
<td>-7052</td>
<td>-0.085%</td>
</tr>
<tr>
<td></td>
<td>30%</td>
<td>1.019E+7</td>
<td>9.639E+6</td>
<td>553209</td>
<td>5.739%</td>
</tr>
<tr>
<td></td>
<td>40%</td>
<td>1.082E+7</td>
<td>1.009E+7</td>
<td>725040</td>
<td>7.185%</td>
</tr>
<tr>
<td></td>
<td>50%</td>
<td>1.217E+7</td>
<td>1.104E+7</td>
<td>1130690</td>
<td>10.239%</td>
</tr>
<tr>
<td>$(s, M)$</td>
<td>10%</td>
<td>6.821E+6</td>
<td>6.403E+6</td>
<td>417907</td>
<td>6.53%</td>
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<tr>
<td></td>
<td>20%</td>
<td>9.980E+6</td>
<td>9.984E+6</td>
<td>-3964</td>
<td>-0.04%</td>
</tr>
<tr>
<td></td>
<td>30%</td>
<td>1.354E+7</td>
<td>1.334E+7</td>
<td>202390</td>
<td>1.52%</td>
</tr>
<tr>
<td></td>
<td>40%</td>
<td>1.612E+7</td>
<td>1.520E+7</td>
<td>923270</td>
<td>6.08%</td>
</tr>
<tr>
<td></td>
<td>50%</td>
<td>2.108E+7</td>
<td>2.002E+7</td>
<td>1055960</td>
<td>5.27%</td>
</tr>
<tr>
<td>$(q, s)$</td>
<td>10%</td>
<td>3.893E+6</td>
<td>3.907E+6</td>
<td>-14525</td>
<td>-0.372%</td>
</tr>
<tr>
<td></td>
<td>20%</td>
<td>4.386E+6</td>
<td>4.315E+6</td>
<td>71683</td>
<td>1.661%</td>
</tr>
<tr>
<td></td>
<td>30%</td>
<td>4.707E+6</td>
<td>4.664E+6</td>
<td>42884</td>
<td>0.920%</td>
</tr>
<tr>
<td></td>
<td>40%</td>
<td>5.413E+6</td>
<td>5.008E+6</td>
<td>405032</td>
<td>8.087%</td>
</tr>
<tr>
<td></td>
<td>50%</td>
<td>5.913E+6</td>
<td>5.469E+6</td>
<td>443510</td>
<td>8.109%</td>
</tr>
<tr>
<td>$(q, M)$</td>
<td>10%</td>
<td>6.168E+6</td>
<td>6.082E+6</td>
<td>86291</td>
<td>1.419%</td>
</tr>
<tr>
<td></td>
<td>20%</td>
<td>9.053E+6</td>
<td>9.165E+6</td>
<td>-111555</td>
<td>-1.217%</td>
</tr>
<tr>
<td></td>
<td>30%</td>
<td>1.234E+7</td>
<td>1.158E+7</td>
<td>758360</td>
<td>6.551%</td>
</tr>
<tr>
<td></td>
<td>40%</td>
<td>1.368E+7</td>
<td>1.312E+7</td>
<td>555690</td>
<td>4.234%</td>
</tr>
<tr>
<td></td>
<td>50%</td>
<td>1.789E+7</td>
<td>1.706E+7</td>
<td>828560</td>
<td>4.857%</td>
</tr>
<tr>
<td>$(d, M)$</td>
<td>10%</td>
<td>7.610E+6</td>
<td>8.064E+6</td>
<td>-454313</td>
<td>-5.634%</td>
</tr>
<tr>
<td></td>
<td>20%</td>
<td>1.094E+7</td>
<td>1.117E+7</td>
<td>-227980</td>
<td>-2.041%</td>
</tr>
<tr>
<td></td>
<td>30%</td>
<td>1.503E+7</td>
<td>1.380E+7</td>
<td>1226920</td>
<td>8.888%</td>
</tr>
<tr>
<td></td>
<td>40%</td>
<td>1.623E+7</td>
<td>1.574E+7</td>
<td>498450</td>
<td>3.168%</td>
</tr>
<tr>
<td></td>
<td>50%</td>
<td>2.032E+7</td>
<td>1.897E+7</td>
<td>1347740</td>
<td>7.103%</td>
</tr>
</tbody>
</table>

Mean of VSS/Sto (%) 3.854%

Table 5.5. Value of deterministic and stochastic problem for the instances having two stochastic parameters
### 5.2.3 Computational Results of Instance with Three Stochastic Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Variability</th>
<th>Deterministic solution</th>
<th>Stochastic solution</th>
<th>VSS</th>
<th>VSS/Sto (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(q, d, s)$</td>
<td>10%</td>
<td>7.053E+6</td>
<td>6.812E+6</td>
<td>240885</td>
<td>3.536%</td>
</tr>
<tr>
<td></td>
<td>20%</td>
<td>8.158E+6</td>
<td>8.054E+6</td>
<td>104208</td>
<td>1.294%</td>
</tr>
<tr>
<td></td>
<td>30%</td>
<td>9.638E+6</td>
<td>9.167E+6</td>
<td>471594</td>
<td>5.145%</td>
</tr>
<tr>
<td></td>
<td>40%</td>
<td>1.094E+7</td>
<td>9.270E+6</td>
<td>1666435</td>
<td>17.977%</td>
</tr>
<tr>
<td></td>
<td>50%</td>
<td>1.343E+7</td>
<td>1.196E+7</td>
<td>1465450</td>
<td>12.253%</td>
</tr>
<tr>
<td>$(d, s, M)$</td>
<td>10%</td>
<td>8.303E+6</td>
<td>8.128E+6</td>
<td>174886</td>
<td>2.152%</td>
</tr>
<tr>
<td></td>
<td>20%</td>
<td>1.099E+7</td>
<td>1.088E+7</td>
<td>108710</td>
<td>0.999%</td>
</tr>
<tr>
<td></td>
<td>30%</td>
<td>1.481E+7</td>
<td>1.416E+7</td>
<td>649830</td>
<td>4.589%</td>
</tr>
<tr>
<td></td>
<td>40%</td>
<td>1.832E+7</td>
<td>1.774E+7</td>
<td>579130</td>
<td>3.265%</td>
</tr>
<tr>
<td></td>
<td>50%</td>
<td>2.026E+7</td>
<td>1.956E+7</td>
<td>701440</td>
<td>3.587%</td>
</tr>
<tr>
<td>$(q, d, M)$</td>
<td>10%</td>
<td>7.987E+6</td>
<td>7.831E+6</td>
<td>155534</td>
<td>1.986%</td>
</tr>
<tr>
<td></td>
<td>20%</td>
<td>1.188E+7</td>
<td>1.125E+7</td>
<td>625130</td>
<td>5.556%</td>
</tr>
<tr>
<td></td>
<td>30%</td>
<td>1.381E+7</td>
<td>1.228E+7</td>
<td>1531840</td>
<td>12.473%</td>
</tr>
<tr>
<td></td>
<td>40%</td>
<td>1.640E+7</td>
<td>1.442E+7</td>
<td>1983840</td>
<td>13.759%</td>
</tr>
<tr>
<td></td>
<td>50%</td>
<td>1.927E+7</td>
<td>1.710E+7</td>
<td>2166380</td>
<td>12.669%</td>
</tr>
<tr>
<td>$(q, s, M)$</td>
<td>10%</td>
<td>6.656E+6</td>
<td>6.627E+6</td>
<td>29467</td>
<td>0.445%</td>
</tr>
<tr>
<td></td>
<td>20%</td>
<td>9.904E+6</td>
<td>9.785E+6</td>
<td>119154</td>
<td>1.218%</td>
</tr>
<tr>
<td></td>
<td>30%</td>
<td>1.410E+7</td>
<td>1.319E+7</td>
<td>913860</td>
<td>6.928%</td>
</tr>
<tr>
<td></td>
<td>40%</td>
<td>1.794E+7</td>
<td>1.654E+7</td>
<td>1400170</td>
<td>8.466%</td>
</tr>
<tr>
<td></td>
<td>50%</td>
<td>1.988E+7</td>
<td>1.834E+7</td>
<td>1539980</td>
<td>8.397%</td>
</tr>
</tbody>
</table>

Mean of VSS/Sto (%) 6.33%

Table 5.6. Value of deterministic and stochastic problem for the instances having three stochastic parameters
5.2.4 Computational Results of Instance with Four Stochastic Parameters

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Variability</th>
<th>Deterministic solution</th>
<th>Stochastic solution</th>
<th>VSS</th>
<th>VSS/Sto (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(q, d, s, M)</td>
<td>10%</td>
<td>7.708E+6</td>
<td>8.013E+6</td>
<td>-305347</td>
<td>-3.81%</td>
</tr>
<tr>
<td></td>
<td>20%</td>
<td>1.162E+7</td>
<td>1.145E+7</td>
<td>167560</td>
<td>1.46%</td>
</tr>
<tr>
<td></td>
<td>30%</td>
<td>1.580E+7</td>
<td>1.485E+7</td>
<td>950500</td>
<td>6.40%</td>
</tr>
<tr>
<td></td>
<td>40%</td>
<td>1.892E+7</td>
<td>1.793E+7</td>
<td>994970</td>
<td>5.55%</td>
</tr>
<tr>
<td></td>
<td>50%</td>
<td>2.198E+7</td>
<td>2.120E+7</td>
<td>783290</td>
<td>3.70%</td>
</tr>
</tbody>
</table>

Mean of VSS/Sto (%) 2.66%

Table 5.7. Value of deterministic and stochastic problem for the instances having four stochastic parameters

Figure 5.3 Effect of variability on the total cost of deterministic (MVDP) and stochastic (SP) problems for the instance of four stochastic parameters, cost-demand-supply-capacity
5.3 Result Analysis

The results presented in the above tables can be discussed and analyzed as follows.

1. It is well known that the computation time of the instances solved by the stochastic problem are higher than the corresponding deterministic ones, due to the number of scenarios and replications involved in solving the stochastic problem (Table 5.3, and Figure 5.1). Although this difference (up to five times on average) for the considered instances may not reflect clearly the complexity and time issues in applying the stochastic programming approach, it gives us an insight on how each stochastic parameter effects on the computation time. For example, according to Figure 5.1, the instances which have the operating cost \( q \) as the stochastic parameter mostly have more computation times than the others within the same category. This can be justified as an effect of the stochastic parameter in the objective function, and the size of the operating cost vector \( q_{odn} \) which carries four dimensions.

2. Stochastic optimization approach outperforms deterministic approach as it is shown in Tables 5.4 – 5.7 for different combinations of uncertainty in parameters. For slight uncertainty in parameters, the value of the stochastic solution \( \text{VSS} \), and the percentage rate of difference over the stochastic solution value \( \frac{\text{VSS}}{\text{Sto}} \) might be small and the two approaches perform quite the same, but by augmenting the variability we see a significant increase in the VSS.

3. The results of the single uncertain parameter from Table 5.4 reveal that the operating cost \( q \) has the least impact among the design parameters to uncertainty. In other words, the results from deterministic and stochastic approaches while the operating costs change do not have significant differences.

4. By increasing the variability of the uncertain parameters, supply \( s \), demand \( d \), and capacity \( M \) we see a significant increase in the objective function values of both deterministic and stochastic problems as it is shown in Tables 5.4 – 5.7. This
is a result of penalizing for outsourcing when we encounter high fluctuations and uncertainty in these parameters. In contrast, increasing the variability for the operating cost ($q$) results in decreasing the objective function value. This trend is also consistent with the results of Santoso et al. (2003). As the operating costs only appear in the objective function, and there is no restricting constraint corresponding to that, the model can encompass fluctuations in costs to lower the objective function value. It is also clear that the stochastic solution is more resilient to the variability of the parameters than the deterministic approach. This resiliency could be improved by increasing the number of scenarios and replications in the SAA problem.
CHAPTER 6

Conclusions and Remarks

In this work, we aimed to apply the stochastic programming approach in designing logistics networks in the presence of uncertainty. Due to the uncertain nature of design parameters and their unpredictable behavior, we need to incorporate these effects into the design process to build a solid, resilient and robust network which can encompass reasonable amounts of fluctuations in parameters. There are several sources of uncertainty involved in network design, and in this study we have considered the uncertainty in operating costs, demand, supply, and capacity. To this end, a two-stage stochastic programming approach was used to formulate and model the problem, and a heuristic solution method called sample average approximation (SAA) was applied to solve the problem.

The logistics network design problem (LNDP) considered in this study is a multi-echelon, multi-product, single-period integrated supply chain model with uncertainty in design parameters. A two-stage stochastic programming approach with a fixed recourse is applied to model this problem. The first-stage decision variables such as network configuration, and supplier selection are made prior to the realization of the uncertain parameters. The second-stage decisions such as flow of commodity through the network are made after the realization of uncertain parameters.

The (SAA) technique generates samples of the stochastic parameters to construct and solve the approximation instead of the exact values. Besides, we have designed an evaluation mechanism in order to compare the results of the stochastic approach and deterministic approach.
Next, we have generated test instances for different combinations of stochastic parameters and levels of variability in order to measure the effect of variability on each of the uncertain parameters. Finally, the results of the stochastic programming and deterministic programing approach have been presented, compared and analyzed, and we have obtained the following insights:

- Stochastic optimization approach outperforms deterministic approach in solving LNDP with uncertainty in design parameters.
- The value of stochastic solution (VSS) increases with the uncertainty in parameters, which suggests to apply stochastic programming approach in highly uncertain environments.
- The stochastic solutions are more resilient to the variability of parameters than deterministic solutions.
- Increasing the variability of demand, supply and capacity increases the value of the objective function for both the deterministic and stochastic approaches.
- Increasing the variability of operating cost results in decreasing the objective function value.

In this thesis, we have limited our test problems to practically small instances with a small number of scenarios in order to make them solvable in a reasonable amount of time. Indeed, incorporating new approaches such as column generation and valid inequalities could improve the performance of the proposed method for larger instances.
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